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MATH323 - A1

Numbers from the 7th Edition

Don't need to do

#	8.9	8.10	8.13	8.14	8.24	8.32	8.39	8.44	8.62	~~8.74~~	8.85	8.101
Page	395	395	395	395	403	405	409	410	418	~~424~~	432	436

8.9 Suppose that $Y_1, Y_2, \ldots, Y_n$ constitute a random sample from a population with probability density function

$$f(y) = \begin{cases} \left(\frac{1}{\theta+1}\right) e^{-y/(\theta+1)}, & y > 0,\ \theta > -1, \\ 0, & \text{elsewhere.} \end{cases}$$

Suggest a suitable statistic to use as an unbiased estimator for θ. [*Hint:* Consider $\overline{Y}$.]

$\overline{Y} = \frac{1}{n}\sum_{i=1}^{n} Y_i$

$\to \hat{\theta}$

Find the mean $\longrightarrow$ $\mu = E[Y] = \int_0^\infty y f(y)\, dy = \int_0^\infty y \cdot \left(\frac{1}{\theta+1}\right) e^{-y/(\theta+1)}\, dy$

$$= \frac{1}{\theta+1} \cdot \int_0^\infty y \cdot e^{-y/(\theta+1)}\, dy$$

$\left\{ u = \frac{y}{\theta+1} \Rightarrow y = u(\theta+1),\quad dy = (\theta+1)\, du \right.$

$$= \frac{1}{\cancel{\theta+1}} \int_0^\infty u \cancel{(\theta+1)} \cdot e^{-u} \cdot (\theta+1)\, du$$

$$= (\theta+1) \cdot \int_0^\infty u \cdot e^{-u}\, du$$

$\mapsto \int u\, dv = uv - \int v\, du$

$$= (\theta+1)\left(\left[-u \cdot e^{-u}\right]_0^\infty + \int_0^\infty e^{-u}\, du \right)$$

(integration by parts)

$$= (\theta+1) \cdot \left(0 + \left[-e^{-u}\right]_0^\infty\right)$$

$* \; e^{-\infty} = 0$

$$= (\theta+1) \cdot (0 - (-1))$$

$$= (\theta+1) \cdot 1 = (\theta+1)$$

$\Rightarrow E[Y] = (\theta+1) \Rightarrow \theta = E[Y] - 1$

$\Rightarrow \hat{\theta} = \overline{Y} - 1$

8.10 The number of breakdowns per week for a type of minicomputer is a random variable Y with a Poisson distribution and mean λ. A random sample $Y_1, Y_2, \ldots, Y_n$ of observations on the weekly number of breakdowns is available.

a Suggest an unbiased estimator for λ.

b The weekly cost of repairing these breakdowns is $C = 3Y + Y^2$. Show that $E(C) = 4\lambda + \lambda^2$.

c Find a function of $Y_1, Y_2, \ldots, Y_n$ that is an unbiased estimator of $E(C)$. [*Hint:* Use what you know about $\overline{Y}$ and $(\overline{Y})^2$.]

a) $\to Y \sim$ Poisson (λ) where $E[Y] = \lambda$

$\to$ for a random sample $Y_1, Y_2, \ldots, Y_n$, the sample mean $\overline{Y} = \frac{1}{n}\sum_{i=1}^{n} Y_i$ has the expectation:

$$E[\overline{Y}] = \frac{1}{n}\sum_{i=1}^{n} E[Y_i] = \lambda$$

$\Rightarrow \hat{\lambda} = \overline{Y}$

b) Weekly cost of repairing: $C = 3Y + Y^2$, NTS: $E[C] = 4\lambda + \lambda^2$

Using linearity of expectation: $E[C] = E[3Y + Y^2] = E[3Y] + E[Y^2]$

$$= 3 \cdot E[Y] + E[Y^2]$$

$$= 3 \cdot \lambda + (\lambda + \lambda^2)$$

$$= 4\lambda + \lambda^2$$

$E[Y] = \lambda$ mean for Poisson variable

$Var(Y) = E[Y^2] - (E[Y])^2$

$E[Y^2] = Var(Y) + (E[Y])^2$

$= \lambda + \lambda^2$

c) $E[C] = 4\lambda + \lambda^2$, substitute $\lambda = \hat{\lambda} = \bar{Y}$

$\Rightarrow \hat{E}[C] = 4 \cdot \bar{Y} + (\bar{Y})^2$ ← estimator

To verify that it is unbiased → $E[\hat{E}[C]] = E[4\bar{Y} + (\bar{Y})^2]$

$$= 4\lambda + \boxed{\frac{\lambda}{n}} + \lambda^2$$

$E[(\bar{Y})^2] = Var(\bar{Y}) + (E[\bar{Y}])^2$

$= \frac{Var(Y)}{n} + \lambda^2$

$= \frac{\lambda}{n} + \lambda^2$

*To correct bias, term $\frac{\lambda}{n}$ must be removed which corresponds to $\frac{\bar{Y}}{n}$

$\Rightarrow \hat{E}[C] = 4\bar{Y} + (\bar{Y})^2 - \frac{\bar{Y}}{n}$

8.13 We have seen that if Y has a binomial distribution with parameters n and p, then Y/n is an unbiased estimator of p. To estimate the variance of Y, we generally use $n(Y/n)(1 - Y/n)$.

a Show that the suggested estimator is a biased estimator of $V(Y)$.

b Modify $n(Y/n)(1 - Y/n)$ slightly to form an unbiased estimator of $V(Y)$.

a) $\hat{V}(Y) = n\left(\frac{Y}{n}\right)\left(1 - \frac{Y}{n}\right) = \frac{Y}{n}(n - Y)$

to check bias, need to compare expected value of $\hat{V}(Y)$ and $V(Y) = np(1-p)$

Binomial distribution

$E[\hat{V}(Y)] = E\left[\frac{Y}{n}(n - Y)\right] = \frac{1}{n} \cdot E[nY - Y^2]$

$$= \frac{1}{n} \cdot \left(n \cdot E[Y] - E[Y^2]\right)$$

$$= \frac{1}{n}\left(n \cdot np - (np(1-p) + (np)^2)\right)$$

$$= \frac{1}{n}\left(n \cdot np - np(1-p) - (np)^2\right)$$

$$= np - p(1-p) - np^2$$

$$= np(1-p) - p(1-p)$$

$$= \boxed{p(1-p)(n-1)}$$

$E[Y] = np$

$Var[Y] = np(1-p)$

$E[Y^2] = Var[Y] + (E[Y])^2$

$= np(1-p) + (np)^2$

$V[Y] = \boxed{np(1-p)}$

$\Rightarrow$ Comparing, $E[\hat{V}(Y)] = \frac{n-1}{n} V(Y)$

$\Rightarrow$ $\hat{V}(Y)$ is biased, with a factor of $\frac{n-1}{n}$

b) To correct bias: $\quad$ *Goal for no bias $\rightarrow E[\hat{V}(Y)] = V(Y)$

$$\hat{V}_{unbiased}(Y) = \frac{n}{n-1} \cdot \hat{V}(Y)$$

$$= \frac{n}{n-1} \cdot n \cdot \frac{Y}{n}\left(1 - \frac{Y}{n}\right)$$

$$= \frac{\cancel{n}}{n-1} \cdot Y\left(\frac{n-Y}{\cancel{n}}\right)$$

$$= \frac{Y(n-Y)}{n-1}$$

8.14 Let $Y_1, Y_2, \ldots, Y_n$ denote a random sample of size n from a population whose density is given by

$$f(y) = \begin{cases} \alpha y^{\alpha-1}/\theta^{\alpha}, & 0 \le y \le \theta, \\ 0, & \text{elsewhere,} \end{cases}$$

where $\alpha > 0$ is a known, fixed value, but θ is unknown. (This is the power family distribution introduced in Exercise 6.17.) Consider the estimator $\hat{\theta} = \max(Y_1, Y_2, \ldots, Y_n)$.

a Show that $\hat{\theta}$ is a biased estimator for θ.

b Find a multiple of $\hat{\theta}$ that is an unbiased estimator of θ.

c Derive MSE($\hat{\theta}$).

a) To check bias, compute $E[\hat{\theta}]$ and compare it to θ

CDF $\rightarrow$ $F_{\hat{\theta}}(t) = P(\hat{\theta} \le t) = P(Y_1 \le t, Y_2 \le t, \ldots, Y_n \le t)$

Since Y_i are independent: $F_{\hat{\theta}}(t) = [F_Y(t)]^n$

CDF of single observation of $Y \rightarrow F_Y(t) = P(Y \le t) = \int_0^t \frac{\alpha y^{\alpha-1}}{\theta^{\alpha}}\, dy$

$$= \frac{\alpha}{\theta^{\alpha}} \int_0^t y^{\alpha-1}\, dy$$

$$= \frac{\alpha}{\theta^{\alpha}} \cdot \left[\frac{y^{\alpha}}{\alpha}\right]_0^t$$

$$= \frac{\cancel{\alpha}}{\theta^{\alpha}} \cdot \frac{t^{\alpha}}{\cancel{\alpha}}$$

$$= \frac{t^{\alpha}}{\theta^{\alpha}} \qquad \text{for } 0 \le t \le \theta$$

PDF of $\hat{\theta}$: $f_{\hat{\theta}}(t) = \frac{d}{dt} F_{\hat{\theta}}(t) = \frac{d}{dt}\left(\frac{t^{\alpha}}{\theta^{\alpha}}\right)^n = n \cdot \frac{\alpha t^{\alpha n-1}}{\theta^{\alpha n}}$

$$E[\hat{\theta}] = \int_0^{\theta} t \cdot f_{\hat{\theta}}(t)\, dt$$

$$= \int_0^{\theta} t \cdot n \cdot \frac{\alpha t^{\alpha n-1}}{\theta^{\alpha n}}\, dt$$

$$= n \cdot \frac{\alpha}{\theta^{\alpha n}} \cdot \int_0^{\theta} t^{\alpha n}\, dt$$

$$= n \cdot \frac{\alpha}{\theta^{\alpha n}} \cdot \left[\frac{t^{\alpha n+1}}{\alpha n+1}\right]_0^{\theta}$$

$$= n \cdot \frac{\alpha}{\cancel{\theta^{\alpha n}}} \cdot \frac{\cancel{\theta^{\alpha n+1}}}{\alpha n+1}$$

$$= \underbrace{\frac{n\alpha}{\alpha n+1}}_{<1} \cdot \theta$$

$\Rightarrow$ $E[\hat{\theta}] = \frac{n\alpha}{\alpha n+1} \cdot \theta$ $\qquad$ $E[\hat{\theta}] \neq \theta$

$\Rightarrow$ biased

b) $\hat{\theta}_{unbiased} = \frac{\alpha n+1}{n\alpha} \cdot \hat{\theta}$

c) $MSE(\hat{\theta}) = Var(\hat{\theta}) + Bias(\hat{\theta})^2$

$$Bias(\hat{\theta}) = E[\hat{\theta}] - \theta = \frac{n\alpha}{\alpha n+1} \cdot \theta - \theta = \theta\left(\frac{n\alpha}{\alpha n+1} - 1\right) = \theta \cdot \left(\frac{\cancel{n\alpha} - (\cancel{\alpha n}+1)}{\alpha n+1}\right) = \frac{-\theta}{\alpha n+1}$$

$$Var(\hat{\theta}) = E[\hat{\theta}^2] - (E[\hat{\theta}])^2$$

$$= \frac{n\alpha\theta^2}{\alpha n+2} - \left(\frac{n\alpha\theta}{\alpha n+1}\right)^2$$

from PDF of $\hat{\theta}$: $E[\hat{\theta}^2] = n \cdot \frac{\alpha}{\theta^{\alpha n}} \int_0^\theta t^2 t^{\alpha n-1} dt$

$$= n \cdot \frac{\alpha}{\theta^{\alpha n}} \int_0^\theta t^{\alpha n+1} dt$$

$$= n \cdot \frac{\alpha}{\cancel{\theta^{\alpha n}}} \cdot \frac{\theta^{\cancel{\alpha n}+2}}{\alpha n+2}$$

$$= \frac{n \cdot \alpha \cdot \theta^2}{\alpha n+2}$$

$MSE(\hat{\theta}) = Var(\hat{\theta}) + Bias(\hat{\theta})^2$

$$= \frac{n\alpha\theta^2}{\alpha n+2} - \left(\frac{n\alpha\theta}{\alpha n+1}\right)^2 + \frac{\theta^2}{(\alpha n+1)^2}$$

$$= \frac{n\alpha\theta^2}{\alpha n+2} - \frac{n^2\alpha^2\theta^2 - \theta^2}{(\alpha n+1)^2}$$

$$= \frac{n\alpha\theta^2}{\alpha n+2} - \frac{\theta^2(n^2\alpha^2-1)}{(\alpha n+1)^2}$$

8.24 (Ebook): Results of a public opinion poll reported on the Internet indicated that 69% of respondents rated the cost of gasoline as a crisis or major problem. The article states that 1001 adults, age 18 years or older, were interviewed and that the results have a sampling error of 3%. How was the 3% calculated, and how should it be interpreted? Can we conclude that a majority of the individuals in the 18+ age group felt that cost of gasoline was a crisis or major problem?

$n = 1001$

ME

* Sampling error is a reflection of the margin error in estimating the population proportion (p) for the sample proportion ($\hat{p}$)

← crit val.

$$ME = z^* \cdot \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

* Since confidence level is not given, we can assume 95%

$$= 1.96 \cdot \sqrt{\frac{0.69 \cdot 0.31}{1001}}$$

$$= 1.96 \cdot \sqrt{\frac{0.2139}{1001}} \approx 0.02866 \rightarrow 2.9\%$$, which aligns with 3%

Interpretation: Sampling error means that the true population is likely with 3% of sample proportion (69%) with 95% confidence
⇒ $CI = (69\% - 3\%, 69\% + 3\%) = (66\%, 72\%)$

Majority Conclusion: Majority means >50%, Since CI (66%, 72%) >50%, we can conclude with 95% confidence that a maj. of individuals in the 18+ group felt that the cost of gasoline was a crisis / major problem

8.32 An auditor randomly samples 20 accounts receivable from among the 500 such accounts of a client's firm. The auditor lists the amount of each account and checks to see if the underlying documents comply with stated procedures. The data are recorded in the accompanying table (amounts are in dollars, Y = yes, and N = no).

Account	Amount	Compliance	Account	Amount	Compliance
1	278	Y	11	188	N
2	192	Y	12	212	N
3	310	Y	13	92	Y
4	94	N	14	56	Y
5	86	Y	15	142	Y
6	335	Y	16	37	Y
7	310	N	17	186	N
8	290	Y	18	221	Y
9	221	Y	19	219	N
10	168	Y	20	305	Y

Estimate the total accounts receivable for the 500 accounts of the firm and place a bound on the error of estimation. Do you think that the *average* account receivable for the firm exceeds $250? Why?

- auditor random sample = 20
- accounts of a client firm =
* Find sample mean, then consider standard deviation

Sample mean: $\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$

$= \frac{278 + 192 + 310 + \dots + 219 + 305}{20} = \frac{3942}{20} = 197.1$

Sample standard deviation: $\sigma = \sqrt{\frac{\sum_{i=1}^{n} y_i^2}{n} - (\bar{y})^2} = \sqrt{\frac{(278)^2 + (192)^2 + \dots + (305^2)}{20} - (197.1)^2} = 90.9$

Confidence interval $\rightarrow \bar{y} \pm 2\frac{\sigma}{\sqrt{n}} = 197.1 \pm 2\left(\frac{90.9}{\sqrt{20}}\right) = (156.45, 237.75)$

$\Rightarrow$ 250\$ is not included in the confidence interval

$\Rightarrow$ Average accounts receivable for the firm doesn't exceed 250\$

8.39 Suppose that the random variable Y has a gamma distribution with parameters $\alpha = 2$ and an unknown β. In Exercise 6.46, you used the method of moment-generating functions to prove a general result implying that $2Y/\beta$ has a χ^2 distribution with 4 degrees of freedom (df). Using $2Y/\beta$ as a pivotal quantity, derive a 90% confidence interval for β.

$\rightarrow Y \sim \text{Gamma}(\alpha = 2, \beta)$

$\rightarrow$ pivotal quantity is $U = \frac{2Y}{\beta} \sim \chi^2_4$ (chi-squared distribution with 4 deg of freedom)

$P(\chi^2_{4,0.05} \le U \le \chi^2_{4,0.95}) = 0.90$

$\chi^2_{4,0.05} \rightarrow 0.711$, $\chi^2_{4,0.95} \rightarrow 9.4877$ (from chi-squared table)

$\Rightarrow P(0.711 \le U \le 9.4877) = 0.90$

$P(0.711 \le \frac{2Y}{\beta} \le 9.4877) = 0.90$

Solve for β: $\frac{2Y}{9.4877} \le \beta \le \frac{2Y}{0.711}$

$\Rightarrow$ confidence interval for β: $\left(\frac{2Y}{9.4877}, \frac{2Y}{0.711}\right)$

8.44 Let Y have probability density function

$$f_Y(y) = \begin{cases} \frac{2(\theta - y)}{\theta^2}, & 0 < y < \theta, \\ 0, & \text{elsewhere.} \end{cases}$$

a Show that Y has distribution function

$$F_Y(y) = \begin{cases} 0, & y \le 0, \\ \frac{2y}{\theta} - \frac{y^2}{\theta^2}, & 0 < y < \theta, \\ 1, & y \ge \theta. \end{cases}$$

b Show that Y/θ is a pivotal quantity.

c Use the pivotal quantity from part (b) to find a 90% lower confidence limit for θ.

a) CDF $\rightarrow F_Y(y) = P(Y \leq y) = \int_{-\infty}^{y} f_Y(t)\,dt$

Case 1: $y \leq 0 \rightarrow f_Y(t) = 0$ (since support of Y is $0 < Y < \theta$)

$$\Rightarrow F_Y(y) = \int_{-\infty}^{y} f_Y(t)\,dt = 0$$

Case 2: $0 < y < \theta \rightarrow f_Y(t) = \frac{2(\theta - t)}{\theta^2}$

$$\Rightarrow F_Y(y) = \int_0^y \frac{2(\theta - t)}{\theta^2}\,dt = \frac{2}{\theta^2}\left(\int_0^y \theta\,dt - \int_0^y t\,dt\right) = \frac{2}{\theta^2}\left(\theta y - \frac{y^2}{2}\right) = \frac{2\theta y - y^2}{\theta^2}$$

Case 3: $y \geq \theta \rightarrow$ entire probability distribution has been accounted for $\longrightarrow F_Y(y) = 1$

$$\Rightarrow F_Y(y) = \begin{cases} 0 & , y \leq 0 \\ \frac{2\theta y - y^2}{\theta^2} & , 0 \leq y \leq \theta \\ 1 & , y \geq \theta \end{cases}$$

$$= \frac{2y}{\theta} - \frac{y^2}{\theta^2}$$

b) Distribution function $U = \frac{Y}{\theta}$:

$$P(U \leq u) = P\left(\frac{Y}{\theta} \leq u\right)$$

$$= P(Y \leq \theta u)$$

$$= F_Y(\theta u)$$

$$= \begin{cases} 0 & , u \leq 0 \\ 2u - u^2 & , 0 < u < 1 \\ 1 & , u \geq 1 \end{cases}$$

$$\frac{2\theta(\theta u) - (\theta u)^2}{\theta^2} = \frac{2\theta^2 u - \theta^2 u^2}{\theta^2} = 2u - u^2$$

$\Rightarrow$ From the distribution function of U, parameter of interest is θ

$\Rightarrow$ pivotal quantity is $U = \frac{Y}{\theta}$

c) Find c such that: $P(U \leq c) = 0.90$

Since $0 < 0.90 < 1$, $F_U(u) = 2u - u^2 \Rightarrow F_U(c) = 0.90$

$$\Rightarrow 2c - c^2 = 0.90$$

$$c^2 - 2c + 0.90 = 0$$

$$c = \frac{2 \pm 0.632}{2}$$

*Take smaller root since $0 < c < 1$

$$c = \frac{2 - 0.632}{2} = 0.684$$

Using pivotal quantity $U = \frac{Y}{\theta} \rightarrow P(U \leq c) = P\left(\frac{Y}{\theta} \leq c\right) = P\left(\theta \geq \frac{Y}{c}\right)$

$$\Rightarrow \theta \geq \frac{Y}{0.684}$$

$\Rightarrow$ 90% lower confidence limit for θ is $\frac{Y}{0.684}$

8.62 The following statistics are the result of an experiment conducted by P. I. Ward to investigate a theory concerning the molting behavior of the male *Gammarus pulex*, a small crustacean.[8] If a male needs to molt while paired with a female, he must release her, and so loses her. The theory is that the male *G. pulex* is able to postpone molting, thereby reducing the possibility of losing his mate. Ward randomly assigned 100 pairs of males and females to two groups of 50 each. Pairs in the first group were maintained together (normal); those in the second group were separated (split). The length of time to molt was recorded for both males and females, and the means, standard deviations, and sample sizes are shown in the accompanying table. (The number of crustaceans in each of the four samples is less than 50 because some in each group did not survive until molting time.)

	Time to Molt (days)		
	Mean	*s*	*n*
Males			
Normal	24.8	7.1	34
Split	21.3	8.1	41
Females			
Normal	8.6	4.8	45
Split	11.6	5.6	48

6. *Source:* "Caught in the Middle," *American Demographics*, July 2001, pp. 14–15.

7. *Source:* Allen L. Shoemaker, "What's Normal? Temperature, Gender and Heart Rate," *Journal of Statistics Education* (1996).

8. *Source:* "*Gammarus pulex* Control Their Moult Timing to Secure Mates," *Animal Behaviour* 32 (1984).

a Find a 99% confidence interval for the difference in mean molt time for "normal" males versus those "split" from their mates.

b Interpret the interval.

$\mu_1 - \mu_2$

a) Confidence interval for difference in means:

$$(\bar{x}_1 - \bar{x}_2) \pm z^* \cdot \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

crit. val

variance of each group

from standard normal table for 99%

$$= (24.8 - 21.3) \pm 2.58 \cdot \sqrt{\frac{(7.1)^2}{34} + \frac{(8.1)^2}{41}}$$

$$= 3.5 \pm 4.527$$

$$= (-1.0279,\ 8.0279)$$

b) Interpretation: 99% confidence interval for diff in mean molt time is $(-1.0279,\ 8.0279)$

↳ 99% confident that the true difference in mean molt times lies within this interval

↳ Since interval includes 0, there is no statistically significant evidence that the mean molt time differ between "Normal" and "Split" males at 99% confidence level

8.85 Two new drugs were given to patients with hypertension. The first drug lowered the blood pressure of 16 patients an average of 11 points, with a standard deviation of 6 points. The second drug lowered the blood pressure of 20 other patients an average of 12 points, with a standard deviation of 8 points. Determine a 95% confidence interval for the difference in the mean reductions in blood pressure, assuming that the measurements are normally distributed with equal variances.

	Drug 1	Drug 2
Sample size n_i	16	20
" mean $\bar{x}_i$	11	12
" sd s_i	6	8

Need to find: 95% CI for $(\mu_1 - \mu_2)$ assuming equal variances

crit value for 95% with $df = n_1 + n_2 - 2$

$$CI = (\bar{x}_1 - \bar{x}_2) \pm t^* \cdot SE$$

$$SE = \sqrt{s_p^2 \left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$$

Since variances are assumed equal, pooled variance:

$$s_p^2 = \frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2} = \frac{(16-1)(6)^2 + (20-1)\cdot(8)^2}{16+20-2} = \frac{1756}{34} \approx 51.65$$

$$SE = \sqrt{s_p^2\left(\frac{1}{n_1}+\frac{1}{n_2}\right)} = \sqrt{51.65\left(\frac{1}{16}+\frac{1}{20}\right)} \approx 2.41$$

for crit t^* value $\rightarrow$ $df = n_1+n_2-2 = 34$

$\rightarrow t^* \approx 2.032$

$$CI = (\bar{x}_1 - \bar{x}_2) \pm t^* \cdot SE$$

$$= (11-12) \pm (2.032)\cdot 2.41$$

$$= -1 \pm 4.9$$

$$= (-5.9, 3.9)$$

8.101 In laboratory work, it is desirable to run careful checks on the variability of readings produced on standard samples. In a study of the amount of calcium in drinking water undertaken as part of a water quality assessment, the same standard sample was run through the laboratory six times at random intervals. The six readings, in parts per million, were 9.54, 9.61, 9.32, 9.48, 9.70, and 9.26. Estimate the population variance σ^2 for readings on this standard, using a 90% confidence interval.

$n = 6$

deg of freedom

$$CI \text{ for } \sigma^2 : \left(\frac{(n-1)s^2}{\chi^2_{upper}}, \frac{(n-1)s^2}{\chi^2_{lower}}\right)$$

sample variance $\rightarrow$ $s^2 = \dfrac{\Sigma(x_i-\bar{x})^2}{n-1}$ $\Bigg\}$ $\bar{x} = \dfrac{\Sigma x_i}{n} = \dfrac{9.54+9.67+\dots+9.26}{6} = \dfrac{56.91}{6} = 9.485$

$$= \frac{(9.54-9.485)^2 + (9.67-9.485)^2 + \dots + (9.26-9.485)^2}{6-1}$$

$$= \frac{0.14275}{5}$$

$$= 0.02855$$

Crit values

↳ for 90% CI with $n-1=5$ df

$\rightarrow \chi^2_{upper} = \chi^2_{0.1,5} = 11.070$

$\rightarrow \chi^2_{lower} = \chi_{0.9,5} = 1.145$

$$\text{Lower limit} = \frac{(n-1)s^2}{\chi^2_{upper}} = \frac{(6-1)\cdot 0.02855}{11.070} \approx 0.013$$

$$\text{Upper limit} = \frac{(n-1)s^2}{\chi^2_{lower}} = \frac{(6-1)\cdot 0.02855}{1.145} \approx 0.125$$

$\Rightarrow$ 90% CI for σ^2 is $(0.013, 0.125)$