Lab 8
Binary Heap

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In this lab, we will learn another data structure called Binary Heap and apply it for Heap Sort.

1. What is Binary Heap?

A Heap is a special Tree-based data structure in which the tree is a **complete binary tree**. Generally, Heaps can be of two types:

- **Max-Heap:** $A[parent(i)] \geq A[i]$
- **Min-Heap:** $A[parent(i)] \leq A[i]$

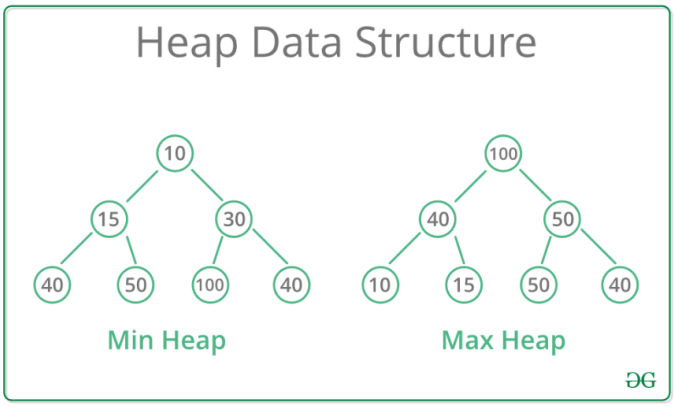
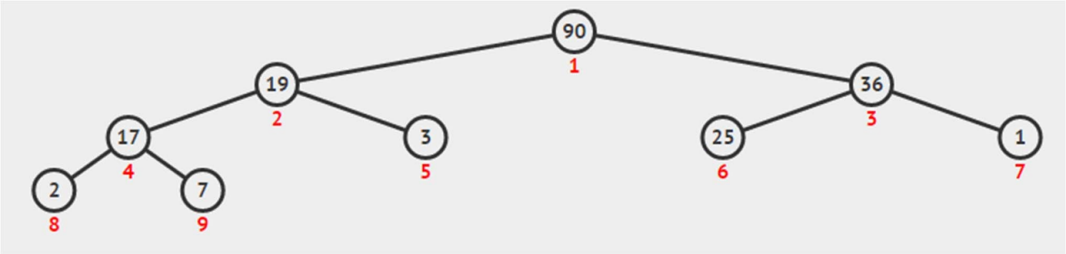


Figure 1: Binary Heap [1]

For storing a **complete binary tree**, we can use an array.

[1] https://www.geeksforgeeks.org/binary-heap/



0	1	2	3	4	5	6	7	8	9	10	11
-	90	19	36	17	3	25	1	2	7	-	-

Figure 2: Using an array to store Binary Heap

Index 0 can be used or not, and the corresponding parent-children relation formulas may be different.

	Root at index 0	**Root at index 1**
Parent(i)	(i-1)/2	i/2
Left(i)	(2*i)+1	2*i
Right(i)	(2*i)+2	(2*i)+1

We will implement Max-heap by using an array that starts at index 1.
Two basic operations of Max-heap are:

- Insert
- Extract Max

2. Construct a Heap class

Here is the code for initializing a heap instance:

```
public class MaxHeap{
    int[] heap;
    int heapSize;
    int maxSize; //maximum size to initialize an heap array

    public MaxHeap(int capity){
        heapSize = 0;
        this.maxSize = capity + 1;
        heap = new int[maxSize];
        heap[0] = -1;
    }
}
```

You need some methods to access the parent and childs index.

```
private int parent(int i){
    return i/2;
}

private int left(int i){
```

```
    //thinking and filling
}

private int right(int i){
    //thinking and filling
}
```

And the method to help us swap two values in an array.

```
private void swap(int i, int j){
    int temp = heap[i];
    heap[i] = heap[j];
    heap[j] = temp;
}
```

After finishing the helper methods, we continue to implement the method to insert a value to a heap.

3. Insert

We have three steps to do:

1. Increase heap size by 1
2. Add a new key at the heap size position.
3. If the new key is smaller than its parent, then we don't need to do anything. If not, **shift it up**.

This is the code of insertion:

```
public void insert(int key){
    if(heapSize == maxSize){
        throw new NoSuchElementException("Overflow Exception"); //
    Remember to import java.util.NoSuchElementException;
    }
    heapSize += 1;
    heap[heapSize] = key;

    shiftUp(heapSize);
}
```

How to "shift up"? This is the answer:

```
private void shiftUp(int i){
    while(i > 1 && heap[parent(i)] < heap[i]){
        swap(parent(i), i); //this method you have defined before
        i = parent(i);
    }
}
```

4. Extract Max

Extract max as known as delete the maximum element (the root of Max-Heap). Insertion needs a shift up method. On the contrary, deletion needs a **shift down** method.

```java
public int extractMax(int i){
    if(heapSize == 0){
        throw new NoSuchElementException("Underflow Exception");
    }
    int max = heap[1];
    heap[1] = heap[heapSize];
    heapSize -= 1;
    shiftDown(1);
    return max;
}
```

This is the code for **shifting down**:

```java
private void shiftDown(int i){
    while(i <= heapSize){
        int max = heap[i];
        int max_id = i;
        if(left(i) <= heapSize && max < heap[left(i)]){
            max = heap[left(i)];
            max_id = left(i);
        }
        if(right(i) <= heapSize && max < heap[right(i)]){
            max = heap[right(i)];
            max_id = right(i);
        }
        if(max_id != i){
            swap(max_id,i);
            i = max_id;
        }
        else{
            break;
        }
    }
}
```

5. Heap Sort

Given the pseudo code:

HeapSort(*array*)
BuildHeap(*array*)
for *i* ←*0* →*n-1* **do**
  A[i] = ExtractMax()
end
return A

Implement this method:

```java
public static void heapSort(int[] arr){
    //code here
}
//This function can be implemented in the class that contains the
    main function.
```

6. Exercise

Exercise 1

Complete the Max-heap class following the instructions in this lab.

Exercise 2

Implement Min-heap of integers.

Exercise 3

Sort the following numbers ascending/descending by using Heap Sort:

$$15, 23, 18, 63, 21, 35, 36, 21, 66, 12, 42, 35, 75, 23, 64, 78, 39$$

Exercise 4

(*) Define the priority queue to queue some people. A person has name and priority number. Given that: higher priority = higher number. Perform these operations:

- Enqueue: (Alex, 3), (Bob, 2), (David, 6), (Susan, 1)
- Dequeue
- Enqueue: (Mike, 5), (Kevin, 4)
- Dequeue
- Dequeue
- Enqueue: (Helen, 0), (Paul, 8), (Iris, 7)
- Dequeue

Show the result of 4 persons will be dequeued.

THE END