Homework 10

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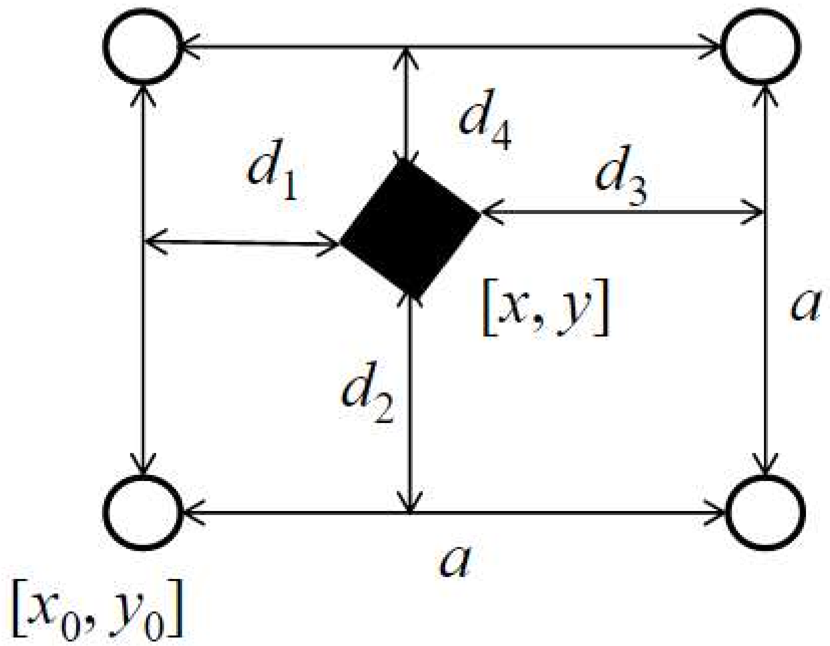
Problem 1



1. Known landmark location and known associations

Since the location of the landmark is known, the EKF SLAM problem can be transfer to EKF location problem

State vector is

$$\mu = [\, x \quad y \,]^T$$

State transfer matrix is

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Control vector is

$$u = [\, S_x \quad S_y \,]^T$$

Control transfer matrix is

$$B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

So, the prediction part of EKF location is:

$$
\begin{aligned}
\bar{\mu}_t &= A\mu_{t-1} + Bu_t \\
\bar{\Sigma}_t &= A\Sigma_{t-1}A^T + Q
\end{aligned}
$$

where

$$
Q = \begin{bmatrix} \sigma_{vx} & 0 \\ 0 & \sigma_{vy} \end{bmatrix}
$$

For the observation, the observation matrix is

$$
z = \begin{bmatrix} d_1 & d_2 & d_3 & d_4 \end{bmatrix}^T
$$

the observation state transfer function is

$$
h(\mu) = \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 0 & -1 \\ 0 & -1 \end{bmatrix} \mu + \begin{bmatrix} -x_0 \\ -y_0 \\ x_0 + a \\ y_0 + a \end{bmatrix}
$$

Thus, the Jacobian matrix is

$$
H = \begin{bmatrix} \frac{\partial h_1(\bar{\mu})}{\partial x} & \frac{\partial h_1(\bar{\mu})}{\partial y} \\ \frac{\partial h_2(\bar{\mu})}{\partial x} & \frac{\partial h_2(\bar{\mu})}{\partial y} \\ \frac{\partial h_3(\bar{\mu})}{\partial x} & \frac{\partial h_3(\bar{\mu})}{\partial y} \\ \frac{\partial h_4(\bar{\mu})}{\partial x} & \frac{\partial h_4(\bar{\mu})}{\partial y} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -1 & 0 \\ 0 & -1 \end{bmatrix}
$$

So, the correction part of EKF location is:

$$
\begin{aligned}
K_t &= \bar{\Sigma}_t H^T (H\bar{\Sigma}_t H^T + R)^{-1} \\
\mu_t &= \bar{\mu}_t + K_t(z - h(\bar{\mu}_t)) \\
\Sigma_t &= (I - K_t H)\bar{\Sigma}_t
\end{aligned}
$$

where

$$
R = \begin{bmatrix} \sigma_{d1} & 0 & 0 & 0 \\ 0 & \sigma_{d2} & 0 & 0 \\ 0 & 0 & \sigma_{d3} & 0 \\ 0 & 0 & 0 & \sigma_{d4} \end{bmatrix}
$$

2. Unknown landmark location and known associations

For SLAM problem, the corresponding state vector is:

$$
\mu = \begin{bmatrix} x & y & x_0 & y_0 & a \end{bmatrix}^T
$$

State transfer matrix is

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Control vector is

$$u = [\, S_x \quad S_y \,]^T$$

Control transfer matrix is

$$B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

So, the prediction part of EKF location is:

$$\begin{aligned} \bar{\mu}_t &= A\mu_{t-1} + Bu_t \\ \bar{\Sigma}_t &= A\Sigma_{t-1}A^T + Q \end{aligned}$$

where

$$Q = \begin{bmatrix} \sigma_{vx} & 0 \\ 0 & \sigma_{vy} \end{bmatrix}$$

For the observation, the observation matrix is

$$z = [\, d_1 \quad d_2 \quad d_3 \quad d_4 \,]^T$$

the observation state transfer function is

$$h(\mu) = \begin{bmatrix} 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 \\ -1 & 0 & 1 & 0 & 1 \\ 0 & -1 & 0 & 1 & 1 \end{bmatrix} \mu$$

Thus, the Jacobian matrix is

$$H = \begin{bmatrix} \frac{\partial h_1(\bar{\mu})}{\partial x} & \frac{\partial h_1(\bar{\mu})}{\partial y} & \frac{\partial h_1(\bar{\mu})}{\partial x_0} & \frac{\partial h_1(\bar{\mu})}{\partial y_0} & \frac{\partial h_1(\bar{\mu})}{\partial a} \\ \frac{\partial h_2(\bar{\mu})}{\partial x} & \frac{\partial h_2(\bar{\mu})}{\partial y} & \frac{\partial h_2(\bar{\mu})}{\partial x_0} & \frac{\partial h_2(\bar{\mu})}{\partial y_0} & \frac{\partial h_2(\bar{\mu})}{\partial a} \\ \frac{\partial h_3(\bar{\mu})}{\partial x} & \frac{\partial h_3(\bar{\mu})}{\partial y} & \frac{\partial h_3(\bar{\mu})}{\partial x_0} & \frac{\partial h_3(\bar{\mu})}{\partial y_0} & \frac{\partial h_3(\bar{\mu})}{\partial a} \\ \frac{\partial h_4(\bar{\mu})}{\partial x} & \frac{\partial h_4(\bar{\mu})}{\partial y} & \frac{\partial h_4(\bar{\mu})}{\partial x_0} & \frac{\partial h_4(\bar{\mu})}{\partial y_0} & \frac{\partial h_4(\bar{\mu})}{\partial a} \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 \\ -1 & 0 & 1 & 0 & 1 \\ 0 & -1 & 0 & 1 & 1 \end{bmatrix}$$

So, the correction part of EKF location is:

$$
\begin{aligned}
K_t &= \bar{\Sigma}_t H^T (H\bar{\Sigma}_t H^T + R)^{-1} \\
\mu_t &= \bar{\mu}_t + K_t(z - h(\bar{\mu}_t)) \\
\Sigma_t &= (I - K_t H)\bar{\Sigma}_t
\end{aligned}
$$

where

$$
R = \begin{bmatrix} \sigma_{d1} & 0 & 0 & 0 \\ 0 & \sigma_{d2} & 0 & 0 \\ 0 & 0 & \sigma_{d3} & 0 \\ 0 & 0 & 0 & \sigma_{d4} \end{bmatrix}
$$

3. Unknown landmark location and unknown associations

In the EKF SLAM unknown association problems, the state vector, observation vector, Jacobian matrix is the same, but the main difference between it and the EKF SLAM known assocations problem is the correction part. Since without the signature, for each observation, it needs to

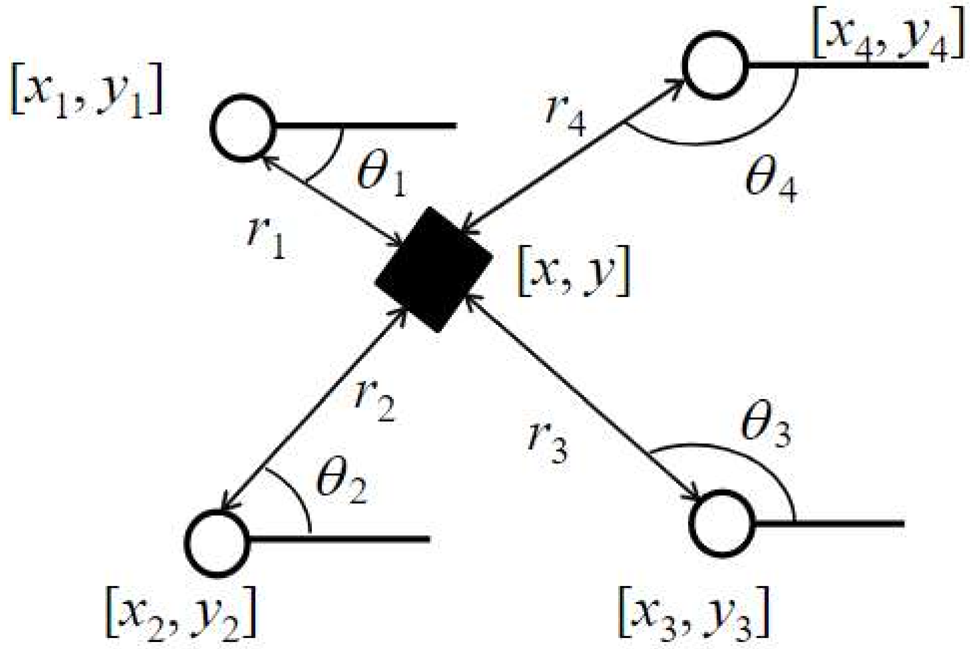
calculate the probablity which the landmark it belongs to:

$$
\begin{aligned}
\Psi_k &= H_{ik}\bar{\Sigma}_t H_{ik}^T + R \\
\pi_k &= (z_{ti} - \hat{z}_{tk})^T \Psi_k (z_{ti} - \hat{z}_{tk})
\end{aligned}
$$

Then

select a k that has the smallest π_k. If π_k beyond the threshold, than make it as the new landmark and continue the correction part.

Problem 2



1. Known landmark location and known associations

Since the location of the landmark is known, the EKF SLAM problem can be transfer to EKF location problem

State vector is

$$\mu = [\, x \quad y \quad \theta \,]^T$$

Control vector is

$$u = [\, \delta_{tran} \quad \delta_{rot1} \quad \delta_{rot2} \,]^T$$

The Jacobian matrix is

$$G_t = \begin{bmatrix} 1 & 0 & -\delta_{tran}\sin(\theta + \delta_{rot1}) \\ 0 & 1 & \delta_{tran}\cos(\theta + \delta_{rot1}) \\ 0 & 0 & 1 \end{bmatrix}$$

So, the prediction part of EKF location is:

$$\bar{\mu}_t = \mu_{t-1} + \begin{bmatrix} \delta_{tran}\cos(\theta + \delta_{rot1}) \\ \delta_{tran}\sin(\theta + \delta_{rot1}) \\ \theta + \delta_{rot1} + \delta_{rot2} \end{bmatrix}$$
$$\bar{\Sigma}_t = G_t \Sigma_{t-1} G_t^T + Q$$

where

$$\begin{bmatrix} \sigma_x & 0 & 0 \\ 0 & \sigma_y & 0 \\ 0 & 0 & \sigma_\theta \end{bmatrix}$$

For the observation, the observation matrix is

$$z = \begin{bmatrix} r_1 & \theta_1 & r_2 & \theta_2 & r_3 & \theta_3 & r_4 & \theta_4 \end{bmatrix}^T$$

the observation state transfer function is

$$h(\mu) = \begin{bmatrix} \sqrt{q_1} \\ atan2((\bar{y} - y_1), (\bar{x} - x_1) \\ \sqrt{q_2} \\ atan2((\bar{y} - y_2), (\bar{x} - x_2) \\ \sqrt{q_3} \\ atan2((\bar{y} - y_3), (\bar{x} - x_3) \\ \sqrt{q_4} \\ atan2((\bar{y} - y_4), (\bar{x} - x_4) \end{bmatrix}$$

where

$$\begin{aligned} q_1 &= (\bar{x} - x_1)^2 + (\bar{y} - y_1)^2 \\ q_2 &= (\bar{x} - x_2)^2 + (\bar{y} - y_2)^2 \\ q_3 &= (\bar{x} - x_3)^2 + (\bar{y} - y_3)^2 \\ q_4 &= (\bar{x} - x_4)^2 + (\bar{y} - y_4)^2 \end{aligned}$$

Thus, the Jacobian matrix is

$$H = \begin{bmatrix} \frac{\partial h_1(\bar{\mu})}{\partial x} & \frac{\partial h_1(\bar{\mu})}{\partial y} & \frac{\partial h_1(\bar{\mu})}{\partial \theta} \\ \frac{\partial h_2(\bar{\mu})}{\partial x} & \frac{\partial h_2(\bar{\mu})}{\partial y} & \frac{\partial h_2(\bar{\mu})}{\partial \theta} \\ \frac{\partial h_3(\bar{\mu})}{\partial x} & \frac{\partial h_3(\bar{\mu})}{\partial y} & \frac{\partial h_3(\bar{\mu})}{\partial \theta} \\ \frac{\partial h_4(\bar{\mu})}{\partial x} & \frac{\partial h_4(\bar{\mu})}{\partial y} & \frac{\partial h_4(\bar{\mu})}{\partial \theta} \end{bmatrix} = \begin{bmatrix} \frac{\bar{x}-x_1}{\sqrt{q_1}} & \frac{\bar{y}-y_1}{\sqrt{q_1}} & 0 \\ -\frac{\bar{y}-y_1}{q_1} & \frac{\bar{x}-x_1}{q_1} & 0 \\ \frac{\bar{x}-x_2}{\sqrt{q_2}} & \frac{\bar{y}-y_2}{\sqrt{q_2}} & 0 \\ -\frac{\bar{y}-y_2}{q_2} & \frac{\bar{x}-x_2}{q_2} & 0 \\ \frac{\bar{x}-x_3}{\sqrt{q_3}} & \frac{\bar{y}-y_3}{\sqrt{q_3}} & 0 \\ -\frac{\bar{y}-y_3}{q_3} & \frac{\bar{x}-x_3}{q_3} & 0 \\ \frac{\bar{x}-x_4}{\sqrt{q_4}} & \frac{\bar{y}-y_4}{\sqrt{q_4}} & 0 \\ -\frac{\bar{y}-y_4}{q_4} & \frac{\bar{x}-x_4}{q_4} & 0 \end{bmatrix}$$

So, the correction part of EKF location is:

$$\begin{aligned} K_t &= \bar{\Sigma}_t H^T (H \bar{\Sigma}_t H^T + R)^{-1} \\ \mu_t &= \bar{\mu}_t + K_t(z - h(\bar{\mu}_t)) \\ \Sigma_t &= (I - K_t H)\bar{\Sigma}_t \end{aligned}$$

where

$$R = \begin{bmatrix} \sigma_{d1} & 0 & 0 & 0 \\ 0 & \sigma_{d2} & 0 & 0 \\ 0 & 0 & \sigma_{d3} & 0 \\ 0 & 0 & 0 & \sigma_{d4} \end{bmatrix}$$

2. Unknown landmark location and known associations

State vector is

$$\mu = \begin{bmatrix} x & y & \theta & x_1 & y_1 & x_2 & y_2 & x_3 & y_3 & x_4 & y_4 \end{bmatrix}^T$$

The Jacobian matrix is

$$G_t = I + F_x^T \begin{bmatrix} 0 & 0 & -\delta_{tran}\sin(\theta + \delta_{rot1}) \\ 0 & 0 & \delta_{tran}\cos(\theta + \delta_{rot1}) \\ 0 & 0 & 0 \end{bmatrix} F_x$$

where

$$F_x = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The prediction part of EKF is

$$\begin{aligned} \bar{\mu}_t &= \mu_{t-1} + F_x^T \begin{bmatrix} 0 & 0 & \delta_{tran}\cos(\theta + \delta_{rot1}) \\ 0 & 0 & \delta_{tran}\sin(\theta + \delta_{rot1}) \\ 0 & 0 & 0 \end{bmatrix} \\ \bar{\Sigma}_t &= G_t \Sigma_{t-1} G_t^T + F_x^T Q F_x \end{aligned}$$

where

$$Q = \begin{bmatrix} \sigma_x & 0 & 0 \\ 0 & \sigma_y & 0 \\ 0 & 0 & \sigma_\theta \end{bmatrix}$$

For the observation, the observation matrix is

$$z = \begin{bmatrix} r_1 & \theta_1 & r_2 & \theta_2 & r_3 & \theta_3 & r_4 & \theta_4 \end{bmatrix}^T$$

the observation state transfer function is

$$h(\mu) = \begin{bmatrix} \sqrt{q_1} \\ atan2((\bar{y} - y_1), (\bar{x} - x_1) \\ \sqrt{q_2} \\ atan2((\bar{y} - y_2), (\bar{x} - x_2) \\ \sqrt{q_3} \\ atan2((\bar{y} - y_3), (\bar{x} - x_3) \\ \sqrt{q_4} \\ atan2((\bar{y} - y_4), (\bar{x} - x_4) \end{bmatrix}$$

where

$$\begin{aligned} q_1 &= (\bar{x} - x_1)^2 + (\bar{y} - y_1)^2 \\ q_2 &= (\bar{x} - x_2)^2 + (\bar{y} - y_2)^2 \\ q_3 &= (\bar{x} - x_3)^2 + (\bar{y} - y_3)^2 \\ q_4 &= (\bar{x} - x_4)^2 + (\bar{y} - y_4)^2 \end{aligned}$$

where j is the index of the landmark

for each observation

$$F_{x,j} = \begin{bmatrix} 1 & 0 & 0 & 0 \cdots 0 & 0 & 0 & 0 & 0 \cdots 0 \\ 0 & 1 & 0 & 0 \cdots 0 & 0 & 0 & 0 & 0 \cdots 0 \\ 0 & 0 & 1 & 0 \cdots 0 & 0 & 0 & 0 & 0 \cdots 0 \\ 0 & 0 & 0 & 0 \cdots 0 & 1 & 0 & 0 & 0 \cdots 0 \\ 0 & 0 & 0 & 0 \cdots 0 & 0 & 1 & 0 & 0 \cdots 0 \\ 0 & 0 & 0 & \underbrace{0 \cdots 0}_{3j-3} & 0 & 0 & 1 & \underbrace{0 \cdots 0}_{N-3j} \end{bmatrix}$$

$$H_i = \frac{1}{q} \begin{bmatrix} -\sqrt{q}\delta_x & -\sqrt{q}\delta_y & 0 & +\sqrt{q}\delta_x & \sqrt{q}\delta_y \\ \delta_y & -\delta_x & -q & -\delta_y & +\delta_x \end{bmatrix} F_j$$

$$H = \begin{bmatrix} H \\ H_i \end{bmatrix}$$

So, the correction part of EKF location is:

$$\begin{aligned} K_t &= \bar{\Sigma}_t H^T (H \bar{\Sigma}_t H^T + R)^{-1} \\ \mu_t &= \bar{\mu}_t + K_t (z - h(\bar{\mu}_t)) \\ \Sigma_t &= (I - K_t H) \bar{\Sigma}_t \end{aligned}$$

where

$$R = \begin{bmatrix} \sigma_{d1} & 0 & 0 & 0 \\ 0 & \sigma_{d2} & 0 & 0 \\ 0 & 0 & \sigma_{d3} & 0 \\ 0 & 0 & 0 & \sigma_{d4} \end{bmatrix}$$

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In the EKF SLAM unknown association problems, the state vector, observation vector, Jacobian matrix is the same, but the main difference between it and the EKF SLAM known assocations problem is the correction part. Since without the signature, for each observation, it needs to

calculate the probablity which the landmark it belongs to:

$$\begin{aligned} \Psi_k &= H_{ik}\bar{\Sigma}_t H_{ik}^T + R \\ \pi_k &= (z_{ti} - \hat{z}_{tk})^T \Psi_k (z_{ti} - \hat{z}_{tk}) \end{aligned}$$

Then

select a k that has the smallest π_k. If π_k beyond the threshold, than make it as the new landmark and continue the correction part.

Particle Filter SLAM

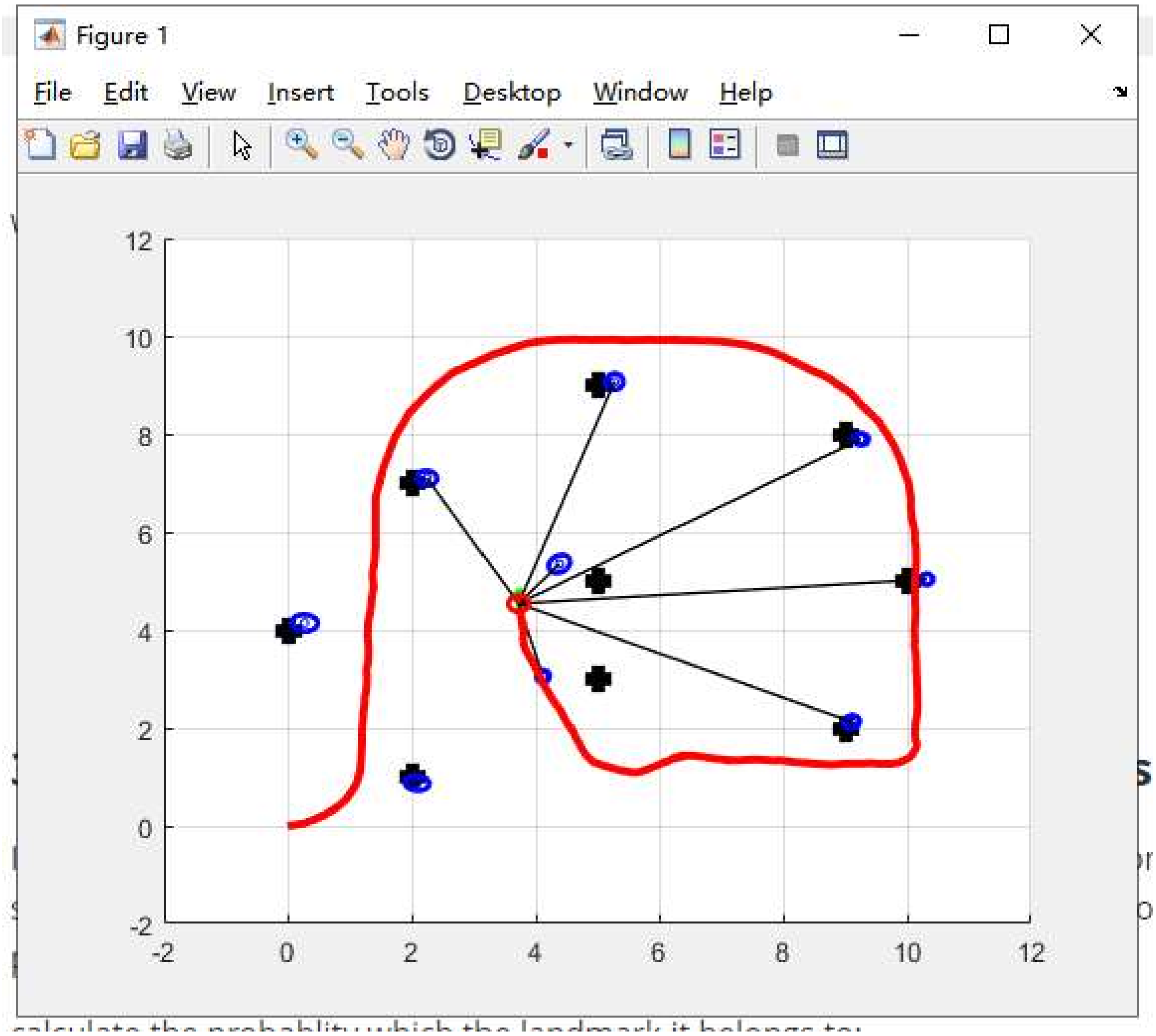
Figure 1
File Edit View Insert Tools Desktop Window Help
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