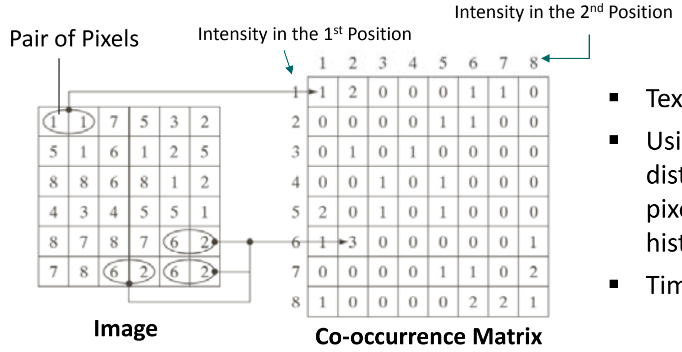
Gray Level Co-occurance Matrix

GLCM

Gray Level Co-occurance Matrix

R. M. Haralick, K. Shanmugam, I. Dinstein (1973)



- Textural feature
- Using joint probability distributions of pairs of pixels (Second-order histogram)
- Time-consuming

GLCM

Gray Level Co-occurance Matrix

Asymmetric GLCM

$$C_{ASYM}(i, j, \Delta x, \Delta y) = \left\{ \begin{array}{l} \text{The number of pixels where} \\ I_Q(x, y) = i \text{ and } I_Q(x+\Delta x, y+\Delta y) = j \end{array} \right\}$$

$I_Q(x, y)$: Quantized Image

i, j : Intensity Levels (after quantization)

(x, y) : Pixel Coordinate

$(\Delta x, \Delta y)$: Offset of Neighboring Pixel

GLCM [Asymmetric]

255	250	152	80	11	5
250	240	200	160	89	52
253	230	180	122	40	2
175	225	190	148	34	12
120	152	205	123	88	15
32	82	153	166	44	4
17	11	9	78	0	2

3	3	2	1	0	0
3	3	3	2	1	0
3	3	2	1	0	0
2	3	2	2	0	0
1	2	3	1	1	0
0	1	2	2	0	0
0	0	0	1	0	0

Label	0	1	2	3
Intensity Range	0-63	64-127	128-191	192-255

STEP 1: Quantize Intensity Level

GLCM [Asymmetric]

y →					
3	3	2	1	0	0
3	3	3	2	1	0
3	3	2	1	0	0
2	3	2	2	0	0
1	2	3	1	1	0
0	1	2	2	0	0
0	0	0	1	0	0

x ↓

$$C_{ASYM}(\Delta x = 0, \Delta y = 1) = \begin{array}{cc} j: \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} & i \\ \begin{bmatrix} \boxed{7} & 2 & 0 & 0 \\ 5 & \square & 2 & \square \\ 2 & \square & 2 & 2 \\ 0 & \square & 4 & 4 \end{bmatrix} & \begin{array}{c} 0 \\ 1 \\ 2 \\ 3 \end{array} \end{array}$$

$$(x, y) = i \;\&\; (x, y+1) = j$$

STEP 2: Count number of occurrence of each pattern of pixel pair

GLCM

y →					
3	3	2	1	0	0
3	3	3	2	1	0
3	3	2	1	0	0
2	3	2	2	0	0
1	2	3	1	1	0
0	1	2	2	0	0
0	0	0	1	0	0

x ↓

$$C_{ASYM}(\Delta x = 0, \Delta y = 1) = \begin{array}{cc} j: \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} & i \\ \begin{bmatrix} 7 & \boxed{2} & 0 & 0 \\ 5 & \square & 2 & \square \\ 2 & \square & 2 & 2 \\ 0 & \square & 4 & 4 \end{bmatrix} & \begin{array}{c} 0 \\ 1 \\ 2 \\ 3 \end{array} \end{array}$$

$$(x, y) = i \;\&\; (x, y+1) = j$$

STEP 2: Count number of occurrence of each pattern of pixel pair

GLCM [Asymmetric]

y →					
3	3	2	1	0	0
3	3	3	2	1	0
3	3	2	1	0	0
2	3	2	2	0	0
1	2	3	1	1	0
0	1	2	2	0	0
0	0	0	1	0	0

x ↓

$$C_{ASYM}(\Delta x = 0, \Delta y = 1) = \begin{array}{cc} j: \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} & i \\ \begin{bmatrix} 7 & 2 & 0 & 0 \\ \boxed{5} & \square & 2 & \square \\ 2 & \square & 2 & 2 \\ 0 & \square & 4 & 4 \end{bmatrix} & \begin{array}{c} 0 \\ 1 \\ 2 \\ 3 \end{array} \end{array}$$

$$(x, y) = i \;\&\; (x, y+1) = j$$

STEP 2: Count number of occurrence of each pattern of pixel pair

GLCM [Asymmetric]

$$C_{ASYM} = \begin{bmatrix} 7 & 2 & 0 & 0 \\ 5 & 1 & 2 & 0 \\ 2 & 3 & 2 & 2 \\ 0 & 1 & 4 & 4 \end{bmatrix} \Rightarrow \frac{1}{\sum_{i,j} C} \begin{bmatrix} 7 & 2 & 0 & 0 \\ 5 & 1 & 2 & 0 \\ 2 & 3 & 2 & 2 \\ 0 & 1 & 4 & 4 \end{bmatrix} = \begin{bmatrix} 0.20 & 0.06 & 0 & 0 \\ 0.14 & 0.03 & 0.06 & 0 \\ 0.06 & 0.09 & 0.06 & 0.06 \\ 0 & 0.03 & 0.11 & 0.11 \end{bmatrix}$$

STEP 3: Normalize with $\sum_{i,j} C$

(optional)

GLCM [Asymmetric]

3	3	2	1	0	0
3	3	3	2	1	0
3	3	2	1	0	0
2	3	2	2	0	0
1	2	3	1	1	0
0	1	2	2	0	0
0	0	0	1	0	0

y (columns), x (rows)

$$C_{ASYM}(\Delta x = 1, \Delta y = -1) = ?$$

$$C_{ASYM}(\Delta x = -1, \Delta y = 1) = ?$$

GLCM

Gray Level Co-occurance Matrix

Symmetric GLCM

$$C_{SYM}(\Delta x, \Delta y) = C_{ASYM}(\Delta x, \Delta y) + C_{ASYM}(-\Delta x, -\Delta y)$$
$$= C_{ASYM}(\Delta x, \Delta y) + [C_{ASYM}(\Delta x, \Delta y)]^T$$

Note that $C_{ASYM}(-\Delta x, -\Delta y) = [C_{ASYM}(\Delta x, \Delta y)]^T$

GLCM [Symmetric]

3	3	2	1	0	0
3	3	3	2	1	0
3	3	2	1	0	0
2	3	2	2	0	0
1	2	3	1	1	0
0	1	2	2	0	0
0	0	0	1	0	0

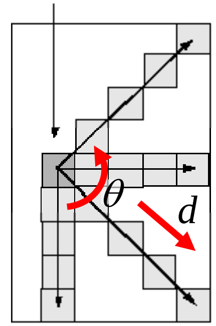
y (columns), x (rows)

$$C_{SYM}(\Delta x = 0, \Delta y = 1)$$
$$= C_{ASYM}(0,1) + [C_{ASYM}(0,1)]^T$$
$$= \begin{bmatrix} 7 & 2 & 0 & 0 \\ 5 & 1 & 2 & 0 \\ 2 & 3 & 2 & 2 \\ 0 & 1 & 4 & 4 \end{bmatrix} + \begin{bmatrix} 7 & 5 & 2 & 0 \\ 2 & 1 & 3 & 1 \\ 0 & 2 & 2 & 4 \\ 0 & 0 & 2 & 4 \end{bmatrix} = \begin{bmatrix} 14 & 7 & 2 & 0 \\ 7 & 2 & 5 & 1 \\ 2 & 5 & 4 & 6 \\ 0 & 1 & 6 & 8 \end{bmatrix}$$
$$= \begin{bmatrix} 0.20 & 0.10 & 0.03 & 0 \\ 0.10 & 0.03 & 0.07 & 0.01 \\ 0.03 & 0.07 & 0.06 & 0.09 \\ 0 & 0.01 & 0.09 & 0.11 \end{bmatrix} \quad \text{(Normalized)}$$

GLCM [Polar Notation]

$$\boxed{C(d, \theta)}$$

Point of interest



$$C(d, \theta = 0^\circ) = C(\Delta x = d, \Delta y = 0)$$
$$C(d, \theta = 45^\circ) = C(\Delta x = d, \Delta y = d)$$
$$C(d, \theta = 90^\circ) = C(\Delta x = 0, \Delta y = d)$$
$$C(d, \theta = 135^\circ) = C(\Delta x = -d, \Delta y = d)$$

Statistical Properties of GLCM

1. Maximum probability $\max_{i,j} C(i, j)$

Measure the strongest response of GLCM.

2. Angular Second Moment $\sum_{i,j} C^2(i, j)$

[Uniformity, Energy]

Measure of uniformity. Uniformity is 1 (maximum) for a constant image

3. Contrast $\sum_{i,j} (i-j)^2 C(i, j)$

Measure of intensity contrast between a pixel and its neighbor.

Note that $C(i, j)$ is a normalized GLCM

Statistical Properties of GLCM

4. Correlation $\sum_{i,j} \frac{(i-\mu_1)(j-\mu_2)C(i, j)}{\sigma_1 \sigma_2}$

Measure of how correlated a pixel is to its neighbor. Range of correlation is [-1,1].

$$C = \begin{bmatrix} 7 & 2 & 0 & 0 \\ 5 & 1 & 2 & 0 \\ 2 & 3 & 2 & 2 \\ 0 & 1 & 4 & 4 \end{bmatrix}$$

$$\mu_1 = \sum_i \left(i \cdot \boxed{\sum_j C(i, j)} \right) \qquad \sigma_1^2 = \sum_i \left((i-\mu_1)^2 \boxed{\sum_j C(i, j)} \right)$$

$$\mu_2 = \sum_j \left(j \cdot \sum_i C(i, j) \right) \qquad \sigma_2^2 = \sum_j \left((j-\mu_2)^2 \sum_i C(i, j) \right)$$

Statistical Properties of GLCM

4. Correlation $\sum_{i,j} \frac{(i-\mu_1)(j-\mu_2)C(i, j)}{\sigma_1 \sigma_2}$

Measure of how correlated a pixel is to its neighbor. Range of correlation is [-1,1].

$$C = \begin{bmatrix} 7 & 2 & 0 & 0 \\ 5 & 1 & 2 & 0 \\ 2 & 3 & 2 & 2 \\ 0 & 1 & 4 & 4 \end{bmatrix}$$

$$\mu_1 = \sum_i \left(i \cdot \sum_j C(i, j) \right) \qquad \sigma_1^2 = \sum_i \left((i-\mu_1)^2 \sum_j C(i, j) \right)$$

$$\mu_2 = \sum_j \left(j \cdot \boxed{\sum_i C(i, j)} \right) \qquad \sigma_2^2 = \sum_j \left((j-\mu_2)^2 \boxed{\sum_i C(i, j)} \right)$$

Statistical Properties of GLCM

5. Homogeneity $\sum_{i,j} \frac{C(i, j)}{1+|i-j|}$

Measure spatial closeness. Homogeneity is maximum when GLCM is a diagonal matrix.

6. Entropy $-\sum_{i,j} C(i, j) \log_2 C(i, j)$

Measure a randomness. Entropy is maximum when all elements in GLCM are equal

Statistical Properties of GLCM

7. Variance

8. Sum Average

9. Sum Variance

10. Sum Entropy

11. Difference Variance

12. Difference Entropy

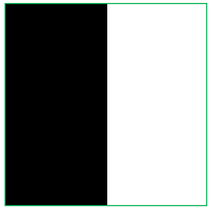
13. Inverse Difference Moment

14. Information Measure of Correlation

15. Maximal Correlation Coefficient

http://www.cis.rit.edu/~cnspci/references/dip/segmentation/haralick1973.pdf

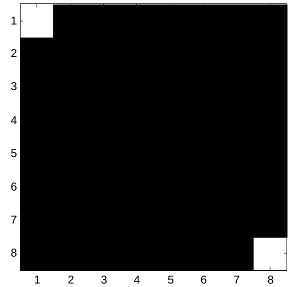
Images



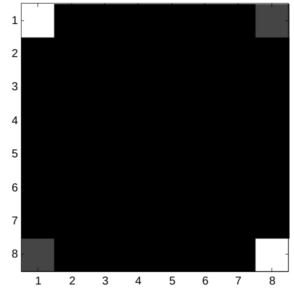
1 $C_{SYM}(\Delta x = 1, \Delta y = 0)$

2 $C_{SYM}(\Delta x = 0, \Delta y = 1)$

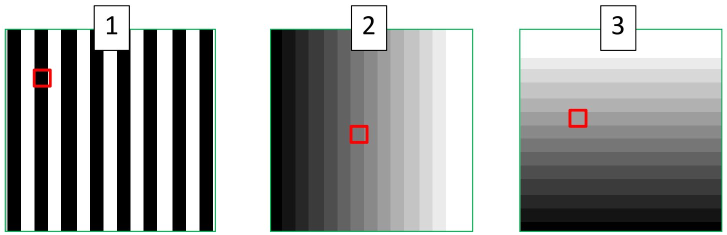
A



B

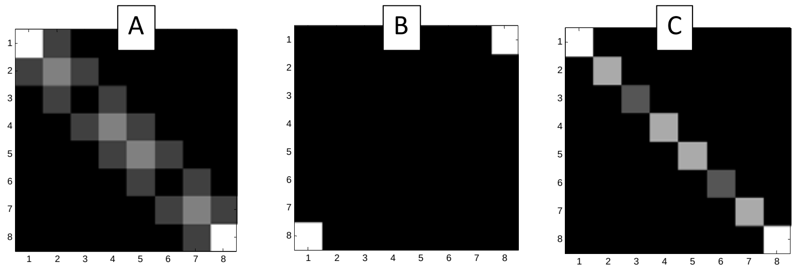


Image

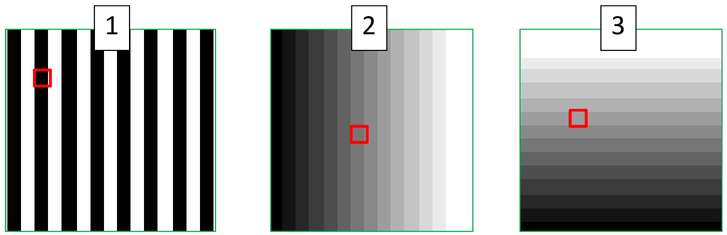


$C_{SYM}(\Delta x = 0, \Delta y = 1)$

GLCM



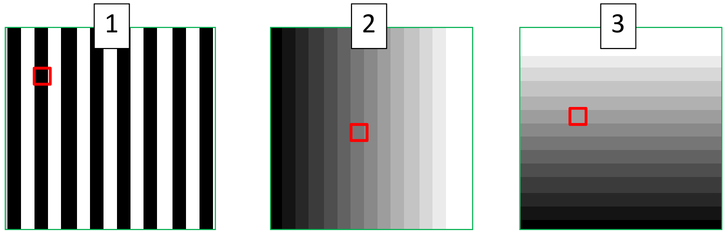
Image



$C_{SYM}(\Delta x = 0, \Delta y = 1)$

Contrast $\square > \square > \square$

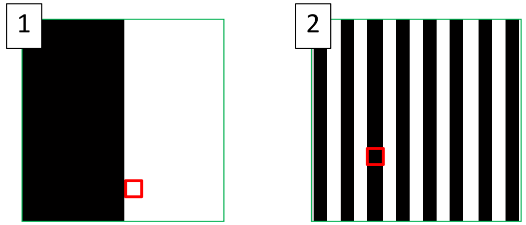
Image



$$C_{SYM}(\Delta x = 0, \Delta y = 1)$$

Entropy ☐ > ☐ > ☐

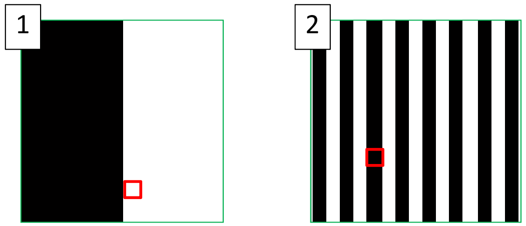
Images



$$C_{SYM}(\Delta x = 1, \Delta y = 0)$$

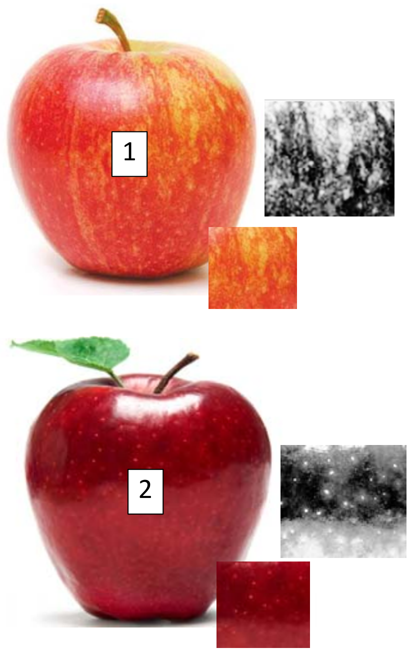
Maximum probability	1	☐	2
Uniformity	1	☐	2
Contrast	1	☐	2
Homogeneity	1	☐	2

Images



$$C_{SYM}(\Delta x = 0, \Delta y = 1)$$

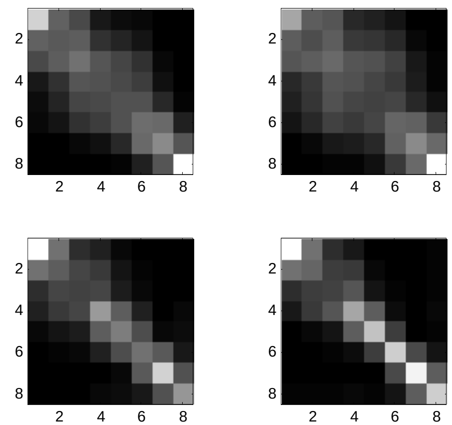
Maximum probability	1	☐	2
Uniformity	1	☐	2
Contrast	1	☐	2
Homogeneity	1	☐	2



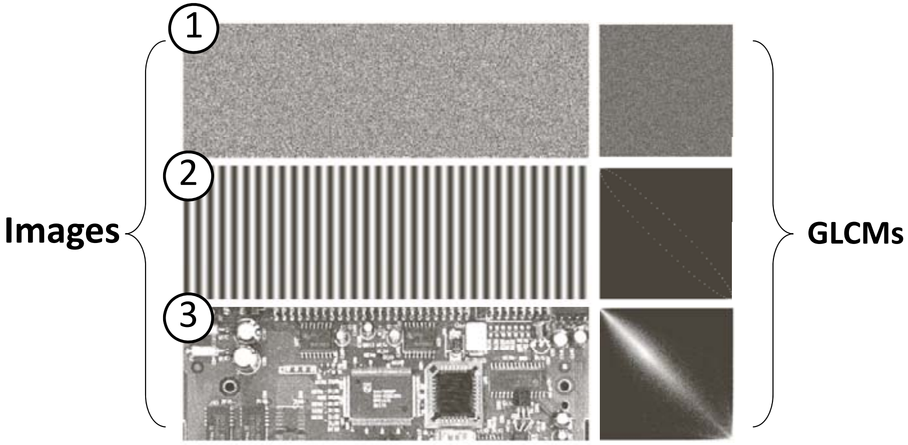
$C_{SYM}(\Delta x = 7, \Delta y = 0)$ $C_{SYM}(\Delta x = 0, \Delta y = 7)$

A

B



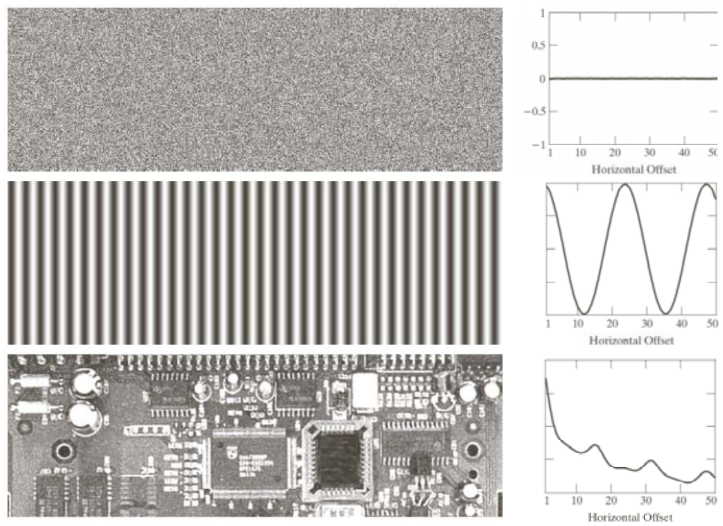
http://www.manager.co.th/daily/viewnews.aspx?NewsID=9560000083911



Normalized Co-occurrence Matrix	Descriptor					
	Max Probability	Correlation	Contrast	Uniformity	Homogeneity	Entropy
①	0.00006	−0.0005	10838	0.00002	0.0366	15.75
②	0.01500	0.9650	570	0.01230	0.0824	6.43
③	0.06860	0.8798	1356	0.00480	0.2048	13.58

Images $\rightarrow C(d, 90^\circ) \rightarrow$ Correlation

(Varying offset d)



GLCM Based Feature Extraction