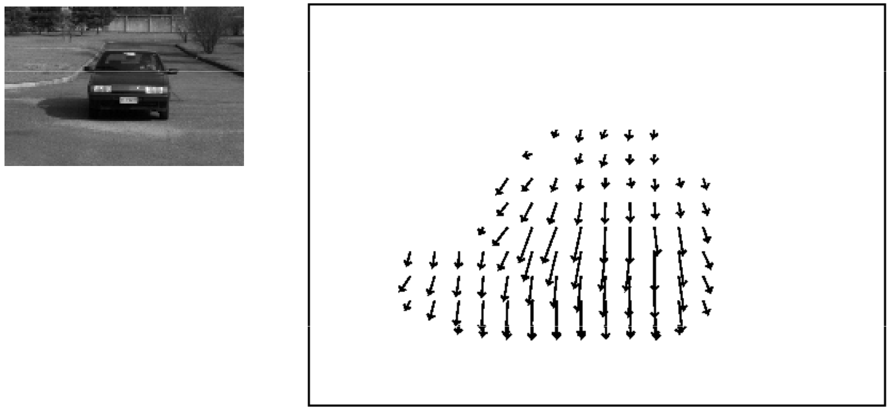
Optical Flow

Optical Flow

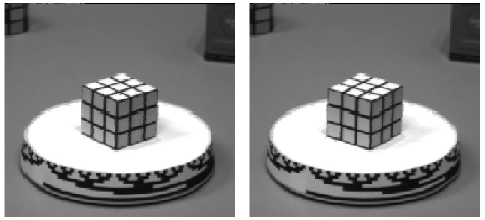
Where did each pixel in image 1 go to in image 2 ?

Optical Flow

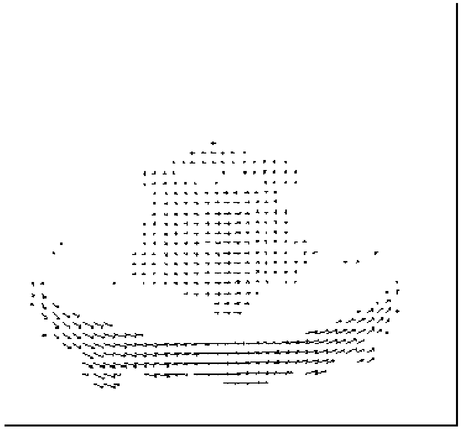


Pierre Kornprobst's Demo

Optical Flow



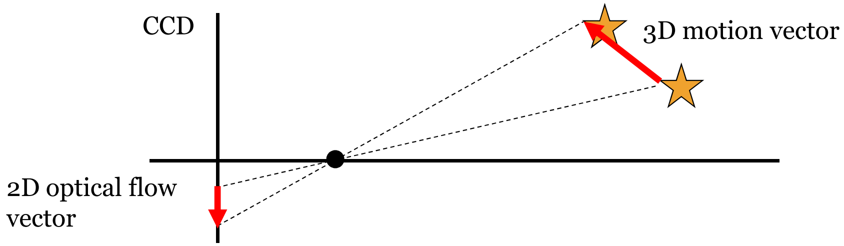
- We are interested in finding the movement of scene objects from time-varying images (videos).



• Will start by estimating motion of ***each pixel*** separately
Then will consider motion of ***entire*** image

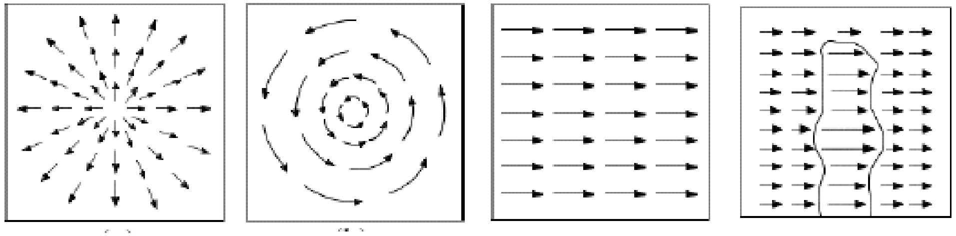
Motion Field & Optical Flow Field

- **Motion Field = Real world 3D motion**
- **Optical Flow Field = Projection of the motion field onto the 2d image**



$$\vec{u} = (u, v)$$

Motion Field & Optical Flow Field



Forward motion

Rotation

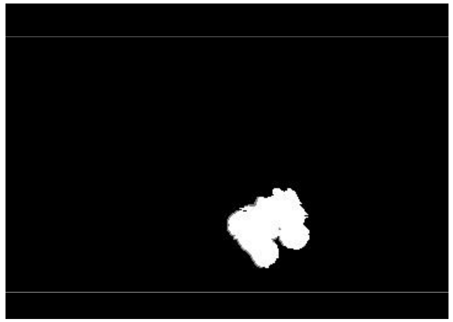
Horizontal translation

Closer objects appear to move faster!!

Scene Interpretation

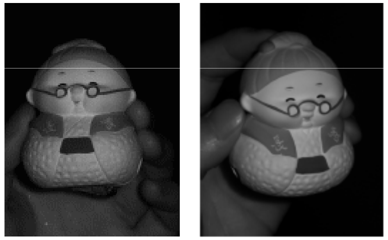
- Given a video sequence with camera/objects moving we can better understand the scene if we find the motions of the camera/objects.
  - How is the camera moving?
  - How many moving objects are there?
  - Which directions are they moving in?
  - How fast are they moving?
  - Can we recognize their type of motion (e.g. walking, running, etc.)?

Motion Segmentation

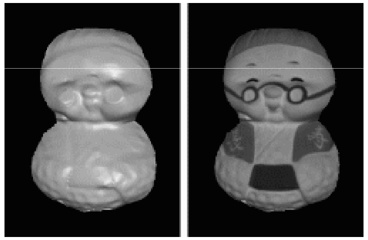


Result by: L.Zelnik-Manor, M.Machline, M.Irani
"*Multi-body Segmentation: Revisiting Motion Consistency*", IJCV 2006

3D Shape Reconstruction



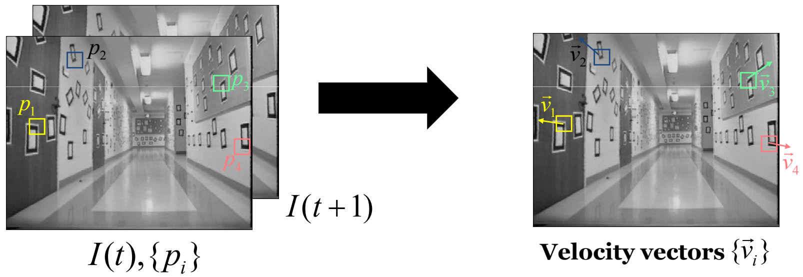
Input



Reconstructed shape

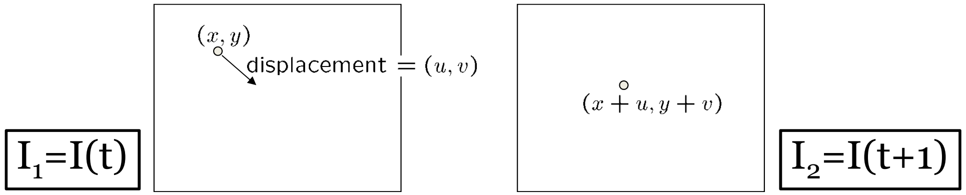
Result by: **L. Zhang, B. Curless, A. Hertzmann, S.M. Seitz**
"Shape and motion under varying illumination: Unifying structure from motion, photometric stereo, and multi-view stereo" ICCV'03

Optical Flow



Goal: *Find for **each pixel** a velocity vector*

Optical Flow



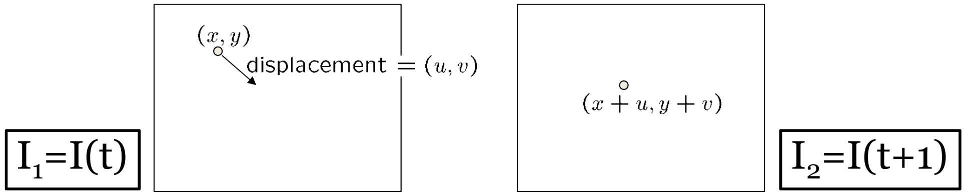
How to estimate pixel motion from image I_1 to image I_2 ?

- Solve pixel correspondence problem
  - given a pixel in I_1, look for nearby pixels of the same color in I_2

Key assumptions

- color constancy: a point in I_1 looks the same in I_2
  - For grayscale images, this is brightness constancy
- small motion: points do not move very far

Optical Flow



Color constancy

$$I_1(x, y) = I_2(x+u, y+v)$$

Small motion

$$I_2(x+u, y+v) \approx I_2(x, y) + \frac{\partial I_2}{\partial x} u + \frac{\partial I_2}{\partial y} v$$

Approximated by Taylor series expansion

Optical Flow

$$I_1(x,y) \approx I_2(x,y) + \frac{\partial I_2}{\partial x} u + \frac{\partial I_2}{\partial y} v$$

$$0 \approx I_2(x,y) - I_1(x,y) + \frac{\partial I_2}{\partial x} u + \frac{\partial I_2}{\partial y} v$$

$$0 \approx I(t+1) - I(t) + \frac{\partial I_2}{\partial x} u + \frac{\partial I_2}{\partial y} v$$

$$0 \approx I_t + I_x \cdot u + I_y \cdot v$$

Optical Flow

We want to determine u and v for each pixel

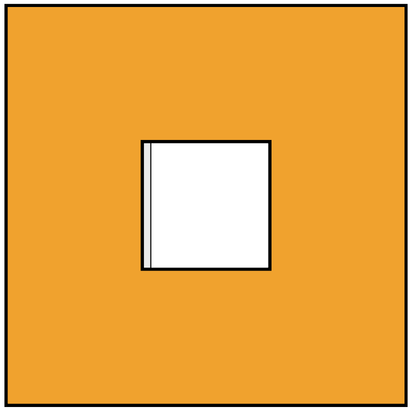
$$0 \approx \underbrace{\boxed{I_t} + \boxed{I_x} \cdot u + \boxed{I_y}}_{\text{This can be calculated from images}} \cdot v$$

2 unknown with only one equation

Need More Constraints

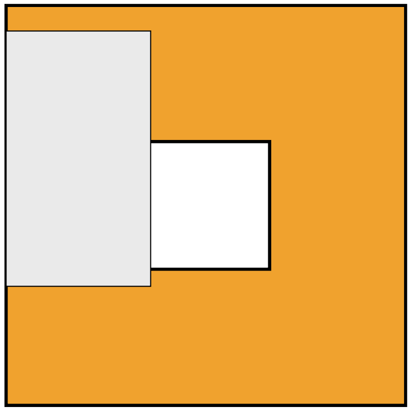
Aperture Problem

- How does this show up visually ?

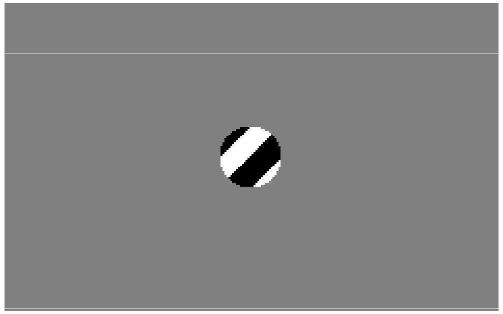


Aperture Problem

- Motion along just an edge is ambiguous

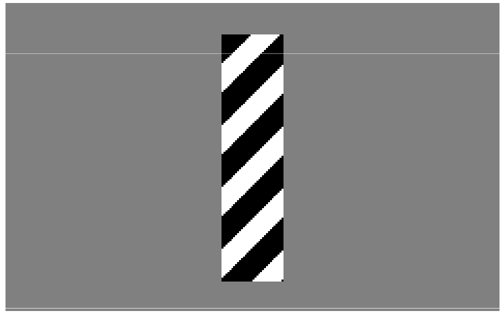


The Barber's Pole Illusion



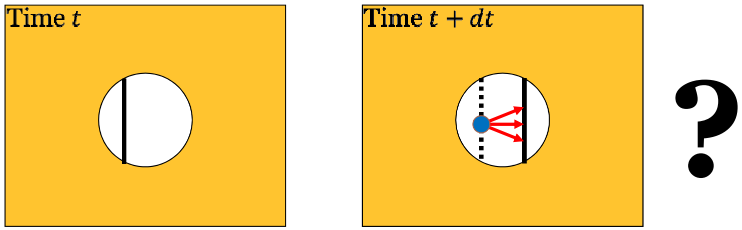
http://en.wikipedia.org/wiki/Barberpole_illusion

The Barber's Pole Illusion



http://en.wikipedia.org/wiki/Barberpole_illusion

Aperture Problem

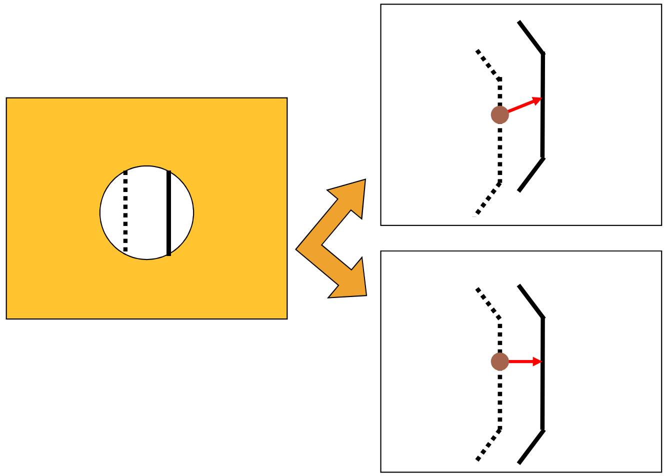


Where did the blue point move to?

We need additional constraints

Aperture Problem

- Sometimes enlarging the aperture can help



Horn-Schunck Method

assume that the flow field is smooth locally

$$E_{HS}(u,v)=\sum_{x,y}\left[\left(I_t+I_x\cdot u+I_y\cdot v\right)^2+\lambda\underbrace{\left(\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial u}{\partial y}\right)^2+\left(\frac{\partial v}{\partial x}\right)^2+\left(\frac{\partial v}{\partial y}\right)^2\right)}_{\text{smoothness in the flow over the whole image}}\right]$$

Regularization parameter

Lucas-Kanade Method

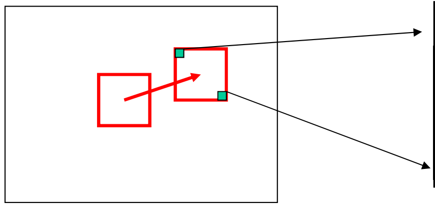
pretend the pixel's neighbors have the same (u, v)

If we use a 5x5 window, that gives us 25 equations per pixel!

$$\left.\begin{aligned}
I_t(x_1,y_1)+I_x(x_1,y_1)\cdot u+I_y(x_1,y_1)\cdot v&\approx 0\\
I_t(x_2,y_2)+I_x(x_2,y_2)\cdot u+I_y(x_2,y_2)\cdot v&\approx 0\\
&\vdots\\
I_t(x_{25},y_{25})+I_x(x_{25},y_{25})\cdot u+I_y(x_{25},y_{25})\cdot v&\approx 0
\end{aligned}\right\}\text{Find best } u, v$$

Lucas-Kanade Method

Assume constant (u,v) in small neighborhood



$$\begin{bmatrix} I_{x1} & I_{y1} \\ I_{x2} & I_{y2} \\ \vdots & \end{bmatrix}\begin{bmatrix} u \\ v \end{bmatrix}=-\begin{bmatrix} I_{t1} \\ I_{t2} \\ \vdots \end{bmatrix}$$

$$\Downarrow$$

$$\mathbf{Au}=\mathbf{b}$$

Lucas-Kanade Method

Find $\mathbf{u}$ that minimize $\left\|\mathbf{Au}-\mathbf{b}\right\|^2$

$$\Downarrow$$

$$\mathbf{u}_{opt}=\left(\mathbf{A}^T\mathbf{A}\right)^{-1}\mathbf{A}^T\mathbf{b}$$

Edge ➔ $\mathbf{A}^T\mathbf{A}$ becomes singular

Homogeneous ➔ $\mathbf{A}^T\mathbf{A}\approx 0$ (low gradient)

High texture ➔ $\mathbf{A}^T\mathbf{A}=$ 

Coarse-to-fine Flow Estimation