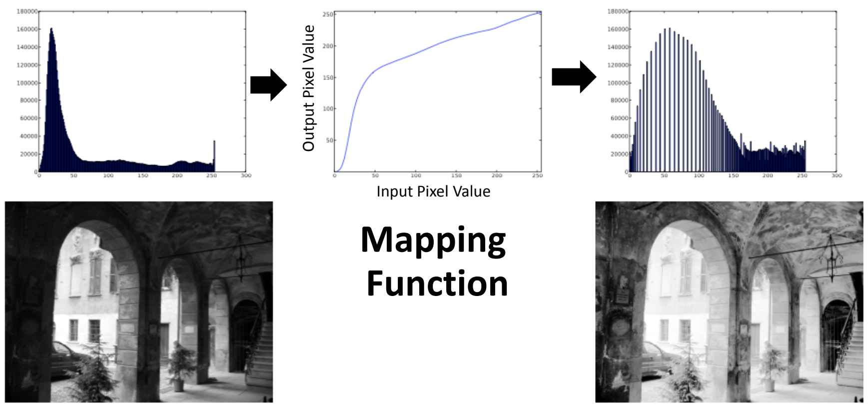
Image Enhancement

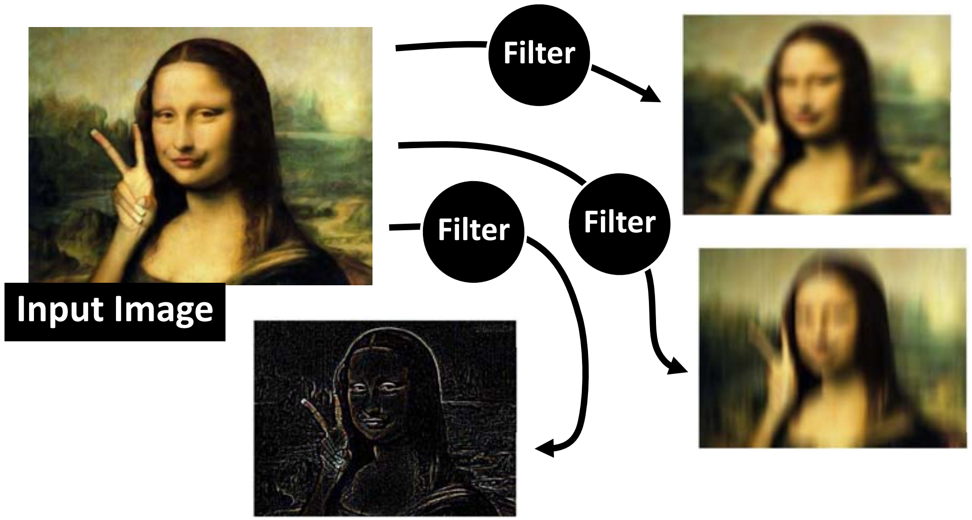
Histogram Equalization



Before **After**

http://www.janeriksolem.net/2009/06/histogram-equalization-with-python-and.html

Image Filtering



Linear Filter

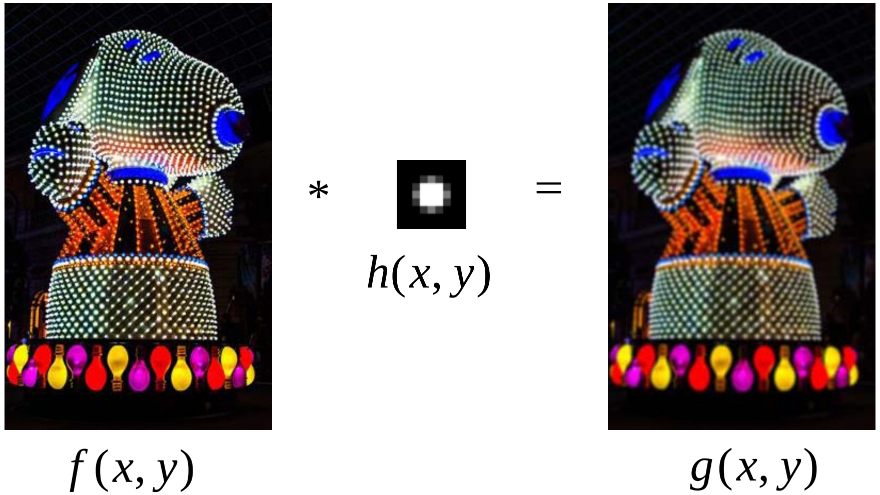
Output Image Input Image

$$g(x,y) = h(x,y) * f(x,y) = \sum_{p,q} h(x-p, y-q) f(p,q)$$

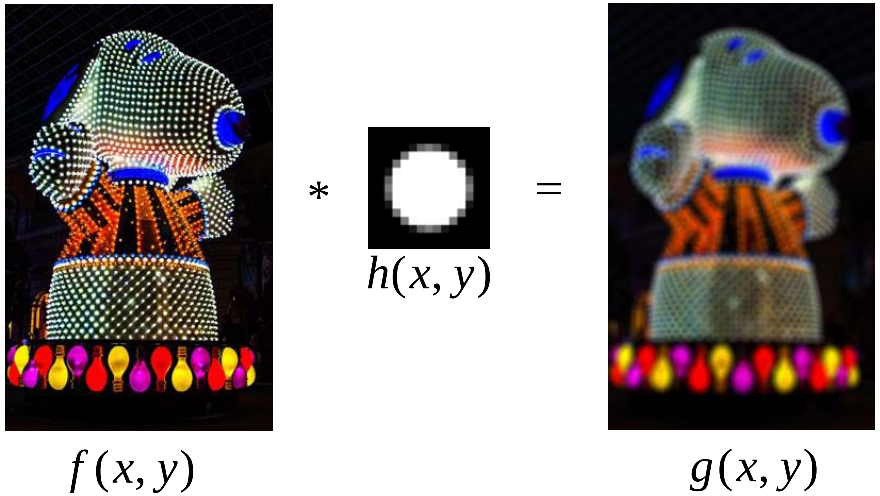
$\Rightarrow h(x,y)$: Filter kernel, Filter mask, Convolution kernel, Convolution mask, Impulse response, Point spread function.

$\Rightarrow *$: Convolution operator

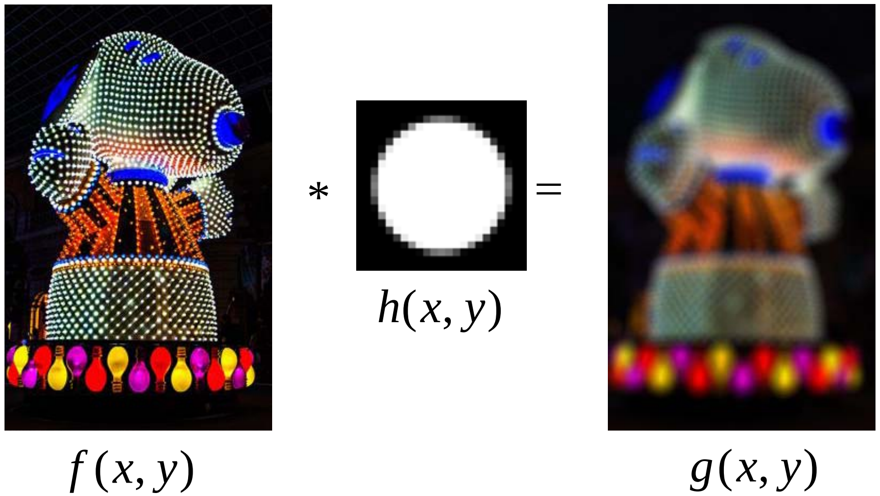
Linear Filter



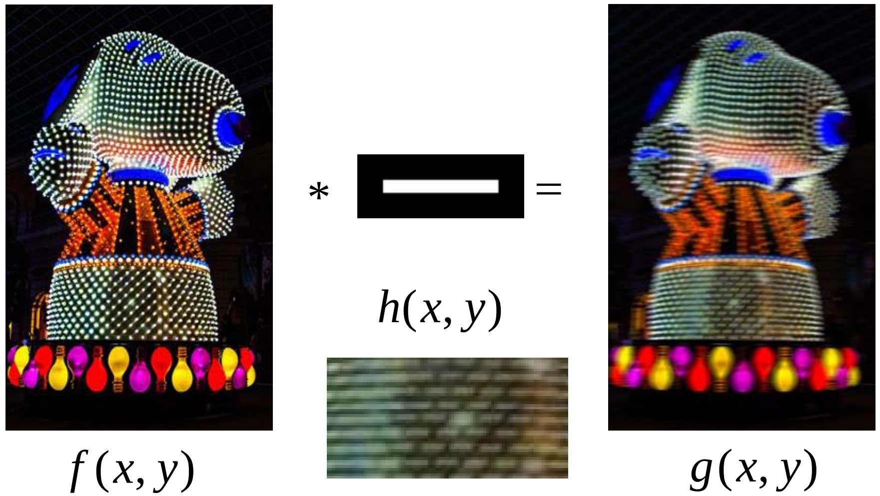
Linear Filter



Linear Filter



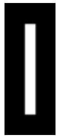
Linear Filter



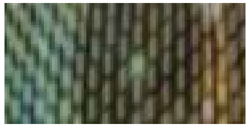
Linear Filter

$f(x,y)$

$*$



$h(x,y)$



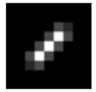
$=$

$g(x,y)$

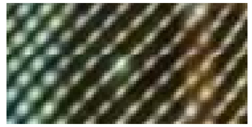
Linear Filter

$f(x,y)$

$*$



$h(x,y)$



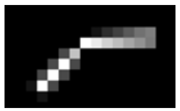
$=$

$g(x,y)$

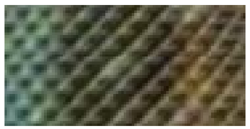
Linear Filter

$f(x,y)$

$*$



$h(x,y)$



$=$

$g(x,y)$

2D Convolution

$$
\begin{array}{|c|c|c|c|c|}\hline
f_{00} & f_{01} & f_{02} & f_{03} & f_{04} \\ \hline
f_{10} & f_{11} & f_{12} & f_{13} & f_{14} \\ \hline
f_{20} & f_{21} & f_{22} & f_{23} & f_{24} \\ \hline
f_{30} & f_{31} & f_{32} & f_{33} & f_{34} \\ \hline
f_{40} & f_{41} & f_{42} & f_{43} & f_{44} \\ \hline
\end{array}
\quad * \quad
\begin{array}{|c|c|c|}\hline
0 & h_{-1,0} & 0 \\ \hline
h_{0,-1} & h_{00} & h_{01} \\ \hline
0 & h_{10} & 0 \\ \hline
\end{array}
$$

Input Image $f(x,y)$

Kernel $h(x,y)$

2D Convolution

Flip $h(x,y)$

0	h_{10}	0
h_{01}	h_{00}	$h_{0,-1}$
0	$h_{-1,0}$	0

$h(-x,-y)$

Find $g(0,0)$

f_{00}	f_{01}	f_{02}	f_{03}	f_{04}
f_{10}	f_{11}	f_{12}	f_{13}	f_{14}
f_{20}	f_{21}	f_{22}	f_{23}	f_{24}
f_{30}	f_{31}	f_{32}	f_{33}	f_{34}
f_{40}	f_{41}	f_{42}	f_{43}	f_{44}

$f(x,y)$

$$g(0,0) = \sum_{p,q} h(-p,-q)\, f(p,q) = h_{00} f_{00} + h_{0,-1} f_{01} + h_{-1,0} f_{10}$$

2D Convolution

Flip $h(x,y)$

0	h_{10}	0
h_{01}	h_{00}	$h_{0,-1}$
0	$h_{-1,0}$	0

$h(-x,-y)$

Find $g(0,1)$

f_{00}	f_{01}	f_{02}	f_{03}	f_{04}
f_{10}	f_{11}	f_{12}	f_{13}	f_{14}
f_{20}	f_{21}	f_{22}	f_{23}	f_{24}
f_{30}	f_{31}	f_{32}	f_{33}	f_{34}
f_{40}	f_{41}	f_{42}	f_{43}	f_{44}

$f(x,y)$

$$g(0,1) = \sum_{p,q} h(-p,1-q)\, f(p,q) = h_{01} f_{00} + h_{00} f_{01} + h_{0,-1} f_{02} + h_{-1,0} f_{11}$$

2D Convolution

Flip $h(x,y)$

0	h_{10}	0
h_{01}	h_{00}	$h_{0,-1}$
0	$h_{-1,0}$	0

$h(-x,-y)$

Find $g(3,1)$

f_{00}	f_{01}	f_{02}	f_{03}	f_{04}
f_{10}	f_{11}	f_{12}	f_{13}	f_{14}
f_{20}	f_{21}	f_{22}	f_{23}	f_{24}
f_{30}	f_{31}	f_{32}	f_{33}	f_{34}
f_{40}	f_{41}	f_{42}	f_{43}	f_{44}

$f(x,y)$

$$g(3,1) = \sum_{p,q} h(3-p,1-q)\, f(p,q) = h_{10} f_{21} + h_{01} f_{30} + h_{00} f_{31} + h_{0.-1} f_{32} + h_{-1,0} f_{41}$$

2D Convolution

0	2	0	3
4	2	1	4
0	1	1	2
2	1	0	1

$f(x,y)$

Filter

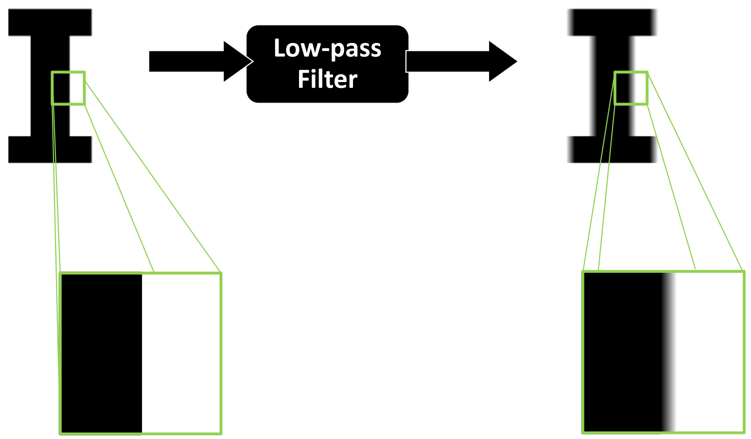
0	1	0
2	0	-2
0	-1	0

$h(x,y)$

	2	3	4
4		5	
0	1		-5
2	-5		-2

$g(x,y) = h * f(x,y)$

Smoothing



Smoothing

Input:

0	0	0	255	255	255
0	0	0	255	255	255
0	0	0	255	255	255
0	0	0	255	255	255
0	0	0	255	255	255

Filter

0	0	0
0	1/2	1/2
0	0	0

Output:

0	0	0	128	255	255
0	0	0	128	255	255
0	0	0	128	255	255
0	0	0	128	255	255
0	0	0	128	255	255

Smoothing

Input:

0	0	0	255	255	255
0	0	0	255	255	255
0	0	0	255	255	255
0	0	0	255	255	255
0	0	0	255	255	255

Filter

0	0	0
1/3	1/3	1/3
0	0	0

Output:

0	0	85	170	255	255
0	0	85	170	255	255
0	0	85	170	255	255
0	0	85	170	255	255
0	0	85	170	255	255

Moving Average Filter

[Mean Filter]

0	0	0
0	1/2	1/2
0	0	0

1 × 2 Kernel

0	0	0
1/3	1/3	1/3
0	0	0

1×3 Kernel

0	1/3	0
0	1/3	0
0	1/3	0

3×1 Kernel

1/21	1/21	1/21	1/21	1/21	1/21	1/21
1/21	1/21	1/21	1/21	1/21	1/21	1/21
1/21	1/21	1/21	1/21	1/21	1/21	1/21

3×7 Kernel

1/25	1/25	1/25	1/25	1/25
1/25	1/25	1/25	1/25	1/25
1/25	1/25	1/25	1/25	1/25
1/25	1/25	1/25	1/25	1/25
1/25	1/25	1/25	1/25	1/25

5×5 Kernel

0	0	1/37	1/37	1/37	0	0
0	1/37	1/37	1/37	1/37	1/37	0
1/37	1/37	1/37	1/37	1/37	1/37	1/37
1/37	1/37	1/37	1/37	1/37	1/37	1/37
1/37	1/37	1/37	1/37	1/37	1/37	1/37
0	1/37	1/37	1/37	1/37	1/37	0
0	0	1/37	1/37	1/37	0	0

Circular Kernel

Moving Average Filter

[Mean Filter]

5×5 Kernel

15×15 Kernel

25×25 Kernel

Derivatives

The 1st Order Derivative

$$\frac{\partial f}{\partial x} = f(x+1, y) - f(x, y)$$

Convolution Kernel for $\frac{\partial f}{\partial x}$

0	1	0
0	−1	0
0	0	0

$$\frac{\partial f}{\partial y} = f(x, y+1) - f(x, y)$$

Convolution Kernel for $\frac{\partial f}{\partial y}$

0	0	0
1	−1	0
0	0	0

Derivatives

y

x

0	0	0	0	0	0
0	0	0	255	255	255
0	0	0	255	255	255
0	0	0	255	255	255
0	0	255	255	255	255
0	255	255	255	255	255

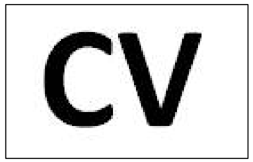
$\frac{\partial f}{\partial x}$ →

0	0	0	255	255	255
0	0	0	0	0	0
0	0	0	0	0	0
0	0	255	0	0	0
0	255	0	0	0	0
0	0	0	0	0	0

$\frac{\partial f}{\partial y}$ →

0	0	0	0	0	0
0	0	255	0	0	0
0	0	255	0	0	0
0	0	255	0	0	0
0	255	0	0	0	0
255	0	0	0	0	0

Derivatives



$\partial f \,/\, \partial x$

$\partial f \,/\, \partial y$

Derivatives

The 2nd Order Derivative

$$\frac{\partial^2 f}{\partial x^2} = f(x+1, y) - 2f(x, y) + f(x-1, y)$$

$$\frac{\partial^2 f}{\partial y^2} = f(x, y+1) - 2f(x, y) + f(x, y-1)$$

Convolution Kernel for $\frac{\partial^2 f}{\partial x^2}$

0	1	0
0	–2	0
0	1	0

Convolution Kernel for $\frac{\partial^2 f}{\partial y^2}$

0	0	0
1	–2	1
0	0	0

Derivatives

y

x

0	0	0	0	0	0
0	0	0	255	255	255
0	0	0	255	255	255
0	0	0	255	255	255
0	0	255	255	255	255
0	255	255	255	255	255

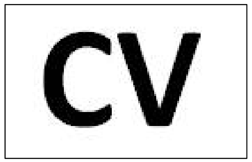
$\frac{\partial^2 f}{\partial x^2}$

0	0	0	255	255	255
0	0	0	-255	-255	-255
0	0	0	0	0	0
0	0	255	0	0	0
0	255	-255	0	0	0
0	-255	0	0	0	0

$\frac{\partial^2 f}{\partial y^2}$

0	0	0	0	0	0
0	0	255	-255	0	0
0	0	255	-255	0	0
0	0	255	-255	0	0
0	255	-255	0	0	0
255	-255	0	0	0	0

Derivatives



$\partial^2 f / \partial x^2$

$\partial^2 f / \partial y^2$

Gradient

Image Gradient = Vector that points in the direction of the greatest rate of increase

$$\nabla \mathbf{f} = \begin{bmatrix} f_x \\ f_y \end{bmatrix} = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial x} \end{bmatrix}$$

Magnitude

$$M(x, y) = \sqrt{f_x^2 + f_y^2} \approx |f_x| + |f_y|$$

Direction

$$\alpha(x, y) = \tan^{-1}\left(\frac{f_y}{f_x}\right)$$

Gradient

The 1-st order derivative

h_x

0	1	0
0	−1	0
0	0	0

h_y

0	0	0
1	−1	0
0	0	0

Prewitt

1	1	1
0	0	0
−1	−1	−1

1	0	−1
1	0	−1
1	0	−1

Sobel

1	2	1
0	0	0
−1	−2	−1

1	0	−1
2	0	−2
1	0	−1

$$f_x = h_x * f \qquad f_y = h_y * f$$

Gradient

The 1st Order Derivatives

y, x

0	0	0	0	0	0
0	0	0	255	255	255
0	0	0	255	255	255
0	0	0	255	255	255
0	0	255	255	255	255
0	255	255	255	255	255

$$\nabla \mathbf{f}(1,5) = \begin{bmatrix} \frac{\partial f}{\partial x}(1,5) \\ \frac{\partial f}{\partial x}(1,5) \end{bmatrix} = \begin{bmatrix} 255 \\ 0 \end{bmatrix}$$

$$M(1,5) = \sqrt{255^2 + 0^2} = 255$$

$$\alpha(1,5) = \tan^{-1}\left(\frac{0}{255}\right) = 0^\circ$$

Gradient

The 1st Order Derivatives

y, x

0	0	0	0	0	0
0	0	0	255	255	255
0	0	0	255	255	255
0	0	0	255	255	255
0	0	255	255	255	255
0	255	255	255	255	255

$$\nabla \mathbf{f}(3,3) = \begin{bmatrix} \frac{\partial f}{\partial x}(3,3) \\ \frac{\partial f}{\partial x}(3,3) \end{bmatrix} = \begin{bmatrix} 0 \\ 255 \end{bmatrix}$$

$$M(3,3) = \sqrt{0^2 + 255^2} = 255$$

$$\alpha(3,3) = \tan^{-1}\left(\frac{255}{0}\right) = 90^\circ$$

Gradient

The 1st Order Derivatives

y, x

0	0	0	0	0	0
0	0	0	255	255	255
0	0	0	255	255	255
0	0	0	255	255	255
0	0	255	255	255	255
0	255	255	255	255	255

$$\nabla \mathbf{f}(5,2) = \begin{bmatrix} \frac{\partial f}{\partial x}(5,2) \\ \frac{\partial f}{\partial x}(5,2) \end{bmatrix} = \begin{bmatrix} 255 \\ 255 \end{bmatrix}$$

$$M(5,2) = \sqrt{255^2 + 255^2} = 361$$

$$\alpha(5,2) = \tan^{-1}\left(\frac{255}{255}\right) = 45^\circ$$

Gradient

The 1st Order Derivatives

y

x

0	0	0	0	0	0
0	0	0	0	0	255
0	0	0	0	200	220
0	0	0	200	150	255
0	0	100	100	200	255
0	100	100	150	255	255

$$\nabla \mathbf{f}(4,4) = \begin{bmatrix} 200 \\ 200 \end{bmatrix}$$

$$M(4,4) = 282 \qquad \alpha(4,4) = 45^\circ$$

$$\nabla \mathbf{f}(5,2) = \begin{bmatrix} 100 \\ 100 \end{bmatrix}$$

$$M(5,2) = 141 \qquad \alpha(3,4) = 45^\circ$$

Gradient

The 1st Order Derivatives

y

x

255	0	0	0	0	0
255	0	0	0	255	255
255	0	0	0	255	255
255	0	0	0	255	255
255	0	255	255	255	255
255	255	255	255	255	255

$$\nabla \mathbf{f}(3,4) = \begin{bmatrix} 0 \\ 255 \end{bmatrix}$$

$$M(3,4) = 255 \qquad \alpha(3,4) = 90^\circ$$

$$\nabla \mathbf{f}(3,2) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$M(3,2) = 0$$

Gradient

y

x

255	0	0	0	0	0
255	0	0	0	255	255
255	0	0	0	255	255
255	0	0	0	255	255
255	0	255	255	255	255
255	255	255	255	255	255

$$\nabla \mathbf{f}(3,4) = \begin{bmatrix} 0 \\ 765 \end{bmatrix}$$

$$M(3,4) = 765$$

$$\alpha(3,4) = 90^\circ$$

$$\nabla \mathbf{f}(3,2) = \begin{bmatrix} 0 \\ -765 \end{bmatrix}$$

$$M(3,2) = 765$$

$$\alpha(3,2) = -90^\circ$$

Prewitt

1	1	1
0	0	0
–1	–1	–1

1	0	–1
1	0	–1
1	0	–1

Gradient

			100	150		150	100			
		100	200	200	150	200	200	100		
		100	200	200	200	200	200	100		
		100	200	200	200	200	200	100		
			100	200	200	200	100			
				100	200	100				
					50					

Prewitt

$$M(x,y) \approx |f_x| + |f_y|$$

Magnitude

0	0	200	400	350	300	350	400	200	0	0
0	200	600	750	700	550	700	750	600	200	0
0	300	700	600	500	300	500	600	700	300	0
0	300	600	300	100	50	100	300	600	300	0
0	300	700	600	200	0	200	600	700	300	0
0	200	600	800	600	200	600	800	600	200	0
0	0	200	600	800	550	800	600	200	0	0
0	0	0	200	550	400	550	200	0	0	0
0	0	0	0	100	50	100	0	0	0	0

Direction

		45	31	-22	0	22	-31	-45		
	45	45	27	-15	0	15	-27	-45	-45	
	63	68	54	-23	0	23	-54	-68	-63	
	90	90	90	-45	0	45	-90	-90	-90	
	117	112	117	135		-135	-117	-112	-117	
	135	135	135	135	180	-135	-135	-135	-135	
		135	135	142	180	-142	-135	-135		
			135	140	180	-140	-135			
				135	180	-135				

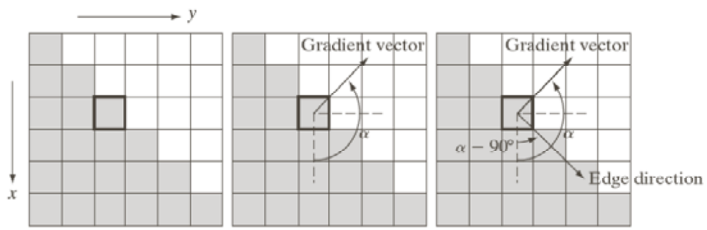
Edge Detection

Basic Edge Detection

Image Smoothing
Average filter, Gaussian filter, Median filter, Morphological operators

Image Gradient
Ordinary 1st order derivatives, Prewitt, Sobel

Thresholding



Gradient magnitude is corresponding to edge strength

Gradient direction is perpendicular to edge direction

Basic Edge Detection

			100	150		150	100			
		100	200	200	150	200	200	100		
		100	200	200	200	200	200	100		
		100	200	200	200	200	200	100		
			100	200	200	200	100			
				100	200	100				
					50					

Magnitude

0	0	200	400	350	300	350	400	200	0	0
0	200	600	750	700	550	700	750	600	200	0
0	300	700	600	500	300	500	600	700	300	0
0	300	600	300	100	50	100	300	600	300	0
0	300	700	600	200	0	200	600	700	300	0
0	200	600	800	600	200	600	800	600	200	0
0	0	200	600	800	550	800	600	200	0	0
0	0	0	200	550	400	550	200	0	0	0
0	0	0	0	100	50	100	0	0	0	0

0	0	200	400	350	300	350	400	200	0	0
0	200	600	**750**	**700**	550	**700**	**750**	600	200	0
0	300	**700**	600	500	300	500	600	**700**	300	0
0	300	600	300	100	50	100	300	600	300	0
0	300	**700**	600	200	0	200	600	**700**	300	0
0	200	600	**800**	600	200	600	**800**	600	200	0
0	0	200	600	**800**	550	**800**	600	200	0	0
0	0	0	200	550	400	550	200	0	0	0
0	0	0	0	100	50	100	0	0	0	0

$$Edge\,Threshold : M(x, y) \geq 700$$

Basic Edge Detection

			100	150		150	100			
		100	200	200	150	200	200	100		
		100	200	200	200	200	200	100		
		100	200	200	200	200	200	100		
			100	200	200	200	100			
				100	200	100				
					50					

Magnitude

0	0	200	400	350	300	350	400	200	0	0
0	200	600	750	700	550	700	750	600	200	0
0	300	700	600	500	300	500	600	700	300	0
0	300	600	300	100	50	100	300	600	300	0
0	300	700	600	200	0	200	600	700	300	0
0	200	600	800	600	200	600	800	600	200	0
0	0	200	600	800	550	800	600	200	0	0
0	0	0	200	550	400	550	200	0	0	0
0	0	0	0	100	50	100	0	0	0	0

0	0	200	400	350	300	350	400	200	0	0
0	200	**600**	**750**	**700**	550	**700**	**750**	**600**	200	0
0	300	**700**	**600**	500	300	500	**600**	**700**	300	0
0	300	**600**	300	100	50	100	300	**600**	300	0
0	300	**700**	**600**	200	0	200	**600**	**700**	300	0
0	200	**600**	**800**	**600**	200	**600**	**800**	**600**	200	0
0	0	200	**600**	**800**	550	**800**	**600**	200	0	0
0	0	0	200	550	400	550	200	0	0	0
0	0	0	0	100	50	100	0	0	0	0

$$Edge\,Threshold : M(x, y) \geq 600$$

Canny Edge Detection

Image Smoothing with $n \times n$ Gaussian Filter

▽ $f_S(x,y)$

Gradient Magnitude and Angle

▽ $M(x,y), \alpha(x,y)$

Nonmaxima Suppression

▽ $g_N(x,y)$

Hysteresis Thresholding and Connectivity Analysis

Canny Edge Detection

Image Smoothing with $n \times n$ Gaussian Filter

$$f_S(x,y) = h(x,y) * f(x,y) \quad \text{where} \quad h(x,y) = e^{-\frac{x^2+y^2}{2\sigma^2}}$$

$h(0,-2) = e^{-\frac{0^2+(-2)^2}{2}}$

0.02	0.08	0.14	0.08	0.02
0.08	0.37	0.61	0.37	0.08
0.14	0.61	1	0.61	0.14
0.08	0.37	0.61	0.37	0.08
0.02	0.08	0.14	0.08	0.02

$h(2,2) = e^{-\frac{2^2+2^2}{2}}$

5×5 Gaussian Filter

$\sigma = 1$

Canny Edge Detection

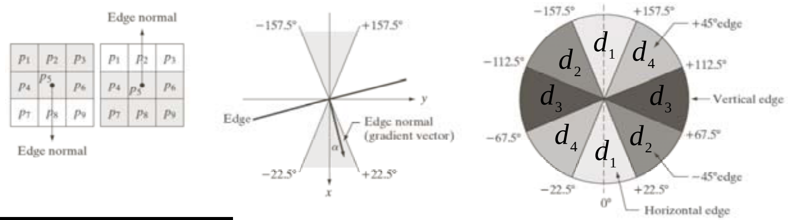
Gradient Magnitude and Direction

$$\nabla \mathbf{f}_S = \begin{bmatrix} f_x \\ f_y \end{bmatrix} = \begin{bmatrix} \dfrac{\partial f_S}{\partial x} \\ \dfrac{\partial f_S}{\partial x} \end{bmatrix}$$

$$M(x,y) = \sqrt{f_x^2 + f_y^2}$$

$$\alpha(x,y) = \tan^{-1}\left(\frac{f_y}{f_x}\right)$$

Canny Edge Detection



Nonmaxima Suppression

d_1, d_2, d_3, d_4 = Four basic edge direction in 3×3 region

For every pixel (x,y); Find the direction d_k that is closest to $\alpha(x,y)$

$$g_N(x,y) = \begin{cases} 0; & M(x,y) \text{ is less than at least one neighbor along } d_k \\ M(x,y); & \text{otherwise} \end{cases}$$

Canny Edge Detection

0	0	0	0	0	0
0	0	7	0	0	0
0	0	6	0	0	0
0	0	2	6	0	0
0	2	4	4	6	0
0	2	7	7	7	0
0	0	0	0	0	0

$f_S(x,y)$

0	7	7	7	0	0
0	6	6	6	0	0
0	-5	1	1	6	0
0	0	4	8		6
0	7	8		8	7
-2	-6	-10	-14	-10	-6
-2	-9	-16	-21	-14	-7

$f_x(x,y)$ (Prewitt)

0	7	0	-7	0	0
0	13	0	-13	0	0
0	15	6	-15	-6	0
2	12	8	-6		-6
4	13	13		-17	-13
4	11	7	2	-11	-13
2	7	5	0	-7	-7

$f_y(x,y)$ (Prewitt)

0	14	7	14	0	0
0	19	6	19	0	0
0	20	7	16	12	0
4	12	12	14		12
6	20	21		25	20
6	17	17	16	21	19
4	16	21	21	21	14

$M(x,y)$

	45	0	-45		
	65	0	-65		
	-72	80	-86	-45	
45	90	63	-37		-45
63	62	58		-65	-62
-63	-61	-35	-8	48	65
-45	-38	-17	0	27	45

$\alpha(x,y)$

0	14	7	14	0	0
0	19	0	19	0	0
0	20	0	16	0	0
0	12	0	0		0
0	20	21		25	0
0	0	17	0	21	0
0	0	21	21	21	0

$g_N(x,y)$

Canny Edge Detection

Hysteresis Thresholding and Connectivity Analysis

$$\text{Strong Edges} \Rightarrow g_{NH}(x,y) = \begin{cases} 1; & g_N(x,y) \ge T_H \\ 0; & \text{otherwise} \end{cases}$$

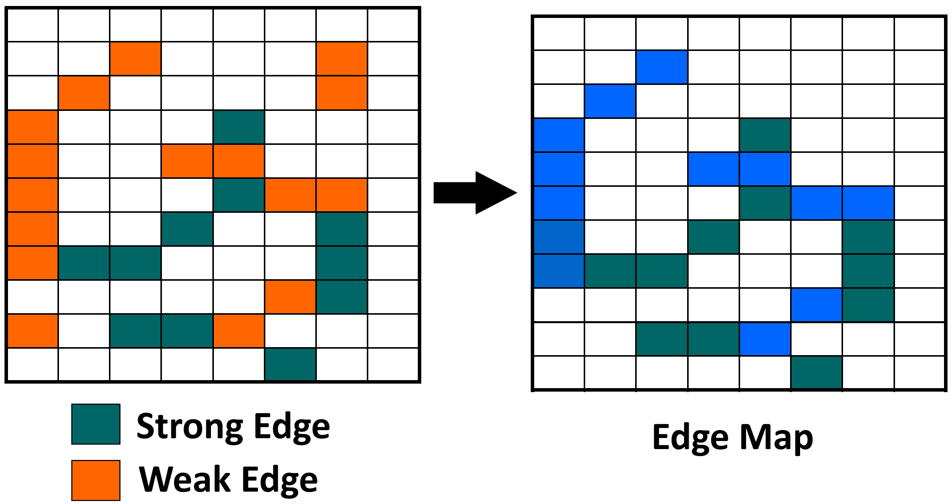
$$\text{Weak Edges} \Rightarrow g_{NL}(x,y) = \begin{cases} 1; & T_L < g_N(x,y) < T_H \\ 0; & \text{otherwise} \end{cases}$$

Edge Pixels includes

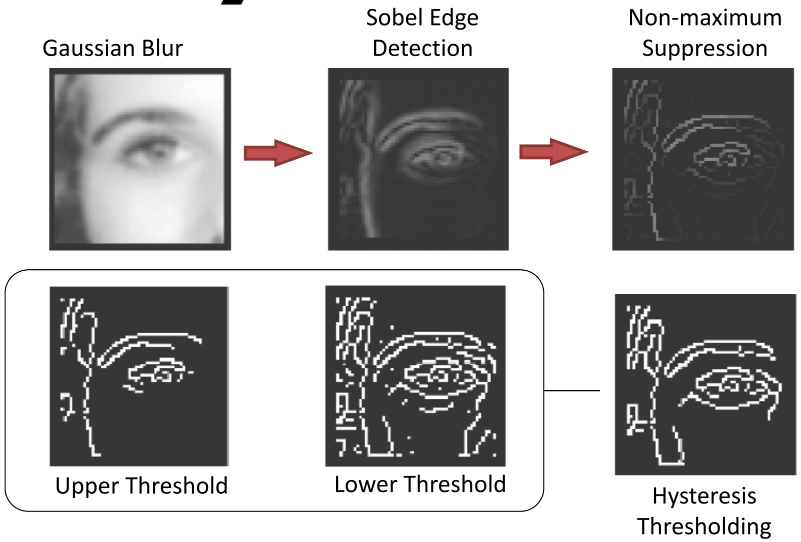
(1) Every pixel belongs to the strong edge ($g_{NH} = 1$)

(2) Pixels belong to the weak edge ($g_{NL} = 1$) that are neighbors of the strong edge (consider as blobs)

Canny Edge Detection



Canny Edge Detection



Median Filter

- Non-linear Filter
- Removing Impulsive Noise (Salt & Pepper Noise)

$$g(x,y) = \underset{(p,q)\in R(x,y)}{\mathrm{MEDIAN}}\{f(p,q)\}$$

$R(x,y)$ is set of neighborhood region around (x,y)

$$g(2,3) = \underset{(p,q)\in R(x,y)}{\mathrm{MEDIAN}}\{f_{12}, f_{13}, f_{14}, f_{22}, f_{23}, f_{24}, f_{32}, f_{33}, f_{34}\}$$

$f(x,y)$

f_{00}	f_{01}	f_{02}	f_{03}	f_{04}
f_{10}	f_{11}	f_{12}	f_{13}	f_{14}
f_{20}	f_{21}	f_{22}	f_{23}	f_{24}
f_{30}	f_{31}	f_{32}	f_{33}	f_{34}
f_{40}	f_{41}	f_{42}	f_{43}	f_{44}

3×3 neighborhood region

Median Filter

0	0	3	2	0	0	0
0	0	1	1	2	0	0
0	0	0	0	47	4	0
0	0	0	10	0	2	0
0	75	65	15	0	0	0
70	80	2	63	0	0	0
0	67	53	55	0	0	0
0	0	72	0	0	0	0

Median Filter 3×3

0	0	0	1	0	0	0
0	0	0		1	0	0
0	0	0	1		0	0
0	0	0	0	2		0
0	2	15	2	0	0	0
0	65		15	0	0	0
0	53	55	2	0	0	0
0	0	0	0	0	0	0

Median Filter

Original Image

Median Filter

Linear Filter

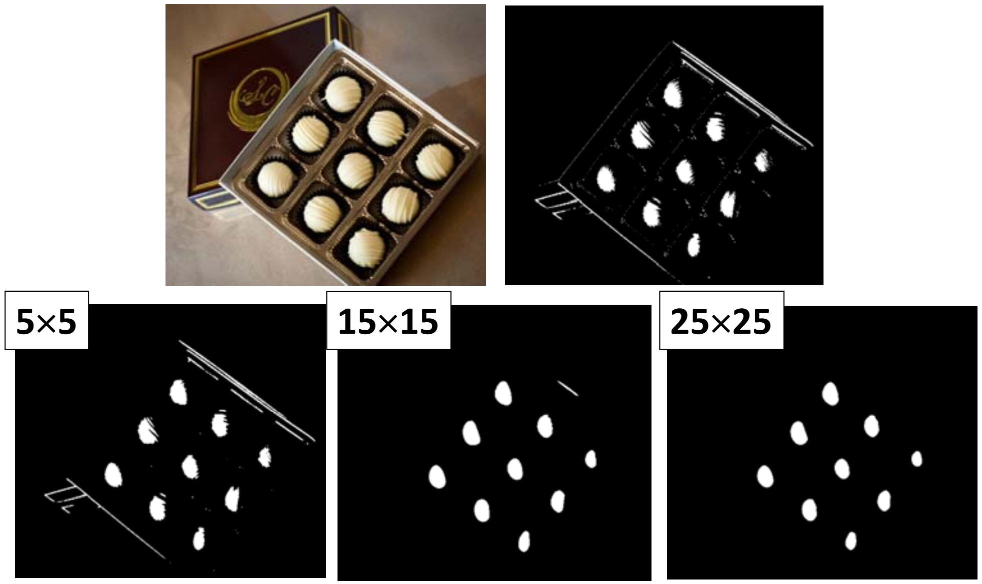
Median Filter

Original Image

Median Filter

Linear Filter

Median Filter



Morphological Operators

Binary Image morphology = Set Operation

- Binary image $f(x,y) \in \{0,1\}$
- Set of all white pixel (foreground)

$$B = \{w = (x,y) \mid f(x,y) = 1\}$$

- The complement of B

$$B^c = \{w = (x,y) \mid f(x,y) = 0\}$$

$f(0,0)$

0	0	0	0	1
0	0	0	1	1
0	1	1	0	0
0	0	0	0	0
1	1	0	0	0

$f(x,y)$

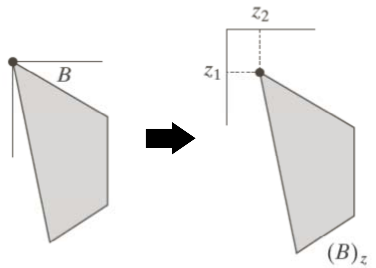
$$B = \{(0,4),(1,3),(1,4),(2,1),(2,2),(4,0),(4,1)\}$$

Morphological Operators

- Translation

$$(B)_z = \{c \mid c = b + z \text{ for } b \in B\}$$

$$z = (z_1, z_2)$$



0	0	0	0	0
0	1	0	1	0
0	1	1	0	0
0	0	0	0	0
0	1	0	0	1

$f(x,y)$

$$(B)_{(-1,2)} = \{(0,3),(0,5),(1,3),(1,4),(3,3),(3,6)\}$$

Dilation

Input Image A

0	0	0	0	0	0	0	0	0
0	1	0	0	0	1	0	0	0
0	0	0	0	0	1	1	0	0
0	0	0	0	0	0	0	0	0
0	0	1	0	0	0	0	0	0
0	0	1	0	0	0	0	0	0
0	1	0	0	0	1	1	1	0
0	0	0	0	0	0	0	0	0

Dilation

Structuring Element B

0	1	0
0	1	1
0	0	0

Output Image $A \oplus B$

0	1	0	0	0	1	0	0	0
0	1	1	0	0	1	1	0	0
0	0	0	0	0	1	1	1	0
0	0	1	0	0	0	0	0	0
0	0	1	1	0	0	0	0	0
0	1	1	1	0	1	1	1	0
0	1	1	0	0	1	1	1	1
0	0	0	0	0	0	0	0	0

Dilation

Input Image A

0	0	0	0	0	0	0	0	0
0	0	0	0	1	1	1	0	0
0	1	1	1	1	0	1	0	0
0	1	0	0	1	1	1	0	0
0	1	0	0	1	1	1	0	0
0	1	0	0	1	1	1	1	0
0	1	1	1	1	1	1	0	0
0	0	0	0	0	0	0	0	0

Dilation

Structuring Element B

1	1	1
1	1	1
1	1	1

Output Image $A \oplus B$

0	0	0	1	1	1	1	1	0
1	1	1	1	1	1	1	1	0
1	1	1	1	1	1	1	1	0
1	1	1	1	1	1	1	1	0
1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	0

Dilation

$$A \oplus B = \bigcup_{b \in B} (A)_b$$

Input Image A

0	0	0	0	0
0	1	0	0	0
0	0	0	0	0
0	0	0	0	0
0	0	1	0	0
0	0	1	0	0

S.E. B

0	1	0
0	1	1
0	0	0

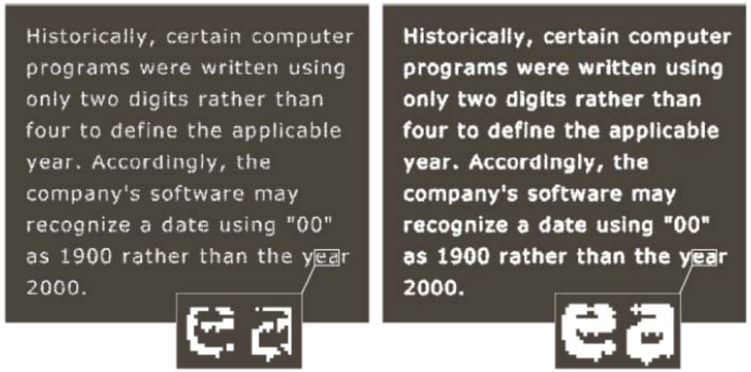
Output $A \oplus B$

$$A = \{(1,1),(4,3),(5,3)\}$$
$$B = \{(0,0),(-1,0),(0,1)\}$$

$$A_{(0,0)} = \{(1,1),(4,3),(5,3)\}$$
$$A_{(-1,0)} = \{(0,1),(3,3),(4,3)\}$$
$$A_{(0,1)} = \{(1,2),(4,4),(5,4)\}$$

$$A \oplus B = A_{(0,0)} \cup A_{(-1,0)} \cup A_{(0,1)}$$
$$= ?$$

Dilation


Historically, certain computer programs were written using only two digits rather than four to define the applicable year. Accordingly, the company's software may recognize a date using "00" as 1900 rather than the year 2000.

Historically, certain computer programs were written using only two digits rather than four to define the applicable year. Accordingly, the company's software may recognize a date using "00" as 1900 rather than the year 2000.

0	1	0
1	1	1
0	1	0

Erosion

Input Image A

0	0	0	0	0	0	0	0	0
0	1	0	1	1	1	0	0	0
0	0	0	1	1	1	1	0	0
0	1	1	1	1	1	1	1	0
0	1	1	1	1	1	1	1	0
0	0	1	1	0	1	0	0	0
0	1	0	0	1	1	1	0	0
0	0	0	0	0	0	0	0	0

Erosion

Structuring Element B

0	1	1
1	1	1
1	1	0

Output Image $A \ominus B$

0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	1	0	0	0	0
0	0	0	1	1	1	0	0	0
0	0	0	1	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0

Erosion

Input Image A

0	0	0	0	0	0	0	0	0
0	1	0	1	1	1	0	0	0
0	0	0	1	1	1	1	0	0
0	1	1	1	1	1	1	1	0
0	1	1	1	1	1	1	1	0
0	0	1	1	0	1	0	0	0
0	1	0	0	1	1	1	0	0
0	0	0	0	0	0	0	0	0

Erosion

Structuring Element B

0	0	1	0	0
0	0	1	0	0
1	1	1	1	1
0	0	1	0	0
0	0	1	0	0

Output Image $A \ominus B$

0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	1	0	1	0	0	0
0	0	0	0	0	1	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0

Erosion

$$A \ominus B = \{z \mid (B)_z \subseteq A\}$$

Input Image A

0	0	0	0	0
0	1	0	0	0
0	1	1	0	0
0	1	1	1	0
0	0	0	0	0
0	0	0	0	0

S.E. B

0	1	0
0	1	1
0	0	0

$A = \{(1,1),(2,1),(2,2),(3,1),(3,2),(3,3)\}$

$B = \{(0,0),(-1,0),(0,1)\}$

Output $A \ominus B$

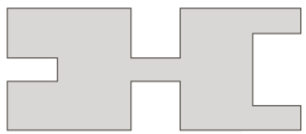
0				
		1		

$B_{(0,0)} = \{(0,0),(-1,0),(0,1)\} \not\subset A$

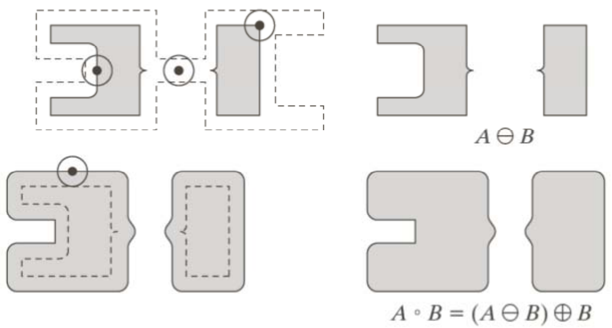
$B_{(3,2)} = \{(3,2),(2,2),(3,3)\} \subset A$

Opening

$$A \circ B = (A \ominus B) \oplus B$$



Input Image A



Erosion

Dilation

Opening

Input Image A

0	0	0	0	0	0	0	0	0
0	1	1	1	0	0	0	0	0
0	0	1	0	0	0	0	1	0
0	0	1	0	0	1	0	1	0
0	0	0	0	0	0	0	1	0
0	0	0	0	0	0	0	0	0
0	0	1	1	1	1	1	0	0
0	0	0	0	0	0	0	0	0

Opening

0	0	0
1	1	1
0	0	0

B

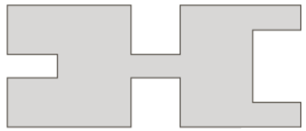
Structuring Element B

Output Image $A \circ B$

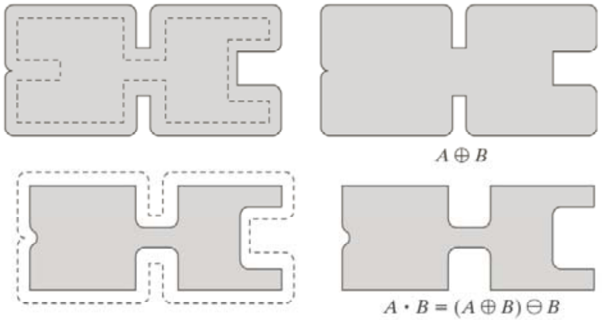
0	0	0	0	0	0	0	0	0
0	1	1	1	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	1	1	1	1	1	0	0
0	0	0	0	0	0	0	0	0

Closing

$$A \bullet B = (A \oplus B) \ominus B$$



Input Image A



Dilation

Erosion

Closing

Input Image A

0	0	0	0	0	0	0	0	0
0	1	0	1	0	1	0	1	0
0	0	0	0	0	0	0	0	0
0	1	0	0	0	0	0	1	0
0	0	0	0	0	0	0	0	0
0	1	0	0	0	0	0	1	0
0	0	1	0	1	0	1	0	0
0	0	0	0	0	0	0	0	0

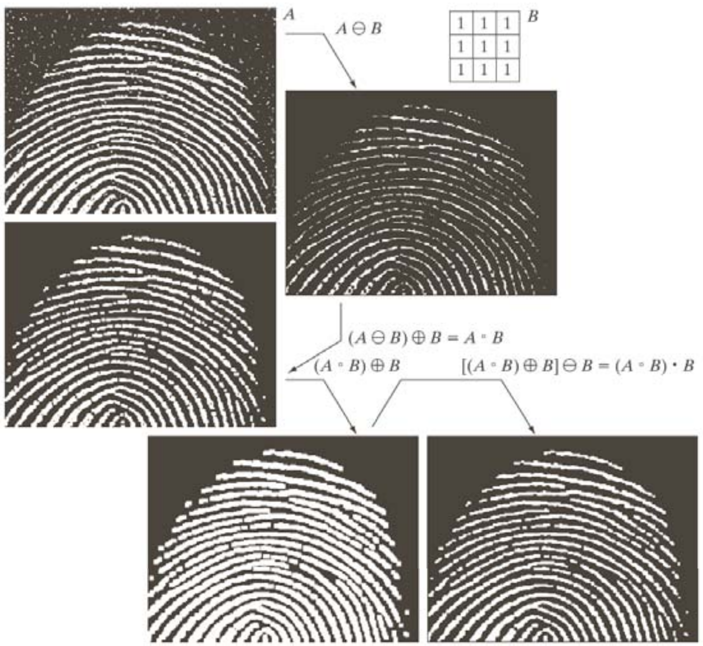
Closing

Structuring Element B

0	0	0
1	1	1
0	0	0

Output Image $A \bullet B$

0	0	0	0	0	0	0	0	0
0	1	1	1	1	1	1	1	0
0	0	0	0	0	0	0	0	0
0	1	0	0	0	0	0	1	0
0	0	0	0	0	0	0	0	0
0	1	0	0	0	0	0	1	0
0	0	1	1	1	1	1	0	0
0	0	0	0	0	0	0	0	0



Morphological Operators

Opening

Closing