

## 作业二

5.

### Doolittle 分解:

根据题意对 A 进行 Doolittle 分解得到

$$A = \begin{bmatrix} 5 & 7 & 9 & 10 \\ 6 & 8 & 10 & 9 \\ 7 & 10 & 8 & 7 \\ 5 & 7 & 6 & 5 \end{bmatrix} = \begin{bmatrix} 1 & & & \\ \frac{6}{5} & 1 & & \\ \frac{7}{5} & -\frac{1}{2} & 1 & \\ 1 & 0 & \frac{3}{5} & 1 \end{bmatrix} \begin{bmatrix} 5 & 7 & 9 & 10 \\ -\frac{2}{5} & -\frac{4}{5} & -3 & \\ -5 & -\frac{17}{2} & & \\ \frac{1}{10} & & & \end{bmatrix}$$

通过  $Ly=b$  即

$$\begin{bmatrix} 1 & & & \\ \frac{6}{5} & 1 & & \\ \frac{7}{5} & -\frac{1}{2} & 1 & \\ 1 & 0 & \frac{3}{5} & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

$$\text{解得 } y = \left[ 1 \quad -\frac{3}{5} \quad -\frac{1}{2} \quad \frac{3}{10} \right]^T$$

通过  $Ux=y$  即

$$\begin{bmatrix} 5 & 7 & 9 & 10 \\ -\frac{2}{5} & -\frac{4}{5} & -3 & \\ -5 & -\frac{17}{2} & & \\ \frac{1}{10} & & & \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix}$$

$$\text{解得 } x = \left[ 20 \quad -12 \quad -5 \quad 3 \right]^T$$

### Crout 分解:

根据题意对 A 进行 Crout 分解得到

$$A = \begin{bmatrix} 5 & 7 & 9 & 10 \\ 6 & 8 & 10 & 9 \\ 7 & 10 & 8 & 7 \\ 5 & 7 & 6 & 5 \end{bmatrix} = \begin{bmatrix} 5 & & & \\ 6 & -\frac{2}{5} & & \\ 7 & \frac{1}{5} & -5 & \\ 5 & 0 & -3 & \frac{1}{10} \end{bmatrix} \begin{bmatrix} 1 & \frac{7}{5} & \frac{9}{5} & 2 \\ & 1 & 2 & \frac{15}{2} \\ & & 1 & \frac{17}{10} \\ & & & 1 \end{bmatrix}$$

通过  $Ly=b$  即

$$\begin{bmatrix} 5 & & & \\ 6 & -\frac{2}{5} & & \\ 7 & \frac{1}{5} & -5 & \\ 5 & 0 & -3 & \frac{1}{10} \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

$$\text{解得 } y = \begin{bmatrix} \frac{1}{5} & \frac{1}{2} & \frac{1}{10} & 3 \end{bmatrix}^T$$

通过  $Ux=y$  即

$$\begin{bmatrix} 1 & \frac{7}{5} & \frac{9}{5} & 2 \\ & 1 & 2 & \frac{15}{2} \\ & & 1 & \frac{17}{10} \\ & & & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix}$$

$$\text{解得 } x = \begin{bmatrix} 20 & -12 & -5 & 3 \end{bmatrix}^T$$

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平方根法：

根据题意将  $A$  进行平方根法分解

$$A = \begin{bmatrix} 4 & -1 & 1 \\ -1 & 4.75 & 2.75 \\ 1 & 2.75 & 3.5 \end{bmatrix} = \begin{bmatrix} 2 & & \\ -\frac{1}{2} & \frac{3\sqrt{2}}{2} & \\ \frac{1}{2} & \sqrt{2} & \frac{\sqrt{5}}{2} \end{bmatrix} \begin{bmatrix} 2 & -\frac{1}{2} & \frac{1}{2} \\ \frac{3\sqrt{2}}{2} & \sqrt{2} & \\ \frac{\sqrt{5}}{2} & & \frac{\sqrt{5}}{2} \end{bmatrix}$$

通过  $\hat{L}y=b$  即

$$\begin{bmatrix} 2 & & \\ -\frac{1}{2} & \frac{3\sqrt{2}}{2} & \\ \frac{1}{2} & \sqrt{2} & \frac{\sqrt{5}}{2} \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 6 \\ 7.25 \end{bmatrix}$$

$$\text{解得 } y = \begin{bmatrix} 2 & 3.2998 & 1.4162 \end{bmatrix}^T$$

通过  $\hat{L}^T x=y$  即

$$\begin{bmatrix} 2 & -\frac{1}{2} & \frac{1}{2} \\ & \frac{3\sqrt{2}}{2} & \sqrt{2} \\ & & \frac{\sqrt{5}}{2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

$$\text{解得 } x = \begin{bmatrix} 0.8611 & 0.7111 & 1.2667 \end{bmatrix}^T$$

### 改进平方根法：

根据题意将A进行改进平方根法分解

$$A = \begin{bmatrix} 4 & -1 & 1 \\ -1 & 4.75 & 2.75 \\ 1 & 2.75 & 3.5 \end{bmatrix} = \begin{bmatrix} 1 & & \\ -\frac{1}{4} & 1 & \\ \frac{1}{4} & \frac{2}{3} & 1 \end{bmatrix} \begin{bmatrix} 4 & & \\ & \frac{9}{2} & \\ & & \frac{5}{4} \end{bmatrix} \begin{bmatrix} 1 & -\frac{1}{4} & \frac{1}{4} \\ & 1 & \frac{2}{3} \\ & & 1 \end{bmatrix}$$

通过 $\hat{L}y=b$ 即

$$\begin{bmatrix} 1 & & \\ -\frac{1}{4} & 1 & \\ \frac{1}{4} & \frac{2}{3} & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 6 \\ 7.25 \end{bmatrix}$$

解得 $y = [4 \ 7 \ 1.5833]^T$

再通过 $L^T x = D^{-1}y$ 即

$$\begin{bmatrix} 1 & -\frac{1}{4} & \frac{1}{4} \\ & 1 & \frac{2}{3} \\ & & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \frac{1}{4} & & \\ & \frac{2}{9} & \\ & & \frac{4}{5} \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

解得 $x = [0.8611 \ 0.7111 \ 1.2667]^T$

8.

(1)根据题意将A进行追逐法分解

$$A = \begin{bmatrix} 2 & 1 & 0 & 0 \\ 1 & 4 & 1 & 0 \\ 0 & 1 & 4 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 2 & & & \\ 1 & \frac{7}{2} & & \\ & 1 & \frac{26}{7} & \\ & & 1 & \frac{45}{26} \end{bmatrix} \begin{bmatrix} 1 & \frac{1}{2} & & \\ & 1 & \frac{2}{7} & \\ & & 1 & \frac{7}{26} \\ & & & 1 \end{bmatrix}$$

通过 $\hat{L}y=b$ 即

$$\begin{bmatrix} 2 \\ 1 & \frac{7}{2} \\ & 1 & \frac{26}{7} \\ & & 1 & \frac{45}{26} \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ 3 \\ 0 \end{bmatrix}$$

$$\text{解得 } y = \left[ \frac{1}{2} \quad -\frac{5}{7} \quad 1 \quad -\frac{26}{45} \right]^T$$

通过  $\hat{U}x=y$  即

$$\begin{bmatrix} 1 & \frac{1}{2} \\ & 1 & \frac{2}{7} \\ & & 1 & \frac{7}{26} \\ & & & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix}$$

$$\text{解得 } x = \left[ \frac{46}{45} \quad -\frac{47}{45} \quad \frac{52}{45} \quad -\frac{26}{45} \right]^T$$

(2) 根据题意将  $A$  进行追逐法分解

$$A = \begin{bmatrix} 4 & -1 & & & \\ -1 & 4 & -1 & & \\ & -1 & 4 & -1 & \\ & & -1 & 4 & -1 \\ & & & -1 & 4 \end{bmatrix} = \begin{bmatrix} 4 & & & & \\ -1 & \frac{15}{4} & & & \\ & -1 & \frac{56}{15} & & \\ & & -1 & \frac{209}{56} & \\ & & & -1 & \frac{780}{209} \end{bmatrix} = \begin{bmatrix} 1 & -\frac{1}{4} & & & \\ & 1 & -\frac{4}{15} & & \\ & & 1 & -\frac{15}{56} & \\ & & & 1 & -\frac{56}{209} \\ & & & & 1 \end{bmatrix}$$

通过  $\hat{L}y=b$  即

$$\begin{bmatrix} 4 & & & & \\ -1 & \frac{15}{4} & & & \\ & -1 & \frac{56}{15} & & \\ & & -1 & \frac{209}{56} & \\ & & & -1 & \frac{780}{209} \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \end{bmatrix} = \begin{bmatrix} 100 \\ 0 \\ 0 \\ 0 \\ 200 \end{bmatrix}$$

$$\text{解得 } y = \left[ 25 \quad \frac{20}{3} \quad \frac{25}{14} \quad \frac{100}{209} \quad \frac{2095}{39} \right]^T$$

通过  $\hat{U}x=y$  即

$$\begin{bmatrix} 1 & -\frac{1}{4} & & & \\ & 1 & -\frac{4}{15} & & \\ & & 1 & -\frac{15}{56} & \\ & & & 1 & -\frac{56}{209} \\ & & & & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \end{bmatrix}$$

解得  $x = [53.7179 \ 14.8718 \ 5.7692 \ 8.2051 \ 27.0513]^T$

10,

根据题意，利用 *Gauss-Jordan* 消元法求矩阵的逆的过程如下

$$\begin{aligned} & \left[ \begin{array}{ccc|ccc} 1 & 1 & -1 & 1 & 0 & 0 \\ 2 & 1 & 0 & 0 & 1 & 0 \\ 1 & -1 & 0 & 0 & 0 & 1 \end{array} \right] \rightarrow \left[ \begin{array}{ccc|ccc} 2 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & -1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 1 \end{array} \right] \rightarrow \left[ \begin{array}{ccc|ccc} 2 & 1 & 0 & 0 & 1 & 0 \\ 0 & \frac{1}{2} & -1 & 1 & -\frac{1}{2} & 0 \\ 0 & -\frac{3}{2} & 0 & 0 & -\frac{1}{2} & 1 \end{array} \right] \rightarrow \left[ \begin{array}{ccc|ccc} 1 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & \frac{1}{2} & -1 & 1 & -\frac{1}{2} & 0 \\ 0 & -\frac{3}{2} & 0 & 0 & -\frac{1}{2} & 1 \end{array} \right] \rightarrow \\ & \left[ \begin{array}{ccc|ccc} 1 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & -\frac{3}{2} & 0 & 0 & -\frac{1}{2} & 1 \\ 0 & \frac{1}{2} & -1 & 1 & -\frac{1}{2} & 0 \end{array} \right] \rightarrow \left[ \begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & \frac{1}{3} & \frac{1}{3} \\ 0 & -\frac{3}{2} & 0 & 0 & -\frac{1}{2} & 1 \\ 0 & 0 & -1 & 1 & -\frac{2}{3} & \frac{1}{3} \end{array} \right] \rightarrow \left[ \begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & \frac{1}{3} & \frac{1}{3} \\ 0 & 1 & 0 & 0 & \frac{1}{3} & -\frac{2}{3} \\ 0 & 0 & -1 & 1 & -\frac{2}{3} & \frac{1}{3} \end{array} \right] \rightarrow \left[ \begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & \frac{1}{3} & \frac{1}{3} \\ 0 & 1 & 0 & 0 & \frac{1}{3} & -\frac{2}{3} \\ 0 & 0 & 1 & -1 & \frac{2}{3} & -\frac{1}{3} \end{array} \right] \end{aligned}$$

故可到矩阵的逆如下

$$A^{-1} = \begin{bmatrix} 0 & \frac{1}{3} & \frac{1}{3} \\ 0 & \frac{1}{3} & -\frac{2}{3} \\ -1 & \frac{2}{3} & -\frac{1}{3} \end{bmatrix}$$