作业 3

2.

(1)根据线性插值条件，选择$x_0 = 0.2, x_1 = 0.3$作为插值点，则$L_1(x)$如下

$$L_1(x) = \frac{x - x_1}{x_0 - x_1} f(x_0) + \frac{x - x_0}{x_1 - x_0} f(x_1)$$

带入$x_0, x_1, f(x_0), f(x_1)$以及$x = 0.23$，得出$f(0.23) = \mathbf{1.25995}$

(2)根据二次插值条件，选择$x_0 = 0.1, x_1 = 0.2, x_3 = 0.3$作为插值点，则$L_2(x)$如下

$$L_2(x) = \frac{(x - x_1)(x - x_2)}{(x_0 - x_1)(x_0 - x_2)} f(x_0) + \frac{(x - x_0)(x - x_2)}{(x_1 - x_0)(x_1 - x_2)} f(x_1) + \frac{(x - x_0)(x - x_1)}{(x_2 - x_0)(x_2 - x_1)} f(x_2)$$

带入$x_0, x_1, x_2, f(x_0), f(x_1), f(x_2)$以及$x = 0.23$，得出$f(0.23) = \mathbf{1.25866}$

7.

(1)根据题意$f(x) = e^x$的等距节点函数表如下

x	-4	-3	-2	-1	0	1	2	3	4
$f(x) = e^x$	0.0183	0.0498	0.1353	0.3679	1.000	2.7183	7.3891	20.0855	54.5982

(2)若用二次插值，取三个样本点$x_0 = x_k - h, x_1 = x_k, x_2 = x_k + h$

则由二次插值余项$R_2(x) = \frac{f^{(3)}(\xi)}{3!}(x - x_0)(x - x_1)(x - x_2)$，

令$F(x) = (x - x_0)(x - x_1)(x - x_2)$，$F'(x) = 3(x - x_k)^2 - h^2$，令$F'(x) = 0$可得

$$max|f(x)| = \left| f\left(x_k + \frac{\sqrt{3}}{3}\right) \right| = \frac{2\sqrt{3}}{9} h^3$$

$\therefore$ 当$-4 \le x \le 4$时，$|R_2(x)| \le \frac{\sqrt{3}}{27} e^4 h^3 \le \varepsilon = 10^{-6}$，故$h < 0.006585$

10.

根据题意，由差商和导数之间的关系

$$f[x_0, x_1, \dots, x_n] = \frac{1}{n!} f^{(n)}(\xi)$$

$\therefore$ 在此题中，$f[2^0 2^1 \cdots 2^6] = \frac{1}{6!} f^{(6)}(\xi), f[2^0 2^1 \cdots 2^7] = \frac{1}{7!} f^{(7)}(\xi)$

而$f^{(6)}(x) = 5 \times 6!\ f^{(7)}(x) = 0$，

$\therefore f[2^0 2^1 \cdots 2^6] = \frac{1}{6!} f^{(6)}(\xi) = \frac{5 \times 6!}{6!} = 5$

而$f[2^0 2^1 \cdots 2^7] = \frac{1}{7!} f^{(7)}(\xi) = 0$

24.

根据题意取等距节点$x_i = a + ih, 0 \le i \le \frac{b-a}{n}$，则任给$x \in [a, b]$，一定$\exists i, s.t. x \in [x_i, x_{i+1}]$

以x_i, x_{i+1}作为插值基点作$f(x)$的线性插值，则有

$$L_1(x) = \frac{x - x_{i+1}}{x_i - x_{i+1}} f(x_i) + \frac{x - x_i}{x_{i+1} - x_i} f(x_{i+1})$$

由于

$$\max_{x \in [x_i, x_{i+1}]} |f''(x)| = 2 \qquad \max_{x \in [x_i, x_{i+1}]} |(x - x_i)(x - x_{i+1})| \le \frac{1}{4} h^2$$

∴该插值函数误差有

$$|f(x) - L_1(x)| \le \frac{1}{2} \max_{x \in [x_i, x_{i+1}]} \left| f''(x) \right| \cdot \max_{x \in [x_i, x_{i+1}]} |(x - x_i)(x - x_{i+1})| \le \frac{1}{4} h^2 = \frac{(b-a)^2}{4n^2}$$