Charged particle motion around magnetized black hole

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Institute of Physics, Silesian University in **Opava**

Opava is located in Upper Silesia, Czech Republic; has a population of 60 000.

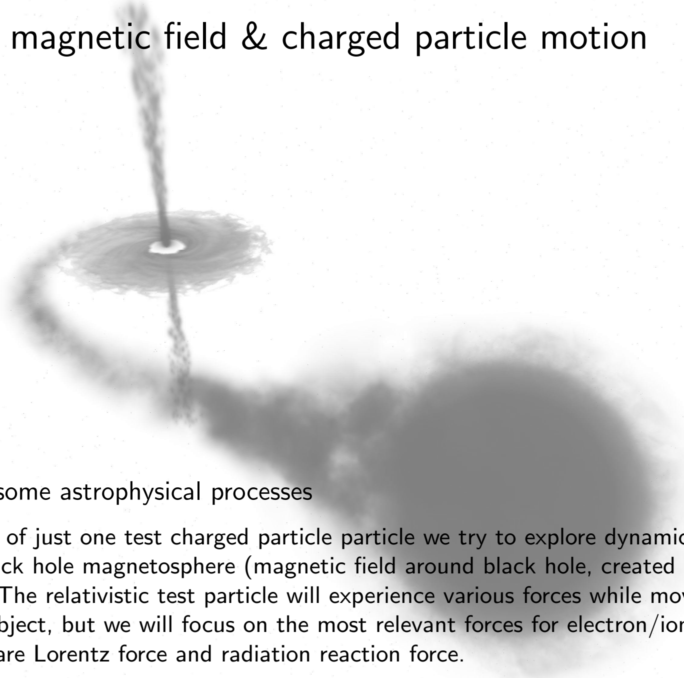
Institute of Physics ~50 people:

- Research Centre of Theoretical Physics and Astrophysics
- Research Centre of Computational Physics and Data Processing

Section 1

Motivation

Black hole, magnetic field & charged particle motion



Motivation

Particle motion

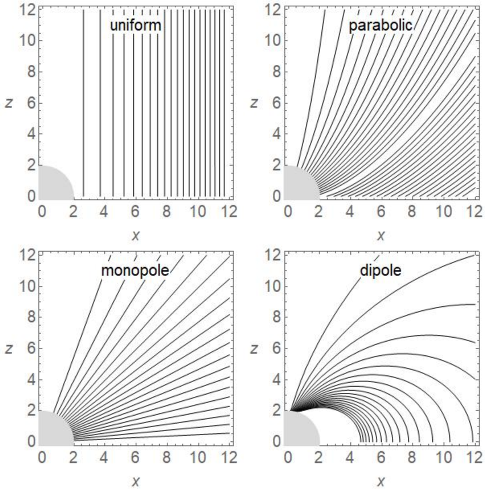
Application to some astrophysical processes

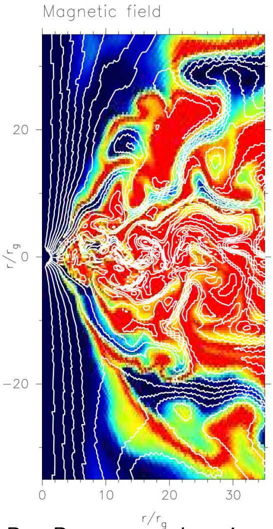
Studying motion of just one test charged particle particle we try to explore dynamic of matter inside black hole magnetosphere (magnetic field around black hole, created by accretion disk). The relativistic test particle will experience various forces while moving around central object, but we will focus on the most relevant forces for electron/ion like particles, which are Lorentz force and radiation reaction force.

Black hole magnetosphere - **gravity** & electro**magnetism**

magnetic field is weak field ($\ll 10^{18}$ Gs) and not contributing to gravity
$\Rightarrow$ solve separately Einstein (rotating BH) and then Maxwell eqns. in curved spacetime

Kerr metric for rotating black hole + magnetic field ???





right fig. from: Agnieszka Janiuk *Microphysics in the Gamma Ray Burst central engine*

The stuff around black hole is plasma

An accretion disk is axially symmetric structure formed by plasma in orbital motion around central black hole. Plasma, the fourth state of matter, is:

- In plasma there are free charge carriers.
- Plasma is quasi-neutral.
- Plasma has collective behavior, it reacts to and also creates electromagnetic fields.

Plasma modeling:

- 3. **N-body problem**
- 2. **Kinetic theory** - distribution function, Boltzmann equation
- 1. **Fluid model** - Magnetohydrodynamics (MHD), fluid, pressure,...
- 0. **Single particle approach** - one particle motion in given magnetic field

$\Rightarrow$ One charged test particle moving in fixed combined magnetic and gravitational field. Gravitational field (Kerr metric) and magnetic field (???) are given and not changing.

$\Rightarrow$ limits of single particle approach:

mean free path inside torus: $(n \sim 10^{16}\ \mathrm{cm}^{-3}, \mathrm{T} \sim 10^{4}\ \mathrm{K})$ $\qquad \lambda \sim 10^{-3}\ \mathrm{cm}$

one orbit around black hole: $(M = 10\ \mathrm{M}_{\odot})$ $\qquad l \sim 10^{7}\ \mathrm{cm}$

Relevant forces for charged particle & dimensional analysis

solar mass BH	$M \sim 10 M_{\odot}$	$B < 10^8$ Gs
supermassive BH	$M \sim 10^9 \mathrm{M}_{\odot}$	$B < 10^4$ Gs

Particle with charge q and mass m orbits around BH with mass M and is under influence of some force F^{μ}

$$\frac{\mathrm{d}u^{\mu}}{\mathrm{d}\tau} + \Gamma^{\mu}_{\alpha\beta} u^{\alpha} u^{\beta} = F^{\mu} \tag{1}$$

where $u^{\mu} = dx^{\mu}/d\tau$ is particle four-velocity, $\Gamma^{\mu}_{\alpha\beta}$ are Christoffel symbols (gravity).

test particle ($m \ll M$)

- elementary particle (electron/ion)
- charged inhomogeneity (hot-spot/star)

gravity	~ 1
particle spin	$\sim 10^{-17}$ -electron$/10 M_{\odot}$ ($\sim 10^{-4}$ -star$/10^8 M_{\odot}$)
Lorentz force	$\sim 10^{7} \cdot \left(\frac{q}{e}\right) \left(\frac{m_e}{m}\right) \left(\frac{B}{10^4 \mathrm{Gs}}\right) \left(\frac{M}{10 M_{\odot}}\right)$
EM radiation reaction	$\sim 10^{-5} \cdot \left(\frac{q}{e}\right)^4 \left(\frac{m_e}{m}\right)^3 \left(\frac{B}{10^4 \mathrm{Gs}}\right)^2 \left(\frac{M}{10 M_{\odot}}\right)$
surrounding medium	?

We will assume the influence of gravity, Lorentz force and radiation reaction force. Space will be consider to be "empty" and effect of surrounding medium will be neglected.

Section 2

Particle motion

(A) Gravity only

Geodesic equation (set of 4 second order ordinary differential equations for t, r, θ, ϕ)

$$\frac{\mathrm{d}u^\mu}{\mathrm{d}\tau} + \Gamma^\mu_{\alpha\beta} u^\alpha u^\beta = 0 \tag{2}$$

where $u^\mu = dx^\mu/d\tau$ is the four-velocity of the particle, $\Gamma^\mu_{\alpha\beta}$ are Christoffel symbols for Kerr black metric $g_{\alpha\beta} = g_{\alpha\beta}(r, \theta)$

$$\Gamma^\mu_{\alpha\beta} = \frac{1}{2} g^{\mu\gamma} \left(g_{\gamma\alpha,\beta} + g_{\gamma\beta,\alpha} - g_{\alpha\beta,\gamma} \right); \ \mathrm{d}s^2 = g_{tt}\mathrm{d}t^2 + 2g_{t\phi}\mathrm{d}t\mathrm{d}\phi + g_{\phi\phi}\mathrm{d}\phi^2 + g_{rr}\mathrm{d}r^2 + g_{\theta\theta}\mathrm{d}\theta^2$$

We can also use super-**Hamiltonian** formalism

$$H = \frac{1}{2} g^{\alpha\beta} p_\alpha p_\beta + \frac{1}{2} m^2 = 0, \tag{3}$$

and equations of motion $\lambda = \tau/m$ (set of 8 first order ordinary differential equations)

$$\begin{aligned} \frac{\mathrm{d}x^\mu}{\mathrm{d}\lambda} &= \frac{\partial H}{\partial p_\mu} \qquad &&| = g^{\mu\nu} p_\nu = p^\mu = m \frac{dx^\mu}{d\tau} \quad \text{four-momentum def.} \\ \frac{\mathrm{d}p_\mu}{\mathrm{d}\lambda} &= -\frac{\partial H}{\partial x^\mu} \qquad &&| = g^{\alpha\beta} \Gamma^\gamma{}_{\mu\alpha} p_\gamma p_\beta \quad \text{geodesic eq. (2)} \end{aligned}$$

(A) Gravity only - symmetries & effective potential

$$\frac{\mathrm{d}x^\mu}{\mathrm{d}\lambda} = \frac{\partial H}{\partial p_\mu}, \quad \frac{\mathrm{d}p_\mu}{\mathrm{d}\lambda} = -\frac{\partial H}{\partial x^\mu} \tag{4}$$

Kerr black metric $g_{\alpha\beta} = g_{\alpha\beta}(r, \theta)$

- static geometry - no time t dependence in geometry
  $\partial H / \partial t = 0 \quad \rightarrow \quad -p_t = E$, energy E is conserved during the motion
- axial symmetry - no ϕ dependence in geometry
  $\partial H / \partial \phi = 0 \quad \rightarrow \quad p_\phi = L$, angular momentum L is conserved

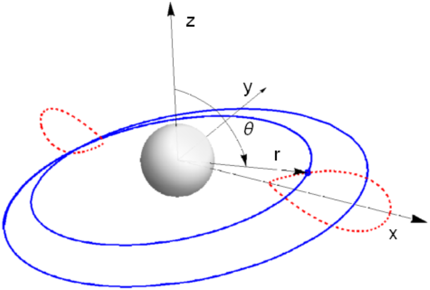
$$\begin{aligned} H &= \frac{1}{2} m^2 + \frac{1}{2} g^{\alpha\beta} p_\alpha p_\beta \\ &= \frac{1}{2} m^2 + \frac{1}{2} g^{tt} E^2 + g^{t\phi} E L + \frac{1}{2} g^{\phi\phi} L^2 + \frac{1}{2} g^{rr} p_r^2 + \frac{1}{2} g^{\theta\theta} p_\theta^2 \end{aligned} \tag{5}$$

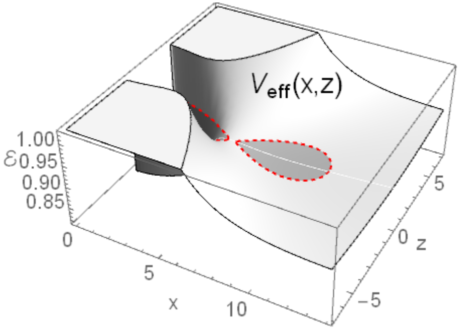
- 8 dimensional phase-space $t, \phi, r, \theta, p^t, p^\phi, p^r, p^\theta$ & symmetry in Hamiltonian
  $\rightarrow$ 4 dimensional phase-space r, θ, p^r, p^θ - motion 2D effective potential

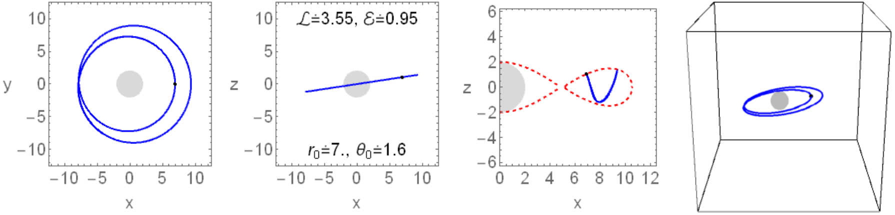
$$V_{\mathrm{eff}}(r, \theta; L) = \frac{g^{t\phi} L - \sqrt{(g^{t\phi} L)^2 - g^{tt}(g^{\phi\phi} L^2 + m^2)}}{g^{tt}}, \tag{6}$$

$g^{\alpha\beta} p_\alpha p_\beta = -m^2$ gives energetic boundary for particle motion $E = V_{\mathrm{eff}}(r, \theta)$

(A) Eff. potential $V_{\text{eff}}(x, z)$ & motion boundary $E = V_{\text{eff}}$

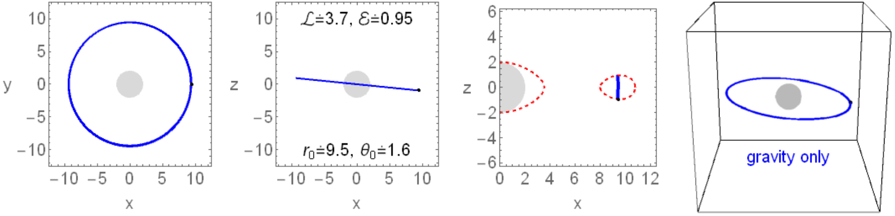




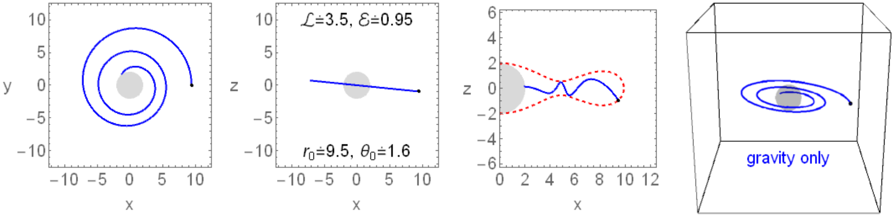


(A) Gravity only

circular orbit



capture by BH - not enough angular momentum $\mathcal{L}$



(B) Gravity + Lorentz force

charged particle motion around magnetized BH can be treated by the **Lorentz equation**

$$\frac{\mathrm{d}u^\mu}{\mathrm{d}\tau} + \Gamma^\mu_{\alpha\beta} u^\alpha u^\beta = \frac{q}{m} g^{\mu\rho} F_{\rho\sigma} u^\sigma \tag{7}$$

where $u^\mu = dx^\mu/d\tau$ and $\Gamma^\mu_{\alpha\beta}$ are Christoffel symbols (gravity) and $F_{\mu\nu}$ is tensor of electromagnetic field constructed from EM four-potential A_ν

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu, \tag{8}$$

Super-Hamiltonian formalism

$$H = \frac{1}{2} g^{\alpha\beta}(\pi_\alpha - qA_\alpha)(\pi_\beta - qA_\beta) + \frac{1}{2}\, m^2, \qquad \frac{\mathrm{d}x^\mu}{\mathrm{d}\zeta} \equiv p^\mu = \frac{\partial H}{\partial \pi_\mu}, \quad \frac{\mathrm{d}\pi_\mu}{\mathrm{d}\zeta} = -\frac{\partial H}{\partial x^\mu},$$

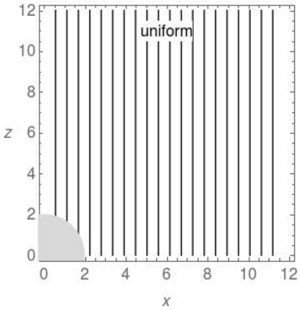
kinematic four-momentum $p_\mu = m\, u_\mu$ vs. generalized four-momentum $\pi_\mu = p_\mu + qA_\mu$

Conserved quantities (if $g_{\alpha\beta} = g_{\alpha\beta}(r,\theta),\ A_t = A_t(r,\theta), A_\phi = A_\phi(r,\theta),\ A_r = A_\theta = 0$)

$$-E = \pi_t = g_{t\alpha} p^\alpha + qA_t, \quad L = \pi_\phi = g_{\phi\alpha} p^\alpha + qA_\phi \tag{9}$$

effective potential $V_{\mathrm{eff}}(r,\theta; L) = \ldots$

(B) Gravity + Lorentz force



simple but relevant configuration:

$$A_\phi = \frac{B}{2}\, g_{\phi\phi}, \quad A_t = \frac{B}{2}\, g_{t\phi} - Ba,$$

uniform magnetic field B - parallel with z axis

Dimensionless quantity $\mathcal{B}$ (magnetic parameter) is related to Lorenz force magnitude:

$$\mathcal{B} = \frac{qBGM}{2mc^4}$$

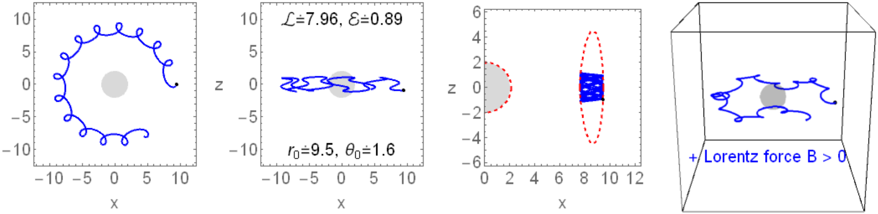
	electron	proton	Fe+	charged dust
$\mathcal{B} = 0.004$	10^{-5} Gs	0.02 Gs	1 Gs	10^9 Gs

For stellar mass black hole $M \approx 10 M_\odot$, we can have one electron e- in the magnetic field $B = 10^{-5}$ Gs or charged dust grain (one electron lost, $m = 2 \times 10^{-16}$ kg) in field $B = 10^9$ Gs - the absolute value of magnetic field parameter is the same in both cases $\mathcal{B} = 0.004$.

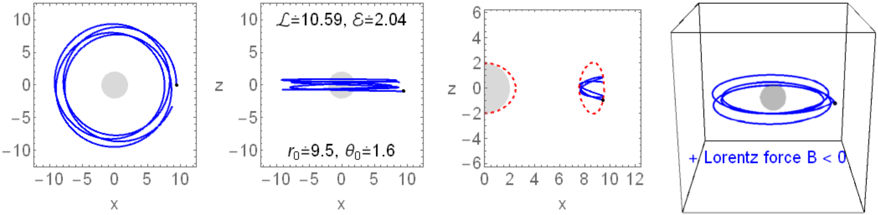
particle trajectories in different magnetic field configurations, code in *Mathematica*®
`https://github.com/XyhwX/particle`

(B) Gravity + Lorentz force

repulsive Lorentz force



attractive Lorentz force



(C) Gravity + Lorentz force + EM radiation reaction

radiation emitted by a charged particle leads to appearance of **back-reaction force**

$$\frac{\mathrm{d}u^\mu}{\mathrm{d}\tau} + \Gamma^\mu_{\alpha\beta} u^\alpha u^\beta = \frac{q}{m} F^\mu{}_\nu u^\nu + \frac{q}{m} \mathcal{F}^\mu{}_\nu u^\nu, \tag{10}$$

Lorentz force is given by EM tenzor $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$; back-reaction force $\frac{q}{m}\mathcal{F}^\mu{}_\nu u^\nu$

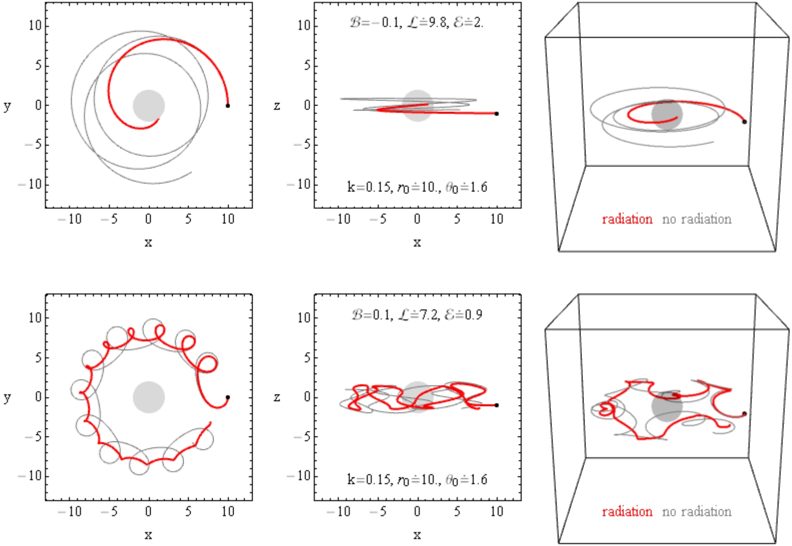
$$\frac{2q^2}{3m}\left(\frac{D^2u^\mu}{d\tau^2} + u^\mu u_\nu \frac{D^2u^\nu}{d\tau^2}\right) + \frac{q^2}{3m}\left(R^\mu{}_\lambda u^\lambda + R^\nu{}_\lambda u_\nu u^\lambda u^\mu\right) + \frac{2q^2}{m} u_\nu \int D^{[\mu} G^{\nu]}_{+\lambda'}(z(\tau), z(\tau'))u^{\lambda'} d\tau'$$

$$= \frac{2q^2}{3m}\left(\frac{DF^\alpha{}_\beta}{dx^\mu} u^\beta u^\mu + \left(F^\alpha{}_\beta F^\beta{}_\mu + F_{\mu\nu} F^\nu{}_\sigma u^\sigma u^\alpha\right) u^\mu\right)$$

- 1st term $d^2u^\mu/d\tau^2$ is problematic- high order for equation of motion but can be substituted - trick from Landau & Lifshitz *Course of Theoretical Physics*
- 2nd term gone zero - Ricci tensor vanishes the vacuum metrics 3rd term - tail integral (non-local nature of radiation reaction) is negligible small
- radiation reaction force act as damping - particle energy and angular momenta are decreasing (not conserved)

• A. Tursunov, M. Kološ, Z. Stuchlík and D. V. Gal'tsov : *Radiation reaction of a charged particle. . .* , The Astro. Journal 861 (1), 16 (2018) [arXiv:1803.09682]

(C) Gravity + Lorentz force + EM radiation reaction

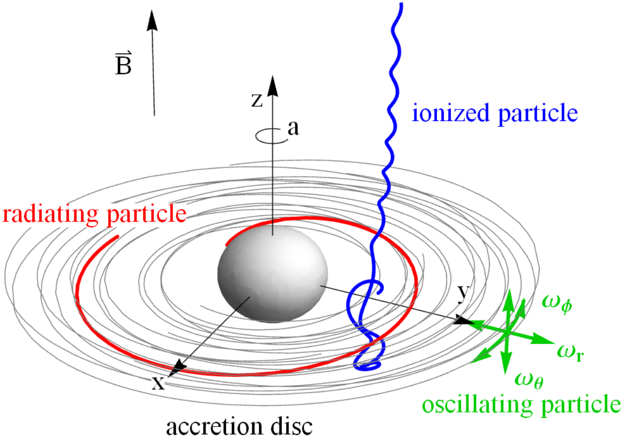


Depending on the orientation of the Lorentz force, the particle either **spirals down** to the BH (first line) or stabilizes the circular orbit by **decaying its oscillations** (second line).

Section 3

Application to some astrophysical processes

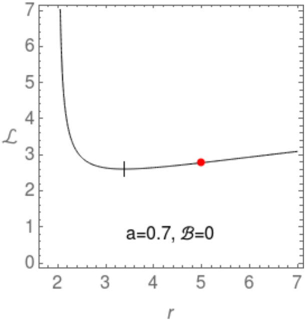
Application to some astrophysical processes



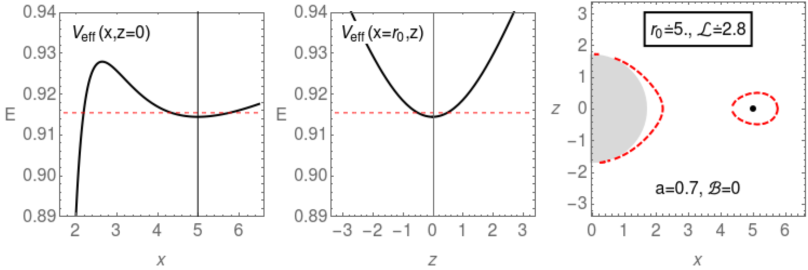
some processes around magnetized black hole:

- (1) Stability of charged particle circular orbit - accretion disk stability
- (2) Charged particle oscillations - observed quasi-periodic oscillations
- (4) Neutral particle ionization - escaping ultra-relativistic particles

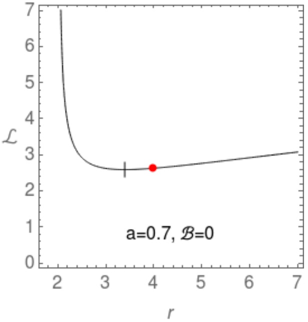
(1) Stability of charged particle circular orbit



- Minima of effective potential $V_{\rm eff}(x, z; \mathcal{L})$ $\Rightarrow$ stable particle circular orbit.
- On left fig. we have angular momenta $\mathcal{L}(r)$ for particle on circular orbit.
- The sum of all circular orbits forms thin Keplerian accretion disc.
- Existence of innermost stable circular orbit - accretion disc inner edge.



(1) Stability of charged particle circular orbit



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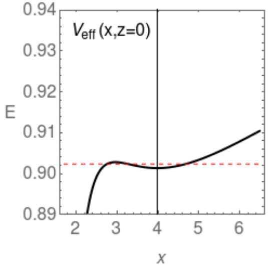
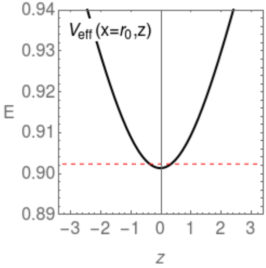
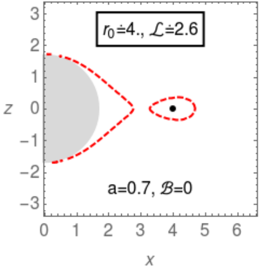
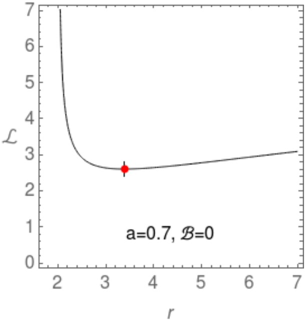






(1) Stability of charged particle circular orbit



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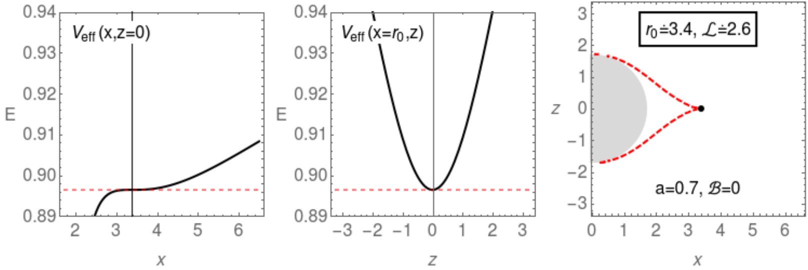
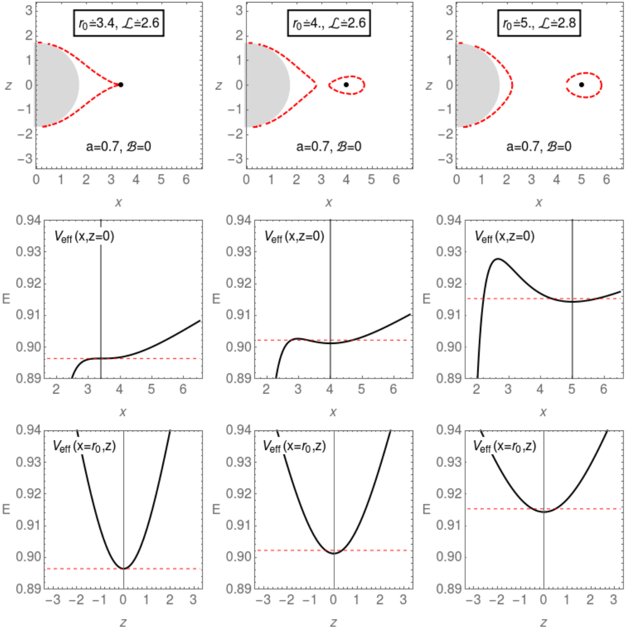
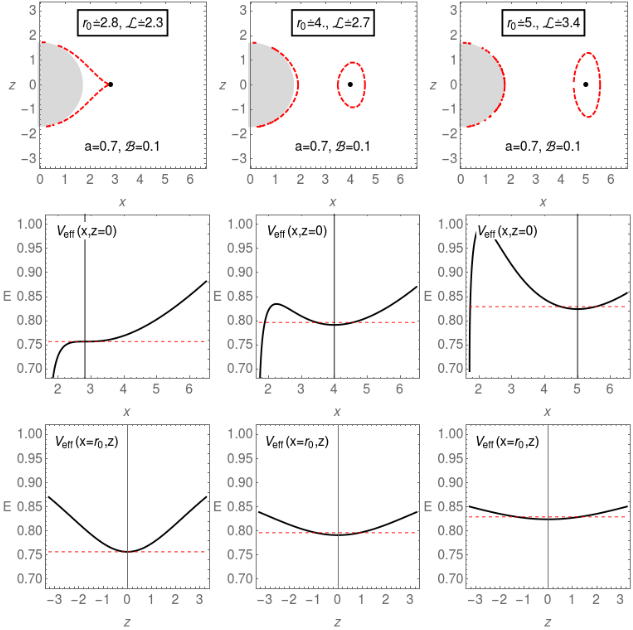


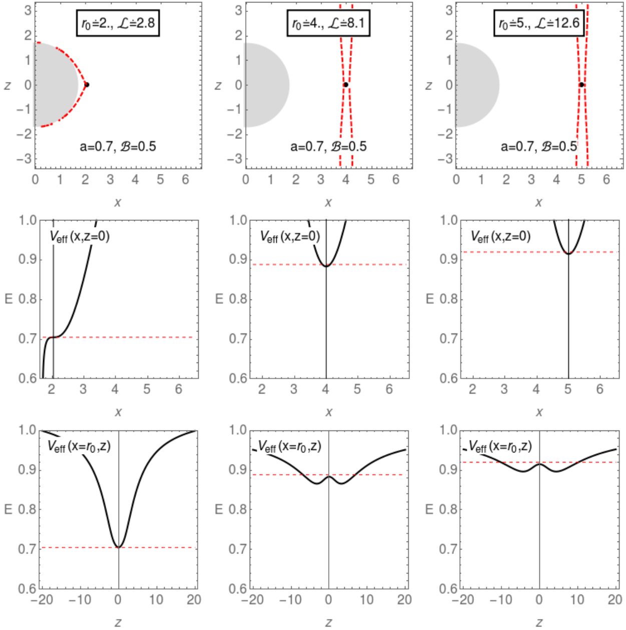
(1) Stability of circular orbit $\mathcal{B} = 0$



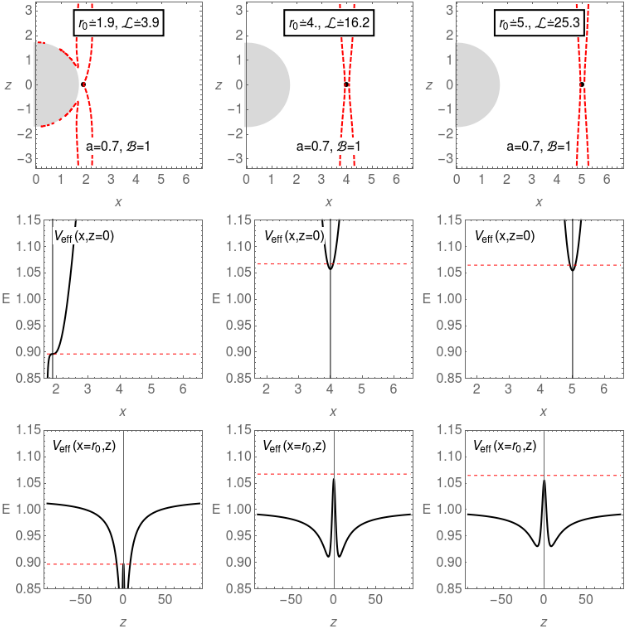
(1) Stability of circular orbit $\quad \mathcal{B} = 0.1$



(1) Stability of circular orbit $\mathcal{B} = 0.5$

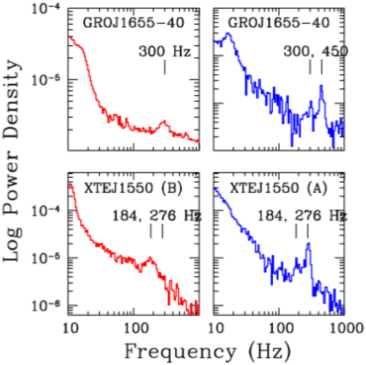


(1) Stability of circular orbit $\mathcal{B} = 1$



(2) Quasi-Periodic Oscillations observed in microquasars

- Microquasars are binary systems composed of a black hole and a companion star.
- Quasi-periodic oscillations (QPOs) of the X-ray power density are observed in microq.
- QPOs frequencies cover the range from few mHz up to 0.5 kHz - high frequency (~ 500 Hz) and low frequency (~ 30 Hz).
- The HF QPOs in BH are detected with the twin peaks which have frequency ratio close to 3:2 - resonance.

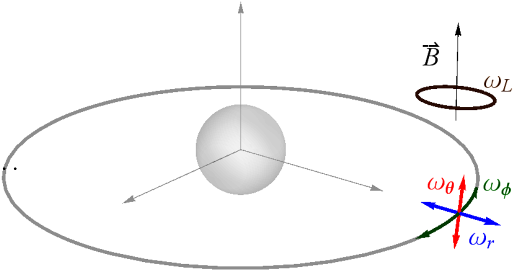


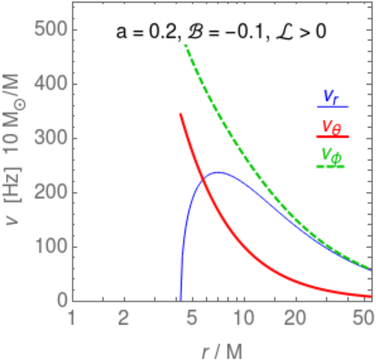
Source	$f_{\rm low}$ [Hz]	$f_{\rm L}$ [Hz]	$f_{\rm U}$ [Hz]	M [$\mathrm{M}_\odot$]	a
GRO 1655-40	18	300	450	6.03–6.57	0.65–0.75
XTE 1550-564	13	184	276	8.5–9.7	0.29–0.52
GRS 1915+105	10	113	168	10.6–14.4	

Observed QPOs data for three microquasars and the restrictions on mass M and spin a of the black holes located in them, based on measurements independent of the QPOs.

(2) Magnetic field and microquasar QPOs

$$m\ddot{x} = F = -V'_x$$
$$\delta x = x - x_0$$
$$V(x) = V(x_0) + V'_x(x_0)\delta x$$
$$+1/2 V''_x(x_0)\delta x^2 + \dots$$
$$m\ddot{x} + V''_x(x_0)\delta x = 0$$
$$\ddot{\delta x} + \omega_x^2 \delta x = 0$$





Perturbation of particle on circular orbit, which is located at minima of effective potential $V_{\rm eff}(r,\theta)$, will leads to particle oscillations with frequencies:

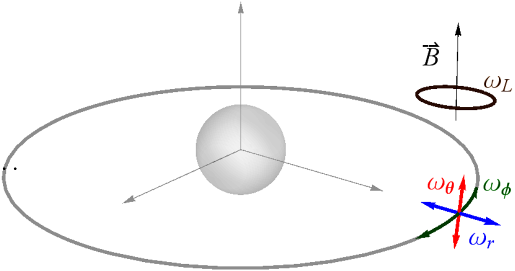
$$\omega_{\rm r}^2 \sim \frac{\partial^2 V_{\rm eff}}{\partial r^2}, \qquad \omega_\phi = \frac{{\rm d}\phi}{{\rm d}\tau} = u^\phi,$$
$$\omega_\theta^2 \sim \frac{\partial^2 V_{\rm eff}}{\partial \theta^2}, \qquad \omega_{\rm L} = \frac{qB}{m}$$

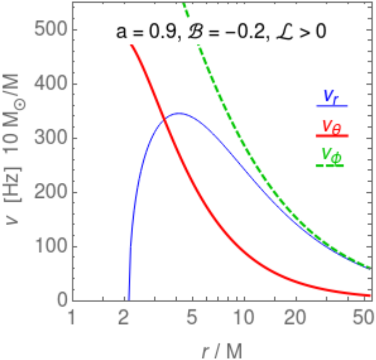
radial $\omega_{\rm r}$, vertical ω_θ, Keplerian ω_ϕ, Larmor $\omega_{\rm L}$

redshifted and in physical units $\boxed{\omega_\alpha \;\Rightarrow\; \nu_\alpha}$

(2) Magnetic field and microquasar QPOs

$$m\ddot{x} = F = -V_x'$$
$$\delta x = x - x_0$$
$$V(x) = V(x_0) + V_x'(x_0)\delta x + 1/2 V_x''(x_0)\delta x^2 + \ldots$$
$$m\ddot{x} + V_x''(x_0)\delta x = 0$$
$$\ddot{\delta x} + \omega_x^2 \delta x = 0$$





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radial $\omega_{\rm r}$, vertical ω_θ, Keplerian ω_ϕ, Larmor $\omega_{\rm L}$

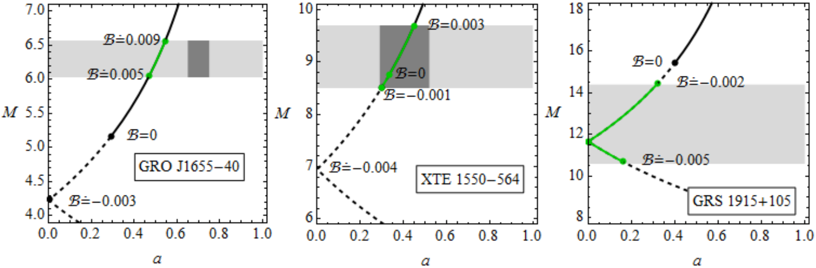
redshifted and in physical units $\boxed{\omega_\alpha \Rightarrow \nu_\alpha}$

(2) Magnetic field influence on the QPOs phenomena

three equations - four variables $r, M, a, \mathcal{B}$ - one free parameter (magnetic field strength $\mathcal{B}$).

$$f_{\rm U} = \nu_{\rm U}(r, M, a, \mathcal{B}), \quad f_{\rm L} = \nu_{\rm L}(r, M, a, \mathcal{B}), \quad f_{\rm low} = \nu_{\rm low}(r, M, a, \mathcal{B}).$$

r resonant radii, BH mass M BH spin a, magnetic field strength $\mathcal{B}$



BH mass M and spin a obtained by another independent method (grey rectangles) compared to the RP QPOs model for charged particle around magnetized BH (thick curves) - $\mathcal{B} \sim 0.004$.

$$\mathcal{B} = \frac{qBGM}{2mc^4}$$

	electron	proton	Fe+	charged dust
$\mathcal{B} = 0.004$	10^{-5} Gs	0.02 Gs	1 Gs	10^9 Gs

• M. Kološ, A. Tursunov and Z. Stuchlík, *Possible signature of magnetic field in microquasar QPOs*, Eur. Phys. J. C 77: 860 (2017) [arXiv:1707.02224]

(3) Observation of Ultra-High-Energy Cosmic Rays

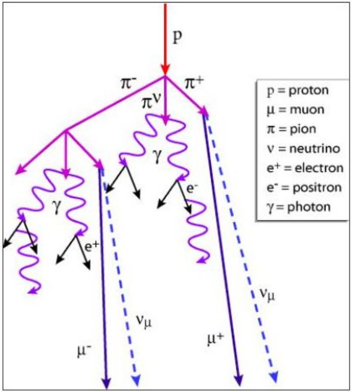
- In UHECR there are particles with energies $E > 10^{18}$ eV.
- Few things we know: UHECR are charged particles, extremely rare, probably proton dominated flux or iron nuclei, extra-Galactic origin
- Energetic accelerator - mechanism is unknown: extra dimensions, new particles. . .
- From supermassive black holes? Black hole thermodynamics: 29% of BHs energy is available for extraction (rotation), for extremely rotating SMBH of 10^9 solar mass the available energy is 10^{74} eV

Let's have one particle with rest mass/energy E_0 it can take energy from BH with efficiency η and got energy

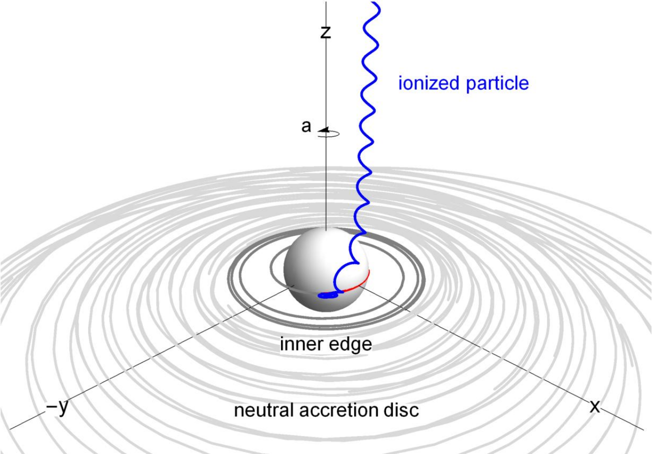
$$E = (1 + \eta) E_0$$

- Penrose (1969) - astro. problematic $\eta \leq 0.2$
- Wagh et al. (1985) astro. relevant $\eta \sim 1$ electromagnetic version of Penrose process
- Blandford & Znajek (1977) BH energy extracted by force-free plasma currents $\eta \sim 3$
- Many other versions with different efficiencies

But for UHECR energies one needs $\eta > 10^8$!



(3) Ionization in vicinity of mag. BH - mode for UHECR



$$\boxed{\pi_{\alpha(1)} = \pi_{\alpha(2)} + \pi_{\alpha(3)}, \qquad E = -\pi_t = -g_{t\alpha}p^\alpha - qA_t} \tag{13}$$

1st neutral particle (black, neutron) has been located on accretion disk inner edge, and starts to fall into BH. It will decay into two ionized particles 2nd (blue, proton) and 3rd (red, electron). While the 3rd particle (red) fall inside BH with negative energy the second (blue) got more energy and escape along magnetic field line.

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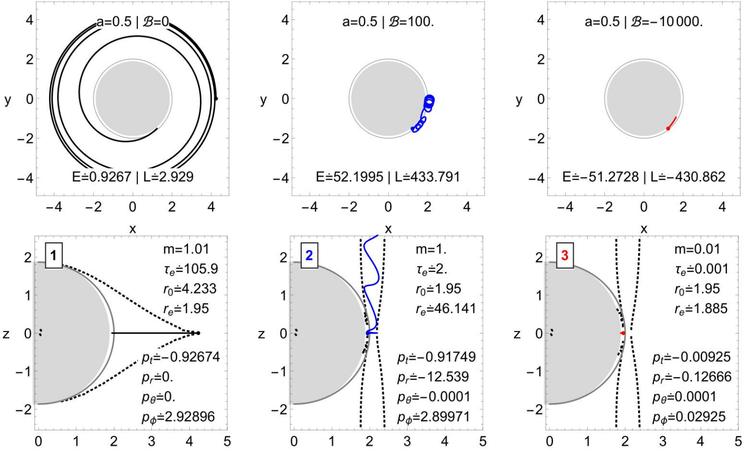

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Summary & What to do next?

One charged particle motion around magnetized black hole is easy to handle.
For empty space the relevant forces are: gravity, Lorentz force (radiation reaction).

We have studied processes around magnetized black hole:

- (1) Stability of circular orbit - particle innermost stable circular orbit is shifting close to the BH horizon, **instability in vertical direction** (escape along z axis)
- (2) Charged particle oscillations - particle frequencies can be well related to the observed microquasar QPOs, magnetic field intensity $\sim 10^{-4}$ Gauss for electron
- (4) Ionized particle acceleration - charged particles are escaping along magnetic field lines, **ultra**-relativistic escape velocity can be reach $> 10^{1}$ Gauss

Future tasks

- one particle: radiation reaction in Kerr, another magnetic field configurations. . .
- one single particle is not enough $\Rightarrow$ GRMHD!

Thank you for your attention