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Discrete Mathematics

GRAPH THEORY

Presidency Group

OVER 40 YEARS OF ACADEMIC WISDOM

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AFFILIATED TO BENGALURU CITY UNIVERSITY, APPROVED BY AICTE, DELHI & RECOGNISED BY THE GOVT. OF KARNATAKA

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HISTORY OF GRAPH THEORY

WHY GRAPH THEORY?

- Graphs used to model pair wise relation between objects.
- Generally a network can be represented by a graph.
- Many practical problems can be easily represented in terms of graph theory.

GRAPH THEORY - HISTORY

- The origin of graph theory can be traced back to Euler's work on the Konigsberg bridges problem (1735), which led to the concept of an Eulerian graph. The study of cycles on polyhedra by the Thomas P. Kirkman (1806-95) and William R. Hamilton (1805-65) led to the concept of a Hamiltonian graph.

Introduction

<u>The role of graphs:</u>

- extremely important in computer science and mathematics
- numerous important applications
- modeling the concept of binary relation

Graphs are extensively and intuitively to convey information in
visual form.
Here we introduce basic mathematical view on graphs.

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GRAPH

Graph (undirected graph) is an ordered pair of sets: *G = (V, E)*, where:

- V is the vertex^1 set
- E is the edge set
- each edge e = {v,w} in E is an unordered pair of vertices from V, called the ends of the edge e.

Vertex can be also called node.

Graph size parameters: n=|*V*|, m=|*E*|

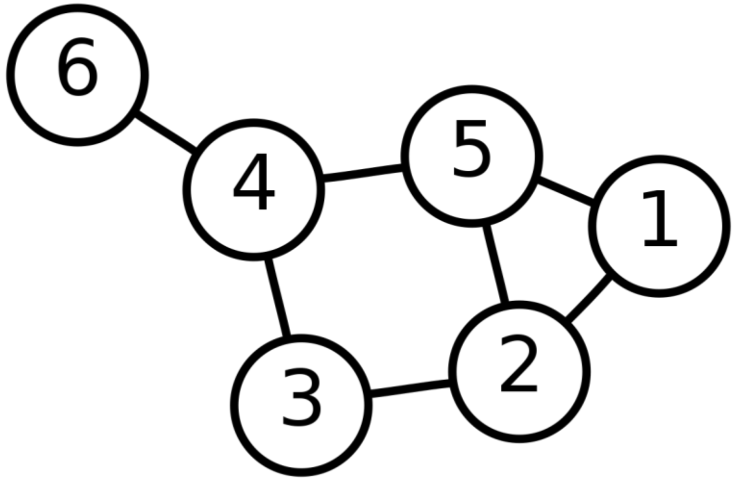
VERTEX AND EDGE

- ## Vertex /Node
  - Basic Element
  - Drawn as a node or a dot.
  - Vertex set of G is usually denoted by V(G), or V or V_1

- ## Edge/Arcs
  - A set of two elements
  - Drawn as a line connecting two vertices, called end vertices, or endpoints.
  - The edge set of G is usually denoted by E(G), or E or Eg

- ## Neighborhood
  - For any node v, the set of nodes it is connected to via an edge iscalled its neighborhood and is represented as N(v)

GRAPH - EXAMPLE



- n = 6 , m = 7
- Vertices(V): ={1,2,3,4,5,6}
- Edge(E): = {1,2},{1,5},{2,3},{2,5},{3,4},{4,5},{4,6}
- N(4): = Neighborhood(4) = {6,5,3}

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EDGE TYPES:

TYPES OF GRAPH

- Undirected –
  - Ex- Distance between two cities, friendships…. Etc.
- Directed – (order pairs of nodes.)
  - Ex- directed edges have a source (head, origin) and target (tail, destination) vertices.
- Weighted –
  - Usually weight is associated.

TYPES

- COMPLETE GRAPH-
  - The complete graph of *n vertices denoted by K_n is the simple graphs,contains exactly one edge between each pair of distinct vertices.*

- *NULL GRAPH-*
  - *A graph which contains only isolated vertices is called a null graph; i.e. the edge set in a null graph is empty and is denoted by N_n.*

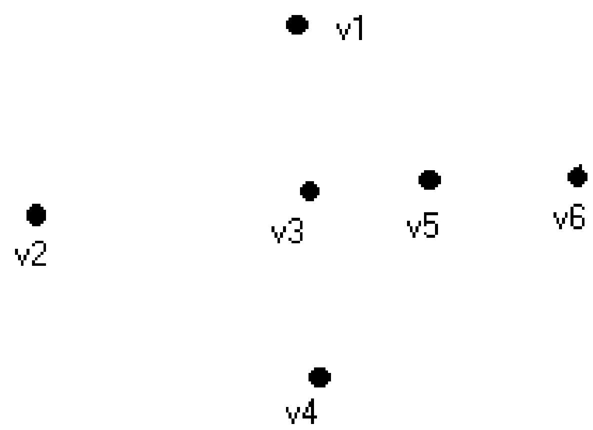
- *REGULAR GRAPH-*
  - *A graph in which all vertices are of equal degree is called a regular graph*

  If the degree of each vertex is r, then the graph is called a regular graph of degee r.
  The complete graph K is regular graph of degree n-1..

- *CYCLE GRAPH-*
  - *The cycle Cn of length n, n>=3, consists of n vertices $V_1, V_2, \ldots, vn$ and edges $(V_1, V_2), (V_2, V_3), \ldots, (V_{n-1}, V_n)$ and (V_n, V_1).*
- *BIPARTITE GRAPH-*
  - *A simple graph G is called bipartite if its vertex set V can be partitioned into two disjoint sets V_1 and V_2 such that every edge in the graph, connects a vertex in V_1 and a vertex in V_2 so that no edge in G connects either two vertices in V1 or two vertices in V_2. When this condition holds we call the pair(V_1, V_2), a bipartition of the vertex set V of G.*
- *WHEEL GRAPH-*
  - *A wheel graph W_n is a graph formed by connecting a single universal vertex to all of vertices of a cycle.*

EMPTY GRAPH / EDGELESS GRAPH

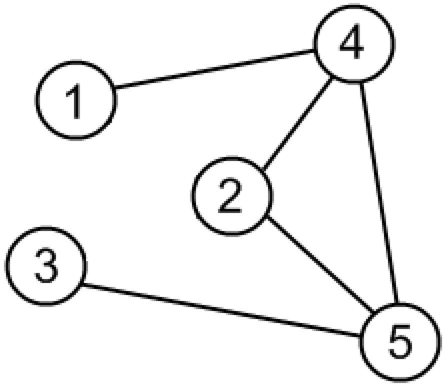
- NO EDGE



- Null graph
  - No nodes
  - Obviously no edge

SIMPLE GRAPH (UNDIRECTED)

- Simple Graph are undirected graphs without loop or multiple edges
- A = AT



For simple graph, $\sum_{vi = ev} \deg(v_i) - 2\,|E|$

MULTIGRAPH AND HYPER GRAPH

If there are possible multiple edges or arcs between the same
pair of vertices we call it a multi-graph.

Notice: in a directed graph (v,w) is a different arc than (w, v)
for v 6= w.

PICTURE OF GRAPH

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A given graph can be depicted on a plane (or other 2-dimensional surface) in multiple ways (example).

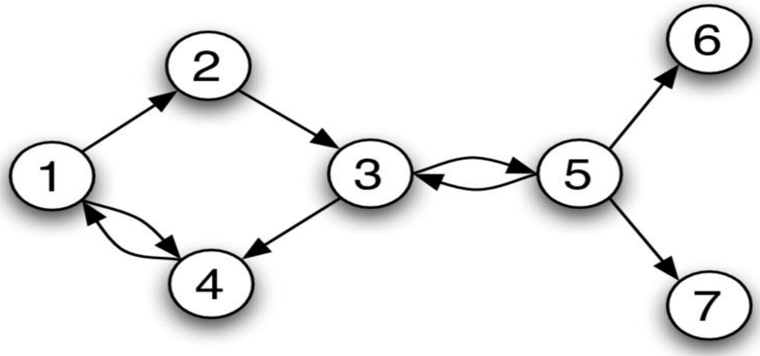
A picture is only a visual form of representation of a graph.

It is necessary to distinguish between an abstract (mathematical) concept of a graph and its picture (visualrepresentation)

DIRECTED GRAPH(DIAGRAPH)

Directed graph (digraph) is an ordered pair: G = (V, E), where:

- V is the set of vertices.
- E is the set of edges(or arc set)
- each edge e = (v,w) in E is an ordered pair of vertices from V, called the tail and head end of the edge,respectively.



DEGREE OF VERTEX

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Degree of a vertex v denoted as deg(v) is the number of edges(or arcs) incident on this vertex.

(note: we assume that each self-loop (v, v) contributes 2 to the degree of the vertex v)

If deg(v) = 0 we call it an isolated vertex.

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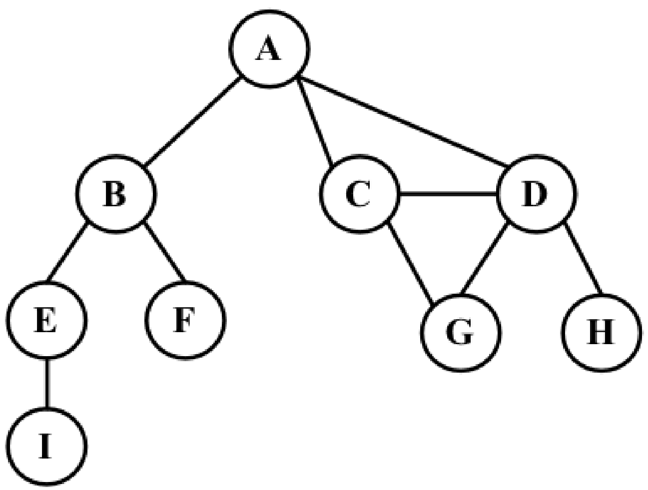
REPRESENTING THE GRAPHS

- *ADJACENCY MATRIX-*
  - *n-by-n matrix with Auv = 1 if (u, v) is an edge.*
  - *Diagonal Entries are self-links or loops.*
  - *Symmetric matrix for undirected graphs.*



	A	B	C	D	E	F	G	H	I
A	0	1	1	1	0	0	0	0	0
B	1	0	0	0	1	1	0	0	0
C	1	0	0	1	0	0	1	0	0
D	1	0	1	0	0	0	1	1	0
E	0	1	0	0	0	0	0	0	1
F	0	1	0	0	0	0	0	0	0
G	0	0	1	1	0	0	0	0	0
H	0	0	0	1	0	0	0	0	0
I	0	0	0	0	1	0	0	0	0

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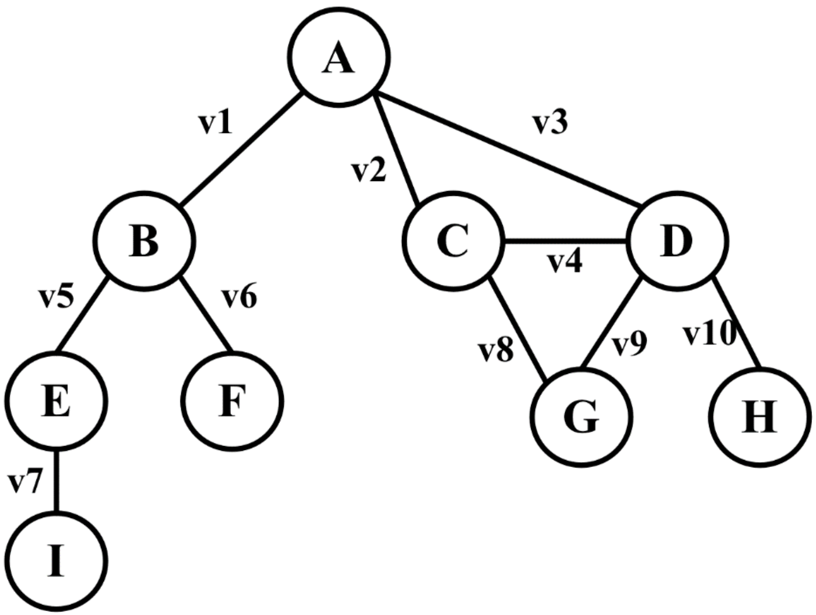
INCIDENCE MATRIX

- V*E
- [Vertex,edges] contains the edge’s data.



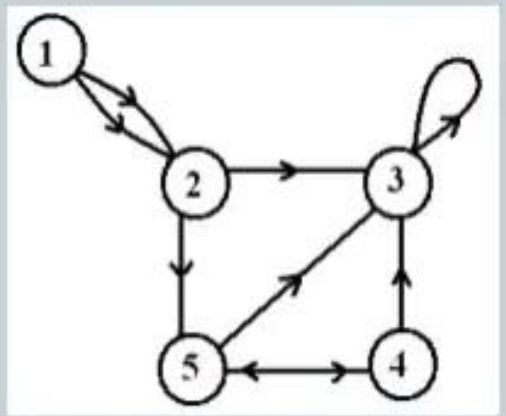
	v1	v2	v3	v4	v5	v6	v7	v8	v9	v10
A	1	1	1	0	0	0	0	0	0	0
B	1	0	0	0	1	1	0	0	0	0
C	0	1	0	1	0	0	0	1	0	0
D	0	0	1	1	0	0	0	0	1	1
E	0	0	0	0	1	0	1	0	0	0
F	0	0	0	0	0	1	0	0	0	0
G	0	0	0	0	0	0	0	1	1	0
H	0	0	0	0	0	0	0	0	0	1
I	0	0	0	0	0	0	1	0	0	0

ADJACENCY LIST

- Edge List

Edge List

1 2
1 2
2 3
2 5
3 3
4 3
4 5
5 3
5 4



- Adjacency List (node list)

Node List

1 2 2
2 3 5
3 3
4 3 5
5 3 4

CONCLUSIONS

Graph theory enables us to study and model networks and solve some difficult problems inherently capable of being modelled using networks.

Various terms e.g. vertex and edge, are associated with graphtheory which gives these terms special meanings. These meanings need to be understood and remembered in order to apply graph theoretic approaches to solving problems.

When solving a problem by developing a graph-basedprogram, careful attention must be given at the design stage tothe structuring of data to help make solving the problemtractable, to enable linkages to be traced efficiently and toavoid duplication of data.