GAIN MORE KNOWLEDGE
REACH GREATER HEIGHTS

PRESIDENCY COLLEGE
(Autonomous)

Reaccredited by NAAC with A+

Presidency Group

OVER 40 YEARS OF ACADEMIC WISDOM

DISCRETE MATHEMATICS

GRAPH THEORY
(HAMILTON'S GRAPH)

PRESIDENCY COLLEGE
(AUTONOMOUS)

AFFILIATED TO BENGALURU CITY UNIVERSITY, APPROVED BY AICTE, DELHI & RECOGNISED BY THE GOVT. OF KARNATAKA

RE-ACCREDITED BY NAAC WITH 'A+' GRADE

GAIN MORE KNOWLEDGE
REACH GREATER HEIGHTS

TEAM-I

GAIN MORE KNOWLEDGE
REACH GREATER HEIGHTS

Presidency College
(Autonomous)

Reaccredited by NAAC with A+

Presidency Group

OVER 40 YEARS OF ACADEMIC WISDOM

- G. Madhav Sharma(team leader)
- Harini.S
- Gursimar Singh Nain
- Sameer Patel
- Jhalak.J

GAIN MORE KNOWLEDGE
REACH GREATER HEIGHTS

Presidency College
(Autonomous)

Reaccredited by NAAC with A+

Presidency Group

OVER 40 YEARS OF ACADEMIC WISDOM

List of topics under Hamilton’s graph

- About sir William rowan Hamilton
- Origin of Hamiltonian graph
- Hamiltonian’s Path
- Hamiltonian’s circuits
- Application of Hamilton's circuit
- Dirac’s theorem
- Ore’s theorem
- Bibliography

ABOUT SIR WILLIAM ROWAN HAMILTON

Presidency College (Autonomous)

GAIN MORE KNOWLEDGE REACH GREATER HEIGHTS

- Born- 4 august 1805 in Dublin, Ireland
- Nationality-Irish
- Died- 2 September 1865 in Dublin, Ireland
- Known for-Hamiltonian mechanics,Cayley-Hamiton theorem and more.

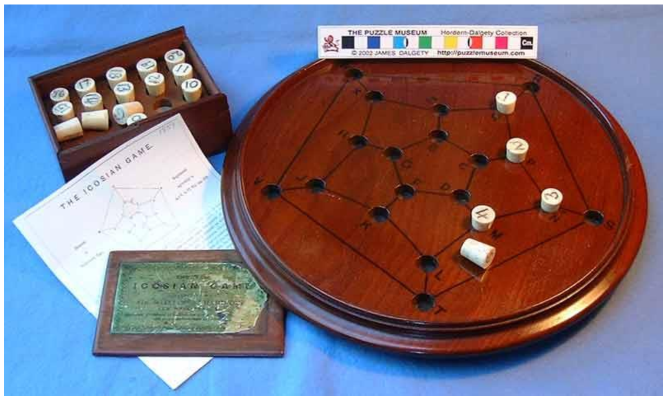
Reaccredited by NAAC with A+

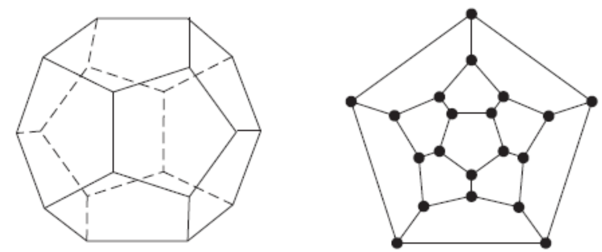
Presidency Group

OVER 40 YEARS OF ACADEMIC WISDOM

Origin of Hamiltonian’s graph

- The Hamilton was derived from a game, called the *Icosian puzzle*, invented in 1857.
- By the Irish mathematician Sir William Rowan Hamilton



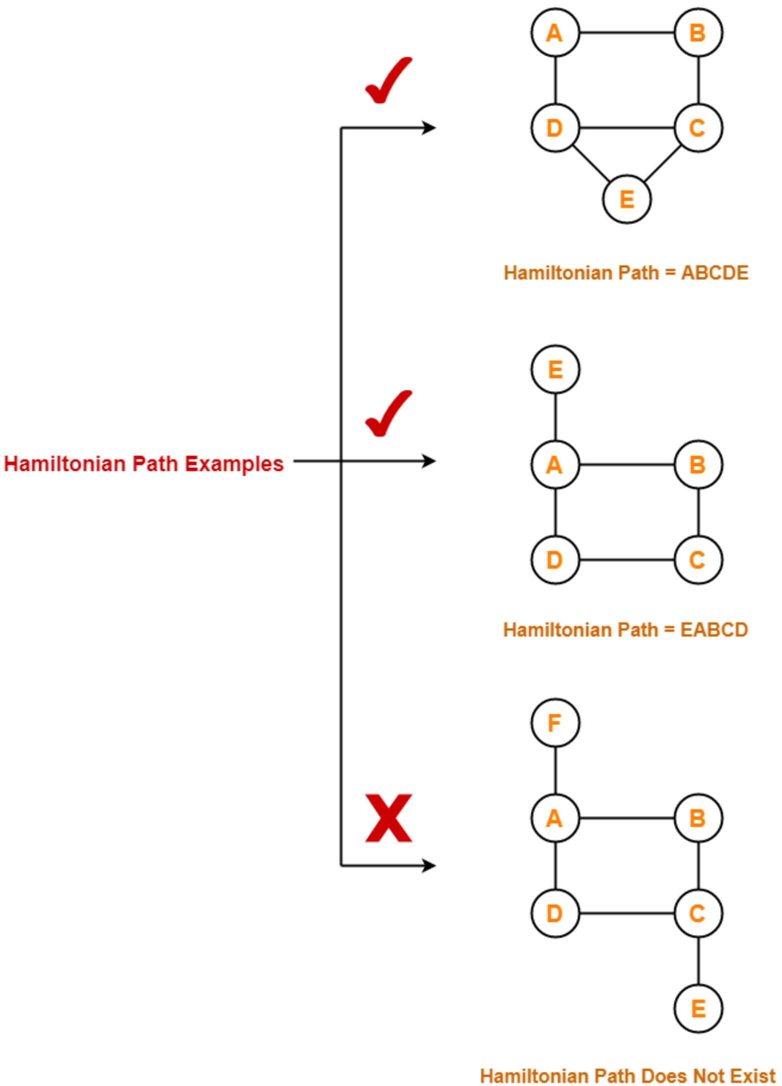


Hamilton’s “A Voyage Round the World” Puzzle.

- It consisted of a wooden dodecahedron (polyhedron with 12 regular pentagons as faces) with a Peg at each vertex of the dodecahedron, and string.
- The 20 vertices of the dodecahedron were labeled with different cities in the world.
- The object of the puzzle was to start at a city and travel along the edges of the dodecahedron, visiting each of the other 19 cities exactly once, and end back at the first city.
- The circuit traveled was marked off using the string and Peg.

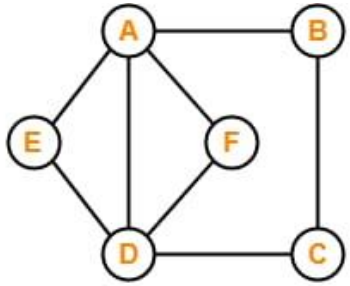
Hamiltonian's path

- Hamiltonian path: It is a path in an directed or undirected graph that visits each vertex of the graph exactly once, without repeating the vertex. Then such a graph is called as Hamiltonian path.
- Example:

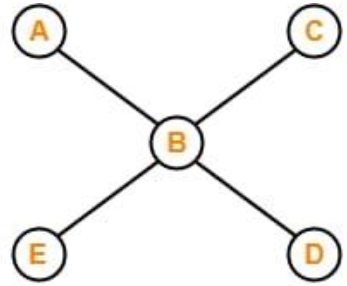


Problems related to Hamiltonian path

A)

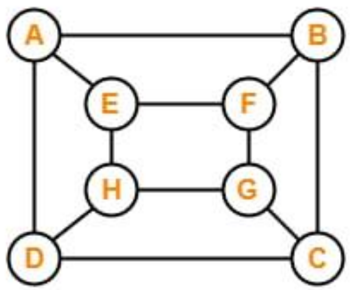


B)

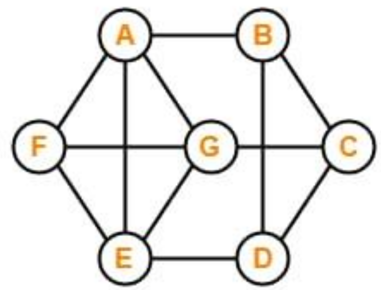


A-no
B-no

C)

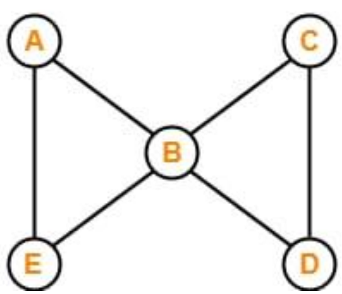


D)

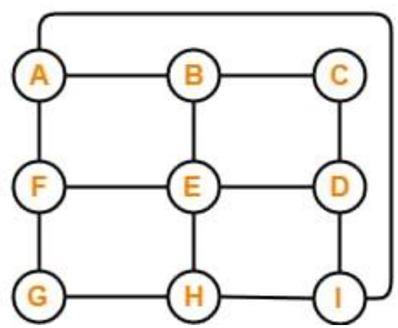


C-yes
D-yes

E)



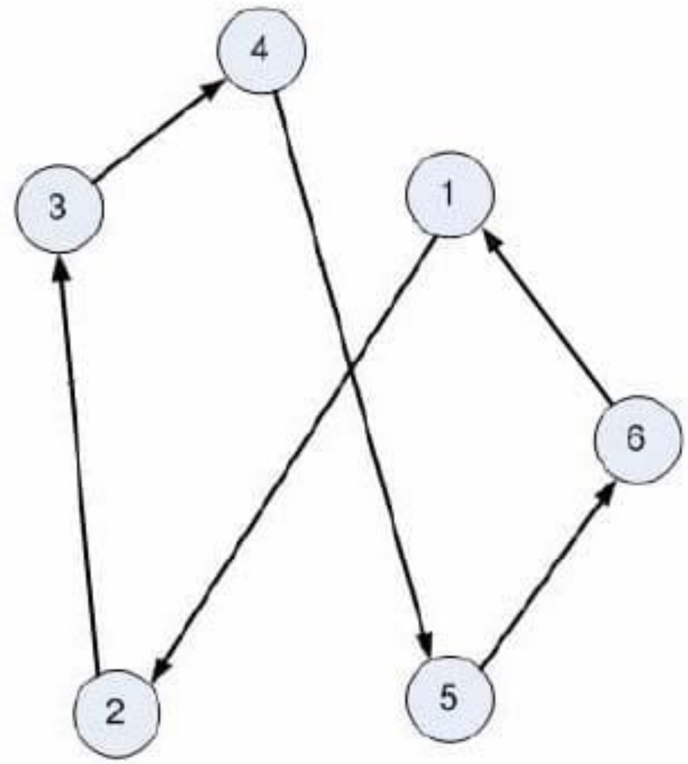
F)



E-yes
F-yes

Hamilton’s circuit

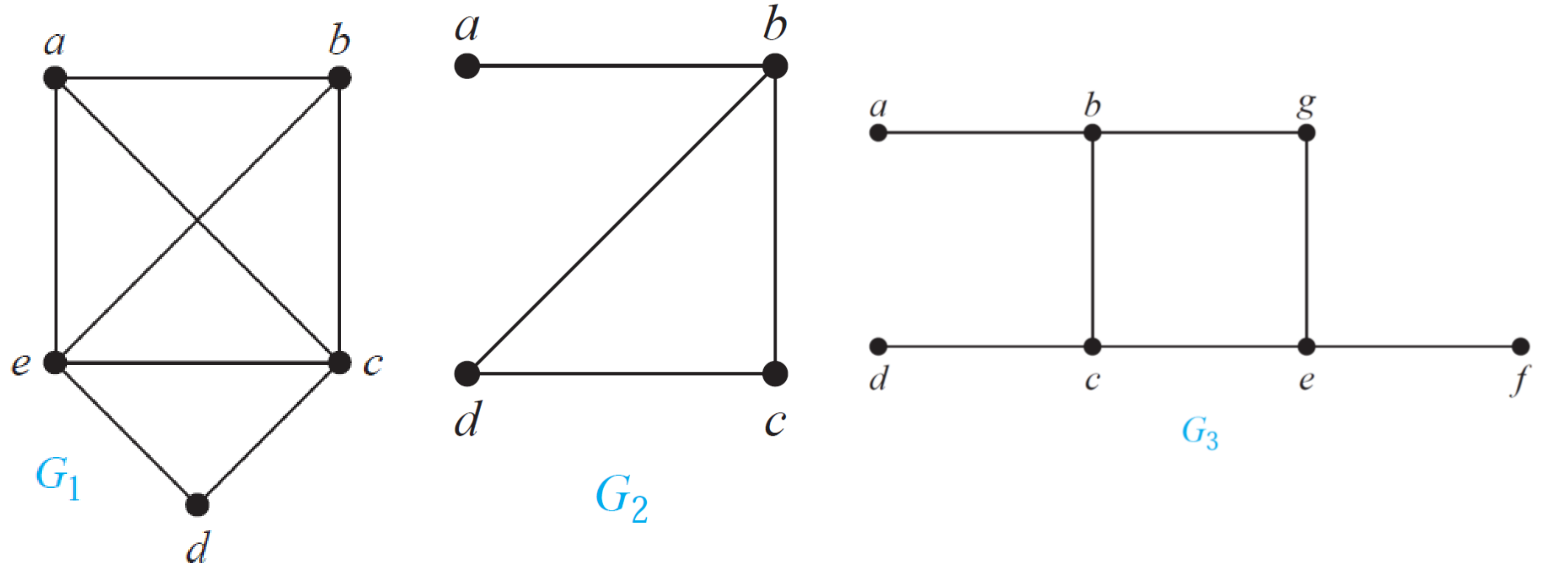
- Hamilton’s circuit: **A graph in which the path begins and ends on the same vertex** (a closed loop) such that each vertex is visited exactly once is known as a Hamiltonian circuit.
- Example:



Important notes

- 1. Any Hamiltonian circuit can be converted to a Hamiltonian path by removing one of its edges.
- 2.Every graph that contains a Hamiltonian circuit also contains a Hamiltonian path but vice versa is not true.
- 3.There may exists more than once Hamiltonian paths and Hamiltonian circuits in a graph.

1-Problems on Hamiltonian circuit

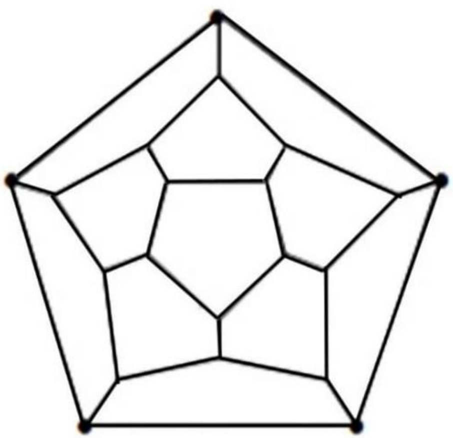


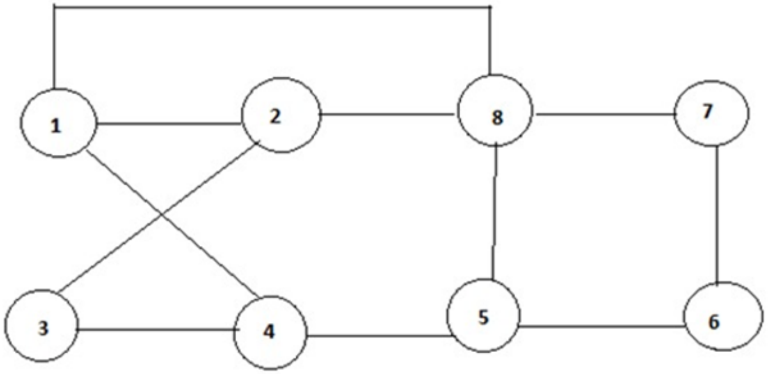
Which of the simple graph Hamiltonian circuits or if not Hamilton path?

Solution

- G1 has a Hamilton circuit because it starts (a) and ends (a) at the same vertex and visits every vertex exactly once.
- G2 does not have a Hamilton circuit because it does not start and end at the same vertex but it is a Hamilton path because every vertex is visited exactly once.
- G3 is neither Hamilton circuit nor Hamilton path.

2-Problems on Hamiltonian circuit



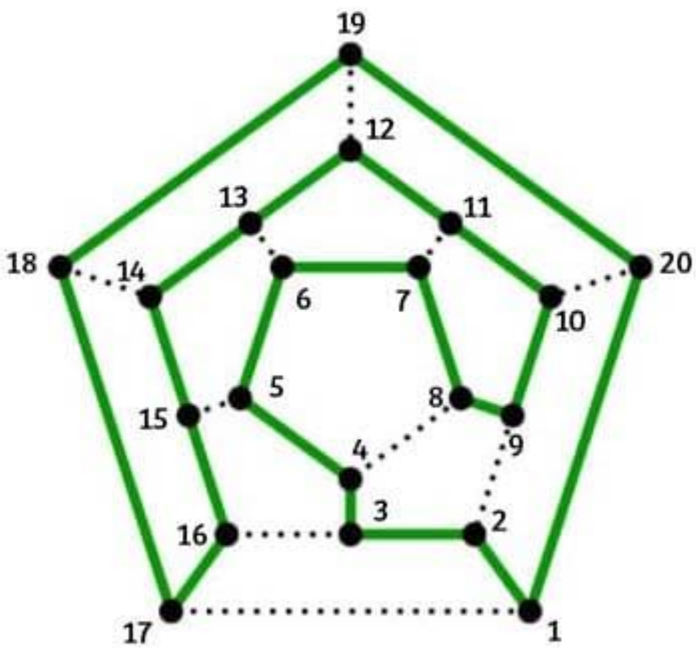


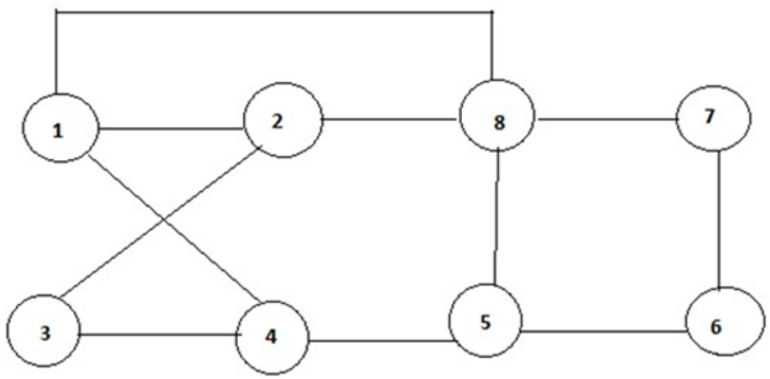
Are these Hamiltonian circuit?

GAIN MORE KNOWLEDGE
REACH GREATER HEIGHTS

Presidency College
(Autonomous)

Reaccredited by NAAC with A+





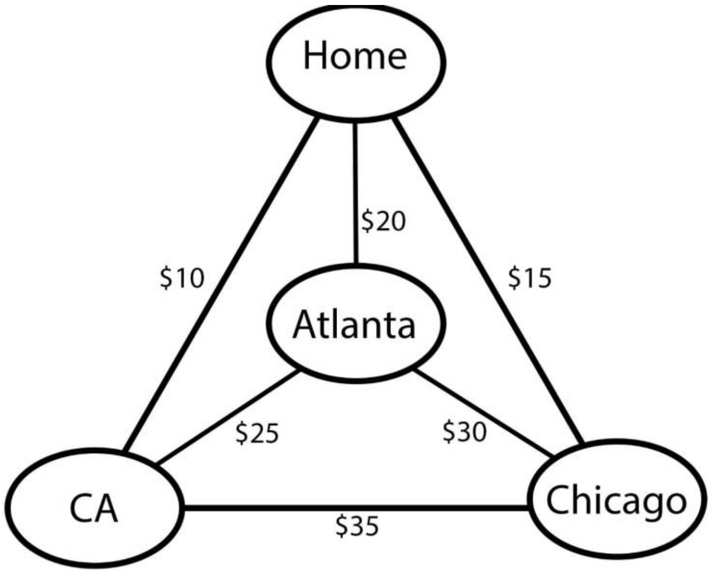
Answer: The above graphs are Hamiltonian circuits as the paths begin and end on the same vertex such that each vertex is visited exactly once.

Application of Hamilton's circuit

Presidency College (Autonomous)

Reaccredited by NAAC with A+

EXAMPLE1:



- Suppose a salesperson needs to give sales pitches in four cities. He looks up the airfares between each city and puts the costs in a graph. In what order should he travel to visit each city once then return home with the lowest cost? A situation like this could be represented with the graph shown at left.

Presidency Group

OVER 40 YEARS OF ACADEMIC WISDOM

Solution

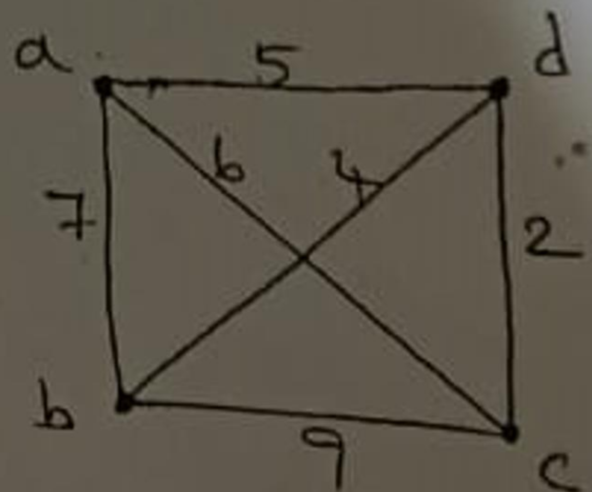
- Hamiltonian circuits path-

Home→CA→Atlanta→Chicago→Home

- The lowest cost of the tour is 10+25+30+15=80

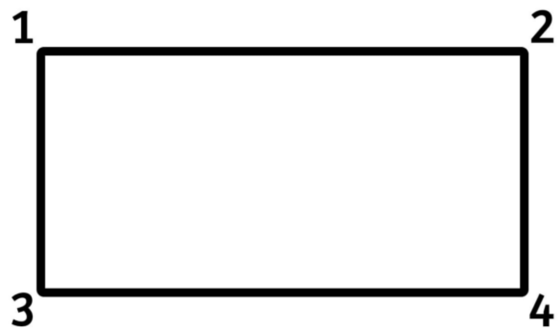
Problems

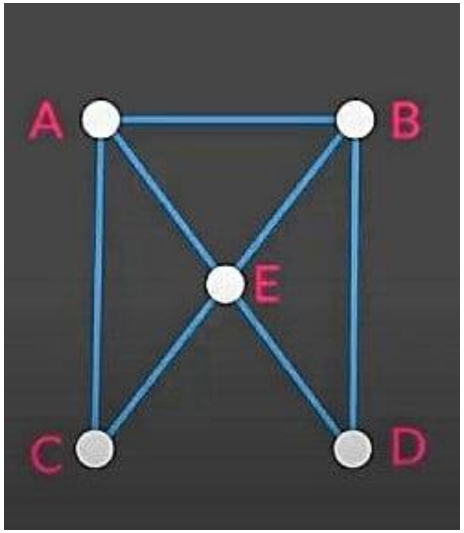
Find three distinct Hamiltonian cycles in the following graph. Also find their weight



Dirac's theorem

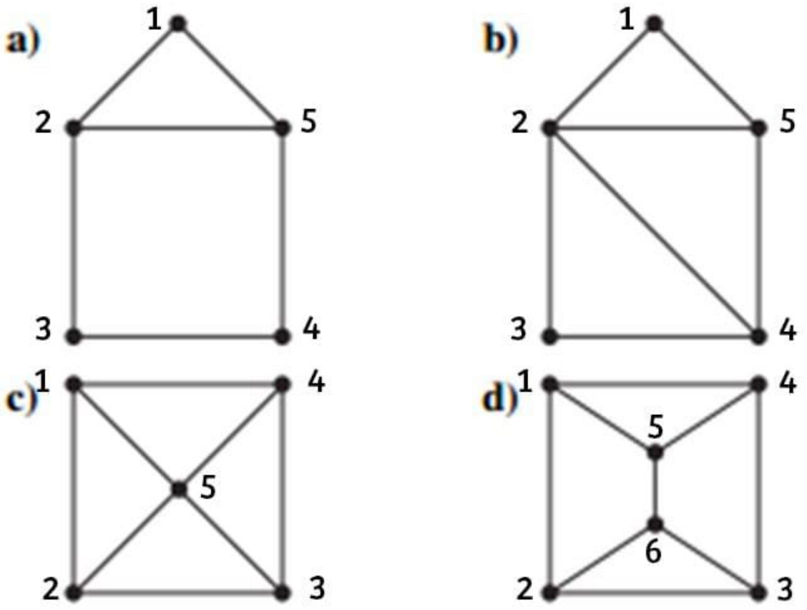
- Definition: If G is a simple graph with n vertices with $n \geq 3$ such that the degree of every vertex in G is at least $n/2$, then G has a Hamilton circuit.
- Example1:





Ore's Theorem

- Definition:If G is a simple graph with n vertices with n ≥ 3 such that degree(u) + degree(v) ≥ n for every pair of nonadjacent vertices u and v in G, then G has a Hamilton circuit.
- Example:



Presidency College (Autonomous)

GAIN MORE KNOWLEDGE REACH GREATER HEIGHTS

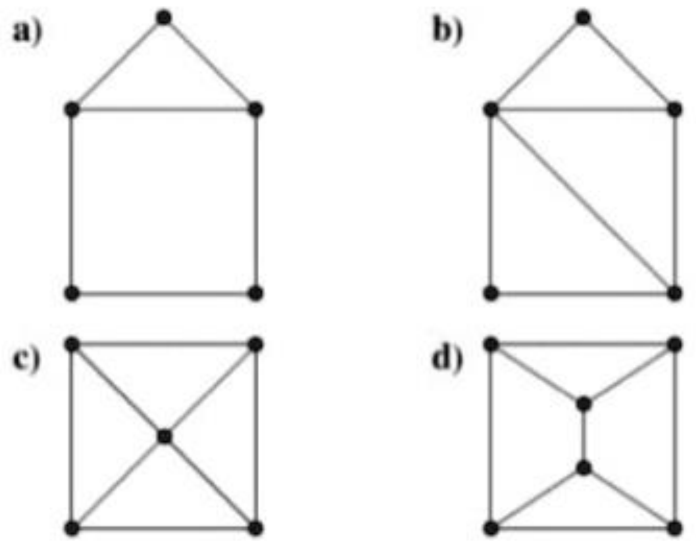
Reaccredited by NAAC with A+

Presidency Group

OVER 40 YEARS OF ACADEMIC WISDOM

PROBLEM’S

1. For each of these graphs, determine (i) whether Dirac's theorem can be used to show that the graph has a Hamilton circuit, (ii) whether Ore's theorem can be used to show that the graph has a Hamilton circuit, and (iii) whether the graph has a Hamilton circuit.



Presidency College (Autonomous)

Reaccredited by NAAC with A+

Presidency Group

Bibliography

- Text books

1. DISCRETE MATHEMATICS AND ITS APPLICATION 7TH EDITION-KH ROSEN
2. GRAPH THEORY AND ITS APPLICATIONS-BY J.A.Bondy and U.S.R.MURTY
3. GRAPH THEORY-ROBIN J.WILSON

GAIN MORE KNOWLEDGE
REACH GREATER HEIGHTS

Presidency College
(Autonomous)

Reaccredited by NAAC with A+

Presidency Group

OVER 40 YEARS OF ACADEMIC WISDOM

THANK YOU