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OVER 40 YEARS OF ACADEMIC WISDOM

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DISCRETE STRUCTURE

GRAPH THEORY

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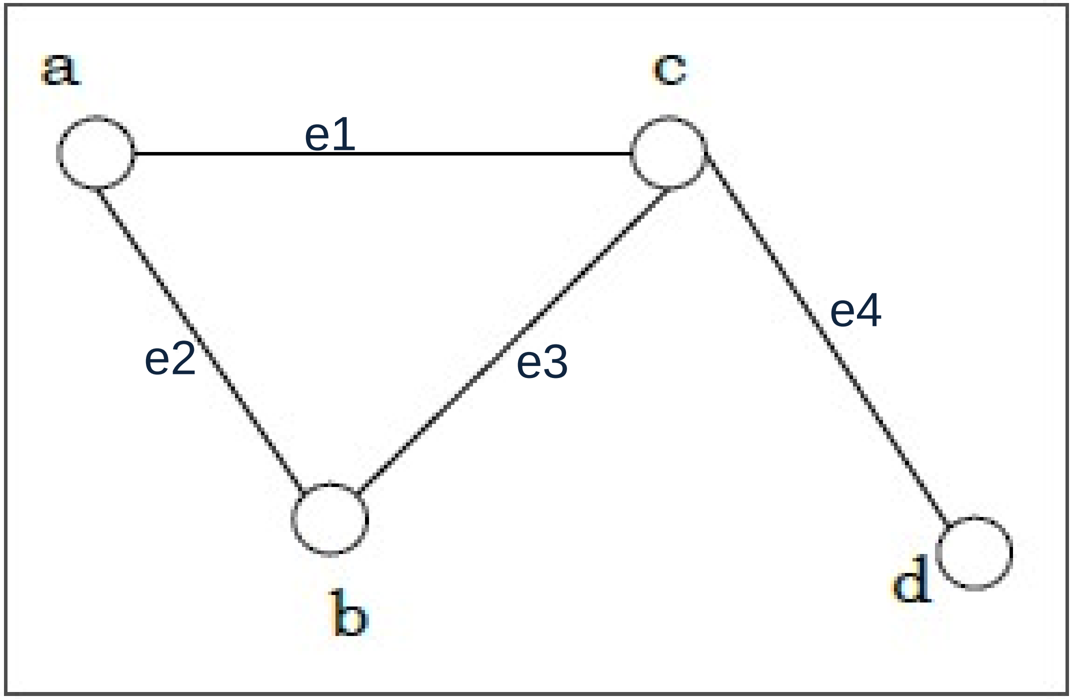
INTRODUCTION TO GRAPH THEORY

WHAT IS A GRAPH?

A graph is a pictorial representation of a set of objects where some pairs of objects are connected by links. The interconnected objects are represented by points termed as **vertices**, and the links that connect the vertices are called ***edges***.

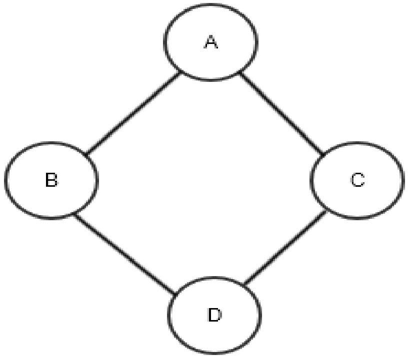
Take a look at the following graph:



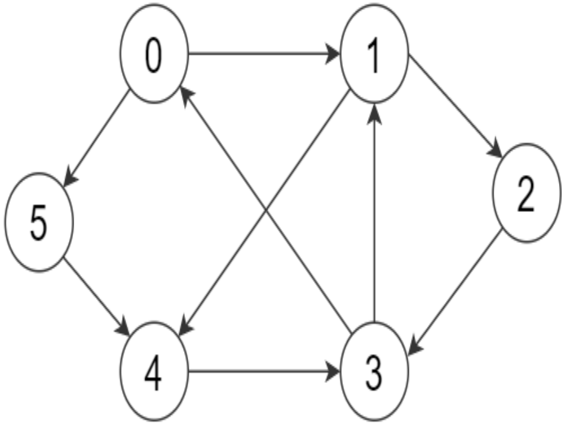
In the above graph,
a, b, c, d - vertices
e1, e2, e3, e4 - edges

Types Of Graphs

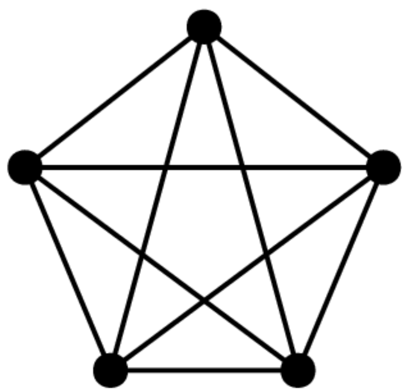
- UNDIRECTED GRAPH



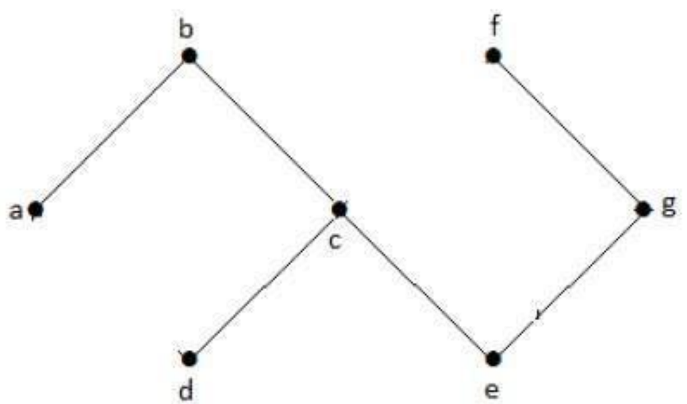
- DIRECTED GRAPH



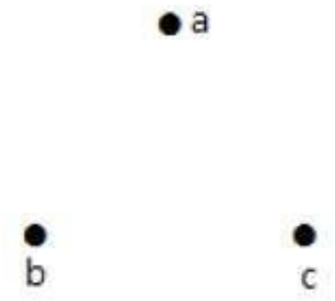
- COMPLETE GRAPH



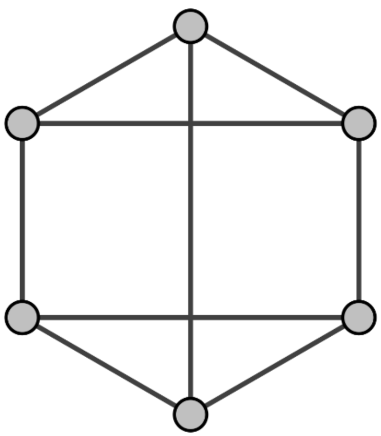
- INCOMPLETE GRAPH



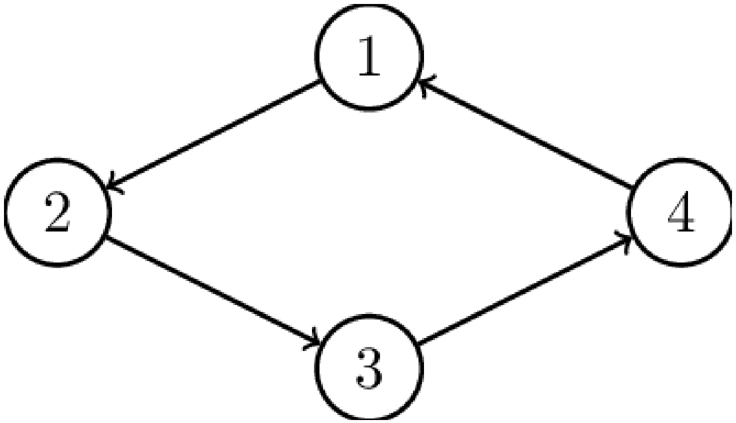
- NULL GRAPH



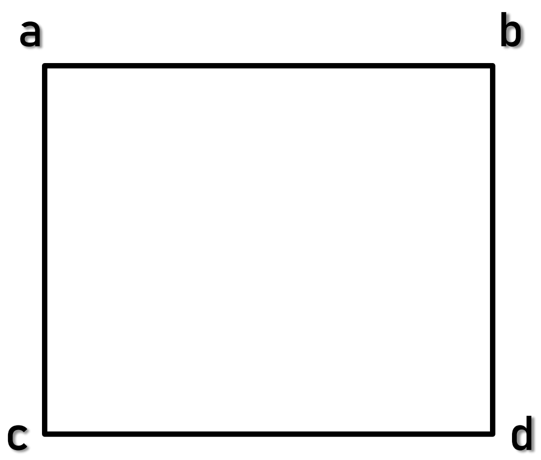
- REGULAR GRAPH



- CYCLIC GRAPH



- SIMPLE GRAPH



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Applications of Graph Theory

Graph Theory has its applications in diverse field of engineering:

- Electrical Engineering

The concepts of graph theory is used extensively in designing circuit connections.

- Computer Science

Graph theory is used for the study of algorithms.

❑ Computer Network

The relationships among interconnected computers in the network follows the principles of graph theory.

❑ Science

The molecular structure and chemical structure of a substance, the DNA structure of an organism, etc., are represented by graphs.

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DISCRETE STRUTCTURE

PLANAR GRAPHS

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INTRODUCTION

What is a planar graph?

A planar graph is a finite set of simple closed arcs, called edges, in the 2- sphere such that any point of intersection of two distinct members of the set is an end of both of them The vertices of a planar graph are the ends of its edges. Clearly any subset of a planar graph is a planar graph.

- In accordance with this definition we say that two planar graphs meet if they have a common edge and are disjoint if they have no common edge. Two disjoint planar graphs may have one or more common vertices.

- The planarity of a graph its ability to be embedded in a plane is a deceptively meaningful property which, through various theorems, can tell us many other things about a graph. Among other properties, planar graphs were famously found to be 4-colorable. Of course, to use such theorems to determine whether a graph has these properties, we must first determine whether that graph is planar.

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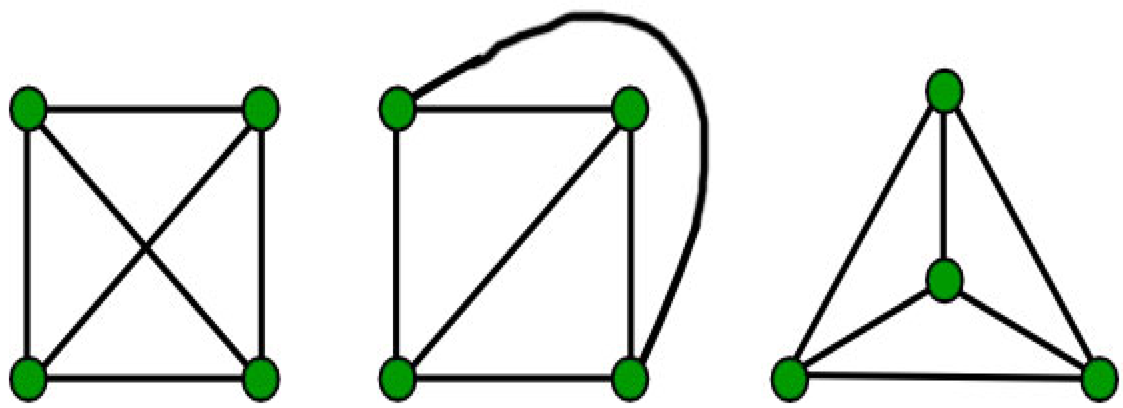
it can be drawn on the plane in such a way that its edges intersect only at their endpoints. In other words, it can be drawn in such a way that no edges cross each other.

Take a look at the following graph:



A graph G is said to be **planar** if there exists some geometric representation of G which can be drawn on a plane such that no two of its edges intersect.

A graph that cannot be drawn on a plane without a crossover between its edges is called **nonplanar**

A drawing of a geometric representation of a graph on any surface such that no edges intersect is called **Embedding.**

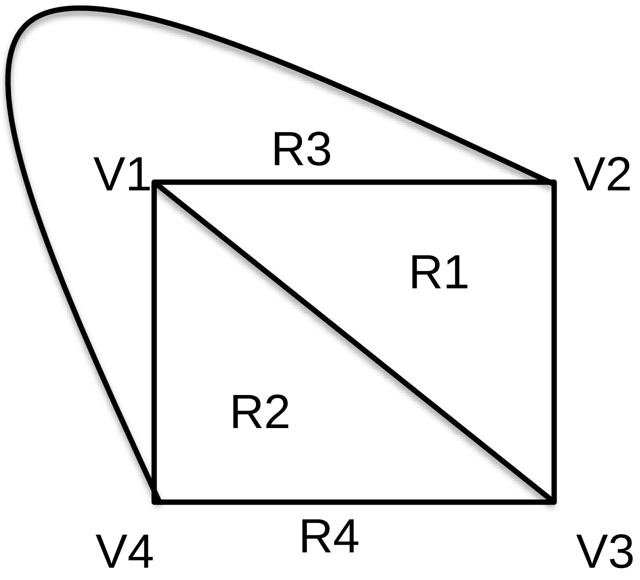
EULER'S FORMULA

Let, G be a planar graph
e be the edges
v be the vertices
r be the no of regions in planar graph

r= e-v+2

According to the figure given beside,
No of edges of the planar graph=6
And vertices=4

Using the above formula,(r=e-v+2)
r=6-4+2
r=4.
Therefore, the no of regions of the planar graph is 4 which is represented by R1, R2, R3, R4 in the figure.



KURATOWSKI'S TWO GRAPHS

Theorem 1 : Kuratowski's First Graph(k_5)

To prove: The complete graph of 5 vertices is non-planar.

Proof: Let the five vertices in the complete graph be named v1, v2, v3, v4, and v5.

We know a complete graph is a simple graph in which every vertex is joined to every other vertex by means of an edge.

This being the case, we must have a circuit going from v1 to v2 to v3 to v4 to v5.(fig. a)

Now let us connect the vertex v1 to v3 by means of an edge, this edge may be drawn inside or outside the graph(without intersecting the five edges drawn previously).

Suppose we choose to draw an edge connecting v1 to v2 from inside the graph.(fig. b)

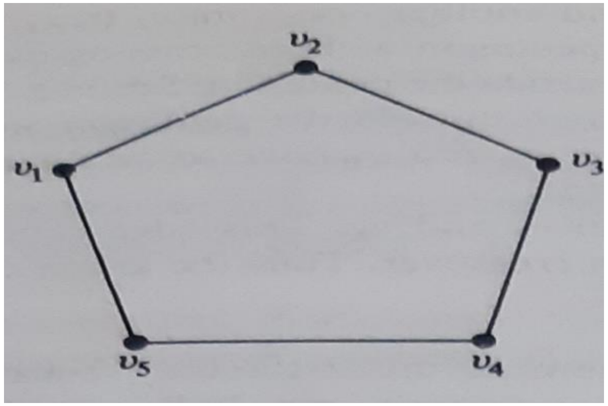


Fig. a

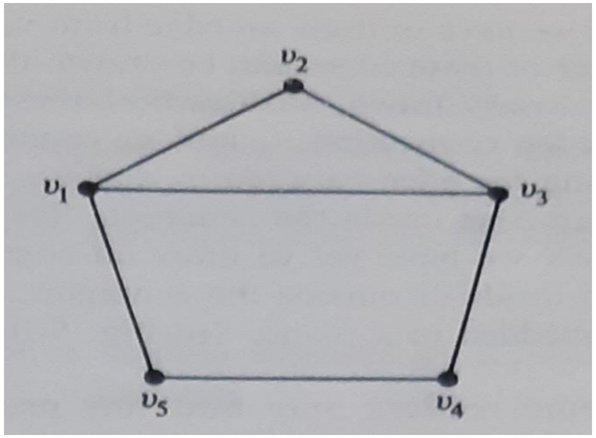


Fig. b

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Now we have to draw an edge from v2 to v4 and another one from v2 to v5. Since neither of these edges can be drawn inside the graph without crossing over the edge(e5) already drawn, we draw both of these edges(e6 and e7) outside the graph(fig. c).

Now, an edge between v3 and v5 cannot be drawn outside the graph without crossing the edge(e6).

Therefore, v3 and v5 have to be connected with an edge(e8) inside the graph.(fig. d).

Now we have yet to draw an edge between v1 and v4. This edge cannot be placed inside or outside the graph without intersecting other edges.

Thus this graph cannot be embedded in a plane which implies that graph is non-planar.

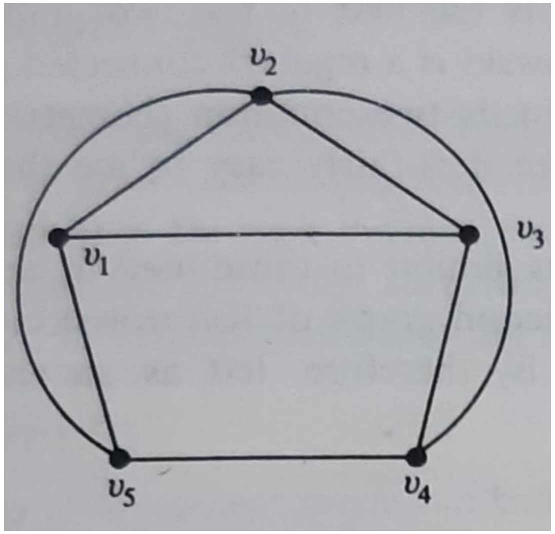

Fig. c

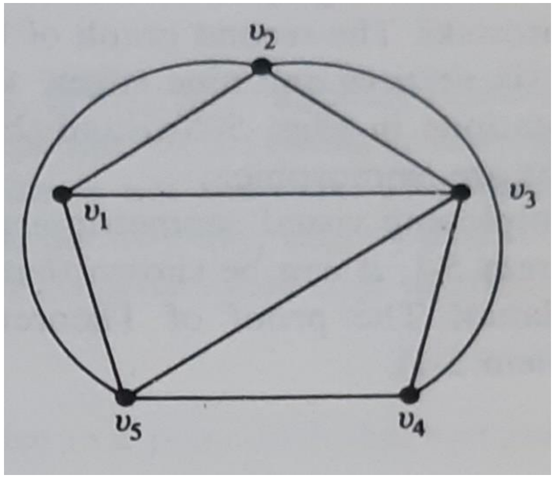

Fig. d

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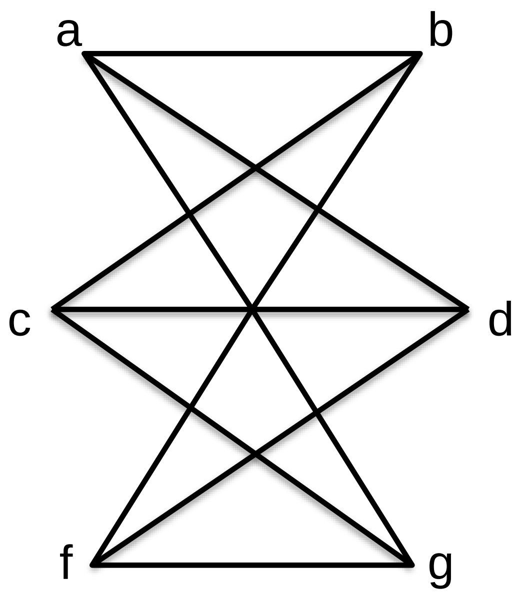
Theorem 2: Kuratowski's Second Graph($k_{3,3}$)

To prove: Kuratowski's second graph($k_{3,3}$) is also nonplanar.



Proof: On the given graph there are 6 vertices and 9 edges.

Assume that $k_{3,3}$ is planar by Euler's formula,

$r=e-v+2$

$r=9-6+2$

$r=5$

Now, every region is bounded by at least 4 edges

So,

$4r<=2e$

$4(5)<=2(9)$

$20<=18$

Since 20 is not smaller than 18,our assumption was false.

Hence, $k_{3,3}$ is non-planar graph.

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