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Discrete Structures Presentation

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Acknowledgement

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I would like to express my special thanks of gratitude to my teacher **MRS. SAVITA GOWDA**, who gave me the golden opportunity to do this wonderful project of **DISCRETE STRUCTURES**.

Who also helped me in completing my project. I come to know about so many new things I am really thankful to them.

Secondly I would like to thank my parents and friends who helped me a lot in finalizing this project within the limited time frame.

Divesh Mandhyan
Ankita Datta
Naniti Bhawna Reddy
Arun AN
Fayeez ul Zama
BCA 'A' 1 Semester

GRAPH THEORY

- Prim's Algorithm & Kruskal's Algorithm (Minimum Spanning Trees)
- Connectivity of trees
- Tournament

Minimum Spanning Trees

A minimum spanning tree in a connected weighted graph is a spanning tree that has the smallest possible sum of weights of its edges.

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Graph Theory

Minimum Spanning Trees
Prim's Algorithm

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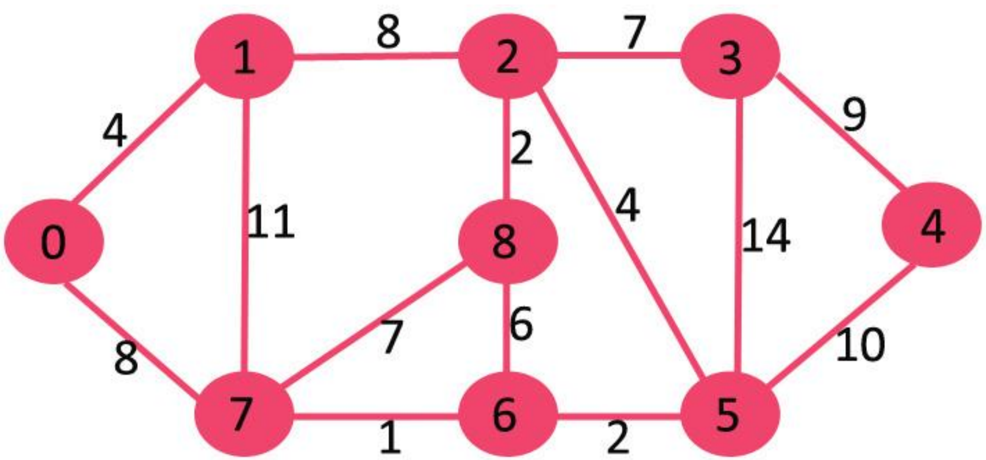
ROBERT CLAY PRIM

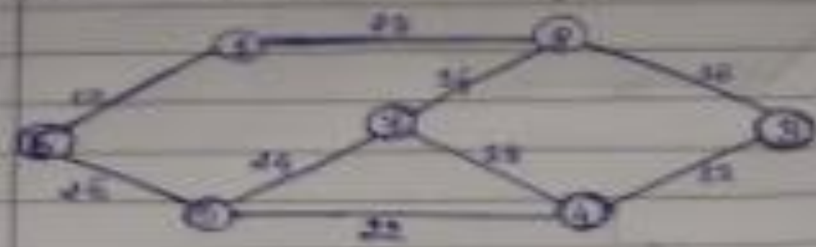
Robert Prim, born in Sweetwater, Texas, received his B.S. in electrical engineering in 1941 and his Ph.D. in mathematics from Princeton University in 1949. He was an engineer at the General Electric Company from 1941 until 1944, an engineer and mathematician at the United States Naval Ordnance Lab from 1944 until 1949, and a research associate at Princeton University from 1948 until 1949. Among the other positions he has held are director of mathematics and mechanics research at Bell Telephone Laboratories from 1958 until 1961 and vice president of research at Sandia Corporation. He is currently retired.

Prim’s Algorithm

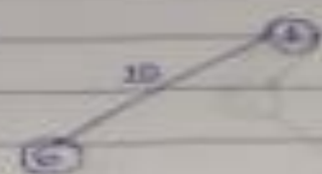
Prim's Algorithm is a greedy algorithm that is used to find the minimum spanning tree from a graph. Prim's algorithm finds the subset of edges that includes every vertex of the graph such that the sum of the weights of the edges can be minimized.



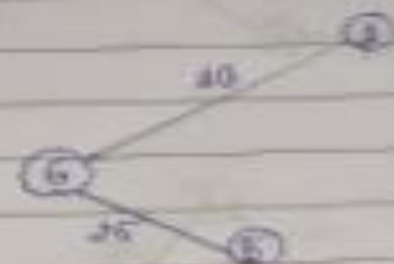
Q: Construct the minimum Spanning for the given graph using prim's algorithm :—



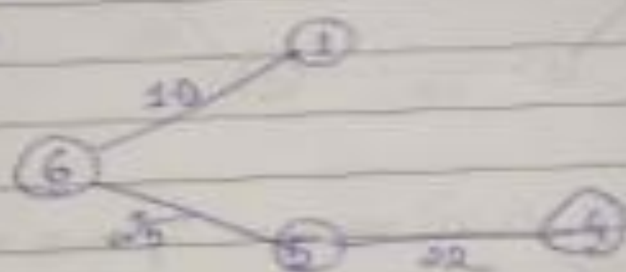
Step 1:—



Step 2:—



Step 3:—



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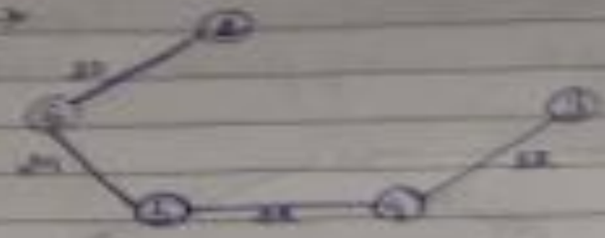
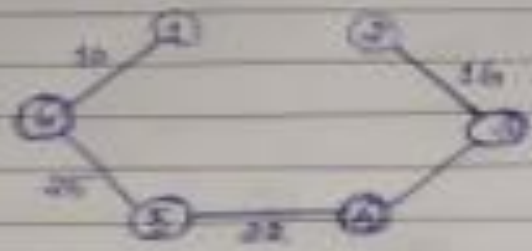
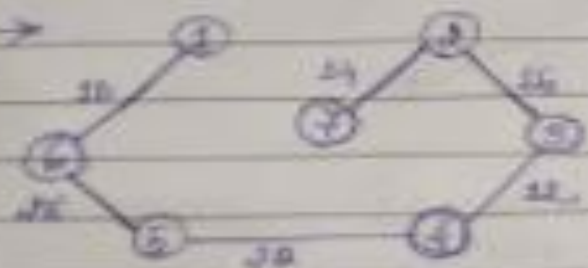
Step 4 →



Step 5 →



Step 6 →



Now, Cost of Minimum Spanning Tree

= Sum of all edges

= 10 + 25 + 22 + 12 + 16 + 14 = 99 units

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Graph Theory

Minimum Spanning Trees
Kruskal Algorithm

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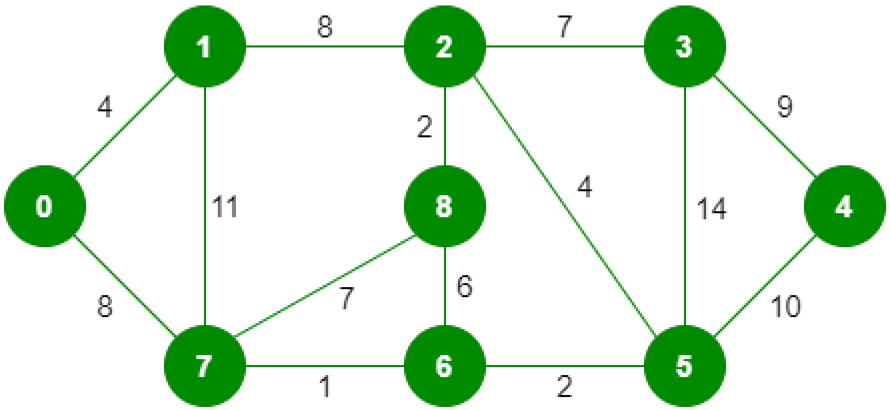
JOSEPH BERNARD KRUSKAL

Joseph Kruskal was born in New York City, where his father was a fur dealer and his mother promoted the art of origami on early television. Kruskal attended the University of Chicago and received his Ph.D from Princeton University in1954. He was an instructor in mathematics at Princeton and at the University of Wisconsin, and later he was an assistant professor at the University of Michigan. In 1959 he became a member of the technical staff at Bell Laboratories, where he worked until his retirement in the late 1990s. Kruskal discovered his algorithm for producing minimum spanning trees when he was a second-year graduate student. He was not sure his 2 1 2 -page paper on this subject was worthy of publication, but was convinced by others to submit it. His research interests included statistical linguistics and psychometrics. Besides his work on minimum spanning trees, Kruskal is also known for contributions to multidimensional scaling. It is noteworthy that Joseph Kruskal's two brothers, Martin and William, also were well known mathematicians.

Kruskal’s Algorthm

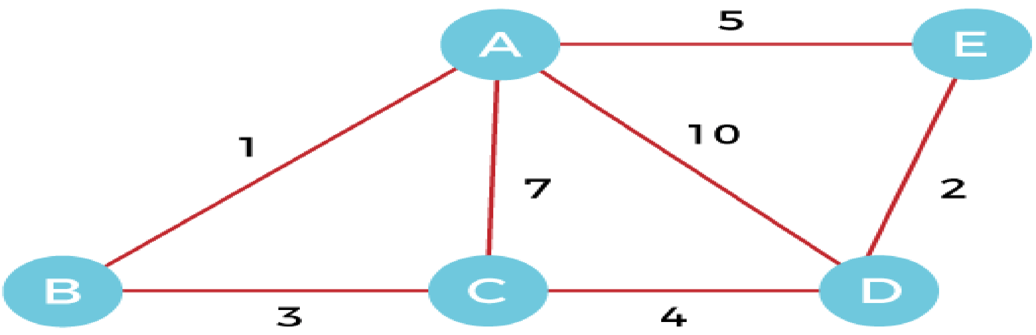


Kruskal's Algorithm is used to find the minimum spanning tree for a connected weighted graph. The main target of the algorithm is to find the subset of edges by using which we can traverse every vertex of the graph. It follows the greedy approach that finds an optimum solution at every stage instead of focusing on a global optimum.

Question of Example – 1

Now, let's see the working of Kruskal's algorithm using an example. It will be easier to understand Kruskal's algorithm using an example.

Suppose a weighted graph is -



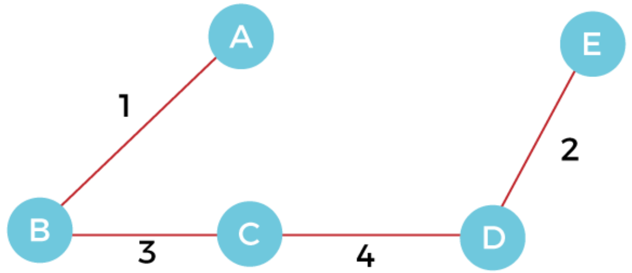
Answer of Example 1

The weight of the edges of the above graph is given in the below table –
Now, sort the edges given above in the ascending order of their weights.

Edge	AB	AC	AD	AE	BC	CD	DE
Weight	1	7	10	5	3	4	2

Now, sort the edges given above in the ascending order of their weights.

Edge	AB	DE	BC	CD	AE	AC	AD
Weight	1	2	3	4	5	7	10



The cost of the MST is = AB + DE + BC + CD = 1 + 2 + 3 + 4 = 10.

Now, the number of edges in the above tree equals the number of vertices minus 1. So, the algorithm stops here.

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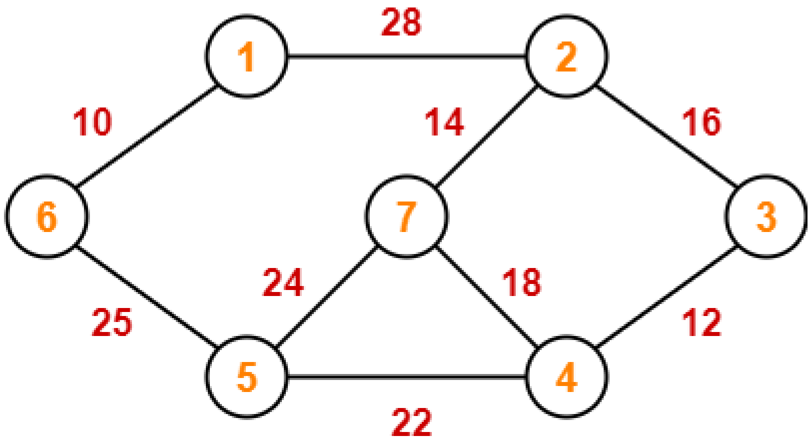
Question of Example – 2

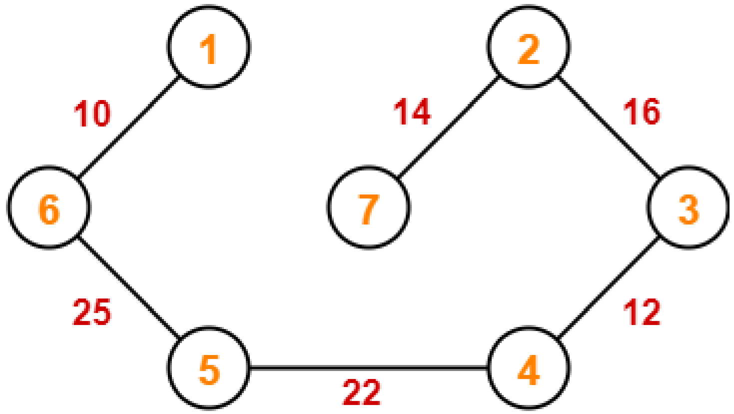
Construct the minimum spanning tree (MST) for the given graph using Kruskal’s Algorithm-



Solution of Example - 2

To construct MST using Kruskal's Algorithm,

- Simply draw all the vertices on the paper.
- Connect these vertices using edges with minimum weights such that no cycle gets formed.



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Graph Theory

Connectivity

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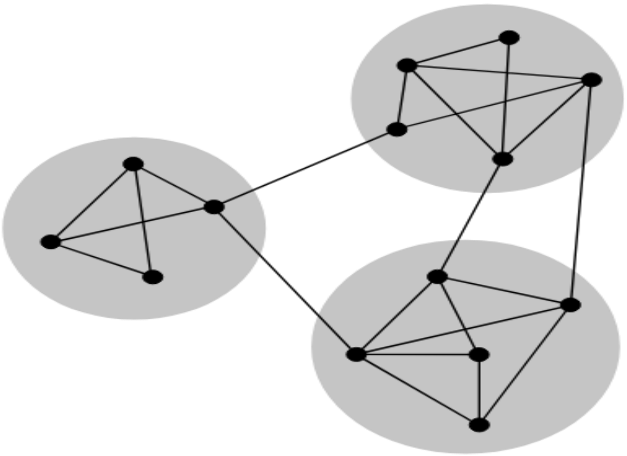
Connectivity

A graph is a connected graph if, for each pair of vertices, there exists at least one single path which joins them. A connected graph may demand a minimum number of edges or vertices which are required to be removed to separate the other vertices from one another. The graph connectivity is the measure of the robustness of the graph as a network.

In a connected graph, if any of the vertices are removed, the graph gets disconnected. Then the graph is called a **vertex-connected graph**. On the other hand, when an edge is removed, the graph becomes disconnected. It is known as an **edge-connected graph.**



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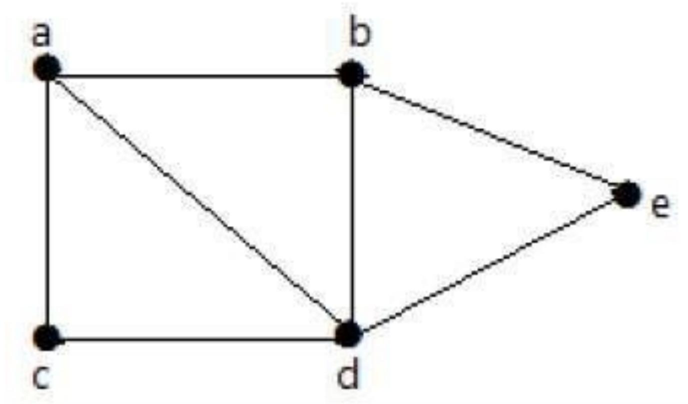
Properties of Connectivity

- The connected graph is called an undirected **graph,** which has at least one path between each pair of vertices.
- The graph that is connected by three vertices is called **1-vertex connected graph** since the removal of any of the vertices will lead to disconnection of the graph.
- If in a connected graph, the removal of one edge leads to the disconnection of the graph, such a graph is called **1-edge connected graph**.
- If there exists a set (say S) of edges (or vertices) in a connected graph, such that by removing all the edges of set S will result in a disconnected graph. Then the set S is called a **cut set**. If S consists of vertices, then it is called a **vertex-cut set**. Similarly, if it has edges, then it is called an **edge-cut set**.
- A bi-connected graph is a connected graph which has two vertices for which there are two disjoint paths between these two vertices.

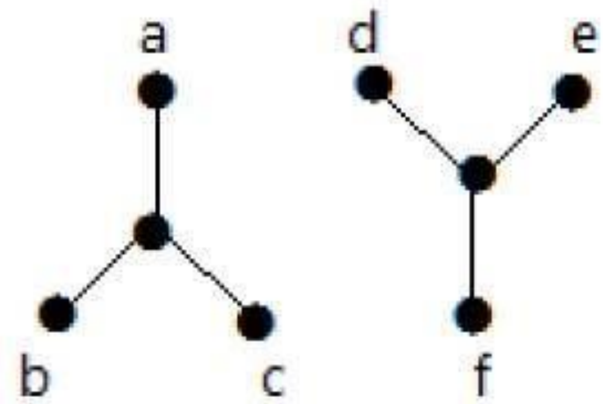
Example – 1 (Connectivity)

In the following graph, it is possible to travel from one vertex to any other vertex. For example, one can traverse from vertex ‘a’ to vertex ‘e’ using the path ‘a-b-e’



Example – 2 (Connectivity)

In the following example, traversing from vertex ‘a’ to vertex ‘f’ is not possible because there is no path between them directly or indirectly. Hence it is a disconnected graph.



Cut Vertex

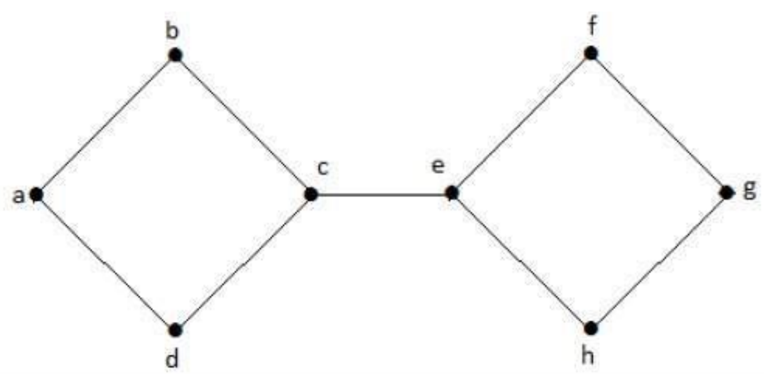
Let ‘G’ be a connected graph. A vertex V ∈ G is called a cut vertex of ‘G’, if ‘G-V’ (Delete ‘V’ from ‘G’) results in a disconnected graph. Removing a cut vertex from a graph breaks it in to two or more graphs.

Note – Removing a cut vertex may render a graph disconnected.

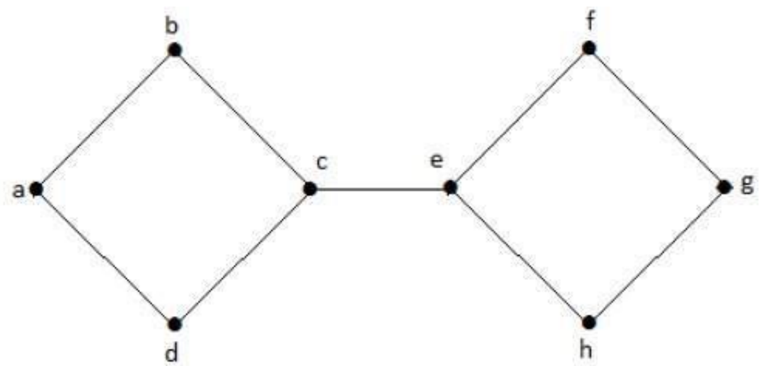
A connected graph ‘G’ may have at most (n–2) cut vertices.

Example

In the following graph, vertices ‘e’ and ‘c’ are the cut vertices.



By removing ‘e’ or ‘c’, the graph will become a disconnected graph.



Without ‘g’, there is no path between vertex ‘c’ and vertex ‘h’ and many other. Hence it is a disconnected graph with cut vertex as ‘e’. Similarly, ‘c’ is also a cut vertex for the above graph.

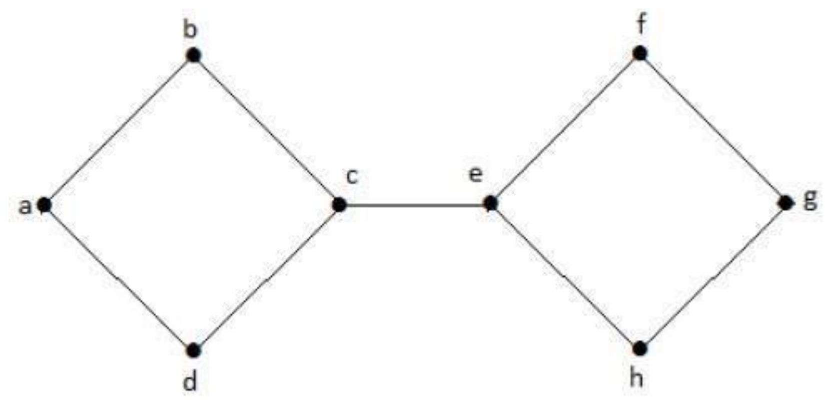
Cut Edge (Bridge)

Let ‘G’ be a connected graph. An edge ‘e’ ∈ G is called a cut edge if ‘G-e’ results in a disconnected graph.

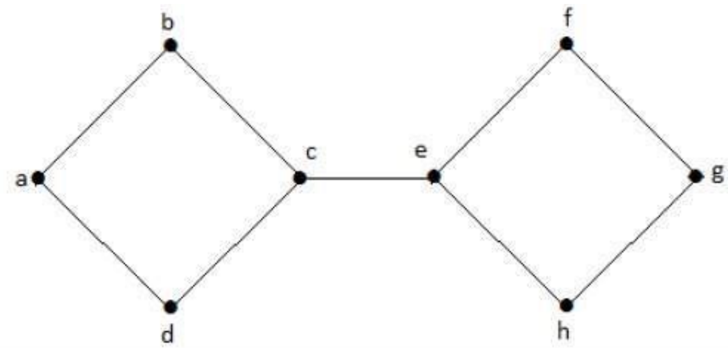
If removing an edge in a graph results in to two or more graphs, then that edge is called a Cut Edge.

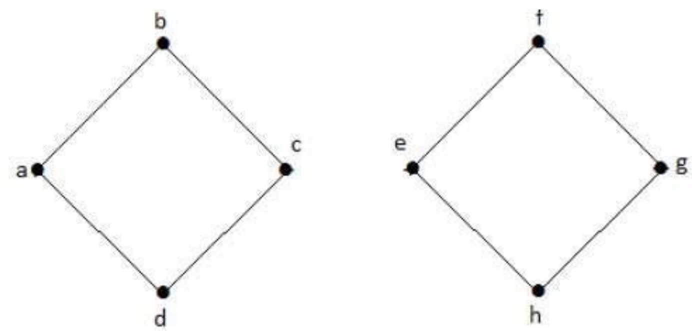
Example

In the following graph, the cut edge is [(c, e)].



By removing the edge (c, e) from the graph, it becomes a disconnected graph.





In the above graph, removing the edge (c, e) breaks the graph into two which is nothing but a disconnected graph. Hence, the edge (c, e) is a cut edge of the graph.

Note − Let 'G' be a connected graph with 'n' vertices, then

- a cut edge e ∈ G if and only if the edge 'e' is not a part of any cycle in G.
- the maximum number of cut edges possible is 'n-1'.
- whenever cut edges exist, cut vertices also exist because at least one vertex of a cut edge is a cut vertex.
- if a cut vertex exists, then a cut edge may or may not exist.

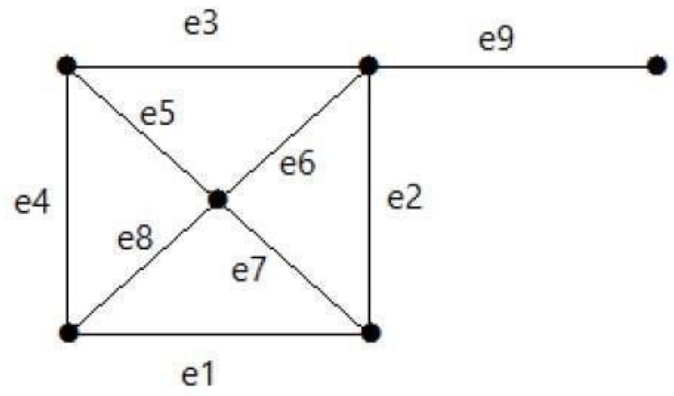
Cut Set of a Graph

Let 'G'= (V, E) be a connected graph. A subset E' of E is called a cut set of G if deletion of all the edges of E' from G makes G disconnect.

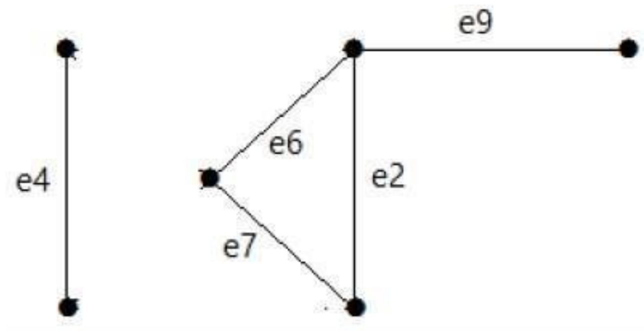
If deleting a certain number of edges from a graph makes it disconnected, then those deleted edges are called the cut set of the graph.

Example

Take a look at the following graph. Its cut set is E1 = {e1, e3, e5, e8}.



After removing the cut set E1 from the graph, it would appear as follows −



Similarly, there are other cut sets that can disconnect the graph −

- E3 = {e9} – Smallest cut set of the graph.
- E4 = {e3, e4, e5}

Edge Connectivity

Let ‘G’ be a connected graph. The minimum number of edges whose removal makes ‘G’ disconnected is called edge connectivity of G.

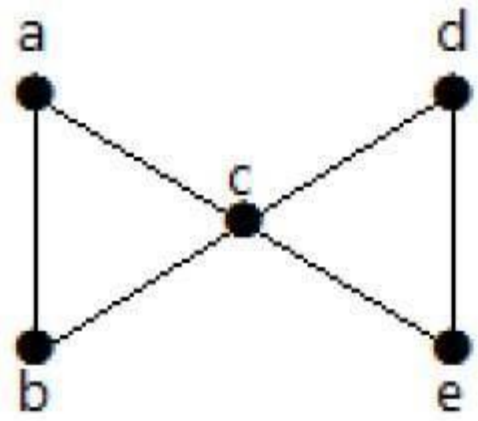
Notation − λ(G)

In other words, the **number of edges in a smallest cut set of G** is called the edge connectivity of G.

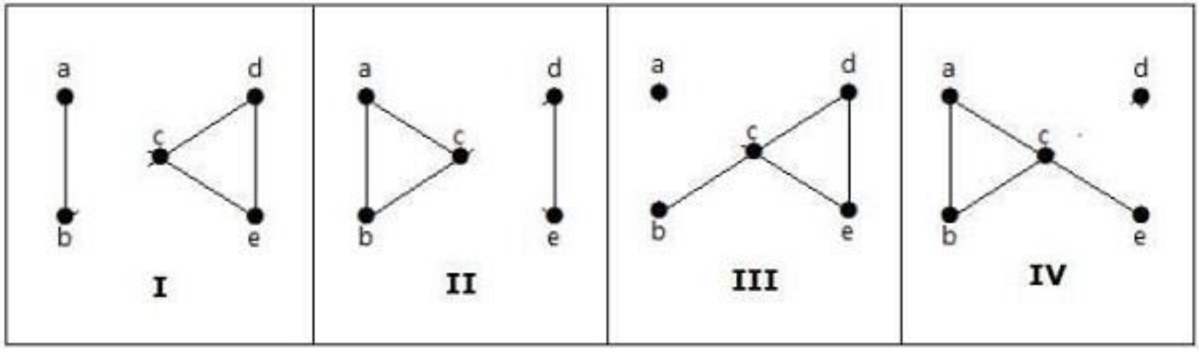
If ‘G’ has a cut edge, then λ(G) is 1. (edge connectivity of G.)

Example

Take a look at the following graph. By removing two minimum edges, the connected graph becomes disconnected. Hence, its edge connectivity (λ(G)) is 2.



Here are the four ways to disconnect the graph by removing two edges −



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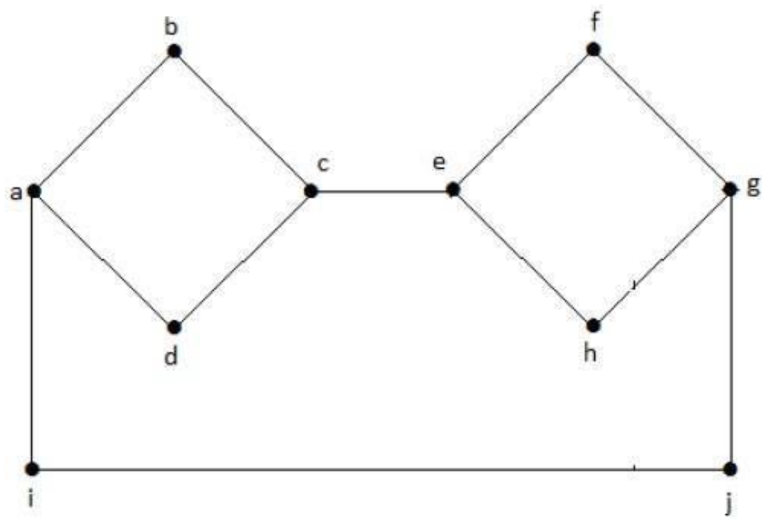
Vertex Connectivity

Let ‘G’ be a connected graph. The minimum number of vertices whose removal makes ‘G’ either disconnected or reduces ‘G’ in to a trivial graph is called its vertex connectivity.

Notation − K(G)

Example

In the above graph, removing the vertices ‘e’ and ‘i’ makes the graph disconnected.



If G has a cut vertex, then $K(G) = 1$.

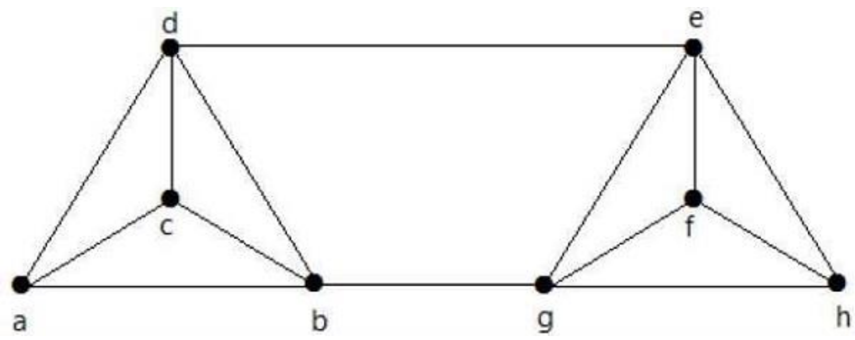
Notation − For any connected graph G,

$K(G) \leq \lambda(G) \leq \delta(G)$

Vertex connectivity ($K(G)$), edge connectivity ($\lambda(G)$), minimum number of degrees of $G(\delta(G))$.

Example

Calculate λ(G) and K(G) for the following graph −



Solution

From the graph,

δ(G) = 3

K(G) ≤ λ(G) ≤ δ(G) = 3 (1)

K(G) ≥ 2 (2)

Deleting the edges {d, e} and {b, h}, we can disconnect G.

Therefore,

λ(G) = 2

2 ≤ λ(G) ≤ δ(G) = 2 (3)

From (2) and (3), vertex connectivity K(G) = 2

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Graph Theory

Tournaments

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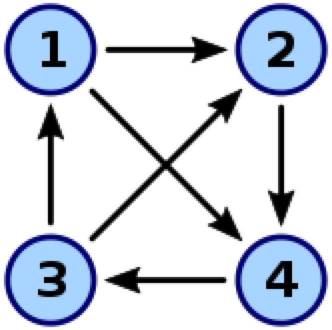
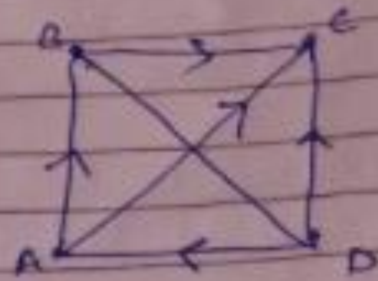
Tournament

A **tournament** is a directed graph (digraph) obtained by assigning a direction for each edge in an undirected complete graph. That is, it is an orientation of a complete graph, or equivalently a directed graph in which every pair of distinct vertices is connected by a directed edge (often, called an **arc**) with any one of the two possible orientations.

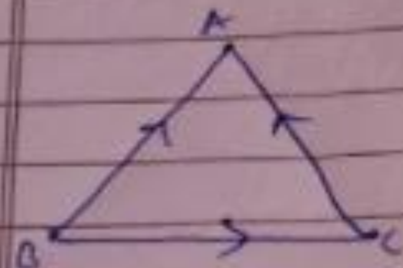
Many of the important properties of tournaments were first investigated by Landau (1953) in order to model dominance relations in flocks of chickens. Current applications of tournaments include the study of voting theory and social choice theory among other things.

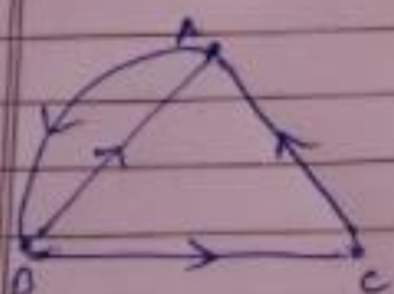


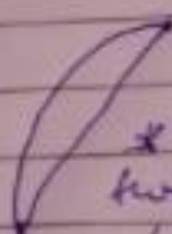
Ex:-



* each pair of vertices has exactly one arc between
* and each edge assign one direction







* so here we got two direction going btw the same vertices

Transitive Tounaments

Let's call your tournament TT, and write a→ba→b when there is an arc from aa to bb. Write a→Ba→B when BB is a vertex set and aa has an arc to all members of BB. Finally write B→aB→a when all members of a set BB have an arc to aa.

1) is straightforward enough. If TT has a cycle of length 3, then it can't be transitive by definition. Conversely, suppose TT is not transitive. Then ∀a,b,c∈V(T)[a→b,b→c⇒a→c]∀a,b,c∈V(T)[a→b,b→c⇒a→c] is false. What do you get by negating this expression ?

2) There's probably a better way for the tougher direction, but here goes. One way is to suppose TT has no Hamiltonian cycle and get that TT is not strongly connected. Let C=c1c2…ckc1C=c1c2…ckc1 be a directed cycle of TT with the largest number of vertices.

Let x∈V(T)x∈V(T) such that x∉Cx∉C. If ci→xci→x for some ci∈Cci∈C, then ci+1→xci+1→x as otherwise, x→ci+1x→ci+1 and C′=c1c2…cixci+1…ckc1C′=c1c2…cixci+1…ckc1 would be a bigger cycle (but CC is a largest cycle by assumption). Applying this reasoning inductively, we have that C→xC→x when some ci→xci→x.

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conditions

(1) A tournament is transitive if and only if it has no cycles

In a directed graph the outdegree of vertex is no of arc going out of the vertex

* outdegree

b = 3

c = 2

d = 1

a = 0

* indegree

b = 0

c = 1

d = 2

a = 3

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Thank You

Source:- Discrete Mathematics (Kenneth H Rosen)