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DISCRETE STRUCTURE

GRAPH ISOMORPHISM

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OVER
40
YEARS
OF ACADEMIC
WISDOM



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AFFILIATED TO BENGALURU CITY UNIVERSITY, APPROVED BY AICTE, DELHI & RECOGNISED BY THE GOVT. OF KARNATAKA
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TEAM B- GROUP NAME

- AMAN SINGH
- ABHINAYA
- ABHISHEK SINGH
- ABRAR AHMED
- ADITI RAJ
- ABHIRUP BISHWAS





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INDEX

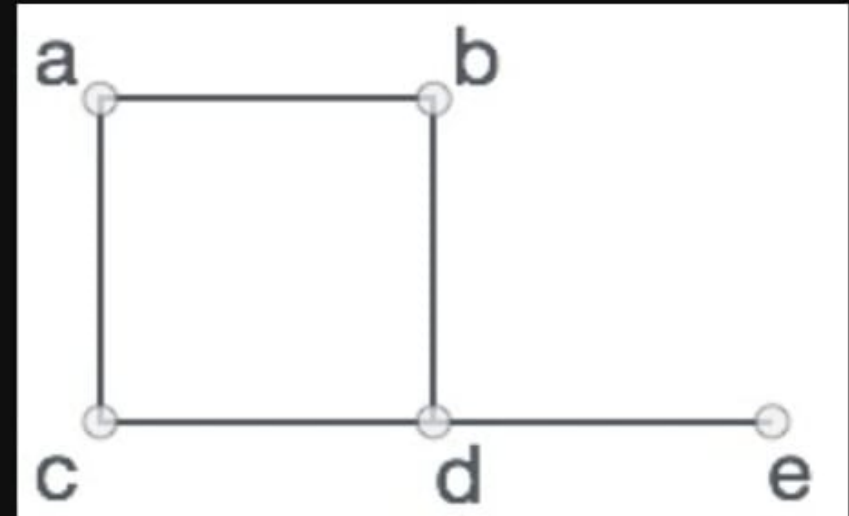
- INTRODUCTION
- WHAT IS GRAPHS
- WHAT IS GRAPHS THEORY
- WHAT IS ISOMORPHISM GRAPHS
- EXAMPLE OF ISOMORPHISM
- QUESTION PAPERS



GRAPH

A graph is a pictorial representation of a set of objects where some pairs of objects are connected by links. The interconnected objects are represented by points termed as **vertices**, and the links that connect the vertices are called **edges**.

Formally, a graph is a pair of sets (**V**, **E**), where **V** is the set of vertices and **E** is the set of edges, connecting the pairs of vertices. Take a look at the following graph –





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In the above graph,

$V = \{a, b, c, d, e\}$

$E = \{ab, ac, bd, cd, de\}$



GRAPH THEORY

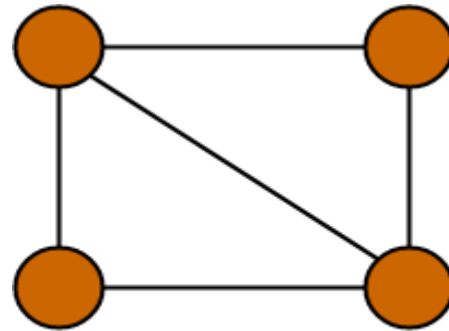
- Graph Theory is the study of points and lines. In Mathematics, it is a sub-field that deals with the study of graphs. It is a pictorial representation that represents the Mathematical truth. Graph theory is the study of relationship between the vertices (nodes) and edges (lines).
- Formally, a graph is denoted as a pair $G(V, E)$.
- Where V represents the finite set vertices and E represents the finite set edges.
- Therefore, we can say a graph includes non-empty set of vertices V and set of edges E .

Example

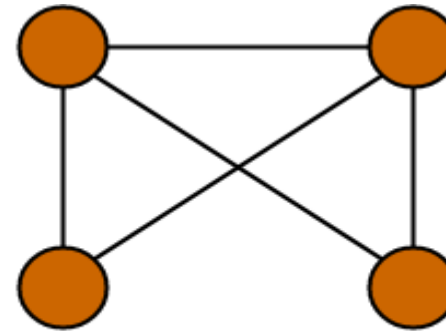
- Suppose, a Graph $G=(V,E)$, where
- Vertices, $V=\{a,b,c,d\}$
- Edges, $E=\{\{a,b\},\{a,c\},\{b,c\},\{c,d\}\}$



ISOMORPHISM



G1



G2

QUESTION: DO THESE TWO GRAPHS ARE SAME?





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- IF YOUR ANSWER NO THEN, YOU NEED TO RETHINK IT.
- THE GRAPHICAL ARRANGEMENT OF THE VERTICES AND EDGES MAKE THEM LOOK DIFFERENT BUT THEY ARE THE SAME GRAPH.
- THE TERM WE USE FOR THIS RELATIONSHIP BETWEEN TWO GRAPHS IS CALLED ISOMORPHIC.





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QUESTION: WHAT DOES IT MEAN TO BE TWO GRAPHS TO BE ISOMORPHIC?

- TWO GRAPHS ARE SAID TO BE ISOMORPHIC IF THERE IS ONE TO ONE CORRESPONDANCE BETWEEN THEIR VERTICES AND EDGES.
- INCIDENCE RELATIONSHIP IS PRESERVED $G_1(V_1E_1)$ AND $G_2(V_2E_2)$ ARE ISOMORPHIC IF THERE IS A BIJECTION FUNCTION TO EACH OTHER.
- $F:V_1(G_1) \rightarrow V_2(G_2)$





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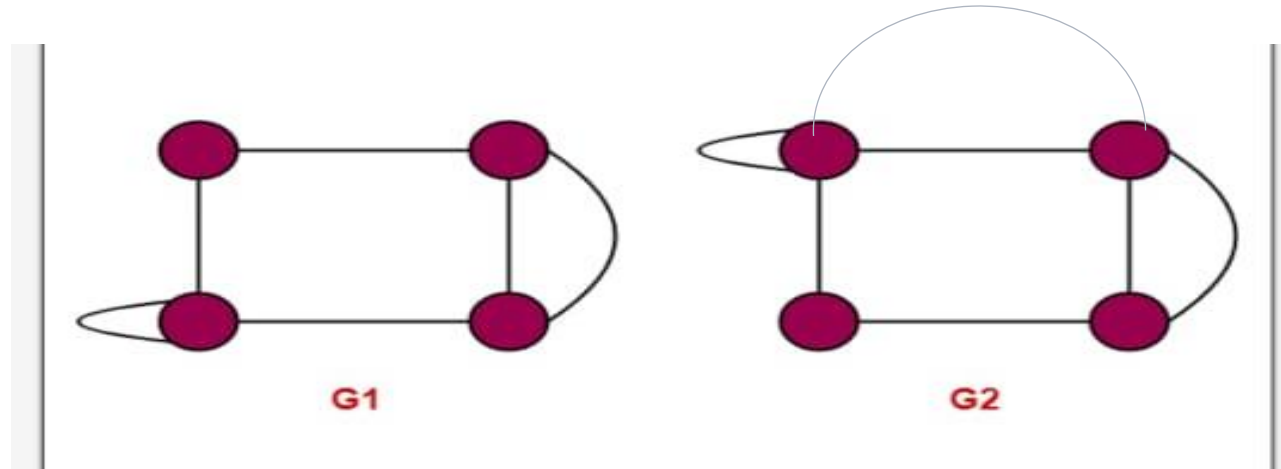
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- TWO GRAPHS ARE SAID TO BE ISOMORPHIC IF THEY ARE PERHAPS THE SAME GRAPH,JUST DRAWN DIFFERENTLY WITH DIFFERENT NAMES.
- i.e THEY HAVE IDENTICAL BEHAVIOUR FOR ANY GRAPH THEORETIC PROPERTIES.



EXAMPLE 1: ARE THE FOLLOWING GRAPHS ISOMORPHIC?



CONDITION-1:

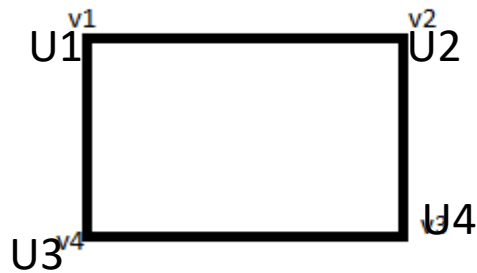
- NUMBER OF VERTICES IN GRAPH $G_1 = 4$
- NUMBER OF VERTICES IN GRAPH $G_2 = 4$
- HERE, BOTH THE GRAPH G_1 AND G_2 HAVE SAME NUMBER OF VERTICES.
- SO, CONDITION-1 SATISFIES.

CONDITION-2:

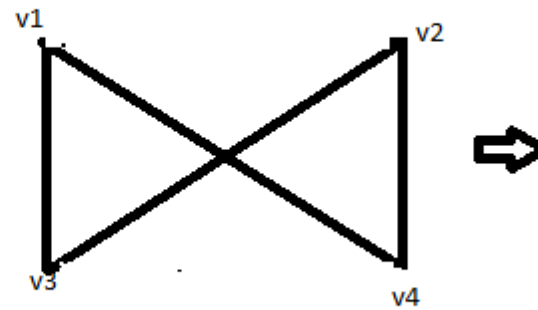
- NUMBER OF EDGES IN GRAPH $G_1 = 5$
- NUMBER OF EDGES IN GRAPH $G_2 = 6$
- HERE, BOTH THE GRAPH G_1 AND G_2 HAVE DIFFERENT NUMBER OF EDGES.
- SO, CONDITION-2 VIOLATES
- SINCE, CONDITION-2 VIOLATES, SO GIVEN GRAPHS CAN NOT BE ISOMORPHIC.
- THEREFORE, G_1 AND G_2 ARE NOT ISOMORPHIC GRAPHS.



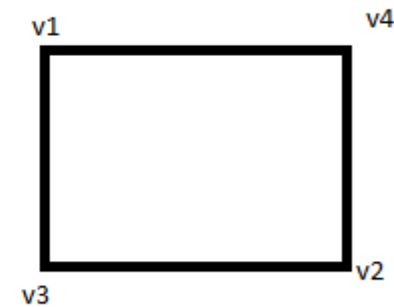
EXAMPLE 2: SHOW THAT G AND H ARE ISOMORPHIC



G



H



H



- NUMBER OF VERTICES IN GRAPH $G = 4$
- NUMBER OF VERTICES IN GRAPH $H = 4$
- HERE, BOTH THE GRAPH G AND H ARE HAVING SAME NUMBER OF VERTICES.
- CONDITION 2
- NUMBER OF EDGES IN GRAPH $G = 4$
- NUMBER OF EDGES IN GRAPH $H = 4$
- BOTH THE GRAPH G AND H HAVE SAME NUMBER OF EDGES.

CONDITION 3

DEGREE SEQUENCE OF GRAPH $G = 2, 2, 2, 2$.

DEGREE .SEQUENCE OF GRAPH $H = 2, 2, 2, 2$

HENCE CONDITION 3 SATISFIED



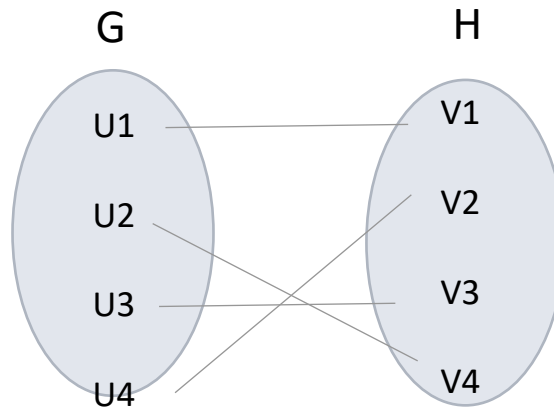
$$F(U1)=V1$$

$$F(U2)=V4$$

$$F(U3)=V3$$

$$F(U4)=V2$$

ONE TO ONE
FUNCTION MAPPING



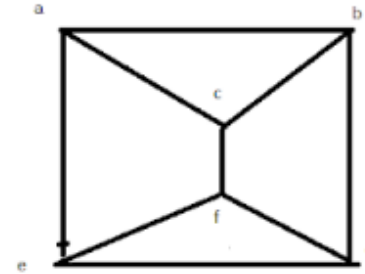
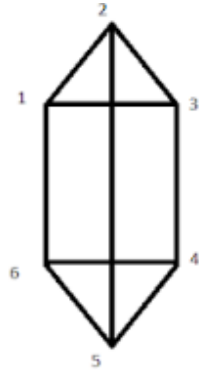
$$(U1, U2) \cong (V1, V4) \quad (U3, U4) \cong (V3, V2)$$

$$(U1, V3) \cong (U1, V3) \quad (U4, U2) \cong (V2, V4)$$

THEREFORE, G AND H ARE ISOMORPHIC. $G \cong H$

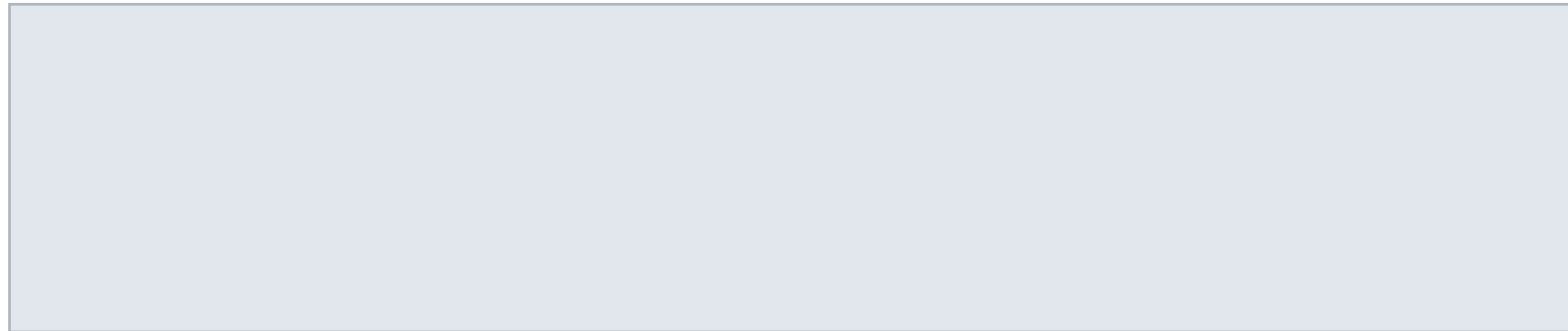


EXAMPLE 3: CHECK G AND H FOR ISOMORPHISM.



G

H



CONDITION 1

- NUMBER OF VERTICES IN $G = 6$
- NUMBER OF VERTICES IN $H = 6$
- HERE, BOTH HAVE SAME NUMBER OF VERTICES.

CONDITION 2

- NUMBER OF EDGES IN $G = 9$
- NUMBER OF EDGES IN $H = 9$
- BOTH HAVE SAME NUMBER OF EDGES.

CONDITION 3

DEGREE SEQUENCE OF GRAPH $G = 3, 3, 3, 3, 3, 3$

DEGREE SEQUENCE OF GRAPH $H = 3, 3, 3, 3, 3, 3$



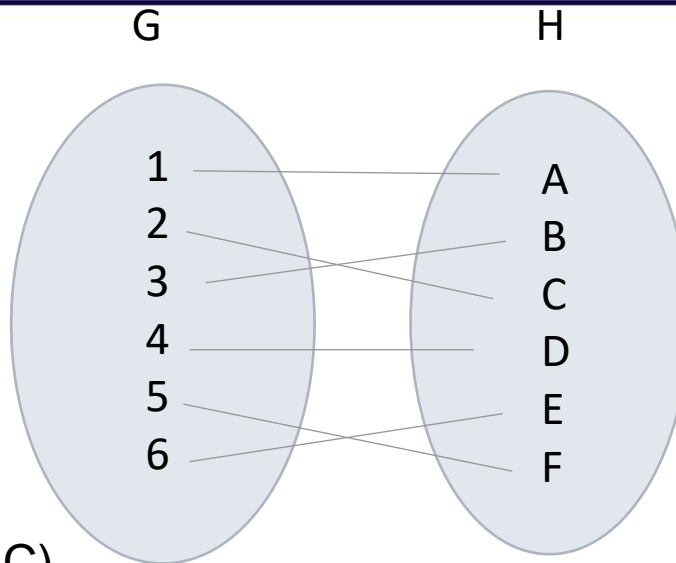
CONDITION 4



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$$(1,2) \cong (A,C)$$

$$(1,3) \cong (A,B)$$

$$(1,6) \cong (A,E)$$

.

.

.

THEREFORE, G AND H ARE ISOMORPHIC.

$$G \cong H$$

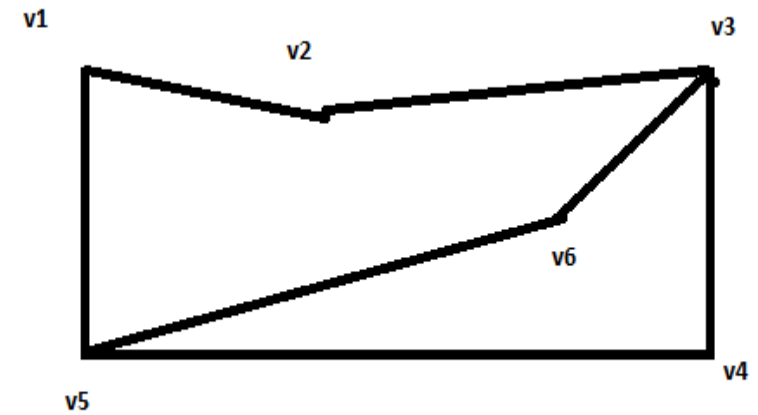
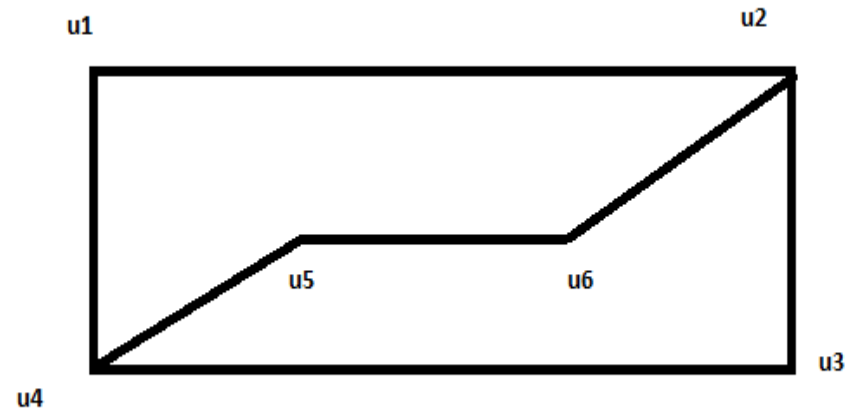
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QUESTIONS FROM PREVIOUS YEARS QUESTION PAPERS

Q1. DEFINE ISOMORPHISM OF TWO GRAPHS. DETERMINE WHETHER THE FOLLOWING GRAPHS ARE ISOMORPHIC OR NOT?



- (i) number of vertices in both the graph =6
- (ii) number of edges in both the graph=7
- (iii) degree

G1		G2
$d(v1) \rightarrow 2$	=	$d(v1) \rightarrow 2$
$d(v2) \rightarrow 3$	=	$d(v3) \rightarrow 3$
$d(v3) \rightarrow 2$	=	$d(v2) \rightarrow 2$
$d(v4) \rightarrow 3$	=	$d(v5) \rightarrow 3$
$d(v5) \rightarrow 2$	=	$d(v4) \rightarrow 2$
$d(v6) \rightarrow 2$	=	$d(v6) \rightarrow 2$

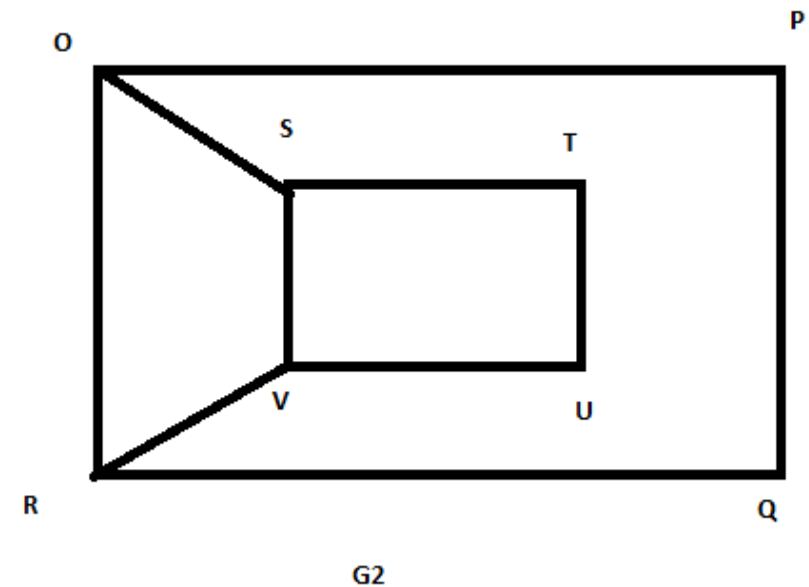
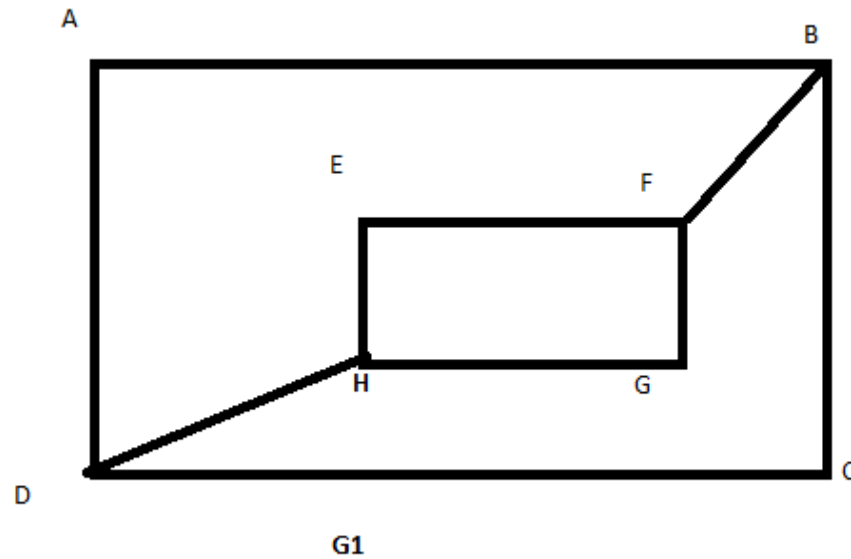
(iv) mapping

$d(v1)=v4$	$d(v3)=v6$	$d(v5)=v1$
$d(v2)=v3$	$d(v4)=v5$	$d(v6)=v2$

HENCE PROVED the given graph is isomorphism



QUESTION 2- EXAMINE WHETHER THE FOLLOWING PAIR OF GRAPHS ARE ISOMORPHIC OR NOT. JUSTIFY YOUR ANSWER.



- (i) number of vertices=8
- (ii) number of edges=10
- (iii) degree

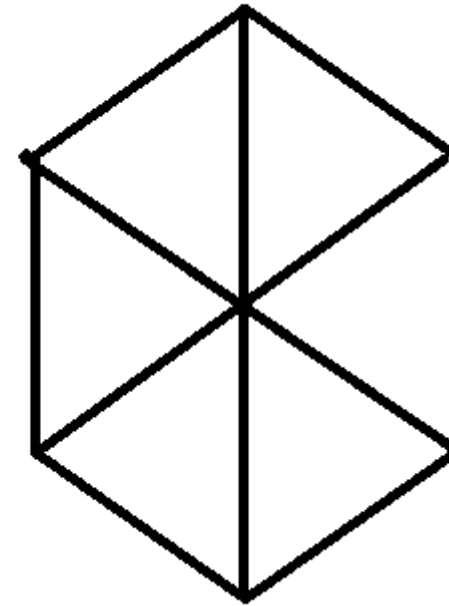
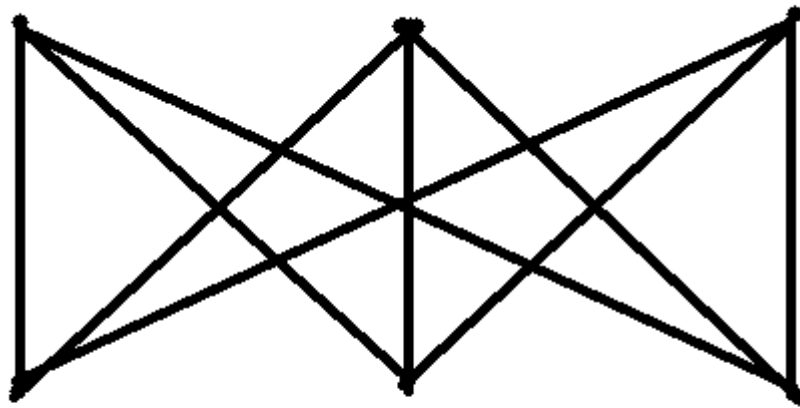
G ₁		G ₂
d(a)--->2	=	d(p)--->2
d(b)--->3	=	d(o)--->3
d(c)--->2	=	d(q)--->2
d(d)--->3	=	d(r)--->3
d(e)--->2	=	d(t)--->2
d(f)--->3	=	d(s)--->3
d(g)--->2	=	d(u)--->2
d(h)--->3	=	d(v)--->3

(iv) here mapping is not able to do because G_1 (A) has 2degree which is connected with 3 and 3 degree so we should see that same thing to be satisfied by G_2 in G_2 (t) has 2degree which is connected with 3 and 2 degree so that 4th condition is not satisfied

HENCE the given graph is not an isomorphism.



- QUESTION-4- DEFINE ISOMORPHISM OF GRAPHS.VERIFY THAT THE 2 GRAPHS SHOWN BELOW ARE ISOMORPHIC.





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• Let's name the graph $G1:u1,u2, u3,u4,u5,u6$

$G2:v1,v2,v3,v4,v5,v6$

1) No of vertices in both the graphs =6

2) No of edges in both the graphs=9

3) Degree

G1	G2
----	----

$u1=3$	$v1=3$
--------	--------

$u2=3$	$v2=3$
--------	--------

$u3=3$	$v3=3$
--------	--------

$u4=3$	$v4=3$
--------	--------

$u5=3$	$v5=3$
--------	--------

$u6=3$	$v6=3$
--------	--------

4) Mapping

$(u1) \rightarrow (v1)$

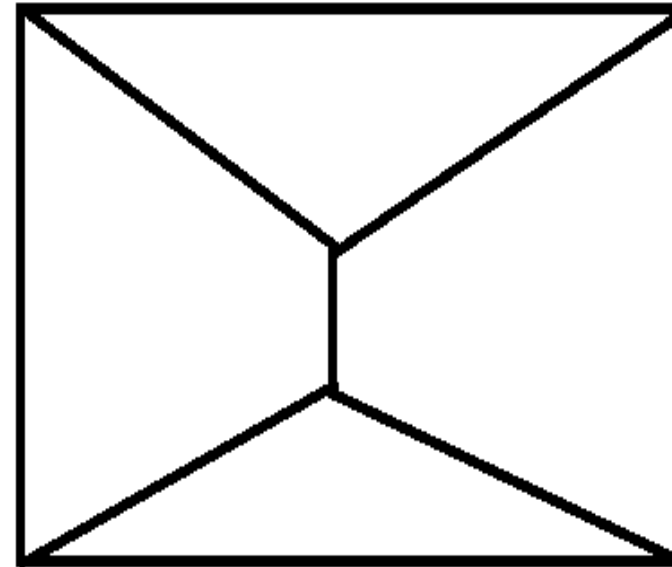
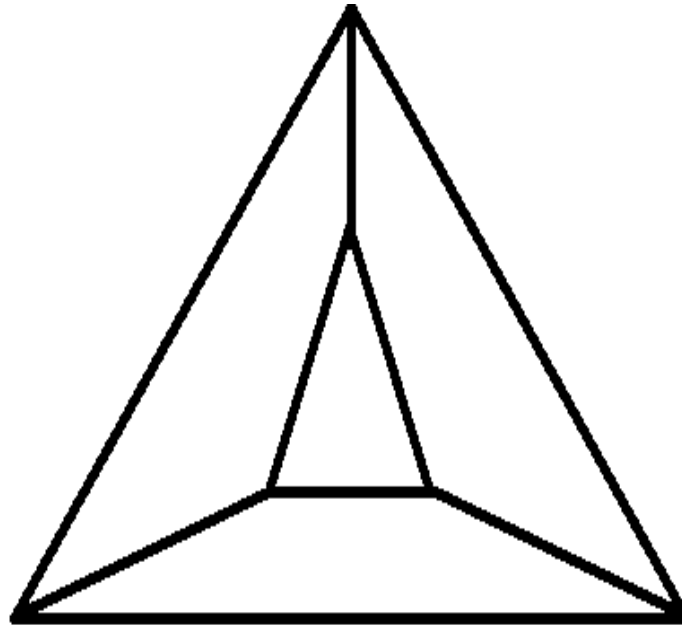
$(u2) \rightarrow (v2)$

$(u3) \rightarrow (v3)$

$(u4) \rightarrow (v4)$



QUESTION-5-DEFINE ISOMORPHIC GRAPHS.VERIFY WHETHER THE FOLLOWING GRAPHS ARE ISOMORPHIC ARE NOT.





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- Let's name the graph G1: a,b,c,d,e,f &
- G2 :p,q,r,s,t,u
- 1)No of vertices in both the graphs are =6
- 2)No of edges in both the graphs are=9
- 3) Degree
- G1 G2
- a=3 p=3
- b=3 q=3
- c=3 r=3
- d=3 s=3
- e=3 t=3
- f=3 u=3
- 4) Mapping
- (a)→(p)
- (b)→(q)
- (c)→(r)
- (d)→(s)
- (e)→(t)





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THANK YOU!

