Introduction to Graph Theory

Lecture 25

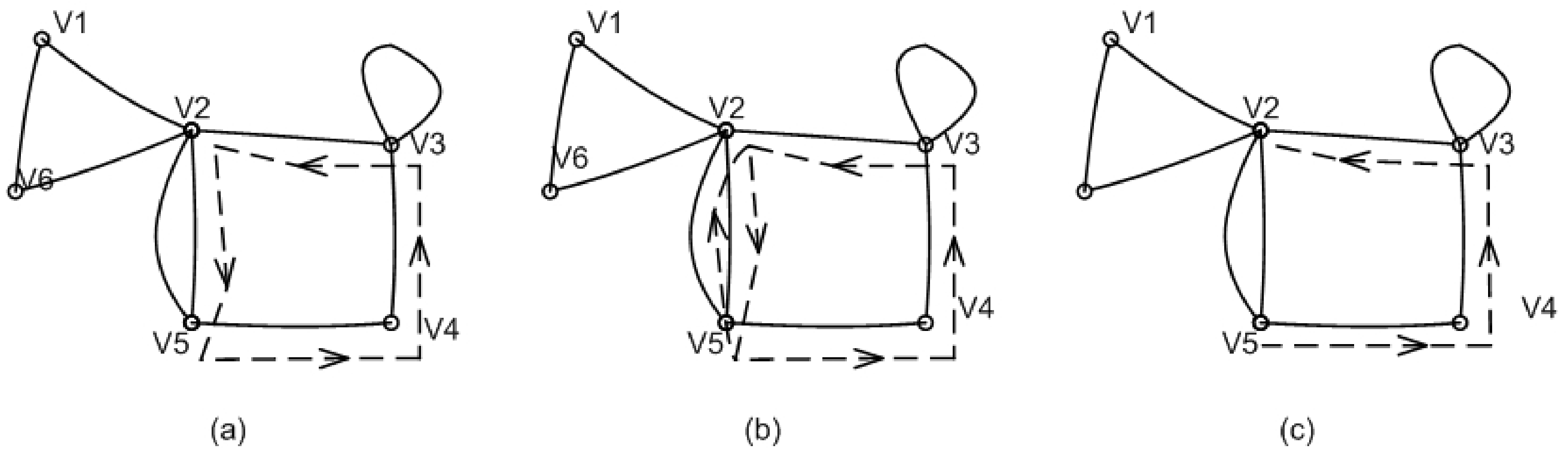
Menu

- Paths
- Connected graphs
- Incidence matrix and adjacency matrix of a graph

Walks, paths and circuits

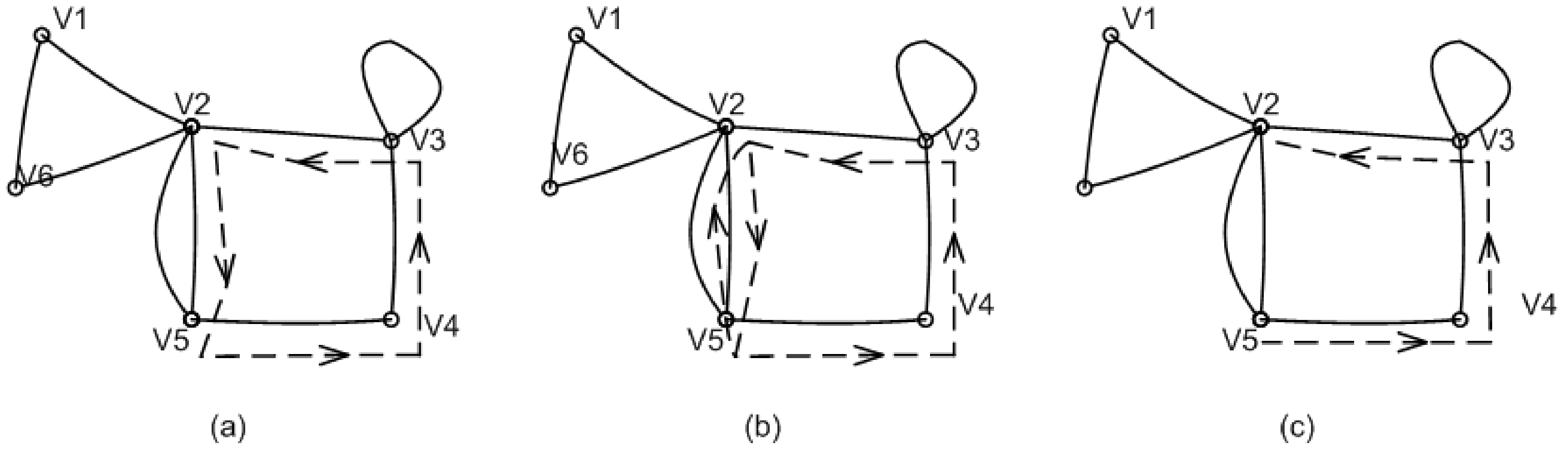
- A sequence of edges of the form
  V_sV_i, V_iV_j, V_jV_k, V_kV_l, V_lV_t is a **walk** from V_s to V_t.
- If these edges are distinct then the walk is called a **trail**, and
- if the vertices are also distinct then the walk is called a **path**.
- A walk or trail is **closed** if $V_s = V_t$.
- A closed walk in which all the vertices are distinct except V_s and V_t, is called a **cycle** or a **circuit**.
- The number of edges in a walk is called its **length**.

<u>Example:</u> Identify whether a path is marked on the graph in each case:



- <u>Solution:</u> (c) is a path, length?

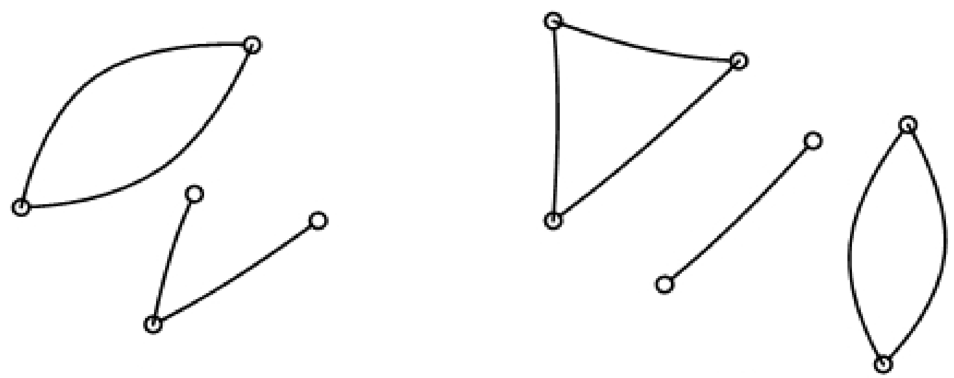
<u>Example:</u> Identify whether a trail, path or circuit is marked on the graph in each case:



- <u>Solution:</u> (a) circuit (b) trail (c) path

Connected graphs

- A graph *G* is **connected** if there is a path from any one of its vertices to any other vertex.
- A **disconnected** graph is said to be made up of **components**.
- <u>Example 5:</u>
- How many components do the following disconnected graphs have?



(a) (b)

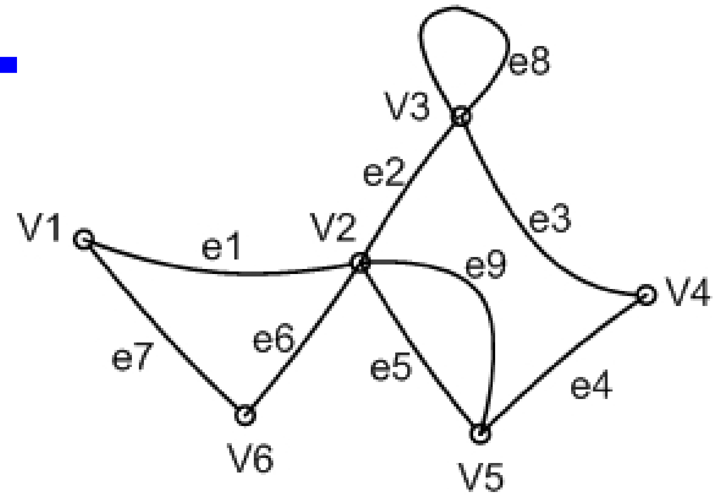
- <u>Solution:</u>
  (a) Two components (b) Three components

Matrix representation of a graph: the incidence matrix

- The **incidence matrix** of a graph G is a $|V| \times |E|$ matrix $\boldsymbol{A}$.

- The element a_{ij} =
- the *number of times* that vertex V_i is incident with the edge e_j

Give the incidence matrix of the graph below:



- The incidence matrix for the graph is given by

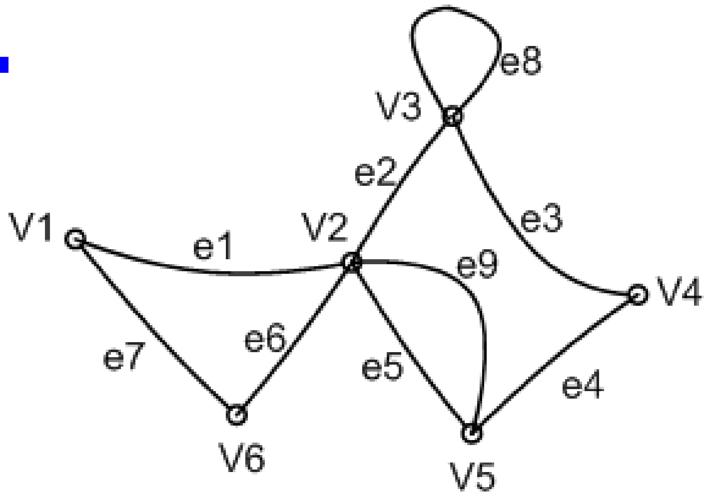
$$
\begin{array}{c}
 \\ V1 \\ V2 \\ V3 \\ V4 \\ V5 \\ V6
\end{array}
\begin{array}{c}
\begin{array}{ccccccccc} e1 & e2 & e3 & e4 & e5 & e6 & e7 & e8 & e9 \end{array} \\
\begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \\
0 & 1 & 1 & 0 & 0 & 0 & 0 & 2 & 0 \\
0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0
\end{pmatrix}
\end{array}
$$

Matrix representation of a graph: the adjacency matrix

- The **adjacency matrix** of a graph G is a $|V| \times |V|$ matrix $\boldsymbol{A}$.

- The element a_{ij} =
- the *number of edges* joining V_i and V_j

Give the adjacency matrix of the graph below:



- The adjacency matrix for the graph is given by

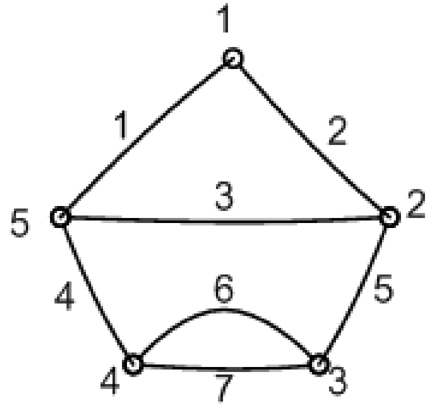
$$
\begin{array}{c}
\\
V1 \\ V2 \\ V3 \\ V4 \\ V5 \\ V6
\end{array}
\begin{array}{c}
\begin{array}{cccccc} V1 & V2 & V3 & V4 & V5 & V6 \end{array} \\
\begin{pmatrix}
0 & 1 & 0 & 0 & 0 & 1 \\
1 & 0 & 1 & 0 & 2 & 1 \\
0 & 1 & 1 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 & 1 & 0 \\
0 & 2 & 0 & 1 & 0 & 0 \\
1 & 1 & 0 & 0 & 0 & 0
\end{pmatrix}
\end{array}
$$

Summary

- Definitions of **paths**
- Definitions of **connected graphs**
  Definitions of **incidence matrix** and **adjacency matrix** of a graph

Q. How would you design data structure for graphs? What type of data structure can we use to store graphs?

- **Self test**
- 1. Write down the adjacency and incidence matrices of the graph below.



- **Self test**
- 2. Draw the graph whose adjacency matrix is given in (a)

$$\begin{pmatrix} 0 & 1 & 1 & 2 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 \\ 2 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{pmatrix}$$

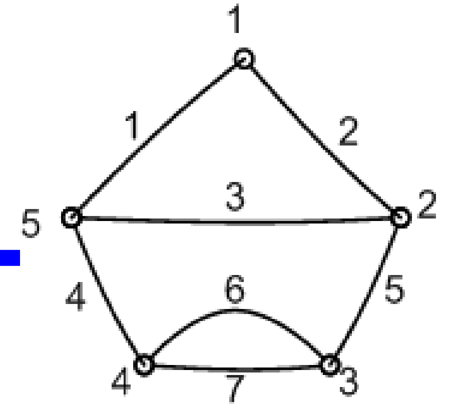
(a)

- **Self test**
- 3. Draw the graph whose incidence matrix is given in (b)

$$\begin{pmatrix} 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

(b)

Answers



- 1. The incidence matrix is

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{array} \\ \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 0 & 0 \end{pmatrix} \end{array}$$

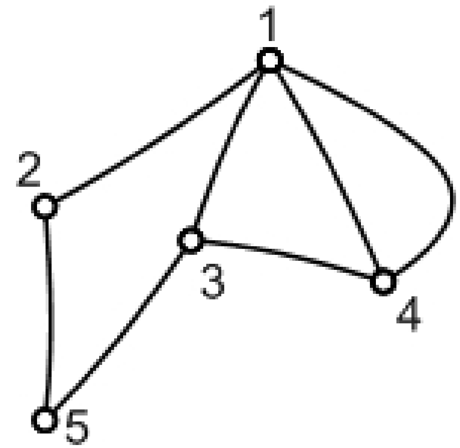
The adjacency matrix is

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{pmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 2 & 0 \\ 0 & 0 & 2 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \end{pmatrix} \end{array}$$

Answers

2.

$$
\begin{array}{c|c}
 & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\
\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{pmatrix} 0 & 1 & 1 & 2 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 \\ 2 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{pmatrix}
\end{array}
$$



$$
\begin{array}{c|cccccccc}
 & a & b & c & d & e & f & g & h \\
\hline
1 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 0 \\
2 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 \\
3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
4 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\
5 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0
\end{array}
$$

3.