(CPT107) - Inclass Exercises - Week 10

Problem 1. Consider the signature *S = {Friend, Female, alice, bob}* consisting of a binary predicate symbol *Friend,* a unary predicate symbol *Female,* and two constant symbols *alice* and *bob.* Assume that these symbols have the following meaning:

- *Friend* means "is a friend of" *(i.e., Friend(x,* y) states: *x* is a friend of y).
- *Female* means "is female" *(i.e., Female(x)* states: *x* is female).
- *alice* and *bob* refer to "Alice" and "Bob", respectively.

Translate the following sentences into S-formulae, that is, for each of the following sentences provide an S-formula that expresses the sentence:

1. Alice is a friend of Bob.
2. Bob has a girlfriend.
3. All friends of Bob are also friends of Alice.
4. Alice has at least two friends.
5. Everybody has a friend.
6. Nobody is a friend of everybody else.

Problem 2. Translate the following statements into S-formulae for an appropriate signature *S.* Before translating the sentences, describe the signature *S* and the meaning of the symbols in it.

1. Liverpool is a city in the UK.
2. There is a city that is larger than Liverpool.
3. London is larger than all other cities in the UK.
4. Some students at University of Liverpool do not live in Liverpool.
5. Not everyone living in Liverpool studies at University of Liverpool.

Inclass Exercises - Week 10 (Solutions)

Solution to Problem 1.

1. $Friend(alice, bob)$
2. $\exists x\, Friend(bob, x) \wedge Female(x)$
3. $\forall x\, Friend(x, bob) \rightarrow Friend(x, alice)$
4. $\exists x \exists y\, Friend(alice, x) \wedge Friend(alice, y) \wedge \neg x = y$
5. $\forall x \exists y Friend(x, y)$
6. $\neg \exists x \forall y\, \neg x = y \rightarrow Friend(x, y)$

Solution to Problem 2. We choose a signature

$$S = \{City, LocatedIn, Larger, UoLStudent, LivesIn, liverpool, london, uk\},$$

where *City* and *UoLStudent* are unary predicate symbols, *LocatedIn*, *Larger*, *LivesIn* are binary predicate symbols, and *liverpool*, *london*, *UK* are constant symbols. We assume that the symbols have the following meaning:

- $City(x)$ means "x is a city";
- $LocatedIn(x, y)$ means "x is located in y";
- $Larger(x, y)$ means "x is larger than y";
- $UoLStudent(x)$ means "x is a student at University of Liverpool";
- $LivesIn(x, y)$ means "x lives in y";
- *liverpool*, *london*, and *uk* stand for "Liverpool", "London", and "UK", respectively.

We are now able to express the five statements:

1. $City(liverpool) \wedge LocatedIn(liverpool, UK)$
2. $\exists x\, City(x) \wedge Larger(x, liverpool)$
3. $\forall x\, (City(x) \wedge LocatedIn(x, uk) \wedge \neg x = london) \rightarrow Larger(london, x)$
4. $\exists x\, UoLStudent(x) \wedge \neg LivesIn(x, liverpool)$
5. $\neg \forall x\, LivesIn(x, liverpool) \rightarrow UoLStudent(x)$