

# Discrete Mathematics and Statistics (CPT 107)

## Tutorial 8

1. Suppose  $p_1$  represents ‘This car is red’,  $p_2$  represents ‘Your house is blue’ and  $p_3$  represents ‘You are tall’.

- What does  $(p_1 \wedge p_2)$  represent?
- What does  $\neg p_3$  represent?
- What does  $\neg(p_2 \wedge p_3)$  represent?

Represent in propositional logic:

- You are not tall.
- If this car is red, then your house is blue.
- This car is not red or you are tall.
- It is not the case that this car is not red.

2. Define  $P \rightarrow Q$ .

3. Let  $P = (p_1 \vee \neg p_2)$  and  $Q = \neg P$ . Let  $I$  be an interpretation such that  $I(p_1) = 0$  and  $I(p_2) = 0$ . Determine  $I(P)$  and  $I(Q)$  using a **truth table**.

4. Let  $P = ((p_1 \wedge \neg p_2) \wedge p_3)$ . Write down the truth table for  $P$ . Determine the interpretations under which  $P$  is true.

5. What is a tautology?

6. Which of the following are tautologies? Check using truth tables.

- (a)  $(p_1 \vee p_1)$ ;
- (b)  $(\neg p_1 \vee (p_2 \wedge p_1))$ ;
- (c)  $(\neg \neg p_1 \leftrightarrow p_1)$ ;
- (d)  $(\neg p_1 \rightarrow \neg p_1)$ .

7. What is a contradiction?

8. Which of the following are contradictions? Check using truth tables.

- (a)  $(\neg p_1 \wedge p_2)$ ;
- (b)  $(p_1 \rightarrow \neg p_1)$ ;
- (c)  $(p_1 \leftrightarrow \neg p_1)$ ;
- (d)  $(p_1 \wedge (\neg p_2 \vee \neg p_1))$ .

9. Define the meaning of  $\Gamma \models P$ .

10. Which of the following are true? Check using truth tables.

- (a)  $\{p_1, (p_1 \rightarrow p_2)\} \models p_2$ ;
- (b)  $\{(p_1 \rightarrow p_2)\} \models (p_2 \rightarrow p_1)$ ;
- (c)  $\{(p_1 \vee \neg p_2)\} \models p_1$ ;

11. Define the relation  $\equiv$  of logical equivalence.

12. Show using truth tables

$$(P \vee (Q \wedge R)) \equiv ((P \vee Q) \wedge (P \vee R)).$$

13. Is  $\neg(P \wedge Q)$  logically equivalent to  $(\neg P \wedge \neg Q)$ ? Discuss the relationship to *De Morgan's Law*.