Discrete Mathematics and Statistics - CPT107

Tutorial 12

1.

- Each outcome is a function from the 4 coins to the set $\{\text{heads}, \text{tails}\}$. There are $2^4 = 16$ functions, and they are all equally likely, so the probability of any one of them is $1/16$. Thus, the probability that the $1p$ and the $5p$ come up heads and the other two coins come up tails is $1/16$.
- Let E_1 be the event that the $1p$ coin comes up tails and E_2 be the event that the $2p$ coin comes up tails.

$$\Pr(E_1 \cup E_2) = \Pr(E_1) + \Pr(E_2) - \Pr(E_1 \cap E_2) = \tfrac{1}{2} + \tfrac{1}{2} - \tfrac{1}{4} = \tfrac{3}{4}.$$

- The event that at least three of the coins comes up tails is the union of five disjoint events: the event that all coins come up tails, and the four events in which a specified coin is the only heads. The probability of each of these five events is $1/16$, so the probability of their union is $5/16$.
- Let F be the event that at least three of the coins come up tails and G be the event that at least one of the low-value coins comes up tails. Then

$$\Pr(F \mid G) = \frac{\Pr(F \cap G)}{\Pr(G)}.$$

  But $F \cap G = F$ so this is $\Pr(F)/\Pr(G) = (5/16)/(3/4) = 5/12$.
- We have shown that $\Pr(F \mid G) = 5/12$ and $\Pr(F) = 5/16$. These are not the same, so events F and G are not independent.
- Let f_1 be the amount of money that I get paid from the flip of the $1p$ coin, and f_2 be the amount of money that I get paid from the flip of the $2p$ coin, and so on. Then $E[f_1] = \frac{1}{2}$ and $E[f_2] = 2 \times \frac{1}{2} = 1$ and $E[f_5] = 5 \times \frac{1}{2} = \frac{5}{2}$ and $E[f_{10}] = 10 \times \frac{1}{2} = 5$. Thus, $E[f_1 + f_2 + f_5 + f_{10}] = \frac{1}{2} + 1 + \frac{5}{2} + 5 = 9$ pence.

2.

- 1/13
- 1/4
- 3/4
- 1/26
- 3/13
- 1/26

3. 100 and “NO”.

4. p(Tom goes out and does homework) or p(Tom does not go out and does his homework) = p(Tom goes out) *x* p(does homework) + p(Tom does not go out) *x* p(does his homework) = (3/4) *x* (1/10) + (1/4) *x* (3/5)

5. Probability that someone in the room has the same birthday as me, denoted by P(B) is 1− probability that no one in the room has the same birthday as me. P(B) = 1 − $(364/365)^n$. We wish P(B) ⩾ 1/2, Taking logs, n ⩾ 253 is obtained.

6.

- Sum of 10: (5,5), (6,4), (4,6): 3/36 (probability)
- Sum of 3: (1,2), (2,1): 2/36 (probability)
- Random number values (winner's gains): -5, 5, 15
- Corresponding probabilities are: 31/36 , 3/36 , 2/36
- E(x) = -5(31/36)+5(3/36)+15(2/36) = (-155+15+30)/36 = -110/36 = -$3.05
- The gambler can expect to lose $3.05 each time he/she plays the game.