TUTORIAL – Week 12 – Q&A

1: The tickets are marked from number 1 to 20. One ticket is chosen at random.
Find the probability that the ticket selected has a digit that is a multiple of number 3 or number 5.

Answer:

The sample space of the above problem is = S = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20}

Multiples of 3 are {3, 6, 9, 12, 15, 18}.

Multiples of 5 are {5, 10, 15, 20}.

P (an event) = count of outcomes in event/ total count of outcomes

P (the ticket selected has a digit that is a multiple of number 3 or number 5) = (multiples of 3 or multiples of 5) / total number of possible outcomes

= {3, 6, 9, 12, 15, 18, 5, 10, 20} / 20 = 9 / 20

2. Three chairs are arranged in a line, and three people randomly take seats.
What is the probability that the person with the middle height ends up in the middle seat?

Solution

Let the people be labeled from tallest to shortest as 1, 2, and 3.
Then the 3! = 6 possible orderings are

1 2 3 1 3 2 2 1 3 2 3 1 3 1 2 3 2 1

We see that two of these (1 2 3 and 3 2 1) have the middle-height person in the middle seat. So, the probability is 2/6 = 1/3.

3. Six chairs are arranged in a line, and three girls and three boys randomly pick seats.
What is the probability that the three girls end up in the three leftmost seats?

Solution

Let's assume that the girls pick their seats first, one at a time.
The first girl has a **3/6** chance of picking one of the three leftmost seats.
Then, given that she is successful, the second girl has a **2/5** chance of success, because only two of the remaining five seats are among the left three.
And finally, given that she too is successful, the third girl has a **1/4** chance of success, because only one of the remaining four seats is among the left three.
If all three girls are successful, then all three boys are guaranteed to end up in the three rightmost seats.
The probability is therefore **3/6 · 2/5 · 1/4 = 1/20.**

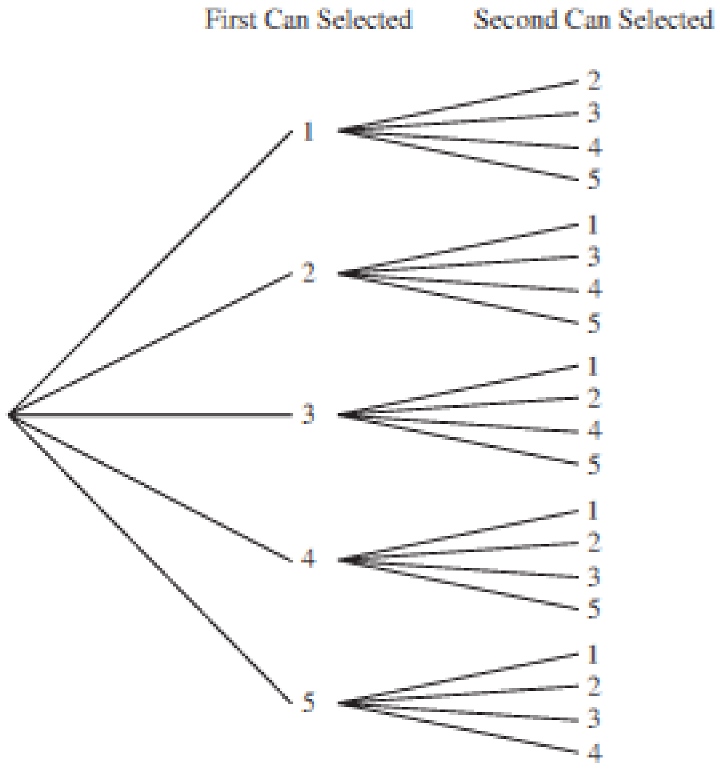
4. Five cans of paint (numbered 1 through 5) were delivered to a professional painter.
Unknown to her, some of the cans (1 and 2) are satin finish and the remaining cans (3, 4, and 5) are glossy finish.
Suppose she selects two cans at random for a particular job.
Let A denote the event that the painter selects the two cans of satin-finish paint, and
let B denote the event that the two cans have different finishes (one of satin and one of glossy).
Find P(A) and P(B).

Solution

This experiment has **20 possible outcomes**.
Because there are five ways to choose the first can and four ways to choose the second can (because there are only four cans left after the first is selected).
These 20 one-element events may be represented in the form {(1, 2)}, {(1, 3)}, and so forth, where the order of the pairs indicates the order of selection.
Because the cans are randomly selected, each of the 20 outcomes has a probability of 1/20.
Thus,

$$\mathbf{P(A) = P(\{(1, 2)\} \cup \{(2, 1)\}) = 2/20 = 0.1}$$

because the probability of the union of disjoint events is equal to the sum of the probabilities of the events in the union.



Similarly,

P(B) = P({(1, 3)} ∪ {(1, 4)} ∪ {(1, 5)} ∪ {(2, 3)} ∪ {(2, 4)} ∪ {(2, 5)} ∪ {(3, 1)} ∪ {(3, 2)} ∪ {(4, 1)} ∪ {(4, 2)} ∪ {(5, 1)} ∪ {(5, 2)}) = **12/20 = 0.6.**

5. A department in a company has 12 members: 8 males and 4 females. To gain greater insight into the employees' views of various benefits, the human resources office plans to form a focus group from members of this department.

Five departmental members will be selected at random from the department's members.

What is the probability that the focus group will only have males?

What is the probability that the focus group will have two males and three females?

Solution

The order in which focus group members is selected is not important; it is the final set of five members that matters.

Thus, the number of ways to select the five focus group members from the 12 departmental members is the combinations of 5 objects selected from 12:

$$\binom{12}{5} = \frac{12!}{5!\,(12-5)!} = \frac{12 \times 11 \times 10 \times 9 \times 8}{5 \times 4 \times 3 \times 2 \times 1} = 792$$

The number of ways to select five males from the eight men in the department is

$$\binom{8}{5} = \frac{8!}{5!\,(8-5)!} = 56$$

The number of ways to choose zero females from the four in the department is

$$\binom{4}{0} = \frac{4!}{0!\,(4-0)!} = 1.$$

The number of ways to select five males and no females from the department's members is

$$\binom{8}{5}\binom{4}{0} = 56 \times 1 = 56.$$

The probability of having five males and no females in the focus group is:

$$\frac{\binom{8}{5}\binom{4}{0}}{\binom{12}{5}} = \frac{56}{792} = \frac{7}{99}.$$

Similar, the probability of the focus group having two males and three females is

$$\frac{\binom{8}{2}\binom{4}{3}}{\binom{12}{5}} = \frac{28 \times 4}{792} = \frac{112}{792} = \frac{14}{99}$$

The probability of two males and three females is twice that of five males and no females.