Discrete Mathematics and Statistics - CPT107

Tutorial week 10

1. Do you think the following are acceptable formulas of propositional logic? If not, why not?

   a. P
   b. $P \,\&\, Q$
   c. $P \oplus Q$
   d. $\forall z.S(z)$
   e. $\neg S(z)$
   f. $\neg(P(a) \vee Q(y)$

2. Identify the predicates needed for expressing the following sentences. Just identify the predicates – don't worry about the logical operators yet.

   a. Iceland is cold but not all countries are cold
   b. No country is a holiday resort unless it is by the sea.
   c. Beautiful holiday resorts are all in European countries.

3. Identify the predicates needed for expressing the following sentences.

   a. Germany is adjacent to Italy.
   b. Two is less than three.
   c. Achilles is faster than the Tortoise, but the Tortoise is smarter than Achilles.

4. Express each of the following statements in predicate logic using the predicates $L(x)$ = "x likes discrete maths" and the constants c = "Charles" and d = "Danica".

   a. Charles likes discrete maths but Danica does not.
   b. Danica likes maths if Charles does too.
   c. Neither Charles nor Danica dislike maths

5. Express each of the following statements in predicate logic using the predicates $P(x, y) =$ "x is the parent of y" and the constants $a =$ "Andrew", $b =$ "Bethany", c = "Carol" and $d =$ "Dakota".

   a. Carol is the child of Andrew and Bethany.
   b. If Carol is the child of Bethany, then Andrew is the parent of Dakota.
   c. Andrew is not the parent of Bethany.
   d. Andrew is not the parent of Andrew.

6. Express the following in colloquial English as precisely as possible.

   a. $\neg\exists x.(\ short(x) \wedge clever(x) \wedge child(x))$
   b. $(\neg\exists x.\ short(x)) \rightarrow \forall x.(\ clever(x) \vee child(x))$
   c. $\forall x.(\ adult(x) \wedge child(x)) \rightarrow (short(j) \wedge tall(j))$, where $j =$ "John"

7. Express the following colloquial English statements using predicate logic. First identify the predicates then the connectives and quantifiers.

   a. All students are clever
   b. Those who do not work hard are lazy
   c. The lazy students are exactly those who do not work hard
   d. Not being lazy is equivalent to being hardworking
   e. Although all students are hardworking there are some who are lazy if they do not work hard

8. With the natural numbers, $\mathbb{N}_{\geq 0}$, as the domain of discourse, and using only the predicates "even" and "odd" and the standard relational and arithmetic operators, express each of the following statements as a predicate logic formula. Which of them are true?

   a. Some odd numbers are greater than 100.
   b. The sum of two odd numbers is an even number.
   c. There is no smallest number.

9. With the integers, $\mathbb{Z}$, as the domain of discourse, write each of the following as a predicate logic formula, using only the standard relational and arithmetic operators; which of them are true?

   a. Some numbers are greater than 100.
   b. The product of two negative numbers is a positive number.
   c. There is no smallest number.

10. Assume that $O(x)$ is the predicate "x is odd", and $L(x)$ is the predicate "x is less than 10". Write an English sentence equivalent to each of the following logic statements. Over the domain of the integers, what is the truth value (i.e. true or false) of each statement?

   a. $\exists x.\ O(x)$
   b. $\forall x.(\ L(x) \rightarrow\ O(x))$
   c. $(\forall x.L(x)) \rightarrow (\forall x.O(x))$