Discrete Mathematics and Statistics - CPT107

Tutorial - week 12

1. Suppose that I have four fair coins with values 1p, 2p, 5p and 10p. I will call the 1p coin and the 2p coin "low-value" coins and the other two coins "high-value coins".
   - What is the probability, when I flip the coins, that the 1p and the 5p come up heads, and the other two coins come up tails?
   - What is the probability that at least one of the low-value coins comes up tails?
   - What is the probability that at least three of the coins come up tails?
   - What is the probability that at least three of the coins come up tails, conditioned on the fact that at least one of the low-value coins comes up tails?
   - Is the event that at least three of the coins come up tails independent of the event that at least one of the low-value coins comes up tails?
   - Suppose that, if a coin comes up heads, I am paid the value of that coin. What is the total value that I expect to receive if I flip all the coins?

2. A card is taken at random from a standard 52-card pack of playing cards. What is the probability that the card is:
   - a five
   - a Diamond
   - not a Spade
   - a red Queen
   - a King, a Queen or a Jack
   - a red Jack?

3. Tom rolls a fair dice 600 times.
   - How many four would you expect him to obtain?
   - Should he be surprised if he obtained 110 fours?

4. If Tom goes out with his friends, then the probability that Tom does his homework is 1/10. Otherwise (if he does not go out), the probability that Tom does his homework is 3/5. We know that the probability that he goes out with his friends is 3/4. What is the probability that Tom does his homework?

5. How many people must there be in a room before the probability that someone has the same birthday as you do is at least 1/2?

6. If a gambler rolls two dice and gets a sum of 10, he wins 10 dollars and if he gets a sum of 3, he wins 20 dollars. The cost to play is 5 dollars. What is the expected value of the game?