

InClass Exercises – Week 11

Solutions

1. If $S = \{a, b, c, d, e\}$ is the set of $n = 5$ objects under consideration and $k=3$, then find all of the possible permutations of 3 elements from S .

Solution

Permutations $P(5, 3) = 5! / (5-3)! = 60$.

2. An instructor has divided the class into seven groups. She wishes to have three of the groups make their presentations today.
In how many ways can she arrange the three presentations?

Solution

The order in which the presentations are made is important (ask the group members!) so

Permutations $P(7, 3) = 7! / (7-3)! = 210$

3. In how many ways can arrange a 3-letter code consisting of three lower case letters?

Solution

There are $26^3 = 17,576$ ways to designate an inventory item by a code consisting of three lower case letters. - Permutation with Repetition

4. How many integers n with $1 \leq n \leq 1500$ have at least two distinct digits?

Solution

There are a total of 1500 integers between 1 and 1500. How many of them don't have at least two distinct digits?

The 9 one-digit numbers, 9 of the two-digit numbers (namely, 11, 22, ..., 99), 9 of the three-digit numbers (namely, 111, 222, ..., 999), and 1 of the four-digit numbers (namely, 1111).

Hence, the total number of integers between 1 and 1500 that have at least two distinct digits is $1500 - (9+9+9+1) = 1,472$.

5. How many four-digit positive integers have a 1, 2, or 3 as their last digit?

What if we also insisted on the digits being distinct?

Solution

We are looking for a four-digit number.

So, we have to make four choices, one for each digit. If we call the thousands digit **a**, the hundreds digit **b**, the tens digit **c**, and the ones digit **d**, then to construct a desired four-digit number, we have to make four choices.

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a b c d

We have 9 choices for **a** (since 0 is not allowed), 10 choices for each of **b** and **c**, and 3 choices for **d**.

The number of these choices is not affected by our other choices; hence the total number of such numbers is $9 \times 10 \times 10 \times 3 = 2,700$.