CPT107 - REVIEW

Final Exam

Topic
Number systems and proof techniques
Set theory and Relations
Functions
Logic
Combinatorics
Probability

The final exam covers the lectures and tutorials of week 1 → 12

NO MCQs in the final exam!

The final exam is CLOSE-BOOK and ON CAMPUS.

FINAL EXAM - Structure

I: Proof Techniques

II: Set Theory

III: Relations

IV: Functions and PHP

V: Logic

VI: Combinatorics and Probability

Revision Guide for CPT107

The exam will have a similar format to the two class assessments. To prepare for the exam, you should study the following materials:

- the slides that were used in lectures (available from the ICE)
- the assigned reading in the books by Haggarty (the assigned reading is given on the first slide of each part of the lecture notes)
- your notes from the lectures themselves
- the things that I wrote on the board during lectures (maybe helpful)
- the problems that you discussed in tutorials

Part 1. Number Systems and Proof Techniques

- Make sure that you know the following sets of numbers:
  - the natural numbers, the integers, the rationals, the real numbers, the prime numbers, and even numbers and odd numbers
- What numbers are in each set? Also, for each of the above, know how to solve problems such as the following: is the set closed under addition, multiplication, subtraction, and division? Is it closed under other operations
- Make sure you know how to use the following proof techniques:
  - proving that a conjecture is false by finding a counter-example
  - proof by contradiction
  - and proof by induction

Part 2. Set Theory

Make sure that you can do the following:

- understand set theory notation
- determine whether two sets are equal
- determine whether a given element is a member of a given set
- determine whether one set is a subset of another
- perform the following operations on sets: union, intersection, relative complement, complement, symmetric difference, power set, Cartesian product
- be able to use, and prove, the laws of the algebra of sets (associative laws, commutative laws, identity laws, distributive laws, complement laws, and De Morgan's laws)
- be able to compute the cardinality of a set, of the union of two sets, of the union of three sets, of the powerset of a set, and of the Cartesian product of sets
- understand characteristic vectors and bit strings

Part 3. Relations

- know what a relation is and be able to write down the ordered pairs in a given relation
- represent binary relations using directed graphs and matrices and be able to transform one representation into another
- know what binary relations and unary relations are
- determine whether a given binary relation is reflexive, symmetric, antisymmetric or transitive
- compute the transitive closure of a binary relation
- determine whether a given binary relation is an equivalence relation; if so, determine its equivalence classes
- determine whether a given binary relation is a partial order
- understand the Hasse diagram of a partial order
- determine whether a given binary relation is a total order

Part 4. Functions

- compute the inverse of a relation
- compose two relations (and understand the notation for compositions)
- determine whether a given relation is a function
- determine the domain, codomain and range of a function
- determine whether a function is injective, surjective, bijective, and invertible
- if a function is invertible, determine its inverse
- compose two functions (and understand the notation for compositions
- use the pigeonhole principle (and the extended pigeonhole principle) to solve counting problems (see the examples in the notes)

Part 5. Propositional Logic and FOL

- understand the notation of propositional logic
- determine whether an expression is a valid formula of propositional logic
- represent statements in propositional logic and understand representations
- determine the truth of a propositional logic formula under an interpretation
- determine under which interpretations the formula is true
- determine whether a propositional logic formula is a tautology or a contradiction
- determine whether two formulas P and Q are logically equivalent
- first-order predicate logic - syntax, semantics and evaluation, a proof system for first-order predicate logic

Part 6. Combinatorics

- understand basic notation for sums, products, and factorials, including the base cases (empty sums and products, 0!)
- be able to count outcomes of unions of disjoint events and sequences of events
- be able to count permutations and k-permutations, and to solve counting problems where these arise
- be able to compute Binomial coefficient and to solve counting problems where these arise
- be able to compute the number of functions from a set A to a set B and to solve counting problems where this arises
- understand the basic notation of discrete probability

Part 7. Probability

- be able to determine the probability of an event, and the probability of unions of events
- be able to compute conditional probabilities and to determine whether events are independent
- be able to compute the expectation of a random variable
- be able to use linearity of expectation to simplify the calculation of expectations