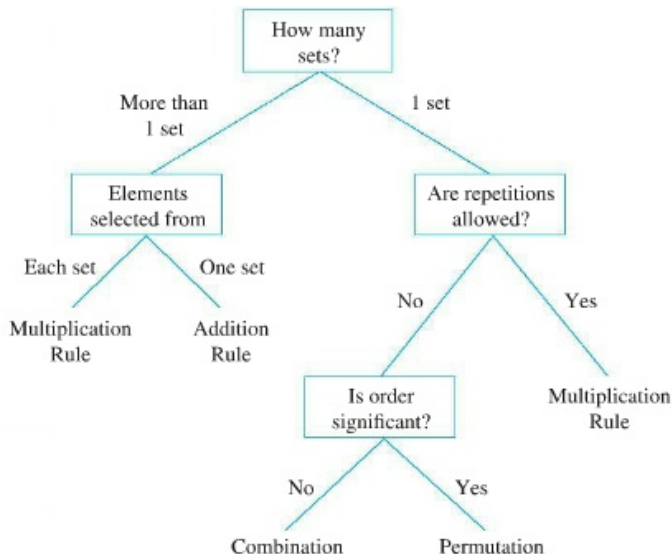


# Mixed Counting Problems

**Problem Solving Strategy:** You may be able to solve a counting problem with a single principle or a problem may be a multilevel problem requiring repeated application of one or several principles. When asked to count the number of objects in a set, it often helps to think of how you might complete the task of constructing an object in the set. It also helps to keep the technique of “overcounting” in mind. The following flowchart from your book may help you decide whether to use the multiplication principle, the addition rule, the formula for the number of permutations or the formula for the number of combinations for a problem or a problem part requiring one of these.

# Mixed Counting Problems



# Mixed Counting Problems

**Example** An experiment consists of rolling a 20 sided die three times. The number on top of each die is recorded. The numbers are written down in the order in which they are observed. How many possible ordered triples of numbers can result from the experiment? (Note the triple  $(17, 10, 3)$  is not the same result as the triple  $(3, 10, 17)$ . )

# Mixed Counting Problems

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There are 20 ways each throw can come up and the order is important so the answer is  $20 \cdot 20 \cdot 20 = 20^3 = 8000$ .

## Mixed Counting Problems

**Example The Hoosier Lottery** When you buy a Powerball ticket, you select 5 different white numbers from among the numbers 1 through 59 (order of selection does not matter), and one red number from among the numbers 1 through 35. How many different Powerball tickets can you buy?

# Mixed Counting Problems

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If you check out the [Powerball web site](#) you will see that you need to select 5 distinct white numbers, so you can do this  $C(59, 5) = 5,006,386$  ways. Then you can pick the red number  $C(35, 1) = 35$  ways so the total number of tickets is  $C(59, 5) \cdot P(35, 1) = 5,006,386 \cdot 35 = 175,223,510$ .

# Mixed Counting Problems

Often problems fit the model of pulling marbles from an urn containing marbles of different colors. For example many of our previous problems involving poker hands fit this model. Polling or taking samples from a population to conduct an observational study or an experiment also fit this model.

**Example: Urn Model:** An urn contains 15 numbered marbles (because they have numbers, two different samples of 4 reds are considered different if the numbers are different, this is useful for calculating probabilities later, when we try to set up an equally likely sample space), of which 10 are red and 5 are white. A sample of 4 marbles is to be selected. **In Parts (a) - (f) of this problem, we are referring to samples of size 4 drawn from the urn described above.**

# Mixed Counting Problems

10 red, 5 white, numbered marbles
-----------------------------------

(a) How many (different) samples (of size 4) are possible?

The order does not matter but the numbers do so we are selecting 4 elements from a set of  $10 + 5$  elements. Hence the answer is  $C(15, 4) = 1,365$ .



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(b) How many samples (of size 4) consist entirely of red marbles?

The order does not matter but the numbers do so we are selecting 4 elements from a set of 10 elements. Hence the answer is  $\mathbf{C(10, 4) = 210}$ .

## Mixed Counting Problems

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(c) How many samples have 2 red and 2 white marbles?

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We can select 2 numbered red marbles in  $\mathbf{C(10, 2)}$  ways and 2 numbered white marbles in  $\mathbf{C(5, 2)}$  ways. Neither choice affects the other so the answer is  $\mathbf{C(10, 2) \cdot C(5, 2) = 45 \cdot 10 = 450}$ .

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(c) How many samples have 2 red and 2 white marbles?

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(d) How many samples (of size 4) have exactly 3 red marbles?

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(d) How many samples (of size 4) have exactly 3 red marbles?

We can select 3 numbered red marbles in  $\mathbf{C(10, 3)}$  ways and 1 numbered white marble in  $\mathbf{C(5, 1)}$  ways. Neither choice affects the other so the answer is

$$\mathbf{C(10, 3) \cdot C(5, 1) = 120 \cdot 5 = 600.}$$

# Mixed Counting Problems

10 red, 5 white, numbered marbles
-----------------------------------

(e) How many samples (of size 4) have at least 3 red?

# Mixed Counting Problems

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The answer is the number of samples with 3 red plus the number of samples with 4 red. We can select 4 numbered red marbles in  $\mathbf{C}(10, 4)$  ways and 0 numbered white marbles in  $\mathbf{C}(5, 0)$  ways. Neither choice affects the other so the answer is  $\mathbf{C}(10, 4) \cdot \mathbf{C}(5, 0) = 210 \cdot 1 = 210$ .

From the last example, there are 600 ways to select samples with exactly 3 red marbles so our answer is  $600 + 210 = 810$ .



## Mixed Counting Problems

10 red, 5 white, numbered marbles
-----------------------------------

(f) How many samples (of size 4) contain at least one red marble?

# Mixed Counting Problems

10 red, 5 white, numbered marbles
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(f) How many samples (of size 4) contain at least one red marble?

One answer is “the number with exactly 1” + “the number with exactly 2” ... “the number with exactly 4”. This is

$$\mathbf{C}(10, 1) \cdot \mathbf{C}(5, 3) + \mathbf{C}(10, 2) \cdot \mathbf{C}(5, 2) + \mathbf{C}(10, 3) \cdot \mathbf{C}(5, 1) + \mathbf{C}(10, 4) \cdot \mathbf{C}(5, 0)$$

which is

$$10 \cdot 10 + 45 \cdot 10 + 120 \cdot 5 + 210 \cdot 1 = 100 + 450 + 600 + 210 = 1,360$$

It is also the total number of samples (1,365) minus the number of samples with no red marbles which is

$$\mathbf{C}(10, 0) \cdot \mathbf{C}(5, 4) = 5.$$

# Mixed Counting Problems

**Example 4:** Recall that a standard deck of cards has 52 cards. The cards can be classified according to suits or denominations. There are 4 suits, hearts, diamonds, spades and clubs. There are 13 cards in each suit. There are 13 denominations, Aces, Kings, Queens, ....., Twos, with 4 cards in each denomination. A poker hand consists of a sample of size 5 drawn from the deck. Poker problems are often like urn problems, with a hitch or two.

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(a) How many poker hands consist of 2 Aces and 3 Kings?

You can pick aces in  $C(4, 2)$  ways and kings in  $C(4, 3)$  ways. Neither choice affects the other so the answer is  $C(4, 2) \cdot C(4, 3) = 6 \cdot 4 = 24$ .

## Mixed Counting Problems

(b) How many poker hands consist of 2 Aces, 2 Kings and a card of a different denomination?

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You can pick the 2 aces, 2 kings in

$C(4, 2) \cdot C(4, 2) = 6 \cdot 6 = 36$  ways. You can pick the remaining card in any of  $52 - 8 = 44$  ways so the answer is  $36 \cdot 44 = 1,584$ .

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There are 13 ways to pick the first denomination. Then are then  $C(4, 3)$  ways to pick 3 cards of that denomination.

There are 12 ways to pick the second denomination and then  $C(4, 2)$  ways to pick 2 cards of that denomination.

Hence there are

$13 \cdot C(4, 3) \cdot 12 \cdot C(4, 2) = 13 \cdot 4 \cdot 12 \cdot 6 = 3,744$ .

## Mixed Counting Problems

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(e) A flush is a hand consisting of five cards from the same suit. How many different flushes are possible?

There are  $C(13, 5)$  ways to get all cards of the same suit so there are  $C(13, 5) \cdot C(4, 1) = 1,287 \cdot 4 = 5,148$  flushes.

## Mixed Counting Problems

Another useful model to keep in mind is that of repeatedly flipping a coin. This is especially useful for counting the number of outcomes of a given type when the experiment involves several repetitions of an experiment with two outcomes. We will explore probabilities for experiments of this type later when we study the Binomial distribution. We have already used this model in taxi cab geometry.

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**Example: Coin Flipping Model** If I flip a coin 20 times, I get a sequence of Heads (H) and tails (T).

(a) How many different sequences of heads and tails are possible?

There are 2 ways the first flip can come up; 2 more for the second and so on. Hence  $2 \cdots 2 = 2^{20} = 1,048,576$ .



## Mixed Counting Problems

(b) How many different sequences of heads and tails have exactly five heads?

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Now we want to keep track of how many heads/tails there are in our sequence. This problem is similar to the taxi cab problem. There are 20 positions which need to be filled with either an 'H' or a 'T'. If we want exactly  $h$  heads in the sequence the answer is  $\mathbf{C(20, h)}$ .

To see we are on the right track recall

$$2^n = \mathbf{C(n, 0)} + \mathbf{C(n, 1)} + \mathbf{C(n, 2)} + \mathbf{C(n, 3)} + \cdots + \mathbf{C(n, n)}$$

so the number of sequences with 0 heads plus the number of sequences with 1 head plus ... plus the number of sequences with 20 heads is all the sequences so should be  $2^{20}$  as in part (a). The actual answer to our problem is  $\mathbf{C(20, 5) = 15,504}$ .

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We did the work in part (b). The answer is

$$\mathbf{C}(20, 0) + \mathbf{C}(20, 1) + \mathbf{C}(20, 2) = 1 + 20 + 190 = 211$$

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$$\mathbf{C(20, 3) + C(20, 4) + \cdots + C(20, 19) + C(20, 20)}.$$

OR

$$\mathbf{2^{20} - (C(20, 0) + C(20, 1) + C(20, 2)) = 1,048,576 - 211 = 1,048,365}$$

## Mixed Counting Problems

**Example** To make a non-vegetarian fajita at Lopez's Grill, you must choose between a flour or corn tortilla. You must then choose one type of meat from 4 types offered. You can then add any combination of extras (including no extras). The extras offered are fajita vegetables, beans, salsa, guacamole, sour cream, cheese and lettuce. How many different fajitas can you make?

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## Extra Problems

**Example** (a) How many different words (including nonsense words) can you make by rearranging the letters of the word

EFFERVESCENCE

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EFFERVESCE

$E \mapsto 5$ ;  $F \mapsto 2$ ;  $R \mapsto 1$ ;  $V \mapsto 1$ ;  $S \mapsto 1$ ;  $C \mapsto 2$ ;  $N \mapsto 1$ . Hence there are  $5 + 2 + 1 + 1 + 1 + 2 + 1 = 13$  letters total and so there are

$$\frac{13!}{2! \cdot 5! \cdot 2! \cdot 1! \cdot 1! \cdot 1! \cdot 1!} = \frac{P(13, 5)}{4} = \frac{51,891,840}{4} = 12,972,960$$

words.

## Extra Problems

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There are 7 distinct letters so if repetitions are not permitted the answer is  $P(7, 4) = 840$ .

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Answer:  $7^4$ . Do not confuse this with the MUCH harder problem of given 13 tiles with the letters in EFFERVESCENCE, how many 4 letter words can be produced? So for example, you could use F twice but not 3 times.

## Extra Problems

**Example** The Notre Dame Model UN club has 20 members. Five are seniors, four are juniors, two are sophomores and nine are freshmen.

(a) In how many ways can the club select a president, a secretary and a treasurer if every member is eligible for each position and no member can hold two positions?

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20 members, 3 officers so  $\mathbf{P}(20, 3)$ . Note you are selecting an ordered subset of 3 distinct elements.

(b) In how many ways can the club choose a group of 5 members to attend the next Model UN meeting in Washington.

Answer:  $\mathbf{C}(20, 5)$ . This time you need a subset of all the members which has 5 elements but the order isn't important.

## Extra Problems

(c) In how many ways can the club choose a group of 5 members to attend the next Model UN meeting in Washington if all members of the group must be freshmen?

Answer:  $C(9, 5)$  since you now must select your subset from the set of 9 freshmen.

## Extra Problems

(d) In how many ways can the group of five be chosen if there must be at least one member from each class?

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There are 5 ways to select a senior, 4 ways to select a junior, 2 ways to select a sophomore and 9 ways to select a freshman. This gives  $5 \cdot 4 \cdot 2 \cdot 9 = 360$  ways to select a subset with 4 elements containing one member of each class. When you have done this there are  $20 - 4 = 16$  members left and you may choose any one of these to round out the group. Hence the answer is  $\frac{360 \cdot 16}{2} = 2,880$ . You must divide by 2 because each set of 5 elements selected by this procedure occurs twice.



## Extra Problems

Here is another approach. Because there are 5 members in the subset and 4 classes, exactly one class occurs twice. If there are 2 seniors, these can be selected in  $C(5, 2)$  ways and the set filled out with 1 junior, 1 sophomore and 1 freshman, hence in  $C(5, 2) \cdot 4 \cdot 2 \cdot 9 = 720$  ways. If there are 2 juniors, these can be selected in  $C(4, 2)$  ways and the set filled out with 1 senior, 1 sophomore and 1 freshman, hence in  $5 \cdot C(4, 2) \cdot 2 \cdot 9 = 540$  ways. If 2 sophomores,  $5 \cdot 4 \cdot C(2, 2) \cdot 9 = 180$ . If 2 freshmen,  $5 \cdot 4 \cdot 2 \cdot C(9, 2) = 1,440$ . Hence the answer is  $720 + 540 + 180 + 1,440 = 2,880$ .

## Extra Problems

**Example** Harry Potter's closet contains 12 numbered brooms, of which 8 are Comet Two Sixty's (numbered 1 - 8) and 4 are Nimbus Two Thousand's (Numbered 9-12). Harry, Ron, George and Fred want to sneak out for a game of Quidditch in the middle of the night. They don't want to turn on the light in case Snape catches them. They reach in the closet and pull out a sample of 4 brooms.

## Extra Problems

(a) How many different samples are possible?

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This is not a well-defined question. Do you want to know how many different sets of brooms you can get or do you want to know how many ways there are if we keep track of which broom Harry gets, which one Ron gets, and so on.

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In other words, do you want **subsets** or **ordered subsets**?

For subsets, the answer is  $\mathbf{C}(12, 4) = 495$ ; for ordered subsets the answer is  $\mathbf{P}(12, 4) = 11,880$ .

## Extra Problems

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The unordered version solution is familiar. There are  $C(8, 1) = 8$  ways to pick the Comet Two Sixty and  $C(4, 3) = 4$  ways to pick the rest so the answer is  $8 \cdot 4 = 32$ .

To do the ordered version, observe that once you have an unordered set of 4 elements, there are  $4! = 24$  ways to order it. Hence the ordered answer is  $32 \cdot 24 = 768$ .

## Extra Problems

(d) How many samples have at least 3 Comet Two Sixty's?

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Figure out how many samples there are with exactly 3; then figure out how many there are with exactly 4 and then add the two answers.

For exactly  $k$  Comet Two Sixty's we have

$C(8, k) \cdot C(4, 4 - k)$  unordered subsets and therefore

$C(8, k) \cdot C(4, 4 - k) \cdot 4!$  ordered ones.