Discrete Mathematics and Statistics (CPT107)
Tutorial I
Part 1. Number Systems and Proof Techniques

1. Give examples of natural numbers x and y such that $x - y$ is not a natural number. Such an example shows that the natural numbers are not "closed under subtraction".

2. Give examples of integers x and y such that x/y is not an integer. Such an example shows that the integers are not "closed under division".

3. Consider an operation which takes numbers x and y and returns $x^2 - y$. Which of the following number systems are closed under this operation?

    - The natural numbers?

    - The positive integers?

    - The integers?

    - The rationals?

4. Show that the rational numbers are closed under subtraction. That is, show that if p and q are rational numbers, then $q - p$ is a rational number.

5. In the notes, we defined a positive integer to be "even" if it has 2 as a factor, and "odd" otherwise. Write down a list of all prime numbers that are even.

6. Show that if positive integers x and y are even, then so is $x + y$.

7. Show that if a positive integer $x > 2$ is even, then $x - 2$ is also an even positive integer.

8. Prove by induction that, for all $n \in Z_+$, $\sum_{i=1}^{n}(-1)^i i^2 = (-1)^n n(n+1)/2$.

$$\sum_{i=1}^{n}(-1)^i i^2 = \frac{(-1)^n n(n+1)}{2}$$

9. Prove that for any real number $x > -1$ and any positive integer n, $(1+x)^n \geq 1 + nx$.

10. Please use the Proof by Contradiction to show that the square root of 3 is irrational.