~~Q~~Question I

1). According to theorem : a function is invertible if and only if it's a bijection.

For Injection, $\exists f(x_1) = f(x_2)$ then $x_1 = x_2$

In this function $f(2) = \frac{2}{1+2^2} = \frac{2}{5}$, $f(\frac{1}{2}) = \frac{\frac{1}{2}}{1+\frac{1}{4}} = \frac{\frac{1}{2}}{\frac{5}{4}} = \frac{2}{5}$

$f(2) = f(\frac{1}{2})$ and $2 \neq \frac{1}{2}$ $\Rightarrow$ this function is not injective $\Rightarrow$ the function isn't bijective

$\Rightarrow$ the function is not invertible

In all, the inverse function does not exist.

2) Proof: ~~f~~ $\forall x \in D$, where D is the set of domain, $f(x) = f(-x)$, $g(x) = g(-x)$.

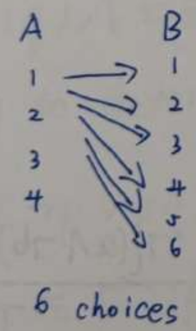
according to the even function defination

$\forall x \in D$,

$(f \circ g)(x) = f(g(x)) = f(g(-x)) = (f \circ g)(-x)$

Therefore, $(f \circ g)$ is also an even function.

3) (a)



As one element will return one element~~s~~ from B based on function, it has $|B| = 6$ choices.

each element from A choices are independent and are adimitted ~~to~~ with replacement.

Therefore we have $6^4 = 1296$ functions from A to B.

(b) "one to one" means once elements from B is choosed by a element from A it can not be choosed by another from A

Therefore we ~~has~~ can consider this as a Permutation without repetition question.

which means we have $P_6^4 = \frac{\overset{3}{\cancel{6}} \times 5 \times \cancel{4} \times \cancel{3}}{\cancel{4} \times \cancel{3} \times \cancel{2} \times 1} = 15$

In all 15 of these are one-to-one.

(c) onto means every B exists a $a \in A$ that $b = f(a)$ but we have $|B| = 6$, $|A| = 4$, $6 > 4$

Therefore it must exist two elements from B don't exist $b = f(a)$

In all, no of them are "onto"

4) Proof: Let 101 integers can be written as $2^k \cdot q$, $q \in \{1, 3, 5, \dots 199\}$

so q has 100 choices, As $101 > 100$

If 101 integers choose different q, there must exist one integer can not choose, which means there must exist a case that $\exists x_1, x_2$, $x_1 = 2^r \cdot q_1$ and $x_2 = 2^s \cdot q_1$.

so ~~$x_2/x_1 \in$~~ Assume that $x_2 > x_1$ then $\frac{x_2}{x_1} = \frac{2^s}{2^r} = 2^{s-r} \geqslant 2^1 = 2$.

which means x_2 can be divided by x_1

In all, if 101 integers are selected from S, there are two integers that one divides the other

Question II

~~1) (a) $q \vee r \Rightarrow p$. $p \Leftarrow q \vee r$~~

~~(b) $q \vee r \Rightarrow p$.~~

1) (a) $p \Rightarrow q \vee r$.

(b) $q \vee r \Rightarrow p$

2) Truth table is below.

a	$\neg a$	b	$\neg b$	$a \wedge b$	$a \vee b$	$\neg a \wedge b$	$a \wedge \neg b$	$\neg[(a \wedge \neg b) \vee (\neg a \wedge b)] \wedge (a$
T	F	T	F	T	T	F	F	T
T	F	F	T	F	T	F	T	F
F	T	T	F	F	T	T	F	F
F	T	F	T	F	F	F	F	F

Based on the defination, logical equivalence of P, Q if $P \equiv Q$.

From truth table, $(a \wedge b) \equiv \neg[(a \wedge \neg b) \vee (\neg a \wedge b)] \wedge (a \vee b)$

Therefore, they are logical equivalent.

(jessie, carol).

(b) $\exists x,\ Female(x) \land Cousin(x, jessie)$.

(c) $\forall x,\ Cousin(x, paul) \rightarrow Cousin(x, carol)$.

(d) $\exists x \exists y,\ Male(x) \land Male(y) \land Cousin(x, jessie) \land Cousin(y, jessie) \land x \neq y$

4) $((P \land \neg q) \Rightarrow \neg q) \equiv (\neg(P \land \neg q) \lor \neg q)$; according to theorem that $\neg(A \land B) \equiv (\neg A \lor \neg B)$

$\equiv ((\neg P \lor q) \lor \neg q)$; according to theorem that $(A \cup B) \cup C \equiv A \cup (B \cup C)$

$\equiv (\neg P \lor (q \lor \neg q))$

$\equiv (\neg P \lor T)$

$\equiv T$

according to excluded middle, ethier q or $\neg q$ is true, so $(q \lor \neg q) \equiv T$

There fore it's a tautology

$(\neg P \lor \neg q) \land (\neg p \land q) \equiv ([(\neg p \lor \neg q) \land q] \land \neg p)$, according to theorem that $(A \land B) \land C \equiv A \land (B \land C)$

$[(\neg p \lor \neg q) \land q] \equiv [q \land (\neg p \lor \neg q)] \equiv (q \land \neg p) \lor (q \land \neg q) \equiv (\neg p \land q) \lor F \equiv (\neg p \land q)$

according to therorem $A \land (B \lor C) \equiv (A \land B) \lor (A \land C)$, $q \land \neg q = F$

Then $[(\neg p \lor \neg q) \land q] \land \neg p \equiv (\neg p \land q) \land \neg p \equiv (\neg p \land q)$

may not $\equiv F$ if $p = F$ and $q = T$

Back to normal, $(\underbrace{\neg F}_{T} \lor \underbrace{\neg T}_{F}) \land (\underbrace{\neg F}_{T} \land T) \equiv T$.

Therefore, it's not a contradiction

5) (a) There exists a real number x, such that x is greater than or to any number.

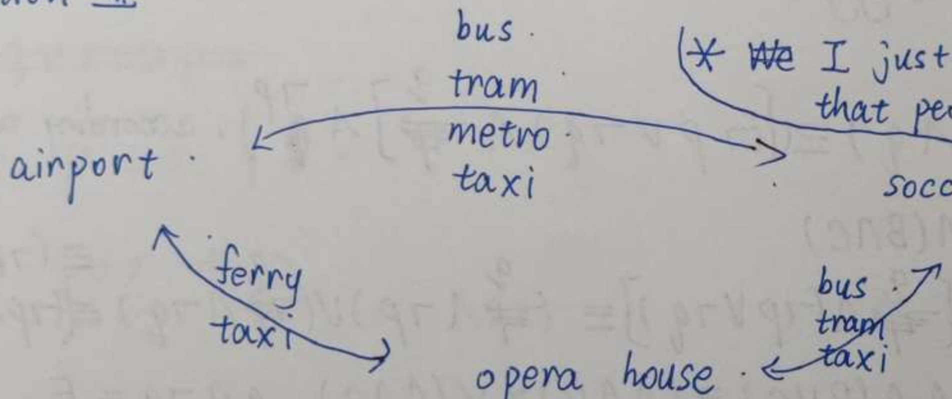
(b) For every real numbers x and y, if x is less than y, then there exists a real number z such that x is less than z and z is less than y.

(c) For every real number x, there exists a real number y that the product of x and y is greater than x

(d) There does not exist a real number x such that for every real number y, x is greater than y.

Question III

1).



(※ ~~We~~ I just calculate the condition that people go will not turn back. like $a \to c \to a \to c \to a$ is not in my consideration if this is true then (a) (b) will be infinite!

(a) directly: 2.

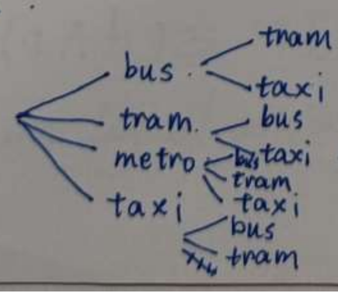
indirectly: use product rule $\prod_{i \in m} |A_i| = 4 \times 3 = 12$.

In all, use sum rule $2 + 12 = 14$.

(b) from (a) we know there are 14 ways to go, which means there are also 14 ways back. Both events are indpendent, use product rule: $14 \times 14 = 196$

(c) directly: 2.

indirectly:



$\Rightarrow 9$

In all, there are 11 ways from airport to opera house with any form of transport at most once directly : 4

indirectly : as there are at most 2 forms of transport, which fulfills the condition, so it's $2 \times 3 = 6$ (product rule) As two ways are indpe

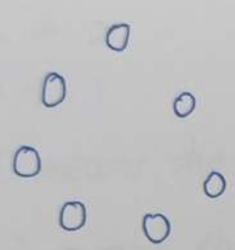
In all $4 + 6 = 10$ (sum rule)

So There are 10 ways to travel from airport to soccer stadium.

2) (a) _ _ _ _ _ _ _
1 2 3 4 5 6 7.

This is a permutations without repetition : $P_7^7 = 7! = 5040$

(b)



firstly, we can consider as the row of chairs which is $7!$.

secondly, we divide the duplicate condition :

_ _ _ _ _ _ _
1 2 3 4 5 6 7.

A B C D E F G.
B C D E F G A
G A B C D E F

7 condition is the same in a circular table

Therefore, $m = \dfrac{7!}{7} = 6! = 720$

3) (a) as every shirt must be hung ~~of~~ on the left of every suit.

we can consider these as two independent events: shirt · suit

As every shirts and suits are different, we use permutation within product rule

$P_6^6 \times P_4^4 = 6! \times 4! = 17280$

therefore, 17280 ways to choose.

(b) firstly Math: $P_8^8 = 8!$

Physics: $P_5^5 = 5!$

Chemistry: $P_7^7 = 7!$

Secondly: M, P, C in Permutation: $P_3^3 = 3!$

As all of them are ~~indpent~~ independent, we use product rule:

$8! \times 5! \times 7! \times 3! \approx 1.46 \times 10^{11}$

Therefore there are approximate to 1.46×10^{11} ways.

4) (a) One application can not catch two positions.

- As position is differet, we suggest as p_1, p_2, p_3, p_4, p_5

$p_1: 60 \rightarrow p_2: 59 \rightarrow p_3: 58 \rightarrow p_4: 57 \rightarrow p_5: 56$.

which is a ~~permanution~~ permutation: $P_{60}^5 = 60 \times 59 \times 58 \times 57 \times 56 = 655381440$

There are 655381440 ways.

(b). we can consider this as a combination question.

firstly choose 3 from 60 to join in. Functionl Analysis: C_{60}^3.

Secondly choose 2 from (60-3) to join in Universal Algebra: C_{57}^2.

Two are independent Can be switch order but with same result!

In all: $C_{60}^3 \times C_{57}^2 = \frac{60 \times 59 \times 58}{3 \times 2 \times 1} \times \frac{57 \times 56}{2} = 54615120$

Therefore, there are 54615120 ways.

...stion IV:

1) $P = \frac{n(s)+n(j)}{N} = \frac{6+3}{5+6+3+2} = \frac{9}{16}$.

The probability that chairperson is a sophomore or junior is $\frac{9}{16}$

2) Let $P(Y)$ means given that sum of spots of the three dice was six prob

$6 = 1+1+4$
$= 1+2+3$
$= 1+3+2$
$= 1+4+1$
$= 2+1+3$
$= 2+2+2$
$= 2+3+1$
$= 3+1+2$
$= 3+2+1$
$= 4+1+1$

$P(Y) = \frac{10}{6\times6\times6} = \frac{10}{216}$

Let $P(X)$ means the probability of three 2s when rolling

$P(X\cap Y)$ means the probability that three 2s are rolling and the sum of spots is 6

$P(X\cap Y) = (\frac{1}{6})^3 = \frac{1}{216}$

According to ~~Combinational~~ Conditional probability $P(X|Y) = \frac{P(X\cap Y)}{P(Y)} = \frac{\frac{1}{216}}{\frac{10}{216}} = \frac{1}{10}$

which is given the sum 6, the prob of each is 2

In all, the probability of getting three twos if it's know that sum of spots of three dice was six is $\frac{1}{10}$

3) (a) Let Thomas can pass all three examinations as $P(\underset{E_1}{x}\cap \underset{E_2}{Y}\cap \underset{E_3}{Z})$.

$$P(Z \mid x\cap Y) = \frac{P(x\cap Y\cap Z)}{\cancel{P(x\cap Y)}\ P(x\cap Y)} = C.$$

$$P(Y\mid x) = \frac{P(x\cap Y)}{P(x)} = B$$

$$P(x) = A$$

$$\Rightarrow P(x\cap Y\cap Z) \cancel{= C \cdot P(x\cap Y) = C\cdot B\cdot P(x) =}$$

$$= \frac{P(x\cap Y\cap Z)}{P(x\cap Y)} \times \frac{P(x\cap Y)}{P(x)} \times P(x)$$

$$= C \times B \times A = ABC$$

In all, the probability that Thomas can pass all three examinations is (ABC).

(b) $P(\hat{Z} \mid x\cap Y) = 1-C = \dfrac{P(x\cap Y\cap \hat{Z})}{P(x\cap Y)}$.

only pass the second means fail at the third one

we need to calculate $P(x\cap Y\cap \hat{Z})$.

As $P(x\cap Y)$ from (a) we know is $P(x)\cdot P(Y\mid x) = AB$.

so $P(x\cap Y\cap \hat{Z}) = P(x\cap Y)\cdot P(\hat{Z}\mid x\cap Y)$.

$$= AB\cdot(1-C).$$

$$= AB - ABC$$

In all, the probability that Thomas only passes second examination is $AB - ABC$.

$x = 1: P_X(x) = \frac{C_5^1}{2^5} = \frac{5}{32}$. (choose 5 question that which one is correct)

$x = 2: P_X(x) = \frac{C_5^2}{2^5} = \frac{10}{32}$.

$x = 3: P_X(x) = \frac{C_5^3}{2^5} = \frac{10}{32}$.

$x = 4: P_X(x) = \frac{C_5^4}{2^5} = \frac{5}{32}$.

$x = 5: P_X(x) = \frac{C_5^5}{2^5} = \frac{1}{32}$

$$E[X] = \sum_{x \in X} x P_X(x) = \cancel{0 \cdot \frac{1}{32}} + 1 \cdot \frac{5}{32} + 2 \cdot \frac{10}{32} + 3 \cdot \frac{10}{32} + 4 \cdot \frac{5}{32} + 5 \cdot \frac{1}{32}$$

$$= \frac{5}{32} + \frac{20}{32} + \frac{30}{32} + \frac{20}{32} + \frac{5}{32}$$

$$= \frac{80}{32} = \frac{10}{4} = 2.5$$

Therefore, the expected number of correct answer is 2.5.