Let R be the relation on the set $\mathbb{R}$ real numbers defined by xRy iff $x - y$ is an integer. Prove that R is an equivalence relation on $\mathbb{R}$.

PROOF.
   I. Reflexive: Suppose $x \in \mathbb{R}$. Then $x - x = 0$, which is an integer. Thus, xRx.
   II. Symmetric: Suppose $x, y \in \mathbb{R}$ and xRy. Then $x - y$ is an integer. Since $y - x = -(x - y)$, $y - x$ is also an integer. Thus, yRx.
   III. Suppose $x, y \in \mathbb{R}$, xRy and yRz. Then $x - y$ and $y - z$ are integers. Thus, the sum $(x - y) + (y - z) = x - z$ is also an integer, and so xRz.

Thus, R is an equivalence relation on $\mathbb{R}$. $\square$

Equality: $x = y$

Answer:

- $x = x$ is True, so equality is reflexive
- If $x = y$, then $y = x$ is True, so equality is symmetric
- If $x = y$ and $y = x$, then $x = y$ is True, so equality is antisymmetric
- If $x = y$ and $y = z$, then $x = z$ is True, so equality is transitive

Not equal: $x \neq y$

Answer:

- $x \neq x$ is False, so not equal is not reflexive
- If $x \neq y$, then $y \neq x$ is True, so not equal is symmetric
- If $x \neq y$ and $y \neq x$, then $x = y$ is False, so equal is not antisymmetric
- If $x \neq y$ and $y \neq z$, then $x \neq z$ is False, so not equality is not transitive

Less than: $x < y$

Answer:

- $x < x$ is False, so less than is not reflexive
- If $x < y$, then $y < x$ is False, so less than is not symmetric
- If $(x < y$ and $y < x)$, then $x = y$ is True, so less than is antisymmetric (The conditional is True because the premise $(x < y$ and $y < x)$ is False)
- If $x < y$ and $y < z$, then $x < z$ is True, so less than is transitive

Greater than or equal: $x \geq y$

Answer:

- $x \geq x$ is True, so greater than or equal is reflexive
- If $x \geq y$, then $y \geq x$ is False, so greater than or equal is not symmetric
- If ($x \geq y$ and $y \geq x$), then $x = y$ is True, so greater than or equal is antisymmetric
- If $x \geq y$ and $y \geq z$, then $x \geq z$ is True, so greater than or equal is transitive