Discrete Mathematics and Statistics (CPT107)
Tutorial 8 - Solutions

1.
   - This car is red and your house is blue.
   - You are not tall.
   - Your house is not blue or you are not tall.

   - $\neg p_3$.
   - $(\neg p_1 \vee p_2)$.
   - $(\neg p_1 \vee p_3)$.
   - $\neg\neg p_1$.

2. $(P \rightarrow Q) = (\neg P \vee Q)$.

3.

p_1	p_2	$\neg p_2$	$(p_1 \vee \neg p_2)$	Q
1	1	0	1	0
1	0	1	1	0
0	1	0	0	1
0	0	1	1	0

Therefore $I(P) = 1$ and $I(Q) = 0$.

4.

p_1	p_2	p_3	$\neg p_2$	$(p_1 \wedge \neg p_2)$	P
1	1	1	0	0	0
1	1	0	0	0	0
1	0	1	1	1	1
1	0	0	1	1	0
0	1	1	0	0	0
0	1	0	0	0	0
0	0	1	1	0	0
0	0	0	1	0	0

P is true under the interpretation I with $I(p_1) = 1$, $I(p_2) = 0$, and $I(p_3) = 1$. No other interpretation makes P true.

5. A formula P is a tautology if and only if $I(P) = 1$ for every interpretation I.

6. (a) $(p_1 \vee p_1)$ is not a tautology. Take I with $I(p_1) = 0$.
   (b) $(\neg p_1 \vee (p_2 \wedge p_1))$ is not a tautology. Take I with $I(p_1) = 1$ and $I(p_2) = 0$.
   (c) $(\neg\neg p_1 \leftrightarrow p_1)$ is a tautology.
   (d) $(\neg p_1 \rightarrow \neg p_1)$ is a tautology.
7. A formula P is a contradiction if and only if $I(P) = 0$ for every interpretation I.
8. (a) $(\neg p_1 \wedge p_2)$ is not a contradiction. Take an I such that $I(p_1) = 0$ and $I(p_2) = 1$.
   (b) $(p_1 \rightarrow \neg p_1)$ is not a contradiction. Take an I such that $I(p_1) = 0$.
   (c) $(p_1 \leftrightarrow \neg p_1)$ is a contradiction.
   (d) $(p_1 \wedge (\neg p_2 \vee \neg p_1))$ is not a contradiction. Take an I such that $I(p_1) = 1$ and $I(p_2) = 0$.
9. $\Gamma \models P$ if and only if, for every interpretation I, the following holds: if $I(Q) = 1$ for all $Q \in \Gamma$, then $I(P) = 1$.
10. (a) $\{p_1, (p_1 \rightarrow p_2)\} \models p_2$ is true.
    (b) $\{(p_1 \rightarrow p_2)\} \models (p_2 \rightarrow p_1)$ is not true. Take I such that $I(p_1) = 0$ and $I(p_2) = 1$. Then $I((p_1 \rightarrow p_2)) = 1$ but $I((p_2 \rightarrow p_1)) = 0$.
    (c) $\{(p_1 \vee \neg p_2)\} \models p_1$ is not true. Take I such that $I(p_1) = 0$ and $I(p_2) = 0$. Then $I((p_1 \vee \neg p_2)) = 1$ but $I(p_1) = 0$.
11. $P \equiv Q$ if and only if for all interpretations I: $I(P) = I(Q)$.
12.

P	Q	R	$(Q \wedge R)$	$(P \vee (Q \wedge R))$	$(P \vee Q)$	$(P \vee R)$	$((P \vee Q) \wedge (P \vee R))$
0	0	0	0	0	0	0	0
0	0	1	0	0	0	1	0
0	1	0	0	0	1	0	0
0	1	1	1	1	1	1	1
1	0	0	0	1	1	1	1
1	0	1	0	1	1	1	1
1	1	0	0	1	1	1	1
1	1	1	1	1	1	1	1

13. No, $\neg(P \wedge Q)$ is not logically equivalent to $(\neg P \wedge \neg Q)$. Take an interpretation I such that $I(P) = 1$ and $I(Q) = 0$. Then $I(\neg(P \wedge Q)) = 1$ but $I(\neg P \wedge \neg Q) = 0$. One of *De Morgan's Laws* states that

$$\neg(P \wedge Q) \equiv (\neg P \vee \neg Q).$$

This is different to the incorrect equivalence in the question because $\wedge$ is replaced by $\vee$.