Discrete Mathematics and Statistics (CPT107)
Tutorial 8 bis - Solutions

1. Let S = it is sunny, C = camping is fun, H = the homework is done, and M = mathematics is easy.

a. Translate the following proposition into the most natural equivalent statement in English. Try to make the sentence as simple and as natural as possible.

$$(M \rightarrow H) \wedge (S \rightarrow C)$$

Solution: Any is done of the when following mathematics solutions is easy, (and similar) and camping are equally is fun valid: when "The it is sunny." "If mathematics homework is easy then the homework is done, and if it is sunny then camping is fun."

b. Translate the following statement into propositional logic.

"Mathematics is easy or camping is fun, as long as it is sunny and the homework is done."

Solution: Either of these two solutions (and logically equivalents) are valid:

$$(S \wedge H) \rightarrow (M \vee C)$$

and

$$(S \wedge H) \leftrightarrow (M \vee C)$$

2. Three boxes are presented to you. One contains gold, the other two are empty. Each box has imprinted on it a clue as to its contents; the clues are
(Box 1) The gold is not here
(Box 2) The gold is not here
(Box 3) The gold is in Box 2
Only one message is true; the other two are false. Which box has the gold? Formalize the puzzle in Propositional Logic and find the solution using a truth table.

Solution *Let B_i with $i \in \{1, 2, 3\}$ stand for "gold is in the i-th box". With this language we can formalize the messages on the boxes as follows:*

$$\boxed{\neg B_1} \qquad \boxed{\neg B_2} \qquad \boxed{B_2}$$

We can also formalize the statements of the problem as follows:

1. *One box contains gold, the other two are empty.*

$$(B_1 \wedge \neg B_2 \wedge \neg B_3) \vee (\neg B_1 \wedge B_2 \wedge \neg B_3) \vee (\neg B_1 \wedge \neg B_2 \wedge B_3) \tag{1}$$

2. *Only one message is true; the other two are false.*

$$(\neg B_1 \wedge \neg\neg B_2 \wedge \neg B_2) \vee (\neg\neg B_1 \wedge \neg B_2 \wedge \neg B_2) \vee (\neg\neg B_1 \wedge \neg\neg B_2 \wedge B_2) \tag{2}$$

(2) is equivalent to:

$$(B_1 \wedge \neg B_2) \vee (B_1 \wedge B_2) \tag{3}$$

Let us compute the truth table for (1) and (3)

B_1	B_2	B_3	(1)	(3)
T	T	T	F	T
T	T	F	F	T
T	F	T	F	T
T	F	F	T	T
F	T	T	F	F
F	T	F	T	F
F	F	T	T	F
F	F	F	F	F

The only assignment I that verifies both (1) and (3) is the one with $I(B_1) = T$ and $I(B_2) = I(B_3) = F$, which implies that the gold is in the first box.