Discrete Mathematics and Statistics (CPT 107)
Tutorial 3

1. List the set of ordered pairs and draw the graphical representation of the relation R between $\{1,2,3,4\}$ and $\{a, b, c\}$ with the matrix:

$$M = \begin{bmatrix} T & F & F \\ F & F & T \\ F & T & F \\ T & F & F \end{bmatrix}$$

2. Let R be the relation on $\{1,2,3,4\}$ given by xRy if and only if $\mathrm{x} - \mathrm{y} = 0$.
   Represent R in the following ways:

   - as a set of ordered pairs;
   - in graphical form;
   - in matrix form.

3. Determine which of the following relations on the set of people is reflexive, symmetric, or transitive:

   (a) 'has the same parents as'
   (b) 'is a brother of'
   (c) 'is at least as clever as'.

4. For each of the following relations on

$$A = \{\mathrm{a} \mid \mathrm{a} \in \mathbb{Z} \text{ and } 1 \leq \mathrm{a} \leq 12\}$$

   list the ordered pairs belonging to:

   - $S_1 = \{(\mathrm{a}, \mathrm{b}) \in \mathrm{A} \times \mathrm{A} \mid \mathrm{a} \cdot \mathrm{b} = 9\}$
   - $S_2 = \{(\mathrm{a}, \mathrm{b}) \in \mathrm{A} \times \mathrm{A} \mid 2\mathrm{a} = 3\mathrm{b}\}$

5. Is there a mistake in the following proof that any transitive and symmetric relation R is reflexive? If so, what is it?
   Let aRb. By symmetry, bRa. By transitivity, if aRb and bRa, then aRa. This proves reflexivity.

6. For each of the following equivalence relations R on a given set A, describe the equivalence classes E_x into which the relation partitions the set A:

   (a) A is the set of books in a library; R is given by xRy if and only if the colour of $x's$ cover is the same as the colour of $y's$ cover.

(b) $A = \mathbb{Z}$; R is given by xRy if and only if $x - y$ is even.
(c) A is the set of people; R is given by xRy if and only if x has the same sex as y.

7. Define an equivalence relation R on $\mathbb{N}$ as follows: xRy if and only if 3 is a divisor of $x - y$. Determine the equivalence classes

   - E_0 of 0;
   - E_1 of 1;
   - E_3 of 3.