

Discrete Mathematics and Statistics (CPT 107)

Tutorial 8

1. Suppose p_1 represents 'This car is red', p_2 represents 'Your house is blue' and p_3 represents 'You are tall'.

- What does $(p_1 \wedge p_2)$ represent?
- What does $\neg p_3$ represent?
- What does $\neg(p_2 \wedge p_3)$ represent?

Represent in propositional logic:

- You are not tall.
- If this car is red, then your house is blue.
- This car is not red or you are tall.
- It is not the case that this car is not red.

2. Define $P \rightarrow Q$.

3. Let $P = (p_1 \vee \neg p_2)$ and $Q = \neg P$. Let I be an interpretation such that $I(p_1) = 0$ and $I(p_2) = 0$. Determine $I(P)$ and $I(Q)$ using a truth table.

4. Let $P = ((p_1 \wedge \neg p_2) \wedge p_3)$. Write down the truth table for P . Determine the interpretations under which P is true.

5. What is a tautology?

6. Which of the following are tautologies? Check using truth tables.

- (a) $(p_1 \vee p_1)$;
- (b) $(\neg p_1 \vee (p_2 \wedge p_1))$;
- (c) $(\neg \neg p_1 \leftrightarrow p_1)$;
- (d) $(\neg p_1 \rightarrow \neg p_1)$.

7. What is a contradiction?

8. Which of the following are contradictions? Check using truth tables.

(a) $(\neg p_1 \wedge p_2)$;

(b) $(p_1 \rightarrow \neg p_1)$;

(c) $(p_1 \leftrightarrow \neg p_1)$;

(d) $(p_1 \wedge (\neg p_2 \vee \neg p_1))$.

9. Define the meaning of $\Gamma \models P$.

10. Which of the following are true? Check using truth tables.

(a) $\{p_1, (p_1 \rightarrow p_2)\} \models p_2$;

(b) $\{(p_1 \rightarrow p_2)\} \models (p_2 \rightarrow p_1)$;

(c) $\{(p_1 \vee \neg p_2)\} \models p_1$;

11. Define the relation \equiv of logical equivalence.

12. Show using truth tables

$$(P \vee (Q \wedge R)) \equiv ((P \vee Q) \wedge (P \vee R)).$$

13. Is $\neg(P \wedge Q)$ logically equivalent to $(\neg P \wedge \neg Q)$? Discuss the relationship to *De Morgan's Law*.