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$$\mathbf{C}(40,7) = \frac{40!}{7! \cdot 33!} = \frac{40 \cdot 39 \cdot 38 \cdot 37 \cdot 36 \cdot 35 \cdot 34}{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} =$$

$$10 \cdot 39 \cdot 38 \cdot 37 \cdot 34 = 18,643,560$$

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$$\mathbf{C}(52,5) = \frac{52!}{5! \cdot 47!} = 2,598,960$$

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There are 26 red cards so $\mathbf{C}(26, 5) = \dfrac{26!}{5! \cdot 21!} = 65,780$

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There are 4 kings and 4 queens. We can select 2 kings in $\mathbf{C}(4,2)$ ways and we can select 3 queens in $\mathbf{C}(4,3)$ ways. We can distinguish kings from queens so the answer is $\mathbf{C}(4,2) \cdot \mathbf{C}(4,3) = 6 \cdot 4 = 24.$

Bonus meditation: Why is

$$\mathbf{C}(8,5) = \mathbf{C}(4,4) \cdot \mathbf{C}(4,1) + \mathbf{C}(4,3) \cdot \mathbf{C}(4,2) + \mathbf{C}(4,2) \cdot \mathbf{C}(4,3) + \mathbf{C}(4,1) \cdot \mathbf{C}(4,4) \ ?$$

Combinations

Example (Quality Control) A factory produces light bulbs and ships them in boxes of 50 to their customers. A quality control inspector checks a box by taking out a sample of size 5 and checking if any of those 5 bulbs are defective. If at least one defective bulb is found the box is not shipped, otherwise the box is shipped. How many different samples of size five can be taken from a box of 50 bulbs?

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$\mathbf{C}(50, 5) = 2, 118, 760.$

Example If a box of 50 light bulbs contains 20 defective light bulbs and 30 non-defective light bulbs, how many samples of size 5 can be drawn from the box so that all of the light bulbs in the sample are good?

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Example If a box of 50 light bulbs contains 20 defective light bulbs and 30 non-defective light bulbs, how many samples of size 5 can be drawn from the box so that all of the light bulbs in the sample are good?

$\mathbf{C}(30, 5) = 142,506.$

Problems using a mixture of counting principles

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There are $\mathbf{C}(4,2)$ ways to get 2 kings and $\mathbf{C}(48,3)$ ways to fill out the hand. Hence there are $\mathbf{C}(4,2)\cdot\mathbf{C}(48,3)$ hands with exactly 2 kings. There are $\mathbf{C}(4,3)\cdot\mathbf{C}(48,2)$ hands with exactly 3 kings and there are $\mathbf{C}(4,4)\cdot\mathbf{C}(48,1)$ hands with exactly 4 kings. Hence there are
$\mathbf{C}(4,2)\cdot\mathbf{C}(48,3)+\mathbf{C}(4,3)\cdot\mathbf{C}(48,2)+\mathbf{C}(4,4)\cdot\mathbf{C}(48,1)$
hands with at least two kings. The number is
$6\cdot 17,296+4\cdot 1,128+1\cdot 48=108,336.$

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Example In the Notre Dame Juggling club, there are 5 graduate students and 7 undergraduate students. All would like to attend a juggling performance in Chicago. However, they only have funding from Student Activities for 5 people to attend. The funding will only apply if at least three of those attending are undergraduates. In how many ways can 5 people be chosen to go to the performance so that the funding will be granted?

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Again it is useful to break the problem up, in this case by number-of-undergraduates. We need to work out how many ways we can get 3 undergraduates, how many ways we can get 4 undergraduates, how many ways we can get 5 undergraduates, and then add these numbers.

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Three undergraduates: $\mathbf{C}(7,3) \cdot \mathbf{C}(5,2)$; Four undergraduates: $\mathbf{C}(7,4) \cdot \mathbf{C}(5,1)$; Five undergraduates: $\mathbf{C}(7,5) \cdot \mathbf{C}(5,0)$. The number is
$35 \cdot 10 + 35 \cdot 5 + 21 \cdot 1 = 546$.

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Remark: $\mathbf{C}(7,3) \cdot \mathbf{C}(9,2) = 1,260$. Why is this NOT the right answer?

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Remark: $\mathbf{C}(7,3)\cdot\mathbf{C}(9,2)=1,260.$ Why is this NOT the right answer? $350+\frac{4!}{3!\cdot 1!}\cdot 175+\frac{5!}{3!\cdot 2!}\cdot 21=1,260.$

Problems using a mixture of counting principles

Example Gino's Pizza Parlor offers 3 three types of crust, 2 types of cheese, 4 vegetable toppings and 3 meat toppings. Pat always chooses one type of crust, one type of cheese, 2 vegetable toppings and two meat toppings. How many different pizzas can Pat create?

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Pat's choices are independent so
$\mathbf{C}(3,1) \cdot \mathbf{C}(2,1) \cdot \mathbf{C}(4,2) \cdot \mathbf{C}(3,2) = 3 \cdot 2 \cdot 6 \cdot 3 = 108.$

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Example How many subsets of a set of size 5 have at least 4 elements?

$\mathbf{C}(5,4) + \mathbf{C}(5,5).$