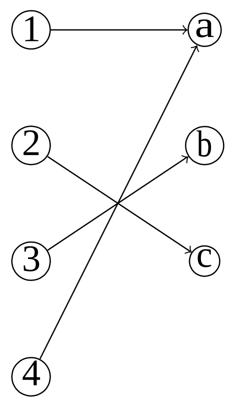
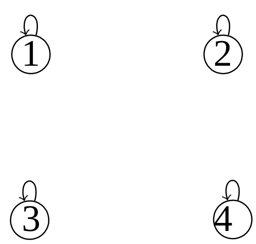
Discrete Mathematics and Statistics (CPT 107)

Tutorial 3 - Solutions

1. - The list of ordered pairs is: $\{(1, a), (2, c), (3, b), (4, a)\}$.
   - The digraph representation is:



2. - As a set of ordered pairs: $\{(1, 1), (2, 2), (3, 3), (4, 4)\}$;
   - As a digraph:



   - As a matrix:

$$M = \begin{bmatrix} T & F & F & F \\ F & T & F & F \\ F & F & T & F \\ F & F & F & T \end{bmatrix}$$

3. (a) reflexive, symmetric and transitive.
   (b) not transitive, not reflexive,
   and not symmetric.
   (c) reflexive and transitive but not symmetric.

4.
   - $S_1 = \{(1, 9), (3, 3), (9, 1)\}$;
   - $S_2 = \{(3, 2), (6, 4), (9, 6), (12, 8)\}$;
   - The transitive closure of S_1 is $S_1 \cup \{(1, 1), (9, 9)\}$;
   - The transitive closure of S_2 is $S_2 \cup \{(9, 4)\}$.

5. Yes, there is a mistake. The following is a counterexample to the above proof: Let $A = \{1, 2, 3\}$ and let $R = \{(1, 1), (1, 2), (2, 1), (2, 2)\}$. Clearly, R in the counterexample is symmetric and transitive but it is not reflexive ($(3, 3) \notin R$).

6. (a) Each equivalence class consists of all those books of a fixed colour.

   (b) There are two equivalence classes, the set of odd integers and the set of even integers.

   (c) There are two equivalence classes, the set of females and the set of males.

7. The equivalence classes are:

   (a) $E_0 = \{0, 3, 6, 9, 12, \ldots\}$;

   (b) $E_1 = \{1, 4, 7, 10, 13, \ldots\}$;

   (c) $E_3 = E_0$.