Xi'an Jiaotong-Liverpool University
西交利物浦大学

Discrete Mathematics and Statistics - CPT107

Week 8 - Tutorial

- Functions: definitions and examples
- Domain, codomain, and range / image
- Injective, surjective, and bijective functions
- Invertible functions
- Compositions of functions

Q.1.

Let $A = \{0, 2, 4,6\}$ and $B = \{1, 3, 5, 7\}$. Determine which of the following relations between A and B forms a function with domain A and codomain B:

(a) $\{(6, 3), (2, 1), (0, 3), (4, 5)\}$,

(b) $\{(2, 3), (4, 7), (0, 1), (6, 5)\}$,

(c) $\{(2, 1), (4, 5), (6, 3)\}$,

(d) $\{(6, 1), (0, 3), (4, 1), (0, 7), (2, 5)\}$.

For those which are functions, which are injective and which are surjective?

Q.2.

Which of the following functions are injective? Which are surjective?

a) $f : \mathbb{Z} \to \mathbb{Z}$ given by $f(x) = x^2 + 1$.
b) $g : \mathbb{N} \to \mathbb{N}$ given by $g(x) = 2^x$.
c) $h : \mathbb{R} \to \mathbb{R}$ given by $h(x) = 5x - 1$

Q.3.

The function $f : A \to B$ is given by $f(x) = 1 + \frac{2}{x}$ where A denotes the set of real numbers excluding 0 and B denotes the set of real numbers excluding 1. Show that f is bijective and determine the inverse function.

Q.4.

Consider the function $f : \mathbb{R} \to \mathbb{R}$ given by $f(x) = 3x$ and the function $g : \mathbb{R} \to \mathbb{R}$ given by $g(x) = x + 9$.

Calculate $g \circ f$, $f \circ g$, $f \circ f$ and $g \circ g$.

Q.5.

Let A, B and C be sets and let $f : A \to B$ and $g : B \to C$ be functions.
Prove that:

a) if f and g are injective, then $g \circ f$ is also injective.
b) if f and g are surjective, then $g \circ f$ is also surjective.
c) if $g \circ f$ is injective, then f is injective.
d) if $g \circ f$ is surjective, then g is surjective.

Q.6.

For some domains and codomains, show the composition:

a) is commutative (e.g. $(g \circ f) = (f \circ g)$);

b) is associative (e.g. $(h \circ g) \circ f = h \circ (g \circ f)$).

Q.7.

Let $f: \mathbb{R} \to \mathbb{R}$ and $g: \mathbb{R} \to \mathbb{R}$. Show that:

a) if f and g are surjective, then fg are surjective.
b) if f and g are injective, then $f+g$ are injective.

Q.8.

Let $m \neq 0$ and b be real numbers and consider the function $f : \mathbb{R} \to \mathbb{R}$ given by $f(x) = mx + b$.

(a) Prove that f is a bijection.

(b) Since f is a bijection, it is invertible. Find its inverse f^{-1}, and show it is an inverse by demonstrating that

$$f^{-1}\big(f(x)\big) = x.$$