Probability

Examples

CONDITIONAL PROBABILITY

Without Replacement

Example 1

In a jar are 5 RED and 5 GREEN jellybeans. What is the probability that the second jellybean drawn from the jar is GREEN given that the first jellybean is RED?

$$P(G \mid R)$$

The second jb is GREEN

If the first one is RED

Starting off there are 5 RED and 5 GREEN jellybeans (10 total)

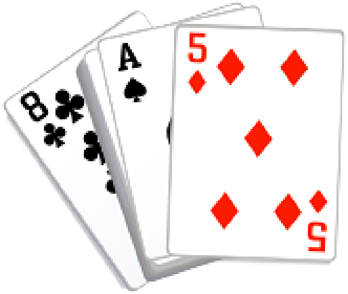
After taking out one red, there are 4 RED and 5 GREEN (9 total)

$$P(G \mid R) = \frac{5}{9} = 0.555$$

Example 2

Two cards are selected in sequence from a standard deck.

Find the probability that the **second card is a queen**, given that the first card is a king.



Solution:

Because the first card is a king and is not replaced, the remaining deck has 51 cards, 4 of which are queens.

$$P(B \mid A) = P(2^{nd} \ card \ is \ a \ Queen \mid 1^{st} \ card \ is \ a \ King) = \frac{4}{51} \approx 0.078$$

Example 3

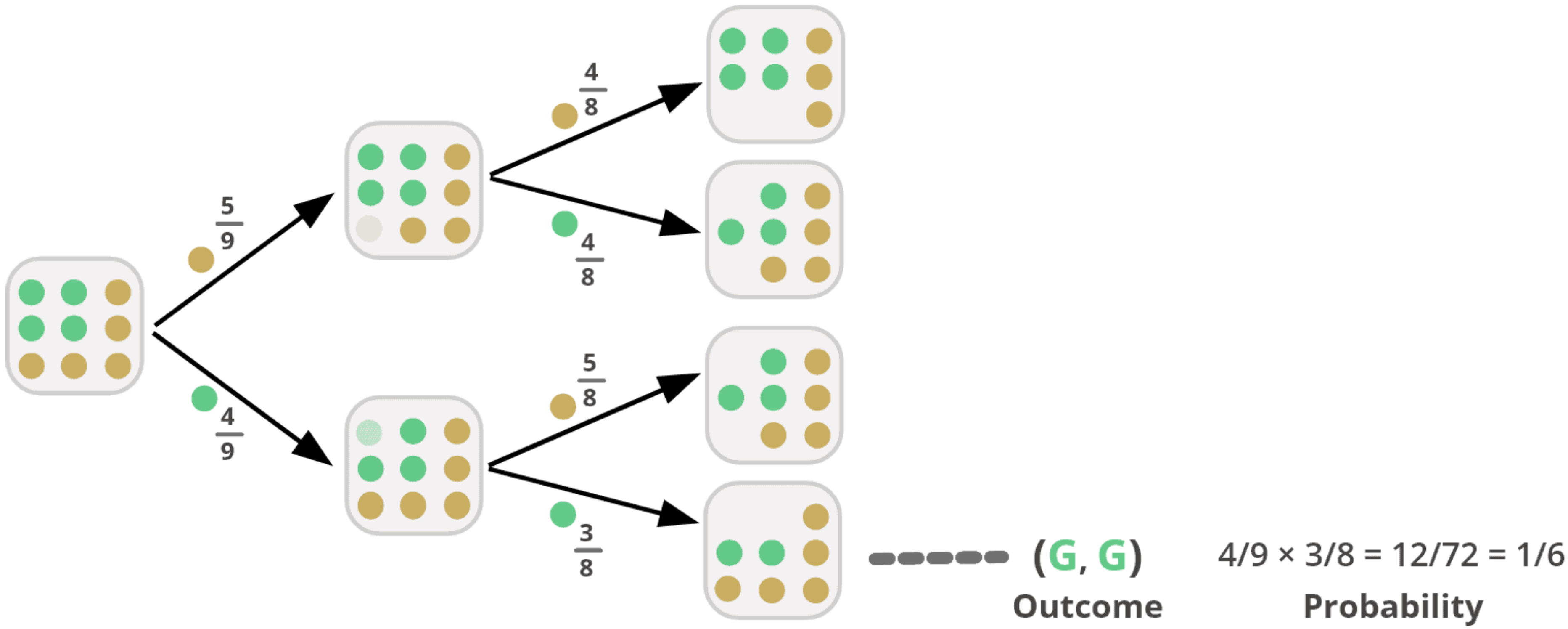
A box contains 4 green candies and 5 brown candies

Let us consider that event A is **getting a green candy**.

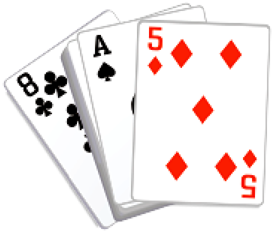
$$P(A) = \frac{\text{number of ways it can happen}}{\text{total number of outcomes}}$$

Determine the **probability of drawing 2 green candies**.

Solution



Example 4



What is the probability of **drawing twice** from a deck and getting the King of diamonds **then** the Queen of diamonds?

Solution:

The probability of drawing twice from the deck: $P(X \text{ and } Y) = P(X) \times P(Y|X)$

$$P(X) = P(Queen\ of\ diamonds) = \frac{1}{52}$$

The occurrence of **X** changes the probability of the occurrence of **Y**

The events are conditional without replacement

$$P(Y|X) = P(2^{nd}\ card\ is\ a\ Queen\ of\ diamonds|1^{st}\ card\ is\ a\ King\ of\ diamonds) = \frac{1}{51}$$

$$P(X \text{ and } Y) = \frac{1}{52} \times \frac{1}{51} = \frac{1}{2652}$$

Example 5

What is the probability of **drawing twice** *(in a raw)* from a deck and getting the king of diamonds and the queen of diamonds **in any order**?

$$P(X \text{ and } Y) = P(X) \times P(Y)$$

$$\frac{2}{52} \times \frac{1}{51} = \frac{2}{2652} = \frac{1}{1326}$$

PROBABILITY

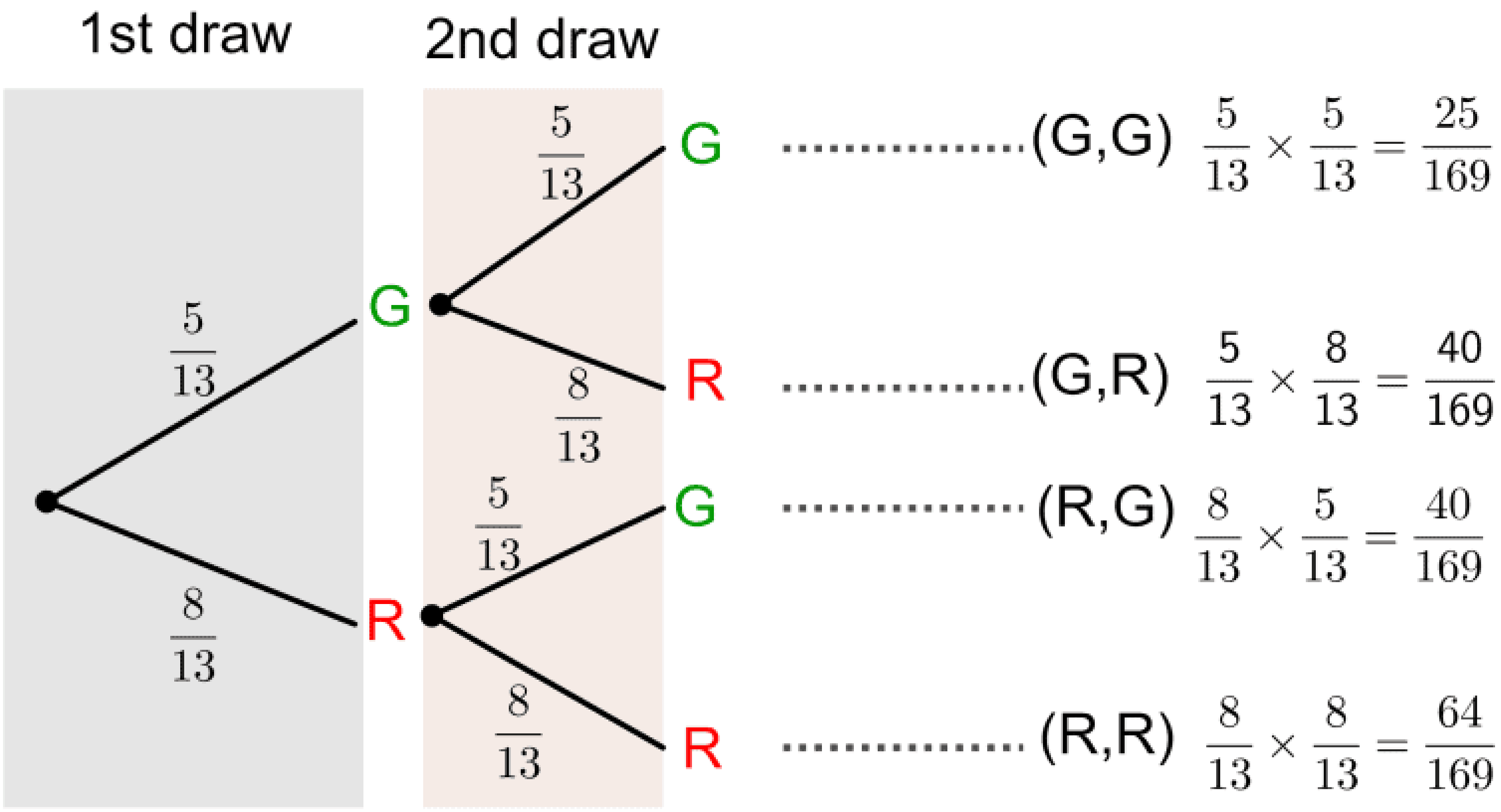
With Replacement

Example 1

Let's suppose there are **thirteen** balls in a box.
Five balls are Green(G), and **eight** balls are Red(R).
If we draw two balls, one at a time, with replacement, find the probability of the following events:

1. Both Balls are Green.
2. Both balls are Red.
3. The first ball is Green and the second is Red.
4. The first ball is Red and the second is Green.

Solution



1st draw
2nd draw
$\frac{5}{13}$
G
$\frac{8}{13}$
R
$\frac{5}{13}$
G
$\frac{8}{13}$
R
$\frac{5}{13}$
G
$\frac{8}{13}$
R
(G,G) $\frac{5}{13} \times \frac{5}{13} = \frac{25}{169}$
(G,R) $\frac{5}{13} \times \frac{8}{13} = \frac{40}{169}$
(R,G) $\frac{8}{13} \times \frac{5}{13} = \frac{40}{169}$
(R,R) $\frac{8}{13} \times \frac{8}{13} = \frac{64}{169}$

Example 2

Four cards are picked randomly, with replacement, from a regular deck of 52 playing cards.
Find the probability that all four are aces.

Solution:

There are four aces in a deck, and as we are replacing after each sample, so

$$P(first\ ace) = P(second\ ace) = P(third\ ace) = P(fourth\ ace) = \frac{4}{52}$$

All four samples are independent, so

$$P(all\ four\ ace) = \frac{4}{52} \cdot \frac{4}{52} \cdot \frac{4}{52} \cdot \frac{4}{52} = \frac{1}{28561}$$

Example – RANDOM VARIABLE

Throw 2 dice

What are the possible outcomes?

1,1	2,1	3,1	4,1	5,1	6,1
1,2	2,2	3,2	4,2	5,2	6,2
1,3	2,3	3,3	4,3	5,3	6,3
1,4	2,4	3,4	4,4	5,4	6,4
1,5	2,5	3,5	4,5	5,5	6,5
1,6	2,6	3,6	4,6	5,6	6,6

What is P(X = 7)?

We can define the *random variable* X to be the sum of the dots on the 2 dice.

{ (1,1), (1,2), (1,3), (1,4), (1,5), (1,6),
(2,1), (2,2), (2,3), (2,4), (2,5), (2,6),
(3,1), (3,2), (3,3), (3,4), (3,5), (3,6),
(4,1), (4,2), (4,3), (4,4), (4,5), (4,6),
(5,1), (5,2), (5,3), (5,4), (5,5), (5,6),
(6,1), (6,2), (6,3), (6,4), (6,5), (6,6) }

Since there are 36 equally likely outcomes, each has a probability of 1/36.

So, since there are 6 outcomes that yield X=7, P(X=7) = 6/36 =1/6