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CPT205 Computer Graphics

Hidden-Surface Removal

Lecture 12
2024-25

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Topics for today

➤ Concepts

- Clipping vs Hidden Surface Removal
- What is hidden-surface removal?
- Why is hidden-surface removal important?

➤ Hidden-surface removal algorithms

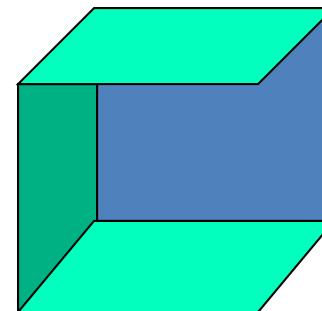
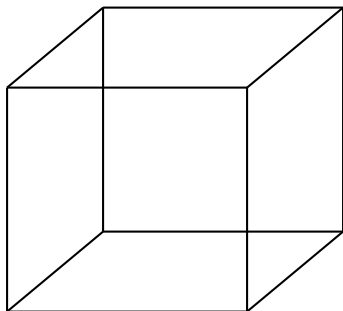
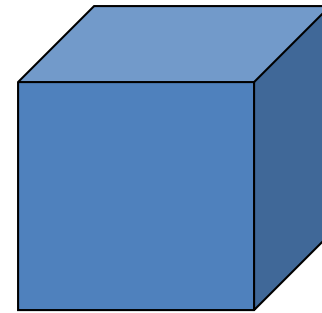
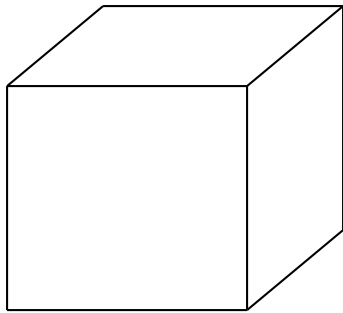
- Object space vs image space
- Painter's algorithm
- Back-face culling
- Z-buffer
- BSP tree

➤ Hidden-surface removal in OpenGL

Clipping and hidden-surface removal

- Clipping has much in common with hidden-surface removal.
- In both cases, we try to remove objects (or parts of objects) that are not visible to the camera.
- Often we can use visibility or occlusion testing early in the process to eliminate as many polygons as possible before going through the entire pipeline (rasterization).

Concepts (1)



Concepts (2)

- Display all visible surfaces, and do not display any occluded surfaces.
- Other names
 - Visible-surface detection
 - Hidden-surface elimination
- Determine which surfaces are visible and which are not
 - Z-buffer is just one of hidden-surface removal algorithms
- We can categorise into
 - Object-space methods
 - Image-space methods

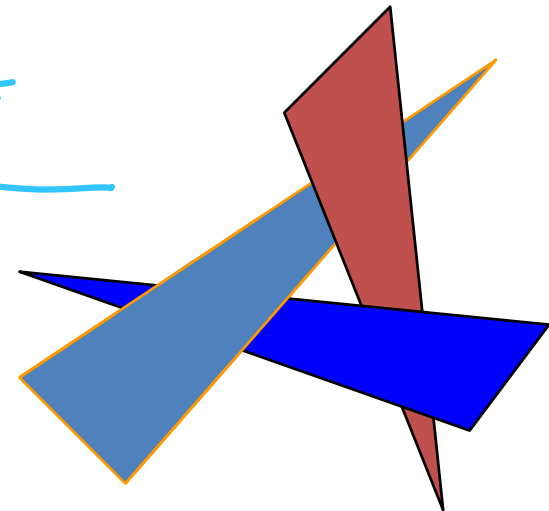
Concepts (3)

➤ Object space algorithms: determine which objects are in front of others

- Resize does not require recalculation;
- Works for static scenes;
- May be difficult / impossible to determine.

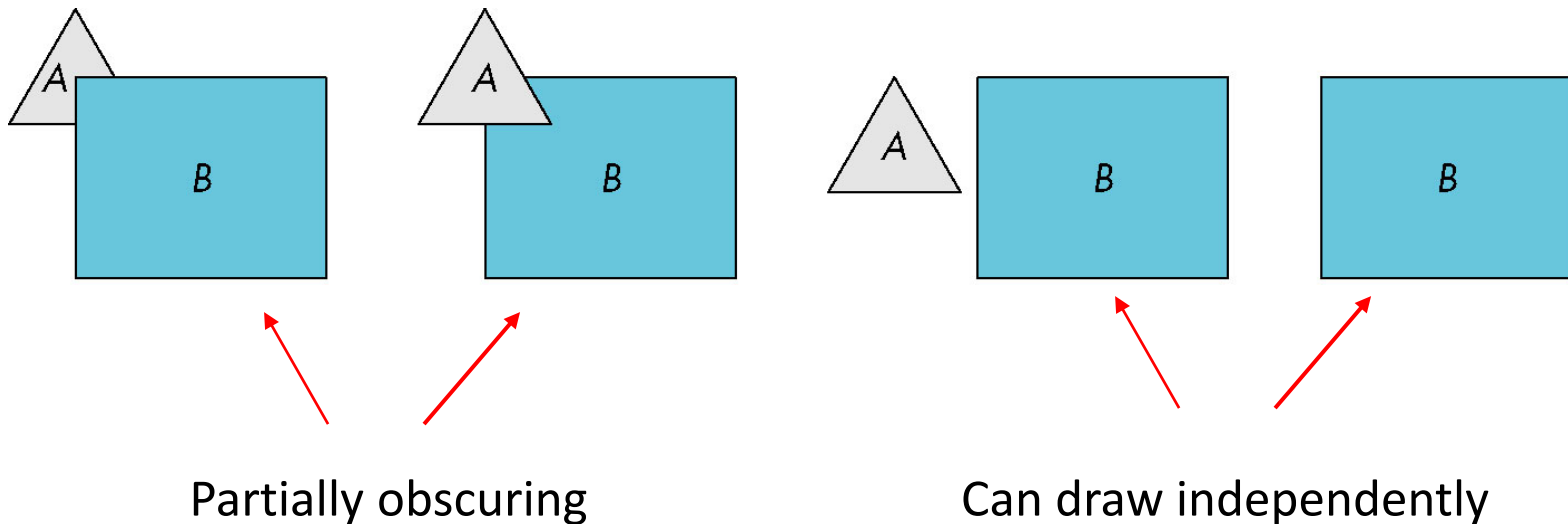
➤ Image space algorithms: determine which object is visible at each pixel

- Resize requires recalculation;
- Works for dynamic scenes.



Object-space approach

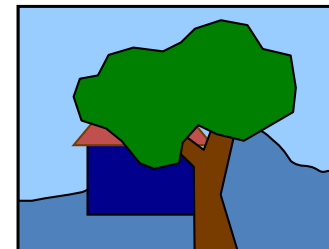
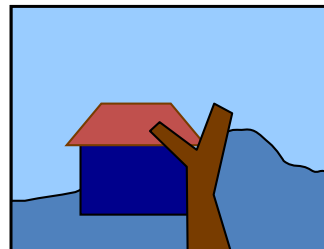
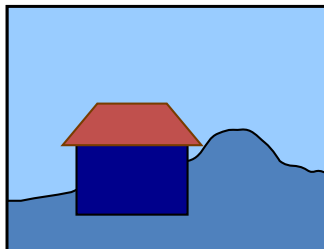
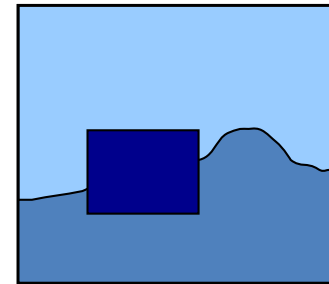
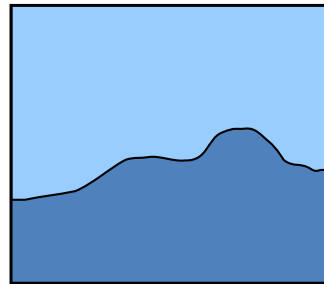
- It uses pairwise testing between polygons (objects).



- Worst case complexity $O(n^2)$ for n polygons.

Painter's algorithm

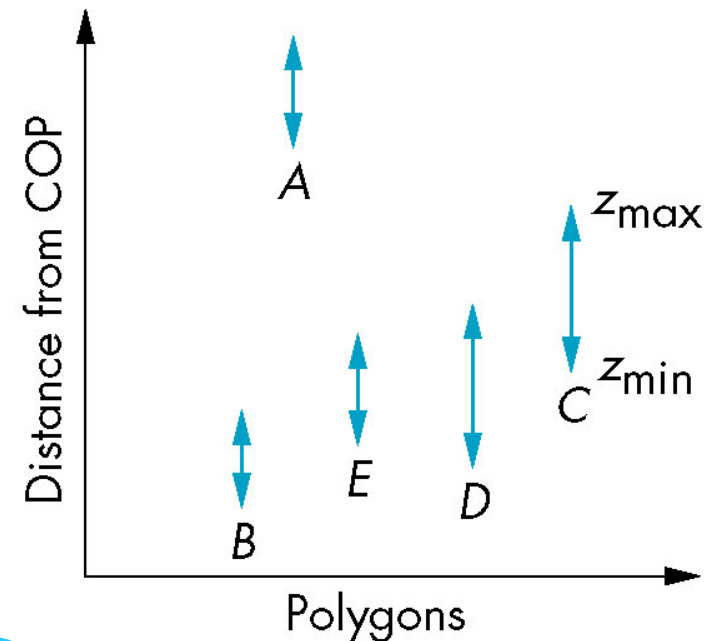
- It is an object-space algorithm.
- Render polygons in the back to front order so that polygons behind others are simply painted over. *从后到前*
- Sort surfaces/polygons by their depth (z value).



Painter's algorithm: Depth sort

- Requires ordering of polygons first
 - $O(n \log n)$ calculations for ordering 先要排序
 - Not every polygon is either in front or behind all other polygons
- Orders polygons and deals with easy cases first and hard later.

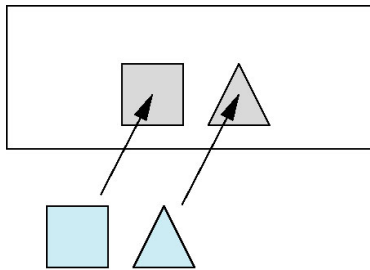
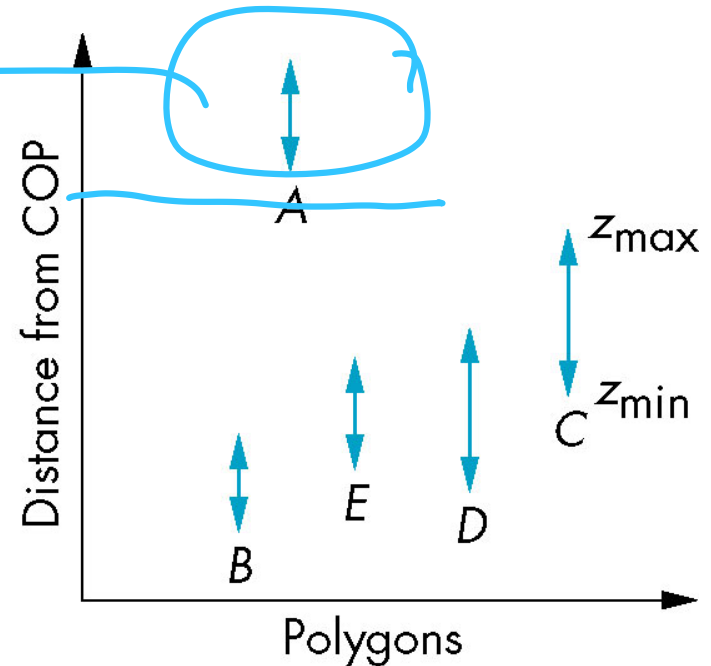
Polygons sorted by distance from
Centre Of Projection (COP)



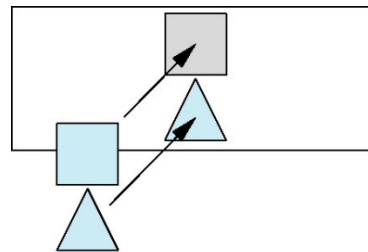
Painter's algorithm: Easy cases

- Polygon A lies entirely behind all other polygons, and can be painted first.

- Polygons which overlap in z (depth) but not in either x or y , can be painted independently.



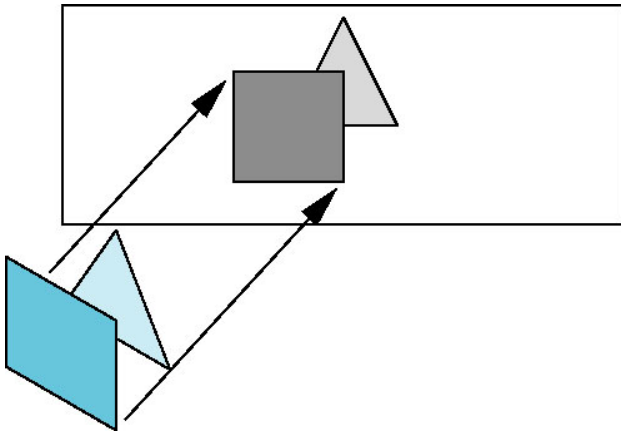
Test of overlap in x



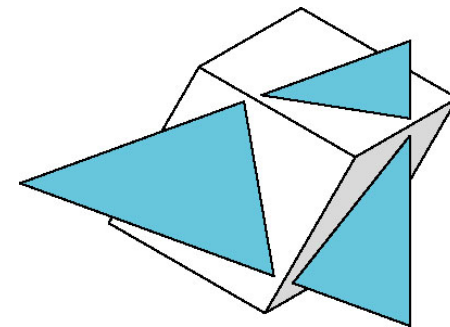
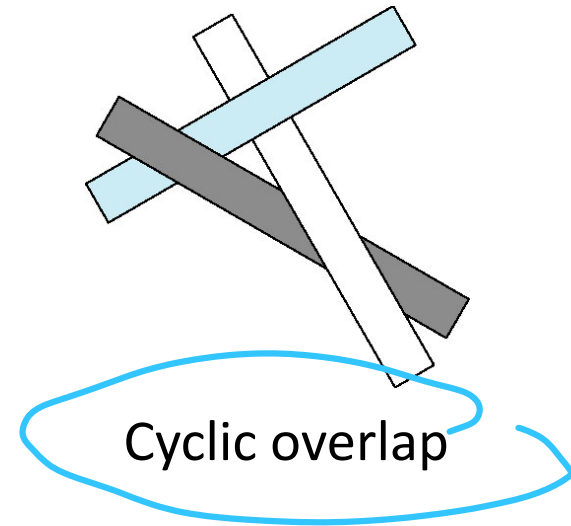
Test of overlap in y

Painter's algorithm: Hard cases 3种

循环重叠 / 穿透 / 在所有方向重叠



Overlap in all directions
(polygons with
overlapping projections)



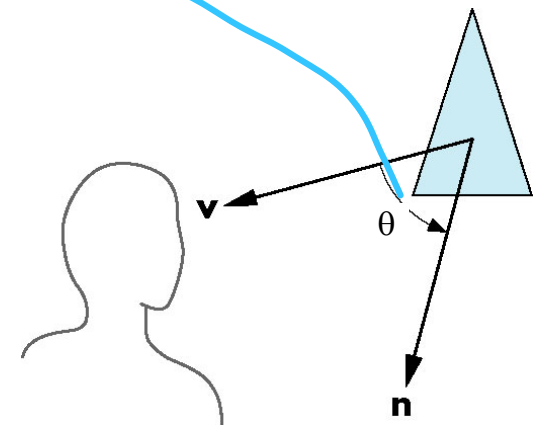
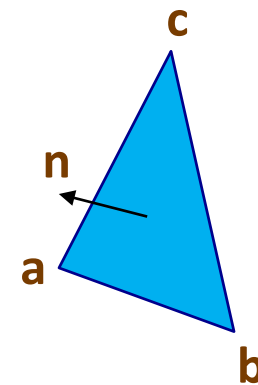
Penetration

Back-face culling (1) 背面删除

- Back-face culling is the process of comparing the position and orientation of polygons against the viewing direction \mathbf{v} , with polygons facing away from the camera eliminated.
- This elimination minimises the amount of computational overhead involved in hidden-surface removal.
- Back-face culling is basically a test for determining the visibility of a polygon, and based on this test the polygon can be removed if not visible – also called back-surface removal.

Back-face culling (2)

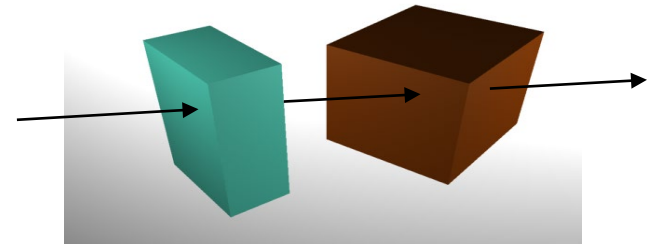
- Assume the object is a solid polyhedron.
- Compute polygon normal \mathbf{n} :
 - Assume a counter-clockwise vertex order
 - For a triangle (\mathbf{abc}) : $\mathbf{n} = (\mathbf{ab}) \times (\mathbf{bc})$
- If $90^\circ \geq \theta \geq -90^\circ$, i.e. $\mathbf{v} \bullet \mathbf{n} \geq 0$, the polygon is a visible face.
- Furthermore, if the object descriptions are converted to projection co-ordinates and the viewing direction is parallel to the viewing z_v axis, i.e. $\mathbf{n} = (0,0,1)^T$, only the sign of the z component of \mathbf{n} is tested.



Back-face culling (3)

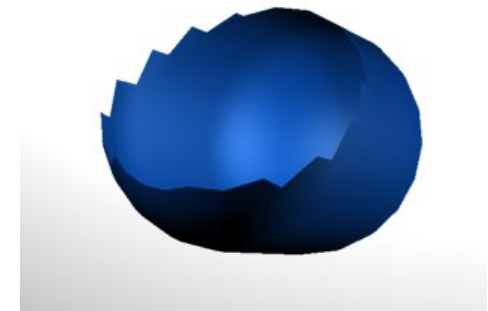
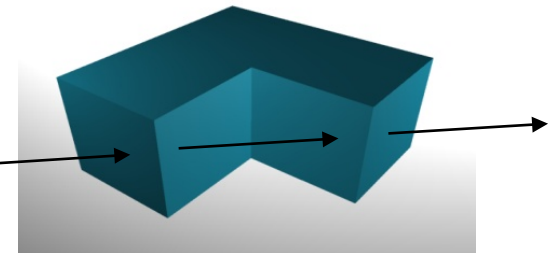
For each polygon P_i

- Find polygon normal \mathbf{n}
- Find viewer direction \mathbf{v}
- If $\mathbf{v} \cdot \mathbf{n} < 0$, then cull P_i

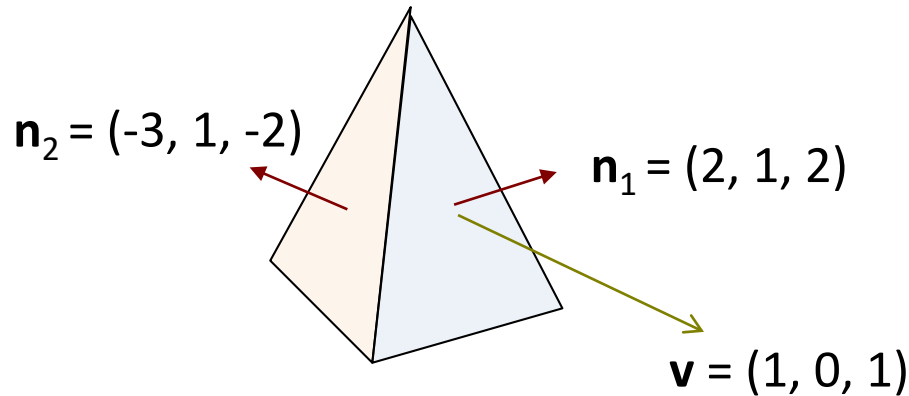


Back-face culling does not work for

- Overlapping front faces due to
 - Multiple objects
 - Concave objects
- Non-polygonal models
- Non-closed objects



Back-face culling: Example



$$\begin{aligned}\mathbf{n}_1 \cdot \mathbf{v} &= (2, 1, 2) \cdot (1, 0, 1) \\ &= 2 + 0 + 2 = 4,\end{aligned}$$

$$\text{i.e. } \mathbf{n}_1 \cdot \mathbf{v} > 0,$$

so face₁ is a front facing polygon.

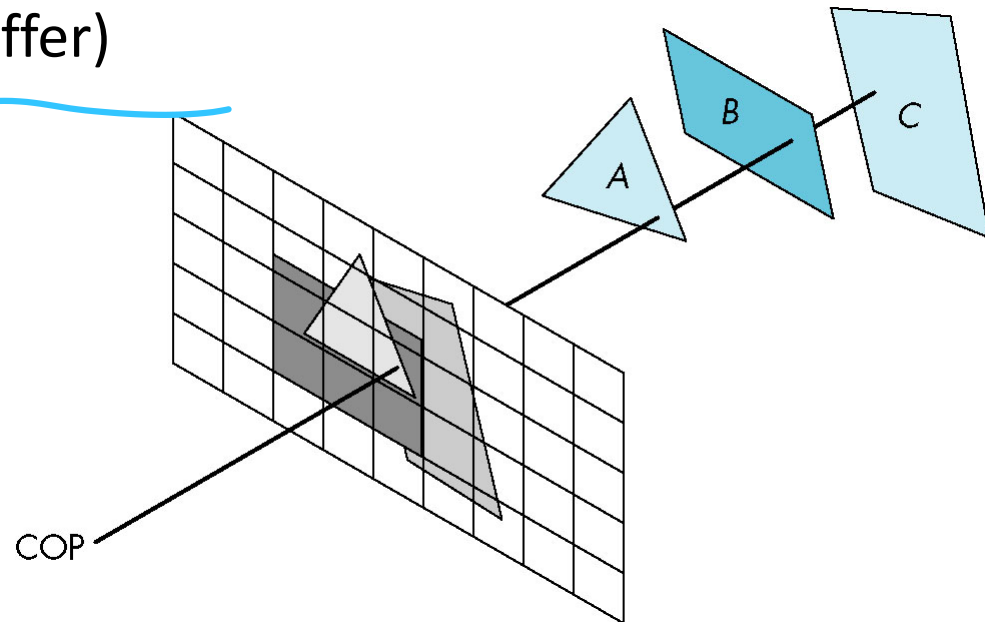
$$\begin{aligned}\mathbf{n}_2 \cdot \mathbf{v} &= (-3, 1, -2) \cdot (1, 0, 1) \\ &= -3 + 0 - 2 = -5\end{aligned}$$

$$\text{i.e. } \mathbf{n}_2 \cdot \mathbf{v} < 0,$$

so face₂ is a back facing polygon.

Image space approach

- Look at each projector ($n*m$ projectors for an $n*m$ framebuffer) and find the closest among k polygons
- Complexity $O(nmk)$
- Ray tracing
- Z-buffer (depth buffer)



Z-buffer (1)

- We now know which pixels contain which object(s).
- However since some pixels may contain two or more objects, we must calculate which of these objects are visible and which are hidden.
- In graphics hardware, hidden-surface removal is generally accomplished using the Z-buffer algorithm.

Z-buffer (2)

留出 $m \times n$ = 二维数组

- In this algorithm, we set aside a two-dimensional array of memory (the Z-buffer) of the same size as the screen (number of rows * number of columns).
- This is in addition to the colour buffer (frame buffer) which we use to store the colour values of pixels to be displayed.
- The Z-buffer will hold values which are depths (quite often z-values).
- The Z-buffer is initialised so that each element has the value of the far clipping plane (the largest possible z-value after clipping is performed).
- The colour buffer is initialised so that each element contains a value which is the background colour.

初始化 每个元素远裁剪平面值 / 颜色缓冲区背景颜色

Z-buffer (3)



- Now for each polygon, we have a set of pixel values which the polygon covers.
- For each of the pixels, we compare its depth (z-value) with the value of the corresponding element already stored in the Z-buffer:
 - If this value is less than the previously stored value, the pixel is nearer the viewer than the previously encountered pixel;
 - Replace the value of the Z- buffer with the z value of the current pixel, and replace the value of the colour buffer with the value of the current pixel.
- Repeat for all polygons in the image.
- The implementation is typically done in normalised co-ordinates so that depth values range from 0 at the near clipping plane to 1.0 at the far clipping plane.

Z-buffer (4)

对画面多边形 k 个循环 k 次.

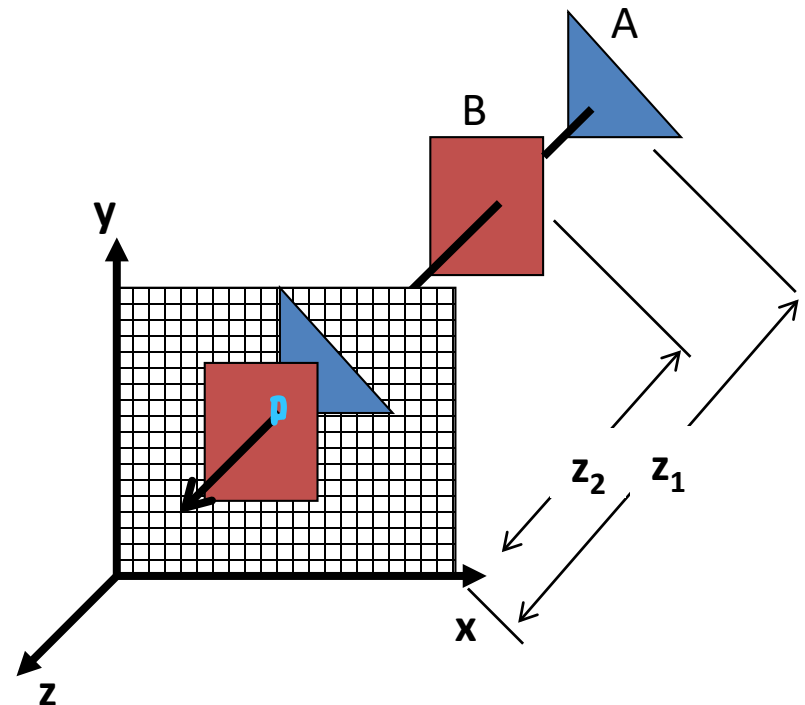
1. Initialise all $depth(x,y)$ to 1.0 and $refresh(x,y)$ to background colour

2. For each pixel
Get current value for $depth(x,y)$
Evaluate depth value z

If ($z < depth(x,y)$)
{
 $depth(x,y) = z$
 $refresh(x,y) = I_s(x,y)$
}

Calculate this using relevant algorithms
/illumination/fill colour/texture

着色



Z-buffer (5)

Advantages

- The most widely used hidden-surface removal algorithm;
- An image-space algorithm which traverses scene and operates per polygon rather than per pixel;
- Rasterize polygon by polygon and determine which (part of) polygons get drawn on the screen;
- Relatively easy to implement in hardware or software.

Memory requirements

- Relies on a secondary buffer called the Z-buffer or depth buffer;
- Z-buffer has the same width and height as the framebuffer;
- Each cell contains the z-value (distance from viewer) of the object at that pixel position.

Scan-line

扫描线
深度变化

If we work scan line by scan line, as we move across a scan line, the depth changes satisfy $a\Delta x + b\Delta y + c\Delta z = 0$, noticing that the plane which contains the polygon is represented by $ax + by + cz + d = 0$

Along scan line $\Delta y = 0$

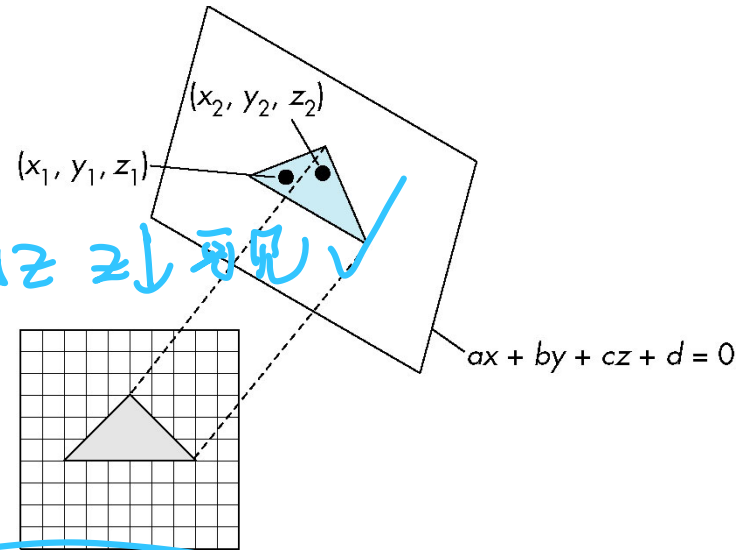
$$\Delta y = 0$$

$$\Delta z = -(a/c) * \Delta x$$

→ 同一像素不同z 无法看见 ✓

In screen space $\Delta x = 1$

Only computed once per polygon.

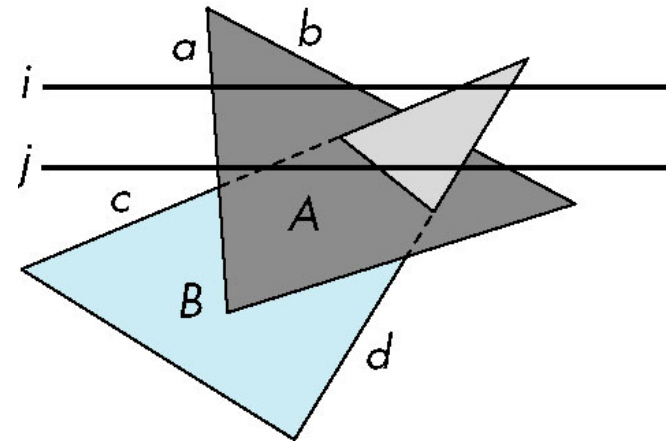


- Worst-case performance of an image-space algorithm is proportional to the number of primitives.
- Performance of z-buffer is proportional to the number of fragments generated by rasterization, which depends on the area of the rasterized polygons.

Scan-line algorithm

We can combine shading and hidden-surface removal through scan line algorithm.

- Scan line i: no need for depth information, can only be in no or one polygon.
- Scan line j: need depth information only when in more than one polygon.



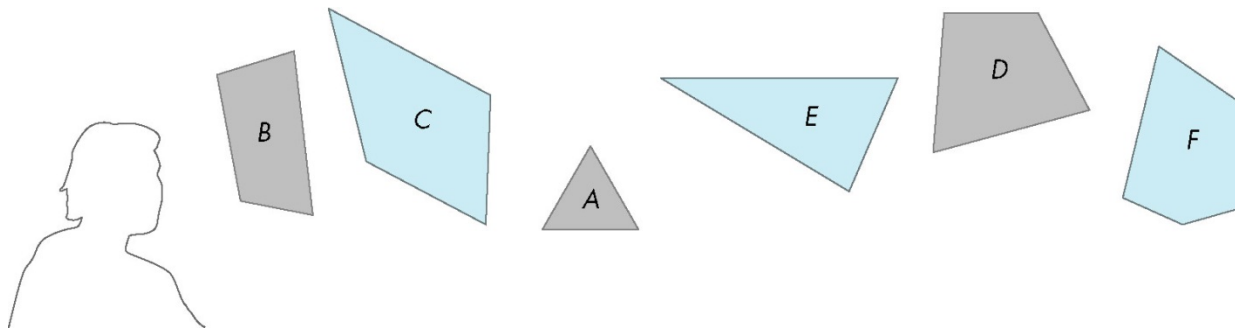
Scan-line algorithm: implementation

Need a data structure to store

- Flag for each polygon (inside/outside)
- Incremental structure for scan lines that stores which edges are encountered
- Parameters for planes

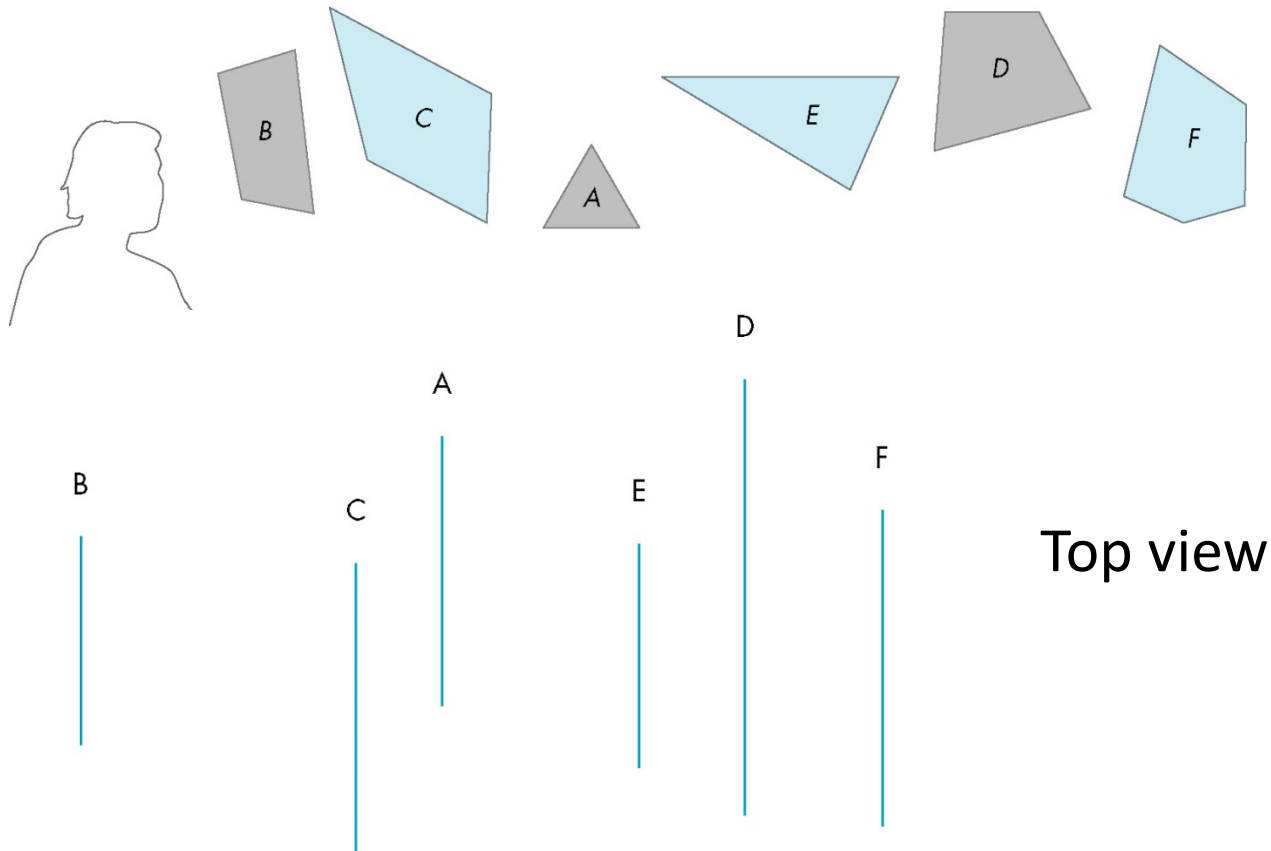
BSP tree (1)

- In many real-time applications, such as games, we want to eliminate as many objects as possible within the application so that we can
 - reduce burden on pipeline
 - reduce traffic on bus
- Partition space with Binary Spatial Partition (BSP) Tree



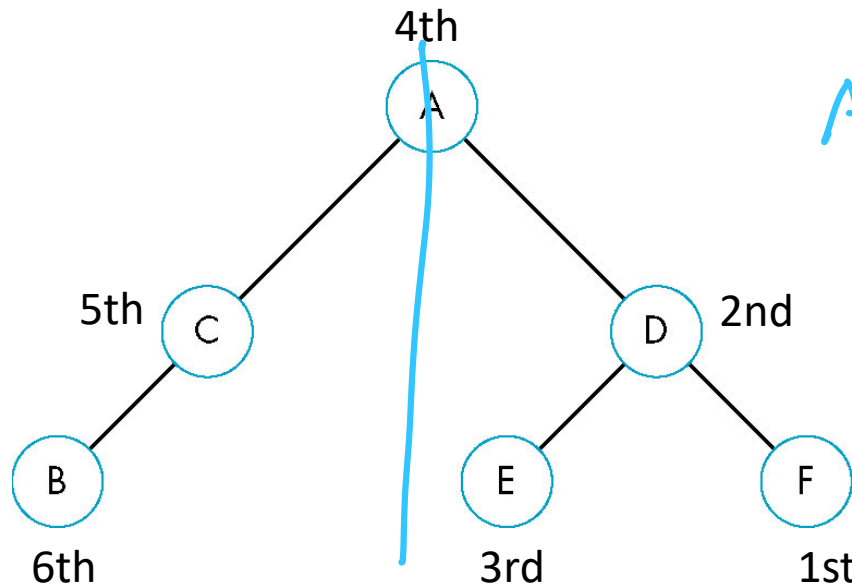
BSP tree (2)

Consider 6 parallel planes. Plane A separates planes B and C from planes D, E and F.



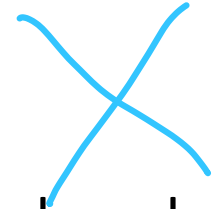
BSP tree (3)

- We can continue recursively
 - plane C separates B from A
 - plane D separates E and F
- We can put this information in a BSP tree
 - use the BSP tree for visibility (inside) and occlusion (outside) testing

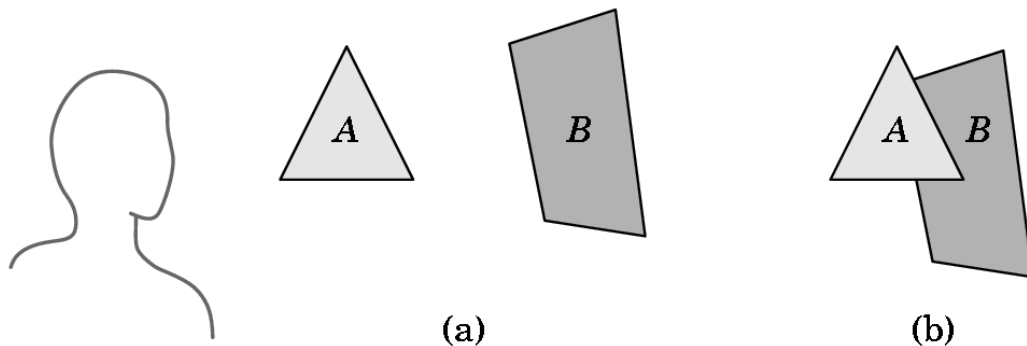


A 隔开 BC 和
DEF

BSP tree



- The *painter's algorithm* for hidden-surface removal works by drawing all faces, from back to front.
- How can we get a listing of the faces in the back-to-front order?
- Put them into a binary tree and traverse the tree, but in what order?



BSP tree

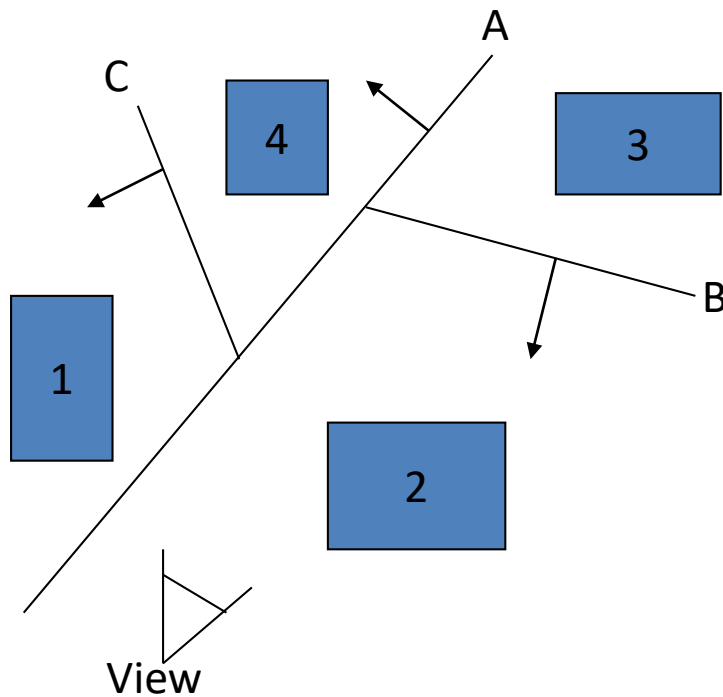
算法

- Choose a polygon (arbitrary)
- Split the cell using the plane on which the polygon lies
 - May have to chop polygons into two if they intersect with the plane (clipping!)
- Continue until each cell contains only one polygon fragment
- Splitting planes could be chosen in other ways, but there is no efficient optimal algorithm for building BSP trees
 - BSP trees are not unique – there can be a number of alternatives for the same set of polygons
 - Optimal means minimum number of polygon fragments in a balanced tree

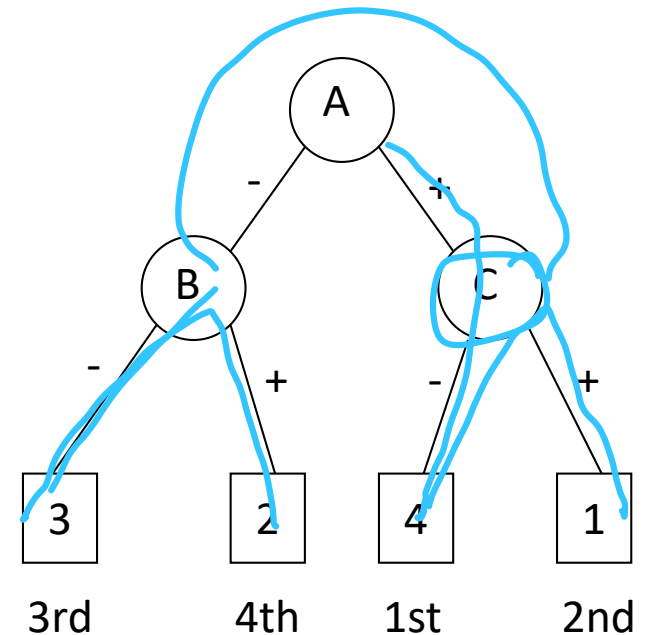
BSP tree: Rendering

- Observation: things on the opposite side of a splitting plane from the viewpoint cannot obscure things on the same side as the viewpoint.
- The rendering is recursive of the BSP tree.
- At each node (for back to front rendering):
 - Recurse down the side of the sub-tree that does not contain the viewpoint – Test viewpoint against the split plane to decide the polygon
 - Draw the polygon in the splitting plane – Paint over whatever has already been drawn
 - Recurse down the side of the tree containing the viewpoint

BSP tree: Rendering example



深搜



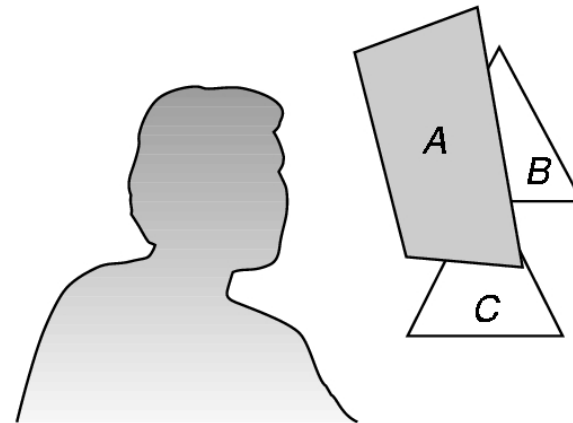
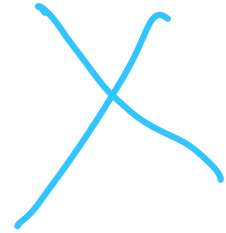
从 $A \rightarrow C \rightarrow 4 \rightarrow C \rightarrow 1 \rightarrow C$
 $\rightarrow A C 4$ 号离最近)

For further explanation, see

<https://www.youtube.com/watch?v=YT8ya4H-Z8Q>

OpenGL functions for hidden-surface removal

- How is hidden-surface removal done in OpenGL?
- OpenGL uses the z-buffer algorithm that saves depth information as objects/triangles are rendered so that only the front objects appear in the image.



OpenGL Z-buffer functions

➤ The algorithm uses an extra buffer, the z-buffer, to store depth information as geometry travels down the pipeline.

➤ It must be

- requested in `main()`

```
glutInitDisplayMode(GLUT_SINGLE |  
                    GLUT_RGB | GLUT_DEPTH)
```

- enabled in `init()`

```
glEnable(GL_DEPTH_TEST)
```

- cleared in the display callback

```
glClear(GL_COLOR_BUFFER_BIT |  
        GL_DEPTH_BUFFER_BIT)
```

OpenGL functions (1)

- OpenGL basic library provides functions for back-face removal and depth-buffer visibility testing.
- In addition, there are hidden-line removal functions for wireframe display, and scenes can be displayed with depth cueing.

OpenGL functions (2)

Face-culling functions - Back-face removal (and back-face culling) is accomplished with the functions

```
glEnable (GL_CULL_FACE) ;  
glCullFace (GLenum) ;
```

- The parameter mode includes GL_BACK, GL_FRONT and GL_FRONT_AND_BACK. So front faces can be removed instead of back faces, or both front and back faces can be removed.
- The default value for **glCullFace ()** is GL_BACK. Therefore, if **glEnable (GL_CULL_FACE)** is called to enable face culling without explicitly calling **glCullFace ()**, back faces in the scene will be removed.

OpenGL functions (3)

Depth-buffer functions

- The first function is about the GLUT initialisation

```
glutInitDisplayMode(GLUT_SINGLE | GLUT_RGB |  
                    GLUT_DEPTH) ;
```

- Depth buffer can then be initialised

```
glClear(GL_DEPTH_BUFFER_BIT) ;
```

- The depth-buffer visibility detection routines are activated / deactivated with

```
glEnable(GL_DEPTH_TEST) ;  
glDisable(GL_DEPTH_TEST) ;
```

OpenGL functions (4)

- The depth values are normalised in the range $[0.0, 1.0]$. But to speed up the surface processing, a maximum depth value, `maxDepth`, can be specified between 0.0 and 1.0

`glClearDepth(maxDepth);`

- Projection co-ordinates are normalised in the range $[-1.0, 1.0]$, and the depth values between the near and far clipping planes are further normalised in the range $[0.0, 1.0]$ where 0.0 and 1.0 correspond to the near and far clipping planes respectively.

OpenGL functions (5)

- The function below allows adjustment of the clipping planes

`glDepthRange(nearNormDepth, farNormDepth);`

where the default values for the two parameters are 0.0 and 1.0 respectively, and they can have a value in the range of [0.0, 1.0].

- Another function for extra options is

`glDepthFunc(testCondition);`

`testCondition` can have the following values: GL_LESS (default), GL_GREATER, GL_EQUAL, GL_NOTEQUAL, GL_LEQUAL, GL_GEQUAL, GL_NEVER (no points are processed) and GL_ALWAYS (all points are processed) for the application.

OpenGL functions (6)

- The status of the depth buffer can be set to read-only or read-write

`glDepthMask(writeStatus) ;`

- When `writeStatus = GL_FALSE`, the write mode in the depth buffer is disabled and only retrieval of values for comparison is possible.
- It is useful when a complex background is used with display of different foreground objects.
 - After storing the background in the depth buffer, the write mode is disabled for processing the foreground, allowing the generation of a series of frames with different foreground objects or with one object in different positions for an animated sequence.
 - It is also useful in displaying transparent objects.

OpenGL functions (7)

- **Wireframe surface-visibility functions** - A wireframe display of an object can be generated with

```
glPolygonMode (GL_FRONT_AND_BACK, GL_LINE) ;  
// In this case, both visible and hidden edges are displayed.
```

- **Depth-cueing functions** - The brightness of an object is varied as a function of its distance to the viewing position

```
glEnable (GL_FOG) ;  
glFog{if}[v] (GL_FOG_MODE, GL_LINEAR) ;  
// linear depth function for colour in [0.0, 1.0]  
glFog{if}[v] (GL_FOG_START, minDepth) ;  
// specifies a different value for  $d_{min}$   
glFog{if}[v] (GL_FOG_END, maxDepth) ;  
// specifies a different value for  $d_{max}$ 
```


Summary

- Concepts of hidden-surface removal
- Clipping vs hidden-surface removal
- Important techniques for hidden-surface removal
 - Object-space and image-space approaches
 - Painter's algorithms
 - Back-face culling
 - Z-buffering
 - BSP tree
- OpenGL functions
`glutInitDisplayMode()`, `glClear()`, `glClearDepth(maxDepth)`,
`glDepthRange(nearNormDepth, farNormDepth)` ;
`glDepthFunc(testCondition)`, `glDepthMask(writeStatus)`,
`glPolygonMode()`, `glEnable(GL_DEPTH_TEST)`, `glEnable(GL_FOG)`,
`glCullFace()`