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CPT205 Computer Graphics

Hierarchical Modelling

Lecture 08

2024-25

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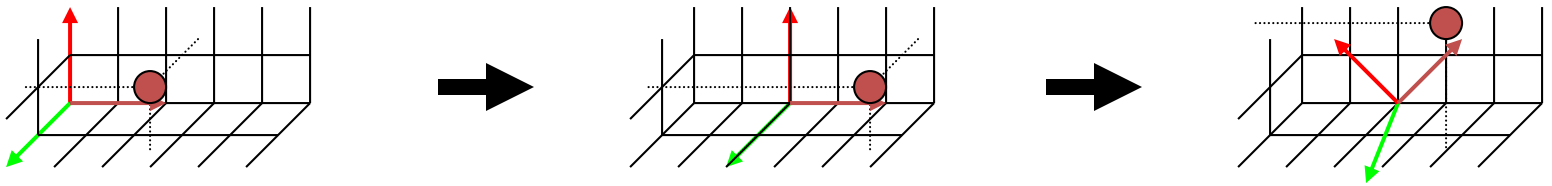
Topics for today

- Local and world co-ordinate frames of reference
- Object transformations
- Linear modelling
  - Symbols
  - Instances
- Hierarchical modelling
  - Hierarchical trees
  - Articulated models
- Examples and code

Local and world frames of reference (1)

- We are used to defining points in space as (x,y,z). But what does that actually mean? Where is (0,0,0)?
- The actual truth is that there is no (0,0,0) in the real world. Objects are always defined *relative* to each other.
- We can *move* (0,0,0) and thus move all the points defined relative to that origin.



Local and world frames of reference (2)

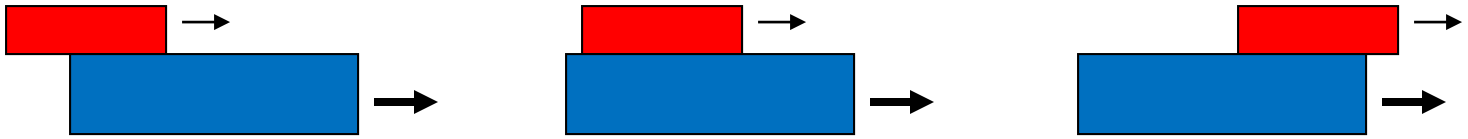
- The following terms are used interchangeably
  - *Local basis*
  - *Local transformation*
  - *Local / model frame of reference*
- Each of these refers to the location, in the greater world, of the (0,0,0) we are working with
  - They also include the concept of the current *local frame*, which is about the x, y, z directions.
  - By rotating the *local frame* of a coordinate system, we can rotate the world it describes.

What does the centre of the world mean?

- A *world frame of reference* is defined for a scene of objects.
- Each object has a *local frame of reference* which is relevant to the world frame.

Relative motion

- Relative motion - a motion takes place relative to a local origin.

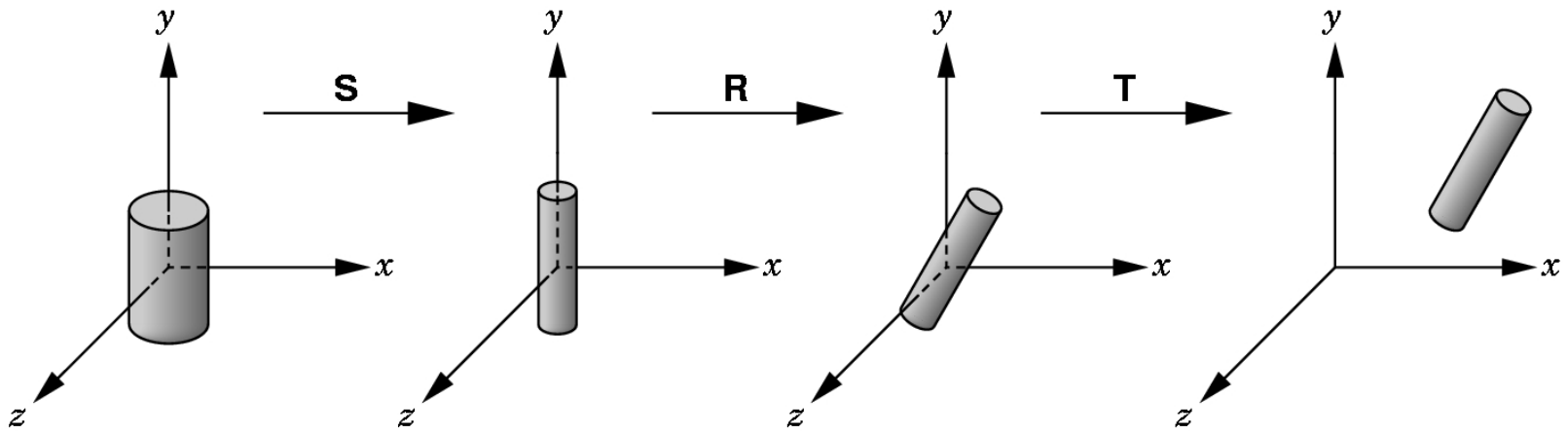


e.g. throwing a ball to a friend as you both ride in a train.

- The term *local origin* refers to the (0,0,0) that is chosen to measure the motion from.
- The local origin may be moving relative to some greater frame of reference.

Linear modelling (1)

- Start with a *symbol* (prototype)
- Each appearance of the object in the scene is an *instance*
  - We must scale, orient and position it to define the instance transformation
  - **M = T · R · S**



Linear modelling (2)

In OpenGL

- Set up appropriate transformations from the model frame (frame of symbols) to the world frame
- Apply it to the MODELVIEW matrix before executing the code

```
glMatrixMode(GL_MODELVIEW); // M = T·R·S
glLoadIdentity();
glTranslatef();
glRotatef();
glScalef();
glutSolidCylinder()          // or other symbol
```

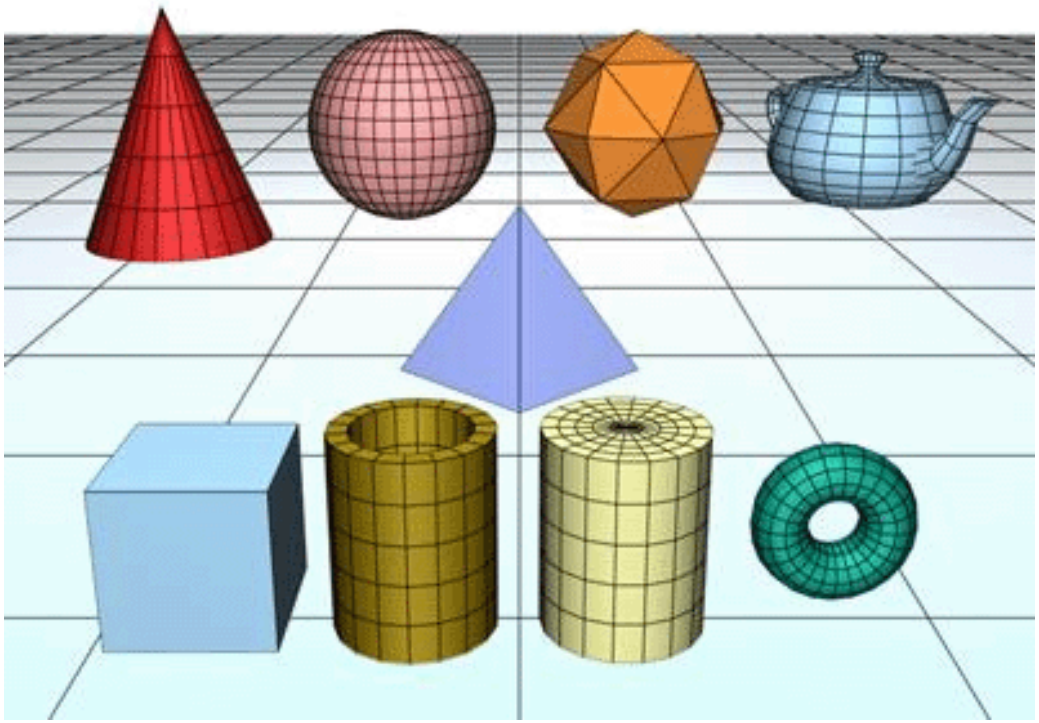
Linear modelling (3)

Example: generating a cylinder

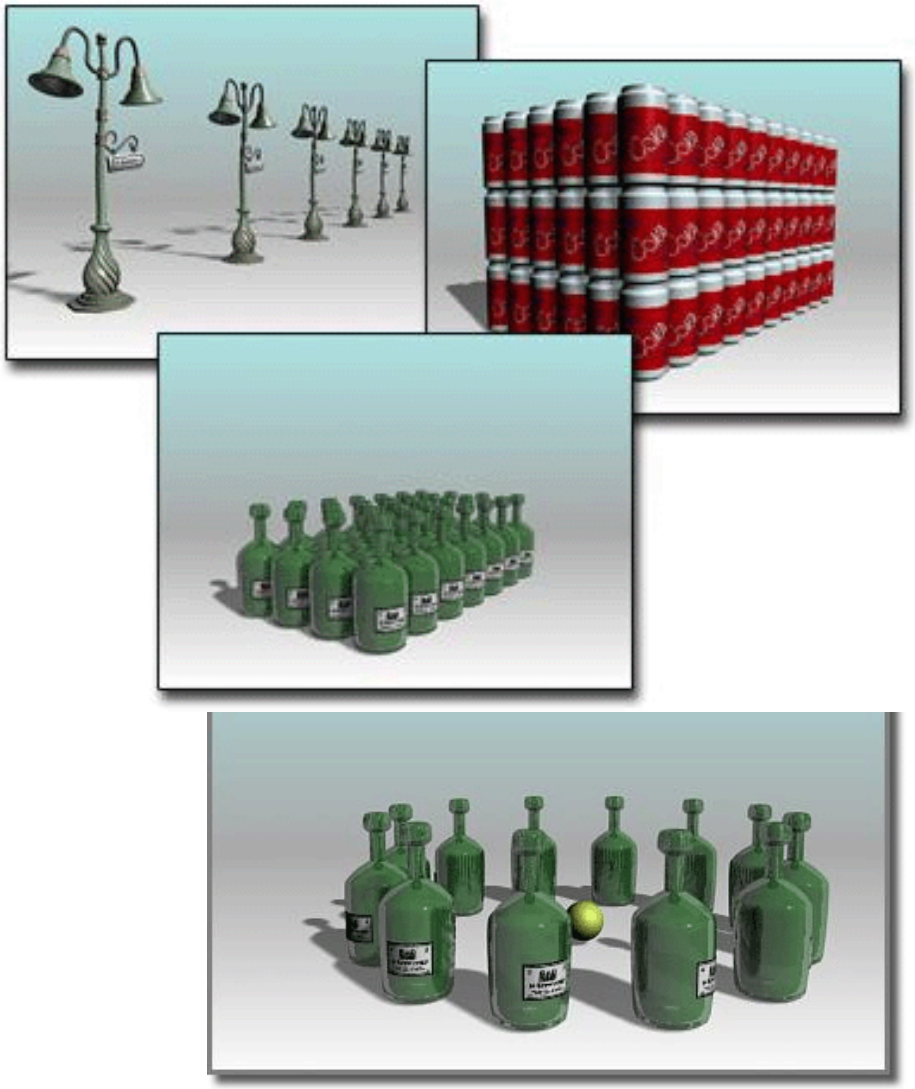
```
glBegin(GL_QUADS);
   For each A = Angles
   {
      glVertex3f(R*cos(A), R*sin(A), 0);
      glVertex3f(R*cos(A+DA), R*sin(A+DA), 0);
      glVertex3f(R*cos(A+DA), R*sin(A+DA), H);
      glVertex3f(R*cos(A), R*sin(A), H);
   }
glEnd();

// Make Polygons for Top/Bottom of cylinder
```

Linear modelling (4)

- Symbols (Primitives)
  Cone, Sphere, GeoSphere, Teapot, Box, Tube, Cylinder, Torus, etc.
- Copy
  Creates a completely separate clone from the original. Modifying one has no effect on the other.
- Instance
  Creates a completely interchangeable clone of the original. Modifying an instanced object is the same as modifying the original.



Linear modelling (5)



- Array: series of clones
  - Linear
    - Select object
    - Define axis
    - Define distance
    - Define number
  - Radial
    - Select object
    - Define axis
    - Define radius
    - Define number

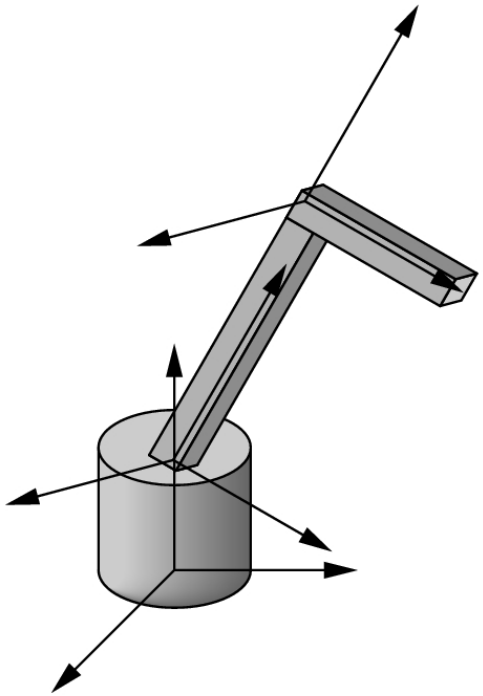
Linear modelling (6)

- Model stored in a table by
  - assigning a number to each symbol and
  - storing the parameters for the instance transformation
- Contains flat information
  but no information on the actual structure
- How to represent complex structures with constraints?
- Each part has its own model frame of co-ordinate system
  but no information of relationships
- How to manipulate with substructures?

Linear modelling (7)

Symbol	Scale	Rotate	Translate
1	s_x, s_y, s_z	u_x, u_y, u_z	d_x, d_y, d_z
2			
3			
1			
1			
.			
.			

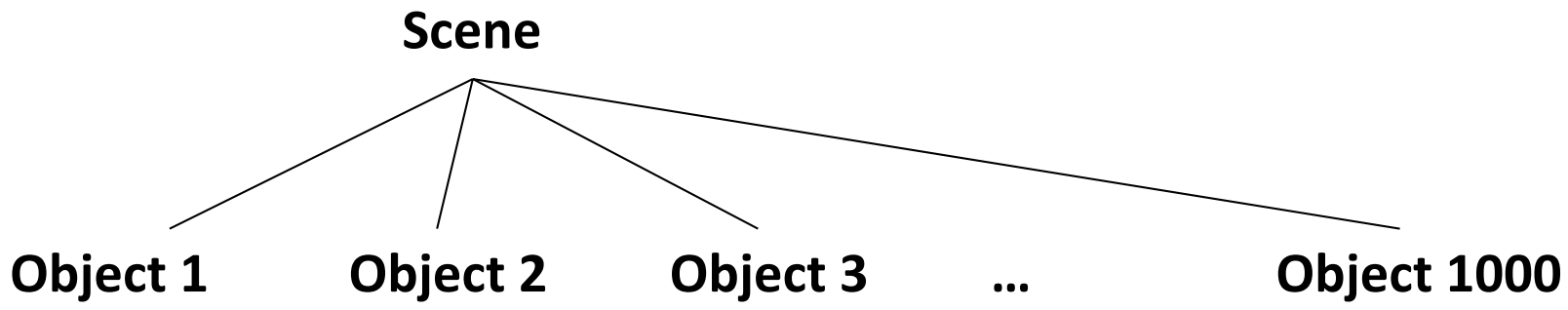
Linear model table



Model with constraints

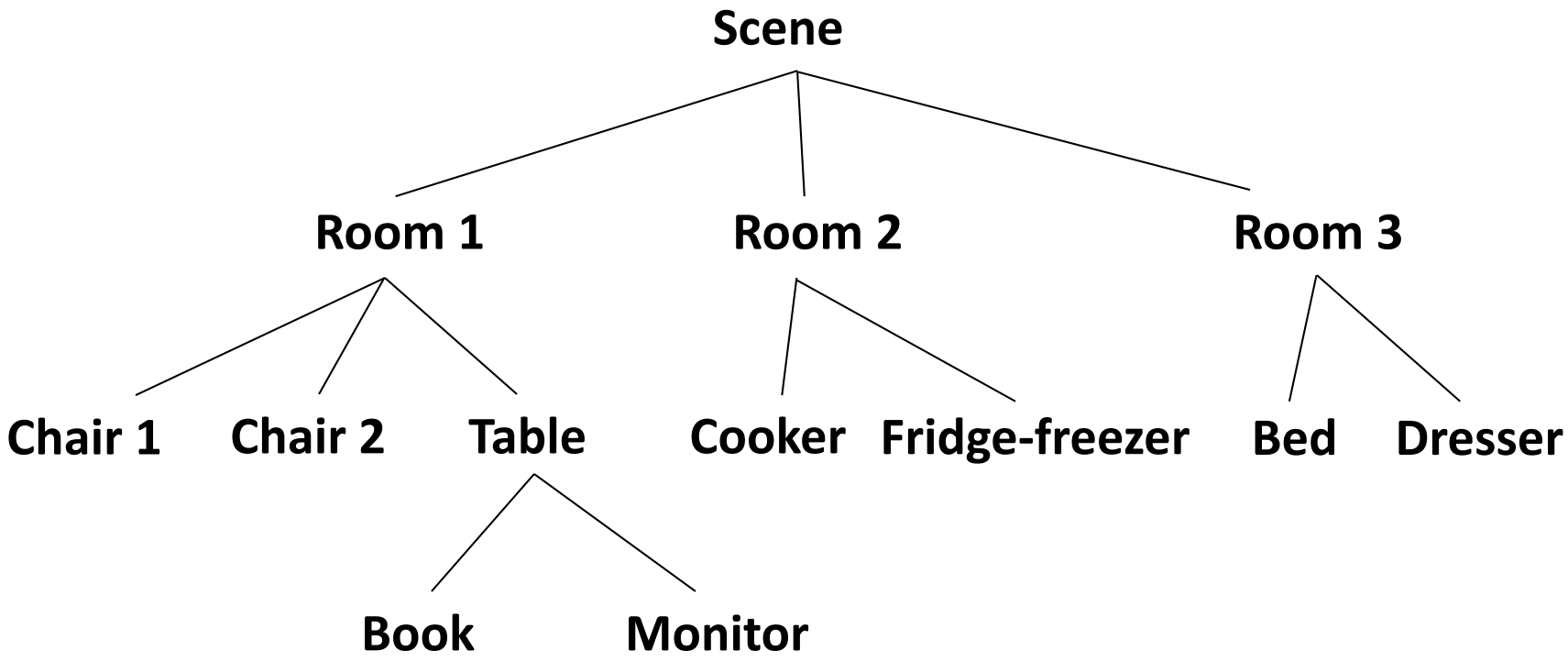
Scene hierarchy (1)

If a scene contains 1000 objects, we might think of a simple organisation like this.



Scene hierarchy (2)

We could also have a hierarchical grouping like this.

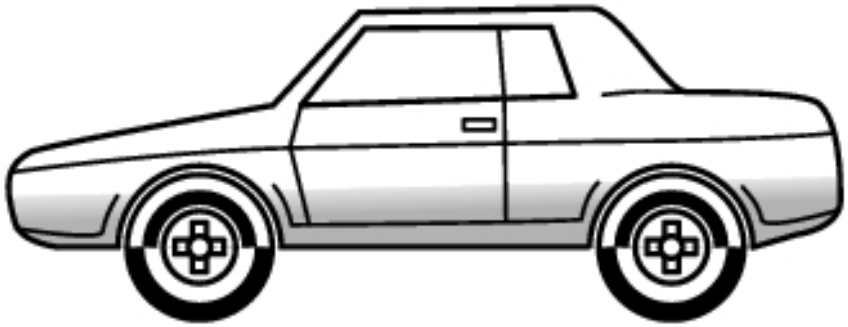


Scene hierarchy (3)

- In a scene, some objects may be grouped together in some way. For example, an *articulated* figure may contain several rigid components connected together in a specified fashion.
  - several objects sitting on a tray that is being carried around
  - a bunch of moons and planets orbiting around in a solar system
  - a hotel with 200 guest rooms, each room containing a bed, table, chairs, etc.
- In each of these cases, the placement of objects is described more easily when we consider their location and orientation relative to each other.

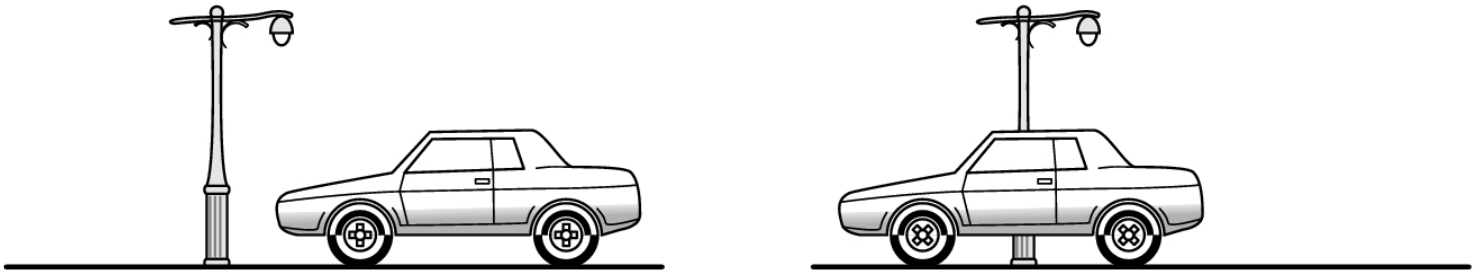
Hierarchical models – a car (1)

- Consider the model of a car
  - Chassis + 4 identical wheels
  - Two symbols
- Speed of the car is actually determined by the rotational speed of wheels or vice versa.



Hierarchical models – a car (2)

```
void main ( );
{   float s = ...;              // speed
    float d[3] = {...};         // direction
    draw_right_front_wheel(s,d);
    draw_left_front_wheel(s,d);
    draw_right_rear_wheel(s,d);
    draw_left_rear_wheel(s,d);
    draw_chassis(s,d);
}   // WE DO NOT WANT THIS!
```



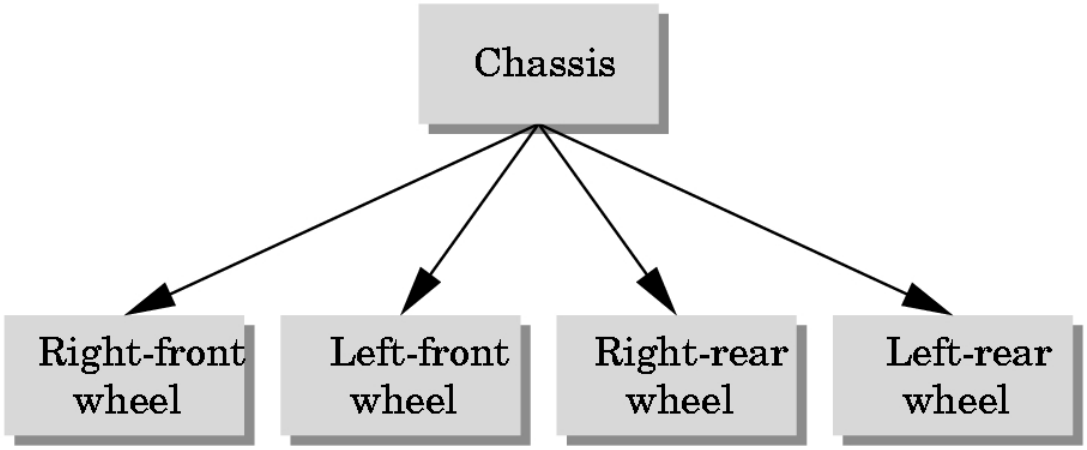
Two frames of reference for animation

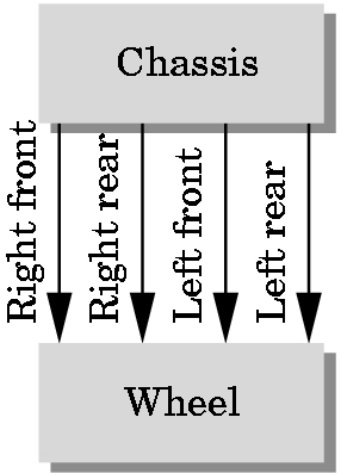
Hierarchical tree (1)

- It is very common in computer graphics to define a complex scene in some sort of hierarchical fashion.
- The individual objects are grouped into a hierarchy that is represented by a tree structure (upside down tree).
  - Each moving part is a single *node* in the tree.
  - The node at the top is the *root node.*
  - Each node (except the root) has exactly one *parent* node which is directly above it.
  - A node may have multiple *children* below it.
  - Nodes with the same parent are called *siblings.*
  - Nodes at the bottom of the tree with no children are called *leaf nodes.*

Hierarchical tree (2)

- Direct Acyclic Graph (DAG) stores a position of each wheel.
- Trees and DAGs – hierarchical methods express the relationships.

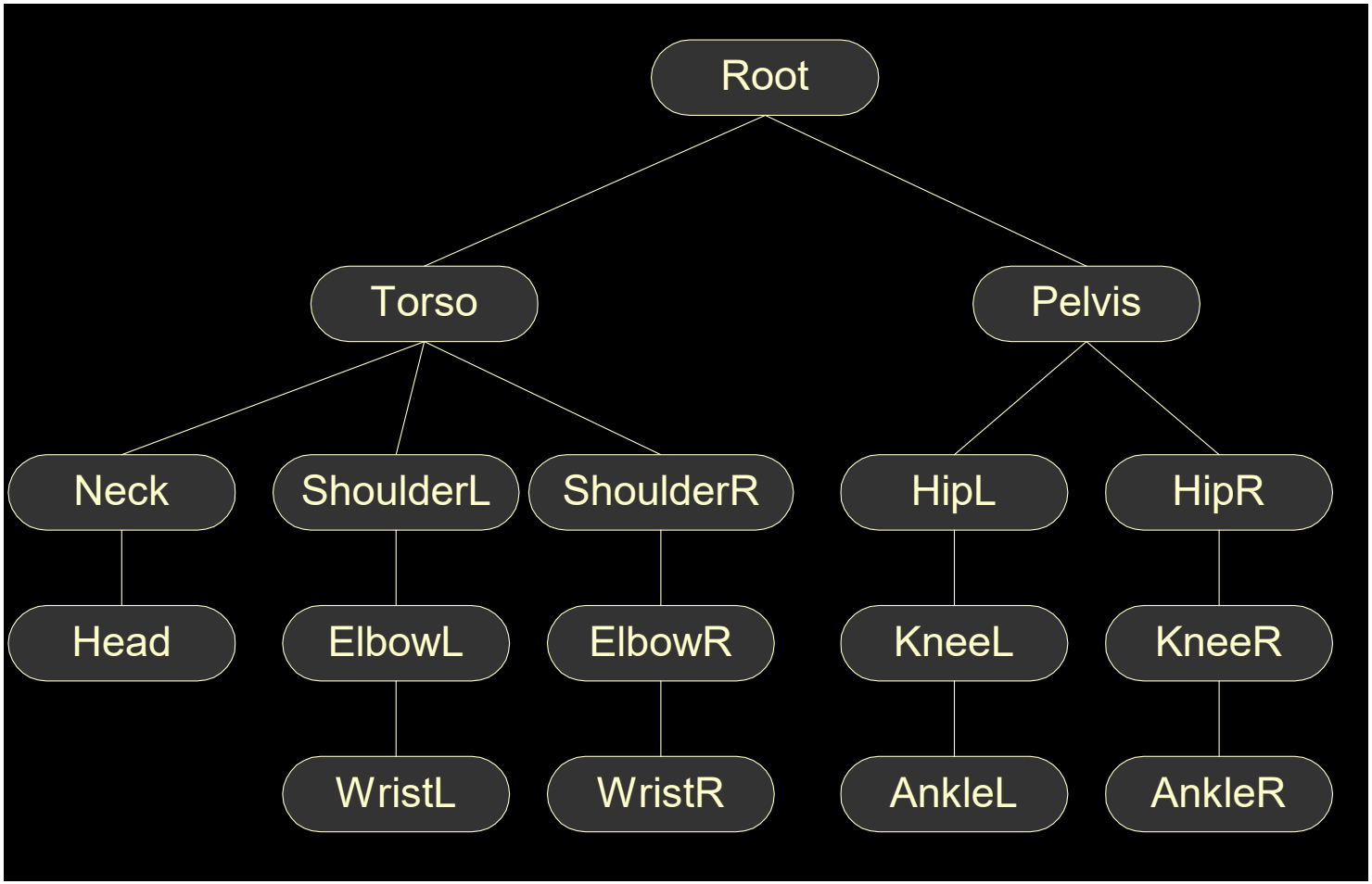




Articulated model

- An articulated model is an example of a hierarchical model consisting of rigid parts and connecting joints.
- The moving parts can be arranged into a tree data structure if we choose some particular piece as the ‘root’.
- For an articulated model (like a biped character), we usually choose the root to be somewhere near the centre of the torso.
- Each joint in the figure has specific allowable *degrees of freedom* (*DOFs*) that define the range of possible poses for the model.

Articulated model – biped character



Hierarchical transformations

- Each *node* in the tree represents an object that has a matrix describing its location and a model describing its geometry.
- When a node up in the tree moves its matrix,
  - it takes its children with it (in other words, rotating a character's shoulder joint will cause the elbow, wrist and fingers to move as well).
  - so child nodes inherit transformations from their parent node.
- Each node in the tree stores a *local matrix* which is its transformation *relative to its parent.*
- To compute a node's *world space matrix*, we need to concatenate its local matrix with its parent's world matrix:
  $\mathbf{M}_{\mathbf{world}} = \mathbf{M}_{\mathbf{parent}} \cdot \mathbf{M}_{\mathbf{local}}$

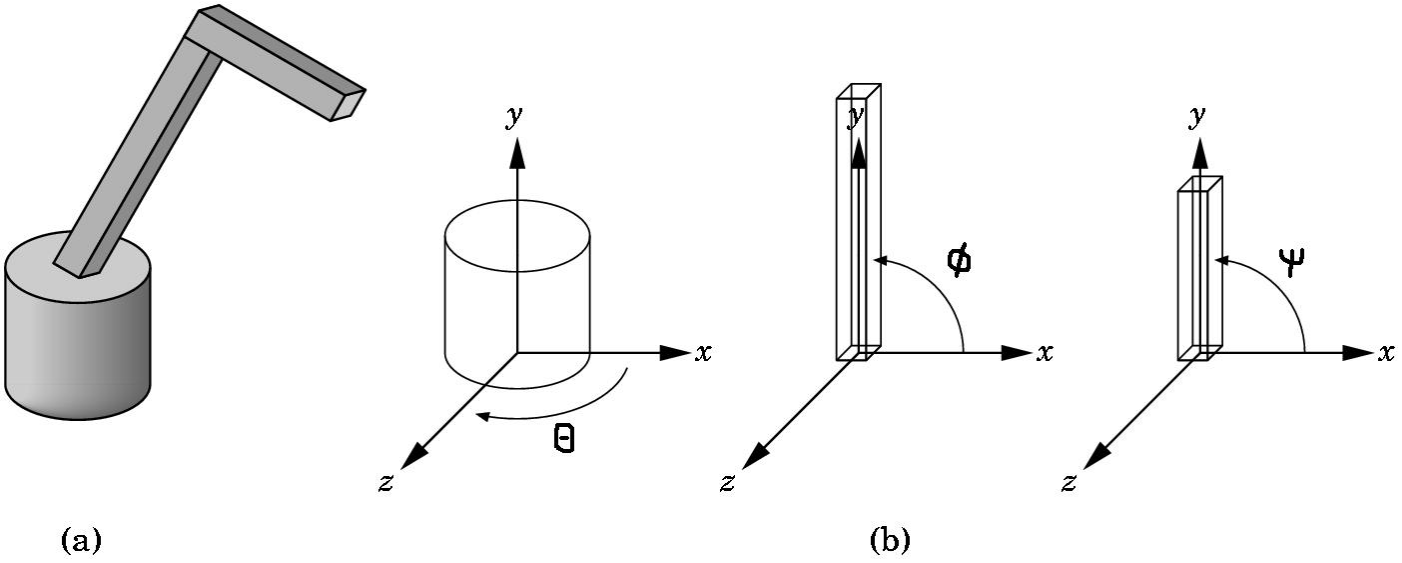
Recursive traversal and OpenGL matrix stacks

- To compute all of the world matrices in the scene, we can traverse the tree in a *depth-first traversal.*
- As each node is traversed, its world space matrix is computed.
- By the time a node is traversed, it is guaranteed that its parent's world matrix is available.
- The GL matrix stack is set up to facilitate the rendering of hierarchical scenes.
- While traversing the tree, we can call **`glPushMatrix()`** when going down a level, and **`glPopMatrix()`** when coming back up.

Articulated model – robot arm (1)

The robot arm is another example of articulated model.

- Parts are connected at joints.
- We can specify the state of model by specifying all joint angles.



Robot arm **Parts in their own frames of reference**

Articulated model – robot arm (2)

- Base rotates independently
  - Single angle determines position
- Lower arm attached to the base
  - Its position depends on the rotation of the base
  - It must also translate relative to the base and rotate around the connecting joint
- Upper arm attached to lower arm
  - Its position depends on both the base and lower arm
  - It must translate relative to the lower arm and rotate around the joint connecting to the lower arm

Articulated model – robot arm (3)

- Base transformation
  - Rotate the base: $\mathbf{R_b}$
  - Apply $\mathbf{M_{b\text{-}w} = R_b}$ to the base
- Lower arm transformation
  - Translate the lower arm relative to the base: $\mathbf{T_{la}}$
  - Rotate the lower arm around the joint: $\mathbf{R_{la}}$
  - Apply $\mathbf{M_{la\text{-}w} = M_{b\text{-}w} \cdot M_{la} = R_b \cdot T_{la} \cdot R_{la}}$ to the lower arm
- Upper arm transformation
  - Translate the upper arm relative to the lower arm: $\mathbf{T_{ua}}$
  - Rotate the upper arm around the joint: $\mathbf{R_{ua}}$
  - Apply $\mathbf{M_{ua\text{-}w} = M_{la\text{-}w} \cdot M_{ua} = R_b \cdot T_{la} \cdot R_{la} \cdot T_{ua} \cdot R_{ua}}$ to the upper arm

Articulated model – robot arm (4)

- Each of the 3 parts has 1 DOF – a joint angle between them.

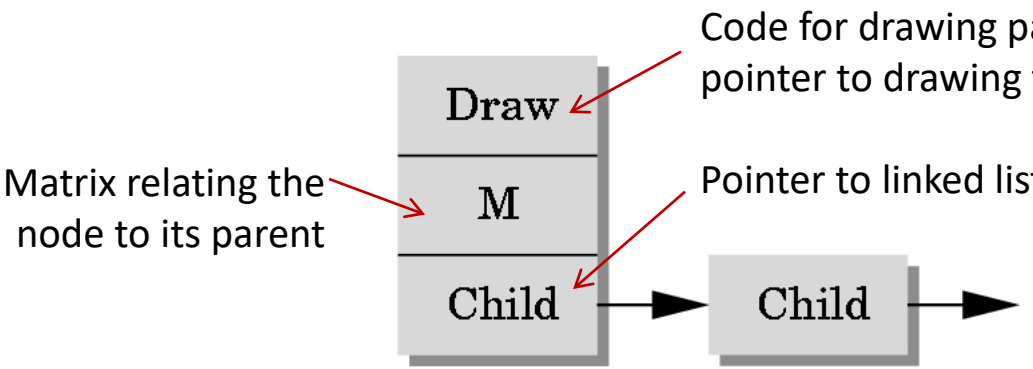
```
void display()
{   glRotatef(theta, 0.0, 1.0, 0.0);
    base();
    glTranslatef(0.0, h1, 0.0);
    glRotatef(phi, 0.0, 0.0, 1.0);
    lower_arm();
    glTranslatef(0.0, h2, 0.0);
    glRotatef(psi, 0.0, 0.0, 1.0);
    upper_arm();
}
```

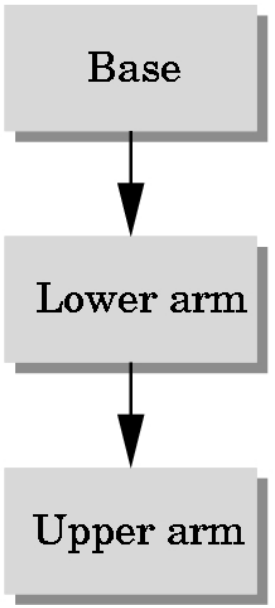
- The code shows relationships between the parts of the model. The appearance can change easily without altering the relationships.
- The MODELVIEW matrix for the upper arm is
  $\mathbf{M_{ua\text{-}w} = R_b(\theta) \cdot T_{la}(h1) \cdot R_{la}(\phi) \cdot T_{ua}(h2) \cdot R_{ua}(\psi)}$

Articulated model – robot arm (5)

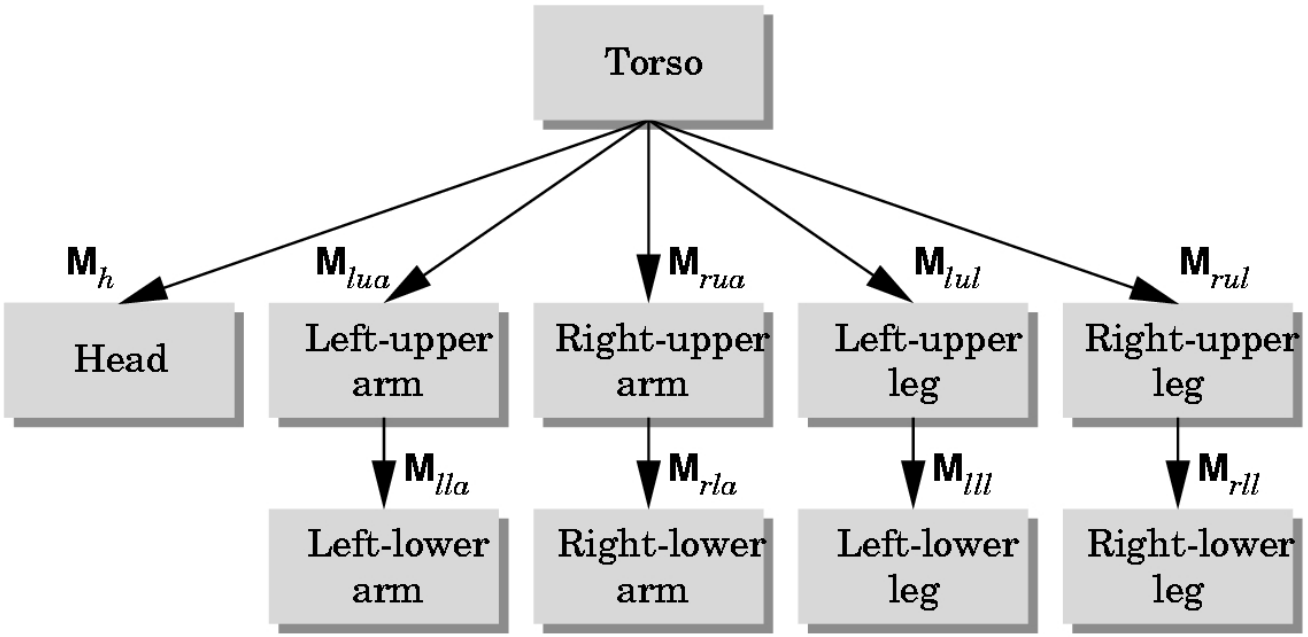
If information is stored in the nodes (not in edges), each node must store at least:

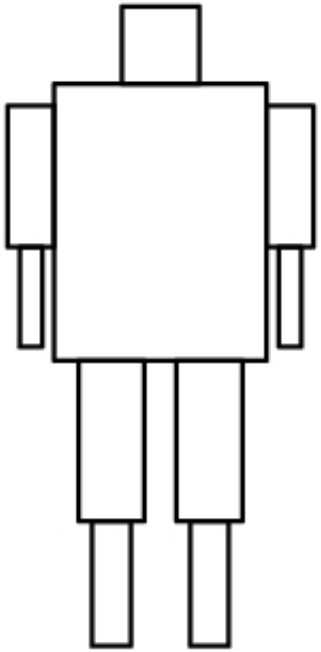
- A pointer to a function that draws the object represented by the node.
- A matrix that positions, orients and scales the object of the node (including its children) relative to the node's parent.
- A pointer to its children.





A humanoid model





A humanoid model – building the model

- We can build a simple implementation using quadrics: ellipsoids and cylinders.
- Access parts through functions such as

  `torso()`

  `left_upper_arm()`
- Matrices describe the position of a node with respect to its parent.

  e.g. $\mathbf{M}_{lla}$ positions left lower arm with respect to left upper arm.

A humanoid model – traversal and display

- The position of the figure is determined by 11 joint angles.
- Display of the tree can be thought of as a graph traversal.
  - Visit each node once.
  - Execute the display function at each node that describes the part associated with the node, applying the correct transformation matrix for position and orientation.

A humanoid model – transformation matrices

There are 10 relevant matrices.

- $\mathbf{M_t}$ positions and orients the entire figure through the torso which is the root node.
- $\mathbf{M_h}$ positions the head with respect to the torso.
- $\mathbf{M_{lua}}$, $\mathbf{M_{rua}}$, $\mathbf{M_{lul}}$, $\mathbf{M_{rul}}$ position the arms and legs with respect to the torso.
- $\mathbf{M_{lla}}$, $\mathbf{M_{rla}}$, $\mathbf{M_{lll}}$, $\mathbf{M_{rll}}$ position the lower parts of the limbs with respect to the corresponding upper limbs (parents).

A humanoid model – tree and traversal

- All matrices are incremental and any traversal algorithm can be used (depth-first or breadth-first). We can traverse from the left to right and depth first.
- Explicit traversal in the code is performed, using stacks to store required matrices and attributes.
- Recursive traversal code is simpler, and the storage of matrices and attributes is made implicitly.

A humanoid model – stack-based traversal

- Set model-view matrix **M** to $\mathbf{M}_t$ and draw the torso.
- Set model-view matrix **M** to $\mathbf{M}_t \cdot \mathbf{M}_h$ and draw the head.
- For the left-upper arm, we need $\mathbf{M}_t \cdot \mathbf{M}_{lua}$ and so on.
- Rather than re-computing $\mathbf{M}_t \cdot \mathbf{M}_{lua}$ from scratch or using an inverse matrix, we can use the matrix stack to store $\mathbf{M}_t \cdot \mathbf{M}_{lua}$ and other matrices as we traverse the tree.

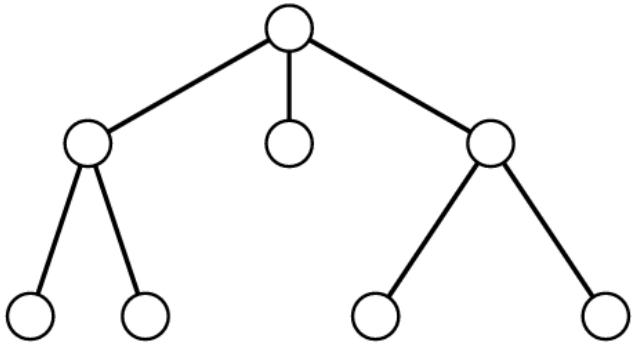
Note that the model-view matrix for the left lower arm is $\mathbf{M}_{lla\text{-}w} = \mathbf{M}_t \cdot \mathbf{M}_{lua} \cdot \mathbf{M}_{lla}$

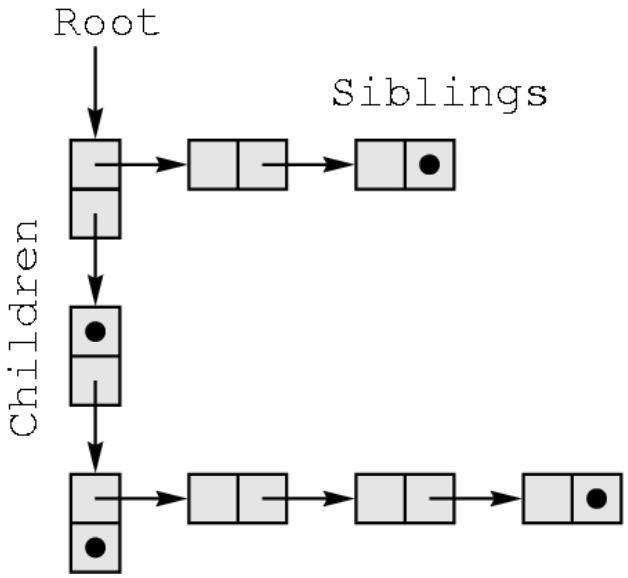
A humanoid model – traversal code

```
void figure() {
   torso();

   glPushMatrix();    // save present MODELVIEW matrix
   glTranslatef();    // update MODELVIEW matrix for the head
   glRotate3();
   head();
   glPopMatrix();     // recover MODELVIEW matrix for the
                      // torso and save the state
   glPushMatrix();
   glTranslatef();    // update MODELVIEW matrix for
   glRotate3();       // the left upper leg
   left_upper_leg();

   glTranslatef();
   glRotate3();       // incremental change for
   left_lower_leg();  // the left_lower_leg
   glPopMatrix();     // recent state recovery
   ...;
}
```

A humanoid model – tree data structure (1)

```
typedef struct treenode
{   GLfloat m[16];
    void(*f)();
    struct treenode *sibling;
    struct treenode *child;
} treenode;

...
treenode torso_node,
         head_node,
         ...;
```





A humanoid model – tree data structure (2)

```
// for the torso
glRotatef(theta[0], 0.0, 1.0, 0.0);
glGetFloatv(GL_MODELVIEW_MATRIX, torso_node.m);
// matrix elements copied to the M of the node

// the torso node has no sibling; and
// the leftmost child is the head node

// rest of the code for the torso node
torso_node.f = torso;
torso_node.sibling = NULL;
torso_node.child = &head_node;
```

A humanoid model – tree data structure (3)

```
// for the upper-arm node
glTranslatef(-(TORSO_RADIUS+UPPER_ARM_RADIUS),
    0.9*TORSO_HEIGHT, 0.0)
glRotatef(theta[3], 1.0, 0.0, 0.0);
glGetFloatv(GL_MODELVIEW_MATRIX, lua_node.m);
// matrix elements copied to the m of the node

lua_node.f = left_upper_arm;
lua_node.sibling = &rua_node;
lua_node.child = &lla_node;
```

A humanoid model – tree data structure (4)

```
// assumption MODELVIEW state
void traverse(treenode* root);
{
    if(root==NULL) return;
    glPushMatrix();
    glMultMatrixf(root->m);
    root->f();
    if(root->child!=NULL) traverse(root->child);
    glPopMatrix();
    if(root->sibling!=NULL) traverse(root->sibling);
} // traversal method is independent of the
  // particular tree!
```

A humanoid model – tree data structure (5)

- We must save model-view matrix (**`glPushMatrix`**) before multiplying it by the node matrix.
- Updated matrix applies to the children of the node.
- But not to its siblings which contain their own matrices; hence we must return to the previous state (**`glPopMatrix`**) before traversing the siblings.
- If we are changing attributes within nodes, we can either push (**`glPushAttrib`**) and pop (**`glPopAttrib`**) attributes within the rendering functions, or push the attributes when we push the model-view matrix.

A humanoid model – tree data structure (6)

```
// generic display callback function
void display(void)
{
    glClear(GL_COLOR_BUFFER_BIT |
            GL_DEPTH_BUFFER_BIT);
    glLoadIdentity();
    traverse(&torso_node);
    glutSwapBuffers();
}
```

Animation can then be implemented by controlling the joint angles (i.e. incremented or decremented) via the mouse or keyboard.

Summary

- Linear modelling does not provide effective ways to retain relationships among the objects of a model.
- Complex models for real world applications can be created and manipulated efficiently with hierarchical modelling.
- Hierarchical structure trees are used to implement hierarchical models in computer graphics.
- The code for the humanoid figure is in C (using Struct) and it would be more efficient to write it in C++ (using Class).

Teaching Plan

Week (c/m)	Lecture	Topic	CW	Lecturer
01 (24.09.09)	Lecture 01 Lecture 02	Introduction and hardware/software Mathematics for computer graphics		Yong Yue
02 (24.09.16)	Lecture 03	Geometric primitives		Nan Xiang
03 (24.09.23)	Lecture 04	Geometric transformations	CW1 out	Nan Xiang
04 (24.10.07)	Lecture 05	Viewing and projection		Nan Xiang
05 (24.10.14)	Lecture 06	Parametric curves and surfaces		Yong Yue
06 (24.10.21)	Lecture 07	3D modelling		Yong Yue
07 (24.10.28)		Reading week	CW1 due	
08 (24.11.04)	Lecture 08	Hierarchical modelling	CW2 out	Yong Yue
09 (24.11.11)	Lecture 09	Lighting and materials		Nan Xiang
10 (24.11.18)	Lecture 10	Texture mapping		Nan Xiang
11 (24.11.25)	Lecture 11	Clipping		Yong Yue
12 (24.12.02)	Lecture 12	Hidden surface removal	CW2 due	Yong Yue
13 (24.12.09)	Revision	Summary and highlights of topics covered / Past exam paper		Nan Xiang / Yong Yue