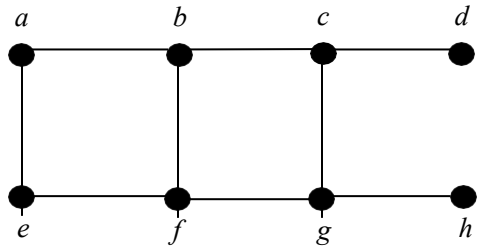
INT102 Algorithmic Foundations
Problem Session 3, Week 6

Location: SC176

Suggested Solutions

Question 1

Consider the following graph G.



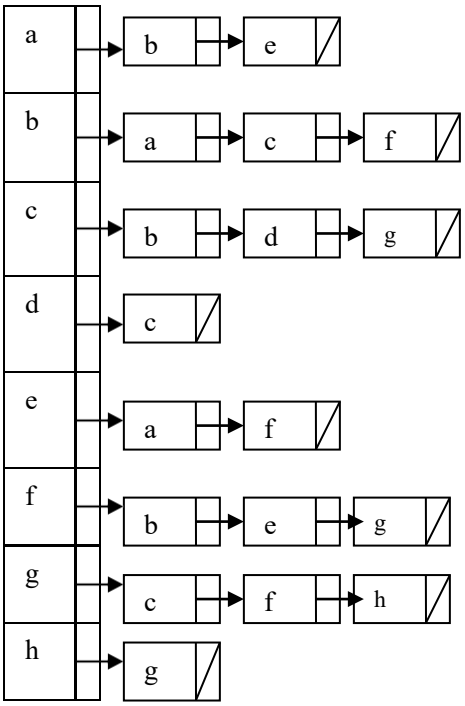
1. Give the adjacency matrix and adjacency list of the graph *G*. **(4 marks)**

Answer:

adjacency matrix (2 marks)

	a	*b*	*c*	*d*	*e*	*f*	*g*	*h*
a	0	1	0	0	1	0	0	0
b	1	0	1	0	0	1	0	0
c	0	1	0	1	0	0	1	0
d	0	0	1	0	0	0	0	0
e	1	0	0	0	0	1	0	0
f	0	1	0	0	1	0	1	0
g	0	0	1	0	0	1	0	1
h	0	0	0	0	0	0	1	0

adjacency list (2 marks)



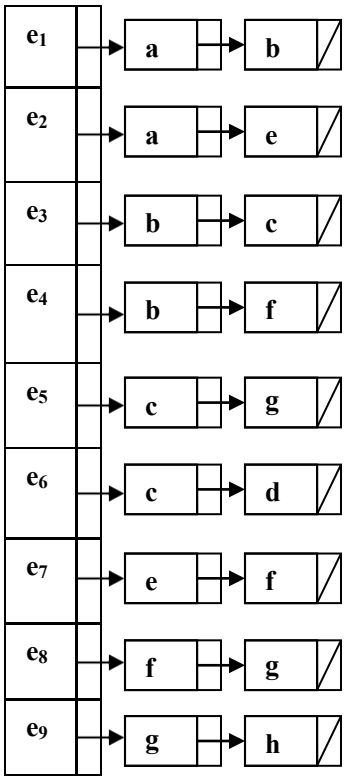
2. Give the incidence matrix and incidence list of the graph *G*. **(4 marks)**

Answer: Let e_1=(a, b), e_2= (a, e), e_3= (b, c) e_4= (b, f), e_5= (c, g), e_6= (c, d), e_7= (e, f), e_8= (f, g), e_9= (g, h)

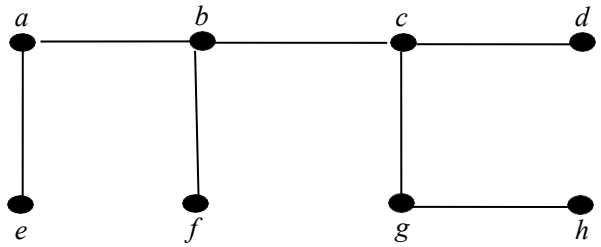
incidence matrix (2 marks)

	a	b	c	d	e	f	g	h
e_1	1	1	0	0	0	0	0	0
e_2	1	0	0	0	1	0	0	0
e_3	0	1	1	0	0	0	0	0
e_4	0	1	0	0	0	1	0	0
e_5	0	0	1	0	0	0	1	0
e_6	0	0	1	1	0	0	0	0
e_7	0	0	0	0	1	1	0	0
e_8	0	0	0	0	0	1	1	0
e_9	0	0	0	0	0	0	1	1

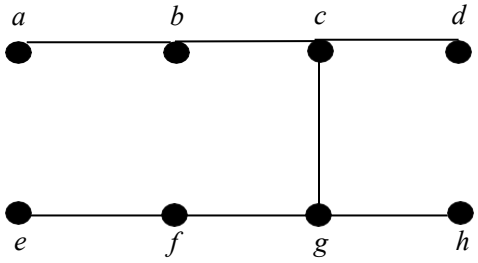
incidence list (2 marks)



3. Starting at the vertex ***a*** and resolving ties by the vertex alphabetical order traverse the graph by breadth-first-search (BFS) and construct the corresponding BFS tree. **(5 marks)**



4. Starting at the vertex ***a*** and resolving ties by the vertex alphabetical order traverse the graph by depth-first-search (DFS) and construct the corresponding BFS tree. **(5 marks)**



Question 2
Consider the following recursive function

$$f(n)=\begin{cases}1 & if\ 0\le n\le 3\\ f(n-1)+f(n-2)+f(n-3)+f(n-4) & if\quad n>4\end{cases}$$

1. Write a recursive (top-down) algorithm to compute it. **(5 marks)**
Answer:

```
Procedure F(n)
    if n==0 or n==1 or n =2 or n =3 then
            return 1
    else
            return F(n-1) + F(n-2) + F(n-3) + F(n-4)
```

2. What is the complexity of your algorithm (in big-O notation)? **(5 marks)**
Answer:

The time complexity of this algorithm is exponential, specifically O(4^n), where n is the input value. This is because for each value of n greater than 4, the function makes four recursive calls to values of n-1, n-2, n-3, and n-4, and each of those calls makes four more recursive calls, and so on. As a result, the number of function calls grows exponentially with the input size, leading to an exponential time complexity.

3. Design and write the pseudo code of a faster nonrecursive (bottom-up) algorithm using the concept of dynamic programming. **(5marks)**

```
Procedure F(n)

    Set A[0] = A[1] = A[2] = A[3] =1

    for i = 4 to n do

            A[i] = A[i-1] + A[i-2] + A[i-3] + A[i-4]

    return A[n]
```

4. What is the time complexity of the faster algorithm (in big-O notation)? (**5 marks**)

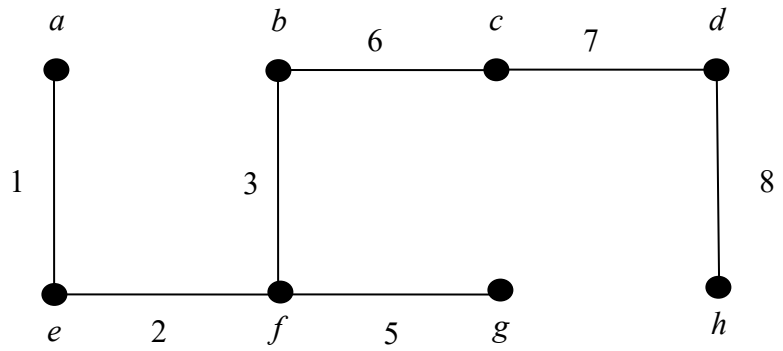
O(n)

Question 3

1. By Prim's algorithm, the order in which the vertices are selected is as follows:
   a, e, f, b, g, c, d, h
   (3 marks for the priority queue)

Order Selected	a(0,-)	b(-,∞)	c(-,∞)	d(-,∞)	e(-,∞)	f(-,∞)	g(-,∞)	h(-,∞)
a(0,-)		b(a,4)	c(-,∞)	d(-,∞)	**e(a,1)**	f(-,∞)	g(-,∞)	h(-,∞)
e(a,1)		b(a,4)	c(-,∞)	d(-,∞)		**f(-,2)**	g(-,∞)	h(-,∞)
f(e,2)		**b(f,3)**	c(f,10)	d(-,∞)			g(f,5)	h(-,∞)
b(f,3)			c(b,6)	d(-,∞)			**g(f,5)**	h(-,∞)
g(f,5)			**c(b,6)**	d(g,14)				h(g,8)
c(b,6)				**d(c,7)**				h(g,8)
d(c,7)								**h(g,8)**
h(g,8)								

The MST is as follows: (3 marks)

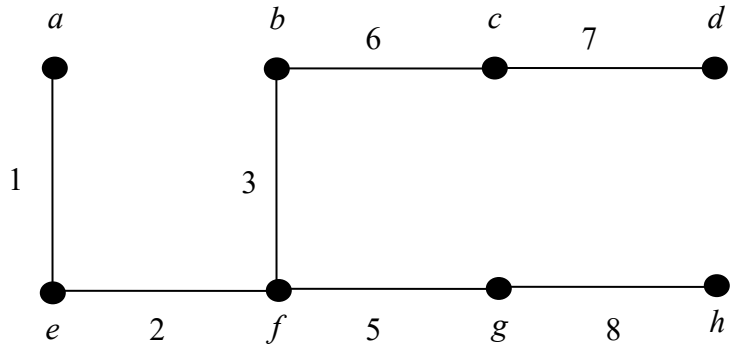


The MST drawn is not unique (1 mark)

2. Sorting edges according to their weights in increasing order:

(3 marks for the order list, 3 marks for the MST)

(a, e)	1
(e, f)	2
(f, b)	3
~~(a, b)~~	~~4~~
(f, g)	5
(b, c)	6
(c, d)	7
(g, h)	8
~~(d, h)~~	~~8~~
~~(c, f)~~	~~10~~
~~(c, g)~~	~~12~~
~~(d, g)~~	~~14~~



The MST is not unique. (1 mark)

3.

Selected order of edges: (a, e), (e, f), (a, b), (f, g), (b, c), (g, h), (c, d)
(3 mark for the priority queue)

Order Selected	a(0,-)	b(-,∞)	c(-,∞)	d(-,∞)	e(-,∞)	f(-,∞)	g(-,∞)	h(-,∞)
a(0,-)		b(a,4)	c(-,∞)	d(-,∞)	**e(a,1)**	f(-,∞)	g(-,∞)	h(-,∞)
e(a,1)		b(a,4)	c(-,∞)	d(-,∞)		**f(e,3)**	g(-,∞)	h(-,∞)
f(e,3)		**b(a,4)**	c(f,13)	d(-,∞)			g(f,8)	h(-,∞)
b(a,4)			c(b,10)	d(-,∞)			**g(f,8)**	h(-,∞)
g(f,8)			**c(b,10)**	d(g,22)				h(g,16)
c(b,10)				d(c,17)				**h(g,16)**
h(g,16)				**d(c,17)**				
d(c,17)								

(3 mark for the shortest paths)