Problem Session Week 13
Wednesday 23 May

Suggested Solutions

Question 1

Given the following instance of the 0/1 Knapsack problem

item	weight	value
1	2	\$12
2	1	\$10
3	3	\$20

The Knapsack Capacity W=3

Let V[i, j] be the value of the most valuable subset of the first i items that fit into the Knapsack of capacity j. Then V[i, j] can be recursively defined as following:

$$V[i, j] = \begin{cases} 0 & \text{if } i = 0 \text{ } or \text{ } j = 0 \\ \max\{V[i-1, j], \quad v_i + V[i-1, j-w_i]\} & \text{if } j - w_i \geq 0 \\ V[i-1, j] & \text{if } j - w_i < 0 \end{cases}$$

1. Using dynamic programming, complete the following table.

capacity j

Item i	0	1	2	3
0				
w_1=2, v_1=12 1				
w_2=1, v_2=10 2				
w_3=3, v_3=20 3				

2. What is the value of the most valuable subset?

3. Give an optimal subset of the instance based on the table.

4. Based on the recurrence relation given above, write a pseudocode of the bottom-up dynamic programming algorithm for the knapsack problem.

5. What is the value of the most valuable subset if the capacity of the knapsack is 2?

Suggested Solutions

1.

Item i	0	1	2	3
0	0	0	0	0
w_1=2, v_1=12 1	0	0	12	12
w_2=1, v_2=10 2	0	10	12	22
w_3=3, v_3=20 3	0	10	12	22

2. 22

3. {Item1, item2}

4. Pseudocode for 0/1 Knapsack algorithm

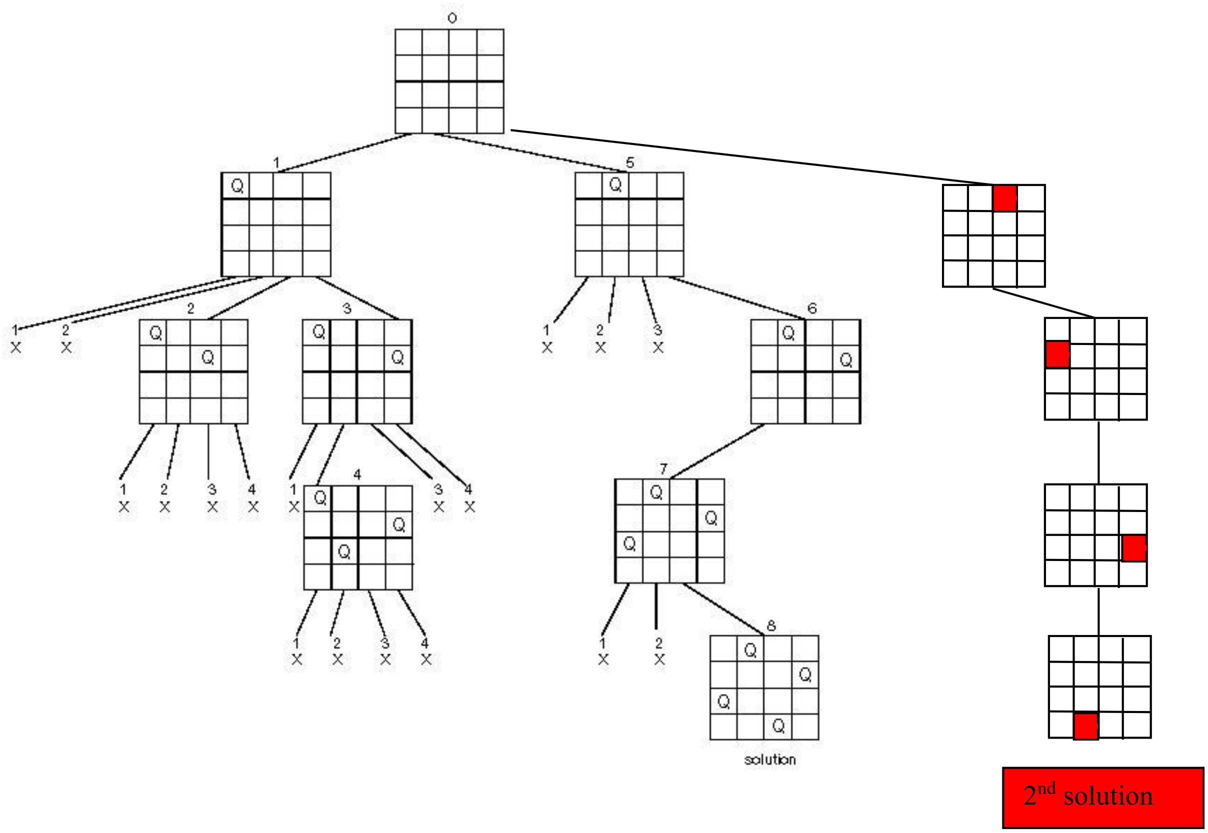
```
For  j = 0  to  W
    V[0, j] = 0
For  i = 1  to  n
    V[i, 0] = 0
For  i = 1  to  n
    For  j = 0  to  W
      if  wi <= j     // item i can be part of the solution
        if  bi + V[i-1, j-wi] > V[i-1, j]
          V[i, j] = bi + V[i-1, j- wi]
        else
          V[i, j] = V[i-1, j]
              else  // wi > j
                V[i, j] = V[i-1,j]
```

5. 12

Question 2

Continue the backtracking search for a solution to the four-queens problem, which was given in this week's lecture, to find the second solution to the problem. Explain how the board's symmetry can be used to find the second solution to the four-queens problem.

Solution:

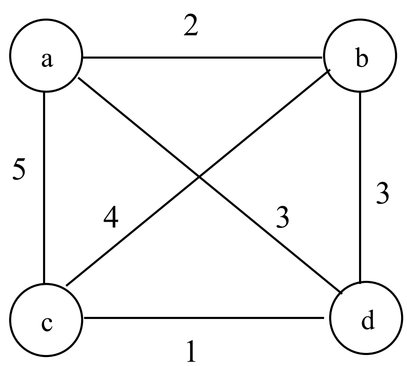


The second solution can be obtained by following way:
If there is a queen at position (i, j), using board's symmetry then in the second solution, it will have a queen at position (i, 4-j+1) (i=1, 2, 3, 4; j=1,2,3,4).

Question 3

1. Is the following graph a Euclidean graph?

2. Apply Twice-Around-the-Tree algorithm to solve the travelling salesman problem for the following graph.

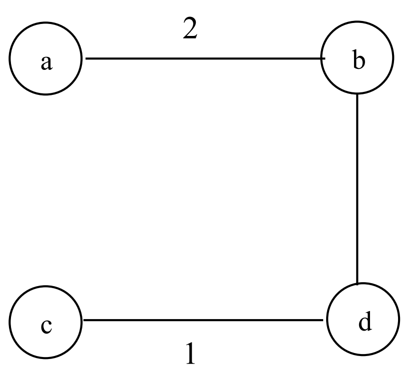


3. What is the optimization solution?
4. What is the accuracy ratio?

Solution:

1. No, as d(a,d) + d(d, c) < d(a, c)

2. A minimum spanning tree of the graph is depicted below. Starting from a, by DFS, we have the following travel: a b d c d b a. By removing the repeat nodes except a, we have the following approximation solution: S_A= a b d c a, with length 11.



3. For this instance, the optimization solution is S*=a b c d a with length 10.

4. The $r(s_a) = f(s_a)/f(s^*) = 11/10=1.1$