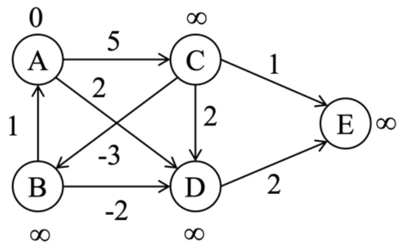
INT102 Algorithmic Foundations
Problem Session 4, Week 8

Location: SC176

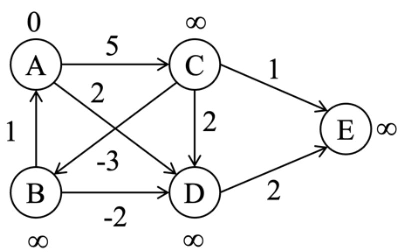
Suggested Solutions

Question 1

Apply Bellman-Ford algorithm to find the shortest paths from the source to all other vertices. G is as following and A is the source vertex.

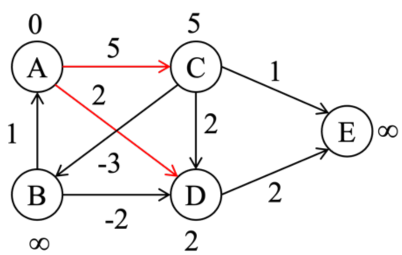


Suggested Solutions:

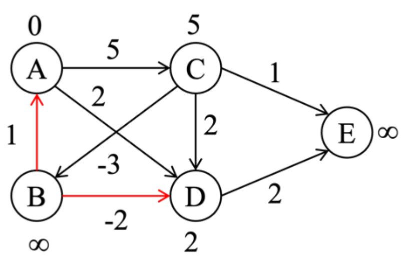


Vertices	A	B	C	D	E
Cost	0	∞	∞	∞	∞
Pre	-	-	-	-	-

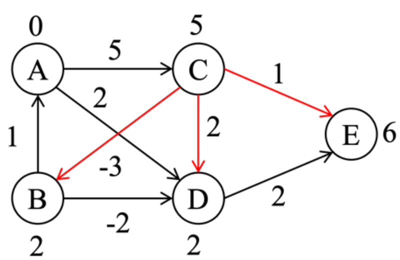
The 1st iteration:



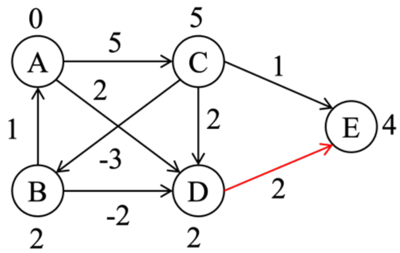
Vertices	A	B	C	D	E
Cost	0	∞	5	2	∞
Pre	-	-	A	A	-



Vertices	A	B	C	D	E
Cost	0	∞	5	2	∞
Pre	-	-	A	A	-

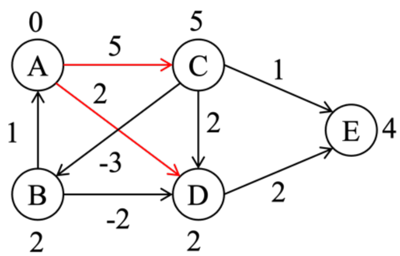


Vertices	A	B	C	D	E
Cost	0	2	5	2	6
Pre	-	C	A	A	C

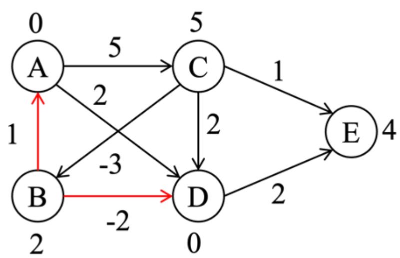


Vertices	A	B	C	D	E
Cost	0	2	5	2	4
Pre	-	C	A	A	D

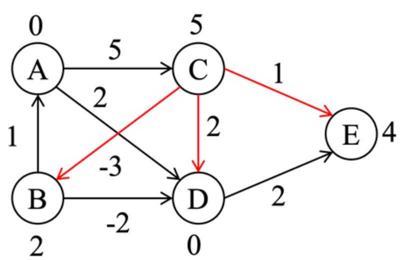
The 2nd iteration:



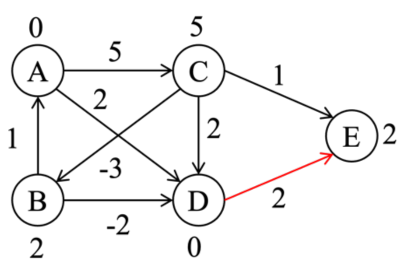
Vertices	A	B	C	D	E
Cost	0	2	5	2	4
Pre	-	C	A	A	D



Vertices	A	B	C	D	E
Cost	0	2	5	0	4
Pre	-	C	A	B	D



Vertices	A	B	C	D	E
Cost	0	2	5	0	4
Pre	-	C	A	B	D



Vertices	A	B	C	D	E
Cost	0	2	5	0	2
Pre	-	C	A	B	D

The 3rd iteration:
No more updating

Question 2
Assuming that the set of possible list is $\{a, b, c, d\}$, sort the following list in alphabetical order by the counting algorithm:

$$b, c, d, c, b, a, a, b$$

Suggested Solutions:

Auxiliary array C:

a	b	c	d
2	3	2	1

a	b	c	d
2	5	7	8

Changes in Output array

1	2	3	4	5	6	7	8
				b			

1	2	3	4	5	6	7	8
	a			b			

1	2	3	4	5	6	7	8
a	a			b			

1	2	3	4	5	6	7	8
a	a		b	b			

1	2	3	4	5	6	7	8
a	a		b	b		c	

1	2	3	4	5	6	7	8
a	a		b	b		c	d

1	2	3	4	5	6	7	8
a	a		b	b	c	c	d

1	2	3	4	5	6	7	8
a	a	b	b	b	c	c	d

Changes in Auxiliary array C

a	b	c	d
2	4	7	8

a	b	c	d
1	4	7	8

a	b	c	d
0	4	7	8

a	b	c	d
0	3	7	8

a	b	c	d
0	3	6	8

a	b	c	d
0	3	6	7

a	b	c	d
0	3	6	7

a	b	c	d
0	3	5	7

Question 3: Consider the problem of searching for genes in DNA sequences using Horspool's algorithm. A DNA sequence is represented by a text on the alphabet {A, C, G, T}, and the gene or a gene segment is a pattern.

3A. Construct the shift table for the following gene segment.

TCCTATTCTT

3B. Apply Horspool' s algorithm to locate the pattern in the following DNA sequence.

TTATAGATCTGGTATTCTTTTATAGATCTCCTATTCTT

Suggested Solutions:

3A. Shift table:

A	C	G	T
5	2	10	1

3B. Pattern searching:

t	t	a	t	a	g	a	t	c	t	g	g	t	a	t	t	c	t	t	t	t	a	t	a	g	a	t	c	t	c	c	t	a	t	t	c	t	t
t	c	c	t	a	t	t	c	t	t																												
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																							t	c	c	t	a	t	t	c	t	t					
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Question 4 (30 marks)
Using a gap penalty of d= -1 and scoring matrix as below

	A	**C**	**G**	**T**
A	**1**	**-3**	**-2**	**-3**
C	**-3**	**1**	**-3**	**-2**
G	**-2**	**-3**	**1**	**-3**
T	**-3**	**-2**	**-3**	**1**

1. Optimal global alignment (15 marks)

   a. Using dynamic programming, fill in the table in computing the score of the optimal global alignment of GAGT and ACATGT.

   b. Based on the table, find all the optimal global alignments of GAGT and ACATGT.

Solution

a.

		0	1	2	3	4	5	6
			A	C	A	T	G	T
0		0	-1 ←	-2 ←	-3 ←	-4 ←	-5 ←	-6 ←
1	G	-1 ↑	↖ -2 ↑ ←	-3 ↑ ←	↖ -4 ↑ ←	-5 ↑ ←	↖ -3	-4 ←
2	A	-2 ↑	↖ 0	-1 ←	↖ -2 ←	-3 ←	-4 ↑ ←	-5 ↑ ←
3	G	-3 ↑	-1 ↑	-2 ↑ ←	-3 ↑ ←	-4 ↑ ←	↖ -2	-3 ←
4	T	-4 ↑	-2 ↑	↖ -3 ↑ ←	-4 ↑ ←	↖ -2	-3 ↑ ←	↖ -1

Marking Schema:

- if all entries (number + arrow) are correct, (a cell may contain both left and up arrows, this is treated as correct) 10 marks
- For each wrong entry, deduce 0.5 mark , maximum 10 marks

b. **Backtrack procedure**: Following the arrows from the bottom right corner to top left corner, each path corresponding to an alignment.

Possible alignments:

1) The path (4,6)(3,5)(2,4)(2,3)(1,2),(1,1),(0,0) corresponds to the following

alignment

A	C	A	T	G	T
G	-	A	-	G	T

2) The path (4,6)(3,5)(2,4)(2,3))(1,2) (1,1)(0,1)(0,0) corresponds to the following alignment

A	-	C	A	T	G	T
-	G	-	A	-	G	T

3) The path (4,6)(3,5)(2,4)(2,3) (1,2) (1,1) (1,0) (0,0) corresponds to the following alignment

-	A	C	A	T	G	T
G	-	-	A	-	G	T

4) The path (4,6)(3,5)(2,4)(2,3)(1,2) (0,2),(0,1) (0,0)corresponds to the following alignment

A	C	-	A	T	G	T
-	-	G	A	-	G	T

5) The path (4,6)(3,5)(2,4)(2,3)(2,2) (2,1),(1,0) (0,0)corresponds to the following alignment

-	A	C	A	T	G	T
G	A	-	-	-	G	T

Marking Schema:

- If the description of the backtrack procedure is correct 1 mark.
- If the one of the above alignments are given 2 marks.
- If corresponding path in the table is given and correct, + 2 marks.

2. Optimal local alignment (15 marks)

a. Using dynamic programming, fill in the table in computing the score of the optimal local alignment of GAGT and ACATGT

b. Based on the table, find all the optimal local alignments of GAGT and ACATGT.

Solution

a.

		0	1	2	3	4	5	6
			A	C	A	T	G	T
0		0	0	0	0	0	0	0
1	G	0	0	0	0	0	↖1	0
2	A	0	↖1	0 ←	↖1	0 ←	0 ↑	0
3	G	0	0 ↑	0	0 ↑	0	↖1	0 ←
4	T	0	0	0	0	1	0 ← ↑	↖2

Marking Schema:

- if all entries (number + arrow) are correct, (a cell may contain both left and up arrows, this is treated as correct) 10 marks
- For each wrong entry, deduce 0.5 mark , maximum 10 marks

b. Backtrack procedure: Start at highest score and trace arrows back to first 0 with no arrow. Each path corresponding to an alignment.

Optimal local alignment (4,6)(3,5)(2,4)(2,3)

A	T	G	T
A	-	G	T

Marking Schema:

- If the description of the backtrack procedure is correct 1 mark.
- If the above alignment is given 2 marks.
- If corresponding path (4,6)(3,5)(2,4)(2,3) in the table is given and correct, 2 marks.

Question 5:
Suppose there are 10 people in a room. Each person shakes hands with some other people in the room. Prove that the number of people having an odd number of handshakes is even.

(Challenge: This puzzle is equivalent to the question in an undirected graph, "prove that the number of vertices with odd degree is even". Try to think why the two questions areequivalent.)

Suggested Solutions:

General case: there are $n>0$ people in a room. Each person shakes hands with some other people in the room. Prove that the number of people having an odd number of handshakes is even.

For such a problem, we construct a graph $G=(V, E)$, where each vertex in V represents one person. (v_i, v_j) is in E if and only if v_i and v_j shake hands. Then $deg(v_i)$ is the number of persons with which v_i shakes hands. With such representation, the problem now can be rephrased as follows:

In the graph $G=(V, E)$, the number of vertices with odd degree is even.

By the fact that the sum of all degrees in a graph is even, the number of vertices with odd degree must be even.