Gaussian RBF network

$F(x) = \sum_{i}^{M} w_i \cdot \varphi(\|x - t_i\|)$ ↓ gauss

Consider a Gaussian RBF network with 3 examples {x,d}, where x denotes the input and d denotes the value. The 3 examples are given as {(1,0), 3}, {(0,1), -1}, {(1,1), 5}. Suppose that all the 3 examples are selected as centers and the $\sigma_i = 1, i = 1,2,3.$

$\psi = \exp(-\frac{\|x-t_i\|^2}{2\sigma^2})$

1) Using Gaussian radial basis functions to compute the hidden layers. (Approximate calculations can be performed using: $\exp(-0.5) \approx 0.6025, \exp(-1) \approx 0.3679$.)

$\phi = \begin{pmatrix} 1 & 0.3679 & 0.6025 \\ 0.3679 & 1 & 0.6025 \\ 0.6025 & 0.6025 & 1 \end{pmatrix}$ N*M

2) Calculate the related weight vector and predict the values of each input. The inverse matrix of $A = \begin{bmatrix} 1.4984 & 1.0988 & 1.4267 \\ 1.0988 & 1.4984 & 1.4267 \\ 1.4267 & 1.4267 & 1.7260 \end{bmatrix}$ is

$\phi \cdot W = d$

① $W = (\phi^T\phi)^{-1}\phi^T d$

$B = \begin{bmatrix} 3.3460 & 0.8432 & -3.4626 \\ 0.8432 & 3.3460 & -3.4626 \\ -3.4626 & -3.4626 & 6.3035 \end{bmatrix}$ $\Rightarrow W = \begin{pmatrix} 0.028 \\ -6.3 \\ 8.7 \end{pmatrix}$

$d = \begin{pmatrix} 3 \\ -1 \\ 5 \end{pmatrix}$

意思是仍有3个点，两个中心!!

3) Suppose that only the two examples, {(1,0), 3}, {(0,1), -1}, are set as centers. Re-calculate the problems of 1) and 2). The inverse matrix of $A = \begin{bmatrix} 1.4984 & 1.0988 \\ 1.0988 & 1.4984 \end{bmatrix}$ is $B = \begin{bmatrix} 1.4439 & -1.0589 \\ -1.0589 & 1.4439 \end{bmatrix}$.

② $W = \varphi^{-1} d$?

见2),3)问

4) For the input vector (0,0), please calculate the hidden layers and final output using the two RBF networks given in 2) and 3).

让你推算 $\phi \cdot W = d$

已知 把(0,0)代入

$\phi = \begin{pmatrix} 1 & 0.3679 \\ 0.3679 & 1 \end{pmatrix}$

$W = \begin{pmatrix} 3.69 \\ -2.63 \end{pmatrix}$ $d = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$

Answer:

(1) The Gaussian radial function: $F(x, c) = \exp(-\frac{|x-c|^2}{2\sigma^2})$

For the input (1,0) $\rho_{11}=1$

$$\rho_{12} = exp\left(-\frac{(1-0)^2+(0-1)^2}{2}\right) = exp(-1)$$

$$\rho_{13} = exp\left(-\frac{(1-1)^2+(0-1)^2}{2}\right) = exp(-0.5)$$

For the input (0,1)

$$\rho_{21} = exp\left(-\frac{(0-1)^2+(1-0)^2}{2}\right) = exp(-1)$$

$$\rho_{22}=1$$

$$\rho_{23} = exp\left(-\frac{(0-1)^2+(1-1)^2}{2}\right) = exp(-0.5)$$

For the input (1,1)

$$\rho_{31} = exp\left(-\frac{(1-1)^2+(1-0)^2}{2}\right) = exp(-0.5)$$

$$\rho_{32} = exp\left(-\frac{(1-0)^2+(1-1)^2}{2}\right) = exp(-0.5)$$

$$\rho_{33}=1$$

The hidden layer can be represented by a approximate matrix:

$$\varphi = \begin{bmatrix} 1 & 0.3679 & 0.6025 \\ 0.3679 & 1 & 0.6025 \\ 0.6025 & 0.6025 & 1 \end{bmatrix}$$

2. The predictions of the examples can be written as:

$$e_i = \sum_{n=1,..,N} \rho_{in} w_n \quad \text{and} \qquad e = \varphi w$$

The real value is $\quad d = \begin{bmatrix} 3 \\ -1 \\ 5 \end{bmatrix}$

Method 1. Minimizing $|e-d|^2 \quad \triangleq \qquad \varphi^T\varphi w \;\; -\varphi^T d = 0$

$$w = (\varphi^T\varphi)^{-1}\varphi^T d$$

$$\varphi^T\varphi = \begin{bmatrix} 1.4984 & 1.0988 & 1.4267 \\ 1.0988 & 1.4984 & 1.4267 \\ 1.4267 & 1.4267 & 1.7260 \end{bmatrix}, \quad (\varphi^T\varphi)^{-1} = \begin{bmatrix} 3.3460 & 0.8432 & -3.4626 \\ 0.8432 & 3.3460 & -3.4626 \\ -3.4626 & -3.4626 & 6.3035 \end{bmatrix}$$

$$w = \begin{bmatrix} 0.0288 \\ -6.3007 \\ 8.7780 \end{bmatrix}$$

Method 2.

$$w = \varphi^{-1} d$$

$\longrightarrow$ 这个不一定存在 对φ得n*n才行

$$\varphi^{-1} = \begin{bmatrix} 1.5700 & -0.0121 & -0.9386 \\ -0.0121 & 1.5700 & -0.9386 \\ -0.9386 & -0.9386 & 2.1311 \end{bmatrix}$$

$$w = \begin{bmatrix} 0.0288 \\ -6.3007 \\ 8.7780 \end{bmatrix}$$

To verify the predictions:

$$e = \varphi w = \begin{bmatrix} 3.0000 \\ -1.0000 \\ 5.0000 \end{bmatrix}$$

3.

The hidden layer can be represented by a approximate matrix:

$$\varphi = \begin{bmatrix} 1 & 0.3679 \\ 0.3679 & 1 \\ 0.6025 & 0.6025 \end{bmatrix}$$

$$\varphi^{\mathrm{T}}\varphi = \begin{bmatrix} 1.4984 & 1.0988 \\ 1.0988 & 1.4984 \end{bmatrix}$$

$$(\varphi^{\mathrm{T}}\varphi)^{-1} = \begin{bmatrix} 1.4439 & -1.0589 \\ -1.0589 & 1.4439 \end{bmatrix}$$

$2\times2 * 2\times3 * 3\times1$

$\Rightarrow 2\times1$

$$\mathrm{w} = \begin{bmatrix} 4.8507 \\ -1.4774 \end{bmatrix}$$

To verify the predictions:

$$e = \varphi w = \begin{bmatrix} 4.3071 \\ 0.3071 \\ 2.0324 \end{bmatrix}$$

由于只用了两个并非精准差值

4. For the first RBF Network (1,0), (0,1), (1,1) are used as centers，the hidden layer for input (0,0) calculate as:

$$\rho_{11} = exp\left(-\frac{(0-1)^2+(0-0)^2}{2}\right) = exp(-0.5)$$

$$\rho_{12} = exp\left(-\frac{(0-0)^2+(0-1)^2}{2}\right) = exp(-0.5)$$

$$\rho_{13} = exp\left(-\frac{(0-1)^2+(0-1)^2}{2}\right) = exp(-1)$$

Therefore, the hidden layer is given by $\varphi = [0.6025, 0.6025, 0.3679]$
The output is

$$d = \varphi w = [0.6025, 0.6025, 0.3679]\begin{bmatrix} 0.0288 \\ -6.3007 \\ 8.7780 \end{bmatrix} = -0.5491$$

For the second RBF Network (1,0), (0,1) are used as centers，the hidden layer for input (0,0) calculate as:

$$\rho_{11} = exp\left(-\frac{(0-1)^2+(0-0)^2}{2}\right) = exp(-0.5)$$

$$\rho_{12} = exp\left(-\frac{(0-0)^2+(0-1)^2}{2}\right) = exp(-0.5)$$

Therefore, the hidden layer is given by $\varphi = [0.6025, 0.6025]$

The output is

$$d = \varphi w = [0.6025, 0.6025]\begin{bmatrix} 4.8507 \\ -1.4774 \end{bmatrix} = 2.0324$$