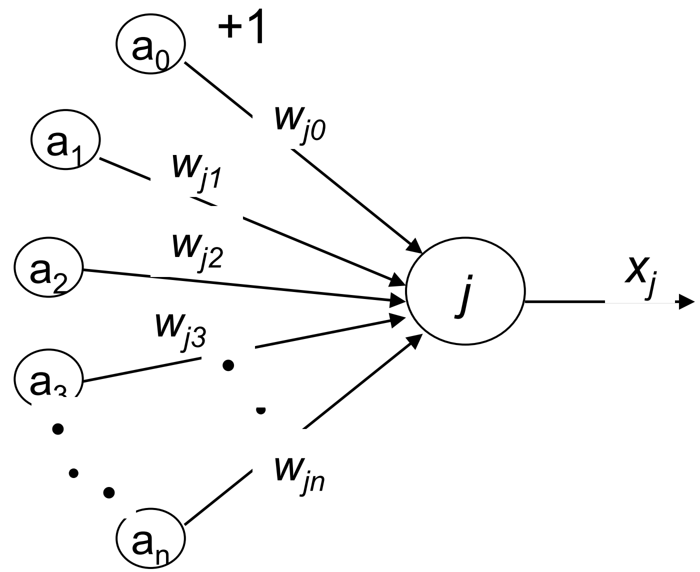
M-P NEURON & HEBB LEARNING

INT301 Bio-computation, Week 1, 2025

Abstract model of a neuron

An abstract neuron *j* with *n* inputs:

- Each input *i* transmits a real value a_i
- Each connection is assigned with the weight w_{ij}



The total input S, *i.e.,* the sum of the products of the inputs with the corresponding weights, is compared with the threshold (equal to 0 in this case), and the outcome X_i is produced consequently

The McCulloch-Pitts Neuron (1943)

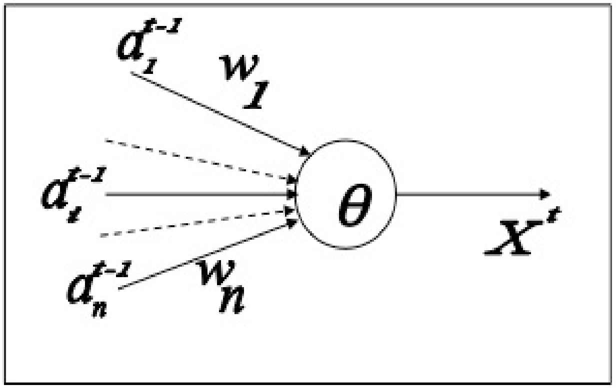
The authors modelled the neuron as

- A binary discrete-time element;
- With *excitatory* and *inhibitory* inputs and an excitation threshold;
- The network of such elements was the first model to tie the study of neural networks to the idea of computation in its modern sense.

The McCulloch-Pitts Neuron

- The input values a_i^t

  from the *i-th* presynaptic neuron at any instant *t* may be equal either to 0 or 1 only



- The weights of connections w_i are

  +1 for excitatory type connection and

  -1 for inhibitory type connection

- There is an excitation threshold θ associated with the neuron.

The McCulloch-Pitts Neuron

- Output x^{t+1} of the neuron at the following instant t+1 is defined according to the rule

$x^{t+1} = 1$ *if and only if* $$S^t = \sum_i w_i a_i^t \geq \theta$$

- In the MP neuron, we shall call the instant total input S^t - ***instant state of the neuron***

The McCulloch-Pitts Neuron

- The **state** S^t of the MP neuron does not depend on the previous state of the neuron itself, but is simply

$$S^t = \sum_i w_{ij} a_i^t = f(t)$$

- The neuron output x^{t+1} is function of its state S^t, therefore the output also can be written as function of discrete time

$$x(t) = g(S^t) = g(f(t))$$

Activation function

- The neuron output x^t can be written as

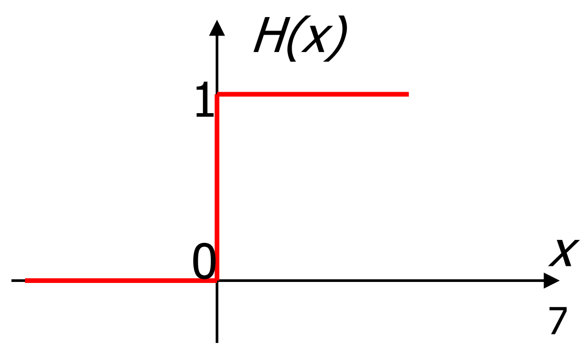
$$x(t) = g(S^t) = g(f(t))$$

where *g* is the ***threshold activation function***

$$g(S^t) = H(S^t - \theta) = \begin{cases} 1, if & S^t \geq \theta; \\ 0, if & S^t < \theta. \end{cases}$$

Here H is the Heaviside (unit step) function:

$$H(X) = \begin{cases} 1, & x \geq 0; \\ 0, & x < 0. \end{cases}$$



MP-neuron vs brain function

- M-P neuron made a base for a machine (network of units) capable of
  - storing information and
  - producing logical and arithmetical operations

- These correspond to the main functions of the brain
  - to store knowledge, and
  - to apply the knowledge stored to solve problems

ANN Learning Rules

- M-P neuron
  - storing information and
  - producing logical and arithmetical operations

↓

- The next step is to realize another important function of the brain, which is

to acquire new knowledge through experience, *i.e.*, ***learning***

ANN learning rules

Learning means

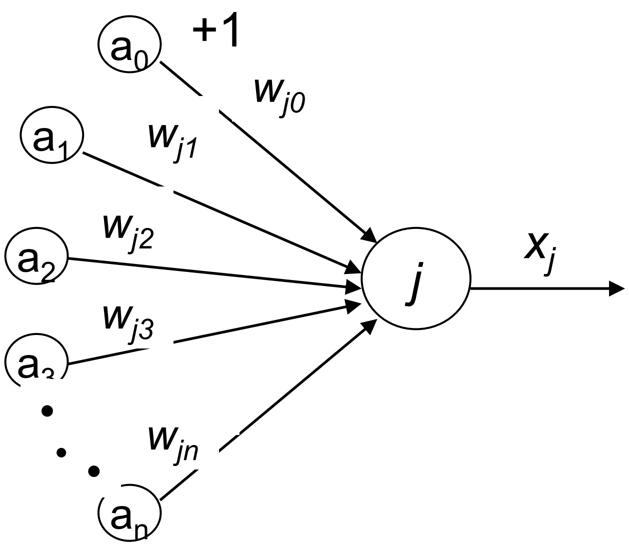
to change in response to experience.

In a network of MP-neurons, binary weights of connections and thresholds are fixed. The only change can be the change of pattern of connections, which is technically expensive.

⇓

Some easily changeable free parameters are needed.

ANN Learning Rules



$$\boxed{S_j = \sum_{i=0}^{n} w_{ji} a_i}$$

$$x_j = \begin{cases} 0 & \text{if } s_j \leq 0 \\ 1 & \text{if } s_j > 0 \end{cases}$$

- The ideal free parameters to adjust, and so to resolve learning without changing patterns of connections, are the weights of connections w_{ji}

ANN Learning Rules

Definition:

- **ANN learning rule**: how to adjust the weights of connections to get desirable output.

- Much work in artificial neural networks focuses on the learning rules that define

how to change the weights of connections between neurons to better adapt a network to serve some overall function.

ANN Learning Rules

- For the first time the problem was formulated in 1940s

  When experimental neuroscience was limited, the classic definitions of these learning rules came not from biology, but from *psychological studies* of **Donald Hebb** and **Frank Rosenblatt**

- Hebb proposed that

  a particular type of use-dependent modification of the connection strength of synapses *might underlie learning in the nervous system*

Hebb's Rule (1949)

Hebb proposed a **neurophysiological postulate**:

" …when an axon of a cell A

- is near enough to excite a cell B and
- repeatedly and persistently takes part in firing it

some growth process or metabolic change takes place in one or both cells

such that A's efficiency as one of the cells firing B, is increased."

Hebb's Rule (1949)

- The simplest formalization of Hebb's rule is

  to increase weight of connection

  at every next instant in the way:

$$w_{ji}^{k+1} = w_{ji}^{k} + \Delta w_{ji}^{k}$$

where

$$\Delta w_{ji}^{k} = C a_i^k x_j^k$$

Hebb's Rule (1949)

$$w_{ji}^{k+1} = w_{ji}^{k} + \Delta w_{ji}^{k}$$

$$\Delta w_{ji}^{k} = C a_i^k x_j^k$$

here

- w_{ji}^{k} is the weight of connection at instant *k*
- w_{ji}^{k+1} is the weight of connection at the following instant *k+1*
- Δw_{ji}^{k} is increment by which the weight of connection is enlarged
- C is positive coefficient which determines *learning rate*
- a_i^k is input value from the presynaptic neuron at instant *k*
- x_j^k is output of the postsynaptic neuron at the same instant *k*

Hebb's Rule (1949)

$$w_{ji}^{k+1} = w_{ji}^{k} + \Delta w_{ji}^{k}$$
$$\Delta w_{ji}^{k} = C a_{i}^{k} x_{j}^{k}$$

- Thus

  the weight of connection changes at the next instant only if both preceding input via this connection and the resulting output simultaneously are not equal to 0

- Equations emphasize the **correlation** nature of a Hebbian synapse

Hebb's Rule (1949)

$$w_{ji}^{k+1} = w_{ji}^{k} + \Delta w_{ji}^{k}$$

$$\Delta w_{ji}^{k} = C a_i^k x_j^k$$

- Hebb's original learning rule
  - referred exclusively to excitatory synapses, and
  - has the <u>unfortunate property</u> that it can only increase synaptic weights, thus washing out the distinctive performance of different neurons in a network, as the connections drive into saturation …

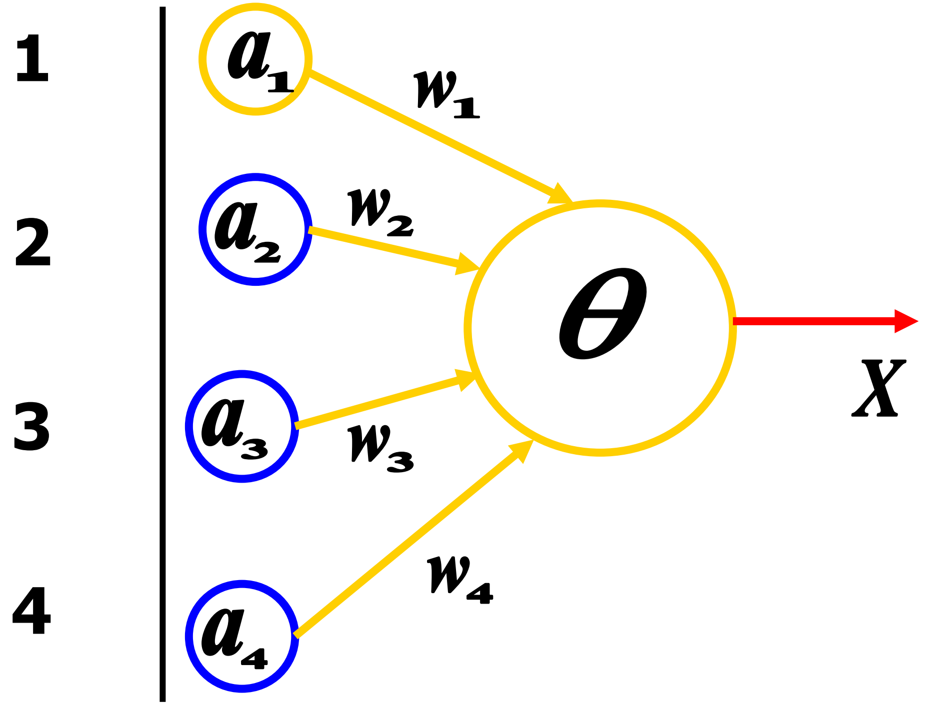
Hebb’s Rule (1949)

- However, when the Hebbian rule is augmented by a formalization rule, e.g., keep constant the total strength of synapses upon a given neuron, it tends to “sharpen” a neuron’s predisposition “without a teacher”, causing its firing to become better correlated with a cluster of stimulus patterns.

- For this reason, Hebb's rule plays an important role in studies of many ANN algorithms, such as unsupervised learning or self-organization, which we will study later.

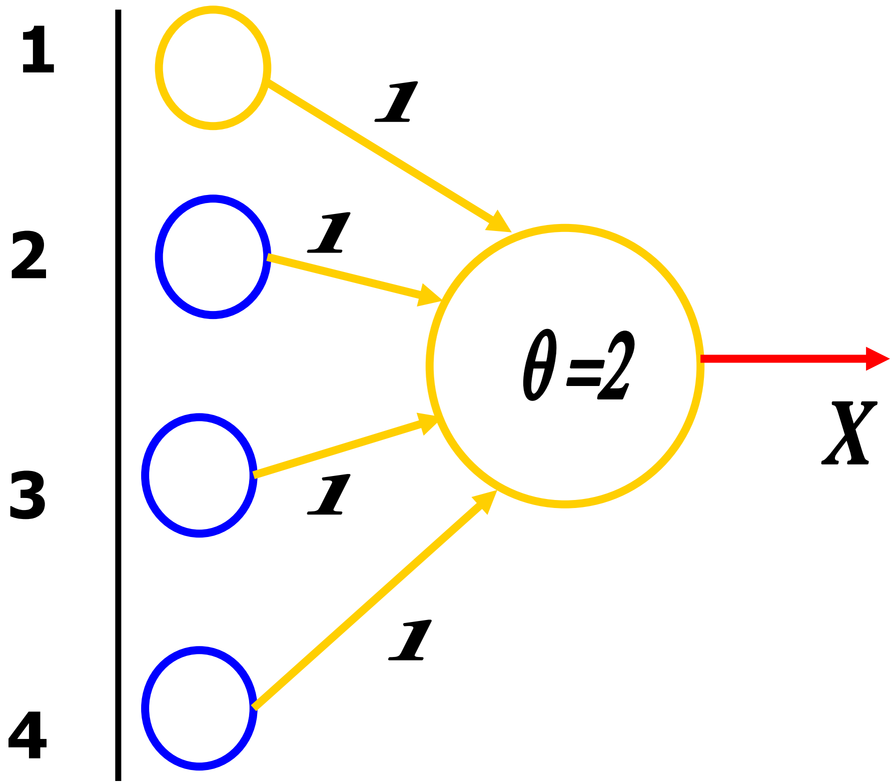
Hebb's rule in practice.

Input unit



Hebb's rule in practice.

Input unit



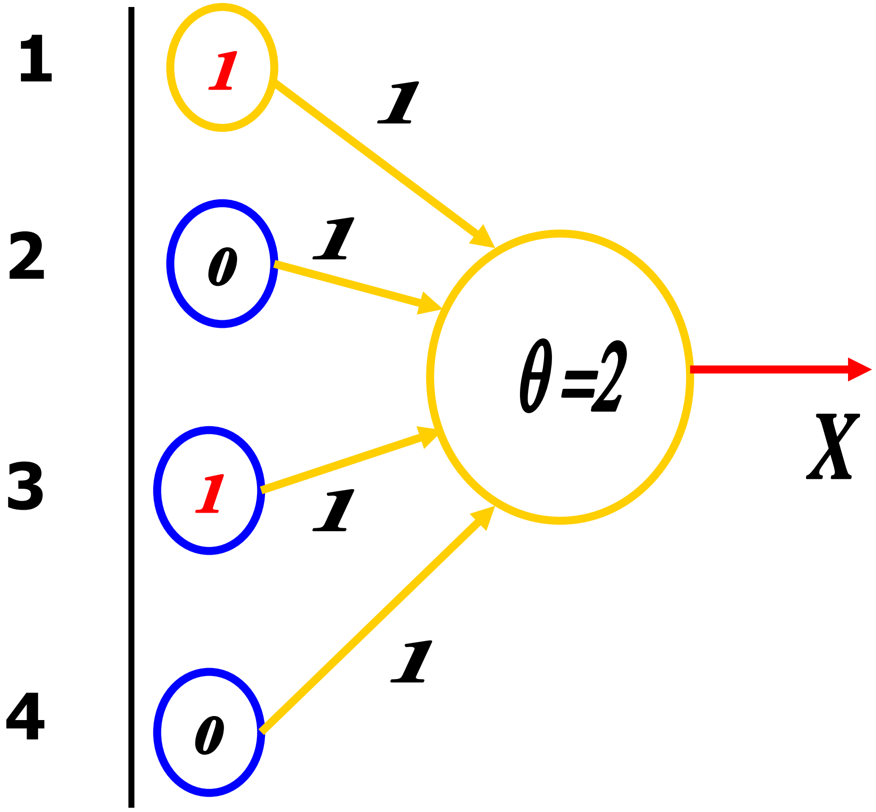
$t=0$ $C=1$

w^0_1	w^0_2	w^0_3	w^0_4
1	1	1	1

Hebb's rule in practice.

Input unit

$\underline{t=0}$ $\underline{C=1}$

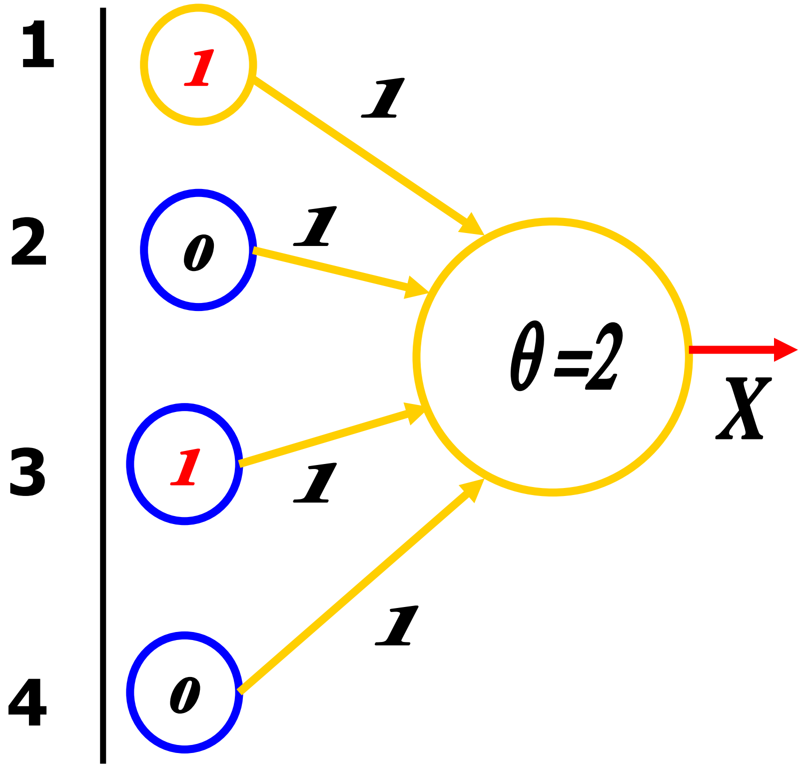


a^0_1	a^0_2	a^0_3	a^0_4
1	**0**	**1**	**0**

w^0_1	w^0_2	w^0_3	w^0_4
1	1	1	1

Hebb's rule in practice.

Input unit



$t=0$ $C=1$

$a^0{}_1$	$a^0{}_2$	$a^0{}_3$	$a^0{}_4$
1	0	1	0

$w^0{}_1$	$w^0{}_2$	$w^0{}_3$	$w^0{}_4$
1	1	1	1

$$S^0 = \sum_{i=1}^{4} a_i^0 w_i^0 =$$

$$= a_1^0 \times w_1^0 + a_2^0 \times w_2^0 + a_3^0 \times w_3^0 + a_4^0 \times w_4^0$$

$$= 1\times 1 + 0\times 1 + 1\times 1 + 0\times 1 = \underline{2 \geq \theta}$$

$$\Downarrow$$

$$X^0 = 1$$

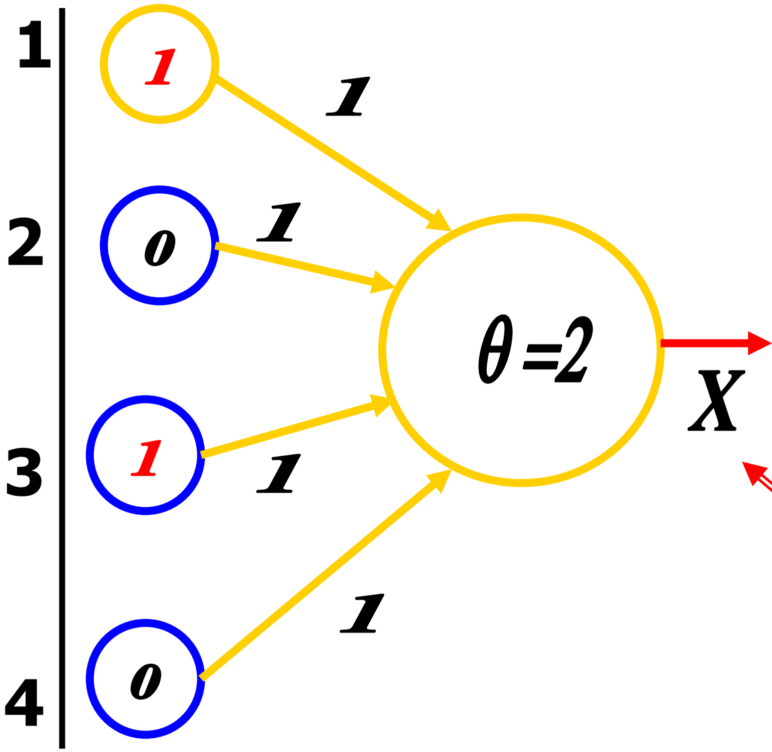
Hebb's rule in practice.

Input unit

$\underline{t=0}$ $\underline{C=1}$

a^0_1	a^0_2	a^0_3	a^0_4
1	0	1	0

w^0_1	w^0_2	w^0_3	w^0_4
1	1	1	1



$$S^0 = \sum_{i=1}^{4} a_i^0 w_i^0 =$$

$$= a_1^0 \times w_1^0 + a_2^0 \times w_2^0 + a_3^0 \times w_3^0 + a_4^0 \times w_4^0$$

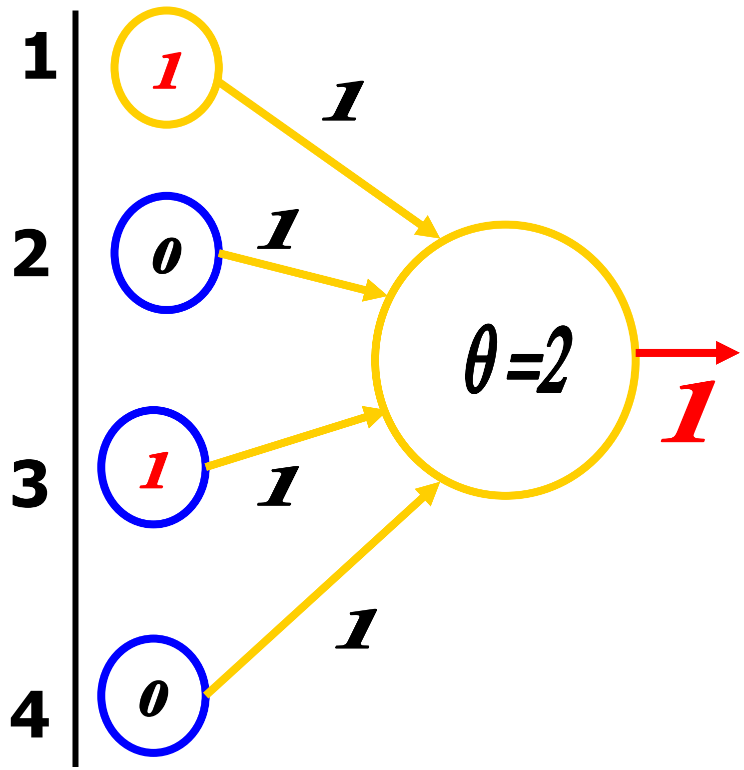
$$= 1\times1 + 0\times1 + 1\times1 + 0\times1 = \underline{2 \ge \theta}$$

$$\Downarrow$$

$$X^0 = 1$$

Hebb's rule in practice.

Input unit



$\underline{t=0}$ $\underline{C=1}$

a^0_1	a^0_2	a^0_3	a^0_4
1	0	1	0

w^0_1	w^0_2	w^0_3	w^0_4
1	1	1	1

$$S^0 = \sum_{i=1}^{4} a_i^0 w_i^0 =$$

$$= a_1^0 \times w_1^0 + a_2^0 \times w_2^0 + a_3^0 \times w_3^0 + a_4^0 \times w_4^0$$

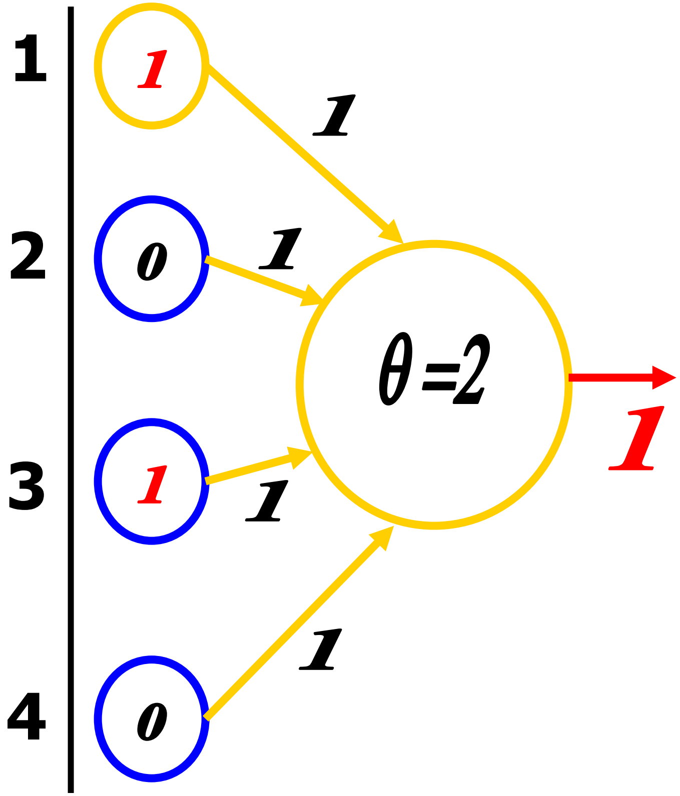
$$= 1 \times 1 + 0 \times 1 + 1 \times 1 + 0 \times 1 = \underline{2 \geq \theta}$$

$$\Downarrow$$

$$X^0 = 1$$

Hebb's rule in practice.

Input unit



t=0 C=1

a^0_1	a^0_2	a^0_3	a^0_4
1	0	1	0

w^0_1	w^0_2	w^0_3	w^0_4
1	1	1	1

$$\Delta w_i^0 = C a_i^0 X^0 \qquad w_i^1 = w_i^0 + \Delta w_i^0$$

$$\Downarrow \qquad\qquad \Downarrow$$

$$\Delta w_1^0 = 1\times1\times1 = \underline{1}, \quad w_1^1 = w_1^0 + \Delta w_1^0 = 1 + \underline{1} = 2;$$

$$\Delta w_2^0 = 1\times0\times1 = 0, \quad w_2^1 = w_2^0 + \Delta w_2^0 = 1 + 0 = 1;$$

$$\Delta w_3^0 = 1\times1\times1 = \underline{1}, \quad w_3^1 = w_3^0 + \Delta w_3^0 = 1 + \underline{1} = 2;$$

$$\Delta w_4^0 = 1\times0\times1 = 0, \quad w_4^1 = w_4^0 + \Delta w_4^0 = 1 + 0 = 1.$$

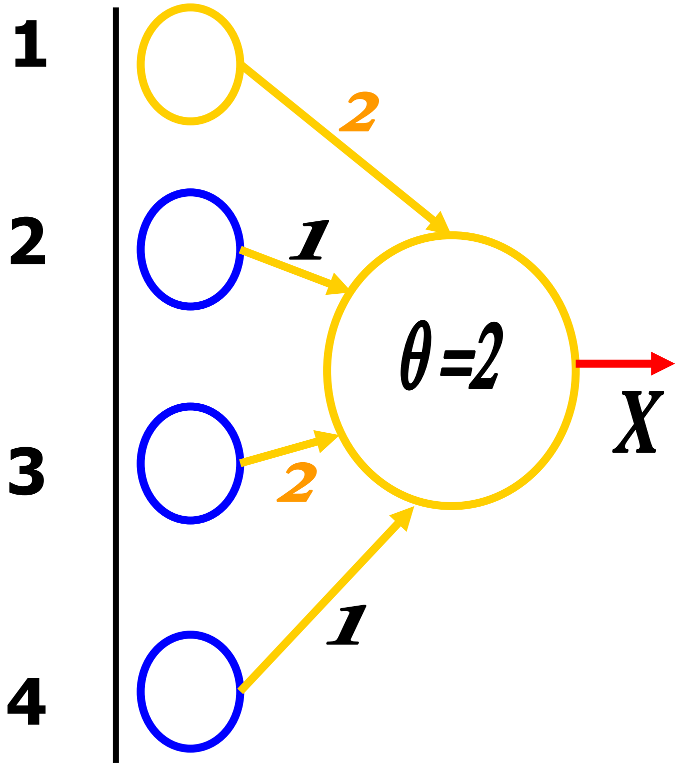
Hebb's rule in practice.

Input unit

$t=1$ $C=1$

a^0_1	a^0_2	a^0_3	a^0_4
1	0	1	0

w^1_1	w^1_2	w^1_3	w^1_4
2	1	2	1



$$w_i^1 = w_i^0 + \Delta w_i^0$$

$$\Downarrow$$

$$w_1^1 = w_1^0 + \Delta w_1^0 = 1 + 1 = \underline{2};$$

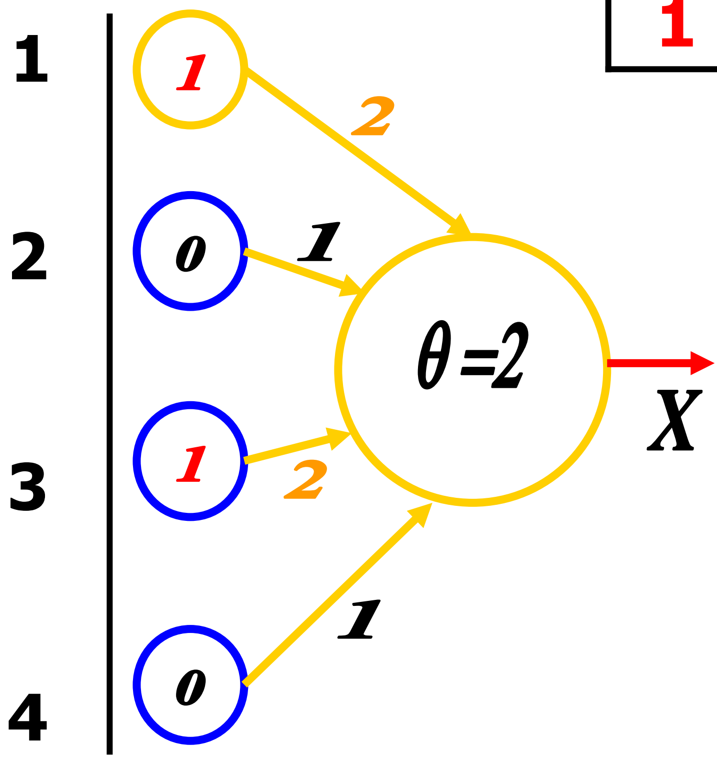
$$w_2^1 = w_2^0 + \Delta w_2^0 = 1 + 0 = 1;$$

$$w_3^1 = w_3^0 + \Delta w_3^0 = 1 + 1 = \underline{2};$$

$$w_4^1 = w_4^0 + \Delta w_4^0 = 1 + 0 = 1.$$

Hebb's rule in practice.

Input unit



$\underline{t=1}$ $\underline{C=1}$

a^1_1	a^1_2	a^1_3	a^1_4
1	0	1	0

w^1_1	w^1_2	w^1_3	w^1_4
2	1	2	1

$$S^1 = \sum_{i=1}^{4} a_i^1 w_i^1 =$$

$$= a_1^1 \times w_1^1 + a_2^1 \times w_2^1 + a_3^1 \times w_3^1 + a_4^1 \times w_4^1$$

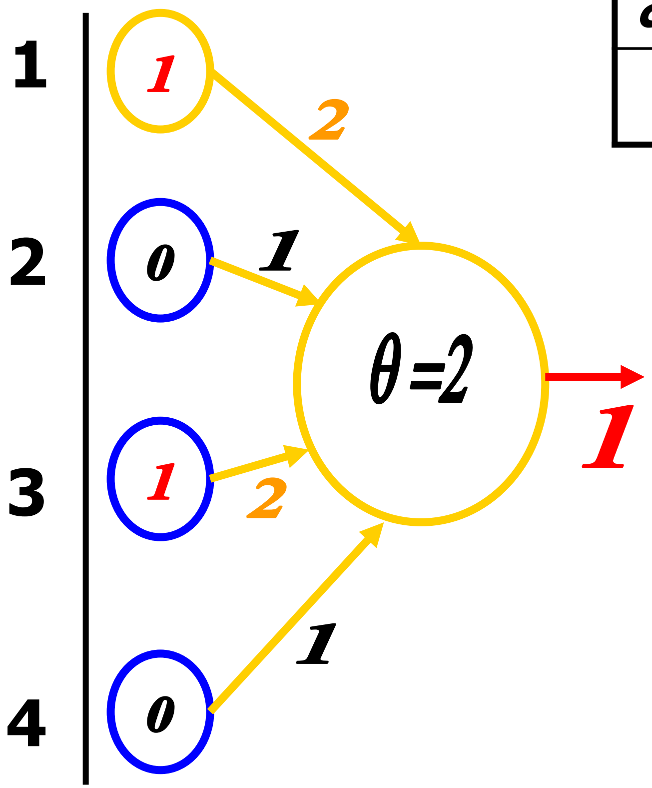
$$= 1 \times 2 + 0 \times 1 + 1 \times 2 + 0 \times 1 = \underline{4 \geq \theta}$$

$$\Downarrow$$

$$X^1 = 1$$

Hebb's rule in practice.

Input unit



$t=1$ $C=1$

a^1_1	a^1_2	a^1_3	a^1_4
1	0	1	0

w^1_1	w^1_2	w^1_3	w^1_4
2	1	2	1

$$\Delta w_i^1 = C a_i^1 X^1 \qquad w_i^2 = w_i^1 + \Delta w_i^1$$

$$\Downarrow \qquad \Downarrow$$

$$\Delta w_1^1 = 1\times 1\times 1 = \underline{1}, \quad w_1^2 = w_1^1 + \Delta w_1^1 = 2 + \underline{1} = 3;$$

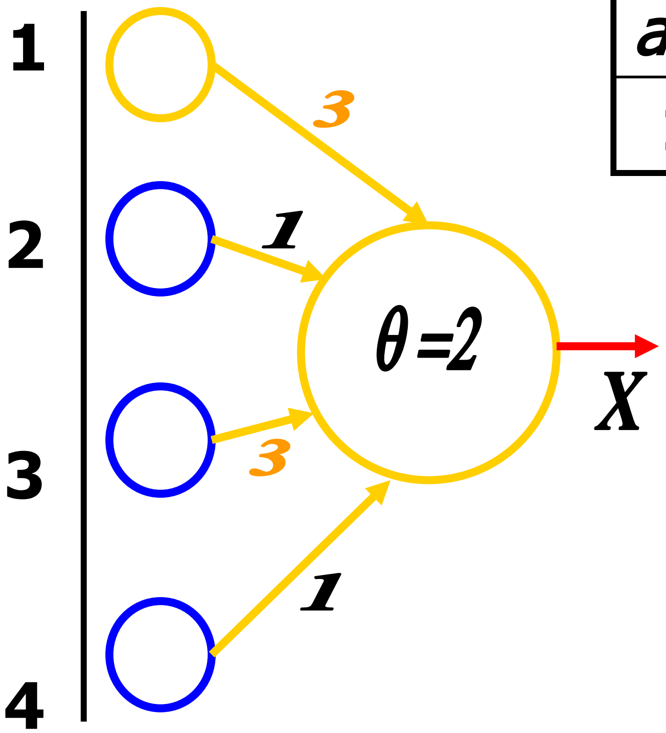
$$\Delta w_2^1 = 1\times 0\times 1 = 0, \quad w_2^2 = w_2^1 + \Delta w_2^1 = 1 + 0 = 1;$$

$$\Delta w_3^1 = 1\times 1\times 1 = \underline{1}, \quad w_3^2 = w_3^1 + \Delta w_3^1 = 2 + \underline{1} = 3;$$

$$\Delta w_4^1 = 1\times 0\times 1 = 0, \quad w_4^2 = w_4^1 + \Delta w_4^1 = 1 + 0 = 1.$$

Hebb's rule in practice.

Input unit



$t=2$ $C=1$

a^1_1	a^1_2	a^1_3	a^1_4
1	0	1	0

w^2_1	w^2_2	w^2_3	w^2_4
3	1	3	1

$$w_i^2 = w_i^1 + \Delta w_i^1$$

$$\Downarrow$$

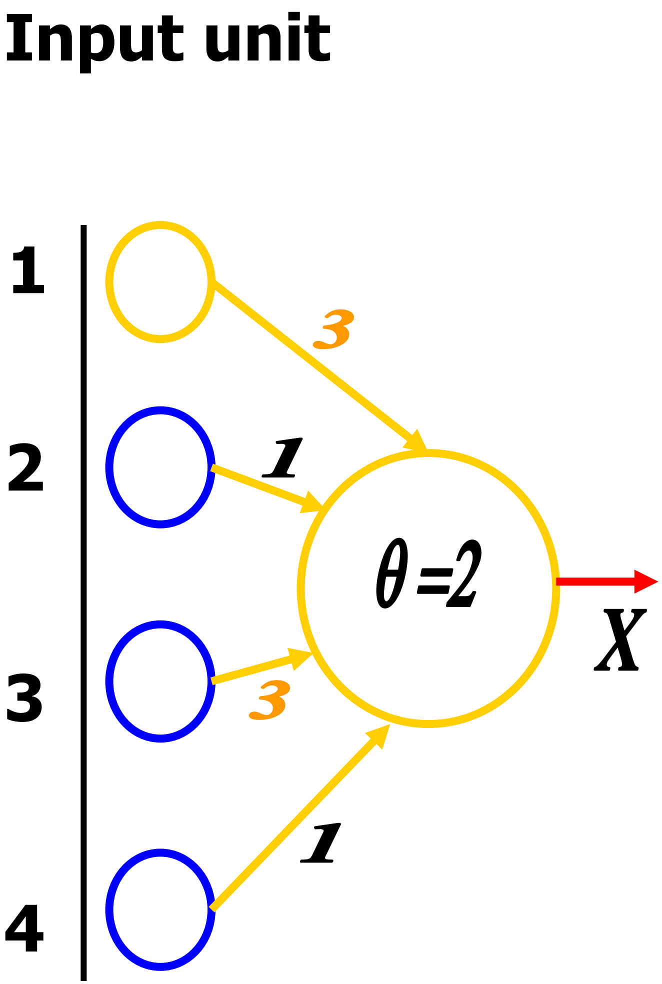
$$w_1^2 = w_1^1 + \Delta w_1^1 = 2 + 1 = \underline{3};$$

$$w_2^2 = w_2^1 + \Delta w_2^1 = 1 + 0 = 1;$$

$$w_3^2 = w_3^1 + \Delta w_3^1 = 2 + 1 = \underline{3};$$

$$w_4^2 = w_4^1 + \Delta w_4^1 = 1 + 0 = 1.$$

Hebb's rule in practice.



$t=2$ $C=1$

w^2_1	w^2_2	w^2_3	w^2_4
3	1	3	1

$$w_i^2 = w_i^1 + \Delta w_i^1$$

$$\Downarrow$$

$$w_1^2 = w_1^1 + \Delta w_1^1 = 2 + 1 = \underline{3};$$

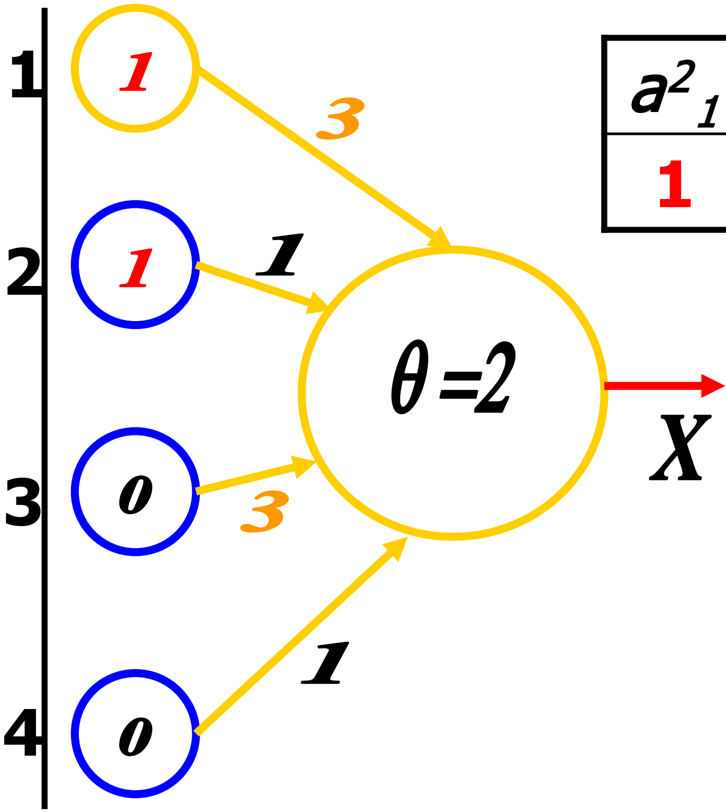
$$w_2^2 = w_2^1 + \Delta w_2^1 = 1 + 0 = 1;$$

$$w_3^2 = w_3^1 + \Delta w_3^1 = 2 + 1 = \underline{3};$$

$$w_4^2 = w_4^1 + \Delta w_4^1 = 1 + 0 = 1.$$

Hebb's rule in practice.

Input unit



t=2 C=1

$a^2{}_1$	$a^2{}_2$	$a^2{}_3$	$a^2{}_4$
1	1	0	0

$w^2{}_1$	$w^2{}_2$	$w^2{}_3$	$w^2{}_4$
3	1	3	1

$$S^2 = \sum_{i=1}^{4} a_i^2 w_i^2 =$$

$$= a_1^2 \times w_1^2 + a_2^2 \times w_2^2 + a_3^2 \times w_3^2 + a_4^2 \times w_4^2$$

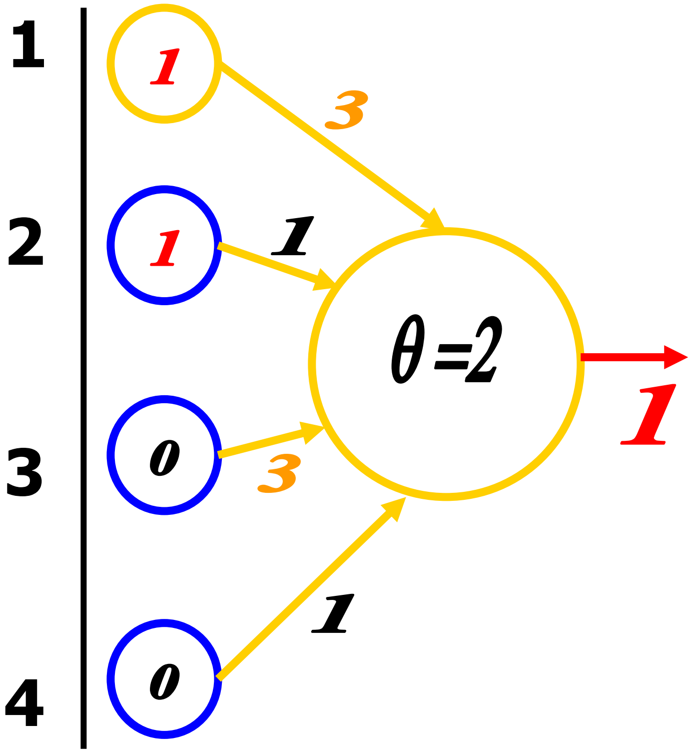
$$= 1 \times 3 + 1 \times 1 + 0 \times 3 + 0 \times 1 = \underline{4 \geq \theta}$$

$$\Downarrow$$

$$X^2 = 1$$

Hebb's rule in practice.

Input unit



t=2 C=1

$a^2{}_1$	$a^2{}_2$	$a^2{}_3$	$a^2{}_4$
1	1	0	0
$w^2{}_1$	$w^2{}_2$	$w^2{}_3$	$w^2{}_4$
3	1	3	1

$$S^2 = \sum_{i=1}^{4} a_i^2 w_i^2 =$$

$$= a_1^2 \times w_1^2 + a_2^2 \times w_2^2 + a_3^2 \times w_3^2 + a_4^2 \times w_4^2$$

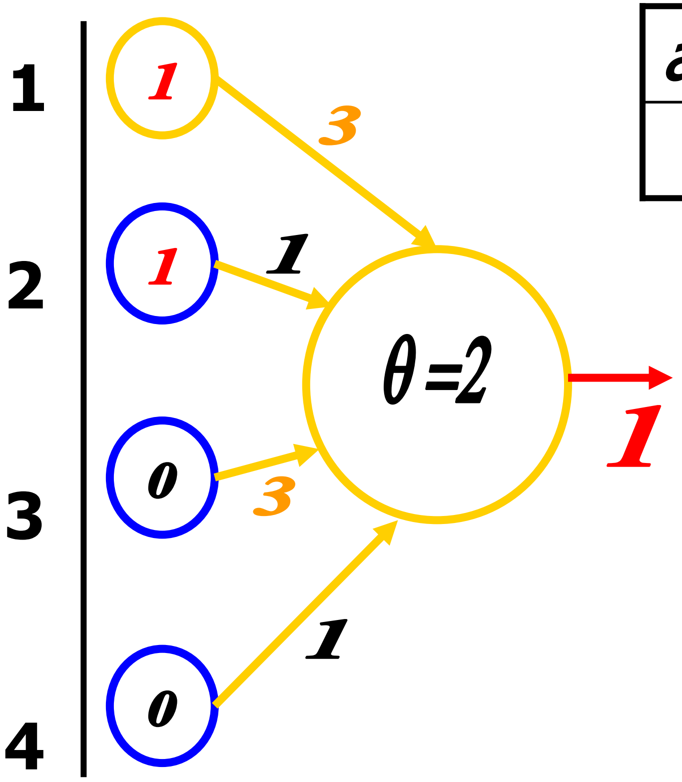
$$= 1 \times 3 + 1 \times 1 + 0 \times 3 + 0 \times 1 = \underline{4 \geq \theta}$$

$$\Downarrow$$

$$X^2 = 1$$

Hebb's rule in practice.

Input unit



t=2 C=1

a^2_1	a^2_2	a^2_3	a^2_4
1	1	0	0

w^2_1	w^2_2	w^2_3	w^2_4
3	1	3	1

$$\Delta w_i^2 = C a_i^2 X^2 \qquad w_i^3 = w_i^3 + \Delta w_i^3$$

$$\Downarrow \qquad \Downarrow$$

$$\Delta w_1^2 = 1\times 1\times 1 = \underline{1}, \quad w_1^3 = w_1^2 + \Delta w_1^2 = 3 + \underline{1} = 4;$$

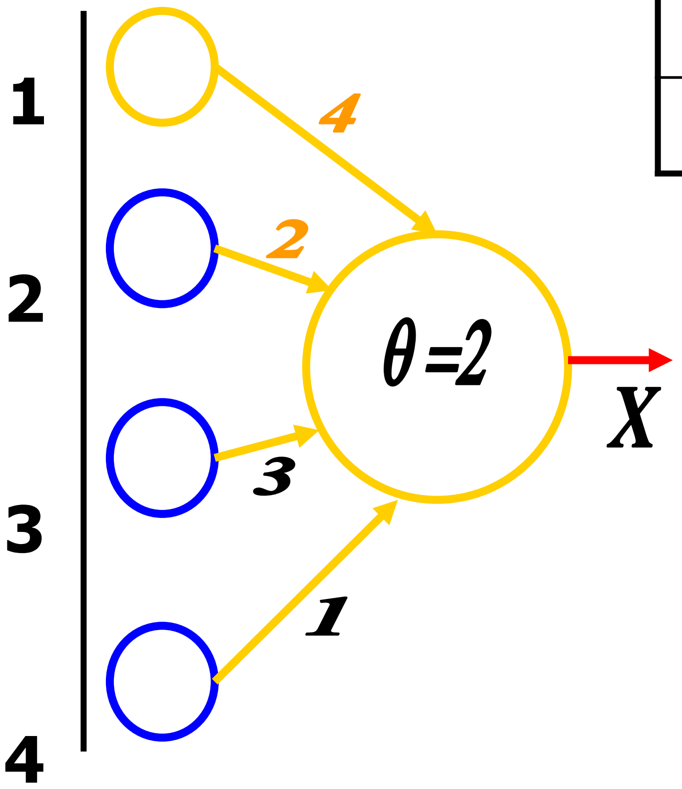
$$\Delta w_2^2 = 1\times 1\times 1 = \underline{1}, \quad w_2^3 = w_2^2 + \Delta w_2^2 = 1 + \underline{1} = 2;$$

$$\Delta w_3^2 = 1\times 0\times 1 = 0, \quad w_3^3 = w_3^2 + \Delta w_3^2 = 3 + 0 = 3;$$

$$\Delta w_4^2 = 1\times 0\times 1 = 0, \quad w_4^3 = w_4^2 + \Delta w_4^2 = 1 + 0 = 1.$$

Hebb’s rule in practice.

Input unit



$t=3$ $C=1$

$a^2{}_1$	$a^2{}_2$	$a^2{}_3$	$a^2{}_4$	$w^3{}_1$	$w^3{}_2$	$w^3{}_3$	$w^3{}_4$
1	1	0	0	4	2	3	1

$$w_i^3 = w_i^3 + \Delta w_i^3$$

$$\Downarrow$$

$$w_1^3 = w_1^2 + \Delta w_1^2 = 3 + \underline{1} = 4;$$

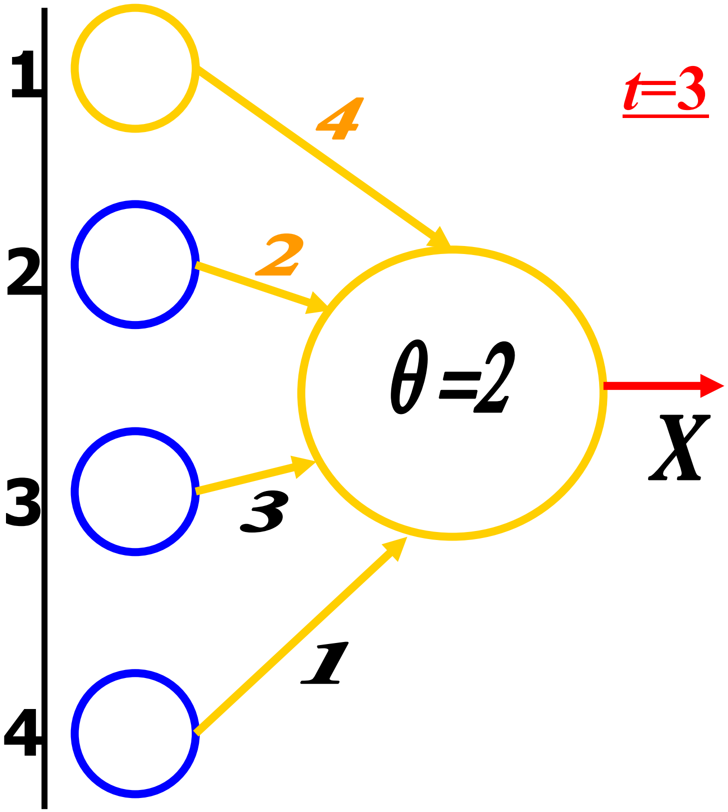
$$w_2^3 = w_2^2 + \Delta w_2^2 = 1 + \underline{1} = 2;$$

$$w_3^3 = w_3^2 + \Delta w_3^2 = 3 + 0 = 3;$$

$$w_4^3 = w_4^2 + \Delta w_4^2 = 1 + 0 = 1.$$

Hebb's rule in practice.

Input unit



t=3 C=1

$w^3{}_1$	$w^3{}_2$	$w^3{}_3$	$w^3{}_4$
4	2	3	1

$$w_i^3 = w_i^3 + \Delta w_i^3$$

$$\Downarrow$$

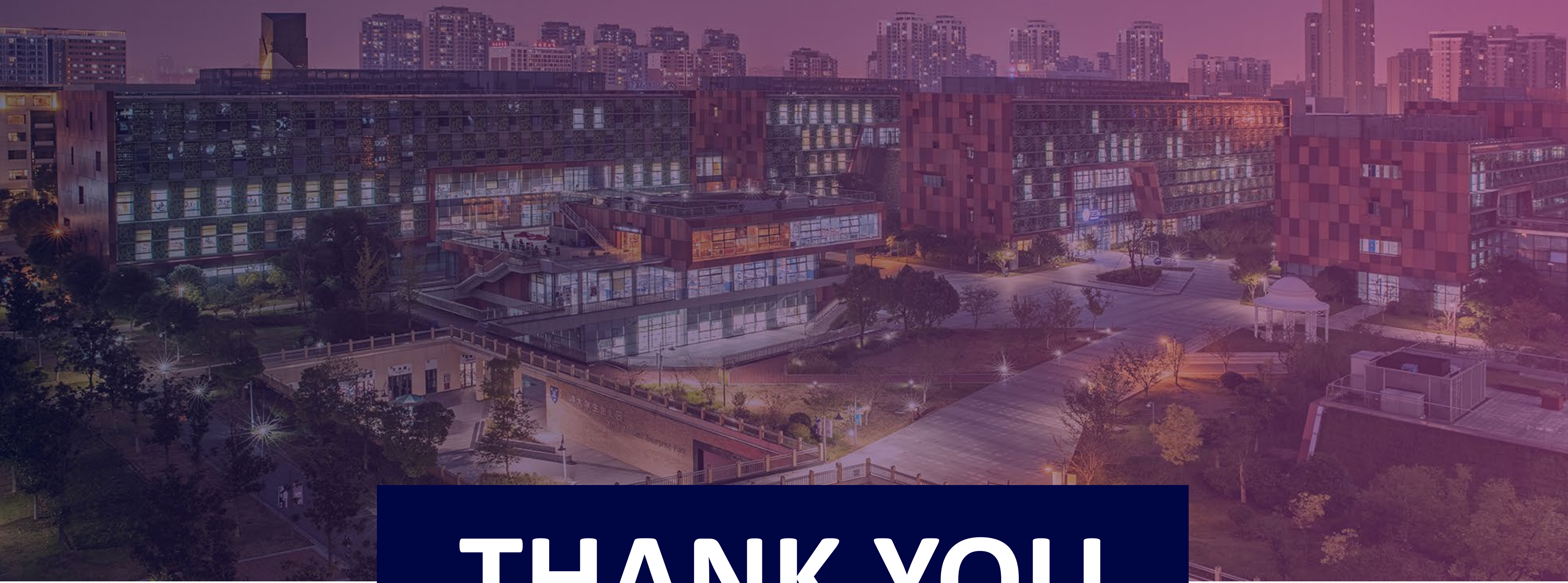
$$w_1^3 = w_1^2 + \Delta w_1^2 = 3 + 1 = 4;$$

$$w_2^3 = w_2^2 + \Delta w_2^2 = 1 + 1 = 2;$$

$$w_3^3 = w_3^2 + \Delta w_3^2 = 3 + 0 = 3;$$

$$w_4^3 = w_4^2 + \Delta w_4^2 = 1 + 0 = 1.$$

And so on…



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