Gaussian RBF network

Consider a Gaussian RBF network with 3 examples {x,d}, where x denotes the input and d denotes the value. The 3 examples are given as {(1,0), 3}, {(0,1), -1}, {(1,1), 5}. Suppose that all the 3 examples are selected as centers and the $\sigma_i = 1, i = 1,2,3$.

1) Using Gaussian radial basis functions to compute the hidden layers. (Approximate calculations can be performed using: $\exp(-0.5) \approx 0.6025, \exp(-1) \approx$0.3679.)

2) Calculate the related weight vector and predict the values of each input. The inverse matrix of $A = \begin{bmatrix} 1.4984 & 1.0988 & 1.4267 \\ 1.0988 & 1.4984 & 1.4267 \\ 1.4267 & 1.4267 & 1.7260 \end{bmatrix}$ is $B=\begin{bmatrix} 3.3460 & 0.8432 & -3.4626 \\ 0.8432 & 3.3460 & -3.4626 \\ -3.4626 & -3.4626 & 6.3035 \end{bmatrix}$

3) Suppose that only the two examples, {(1,0), 3}, {(0,1), -1}, are set as centers. Re-calculate the problems of 1) and 2). The inverse matrix of $A = \begin{bmatrix} 1.4984 & 1.0988 \\ 1.0988 & 1.4984 \end{bmatrix}$ is $B = \begin{bmatrix} 1.4439 & -1.0589 \\ -1.0589 & 1.4439 \end{bmatrix}$.

4) For the input vector (0,0), please calculate the hidden layers and final output using the two RBF networks given in 2) and 3).

Answer:

(1) The Gaussian radial function: $F(x,c) = \exp(-\frac{|x-c|^2}{2\sigma^2})$

For the input (1,0) $\quad \rho_{11}$=1

$$\rho_{12} = exp\left(-\frac{(1-0)^2+(0-1)^2}{2}\right) = exp(-1)$$

$$\rho_{13} = exp\left(-\frac{(1-1)^2+(0-1)^2}{2}\right) = exp(-0.5)$$

For the input (0,1)

$$\rho_{21} = exp\left(-\frac{(0-1)^2+(1-0)^2}{2}\right) = exp(-1)$$

ρ_{22}=1

$$\rho_{23} = exp\left(-\frac{(0-1)^2+(1-1)^2}{2}\right) = exp(-0.5)$$

For the input (1,1)

$$\rho_{31} = exp\left(-\frac{(1-1)^2+(1-0)^2}{2}\right) = exp(-0.5)$$

$$\rho_{32} = exp\left(-\frac{(1-0)^2+(1-1)^2}{2}\right) = exp(-0.5)$$

$$\rho_{33}=1$$

The hidden layer can be represented by a approximate matrix:

$$\varphi = \begin{bmatrix} 1 & 0.3679 & 0.6025 \\ 0.3679 & 1 & 0.6025 \\ 0.6025 & 0.6025 & 1 \end{bmatrix}$$

2. The predictions of the examples can be written as:

$$e_i = \sum_{n=1,..,N} \rho_{in} w_n \text{ and } \quad e = \varphi w$$

The real value is $\quad d = \begin{bmatrix} 3 \\ -1 \\ 5 \end{bmatrix}$

Method 1. Minimizing $|e-d|^2 \quad \triangleq \quad \varphi^T \varphi w \ \text{-}\varphi^T d = 0$

$$w = (\varphi^T\varphi)^{-1}\varphi^T d$$

$$\varphi^T\varphi = \begin{bmatrix} 1.4984 & 1.0988 & 1.4267 \\ 1.0988 & 1.4984 & 1.4267 \\ 1.4267 & 1.4267 & 1.7260 \end{bmatrix}, \quad (\varphi^T\varphi)^{-1} = \begin{bmatrix} 3.3460 & 0.8432 & -3.4626 \\ 0.8432 & 3.3460 & -3.4626 \\ -3.4626 & -3.4626 & 6.3035 \end{bmatrix}$$

$$w = \begin{bmatrix} 0.0288 \\ -6.3007 \\ 8.7780 \end{bmatrix}$$

Method 2.

$$w = \varphi^{-1} d$$

$$\varphi^{-1} = \begin{bmatrix} 1.5700 & -0.0121 & -0.9386 \\ -0.0121 & 1.5700 & -0.9386 \\ -0.9386 & -0.9386 & 2.1311 \end{bmatrix}$$

$$w = \begin{bmatrix} 0.0288 \\ -6.3007 \\ 8.7780 \end{bmatrix}$$

To verify the predictions:

$$e = \varphi w = \begin{bmatrix} 3.0000 \\ -1.0000 \\ 5.0000 \end{bmatrix}$$

3.

The hidden layer can be represented by a approximate matrix:

$$\varphi = \begin{bmatrix} 1 & 0.3679 \\ 0.3679 & 1 \\ 0.6025 & 0.6025 \end{bmatrix}$$

$$\varphi^{\mathrm{T}}\varphi = \begin{bmatrix} 1.4984 & 1.0988 \\ 1.0988 & 1.4984 \end{bmatrix}$$

$$(\varphi^{\mathrm{T}}\varphi)^{-1} = \begin{bmatrix} 1.4439 & -1.0589 \\ -1.0589 & 1.4439 \end{bmatrix}$$

$$\mathrm{w} = \begin{bmatrix} 4.8507 \\ -1.4774 \end{bmatrix}$$

To verify the predictions:

$$e = \varphi w = \begin{bmatrix} 4.3071 \\ 0.3071 \\ 2.0324 \end{bmatrix}$$

4. For the first RBF Network (1,0), (0,1), (1,1) are used as centers，the hidden layer for input (0,0) calculate as:

$$\rho_{11} = exp\left(-\frac{(0-1)^2+(0-0)^2}{2}\right) = exp(-0.5)$$

$$\rho_{12} = exp\left(-\frac{(0-0)^2+(0-1)^2}{2}\right) = exp(-0.5)$$

$$\rho_{13} = exp\left(-\frac{(0-1)^2+(0-1)^2}{2}\right) = exp(-1)$$

Therefore, the hidden layer is given by $\varphi = [0.6025, 0.6025, 0.3679]$

The output is

$$d = \varphi w = [0.6025, 0.6025, 0.3679] \begin{bmatrix} 0.0288 \\ -6.3007 \\ 8.7780 \end{bmatrix} = -0.5491$$

For the second RBF Network (1,0), (0,1) are used as centers，the hidden layer for input (0,0) calculate as:

$$\rho_{11} = exp\left(-\frac{(0-1)^2+(0-0)^2}{2}\right) = exp(-0.5)$$

$$\rho_{12} = exp\left(-\frac{(0-0)^2+(0-1)^2}{2}\right) = exp(-0.5)$$

Therefore, the hidden layer is given by $\varphi = [0.6025, 0.6025]$

The output is

$$d = \varphi w = [0.6025, 0.6025] \begin{bmatrix} 4.8507 \\ -1.4774 \end{bmatrix} = 2.0324$$