A BRIEF ILLUSTRATION OF PCA

INT301 Bio-computation, Week 10, 2025

Data Simulation

```
%%  generate the data
%%  define the direction of the real data
rd          =       [1;1];
rd          =       rd/norm(rd);        %normlize the data direction
rd_std      =       1.5;                %define the standard deviation of the real data
rd_num      =       500;                %define the number of data
noi_std     =       0.05;                %define the standard deviation of the noise

%%  simulate the observations
rd_std_vec          =       randn([1,rd_num])*rd_std;
for k =1:rd_num
    rd_data(:,k)    =       rd*rd_std_vec(k);
end
noi_data            =       randn([length(rd),rd_num])*noi_std;
ob_data             =       rd_data + noi_data;

%%  show the data
figure;
plot(ob_data(1,:), ob_data(2,:),'b*')
legend('Observations')
disp('The direction of the data distribution:')
disp(rd)
```

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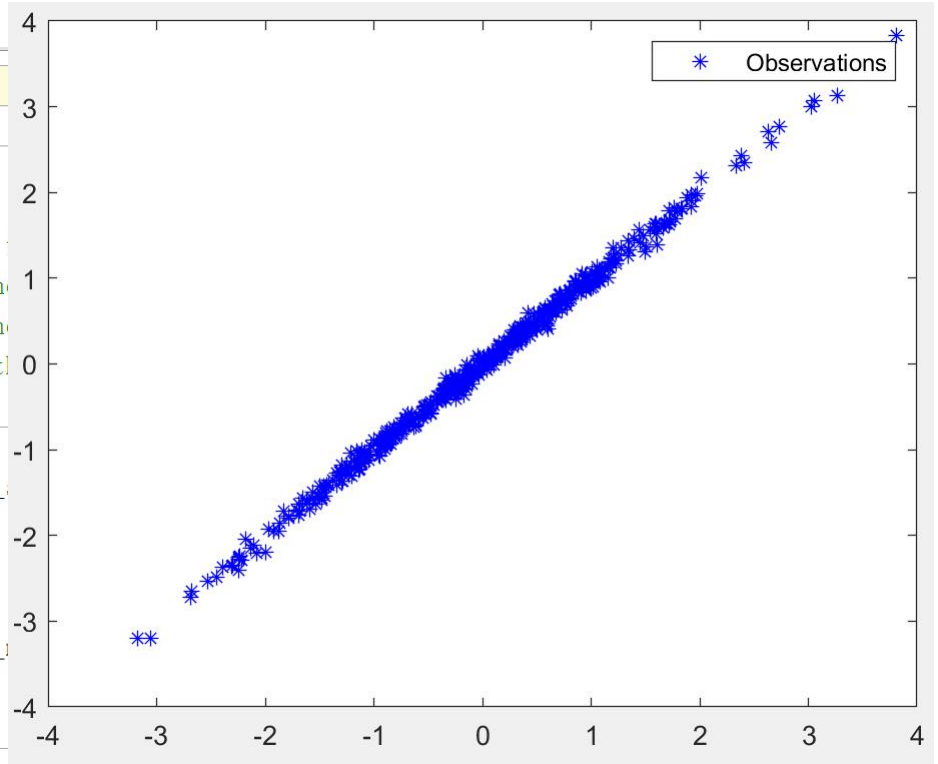


```
>> PCA_demo
The direction of the data distribution:
    0.7071
    0.7071
```

SVD

```
%%  SVD analysis
[U, S, V]          =       svd(ob_data);
singular_vec       =       diag(S);
disp('The eigenvectors:')
disp(U)

disp('The singular values are given:')
disp(singular_vec(1:2))

disp('The estimations of std:')
disp(singular_vec(1:2)/sqrt(rd_num))
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```
The eigenvectors:
   -0.7065    0.7077
   -0.7077   -0.7065

The singular values are given:
   33.6941
    1.0605

The estimations of std:
    1.5068
    0.0474
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Diagonal decomposition of the correlation matrix

```
%%  Diagonal decomposition of the correlation matrix
R_Mat               = ob_data*ob_data';
disp('the correlation matrix of the data is given:');
disp(R_Mat)

[V, D]              = eig(R_Mat);
disp('The eigenvectors are given:')
disp(V)
disp('The eigenvalues are given:')
disp(diag(D))
disp('The singular vaules are given in another form:')
disp(sqrt(diag(D)))
```

```
the correlation matrix of the data is given:
  567.1940  567.0836
  567.0836  569.2243

The eigenvectors are given:
   -0.7077    0.7065
    0.7065    0.7077

The eigenvalues are given:
   1.0e+03 *

    0.0011
    1.1353

The singular vaules are given in another form:
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Projection and reconstruction

```
%%  Reconstruction with eigenvectors
%%  Projection with the maximum eignvalue related eigenvector
ob_pro                =  V(:,2)'*ob_data;
pro_std               =  std(ob_pro)*sqrt(rd_num);
disp('the std of the projections');
disp(pro_std)

%% reconstructions with the maximum eignvalue related eigenvector
pro_data               = V(:,2)*V(:,2)'*ob_data;
figure;
plot(rd_data(1,:), rd_data(2,:),'b*')
hold on
plot(pro_data(1,:), pro_data(2,:),'ro')
legend('real data', 'reconstructions')
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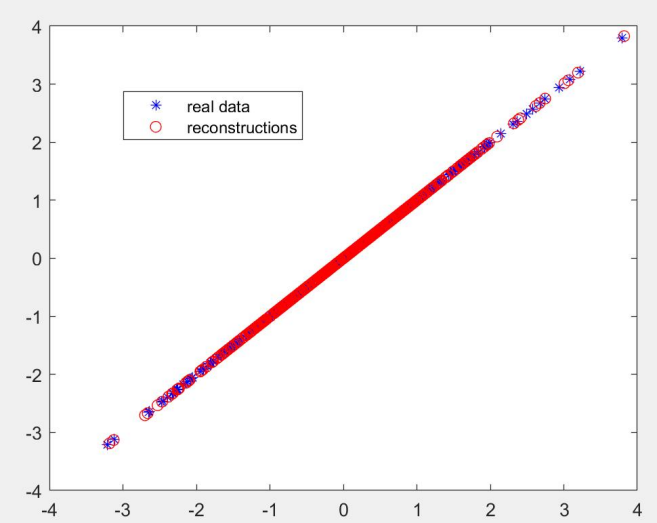

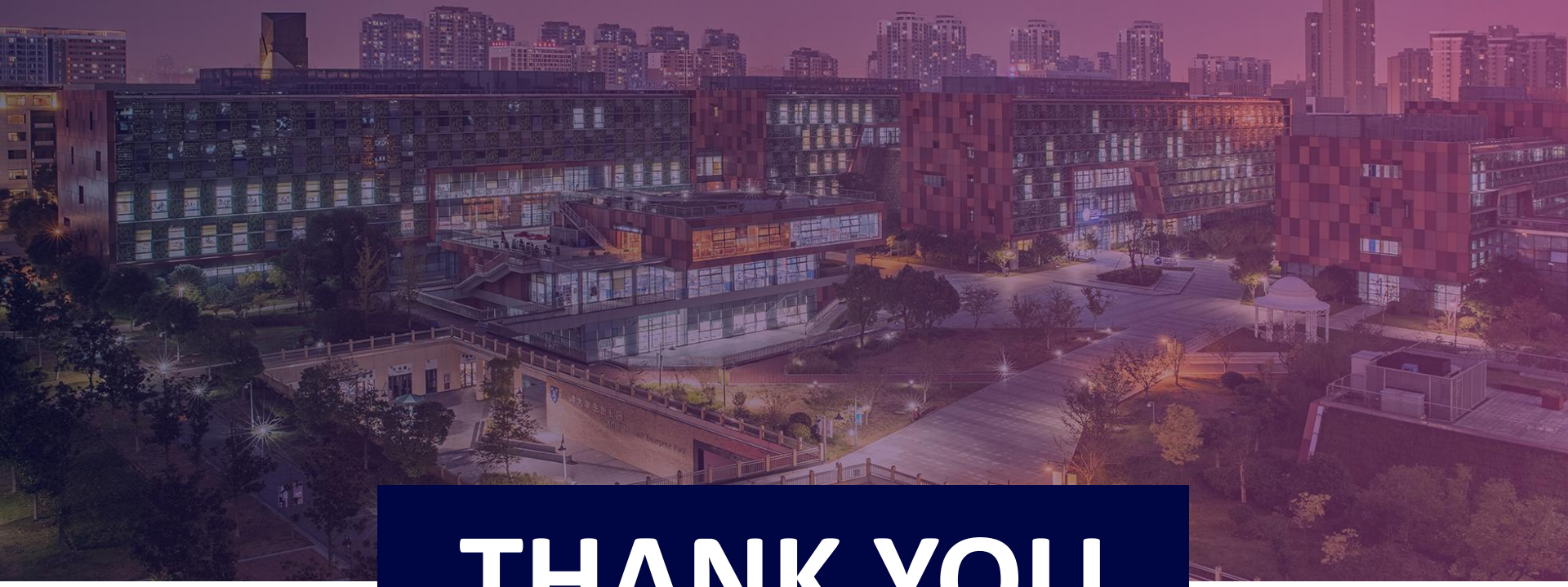

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