# DECISION MAKING IN COMPLEX ENVIRONMENTS

**The Analytic Hierarchy Process (AHP)
for Decision Making
and
The Analytic Network Process (ANP)
for Decision Making with
Dependence and Feedback**

Rozann W. Saaty

**SUPER DECISIONS**

*Software for Decision Making with Dependence and Feedback*

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**Biography**

Thomas Saaty holds the Chair of University Professor at the University of Pittsburgh. He received his Ph.D. in mathematics with a minor in physics from Yale University. From 1963-1969, he worked at the Arms Control and Disarmament Agency in Washington. From 1969-1979, he was a professor at the Wharton School of the University of Pennsylvania where he developed the Analytic Hierarchy Process for decision-making. Inspired by world events that took place during his time at the Arms Control agency, the AHP was his answer for how to deal with weapons tradeoffs, resource allocation and decision-making. He has written eight books devoted to the theory and applications of the Analytic Hierarchy Process (AHP) and its generalization, the Analytic Network Process (ANP) for decision making with dependence and feedback. He has also written books on Modern Nonlinear Equations, Nonlinear Mathematics, Graph Theory, The Four Color Problem, Behavioral Mathematics, Queuing Theory, Optimization in Integers, and Embracing the Future, as well as numerous papers. He has also served as a consultant to many corporations and governments. His research interests include decision-making, planning, and the analysis of neural functions. His most recent book, published in 2001, is *Creative Thinking, Problem Solving & Decision Making*. It includes a powerpoint slide presentation on a CD. The book is a rich collection of ideas, incorporating research by a growing body of researchers and practitioners, profiles of creative people, projects and products, theory, philosophy, physics and metaphysics…all explained with a liberal dose of humor. Dr. Saaty is the co-developer of AHP and ANP software and of the ANP software *SuperDecisions* for decision-making with dependence and feedback.

# FOREWORD

We want this book to help people be exemplary survivors, persistent and durable, creative decision-makers and leaders, effective negotiators, doers and implementers of ideas and plans.

The Analytic Network Process (ANP) is the most comprehensive framework for the analysis of societal, governmental and corporate decisions that is available today to the decision-maker. It allows one to include all the factors and criteria, tangible and intangible that have bearing on making a best decision. The Analytic Network Process allows both interaction and feedback within clusters of elements (inner dependence) and between clusters (outer dependence). Such feedback best captures the complex effects of interplay in human society, especially when risk and uncertainty are involved.

The ANP, developed by Thomas L. Saaty, provides a way to input judgments and measurements to derive ratio scale priorities for the distribution of influence among the factors and groups of factors in the decision. Ratio scales make possible proportionate allocation of resources according to derived priorities. The well-known decision theory, the Analytic Hierarchy Process (AHP) is a special case of the ANP. Both the AHP and the ANP derive ratio scale priorities by making paired comparisons of elements on a common property or criterion. Although many decision problems are best studied through the ANP, one may wish to compare the results obtained with it to those obtained using the AHP or any other decision approach with respect to the time it took to obtain the results, the effort involved in making the judgments, and the relevance and accuracy of the results.

ANP models have three parts: the first is a strategic criteria in terms of which a decision is evaluated according to its merits of Benefits, Opportunities, Costs and Risk. Each merit provides control criteria for the second part of the decision and with each control criterion there is associated a network of influences that determine the priorities of the alternatives of the decision for that control criterion. The priorities of the merits and those of the control criteria are then used to synthesize the priorities of the alternatives to obtain the final best answer. The supermatrix and its powers are the fundamental tools needed to lay out the workings of the ANP

The ANP has been applied to a large variety of decisions: marketing, medical, political, military, social, forecasting and prediction and many others. Its accuracy of prediction is impressive in applications that have been made to economic trends, sports and other events for which the outcome later became known. Detailed case studies of applications are included in this book and also in the book, *The Analytic Network Process: Decision Making with Dependence and Feedback*, by Thomas L. Saaty.

# TABLE OF CONTENTS

# Tutorial for the SuperDecisions Software

## PART 1. HOW TO USE *SUPERDECISIONS* TO BUILD AHP HIERARCHICAL DECISION MODELS

This tutorial has two parts. The first is on how to create hierarchical AHP decision models using the *SuperDecisions* software. The second is on how to create network ANP decision models for decision making with dependence and feedback. It covers the commands and concepts used in the software. We start by showing how to build an Analytic Hierarchy Process (AHP) hierarchical decision model.

### INSTALLING THE SUPERDECISIONS SOFTWARE

- o **To Install the *SuperDecisions* software insert the CD in the CD drive on your computer. Click on Start, then on Run and run the file Setup.exe to install the software. If this file does does not automatically appear, click on the My Computer icon on the desktop, then click on the icon for the CD drive and on the file Setup.exe. No serial number is necessary to install the software. You may download an updated version free from www. superdecisions.com when this software expires after 9 months.**
- o **To start the program click on the icon that has appeared on the desktop of your computer or go to Start/Programs/Super Decisions:**



### INTRODUCTION TO BASIC CONCEPTS

This lesson describes how to build the simplest decision model that has a goal, criteria and alternatives, make judgments (paired comparisons), and compute the results to find the best alternative.

## CLUSTERS AND ELEMENTS

A hierarchical decision model has a goal, criteria that are evaluated for their importance to the goal, and alternatives that are evaluated for how preferred they are with respect to each criterion. An abstract view of such a hierarchy is shown in Figure 1. The goal, the criteria and the alternatives are all elements in the decision problem, or nodes in the model. The lines connecting the goal to each criterion means that the criteria must be pairwise compared for their importance with respect to the goal. Similarly, the lines connecting each criterion to the alternatives mean the alternatives are pairwise compared as to which is more preferred for that criterion. Thus in the hierarchy that is shown there are six sets of pairwise comparisons, one for the criteria with respect to the goal and 5 for the alternatives with respect to the 5 criteria.

A *SuperDecisions* model consists of clusters of elements (or nodes), rather than elements (or nodes) arranged in levels. The simplest hierarchical model has a goal cluster containing the goal element, a criteria cluster containing the criteria elements and an alternatives cluster containing the alternative elements as shown in Figure 1. When clusters are connected by a line it means nodes in them are connected. The cluster containing the alternatives of the decision must be named A*lternatives*. Nodes and Clusters are organized alphabetically in the calculations, so an easy way to control the order is to preface the names with numbers.
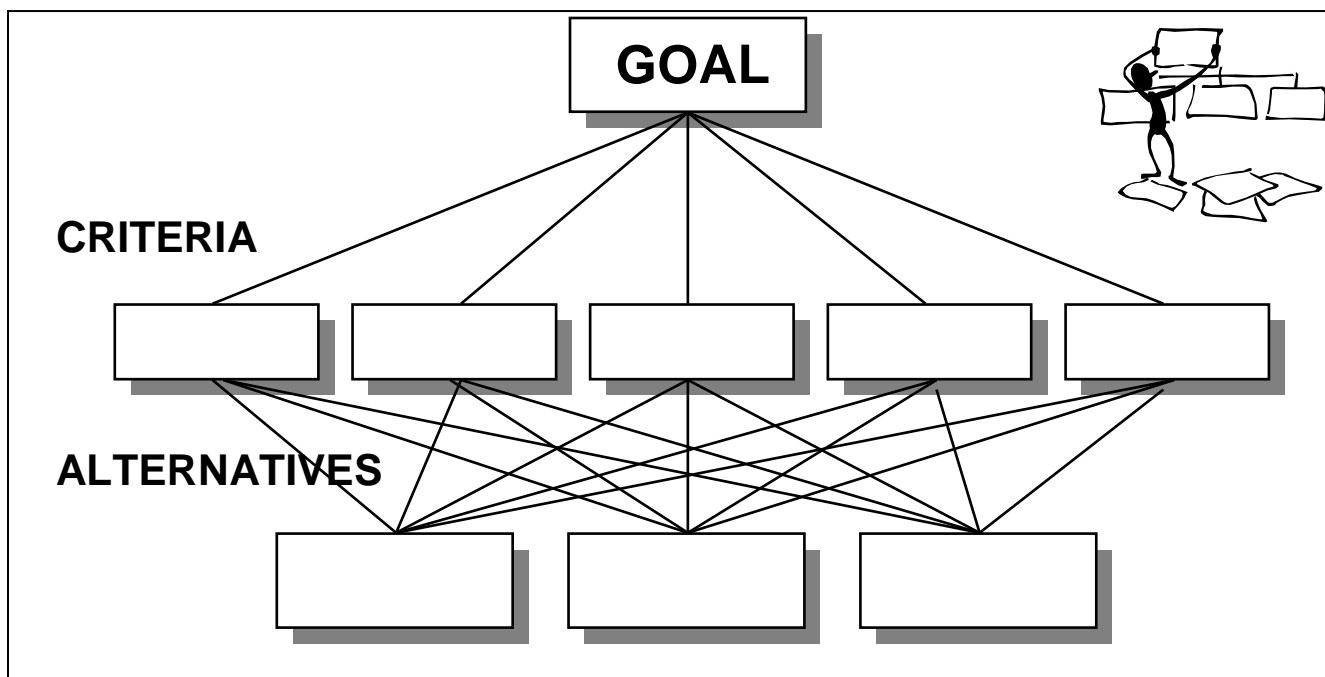


**Figure 1.** Abstract Representation of a Decision Hierarchy
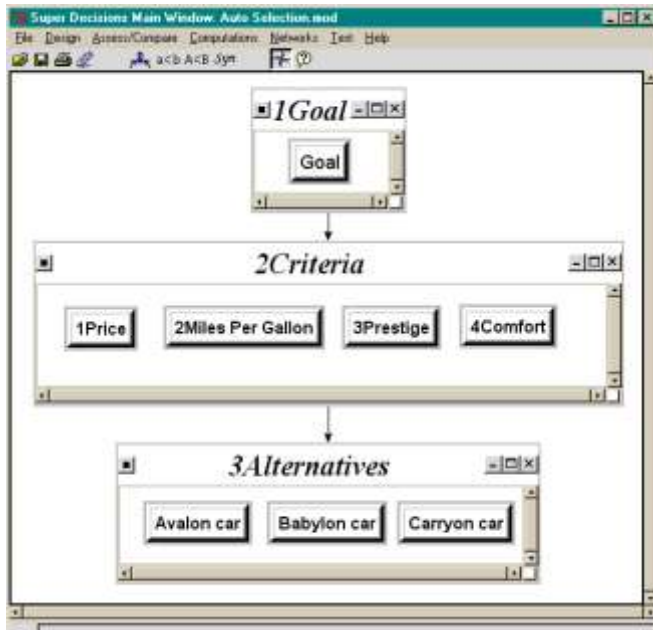
*We are indebted to Professor Saul Gass for this model. He came up with these fictitious car names to avoid any complaints from actual car manufacturers. Since then, however, an actual car named Avalon has come to the market. We stick by our guns. These names are not intended to represent any actual car that is currently being sold or ever was. Any resemblance is entirely coincidental.*

**Figure 2.** *SuperDecisions* **Hierarchical Model for Selecting a Car.**

The model in Figure 2, *Car Hierarchy.mod,* is included with the sample models for the *SuperDecisions* software.

- o To load this model click on the Help command, then on Sample Models and select Car Hierarchy.mod.

When clusters are shown connected with a line, it means elements in them are connected. To see which nodes are connected from a source or "parent" node, turn on the "Show Connections" icon and place the cursor over the parent node. The nodes to which it is connected will then be outlined in red as shown in Figure 3.
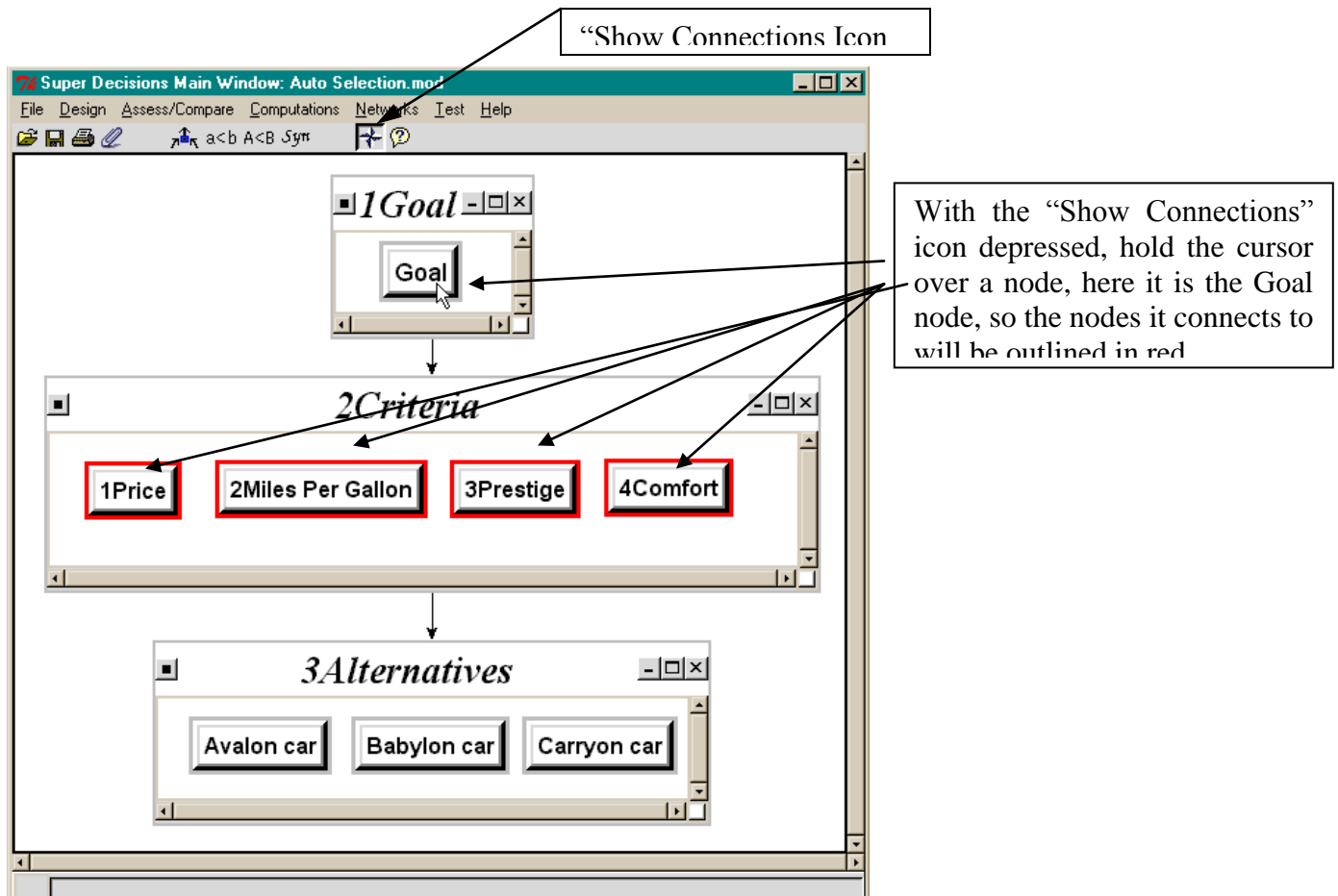
**Figure 3.** Turn on "Show Connections" Icon to see Element Connections.

Each criterion node is also connected to the alternatives. Place the cursor over each criterion in turn to highlight the alternatives for each.

In a hierarchical *SuperDecisions* model clusters are connected by arrows going in one direction from top to bottom. In network models, covered in the next chapter, clusters may be connected with arrows going both ways and also may be connected to themselves with a loop.

In a hierarchical structure like that shown in Figure 3, each comparison set is made up of a parent node and the nodes it connects to in the cluster below. There are five sets of pairwise comparisons to do for this model: the criteria with respect to the goal, and the alternatives with respect to each of the four criteria.

## SHOWING THE PAIRWISE COMPARISONS FOR THE CRITERIA WITH RESPECT TO THE GOAL

In this model the pairwise comparisons have already been completed.

- o To see the pairwise comparisons for the criteria with respect to the goal, left-click on the Goal node inside the Goal cluster to select it.

- ○ Select the Assess/Compare, Node Comparisons command
- ○ Click on the button "Do Comparison" to show Figure 4.

**Figure 4.**  The Questionnaire Comparison Screen

- ○ Click on the Matrix buttom to see the equivalent Matrix comparison screen shown in Figure 5.   Click on Graphic, then Verbal.

**Figure 5.**  The Matrix Pairwise Comparison Screen

A number in the matrix is a dominance judgment.  Blue indicates the element listed at the left is dominant (more important, more preferred,…) than the element listed at the top.  Red indicates that the element listed at the top is dominant.  A judgment of 1.0 means they are equal, a judgment of 3.0 means moderately or three times as much (if you are dealing with measureables), and 9.0 means nine times as much.  You should group your elements into homogeneous clusters so that it is not necessary to use a number larger than 9.  The Fundamental Scale for judgments is shown in Table 1.

**Table 1.** The Fundamental Scale for Making Judgments

| | |
|---|---|
| 1 | Equal |
| 2 | Between Equal and Moderate |
| 3 | Moderate |
| 4 | Between Moderate and Strong |
| 5 | Strong |
| 6 | Between Strong and Very Strong |
| 7 | Very Strong |
| 8 | Between Very Strong and Extreme |
| 9 | Extreme |
| | Decimal judgments, such as 3.5, are allowed for fine tuning, and judgments greater than 9 may be entered, though it is suggested that they be avoided. |

When a number greater than 9 is suggested by the inconsistency checking, this means that the elements you have grouped together are too disparate. You may input a number greater than 9, but perhaps you should re-organize your structure so that such a comparison is not required. It will do no great damage to allow numbers up to 12 or 13, but you should not go much beyond that.

- o Click on the Computations, Show New Priorities command to see the results of this pairwise comparison shown in Figure 6.



The inconsistency index is shown here. At 0.065 it is less than 0.10 so no correction of judgments is needed.
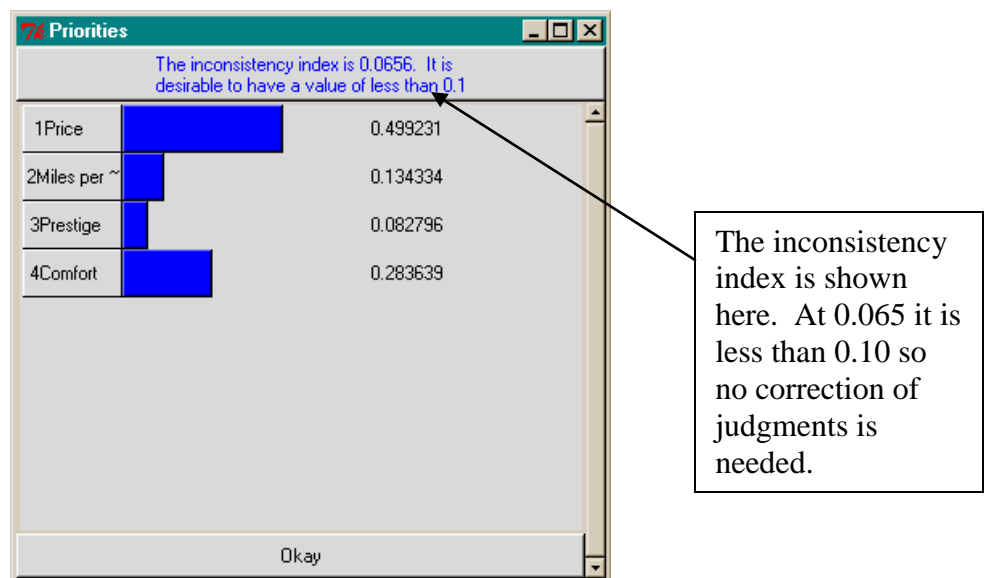
**Figure 6.** The Results of the Pairwise Comparisons

Here the inconsistency is 0.0656, so it is not necessary to correct any judgments.

- o To see the judgment that is most inconsistent click on the Computations, Most Inconsistent (ala Expert Choice) command.

Building a hierarchy is as much an art as it is a science.  Following are some guidelines:
Guideline 1: Try not to include more than seven to nine elements in any cluster or grouping of elements because experiments have shown that it is cognitively challenging for human beings to deal with more than nine factors at one time and this can result in less accurate priorities.

Guideline 2: Try to cluster elements so that they include elements that are "comparable", or do not differ by orders of magnitude.  In other words, try not to include items of very small significance in the same cluster as items of greater significance. The purpose of a hierarchy is to cluster the more important elements together and the less important elements together.

By keeping these two simple guidelines in mind, you will be able to model complex decisions correctly and efficiently.

### DIFFERENTIATING BETWEEN OBJECTIVES AND CRITERIA

Often the words criteria and objectives are used interchangeably.  A criterion is a principle or a standard that things are judged by while an objective is something that is sought or aimed for. The elements in a cluster may be thought of as objectives, or as criteria, depending on the model you are creating.

# EXERCISE ONE - BUILDING A HIERARCHICAL MODEL

In this exercise you will build a model and perform pairwise assessments throughout.  You will then synthesize to get your results and perform sensitivity analyses.

Your goal will be to find the best vacation place.  We suggest you limit the number of criteria to four, and the places to three or four.  You may enter your own criteria and vacation places. Criteria might be cost, night life, friends, shopping, ocean, scuba diving, hiking, golfing, ease of getting to, climate, attractions, etc.

## CREATING A MODEL TO PICK THE BEST VACATION PLACE

Double click the *SuperDecisions* shortcut icon on the desktop:



or click the Windows Start key, select Programs, select *SuperDecisions* program group and then select *SuperDecisions*.  A blank Main Window will appear

## CREATE GOAL, CRITERIA AND ALTERNATIVE CLUSTERS

- From the Main Window menu, select Design, Cluster, New to create the first cluster. The new cluster dialogue box will appear as shown in             Figure 7.
    - Shortcut: Press <Shift> <n> simultaneously,  that is, capital letter N, with your

cursor located on the background of the Main Window.

- Start the names of the clusters with numbers to control their order as they are displayed in alphabetical order in the supermatrices.

- Type *1Goal* for the name and "This is the goal cluster, the top level in a hierarchical model" for the description. Select the "Create Another" button to create the next cluster. To change the font click on the buttons under Main Font and make your choices from the dropdown menus. For the model to appear as it does in Figure 3, set the main font to New Times Roman, 28, Bold Italic.
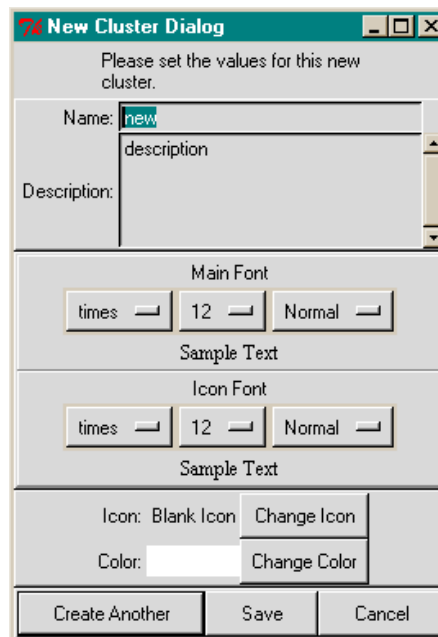


**Figure 7.**  New Cluster Dialogue Box.

- Type *2Criteria* in the new cluster dialogue box with the description "Criteria for selecting a vacation place," and again select the "Create Another" button to create the next cluster.

- Type *3Alternatives* with the description "Alternative vacation places" and click the "Save" button to save and terminate the process of adding new clusters.

Arrange the clusters as shown below by clicking on the title bar and dragging.
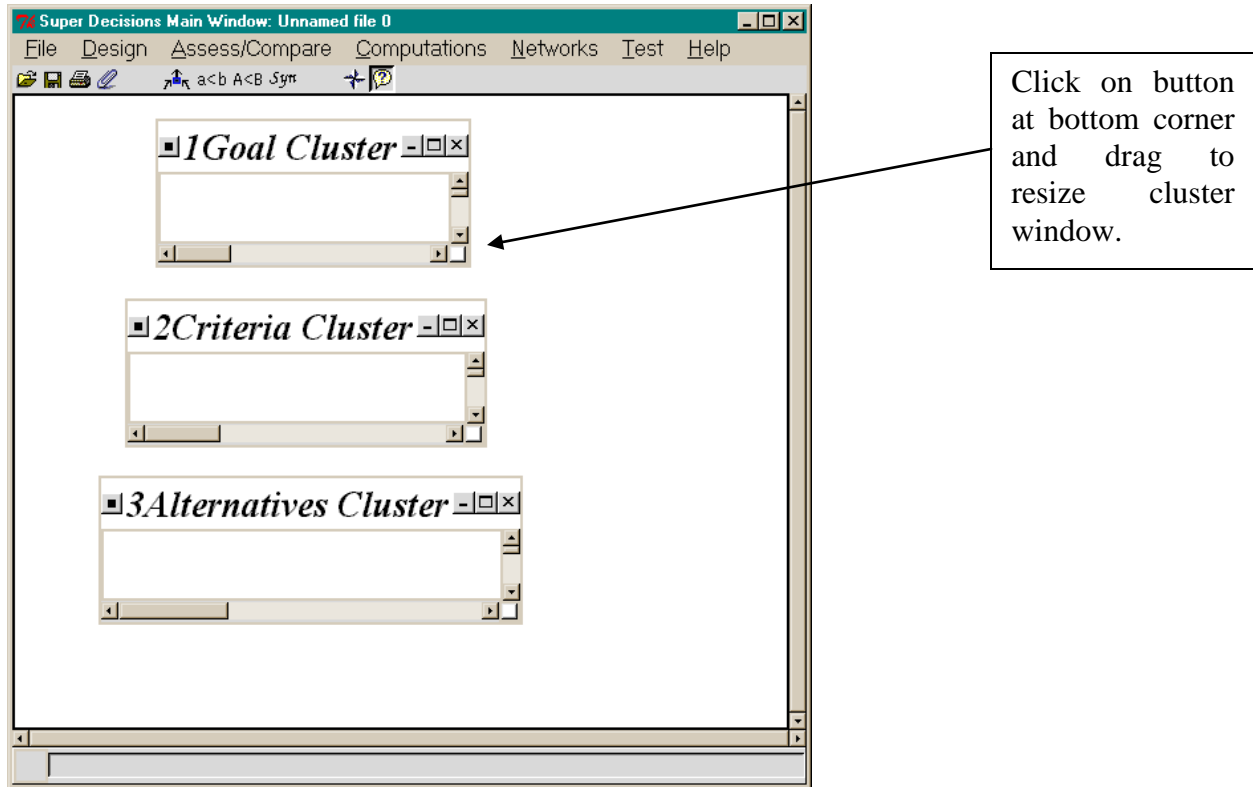
## CREATE THE ELEMENTS INSIDE THE CLUSTERS

- Select Design, Node, New then select *1Goal* from the list of clusters that appears to show you wish to enter the element in that cluster. Enter *Goal Node* in the Name field and a description in the Description field. To pick a background color for the node click on the "Change Color" button and select a color. Select the font you prefer. Press Save to end the process of entering nodes in this cluster.
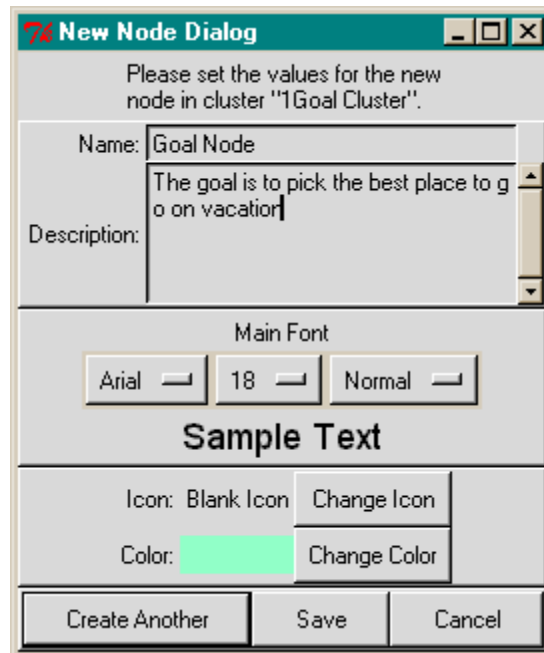
- *Shortcut to create new node: Locate your cursor on the background of the goal cluster window and press the letter <n>.*
- *Another shortcut to create new node: Right click on the background of the cluster to get a drop down menu of commands and select "create node in cluster"*

## ADD THE NODESAND THEIR DESCRIPTIONS TO THE OTHER CLUSTERS:

- **2Criteria Cluster**
  - 1Activities: Activities one can engage in
  - 2Nightlife:  Things to do at night
  - 3Sightseeinge: Sightseeing
  - 4Cost: Cost including travel, accommodations and activities
- **3Alternatives Cluster**
  - *1Orlando*: Orlando has excellent activities, decent nightlife and is moderate in cost.
  - *2San Francisco*: San Francisco has moderate activities, above average night life, great sightseeing and can be costly.
  - *3New York*: New York has excellent activities, excellent nightlife, very good sightseeing and can be very costly.
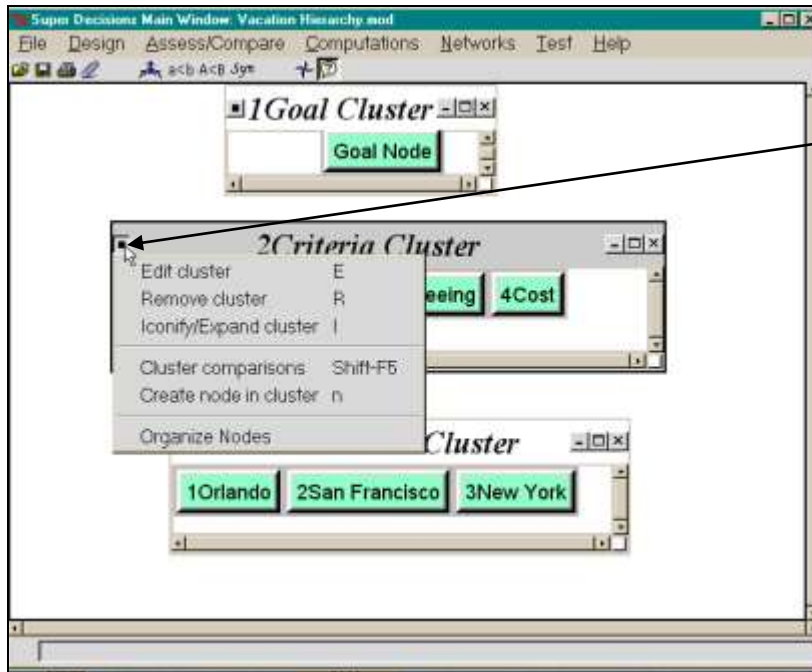
Click on top left corner for menu. Select Organize Nodes horizontally to line up the nodes and find any that may have disappeared from view.

## CONNECT THE ELEMENTS

No lines connect the clusters yet because the elements within them are not connected.

### TO CONNECT THE GOAL NODE TO THE CRITERIA NODES:

- Select the Design, "Node connections from" command. In the list that appears click on the node named Goal that is to be the parent or source node. In the list that now appears, left click on the criteria nodes to select them.

  - *Shortcut: To connect using the "Make Connections" icon*  *, left click on the icon to depress it and turn on the connections mode, then left click on the source node and right click on each of the nodes it connects to. To turn off the connections mode, right click the icon again to un-depress it.*
  - *Shortcut: To disconnect, left click on the source node and right click on any of the nodes that are outlined in red (that were previously connected to it) to disconnect.*

### TO CONNECT THE CRITERIA TO THE ALTERNATIVES:

- Select the Design, "Node connections from" command. In the list that appears click on the node named 1Price that is to be the parent or source node. In the list that now appears, left click on all the alternatives to select them.
- Connect the rest of the criterion nodes to the Alternative nodes in turn.

### SHORTCUT FOR CONNECTING ALL THE NODES IN A CLUSTER AT ONCE:

- Press the <Shift> key and hold it down while left clicking on one of the nodes in the cluster. This selects all the nodes in that cluster as source nodes. *(Make sure the "Make Connections" icon is turned on, that is, depressed).*

- Move to the cluster with nodes to be connected from the source nodes, press the <Shift> key and hold it down while right clicking on one of the nodes in the "to" cluster. This selects all the nodes and connects all the previously selected source nodes to them.

## THE COMPLETED MODEL
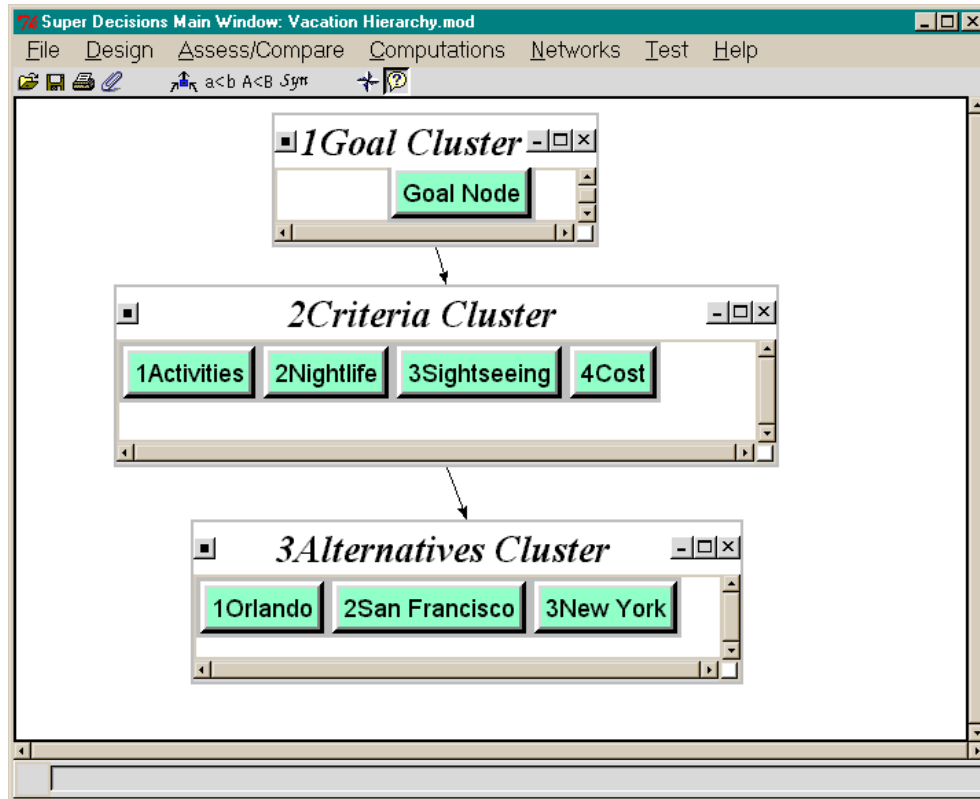


**THE NODE MENU FOR RENAMING NODES, REMOVING THEM, MAKING AND HIGHLIGHTING CONNECTIONS, AND ENTERING THE COMPARISON MODE:**

- Right click on a node to get its dropdown menu shown below from which you can perform operations on that node:



## SAVING YOUR MODEL

These commands for saving a model are located on the File menu.

- **File, Save**  saves a copy of your model in the directory you designate as a file with the extension .ahp
- **File, Save As** is used to save your model under a new name.  The old file is closed and you continue working in the new file.  It is useful to save successive versions such as car1.ahp, car2.ahp, car3.ahp and so on about every 30 minutes so you always have a recent copy you can restore in case of an unforeseen problem.
- **File, Advanced Save** can be used to compress models to a smaller size file with the extension .mod.gz.  Such models can be opened in the usual way with the File, Open command, but it may take a little longer.
- **File, Configure, General tab**  Go to the General tab in the configure dialogue box to set the automatic saving interval for models.  The un-do command is currently disabled.

Next you will learn how to make judgments or pairwise comparisons about the criteria and alternatives in your model.

## ASSESSMENTS / PAIRWISE COMPARISONS OVERVIEW

One of the major strengths of the AHP is the use of pairwise comparisons to derive accurate ratio scale priorities, as opposed to using traditional approaches of "assigning weights" which can also be difficult to justify.  Pairwise comparison is the process of comparing the relative importance, preference, or likelihood of two elements (for example, criteria) with respect to another element (for example, the goal) in the level above to establish priorities for the elements being compared. Pairwise comparisons are carried out for all the parent/children sets of nodes.

The nodes that are to be pairwise compared are always all in the same cluster and are compared with respect to their parent element, the node from which they are connected.  This results in "local priorities" of the children nodes with respect to the parent.

Select the Assess/Compare, Node comparisons command to start the comparison process.  If a node has been selected by clicking on it prior to starting the comparisons, that node will be the parent node in the first set of comparisons to come up.  If no node has been selected, the first node in the first cluster will be the parent.  The parent node and the cluster where the children nodes are located are specified in the node comparison dialogue box that appears.

Click on the Do Comparison button to start the comparison process.  The pairwise comparison process starts in the mode that was last in use, or in the Questionnaire mode the first time.  There are four pairwise comparison assessment modes.  To switch from one mode to the other click on the tab at the top.   When a judgment entered in one mode is recorded in one mode, the same judgment is recorded in all the modes.  Calculations are based on the numbers in the Matrix mode (the Questionnaire mode always shows whole numbers, and no numbers are shown in the Graphic and Verbal mode)

.

Comparisons are made with respect to the parent node listed here.

The children nodes that are to be pairwise compared are in the cluster listed here.
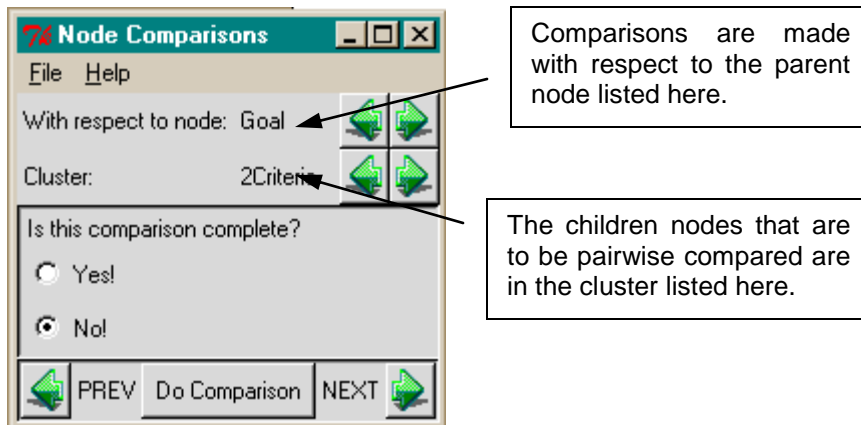
- **Questionnaire** In this mode, select the judgment on the side of the most dominant member of the pair, that is, the most important, if it is a criterion as shown below, or the most preferred, in the case of alternatives. To invert the dominance order click on the same judgment on the other side of the 1 in the row, nearer to the more important, or more preferred node.

### QUESTIONNAIRE MODE



To show the priorities derived from any of the comparison modes, select Computations, Show new Priorities from the comparison screen menu.
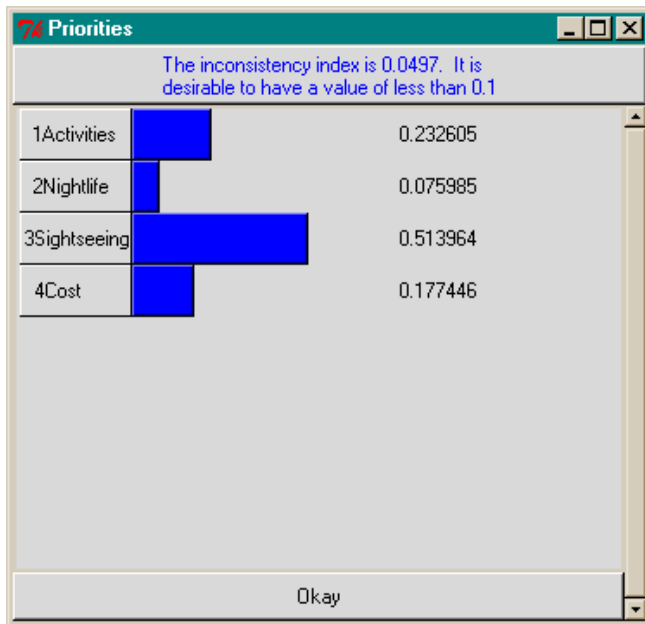
To exit from the comparison mode, click on the x in the upper right hand corner, or select the File Close command.

- **Numerical judgments** are made in a matrix using a nine-point scale that represent how many times one element is more important than another. The arrow at the left of the entry points to the dominant element. To switch which element is dominant, left double-click on the arrow.

**NUMERICAL OR MATRIX MODE**



The current judgment is highlighted. Arrow points to dominant element.

Double click arrow to change dominant criterion from price to comfort.
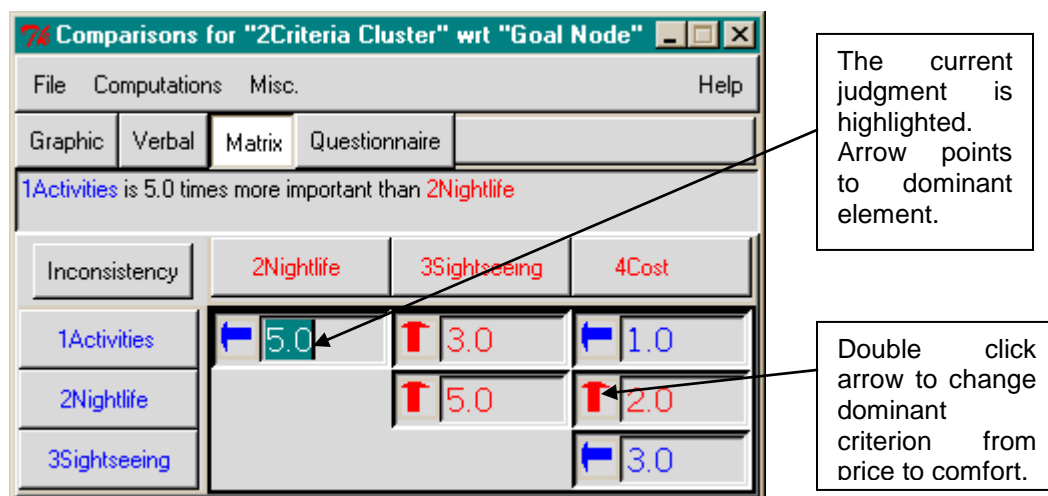
- **Verbal judgments** are used to compare factors using the words Equal, Moderate, Strong, Very Strong, Extreme. Equal requires no explanation. Extreme means an order of magnitude − about 9 or 10 to 1. Judgments between these words, such as Moderate to Strong are also possible. In the verbal mode the node mentioned first is the dominant one. To invert click the "Invert Comparison" button at the bottom of the screen.

**VERBAL JUDGMENT MODE**
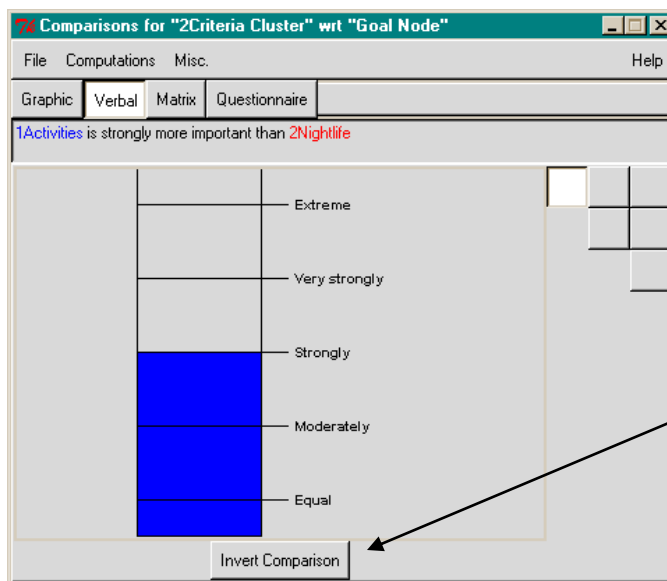


> To invert judgment click on the "Invert Comparison" button

- **Graphical judgments** are made by clicking and dragging the pie slice to change the relative size of the pie slice to the circle and the relative length of the two bars until it represents how many times more important one element is than the other.

**GRAPHICAL JUDGMENT MODE**
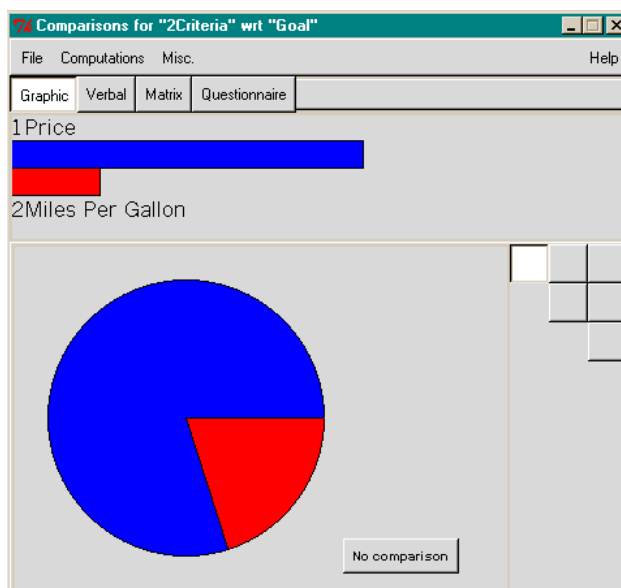
**MISCELLANEOUS MENU IN COMPARISONS WINDOW**
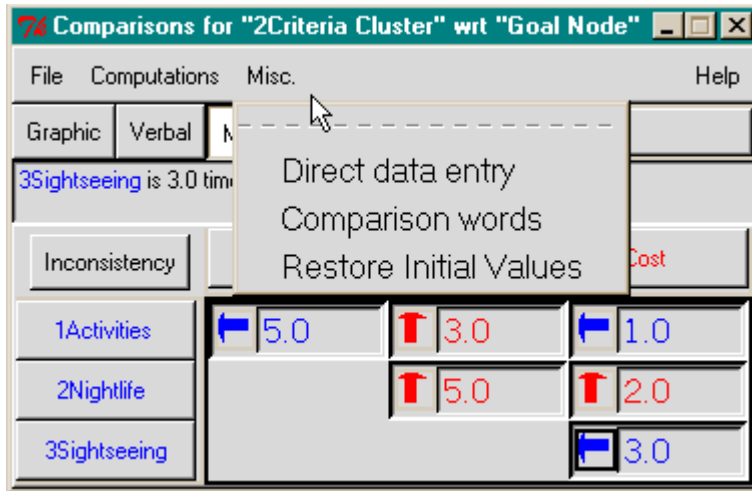


- Select the command ***Direct data entry*** to directly enter data values that are converted to priorities by summing and dividing each entry by the total (normalizing to 1).

**ENTERING DIRECT PRIORITIES**
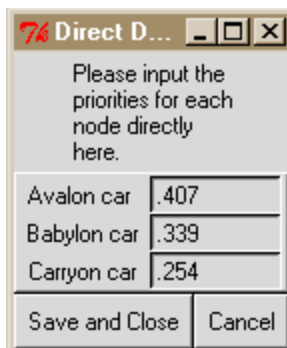


When entering numbers for preference and higher numbers are better, for example if you were comparing cars for price, enter the data itself. If higher numbers are less preferred, for example, for cost, higher numbers (prices) are less desirable. In this case invert the priorities as shown below. A spreadsheet program, Excel, was used to perform the calculations and create the table below (E-05 means $10^{-5}$).

| Car | Price | 1/Price | (1/Price)/Total (final priorities) |
|---|---|---|---|
| Avalon | 15000 | 6.667E-05 | 0.407 |
| Babylon | 18000 | 5.556E-05 | 0.339 |
| Carryon | 24000 | 4.167E-05 | 0.254 |
| | Total | 0.000164 | |

Tip: Even when you have data, as with car prices, it is often better to interpret what the numbers mean to you and use the Questionnaire mode so the final results match your personal values or your personal interpretation of what the numbers mean to you.

Select the command Computations/Priorities to see the results from pairwise comparisons or from direct data such as prices of cars.  Prices need to be "inverted" as shown above.

### CHANGING THE PAIRED COMPARISON TYPE

There are three different paired comparison types, **Importance**, **Preference** and **Likelihood**.  Or you can select **Other** and enter a word of your own choosing – "beauty" for example. Importance is most appropriate when comparing criteria or criteria.  Preference is appropriate when comparing alternatives with respect to a criterion.   Likelihood is appropriate when comparing the likelihood of uncertain events or scenarios, such as in risk analysis.

- Select the command Misc./ Comparison words to change the comparison type.
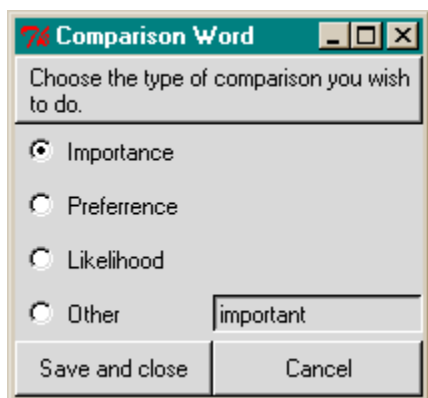


### CHECKING INCONSISTENCY

The inconsistency measure is useful for identifying possible errors in judgments as well as actual inconsistencies in the judgments themselves; this is accessed from the Priorities Window.

Inconsistency measures the logical inconsistency of your judgments. For example, if you were to say that A is more important than B and B is more important than C and then say that C is more important than A you are not being consistent.   A somewhat less inconsistent situation would arise if you would say that A is 3 times more important than B, B is 2 times more important than C, and that C is 8 times more important than A.

In general, the inconsistency ratio should be less than 0.1 or so to be considered reasonably consistent.   To find out which judgment is most inconsistent select the Computations/Most inconsistent (ala Tom) to have it highlighted. Regardless of the mode you are in when you select that command, you will be taken to the Matrix mode. You will then be asked if you want to see the value that would make that judgment most consistent with the rest.

**MENU FOR CHECKING INCONSISTENCY IN COMPARISON WINDOW**
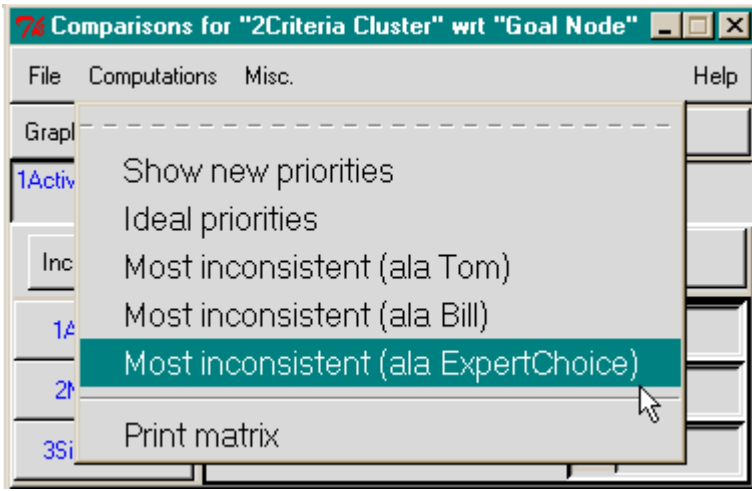


You may then "Show the Best Value". You can enter the suggested value, or some other judgment that you think fits better, by typing it in, or leave it as it is.

# MAKING THE JUDGMENTS

Since judgments about the relative importance of the criteria may depend on the alternatives being considered, by making the judgments "bottom up", that is first for the alternatives, then for the criteria, you can be more informed on how important the criteria really are. However, for illustrative purposes, we will make judgments "top down" in this tutorial.

**JUDGE THE CRITERIA WITH RESPECT TO THE GOAL NODE**

1. From the Main Screen, left click the Goal (until depressed).
2. Select Assess/Compare, Node from the menu or `a<b` on the shortcut bar .
3. Click the "Do Comparisons" button to get into the comparison mode. You may use any of the modes: questionnaire, verbal, matrix or graphical.
4. After making the judgments select Computations, Show new priorities.
5. If your inconsistency is more than 0.10, select Computations, Most inconsistent ala Expert Choice to find out where it is. Then modify your judgments to improve your inconsistency.
6. Close the comparison model to return to the comparison navigator
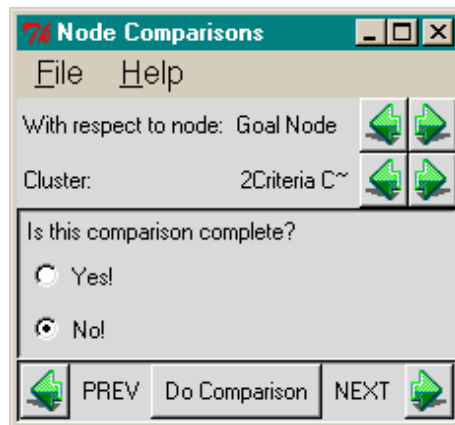
**JUDGE THE ALTERNATIVES WITH RESPECT TO EACH OF THE CRITERIA NODES**

1.  Click "Yes" to mark the criteria comparisons with respect to the goal as complete. Click the Next button to move to the first criterion, and click the Do Comparison button to get into the comparison mode for the vacation places with respect to the first criterion.
2.  Continue the process until all the judgments are finished.

**CHECKING COMPLETED COMPARISONS**

To show which comparisons have been completed left click the "Show Node Comparisons" icon

 and hold the cursor over the "parent node", in this case the Goal, to have clusters with completed comparisons outlined in red.

**VIEW WITH "SHOW CONNECTIONS" ICON TURNED ON**



Turn on "Show Connections" icon  by clicking on it and hold cursor over "parent" node to have clusters containing its children nodes for which comparisons have been completed outlined in red.
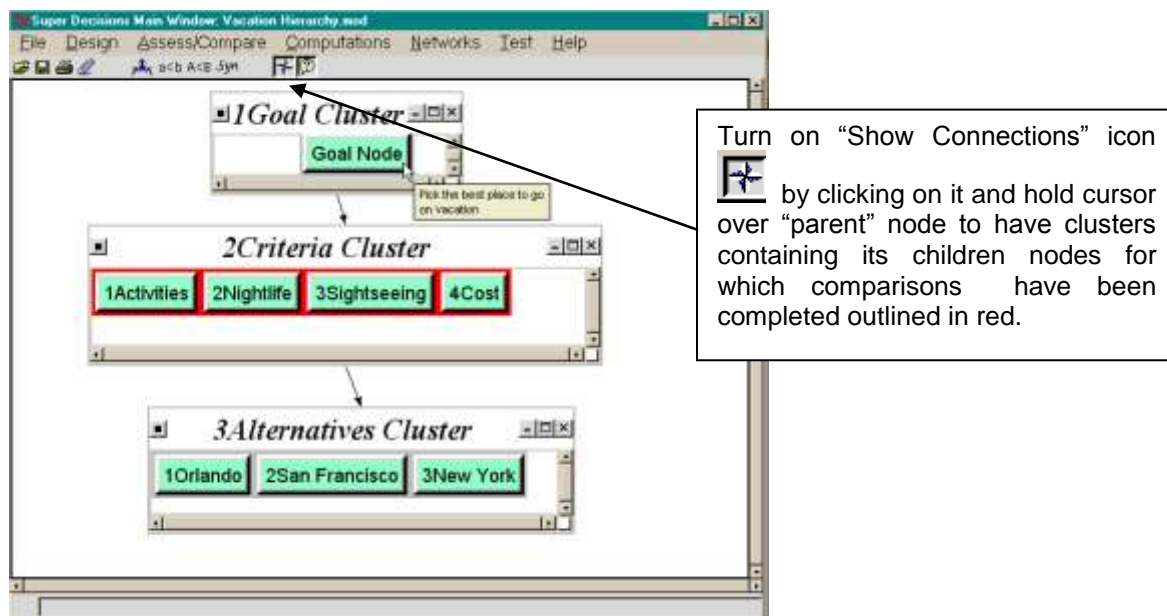
## SYNTHESIS - GETTING THE RESULTS

The results for the alternatives are obtained with the Synthesis command in the Main Model View.  Select the Computations/Synthesize command, or click the shortcut icon  *Syn*   to see the final results:

### RESULTS FROM THE SYNTHESIS COMMAND
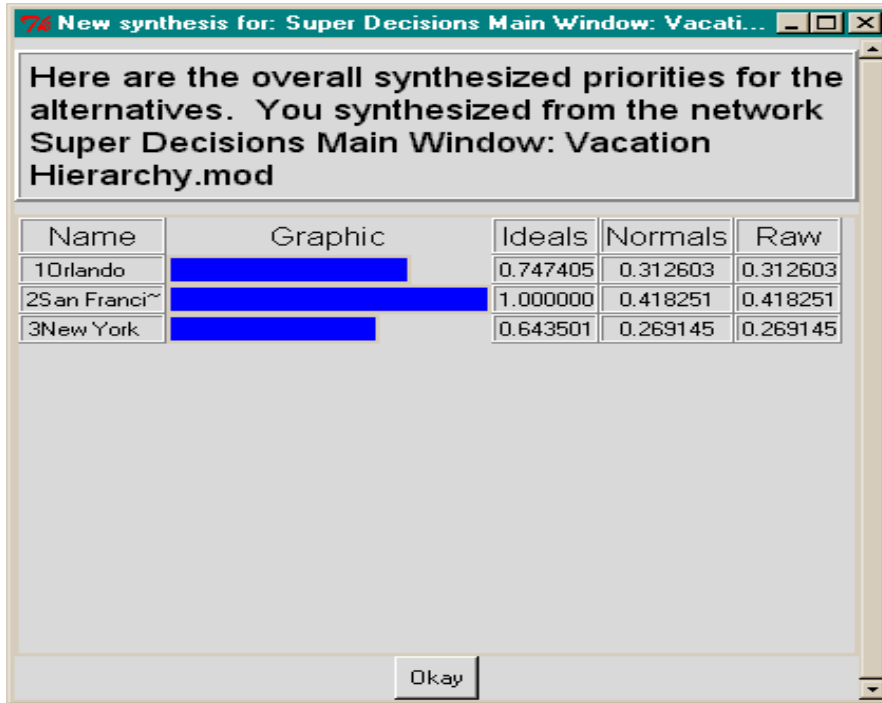


The *Normals* column presents the results in the form of priorities.  This is the usual way to report on results.  The *Ideals* column is obtained from the *Normals* column by dividing each of its entries by the largest value in the column.  The *Raw* column is read directly from the Limit Supermatrix.  In a hierarchical model such as this one the *Raw* column and the *Normals* column are the same.

These results show that the San Francisco would be the best choice for this decision maker.  The "Ideal" column shows the results divided by the largest value so that the best choice has a priority of 1.0.  The others are in the same proportion as in "Normals" and are interpreted this way: Orlando is 74.7 % as good as San Francisco and New York is 64.3% as good as San Francisco.

This answer reflects the preferences of the person who made the judgments, incorporating their personal values and desires, and is not an objective assessment of the relative value as a vacation place of Orlando, San Francisco and New York.  This result also matches our intuition as we know a high priority was put on activities.

# SENSITIVITY

Select the Computations Sensitivity command to get into the sensitivity module.

It is necessary to set the independent variable to see a meaningful sensitivity graph.
There is one line for each alternative in sensitivity windows. In the software they are color coded so that it is easy to see which line corresponds to each alternative, but here in black and white they look the same.

**OPENING SCREEN IN SENSITIVITY MODULE FOR VACATION EXERCISE**
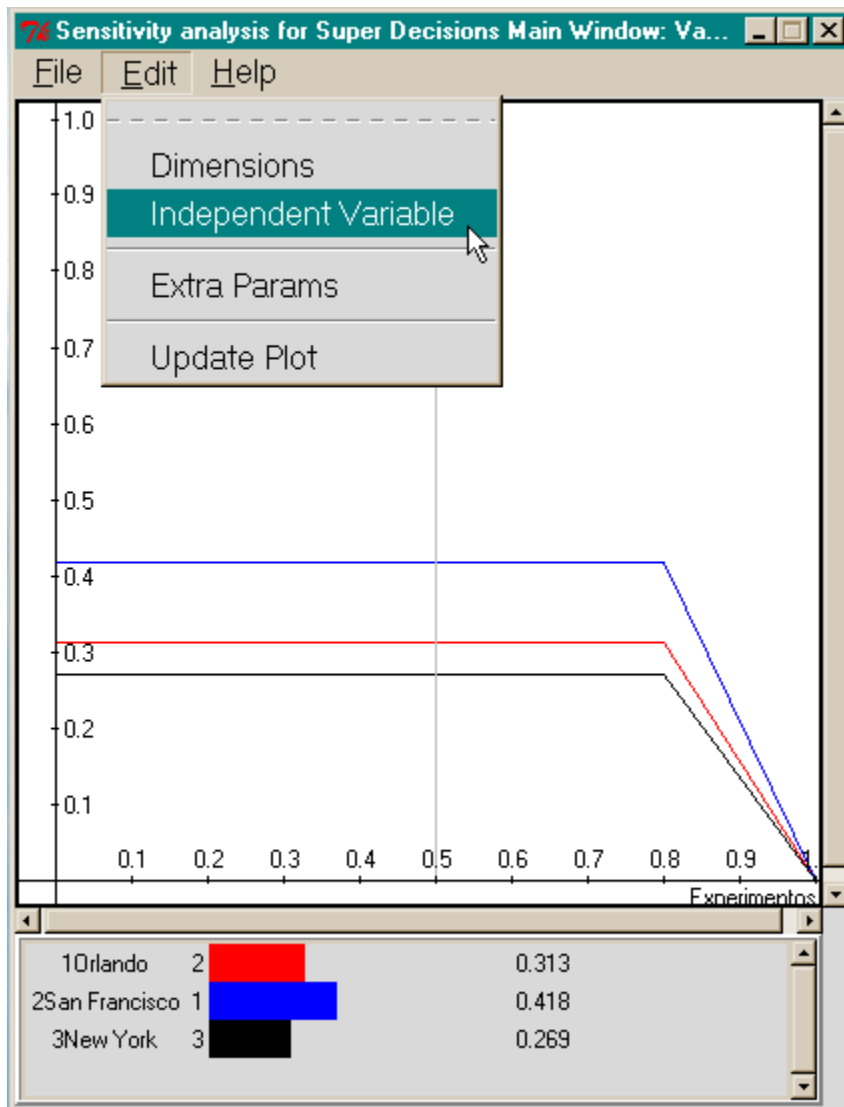
**SENSITIVITY INPUT SETTINGS**
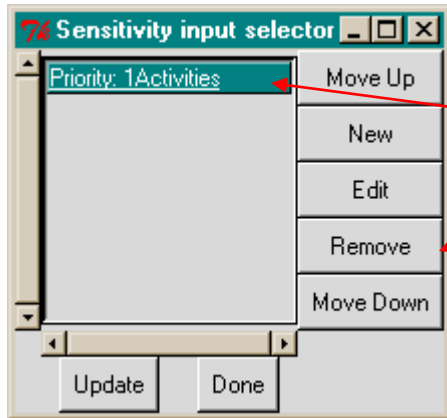


Remove Activities, the automatically selected Independent Variable by first selecting it, then clicking the Remove button.

Click the New button in the Sensitivity input selector. The New Parameter dialogue box shown below will appear. Set the new parameters as shown below. Click on Parameter Type button and choose Supermatrix (type 2). The Network is correctly set to 0 for a hierarchical model. Click on the Wrt (With respect to) Node and select Goal. Set Start to .01 and end to .99. This is necessary only for hierarchical models as including the actual endpoints of the interval, 0 and 1, gives a meaningless graph near the endpoints. Click the Done button on the New parameter box, then click the Update button on the Sensitivity input selector.

**SELECTING THE INDEPENDENT VARIABLE AND SETTING THE OTHER PARAMETERS**

| Before Setting Parameters | After Setting Parameters | Final Selected Parameters |
|---|---|---|
|  |  |  |

### THE SENSITIVITY GRAPH FOR ACTIVITIES
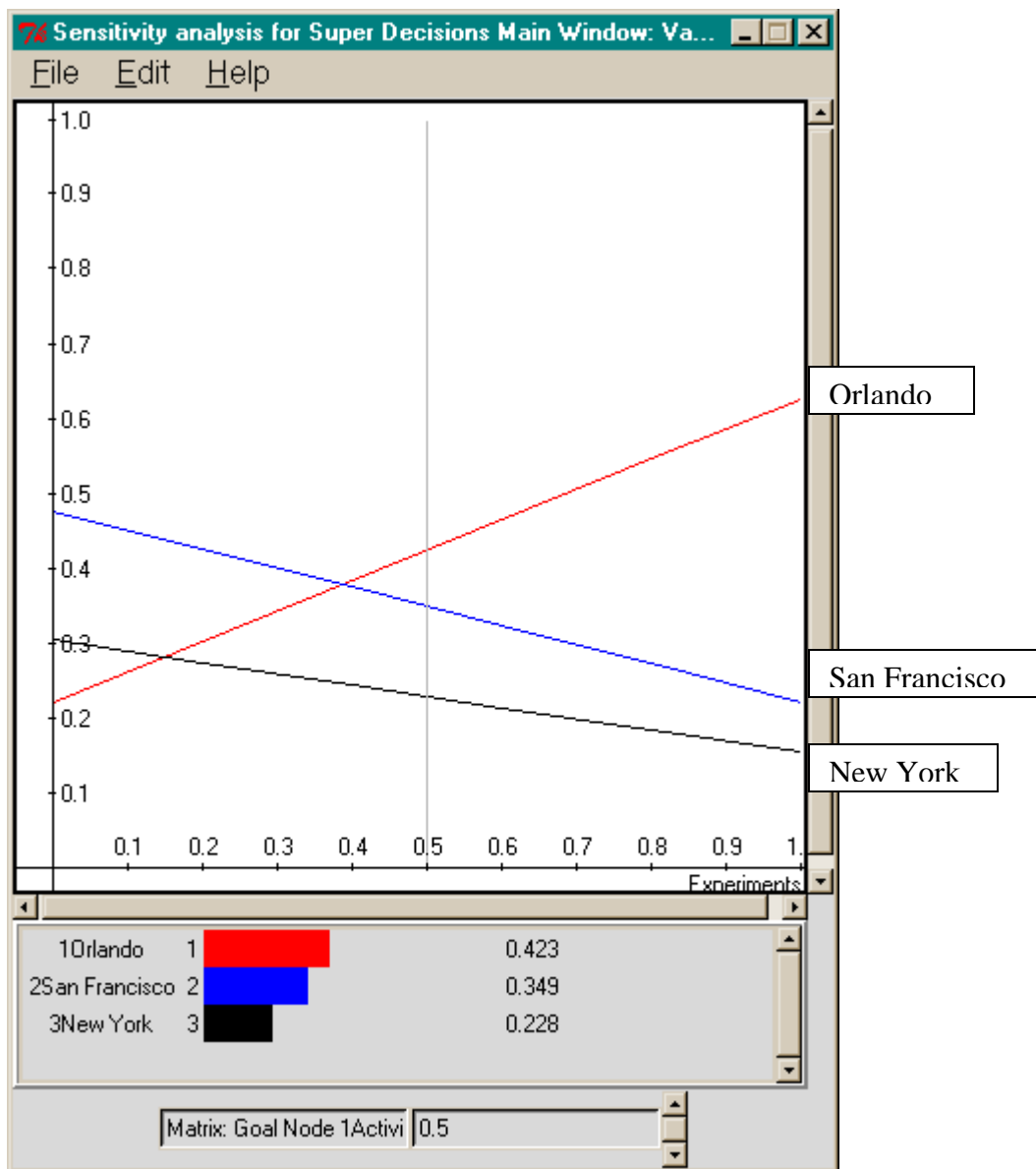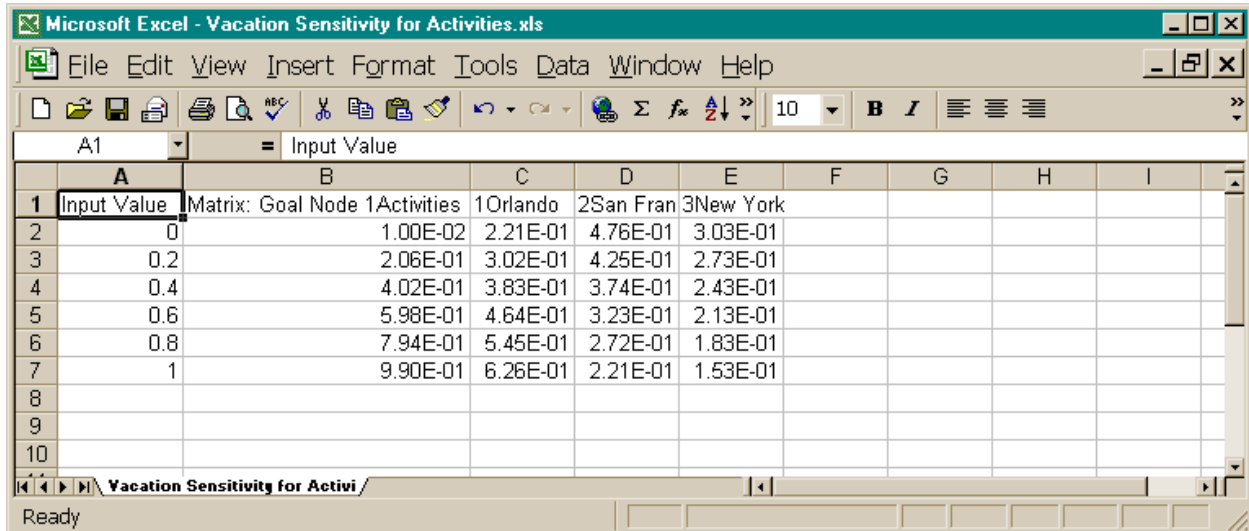


The priority of Activities is plotted on the x axis and the priorities of the alternatives are plotted on the y axis. Click on the blue vertical line and drag it to change the priority of activities. You can then read the corresponding values for the alternatives from the intercept points of the vertical line with the alternative lines. At the point Activities = 0.5, Orlando is about .42, San Fancisco is about .35 and New York is about .25. What this graph is telling us is that if the priority of Activities is greater than about 0.4, Orlando becomes the preferred choice. Before that San Francisco is the best alternative.

You can explore sensitivity for the other criteria by returning to the Parameter Selection dialogue boxes and resetting the parameters.

Select the File Save command and enter a name such as *ActivitySensitivity.xls* to save the data from which the graph is constructed. Load Excel and select File Open to import the file using the Excel Wizard. Keep clicking okay til the import finishes.

**THE ACTIVITIES SENSITIVITY GRAPH DATA POINTS AS SHOWN IN EXCEL**

| | A | B | C | D | E | F | G | H | I |
|---|---|---|---|---|---|---|---|---|---|
| 1 | Input Value | Matrix: Goal Node 1Activities | 1Orlando | 2San Fran | 3New York | | | | |
| 2 | 0 | 1.00E-02 | 2.21E-01 | 4.76E-01 | 3.03E-01 | | | | |
| 3 | 0.2 | 2.06E-01 | 3.02E-01 | 4.25E-01 | 2.73E-01 | | | | |
| 4 | 0.4 | 4.02E-01 | 3.83E-01 | 3.74E-01 | 2.43E-01 | | | | |
| 5 | 0.6 | 5.98E-01 | 4.64E-01 | 3.23E-01 | 2.13E-01 | | | | |
| 6 | 0.8 | 7.94E-01 | 5.45E-01 | 2.72E-01 | 1.83E-01 | | | | |
| 7 | 1 | 9.90E-01 | 6.26E-01 | 2.21E-01 | 1.53E-01 | | | | |

The interpretation of the data shown above is that as the input value, the priority of Activities under the Goal node in the Unweighted Supermatrix, changes from 0 to 1 in six steps the corresponding priorities of the alternatives, shown above, are computed from the Limit Supermatrix.

## THE SUPERMATRICES

The priorities derived from the pairwise comparisons are entered in the Unweighted Supermatrix. Select the Computations/Unweighted SuperMatrix/ Graphical to see how the priorities that were derived in the comparison process appear there.

**UNWEIGHTED SUPERMATRIX**

| Cluster Node Labels | | 1Goal Cluster | 2Criteria Cluster | | | | 3Alternatives Cluster | | |
|---|---|---|---|---|---|---|---|---|---|
| | | Goal Node | 1Activities | 2Nightlife | 3Sightseeing | 4Cost | 1Orlando | 2San Francisco | 3New York |
| 1Goal Cluster | Goal Node | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 |
| 2Criteria Cluster | 1Activities | 0.232606 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 |
| | 2Nightlife | 0.075985 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 |
| | 3Sightseeing | 0.513961 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 |
| | 4Cost | 0.177449 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 |
| 3Alternatives Cluster | 1Orlando | 0.000000 | 0.630097 | 0.163424 | 0.075057 | 0.648329 | 0.000000 | 0.000000 | 0.000000 |
| | 2San Francisco | 0.000000 | 0.218442 | 0.296961 | 0.591727 | 0.229651 | 0.000000 | 0.000000 | 0.000000 |
| | 3New York | 0.000000 | 0.151461 | 0.539614 | 0.333216 | 0.122020 | 0.000000 | 0.000000 | 0.000000 |

Done

In a hierarchical model like this, the Weighted Supermatrix is the same as the Unweighted Supermatrix because the clusters are not weighted.  Raising the Weighted Supermatrix to powers yields the Limit Matrix from which the final answers are extracted.  The final priorities for the Alternatives are in the column under the Goal.

**THE LIMIT MATRIX IS THE FINAL SUPERMATRIX CONTAINING THE ANSWERS**

| Cluster Node Labels | | 1Goal Cluster | 2Criteria Cluster | | | | 3Alternatives Cluster | | |
|---|---|---|---|---|---|---|---|---|---|
| | | Goal Node | 1Activities | 2Nightlife | 3Sightseeing | 4Cost | 1Orlando | 2San Francisco | 3New York |
| 1Goal Cluster | Goal Node | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 |
| 2Criteria Cluster | 1Activities | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 |
| | 2Nightlife | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 |
| | 3Sightseeing | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 |
| | 4Cost | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 |
| 3Alternatives Cluster | 1Orlando | 0.312603 | 0.630097 | 0.163424 | 0.075057 | 0.648329 | 1.000000 | 0.000000 | 0.000000 |
| | 2San Francisco | 0.418251 | 0.218442 | 0.296961 | 0.591727 | 0.229651 | 0.000000 | 1.000000 | 0.000000 |
| | 3New York | 0.269145 | 0.151461 | 0.539614 | 0.333216 | 0.122020 | 0.000000 | 0.000000 | 1.000000 |

Done

By selecting the Computation Limit Matrix Option and setting the computation method to either of the New Hierarchy options, the Limit Matrix will display the intermediate priorities under every node in the model. This option must be selected each time the model is opened. The default value is the first, Calculus type, which always ends up with only the priorities for the alternatives in each column.for a model that has a hierarchical structure. It is handy to be able to read the priorities of the criteria with respect to the Goal in the first column.

**THE LIMIT MATRIX WITH HIERARCHICAL OPTION CALCULATIONS**

> Priorities for intermediate nodes now appear in the Limit Matrix.

**Super Decisions Main Window: Vacation Hierarchy.mod: Limit Matrix**

| Cluster Node Labels | | 1Goal Cluster | 2Criteria Cluster | | | | 3Alternatives Cluster | | |
|---|---|---|---|---|---|---|---|---|---|
| | | Goal Node | 1Activities | 2Nightlife | 3Sightseeing | 4Cost | 1Orlando | 2San Francisco | 3New York |
| 1Goal Cluster | Goal Node | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 |
| 2Criteria Cluster | 1Activities | 0.116303 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 |
| | 2Nightlife | 0.037992 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 |
| | 3Sightseeing | 0.256980 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 |
| | 4Cost | 0.088724 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.000000 |
| 3Alternatives Cluster | 1Orlando | 0.156302 | 0.630097 | 0.163424 | 0.075057 | 0.648329 | 0.000000 | 0.000000 | 0.000000 |
| | 2San Francisco | 0.209126 | 0.218442 | 0.296961 | 0.591727 | 0.229651 | 0.000000 | 0.000000 | 0.000000 |
| | 3New York | 0.134573 | 0.151461 | 0.539614 | 0.333216 | 0.122020 | 0.000000 | 0.000000 | 0.000000 |

Done

# DEMO OF A RATINGS MODEL

People often "rate" alternatives using words such as High, Medium and Low; or Excellent, Very Good, Good, Fair and Poor with respect to some characteristic they have. We use the same idea and rate alternatives with respect to the criteria. We add one twist and prioritize the ratings words themselves, so that a "High" rating, for example, is twice as good as a "Medium" rating.

We will now have you load a previously created model set up to do ratings. The model to select a vacation place was converted into a Ratings model named *Vacation Places Rated.mod*. We will have you load this prepared model and use it to rate some vacation places. It is located in Sample Models under the Help command.

In this model the vacation places do not appear in the main model. They are in the Ratings module attached to the model. You can see from the title bar that this is a ratings type of model. The advantage of a ratings model is that the evaluation structure is set up and each alternative is evaluated as to how it performs on each criterion. This dramatically shortens the number of judgments required. However, ratings models are only appropriate in very well understood

evaluation settings where knowledgeable people or experts have provided the evaluation structure.  They should not be used in new or one-of-a-kind decisions.

**VACATION PLACES RATED MODEL – MAIN SCREEN**
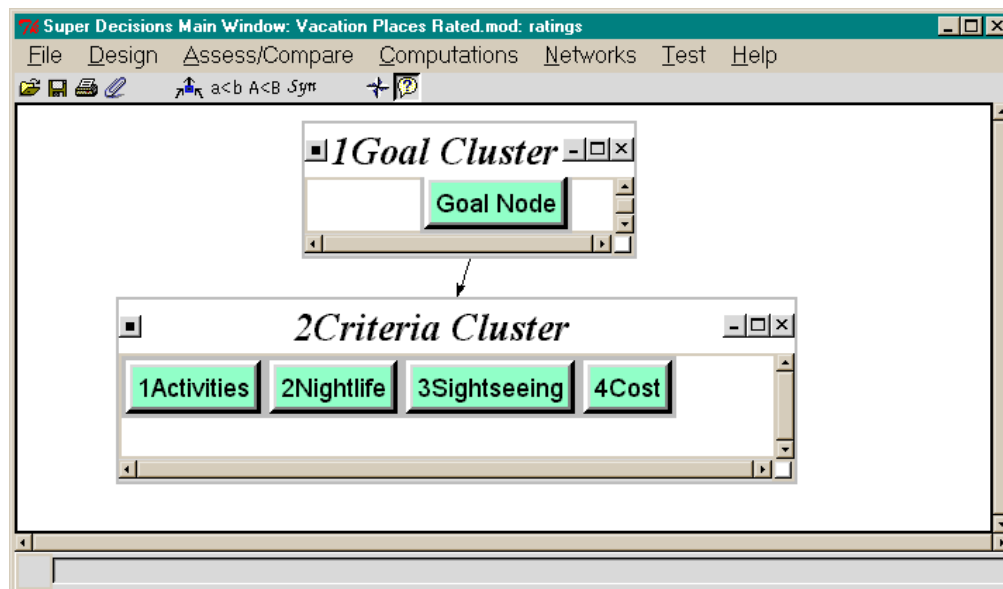


To get into the Ratings module shown below select the Assess/Compare, Ratings command, or select the Design, Ratings command – both do the same thing.  When the ratings module is open, the main screen view is locked.  To return to it, close the Ratings window.

**VACATION PLACES RATED MODEL – RATINGS SCREEN**

The results in a ratings model are obtained with the Calculations, Priorities command. The Synthesis results, in which the Normals column is the same as the Priorities below, can also be obtained in the ratings module or back in the main model with the Computations, Synthesis command.

**RESULTS FOR THE ALTERNATIVES DERIVED IN THE RATINGS MODULE**

| Results as Priorities | Results as Totals |
|---|---|
|  |  |
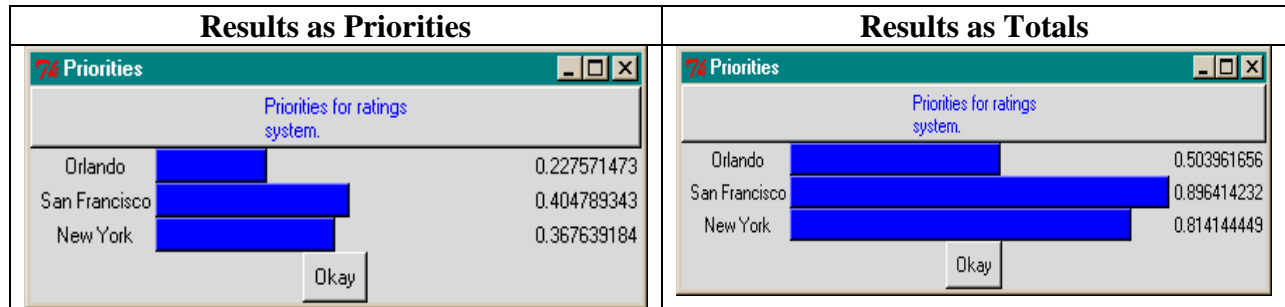
When shown as Totals the results may be interpreted this way: San Francisco, with a priority of .896, is the best, but is still only 89.6% as good as a perfect vacation place would be while New York has %81.4. The Priorities are obtained from the Totals by normalizing, that is, summing the Totals values and dividing by the sum. When shown as priorities the results sum to 1.0.

**NOW TRY ADDING SOME VACATION PLACES:**
1. Use the command Edit Alternative New to add a new alternative.
2. Click on a cell to show the possible ratings and select the one you want as shown below.
3. After you complete the ratings select the Calcuations command and look at both Priorities and Totals to see your results.
4. Close the Ratings window and return to the main screen. Try the Synthesize command to see that the results are the same.
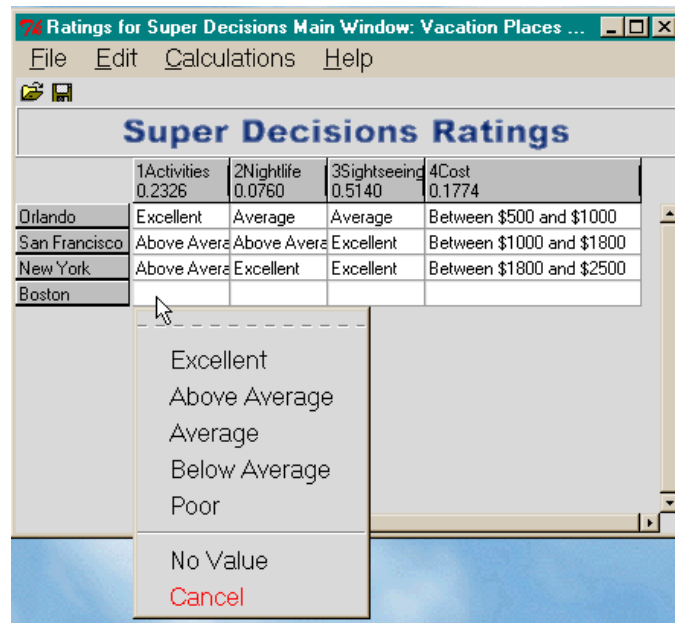
# EXERCISE TWO - CREATING A RATINGS MODEL

- **Creating a Ratings Model** We shall show how a Ratings model is created by starting with the previous Vacation example and converting it from a relative model using only pairwise comparisons to a Ratings model where standard categories or "Ratings" are set up for each criterion.  The alternatives, the vacation places, are then evaluated or rated against these standards by selecting for each the appropriate category for each criterion.

- First, save a copy of the vacation model you created in exercise one by selecting the File/Save As command and name it "Vacation Ratings Model".  Remove the Alternatives cluster by right clicking on the title bar and deleting it.  Select the Assess/Compare Ratings command to create the Ratings spreadsheet.  Once the Ratings screen appears, shown below, the Main Model View will disappear and remain "locked out" until the Ratings screen is closed.

- **Add Criteria** Select the Edit Criteria New command and click on the nodes from the main model that will be the criteria in the Ratings spreadsheet.  Selecting all four criteria brings them into the Ratings spreadsheet.  The priorities for the criteria, shown below the names, are the same ones that were established in the main model though pairwise comparison.

| Startup Ratings Window | Ratings Window after Criteria Selected |
|---|---|
|  |  |
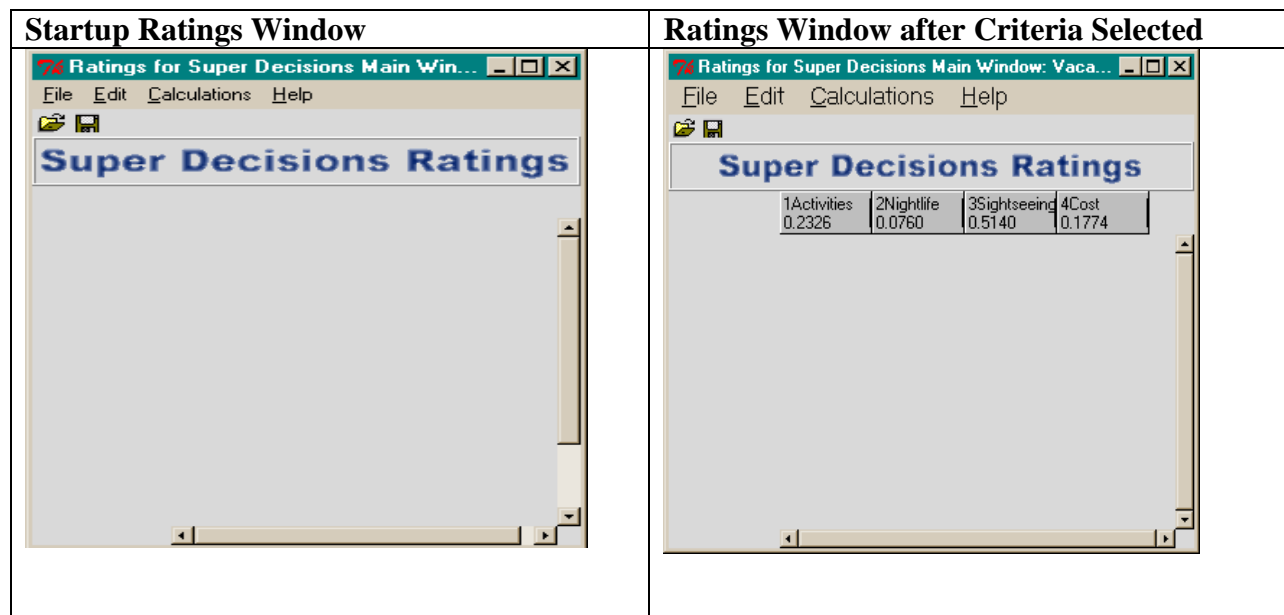
- **Add Alternatives** Next, select the Edit Alternatives New command and type the names of the alternatives Orlando, San Francisco, New York.  This results in the Super Decisions Ratings table with alternatives added.

**Ratings Window with Alternatives Added**
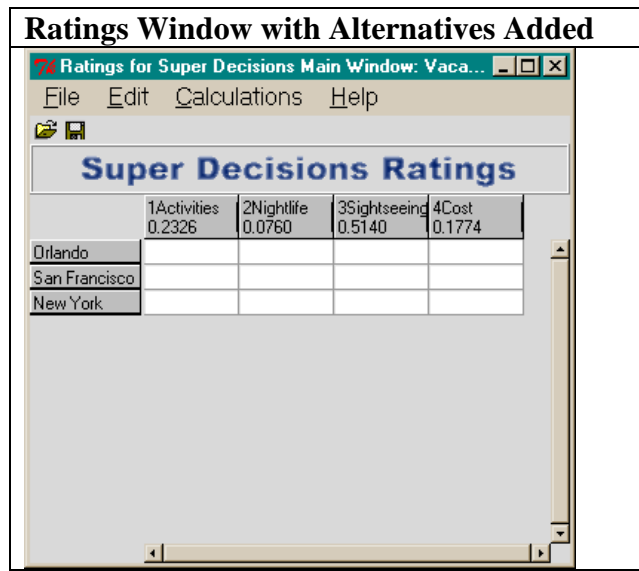


- **Creating and Prioritizing the Ratings Categories** This step must be done before the alternatives can be rated. Select the Edit Criteria Categories command, choose *Activities* and create categories for it (below):

**CATEGORY EDITOR FOR CREATING CRITERION CATEGORIES**
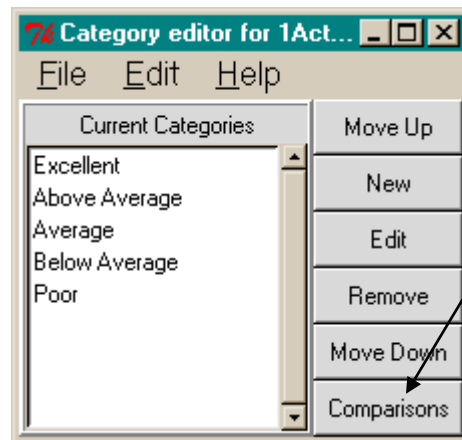


Click the Comparisons button to pairwise compare categories and establish priorities for them.

Clicking the Comparison button will bring up the following comparison screen where we pairwise compare to establish priorities for the categories. Select the command Computations/Ideal Priorities to see the resulting priorities.

**COMPARE THE RATING CATEGORIES FOR ACTIVITIES**



| **Priorities for Activities** | | **Idealized Priorities for Activities** | |
|---|---|---|---|



The idealized priorities shown above are obtained from the priorities in the first column above by dividing each by the largest (0.462 in this case) and are used in the Ratings spreadsheet. For example, an Excellent rating means place the number 1.000 in the cell, for each cell multiply the cell idealized value times the column priority listed at the top of the column and add across to get the total for that row.
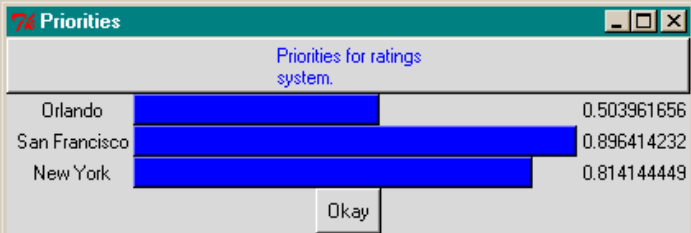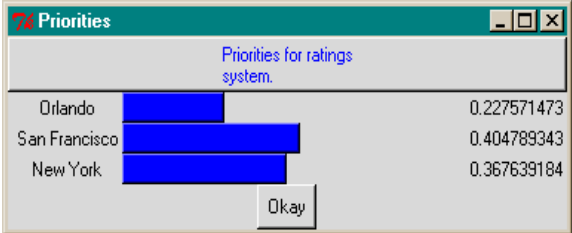
Close all open windows to return to the Ratings spreadsheet. Right or left click on the (Orlando, Activities) cell to get the dropdown menu for selecting the appropriate category.

Create categories for the rest of the criteria and prioritize them.  The rest of the categories that were established in this model and their priorities are shown in the table below.  It is not necessary to have the same number of rating categories for each column (there are 3 for Prestige).

| Activities | Nightlife | Sightseeing | Cost |
|---|---|---|---|
| Excellent  1.000 | Excellent  1.000 | Exceptional 1.000 | $500-$1000    0.511 |
| Above Average  0.664 | Above Average  0.664 | Above Average  0.337 | $1000-$1800    1.000 |
| Average  0.306 | Average  0.306 | Average  0.148 | $1800-$2500   0.393 |
| Below Average 0.126 | Below Average 0.126 | Poor  0.0652 | > $2500   0.098 |
| Poor  0.06 | Poor  0.06 | | |

### VIEWING THE RESULTS IN RATINGS

- Select the Calculations/Totals command to see the evaluation scores before normalizing.  The Total for each row is obtained by multiplying the values for the ratings in the cells times the column priorities and summing.

- Select the Calculations/Priorities command to see the results as priorities that sum to 1.

| RESULTS SHOWN AS TOTALS IN THE RATINGS SPREADSHEET | TOTALS CONVERTED TO PRIORITIES IN THE RATINGS SPREADSHEET |
|---|---|
|  |  |

- Select the Calculations/Matrix Priorities command to see the ideal priority values in each cell that correspond to the verbal ratings.  These values can be blocked, copied with Ctrl C and pasted with Ctrl V into an Excel Spreadsheet for further processing.

```
┌─ 7⁄ Ratings Priorities Matrix ──────────────────── _ □ ✕ ┐
│      0.232605        0.075985        0.513961      0.177449        ▲ │
│      1Activities     2Nightlife      3Sightseeing  4Cost             │
│ Orlando          1.000000       0.306142       0.306142    0.511411  │
│                                                                      │
│ San Francis~     0.664326       0.664326       1.000000    1.000000  │
│                                                                      │
│ New York         0.664326       1.000000       1.000000    0.392636  │
│                                                                   ▼  │
│ ◄                                                              ►     │
└──────────────────────────────────────────────────────────────────┘
```

# HIERARCHICAL MODELS WITH SUB-CRITERIA

When the criteria in a hierarchical model are broken down into sub-criteria one way to handle it, though not the only one is to put the criteria in a cluster, then create a cluster for the subcriteria of each criterion. A second way would be to put all the subcriteria in a level into the same cluster, but then it would be difficult to see at a glance the subcriteria that belong to a criterion.
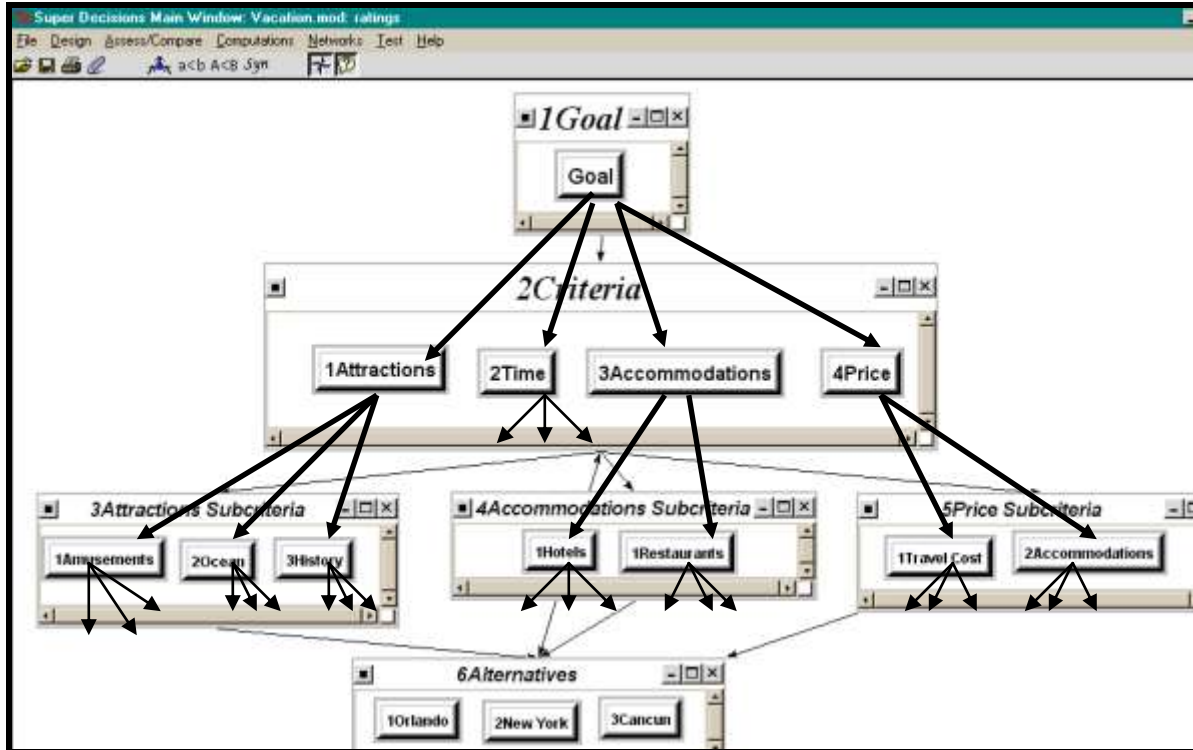
In the figure above, one main criterion node, Time, connects directly to the alternatives. The other criteria connect to subcriteria that then connect to the alternatives. The lowest level of criteria that connect to the alternatives are called the "covering criteria". Here the covering criteria are the main criterion Time plus the subcriteria: Amusements, Oceans, History, Hotels, Restaurants, Travel Cost, Accommodations. The pairwise comparisons are done in the usual way for this model. It can also be done as a Ratings model by selecting the bottom level or "covering criteria" as the criteria in Ratings.

## COMPLETE HIERARCHICAL MODELS

In a complete hierarchical model, all the nodes in a level connect to all the nodes in the next lower level. The model shown in Figure 8 below has a "complete hierarchy" structure except for the next to last level between "Groups Affected" and "Objectives". The Farmers were interested only in Irrigation and Flood Control on the next level, the Recreationists only in Flat Dam (dam full of water) and White Dam (low water level in dam), the Power Users only in power and the Environmentalists only in the Environment. The clusters in the ANP model shown in Figure 9 correspond to the levels in hierarchic drawing in Figure 8.

**Focus:**
**(1)**

At what level should the Dam be kept:  Full or Half-Full

**Decision Criteria:**
**(2)**

Financial

Political

Env't Protection

Social Protection

**Decision Makers:**
**(3)**

Congress

Dept.Inter.

Courts

State

Lobbys

**Factors:**
**(4)**

Clout

Legal Position

Potential Financial Loss

Irreversibility of the Env't

Archeo-logical Problems

Current Financial Resources

**Groups Affected:**
**(5)**

Farmers

Recreation

Power Users

Environmentalists

**Objec-tives:**
**(6)**

Irrigation

Flood Control

Flat Dam

White Dam

Cheap Power

Protect Environment

**Alterna-tives:**
**(7)**

Half-Full Dam

Full Dam

**Figure 8.**  Hierarchy  for Deciding on Water Level  in a Dam
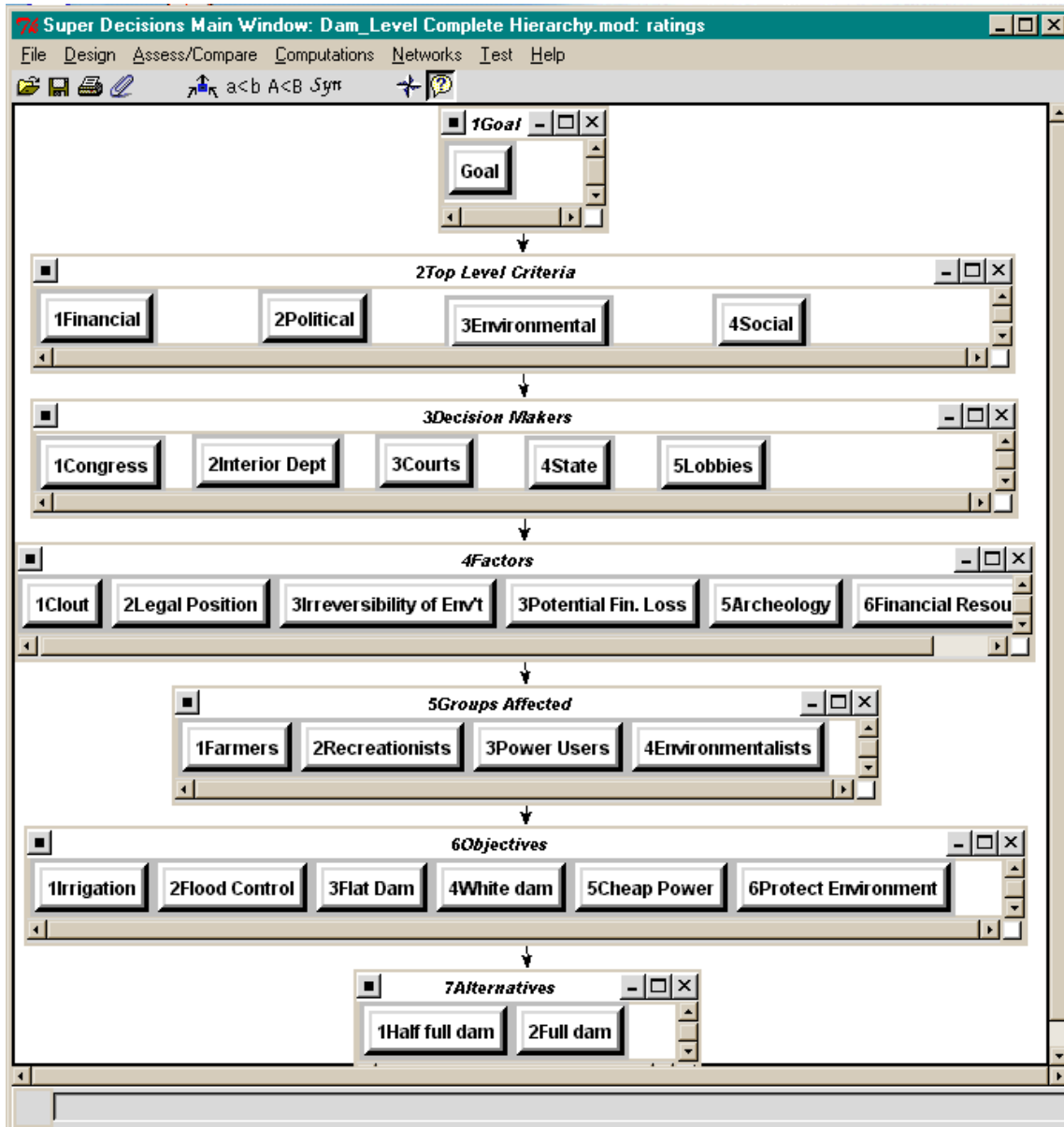
**Figure 9.** The ANP Model for that Corresponds to the Dam Problem shown in Figure 8

In the *SuperDecisions* software, a connection from one cluster to another means at least one node in the source cluster is connected to one node in the target cluster. To see exactly which nodes are connected in the software click on the icon ⊹ to show the node connections.

# PART 2. HOW TO USE *SUPERDECISIONS* TO BUILD NETWORK ANP MODELS

## Summary

An Analytic Network Model of a problem may consist of a single network or a number of networks. To build an Analytic Network Process (ANP) network, you need to:

1. Think about the elements in it and decide what kind of logical groupings of nodes and clusters would best describe the problem;
2. Build a cluster first, then create the nodes within it
3. Select one node as a potential parent node and examine all the clusters in turn to determine if they have nodes that the parent node either influences or is influenced by to select its children nodes in that cluster.
4. Create the links between the parent node and all its children nodes in each cluster – this is how the comparison sets of nodes are created.
5. Clusters are linked automatically when nodes are linked.
6. Make sure the *influences* or *is influenced by* question is posed in a consistent way throughout this network. Make pairwise comparison judgments on nodes and clusters and synthesize.

The following topics are covered in the software tutorial:

- Installing the *ANP Software*
- A demonstration of a single network system (*Hamburger* model)
- A demonstration of a two-layer system (*Car Purchase BCR* model with Benefits Costs and Risks subnets) and of a three-layer system (*National Missile Defense* model) with its decision subnets in the bottom layer.
- Step-by-step walkthrough of building a simple single-network model
- Step-by-step walkthrough of building a complex model
- Special *SuperDecisions* software functions and commands

# INTRODUCTION TO DECISION MAKING USING THE ANALYTIC NETWORK PROCESS (ANP)

The power of the Analytic Network Process (ANP) lies in its use of ratio scales to capture all kinds of interactions and make accurate predictions, and, even further, to make better decisions. So far, it has proven itself to be a success when expert knowledge was used with it to predict sports outcomes, economic turns, business, social and political events.

The ANP is the first mathematical theory that makes it possible for us to deal systematically with all kinds of dependence and feedback. The reason for its success is the way it elicits judgments and uses measurement to derive ratio scales. Priorities as ratio scales are a fundamental kind of number amenable to performing the basic arithmetic operations of adding within the same scale and multiplying different scales meaningfully as required by the ANP.

The Analytic Network Process (ANP) is a new theory that extends the AHP to cases of dependence and feedback and generalizes on the supermatrix approach introduced in Thomas Saaty's 1980 book on the Analytic Hierarchy Process. It allows interactions and feedback within clusters (inner dependence) and between clusters (outer dependence). Feedback can better capture the complex effects of interplay in human society. The ANP provides a thorough framework to include clusters of elements connected in any desired way to investigate the process of deriving ratio scales priorities from the distribution of influence among elements and among clusters. The AHP becomes a special case of the ANP. Although many decision problems are best studied through the ANP, it is not true that forcing an ANP model always yields better results than using the hierarchies of the AHP. There are examples to justify the use of both. We have yet to learn when the shortcut of the hierarchy is justified, not simply on grounds of expediency and efficiency, but also for reasons of validity of the outcome.

The ANP is implemented in the software *SuperDecisions* and has been applied to various problems both to deal with decisions and to illustrate the uses of the new theory. The ANP is a coupling of two parts. The first consists of a control hierarchy or network of criteria and subcriteria that control the interactions in the system under study. The second is a network of influences among the elements and clusters. The network varies from criterion to criterion and a supermatrix of limiting influence is computed for each control criterion. Finally, each of these supermatrices is weighted by the priority of its control criterion and the results are synthesized through addition for all the control criteria.

In addition, a problem is often studied through a control hierarchy or system of benefits, a second for costs, a third for opportunities, and a fourth for risks. The synthesized results of the four control systems are combined by taking the quotient of the benefits times the opportunities to the costs times the risks to determine the best outcome. Other formulas may be employed at times to combine results. Here is a rough outline of the steps of the ANP followed by two lists of concepts of both the AHP and the ANP.

# CONCEPTS OF THE ANALYTIC HIERARCHY PROCESS

1. Elements of the problem, goal, subgoals, time horizons, scenarios, actors and stakeholders, their objectives and policies, criteria, subcriteria, attributes, and alternatives.
2. Hierarchic structure.
3. Judgments - absolute numbers, homogeneity, clustering, pivot elements, tangibles and intangibles.
4. Comparisons, dominance and reciprocity with respect to an attribute, inconsistency and the eigenvector, use of actual measurements.
5. The number of judgments; how to take fewer judgments.
6. Derived ratio scales - in AHP the priorities are derived and are proven to belong to a ratio scale.
7. Interval judgments, stochastic judgments.
8. Synthesis - multilinear forms - density.
9. Rank - the dominance mode, the performance mode with respect to an ideal.
10. Absolute measurement - rating alternatives one at a time.
11. Benefits, opportunities, costs and risks hierarchies.
12. Parallel with human thinking - neural firing creates awareness and intensity of stimuli for both tangibles and intangibles. Measurements are data to be interpreted.
13. Group Decision Making and the reciprocal property; Pareto optimality: if each prefers A to B, then the group does.
14. Sensitivity Analysis.
15. Learning and revision as a process.

# CONCEPTS OF THE ANALYTIC NETWORK PROCESS

1. Feedback, inner and outer dependence.
2. Influence with respect to a criterion.
3. The control hierarchy or system.
4. The supermatrix.
5. The limiting supermatrix and limiting priorities.
6. Primitivity, irreducibility, cyclicity.
7. Make the limiting supermatrix stochastic: why clusters must be compared.
8. Synthesis for the criteria of a control hierarchy or a control system.
9. Synthesis for benefits, costs, opportunities, and risks control hierarchies.
10. Formulation to compute the limit.
11. Relation to Neural Network Firing - the continuous case.
12. The density of neural firing and distributions and their applications to reproduce visual images and symphonic compositions. Further research in the area is needed.

## TWO WAYS TO FRAME THE QUESTION WHEN MAKING ANP COMPARISONS

When making pairwise comparisons in an ANP model the questions are formulated in terms of dominance or influence.Given a parent element, which of two elements being compared with respect to it has greater influence (is more dominant) with respect to that parent element? Or, which is influenced more with respect to that parent element? You want to avoid changing perspective. For example, in comparing A to B with respect to a criterion, you ask whether the criterion influences A or B more. Then if for the next comparison involving A and C you ask whether A or C influences the criterion more, this would be a change in perspective that would undermine the whole exercise. You must keep in mind whether the influence is flowing from the parent element to the elements being compared, or the other way around.

Use one of the following two questions throughout an exercise:

1.  Given a parent element and comparing elements A and B under it, which element has greater influence *on* the parent element?
2.  Given a parent element and comparing elements A and B, which element is influenced more *by* the parent element?

# *SUPERDECISIONS* SOFTWARE TUTORIAL

## INTRODUCTION TO THE *SUPERDECISIONS* SOFTWARE

In this introduction we review the ANP process and the *SuperDecisions* software and show some applications. Applications may be simple, consisting of a single network, or complex, consisting of a main network and two or more layers of sub-networks. Each network and sub-network is created in its own window.

In practice an application consists of:

*   **A Simple Network –** *HamburgerModel* - All the clusters and their nodes are in a single window. An example of a simple network would be a "market share" application such as the Hamburger model. The simple network itself is the decision network because it contains the cluster of nodes that serve as the alternatives of the decision. In a market share application they are the competitors for whom market share is being predicted; for example, in the Hamburger model they are McDonald's, Burger King and Wendy's.
*   **A Two-level Network -** *Car Purchase BCR* -There is a top-level network with Merit nodes such as Benefits, Opportunities, Costs and Risks, each of which has a sub-network. The alternatives cluster is in each of the sub-networks. The sub-networks are the decision networks because they contain the alternatives.
*   **A Complex Network** - *National Missile Defense Model* -There is a main network of Merits nodes (Benefits, Opportunities, Costs and Risks), each having an attached sub-network that contains among others nodes that will serve as control criteria. The nodes selected to serve as control criteria, the high priority nodes in the network, have decision networks containing the alternatives attached to them. In practice this is the most

complex system we work with, though there is no theoretical limitation on the number of levels of sub-networks.

A model is contained in a network system that physically is a single file. All the networks and sub-networks are in the same file. The file has the extension *.mod*; for example, the *Hamburger* model is in the file named *Hamburger.mod*. If you have the *SuperDecisions* software installed an application can be launched by launched by double-clicking on it within Windows Explorer. However, the files for the Encyclicon applications have been saved in an advanced zipped format to reduce their size. These files end with the .mod.gz extension. Such a zipped file cannot be launched by double-clicking. Instead you must load the *SuperDecisions* software first, then open the file using the File Open command. Working with zipped files takes a little longer when loading and saving, but the space saved is considerable.

There are four applications that we will discuss in the introduction: *Bridge*, *Hamburger*, *Car Purchase BCR,* and *National Missile Defense*. The *Bridge* example is a simple network used to illustrate the idea of feedback. The *Hamburger* example is a simple network used to estimate market share of fast food restaurants. The *Car Purchase BCR* model is a two-level network with Benefits, Costs and Risks nodes in the top-level network, and the alternatives in the sub-networks attached to them. They range from the simplest example of feedback to a complex multi-level network structure. All of these examples are included in the sample models of the *SuperDecisions* software. To load a sample model use the Help, Sample Models command in the software and select the one you want. Sample models are located in the c:/program files/super decisions/samples directory.

*Bridge* is the simplest example in a single network and we use it to demonstrate the supermatrix idea.

*Hamburger* is used to estimate the market share of three fast-food hamburger places. The model consists of a single network containing the factors that consumers consider when choosing a fast-food restaurant. Some of its clusters are inner dependent with the nodes in them being compared with respect to other nodes in the same cluster. It has cluster comparisons as well as node comparisons. We use this model to explain more complicated supermatrices, inner and outer dependence and the motivation behind doing cluster comparisons. It is a simple network model in a single window.

*Car Purchase BCR* is a model for selecting the best kind of car to buy: Japanese, European or American, by considering the benefits, costs and risks of each type of car. It is a complex two-layer model with three sub-networks. The top-level network contains a benefits node, a costs node and a risks node, each of which has a sub-network where the alternatives are located. Judgments in a sub-network are made from the perspective of its controlling node in the network above.

*National Missile Defense* is the most complex kind of application with a top-level control model in which the priorities of the BOCR have been obtained by rating them against the US's

national objectives.  It has control criteria sub-networks under that and finally decision networks containing the alternatives at the bottom.

## DEMONSTRATION OF THE SIMPLEST TYPE OF FEEDBACK NETWORK, THE BRIDGE MODEL

*Bridge* is a decision problem to pick the best of two bridge designs. It is a simple network of one level that contains only two clusters, with two nodes in each cluster, and links between the nodes.  A network is structured of clusters, nodes and links. We use this model to show how feedback arises in a network decision structure and how the pairwise comparison questions are formulated when there is feedback. Here the clusters are outer dependent, that is, nodes in a cluster are compared only with respect to nodes in the other cluster.

Load the *Bridge* model by selecting Help, Sample Models from the main menu and selecting *bridge.mod*

The *Bridge* model, a simple network model, is shown in Figure 10.  Clusters may be re-sized by left-clicking on the small button at the lower right hand corner and dragging.  To select a cluster left-click on the title bar.  A cluster is selected when its title bar is highlighted.  Left-click on the title bar of a cluster window and drag to move it to a different location.
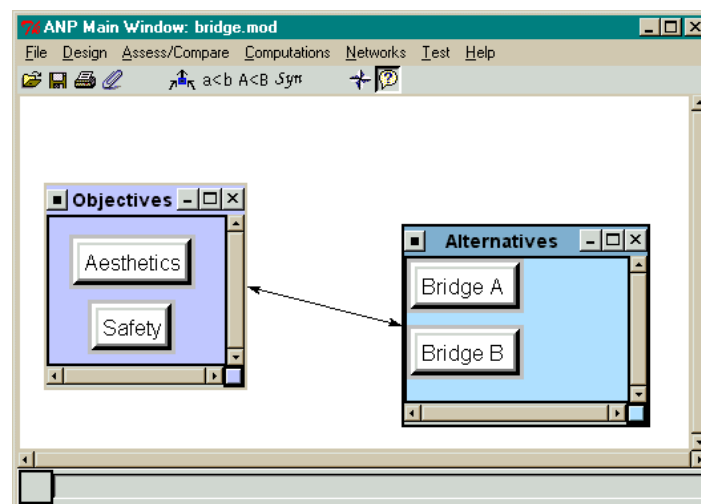


**Figure 10.** The *Bridge* Model: a Simple Network.

The decision in this model is to select the best bridge.  The objectives are to have a safe bridge and an aesthetically pleasing bridge.  If one were doing the model from the top down as in a hierarchy, there would be a goal with Aesthetics and Safety as the criteria and Bridge A and Bridge B as the alternatives.  One would compare Aesthetics to Safety, Safety would likely be perceived as extremely more important, so the safest bridge would be the "best" choice.

But in a feedback network one compares the bridges for preference with respect to Aesthetics and to Safety, and one also compares the prevalence of Aesthetics versus Safety for each bridge. The net result of this is that priorities are obtained for all four nodes in the system.  Suppose the safest bridge, B, is unattractive, and the nicer looking bridge, A, is very safe, though not as safe

as B.  The priorities of the criteria depend on the bridges available and since both are quite safe, the priority of safety in the feedback system ends up less than it would be in a hierarchy where one compares Safety to Aesthetics in the abstract and apart from any specific bridge.  It makes common sense that if both bridges are very safe, one should pick the better-looking bridge, even though one bridge is far safer than the other.  The Analytic Network Process through feedback guides us to the best choice in a way that matches our common sense.

The Aesthetics node in the Objectives cluster is linked to Bridge A and to Bridge B, and because there is a link between nodes, a link appears from the Objectives cluster to the Alternatives cluster. Because at least one node in the Objectives cluster is linked to nodes in the Alternatives cluster, a link appears automatically from the Objectives cluster to the Alternatives cluster. Aesthetics is the parent node and Bridge A will be compared to Bridge B with respect to it. The node Safety is also linked to Bridge A and Bridge B, and they will be compared for preference with respect to safety.

To turn on the "show connections" mode, as shown in Figure 11 click on the star-shaped icon ![icon].  When you place your cursor over a node when this icon is depressed, the nodes connected from it will be outlined in red.  Try this by placing the cursor over the Bridge A node and the Aesthetics and Safety nodes will be outlined in red.

When a node has had the comparisons marked as completed for nodes within a cluster that are connected to it, the cluster window of these nodes will also be outlined in red.  Both bridges are also connected to the nodes in the Objectives cluster, so holding the cursor over the Bridge A node will show Aesthetics and Safety outlined in red, and the Objectives cluster being outlined in red indicates that the comparison of these nodes with respect to Bridge A is complete.
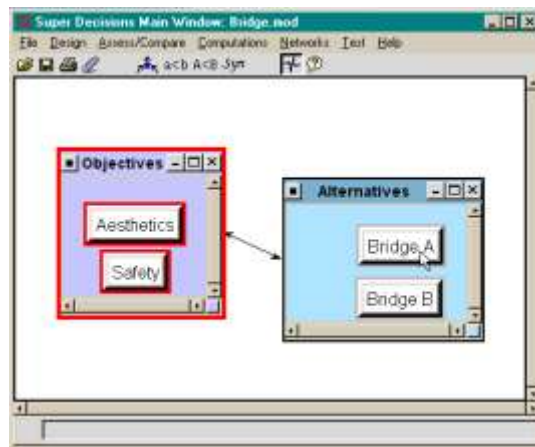


**Figure 11.**  The Aesthetics Node is connected to Bridge A and Bridge B.

## FEEDBACK LINKS

It is easy for those who have used the Analytic Hierarchy Process to understand how to pairwise compare Bridge A and Bridge B with respect to Aesthetics.  Bridge A would be highly preferred. But what may be new is the idea that criteria may be compared with respect to an alternative.

What does that mean? When comparing, for example, Aesthetics and Safety with respect to Bridge A, the question is: What is more a more pronounced or prevalent characteristic of Bridge A, its aesthetics or its safety? Bridge A is beautiful and that is what we like best about it. Its safety, though quite adequate, is nothing notable. So we strongly prefer its aesthetics to its safety.

For Bridge B what is its more preferable characteristic, aesthetics or safety? Since it is quite ugly, the answer is that the Safety of Bridge B is extremely preferable to its Aesthetics. These kinds of preference questions and answers, both directions, help us establish our true priorities for *all* the elements in the problem.

## THE SUPERMATRIX

The comparison process will be covered in the next demonstration. Here we will show the various computations involving the supermatrix. To show the three different supermatrices, select the Computations command from the menu shown in Figure 12.
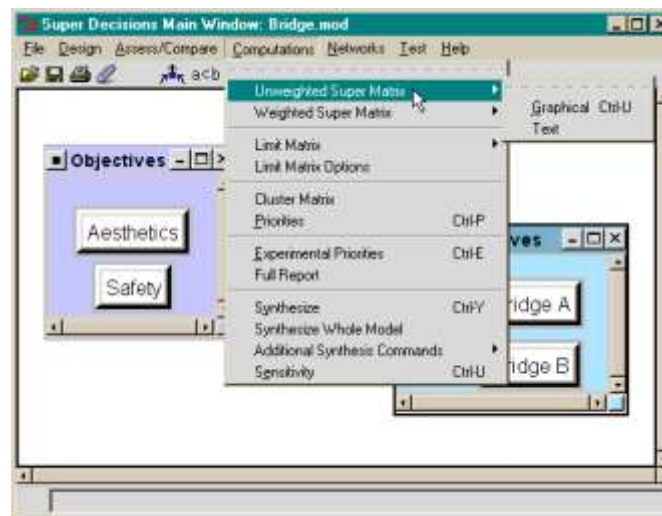


**Figure 12.** The Computations Menu.

## THE UN-WEIGHTED, WEIGHTED AND LIMIT SUPERMATRICES

There are three supermatrices associated with each network: the Unweighted Supermatrix, the Weighted Supermatrix and the Limit Supermatrix. Supermatrices are arranged with the clusters in alphabetical order across the top and down the left side, and with the elements within each cluster in alphabetical order across the top and down the left side. To change the ordering in a supermatrix, you need only re-name the clusters and/or the elements, so their alphabetical order will be the order you want. Changing names after building a model and making comparisons is permitted and will correctly preserve any judgments that have been made.

The unweighted supermatrix contains the local priorities derived from the pairwise comparisons throughout the network as shown in Figure 13. For example, the priorities of the elements Aesthetics and Safety, with respect to Bridge A are shown in the two bottom cells of the first column, 0.857143 and 0.142857. This may be interpreted with the statement, "The Aesthetics

characteristic of Bridge A is between strongly and very strongly, or 6 times, more its dominant preferred characteristic than its Safety aspect." All the local priority information can be read directly from the unweighted Supermatrix.

| Cluster Node Labels | | Alternatives | | Objectives | |
|---|---|---|---|---|---|
| | | Bridge A | Bridge B | Aesthetics | Safety |
| Alternatives | Bridge A | 0.000000 | 0.000000 | 0.875000 | 0.333333 |
| | Bridge B | 0.000000 | 0.000000 | 0.125000 | 0.666667 |
| Objectives | Aesthetics | 0.857143 | 0.100000 | 0.000000 | 0.000000 |
| | Safety | 0.142857 | 0.900000 | 0.000000 | 0.000000 |

**Figure 13.** The Unweighted Supermatrix for the Bridge Model.

Definition of Component: A component in a supermatrix is the block defined by a cluster name at the left and a cluster name at the top. For example, the (Alternatives, Alternatives) component in Figure 13 is composed of the block of four zeros in the upper left-hand corner shown in the screen clip below.

**Detail of (Alternatives, Alternatives) Component from Figure 13**

| Cluster Node Labels | | Alternatives | |
|---|---|---|---|
| | | Bridge A | Bridge B |
| Alternatives | Bridge A | 0.000000 | 0.000000 |
| | Bridge B | 0.000000 | 0.000000 |
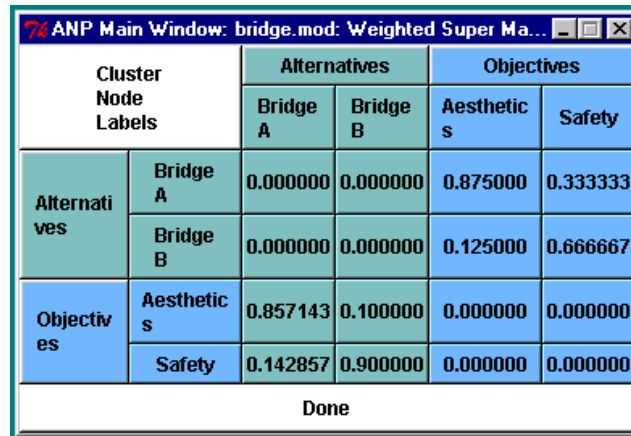
The (Alternatives, Objectives) component is the block of four numbers in the left hand bottom corner of Figure 13 as shown in the screen clip below:

**Detail of (Objectives, Alternatives) Component from Figure 13**

| Objectives | Aesthetics | 0.857143 | 0.100000 |
|---|---|---|---|
| | Safety | 0.142857 | 0.900000 |

The weighted supermatrix is obtained by multiplying all the elements in a component of the unweighted supermatrix by the corresponding cluster weight. We will say more about cluster weights when we demonstrate the *Hamburger* model later. In this example there were no cluster comparisons, since there are only two clusters and cluster comparisons cannot be made when there are only two. The weighted and unweighted supermatrices are the same in this
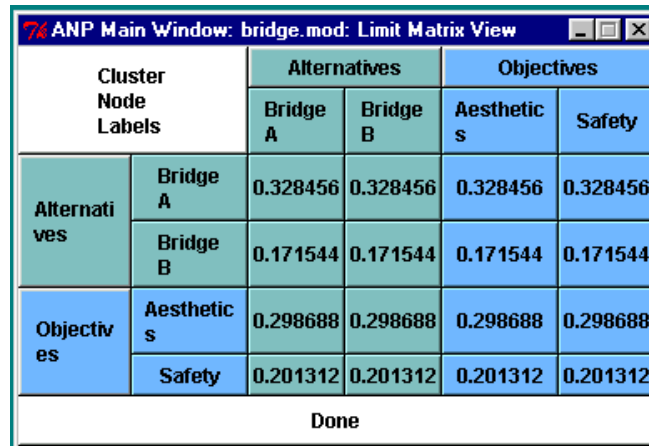
example and are shown in Figure 14. Notice that as the columns already sum to one in the unweighted supermatrix there is no need to weight the components to make the columns sum to one in the weighted supermatrix.

| Cluster Node Labels | | Alternatives | | Objectives | |
|---|---|---|---|---|---|
| | | Bridge A | Bridge B | Aesthetics | Safety |
| Alternatives | Bridge A | 0.000000 | 0.000000 | 0.875000 | 0.333333 |
| | Bridge B | 0.000000 | 0.000000 | 0.125000 | 0.666667 |
| Objectives | Aesthetics | 0.857143 | 0.100000 | 0.000000 | 0.000000 |
| | Safety | 0.142857 | 0.900000 | 0.000000 | 0.000000 |

**Figure 14.** The Weighted (same as Unweighted) Supermatrix for the Bridge Model.

The limit supermatrix is obtained by raising the weighted supermatrix to powers by multiplying it times itself. When the column of numbers is the same for every column, the limit matrix has been reached and the matrix multiplication process is halted. The limit supermatrix for the *Bridge* Model is shown in Figure 15.

| Cluster Node Labels | | Alternatives | | Objectives | |
|---|---|---|---|---|---|
| | | Bridge A | Bridge B | Aesthetics | Safety |
| Alternatives | Bridge A | 0.328456 | 0.328456 | 0.328456 | 0.328456 |
| | Bridge B | 0.171544 | 0.171544 | 0.171544 | 0.171544 |
| Objectives | Aesthetics | 0.298688 | 0.298688 | 0.298688 | 0.298688 |
| | Safety | 0.201312 | 0.201312 | 0.201312 | 0.201312 |

**Figure 15.** The Limit Supermatrix for the Bridge Model.

The Computations Priorities command on the menu displays the priorities in two ways: as they appear in the supermatrix, and with the priorities normalized by cluster as shown in Figure 16. The columns of the limit supermatrix are all the same, so the priorities for all the nodes can be read from any column.
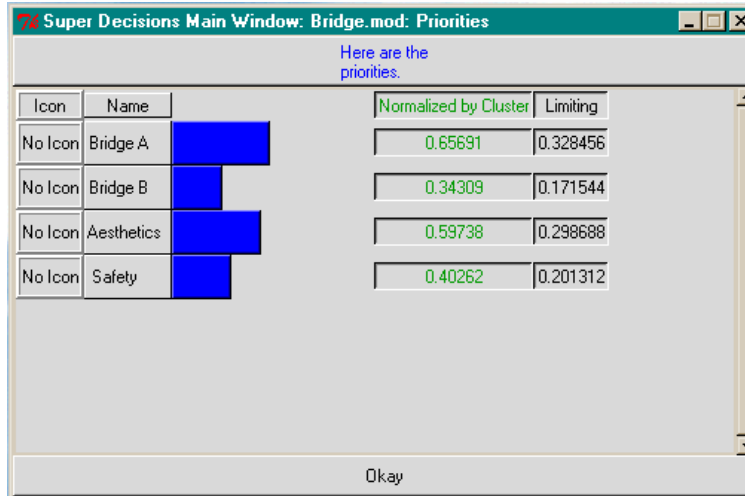
**Figure 16.** The Priorities from the Limit Supermatrix

The Computations Synthesize command displays the final results in three ways as shown in Figure 17. The *Raw* column gives the priorities from the limiting supermatrix (which also appear in the *Limiting* column above), the *Normals* column shows the results normalized for each component (which also appear in the *Normalized by Cluster* column above) and the *Ideals* column shows the results obtained by dividing the values in either the normalized or limiting columns by the largest value in the column.
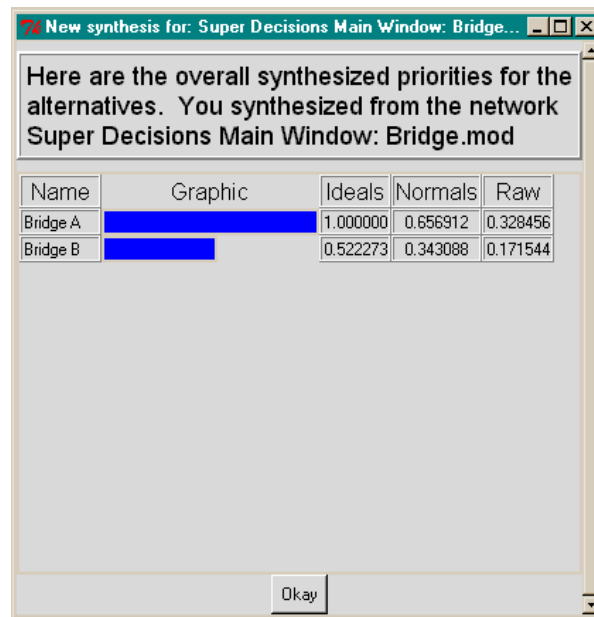


**Figure 17.** The Synthesized Values – the Results for the Alternatives.

The results show that Bridge A is best. This matches our intuition: "Bridge A is very safe and also the best looking, so choose it. Bridge B has overkill so far as safety is concerned, and it is not good looking – so it is not the best bridge overall. Do not choose it."

49

# DEMO OF THE HAMBURGER MODEL - A SIMPLE NETWORK MODEL

The purpose of the *Hamburger* model, a simple network application shown in Figure 18, is to estimate the market share of three fast-food hamburger joints. A simple network has all the clusters and their nodes in a single window. There are no sub-networks. All the comparison questions are asked from the perspective of what is more important or preferred with respect to market share. In this demo we will explain the motivation behind doing cluster comparisons.
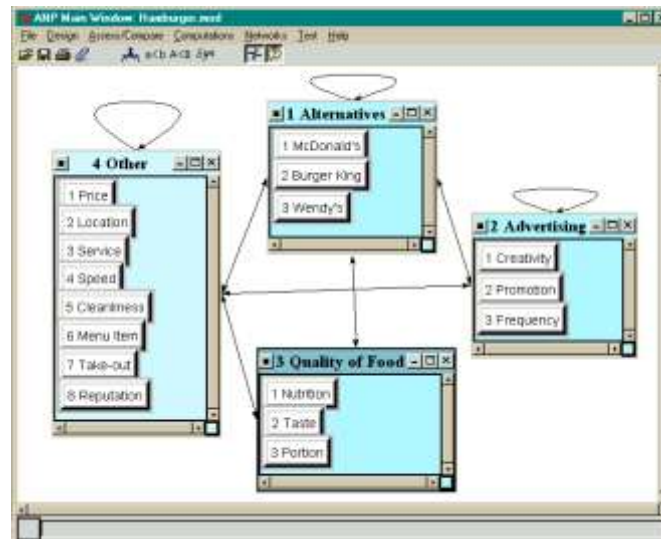


**Figure 18.** The *Hamburger* Model for Predicting Market Share.

In this model the loops indicate inner dependence among the elements in the cluster. In Figure 19 is a view of the model with icons instead of cluster windows. To switch back and forth from a cluster icon to a cluster window double left click with your mouse on it.
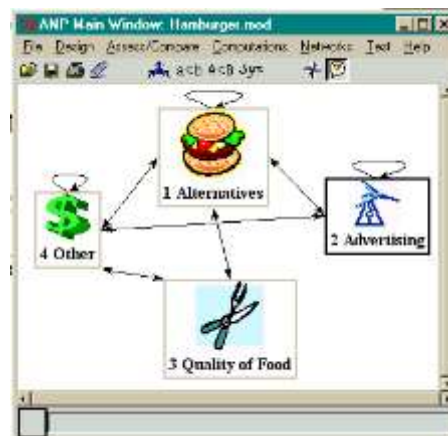


**Figure 19**. An Iconized View of the Hamburger Model.

*Shortcut to Cluster menu: Right click on the background within any cluster to get a drop-down menu of cluster commands as shown in Figure 20 below.*
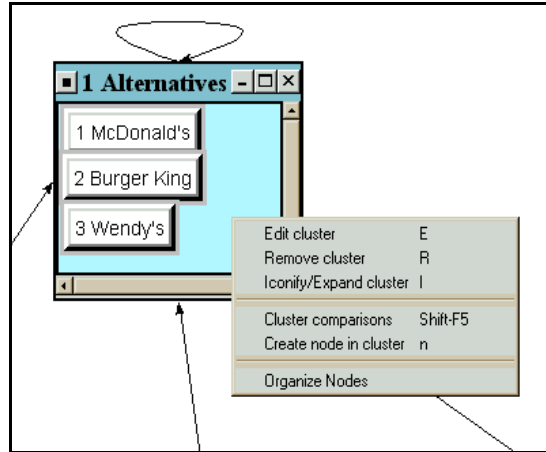
**Figure 20.**  Drop-down Menu of Cluster Commands.

Select the Edit cluster command to change fonts, choose colors or icons, or type definitions.  The Organize Nodes  command can be used to find missing nodes that may have accidentally scrolled off the window.

## COMPARISONS

Pairwise comparisons for the nodes in each cluster that belong to a parent node are carried out for all the parent nodes in the model.  Select the Assess/Compare command, then select the cluster and the node to serve as the parent node.  Select the cluster containing nodes to be compared with respect to the parent node.  You can also start the comparison process by using the drop down node menu.

*Shortcut to Node menu:  Right click on a node, McDonald's, for instance, for the drop down menu with commands relating to that node to appear as shown in Figure 21.*
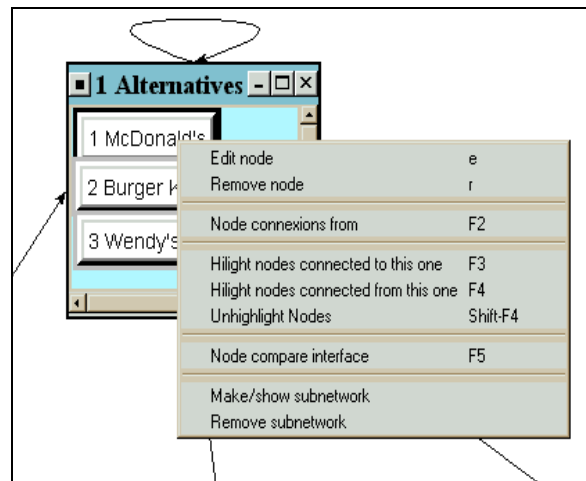


**Figure 21.**  The Dropdown Menu for a Node.

To initiate comparisons with respect to a selected node select the Node Compare Interface command from the drop-down menu, then select the cluster having the nodes you want to compare with respect to McDonald's.  This will bring up the comparisons screen in the

Questionnaire mode as shown in Figure 22. You can select from several ways to do comparisons: Graphic, Verbal, Matrix, and Questionnaire listed on the tabs at the top.



**Figure 22.** The Questionnaire Mode for Comparing Nodes in the "Other" Cluster with respect to McDonald's

To switch from the Questionnaire mode to the Matrix mode shown in Figure 23 click on the Matrix tab.



**Figure 23.** The Matrix Mode for Comparing Nodes in the "Other" Cluster with respect to McDonald's

To show all the judgments in the matrix you must use the scroll bars at the right side and bottom of the window. A judgment is entered in each cell. A cell contains the comparison for the pair listed at the top and at the side. The arrows in the Matrix mode point toward the preferred node of the pair. The top node is preferred when the arrow is red, the side node when the arrow is blue. To toggle a comparison between red and blue, double-click on the arrow button. This inverts the comparison so that the other node is preferred.

*Tip: To invert a judgment, double-click on the arrow button associated with it.*

To compute the local priorities associated with these judgments, select the Computations, Show New Priorities command.   The priorities of the nodes in the Other cluster with respect to McDonald's will be displayed as shown in Figure 24.  Consistency can also be improved from the Computations menu.   Select the Okay bar at the bottom of the window to return to the Comparison mode.  Select File, Save Changes and File, Close to return to the main view.
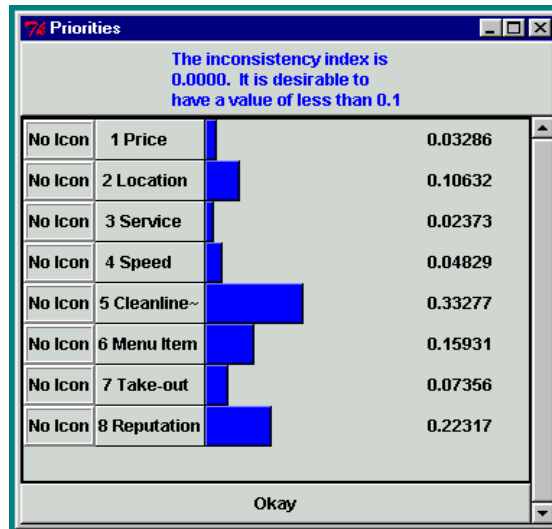


**Figure 24.**  The Local Priorities for Nodes in "Other", Compared with Respect to McDonald's.

The results of all the pairwise comparisons are entered in the unweighted supermatrix.   To display it select Computations, Unweighted Supermatrix.   To print any of the supermatrices select the command File, Export, and type of supermatrix to be exported, for example, select Unweighted  to export to the file *hamburger.unweighted.txt*.  This file can then be imported into Excel to format and print as you like by opening Excel and selecting File, Import and enter the name of the text file that was exported.  The Excel Text Import Wizard will appear and help you through the process.  Select Delimited, then select Tab with the Text Qualifier field set to ", accept the General Data format for columns and finish.  The unweighted supermatrix will then be loaded into Excel and will include row and column headings of both clusters and nodes.

Examples are shown below of such imported tables.  They were obtained by first exporting from the *SuperDecisions* program, then importing to Excel, formatting within Excel, then importing into Word.  The first table gives the cluster weights matrix.  The values in the cluster matrix are used to weight the unweighted supermatrix by multiplying the value in the (Alternatives, Alternatives) cell of the cluster matrix times the value in each cell in the (Alternatives, Alternatives) component of the unweighted supermatrix to produce the weighted supermatrix. Every component is weighted with its corresponding Cluster Matrix weight in this way.

**Table 2.** Cluster Weights Matrix

|  | Alternatives | Advertising | Quality | Other |
|---|---|---|---|---|
| 1. Alternatives | 0.2128 | 0.2956 | 0.5000 | 0.1304 |
| 2. Advertising | 0.5319 | 0.2571 | 0.0000 | 0.6079 |
| 3. Quality | 0.0659 | 0.0000 | 0.0000 | 0.0655 |
| 4. Other | 0.1893 | 0.4473 | 0.5000 | 0.1969 |

**Table 3.** Supermatrix of Unweighted Priorities (shown in two parts)

|  |  | 1 Alternatives | | | 2 Advertising | | | 3 Quality of Food | | |
|---|---|---|---|---|---|---|---|---|---|---|
|  |  | 1 McDonald's | 2 Burger King | 3 Wendy's | 1 Creativity | 2 Promotion | 3 Frequency | 1 Nutrition | 2 Taste | 3 Portion |
| 1 Alternati~ | 1 McDonald's | 0.0000 | 0.8333 | 0.7500 | 0.6141 | 0.7174 | 0.7174 | 0.2488 | 0.2899 | 0.5989 |
|  | 2 Burger King | 0.8000 | 0.0000 | 0.2500 | 0.2685 | 0.1942 | 0.1942 | 0.1561 | 0.1040 | 0.1262 |
|  | 3 Wendy's | 0.2000 | 0.1667 | 0.0000 | 0.1174 | 0.0884 | 0.0884 | 0.5951 | 0.6061 | 0.2749 |
| 2 Advertisi~ | 1 Creativity | 0.2074 | 0.1783 | 0.2810 | 0.0000 | 0.3333 | 0.5000 | 0.0000 | 0.0000 | 0.0000 |
|  | 2 Promotion | 0.1298 | 0.1120 | 0.0720 | 0.1250 | 0.0000 | 0.5000 | 0.0000 | 0.0000 | 0.0000 |
|  | 3 Frequency | 0.6628 | 0.7096 | 0.6470 | 0.8750 | 0.6667 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 3 Quality o~ | 1 Nutrition | 0.3319 | 0.2810 | 0.6241 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
|  | 2 Taste | 0.1388 | 0.0720 | 0.2823 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
|  | 3 Portion | 0.5293 | 0.6470 | 0.0936 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 4 Other | 1 Price | 0.0329 | 0.2408 | 0.0300 | 0.0000 | 0.8333 | 0.0000 | 0.0000 | 0.0000 | 0.8571 |
|  | 2 Location | 0.1063 | 0.2231 | 0.1417 | 0.7095 | 0.0000 | 0.1958 | 0.0000 | 0.0000 | 0.0000 |
|  | 3 Service | 0.0237 | 0.1418 | 0.0648 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
|  | 4 Speed | 0.0483 | 0.1407 | 0.0641 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
|  | 5 Cleanliness | 0.3328 | 0.1096 | 0.2756 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
|  | 6 Menu Item | 0.1593 | 0.0512 | 0.1571 | 0.1377 | 0.1667 | 0.3108 | 0.0000 | 0.0000 | 0.0000 |
|  | 7 Take-out | 0.0736 | 0.0506 | 0.0589 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.1429 |
|  | 8 Reputation | 0.2232 | 0.0422 | 0.2078 | 0.1528 | 0.0000 | 0.4934 | 0.0000 | 0.0000 | 0.0000 |

|  |  | 4 Other | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
|  |  | 1 Price | 2 Location | 3 Service | 4 Speed | 5 Cleanliness | 6 Menu Item | 7 Take-out | 8 Reputation |
| 1 Alternati~ | 1 McDonald's | 0.6531 | 0.6531 | 0.3319 | 0.5387 | 0.2500 | 0.4934 | 0.4837 | 0.6749 |
|  | 2 Burger Ki~ | 0.2507 | 0.2507 | 0.1388 | 0.3624 | 0.2500 | 0.1958 | 0.3133 | 0.2238 |
|  | 3 Wendy's | 0.0962 | 0.0962 | 0.5293 | 0.0989 | 0.5000 | 0.3108 | 0.2029 | 0.1012 |
| 2 Advertisi~ | 1 Creativity | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0780 | 0.0000 | 0.0819 |
|  | 2 Promotion | 0.8333 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.1711 | 0.0000 | 0.3678 |
|  | 3 Frequency | 0.1667 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.7509 | 0.0000 | 0.5503 |
| 3 Quality o~ | 1 Nutrition | 0.1667 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0756 | 0.0000 | 0.0936 |
|  | 2 Taste | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.6952 | 0.0000 | 0.6241 |
|  | 3 Portion | 0.8333 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.2292 | 0.0000 | 0.2823 |
| 4 Other | 1 Price | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.1153 | 0.0000 | 0.0627 |
|  | 2 Location | 0.5000 | 0.0000 | 0.0981 | 0.0000 | 0.1711 | 0.0526 | 0.6572 | 0.2653 |
|  | 3 Service | 0.0000 | 0.0000 | 0.0000 | 0.1873 | 0.0780 | 0.0000 | 0.0548 | 0.0444 |
|  | 4 Speed | 0.0000 | 0.0000 | 0.2857 | 0.0000 | 0.7509 | 0.1946 | 0.2880 | 0.0835 |
|  | 5 Cleanline~ | 0.0000 | 0.0000 | 0.5181 | 0.0000 | 0.0000 | 0.6375 | 0.0000 | 0.2378 |
|  | 6 Menu Item | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.1929 |
|  | 7 Take-out | 0.5000 | 0.0000 | 0.0000 | 0.7313 | 0.0000 | 0.0000 | 0.0000 | 0.0567 |
|  | 8 Reputation | 0.0000 | 0.0000 | 0.0981 | 0.0814 | 0.0000 | 0.0000 | 0.0000 | 0.0567 |

**Table 4.** Weighted Supermatrix

| | | 1 Alternatives | | | 2 Advertising | | | 3 Quality of food | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | 1 McDonald's | 2 Burger King | 3 Wendy's | 1 Creativity | 2 Promotion | 3 Frequency | 1 Nutrition | 2 Taste | 3 Portion |
| 1 Alternati~ | 1 McDonald's | 0.0000 | 0.1774 | 0.1596 | 0.1815 | 0.2121 | 0.2121 | 0.2488 | 0.2899 | 0.2995 |
| | 2 Burger Ki~ | 0.1703 | 0.0000 | 0.0532 | 0.0794 | 0.0574 | 0.0574 | 0.1561 | 0.1040 | 0.0631 |
| | 3 Wendy's | 0.0426 | 0.0355 | 0.0000 | 0.0347 | 0.0261 | 0.0261 | 0.5951 | 0.6061 | 0.1375 |
| 2 Advertisi~ | 1 Creativity | 0.1103 | 0.0949 | 0.1495 | 0.0000 | 0.0857 | 0.1286 | 0.0000 | 0.0000 | 0.0000 |
| | 2 Promotion | 0.0690 | 0.0596 | 0.0383 | 0.0321 | 0.0000 | 0.1286 | 0.0000 | 0.0000 | 0.0000 |
| | 3 Frequency | 0.3526 | 0.3775 | 0.3442 | 0.2250 | 0.1714 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 3 Quality o~ | 1 Nutrition | 0.0219 | 0.0185 | 0.0411 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| | 2 Taste | 0.0091 | 0.0047 | 0.0186 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| | 3 Portion | 0.0349 | 0.0427 | 0.0062 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| 4 Other | 1 Price | 0.0062 | 0.0456 | 0.0057 | 0.0000 | 0.3727 | 0.0000 | 0.0000 | 0.0000 | 0.4286 |
| | 2 Location | 0.0201 | 0.0422 | 0.0268 | 0.3173 | 0.0000 | 0.0876 | 0.0000 | 0.0000 | 0.0000 |
| | 3 Service | 0.0045 | 0.0268 | 0.0123 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| | 4 Speed | 0.0091 | 0.0266 | 0.0121 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| | 5 Cleanline~ | 0.0630 | 0.0207 | 0.0522 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
| | 6 Menu Item | 0.0302 | 0.0097 | 0.0297 | 0.0616 | 0.0745 | 0.1390 | 0.0000 | 0.0000 | 0.0000 |
| | 7 Take-out | 0.0139 | 0.0096 | 0.0112 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0714 |
| | 8 Reputation | 0.0422 | 0.0080 | 0.0393 | 0.0684 | 0.0000 | 0.2207 | 0.0000 | 0.0000 | 0.0000 |

| | | 4 Other | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | 1 Price | 2 Location | 3 Service | 4 Speed | 5 Cleanliness | 6 Menu Item | 7 Take-out | 8 Reputation |
| 1 Alternati~ | 1 McDonald's | 0.0852 | 0.6531 | 0.1326 | 0.2151 | 0.0998 | 0.0643 | 0.1932 | 0.0880 |
| | 2 Burger Ki~ | 0.0327 | 0.2507 | 0.0554 | 0.1447 | 0.0998 | 0.0255 | 0.1251 | 0.0292 |
| | 3 Wendy's | 0.0125 | 0.0962 | 0.2114 | 0.0395 | 0.1997 | 0.0405 | 0.0810 | 0.0132 |
| 2 Advertisi~ | 1 Creativity | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0474 | 0.0000 | 0.0498 |
| | 2 Promotion | 0.5066 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.1040 | 0.0000 | 0.2236 |
| | 3 Frequency | 0.1013 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.4565 | 0.0000 | 0.3345 |
| 3 Quality o~ | 1 Nutrition | 0.0109 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0050 | 0.0000 | 0.0061 |
| | 2 Taste | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0456 | 0.0000 | 0.0409 |
| | 3 Portion | 0.0546 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0150 | 0.0000 | 0.0185 |
| 4 Other | 1 Price | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0226 | 0.0000 | 0.0123 |
| | 2 Location | 0.0981 | 0.0000 | 0.0589 | 0.0000 | 0.1028 | 0.0103 | 0.3947 | 0.0520 |
| | 3 Service | 0.0000 | 0.0000 | 0.0000 | 0.1125 | 0.0469 | 0.0000 | 0.0329 | 0.0087 |
| | 4 Speed | 0.0000 | 0.0000 | 0.1716 | 0.0000 | 0.4510 | 0.0382 | 0.1730 | 0.0164 |
| | 5 Cleanline~ | 0.0000 | 0.0000 | 0.3112 | 0.0000 | 0.0000 | 0.1250 | 0.0000 | 0.0466 |
| | 6 Menu Item | 0.0000 | 0.1000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0378 |
| | 7 Take-out | 0.0981 | 0.0000 | 0.0000 | 0.4393 | 0.0000 | 0.0000 | 0.0000 | 0.0111 |
| | 8 Reputation | 0.0000 | 0.0000 | 0.0589 | 0.0489 | 0.0000 | 0.0000 | 0.0000 | 0.0111 |

55

**Table 5**. Limiting Supermatrix

| | | 1 Alternatives | | | 2 Advertising | | | 3 Quality of food | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | 1 McDonald's | 2 Burger King | 3 Wendy's | 1 Creativity | 2 Promotion | 3 Frequency | 1 Nutrition | 2 Taste | 3 Portion |
| 1 Alternati~ | 1 McDonald's | 0.1749 | 0.1749 | 0.1749 | 0.1749 | 0.1749 | 0.1749 | 0.1749 | 0.1749 | 0.1749 |
| | 2 Burger Ki~ | 0.0883 | 0.0883 | 0.0883 | 0.0883 | 0.0883 | 0.0883 | 0.0883 | 0.0883 | 0.0883 |
| | 3 Wendy's | 0.0520 | 0.0520 | 0.0520 | 0.0520 | 0.0520 | 0.0520 | 0.0520 | 0.0520 | 0.0520 |
| 2 Advertisi~ | 1 Creativity | 0.0727 | 0.0727 | 0.0727 | 0.0727 | 0.0727 | 0.0727 | 0.0727 | 0.0727 | 0.0727 |
| | 2 Promotion | 0.0878 | 0.0878 | 0.0878 | 0.0878 | 0.0878 | 0.0878 | 0.0878 | 0.0878 | 0.0878 |
| | 3 Frequency | 0.1905 | 0.1905 | 0.1905 | 0.1905 | 0.1905 | 0.1905 | 0.1905 | 0.1905 | 0.1905 |
| 3 Quality o~ | 1 Nutrition | 0.0087 | 0.0087 | 0.0087 | 0.0087 | 0.0087 | 0.0087 | 0.0087 | 0.0087 | 0.0087 |
| | 2 Taste | 0.0076 | 0.0076 | 0.0076 | 0.0076 | 0.0076 | 0.0076 | 0.0076 | 0.0076 | 0.0076 |
| | 3 Portion | 0.0145 | 0.0145 | 0.0145 | 0.0145 | 0.0145 | 0.0145 | 0.0145 | 0.0145 | 0.0145 |
| 4 Other | 1 Price | 0.0462 | 0.0462 | 0.0462 | 0.0462 | 0.0462 | 0.0462 | 0.0462 | 0.0462 | 0.0462 |
| | 2 Location | 0.0681 | 0.0681 | 0.0681 | 0.0681 | 0.0681 | 0.0681 | 0.0681 | 0.0681 | 0.0681 |
| | 3 Service | 0.0091 | 0.0091 | 0.0091 | 0.0091 | 0.0091 | 0.0091 | 0.0091 | 0.0091 | 0.0091 |
| | 4 Speed | 0.0248 | 0.0248 | 0.0248 | 0.0248 | 0.0248 | 0.0248 | 0.0248 | 0.0248 | 0.0248 |
| | 5 Cleanline~ | 0.0271 | 0.0271 | 0.0271 | 0.0271 | 0.0271 | 0.0271 | 0.0271 | 0.0271 | 0.0271 |
| | 6 Menu Item | 0.0474 | 0.0474 | 0.0474 | 0.0474 | 0.0474 | 0.0474 | 0.0474 | 0.0474 | 0.0474 |
| | 7 Take-out | 0.0210 | 0.0210 | 0.0210 | 0.0210 | 0.0210 | 0.0210 | 0.0210 | 0.0210 | 0.0210 |
| | 8 Reputation | 0.0596 | 0.0596 | 0.0596 | 0.0596 | 0.0596 | 0.0596 | 0.0596 | 0.0596 | 0.0596 |

| | | 4 Other | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | 1 Price | 2 Location | 3 Service | 4 Speed | 5 Cleanliness | 6 Menu Item | 7 Take-out | 8 Reputation |
| 1 Alternati~ | 1 McDonald's | 0.1749 | 0.1749 | 0.1749 | 0.1749 | 0.1749 | 0.1749 | 0.1749 | 0.1749 |
| | 2 Burger Ki~ | 0.0883 | 0.0883 | 0.0883 | 0.0883 | 0.0883 | 0.0883 | 0.0883 | 0.0883 |
| | 3 Wendy's | 0.0520 | 0.0520 | 0.0520 | 0.0520 | 0.0520 | 0.0520 | 0.0520 | 0.0520 |
| 2 Advertisi~ | 1 Creativity | 0.0727 | 0.0727 | 0.0727 | 0.0727 | 0.0727 | 0.0727 | 0.0727 | 0.0727 |
| | 2 Promotion | 0.0878 | 0.0878 | 0.0878 | 0.0878 | 0.0878 | 0.0878 | 0.0878 | 0.0878 |
| | 3 Frequency | 0.1905 | 0.1905 | 0.1905 | 0.1905 | 0.1905 | 0.1905 | 0.1905 | 0.1905 |
| 3 Quality o~ | 1 Nutrition | 0.0087 | 0.0087 | 0.0087 | 0.0087 | 0.0087 | 0.0087 | 0.0087 | 0.0087 |
| | 2 Taste | 0.0076 | 0.0076 | 0.0076 | 0.0076 | 0.0076 | 0.0076 | 0.0076 | 0.0076 |
| | 3 Portion | 0.0145 | 0.0145 | 0.0145 | 0.0145 | 0.0145 | 0.0145 | 0.0145 | 0.0145 |
| 4 Other | 1 Price | 0.0462 | 0.0462 | 0.0462 | 0.0462 | 0.0462 | 0.0462 | 0.0462 | 0.0462 |
| | 2 Location | 0.0681 | 0.0681 | 0.0681 | 0.0681 | 0.0681 | 0.0681 | 0.0681 | 0.0681 |
| | 3 Service | 0.0091 | 0.0091 | 0.0091 | 0.0091 | 0.0091 | 0.0091 | 0.0091 | 0.0091 |
| | 4 Speed | 0.0248 | 0.0248 | 0.0248 | 0.0248 | 0.0248 | 0.0248 | 0.0248 | 0.0248 |
| | 5 Cleanline~ | 0.0271 | 0.0271 | 0.0271 | 0.0271 | 0.0271 | 0.0271 | 0.0271 | 0.0271 |
| | 6 Menu Item | 0.0474 | 0.0474 | 0.0474 | 0.0474 | 0.0474 | 0.0474 | 0.0474 | 0.0474 |
| | 7 Take-out | 0.0210 | 0.0210 | 0.0210 | 0.0210 | 0.0210 | 0.0210 | 0.0210 | 0.0210 |
| | 8 Reputation | 0.0596 | 0.0596 | 0.0596 | 0.0596 | 0.0596 | 0.0596 | 0.0596 | 0.0596 |

**Table 6.** Limiting Priorities and Normalized by Cluster Priorities

|  |  | Priorities from Limiting Matrix | Priorities Normalized by Cluster |
|---|---|---|---|
| 1 Alternatives | 1 McDonald's | 0.1749 | 0.5549 |
|  | 2 Burger King | 0.0883 | 0.2801 |
|  | 3 Wendy's | 0.0520 | 0.1650 |
| 2 Advertising | 1 Creativity | 0.0727 | 0.2071 |
|  | 2 Promotion | 0.0878 | 0.2501 |
|  | 3 Frequency | 0.1905 | 0.5427 |
| 3 Quality of food | 1 Nutrition | 0.0087 | 0.2825 |
|  | 2 Taste | 0.0076 | 0.2468 |
|  | 3 Portion | 0.0145 | 0.4708 |
| 4 Other | 1 Price | 0.0462 | 0.1523 |
|  | 2 Location | 0.0681 | 0.2245 |
|  | 3 Service | 0.0091 | 0.0300 |
|  | 4 Speed | 0.0248 | 0.0818 |
|  | 5 Cleanliness | 0.0271 | 0.0894 |
|  | 6 Menu Item | 0.0474 | 0.1563 |
|  | 7 Take-out | 0.0210 | 0.0692 |
|  | 8 Reputation | 0.0596 | 0.1965 |

## MAKING CLUSTER COMPARISONS

To compare clusters take each cluster in turn (as the parent) and pairwise compare all the clusters it connects to for importance with respect to their influence on it. This is how the Cluster Matrix is generated. Keep in mind that the overall goal here is Market Share. For example, select Assess, Compare, Cluster comparisons and choose the Alternatives cluster. The comparison process now is used to pairwise compare the clusters for influence to which the parent cluster connects. Here is an example of how you formulate a cluster comparison question: "Which influences the market share of the Alternatives more, Advertising or Quality of Food?" The judgment is made that advertising is between very strongly and extremely more important than the quality of food in influencing market share, and a value of 8.1 is entered.

### MAKING INNER DEPENDENT CLUSTER COMPARISONS

As the Alternatives cluster is inner dependent, it is connected to itself, so it is one of the four clusters being pairwise compared with respect to Alternatives. The only time you have to ask such a question, comparing a cluster against another cluster with respect to itself as the parent cluster, is when it exhibits inner dependence. Since nodes in the Alternatives cluster are connected to other nodes in that cluster, it must influence itself. If it makes sense to ask the question: "Does Wendy's or Burger King influence McDonald's market share more?" so it needs to be compared for how important its influence is on itself compared to the other clusters it links to. The Wendy's versus Burger King question makes sense. We are really asking which one is a

stronger competitor of McDonald's. A judgment of 4 was entered for Burger King over Wendy's; that is, Burger King is a stronger competitor, and that seems reasonable as Burger King has the second largest market share of the three alternatives.

## WHY MAKE CLUSTER COMPARISONS?

If all the clusters are equally important it is not necessary to make cluster comparisons, and the cluster weights are set to 1/n in the cluster matrix. The value of n is equal to the number of non-zero components beneath each component across the top of the unweighted supermatrix.

However, the clusters in a network may not be equally important. Then they need to be compared to establish the weights in the cluster matrix. Weighting all the elements in each unweighted supermatrix component by the corresponding cluster matrix cell, whether set by the default value of 1/n described above, or by comparing the clusters and using the derived values, causes the matrix to be column stochastic, that is, each column sums to one. In Table 2 for the unweighted supermatrix the first column sums to 4.000. In Table 3 for the weighted supermatrix it sums to 1.000.

It is essential in real life problems that one know the importance of the groups or clusters to which the elements belong because the final priorities do (and should) depend on that. For example, a society of astronomers is not as important to immediate human survival as the society of farmers, although on the face of it an astronomer may seem more important than a farmer because there are a much smaller number of them (and some people say the greatest potential disaster is a comet crashing into the earth – so astronomers might help avoid that).

Select the Computations, Cluster Matrix command to display the Cluster matrix for the *Hamburger* model shown in Figure 25.



| Cluster Node Labels | 1 Alternatives | 2 Advertising | 3 Quality of Food | 4 Other |
|---|---|---|---|---|
| 1 Alternatives | 0.212821 | 0.295623 | 0.500000 | 0.130395 |
| 2 Advertising | 0.531948 | 0.257117 | 0.000000 | 0.607943 |
| 3 Quality of Food | 0.065917 | 0.000000 | 0.000000 | 0.065549 |
| 4 Other | 0.189315 | 0.447260 | 0.500000 | 0.196114 |

**Figure 25.** The Cluster Matrix

## THE FINAL RESULTS

The final results for the Hamburger model are obtained by selecting the command Computations, Synthesize. The results are shown in Table 5: McDonald's has 55.49% of the market share,

Burger King has 28.01% and Wendy's has 16.50%. At the time the model was done the actual values published in the *Market Share Reporter* of 1994 were: 58.23%, 28.57% and 13.20%.

# DEMONSTRATION OF A TWO-LAYER SYSTEM, THE CAR PURCHASE BCR MODEL

This model will be used to show a 2-level model with a top-level control network and three sub-networks. It is a model to pick the best type of car: European, Japanese or American. The top-level control network is actually a hierarchy with three control criteria: Benefits, Costs and Risks, and there is a sub-network associated with each. Every sub-network must contain a cluster with the alternatives in it, and these sub-networks do.
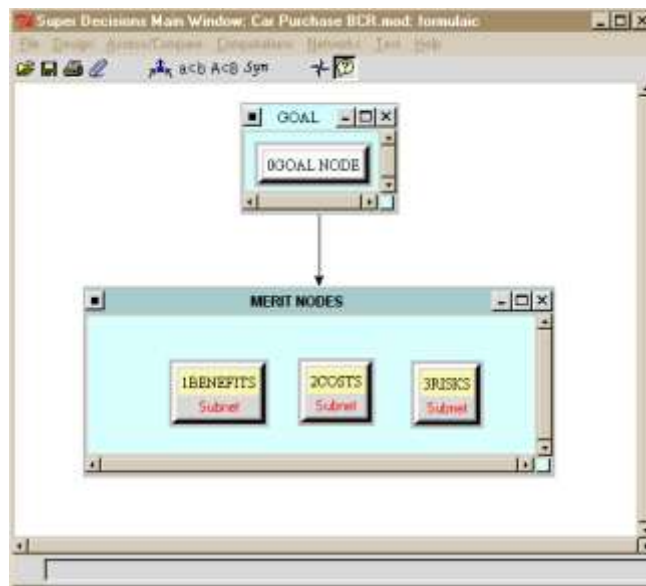


**Figure 26.** The Top Level Network for the *Car Purchase BCR* Model.

### MAKE/SHOW SUBNETS

To make or show a sub-network for a node, right click on the background of the node and select the command Make/show sub-network as shown in the screen clip below. If there is an existing sub-network, it will open. If not, a blank window will appear. When a node has an attached sub-network, the word Subnet is tacked onto the node name. After creating a sub-network, you can open it by left double-clicking on the node name of the node it is attached to. You *must* click on the node name and *not* the red area with the word Subnet on it to bring up the sub-network.

### NODE MENU

To have a drop-down node menu appear, right-click on any node, for example the Risks node, as shown in Figure 27 below. Commands invoked from this menu will apply to the Risk node.
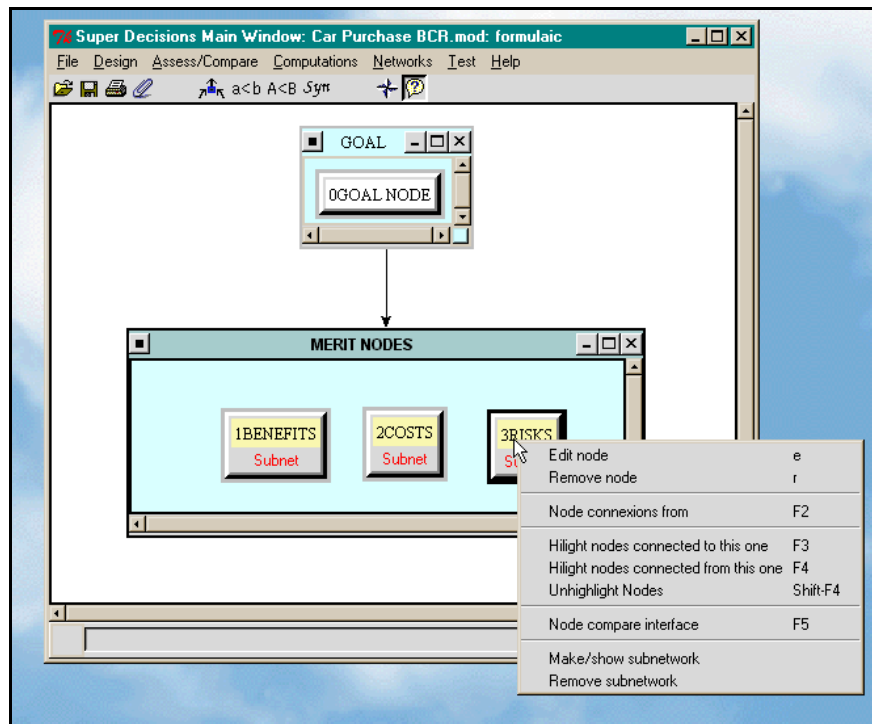
**Figure 27.** Node Menu Commands

The sub-networks are contained in separate Windows as shown in the view of the entire model in Figure 28 below. To open the subnet attached to the Benefits node, for example, click on the word Benefits (*not* on the red word Subnet beneath it!) Note that the subnets here are decision subnets as each has an Alternatives cluster in it. The command structure in a subnet window is the same as in the top-level network window.
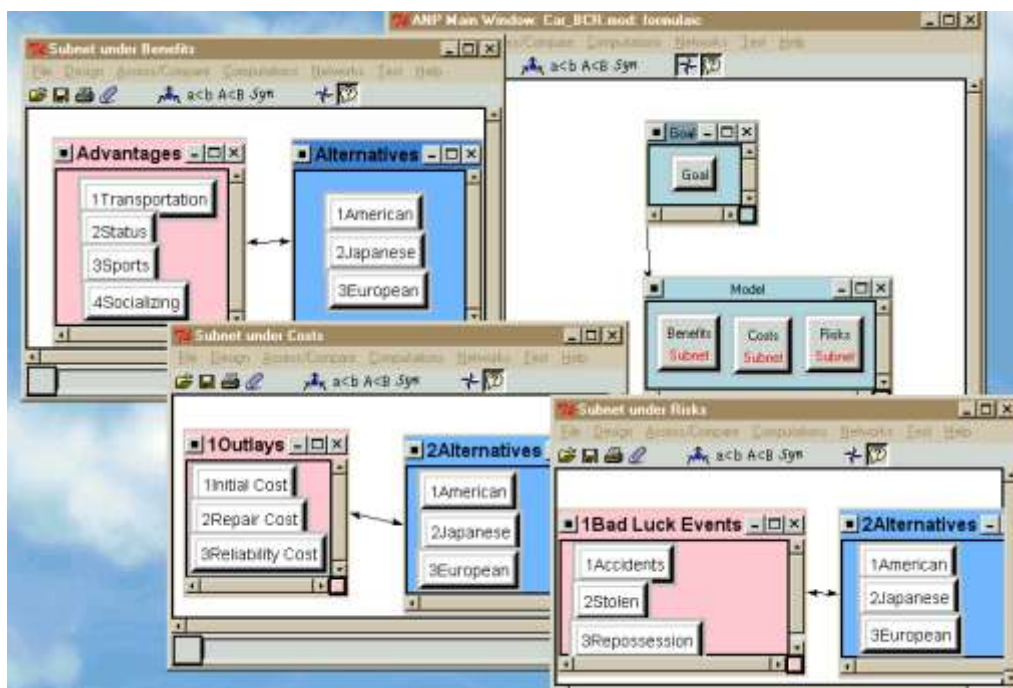
**Figure 28.** *Car Purchase BCR* Model View of All Networks.

*Tip: In handling the subnet windows if one disappears it may have become minimized and be down on the taskbar at the bottom of the desktop screen. Left-click on it to restore it.*

*Tip on Saving: You can save a complete complex model by selecting the File, Save command from either the top-level network or from a subnet. You can also opt to save only a subnet. You can save subnets and use them as templates later by opening in the blank window for a new subnet.*

### SYNTHESIZING TO SHOW PRIORITIES IN A SUB-NETWORK

Open the Benefits subnet by double-clicking on the word Benefits. Select the Computations, Synthesize command in the Benefits Subnet to show the results under Benefits. You see that the European car has the greatest benefits as shown in Figure 29.
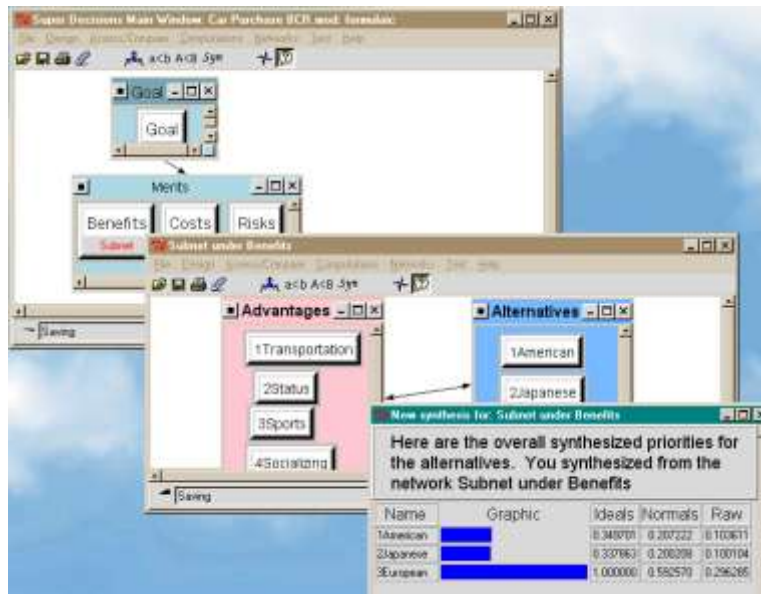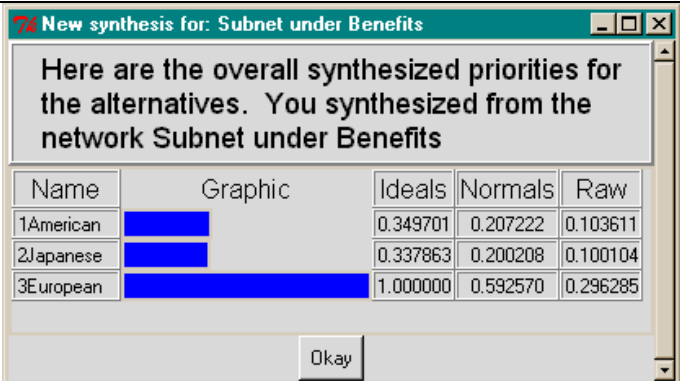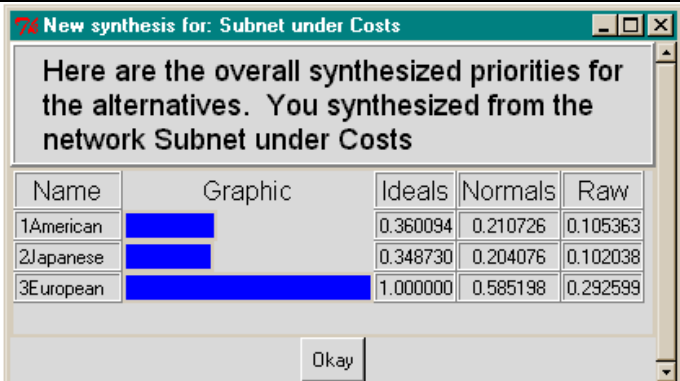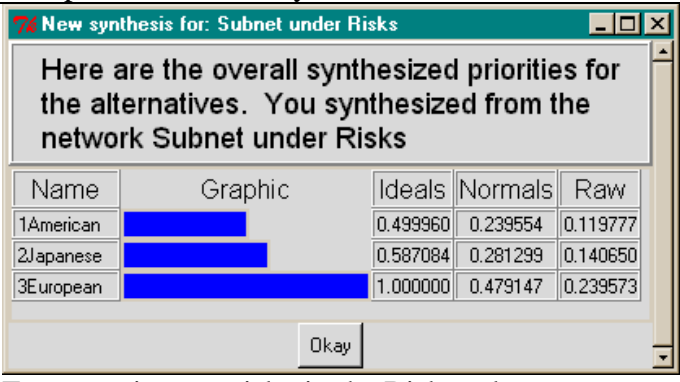


**Figure 29.** The Synthesis Results in the Benefits Subnet.

The synthesized priorities for each of the three subnets are shown in Table 6 below. In the Costs network and the Risks network the comparison questions are phrased this way: "Which is the most costly?" and "Which is the most risky?" Thus the European car has a 1 in the Ideals column for COSTS in Table 7 which means it is the *most* costly to buy. It also has a 1 in the Ideals column for RISKS so it is also the most risky. The sub-network results are shown in Table 7, and the results as combined in the top-level network in Table 8.

**Table 7.** Results in the Subnets of the Car Purchase BCR model.

| BENEFITS | |
|---|---|
| | New synthesis for: Subnet under Benefits<br><br>Here are the overall synthesized priorities for the alternatives. You synthesized from the network Subnet under Benefits<br><br>| Name | Graphic | Ideals | Normals | Raw |<br>| 1American | | 0.349701 | 0.207222 | 0.103611 |<br>| 2Japanese | | 0.337863 | 0.200208 | 0.100104 |<br>| 3European | | 1.000000 | 0.592570 | 0.296285 |<br><br>Okay |
| | European is best in the Benefits subnet |
| COSTS | |
| | New synthesis for: Subnet under Costs<br><br>Here are the overall synthesized priorities for the alternatives. You synthesized from the network Subnet under Costs<br><br>| Name | Graphic | Ideals | Normals | Raw |<br>| 1American | | 0.360094 | 0.210726 | 0.105363 |<br>| 2Japanese | | 0.348730 | 0.204076 | 0.102038 |<br>| 3European | | 1.000000 | 0.585198 | 0.292599 |<br><br>Okay |
| | European is most costly in the Costs subnet. |
| RISKS | |
| | New synthesis for: Subnet under Risks<br><br>Here are the overall synthesized priorities for the alternatives. You synthesized from the network Subnet under Risks<br><br>| Name | Graphic | Ideals | Normals | Raw |<br>| 1American | | 0.499960 | 0.239554 | 0.119777 |<br>| 2Japanese | | 0.587084 | 0.281299 | 0.140650 |<br>| 3European | | 1.000000 | 0.479147 | 0.239573 |<br><br>Okay |
| | European is most risky in the Risks subnet. |

The overall results are obtained by synthesizing in the top-level network using the Computations, Synthesize command as shown in Figure 30. This combines the results from the subnets according to the formula (Benefits/(Costs*Risks) for each alternative.
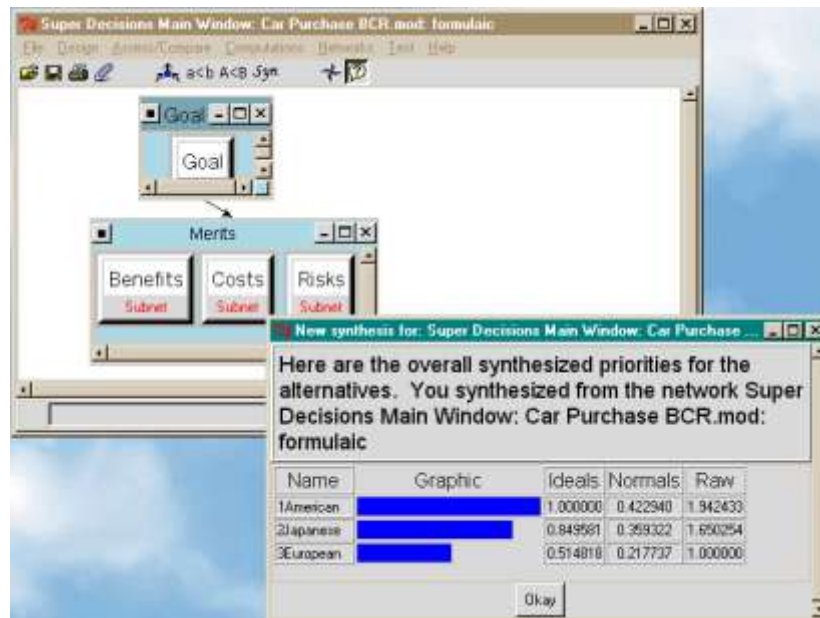
**Figure 30.** The *Car Purchase BCR* Overall Results.

Select the command Design, Add/Edit Formula shown in Figure 31. This opens the Formula dialogue box shown in Figure 32.
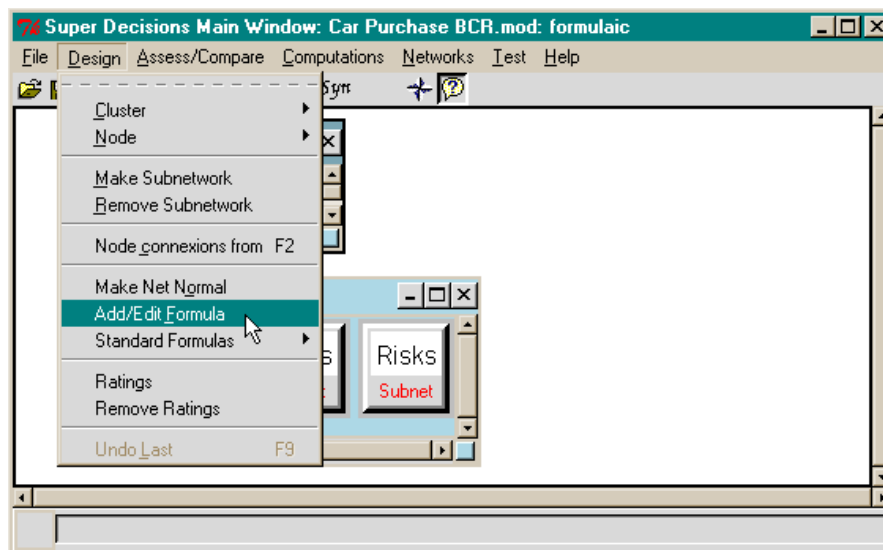


**Figure 31**. The Menu Command that Shows the Formula.
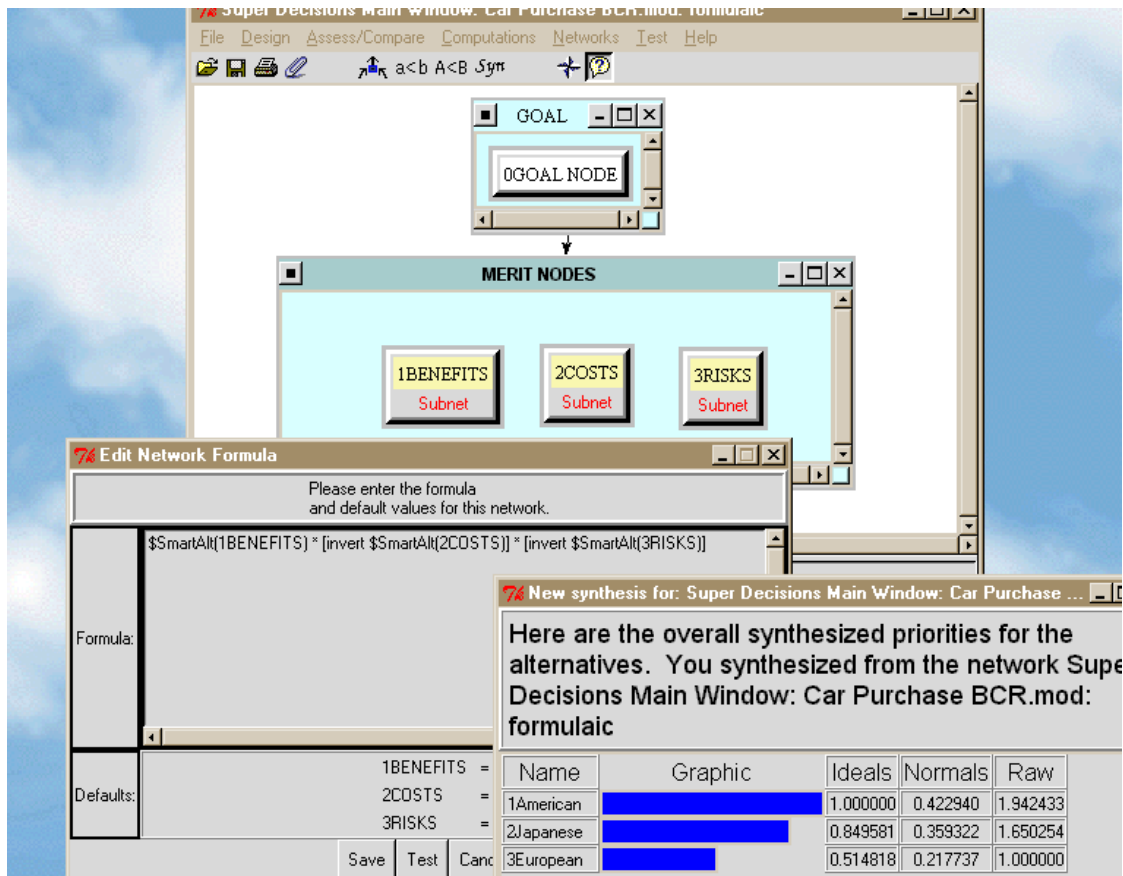
**Figure 32.** The Multiplicative Formula for Combining Results from Subnets in *Car Purchase BCR* Model

In the term in the formula, $SmartAlt(Benefits), the $ prefix indicates that the term is to be calculated for each alternative and SmartAlt(Benefits) means to use the appropriate values from the network beneath the Benefits node. This gives the results shown in the column under BENEFITS in Table 8.

The Multiplicative Formula is used in Figure 32. The Computations Synthesize command displays results for the alternatives in three ways: as Ideal, Normal and Raw numbers. Smart means use the Raw numbers if the network has more networks beneath it; and use the Ideal numbers if the network is either a bottom level network, or a network that has Ratings. So in this case Smart is choosing the Ideal numbers. The *invert* function in the Costs and Risks terms inverts the values; for example, Costs is inverted to 1/Costs. For more discussion of formulas see the section on Formulas at the end of this chapter.

Now change to the additive formula by selecting the command Design, Standard Formulas, Additive. Synthesis now gives the results shown in Figure 33.
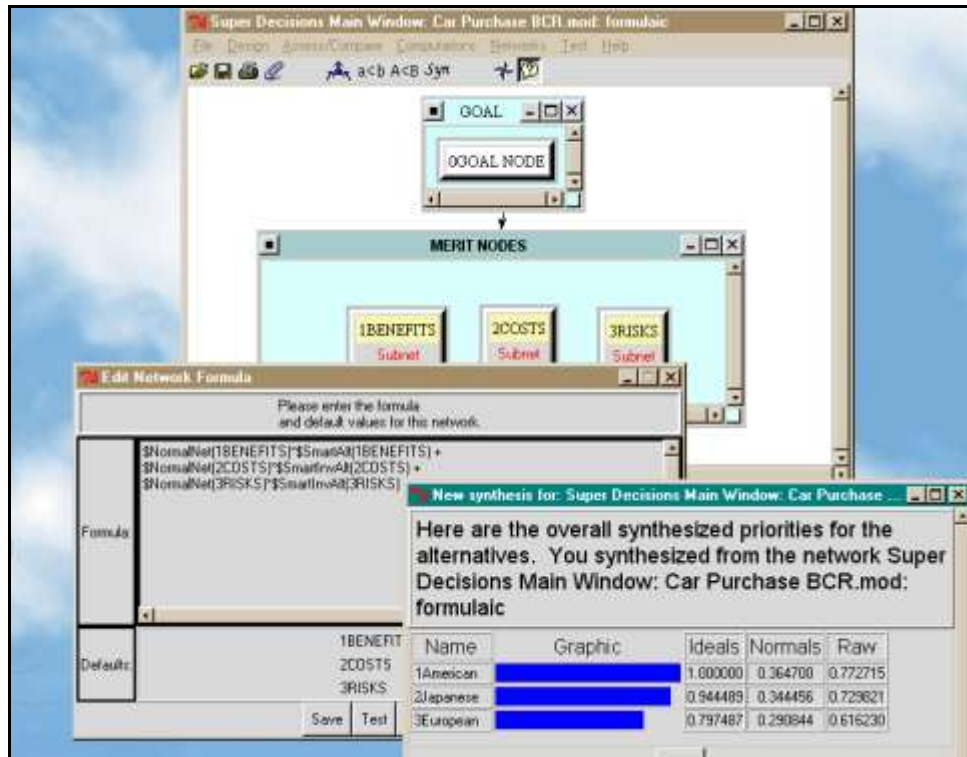
**Figure 33.** Synthesis Using the Additive Formula

The results are shown in two ways in Table 8: using the multiplicative formula and using the additive formula. In the additive formula the priorities in the Costs and Risks subnets have to be flipped from the most costly (risky) alternative having the highest value to the least costly (risky) alternative having the highest value. See the section on Formulas at the end of the chapter for how to do that.

**Table 8.** Combining Priorities in the Top-Level Network in Two Ways

|  | BENEFITS | COSTS | RISKS | B/(C*R) MULTIP. FORMULA | B+1/C+1/R ADDITIVE FORMULA |
|---|---|---|---|---|---|
| American Car | .104 | .105 | .120 | . 423 | .365 |
| Japanese Car | .100 | .102 | .141 | .359 | .344 |
| European Car | .296 | .293 | .240 | .218 | .291 |

## ANALYSIS OF RESULTS FOR THE CAR PURCHASE BCR MODEL

To analyze the outcome for the *Car Purchase BCR* model shown in Table 8, we can observe that the European car has the most benefits by far. But it is also the most costly, both to purchase and repair, and the most risky, because of its greater likelihood of being stolen. It has the lowest overall priority. Netting it all out the American car is best over all, mostly because of its lower risks. The ranks are the same for the cars using either formula for synthesis.

## SENSITIVITY GRAPH

Now we will show how the results would change if the importance of the Benefits node were changed. Make sure the additive formula by taking a look at the formula using the Design Add/Edit Formula command. See the sensitivity graph in Figure 34 with Benefits selected as the Independent Parameter. The sensitivity graph shows that at a Benefits value (on the x-axis) of about .46 the best car changes from American (the top line on the left axis) to Japanese (the second line from the top on the left axis). Sensitivity can be performed only when the additive formula is selected because in the multiplicative formula the weights of B, C and R are the same for each alternative and cancel out. See the Formulas section at the end of the chapter for more about this.
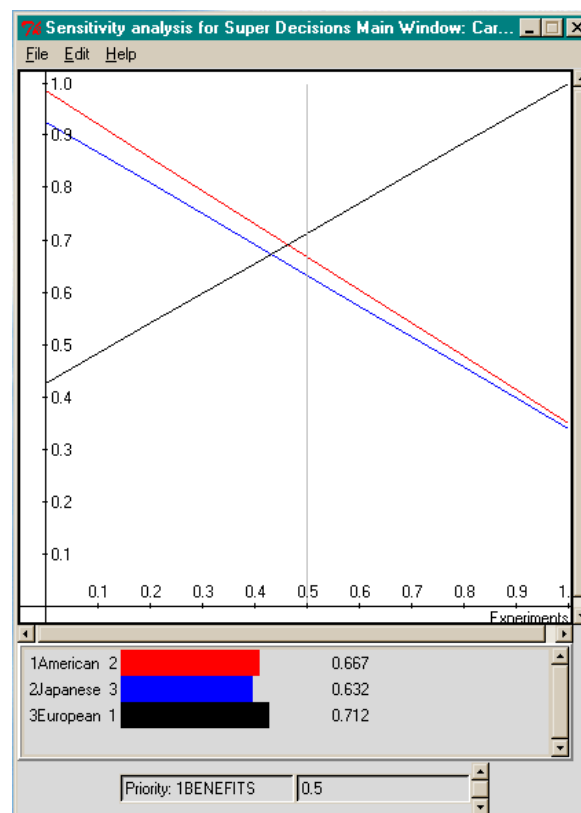


**Figure 34.** *Car Purchase* Sensitivity Graph for Benefits (Additive Formula)

# DEMONSTRATION OF A COMPLEX THREE-LAYER SYSTEM: THE NATIONAL MISSILE DEFENSE MODEL (NMD)

This application is available for examination in the sample models of the *SuperDecisions* software. The file is called *NationalMissileDefense.mod*. This model was presented by Thomas L. Saaty at the 6th International Symposium on the AHP in Bern, Switzerland, 2001. This model is a complete multi-layer structure with Benefits, Opportunities, Costs and Risks (BOCR) merit nodes in the top-level network, control criteria in their attached subnets, and bottom level decision subnets containing the alternatives that are attached to the control criteria. The top-level

model also has an attached Ratings component for evaluating the importance of the BOCR. An overview of such a complete model structure is shown in Figure 35.

The United States government faces the crucial decision of whether or not to commit itself to the deployment of a National Missile Defense (NMD) system. Many experts in politics, the military, and academia have expressed different views regarding this decision. The most important rationale behind supporters of the NMD system is protecting the US from potential threats said to come from countries such as North Korea, Iran and Iraq. According to the Central Intelligence Agency, North Korea's Taepo Dong long-range missile tests were successful, and it has been developing a second generation capable of reaching the US. Iran also tested its medium-range missile Shahab-3 in July 2000. Opponents express doubts about the technical feasibility, high costs (estimated at $60 billion), political damage, possible arms race, and the exacerbation of foreign relations.

The current plan for NMD originated with President Reagan's Strategic Defense Initiative (SDI) in the 1980s. SDI investigated technologies for destroying incoming missiles. The controversies surrounding the project were intensified with the National Missile Defense Act of 1996, introduced by Senator Sam Nunn (D-GA) in June 25, 1996. The bill required Congress to make a decision on whether the US should deploy the NMD system by 2000. The bill also targeted the end of 2003 as the time for the US to be capable of deploying NMD.

The next year the Senate Armed Services Committee approved the National Missile Defense Act of 1997 by winning 10 votes out of 18, along party lines. This Act mandated deployment of an anti-missile system, consisting of 100 ground-based interceptor missiles at a single site, plus ground-based radars and space-based sensors. The intelligence community estimated a shortened warning time for the US against intercontinental ballistic missiles (ICBMs) deployment. However, the deployment of NMD by 2003, analyzed by an independent Commission to Assess the Ballistic Missile Threat to the United States, concluded that it would generate high risks and possible failure. Accordingly, the Administration adjusted its plan to deploy an NMD in 2005.

## SCHEMATIC OF A COMPLEX MODEL

This is the type of model generated when you select the File, New command and Full Model template.

**THE STRUCTURE OF COMPLEX DECIS**

**PERSONAL OR GROUP CRITERIA FOR RATING**
**BOCR NODES** (SUBJECTIVE VALUES)

Satisfaction     Prosperity     Security     Growth

**THE BOCR MERIT CONTROL NODES**
(LINK FROM SUBJECTIVE TO OBJECTIVES)

BENEFITS     OPPORTUNITIES     COSTS     RISK

Several control criteria for each of the four BOCR whose priorities are
a hierarchy or a network.

**FEEDBACK NETWORKS**
(OBJECTIVE VALUES)

Decision networks containing alternatives-one for each BOCR control cr
*1. Economic benefits*     *2. Political benefits*     *3. Social benefits*     *4. Te*

**Figure 35.** The Structure of a Full Model for Complex Decisions

68

## THE PRIORITIZATION OF COMPLEX DECISI

### PERSONAL  OR GROUP CRITERIA FOR RATING
### BOCR NODES (SUBJECTIVE VALUES)

**1**. Identify and  prioritize personal or group criteria and subcriteria appl
you make.

**2**. Establish intensities and prioritize them for each lowest level criterio

**3**. Rate the Benefits, Opportunities, Costs and the Risks one at a time o
then normalize.

### THE BOCR MERIT CONTROL NODES
### (LINK FROM SUBJECTIVE TO OBJECTIVES)

○          ○          ○          ○

BENEFITS     OPPORTUNITIES          COSTS          RISKS

Identify and prioritize the control criteria and subcriteria for each of the fo

### FEEDBACK NETWORKS
### (OBJECTIVE VALUES)

Decision networks containing alternatives-one for each BOCR contro

**1.** For each network corresponding to one of the several control criteri
derive priorities from paired comparison matrices and use them in a
the same for the criteria under the other three BOCR merits.

**2.** Pairwise compare the impact of the components on each component
with respect to the control criterion, and use these priorities to weigh
corresponding blocks of the super matrix. Obtain the limiting supern
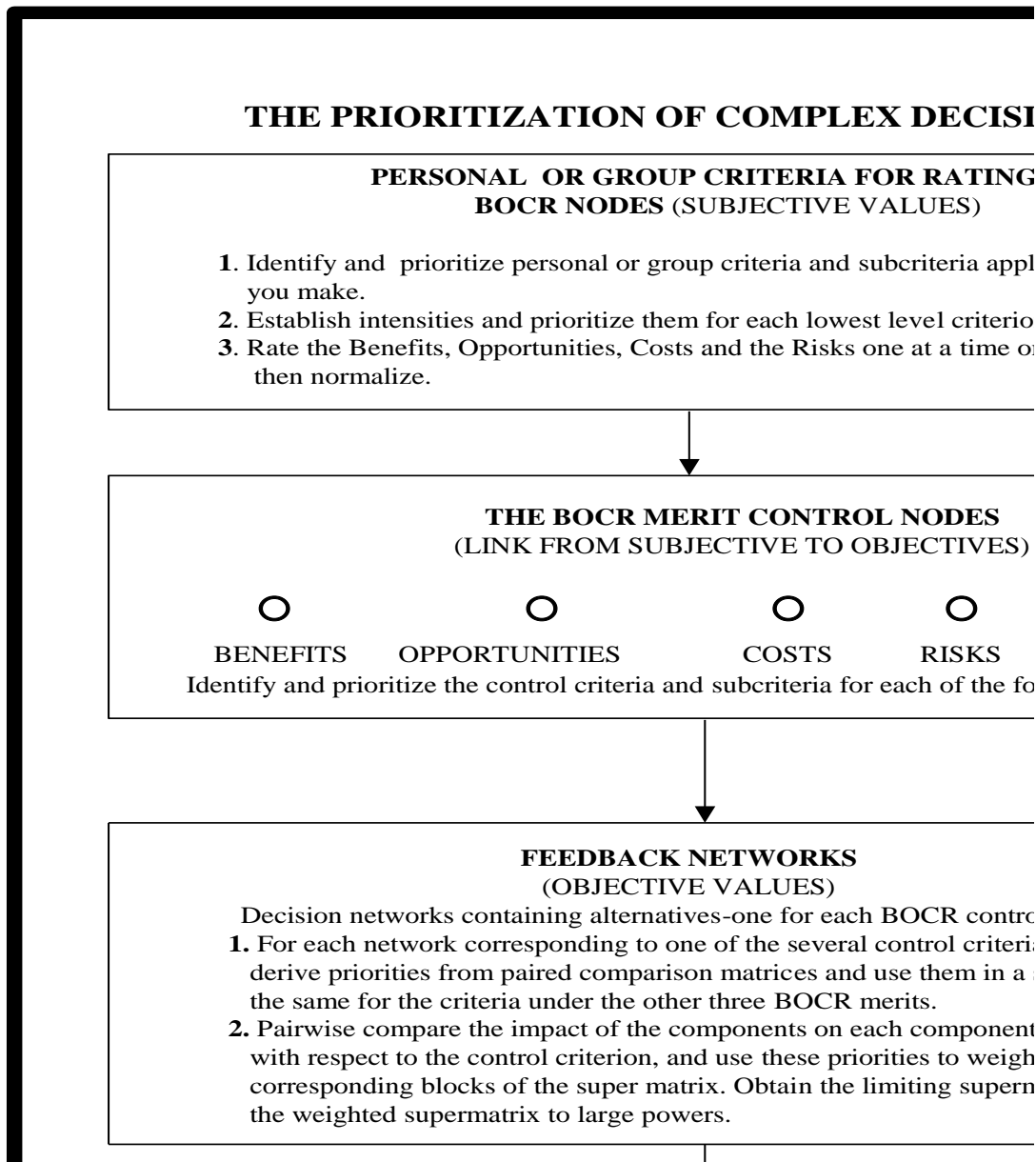the weighted supermatrix to large powers.

**Figure 36.** The Prioritization of Decisions

The deployment of NMD is not solely based on technological development. The next president of the US has to deal with international politics. The Anti-Ballistic Missile (ABM) treaty signed by the US and the former Soviet Union in 1972 would ban NMD, and the next president should be able to persuade or renegotiate the ABM treaty with Russia's president, Vladimir Putin, who strongly opposes the plan. How to deal with the reactions of China and NATO is another issue for the president after Mr. Clinton to consider. This analysis was done in October 2000 when Mr. Clinton was still President. Since then, George W. Bush has become President, and his Secretary of State, Colin Powell, has made statements supporting the NMD deployment.

## OVERVIEW OF THE NMD MODEL

Under the situation in October 2000, what is the best direction for NMD to take? The following alternatives and criteria for evaluating the decision were identified.
1. **Deploy NMD.** Fully deploying the NMD program
2. **Global Defense.** Amending the ABM treaty to be more restrictive to more countries by using any economic, political, and diplomatic means as well as implementing joint-development of a worldwide defense system.
3. **R & D.** This alternative is not concerned with deployment, but proceeds with research and development of missile defense technology.
4. **Termination of the NMD program.** Disregarding any further R&D and deployment plan.

The evaluation criteria are categorized into benefits, costs, opportunities, and risks. A three-layer network model was developed, using the *SuperDecisions* software:
- **Top-level Network** This is a single network that has in it the benefits, opportunities, costs, and risks nodes (the BOCR nodes) and the strategic criteria used to evaluate their importance for this decision. This network has an associated Ratings spreadsheet that is used to evaluate the BOCR under the strategic criteria.
- **Control Criteria Networks** Each of the BOCR has a subnet attached to it containing its control criteria. Usually the structure in these subnets is hierarchical. The most important control criteria are selected to have decision subnets created for them.
- **Decision Networks.** A decision subnet is created for each high value control criterion. The alternatives of the decision appear in a cluster in each decision subnet, but other than that the clusters and nodes may be different.

## THE TOP-LEVEL NETWORK

The assessment criteria used to determine the priorities of the BOCR merits are shown in Figure 37. The decision on the NMD project is reviewed in the context of the three criteria that are used to evaluate the merits. These are World Peace, Human Well-being, and International Politics. The three sub-criteria, Adversary Countries, Security Dilemma, and Terrorism cover all the causes disturbing or stabilizing peace in the world. The first sub-criterion, Adversary Countries concerns the potential threats by adversary countries. The second criterion, Security Dilemma means that increasing one country's security inevitably decreases other countries' security. Terrorism indicates any possibility of the rise or decline of terrorism in the world. Human Well-being includes Technological Advancement and Market Creation. Technological Advancement

driven by the NMD research and development process can ultimately benefit all people, particularly in providing possible space exploration that can lead to the creation of new markets. Moreover, the 21st century is characterized as a post-industrialization era. Service industries in communication and transportation will benefit not only businesses associated with these industries, but also consumers who can enjoy the products from the new market. The last criterion is International Politics. It is composed of two sub-criteria, Military Relations and Diplomatic Relations. Military Relations refer to the impact of NMD on relations with US allies for better or for worse. Also, the impact of NMD on diplomatic relations among all countries should be considered.
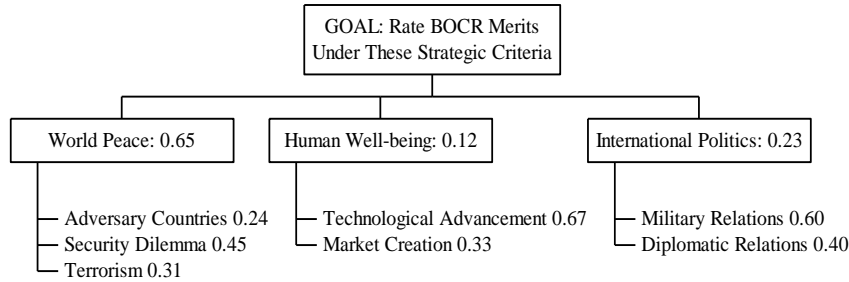


**Figure 37.** Strategic Criteria for Rating Benefits, Opportunities, Costs and Risks (BOCR)

We will now turn to the view in the *SuperDecisions* software. The strategic criteria and the BOCR nodes are in the top-level network shown in Figure 38. The nodes in the Criteria cluster are pairwise compared with respect to the goal in the Strategic Criteria. The file name for this model is *National Missile Defense.mod* and it is included in the sample models. The file name is shown on the title bar of the top-level network window. The title bar also includes the word *formulaic*, indicating that a formula is associated with this network for combining the results from the subnets underneath the BOCR nodes. And, finally, the word *ratings* appears on the title bar to show that a ratings spreadsheet is associated with this network.
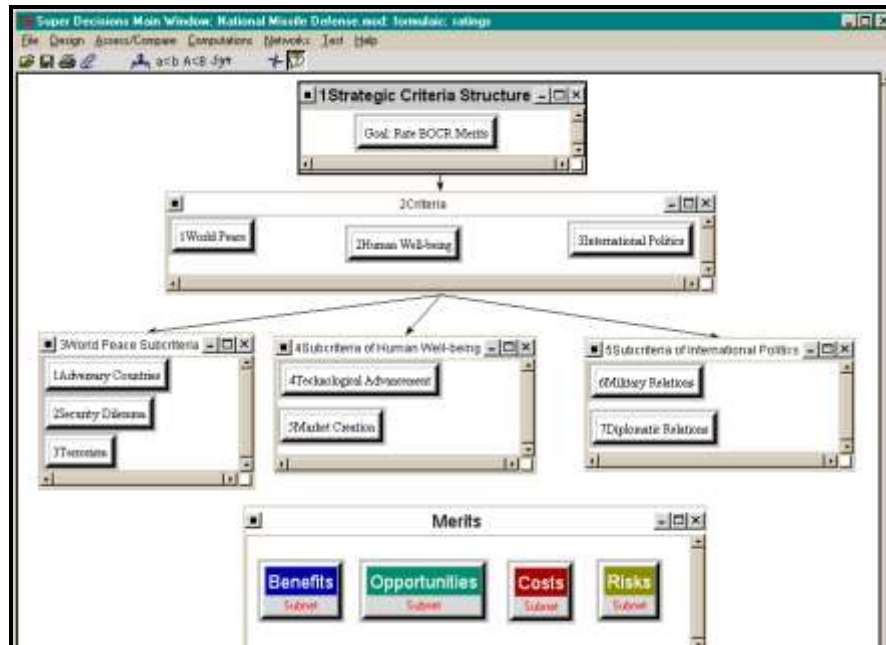
**Figure 38.** Top-level Network of the *National Missile Defense* Model.

The ratings spreadsheet for rating the BOCR is shown in Figure 39 along with the priorities derived for them. To access it use the command Assess/Compare, Ratings in the network to which it is attached. To create it in the first place, use the command Design, Ratings. This will also access it if it was previously created. The values for the subcriteria nodes in Ratings can be read in the main network from the Computations, Limit Matrix in the column headed Goal.
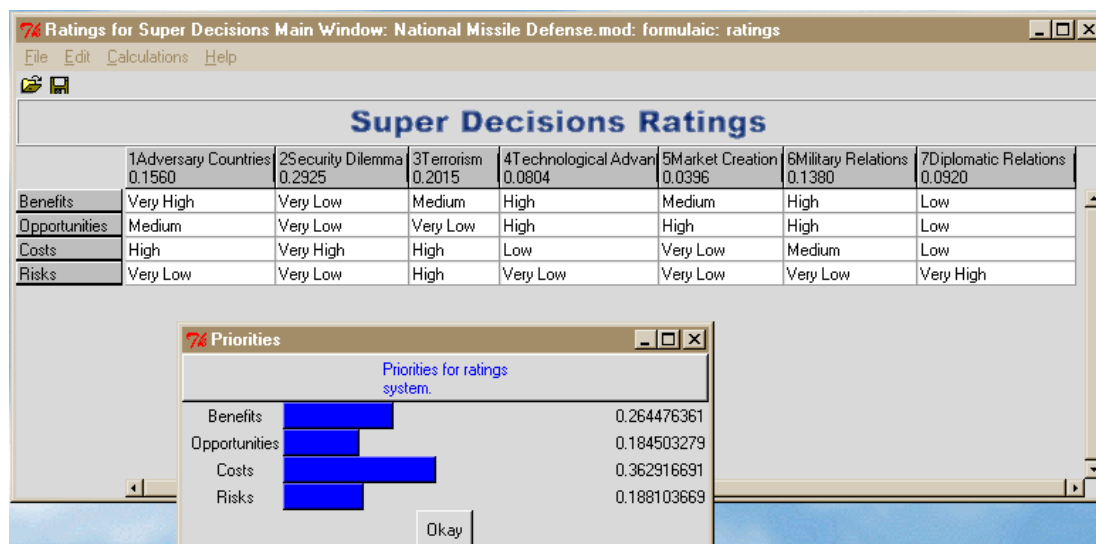


**Figure 39.** Ratings Spreadsheet and Resulting BOCR Priorities for Top-Level of NMD Model.

To complete the story, the synthesis command uses the formula for the top-level network shown in Figure 40. Do not be alarmed by its seeming complexity. It simply specifies that the values for the BOCR as determined in Ratings are used to weight the values for the alternatives coming up from the subnets under the BOCR and the four BOCR terms are then added and normalized to

obtain the synthesized results for each alternative. Formulas are discussed in more detail at the end of this chapter.



**Figure 40.** The Formula Associated with the NMD Top-level Network.

## THE CONTROL CRITERIA NETWORKS

Each of the BOCR has an attached control criteria network. A partial view of the Benefits subnet is shown in Figure 41.
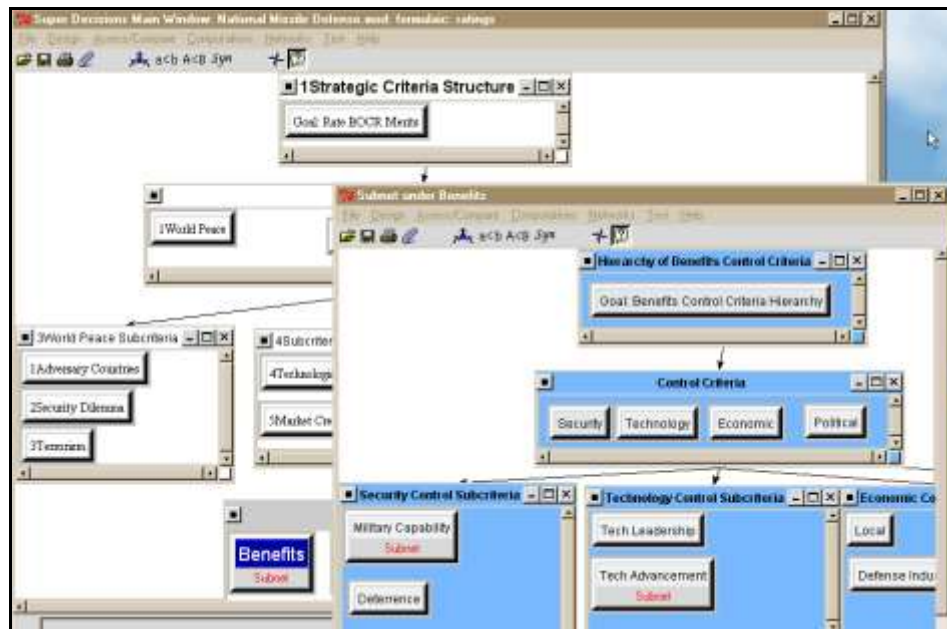


**Figure 41.** The Control Criteria Subnet belonging to Benefits

In this Benefits subnet the four main criteria of Security, Technology, Economic and Political are pairwise compared with respect to the Goal. Then the subcriteria of Security, Military Capability and Deterrence are pairwise compared with respect to Security, and so on. The most important potential control criteria are Military Capability (.28) and Tech Advancement (.24), together

having .52 or more than half of the priority. These values can be read from the Computations, Limit Matrix in the column headed Goal.

## THE DECISION NETWORKS

We will examine one bottom level decision network, the one attached to the Military Capability control criterion, shown in Figure 42 in detail. The decision makers: Congress, and the executive branch consisting of the President and the Department of Defense (the node name is military). Technical experts and the Defense Industry influence Congress and the executive branch by providing their professional expertise and technical information.

Congress, Military, Defense Industry, and Technical Experts all have a say about the extent to which the alternatives contribute to the Military Capability of the US. The results are that Deploy NMD will increase the military capability followed by Global Defense, R&D and Termination but to very different degrees. Table 3 shows the 23 control criteria under the benefits, opportunities costs and risks and their priorities.

**Table 3** Criteria and Their Priorities

| Merits | Criteria | Subcriteria | Local Priorities | Global Priorities |
|---|---|---|---|---|
| Benefits (0.264) | Economic (0.157) | Local Economy | 0.141 | 0.006 |
| | | Defense Industry | 0.859 | 0.036 |
| | Political (0.074) | Bargaining Power | 0.859 | 0.017 |
| | | U.S. Military Leadership | 0.141 | 0.003 |
| | Security (0.481) | Deterrence | 0.267 | 0.034 |
| | | Military Capability | 0.590 | **0.076** |
| | | Anti-terrorism | 0.143 | 0.018 |
| | Technology (0.288) | Tech. Advancement | 0.834 | **0.064** |
| | | Tech. Leadership | 0.166 | 0.013 |
| Opportunities (0.185) | | Arms Sales | 0.520 | **0.094** |
| | | Spin- off | 0.326 | **0.059** |
| | | Space Development | 0.051 | 0.009 |
| | | Protection of Allies | 0.103 | 0.019 |
| Costs (0.363) | Security (0.687) | Security Threat (Vulnerability to the security threat) | 1.000 | **0.248** |
| | Economic (0.228) | Sunk Cost | 0.539 | **0.044** |
| | | Further Investment | 0.461 | **0.038** |
| | Political (0.085) | ABM Treaty | 0.589 | 0.018 |
| | | Foreign Relations | 0.411 | 0.013 |
| Risks (0.188) | | Technical Failure | 0.430 | **0.082** |
| | | Arms Race | 0.268 | **0.051** |

| | | | |
|---|---|---|---|
| | Increased Terrorism | 0.052 | 0.010 |
| | Environmental Damage | 0.080 | 0.015 |
| | U.S. Reputation | 0.170 | 0.032 |

The Merits values in the first column are obtained by using the Calculations, Priorities command in Ratings. The criteria and subcriteria values are obtained by going into the control criteria subnet (for Benefits for example), using the Computations, Unweighted Supermatrix command and reading the values from it. The Global Priorities values are obtained using Excel. For example, the first one, 0.006, is obtained by multiplying 0.264 x 0.157 x 0.141.

The 23 criteria are in the control criteria subnets attached to the BOCR. They were prioritized through pairwise comparisons. Among these 23 criteria, the sum of the priorities of 9 of them, security threat, arms sales, technical failure, military capability, technological advancement, sunk cost, spin off, arms race, and further investment account for over 0.76 of the total. To economize effort, we used 9 to do the analysis. We renormalize their priorities within their respective merits and continue. One of these sub-networks is shown in Figure 42.
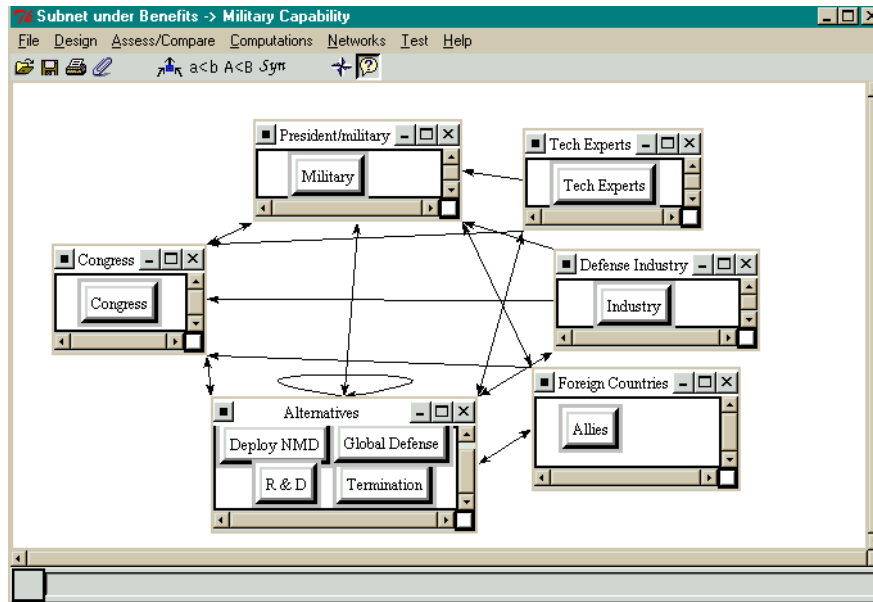


**Figure 42.** Military Capability Sub-network Under Benefits

Table 5 shows the final synthesis of the alternatives for each of the BOCR merits and the overall result, using the reciprocals of the synthesized priorities of costs and risks.

**Table 5** Final Outcome

| | Benefits (0.264) | Opportunities (0.184) | invCosts (0.363) | invRisks (0.188) | Final Outcome using Additive (norm.) | Final Outcome using Multiplicative (norm.) |
|---|---|---|---|---|---|---|
| Deploy NMD | 0.434 | 0.473 | 0.306 | 0.116 | **0.331** | **0.493** |

| Global Defense | 0.357 | 0.290 | 0.305 | 0.178 | 0.291 | 0.379 |
|---|---|---|---|---|---|---|
| R & D | 0.161 | 0.151 | 0.236 | 0.289 | 0.212 | 0.110 |
| Termination | 0.049 | 0.085 | 0.153 | 0.417 | 0.165 | 0.018 |

The final outcome is calculated in two ways: using an additive formula and a multiplicative formula. The effect of using the multiplicative formula is that the weights of the BOCR cancel out due to the form of the formula, so in effect they are equal at 0.25. The reason the Deploy option is so much better under the multiplicative formula is that costs are very high for that alternative, thus when costs are weighted by .363 as they are in the additive formula, it drags down the value of the Deploy option.

## OVERALL OUTCOME AND SENSITIVITY ANALYSES

Let us look at sensitivity for results using the additive formula. Deploy NMD at 0.331 is the best option. It is a comprehensive result that takes into consideration all benefits, opportunities, costs, and risks. First, we let us examine sensitivity if the value of Benefits changes. No matter what the value of Benefits, the Deploy option (the top line) is dominant. There is one line for each alternative in sensitivity windows. In the software they are color coded so that it is easy to see which line corresponds to each alternative, but here in black and white they look the same.
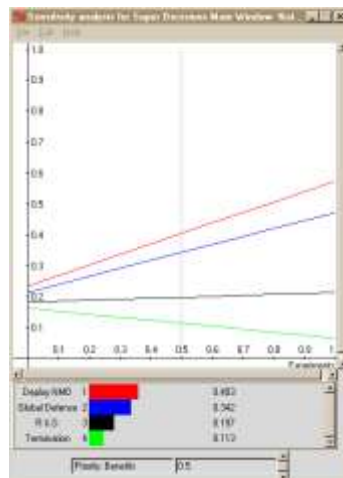


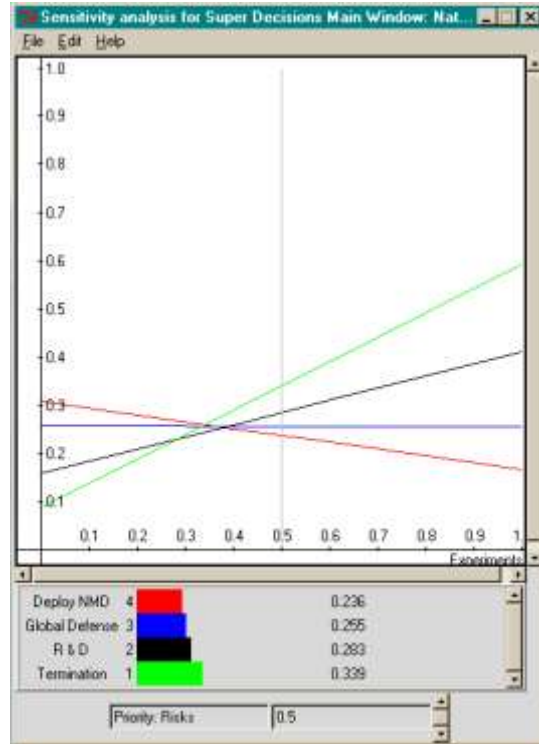**Figure 43**. Sensitivity Analysis: Benefits

**Figure 44.** Sensitivity Analysis: Risks

We did similar sensitivity tests for some of the control criteria in the subnets as shown below. We found that the outcome was very stable and did not change the overall ranks except for changes of the 3 criteria, security threat, further investment, and sunk cost, all under costs. When the priority of the security threat decreases to 0.154 from 0.248 or the priority of the further investment increases to 0.774 or the priority of the sunk cost increases to 0.803, then the rank of termination changes from third to second to first.
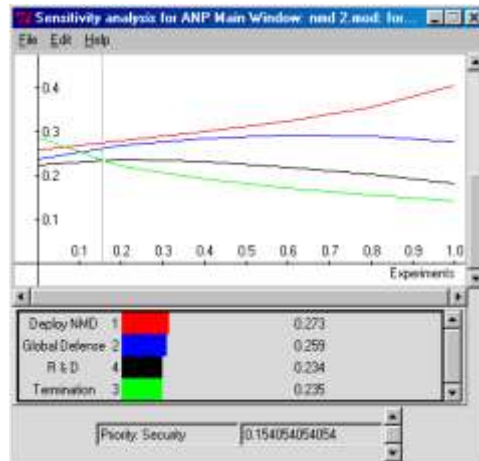


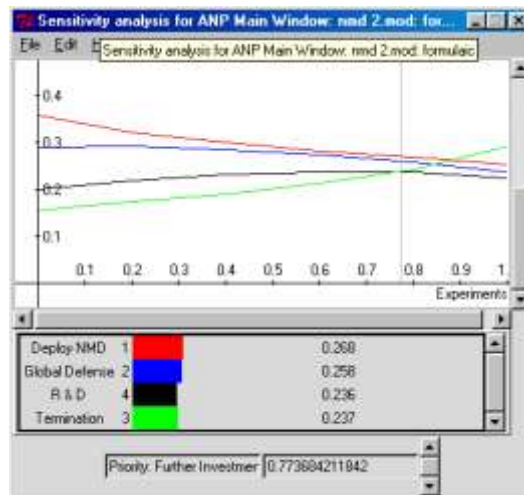**Figure 45.** Sensitivity Analysis: Security Threat

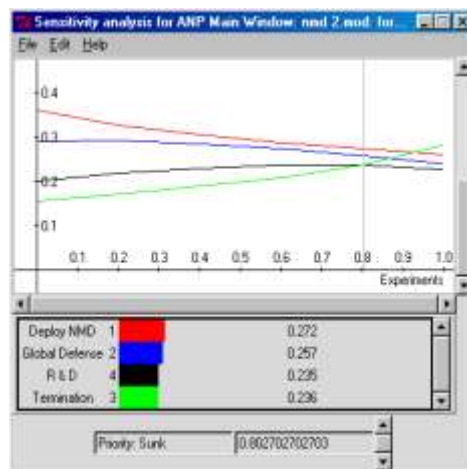**Figure 46.** Sensitivity Analysis**:** Further Investment

**Figure 47.** Sensitivity Analysis: Sunk Cost

## OVERVIEW OF FORMULAS FOR MULTI-LAYER MODELS

In general formulas are used only in the top-level network of a multi-layer system for synthesizing the results as they feed up from the subnets. No formulas are used in the bottom level subnets as they simply feed the synthesized values for the alternatives upward, and they continue to rise up through any number of intervening sub-networks until they reach the top. See the section on Formulas at the end of the chapter for more information on how to select and input a formula. The Additive formula was used in the NMD model.

## HOW TO BUILD AND GET RESULTS IN A MODEL WITH SUBNETS- A WALKTHROUGH

Here we will show you step by step how to build the car model that was demonstrated earlier in the chapter. We shall name the new one the *aejcar.mod* sample model. The end result will be the same as the *Car Purchase BCR* model if you use our inputs, but use your own if you wish.

The model you will build contains a control network and three sub-networks associated with the Benefits, Costs and Risks control criteria in the control network. Networks are built of clusters that contain nodes with links between the clusters and links between the nodes. The nodes in a cluster have something in common, without being very specific about what it is, that makes them related in some sense. This model is to help a car buyer prioritize, in terms of overall satisfaction, the type of car he or she prefers, American, Japanese or European. The types of cars or carmakers (American, Japanese, or European) are the alternatives. One requirement for an ANP model is that there must be a cluster labeled "Alternatives" in each sub-network that contains the alternatives of the problem as nodes. Start the model by entering the three control models, their clusters and the elements in the clusters as shown in Table 9 .

**Table 9.** The Nodes You Will Create in the *aejcar* Model.

| Control Criterion | Clusters in Sub-Model | Nodes in Clusters |
|---|---|---|
| **Benefits criterion** | Advantages | Transportation, Status, Sports, Socializing |
| | Alternatives | American, Japanese European |
| **Costs criterion** | Outlays | Initial Cost, Repair Cost, Reliability Cost |
| | Alternatives | American, Japanese European |
| **Risks criterion** | Bad Luck Events | Accidents, Stolen, Repossession |
| | Alternatives | American, Japanese European |

# CREATING A MODEL

Load the ANP software. The opening screen is shown in Figure 48 below. You may build a simple network in this window that comprises the whole model, or you may create some clusters with nodes in them and create sub-networks for some of the nodes. You then create clusters and nodes in the sub-networks and for some of the nodes there create sub-sub-networks. In theory there is no limit to the number of layers. The top-level model is the first layer, the sub-networks are the second layer and the sub-sub-networks are the third layer.
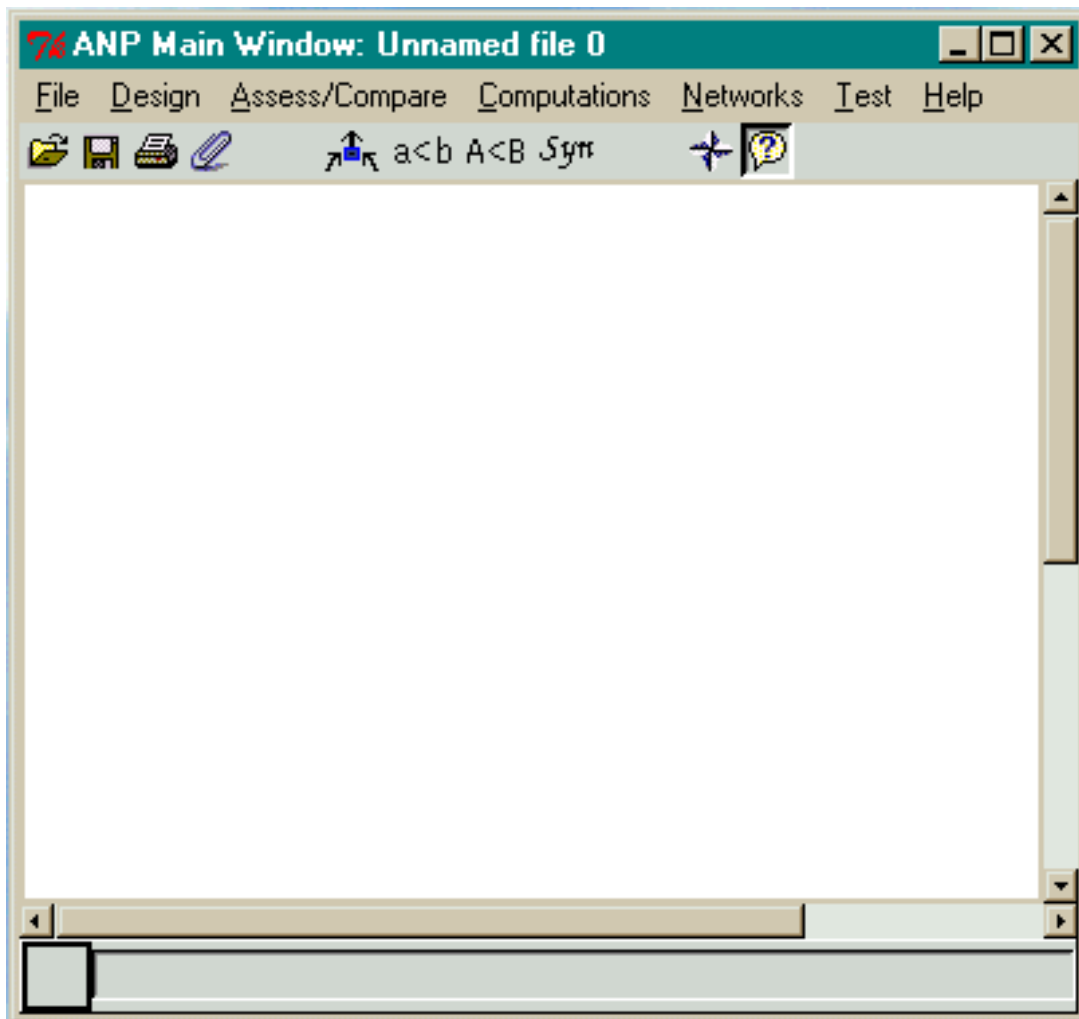
**Figure 48.** The Starting ANP Main or Top-level Window.

## STARTING A NEW MODEL

In this section we will walk you through creating the *Car Purchase* model demonstrated earlier. Name the new model *aejcar.mod* and create it using the Small Template. To start a new model using a template select the *File New* command as shown in Figure 49 below.
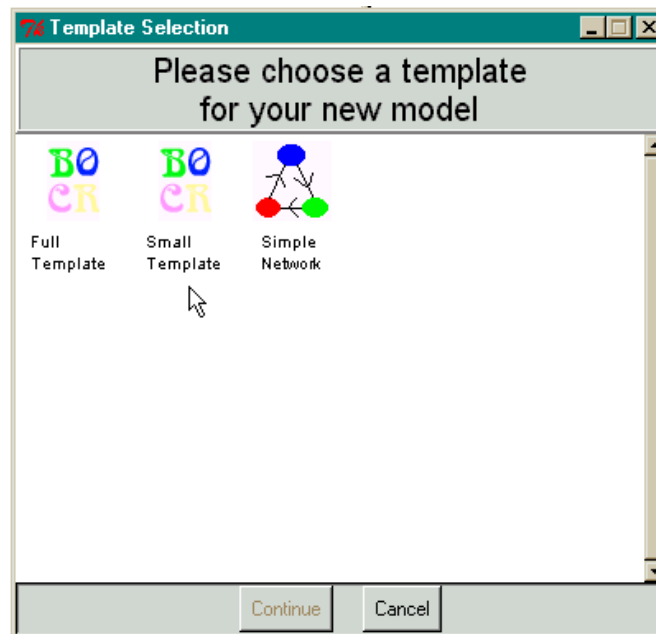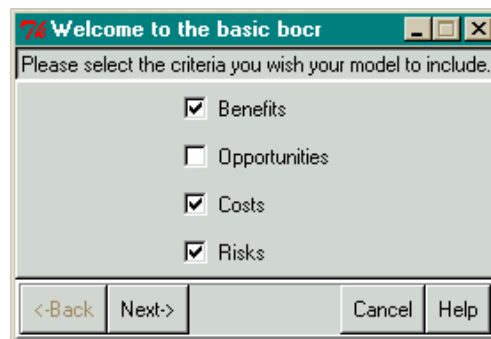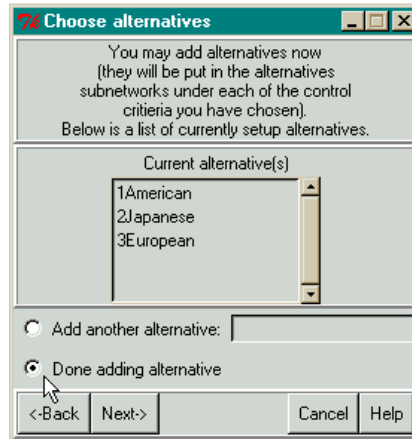
**Figure 49.** Select <u>F</u>ile <u>N</u>ew, then Small Template to Start the Car Model.

## STARTING A MODEL USING THE SMALL TEMPLATE

1. Select Benefits, Costs and Risks



2. Add Alternatives – Select Done and Next when Finished

The automatically generated control network for a BCR model will appear in the ANP main window as shown in Figure 50 below. It has what amounts to a hierarchy with a goal in the goal cluster and the BCR merits in the cluster named Model. You could have used the *Design Cluster* and *Design Nodes* commands and built the same model yourself.
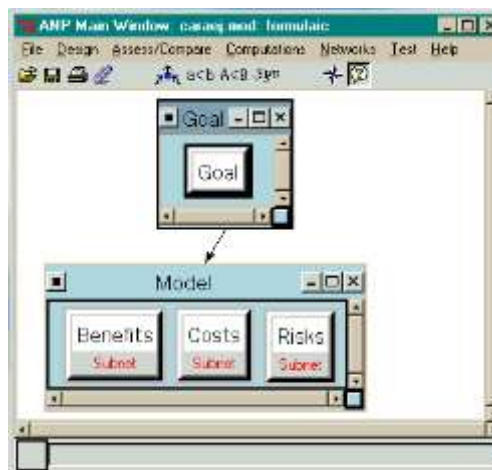


**Figure 50.** The Top Level Network that was Automatically Generated for the *aejcar* Model.

The Benefits, Costs and Risks sub-networks are started for you with the alternative cluster already in place. See Figure 51 below. You need to add the other clusters.
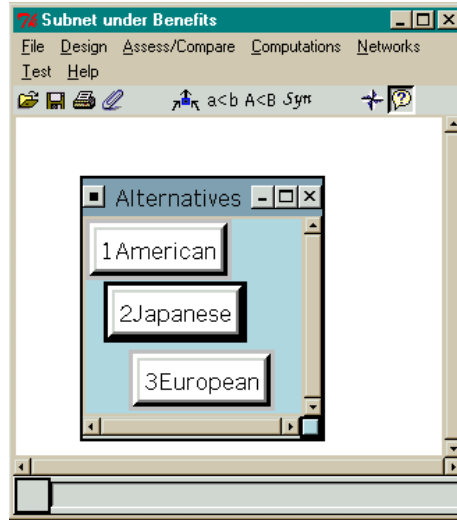
**Figure 51.** The Subnets Created by the Template under Benefits, Costs and Risks are the Same.

To reach a subnet, left double-click on its control node in the network immediately above the subnet, or you can use the node drop-down menu and select the *Make/show Subnetwork* command.

The clusters look like windows with scroll bars. To change the size of a cluster window click on the blue button at the lower left corner of the window and drag. To drop down a cluster menu left-click on the left top corner of the cluster as shown in Figure 52. To minimize it by changing it into an icon, click on the minus sign at the top right, or the *X*. To change it back from an icon, left double-click on the icon. To enlarge the cluster left-click on the "maximize" square at the top right.
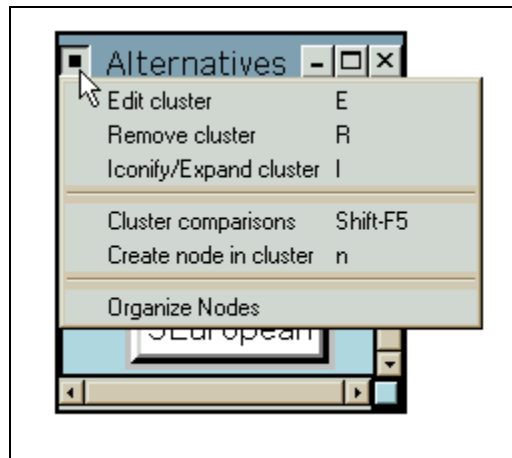


**Figure 52.** View of Cluster Window Icons and Menu

The Editing Cluster dialogue box is shown in Figure 53 below. You can change many things about the appearance of a cluster using the properties here.
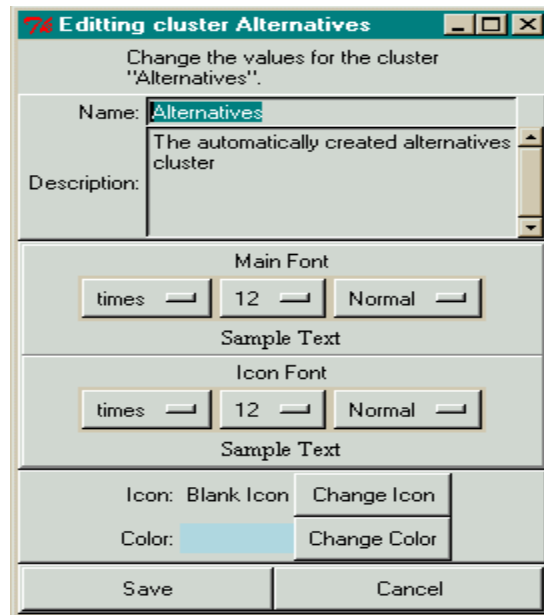
**Figure 53**.  The Editing Cluster Dialog Box

*Tip:  If you want to choose a particular default font, style, and background color for clusters and for nodes before you start a new model, select the Ｆile Ｃonfigure command and set them using the dialog box shown in* Figure 54 *before creating the control network.*
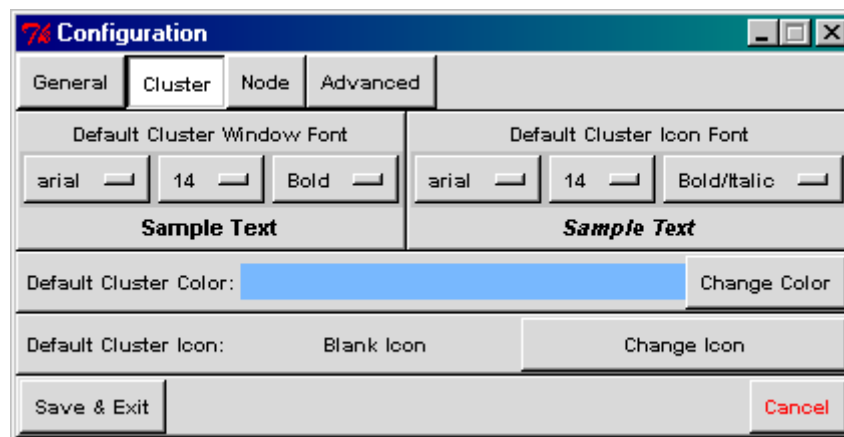


**Figure 54.**  The Configuration Box for Customizing Cluster and Node Fonts.

## FORMULAS

Using a template will assure that the formula for combining the results in the top-level network as they feed up from the subnets is correct.  The only formula in a two-level model is in the top network.  No formula is used in a bottom level subnet as it simply feeds the synthesized values for the alternatives upward, and they continue to pour upwards through any number of intervening sub-networks until they reach the top.  The formula used in the top-level network is automatically generated when you use the Small Template wizard to start your model.  Select the *Design, Add/Edit Formula* command to see the network formula that was generated, shown in Figure 55 below.

**Figure 55.** Subtractive Formula Produced by Template for Two-level Model.

## MULTIPLICATIVE FORMULA

The standard formula that is automatically generated automatically uses the names of the control nodes that have been chosen in the template: in this case they are Benefits, Costs and Risks. It is the standard subtractive formula. See the section on formulas at the end of this chapter for more information. The formula can be changed by the user simply typing in a new one, or by selecting a different standard formula. Change the formula to the multiplicative formula using the following commands.

1. Select the *Design, Add/Edit Formula* command to see the automatically generated formula.
2. Close that and select the *Design, Standard Formulas, Multiplicative* to change the formula to the one shown in Figure 56.
3. Check the new formula by selecting the *Design, Add/Edit Formula* command again.



**Figure 56.** The Multiplicative Formula

85

Multiplying by the inverse of Risks and Costs ( B * (1/Risks) *(1/Costs))is the same as dividing Benefits by Risks and Costs. The questions in the Costs and Risks subnets must be phrased by asking which is the more costly or more risky, so that the alternatives with the most cost and most risk end up with the highest priorities. The higher the Risk and Cost numbers for an alternative, the lower the B/CR result for that alternative, which is what you would want.

## LINKING NODES

The simplest way to link a node is to turn on the make connections node by clicking on the ⬆ Icon to depress it. Then left-click on the "parent" node and right-click on all the descendants of it (which may be in different clusters – you can connect them all in one process). You can also use the Node connections from command on the dropdown node menu shown in Figure 57.
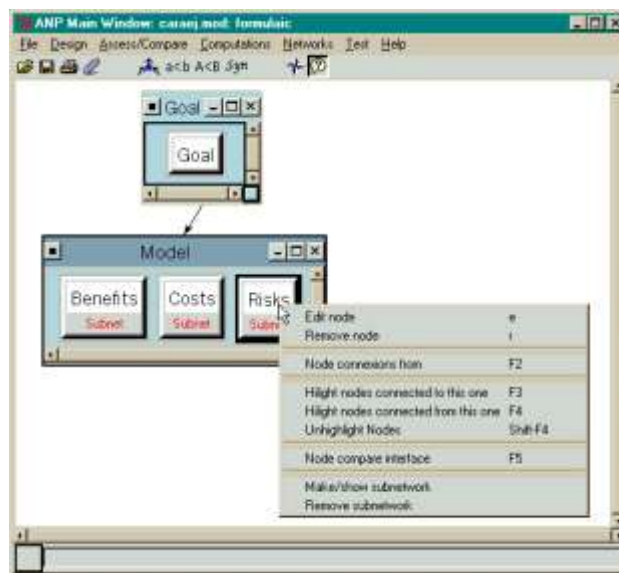


**Figure 57.** The Dropdown Node Menu obtained by Right-clicking with the Cursor located over a Node.

## CREATING A SUB-NETWORK

It is not necessary for the car model to create any subnet as they were automatically created by the template wizard. However, if you need to create a sub-network, right-click on the node to have a subnet built for it, for example, the Benefits criterion node. Select the *Make/display sub-network* menu command to open a new window for the sub-network. Nodes with sub-networks attached to them have the word subnet displayed in red at their bottom.

Tip: It is also possible to save a subnet in a separate file and open it in a blank sub-network. This will save you time entering node names if you plan the sub-network template properly. For example, after adding the Alternatives cluster to a subnet when you are constructing your own subnets, save it, then open it in the succeeding blank subnet windows.

## COMPLETING THE SUB-NETWORKS

To create the sub-network for the Benefits Control Criterion:

- Place the cursor over the Benefits criterion node in the Benefits Criteria cluster and click with your right mouse button to drop down its node command menu.
- Select the Make/show network command. The window titled Subnet under Benefits criterion appears. It will already have the alternatives cluster in it. Finish the subnet so it contains the clusters and nodes shown below:

| Clusters | Nodes |
|---|---|
| Advantages | Transportation, Status, Sports, Socializing |
| Alternatives | American, Japanese European |

Select the <u>D</u>esign <u>C</u>luster <u>N</u>ew command from the menu of the window titled Subnet under Benefits criterion and enter the name of the cluster: **1Advantages** in the New Cluster Dialogue box shown in Figure 58.

To advance from one field to the next in the New Cluster Dialog box press the `<Tab>` key. From this dialogue box you can:
- Enter the name of the node (spaces are allowed)
- Enter the description of the node
- Select the font for the cluster when displayed as a window
- Select the font for the cluster when displayed as an icon
- Change the background color of the cluster window
- Select an icon for the cluster by double-clicking on your selection
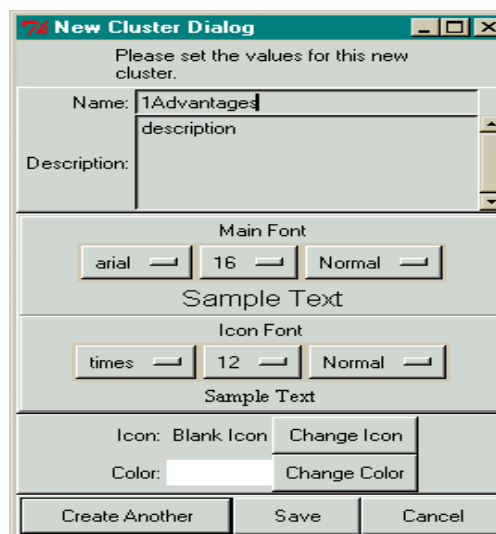- You may either save and stop the cluster creation process, or create another



**Figure 58.** The New Cluster Dialog Box for Creating Clusters.

Caution: When you create clusters (and nodes) one after the other using the Create Another button they should cascade. However, if you stop the process with the Save button and later start again, the first new cluster will be created in the upper left-hand corner of the window and may land on top of a previously created cluster in that position. Left click on the top one and drag it somewhere else if this happens.

Mouse Shortcut for Creating Clusters: Press the <Shift> key and left and click with the mouse anywhere on the background of the main window. The new cluster will appear wherever the mouse cursor was at the time.

Keyboard Shortcut for Creating Clusters: Press <Shift> <n>, that is, a capital N. The new cluster will appear wherever the mouse cursor was at the time.

Once an icon has been selected for a cluster and you have saved it you can toggle between the cluster window and icon view by double-clicking with the left mouse button on the cluster or icon.

If a dialogue box disappears before you have saved, is has probably been minimized and is on the Windows menu bar at the bottom of your screen. Left-click on it to restore it and proceed with the Save.

### CREATING NODES IN CLUSTERS

To create the nodes select the Design Node New command. Select the cluster into which you wish to add the nodes and type the names in the Create New Node dialog box just as you did with the clusters. Select the Create Another button to add another node until you are finished then select the button. Alternately, you may right-click on the background of the cluster into which you wish to place the nodes, and select the command Create node in cluster from the node menu that appears. The Node Dialog Box for Editing and Creating New Nodes is like the cluster dialog box, so those screen clips will not be repeated here.

*Tip: Both Clusters and Nodes are organized for making comparisons and placed into the supermatrix in alphabetical order. To make the nodes appear in the order you like, preface the names with 1,2,3,….*

Enter the node names in the 1Advantages cluster: 1Transportation, 2Status, 3Sports, 4Socializing. Enter the node names 1American, 2Japanese and 3European in the Alternatives cluster. The resulting completed subnet titled Subnet under Benefits criterion is shown in Figure 59 below.
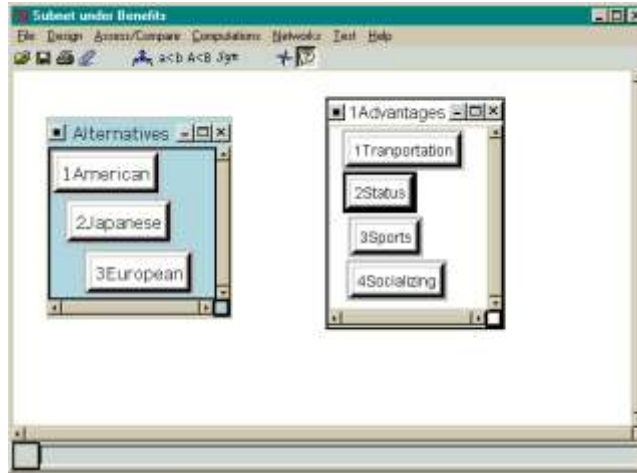
**Figure 59.** The Completed Subnet for the *Benefits criterion* Node.

*Keyboard Shortcut: Add nodes in a cluster by placing the mouse in the cluster then pressing the letter <n>. The new node will be added where the mouse cursor is.*

*Another Shortcut for Adding Nodes: Select the <Shift> key and left-click over the background of a cluster to add a new node where the mouse cursor was when you left-clicked.*

See the node creating/editing dialogue box below that appears when you select the command Create node in cluster shown above.

Create the Costs and Risks subnets by adding the Outlays cluster in Costs and the Bad Luck Events cluster in Risks. The model now has all the clusters and nodes in it as shown in Figure 60. The links now need to be made and the judgments entered.

We shall now describe the node dropdown menu. Click with your right mouse button on the Benefits criterion node to drop down its node menu. The commands on the menu apply to this node and are described below.
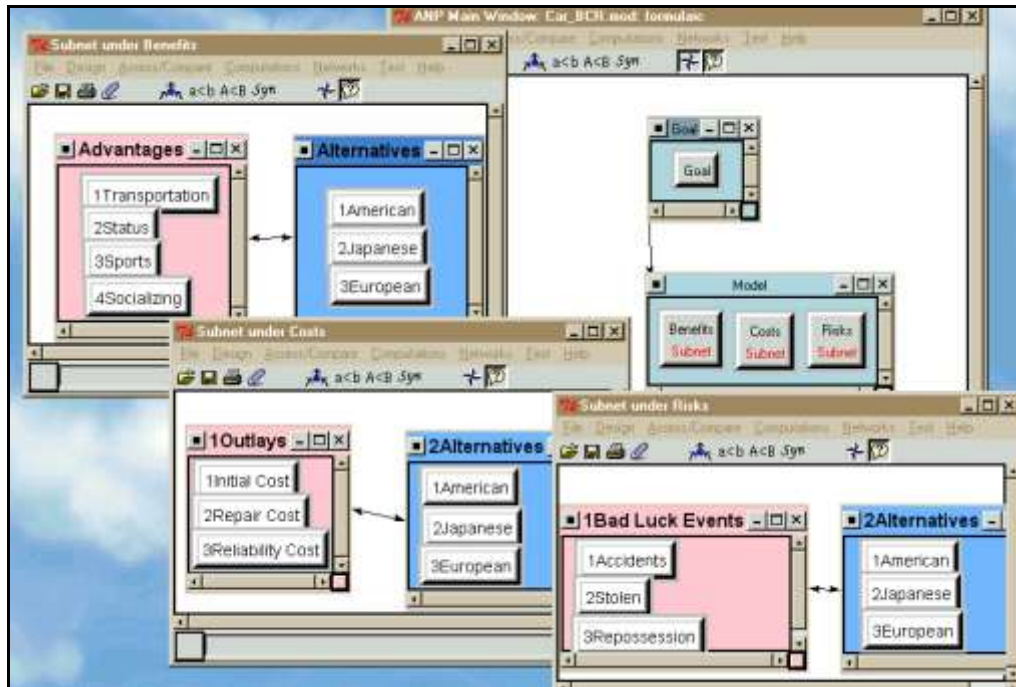
**Figure 60.** Completed Model: Top level and 3 Subnets.

### THE NODE MENU COMMANDS

The commands in the drop down node menu are:

- *Edit node* – change the appearance of the fonts, change the title and description of the node, and select an icon for the node.
- *Remove node* – delete the node
- *Node connexions from* – create connections from this node to other nodes
- *Hilight nodes connected to this one* – highlight all the nodes in other clusters connected to this node. These would be parent nodes for comparison groups that this node belongs to.
- *Hilight nodes connected from this one* – highlight all the nodes in other clusters connected from this node. These would be nodes belonging to comparison groups for which this node is a parent.
- *Unhighlight Nodes* – turn off the highlighting that was previously turned on. This command is the only way to do that.
- *Node compare interface* – compare a group of nodes of which this node is the parent. There may be such groups in more than one cluster, so you will be asked to select the cluster the group is in. Links or connections have to be established before you can make comparisons.
- *Make/show sub-network* – create the sub-network for this node or open it if it has already been created. When first created the sub-network is an empty window. You may start a new network by creating clusters and nodes with the Design command; or you may import a network from a Word document file using the File Import command. You may also open a previously created network using the File Open command. Use the File Save command to save the new network within the current main model's file. The new network will contain all the clusters, nodes, links and judgments of the original one.

- *Remove sub-network* – this command will detach an existing sub-network from this node and it will be deleted if you do not save it or have not previous saved it under a name of its own.

# CREATING CONNECTIONS OR LINKS BETWEEN NODES

The process of linking nodes is as follows. Pick a node that is a potential parent node. Look at another cluster and ask if there are nodes in that cluster you wish to compare with respect to the node you have picked. If there are, create links to them. There must be at least two nodes in the other cluster to form a group that can be compared with respect to the prospective parent node. Examine the next cluster for nodes that should be compared with respect to the parent node. Finally examine the node's own cluster to see if there is interdependence which means you want to compare other nodes in its own cluster with respect to it.

To make the connections, turn on the connection mode by clicking on this icon:



Left-click on the node that will serve as the parent and right-click on the nodes it is to be connected to.

In this example, every node serves as the parent to all the nodes in the other cluster. This is not generally the case. The links between the clusters therefore have arrows at both ends as the nodes are linked both ways. Create the links for all three subnets. To show what nodes are connected, turn on the "Show Connections" icon:



When you pass the mouse across a node, all the other nodes in the entire network that it is connected to will appear with a red outline around them. And if you have marked any comparisons that were down as completed when you exited the comparison mode, the component boxes containing nodes with completed comparisons with respect to the node your mouse is resting over will be outlined in red.

### INNER AND OUTER DEPENDENCE

When elements are linked only to elements in another cluster, we say the model exhibits only outer dependence. When elements are linked to elements in their own cluster, we say there is inner dependence. There is only outer dependence in this example. All three subnets are totally connected

Notice that the Alternatives clusters contain the same nodes in all three sub-networks. Every sub-network must have a cluster named Alternatives, or some variation of the name such as 1Alternatives. The software can recognize most forms of the word Alternatives. It uses this word to synthesize the priorities for the alternatives properly. The building of the model is now complete. Next the judgments must be made.

# MAKING JUDGMENTS OR PERFORMING ASSESSMENTS OR COMPARISONS

Now you are ready to begin making judgments in the model. You must go through the model selecting one node at a time and pairwise comparing or putting in data for each of the comparison groups linked to it. Make sure you have entered judgments for every parent node throughout each sub-network.

## ENTERING JUDGMENTS

We will demonstrate the comparison process with the subnet for the Benefits criterion. Show the subnet by right-clicking on the background of the Benefits criterion in the control network. Select the command Make/Show subnetwork. Within the sub-network right-click on the node Status, and select the *Node compare interface* command, then select the cluster 2Alternatives containing the nodes to be compared to Status. Alternately select the *Assess/compare* command and *Node Comparisons* from the menu at the top of the Window and select the cluster containing the node you want to compare with respect to, 1Advantages, then select the node Status. Finally select the cluster containing the nodes to be compared, 2Alternatives. The comparison window will appear for the node Status in the Questionnaire mode Figure 61.

*Tip: If you will be comparing nodes in multiple clusters for the given parent node, you may select them all at once. The comparison process will then automatically cycle through offering comparisons for nodes in one cluster after another. If nodes in only one cluster will be compared, you may double-click on that cluster in the cluster selector box and go directly into the comparisons.*



**Figure 61.** Comparisons in the Questionnaire Mode for the Alternatives under Status.

Pairwise comparisons are made between two elements in one cluster with respect to a "parent" element in another cluster. For example, the elements American and Japanese in the "Alternatives" cluster are pairwise compared with respect to Status, which is in the "Advantages" cluster. For this comparison the link is from the parent element, Status, to the elements being compared, American and Japanese. The judgment of 4 preferring Japanese to American for Status has been entered by left clicking on the 4 in the Questionnaire mode shown above. Since Japanese is preferred over American, select the 4 nearer the Japanese label on the right.

The Matrix mode for entering judgments is shown in Figure 62. These are equivalent to the judgments shown in the Questionnaire Mode. Note that the arrow next to the judgment points to the preferred member of the pair. Double-click on the button the arrow is on to invert the judgment. The button indicators at the right show which judgment you are on. In the verbal mode they are important to know how far you have come in making comparisons.
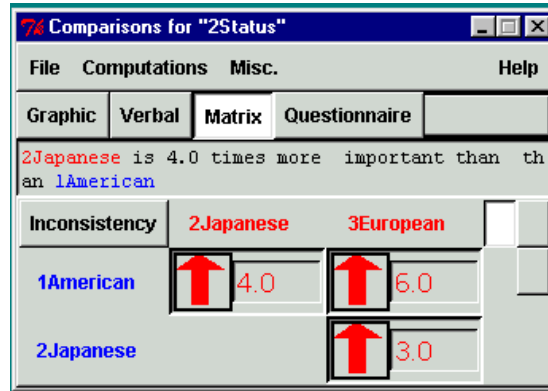


**Figure 62.** The Matrix Comparison Mode for Comparing the
Alternatives with respect to Status.

The equivalent Verbal Comparison mode for comparing the alternatives with respect to Status is shown in Figure 63. Note the button at the bottom of the screen labelled "Invert Comparison". Click on it to invert the preference order.
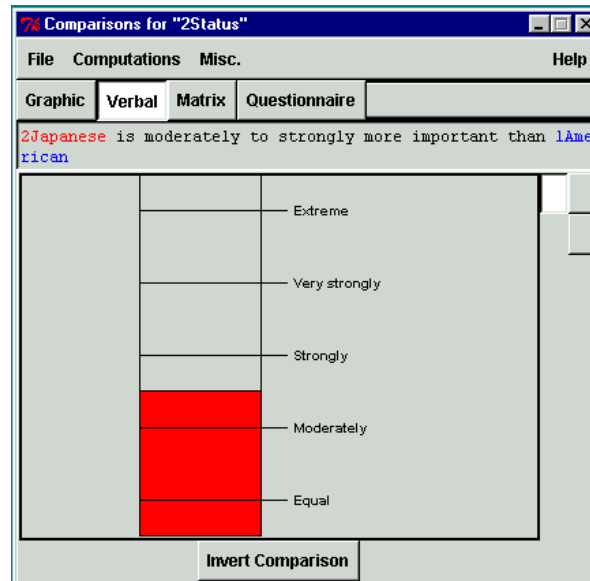


**Figure 63.** The Verbal Comparison Mode for the Alternatives
with respect to Status.

The three carmakers will rank differently on Status depending on who is making the judgments. This is a subjective personal opinion. Different individuals have different answers and both are "right" for that individual. One's prior experience, word-of-mouth information, and actual data all affect one's subjective judgments. The purpose in decision-making is to come up with one's own best choice in the sense that it is the most preferred by the individual making the decision.

Suppose for the individual here that is making the judgment, Japanese carmakers are preferred to American carmakers for Status. The judgment entered is 4, equivalent to the verbal judgment of "between moderately and strongly". The arrow on the 4 in the Matrix Comparison Mode points upward toward Japanese to indicate that it is preferred over American.

The comparative word used in the comparison phrase can be changed from the default word of importance by selecting it in the dialogue box shown in

Figure 64 below:
- Select the menu command *misc.*, *Comparison words* to bring up the dialog box
- Select a new comparative word, for example, preference.
- Or type a new comparative word of your own in the field the arrow is pointing to, `"beautiful",` for example.
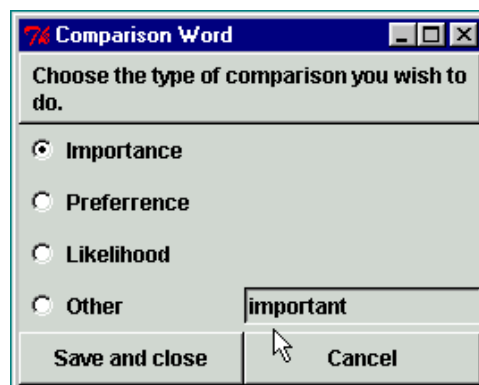


**Figure 64.** Dialog Box for Changing the Word used for Expressing Comparison preferences.

*TIP: When making the pairwise comparisons, one should keep in mind that the control criterion is Benefits. Which of the elements being compared yields the greater **benefit** with respect to Status, American or Japanese carmakers?*

The modes for making assessments are:
- Graphic
- Verbal
- Matrix
- Questionnaire
- Direct Data Entry (Select this mode under the Misc. command)

The first four modes are based on making judgments using the Fundamental Scale of the AHP from 1 to 9. Switch among these four modes by selecting the tab for the one you want. The fifth mode, Direct Data Entry is accessed under the Misc. command). In this mode the pairwise comparison entries are calculated by taking the ratio of the data for the pair being compared and entering it in the Matrix mode. If the ratio is less than 1.0, the inverse of the ratio is computed and entered as a number greater than 1 in the matrix mode with the arrow indicating which of the pair had the larger original value. All computations are based on the values currently appearing in the Matrix mode.

*Tip: If after entering Direct Data you change to another comparison mode and enter one or two values in the new mode, these values will be changed in the Matrix mode while the other values will be the original ratios from the Direct Data mode. So the computations will be based on a mix of data and judgments. This is okay so long as you understand the results you are getting.*

### CALCULATING PRIORITIES IN THE COMPARISON MODE

To calculate the priorities based on the judgments you have entered, select the Computations, Show new priorities command in the comparison mode menu. The priorities with respect to Status are as shown in Figure 65. Priorities such as these that are the result of doing comparisons are referred to as local priorities.
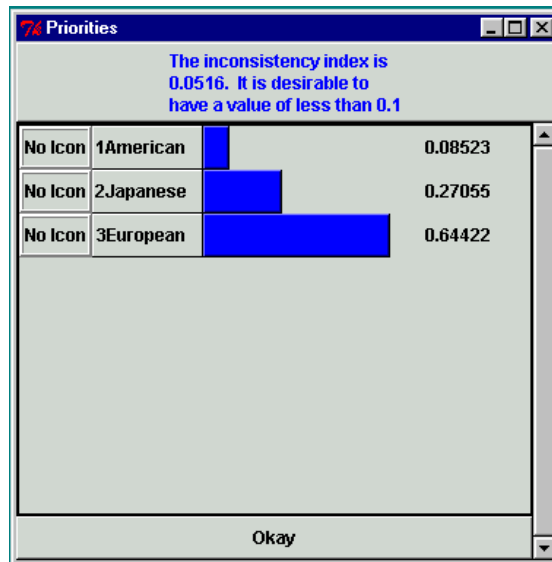


**Figure 65.** The Priorities for the Alternatives with respect to Status

### IMPROVING INCONSISTENCY

The inconsistency ratio is shown at the top of the Priorities screen. It is 0.0516 in this case. A ratio of less than 0.1 is desirable, so this is an acceptable level of inconsistency. If the ratio is higher, select the Computations, Most inconsistent command from the menu. Inconsistency can only be improved using the Matrix mode, so if you select that command from another mode you will be taken to the matrix mode where you can improve consistency. The most inconsistent judgment will be highlighted and you can request the Best Value for it. You can change it, or not, as you wish.

To use the suggested judgment, simply type it in the highlighted cell. The cell arrow will automatically invert to keep it correct. If a judgment such as .5 was suggested, after you type it in it will be converted and appear as a 2, with a red arrow pointing up.

## THE SUPERMATRIX AND CALCULATING RESULTS

There are seven groups to be pairwise compared in the Benefits criterion sub-network because each of the elements serves as a parent node for comparing all the nodes in the other cluster. For example, Style is a parent node for comparing the Alternatives: American, Japanese and European; and American, in turn, is a parent node for comparing the Advantages nodes Transportation, Status, Sports and Socializing.

Thus there are seven local priority vectors in the Benefits sub-network and these get entered into the unweighted Supermatrix. The Computations command shown below in Figure 66 is used for the results, both in a sub-network and in a control network. It is not necessary to use the command in a sub-network, sub-network results are fed up to the control network in any case, but it is often helpful to see these intermediate results and they may be useful in analyzing the final results.
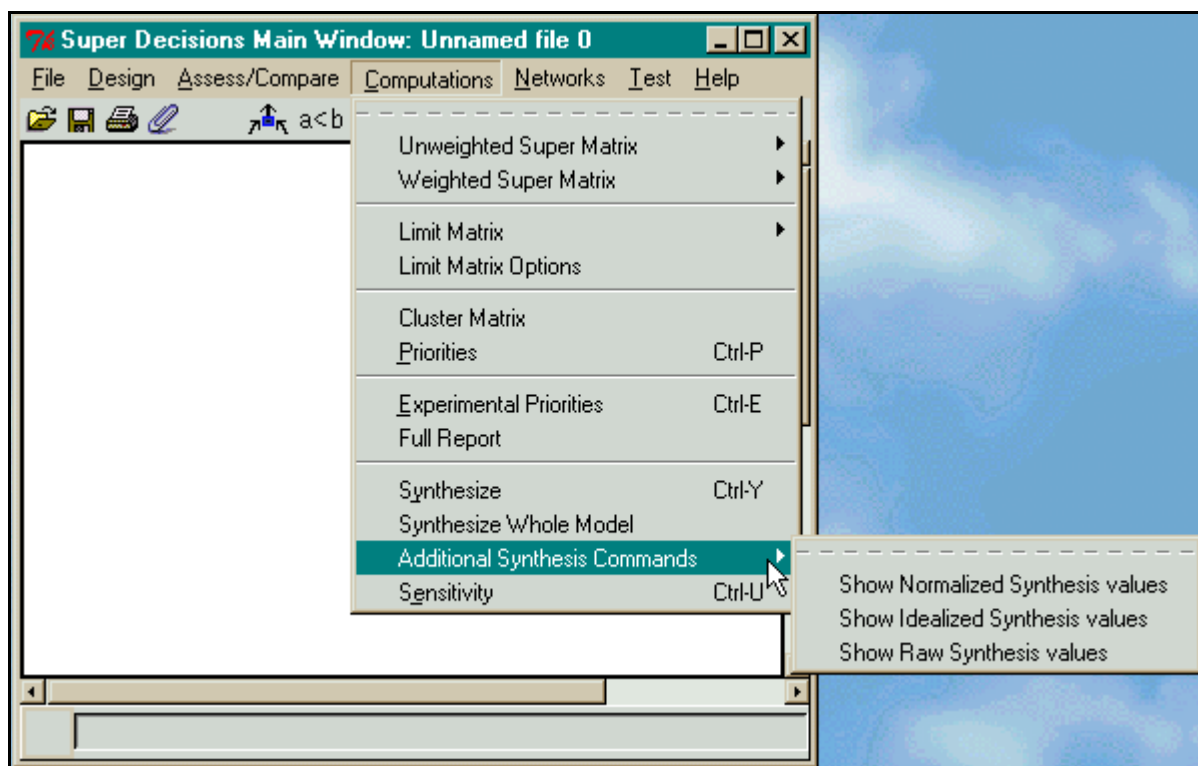


**Figure 66.** The Computations Menu showing the Synthesize Command

## GETTING RESULTS

Here we briefly described the commands on the Computations menu:

- *Unweighted Super Matrix* – contains all the local priority vectors of the comparison groups in the network.
- *Weighted Super Matrix* – the cluster weights have been multiplied times the local priority vectors in the unweighted supermatrix to make each column stochastic (that is, sum to 1)
- *Limit Matrix* – the weighted super matrix has been raised to powers until it stabilizes - that is, all the columns in the matrix have the same values.

- *Limit Matrix Options* – these are some experimental ways to calculate the limit matrix. You should use the already selected "Calculus Option".
- *Cluster Matrix* – The matrix of cluster weights is displayed. The number in each cell of the cluster matrix is multiplied times all the cells in the unweighted supermatrix in the corresponding component.
- *Priorities* – this command gives the priorities of all the nodes in a sub-network.
- *Experimental Priorities* is for research purposes and will soon be removed.
- *Full Report* – this command gives a very comprehensive report about the structure of the model and the various partial results, such as synthesis of alternative values from a sub-network. This report may be previewed or printed to an HTML file. You may then modify it, or extract the parts you need.
- *Synthesize* – gives the priority vector for the Alternatives in the sub-network, when the calculations are being done in the sub-network, and gives the synthesized priority vector for the Alternatives over *all* the sub-networks when the calculations are being done in the control network.
- *Sensitivity* – This is a what-if type of sensitivity that allows you to select any combination of independent variables. They may be nodes, supermatrix entries, or judgments. The priorities of the alternatives are graphed for each point on the x-axis of the graph, labeled "experiments". Sensitivity is often performed for the BOCR; it will not give meaningful results if the Multiplicative Formula is being used for synthesis, as changing the priority of any one of them will not change the answer.

We shall now show the three forms of the supermatrix in the *Benefits criterion* sub-network (remember, there are no supermatrices in the Control network). The first is the Unweighted Super Matrix in Figure 67.

| Cluster Node Labels | | Advantages | | | | Alternatives | | |
|---|---|---|---|---|---|---|---|---|
| | | 1Transportation | 2Status | 3Sports | 4Socializing | 1American | 2Japanese | 3European |
| Advantages | 1Transportation | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.489240 | 0.141216 | 0.058860 |
| | 2Status | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.065218 | 0.202572 | 0.494965 |
| | 3Sports | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.098394 | 0.485005 | 0.361038 |
| | 4Socializing | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.347148 | 0.171207 | 0.085137 |
| Alternatives | 1American | 0.614411 | 0.085220 | 0.111111 | 0.253428 | 0.000000 | 0.000000 | 0.000000 |
| | 2Japanese | 0.117221 | 0.270557 | 0.222222 | 0.084803 | 0.000000 | 0.000000 | 0.000000 |
| | 3European | 0.268368 | 0.644223 | 0.666667 | 0.661769 | 0.000000 | 0.000000 | 0.000000 |

**Figure 67.** The Unweighted Supermatrix for the Subnet under the Benefits Criterion Node.

| Cluster Node Labels | | Advantages | | | | Alternatives | | |
|---|---|---|---|---|---|---|---|---|
| | | 1Transportation | 2Status | 3Sports | 4Socializing | 1American | 2Japanese | 3European |
| Advantages | 1Transportation | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.489240 | 0.141216 | 0.058860 |
| | 2Status | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.065218 | 0.202572 | 0.494965 |
| | 3Sports | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.098394 | 0.485005 | 0.361038 |
| | 4Socializing | 0.000000 | 0.000000 | 0.000000 | 0.000000 | 0.347148 | 0.171207 | 0.085137 |
| Alternatives | 1American | 0.614411 | 0.085220 | 0.111111 | 0.253428 | 0.000000 | 0.000000 | 0.000000 |
| | 2Japanese | 0.117221 | 0.270557 | 0.222222 | 0.084803 | 0.000000 | 0.000000 | 0.000000 |
| | 3European | 0.268368 | 0.644223 | 0.666667 | 0.661769 | 0.000000 | 0.000000 | 0.000000 |

Done

**Figure 68.** The Weighted Supermatrix for the Benefits criterion Subnet; it is the Same as the Unweighted Supermatrix in this Case.

Note that the values are exactly the same for the Weighted Super Matrix shown in Figure 68 as they were for the unweighted supermatrix. This is because no cluster comparisons were done – nor were they necessary because the columns in the unweighted supermatrix already sum to 1. It would have been possible to do cluster comparisons – even though there are only two clusters – if one of the clusters had been inner dependent and linked to itself.

The Cluster Matrix is shown below in Figure 69. No cluster comparisons were made, so it has the default values of 1.0 for the non-zero components in the unweighted supermatrix: the (Alternatives, Advantages) component and the (Advantages, Alternatives) component. And it has the default values of 0.0 for the zero components which are the (Advantages, Advantages) component and the (Alternatives, Alternatives) component.

| Cluster Node Labels | Advantages | Alternatives |
|---|---|---|
| Advantages | 0.000000 | 1.000000 |
| Alternatives | 1.000000 | 0.000000 |

Done

**Figure 69.** The Cluster Matrix for the *Benefits criterion* Subnet.

| Cluster Node Labels | | Advantages | | | | Alternatives | | |
|---|---|---|---|---|---|---|---|---|
| | | 1Transport ation | 2Status | 3Sport s | 4Socializin g | 1America n | 2Japanes e | 3Europea n |
| Advanta ges | 1Transport ation | 0.082266 | 0.082266 | 0.082266 | 0.082266 | 0.082266 | 0.082266 | 0.082266 |
| | 2Status | 0.173686 | 0.173686 | 0.173686 | 0.173686 | 0.173686 | 0.173686 | 0.173686 |
| | 3Sport s | 0.165716 | 0.165716 | 0.165716 | 0.165716 | 0.165716 | 0.165716 | 0.165716 |
| | 4Socializin g | 0.078332 | 0.078332 | 0.078332 | 0.078332 | 0.078332 | 0.078332 | 0.078332 |
| Alternati ves | 1America n | 0.103611 | 0.103611 | 0.103611 | 0.103611 | 0.103611 | 0.103611 | 0.103611 |
| | 2Japanes e | 0.100104 | 0.100104 | 0.100104 | 0.100104 | 0.100104 | 0.100104 | 0.100104 |
| | 3Europea n | 0.296285 | 0.296285 | 0.296285 | 0.296285 | 0.296285 | 0.296285 | 0.296285 |

Done

**Figure 70.** The Limit Matrix for the *Benefits criterion* Subnet.

Figure 70 shows the limit matrix to which the weighted matrix converged when raised to powers. The limit matrix has been reached when all the columns are the same.

Elect the Computations Priorities command to get the view shown in Figure 71 of the priorities for all the nodes in the Benefit criterion subnet. These are simply the values from any column of the Limit Matrix. They are shown in two ways: the Limiting values are from the Limit Matrix. The Normalized by Cluster values are obtained by normalizing the priorities in each component so they sum to 1.0.

| Icon | Name | | Normalized by Cluster | Limiting |
|---|---|---|---|---|
| No Icon | 1Transporta~ | | 0.16453 | 0.082266 |
| No Icon | 2Status | | 0.34737 | 0.173686 |
| No Icon | 3Sports | | 0.33143 | 0.165716 |
| No Icon | 4Socializing | | 0.15666 | 0.078332 |
| No Icon | 1American | | 0.20722 | 0.103611 |
| No Icon | 2Japanese | | 0.20021 | 0.100104 |
| No Icon | 3European | | 0.59257 | 0.296285 |

Okay

**Figure 71.** The Priorities for the *Benefit criterion* Subnet.

Finally, select the Computations Synthesize command to get the synthesized priorities for the nodes in the Alternatives cluster in the *Benefits criterion* subnet. They are shown in Figure 72 below. The analysis that one might apply here is that for benefits the European car is preferred, almost 3 to 1, to either the Japanese car or the American car. Why don't we all drive European cars? Well, there are drawbacks as will be seen when we examine the costs and risks subnets.
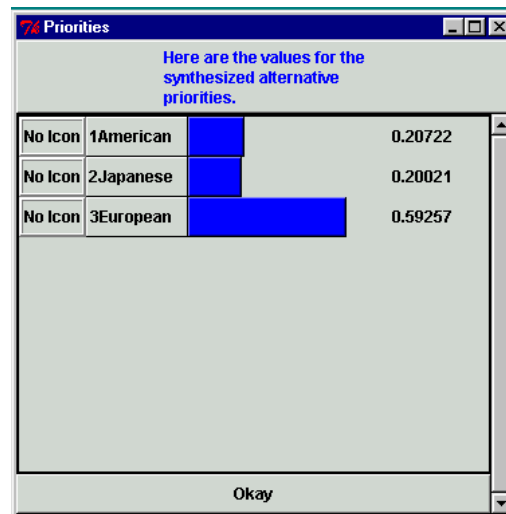
**Figure 72.** The Synthesized Priorities for the Alternatives in the *Benefits criterion* Subnet.

You may complete this model by making judgments in all of the subnets. After you have finished that, go to the top level Control network and select the Computations, Synthesize command to synthesize the results for the entire model as shown in Figure 73. These results show that overall the American car is slightly the best choice. Recall that the overall synthesis in the top-level control model used the additive form of the BOCR equation.



**Figure 73.** Overall Synthesized Priorities

Note that the Computations, Priorities command will give default equal priorities of .333 for each of the BCR nodes in the Control model, because though they are linked to the goal, no comparisons were made or distinguishing data entered as to their importance in this decision.

# SENSITIVITY

To display a sensitivity graph select Computations, Sensitivity from the menu of the top level model. (You can also perform sensitivity from lower levels.) You cannot perform sensitivity 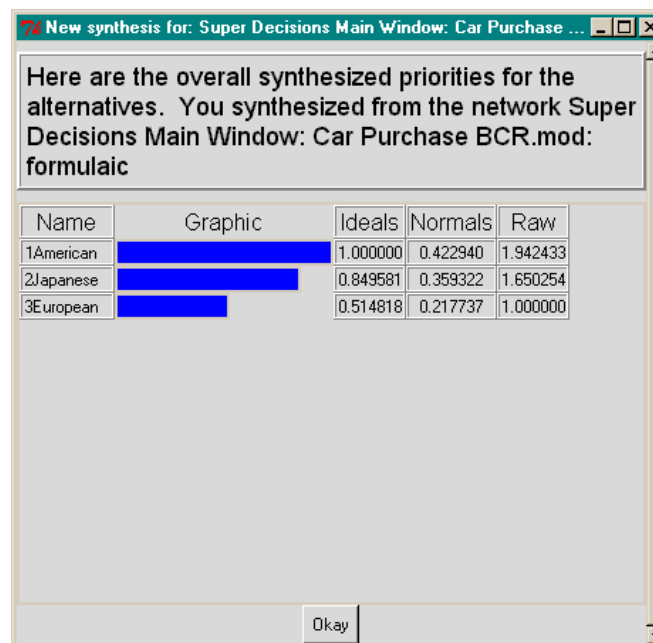for the merits of BOCR if the Multiplicative formula is selected, because the effects of changing their weights cancel out due to the formula. You must select one of the other formulas, for example, the Additive formula.

This is a what-if type of sensitivity that allows you to select any combination of independent variables. The initial sensitivity screen that appears always starts with one node selected, the first alphabetically in the network where the command was issued. The startup sensitivity screen for the Car Purchase model has the Goal Node selected as shown in Figure 74.
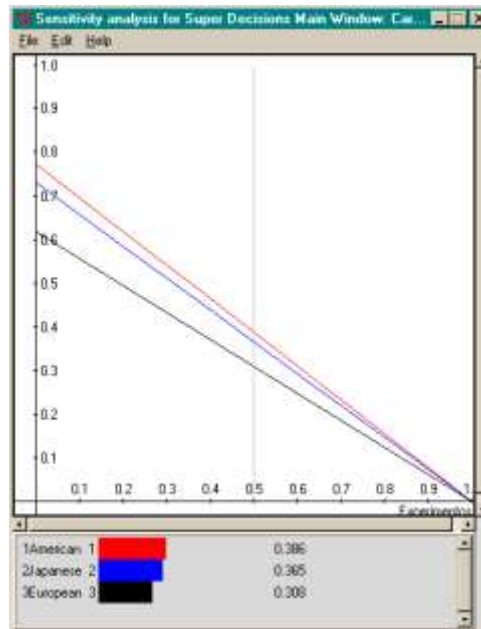


**Figure 74.** Startup Sensitivity Screen (with Goal as Selected Node) for *Car Purchase* Model

You need to change it to the Benefits node, by selecting the Edit, Independent Variable command on the Sensitivity analysis screen. The Sensitivity input selector will appear as shown in Figure 75.
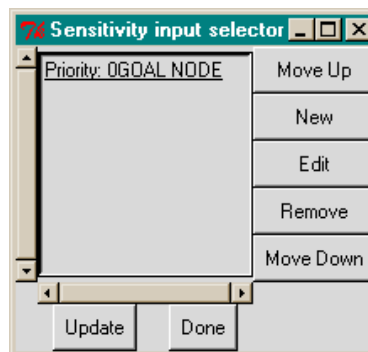


**Figure 75.** Sensitivity Input Selector

Remove the Goal node and select the New button to choose a new independent variable. From the New Parameter screen select the parameter type, the network and the independent variable node. You may also select Start and End values on the x-axis and the number of steps as shown in Figure 76:

| *Parameter Type* Options | *Network* selection Options | *Wrt Node* Options |
|---|---|---|
|  |  |  |
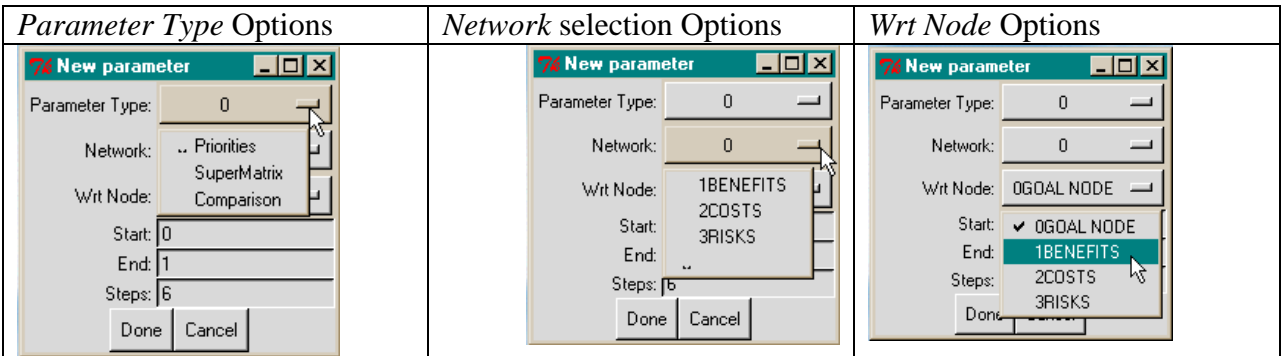
**Figure 76.** Selecting the Independent Variable from the New Parameter Screen.

Selecting Priorities sets the *Parameter Type* to zero. For *Network* the top-level network, network 0, is always automatically selected when this screen appears. For *Wrt Node* you need to select the node with respect to which you want to perform sensitivity, Benefits in this case. Leave the number of steps as they are, click the Done button, then click the Update button on the Sensitivity input selector screen to get the sensitivity graph shown in Figure 77 with the Benefits node selected.
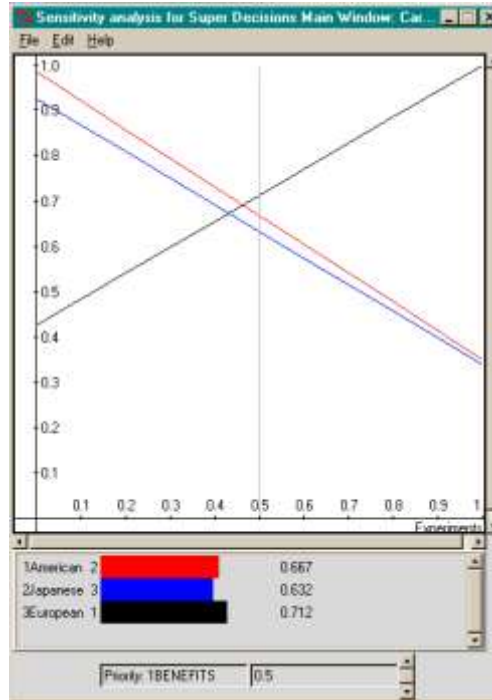
**Figure 77.** The Sensitivity Graph with Benefits Selected as the Dependent Variable

You may select more than one independent variable and perform sensitivity on two variables at a time. For each independent variable a range of values, and the number of steps, can be selected. Many points are generated for all the possible combinations of independent variable values. If there is a single independent variable selected, with a range of priority for that variable from .2 to .4 selected, with 4 steps (5 points of calculation) for sensitivity examination, there would be points for the corresponding alternative values plotted for the independent variable when its value is .2, .25, .30, .35, and .4. These points would be spread equally across the x-axis. The numbers on the x-axis relate only indirectly: the .30 value of Benefits would be plotted at 0.5. The lines for the alternatives are obtained by joining the points relating to a given alternative. Move the blue line back and forth by clicking and dragging and read the value of benefits and the alternative values (under the graph) for the current location of the blue line.

If there are three independent variables selected, with 4 steps, there would be $5^3$ different combinations of dependent variable values plotted. These are "experiments". The results of the 125 experiments are plotted along the x-axis (and simply joined to make them look like continuous lines). The Dimensions command is especially useful to stretch out the graph along the x-axis so you can examine more closely where the rank switches occur. Try using 800 or 1200 for the x-axis dimension.

Select Independent Variable from the Edit menu in Sensitivity Analysis to see the Sensitivity input selector shown in Figure 78. Note that the priority of Costs has been added in this figure.
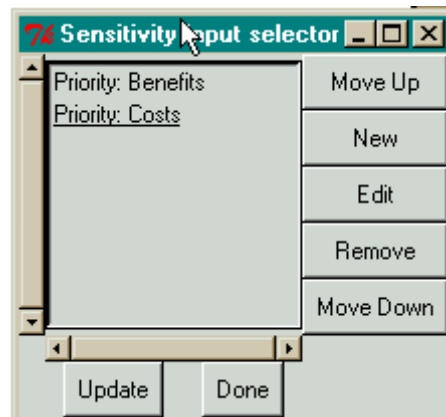
**Figure 78.** Sensitivity Dependent Variable Input Selector

Don't get carried away with the number of steps you select. The calculations can get huge if you have selected too many independent variables and lots of steps. Selecting Benefits, then Costs leads to the graph shown in Figure 79.

The vertical blue line is at .5 shows the results half-way through the 27 combinations of values of Benefits and Costs that were calculated and plotted. At that particular point in the combinations, the values for the independent variables and the alternatives are shown below the graph. You may move the blue line by clicking and dragging, then read off the intersect values from the graph. You may also elect to have the corresponding values of other parameters shown by selecting the Extra Params command on the Sensitivity Analysis Edit menu.
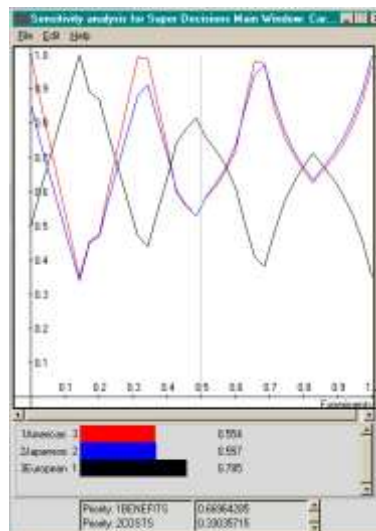


**Figure 79.** Sensitivity Graph with Benefits then Costs Selected as Dependent Variables

## EXPORTING SENSITIVITY DATA

To export the data points of the sensitivity plot to Excel, select File Save from the Sensitivity Analysis menu. Use a name ending in .txt when asked for a file name. Then load Excel, select the File, Open command and open the file. Be sure to change the type of file to *.* to display names of all files so the .txt file will show up. The Excel import wizard will then come up. Step through it selecting OK or Next to Finish at the end. The Excel spreadsheet containing the

sensitivity data points will then appear.  You can use it then for displaying the data in more ways and for data manipulation – such as arranging one of the alternative columns in decreasing order, so you can see the maximum value obtained.

# FILE BACKUP AND AUTOMATIC SAVING

The normal way to save is to save a complete model in a file with a .mod extension. You may also save parts of a model:
1) the current network only;
2) the current network and its sub-networks.

You may also save models in a compressed format.  Compressed models have file names with the double extension .mod.gz. Models saved in a compressed format take 2 or 3 seconds longer to save and to load, but are usually considerably smaller, depending on what attached files are included in it that may not compress so well. Models that have been saved in compressed format are opened in the same way as regularly saved models using the File Open command.

Use the *File Save* command to save the currently loaded model as a file with a .mod extension.

Use the *File Save As* command to save the currently loaded model under a new name.

Use the *Advanced Save* command to save the currently loaded model in a compressed format, or to save only portions of the currently loaded model. Using the Advanced Save Options you can select how much of the model you wish to save, and whether or not to compress:

## SAVING OPTIONS
- Whole Model
- Only this network and its subnets
- Only this network (no sub-networks of it)

**Compression Options:**
- No compression
- Compress this model only
- Compress models from now on

*Tip:  The File Save command may used to create a template of a subnet and start every subnet using the template.  As subnets always have the same cluster of alternatives, and often even other clusters are simila,r this may be a time-saver.  Select the File Open command in a subnet that has just been created and open the saved subnet which is serving as a template.  By doing File Save and selecting the entire model, you will save the newly created sub-network within the whole model and the original subnet will be preserved to use again.*

When sub-networks are minimized, they appear on the Windows menu bar at the bottom of the screen as open applications.  Simply click on them to restore them to full size. You may have all

the windows in a model open at once, the control network and all the sub-networks. You may also have other models open at the same time, and create other models.

Saving is automatic and you can change the interval at which the automatic save occurs from every 10 seconds to every 30 minutes by selecting the File, Configure command and the General tab.

# SPECIAL ANP SOFTWARE FUNCTIONS AND COMMANDS

## FORMULAS

Formulas are used to synthesize results for the alternatives. Generally only the top-level network has a formula, but a formula may be entered for each network in an entire multi-layer system.

### FORMULAS FOR COMPUTING SYNTHESIS RESULTS

It is not necessary to have a formula for a single network system as the results, for the nodes in the Alternatives cluster, come directly from the supermatrix of the network. A formula in the top-level network of a multi-layer system is used to combine results from the subnets for the alternatives. A formula gives the user control over how synthesis is done: additive, multiplicative and so on. If there is no formula, the synthesize command sums the results using simple addition. This is right only when there are no nodes with subnets that are "against" you such as costs and risks. There will be more about this when the various options for formulas are given at the end of this section. Synthesis results are presented in three forms as shown in Figure 80:

- **Ideal** The ideal form is obtained by dividing the raw value for each alternative by that of the largest alternative.
- **Normal** The normal form is obtained by summing the raw values for the alternatives and dividing each by the sum.
- **Raw** The raw numbers are obtained either from the supermatrix or by combining ideal values from subnets depending on the network where the synthesis command is used.
    - **Simple Network Model** If you synthesize in a model that consists of a single network with a cluster named Alternatives then the raw scores come directly from the supermatrix.
    - **Bottom Level Decision Network** If you synthesize from a bottom level decision network, the raw scores come directly from the supermatrix, because there is an alternatives cluster in the network and an alternatives component in the supermatrix.
    - **Intermediate Level Network** For intermediate networks there are no clusters named Alternatives (they are in bottom level decision networks), so alternative values are gotten only from nodes that have subnets that are feeding them up. The priorities of just those nodes that have subnets are re-computed by normalizing so they add up to one. This new priority for each node is multiplied times the ideal value for the alternatives in its subnet, and they are summed for all nodes with subnets. If the same alternative is best in all the subnets it will have a

> weight of one in the ideal form in the intermediate level network, but if it not the weight will be less than one.

The rationale for doing it this way is that the most information is retained by using the ideal values, for example, an alternative with an ideal score of .75 in a decision network is 75% as good as a perfect alternative. So making the raw values in intermediate networks ideal-based retains the information about how an alternative scores against an ideal. If you use normalized values this information would be lost.



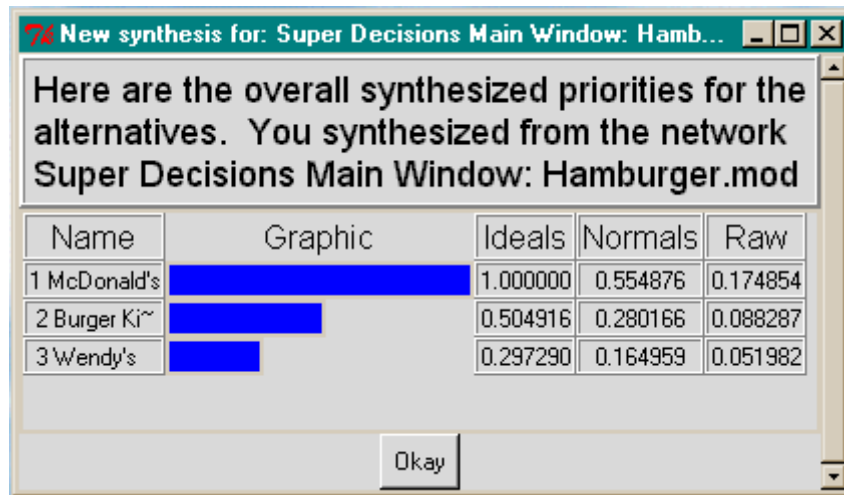**Figure 80.** Example of How Synthesis Results are Displayed in each Network – from the *Hamburger* Model.

Formula commands are under the Design, Standard Formulas menu shown in Figure 81. A formula may be set using the Design, Add/Edit formula and typing a formula into the Edit Network Formula dialog box shown in Figure 82.
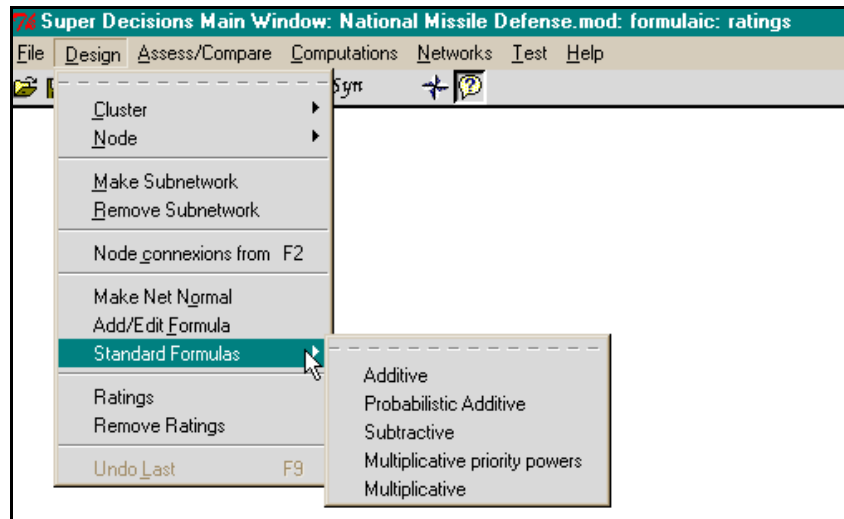


**Figure 81.** Commands for Editing and Selecting Formulas on the Design Menu.

**Figure 82.** The Edit Network Formula Dialog Box for Editing/Showing Formula for Current Network.

The formula shown in Figure 82 is the one for the top-level network in the *National Missile Defense* model. It is the Additive Formula and was selected from the list of standard formulas under the Design menu command. The title bar of the network window has the word Formulaic on it, meaning the formula that appears in the dialog box is being used in the synthesis process.

## THE STANDARD FORMULAS

You can select a formula from the Standard formula list, and you can change it at any time. If no formula is selected, it means use the Alternatives' priorities from the limit supermatrix if you have a single network model, or add the results for the Alternatives coming up from subnets for any nodes that have subnets. This formula is somewhat mindless and simply adds regardless of whether the nodes with subnets are "for" you or "against" you. Smarter formulas invert the values under nodes named Costs and Risks, so that the worst alternatives get smaller values. In general you should select a formula if you have a multi-layer model.

- **Additive:** This formula uses un-inverted values for the alternative values coming up from subnets under Benefits and Opportunities and inverted values for those coming up from Costs and Risks subnets. The inverted values are "flipped" so that the most costly alternative, for example, now has the lowest priority. This is accomplished by taking the inverse of each alternative's priority. If the priority is $a_1$ for example, its inverse is $1/a_1$. Sum these inverses for all the alternatives, and divide each inverse by the sum. Then re-normalize. This will completely flip the priorities the other way around, and they will maintain their proportionate values. Weight the alternative values appropriately by the respective priorities of the BOCR in the top-level network, use priorities obtained from the Ratings spreadsheet if there is one, add and normalize to get the final result. The formula is:

$NormalNet(Benefits)*$SmartAlt(Benefits) + $NormalNet(Costs)*$SmartInvAlt(Costs) + $NormalNet(Opportunities)*$SmartAlt(Opportunities) + $NormalNet(Risks)*$SmartInvAlt(Risks)

- **Probabilistic Additive:** This formula treats the Costs and Risks values from the subnets something like probabilities. If a likelihood of an occurrence is p, then the likelihood of not having an occurrence is 1-p. Since the high cost and high risk alternatives have the highest priorities, using the formula one minus the value coming up from the subnet changes the highest priorities from most costly (risky) to least costly (risky), and adding is appropriate.

  $NormalNet(Benefits)*$SmartAlt(Benefits) + $NormalNet(Costs)*(1-$SmartAlt(Costs)) + $NormalNet(Opportunities)*$SmartAlt(Opportunities) + $NormalNet(Risks)*(1-$SmartAlt(Risks))

- **Subtractive:** This formula leaves the most costly and risky alternatives with the highest priorities, as they come up from the subnets, but subtracts from the benefits and opportunities. It can end up giving negative results. If resource allocation is the objective, Alternatives with negative results probably should not receive any investment.

  $NormalNet(Benefits)*$SmartAlt(Benefits) + $NormalNet(Costs)*(-$SmartAlt(Costs)) + $NormalNet(Opportunities)*$SmartAlt(Opportunities) + $NormalNet(Risks)*(-$SmartAlt(Risks))

- **Multiplicative Priority Powers:** Here the alternative values coming up from the subnets are raised to the power of the BO and CR (in the main network, or from ratings if they were done that way), where the C and R powers are negative, that is the BO product is divided by the CR product.

  [root $SmartAlt(Benefits) $IdealNet(Benefits)] * [neg_root $SmartAlt(Costs) $IdealNet(Costs)] * [root $SmartAlt(Opportunities) $IdealNet(Opportunities)] * [neg_root $SmartAlt(Risks) $IdealNet(Risks)]

- **Multiplicative:** This is the original BO/CR formula. The alternative values coming up from the subnets for B and O are multiplied and the result is divided by the product of the alternative values coming up from the subnets for C and R. In this case, even if the BOCR are weighted, it would not change the result from leaving them equal, as their weights would cancel out because of the formula. This is a marginal utility type of formula.

  $SmartAlt(Benefits) * [invert $SmartAlt(Costs)] * $SmartAlt(Opportunities) * [invert $SmartAlt(Risks)]

Some definitions of the terms used in the formulas follow:
- **Alt(Node name)** refers to the alternative values in the subnet belonging to Node name
- **SmartAlt(Node name)** means to pick the best form of the alternative values to pass up: if the subnet is a bottom level one, pass up the Ideal values from synthesis in the subnet;

if it is a bottom level subnet where the alternatives' values come from ratings, or if it is an intermediate level subnet, pass up the Raw values from synthesis in the subnet.

- **Net(Node name)** means to use the priority value of Node name in the current network
- **IdealNet(Node name)** means to use the ideal value of Node name in the current network
- **NormalNet(Node name)** means to use the normalized value of Node name in the current network. If Node name appears in an attached ratings spreadsheet use its normalized value from there.
- **InvAlt(Node name)** means to flip the values of the nodes in the subnet so that the node formerly having the largest value (most costly, most risky, for example), now has the smallest value (least costly, least risky). This is done by inverting the value of each alternative, summing, dividing the inverted value by the sum and re-normalizing.
- **invert (Node name)** means to invert the value of Node name by taking 1/ Node name. It is not the same as the flipping described above.

The five formulas lead to the following synthesis results, shown in normalized form:

| Alternatives | Additive | Probabilistic Additive | Subtractive | Multiplicative Priority Powers | Multiplicative |
|---|---|---|---|---|---|
| *Deploy* | .331 | .333 | .201 | .556 | .493 |
| *Global Defense* | .291 | .307 | .107 | .371 | .379 |
| *R&D* | .212 | .223 | -.193 | .067 | .220 |
| *Termination* | .165 | .137 | -.500 | .006 | .018 |

Notice that the ranks of the alternatives are the same regardless of which formula is used, though this is not always necessarily so. The formula one picks depends on the use one wants to make of the results. If the purpose is to simply pick the best alternative, any one of the five formulas will do.

You can try out various formulas after your model is completed. Simply change the formula by selecting the new one you want and synthesize.

The **meaning of $ when used in a formula**: The **$** in the formula means to compute the results for each alternative in turn.

## SYNTHESIS RESULTS

The **synthesis results** for the alternatives can be displayed from any network or sub-network. The results for any network are obtained by synthesizing all subnets (if there are any) up through the network where the command is issued. Results are presented in three different ways:

**Raw** If the network is a bottom level decision network the values for each alternative are drawn directly from the Limit Matrix for that network. If the network is an intermediate network containing control criteria, the nodes that are the control criteria are normalized and multiplied times the Ideal values of the alternative vectors coming from the networks below each of them. These weighted vectors are then added to yield the Raw score vector. When the word Totals is used in a formula, it means use the Raw numbers from synthesis in the subnet below.

**Normal**  A vector of priorities for the alternatives is normalized by adding the elements in it, and dividing each element by the sum to yield the normalized vector.  The sum of the numbers in the normalized vector is 1.

**Ideal** The Totals priority vectors above are converted to Idealized values by dividing each element in the vector by the largest, so that the best alternative gets a priority of 1 and the others get their proper proportion less than 1.

## ATTACHMENTS

Other files that relate to a model may be attached within it using the paperclip attachment icon shown below.

For example, a model to determine Ford's best strategy with respect to the Ford Explorer/Firestone Tire problem in which the tires were suspected of contributing to rollover accidents in Explorer SUV's (Sport Utility Vehicles) is shown in Figure 83.  It had several associated files attached within it: a text file, a powerpoint slide show, an Expert Choice AHP model and a report document written in Word.   The files become permanently attached to the model.

To launch an attached file, double-click on it.  When the attached file is open, you cannot return to the *SuperDecisions* software.  You must close the attached file to return to the model.
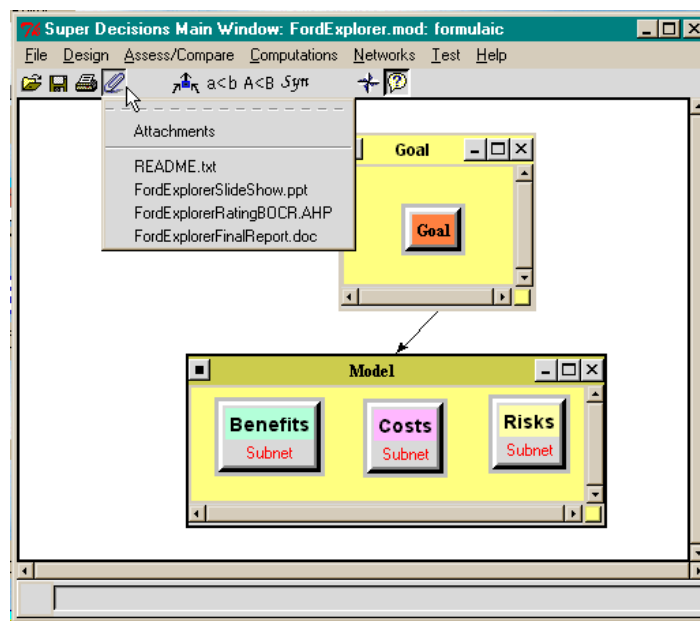


**Figure 83.**  Attached Files within a Model

## PRINTING

Printing is done from the Computations Full Report menu. One can also use screen captures. Display what you wish to print in a window or dialogue box and press <Alt> and <PrtSc> simultaneously to capture the currently active window. Then go to a Word document and select the command Edit Paste and paste the entire window into the document. It is difficult to display large supermatrices in this way. In that case it may be better to export as a text file to Excel.

Select the File Export command to export the Unweighted Super Matrix, the Weighted Super Matrix, the Limit Matrix and the Cluster Matrix as text files. You can then import them into Excel and use Excel's formatting and printing capabilities. For example, the Unweighted Super Matrix for this model will be exported to the text file: *aejcar.unweighted.txt.* To import it into Excel, launch Excel, select the File Open command and select the .txt file from the subdirectory where it is. Remember to set the Excel file type to all files (\*.\*) so it will appear. Then use the Excel import wizard. Just click through the wizard using the default types and the file will open as an Excel spreadsheet with cluster and element labels.

## SOME CHARACTERISTICS OF SUB-NETS

- They can be different in structure, connections and factors, except for the alternatives, which must be the same.
- They may, of course, be different with regard to their judgments.
- It is critical to observe the exact spelling of the common set of alternatives.

The only requirement in building the control model relates to the final result. As the various ratios have to be computed for the alternatives across several sub-networks they need to be identified to the software by being in a cluster labeled Alternatives in each sub-network. The elements we identify as being alternatives are the focus of our concern for which we would like to know the BOCR or general results. So whenever the Computations Synthesize command is used, it gives the results for whatever is in the component we have labeled as Alternatives.

# QUICK STEPS FOR DOING A COMPLEX MODEL WITH RATINGS

1. Create the top-level model with the BOCR in a cluster together. Build the strategic criteria structure for rating the BOCR; use a goal cluster with a goal node, a strategic criteria cluster with the main criteria and a cluster for each of the main criteria containing their strategic subcriteria. Pairwise compare the main criteria with respect to the goal and pairwise the strategic subcriteria with respect to their strategic criterion.
2. Create a Ratings structure attached to this network using the Build Ratings command.
   - Use the Edit Criteria command in the Ratings spreadsheet and select the strategic subcriteria you want to use to rate the BOCR;
   - Use the Edit Alternatives, New command to create the alternatives. Create alternatives named Benefits, Opportunities, Costs and Risks.

- Create categories for each of the strategic subcriteria, (a category is, for example, Hi, Med, Low).
- Select the comparisons button on the category creation screen and pairwise compare Hi, Med, Low to establish priorities for them. You can save and re-use categories.
- Rate the BOCR by selecting the appropriate category in each cell in the spreadsheet.
- Use the Calculations Totals command in Ratings to obtain the raw total scores for each of the BOCR; use the Calculations Priorities command in Ratings to normalize the raw scores. The normalized values are used for the priorities of the BOCR to do synthesis in the top-level network.

3. Create subnets for each of the BOCR. These are called the control criteria subnets.
   - Build a hierarchical structure in each subnet, with a goal cluster having a goal node, a cluster of main criteria, and a cluster for the subcriteria of each of the criteria.
   - Link the goal to the criteria, and each criterion to its subcriteria. Pairwise compare throughout for importance to establish priorities for the subcriteria. These subcriteria are your potential "control criteria".
   - Do the Computations, Priorities command and look for the high priority subcriteria in the Limiting column. Pick some of them to become your control criteria. You should have enough of them to cover about 70% to 80% of the total priority (for the subcriteria).

4. Build subnets for each of the selected control criteria.
   - These are called the decision subnets. There must be an alternatives cluster in every lowest level subnet along with other clusters. Subnets often look similar, so it is often useful to make a template at this point and reuse it as you build the rest of the subnets.
   - Link the nodes appropriately and do the pairwise comparisons throughout. In Benefits decision subnets ask the pairwise comparison question as to which is most beneficial. Similarly for Opportunities. In Risks and Costs decision subnets ask the pairwise comparison question as to which is riskiest or costliest. The worst alternative gets the highest priority for Risks and Costs.

5. Get results from the top-level model by using the synthesize command.
   - Set your formula using the Design Standard Formulas command and selecting the one you want. Start with the Multiplicative formula and record the results for your report; then use the Additive formula and the Subtractive formula.

6. Perform sensitivity by selecting the Additive formula and do the sensitivity graph for each of the BOCR nodes. You cannot perform sensitivity using the Multiplicative or Subtractive formulas. You can also do sensitivity for other nodes, for example, control criteria nodes in the subnets. All sensitivity graphs should be done from the top-level network. Pick the node you want to do sensitivity for by selecting it using the New Parameter selector within the sensitivity process.

# REFERENCES

The following list of books about the Analytic Hierarchy Process and the Analytic Network Process are available from RWS Publications, 4922 Ellsworth Avenue, Pittsburgh, PA 15213, www.rwspublications.com, phone: 412-621-4492; fax: 412-681-4510,.

*The Analytic Network Process: Decision Making with Dependence and Feedback* Thomas L. Saaty, 386 pp., RWS Publ., 2001.  ISBN 0-9620317-9-8. This book shows how to make decisions when alternatives depend on criteria, but also the criteria depend on the alternatives.  It  shows how to cope with dependence between different groups of people, goals and criteria.  The Analytic Network Process is particularly useful to project the future of a group or company considering all the influences and risks: economic, social, political, technological, environmental, and others.  Accompanying ANP software is under development.  Softcover.

*The Brain: Unraveling the Mystery of How it Works, The Neural Network Process*  T.L. Saaty, 481 pp., RWS Publ.,  1999. ISBN 1-888603-02-X. This work confirms what many contemporary thinkers have claimed: that all human actions, sensations, thoughts and even emotions are derived from the synthesis of neural firings in the brain.  How this happens and the precise nature of their interaction, feedback, and synthesis, however, has not been fully described – until now.  Hardcover.

*Creative Thinking, Problem Solving & Decision Making*  T.L. Saaty, 267 pp., RWS Publ.,  2001.  ISBN-1-888603-03-8. Includes powerpoint slide presentation on CD. Creative Thinking, Problem Solving and Decision Making is a rich collection of ideas, incorporating research by a growing body of researchers and practitioners, profiles of creative people, projects and products, theory, philosophy, physics and metaphysics…all explained with a liberal dose of humor.  Exercises at the end of each chapter help readers build their creative "muscles' and develop skills and the confidence to successfully meet challenges. Hardcover.

*Decision Making in Economic, Social and Technological Environments*  Vol. VII, AHP Series, Thomas L. Saaty and Luis G. Vargas, 330 pp., RWS Publ., 1994.  ISBN 0-9620317-7-1.  This book is a collection of selected applications of the AHP on economics, social and political sciences, and technological design.  This volume along with other volumes on decision making, planning, conflict resolution and forecasting, rounds out the diversity of application areas.  Softcover.

***Decision Making for Leaders*** Vol. II, AHP Series, Thomas L. Saaty, 315 pp., RWS Publ., 2001 (new ed.). ISBN 0-9620317-8-X. This is a book of case studies in multicriteria decision-making using the Analytic Hierarchy Process. The basics of the theory are described in a clear, non-technical manner with many examples. Suitable for business leaders and also the best book for introducing the AHP to students. Softcover.
***Toma De Decisiones Para Líderes*** Spanish translation, RWS Publ., 1997. ISBN 0-888603-01-1. Softcover.

***The Fundamentals of Decision Making and Priority Theory with the Analytic Hierarchy Process*** Vol. VI, AHP Series, Thomas L. Saaty, 478 pp., RWS Publ., 2000 (revised). ISBN 0-888603-01-1. This book is a comprehensive summary, primarily of the author's own thinking and research, about the Analytic Hierarchy Process and decision making. It includes advanced mathematical theory and diverse applications, plus 70 pages of references to articles on the AHP. *Fundamentals of Decision Making* has all the latest theoretical developments in the AHP and new theoretical material not published elsewhere. Softcover.

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