Complex Module Formulas

- **Forms of Complex Number:**

$z = x + iy \rightarrow$ Cartesion Form

$z = r\,[\cos\theta + i\sin\theta] \rightarrow$ Polar Form

$z = re^{i\theta} \rightarrow$ Exponential Form

- **Short cuts:**

i) $z \cdot \bar{z} = x^2 + y^2$

ii) $z + \bar{z} = 2\,Re(z)$

iii) $z - \bar{z} = 2i\;Im(z)$

- **Modulus & Argument:**

i) Modulus $= |z| = r = \sqrt{x^2+y^2}$

ii) Argument $= \theta = \tan^{-1}\left(\dfrac{y}{x}\right)$

$$= \tan^{-1}\left(\frac{Im(z)}{Re(z)}\right)$$

- Exponential Form :

i) $e^{i\theta} = \cos\theta + i\sin\theta$

ii) $e^{-i\theta} = \cos\theta - i\sin\theta$

iii) $e^{i\theta} + e^{-i\theta} = 2\cos\theta$

iv) $e^{i\theta} - e^{-i\theta} = 2i\sin\theta$

- Argument Tricks :

$\arg(z \cdot z_2) = \arg z_1 + \arg . z_2$

$\arg\left(\frac{z_1}{z_2}\right) = \arg z_1 - \arg z_2$

- De - Moivre's Theorem :

i) $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta = z^n$

ii) $(\cos\theta + i\sin\theta)^n = \cos n\theta - i\sin n\theta = z^{-n}$

iii) $\cos n\theta = \frac{1}{2}(z^n + z^{-n})$

iv) $\sin n\theta = \frac{1}{2i}(z^n - z^{-n})$

v) $x^n + \frac{1}{x^n} = 2\cos n\theta$

vi) $x^n - \frac{1}{x^n} = 2i\sin n\theta$

PAGE No.
DATE / /

vii) Roots of Complex number:

$$(\cos\theta + i\sin\theta)^{1/n} = \cos\left(\frac{2K\pi+\theta}{n}\right) + i\sin\left(\frac{2K\pi+\theta}{n}\right)$$

By putting $K = 0, 1, 2, \ldots, n-1$
We get n roots.

- Trignometric Functions in terms of Exponential

i) $\cos\theta = \dfrac{e^{i\theta} + e^{-i\theta}}{2}$

ii) $\sin\theta = \dfrac{e^{i\theta} - e^{-i\theta}}{2i}$

- Hyperbolic Functions
- Basic Formula:

i) $\sinh x = \dfrac{e^{x} - e^{-x}}{2}$

ii) $\cosh x = \dfrac{e^{x} + e^{-x}}{2}$

iii) $\tanh x = \dfrac{e^{x} - e^{-x}}{e^{x} + e^{-x}}$

iv) $e^{x} = \cosh(x) + \sinh(x)$

- Relationship between Circular and Hyperbolic :

i) $\sin(ix) = i\sinh(x)$ ii) $\sinh(ix) = i\sin(x)$

iii) $\cos(ix) = \cosh(x)$ iv) $\cosh(ix) = \cos(x)$

v) $\tanh(ix) = i\tan(x)$ vi) $\tan(x) = -i\tanh(ix)$

- Identities :

i) $\cosh^2(x) - \sinh^2(x) = 1$

ii) $\text{sech}^2(x) + \tanh^2(x) = 1$

iii) $\coth^2(x) - \text{cosech}^2(x) = 1$

- Important Expansions

i) $\sinh(x \pm y) = \sinh(x)\cdot\cosh(y) \pm \cosh(x)\cdot\sinh(y)$

ii) $\cosh(x \pm y) = \cosh(x)\cdot\cosh(y) \pm \sinh(x)\cdot\sinh(y)$

iii) $\tanh(x \pm y) = \dfrac{\tanh(x) \pm \tanh(y)}{1 \pm \tanh(x)\cdot\tanh(y)}$

- Formulae for $2x$ and $3x$

i) $\sinh(2x) = 2\sinh(x)\cdot\cosh(x)$

ii) $\cosh(2x) = \cosh^2(x) + \sinh^2(x)$
$= 2\cosh^2(x) - 1$
$= 2\sinh^2(x) + 1$

iii) $\tanh(2x) = \dfrac{2\tanh(x)}{1+\tanh^2(x)}$

iv) $\sinh(3x) = 3\sinh(x) + 4\sinh^3(x)$

v) $\cosh(3x) = 4\cosh^3(x) - 3\cosh(x)$

vi) $\tanh(3x) = \dfrac{3\tanh(x) + \tanh^3(x)}{1 + 3\tanh^2(x)}$

vii) $\sinh(x) = \dfrac{2\tanh(x/2)}{1 - \tanh^2(x/2)}$

viii) $\cosh(x) = \dfrac{1+\tanh^2(x/2)}{1-\tanh^2(x/2)}$

ix) $\tanh(x) = \dfrac{2\tanh(x)}{1+\tanh^2(x/2)}$

- Product Formulae :

i) $\sinh(x+y) + \sinh(x-y) = 2\sinh(x)\cdot\cosh(y)$

ii) $\sinh(x+y) - \sinh(x-y) = 2\cosh(x)\cdot\sinh(y)$

iii) $\cosh(x+y) + \cosh(x-y) = 2\cosh(x)\cdot\cosh(y)$

iv) $\cosh(x+y) - \cosh(x-y) = 2\sinh(x)\cdot\sinh(y)$

v) $\sinh(x) + \sin(y) = 2\sinh\left(\dfrac{x+y}{2}\right)\cdot\cosh\left(\dfrac{x+y}{2}\right)$

vi) $\sinh(x) - \sinh(y) = 2\cosh\left(\frac{x+y}{2}\right)\cdot \cancel{\cosh}\,\sinh\left(\frac{x-y}{2}\right)$

vii) $\cosh(x) + \cosh(y) = 2\cosh\left(\frac{x+y}{2}\right)\cdot \cosh\left(\frac{x-y}{2}\right)$

viii) $\cosh(x) - \cosh(y) = 2\sinh\left(\frac{x+y}{2}\right)\cdot \sinh\left(\frac{x-y}{2}\right)$

- Differentiation and Integration Formula

i) $\frac{d(\sinh(x))}{dx} = \cosh(x)$

ii) $\frac{d(\cosh(x))}{dx} = \sinh(x)$

iii) $\frac{d(\tanh(x))}{dx} = \text{sech}^2(x)$

iv) $\int \cosh(x)\cdot dx = \sinh(x)$

v) $\int \sinh(x)\cdot dx = \cosh(x)$

vi) $\int \text{sech}^2(x)\cdot dx = \tanh(x)$

PAGE No.
DATE / /

- Inverse Hyperbolic Functions :-

$$\sinh^{-1}(z) = \log(z+\sqrt{z^2+1})$$

$$\cosh^{-1}(z) = \log(z+\sqrt{z^2-1})$$

$$\tanh^{-1}(z) = \frac{1}{2}\log\left(\frac{1+z}{1-z}\right)$$

- Inverse Hyperbolic ke Integration

$$\int \frac{dx}{\sqrt{x^2+a^2}} = \sinh^{-1}\left(\frac{x}{a}\right)$$

$$\int \frac{dx}{\sqrt{x^2-a^2}} = \cosh^{-1}\left(\frac{x}{a}\right)$$

$$\int \frac{dx}{a^2-x^2} = \frac{1}{a}\tanh^{-1}\left(\frac{x}{a}\right)$$

↑

Differentiation ke liye isko ulta kro

(waise kam nahi ayga)

PAGE No.
DATE / /

- Logarithm of Complex Number

i) $\log(x+iy) = \log r + i\theta$

ii) $\log(x+iy) = \frac{1}{2}\log(x^2+y^2) + i\tan^{-1}\left(\frac{y}{x}\right)$

$\hookrightarrow$ Principal value

iii) $\text{Log}(x+iy) = 2n\pi i + \log(x+iy)$

iv) $\text{Log}(x+iy) = \log r + i(2n\pi + \theta)$

$\hookrightarrow$ General value

Log $\neq$ log. log is used for principal
Log is for general