Assignment Problem Sheet

DEPARTMENT OF MATHEMATICS

NIT PATNA

Submit the assignment with the answers to any ten questions.

1. Find the solution of the differential equation $(6x - 4y + 1)dy - (3x - 2y + 1)dx = 0$.

2. Solve $y' + 2y\tan x = \sin x$ given that $y = 0$ when $x = \frac{\pi}{3}$.

3. Show that the following equations are exact differential equations and solve them

   a. $x(x^2 + y^2 - a^2)dx + y(x^2 - y^2 - b^2)dy = 0$.

   b. $(x - 2e^y)dy + (y + x\sin x)dx = 0$.

4. Solve the equations $(2x + 1)^2 y'' - 2(2x + 1)y' - 12y = 6x$.

5. Find the extremum value of the function $u = x^m y^n z^p$, subject to the condition $x + y + z = a$.

6. Let $y = \sin(m\sin^{-1} x)$. Then prove that

$$(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} + (m^2 - n^2)y_n = 0.$$

7. Expand the function $f(x, y) = \sin xy$ around the point $(1, \frac{\pi}{2})$ in Taylor's series up to the second degree terms.

8. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be defined as

$$f(x, y) = \begin{cases} \dfrac{xy^2}{x^2 + y^4}, & x \neq 0 \\ 0 & x = 0 \end{cases}.$$

   Prove that f possesses first order partial derivatives at $(0, 0)$, but it is not differentiable at $(0, 0)$.

9. Let $y = F(x, t)$ where F is differentiable function of two independent variables x and t which are related to two variablesu, v by the relations:

$$u = x + ct, \quad v = x - ct.$$

   Prove that the equation,

$$\frac{\partial^2 y}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 y}{\partial t^2} = 0$$

   can be transformed into $\frac{\partial^2 y}{\partial u \partial v} = 0$.

10. Solve $y' + 2y\tan x = \sin(x)$ given that $y = 0$ when $x = \frac{\pi}{4}$

11. Solve $\frac{d^2y}{dx^2} + y = \tan x$.

12. Find the general solution of the following equation:

$$x^2\frac{d^2y}{dx^2} + 4x\frac{dy}{dx} + 2y = x\log x$$

13. Solve the following equation using the method of variation of parameters:

$$\frac{d^2y}{dx^2} + y = \sec x$$

14. Find the third order Taylor polynomial at the point (1,-1,1) of $f(x, y, z) = 2xy^2 + xz^3$

15. If $u = x\phi_1\left(\frac{y}{x}\right) + \phi_2\left(\frac{y}{x}\right)$ then show that

$$x^2 u_{xx} + 2xy u_{xy} + y^2 u_{yy} = 0$$

16. Expand $f(x, y) = e^x \log(1 + y)$ as a Maclaurin series with terms upto third degree terms of x and y.

17. Solve the following equation:

$$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 13y = 8e^{3x}\sin 2x$$

18. Find the general solution of the following equation by converting it into linear equation:

$$x\frac{dy}{dx} + y = y^2 \log x$$

19. Check whether the following equation is exact or not and then solve it:

$$(3y + 4xy^2)dx + (2x + 3x^2y)dy = 0$$

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