National Institute of Technology Patna
Department of Mathematics
Mid Semester Examination : March 2022

Course Name: **Engineering Mathematics-I** Course Code: MA14102
Program: CSE-I.
Duration: 2 Hrs Full Marks: 30

Answer all the questions

1. For the matrix $A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 0 & -1 & -1 \end{bmatrix}$, find non-singular matrices P and Q such that PAQ reduces to the normal form and hence find its rank.

[4]

2. Show that the matrix $A = \begin{bmatrix} 3 & 1 & -1 \\ -2 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}$ is diagonalizable. Hence, find P such that $P^{-1}AP$ is a diagonal matrix. Then, obtain the matrix $B = A^2 + 5A + 3I$.

[4]

3. If W be the subspace of $\mathbb{R}^4$ generated by the vectors $(1, -2, 5, -3)$, $(2, 3, 1, -4)$, and $(3, 8, -3, -5)$, find a basis and the dimension of W.

[4]

4. If V be the vecor space of all polynomials of degree ≤ 3, determine whether or not the set $S = \{t^3, t^2 + t, t^3 + t + 1\}$ spans V?

[4]

5. Using $\delta - \epsilon$ approach show that

$$\lim_{(x,y,z)\to(0,0,0)} \left(\frac{xy + yz + zx}{\sqrt{x^2 + y^2 + z^2}} \right) = 0.$$

[4]

6. Show that the function

$$f(x,y) = \begin{cases} \frac{xy}{x^2+2y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$$

is not continuous at $(0,0)$ but its partial derivatives f_x and f_y exist at $(0,0)$.

[4]

7. If $u(x,y) = \cos^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, $0 < x, y < 1$, then prove that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = -\frac{1}{2}\cot u.$$

[3]

8. Solve the differential equation

$$(x^2y - 2xy^2)dx - (x^3 - 3x^2y)dy = 0.$$

[3]

***** All the Best *****