LABORATORY MANUAL FOR KATER'S PENDULUM: TO FIND g AT A PLACE

Experiment No-01
Kater's Pendulum

AIM

To determine *g*, the acceleration of gravity at a particular location.

APPARATUS

1. Kater's pendulum,
2. Stopwatch,
3. Meter scale and
4. Knife edges (Installed on a table).

THEORY

Kater's pendulum, shown in Fig. 1, is a physical pendulum composed of a metal rod 1.20 m in length, upon which are mounted a sliding metal weight W_1, a sliding wooden weight W_2, a small sliding metal cylinder w, and two sliding knife edges K_1 and K_2 that face each other. Each of the sliding objects can be clamped in place on the rod. The pendulum can suspended and set swinging by resting either knife edge on a flat, level surface. The wooden weight W_2 is the same size and shape as the metal weight W_1. Its function is to provide as near equal air resistance to swinging as possible in either suspension, which happens if W_1 and W_2, and separately K_1 and K_2, are constrained to be equidistant from the ends of the metal rod. The centre of mass G can be located by balancing the pendulum on an external knife edge. Due to the difference in mass between the metal and wooden weights W_1 and W_2, G is not at the centre of the rod, and the distances h_1 and h_2 from G to the suspension points O_1 and O_2 at the knife edges K_1 and K_2 are not equal. Fine adjustments in the position of G, and thus in h_1 and h_2, can be made by moving the small metal cylinder w.

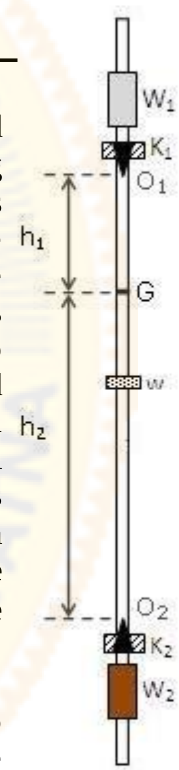


Figure 1

In Fig. 1, we consider the force of gravity to be acting at G. If h_i is the distance to G from the suspension point O_i at the knife edge K_i, the equation of motion of the pendulum is

$$I_i\ddot{\theta} = -Mgh_i \sin\theta$$

where I_i is the moment of inertia of the pendulum about the suspension point O_i, and i can be 1 or 2. Comparing to the equation of motion for a simple pendulum

$$Ml_i^2\ddot{\theta} = -Mgl_i \sin\theta$$

we see that the two equations of motion are the same if we take

$$\frac{Mgh_i}{l_i} = \frac{g}{l_i} \qquad (1)$$

It is convenient to define the radius of gyration of a compound pendulum such that if all its mass M were at a distance from O_i, the moment of inertia about O_i would be I_i, which we do by writing

$$I_i = Mk_i^2$$

Inserting this definition into equation (1) shows that

$$k_i^2 = h_i l_i \qquad (2)$$

If I_G is the moment of inertia of the pendulum about its centre of mass G, we can also define the radius of gyration about the centre of mass by writing

$$I_G = Mk_G^2$$

The parallel axis theorem gives us $\quad k_i^2 = h_i^2 + k_G^2$

so that, using (2), we have $\quad l_i = \frac{h_i^2+k_G^2}{h_i}$

The period of the pendulum from either suspension point is then

$$T_i = 2\pi\sqrt{\frac{l_i}{g}} = 2\pi\sqrt{\frac{h_i^2+k_G^2}{gh_i}} \quad ----------------(3)$$

Squaring (3), one can show that

$$h_1T_1^2 - h_2T_2^2 = \frac{4\pi^2}{g}(h_1^2 - h_2^2)$$

and in turn,

$$\frac{4\pi^2}{g} = \frac{h_1T_1^2 - h_2T_2^2}{(h_1+h_2)(h_1-h_2)} = \frac{T_1^2+T_2^2}{2(h_1+h_2)} + \frac{T_1^2-T_2^2}{2(h_1-h_2)}$$

which allows us to calculate g,

$$\boxed{\mathbf{g} = 8\pi^2\left[\frac{T_1^2+T_2^2}{h_1+h_2} + \frac{T_1^2-T_2^2}{h_1-h_2}\right]^{-1}}$$

Here, T_1: time periods of the oscillating pendulum from knife-edge K_1

T_2: time periods of the oscillating pendulum from knife-edge K_2

h_1: distances between knife-edges K_1 and CG of the pendulum

h_2: distances between knife-edges K_2 and CG of the pendulum

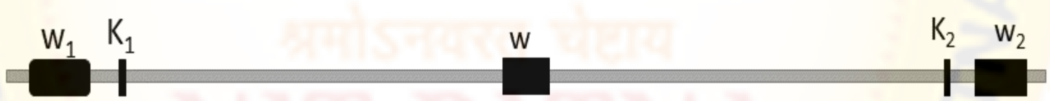
PROCEDURE

- Balance the pendulum on a sharp wedge and mark the position of its center of gravity. Measure the distance of the knife-edge K_1 as h_1 and that of K_2 as h_2 from the center of gravity.
- Choose the position of knife edge, steel and wood cylinder by changing the sliders for it.
- Shift the weight W_1 to one end of Kater's pendulum and fix it. Fix the knife edge K_1 just below it.

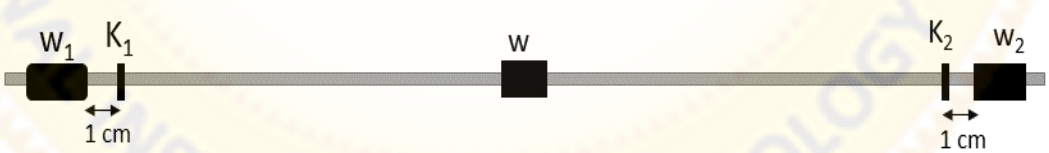
- Keep the knife edge K_2 at the other end and fix the wooden weight W_2 symmetrical to other end. Keep the small weight 'w' near to center and fix the position. Do not move the positions of W_1, W_2 and W during your experiments.
- Suspend the pendulum about the knife edge 1 and take the time for about 10 oscillations. Note down the time t_1 using a stopwatch and calculate its time period using equation $T_1=t_1/10$.
- Now suspend about knife edge K_2 by inverting the pendulum and note the time t_2 for 10 oscillations. Calculate T_2 also.
- If $T_1 \neq T_2$, adjust the position of knife edge K_2 so that $T_1 \neq T_2$
- Repeat the experiment by changing the values of h_1 and h_2 which can be done by varying the distance between $K_1 - W_1$ and $K_2 - W_2$ as 1 cm, 2 cm, 3 cm.
- In each case note down the time difference between T_1 and T_2. Comment on which case your determined value of g is more close to the actual value.

Balance the Kater's pendulum on a knife edge fixed on a table to find the center of gravity.

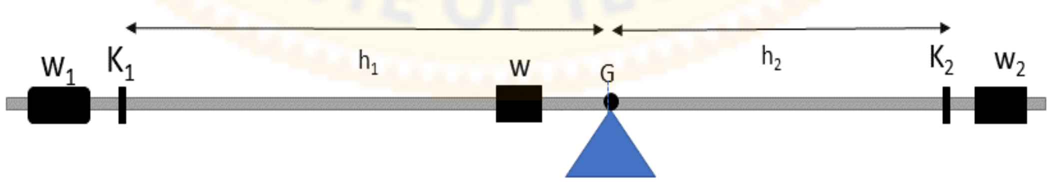
Step 1: Move the mass W at the middle of the rod.



Step 2: Set the distance between W_1-K_1 and W_2-K_2 as 1 cm on both sides. And make sure the apparatus is symmetric from both end.



Step 3: Now balance the whole apparatus on a knife edge fixed on a table to find center of gravity. Mark the point G with a marker/chalk. This is your center of gravity. Find the distances h_1 and h_2 and note down



Step 4: Find the time period T1 and T2 with this configuration. Repeat the process with the distance between W_1-K_1 and W_2-K_2 set as 2 cm and 3 cm.

OBSERVATION

Least count of the scale used fro measuring h_1 and h_2 = …
Distance of K_1 from C.G, $\boldsymbol{h_1}$ = ... m.
Distance of K_2 from C.G, $\boldsymbol{h_2}$ = ... m.

TABLE 1: Determination of T_1 and T_2

Least count of the stop watch: …. sec

KNIFE EDGE	Time for 10 oscillations (sec)			Time period $T = \frac{t}{10}$ in sec
	1 (s)	2 (s)	Mean time t (s)	
K_1				**(T_1)**
K_2				**(T_2)**

Repeat the procedure for two other different values of $\boldsymbol{h_1}$ & $\boldsymbol{h_2}$

TABLE 2: Determination of T_1 and T_2

Distance of K_1 from C.G, $\boldsymbol{h_1}$ = ... m.
Distance of K_2 from C.G, $\boldsymbol{h_2}$ = ... m.

Least count of the stop watch: …. sec

KNIFE EDGE	Time for 10 oscillations (sec)			Time period $T = \frac{t}{10}$ in sec
	1 (s)	2 (s)	Mean time t (s)	
K_1				**(T_1)**
K_2				**(T_2)**

TABLE 3: Determination of T_1 and T_2

Distance of K_1 from C.G, h_1 = ... m.
Distance of K_2 from C.G, h_2 = ... m.

Least count of the stop watch: …. sec

KNIFE EDGE	Time for 10 oscillations (sec)			Time period $T = \frac{t}{10}$ in sec
	1 (s)	2 (s)	Mean time t (s)	
K_1				**(T_1)**
K_2				**(T_2)**

CALCULATION

Distance of K_1 from C.G, h_1 = ... m.
Distance of K_2 from C.G, h_2 = ... m.
Time period T_1 = ….. s
Time period T_2 = ….. s

$$\therefore g = 8\pi^2 \left[\frac{T_1^2 + T_2^2}{h_1 + h_2} + \frac{T_1^2 - T_2^2}{h_1 - h_2}\right]^{-1}$$

Acceleration due to gravity, g = ... ms^{-2}.

RESULT

The acceleration due to gravity at a given place is found to be =...............ms^{-2}.

Among all of the three cases, the value of g which is closer to 9.81 m/s^2 is to be considered for the final value.

EROR CALCULATION

$$g = 8\pi^2 \left[\frac{T_1^2 + T_2^2}{h_1 + h_2} + \frac{T_1^2 - T_2^2}{h_1 - h_2} \right]^{-1}$$

Using the above formula do the error calculation and find
$\Delta g. The\ formula\ can\ be\ simplified\ by\ assumimng\ T_1 \approx T_2$

$$\mathrm{g} = 8\pi^2 \left[\frac{T_1^2 + T_2^2}{h_1 + h_2} \right]^{-1}$$

$\therefore g' = g \pm \Delta g.$

Aslo, calculate the standard error in g from actual value of $\boldsymbol{g}$ = 9.81 m/s^2

$\frac{\Delta g}{g} \times \mathbf{100} = \frac{g_{\ standard} - g_{\ measured}}{g_{\ standard}} \times \mathbf{100} = \ldots.. \%.$

PRECAUTIONS

1. The two knife-edges should be parallel to each other.
2. The amplitude of vibration should be small so that the pendulum satisfies the condition of simple harmonic motion.
3. To avoid any irregularity of motion the time period should be noted after the pendulum has made a few oscillation.
4. To avoid friction there should be glass surface on the rigid support.