# 3 Wave Optics: Interference

## LEARNING OBJECTIVES

After reading this chapter you will be able to understand:

- Interference of light.
- Interference in thin films (parallel and wedge shaped film).
- Newton's rings.

## 3.1 Introduction

Optics is the branch of physics in which we study the nature of light and the phenomenon exhibited by it. Before 19th century many theories have been put forward in this direction. The most famous among them were Newton's corpuscular theory, Huygen's wave theory, Maxwell's electromagnetic theory, Planck quantum theory of light, etc.

According to corpuscular theory, light consists of very small, weightless and perfectly elastic particles called corpuscles. Reflection, refraction and rectilinear propagation can be explained by this theory but it fails to explain interference, diffraction and polarization.

Huygen's suggested that light creates periodic disturbance which travels as waves in a manner very similar to that of sound waves. It explains reflection, refraction, interference and diffraction but fails to explain polarization and rectilinear propagation of light.

Thereafter, Maxwell's considered the light to be electromagnetic in nature. Therefore, no material medium is required for its propagation. In this reference, Planck assumed that light consists of small particles in the form of discrete bundles of energy called quanta or photons and the energy of one photon is equal to $h\upsilon$ where $h$ is Planck constant and $\upsilon$ is frequency.

Presently, it is assumed that light behaves in a dual nature: one is particle and other wave nature. Therefore, on the basis of wave nature we can explain the phenomena of interference, diffraction and polarization (also called wave optics). However, photoelectric effect and Compton effect can be explained by particle nature. In this section, we limit ourselves to the wave nature of light.

## 3.2 Interference of Light

When two waves of same frequency and constant phase difference travel simultaneously in the same direction, then there is a change in the intensity of the waves due to superposition of two waves. This change in the intensity is said to be interference.

The points where change in intensity is greater than that of the sum of the intensities due to the individual waves are called constructive interference whereas some other points where change in intensities is less than that of the sum of the intensities due to individual waves are called destructive interference.

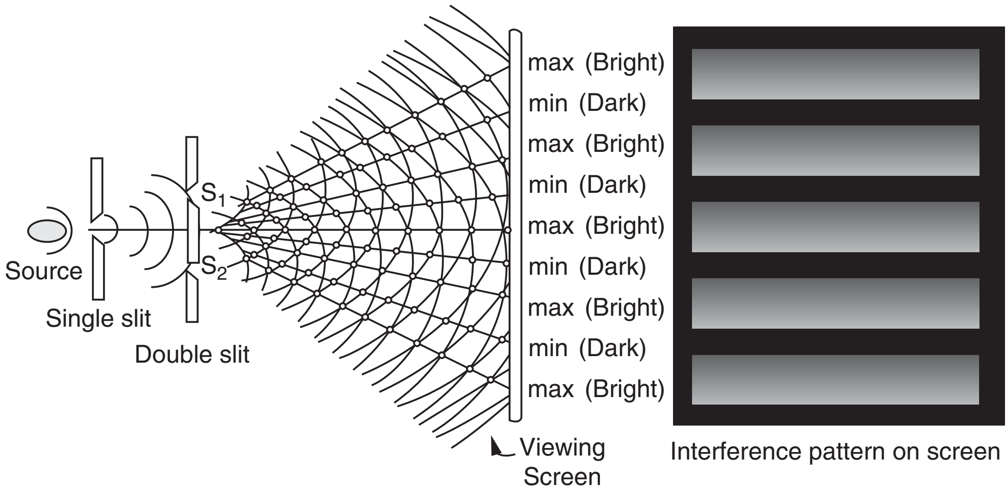


**Figure 1** Fringe pattern.

In other words, one can say that interference is nothing but the redistribution of energy. After interference, we get interference fringes which are alternate dark and bright bands of regular or irregular shape (see Fig. 1).

## 3.3 Superposition

We have already discussed in the previous section that when two waves travel in a medium, there is a modification in intensity of the waves due to superposition of two waves. As a result a new wave is formed whose amplitude is determined by superposition principle. Let $A$ be the resultant amplitude and $A_1$, $A_2$, $A_3$, $A_4$, … be the amplitudes of the individual waves. Then

$$A = A_1 \pm A_2 \pm A_3 \pm A_4 \pm \cdots$$

where '+ve' sign stands for amplitude of waves in same direction whereas '–ve' sign stands for opposite direction. Thus, *the resultant amplitude at a point and at any instant of time is the algebraic sum of the amplitudes of the individual waves*. This is known as a principle of superposition.

## 3.4 Types of Interference

1. **Division of wave fronts:** In this case, the wave front is divided into two parts to produce interference fringes. For example, laser, Fresnel biprism, Young's double slit experiment, Fresnel mirrors, etc.
2. **Division of amplitude:** Under this category, amplitude of the incident light is divided into two parts due to reflection or refraction to produce interference fringes. For example, thin films, Newton's ring, Michelson interferometer, etc.

## 3.5 Theory of Interference

To understand the phenomena of interference and derivation of an expression for the change in intensity at any point on the screen, consider a monochromatic light source S emitting waves of wavelength $\lambda$. $S_1$ and $S_2$ are two narrow slits close together and equidistant from S as shown in Fig. 2.

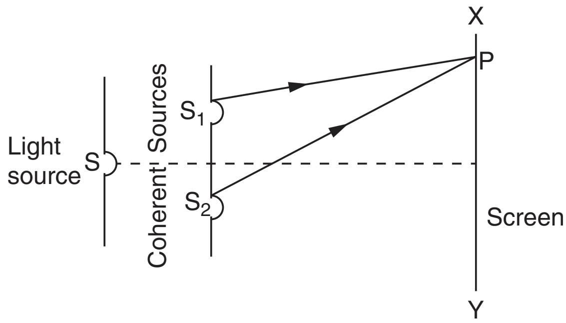


**Figure 2** Theory of interference.

Let $a$ and $b$ be amplitudes of two waves at P from $S_1$ and $S_2$, respectively and $y_1$ and $y_2$ be the corresponding displacements. Then

$$y_1 = a\sin(\omega t) \tag{3.1}$$

$$y_2 = a\sin(\omega t + \delta) \tag{3.2}$$

where $\delta$ is the phase difference between the two waves. Using the phenomena of superposition, the resultant displacement is

$$\begin{aligned} y &= y_1 + y_2 \\ &= a\sin(\omega t) + b\sin(\omega t + \delta) \\ &= a\sin(\omega t) + b\sin\omega t\cos\delta + b\cos\omega t\sin\delta \\ &= (a + b\cos\delta)\sin\omega t + b\cos\omega t\sin\delta \end{aligned} \tag{3.3}$$

Let

$$(a + b\cos\delta) = R\cos\theta \tag{3.4}$$

$$b\sin\delta = R\sin\theta \tag{3.5}$$

where $R$ and $\delta$ are the new constants. Now substituting the values of Eqs. (3.4) and (3.5) in Eq. (3.3) we get

$$y = \sin\omega t R\cos\theta + \cos\omega t R\sin\theta = R\,\sin(\omega t + \theta) \tag{3.6}$$

Thus, the resultant displacement at point P on screen is simple harmonic of amplitude $R$ and phase $\theta$. Squaring and adding Eqs. (3.4) and (3.5) we get

$$\begin{aligned} R^2\cos^2\theta + R^2\sin^2\theta &= (a + b\cos\delta)^2 + (b\sin\delta)^2 \\ \Rightarrow R^2 &= a^2 + b^2\cos^2\delta + 2ab\cos\delta + b^2\sin^2\delta = a^2 + b^2 + 2ab\cos\delta \end{aligned} \tag{3.7}$$

Since, we know that intensity $I$ is proportional to the square of the amplitude $R^2$. So,

$$I = R^2 = a^2 + b^2 + 2ab\cos\delta \tag{3.8}$$

Thus from Eq. (3.8) we can conclude that resultant intensity is different than the sum of the intensities due to individual waves.

### 3.5.1 Constructive Interference or Maxima

We know that maximum and minimum value of cosine is +1 and −1 respectively. So

$$\cos\delta = +1 \quad \text{or} \quad \delta = 0, 2\pi, 4\pi, 6\pi, \ldots$$

Generally, we can write

$$\delta = 2n\pi, \quad n = 0, 1, 2, 3, \ldots$$

Since

$$\text{Path difference} = \frac{\lambda}{2\pi}\text{Phase difference}$$

therefore we have

$$\text{Path difference} = \frac{\lambda}{2\pi}\times 2n\pi = n\lambda \tag{3.9}$$

Thus, the path difference between two interfering waves is equal to integral multiples of $\lambda$.
Putting the value of $\cos\delta = +1$ in Eq. (3.8) we get

$$I_{\max} = R^2_{\max} = a^2 + b^2 + 2ab = (a+b)^2 \tag{3.10}$$

Equation (3.10) clearly indicates that the maximum intensity is greater than the sum of the intensities due to two individual waves.

### 3.5.2 Destructive Interference or Minima

We know that

$$\cos\delta = -1 \quad \text{or} \quad \delta = \pi, 3\pi, 5\pi, \ldots$$

Generally, we can write

$$\delta = (2n-1)\,\pi, \quad n = 1, 2, 3, \ldots$$

or

$$\delta = (2n+1)\,\pi, \quad n = 0, 1, 2, 3, \ldots$$

Since

$$\text{Path difference} = \frac{\lambda}{2\pi}\text{Phase difference}$$

Therefore

$$\text{Path difference} = \frac{\lambda}{2\pi}\times(2n-1)\,\pi = (2n-1)\frac{\lambda}{2}$$

or

$$= \frac{\lambda}{2\pi}\times(2n+1)\,\pi = (2n+1)\frac{\lambda}{2} \tag{3.11}$$

Thus, path difference between two interfering waves is equal to odd multiples of $\lambda/2$. Putting the value of $\cos\delta = -1$ in Eq. (3.8) we get

$$I_{\min} = R^2_{\min} = a^2 + b^2 - 2ab = (a-b)^2 \tag{3.12}$$

Equation (3.12) clearly indicates that the minimum intensity is less than the sum of the intensities due to the individual intensities.

Let us see what happens if the amplitude of two waves is same, that is, $a = b$:

$$I_{max} = a^2 + a^2 + 2aa = 4a^2 \tag{3.13}$$

$$I_{min} = a^2 + a^2 - 2aa = 0 \tag{3.14}$$

From Eq. (3.13) one can conclude that the resultant intensity is maximum or brightness occurs at points on which $\delta = 2n\pi$ and is $4a^2$; however, it is minimum or darkness appears at points on which $\delta = (2n-1)\pi$ or $(2n+1)\pi$ and is zero. We plot the curve between maximum and minimum intensities with phase difference $\delta$. The intensity varies between zero and $4a^2$ depending upon the phase difference $\delta$ between two interfering waves as shown in Fig. 3; it is called the *intensity distribution curve*. The law of conservation of energy is also true in the formation of interference fringes.

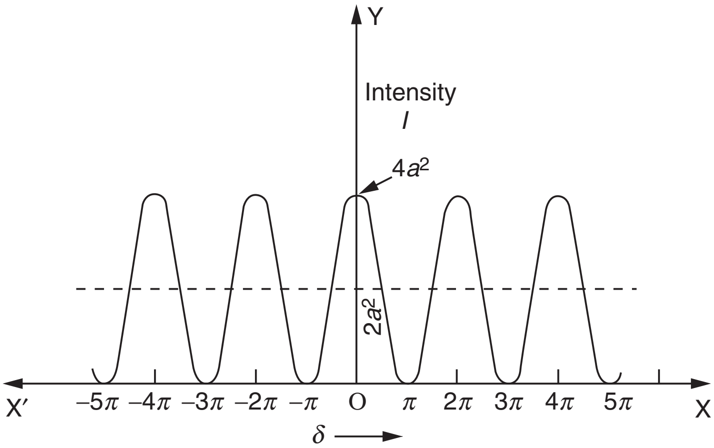


**Figure 3** Variation of intensity with $\delta$.

**Average Intensity:** It is the average of the minimum and maximum intensities and it is given by

$$I_{av} = \frac{\int_0^{2\pi} I\,d\delta}{\int_0^{2\pi} d\delta} = \frac{\int_0^{2\pi} (a^2 + b^2 + 2ab\cos\delta)\,d\delta}{\int_0^{2\pi} d\delta} = \frac{[a^2\delta + b^2\delta + 2ab\sin\delta]_0^{2\pi}}{[\delta]_0^{2\pi}} = \frac{[a^2 + b^2]\,2\pi}{2\pi} = 2a^2 \quad (\because a = b)$$

The average intensity is equal to the sum of the separate intensities, that is, energy is neither created nor destroyed but it merely redistributes in the interference pattern. Thus, we prove that the phenomenon of interference is in accordance with the law of conservation of energy.

## 3.6 Coherent Sources

Coherent sources are nothing but two light waves of same frequency of wavelength having same amplitude and always a constant phase difference between them. In actual practice two independent sources cannot be coherent because they cannot maintain a constant phase difference between them. But for experimental purposes two virtual sources obtained from a single source can act as coherent.

### 3.6.1 Condition for the Interference or Permanent or Sustained Interference

The conditions for sustained interference are as follows:

1. The first and foremost condition is that the two interfering sources must be coherent, that is, they always should maintain constant phase difference.
2. The wavelength and time period of the two interfering sources must be the same.
3. The amplitude or intensities must be equal or very nearly equal.
4. The separation between the two coherent sources must be as small as possible.
5. Two sources should be narrow.
6. The distance between two sources and screen should be as large as possible.

## 3.7 Fringe Width

Let us consider S to be the source illuminated with monochromatic light having wavelength $\lambda$ (see Fig. 4). Let $S_1$ and $S_2$ be the two equidistant coherent sources from S. Let $2d$ be the separation between two narrow slits $S_1$ and $S_2$ and $D$ be the distance of screen from two coherent sources. Let us consider P to be the point on the screen at a distance $x$ from O where bright or dark bands are located. Thus

$$\text{Path difference} = S_2P - S_1P$$

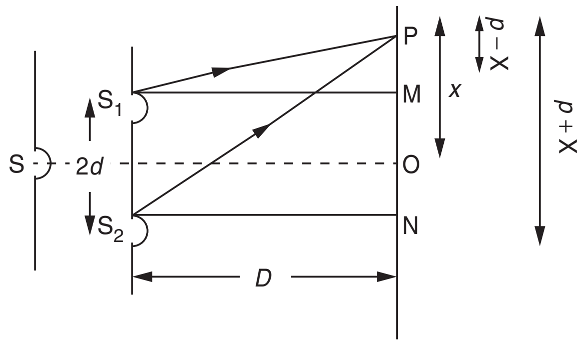


**Figure 4** Young double slit experiment.

To determine the values of $S_2P$ and $S_1P$, we consider the triangles $S_2NP$ and $S_1MP$ in Fig. 4. Now

$$S_2P - S_1P = \sqrt{[(S_2N)^2 + (NP)^2} - \sqrt{(S_1M)^2 + (MP)^2]}$$

$$= \sqrt{[(D)^2 + (x+d)^2} - \sqrt{(D)^2 + (x-d)^2]}$$

$$= D\left(1 + \frac{(x+d)^2}{D^2}\right)^{1/2} - D\left(1 + \frac{(x-d)^2}{D^2}\right)^{1/2}$$

Using Binomial theorem and neglecting higher terms, we get

$$S_2P - S_1P = D\left(1 + \frac{(x+d)^2}{2D^2}\right) - D\left(1 + \frac{(x-d)^2}{2D^2}\right) = \frac{1}{2D}[(x+d)^2 - (x-d)^2] = \frac{2xd}{D} \qquad (3.15)$$

If $\delta$ is the corresponding phase difference, then

$$\delta = \frac{2\pi}{\lambda} \times \frac{2xd}{D}$$

### 3.7.1 Bright Fringe or Maxima

For maxima, path difference should be $n\lambda$. Therefore

$$\frac{2xd}{D} = n\lambda \Rightarrow x = \frac{nD\lambda}{2d}$$

If $x_n$ is the position of $n$th bright fringe, then

$$x_n = \frac{nD\lambda}{2d}, \quad n = 0, 1, 2, 3, \ldots$$

**Fringe width:** It is defined as the separation between two consecutive bright fringes. Hence, fringe width ($\omega$) is given by

$$\omega = x_n - x_{n-1} = \frac{nD\lambda}{2d} - \frac{(n-1)D\lambda}{2d} = \frac{D\lambda}{2d} \tag{3.16}$$

### 3.7.2 Dark Fringe or Minima

For minima, path difference should be $(2n-1)\lambda/2$. Therefore

$$\frac{2xd}{D} = (2n-1)\frac{\lambda}{2} \quad \Rightarrow x = \frac{(2n-1)\,D\lambda}{4d}$$

If $x_n$ is the position of the $n$th dark fringe, then

$$x_n = \frac{(2n-1)\,D\lambda}{4d}, \quad n = 1, 2, 3, \ldots$$

**Fringe width:** It is defined as the separation between two consecutive dark fringes. Hence, fringe width ($\omega$) is given by

$$\omega = x_n - x_{n-1} = \frac{(2n-1)\,D\lambda}{4d} - \frac{[2(n-1)-1]\,D\lambda}{4d} = \frac{D\lambda}{2d} \tag{3.17}$$

From Eqs. (3.16) and (3.17), it is clear that fringe width varies directly with $D$ and $\lambda$, and inversely with $2d$. We can also conclude that all bright and dark fringes are of equal width.

## 3.8 Interference in Thin Films

We know that when white light falls on a thin film of oil spread on the surface of water, beautiful colors are seen. Similar colors are also produced by the thin film of soap bubble. This phenomenon can be explained on the basis of interference in thin films.

### 3.8.1 Interference in Thin Film Due to Reflected Light

Consider a thin film of refractive index $\mu$ and thickness $t$ (see Fig. 5). Let a ray SA fall on the upper surface of the film at incident angle $i$. The ray is partly reflected along AE and partly refracted along AB at angle $r$. Lower surface also reflects the ray along BC and finally, the ray emerges out from the upper surface of the film along CD. To evaluate the path difference between AE and CD, we draw perpendiculars CF and AG on AE and BC, respectively.

The optical path difference between AE and CD is

$$\Delta = \mu(\text{AB} + \text{BC}) - \text{AF} \tag{3.18}$$

$$\Rightarrow \lambda = \frac{D_{n+p}^2 - D_n^2}{4pR} \tag{3.43}$$

Equation (3.43) represents the wavelength of light used.

## 3.13 Determination of the Refractive Index of a Liquid

In order to determine the refractive index of given liquid, first the experiment is performed in air. So

$$(D_{n+p}^2 - D_n^2)_{\text{air}} = 4p\lambda R \tag{3.44}$$

Now the liquid whose refractive index ($\mu$) is to be determined is introduced between the plano-convex lens and plane glass plate. Hence,

$$(D_{n+p}^2 - D_n^2)_{\text{liquid}} = 4p\lambda R/\mu \tag{3.45}$$

From Eqs. (3.44) and (3.45), we have

$$\mu = \frac{(D_{n+p}^2 - D_n^2)_{\text{air}}}{(D_{n+p}^2 - D_n^2)_{\text{liquid}}}$$

This is required expression for refractive index of liquid.

## Solved Examples

### Example 1

Prove that $\dfrac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}} = \dfrac{2\sqrt{\alpha}}{1+\alpha}$, where $\alpha$ is the ratio of two intensities.

**Solution:** Let us consider $I_1$ and $I_2$ to be the intensities and $a$ and $b$ the amplitudes of the two coherent sources. Then according to the question

$$\frac{I_1}{I_2} = \frac{a^2}{b^2} \qquad \Rightarrow \frac{a}{b} = \sqrt{\frac{I_1}{I_2}} = \sqrt{\alpha}$$

Now

$$I_{\max} = (a+b)^2 \quad \text{and} \quad I_{\min} = (a-b)^2$$

So

$$\frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}} = \frac{(a+b)^2 - (a-b)^2}{(a+b)^2 + (a-b)^2} = \frac{2ab}{a^2 + b^2} = \frac{2\left(\dfrac{a}{b}\right)}{\left(\dfrac{a}{b}\right)^2 + 1} = \frac{2\sqrt{\alpha}}{1+\alpha}$$

## Example 2

In an interference pattern with two coherent sources the amplitude of the intensity variation is found to be 5% of the average intensity. Calculate the relative intensities of the interfering sources.

**Solution:** Consider the amplitude ratio of the two sources to be $a$:1. Then $I_{\max}$ and $I_{\min}$ is $(a + 1)^2$ and $(a - 1)^2$, respectively,

$$\frac{I_{\max}}{I_{\min}} = \frac{(a+1)^2}{(a-1)^2}$$

Since, intensity variation is found at 5%, then the maximum is 105 and minimum is 95. So

$$\frac{I_{\max}}{I_{\min}} = \frac{(a+1)^2}{(a-1)^2} = \frac{(105)^2}{(95)^2}$$

$$\Rightarrow \frac{a+1}{a-1} = 1.05 \Rightarrow a \cong 40$$

Therefore,

$$\frac{I_1}{I_2} = \frac{a^2}{1} = \frac{(40)^2}{1} = \frac{1600}{1}$$

## Example 3

Light of wavelength 5893 Å is reflected at nearly normal incidence from a soap film of refractive index 1.42. What is the least thickness of the film that will appear black?

**Solution:** We know that the condition for dark ring or black in reflected light is

$$2\mu t \cos r = n\lambda$$

According to the question, $n = 1$ (for least thickness), $\lambda = 5893$ Å, $\mu = 1.42$. For normal incidence $r = 0$. Hence

$$t = \frac{1 \times 5893}{2 \times 1.42 \times 1} = 2075 \text{ Å}$$

## Example 4

Two glass plates enclose a wedge-shaped air film, touching at one edge and separated by a wire of 0.05 mm diameter at a distance of 10 cm from the edge. Calculate the fringe width of $\lambda = 5500$ Å from a broad source that falls normally on the film.

**Solution:** Fringe width

$$\omega = \frac{\lambda}{2\mu\theta_n}$$

We have $\lambda = 5500\ \mathring{A} = 5500 \times 10^{-8}$ cm, $t = 0.05$ mm = 0.005 cm, distance ($x$) = 10 cm and for air film $\mu = 1$.

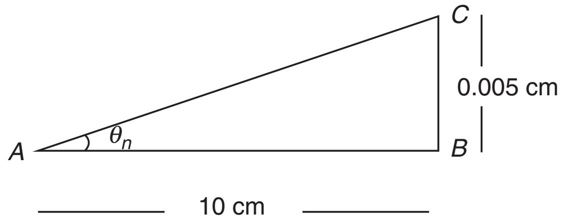


**Figure 12** Wedge-shaped film.

From Fig. 12

$$\theta_n = \frac{BC}{AB} = \frac{0.005}{10}$$

So

$$\omega = \frac{\lambda}{2\mu\theta_n} = \frac{5500 \times 10^{-8} \times 10}{2 \times 0.005} = 5.5 \times 10^{-3} \text{ cm}$$

## Example 5

If the angle of wedge is 0.15° of arc and the wavelength of sodium D lines are 5890 Å and 5896 Å, find the distance from the apex of the wedge at which the maximum due to each wavelength first coincide.

**Solution:** If the two wavelengths are coincide and $t$ is the thickness of the film. Thus,

$$2t = (2n + 1)\lambda_1/2 = (2n + 3)\lambda_2/2$$

So,

$$n = \frac{(3\lambda_2 - \lambda_1)}{2(\lambda_1 - \lambda_2)}$$

Now

$$2t = \frac{\lambda_1 \lambda_2}{\lambda_1 - \lambda_2}$$

Suppose $\theta_n$ is the angle of wedge and $x$ is the distance from the apex of the wedge. Then $t = x\,\theta_n$. So

$$2x\,\theta_n = \frac{\lambda_1 \lambda_2}{\lambda_1 - \lambda_2}$$

Substituting $\lambda_1 = 5896 \times 10^{-8}$ cm, $\lambda_2 = 5896 \times 10^{-8}$ cm and $\theta_n = 0.15° = 0.15\ \pi/180$ radian in above equation, we get

$$x = \frac{5896 \times 10^{-8} \times 5890 \times 10^{-8} \times 180}{(5896 \times 10^{-8} - 5890 \times 10^{-8}) \times 2 \times 0.15 \times 3.14} = 11.05 \text{ cm}$$

# 4 Diffraction of Light

## LEARNING OBJECTIVES

After reading this chapter, you will be able to understand:

- Single, double and *N*-slit diffraction.
- Diffraction grating.
- Grating spectra.
- Dispersive power.
- Rayleigh's criterion.
- Resolving power of grating.

## 4.1 Introduction

We have seen that the sunlight comes in a dark room through a hole in the window in straight line. Similarly, a sharp shadow of an opaque object implies rectilinear propagation of light. But in 1665, Grimaldi observed that when a beam of light passes through a small aperture or a narrow slit, it does not follow rectilinear path but bends around the corners of the obstacles (slit or aperture). This bending of light depends on the size of slit or aperture and wavelength of light wave. This bending or deviation is extremely small when the wavelength is small in comparison to size of slit or aperture and much more if wavelength is comparable to size of slit or aperture. Thus the diffraction phenomenon is the bending of light from the edges or corners of slit or obstacle and spreading in the region or geometrical shadow and distribution of intensity in the form of bright and dark fringes on screen which is called diffraction pattern (see Fig. 1).

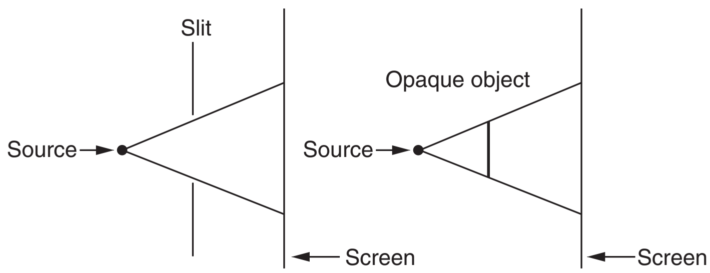


**Figure 1** The bending of light round corners of an object.

## 4.2 Classification of Diffraction

The phenomena of diffraction can be classified into two following categories:

1. **Fresnel diffraction:** In Fresnel diffraction, the source or screen or both are at finite distance from obstacles.
2. **Fraunhofer diffraction:** In Fraunhofer diffraction, the source or screen or both are at infinite distance from obstacles.

The differences between Fresnel and Fraunhofer diffractions are given in Table 1.

**Table 1** Differences between Fresnel and Fraunhofer diffractions

| *S. No.* | *Fresnel Diffraction* | *Fraunhofer Diffraction* |
|---|---|---|
| 1. | Source or screen or both are at finite distance from obstacle. | Source or screen or both are at infinite distance from obstacle. |
| 2. | No lens is used. | Combination of lenses is used. |
| 3. | Incident wave front is generally spherical or cylindrical. | Incident wave front is plane. |
| 4. | Diffraction pattern is a shadow of obstacle. | Diffraction pattern is an image of obstacle. |
| 5. | Central point in diffraction pattern is either dark or bright depending on the number of Fresnel zones. | Central point is always bright. |

## 4.3 An Important Mathematical Analysis

This mathematical treatment is required in the formulation of intensity variation relation in single slit which in turn helps in analysis of double slit and multiple slit (grating) diffraction. Also it gives a glimpse of how mathematical relation having physical reality makes the understanding of a system easy.

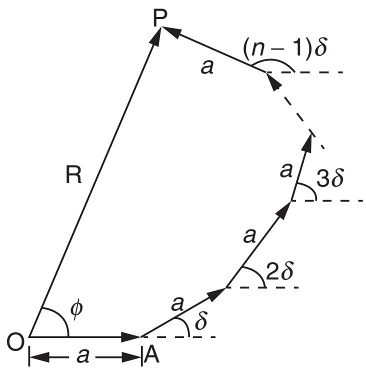


**Figure 2** Determination of resultant amplitude and phase.

Here we consider waves having equal amplitude '$a$' and having common phase difference $\delta$ between successive waves, that is, phase increases in AP. To find the resultant amplitude $R$ and phase $\theta$, a polygon is constructed as shown in Fig. 2. Resolving $a$ along and perpendicular to the first side, we get

$$R\cos\theta = a + a\cos\delta + a\cos 2\delta + a\cos 3\delta + \cdots + a\cos(n-1)\delta \qquad (4.1)$$

$$R\sin\theta = 0 + a\sin\delta + a\sin 2\delta + a\sin 3\delta + \cdots + a\sin(n-1)\delta \qquad (4.2)$$

Multiplying Eq. (4.1) by 2 $\sin\delta/2$ and using trigonometric identities, we get

$$2R\cos\theta\sin\delta/2 = a\,[2\sin(\delta/2) + 2\sin(\delta/2)\cos\delta + 2\sin(\delta/2)\cos 2\delta + \cdots + 2\sin(\delta/2)\cos(n-1)\delta]$$

$$= a\,[2\sin(\delta/2) + \{\sin(3\delta/2) - \sin(\delta/2)\} + \{\sin(5\delta/2) - \sin(3\delta/2)\} + \cdots + \{\sin(n-1/2)\delta - \sin(n-3/2)\delta\}]$$

$$= a\,[\sin(\delta/2) + \sin(n-1/2)\delta] = 2a\sin(n\delta/2)\cos(n-1/2)\delta$$

So

$$R\cos\theta = [a\sin(n\delta/2)\cos(n-1)\delta/2]/\sin\delta/2$$

Similarly,

$$R\sin\theta = [a\sin(n\delta/2)\sin(n-1)\delta/2]/\sin\delta/2$$

Squaring and adding the above equations, we have

$$R = a\sin(n\delta/2)/\sin(\delta/2)$$

and $$\tan\theta = \tan(n-1)\delta/2 \Rightarrow \theta = (n-1)\delta/2$$

Let $n\delta = 2\alpha$. Then $R = a\sin\alpha/\sin(\alpha/n)$. $\alpha/n$ is very small as $n$ is infinitely large. Hence

$$R = a\sin\alpha/(\alpha/n) = na\sin\alpha/\alpha = A\sin\alpha/\alpha$$

and $$\theta = (n-1)\delta/2 \approx n\,\delta/2 = \alpha$$

As $n$ is very large then $n \approx n-1$. Since Fraunhofer diffraction is much more important than Fresnel diffraction, therefore in the next section we will discuss the simple diffraction phenomena.

## 4.4 Fraunhofer Diffraction at a Single Slit

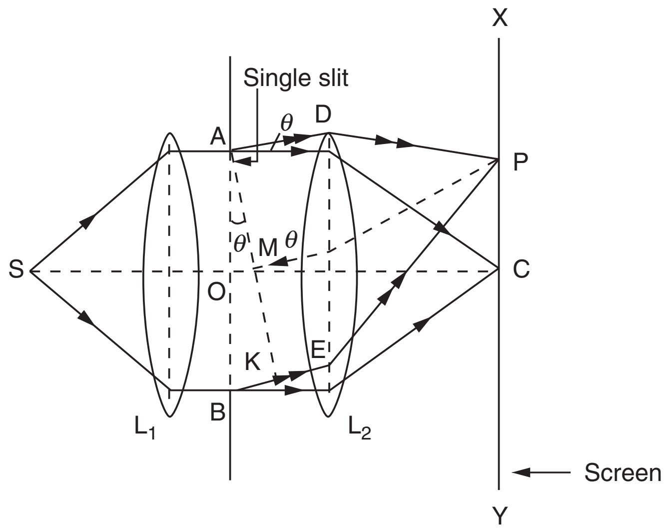


**Figure 3** Fraunhofer diffraction at a single slit.

Let a monochromatic light of wavelength $\lambda$ be incident on collimating lens $L_1$ (see Fig. 3). A parallel beam of light emerges out from $L_1$ and normally falls upon a slit AB, whose length is large compared to its width $a$. The diffracted light is focused by another lens $L_2$. The diffraction pattern obtained on the screen consists of a central bright band having alternate dark and bright bands of decreasing intensity on both sides.

The rays diffracted along the direction of incident rays are focused at C while those diffracted at an angle $\theta$ are focused at P. Since all the wavelets from AB reach C in the same phase, hence intensity at C is maximum whereas the wavelets reach P at different times due to unequal distance. Hence, they have different path and phase.

Let AK be perpendicular to BK. The path difference between the rays originating from extreme points A and B is given by

$$BK = AB\sin\theta = a\sin\theta \tag{4.3}$$

where $a$ is the width of slit AB. Now, the corresponding phase difference between the rays originating from extreme points A and B is

$$\frac{2\pi}{\lambda}\times a\sin\theta$$

Let the aperture AB be divided into a large number $n$ of equal parts, each part being the source of secondary wavelets. The amplitude of vibrations at P due to each part will be the same, say $a$, but their phase will vary gradually from 0 to $(2\pi/\lambda)\times a\sin\theta$. The phase difference between the waves from two consecutive parts is

$$\delta=\frac{1}{n}\frac{2\pi}{\lambda}\times a\sin\theta \tag{4.4}$$

where $n$ is the number of vibrations. The resultant amplitude at P is given by

$$R=a\frac{\sin(n\delta/2)}{\sin(\delta/2)}=a\frac{\sin(\pi a\sin\theta/\lambda)}{\sin(\pi a\sin\theta/n\lambda)} \tag{4.5}$$

$$\Rightarrow \qquad R=a\frac{\sin\alpha}{\sin(\alpha/n)} \tag{4.6}$$

where $\alpha=(\pi a\sin\theta)/\lambda$. Since $\alpha/n$ is very small, therefore, $\sin(\alpha/n)=(\alpha/n)$. So we have

$$R=a\frac{\sin\alpha}{\alpha/n}=na\frac{\sin\alpha}{\alpha} \quad\Rightarrow\quad R=A\frac{\sin\alpha}{\alpha} \tag{4.7}$$

where $A=na$ is the amplitude of all the vibrations in same phase. Now intensity at P is

$$I=R^2=A^2\frac{\sin^2\alpha}{\alpha^2} \tag{4.8}$$

For intensity to be maximum or minimum

$$\frac{dI}{d\alpha}=0\Rightarrow\frac{d}{d\alpha}\left(A^2\frac{\sin^2\alpha}{\alpha^2}\right)=0\Rightarrow A^2\left(\frac{\alpha^2\,2\sin\alpha\cos\alpha-\sin^2\alpha\;2\alpha}{\alpha^4}\right)=0$$

$$\Rightarrow A^2\left(\frac{2\sin\alpha\cos\alpha}{\alpha^2}-\frac{2\sin^2\alpha}{\alpha^3}\right)=0\Rightarrow A^2\,\frac{2\sin\alpha}{\alpha}\left(\frac{\alpha\cos\alpha-\sin\alpha}{\alpha^2}\right)=0$$

Now,

$$\text{Either }\frac{\sin\alpha}{\alpha}=0\quad\text{or}\quad\left(\frac{\alpha\cos\alpha-\sin\alpha}{\alpha^2}\right)=0$$

$$\Rightarrow\text{Either }\sin\alpha=0\quad\text{or}\quad\alpha\cos\alpha-\sin\alpha=0$$

$$\Rightarrow\ \text{(a) }\sin\alpha=0\quad\text{or}\quad\text{(b) }\alpha=\tan\alpha$$

**Condition for Minimum Intensity:** When $\sin\alpha/\alpha=0$, then intensity is zero. So

$$\sin\alpha=0\Rightarrow\alpha=\pm m\pi\Rightarrow\frac{\pi a\sin\theta}{\lambda}=\pm m\pi$$

$$\Rightarrow \qquad a\sin\theta=\pm m\lambda \tag{4.9}$$

Equation (4.9) gives the position of 1st, 2nd, 3rd,… minima corresponding to $m$ = 1, 2, 3 and so on. Here $m \neq 0$ because when $m = 0$ then $a\sin\theta = 0$ which is the condition of maximum intensity.

**Condition for Maximum Intensity:** For maximum intensity $\alpha = \tan\alpha$. This equation can be solved graphically by plotting the curves $y = \alpha$ and $y = \tan\alpha$ as shown in Fig. 4.

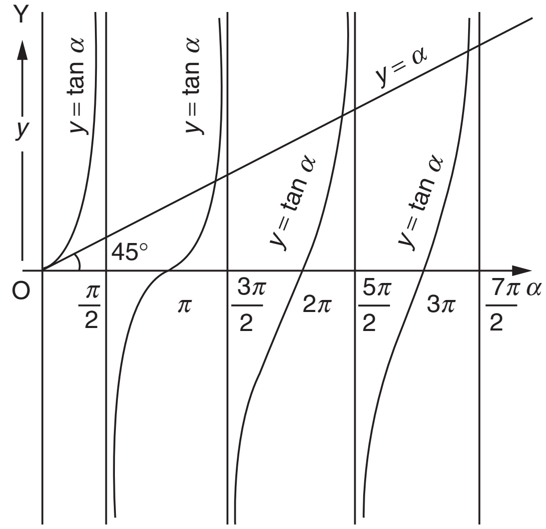


**Figure 4** Curves between $y = \alpha$ and $y = \tan\alpha$.

The abscissa of the points of intersection of these curves gives the required value of $\alpha$ for which intensity is maximum.

**Principal maximum** For central maximum $\alpha = 0$ so $(\sin\alpha)/\alpha \rightarrow 1$. From Eq. (4.8) we have

$$I = A^2 = I_0 \text{ (say)}$$

**Secondary maxima** Secondary maxima falls at the points of intersection of two curves which are nearly at $\alpha$ = odd number multiples of $\lambda/2$:

$$\alpha = \pm\frac{(2m+1)\lambda}{2} \tag{4.10}$$

$$\Rightarrow \frac{\pi a\sin\theta}{\lambda} = \pm\frac{(2m+1)\pi}{2} \Rightarrow a\sin\theta = \pm\frac{(2m+1)\lambda}{2} \tag{4.11}$$

If we put $m$ = 1, 2, 3, …we get the position of 1st, 2nd, 3rd, … secondary maxima, respectively. Now putting $m$ =1, 2, 3, … in Eq. (4.11) we get

$$\alpha = \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, \ldots$$

Intensity of 1st secondary maxima

$$I_1 = A^2\frac{\sin^2(3\pi/2)}{(3\pi/2)^2} = A^2\frac{4}{9\pi^2} = \frac{4}{9\pi^2}I_0 \text{ i.e. } 4.5\% \text{ of } I_0$$

Intensity of 2nd secondary maxima

$$I_2 = A^2\frac{\sin^2(5\pi/2)}{(5\pi/2)^2} = A^2\frac{4}{25\pi^2} = \frac{4}{25\pi^2}I_0 \text{ i.e. } 1.5\% \text{ of } I_0$$

Intensity of 3rd secondary maxima

$$I_3 = A^2 \frac{\sin^2(7\pi/2)}{(7\pi/2)^2} = A^2 \frac{4}{49\pi^2} = \frac{4}{49\pi^2} I_0$$

Thus the relative intensities of successive maxima are

$$I_0 : I_1 : I_2 : I_3 \ \ldots = 1 : \frac{4}{9\pi^2} : \frac{4}{25\pi^2} : \frac{4}{49\pi^2} \ldots$$

Thus, most of the light is concentrated in the central maxima and intensity of secondary maxima goes on decreasing. In short, we can say that the diffraction pattern consists of a bright central maxima surrounded alternately by minima of zero intensity and feeble secondary maxima of rapidly decreasing intensities.

The intensity distribution curve is shown in Fig. 5.

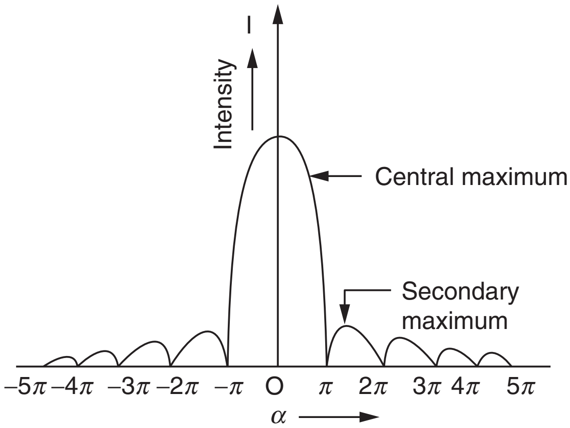


**Figure 5** The intensity distribution curve.

## 4.5 Fraunhofer Diffraction due to Double Slit

Let AB and GH be the two parallel slits of equal width $a$ and separated by an opaque distance $b$. Let a plane wave front be incident normally upon the slits. The light diffracted from these slits is focused by lens $L_2$ on the screen XY as shown in Fig. 6.

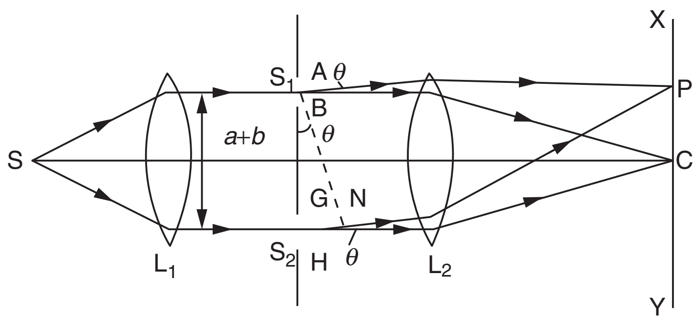


**Figure 6** Fraunhofer diffraction due to double slit.

***The diffraction at double slit is a case of diffraction as well as interference.*** The pattern obtained on the screen consists of equally spaced interference fringes in the region normally occupied by the central maxima

$$a\sin\theta = \pm m\lambda$$

According to the question $\lambda = 6000$ Å $= 6 \times 10^{-5}$cm, $a = 0.1$ mm $= 0.01$ cm and $m = 1$. Hence,

$$\theta = \sin^{-1}\left(\frac{m\lambda}{a}\right) = \frac{1\times 6\times 10^{-5}}{0.01} = 0.344°$$

Total angular width of the central maxima $= 2\,\theta = 2 \times 0.344° = 0.688°$

Total linear width of central maxima $= \dfrac{2D\lambda}{a} = \dfrac{2\times 100\times 6\times 10^{-5}}{0.01} = 1.2$ cm

## Example 3

Find the angular separation between the first-order minima on either side of central maxima when slit is $6 \times 10^{-4}$ cm wide. Given wavelength of light $\lambda = 6000$ Å.

**Solution:** We know that the minimum intensity in single slit is

$$a\sin\theta = \pm m\lambda$$

According to the question $\lambda = 6000$ Å $= 6 \times 10^{-5}$ cm, $a = 6 \times 10^{-4}$ cm and $m = 1$. Hence,

$$\theta = \sin^{-1}\left(\frac{m\lambda}{a}\right) = \frac{1\times 6\times 10^{-5}}{6\times 10^{-4}} = \sin^{-1}(0.1) = 5°44'21''$$

Angular separation between the first-order minima $2\theta = 2\times 5°44'21'' = 11°28'42''$.

## Example 4

A diffraction grating used at normal incidence gives a green line (5400 Å) in a certain order superimposed on the violet line (4050 Å) of the next higher order. If the angle of diffraction is 30°, how many lines per cm are there in grating?

**Solution:** We know that the $n$th order principal maxima is

$$(a + b)\sin\theta = n\lambda$$

According to the question, for the green light $\lambda = 5400$ Å , $\theta = 30°$ and the order is $n$. So

$$(a + b)\sin 30° = 5400\, n \tag{4.36}$$