$\vec{\nabla} = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$

$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$

$\vec{\nabla}\cdot\vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$

let, $\vec{A} = 5x^2\hat{i} + 3y^3x\hat{j} + 4x^2y^2z^2\hat{k}$

$\vec{\nabla}\cdot\vec{A} = 10x + 9xy^2 + 8x^2y^2z$

$\vec{\nabla}\cdot\vec{A}\Big|_{(1,2,1)} = 10 + 9(4) + 8(4)(3)$

$= 10 + 36 + 96$

$= 142$

$\underset{\text{vector}}{\vec{\nabla}}\cdot\vec{A} \rightarrow$ divergence of $\vec{A}$ $\rightarrow$ number (flux of $\vec{A}$)

$\vec{\nabla}(\underset{\text{scalar}}{F}) \rightarrow$ vector $\rightarrow$ gradient (i.e. slope)

$\vec{\nabla}\times\vec{A} \rightarrow$ curl $(\vec{A})$

$\vec{A} = 3x^2\hat{i} + 4xy\hat{j} + 7xyz\hat{k}$

$$\vec{\nabla}\times\vec{A} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 3x^2 & 4xy & 7xyz \end{vmatrix} = \hat{i}(7xz - \cancel{0}) - \hat{j}(7yz - 0) + \hat{k}(4y - 0)$$

$$= 7xz\,\hat{i} - 7yz\,\hat{j} + 4y\,\hat{k}$$

- if curl of any vector is non-zero, it is rotating.

$\underbrace{\vec{\nabla}\times\vec{B} \neq 0}_{\text{always a rotating vector}}$ $\qquad$ $\underbrace{\vec{\nabla}\times\vec{B} = 0}_{\text{irrotating vector}}$

- if $\vec{\nabla}\cdot\vec{A} = 0$, then $\vec{A}$ is solenoidal vector (flux through $\vec{A} = 0$)

Cartesian Coordinates (x, y, z) $\nabla = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$

Spherical " (r, θ, ϕ) $\vec{\nabla} = \hat{r}\frac{\partial}{\partial r} + \hat{\theta}\left(\frac{1}{r}\right)\frac{\partial}{\partial \theta} + \hat{\phi}\left(\frac{1}{r\sin\theta}\right)\frac{\partial}{\partial \phi}$

Cylindrical " (s, ϕ, z) $\vec{\nabla} = \hat{s}\frac{\partial}{\partial s} + \hat{\phi}\frac{\partial}{\partial \phi} + \hat{z}\frac{\partial}{\partial z}$

(i) Gradient: $\vec{\nabla}F \rightarrow$ slope

(ii) Divergence: $\vec{\nabla}\cdot\vec{A} \rightarrow$ number $\rightarrow$ solenoidal character of $\vec{A}$

(iii) Curl: $\vec{\nabla}\times\vec{A} \rightarrow$ vector $\rightarrow$ rot. nature of $\vec{A}$

1. Gauss' (Green's) Theorem of Vector Algebra

$$\int_V (\vec{\nabla}\cdot\vec{F})\,dV = \int_S \vec{F}\,ds$$

- divergence of a vector $\vec{F}$ over a volume 'V' is surface integral of $\vec{F}$

2. Stoke's Theorem of Vector Algebra

$$\int_S (\vec{\nabla}\times\vec{F})\,ds = \int_C \vec{F}\,dl$$

- curl of a vector $\vec{F}$ over a surface 'S' is line/curl integral of $\vec{F}$

Q) $\vec{A} = x^2\hat{i} + y^2\hat{j} + z^2\hat{k}$

Prove (i) Is the vector field A solenoidal?

(ii) " " " " A irrotational?

$\vec{\nabla}\cdot\vec{A} = 2x + 2y + 2z \rightarrow \therefore$ Non-solenoidal

$$\vec{\nabla}\times\vec{A} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2 & y^2 & z^2 \end{vmatrix} = \hat{i}(0-0) - \hat{j}(0-0) + \hat{k}(0-0) = 0$$

$\therefore$ Irrotational

$\vec{B} = \frac{2z^2y}{x^3}\hat{i} + \frac{x^2}{z^2}\hat{j} + \frac{2yz}{x^2}\hat{k}$

$\vec{\nabla}\cdot\vec{B} = -\frac{6z^2y}{x^4} + 0 + \frac{2y}{x^2} \longrightarrow$ Non-solenoidal

$$\vec{\nabla}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{2z^2y}{x^3} & \frac{x^2}{z^2} & \frac{2yz}{x^2} \end{vmatrix} = \hat{i}\left(\frac{2z}{x^2} - \frac{2x^2}{z^3}\right) - \hat{j}\left(-\frac{4yz}{x^3} - \frac{4yz}{x^3}\right) + \hat{k}\left(\frac{2x}{z^2} - \frac{2z^2}{x^3}\right)$$

$\therefore$ Irrotational

Q) $\phi = \frac{e^{-a/r}}{r^2}$

Find gradient of ϕ

$$\text{grad}(\phi) = \nabla\phi$$
$$= \frac{\partial}{\partial r}\left(\frac{e^{-a/r}}{r^2}\right)\hat{r}$$
$$= \frac{\left(e^{-a/r}\cdot\left(\frac{a}{r^2}\right)\cdot r^2 - e^{-a/r}\cdot 2r\right)}{r^4}\hat{r}$$
$$= \frac{e^{-a/r}(a-2r)}{r^4}\hat{r}$$

Q) Find the value of 'a' of $\bar{A} = (x+3y)\hat{i} + (2y+3z)\hat{j} + (x+az)\hat{k}$, if $\bar{A}$ is solenoidal.

$$\bar{\nabla}\cdot\bar{A} = 0$$
$$\Rightarrow 1+2+a = 0$$
$$\Rightarrow \boxed{a=-3}$$

Important Entities

(I) $\text{Div}(\text{Grad}\,\phi) = \bar{\nabla}\cdot(\bar{\nabla}\phi) = \nabla^2\phi$

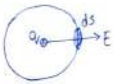
(II) $\text{Curl}(\text{Grad}\,\phi) = \bar{\nabla}\times(\bar{\nabla}\phi) = 0$

(III) $\text{Div}(\text{Curl}\,\bar{A}) = \bar{\nabla}\cdot(\bar{\nabla}\times\bar{A}) = 0$

(IV) $\text{Curl}(\text{Curl}\,\bar{A}) = \bar{\nabla}\times(\bar{\nabla}\times\bar{A})$
$$= \bar{\nabla}(\bar{\nabla}\cdot\bar{A}) - \bar{\nabla}^2 A$$

(V) $\text{Div}(\bar{A}\times\bar{B}) = \bar{\nabla}\cdot(\bar{A}\times\bar{B})$
$$= \bar{B}\cdot(\bar{\nabla}\times\bar{A}) - \bar{A}\cdot(\bar{\nabla}\times\bar{B})$$

Gauss Theorem of Electricity



$$\phi = \int_S E\cdot ds$$
$$= \int_S E\,ds\cos\theta$$
$$\phi_{max} = \int_S E\,ds$$
$$= \frac{1}{4\pi\epsilon_0}\cdot\frac{q}{r^2}\int_S ds$$
$$= \frac{q}{4\pi\epsilon_0 r^2}\times 4\pi r^2$$
$$= \frac{q}{\epsilon_0}$$

$$\boxed{\phi = \int_S E\cdot ds = \frac{q_{enclosed}}{\epsilon_0}}$$ (integral form)

$$\downarrow$$

$$\int_V \bar{\nabla}\cdot\bar{E}\,dV = \frac{1}{\epsilon_0}\int_V \rho\,dV$$

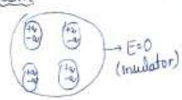
$$\boxed{\bar{\nabla}\cdot\bar{E} = \frac{\rho}{\epsilon_0}}$$ (differential form)

$$\bar{\nabla}\cdot(\epsilon_0\bar{E}) = \rho$$

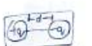
$$\boxed{\bar{\nabla}\cdot\bar{D} = \rho}$$

- Charge should be enclosed by surface and static (NOT dynamic)
- Applicable for conductors

Dielectric



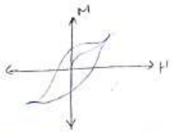
$E = 10\,V$

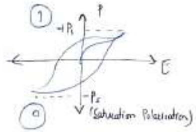


induced dipole

$$\boxed{P = \frac{qd}{V}}$$ (polarization)

Energy density $= \frac{1}{2}\epsilon_0 E^2 = \frac{1}{2}\mu H^2$





- More the area, more the energy absorbed by the material

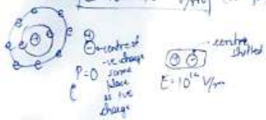
- Mica - least dielectric material (Insulator)

Types of Polarisation (P)

1. Electronic Polarization (P_E)
2. Ionic Polarization (P_I)
3. Orientational Polarization (P_o)
4. Space - charge Polarization (P_s)

Electronic Polarization

$\boxed{E \cdot 10^{12} - 10^{14} \ V/m}$ (10^{14} V/m)



Ionic Polarization

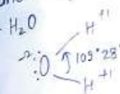
-- Na^+ -- Cl^- -- Na^+ - Cl^- --
-- Cl^- -- Na^+ -- Cl^- -- Na^+ --

$\boxed{E = 10^{8} - 10^{11} \ V/m}$

Orientational Polarization

- Permanent dipole moment

e.g. - H_2O

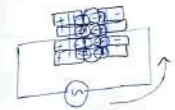


$E = 0 \Rightarrow$ dipole moment = 0 (distributed randomly making net dipole moment ≠ 0)

$E \neq 0 \Rightarrow$ all oriented in dir of $\bar{E}$

($10^6 - 10^9$ V/m)

Space - Charge Polarization



For P_s, $\boxed{E = 10^{2} - 10^{6} \ V/m}$

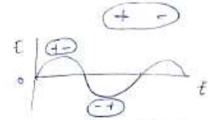
$\boxed{\vec{P} = P_E + P_I + P_o + P_s}$

└ vector

- Those material that have high dielectric constant are used in memory devices

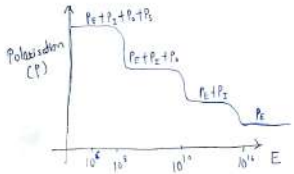
- The system that shows P_E will show P_I, P_o, P_s as well, but vice-versa is not true.

Effect of (AC) electric field on Polarisation or dielectric constant



$$\boxed{E = E_0 \sin \omega t}$$

- Time required for flipping of dipole moment in applied ext. AC field : <u>relaxation time</u> (electric)

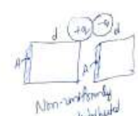


- The material that has P_E can possess all other polarizations.

Dielectric breakdown: when applied field exceeds the critical value of a material, the dipoles break and the dielectric starts to behave as a conductor.

$\epsilon \longrightarrow$ tensor quantity
(dielectric constant)

$$\boxed{\text{Polarisation} = \frac{\text{dipole moment}}{\text{Volume}}}$$



$$\vec{P} = \frac{p}{A.d} = \frac{q.d}{A\,d} = \frac{q}{A}$$

$\boxed{\vec{P} = \vec{\sigma_b}}$ (surface bound-charge density)

$$-Q = \int_S P.ds$$

$$-\int_V \rho\, dV = \int_S P\, ds$$

$$-\int_V \rho\, dV = \int_V \overline{\nabla}.\overline{P}\, dV$$

$$-\int_V \rho_b\, dV = \int_V \overline{\nabla}.\overline{P}\, dV$$

ρ_b → bound volume charge density

$$\boxed{\rho_b = -\overline{\nabla}.\overline{P}}$$

Gauss Theorem: $\overline{\nabla}.\overline{E} = \rho/\epsilon_0$

$$\overline{\nabla}.(\epsilon_0\overline{E}) = \rho = \rho_{free} + \rho_b$$

$$\Rightarrow \overline{\nabla}.(\epsilon_0\overline{E}) = \rho_{free} - \overline{\nabla}.\overline{P}$$

$$\Rightarrow \overline{\nabla}(\epsilon_0\overline{E} + \overline{P}) = \rho_{free}$$

$\overline{\nabla}.\overline{D} = \rho_{free}$ (we know that)

$$\therefore \boxed{\overline{D} = \epsilon_0\overline{E} + \overline{P}}$$

$\boxed{\overline{\nabla}.\overline{D} = \rho_{free}}$

$\vec{D}$ → displacement vector
$(\overline{D} = \epsilon_0\overline{E})$

$$\overline{P} \propto \overline{E}$$

$$\boxed{\overline{P} = \epsilon_0 \chi_e \overline{E}}$$

χ_e → electrical susceptibility — it tells the extent to which a material can be polarized.

ϵ_0 → permittivity of free space

$\boxed{\epsilon_0 = 8.85 \times 10^{-12}\ C^2/m^2N}$

$$\overline{D} = \epsilon_0\overline{E} + \epsilon_0\chi_e\overline{E}$$

$$= \epsilon_0\overline{E}(1+\chi_e)$$

$\boxed{\overline{D} = \epsilon_0\epsilon_r\overline{E}}$

$\boxed{\overline{D} = \epsilon\overline{E}}$

$\boxed{\epsilon_r = \frac{\epsilon}{\epsilon_0}}$

$\boxed{\epsilon_r = 1 + \chi_e}$

Q.) An isotropic material of relative permittivity 'ϵ_r' is placed normal to a uniform ext. electric field with an electric displacement vector of mag. 5×10^{-4} C/m². If the vol of the slab is $0.5\ m^3$ and mag. of polarisation 4×10^{-4} C/m². Find value of ϵ_r and total dipole moment of slab.

$$P = \frac{\text{dipole moment}}{\text{Volume}}$$

$$\Rightarrow \text{dipole moment} = 4 \times 10^{-4} \times 0.5 \quad C\text{-}m$$
$$= 2 \times 10^{-4}\ C\text{-}m$$

$$\bar{D} = \epsilon_0 \bar{E} + \bar{P}$$
$$\Rightarrow \bar{D} = \epsilon_0 \epsilon_r \bar{E}$$
$$\Rightarrow \epsilon_r = \frac{D}{\epsilon_0 E} = \frac{5 \times 10^{-4}}{\cancel{\epsilon_0} \times \frac{10^{-4}}{\cancel{\epsilon_0}}} = 5$$
$$\cancel{= \frac{5 \times 10^{-4}}{\frac{\epsilon_0 \sqrt{41 \times 10^{-8}}}{\epsilon_0}}}$$
$$\cancel{= \frac{5 \times 10^{-4}}{\sqrt{41} \times 10^{-4}}}$$
$$\cancel{= \frac{5}{\sqrt{41}}}$$

$$\cancel{\bar{E} = \frac{\bar{D} + (-\bar{P})}{\epsilon_0}}$$
$$\cancel{= \frac{\sqrt{(5 \times 10^{-4})^2 + (-4 \times 10^{-4})^2}}{\epsilon_0}}$$
$$\cancel{= \frac{\sqrt{41 \times 10^{-8}}}{\epsilon_0}}$$
$$E = \frac{D - P}{\epsilon_0}$$
$$= \frac{10^{-4}}{\epsilon_0}$$

$$\boxed{\bar{\nabla} \cdot \bar{D} = \rho_{free}}$$ ← differential form of Gauss theorem (when polarisation exists)

$$\int_V (\bar{\nabla} \cdot \bar{D})\, dV = \int_V \rho\, dV$$

$$\boxed{\oint_S \bar{D} \cdot d\bar{s} = Q_{free}}$$

Q.) Two parallel plates of capacitors having equal and opposite charges are separated by 6 mm thick dielectric material of dielectric constant 2.8. If the electric field strength inside is 10^5 V/m. Determine $\bar{P}$, $\bar{D}$ and energy density in the dielectrics

$$\bar{D} = \epsilon_0 \epsilon_r \bar{E}$$
$$= 8.85 \times 10^{-12} \times 2.8 \times 10^{5}\ C/m^2$$
$$= 24.78 \times 10^{-7}\ C/m^2 = 2.478 \times 10^{-6}\ C/m^2$$

$$\bar{P} = \epsilon_0 \chi_e \bar{E}$$
$$= \epsilon_0 (\epsilon_r - 1) \bar{E}$$
$$= 8.85 \times 10^{-12} \times 1.8 \times 10^{5}\ C/m^2$$
$$= 15.930 \times 10^{-7}\ C/m^2$$
$$= 1.5930 \times 10^{-6}\ C/m^2$$

$$\text{Energy density} = \frac{1}{2} \epsilon E^2$$
$$= \frac{1}{2} \epsilon_0 \epsilon_r E^2$$
$$= \frac{1}{\cancel{2}} \times 8.85 \times 10^{-12} \times \cancel{2.8}^{1.4} \times 10^{10}\ J/m^3$$
$$= 12.390 \times 10^{-2}\ J/m^3$$
$$= 1.2390 \times 10^{-1}\ J/m^3$$

$885 \times 28 = 24780$

$885 \times 18 = 15930$

$885 \times 14 = 12390$

Q) Dielectric constant of a gas at NTP is 1.00074. Calculate the polarisation density of each atom of gas when it is held in ext. elec field 3×10^4 V/m

$$\bar{P} = \epsilon_0 \chi_e \bar{E}$$
$$= \epsilon_0(\epsilon_r - 1)E$$
$$= 8.85\times10^{-12} \times 0.00074 \times 3\times10^4 \ C/m^2$$
$$= 0.01965\times10^{-8} \ C/m^2$$
$$= 1.965\times10^{-10} \ C/m^2$$

885 × 74 = 3540 + 6195 = 65500; 65500 × 3 = 196500

Q) Two parallel plates having equal and opposite charges separated by 2 cm thick slab of $\epsilon_r = 3$. If E inside 10^6 V/m. Calculate $\bar{D}$ and $\bar{P}$.

$$\bar{P} = \epsilon_0 \chi_e \bar{E}$$
$$= \epsilon_0(\epsilon_r - 1)E$$
$$= 8.85\times10^{-12} \times 2\times10^6 \ C/m^2$$
$$= 17.7\times10^{-6} \ C/m^2$$
$$= 1.77\times10^{-5} \ C/m^2$$

885 × 2 = 1770

$$\bar{D} = \epsilon_0 \epsilon_r \bar{E}$$
$$= 8.85\times10^{-12} \times 3\times10^6$$
$$= 26.55\times10^{-6} \ C/m^2$$
$$= 2.655\times10^{-5} \ C/m^2$$

885 × 3 = 2655

Magnetostatics

$\phi = B.A$ — B: magnetic flux density; H: mag field intensity; M: magnetisation $= \frac{\text{mag. moment}}{\text{volume}}$

$$B = \frac{\mu_0}{4\pi}\frac{2I}{r} = \frac{\mu_0 I}{2\pi r}$$

Origin of Magnetism

(i) Orbital motion of e^- → very weak mag moment

(ii) Spin motion of e^-

Types of magnetic material

(i) Diamagnetic → $\chi_M < 0$ → only orbital motion

(ii) Paramagnetic → $\chi_M = 0$ (absence of H), $\chi_M > 0$ (small +ve) (presence of H)

(iii) Ferromagnetic → $\chi_M >> 0$

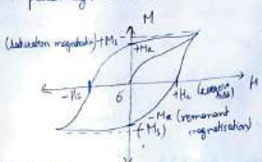
(iv) Anti-ferromagnetic

(v) Ferrimagnetic

$$\boxed{\chi_M = \frac{M}{H}}$$

χ_M: mag. susceptibility

- In the absence of applied mag. field (H), the mag moment/magnetization = 0 in paramagnetic material.

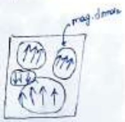


"Hysteresis loop" (ferromagnetic)
↳ energy absorbed by material under mag field

$3d^5$ → Fe, Co, Ni

↑	↑	↑	↑	↑

(all unpaired e^-)



Ferromagnetic

- each domain has mag. moment even in absence of mag. field but net = 0
- when H is applied, all domains oriented in that dir^n giving large value of χ_M.

$T_c \rightarrow$ curie temp.
(ferromagnetic $\rightarrow$ paramagnetic)

Anti-ferro $\rightarrow$ | ↑↓ | ↑↓ | ↑↓ | ↑↓ | ↑↓ | (absence of field H)

$M = 0$

· On applying field, it shows 'M'.

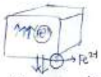
T_N (Néel temp.)
(anti-ferro $\rightarrow$ para)

e.g.- MnO

Ferri $\rightarrow$ residual of spin/mag. moment due to structure

· spinal
· hexa ferrite

e.g.- AB_2O_4, $NiFe_2O_4$, $FeFe_2O_4$, Fe_3O_4
$\downarrow$
$(FeO)(Fe_2O_3)$



· 2 get cancelled
· 1 residual remains

Biot-Savart law

$B = \frac{\mu_0 I}{2\pi r}$

$\oint B \cdot dl = \oint B\, dl \cos\theta$

$\theta = 0$

$= B \oint_c dl$

$= B(2\pi r)$

$= \frac{\mu_0 I}{2\pi r}(2\pi r)$

$\boxed{\oint_c B \cdot dl = \mu_0 I_{enclosed}}$ $\rightarrow$ "Ampere's circuital law" (integral form)

$\oint_c B \cdot dl = \mu_0 I_{enclosed}$

By Stoke's theorem of vector algebra,

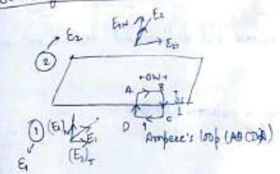
$J = \frac{I}{A}$

$I = \int J \cdot ds$

$\int_S (\vec{\nabla} \times \vec{B}) ds = \mu_0 \int J\, ds$

$\boxed{\vec{\nabla} \times \vec{B} = \mu_0 J}$ $\rightarrow$ differential form of Ampere's circuital law

Boundary Conditions



Let us consider two mediums of permittivity ε_1 and ε_2 and ABCD be a Ampere's loop through both mediums having width ΔW and thickness ΔS

Let E_1 be electric field in med. (I)

E_2 be electric field in med. (II)

having tangential and normal components

$E_1 = E_{1N} + E_{1T}$

$E_2 = E_{2N} + E_{2T}$

· The condition should be satisfied by the field at the interface separating the two med. and at the common boundary of these medium, are called as "boundary condition".

For closed loop, $\int B \cdot dS = 0$

Analogically, $\int E \cdot dl = 0$ (for closed path ABCDA whose width ΔW and thickness ΔS)

$0 = E_{2T} \cdot \Delta W + \left[-\cancel{E_{2N}\frac{\Delta S}{2}} - E_{1N}\frac{\Delta S}{2}\right] + \left[-E_{1T} \cdot \Delta W\right] + \left[E_{1N}\frac{\Delta S}{2} + \cancel{E_{2N}\frac{\Delta S}{2}}\right]$

$\Rightarrow E_{1T}\Delta W = E_{2T}\Delta W \Rightarrow \boxed{E_{1T} = E_{2T}}$ "boundary condition"

* Tangential component of electric field will be always same

$$\boxed{E_{1T} = E_{2T}} \quad \text{———— (I)}$$

$$D_{2T} = \epsilon_2 E_{2T}$$

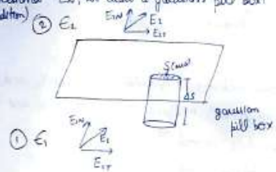
$$\Rightarrow E_{2T} = \frac{D_{2T}}{\epsilon_2}$$

Similarly, $E_{1T} = \frac{D_{1T}}{\epsilon_1}$

$$\therefore \boxed{\frac{D_{2T}}{\epsilon_2} = \frac{D_{1T}}{\epsilon_1}} \quad \text{———— (II)}$$

"E_T is always continuous but displacement vector is not in case of different permittivity"

* To understand E_N, we draw a gaussian pill box:
(find condition)



We know that,

$$\oint D \cdot ds = q \qquad \text{(Gauss Theorem of electricity)}$$

Under close limit $\Delta s \to 0$

$$D_{2N} S - D_{1N} S = \sigma S$$

$$\boxed{D_{2N} - D_{1N} = \sigma}$$

• Normal component of D is discontinuous, which amounts to free charge density (σ)

If $\sigma = 0$, $\boxed{D_{2N} = D_{1N}}$ → D_N becomes continuous ———— (III)

$\boxed{\epsilon_2 E_{2N} = \epsilon_1 E_{1N}}$ → E_N is always discontinuous ———— (IV)

(I), (II), (III), (IV) are boundary conditions of $\bar{D}$ and $\bar{E}$ when separated by inte mediums of permittivities ϵ_1 and ϵ_2

$$B = \frac{\mu_0 I}{2\pi r} = \frac{\mu_0 I r}{2(\pi r^2)} = \frac{\mu_0 J r}{2}$$

$$B = \frac{\mu_0}{4\pi}\frac{I}{r^2}$$

$$B(\bar{r}) = \frac{\mu_0}{4\pi}\int J(r) \times \frac{\hat{r}}{r^2}\, d\tau$$

$$\boxed{B = \frac{\mu_0}{4\pi}\int \frac{I\, dl\, \hat{r}}{r^2}}$$

$\boxed{\bar{\nabla}\cdot\bar{B} = 0}$ ⇒ magnetic monopoles can't exist

Maxwell's eqⁿ (for electrostatic/magnetostatic charges)

"Steady state" → no time factor

$$\boxed{\nabla\cdot E = \frac{\rho}{\epsilon_0}} \equiv \boxed{\bar{\nabla}\cdot\bar{D} = \rho}$$

$$\boxed{\bar{\nabla}\cdot\bar{B} = 0}$$

$$\boxed{\bar{\nabla}\times\bar{E} = 0}$$

$$\boxed{\bar{\nabla}\times\bar{B} = \mu_0 \bar{J}} \equiv \boxed{\bar{\nabla}\times\bar{H} = \bar{J}}$$

Continuity eqⁿ: $\boxed{\frac{\partial\rho}{\partial t} + \bar{\nabla}\cdot\bar{J} = 0}$

$\bar{\nabla}\cdot\bar{B} = 0$

$\Rightarrow \bar{B} = \bar{\nabla}\times\bar{A}$

vector potential

$\bar{\nabla}\times(\bar{\nabla}\times\bar{A}) = \mu_0\bar{J}$

$\bar{\nabla}(\bar{\nabla}\cdot\bar{A}) = \nabla^2 A = \mu_0 J$

0

$\bar{\nabla}\cdot\bar{A} = 0$

$\boxed{\nabla^2 A = -\mu_0 J}$

Poisson eq^n

$\rho = \rho_{free} + \rho_{bound}$

$J = J_{free} + J_{bound}$

$\boxed{J_{bound} = \bar{\nabla}\times\bar{M}}$

magnetisation

$\bar{\nabla}\times\bar{B} = \mu_0(J_f + J_b)$

$\Rightarrow \bar{\nabla}\times\dfrac{\bar{B}}{\mu_0} = J_f + (\bar{\nabla}\times\bar{M})$

$\Rightarrow \bar{\nabla}\times\underbrace{\left(\dfrac{\bar{B}}{\mu_0} - \bar{M}\right)}_{\bar{H}} = J_f$

$\Rightarrow \bar{\nabla}\times\bar{H} = \bar{J}$

$\nabla^2\phi = 0$

scalar potential

$\bar{\nabla}\times\bar{E} = 0$

$\Rightarrow \bar{E} = -\bar{\nabla}\phi$

$\boxed{\nabla^2\phi = \rho/\epsilon_0}$

$\dfrac{B}{\mu_0} - M = H$

$\Rightarrow \boxed{\bar{B} = \mu_0(\bar{M} + \bar{H})}$

$\bar{H}$ (in magnetostatics) $\equiv \bar{D}$ (in electrostatics)

displacement vector

$\boxed{\chi_M = \dfrac{\bar{M}}{\bar{H}}}$

$\boxed{\bar{M} \propto \bar{H}} \Rightarrow \boxed{\bar{M} = \bar{H}\chi_m}$

$\Rightarrow \boxed{\bar{B} = \mu\bar{H}}$

$\bar{B} = \mu_0 \underbrace{(1+\chi_m)}_{\mu_r}\bar{H}$

$\bar{E} \equiv \bar{B}$

$\bar{P} \equiv \bar{M}$

$\bar{D} \equiv \bar{H}$

$\boxed{1+\chi_m = \mu_r}$

$\boxed{\mu_0\mu_r = \mu}$

Continuity Eq^n

"It says that the total current flowing out of some volume must be equal to the rate of decrease of the charge enclosed within that volume if charge is neither being created nor being destroyed."

$$I = \frac{-dQ}{dt}$$

$I \rightarrow$ total current flowing out of the volume

$-\dfrac{dQ}{dt} \rightarrow$ rate of decrease of charge

$$I = \int_S J\cdot dS \qquad Q = \int_V \rho\, dV$$

$$\int_S \bar{J}\cdot ds = \frac{-d}{dt}\int_V \rho\, dV$$

By Gauss Theorem of Vector Algebra

$$\int_V (\bar{\nabla}\cdot\bar{J})dV = -\int_V \frac{\partial\rho}{\partial t}\, dV$$

$$\boxed{\frac{\partial\rho}{\partial t} + \bar{\nabla}\cdot\bar{J} = 0} \Rightarrow \boxed{\frac{\partial\rho}{\partial t} = -\bar{\nabla}\cdot\bar{J}}$$

(continuity eq^n)

Maxwell's fourth eq^n:

$\bar{\nabla}\times\bar{H} = \bar{J}$

Taking dot product with $\bar{\nabla}$ on both sides:

$\bar{\nabla}\cdot(\bar{\nabla}\times\bar{H}) = \bar{\nabla}\cdot\bar{J}$

$\boxed{\bar{\nabla}\cdot\bar{J} = 0} \rightarrow \times\times\times$

Maxwell's correction:

$\bar{\nabla}\times\bar{H} = \bar{J} + \bar{J}'$

Faraday's law

$$\mathcal{E}_{emf} = \frac{-\partial \phi_B}{\partial t} \qquad \phi_B = \int_s \bar{B}\cdot ds$$

$$\mathcal{E}_{emf} = \int_c \bar{E}\cdot dl = \frac{-\partial \phi}{\partial t}$$

By stokes theorem of vector algebra

$$\int_s (\bar{\nabla}\times\bar{E})\, ds = \int_s \left(-\frac{\partial B}{\partial t}\right) ds$$

$$\Rightarrow \boxed{\bar{\nabla}\times\bar{E} = \frac{-\partial B}{\partial t}}$$ ← Maxwell's third law (Faraday's law)

Maxwell's Eq^n

(I) $\bar{\nabla}\cdot\bar{E} = \frac{\rho}{\epsilon_0}$ → Gauss law of electrostatics

(II) $\bar{\nabla}\cdot\bar{B} = 0$ → Non-existence of monopoles

(III) $\bar{\nabla}\times\bar{E} = \frac{-\partial \bar{B}}{\partial t}$ → Faraday's law

(IV) $\bar{\nabla}\times\bar{H} = \bar{J} + \bar{J}'$ → modified Ampere's law

Stationary charge

$$\bar{\nabla}\cdot\bar{J} = 0$$

$$\frac{\partial \rho}{\partial t} = 0$$

In general,

$$\bar{\nabla}\cdot\bar{J} = \frac{-\partial \rho}{\partial t} \qquad \bar{J} = \sigma\bar{E}$$

$$\bar{\nabla}\cdot(\sigma\bar{E}) = \frac{-\partial \rho}{\partial t} \qquad \bar{D} = \epsilon\bar{E}$$

$$\bar{\nabla}\cdot\left(\frac{\sigma}{\epsilon}\bar{D}\right) = \frac{-\partial \rho}{\partial t}$$

$$\underbrace{\bar{\nabla}\cdot\bar{D}}_{\rho} = -\frac{\epsilon}{\sigma}\left(\frac{\partial \rho}{\partial t}\right)$$

$$\Rightarrow \rho = -\frac{\epsilon}{\sigma}\left(\frac{\partial \rho}{\partial t}\right) \Rightarrow \boxed{\rho = \rho_0 e^{-t/\tau}}$$

$\tau \rightarrow$ time constant/relaxation time

Maxwell's Fourth eq^n

$$\bar{\nabla}\times\bar{H} = \bar{J} + \bar{J}'$$

$$\Rightarrow \bar{\nabla}\cdot(\bar{\nabla}\times\bar{H}) = \bar{\nabla}\cdot\bar{J} + \bar{\nabla}\cdot\bar{J}'$$

$$\Rightarrow \bar{\nabla}\cdot\bar{J}' = -\bar{\nabla}\cdot\bar{J}$$

$$\Rightarrow \bar{\nabla}\cdot\bar{J}' = \frac{\partial \rho}{\partial t}$$

$$\Rightarrow \bar{\nabla}\cdot\bar{J}' = \frac{\partial}{\partial t}(\bar{\nabla}\cdot\bar{D})$$

$$\Rightarrow \bar{\nabla}\cdot\bar{J}' = \bar{\nabla}\cdot\left(\frac{\partial \bar{D}}{\partial t}\right)$$

$$\Rightarrow \boxed{\bar{J}' = \frac{\partial \bar{D}}{\partial t}}$$ → displacement current density

$$\boxed{\bar{\nabla}\times\bar{H} = \bar{J} + \frac{\partial \bar{D}}{\partial t}}$$

Maxwell's Eq^n

(I) $\bar{\nabla}\cdot\bar{D} = \rho$ → Gauss law of electrostatics

(II) $\bar{\nabla}\cdot\bar{B} = 0$ → Non-existence of monopoles

(III) $\bar{\nabla}\times\bar{E} = \frac{-\partial \bar{B}}{\partial t}$ → Faraday's law

(IV) $\bar{\nabla}\times\bar{H} = \bar{J} + \frac{\partial \bar{D}}{\partial t}$ → modified Ampere's law

$$\bar{\nabla}\cdot(\bar{\nabla}\times\bar{H}) = \bar{\nabla}\cdot\bar{J} + \bar{\nabla}\cdot\frac{\partial \bar{D}}{\partial t}$$

$$\Rightarrow 0 = \bar{\nabla}\cdot\bar{J} + \bar{\nabla}\cdot\frac{\partial \bar{D}}{\partial t}$$

$$\Rightarrow \bar{\nabla}\cdot\bar{J} = -\bar{\nabla}\cdot\frac{\partial \bar{D}}{\partial t}$$

$$\Rightarrow \cancel{\bar{J} = \frac{-\partial \bar{D}}{\partial t}} \Rightarrow \bar{\nabla}\cdot\bar{J} + \frac{\partial}{\partial t}(\bar{\nabla}\cdot\bar{D}) = 0$$

$$\Rightarrow \cancel{\bar{J} + \frac{\partial \bar{D}}{\partial t} = 0} \Rightarrow \bar{\nabla}\cdot\bar{J} + \frac{\partial \rho}{\partial t} = 0$$ → continuity eq^n

(i) Flux of ele field of a circuit if charge is enclosed

(ii) Flux of mag. field is zero

(iii) Changing mag flux can induce ele. field

(iv) For steady state, mag. field can be found using $\bar{\nabla} \times \bar{B} = \mu \bar{J}$
(charge not varying with time)

if varying, $\bar{\nabla} \times \bar{H} = \mu(\bar{J} + \epsilon \dots)$ we use Maxwell's correction.

Demonstration of Displacement Current

Maxwell's Eq^n in free space

$\mu \rightarrow \mu_0$

$\epsilon \rightarrow \epsilon_0$

$\rho = 0 \quad J = 0$ (let)

(i) $\bar{\nabla} \cdot \bar{E} = 0$

(ii) $\bar{\nabla} \cdot \bar{B} = 0$

(iii) $\bar{\nabla} \times \bar{E} = \frac{-\partial \bar{B}}{\partial t}$

(iv) $\bar{\nabla} \times \bar{H} = \frac{\partial \bar{D}}{\partial t}$

$\boxed{\bar{B} = \mu_0 \bar{H}}$

$\boxed{\bar{D} = \epsilon_0 \bar{E}}$

(iii) $\bar{\nabla} \times \bar{E} = -\frac{\partial \bar{B}}{\partial t}$

$\bar{\nabla} \times (\bar{\nabla} \times \bar{E}) = -\left(\bar{\nabla} \times \frac{\partial \bar{B}}{\partial t}\right)$

$\left(\bar{\nabla}(\bar{\nabla} \cdot \bar{E}) - \bar{\nabla}^2 \bar{E}\right) = -\frac{\partial}{\partial t}(\bar{\nabla} \times \bar{B})$

$0 - \nabla^2 E = \frac{-\partial}{\partial t}(\bar{\nabla} \times \bar{B})$

$\nabla^2 E = \frac{\partial}{\partial t} \mu_0 \left(\frac{\partial D}{\partial t}\right)$

$\nabla^2 E = \mu_0 \left(\frac{\partial^2 D}{\partial t^2}\right)$

$\nabla^2 E = \mu_0 \epsilon_0 \left(\frac{\partial^2 E}{\partial t^2}\right)$

$$\boxed{\nabla^2 E - \mu_0 \epsilon_0 \frac{\partial^2 E}{\partial t^2} = 0}$$

(iv) $\bar{\nabla} \times \bar{H} = \frac{\partial \bar{D}}{\partial t}$

$\bar{\nabla} \times (\bar{\nabla} \times \bar{H}) = \bar{\nabla} \times \frac{\partial \bar{D}}{\partial t}$

$\bar{\nabla}(\bar{\nabla} \cdot \bar{H}) - \bar{\nabla}^2 \bar{H} = \frac{\partial}{\partial t}(\bar{\nabla} \times \bar{D})$

$\frac{\bar{\nabla}(\bar{\nabla} \cdot \bar{B})}{\mu_0} - \nabla^2 H = \epsilon_0 \frac{\partial}{\partial t}(\bar{\nabla} \times \bar{E})$

$0 - \nabla^2 H = -\epsilon_0 \frac{\partial^2 \bar{B}}{\partial t^2}$

$$\boxed{\nabla^2 \bar{B} = \mu_0 \epsilon_0 \frac{\partial^2 \bar{B}}{\partial t^2}}$$

$$\boxed{\nabla^2 \bar{B} - \mu_0 \epsilon_0 \frac{\partial^2 B}{\partial t^2} = 0}$$

$\nabla^2 u - \frac{1}{v^2} \frac{\partial^2 u}{\partial t^2} = 0$

$\therefore \frac{1}{v^2} = \mu_0 \epsilon_0$

$v = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = \frac{1}{\sqrt{4\pi \times 10^{-7} \times 8.85 \times 10^{-12}}}$ m/s

$= \frac{1}{\sqrt{4\pi \times 8.85 \times 10^{-19}}}$ m/s

$\simeq 3 \times 10^8$ m/s

Plane wave eq^n

$$\bar{E}(r,t) = \bar{E}_0 e^{i(\bar{k}\cdot\bar{r} - \omega t)}$$

$$\bar{H}(r,t) = \bar{H}_0 e^{i(\bar{k}\cdot\bar{r} - \omega t)}$$

$\frac{\omega}{k} \rightarrow$ phase velocity

$$\bar{k} = k_x\hat{i} + k_y\hat{j} + k_z\hat{k}$$

$$\bar{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$\bar{k}\cdot\bar{r} = k_x x + k_y y + k_z z$$

$$\bar{E}_0 = E_{0x}\hat{i} + E_{0y}\hat{j} + E_{0z}\hat{k} \qquad \bar{H}_0 = H_{0x}\hat{i} + H_{0y}\hat{j} + H_{0z}\hat{k}$$

$$\cancel{\bar{\nabla}\times\bar{E} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & E_z \end{vmatrix} e^{i(\bar{k}\cdot\bar{r} - \omega t)}}$$

$$\cancel{= \left[\hat{i}\left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\right) - \hat{j}\left(\frac{\partial E_z}{\partial x} - \frac{\partial E_x}{\partial z}\right) + \hat{k}\left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right)\right] e^{i(\bar{k}\cdot\bar{r} - \omega t)}}$$

$$\cancel{= \hat{i}}$$

$$\boxed{\bar{\nabla}\times\bar{E} = i[\bar{k}\times\bar{E}]}$$

$$= -\mu_0 \frac{\partial \bar{H}}{\partial t}$$

$$= -\mu_0(-i\omega\bar{H})$$

$$\boxed{\bar{k}\times\bar{E} = \omega\bar{B}}$$

$$\bar{\nabla}\times\bar{E} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_{0x} e^{i(\bar{k}\cdot\bar{r} - \omega t)} & E_{0y} e^{i(\bar{k}\cdot\bar{r} - \omega t)} & E_{0z} e^{i(\bar{k}\cdot\bar{r} - \omega t)} \end{vmatrix}$$

$$= \hat{i}\left(E_{0z}\cdot e^{i(\bar{k}\cdot\bar{r} - \omega t)}\cdot i\cdot k_y - E_{0y} e^{i(\bar{k}\cdot\bar{r} - \omega t)}\cdot i k_z\right) - \hat{j}\left(E_{0z}\cdot e^{i(\bar{k}\cdot\bar{r} - \omega t)}\cdot i\cdot k_x - E_{0x}\cdot e^{i(\bar{k}\cdot\bar{r} - \omega t)}\cdot i\cdot k_z\right)$$

$$+ \hat{k}\left(E_{0y}\cdot e^{i(\bar{k}\cdot\bar{r} - \omega t)}\cdot i k_x - E_{0x}\cdot e^{i(\bar{k}\cdot\bar{r} - \omega t)}\cdot i k_y\right)$$

$$= i\cdot e^{i(\bar{k}\cdot\bar{r} - \omega t)}\left[(k_y E_{0z} - k_z E_{0y})\hat{i} - (k_x E_{0z} - k_z E_{0x})\hat{j} + (k_x E_{0y} - k_y E_{0x})\hat{k}\right]$$

$$= i[\bar{k}\times\bar{E}]$$

Similarly,

$$\bar{\nabla}\times\bar{H} = i\cdot e^{i(\bar{k}\cdot\bar{r} - \omega t)}\left[(k_y H_{0z} - k_z H_{0y})\hat{i} - (k_x H_{0z} - k_z H_{0x})\hat{j} + (k_x H_{0y} - k_y H_{0x})\hat{k}\right]$$

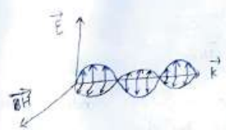
$$= i[\bar{k}\times\bar{H}]$$

$$\bar{\nabla}\times\bar{H} = \epsilon_0 \frac{\partial \bar{E}}{\partial t}$$

$$i[\bar{k}\times\bar{H}] = \epsilon_0(-i\omega E)$$

$$\Rightarrow \boxed{\bar{k}\times\bar{H} = -\omega\bar{D}}$$

- $\bar{E}$ and $\bar{B}$ are normal to each other and EM waves is normal to both.



- $\vec{E}, \vec{H}, \vec{k}$ form a set of orthogonal vectors

Maxwell's eqⁿ in isotropic medium

$\epsilon \rightarrow$ permittivity $\qquad \bar{D} = \epsilon \bar{E}$

$\mu \rightarrow$ permeability $\qquad \bar{B} = \mu \bar{H}$

$$\rho = 0, \bar{J} = 0$$

$$\bar{E} = \bar{E}_0 e^{i(\bar{k}\cdot\bar{r} - \omega t)}$$

$$\bar{H} = \bar{H}_0 e^{i(\bar{k}\cdot\bar{r} - \omega t)}$$

From eqⁿ (iii),

$$\bar{\nabla} \times \bar{E} = -\frac{\partial \bar{B}}{\partial t}$$

Taking $\bar{\nabla} \times$ on both sides,

$$\bar{\nabla} \times (\bar{\nabla} \times \bar{E}) = \frac{-\partial}{\partial t}(\bar{\nabla} \times \bar{B})$$

$$\Rightarrow \bar{\nabla}(\underbrace{\bar{\nabla}\cdot\bar{E}}_{0}) - \nabla^2 \bar{E} = -\mu\frac{\partial}{\partial t}(\bar{\nabla} \times \bar{H})$$

$$\Rightarrow 0 - \nabla^2 \bar{E} = -\mu\epsilon \frac{\partial^2 \bar{E}}{\partial t^2}$$

$$\Rightarrow \cancel{\boxed{\frac{\partial \bar{E}}{\partial t} - \frac{1}{\mu\epsilon}\frac{\partial^2 \bar{E}}{\partial t^2} = 0}} \qquad \boxed{\nabla^2 \bar{E} - \mu\epsilon \frac{\partial^2 \bar{E}}{\partial t^2} = 0}$$

Similarly,

$$\boxed{\nabla^2 \bar{H} - \mu\epsilon \frac{\partial^2 \bar{H}}{\partial t^2}}$$

$$\therefore \frac{1}{v^2} = \mu\epsilon$$

$$\Rightarrow \boxed{v = \frac{1}{\sqrt{\mu\epsilon}} = \frac{c}{\sqrt{\mu_r \epsilon_r}}}$$

- ~~Velocity~~ Speed of E.M wave in isotropic dielectric medium is always smaller than that of in free space.

$$\boxed{\text{Refractive index} = \frac{c}{v} = \sqrt{\mu_r \epsilon_r}}$$

- For non-magnetic medium, $\mu_r \approx 1$

$$\boxed{\text{Refractive index} = \sqrt{\epsilon_r}}$$

$$\boxed{\bar{k} \times \bar{E} = \mu_0 \omega \bar{H}} \qquad \boxed{\bar{k} \times \bar{H} = -\epsilon_0 \omega \bar{E}}$$

Taking "$\bar{k}\times$" on both sides,

$$\bar{k} \times (\bar{k} \times \bar{E}) = \mu_0 \omega (\bar{k} \times \bar{H})$$

$$\Rightarrow \bar{k}(\underbrace{\bar{k}\cdot\bar{E}}_{0}) - \bar{E}(\bar{k}\cdot\bar{k}) = \mu_0\omega(-\epsilon_0 \omega \bar{E})$$

$$\Rightarrow -k^2 \bar{E} + \mu_0 \epsilon_0 \omega^2 \bar{E} = 0$$

$$\Rightarrow (\mu_0 \epsilon_0 \omega^2 - k^2) = 0$$

$$\Rightarrow k^2 = \mu_0 \epsilon_0 \omega^2$$

$$\Rightarrow \left(\frac{\omega}{k}\right)^2 = \frac{1}{\mu_0 \epsilon_0}$$

$$\Rightarrow \left(\frac{\omega}{k}\right) = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

$$\therefore \text{Phase velocity } \left(\frac{\omega}{k}\right) = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = c \approx 3 \times 10^8 \text{ m/s}$$

$$\bar{k} \times \bar{E} = \mu_0 \omega \bar{H}$$

Taking mod on both sides,

$$|\bar{k} \times \bar{E}| = |\mu_0 \omega \bar{H}|$$

$$\Rightarrow |\bar{k}||\bar{E}| = \mu_0 \omega |\bar{H}|$$

$$\Rightarrow \frac{E}{H} = \frac{\mu_0 \omega}{k}$$

$$\Rightarrow \frac{E}{H} = \frac{\mu_0}{\sqrt{\mu_0 \epsilon_0}}$$

$$\Rightarrow \frac{E}{H} = \sqrt{\frac{\mu_0}{\epsilon_0}} = 3.77 \times 10^2 \frac{\text{N-m}}{\text{A}^2\text{-s}}$$

$$\Rightarrow \frac{E}{H} \approx 377\,\Omega$$

characteristic impedance

["min" impedance of a EM wave due to its existence]

"EM wave is transverse in nature"

Proof:

$\vec{\nabla}\cdot\vec{E}=0$

$\vec{\nabla}\cdot\vec{H}=0$

$\vec{E}=(E_{0x}\hat{i}+E_{0y}\hat{j}+E_{0z}\hat{k})\,e^{i(\vec{k}\cdot\vec{r}-\omega t)}$

$\vec{H}=(H_{0x}\hat{i}+H_{0y}\hat{j}+H_{0z}\hat{k})\,e^{i(\vec{k}\cdot\vec{r}-\omega t)}$

$\vec{k}\cdot\vec{r}=k_x x+k_y y+k_z z$

$\vec{\nabla}=\hat{i}\frac{\partial}{\partial x}+\hat{j}\frac{\partial}{\partial y}+\hat{k}\frac{\partial}{\partial z}$

$\vec{\nabla}\cdot\vec{E}=i\,e^{i(\vec{k}\cdot\vec{r}-\omega t)}\cdot(E_{0x}k_x+E_{0y}k_y+E_{0z}k_z)$

$\vec{\nabla}\cdot\vec{H}=i\,e^{i(\vec{k}\cdot\vec{r}-\omega t)}\cdot(H_{0x}k_x+H_{0y}k_y+H_{0z}k_z)$

$\vec{\nabla}\cdot\vec{E}=0$ | $\vec{\nabla}\cdot\vec{H}=0$

$\Rightarrow i(\vec{k}\cdot\vec{E})=0$ | $\Rightarrow i(\vec{k}\cdot\vec{H})=0$

$\Rightarrow \vec{k}\perp\vec{E}$ | $\Rightarrow \vec{k}\perp\vec{H}$

Hence, EM wave is transverse in nature

EM wave in conducting medium

charge on surface ($\rho=0$)

Maxwell Eq^n: $\boxed{\vec{J}=\sigma\vec{E}}$

1. $\vec{\nabla}\cdot\vec{E}=0$
2. $\vec{\nabla}\cdot\vec{H}=0$
3. $\vec{\nabla}\times\vec{E}=-\mu\frac{\partial\vec{H}}{\partial t}$
4. $\vec{\nabla}\times\vec{H}=\sigma\vec{E}+\epsilon\frac{\partial\vec{E}}{\partial t}$

Taking cross product with $\vec{\nabla}$ on eq^n (IV),

$\vec{\nabla}\times(\vec{\nabla}\times\vec{H})=(\vec{\nabla}\times\vec{E})\sigma+\epsilon\frac{\partial}{\partial t}(\vec{\nabla}\times\vec{E})$

$\Rightarrow \vec{\nabla}(\underbrace{\vec{\nabla}\cdot\vec{H}}_{0})-\nabla^2 H=-\sigma\mu\frac{\partial\vec{H}}{\partial t}-(\epsilon\mu)\frac{\partial^2\vec{H}}{\partial t^2}$

$\Rightarrow \boxed{\nabla^2 H=\sigma\mu\frac{\partial\vec{H}}{\partial t}+\epsilon\mu\frac{\partial^2\vec{H}}{\partial t^2}}$

Similarly, taking cross product with $\vec{\nabla}$ on eq^n (III),

$\boxed{\nabla^2\vec{E}=\mu\sigma\frac{\partial}{\partial t}\vec{E}+\mu\epsilon\frac{\partial^2\vec{E}}{\partial t^2}}$

If $\sigma=0$, $\boxed{\nabla^2\vec{E}=\mu\epsilon\frac{\partial^2\vec{E}}{\partial t^2}}$ and, $\boxed{\nabla^2\vec{H}=\mu\epsilon\frac{\partial^2\vec{H}}{\partial t^2}}$

If $\mu=\mu_0$ and $\epsilon=\epsilon_0$

$\boxed{\nabla^2\vec{E}=\mu_0\epsilon_0\frac{\partial^2\vec{E}}{\partial t^2}}$ and, $\boxed{\nabla^2\vec{H}=\epsilon_0\mu_0\frac{\partial^2\vec{H}}{\partial t^2}}$

$\left.\begin{array}{l}\cdot\ \mu\sigma\frac{\partial\vec{E}}{\partial t}\\ \cdot\ \mu\sigma\frac{\partial\vec{H}}{\partial t}\end{array}\right\}$ loss factor / dissipating factor

Q) Show that the eq^n of continuity is contained in Maxwell's eq^n.

Q) Show that the electric and magnetic energy densities in a free travelling wave are equal. Also prove that the total energy density is $U=\epsilon_0 E^2$ or $=\mu_0 H^2$.

A: $U_E=\frac{1}{2}\epsilon_0 E^2$ | $U_H=\frac{1}{2}\mu_0 H^2$

$=\frac{1}{2}\mu_0 H^2$ | $=\frac{1}{2}\epsilon_0 E^2$

$\because U_E=U_H$

$\therefore U=U_E+U_H$

$=\epsilon_0 E^2$

$=\mu_0 H^2$

Q) $\vec{E}_1=k(xy\,\hat{i}+2yz\,\hat{j}+3xz\,\hat{k})$ and $\vec{E}_2=k(y^2\hat{i}+(2xy+z^2)\hat{j}+2yz\,\hat{k})$. Which field is possible and which is not?

$\left.\begin{array}{l}\vec{\nabla}\cdot\vec{E}_1=k(y+2z+3x)\\ \vec{\nabla}\cdot\vec{E}_2=k(0+2x+2y)\end{array}\right\}$ Both possible

$\cdot$ If $\vec{\nabla}\cdot\vec{E}=0$, then NOT possible.

Q) $\vec{B}_1=k(3x^2z\,\hat{i}+5xy^2\,\hat{j}+3x^4z^3\,\hat{k})$

$\vec{B}_2=k(y^4\hat{i}+(2x^2y^2+z^2)\hat{j}+2y^3z^4\hat{k})$

$\left.\begin{array}{l}\vec{\nabla}\cdot\vec{B}_1=k(6xz+10xy+9x^4z^2)\\ \vec{\nabla}\cdot\vec{B}_2=k(4x^2y+8y^3z^3)\end{array}\right\}$ NOT possible

Poynting Theorem/Work-Energy Theorem/Power Loss Theorem

· To find the instantaneous power

$$\mathrm{Div}(\bar{A}\times\bar{B}) = \bar{B}\cdot(\vec{\nabla}\times\vec{A}) - \vec{A}\cdot(\vec{\nabla}\times\bar{B})$$

$$\vec{\nabla}\cdot(\bar{A}\times\vec{B})$$

$$\vec{\nabla}\cdot(\vec{E}\times\vec{H}) = \vec{H}\cdot(\vec{\nabla}\times\vec{E}) - \vec{E}\cdot(\vec{\nabla}\times\vec{H})$$

From Maxwell's eq^n (IV),

$$\vec{\nabla}\times\vec{H} = \vec{J} + \frac{\partial\vec{D}}{\partial t}$$

$$\Rightarrow \vec{E}\cdot(\vec{\nabla}\times\vec{H}) = \vec{E}\cdot\vec{J} + E\cdot\frac{\partial\vec{D}}{\partial t}$$

$\boxed{\vec{J} = \sigma\vec{E}}$

$\boxed{\vec{E}\cdot\vec{J} = \sigma E^2}$

$\boxed{\vec{B} = \mu\vec{H}}$

$\boxed{D = \epsilon\vec{E}}$

From Maxwell's eq^n (III),

$$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$

$$\therefore \vec{\nabla}\cdot(\vec{E}\times\vec{H}) = \vec{H}\cdot\left(-\frac{\partial\vec{B}}{\partial t}\right) - \left[\vec{E}\cdot\vec{J} + \vec{E}\cdot\frac{\partial\vec{D}}{\partial t}\right]$$

$$\Rightarrow -\bar{\nabla}\cdot(\bar{E}\times\bar{H}) = \frac{2}{2}\mu\bar{H}\cdot\frac{\partial\bar{H}}{\partial t} + \bar{E}\cdot\bar{J} + \frac{2}{2}\epsilon\bar{E}\cdot\frac{\partial\bar{E}}{\partial t}$$

$$= \frac{1}{2}\mu\frac{\partial H^2}{\partial t} + \bar{E}\cdot\bar{J} + \frac{1}{2}\epsilon\frac{\partial E^2}{\partial t}$$

$$= \bar{E}\cdot\bar{J} + \frac{\partial}{\partial t}\left(\frac{1}{2}\mu H^2\right) + \frac{\partial}{\partial t}\left(\frac{1}{2}\epsilon E^2\right)$$

$$\Rightarrow \bar{\nabla}\cdot(\bar{E}\times\bar{H}) = -\bar{E}\cdot\bar{J} - \frac{\partial}{\partial t}\underbrace{\left(\frac{1}{2}\mu H^2\right)}_{U_H} - \frac{\partial}{\partial t}\underbrace{\left(\frac{1}{2}\epsilon E^2\right)}_{U_E}$$

$$-\int_V \bar{\nabla}\cdot(\bar{E}\times\bar{H})\,dV = \int_V(\bar{E}\cdot\bar{J})\,dV + \int_V\frac{\partial}{\partial t}(U_E + U_H)\,dV$$

$$\Rightarrow \boxed{-\oint_S(\bar{E}\times\bar{H})\cdot dS = \int_V(\bar{E}\cdot\bar{J})\,dV + \int_V\frac{\partial}{\partial t}(U_E + U_H)\,dV}$$ ← integral form of Poynting Theorem

•"Integral form of Poynting Theorem states that ~~internal~~ net inwards flux of the poynting vector through some closed surface is sum of power dissipated in the vol enclosed by the surface and the rate of change of energy stored in the vol enclosed by the surface."

· Poynting Vector $\boxed{\vec{S} = \vec{E}\times\vec{H}}$

$\boxed{\text{Unit: } W/m^2}$

Q) Calculate the value of poynting vector for a 60 W bulb at a distance of 0.5 m from it.

$$\therefore S = \frac{P}{A} = \frac{60}{4\pi(0.5)^2}\ W/m^2$$

$$= \frac{60}{4\pi\times 25}\times 10^2\ W/m^2$$

$$= \frac{240}{4\pi}\ W/m^2$$

$$= \frac{60}{\pi}\ W/m^2$$

Q) Calculate the value of poynting vector at surface of sun if power radiated is 3.8×10^{26} W and 7×10^8 m is its radius

$$\therefore S = \frac{P}{A} = \frac{3.8\times10^{26}}{4\pi\times7\times7\times10^{16}}$$

$$= \frac{3.8}{4\times22\times7}\times\frac{10^{11}}{10}\ W/m^2$$

Q) A plane wave travelling in free space has an amplitude of electric field $E_y = 100$ V/m and frequency 1 GHz.

(i) Find the phase velocity, wavelength and propagation constant

(ii) Determine the characteristic impedance of free space

(iii) Find mag. and dir^n of mag. field intensity

(i) $\text{speed} = \frac{1}{\sqrt{\mu_0\epsilon_0}} = c = 3\times10^8$ m/s

$v = f\lambda$

$\Rightarrow \lambda = \frac{c}{f} = \frac{3\times10^8}{10^9} = 0.3$ m

$c = \frac{\omega}{k}$

$\Rightarrow k = \frac{\omega}{c} = \frac{2\pi f}{c} = \frac{2\pi c}{\lambda c} = \frac{2\pi}{\lambda} = \frac{2\times3.14}{0.3015} \approx 2.93\ m^{-1}$

(ii) $Z = \frac{E}{H} = \sqrt{\frac{\mu_0}{\epsilon_0}} = 377\,\Omega$

(iii) $\vec{H}$ (A/m) is in z-dir^n as E is in y and k is in x

$B = \frac{E}{c} = \frac{100}{3\times10^8} = 33.33\times10^{-8}$ T

$= 3.333\times10^{-7}$ T

$H = \frac{B}{\mu} =$

Q) A plane wave travelling in (+ve) x-dirⁿ in a lossless unbounded medium having permeability $\mu = \frac{\mu_0}{3}$ and permittivity $\epsilon = 3\epsilon_0$.

(i) Find the velocity of wave

(ii) If $E_y = 10$ V/m and $E_z = 0$ V/m. Find amplitude and dirⁿ of mag field intensity (H)

(i) $v = \frac{1}{\sqrt{\mu\epsilon}} = \frac{1}{\sqrt{\frac{\mu_0}{3}\epsilon_0 \times 3}} = c = 3 \times 10^8$ m/s

(ii) H is in z-dirⁿ

$$\frac{E}{H} = \sqrt{\frac{\mu_0}{9\epsilon_0}} = \frac{1}{3} \times 377\,\Omega$$

$$\Rightarrow H_z = \frac{10 \times \cancel{3}}{\cancel{377}} = \frac{30}{377} = 0.079 \text{ A/m}$$

Q) A plane travelling wave in free space has an avg. poynting vector with a magnitude of 3 $\frac{J}{m^2 s}$. Find the avg. energy density of wave.

$P = 3$ J/m²s

$U = \epsilon_0 E^2 = \mu_0 H^2$

$$\therefore \frac{P}{U} = \frac{EH}{\mu_0 H^2}$$

$$= \frac{1}{\mu_0}\frac{E}{H}$$

$$= \frac{1}{\mu_0}\sqrt{\frac{\mu_0}{\epsilon_0}}$$

$$= \frac{1}{\sqrt{\mu_0\epsilon_0}}$$

$$= c$$

$$\therefore \frac{P}{c} = \frac{3}{3 \times 10^8} = 10^{-8} \text{ J/m}^3$$

Properties of Matter

Simple Harmonic Motion

An object oscillates periodically and always tends to its mean position such that total energy is constant.

Types:

(1) Linear SHM $\rightarrow F \propto (-x)$

(2) Angular SHM $\rightarrow \tau \propto (-\theta)$

$$F \propto -x$$

$$\boxed{F = -kx}$$

$$F = m\frac{d^2x}{dt^2}$$

$$\Rightarrow m\frac{d^2x}{dt^2} + kx = 0$$

$$\Rightarrow \frac{d^2x}{dt^2} + \left(\frac{k}{m}\right)x = 0$$

$$\Rightarrow \boxed{\frac{d^2x}{dt^2} + \omega^2 x = 0} \quad \text{—(1)}, \quad \omega^2 = \frac{k}{m}$$

$$\Rightarrow \boxed{\omega = \sqrt{\frac{k}{m}}}$$

$$\Rightarrow \frac{2\pi}{T} = \sqrt{\frac{k}{m}}$$

$$\Rightarrow \boxed{T = 2\pi\sqrt{\frac{m}{k}}}$$

" A particle of mass 'm' executing SHM with displacement from its mean position 'x' along a straight line at any time 't', then the force acting on it is $\boxed{F = -kx}$ "

let us assume,

$$x = Be^{\alpha t}$$

$$\Rightarrow \frac{dx}{dt} = \alpha Be^{\alpha t}$$

$$\Rightarrow \frac{d^2x}{dt^2} = \alpha^2 Be^{\alpha t} = \alpha^2 x$$

$$\Rightarrow \frac{d^2x}{dt^2} - \alpha^2 x = 0$$

Putting it in eqⁿ (1),

$$\alpha^2 x + \omega^2 x = 0$$

$$\Rightarrow x(\alpha^2 + \omega^2) = 0 \Rightarrow \underbrace{Be^{\alpha t}}_{\text{can't be zero}}(\alpha^2 + \omega^2) = 0 \Rightarrow \boxed{\alpha^2 = -\omega^2} \Rightarrow \boxed{\alpha = \pm i\omega}$$

$\therefore \boxed{x = Be^{\pm i\omega t}}$

Linear combination for general solⁿ:

$\boxed{x = B_1 e^{i\omega t} + B_2 e^{-i\omega t}}$, where $B_1, B_2 \rightarrow$ constants

$$x = B_1\left[\cos(\omega t) + i\sin(\omega t)\right] + B_2\left[\cos(\omega t) - i\sin(\omega t)\right]$$

$$= \underbrace{(B_1 + B_2)}_{A\sin\delta} \cdot \cos(\omega t) + \underbrace{(B_1 - B_2)\, i}_{A\cos\delta} \cdot \sin(\omega t)$$

$$= A\sin\delta \cdot \cos\omega t + A\cos\delta \sin\omega t$$

$\boxed{x = A\sin(\omega t + \delta)} \longleftarrow$ solⁿ of 'x'

- It gives displacement of a particle executing SHM at any instant time 't' with 'A' as the maxᵐ (amplitude) displacement of the particle.
- if we put $\boxed{t \rightarrow t + \frac{2\pi}{\omega}}$, we get same eqⁿ.

$$x = A\sin\left[\omega\left(t + \frac{2\pi}{\omega}\right) + \delta\right]$$

$$= A\sin\left[2\pi + \omega t + \delta\right]$$

$$= A\sin\left[\omega t + \delta\right]$$

$T \rightarrow$ timeperiod $(\omega t + \delta) \rightarrow$ phase (constant) with which the particle executes SHM

$\boxed{T = \frac{2\pi}{\omega}}$

- Particle executing SHM $\rightarrow$ "harmonic oscillator"

$$x = A\sin(\omega t + \delta)$$

$$\boxed{v = \frac{dx}{dt} = A\omega\cos(\omega t + \delta)}$$

$$\boxed{v = \omega\sqrt{A^2 - x^2}}$$

at $x = 0$, $\boxed{v = A\omega} \rightarrow v_{max}$ at mean position

$\boxed{a = \frac{dv}{dt} = \frac{d^2x}{dt^2} = -A\omega^2\sin(\omega t + \delta) = -\omega^2 x}$. $\boxed{a_{max} = -\omega^2 A}$

$\boxed{\text{Total Energy} = \frac{1}{2}m\omega^2 A^2}$

At mean position, KE → maxᵐ and PE = 0

$$KE = \frac{1}{2}mv^2$$

$$= \frac{1}{2}m\omega^2(A^2 - x^2)$$

$\because x = 0$

$\therefore KE = \frac{1}{2}m\omega^2 A^2$

$$T.E = KE + PE$$

$$= \frac{1}{2}m\omega^2 A^2 + 0$$

$$= \frac{1}{2}m\omega^2 A^2$$

At extreme position, KE = 0 ($\because v = 0$) and PE → maxᵐ

and $F = -m\omega^2 A$

$\therefore \cancel{PE} = -\int F\,\cancel{dx}$

In general,

$$F = -kx$$

$$PE = -\int F\,dx$$

$$= \frac{1}{2}kx^2$$

$$KE = \frac{1}{2}mv^2$$

$$= \frac{1}{2}m\omega^2(A^2 - x^2)$$

$$\therefore T.E = \frac{1}{2}m\omega^2(A^2 - x^2) + \frac{1}{2}m\omega^2 x^2$$

$$= \frac{1}{2}m\omega^2 A^2$$

Q.> Find the max^n velocity and acceleration of SHM of $T = 10\pi$ and amplitude $A = 5\times10^{-2}$ m.

$v = \omega\sqrt{A^2 - x^2}$

$v_{max} = \omega A$

$= \frac{2\pi}{T} A$

$= \frac{2\pi}{10\pi} \times 5 \times 10^{-2}$ m/s

$= 10^{-2}$ m/s

$a = -\omega^2 x$

$a_{max} = -\omega^2 A$

$= -\left(\frac{2\pi}{10\pi}\right)^2 \times 5\times10^{-2}$ m/s^2

$= -\frac{1}{5}\times10^{-2}$ m/s^2

Q.> A particle of mass m = 100 g is placed in a field of potential $U = 5x^2 + 10$ erg/gm. Find the frequency.

$U = 5x^2 + 10$ erg/gm $mU = 500x^2 + 1000$ erg

$F = -\frac{dU}{dx} = -10x$ dynes

$\Rightarrow 100\,\frac{d^2x}{dt^2} = -10x$

$\Rightarrow \frac{d^2x}{dt^2} = -\frac{1}{10}x$

$\omega^2 = \frac{1}{10}$

$\omega = \pm\sqrt{\frac{1}{10}}$

$\Rightarrow 2\pi f = \sqrt{\frac{1}{10}}$

$\Rightarrow f = \frac{1}{2\pi}\sqrt{\frac{1}{10}}$ Hz

Q.> A mass of 1 kg is attached to a spring of stiffness constant 16 N/m. Find its natural frequency.

$T = 2\pi\sqrt{\frac{m}{K}}$

$f = \frac{1}{2\pi}\sqrt{\frac{K}{m}} = \frac{1}{2\pi}\sqrt{\frac{16}{1}}$

$= \frac{4}{2\pi}$

$= \frac{2}{\pi}$ Hz

A particle executing SHM along a straight line has vel. 16 cm/s and 12 cm/s when passing through points 3 cm and 4 cm from its mean position respectively. Find (i) amplitude (ii) time period

$v = \omega\sqrt{A^2 - x^2}$

$16 = \omega\sqrt{A^2 - 3^2}$ — (i) $12 = \omega\sqrt{A^2 - 4^2}$ — (ii)

eq^n (i) ÷ (ii),

$\frac{16}{12} = \frac{\omega\sqrt{A^2 - 3^2}}{\omega\sqrt{A^2 - 4^2}}$

$\Rightarrow \frac{A^2 - 9}{A^2 - 16} = \frac{16}{9}$

$\Rightarrow 9A^2 - 81 = 16A^2 - 256$

$\Rightarrow 7A^2 = 175$

$\Rightarrow A = \sqrt{\frac{175}{7}}$ cm $= 5$ cm

$\therefore 16 = \omega\sqrt{5^2 - 3^2}$

$\Rightarrow 16 = 4\omega$

$\Rightarrow \omega = 4$ rad/s

$\Rightarrow \frac{2\pi}{T} = 4$

$\Rightarrow T = \frac{\pi}{2}$ s

-256
81
175

$T = \frac{\pi}{2}$ s

Find the eq^n of a wave of amplitude 2 cm, time period 0.5 s and velocity is 200 cm/s along x-axis.

$y = A\sin(\omega t - kx + \phi)$

$= \frac{2}{100}\sin(4\pi t - 2\pi x)$

$v = \frac{\omega}{K}$

$\Rightarrow K = \frac{\omega}{v} = \frac{2\pi}{Tv} = \frac{2\pi}{\left(\frac{1}{2}\right)\times200}$

$= \frac{\pi}{50}$ cm^{-1}

$= 2\pi$ m^{-1}

$\omega = \frac{2\pi}{T} = \frac{2\pi}{\left(\frac{1}{2}\right)} = 4\pi$ s^{-1}

Q.) The displacement of a sound wave is given by $u(x,t) = 1.5\times10^{-3}\sin\left(\frac{2\pi}{8}x - 80\pi t\right)$, where $x \rightarrow$ metres and $t \rightarrow$ seconds. Find out amplitude, wavelength and frequency of wave

$$u(x,t) = 1.5\times10^{-3}\sin\left(\frac{2\pi}{8}x - 80\pi t\right)$$

$$\boxed{A = 1.5\times10^{-3}}$$

$$\cancel{\omega = \frac{2\pi}{8}}$$

$$\Rightarrow \frac{2\pi}{T} = \cancel{\frac{2\pi}{8}}\ 80\pi$$

$$\Rightarrow \boxed{\cancel{T = 8s}}\quad \boxed{T = \frac{1}{40}s = 0.025s}\quad \boxed{f = 40\ Hz}$$

$$k = \frac{2\pi}{\lambda} \Rightarrow \lambda = \frac{2\pi}{2\pi}\cdot 8\ m = 8m$$

$$\cancel{\Rightarrow \lambda = \frac{2\pi}{80\pi} = \frac{1}{40}\ m = 0.025\ m}$$

Q.) The visible region of EM spectra is 400nm − 700nm. Find equivalent frequency range.

$$v = f\lambda$$

$$\Rightarrow 3\times10^{8} = f\times400\times10^{-9}$$

$$\Rightarrow f = \frac{3\times10^{8}}{400}\times10^{9} = 0.75\times10^{15}\ Hz$$

$$v = f\lambda$$

$$\Rightarrow 3\times10^{8} = f\times700\times10^{-9}$$

$$\Rightarrow f = \frac{3\times10^{8}\times10^{9}}{700} = \frac{3}{7}\times10^{15}\ Hz$$

Q.) A spring stretches 0.15m when a 0.3 kg mass hangs from it. The spring is then stretched an addition 0.1 m and then released. Determine (i) spring constant (ii) amplitude of oscillation 'A' and (iii) V_{max}

(i) $mg = kx$

$$\Rightarrow 0.3\times10 = k\times0.15$$

$$\Rightarrow k = \frac{3\times100}{0.15} = 20\ N/m$$

(ii) $A = 0.1\ m$

(iii) $V_{max} = \omega A$

$$= \sqrt{\frac{k}{m}}\cdot A$$

$$= \sqrt{\frac{200}{0.3}}\times0.1\ m/s$$

$$= \frac{1}{10}\sqrt{\frac{200}{3}}\ m/s \approx 0.81\ m/s$$

Damped Oscillation (Damped Harmonic Motion)

spring constant $= k$

x' = displacement of body from the equilibrium state at any inst. of time 't'

$\frac{dx'}{dt}$ = inst. velocity

Two types of forces:

(i) Restoring force $= -kx \rightarrow$ act in the opposite dirⁿ

(ii) Damping force $= -q'\frac{dx'}{dt} \rightarrow$ opposite dirⁿ to the motion (velocity)

$$m\frac{d^2x'}{dt^2} = -kx' - q'\frac{dx'}{dt}$$

$$\boxed{\frac{d^2x'}{dt^2} + \frac{k}{m}x' + \frac{q'}{m}\frac{dx'}{dt} = 0} \quad \text{—— (i)}$$

2nd order differential eqⁿ

$$\boxed{\frac{d^2x'}{dt^2} + 2s\frac{dx'}{dt} + \omega^2x' = 0} \quad \text{—— (ii)}$$

Assume, $x' = Ae^{\alpha t}$

Then $\frac{dx'}{dt} = A\alpha e^{\alpha t} = \alpha x'$

$$\frac{d^2x'}{dt^2} = \alpha^2 x'$$

Putting it in eqⁿ (ii),

$$\alpha^2x' + 2s\alpha x' + \omega^2x' = 0$$

$$\Rightarrow x'(\alpha^2 + 2s\alpha + \omega^2) = 0$$

$$\Rightarrow \underbrace{Ae^{\alpha t}}_{\text{can't be zero}}(\alpha^2 + 2s\alpha + \omega^2) = 0$$

$$\alpha^2 + 2s\alpha + \omega^2 = 0$$

$$\alpha = \frac{-b\pm\sqrt{b^2-4ac}}{2a} = \frac{-2s\pm\sqrt{4s^2-4\omega^2}}{2} = -s\pm\sqrt{s^2-\omega^2}$$

$$\alpha_1 = -s+\sqrt{s^2-\omega^2} \qquad \alpha_2 = -s-\sqrt{s^2-\omega^2}$$

$$\boxed{\therefore x' = A_1 e^{(-s+\sqrt{s^2-\omega^2})t} + A_2 e^{(-s-\sqrt{s^2-\omega^2})t}} \rightarrow \text{General sol}^n \text{ of eq}^n \text{ (ii)}$$

— (iii)

$A_1, A_2 \rightarrow$ arbitrary constant

$\therefore s, \omega \leftarrow$ sol^n depends on

(i) $s^2 > \omega^2$ (ii) $s^2 = \omega^2$ (iii) $s^2 < \omega^2$

Case (i): $s^2 > \omega^2$ — "Overdamped"

$\because s^2 > \omega^2$

$\therefore \sqrt{s^2-\omega^2} \rightarrow$ real, (+ve), less than 's'

- Hence, both components (exponents) in (iii) are (-ve), and the displacement x' decreases exponentially without performing any oscillation (continuously to zero)

↳ "Overdamped"

Case (ii): $s^2 = \omega^2$ — "Critically damped"

$\therefore \sqrt{s^2-\omega^2} = \beta$ (we assume it's not exactly zero instead a very small quantity)

From eq^n (iii),

$$\boxed{x' = A_1 e^{(-s+\beta)t} + A_2 e^{(-s-\beta)t}}$$

$$x' = e^{-st}\left(A_1 e^{\beta t} + A_2 e^{-\beta t}\right)$$

$$x' = e^{-st}\left[A_1\left(1+\beta t+\frac{(\beta t)^2}{2!}+\ldots\right) + A_2\left(1+(-\beta t)+\frac{(-\beta t)^2}{2!}+\ldots\right)\right]$$

$$= e^{-st}\left[(A_1+A_2)+\beta t(A_1-A_2)\right]$$

let $P' = A_1 + A_2$

$Q' = (A_1 - A_2)\beta$

$$= e^{-st}(P' + t\,Q')$$

if $t \rightarrow \uparrow$, $P' + Q't \rightarrow \uparrow$ and $e^{-st} \rightarrow$ dec. and finally tends to zero

↳ "Critical damped motion"

eg:- Voltmeter and Ammeter, the pointer moves to ~~correct~~ correct position & comes to rest without oscillation

Case (iii): $s^2 < \omega^2$ — "Oscillatory Motion" / "Damped harmonic oscillator"

$\therefore \sqrt{s^2-\omega^2} \rightarrow$ imaginary

$= i\sqrt{\omega^2-s^2}$

$= i\beta' \quad (\beta' = \sqrt{(\omega^2-s^2)})$

From eq^n (iii),

$$x' = A_1 e^{(-s+i\beta')t} + A_2 e^{(-s-i\beta')t}$$

$$= e^{-st}\left(A_1 e^{i\beta' t} + A_2 e^{-i\beta' t}\right)$$

$$= e^{-st}\left[A_1(\cos(\beta' t) + i\sin(\beta' t)) + A_2(\cos(-\beta' t) + i\sin(-\beta' t))\right]$$

$$= e^{-st}\left[(A_1\cos(\beta' t) + i\sin(\beta' t)) + (A_2\cos(\beta' t) - i\sin(\beta' t))\right]$$

$$= e^{-st}\left[\cos(\beta' t)\cdot\underbrace{(A_1+A_2)}_{A\sin\delta} + \sin(\beta' t)\,\underbrace{i(A_1-A_2)}_{A\cos\delta}\right]$$

$$= e^{-st}\left[A\sin\delta\cdot\cos\beta' t + A\cos\delta\,\sin\beta' t\right]$$

$$= Ae^{-st}\sin(\delta + \beta' t)$$

$$\boxed{x' = Ae^{-st}\cdot\sin\left(t\sqrt{\omega^2-s^2}+\delta\right)}$$

↳ "Oscillatory Motion" → The oscillations are not simple harmonic because amplitude Ae^{-st} is not constant but dec. with time

Graph:

Forced Oscillation

$$\underbrace{m\frac{d^2x}{dt^2}}_{} + \underbrace{b\frac{dx}{dt}}_{\text{Damping force}} + \underbrace{kx}_{\text{Restoring force}} = \underbrace{F_0 \sin \omega'' t}_{\text{Applied force}}$$

$$\frac{d^2x}{dt^2} + \frac{b}{m}\frac{dx}{dt} + \frac{k}{m}x = \frac{F_0}{m}\sin \omega'' t$$

$$\frac{d^2x}{dt^2} + 2\lambda\frac{dx}{dt} + \omega^2 x = f_0 \sin \omega'' t$$

where, $\omega^2 = \frac{k}{m}$, $\underset{\text{Damping constant}}{2\lambda} = \frac{b}{m}$; $f_0 = \frac{F_0}{m}$

$\Rightarrow \underset{\text{natural frequency}}{\omega} = \sqrt{\frac{k}{m}}$

Let $x = A\sin(\omega'' t - \theta)$

$$\frac{dx}{dt} = A\omega'' \cos(\omega'' t - \theta)$$

$$\frac{d^2x}{dt^2} = -A(\omega'')^2 \sin(\omega'' t - \theta)$$

$$= -(\omega'')^2 x$$

$$\frac{d^2x}{dt^2} + 2\lambda\frac{dx}{dt} + \omega^2 x = f_0 \sin(\omega'' t)$$

$\Rightarrow -(\omega'')^2 x + 2\lambda A\omega'' \cos(\omega'' t - \theta) + \omega^2 A\sin(\omega'' t - \theta) = f_0 \sin(\omega'' t)$

$\Rightarrow -(\omega'')^2 A\sin(\omega'' t - \theta) + 2\lambda A\omega'' \cos(\omega'' t - \theta) + \omega^2 A\sin(\omega'' t - \theta) = f_0 \sin \omega'' t$

$\Rightarrow -(\omega'')^2 A\sin(\omega'' t - \theta) + \omega^2 A\sin(\omega'' t - \theta) - f_0 \sin(\omega'' t - \theta)\cos\theta$
$= f_0 \cos(\omega'' t - \theta)\sin\theta - 2\lambda A\omega'' \cos(\omega'' t - \theta)$

$\Rightarrow \sin(\omega'' t - \theta)\,[-(\omega'')^2 A + \omega^2 A - f_0\cos\theta] = \cos(\omega'' t - \theta)\,[f_0 \sin\theta - 2\lambda A\omega'']$

$\Rightarrow [-(\omega'')^2 A + \omega^2 A - f_0\cos\theta]\sin(\omega'' t - \theta) = [f_0\sin\theta - 2\lambda A\omega'']\cos(\omega'' t - \theta)$

$\Rightarrow [-(\omega'')^2 A + \omega^2 A - f_0\cos\theta]\sin(\omega'' t - \theta) - [f_0\sin\theta - 2\lambda A\omega'']\cos(\omega'' t - \theta) = 0$

$\Rightarrow [-(\omega'')^2 A + \omega^2 A - f_0\cos\theta]$

$-A(\omega'')^2 - f_0\cos\theta + \omega^2 A = 0$ and $f_0\sin\theta - 2\lambda A\omega'' = 0$

$\Rightarrow f_0(\cos\theta) = A\omega^2 - A(\omega'')^2$ — (i) $\quad \Rightarrow f_0 \sin\theta = 2\lambda A\omega''$ — (ii)

(i)^2 + (ii)^2,

$$\Rightarrow f_0^2 = [A\omega^2 - A(\omega'')^2]^2 + [2\lambda A\omega'']^2$$

$$\Rightarrow f_0^2 = A^2\omega^4 + A^2(\omega'')^4 - 2A^2\omega^2(\omega'')^2 + 4\lambda^2 A^2(\omega'')^2$$

$$\Rightarrow f_0^2 = A^2[\omega^4 + (\omega'')^4 - 2\omega^2(\omega'')^2 + 4\lambda^2(\omega'')^2]$$

$$\Rightarrow A^2 = \frac{f_0^2}{\omega^4 + (\omega'')^4 - 2\omega^2(\omega'')^2 + 4\lambda^2(\omega'')^2}$$

$$\Rightarrow A = \pm\frac{f_0}{\sqrt{\omega^4 + (\omega'')^4 - 2\omega^2(\omega'')^2 + 4\lambda^2(\omega'')^2}}$$

$$\Rightarrow \boxed{A = \frac{f_0}{\sqrt{[\omega^2 - (\omega'')^2]^2 + 4\lambda^2(\omega'')^2}}}$$

$$\boxed{\tan\theta = \frac{2\lambda\omega''}{(\omega^2 - (\omega'')^2)}} \quad \text{(ii} \div \text{i)}$$

$$\therefore \boxed{x' = \frac{f_0}{\sqrt{[\omega^2 - (\omega'')^2]^2 + 4\lambda^2(\omega'')^2}}\sin(\omega'' t - \theta)}$$

x_{max} when $\underset{\text{"Resonant frequency"}}{\boxed{\omega = \omega''}}$

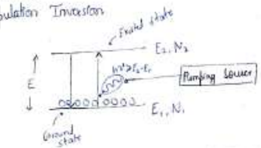
The system is undamped and vibrate with frequency ω'' but will have a phase lag of $\boxed{\theta = \tan^{-1}\frac{2\lambda\omega''}{\omega^2 - (\omega'')^2}}$.

LASER

(Light Amplification by Stimulated Emission of Radiation)

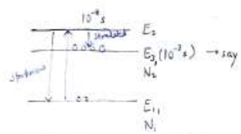
(i) Absorption
(ii) Spontaneous Emission --radiationless
(iii) Stimulated Emission

→ Pumping
→ Population Inversion



$$\boxed{h\nu \geq E_2 - E_1}$$

- e^- stay in excited state for 10^{-8} s.
- For LASER, lifetime of e^- in excited state should be $> 10^{-8}$ s



- If $\boxed{N_2 > N_1}$, it is called "population inversion", i.e higher energy state has more atoms than ground state.

- Only the stimulated emission atoms contribute to LASER.

(i) Absorption

$P_{12} \propto u(\nu)$
$\quad$ └ number/energy density

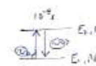
- Probability of occurrence of absorption from state 1 to state 2 is proportional to energy density $u(\nu)$ of the radiation

$$\boxed{P_{12} = B_{12} \cdot u(\nu)}$$

- $B_{12} \to$ proportionality constant
  known as "Einstein's coefficient of absorption of radiation"

(ii) Spontaneous Emission

- Probability of occurrence of spontaneous emission — transition from state 2 to 1 depends only on the property of state 2 and 1



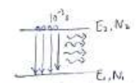
$$\boxed{P'_{21} = A_{21}}$$

- $A_{21} \to$ "Einstein's coefficient of spontaneous emission of radiation"

(iii) Stimulated Emission $(N_2 > N_1)$

- Probability of occurrence of stimulated emission transition from upper level to state is proportional to number density $u(\nu)$

$P''_{21} \propto u(\nu)$



$$\boxed{P''_{21} = B_{21} \, u(\nu)}$$

- $B_{21} \to$ "Einstein's coefficient of stimulated emission of radiation"

$\boxed{P_{21} = P'_{21} + P''_{21}}$ (Total probability of emission)

$N_1 \times$ total absorption $= N_2 \times$ total emission

$\boxed{N_1 \cdot P_{12} = N_2 \cdot P_{21}}$ (at thermal equilibrium) $\Rightarrow N_1 \cdot B_{12} \cdot u(\nu) = N_2 [A_{21} + B_{21} u(\nu)]$

$$N_1 \cdot P_{12} = N_2 \cdot P_{21}$$

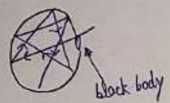
$$\Rightarrow N_1 \cdot B_{12} \cdot u(\nu) = N_2 \left[A_{21} + B_{21} \cdot u(\nu)\right]$$

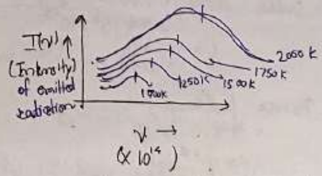
$$\Rightarrow u(\nu) \left[N_1 B_{12} - N_2 \cdot B_{21}\right] = N_2 \cdot A_{21}$$

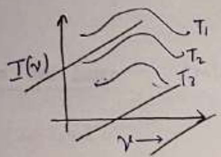
$$\Rightarrow \boxed{u(\nu) = \frac{N_2 \cdot A_{21}}{N_1 \cdot B_{12} - N_2 \cdot B_{21}}}$$

Quantum Mechanics

Black Body Radiation



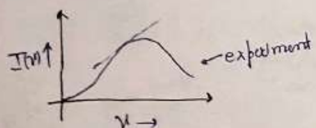




- If temp ↑es, total amount of emitted radiation ↑es
- The position of maxima shifts towards higher frequency region with temperature.

A/c Rayleigh Jean law,

$$I(\nu) = \frac{8\pi\nu^2}{c^3} kT\, d\nu$$



- Satisfied at lower frequency region, $\underbrace{\text{not at higher frequencies}}_{\text{"UV catastrophe"}}$

A/c Planck's modification,

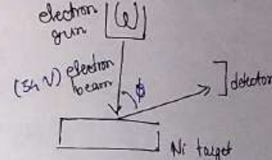
$$I(\nu) = \frac{8\pi h\nu^3}{c^3}\left(\frac{1}{e^{h\nu/kT}-1}\right)$$

Case I : $h\nu \gg kT$
Case II : $h\nu = kT$
Case III : $h\nu < kT$

- It explained black body radiation (partially)

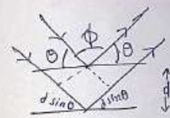
Davisson - Germer Experiment

- Demonstration of matter wave



- Max^{m} intensity at $\phi = 50°$

Bragg's Law



$\Delta x = 2d\sin\theta$

$$\boxed{2d\sin\theta = n\lambda}$$

$2\theta + \phi = 180°$

$\Rightarrow 2\theta + 50° = 180°$

$\Rightarrow \boxed{\theta = 65°}$

$\boxed{d_{Ni} = 0.91\ \text{Å}}$

Let, $n = 1$

$2d\sin\theta = \lambda$

$\Rightarrow \lambda = 2 \times 0.91 \times \sin 65°\ \text{Å}$

$= 1.65\ \text{Å}$ (when considered wave)

Generally, $\boxed{\frac{1}{2}mv^2 = eV}$ Also, $\boxed{\lambda = \frac{h}{p}}$

$$\boxed{\lambda = \sqrt{\frac{150}{V}}}$$

$\Rightarrow \lambda = 1.67\ \text{Å}$ (when considered wave particle)

Wave → interference, diffraction, frequency, wavelength, polarisation

Particle → mass, velocity, momentum, blackbody radiation, PE effect

De-Broglie Hypothesis

· Wave-Particle duality

$$\lambda = \frac{h}{mv} = \frac{h}{p}$$

wave property — particle property

$\boxed{E = h\nu}$ $\boxed{E = mc^2}$

$$\boxed{\text{Kinetic Energy} = \frac{1}{2}mv^2 = \frac{p^2}{2m}}$$

$$\boxed{p = \sqrt{2mE}}$$

$$\boxed{\lambda = \frac{h}{\sqrt{2mE}}}$$

Velocity of De-Broglie Wave

$$\boxed{\lambda = \sqrt{\frac{150}{V}}}$$

potential

$$\boxed{\lambda = \frac{12.25}{\sqrt{V}}}$$

$$eV = \frac{1}{2}mv^2$$

$$\lambda = \frac{h}{mv}$$

$$\Rightarrow \lambda = \frac{h}{\sqrt{2meV}} = \sqrt{\frac{150}{V}} = \frac{12.25}{\sqrt{V}}$$

$h = 6.63 \times 10^{-34}$ Js

$e = 1.6 \times 10^{-19}$ C

$m_e = 9 \times 10^{-31}$ kg

$V = 100$ V

$$\boxed{\lambda_e = 1.226\ A^\circ}$$

• Particle heavier ⇒ 'λ' is less

$m_p = 1.67 \times 10^{-27}$ kg

$$\boxed{\lambda_p = 0.28\ A^\circ}$$

Phase and Group Velocities

$$y = a \sin(\omega t - kx)$$

Here, $\omega t - kx \rightarrow$ constant

$$\boxed{\frac{dx}{dt} = \frac{\omega}{k} = v_p}$$

$\boxed{\lambda = \frac{h}{p}}$ $\boxed{E = h\nu}$ $\boxed{v = \lambda\nu}$

$$v_p = \frac{E}{h} \times \frac{h}{p} = \frac{E}{mv} = \frac{mc^2}{mv}$$

$$\boxed{v_p v = c^2}$$

if $c >> v$ / $\boxed{v_p > c}$

The phase vel. of a de-broglie wave associated with particle moving with vel. 'v' is greater than vel. of light 'c'.

Group Velocities

$$y_1 = a \sin(\omega_1 t - k_1 x)$$

$$y_2 = a \sin(\omega_2 t - k_2 x)$$

After superposition,

$$y = y_1 + y_2$$

$$= a\left[2 \sin\left(\frac{(\omega_1+\omega_2)t - (k_1+k_2)x}{2}\right) \cos\left(\frac{(\omega_1-\omega_2)t - (k_1-k_2)x}{2}\right)\right]$$

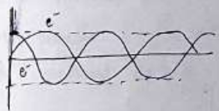
Let, $\left(\omega = \frac{\omega_1+\omega_2}{2}\right)$; $k = \left(\frac{k_1+k_2}{2}\right)$

$$= 2a \cos(\omega t - kx) \cdot \sin\left(\frac{\Delta\omega}{2}t - \frac{\Delta k}{2}x\right)$$

$$\boxed{v_g = \frac{\Delta\omega/2}{\Delta k/2} = \frac{\Delta\omega}{\Delta k}}$$

$$v_g = \frac{\partial\omega}{\partial k} = \frac{\partial(2\pi\nu)}{\partial(2\pi/\lambda)} = \lambda^2 \frac{\partial\nu}{\partial\lambda}$$

$$\boxed{v_g = -\frac{\lambda^2}{2\pi}\frac{\partial\omega}{\partial\lambda}}$$



· maxima/minima related to v_g.

Relationship b/w v_g and v_p

$$v_g = \frac{\partial\omega}{\partial k} \qquad v_p = \frac{\omega}{k}$$

$$= \frac{\partial}{\partial k}(v_p k) = v_p + k\frac{dv_p}{dk}$$

$$k = \frac{2\pi}{\lambda} \Rightarrow dk = -\frac{2\pi}{\lambda^2}d\lambda \Rightarrow \frac{k}{dk} = \frac{-\lambda}{d\lambda}$$

$$\boxed{v_g = v_p - \lambda\frac{dv_p}{d\lambda}}$$

↳ $v_g << v_p$

Heisenberg's Uncertainty Principle

- It is impossible to determine the exact position and momentum of a small moving particle (like e^-) simultaneously.

$$\boxed{\Delta x . \Delta p \geqslant \frac{h}{4\pi}}$$

$$\boxed{\Delta x . \Delta p \approx \hbar/2} \quad ; \hbar = \frac{h}{2\pi}$$

$$\boxed{\Delta E . \Delta t \approx \hbar/2}$$

Applications:

(i) Non-existence of e^- in the nucleus.

$$\Delta p = \frac{h}{2\pi \Delta x}$$

$$= \frac{6.63 \times 10^{-34}}{2 \times 3.14 \times 10^{-14}}$$

$$= 5.275 \times 10^{-21} \text{ kgm/s}$$

$$E = \frac{1}{2}mv^2 = \frac{p^2}{2m} = \frac{(5.275 \times 10^{-21})^2}{2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-13}} \text{ eV}$$

$$= 9.6 \text{ MeV}$$

- The max^m energy that e^- can possess ~~to exist in nucleus~~ is 4 MeV. but it requires 9.6 MeV to reside inside the nucleus.

(ii) Radius of Bohr's 1st orbit

$$\Delta x . \Delta p \approx \hbar$$

$$KE = \Delta T = \frac{(\Delta p)^2}{2m} = \frac{1}{2m}\left(\frac{\hbar}{\Delta x}\right)^2$$

$$PE = \frac{1}{4\pi\epsilon_0} \cdot \frac{(Ze)(-e)}{\Delta x}$$

Total energy $\Delta E = KE + PE$

$$= \frac{1}{2m}\left(\frac{\hbar}{\Delta x}\right)^2 - \frac{Ze^2}{4\pi\epsilon_0 \Delta x}$$

$$\boxed{\Delta x = \frac{\epsilon_0 h^2}{\pi m Z e^2}}$$

For max^m/min^m; $\frac{\partial(\Delta E)}{\partial(\Delta x)} = 0$

(iii) Energy in a box of a particle infinite potential wall (of a particle)

(iv) Ground State Energy of a linear harmonic oscillator.

(iii) Energy of a particle in a box (Infinite potential well)

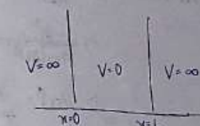
- The particle is confined in a box of length 'L',

$$\therefore \Delta x \approx L$$

Now, $\Delta x . \Delta p \geqslant \frac{\hbar}{2}$ (Uncertainty Principle)

$$\Rightarrow \Delta p \geqslant \frac{\hbar}{2L}$$

Also potential $V(x) = \begin{cases} 0 & , 0 < x < L \\ \infty & , x < 0 \text{ (or) } x > L \end{cases}$



i.e. PE = 0 inside the box

$$KE = \frac{p^2}{2m}$$

Taking $p \approx \Delta p$

$$E_{min} = \frac{(\Delta p)^2}{2m}$$

$$\Rightarrow E_{min} \left(\frac{\hbar}{2L}\right)^2\left(\frac{1}{2m}\right) = \frac{\hbar^2}{8mL}$$

* The existence of non-zero ground state energy $\boxed{E = \frac{\hbar^2}{8mL}}$ is a direct consequence of the uncertainty principle. If this energy were zero, then momentum will be exactly zero which'll contradict $\Delta p \geqslant \frac{\hbar}{2L}$.

(IV) Ground State Energy of a linear harmonic oscillator

$$\Delta x \cdot \Delta p \approx \frac{\hbar}{2} \Rightarrow \Delta p \approx \frac{\hbar}{2\Delta x}$$

$$KE = \frac{(\Delta p)^2}{2m} = \left(\frac{\hbar}{2\Delta x}\right)^2\left(\frac{1}{2m}\right) = \frac{\hbar^2}{8m(\Delta x)^2}$$

$$PE = \frac{1}{2}k(\Delta x)^2$$

$$E = KE + PE$$

$$= \frac{\hbar^2}{8m(\Delta x)^2} + \frac{1}{2}k(\Delta x)^2 \quad \text{——} (1)$$

For min^m energy, $\frac{\partial E}{\partial(\Delta x)} = 0$

$$\Rightarrow \frac{-\hbar^2}{4m(\Delta x)^3} + k(\Delta x) = 0$$

$$\Rightarrow (\Delta x)^4 = \frac{\hbar^2}{4mk}$$

$$\Rightarrow \Delta x = \left(\frac{\hbar^2}{4mk}\right)^{1/4} \quad \text{——} (II)$$

From (1) and (II),

$\boxed{E_{min} = \frac{1}{2}\hbar\omega}$, where $\omega = \sqrt{\frac{K}{m}}$ (angular frequency of oscillator)

↳ "Zero-point energy"

- The existence of the zero-point energy is a direct consequence of the uncertainty principle because if this ground state energy were zero, both Δx and Δp would be precisely zero, contradicting the uncertainty principle

Wave Function

$\psi(x,y,z,t) \rightarrow$ has both characters (particle & wave)

- has no meaning by itself

$|\psi|^2 \rightarrow$ probability of finding a particle,

$$\boxed{\int_{-\infty}^{\infty} |\psi|^2 \, dV = 1} \text{ (Normalization condition)}$$

Properties of $\psi(x,y,z,t)$:

(I) · Continuous
· Finite
· Single-valued

(II) $\frac{\partial \psi}{\partial x}, \frac{\partial \psi}{\partial y}, \frac{\partial \psi}{\partial z}$ be continuous, finite, single-valued

(III) ψ should be normalized

- ψ is complex $(a+ib)$ in nature.

Schrodinger's Equation

$\psi = a+ib$

$\psi^* = a-ib$, (conjugate)

Time Independent:

By Newtonian mechanics,

$$\frac{\partial^2 \psi}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 \psi}{\partial t^2}$$

Same analogy,

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} = \frac{1}{v^2}\frac{\partial^2 \psi}{\partial t^2}$$

$v \rightarrow$ velocity of particle associated with wave.

$$\psi(x,y,z,t) = \underbrace{\psi_0(x,y,z)}_{\text{time-independent amplitude}} \cdot e^{-i\omega t} = \psi_0(r) \cdot e^{-i\omega t}$$

$$\frac{\partial^2 \psi}{\partial t^2} = (-i\omega)(-i\omega)\,\psi_0 \, e^{-i\omega t}$$

$$= -\omega^2 \psi_0 \cdot e^{-i\omega t}$$

$$= -\omega^2 \psi$$

Now, $\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2} + \frac{\omega^2}{v^2}\psi = 0$, $\omega = 2\pi\nu = \frac{2\pi v}{\lambda}$

$$\nabla^2\psi + \frac{\omega^2}{v^2}\psi = 0$$ $\Rightarrow \frac{\omega}{v} = \frac{2\pi}{\lambda}$

$$\Rightarrow \boxed{\nabla^2\psi + \frac{4\pi^2}{\lambda^2}\psi = 0}$$

Now $\lambda = \frac{h}{p} = \frac{h}{mv}$

$$\Rightarrow \nabla^2\psi + \frac{4\pi^2 m^2 v^2}{h^2}\psi = 0$$

Also, Energy = KE + PE
(E) (V)

$$\Rightarrow (E - V) = \frac{1}{2}mv^2$$

$$\Rightarrow \nabla^2\psi + \frac{8\pi^2 m}{h^2}(E - V)\psi = 0$$

$$\Rightarrow \boxed{\nabla^2\psi + \frac{2m}{\hbar^2}(E - V)\psi = 0}$$

$$\boxed{\nabla^2\psi + \frac{2mE}{\hbar^2}\psi = 0}$$ (For free particle $V = 0$)

Time-Dependent Schrodinger's eqⁿ

$$\psi = \psi_0 e^{-i\omega t}$$ $E = h\nu \Rightarrow \nu = \frac{E}{h}$

$$\frac{\partial\psi}{\partial t} = -i\omega\psi_0 e^{-i\omega t}$$

$$= -i\,2\pi\nu\psi$$

$$= -\frac{i\,2\pi E}{h}\psi$$

$$= -\frac{iE}{\hbar}\psi$$

$$= \frac{E}{i\hbar}\psi$$

$$\frac{\partial \psi}{\partial t} = \frac{E}{i\hbar}\psi$$

$$\Rightarrow E\psi = i\hbar\frac{\partial \psi}{\partial t}$$

Time independent eqn :

$$\nabla^2\psi + \frac{2m}{\hbar^2}(E\psi - V\psi) = 0$$

$$\nabla^2\psi + \frac{2m}{\hbar^2}\left(i\hbar\frac{\partial \psi}{\partial t} - V\psi\right) = 0$$

$$\Rightarrow \boxed{-\frac{\hbar^2}{2m}\nabla^2\psi + V\psi = i\hbar\frac{\partial \psi}{\partial t}}$$

$$\Rightarrow \left(-\frac{\hbar^2}{2m}\nabla^2 + V\right)\psi = i\hbar\frac{\partial \psi}{\partial t}$$

$$\boxed{-\frac{\hbar^2}{2m}\nabla^2 + V = H \text{ (hamiltonian)}}$$

$$\boxed{H\psi = \cancel{\hbar}E\psi}$$

eigenfunction

$$A\psi = a\psi$$

operator — eigen-value (energy value)

e.g. $A = \frac{\partial^2}{\partial x^2}$ and $\psi = e^{2x}$, find eigen-values

$$A\psi = \frac{\partial^2}{\partial x^2}(e^{2x})$$

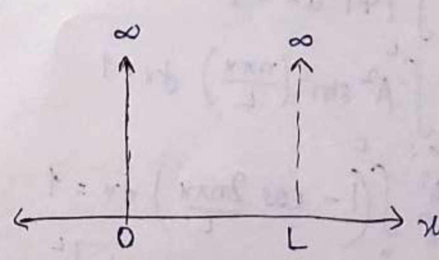
$$= 4e^{2x}$$

$\therefore$ Eigenvalue $(a) = 4$

<u>Particle in box (having infinite potential well)</u>

- Particle is free $(V = 0)$

$$V = \begin{cases} 0, & 0 \leq x \leq L \\ \infty, & x < 0 \text{ and } x > L \end{cases}$$



$$\nabla^2\psi + \frac{2m}{\hbar^2}(E - V)\psi = 0$$

$\because V = 0$

$$\therefore \nabla^2\psi + \frac{2mE}{\hbar^2}\psi = 0$$

$$\Rightarrow \boxed{\nabla^2\psi + k^2\psi = 0} \quad \text{where, } k^2 = \frac{2mE}{\hbar^2}$$

Let, $\psi = A\sin(kx) + B\cos(kx)$

at $x=0,\ \psi=0$

$x=L,\ \psi=0$

(Particle can't exist at walls as position will be exactly known)

at $x=0,\ \psi=0$

$0 = 0 + B\cos k$

$\Rightarrow \boxed{B=0}$

at $x=L,\ \psi=0$

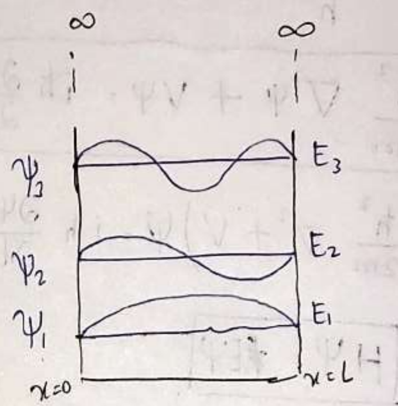
$0 = A\sin(kL)$

$\Rightarrow \boxed{kL = n\pi}$, $n \in \mathbb{Z}$

$\Rightarrow k^2L^2 = n^2\pi^2$

$\Rightarrow \dfrac{2mE}{\hbar^2} = \dfrac{n^2\pi^2}{L^2}$

$\Rightarrow \boxed{E_n = \dfrac{\hbar^2 n^2 \pi^2}{2mL^2}}$



$E_1 = \dfrac{\hbar^2\pi^2}{2mL^2}$, $E_2 = 4\dfrac{\hbar^2\pi^2}{2mL^2}$, $E_3 = 9\dfrac{\hbar^2\pi^2}{2mL^2}$, ...

$\therefore$ Energy is quantized (discrete energy levels)

if $\psi \rightarrow$ normalized

then, $\int |\psi|^2 dV = 1$

$\psi = A\sin(kx)$

$k = \dfrac{n\pi}{L}$

$\psi = A\sin\left(\dfrac{n\pi x}{L}\right)$

In 1-D,

$\int |\psi|^2 dx = 1$

$\Rightarrow \int_0^L A^2 \sin^2\left(\dfrac{n\pi x}{L}\right) dx = 1$

$\Rightarrow \dfrac{A^2}{2}\int_0^L \left(1 - \cos\dfrac{2n\pi x}{L}\right) dx = 1$

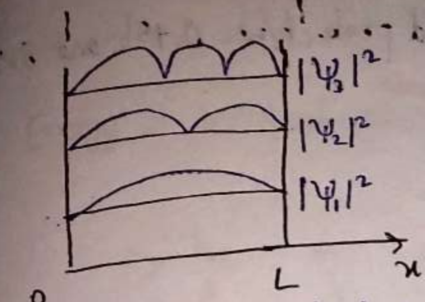
$\Rightarrow \dfrac{A^2}{2}\left[x - \dfrac{\sin\left(\frac{2n\pi x}{L}\right)}{\left(\frac{2n\pi}{L}\right)}\right]_0^L = 1$

$\Rightarrow \dfrac{A^2}{2}\left[L - \dfrac{L}{2n\pi}(0-0)\right] = 1$

$\Rightarrow A = \pm\sqrt{\dfrac{2}{L}}$

$\therefore \boxed{\psi_n = \sqrt{\dfrac{2}{L}}\sin\left(\dfrac{n\pi}{L}x\right)}$ ← normalized wave function

$\boxed{E_n = \dfrac{\hbar^2 n^2 \pi^2}{2mL^2}}$



$|\psi|^2 \rightarrow$ probability of finding particle

- if $E = E_1$, then max^m probability of finding particle $= L/2$

$E = E_2$, " " " " " " $= \frac{L}{4}, \frac{3L}{4}$

Q.) Find the probability that particle present in a box 'L' wide can be found b/w 0.45 L and 0.55 L for ground state and first excited state.

Sol^n: For ground state,

$$\psi_1 = \sqrt{\frac{2}{L}} \sin\left(\frac{\pi}{L}x\right)$$

$$\int_{0.45L}^{0.55L} |\psi_1|^2 \, dx = \frac{2}{L} \int_{0.45L}^{0.55L} \sin^2\left(\frac{\pi}{L}x\right) dx$$

$$= \frac{2}{L} \int_{0.45L}^{0.55L} \left(1 - \cos\frac{2\pi x}{L}\right) dx$$

$$= \frac{2}{L}\left[(0.10L) - \frac{L}{2\pi}\left(\sin\left(\frac{2\pi}{\cancel{L}}(0.55)\cancel{L}\right) - \sin\left(\frac{2\pi}{\cancel{L}}(0.45)\cancel{L}\right)\right)\right]$$

$=$

For $n = 2$,

Q.) If $\psi = ax$ and $x=0$ to $x=1$, then find prob. b/w 0.45 and 0.55.

$$\int_0^1 |\psi|^2 dx = 1$$

$$\Rightarrow a^2 \int_0^1 x^2 dx = 1$$

$$\Rightarrow \frac{a^2}{3} = 1$$

$$\Rightarrow a = \pm\sqrt{3}$$

Now,

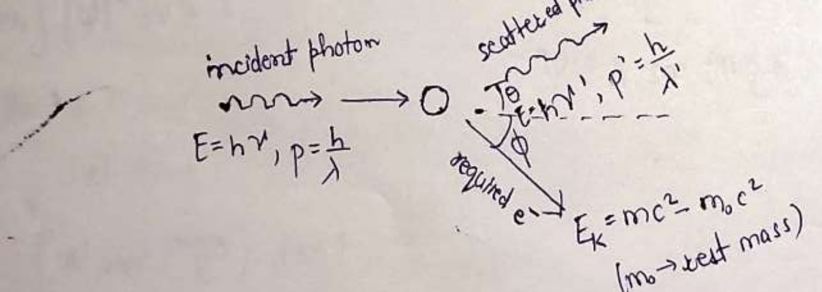
$$\int_{0.45}^{0.55} |\psi|^2 dx = 3\int_{0.45}^{0.55} x^2 dx$$

$$= \frac{3}{3}\left[x^3\right]_{0.45}^{0.55}$$

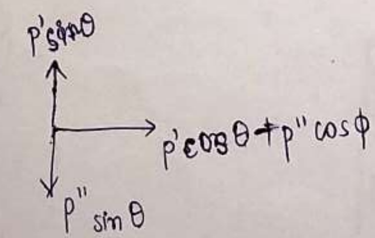
$$=$$

<u>Compton Scattering</u>

- coherent scattering: energy absorbed = energy released (higher wavelengths)
- incoherent scattering: energy absorbed > energy released (lower wavelengths)



$$\boxed{m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}}$$



A/c conservation of energy,

Energy of incident photon = Energy of scattered photon + Energy of required e^-

$$\Rightarrow \frac{hc}{\lambda} = \frac{hc}{\lambda'} + (m-m_0)c^2$$

$$\Rightarrow hc\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right) = (m-m_0)c^2$$

$$\Rightarrow h\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right) = (m-m_0)c \quad \text{——— (I)}$$

A/c conservation of momentum,

$$\frac{h}{\lambda'}\sin\theta = \frac{m_0 v}{\sqrt{1-\frac{v^2}{c^2}}} \quad \text{——— (II)}$$

$$\frac{h}{\lambda} = \frac{h}{\lambda'}\cos\theta + \frac{m_0 v}{\sqrt{1-\frac{v^2}{c^2}}}\cos\phi \quad \text{——— (III)}$$

$$\boxed{\Delta\lambda = \lambda' - \lambda = \frac{h}{m_0 c}(1-\cos\theta)} \qquad \boxed{\frac{h}{m_0 c} \approx 0.0242\ A^\circ}$$

$\theta = 0^\circ \Rightarrow \Delta\lambda = 0$

$\theta = 90^\circ \Rightarrow \Delta\lambda = \frac{h}{m_0 c}$ $\qquad$ $\Delta\lambda \rightarrow$ compton wavelength shift

$\theta = 180^\circ \Rightarrow \Delta\lambda = \frac{2h}{m_0 c}$

- % compton shift for $\lambda = 4000\ A^\circ$ (visible region) $= \frac{(\Delta\lambda)_{max}}{\lambda} \times 100\%$

$= 0.001\%$ (Not observed)

- Significant for X-rays ($\lambda \approx 1.5\ A^\circ$)

Optics

Spatial And Temporal Coherences :-

Spatial coherence describes the correlation between waves at different point in space. While temporal Coherence describe the Correlation between waves observed at different moment of time.

Spatial Coherence :-

- Refers to the correlation or predictable relationship between the electric field and at different locations across a beam profile
- Essentially, it describes how well the phase of wave is maintained across different point in space.
- A high degree of spatial coherence means the waves phase is consistence across the beam while a low degree of spatial coherence means the phase varies randomly.
- Example of phenomena where spatial coherence is importance include interference pattern in Young's double slit experiment and the formation of holograms.

Temporal Coherence :-

- Refers to the correlation between the electrical field at one location but at different time.
- It describes how long the phase of a wave remains consistent over time.

- A high degree of temporal coherence means the wave's phase remains consistence for a longer duration, while a low degree of temporal coherence means the phase varies rapidly.

- Example of phenomena where temporal coherence is important include the sharpness of interference fringes in Michelson interferometer fringes in Michelson interferometer and the wavelength of light emitted by a laser.

Interference :-

Interference of light is a phenomenon that occurs when two or more light waves interact with each other. It can result in constructive or destructive interference.

- Light wave can interfere constructively or destructively depending on their path difference.
- Constructive interference occurs when the crests of two light wave meets, causing them to combine and form a wave of greater amplitude or brightness.
- Destructive interference plays a crucial role in the diffraction and interference patterns observed in optics.

Condition for Light Interference :-

- The light source must be coherent.
- The light source must be monochromatic.
- The source must have equal amplitude and intensities.
- The source must be close enough to produce wide fridge.

Interference due to divison of Amplitude :-

Interference can be produced by two methods : (i) Division of wavefront and (ii) Division of amplitude. In division of amplitude, the amplitude of the incoming beam is divided into two parts, either by parallel reflection or by refraction. These two parts, either by travel unequal distance or reunite to produce interference. for example:- Newton's rings and Michelson's interferometer.

Stokes Phase Law :

This law states that a light wave When get reflected from a denser medium suffers a phase change of 180° or π.

Proof:- Let r_1, t be reflection and transmission Coefficient of rarer medium (air). and r', t' be reflection and transmission constant of denser medium. denser medium is (water).

region ①

$$a = r(ra) + t'(ta)$$

$$a = (r^2 + t't)a$$

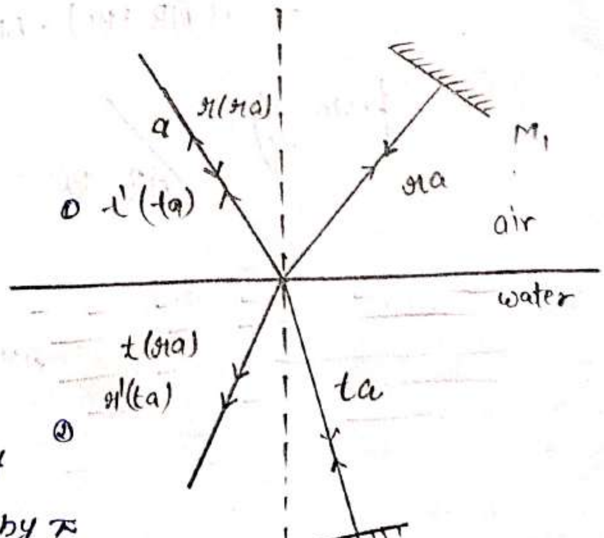
$$1 = r^2 + t't \quad \rightarrow ①$$

region②

$$0 = (ta)r' + t(ra)$$

$$0 = tr' + rt$$

$$-r' = r$$



from this eqn we could say that the light wave suffers phase change by π after reflection.

for eq^n ①.

$$tt' = 1 - r^2 \rightarrow (i)$$

$$r^2 + t^2 = 1$$

$$t^2 = 1 - r^2 \rightarrow (ii)$$

On Comparing (i) and (ii)

$$tt' = t^2$$

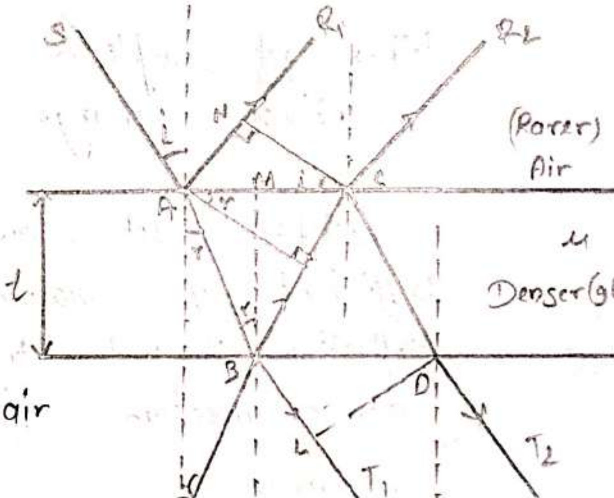
$$\boxed{t' = t}$$

this will not show any phase change.

Interference in thin film

A ray of monochromatic light SA be incident at an angle i on a parallel-sided transparent thin film of thickness t and refractive index ($\mu > 1$). Lets see in figure.

Let CN and BM be perpendicular to AR_1 and AC. As the path of the rays AR_1 and CR_2 beyond CN is equal, the path difference between them is.



Interference in this films

$$\Delta = \text{path ABC in film} - \text{path AN in air}$$

$$= \mu(AB + BC) - AN$$

from fig :-

$$AB = BC = \frac{BM}{\cos r} = \frac{t}{\cos r}$$

and

$$AN = AC \sin i$$

$$= (AM + MC) \sin i$$

$$= (BM \tan r + BM \tan r) \sin i$$

$$= 2t \tan r \sin i$$

$$= 2t \frac{\sin r}{\cos r} \sin i$$

$$= 2t \frac{\sin r}{\cos r} (\mu \sin r)$$

$$AN = 2\mu t \frac{\sin^2 r}{\cos r}$$

Substituting the value of AB, BC and AN in Eq. (6.19), we get

$$\Delta = \mu \left[\frac{t}{\cos r} + \frac{t}{\cos r} \right] - 2\mu t \frac{\sin^2 r}{\cos r}$$

$$= \frac{2\mu t}{\cos r} (1 - \sin^2 r)$$

$$= 2\mu t \cos r$$

The ray AR, having suffered a reflection at the surface of a denser medium undergoes a phase change of π, which is equivalent to a path different of λ. Hence, the effective path difference between AR, and CR_2 is $2\mu t \cos r - (\lambda/2)$

Interference Due to Reflected Light:-

For maxima :-

$$2\mu t \cos r - \frac{\lambda}{2} = n\lambda$$

$$2\mu t \cos r = (2n+1)\frac{\lambda}{2}$$

for minima :-

$$2\mu t \cos r - \frac{\lambda}{2} = (2n-1)\frac{\lambda}{2}$$

$$2\mu t \cos r = n\lambda$$

Interference Due to Transmitted Light :-

Similarly, the path difference between the transmitted rays BT_1 and DT_2 is given by :

$$\Delta = \mu(BC + CD) - BL$$
$$= 2\mu t \cos r$$

Hence, the effective path difference between BT_1 and DT_2 is also $2\mu t \cos r$.

for maxima

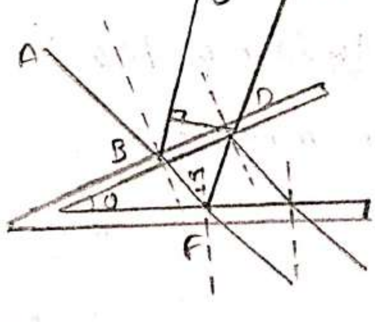
$$2\mu t \cos r = n\lambda$$

for minima

$$2\mu t \cos r = (2n+1)\frac{\lambda}{2}$$

Interference In Wedge-Shaped Thin Films :-

As we have already find the phase difference in thin film similarly :-



$$\Delta = 2\mu t \cos r$$

for Constructive interference (maxima)

$$2\mu t \cos r - \frac{\lambda}{2} = 2n$$

$$2\mu t \cos r = (2n+1)\lambda/2$$

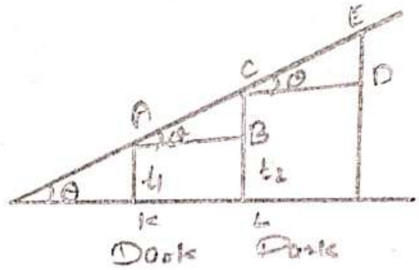
For destructive Interference :-

$$2\mu t \cos r - \frac{\lambda}{2} = (2n-1)\frac{\lambda}{2}$$

$$2\mu t \cos r = n\lambda$$

<u>Frindge width :-</u>

We are trying to find the dark frindges :. lets assume the incident is normal. $(i=0, r=0)$ and then $(\cos r = 1)$.



Let if thickness of film at A is denoted by t. then at A

$$2\mu t_1 = m\lambda$$

Let the next dark frindge will occurs at C. at that point the thickness = t_2. Then at C.

$$2\mu t_2 = (m+1)\lambda$$

Now subtracting :-

$$2\mu(t_2 - t_1) = m\lambda + \lambda - m\lambda$$

$$2\mu(t_2 - t_1) = \lambda$$

$$t_2 - t_1 = BC$$

$$2\mu(BC) = \lambda$$

$$BC = \frac{\lambda}{2\mu}$$

In ΔABC

$$\tan\theta = \frac{BC}{AB}$$

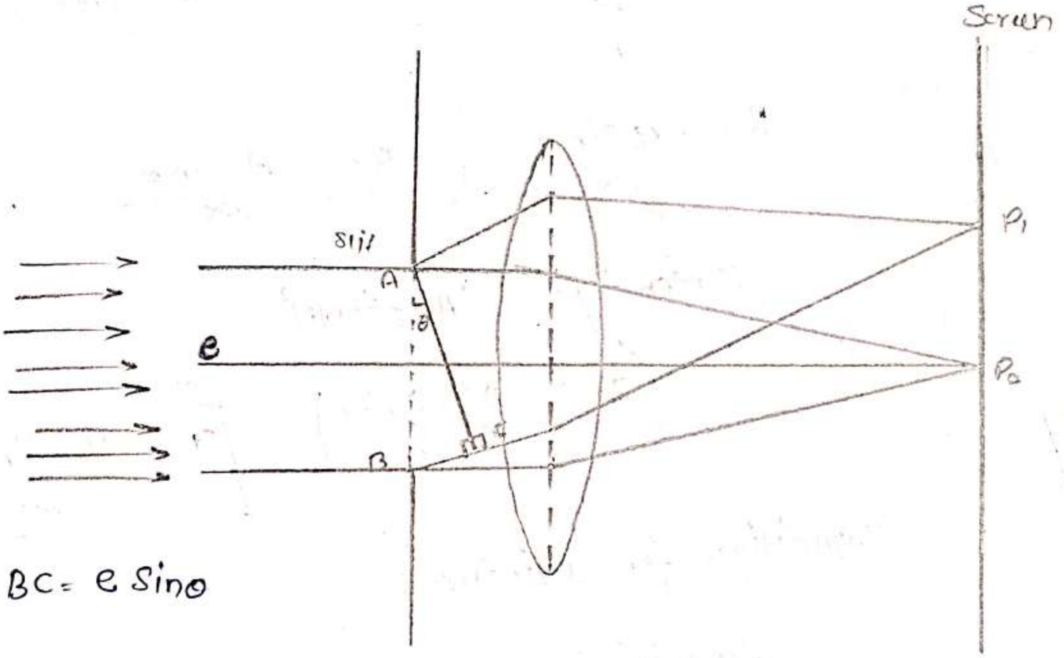
$$BC = AB\tan\theta$$

$$\left(AB\tan\theta = \frac{\lambda}{2\mu}\right)$$

Now we know $\tan\theta \sim \theta$ for small angle.

$$BC = \frac{\lambda}{2\mu\theta}$$

Franhoffers diffraction by single slit :-



Path difference = BC = $e \sin\theta$

Phase difference = $\frac{2\pi}{\lambda}$ (path diff.)

$$\delta = \frac{2\pi}{\lambda} e\sin\theta$$

Effective path difference = $\frac{1}{n}\frac{2\pi}{\lambda} e\sin\theta$

Resultant Amplitude $\Rightarrow$ $R = \dfrac{a \sin \frac{n\delta}{2}}{\sin\frac{\delta}{2}}$

$$R = \frac{a \sin \frac{n}{2}\left(\frac{2\pi}{\lambda n} e\sin\theta\right)}{\sin \frac{1}{2n}\left(\frac{2\pi}{\lambda} e\sin\theta\right)}$$

$$= \frac{a \sin\left(\frac{\pi}{\lambda} e\sin\theta\right)}{\sin\left(\frac{\pi}{n\lambda} e\sin\theta\right)}$$

Let us Considerer = $\left(\frac{\pi}{\lambda} e\sin\theta = \alpha\right)$

$$R = \frac{a \sin\alpha}{\sin\left(\frac{\alpha}{n}\right)}$$

$$R = \frac{a \sin\alpha}{\frac{\alpha}{n}} \quad \text{because } \sin\left(\frac{\alpha}{n}\right) \approx \frac{\alpha}{n} \quad \text{as} \left(\frac{\alpha}{n} <<< 1\right)$$

$$R = \frac{na \sin\alpha}{\alpha} \rightarrow R = \frac{A \sin\alpha}{\alpha}$$

Intensity = $(\text{Amplitude})^2$

$$\left(I = \frac{A^2 \sin^2\alpha}{\alpha^2}\right) \quad * \ \boxed{I = \frac{I_0 \sin^2\alpha}{\alpha^2}} \quad \left(\alpha = \frac{\pi}{d} e \sin\theta\right)$$

Condition for Minima :-

for minima $I = 0$ and $(A \neq 0)$ $(\alpha \neq 0)$

So. $\sin\alpha = 0$

$\alpha = \pm n\pi$

Hence for the minima position = $n\pi$, 1π, 2π. etc

Condition for Central Maxima :-

for maximum value of $(I = \infty)$

for achieving maximum $(\alpha = 0)$

then $\left(\frac{e\pi}{d} \sin\theta = 0\right)$.

That means, when $\underline{\underline{\theta = 0}}$ then we get the central maxima

for secondary maxima :-

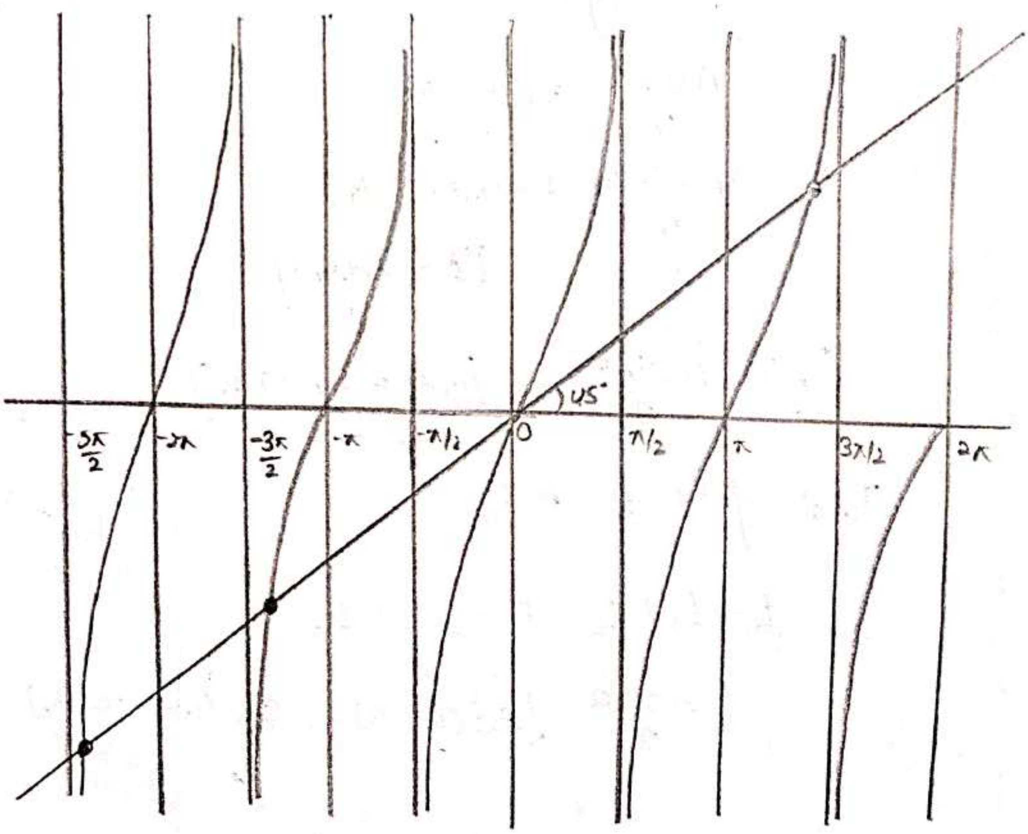
for secondary maxima we should differentiate intensity by with respect to α.

$$\frac{dI}{d\alpha} = \frac{d}{d\alpha}\left[A^2\left(\frac{\sin^2\alpha}{\alpha}\right)^2\right] = 0$$

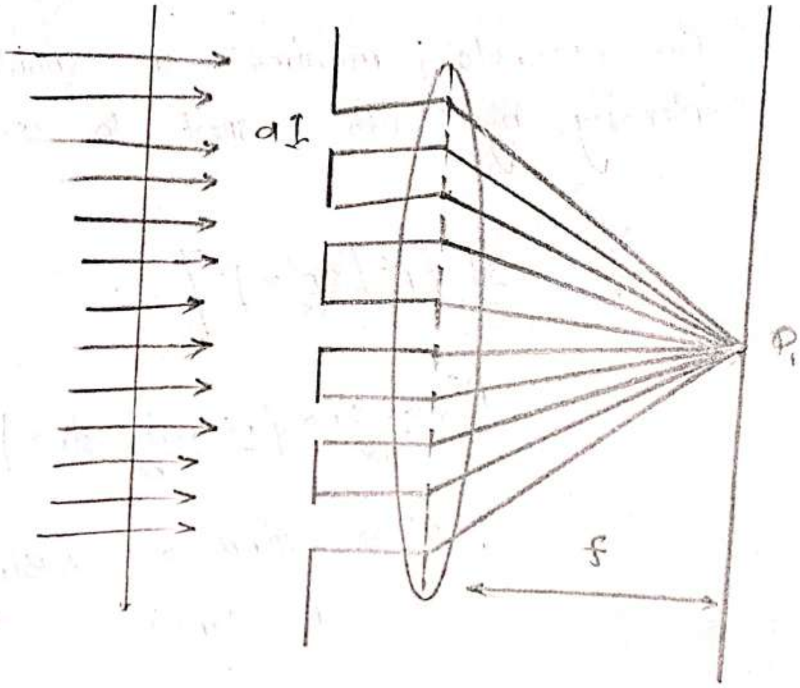
$$= A^2 \times \frac{2\sin\alpha}{\alpha}\left(\frac{\alpha\cos\alpha - \sin\alpha}{\alpha}\right) = 0$$

$$\alpha\cos\alpha - \sin\alpha = 0 \quad \& \quad 2\sin\alpha = 0$$

$$(\alpha = \tan\alpha) \qquad \alpha = \pm n\pi$$



Fraunhofer diffraction by N slit (grating)



Due to single slit :-

$$E_1 = \frac{A\sin\beta}{\beta}\cos(\omega t-\beta)$$

$$E_2 = \frac{A\sin\beta}{\beta}\cos(\omega t-\beta-\phi_1)$$

$$\vdots \qquad \left[\phi_1 = \frac{\pi d\sin\theta}{\lambda}\right]$$

$$E_n = \frac{A\sin\beta}{\beta}\cos\left[(\omega t-\beta-(n-1)\phi_1\right]$$

Total field at P is

$$E = E_1 + E_2 + E_3 + \_\_\_ + E_3$$

$$= \frac{A\sin\beta}{\beta}\left[\cos(\omega t-\beta) + \cos(\omega t-\beta-\phi_1) + \cos(\omega t-\beta-2\phi_1) + \text{-------} + \cos(\omega t-\beta-(n-1)\phi_1)\right]$$

$=$

$$= \frac{A \sin\beta}{\beta} \cdot \frac{\sin N\alpha}{\alpha} \cos\left[\omega t - \beta - \frac{(N-1)}{2}\phi_1\right]$$

where $\left(\alpha = \frac{\phi_1}{2}\right)$

Now, the resultant intensity at P is I_p.

$$I_p = A^2$$

$$I_p = I_0 \frac{\sin^2\beta}{\beta^2} \cdot \frac{\sin^2 N\alpha}{\sin^2\alpha}$$

Where $I_0 \frac{\sin^2\beta}{\beta^2}$ → this is due to intensity of first slit

$\frac{\sin^2 N\alpha}{\sin^2\alpha}$ → Interference pattern due to secondary width coming from different point source of different slit