Optics

Spatial And Temporal Coherences :-

Spatial coherence describes the correlation between waves at different point in space. While temporal Coherence describe the Correlation between waves observed at different moment of time.

Spatial Coherence :-

- Refers to the correlation or predictable relationship between the electric field and at different locations across a beam profile.
- Essentially, it describes how well the phase of wave is maintained across different point in space.
- A high degree of spatial coherence means the wave's phase is consistence across the beam while a low degree of spatial coherence means the phase varies randomly.
- Example of phenomena where spatial coherence is importance include interference pattern in Young's double slit experiment and the formation of holograms.

Temporal Coherence :-

- Refers to the correlation between the electrical field at one location but at different time.
- It describes how long the phase of a wave remains consistent over time.

•). A high degree of temporal coherence means the wave's phase remains consistence for a longer duration, while a low degree of temporal coherence means the phase varies rapidly.

•). Example of phenomena where temporal coherence is important include the sharpness of interference fringes in Michelson interferometer fringes in Michelson interferometer and the wavelength of light emitted by a laser.

Interference :-

Interference of light is a phenomenon that occurs when two or more light waves interact with each other. It can result in constructive or destructive interference.

•). Light wave can interfere constructively or destructively depending on their path difference.

•). Constructive interference occurs when the crests of two light wave meets, causing them to combine and form a wave of greater amplitude or brightness.

•). Destructive interference plays a crucial role in the diffraction and interference patterns observed in optics.

Condition for Light Interference :-

•). The light source must be coherent.

•). The light source must be monochromatic.

•). The source must have equal amplitude and intensities.

•). The source must be close enough to produce wide fridge.

Interference due to divison of Amplitude :-

Interference can be produced by two methods : (i) Division of wavefront and (ii) Division of amplitude. In division of amplitude, the amplitude of the incoming beam is divided into two parts, either by parallel reflection or by refraction. These two parts, either by travel unequal distance or reunite to produce interference. for example:- Newton's rings and Michelson's interferometer.

Stokes Phase Law :-

This law states that a light wave When get reflected from a denser medium suffers a phase change of 180° or π.

Proof:- Let r, t be reflection and transmission Coefficient of rarer medium (air). and r', t' be reflection and transmission constant of denser medium. denser medium is (water).

region ①

$$a = r(ra) + t'(ta)$$

$$\cancel{a} = (r^2 + t't)\cancel{a}$$

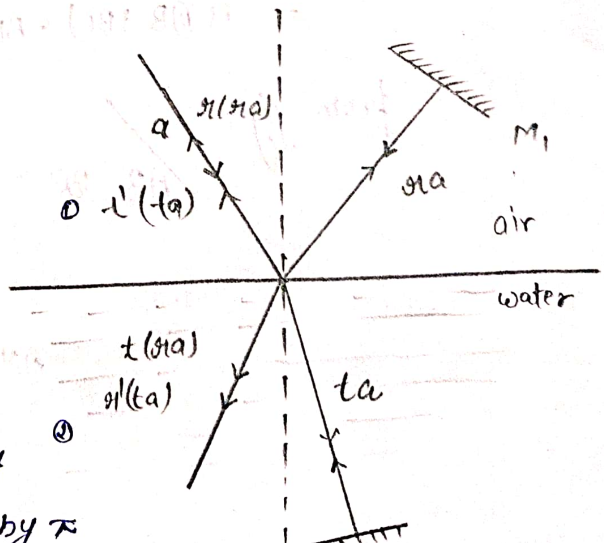
$$1 = r^2 + t't \longrightarrow ①$$

region②

$$0 = (ta)r' + t(ra)$$

$$0 = tr' + rt$$

$$\boxed{-r' = r}$$



from this eqⁿ we could say that the light wave suffers phase change by π after reflection.

for eq^n (1).

$$tt' = 1 - r^2 \rightarrow (i)$$

$$r^2 + t^2 = 1$$

$$t^2 = 1 - r^2 \rightarrow (ii)$$

On Comparing (i) and (ii)

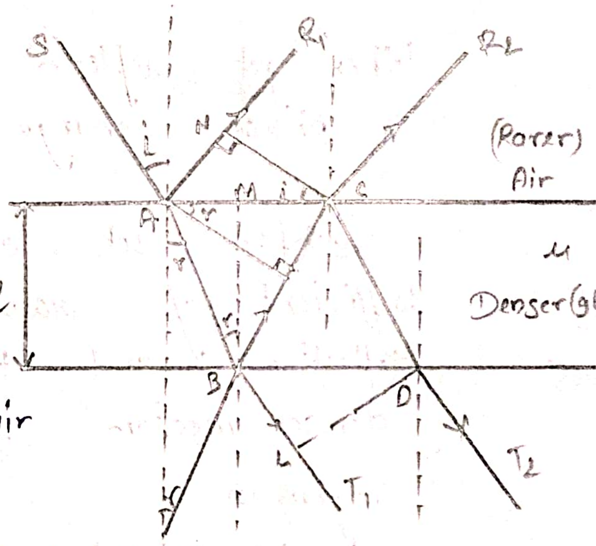
$$tt' = t^2$$

$$\boxed{t' = t}$$

this will not show any phase change.

Interference in thin film

A ray of monochromatic light SA be incident at an angle i on a parallel-sided transparent thin film of thickness t and refractive index ($\mu > 1$). Lets see in figure.



Interference in this films

Let CN and BM be perpendicular to AR_1 and AC. As the path of the rays AR_1 and CR_2 beyond CN is equal, the path difference between them is.

$$\Delta = \text{path ABC in film} - \text{path AN in air}$$

$$= \mu(AB + BC) - AN$$

from fig :-

$$AB = BC = \frac{BM}{\cos r} = \frac{t}{\cos r}$$

and

$$AN = AC \sin i$$

$$= (AM + MC) \sin i$$

$$= (BM \tan r + BM \tan r) \sin i$$

$$= 2t \tan r \sin i$$

$$= 2t \frac{\sin r}{\cos r} \sin i$$

$$= 2t \frac{\sin r}{\cos r} (\mu \sin r)$$

$$AN = 2\mu t \frac{\sin^2 r}{\cos r}$$

Substituting the value of AB, BC and AN in Eq. (6.19), we get

$$\Delta = \mu \left[\frac{t}{\cos r} + \frac{t}{\cos r} \right] - 2\mu t \frac{\sin^2 r}{\cos r}$$

$$= \frac{2\mu t}{\cos r} (1 - \sin^2 r)$$

$$= 2\mu t \cos r$$

The ray AR, having suffered a reflection at the surface of a denser medium undergoes a phase change of π, which is equivalent to a path different of λ. Hence, the effective path difference between AR, and CR_2 is $2\mu t \cos r - (\lambda/2)$

Interference Due to Reflected Light :-

For maxima :-

$$2\mu t \cos r - \frac{\lambda}{2} = n\lambda$$

$$2\mu t \cos r = (2n+1)\frac{\lambda}{2}$$

for minima :-

$$2\mu t \cos r - \frac{\lambda}{2} = (2n-1)\frac{\lambda}{2}$$

$$2\mu t \cos r = n\lambda$$

Interference Due to Transmitted Light :-

Similarly, the path difference between the transmitted rays BT_1 and DT_2 is given by.

$$\Delta = \mu(BC + CD) - BL$$

$$= 2\mu t \cos r$$

Hence, the effective path difference between BT_1 and DT_2 is also $2\mu t \cos r$.

for maxima

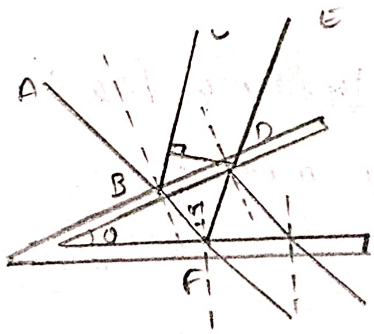
$$2\mu t \cos r = n\lambda$$

for minima

$$2\mu t \cos r = (2n+1)\frac{\lambda}{2}$$

Interference In Wedge-Shaped Thin Films :-

As we have already find the phase difference in thin film similarly :-



$$\Delta = 2\mu t \cos r$$

for Constructive interference (maxima)

$$2\mu t \cos r - \frac{\lambda}{2} = 2n$$

$$2\mu t \cos r = (2n+1)\lambda/2$$

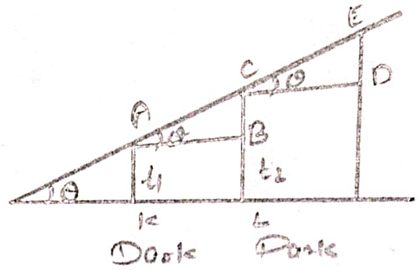
For destructive Interference :-

$$2\mu t \cos r - \frac{\lambda}{2} = (2n-1)\frac{\lambda}{2}$$

$$2\mu t \cos r = n\lambda$$

Frindge width :-

We are trying to find the dark frindges. lets assume the incident is normal, $(i=0,\ r=0)$ and then $(\cos r = 1)$.



let if thickness of film at A is denoted by t. then at A

$$2\mu t_1 = m\lambda$$

Let the next dark frindge will occurs at C. at that point the thickness = t_2. Then at C.

$$2\mu t_2 = (m+1)\lambda$$

Now subtracting :-

$$2\mu(t_2 - t_1) = m\lambda + \lambda - m\lambda$$

$$2\mu(t_2 - t_1) = \lambda$$

$$t_2 - t_1 = BC$$

$$2\mu(BC) = \lambda$$

$$BC = \frac{\lambda}{2\mu}$$

In ΔABC

$$\tan\theta = \frac{BC}{AB}$$

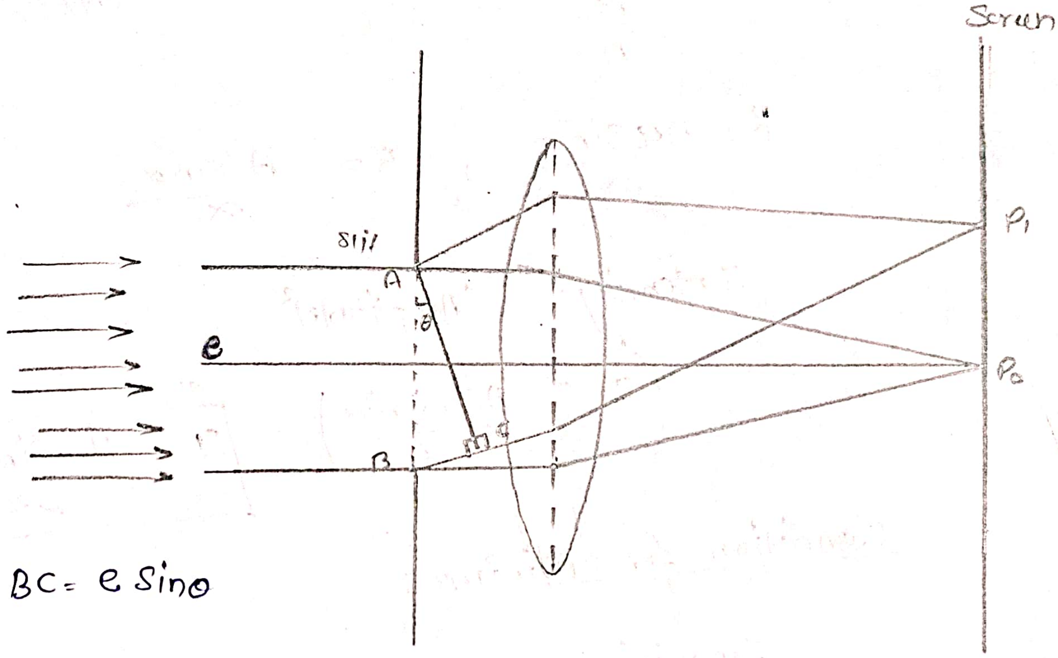
$$BC = AB\tan\theta$$

$$\left(AB\tan\theta = \frac{\lambda}{2\mu}\right)$$

Now we know $\tan\theta \sim \theta$ for small angle.

$$BC = \frac{\lambda}{2\mu\theta}$$

Fraunhoffers diffraction by single slit :-



Path difference = $BC = e\sin\theta$

Phase difference = $\frac{2\pi}{\lambda}$ (path diff.)

$\delta = \frac{2\pi}{\lambda} e\sin\theta$ Effective path difference = $\frac{1}{n}\frac{2\pi}{\lambda} e\sin\theta$

Resultant Amplitude $\Rightarrow$ $R = \frac{a\sin\frac{n\delta}{2}}{\sin\frac{\delta}{2}}$

$$R = \frac{a\sin\frac{n}{2}\left(\frac{2\pi}{\lambda n} e\sin\theta\right)}{\sin\frac{1}{2n}\left(\frac{2\pi}{\lambda} e\sin\theta\right)}$$

$$= \frac{a\sin\left(\frac{\pi}{\lambda} e\sin\theta\right)}{\sin\left(\frac{\pi}{n\lambda} e\sin\theta\right)}$$

Let us considerer = $\left(\frac{\pi}{\lambda} e\sin\theta = \alpha\right)$

$$R = \frac{a\sin\alpha}{\sin\left(\frac{\alpha}{n}\right)}$$

$$R = \frac{a \sin\alpha}{\frac{\alpha}{n}}$$ because $\sin\left(\frac{\alpha}{n}\right) \sim \frac{\alpha}{n}$ as $\left(\frac{\alpha}{n} <<< 1\right)$

$$R = \frac{na \sin\alpha}{\alpha} \rightarrow R = \frac{A \sin\alpha}{\alpha}$$

Intensity = $(\text{Amplitude})^2$

$$\left(I = \frac{A^2 \sin^2\alpha}{\alpha^2}\right)$$

*

$$\boxed{I = \frac{I_0 \sin^2\alpha}{\alpha^2}} \quad \left(\alpha = \frac{\pi}{\lambda} e \sin\theta\right)$$

Condition for minima :-

for minima $I = 0$ and $(A \neq 0)$ $(\alpha \neq 0)$

So. $\sin\alpha = 0$

$\alpha = \pm n\pi$

Hence for the minima position = $n\pi$, $1\pi, 2\pi$. etc

Condition for central Maxima :-

for maximum value of $(I = \infty)$

for achieving maximum $(\alpha = 0)$

then $\left(\frac{e\pi}{\lambda} \sin\theta = 0\right)$.

That means, when $\underline{\underline{\theta = 0}}$ then we get the central maxima

For secondary maxima :-

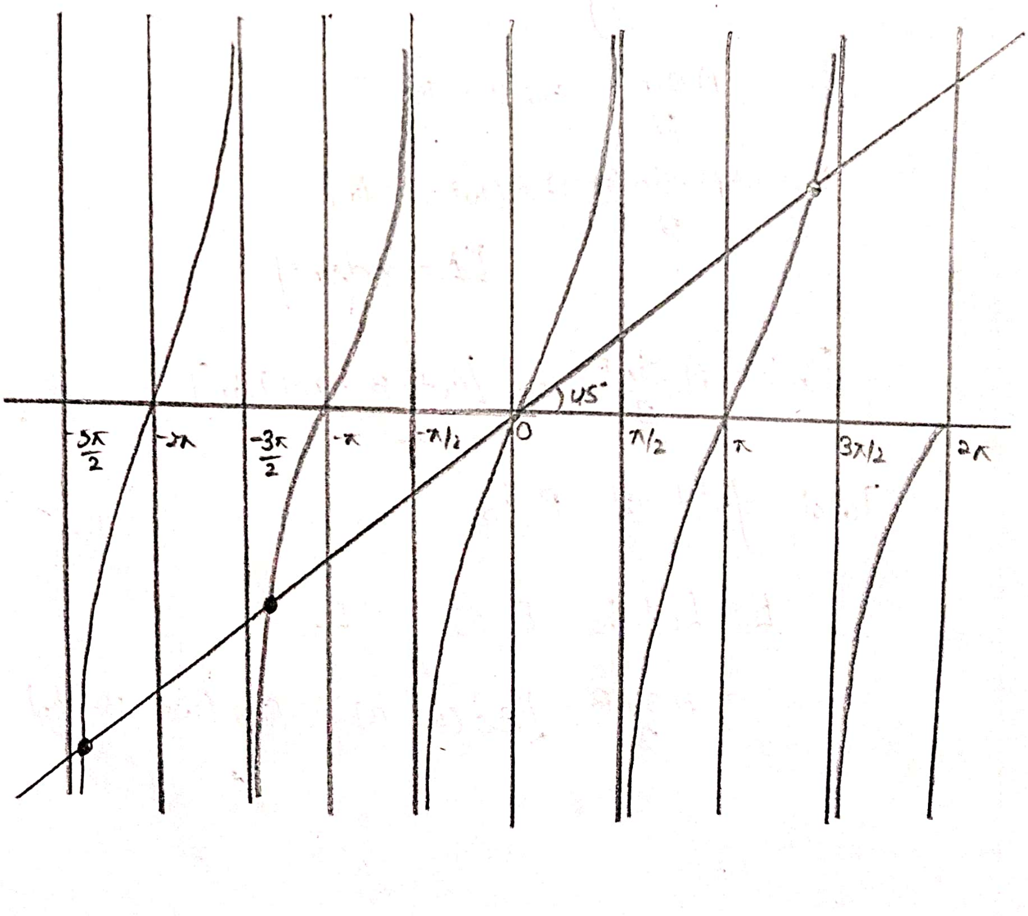
For secondary maxima we should differentiate intensity by with respect to α.

$$\frac{dI}{d\alpha} = \frac{d}{d\alpha}\left[A^2\left(\frac{\sin^2\alpha}{\alpha}\right)^2\right] = 0$$

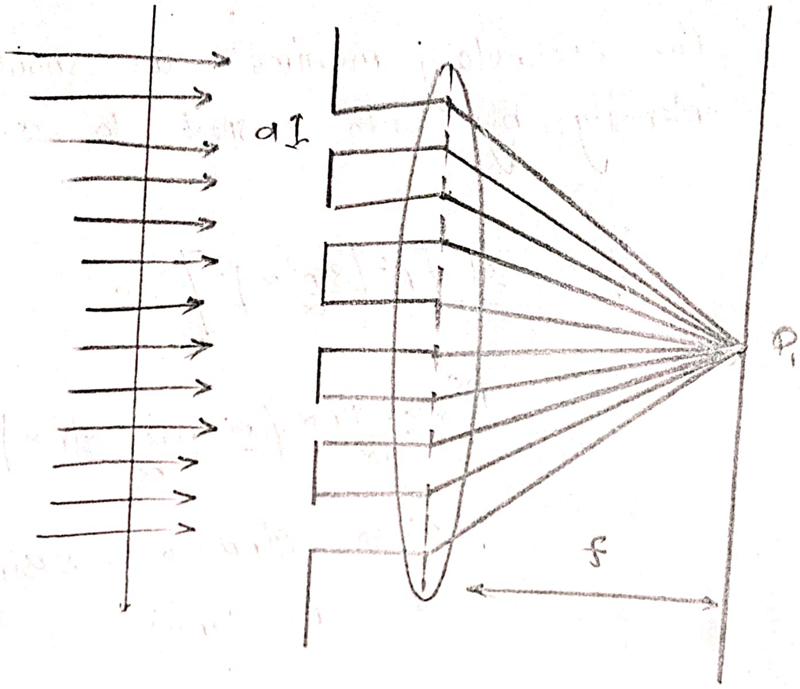
$$= A^2 \times \frac{2\sin\alpha}{\alpha}\left(\frac{\alpha\cos\alpha - \sin\alpha}{\alpha}\right) = 0$$

$$\alpha\cos\alpha - \sin\alpha = 0 \quad \& \quad 2\sin\alpha = 0$$

$$(\alpha = \tan\alpha) \qquad \alpha = \pm n\pi$$



Fraunhofer diffraction by N slit (grating).



Due to single slit :-

$$E_1 = \frac{A \sin\beta}{\beta} \cos(\omega t - \beta)$$

$$E_2 = \frac{A \sin\beta}{\beta} \cos(\omega t - \beta - \phi_1)$$

$$\left[\phi_1 = \frac{\pi d \sin\theta}{\lambda}\right]$$

$$\vdots$$

$$E_n = \frac{A \sin\beta}{\beta} \cos[(\omega t - \beta - (n-1)\phi_1]$$

Total field at P is

$$E = E_1 + E_2 + E_3 + \_\_\_ + E_3$$

$$= \frac{A \sin\beta}{\beta} [\cos(\omega t - \beta) + \cos(\omega t - \beta - \phi_1) + \cos(\omega t - \beta - 2\phi_1) + \_\_\_\_\_ + \cos(\omega t - \beta - (n-1)\phi_1)]$$

$=$

$$= \frac{A\sin\beta}{\beta} \cdot \frac{\sin N\alpha}{\alpha} \cos\left[\omega t - \beta - \frac{(N-1)\phi_1}{2}\right]$$

where $\left(\alpha = \frac{\phi_1}{2}\right)$

Now, the resultant intensity at P is I_p.

$$I_P = A^2$$

$$I_p = I_o \frac{\sin^2\beta}{\beta^2} \cdot \frac{\sin^2 N\alpha}{\sin^2\alpha}$$

Where $I_o \frac{\sin^2\beta}{\beta^2}$ → this is due to intensity of first slit

$\frac{\sin^2 N\alpha}{\sin^2\alpha}$ → Interference pattern due to secondary width coming from different point source of different slit