LABORATORY MANUAL FOR COMPOUND PENDULUM

Compound pendulum- To find 'g' and radius of gyration

Aim: To determine (a) the value of acceleration due to gravity 'g' at the given place by using a compound pendulum, (b) the radius of gyration and hence the moment of inertia of the compound pendulum about an axis passing through its centre of mass.

Apparatus: The compound pendulum, stop watch, etc.

Theory

A compound pendulum, also known as a physical pendulum, is a body of any arbitrary shape pivoted at any point so that it can oscillate in a plane when its centre of mass is slightly displaced on one side and is released.

In the figure *S* is the suspension centre and *G* is the centre of gravity of the body. Let the vertical distance *SG* be l when the body is in its normal position of rest. If the body is oscillated through an angle θ about an axis passing through *S* and perpendicular to the vertical plane of the body, its centre of gravity takes the position *G'*. The torque acting on the body due to its weight *mg* is given by,

$$\Gamma = -Mgl\sin\theta$$

The negative sign indicates that the torque acts opposite to the direction of increase of θ. If I is the moment of inertia of the body about the axis of rotation, then the torque is also given as,

$$\Gamma = I\alpha = I\frac{d^2\theta}{dt^2}$$

i.e. $$I\frac{d^2\theta}{dt^2} = -Mgl\sin\theta$$

If the angular displacement θ is very small, $\sin\theta = \theta$. Then the equation of motion becomes,

$$\frac{d^2\theta}{dt^2} + \frac{Mgl}{I}\theta = 0 \qquad (1)$$

Eqn.1 shows shat the motion of the pendulum is simple harmonic with an angular frequency, $\omega_0 = \sqrt{\frac{Mgl}{I}}$. Its period of oscillation is given by,

$$T = \frac{2\pi}{\omega_0} = 2\pi\sqrt{\frac{I}{Mgl}} \qquad (2)$$

Now we define $$L = \frac{I}{Ml} \qquad (3)$$

Then, $$T = 2\pi\sqrt{\frac{L}{g}} \qquad (4)$$

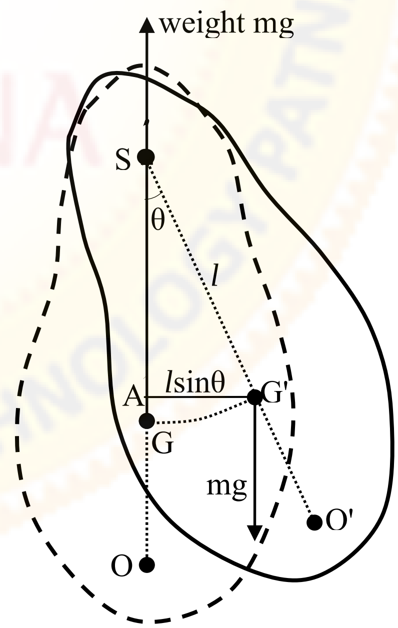

Fig.a

where, L is called the length of an equivalent simple pendulum.

If K is the radius of gyration of the compound pendulum about an axis through the centre of mass, the moment of inertia is,

$$I_{CM} = MK^2 \qquad (5)$$

Applying the parallel axes theorem the moment of inertia around the pivot is given by,

$$I = I_{CM} + Ml^2 = MK^2 + Ml^2 = M\left(K^2 + l^2\right) \qquad (6)$$

Hence from eqn.2 we get,

$$T = 2\pi\sqrt{\frac{K^2 + l^2}{gl}} = 2\pi\sqrt{\frac{L}{g}} \qquad (7)$$

where, $$L = \frac{I}{Ml} = \frac{K^2 + l^2}{l} \qquad (8)$$

Thus, if we know the radius of gyration of an irregular body around an axis through the centre of mass, the time period of oscillation of the body for different points of pivoting can be calculated. Fig.b shows the graph between the time period T in the Y axis and the distance of the point of suspension (axis of rotation) from one end of the bar in the X axis.

Centres of suspension and oscillation are mutually interchangeable: In fig.a consider the point O' on the line joining the centre of suspension 'S' and centre of gravity G' at a distance $\left(\frac{K^2}{l} + l\right)$ from 'S' or $\frac{K^2}{l}$ from G'. This point is called the *centre of oscillation*. An axis passing through the centre of oscillation and parallel to the axis of suspension is called *axis of oscillation*.

Let $SG' = l_1$ and $G'O' = l_2 = \frac{K^2}{l_1}$.

Let T_1 be the time period with 'S' as point of suspension. Now we find out the period of oscillation T_2 with O' as point of suspension. Then,

$$T_2 = \sqrt{\frac{4\pi^2}{g}\left(\frac{K^2}{l_2} + l_2\right)}$$

But, $$l_2 = \frac{K^2}{l_1} \qquad (9)$$

Then, $$T_2 = \sqrt{\frac{4\pi^2}{g}\left(\frac{K^2}{l_1} + l_1\right)} = T_1$$

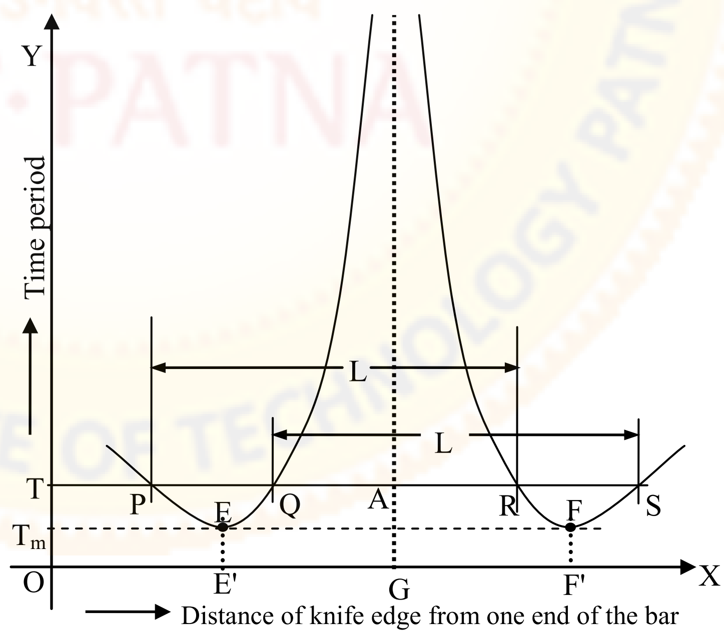

Fig.b

Thus the axes of suspension and oscillation are interchangeable. And if 'L' is the distance between them we can write,

$$L = \frac{K^2}{l_1} + l_1 = \frac{K^2}{l_2} + l_2 \quad (10)$$

And $$T = T_1 = T_2 = 2\pi\sqrt{\frac{L}{g}}$$

Thus by knowing L and T value of acceleration due to gravity g can be obtained as,

$$g = \frac{4\pi^2 L}{T^2} \quad (11)$$

To determine L and K: Draw the graph between the time period T in the X axis and the distance of the point of suspension (axis of rotation) from one end of the bar as shown in fig.b. From the graph, for a given T,

$$L = \frac{PR + QS}{2} \quad (12)$$

By eqn.8, $$K = \sqrt{l_1 l_2} = \sqrt{PA \times AR} = \sqrt{QA \times AS}$$

Thus, $$K = \frac{\sqrt{PA \times AR} + \sqrt{QA \times AS}}{2} \quad (13)$$

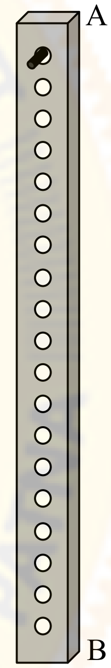

Fig.c

Procedure: In our experiment we use a *symmetric compound pendulum* as shown in fig.c. The compound pendulum is suspended on a knife edge passing through the first hole near one of the ends, say, A. The pendulum is pulled aside slightly and is released so that the pendulum oscillates with small amplitude. The time for 20 oscillations is determined twice and the average is calculated. From this, the period of oscillation T of the symmetric pendulum is found out. Similarly, the time periods of the pendulum by suspending the pendulum in successive holes till the hole near the other end B. (For holes beyond the centre of gravity, the pendulum gets inverted). The distances ‘x’ from the end A to the edge of the holes at which the knife edge touches are measured by a metre scale.

The centre of gravity of the bar is determined by balancing it on a knife edge. The position of centre of gravity from the end A is also measured. The mass of the bar (including the knife edge if it is attached to the bar) is measured using a balance.

A graph is drawn taking the distances ‘x’ of the holes from the end A along the X-axis and the time periods T along the Y-axis as shown in fig.b.

To determine the length of the equivalent simple pendulum and the radius of gyration K about the axis passing through the centre of gravity from the graph, draw lines parallel to the X-axis for particular values of T. Determine PR and QS and from these L is calculated. Also determine PA, AR, QA and AS and from these K is calculated. Finally, using eqn.11 the value of acceleration due to gravity ‘g’ is calculated and the moment of inertia of the bar about an axis through the centre of mass (centre of gravity) using eqn.5. We can also calculate the moment of inertia of the bar about an axis at a distance ‘a’ from the end A and perpendicular to the bar by applying the parallel axes theorem, $I = MK^2 + Ma^2$.

- Distances ‘x’ from the end A depends on how the knife edge is fixed in the holes. It may be the top end, bottom end or centre of the hole. The inversion of the bar also is taken into account in this case.

Observation and tabulation

Mass of the bar, M = …….. kg.

Position of centre of gravity G from the end A = ……. m

Distance ‘x’ from the end A in metre	Time for 20 oscillations in sec.			Period T sec
	1	2	Mean	

To determine acceleration due to gravity (Observations from graph)

Sl.No.	T sec	PR m	QS m	$L = \frac{PR + QS}{2}$ m	$\frac{L}{T^2}$ ms^{-2}	Mean $\frac{L}{T^2}$ ms^{-2}	$g = 4\pi^2 \left(\frac{L}{T^2}\right)$ ms^{-2}

To find radius of gyration and moment of inertia (Observations from graph)

Sl.No.	T sec	PA m	AR m	QA m	AS m	$K = \frac{\sqrt{PA \times AR} + \sqrt{QA \times AS}}{2}$ m	Mean K m	$I_{CM} = MK^2$ $kg.m^2$

Result

Acceleration due to gravity at the place, ‘g’ = ……… ms^{-2}

Radius of gyration about an axis through the centre of mass, K = ………. m

Moment of inertia about an axis through the centre of mass, I_{CM} = ………. $kg.m^2$