# Determination of modulus of rigidity and Young's modulus using Searle's Arrangement

# Experiment No. 11
# Determination of modulus of rigidity and Young's modulus using Searle's Arrangement

## AIM

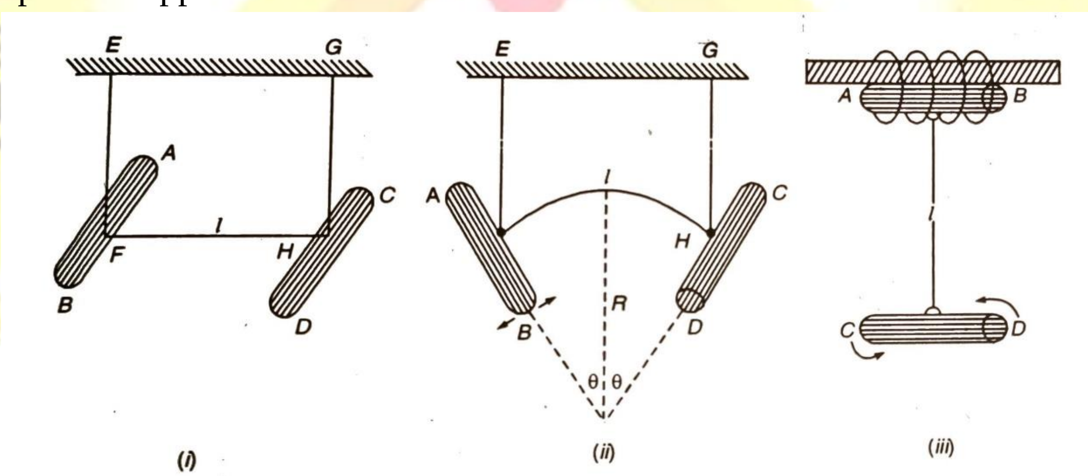
To determine Young's modulus, modulus of rigidity and Poisson's ratio for the material of wire by Searle's arrangement.

## APPARATUS

1. Searle's arrangement (vertical stand and two identical bars including stand), Experimental Wire, screw gauge, Vernier calipers, meter scale, candle, thread and match box.

## THEORY

Description of apparatus:




To find the Young modulus of the wire (i) the ends B and D are drawn slightly towards each other in a horizontal plane. The wire FH gets bent into a circular arc. The ends B and D are then released. The rods begin to execute torsional vibrations, their middle points F and H, remaining practically at rest. This is shown by Fig. (b). If r be the radius of the wire, l be the effective length of the wire between the two rods and Y be the young modulus of the rod, then the bending moment of the wire = couple exerted by it on each road = $\frac{Y\pi r^4}{l} \cdot 2\theta$. This couple produces an angular acceleration α in the rods.

If $I_1$ be the moment of inertia of a rod about its axis of suspension,

$$I_1\alpha = -\frac{Y\pi r^4}{l}\cdot\theta$$

$$\alpha = -\frac{Y\pi r^4}{2lI_1}\theta$$

So, the motion is simple harmonic and the time period is

$$T_1 = 2\pi\sqrt{\frac{2lI_1}{Y\pi r^4}} \text{ and } Y = \frac{8\pi l I_1}{r^4 T_1^2}$$

Now similarly in the case of second configuration AB is fixed horizontally to a rigid support. The wire now hangs vertically with CD suspended horizontally. At its lower end. If CD is twisted horizontally through a small angle and released, it begins to execute torsional vibrations of time period $T_2 = 2\pi\sqrt{\frac{I_1}{c}},\ c = \frac{\pi\eta r^4}{2l}$

$$\therefore T_2 = 2\pi\sqrt{\frac{2lI_1}{\pi\eta r^4}} \text{ and } \eta = \frac{8\pi l I_1}{r^4 T_2^2}$$

$$I_1 = \text{Moment of Inertia of the rectangular bar} = \frac{M(B^2 + L^2)}{12}$$

$$\text{Where b and L are breadth and length of the rod.}$$

- Now Poisson's ratio $\sigma = \frac{Y}{2\eta} - 1$
- $\text{and bulk modulus}, K = \frac{Y}{3(1-2\sigma)}$

Hence, with the Searle's arrangement, we can determine $Y, \eta, \sigma$ **and** $K$ of the wire.

## PROCEDURE

- Set up an arrangement as shown in Fig. (a) so that AB and CD Are in the same horizontal plane.
- Pass a cotton loop round the ends *B* and *D* and thus draw them a little towards each other. Burn the loop, the ends *B* and *D* become free and begin to vibrate. Note the time, $t_1$, for each 20 vibrations of the end *B* for three times.
- Remove the two rods from the suspensions. Set up *AB* horizontal, Fig. (c) and fix it to a rigid support. Twist *CD* horizontally through a small angle and determine the time $t_2$ for 20 vibrations (looking at one end *D*) thrice.

- Measure the length of the wire and measure its diameter at a number of places in two mutually perpendicular directions at each place. Determine the mass, M of rod CD (M may be supplied), measure its breadth a and width b.

## OBSERVATION

Least count of the stop watch = 0.01 sec

- **TABLE 1: Determination of T1 and T2**

| Configuration | Time for 20 oscillations (sec) | | | | Time period $T = \frac{t}{20}$ in sec |
|---|---|---|---|---|---|
| | 1 (s) | 2 (s) | 3 (s) | Mean time $t_1$ (s) | |
| **Configuration as (ii)** | | | | $t_1$ | $\left(T_1 = \frac{t_1}{20}\right)$ |
| **Configuration as (iii)** | | | | $t_2$ | $\left(T_2 = \frac{t_2}{20}\right)$ |

- **TABLE 2: Determination radius (r) of the wire using screw gauge.**

Pitch of the screw gauge =

Least count of the screw gauge ($\boldsymbol{l}$.**c**) = $\frac{pitch}{Total\ no\ of\ div.\ in\ circular\ scale}$ =

| SL. No. | Main Scale reading (M) (mm) | Circular Scale reading (CSR) | Total Reading = M+ CSR× $\boldsymbol{l}$.**c** (mm) | Mean diameter (d) in mm |
|---|---|---|---|---|
| **1.** | | | **($d_1$)** | |
| **2.** | | | **($d_2$)** | $d = \frac{d_1+d_2+d_3}{3}$ |
| **3.** | | | **($d_3$)** | |

Mean diameter (d) = …. in m

**(i) Radius (r) = …. in m**

**(ii) Effective Length of the wire ($l$) = …. cm = ….. m (using scale)**

- **TABLE 3: Determination of Moment of Inertia ($I_1$) of the rectangular bar**
  Least count of the Vernier calipers (**$l$.c**) = …. cm
  Least count of the scale used = … cm

| SL. No. | Determination of breadth (b) of the rectangular bar | | | Mean breadth (b) in cm | Length of the bar (L) in cm |
|---|---|---|---|---|---|
| | Main Scale reading (M) (cm) | Vernier Scale reading (VSR) | Total Reading = M+ VSR× $l$.c (cm) | | |
| **1.** | | | **($B_1$)** | $B = \frac{B_1 + B_2 + B_3}{3}$ | |
| **2.** | | | **($B_2$)** | | |
| **3.** | | | **($B_3$)** | | |

So, breadth of the bar (d) = …. cm = …. M and length of the bar (L) = … cm = … m

**(iii) Mass of the rod CD, M = …. g = ….. kg (supplied)**

**(iv)** $\therefore\ I_1 = \textit{Moment of Inertia of the rectangular bar} = \frac{M(B^2+L^2)}{12}$ **kg. m²**

## CALCULATIOS

- $Y = \frac{8\pi l I_1}{r^4 T_1^2} = \ldots . N/m^2$
- $\eta = \frac{8\pi l I_1}{r^4 T_2^2} = \ldots . N/m^2$
- Now, Poisson's ratio $\sigma = \frac{Y}{2\eta} - 1$
- $\textit{and bulk modulus}, K = \frac{Y}{3(1-2\sigma)} = \ldots . N/m^2$

## RESULT

So, Young modulus ($\mathbf{Y}$) = … $N/m^2$, modulus of rigidity ($\boldsymbol{\eta}$) = $N/m^2$, Poisson's ratio ($\sigma$) = … and the bulk modulus of the rod is (K) = … $N/m^2$.

## EROR CALCULATION

Show the error calculations for Young modulus ($\mathbf{Y}$) and modulus of rigidity ($\boldsymbol{\eta}$).

## PRECAUTIONS

1. The two rods AB and CD should hang horizontal in the same plane and they should be twisted through equal angles, $\boldsymbol{\theta}$, before being released.
2. The ends of the wire should be firmly secured into the rods. The rods should execute torsional oscillations of small amplitude and their middle points should have the least possible movement.
3. The diameter of the wire should be determined very carefully and it should be very small as compared to its length.
4. Do not bend the wire force fully.