Improved DOA Estimation in KR-MUSIC Algorithm with Non-Uniform Noise

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Abstract

The Khatri-Rao (KR)-MUSIC[1] algorithm for Direction of Arrival (DOA) estimation of quasi-stationary signals in the presence of non-uniform noise doesn't explicitly estimate the noise covariance matrix. Hence, the method performs poorly in low SNR environment as the effect of covariance between noise and signal vectors increases on the data covariance matrix. The uniform linear array DOA model in KR-subspace follows the sparse representation in spatial domain for all frames of the quasi-stationary signals. In this paper, we propose a modified denoising paradigm to improve the DOA estimation of the algorithm by using this sparse representation in KR-subspace. The sparse representation is utilized to perform covariance fitting to eliminate noise from the data model. Numerical experiment further demonstrate the effectiveness of the proposed method. An implementation of the proposed algorithm can be found in the following link: http://tinyurl.com/ee260-aich

1 Introduction

For several decades, array signal processing has been of keen interest in variety of applications, such as radar, sonar, radio astronomy, etc. [1]. Major state-of-the-art classical algorithms have been proposed over the past few decades, such as MUltiple SIgnal Classification (MUSIC) [2] and Estimation of Signal Parameters via Rotational Invariance Techniques (ESPRIT) [3]. Such algorithms require a compact sensor array in order to avoid the problem of angle ambiguity as these algorithms use noise/signal subspace analysis for DOA estimation. However, these compact sensor array configurations suffers from less degrees of freedom, disturbances due to adjacent sensors. One recent method to tackle these issues is the use of sparse representation framework. It has attracted a significant interest in DOA estimation as this problem has signals being sparse in the spatial domain. Specifically, sparse linear array configurations such as co-prime array systems and nested array systems, with their variants have been major frameworks in focus [4, 5]. These array frameworks provide enlarged degrees of freedom which is capable of handling more sources than sensors and, hence result in higher resolution in DOA estimation. Recent works, such as ℓ_1-based singular value decomposition (ℓ_1-SVD) algorithm [6] deal with DOA estimation problem by representing the sensor array output with an over-complete basis. Another class of methods [7] utilise second-order statistics of the sensor array output to tackle the underlying DOA parameter estimation problem.

One major underlying assumption about these frameworks has been on the noise being uniform. This makes these algorithms difficult to produce accurate results in practical applications. This assumption may not always hold in real-life scenarios because of non-uniformity of the individual sensor response, the mutual coupling between sensor elements, and similar model errors, apart from

[1]MUltiple SIgnal Classification

case that the additive noise is non-uniform in nature [8]. This brings us to the focus of the paper, on how we can denoise the sensor array output in case of quasi-stationary signals in KR subspace. The KR-MUSIC algorithm [9] is a DOA estimation algorithm for quasi-stationary uses the subspace characteristics of the KR product of the sensor array manifold vectors. It assumes that the sensor array structure is set up as an uniform linear array which makes this KR subspace approach an one-dimensional search problem based on the MUSIC algorithm. The noise elimination process followed in this work explicitly makes an assumption that the noise is statistically independent of the source signals and hence, this noise can be eliminated using an orthogonal projection of $\mathbf{1}_M$ where M is the total number of frames. However this assumption, doesn't fit well because the true covariance matrix cannot be computed accurately due to limited number of time snapshots and hence, the noise elimination is poor. This motivates us to explore another technique of denoising based on the second-order statistics of covariance matrix for the non-uniform noise case. We basically perform a covariance fitting based on the asymptotic property of a vectorized covariance matrix originally proposed in [10], for denoising before the KR-MUSIC algorithm is applied.

Notations: Vectors and matrices are denoted with bold lowercase and uppercase letters, respectively. Superscript $^\top$ denotes transpose, whereas superscript $^\mathsf{H}$ denotes transpose conjugate. The symbol $\odot$ denotes the Khatri-Rao product [9] between two matrices of appropriate size. $\text{vec}(\cdot)$ represents the vectorization operation on the given matrix. $\mathcal{R}(\cdot)$ denotes the range space operator. The symbol $\otimes$ denotes the Kronecker product.

2 Problem Formulation

2.1 KR-MUSIC algorithm

Consider the DOA model based on quasi-stationary signals from K far-field sources impinging on an Uniform Linear Array (ULA) with N sensors. Denote the observed signal from nth sensor to be $x_n(t)$, signal transmitted by kth source to be $s_k(t)$, and noise in the observed signal to be $w_n(t)$. Further, define $\mathbf{A} = [\mathbf{a}(\theta_1), \mathbf{a}(\theta_2), \cdots, \mathbf{a}(\theta_K)] \in \mathbb{C}^{N\times K}$ as the array response matrix composed of steering vectors $\mathbf{a}(\theta_k) = [1, \exp(-j\frac{2\pi d}{\lambda}\sin(\theta_k)), \cdots, \exp(-j\frac{2\pi d}{\lambda}(N-1)\sin(\theta_k))]^\top$ with parameters d and λ being the inter-sensor spacing and the signal wavelength, respectively, for each kth source ($\theta_k \in [-\pi/2, \pi/2]$). Keeping $\mathbf{x}(t) = [x_1(t), x_2(t), \cdots, x_N(t)]^\top$, $\mathbf{s}(t) = [s_1(t), s_2(t), \cdots, s_K(t)]^\top$, and $\mathbf{w}(t)$ defined same as $\mathbf{x}(t)$, the received data from the ULA can be modelled as

$$\mathbf{x}(t) = \mathbf{A}\mathbf{s}(t) + \mathbf{w}(t), \quad t = 0, 1, 2, \cdots \tag{1}$$

The following assumptions are made on the data model:

(A1.) Source signals $\{s_k(t)\}_{k=1}^K$ are mutually uncorrelated and have zero-mean.

(A2.) All $\{\theta_k\}_{k=1}^K$ are distinct.

(A3.) The noise $\mathbf{w}(t)$ is zero-mean wide-sense stationary (WSS) with covariance matrix, $\mathbf{W} \triangleq \mathbb{E}[\mathbf{w}(t)\mathbf{w}^\mathsf{H}(t)] = \text{diag}(\sigma_1, \sigma_2, \cdots, \sigma_N)$.

(A4.) All $\{s_k(t)\}_{k=1}^K$ are wide-sense quasi-stationary with frame length L, such that for frame number $m = 1, 2, \cdots, M$, $\mathbb{E}[|s_k(t)|^2] = \gamma_{mk}, \ \forall t \in [(m-1)L, mL-1]$.

The local (for each frame) covariance matrix is defined as $\mathbf{R}_m \triangleq \mathbb{E}[\mathbf{x}(t)\mathbf{x}^\mathsf{H}(t)] = \mathbf{A}\mathbf{\Gamma}_m\mathbf{A}^\mathsf{H} + \mathbf{W}$ (where $\mathbf{\Gamma}_m = \text{diag}(\gamma_{m1}, \gamma_{m2}, \cdots, \gamma_{mK})$. Vectorize this covariance matrix and formulate as follows

$$\begin{aligned}\mathbf{y}_m &\triangleq \text{vec}(\mathbf{R}_m) = \text{vec}(\mathbf{A}\mathbf{\Gamma}_m\mathbf{A}^\mathsf{H}) + \text{vec}(\mathbf{W}) \\ &= (\mathbf{A}\odot\mathbf{A})\boldsymbol{\gamma}_m + \text{vec}(\mathbf{W})\end{aligned} \tag{2}$$

where $\boldsymbol{\gamma}_m = [\gamma_{m1}, \gamma_{m2}, \cdots, \gamma_{mK}]^\top$. Using (2), stack all frames together to form

$$\begin{aligned}\mathbf{Y} &\triangleq [\mathbf{y}_1, \mathbf{y}_2, \cdots, \mathbf{y}_M] \\ &= (\mathbf{A}\odot\mathbf{A})\mathbf{\Psi}^\top + \text{vec}(\mathbf{W})\mathbf{1}_M^\top\end{aligned}$$

where $\mathbf{\Psi} = [\boldsymbol{\gamma}_1, \boldsymbol{\gamma}_2, \cdots, \boldsymbol{\gamma}_M]^\top$. Next, with the assumption that $[\mathbf{\Psi}, \mathbf{1}_M] \in \mathbb{R}^{M\times(K+1)}$ is full column rank, we can eliminate noise as follows. Let $\mathbf{P}^\perp = \mathbf{I}_M - \frac{1}{M}\mathbf{1}_M\mathbf{1}_M^\top$ be the orthogonal projector of $\mathbf{1}_M$. Then,

$$\mathbf{Y}\mathbf{P}^\perp = (\mathbf{A}\odot\mathbf{A})(\mathbf{P}^\perp\mathbf{\Psi})^\top \tag{3}$$

which is now ideally a noise free model. Assuming $(\mathbf{A}\odot\mathbf{A})$ is full column rank, we have $\mathcal{R}(\mathbf{A}\odot\mathbf{A}) = \mathcal{R}(\mathbf{Y}\mathbf{P}^{\perp})$. Apply SVD on $\mathbf{Y}\mathbf{P}^{\perp}$:

$$\mathbf{Y}\mathbf{P}^{\perp} = [\mathbf{U}_s \quad \mathbf{U}_n] \begin{bmatrix} \mathbf{\Sigma}_s & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{V}_s^{\mathsf{H}} \\ \mathbf{V}_n^{\mathsf{H}} \end{bmatrix} \tag{4}$$

where $\mathbf{U}_s \in \mathbb{C}^{N^2 \times K}$ and $\mathbf{V}_s \in \mathbb{C}^{M \times K}$ are respectively the left and right singular matrices associated with the non-zero singular values, $\mathbf{U}_n \in \mathbb{C}^{N^2 \times (N^2-K)}$ and $\mathbf{V}_n \in \mathbb{C}^{M \times (N^2-K)}$ are the counterparts for the zero singular values, and $\mathbf{\Sigma}_s \in \mathbb{R}^{K \times K}$ is the nonzero singular values matrix. Further, $\mathcal{R}(\mathbf{A}\odot\mathbf{A}) = \mathcal{R}(\mathbf{U}_s)$ and $\mathbf{U}_n^{\mathsf{H}}\mathbf{U}_s = \mathbf{0}$, which gives

$$\mathbf{U}_n^{\mathsf{H}}[\mathbf{A}\odot\mathbf{A}]_k = \mathbf{0} \quad \forall\, k = 1, 2, \cdots, K \tag{5}$$

This gives the KR subspace criterion for DOA estimation of quasi-stationary sources:

$$\text{find} \quad \theta \quad \text{s.t.} \quad \mathbf{U}_n^{\mathsf{H}}[\mathbf{A}\odot\mathbf{A}] = 0 \tag{6}$$

We can now apply the MUSIC algorithm over this denoised data, to obtain the DOA estimates over the search grid $\theta \in [-\pi/2, \pi/2]$.

2.2 Inefficient denoising procedure

In practice, the local covariances are computed by local time averaging:

$$\hat{\mathbf{R}}_m = \frac{1}{L}\sum_{t=(m-1)L}^{mL-1} \mathbf{x}(t)\mathbf{x}^{\mathsf{H}}(t) = \mathbf{A}(\theta)\hat{\mathbf{\Gamma}}_m\mathbf{A}^{\mathsf{H}}(\theta) + \hat{\mathbf{W}} + \mathbf{A}(\theta)\overbrace{\left\{\frac{1}{L}\sum_{t=(m-1)L}^{mL-1}\mathbf{s}(t)\mathbf{n}^{\mathsf{H}}(t)\right\}}^{\mathbf{I}}$$
$$+ \overbrace{\left\{\frac{1}{L}\sum_{t=(m-1)L}^{mL-1}\mathbf{n}(t)\mathbf{s}^{\mathsf{H}}(t)\right\}}^{\mathbf{II}}\mathbf{A}(\theta) \tag{7}$$

This is because we have only finite samples available $\left(\lim_{L\to\infty}\hat{\mathbf{R}}_m = \mathbf{R}_m\right)$. This means $\hat{\mathbf{y}} = \mathrm{vec}(\hat{\mathbf{R}}_m)$ will have error w.r.t. $\mathbf{y}$. Define $\Delta\mathbf{y} = \mathbf{y} - \hat{\mathbf{y}}$, termed as estimation error, represented by terms **I** and **II** in (7). **I** and **II** become dominant when time snapshots are limited. Hence, the orthogonal operation in (4) will not denoise the vectorized efficiently. This situation worsens especially in the low SNR environment. Next, we devise a method to effectively denoise $\hat{\mathbf{y}}$, when the number of samples are limited, to make the KR-MUSIC algorithm robust.

3 Proposed denoising and DOA estimation procedure

Comparing (1) to (2) for mth frame, it can be seen that $\mathbf{y}_m$ and $\boldsymbol{\gamma}_m$ are the new observation vector and the equivalent source vector, respectively. $(\mathbf{A}\odot\mathbf{A})$ is the new virtual manifold matrix with its dimension N^2, which significantly increases the degrees of freedom of the ULA and shows an extended-aperture, thereby providing the capability of processing the under-determined DOA estimation. Next, define all potential DOAs from $[-\pi/2, \pi/2]$ as $\tilde{\boldsymbol{\theta}} = [\tilde{\theta}_1, \tilde{\theta}_2, \cdots, \tilde{\theta}_Q]$ where $Q \gg N^2$. Assuming the target DOAs lie on the defined grid, we can rewrite (2) in the following form

$$\mathbf{y}_m = \mathbf{B}(\tilde{\theta})\mathbf{u}_m \tag{8}$$

where $\mathbf{B}(\tilde{\theta}) = [\mathbf{b}(\tilde{\theta}_1), \mathbf{b}(\tilde{\theta}_2), \cdots, \mathbf{b}(\tilde{\theta}_Q)] \in \mathbb{C}^{N^2 \times Q}$ is the over-complete basis. $\mathbf{u}_m \in \mathbb{C}^Q$ now ideally is a K-sparse vector whose non-zero elements are equal to $\boldsymbol{\gamma}_m$ and correspond to DOAs (θ_k) in $\mathbf{B}(\tilde{\theta})$. Hence, the DOA estimation can be reduced to the detection of only the non-zero elements of $\mathbf{u}_m$. For all M frames, we can write

$$\mathbf{Y} \triangleq [\mathbf{y}_1, \mathbf{y}_2, \cdots, \mathbf{y}_M] = \mathbf{B}(\tilde{\theta})\mathbf{\Phi}^{\top}$$

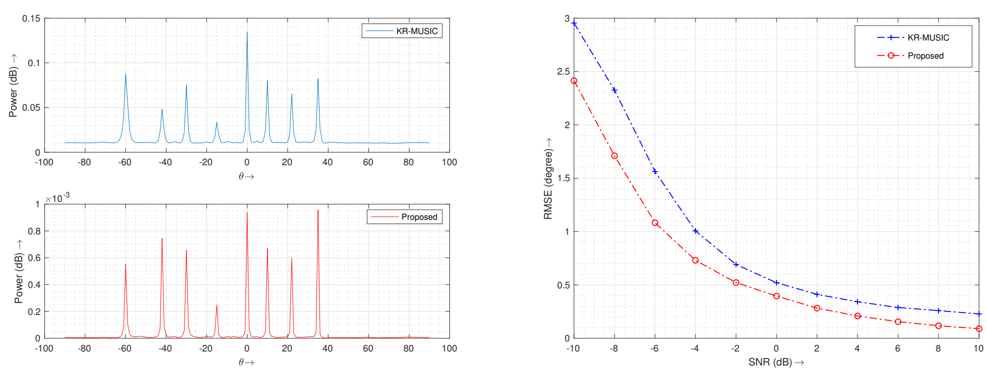
where $\mathbf{\Phi} = [\mathbf{u}_1, \mathbf{u}_2, \cdots, \mathbf{u}_M] \in \mathbb{C}^{Q\times M}$. Continue with the same denoising as before, and multiply $\mathbf{P}^\perp$ to $\mathbf{Y}$ to deal with the noise covariance matrix $\hat{\mathbf{W}}$. Now, the authors in [10] have showed that $\Delta\mathbf{y} \sim \mathrm{AsN}\Big(\mathbf{0}_{N^2}, \frac{\mathbf{R}\otimes\mathbf{R}}{N}\Big)$ where $\mathrm{AsN}(\cdot)$ represents the asymptotically normal distribution. This gives

$$\mathbf{F}^{-1/2}(\Delta\mathbf{y}) \sim \mathrm{AsN}\Big(\mathbf{0}_{N^2}, \mathbf{I}_{N^2}\Big) \tag{9}$$

where $\mathbf{F}^{-1/2}$ is the Hermitian square root of $\mathbf{F}^{-1}$, with $\mathbf{F} = \frac{\hat{\mathbf{R}}\otimes\hat{\mathbf{R}}}{N}$. The denoising process based on (9) for all M frames yields to be $\mathbf{F}^{-1/2}\mathbf{Y}\mathbf{P}^\perp$ and hence, the grid search can be performed using the compressive sensing based MUSIC algorithm as in [11], with the virtual manifold vectors as columns of $\mathbf{B}(\tilde{\theta})$ for DOA estimation.

4 Simulation results

In this section, we present the results for numerical experiments conducted to examine the performance of the proposed denoising procedure in the KR-MUSIC algorithm. The experiment environment was set up as follows. A two-level nested array of six sensors (ULA arrangement of three sensors in each level) element positions were set up. 8 sources were considered to be from directions $[60°, -42°, -30°, -15°, 0°, 10°, 22°, 35°]$. All the sources were assumed to be quasi-stationary, divided into 16 frames with 200 snapshots each. Next, the noise covariance matrix was set as $\mathbf{N} = \mathrm{diag}(10.2,\ 5.6,\ 8.5,\ 11.2,\ 7.8,\ 9.5,\ 11.2,\ 7.8,\ 9.5,\ 3.1)$ with input SNR being computed as $\frac{\mathbf{As}}{\|\mathbf{N}\|^2}$. Based on these experimental conditions, the simulation results on the DOA spectrum and the RMSE versus SNR are depicted in Fig. 1a and Fig. 1b, respectively. The root mean square error (RMSE) result was obtained from 1000 independent trials with $\mathrm{RMSE} = \left(\frac{1}{1000K}\sum_{k=1}^{K}\sum_{i=1}^{1000}(\hat{\theta}_k - \theta_k)^2\right)^{1/2}$. It can be observed from Fig. 1a that the DOA spectrum detects all the sources and agrees well with



(a) DOA spectrum comparison

(b) RMSE vs SNR curves

the original KR-MUSIC algorithm. From Fig. 1b, it can be observed that the RMSE curve shows an improvement as the detection error is comparatively lower in case of proposed method. This shows that the proposed denoising process is better at eliminating noise compared to the original method, in complete range of SNRs.

5 Conclusions

In this work, we have presented a new denoising method for DOA estimation method KR-MUSIC algorithm. This denoising procedure uses the covariance-based sparse representation framework and performs a covariance fitting to eliminate noise. The numerical simulations confirms with our proposed denoising which makes KR-MUSIC algorithm perform better compared to its original denoising procedure.

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