Claim: If the xor of matchsticks in all rows in zero, only then the first player loses.

Proof:

If only one row is present , the claim holds trivially.

For #rows > 1, we need to show that

i) if xor of all the numbers is zero then all the possible states after 1 move must have non-zero xor.

Since one number has been reduced, it means at least one bit has been changed in binary representation of that number and so the corresponding bit in the xor of numbers is 1. So in all the possible states the xor of numbers is non-zero.

ii) if xor of numbers is non-zero then there always exists a move such that the xor becomes zero.

In the xor or the numbers consider the most significant bit which equals one. There has to be at least one number which has 1 in the corresponding position (If all bits in that position were 0 then it would be 0 in xor too).

Let that number be T = a1 a2 … ak 1 b1  b2 b3 …. bn

Let the xor be X = 0 0 ….. 0 1 c1 c2 c3 …. cn

Change that number to T’ = a1 a2 … ak 0 b1^c1 b2^c2 b3^ c3 … bn^cn

Xor of rest of numbers = X^T

New xor = X^T^T’ = (T^(X^T’)) [ The property that ( bj ^ c j) ^ cj = bj has been used]

= ( T^T ) = 0