CS 215 : Data Analysis and Interpretation

(Instructor : Suyash P. Awate)

Quiz (Closed Book, Notes, all.)

Roll Number: ______________________________

Name: ______________________________

For all questions, if you feel that some information is missing, make justifiable assumptions, state them clearly, and answer the question.

Relevant Formulae

- Poisson: $P(k|\lambda) := \lambda^k \exp(-\lambda)/(k!)$
- Exponential: $P(x;\lambda) = \lambda \exp(-\lambda x); \forall x > 0$
- Univariate Gaussian:

$$P(x;\mu,\sigma^2) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-0.5\frac{(x-\mu)^2}{\sigma^2}\right)$$

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1. [15 points]

   State all modeling assumptions clearly. Don't use any property or theorem unless you prove it from basic definitions.

   Consider $\{X_i\}_{i=1}^{8}$ as i.i.d. Bernoulli random variables.

   • (3 points) Consider random variables $\{Y_j := X_{2j-1} + X_{2j}\}_{j=1}^{4}$. State the probability mass function (PMF) of the random variables $\{Y_j\}_{j=1}^{4}$. Are the random variables $\{Y_j\}_{j=1}^{4}$ independent ?

   Binomial(2,p). Independent.

   • (3 points) Consider random variables $\{Z_k := Y_{2k-1} + Y_{2k}\}_{k=1}^{2}$. State the probability mass function (PMF) of the random variables $\{Z_k\}_{k=1}^{2}$. Are the random variables $\{Z_k\}_{k=1}^{2}$ independent ?

   Binomial(4,p). Independent.

   • (3 points) Mathematically derive the mean and variance of random variable Z_1.

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   mean $= 4p$. variance $= 4pq$.

   • (6 points) Consider a random variable $W := 1 + 0.3 \times (Z_1 + X_8)$. Describe the probability mass function (PMF) of the random variable W. Mathematically derive its mean and variance.

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   same probabilities as in Binomial(5,p) for outcomes $\{1, 1.3, 1.6, 1.9, 2.2, 2.5\}$.

   mean $= 1 + 1.5p$. one way is to derive and then use properties of linearity of expectation.

   variance $= 0.45pq$. one way is to derive and then use properties of variance under addition and scaling.

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2. [10 points]

   State all modeling assumptions clearly. Don't use any property or theorem unless you prove it from basic definitions.

   Let random variable X model the number of coin tosses made to get the first "heads".

   • (2 points) State the probability mass function (PMF) of X.

   Geometric PMF. Please see the class slides.

   • (4 points) Does X satisfy the memoryless property ? If so, mathematically prove why so. If not, mathematically prove why not.

   Please see the class slides.

• (4 points) Let continuous random varible Y have a uniform probability density function (PDF) over (a, b). Does Y satisfy the memoryless property ? If so, mathematically prove why so. If not, mathematically prove why not.

It doesn't. Counter example: Let $a = 0, b = 1$. Then, $0.5 = P(X > 0.5 + 0.25|X > 0.5) \neq P(X > 0.25) = 0.75$

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3. [10 points]

   State all modeling assumptions clearly. Don't use any property or theorem unless you prove it from basic definitions.

   Let random variable X have a normal probability density function (PDF) with mean zero and variance σ^2.

   Define a random variable $Y := |X|$.

   • Mathematically derive the maximum likelihood (ML) estimate for the parameter associated with the PDF of Y.

Please see the class slides for the ML estimation for the half-normal random variable.

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4. [20 points]

   State all modeling assumptions clearly. Don't use any property or theorem unless you prove it from basic definitions.

   • (3 points) Is the sample mean an unbiased estimator ? Mathematically prove or disprove. Mathematically derive its bias.

Unbiased. Please see the class slides.

• (5 points) Mathematically derive the variance of the sample mean.

Please see the class slides.

• (5 points) Is the sample variance an unbiased estimator ? Mathematically prove or disprove. Mathematically derive its bias.

Please see the class slides.

• (7 points) Is the sample covariance an unbiased estimator ? Mathematically prove or disprove. Mathematically derive its bias.

Please see the class slides.
