Logic in CS Fall 2025

Problem Sheet 1

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1. There was a course assignment and there are three suspects for plagiarism: Dion, Thomas, and Yuvraj. You overheard the following statements:

   - Dion says: "If Thomas did it, then Yuvraj helped him."
   - Thomas says: "I didn't do it. Also I am not lying."
   - Yuvraj says: "I'm innocent, and Dion is telling the truth."

   Only one person has plagged. Assume only the guilty person is lying and the two innocent ones are telling the truth. The issue escalated to DADAC and now you are tasked with the duty to determine who committed the plagiarism. Encode the statements into propositional logic and deduce the answer.

   Note. If you help someone in plag, you are also guilty! Also, plagiarism has undesirable consequences!

2. You are participating in a logic game show. There are two doors and behind one of them is a very prestigious prize. The signs on the doors read:

   - Door 1: "The prize is behind Door 2."
   - Door 2: "Exactly one of these signs is true."

   You are told exactly one sign is true. Which door hides the prize? Deduce the answer by encoding this using propositional logic.

3. The Pigeon Hole Principle states that if there are $n+1$ pigeons sitting amongst n holes then there is atleast one hole with more than one pigeon sitting in it. For $i \in \{1, 2, .., n+1\}$ and $j \in \{1, 2, ..., n\}$, let the atomic proposition $P(i, j)$ indicate that the i-th pigeon is sitting in the j-th hole.

   Write out a propositional logic formula that states the Pigeon Hole Principle.

4. A perfect matching in an undirected graph $G = (V, E)$ is a subset of edges $M \subseteq E$ such that every vertex $v \in V$ is an endpoint of exactly one edge in M. Given a finite graph G, describe how to obtain a propositional logic formula φ_G such that φ_G is satisfiable iff G has a perfect matching. φ_G must be computable in time polynomial in $|V|$.

5. Let p and q be atomic propositions that take values from the set {`true`, `false`}. Consider the following two formulae:

$$\phi_1 = (p \to \neg\phi_2) \quad \text{and} \quad \phi_2 = (q \to \neg\phi_1)$$

Show using natural deduction that $\vdash \phi_1 \vee \phi_2$. You are allowed to use the Law of Excluded Middle (LEM) at most once in the proof.

6. Prove formally $\vdash [(p \to q) \to q] \to [(q \to p) \to p]$

7. Let $\mathcal{H}$ be a given set of premises. If $\mathcal{H} \vdash (A \to B)$ and $\mathcal{H} \vdash (C \vee A)$, then show that $\mathcal{H} \vdash (B \vee C)$ where A, B, C are wffs.

8. Let $\mathcal{H}$ be a given set of premises. If $\mathcal{H} \vdash (A \to C)$ and $\mathcal{H} \vdash (B \to C)$, then show that $\mathcal{H} \vdash ((A \vee B) \to C)$. Here, A, B and C are wffs.

9. Let $\mathcal{L}$ be a formulation of propositional logic in which the sole connectives are negation and disjunction. The rules of natural deduction corresponding to disjunction and negation (also includes double negation) are available. For any wffs A, B and C, let $\neg(A \vee B) \vee (B \vee C)$ be an axiom of $\mathcal{L}$. Show that any wff of $\mathcal{L}$ is a theorem of $\mathcal{L}$.

10. Let $\mathcal{P}$ denote propositional logic. Suppose we add to $\mathcal{P}$ the axiom schema $(A \to B)$ for wffs A, B of $\mathcal{P}$. Comment on the consistency of the resulting logical system obtained. A logic system $\mathcal{P}$ is inconsistent if it is capable of producing $\perp$ using the rules of natural deduction.