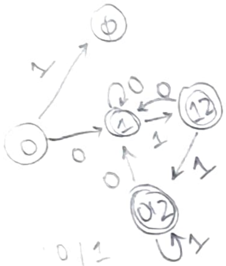
CS 228 Spring 2023 Part 2, Quiz 1

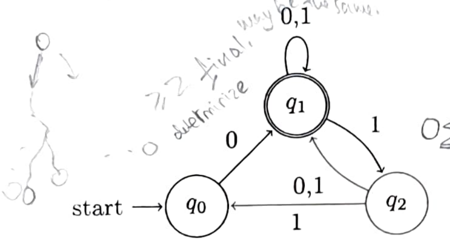
23-03-2023 Total Marks:

- *This is an open class notes exam.*
- *If you need to make any assumptions, state them clearly.*
- *Write your answers in the spaces provided in the question booklet.*
- *If needed, you may cite results/proofs covered in class without reproducing them.*
- *Penalty for Copying: FR grade*



	0	1
0	1	ϕ
1	1	12
12	1	012
012	1	012
ϕ	ϕ	ϕ

1. *[8 marks]*
Consider the NFA M in the figure. Define $L^{(k)}(M)$ as the set of all strings $w \in L(M)$ such that w has atleast k-accepting runs in the NFA M. Obviously $L^{(1)}(M) = L(M)$, and hence $L^{(1)}(M)$ is regular.



$0\Sigma^*$ has 1 run

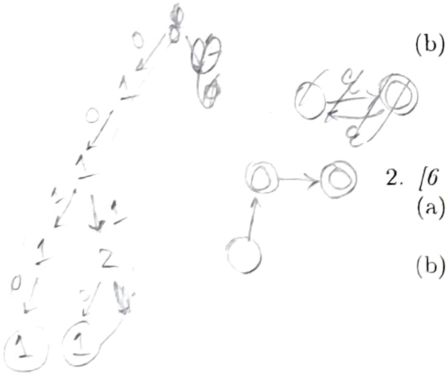
$0\Sigma^*1\Sigma^+$ $0\Sigma^*1B\Sigma^*$ has 2 runs

01 $\in L^{(2)}$

00‥1( )

(a) For the M given above, show that $L^{(2)}(M)$ is regular by constructing a NFA/DFA. *[3 marks]*
(b) Prove or Disprove: $L^{(k)}(N)$ is regular for all NFA N (N is an NFA with $\varepsilon-$ transitions). Justify your arguments. *[5 marks]*

$L^{(k)}(N) = \emptyset \cup (L_{s,q_1} \cap L)$



2. *[6 marks]*
(a) Write an MSO sentence ψ over the graph signature $\tau = \{E\}$ which is satisfied only by bipartite graphs. *[3 marks]*
(b) Write a FO sentence φ over the word signature $\tau = \{Q_a, Q_b, Q_c, S, <\}$ which captures all words having at least one c, we see only b's after the last c. *[3 marks]*

$\exists x\,(Q_c(x) \wedge (\forall y\,(y>x \rightarrow Q_b(x)))$

3. *[6 marks]* Consider the formula $\varphi_1 = \forall x.\forall y.(Q_a(x) \wedge x < y \Rightarrow Q_b(y))$. Draw the equivalent NFA/DFA accepting $L(\varphi_1)$, following all steps done in class.

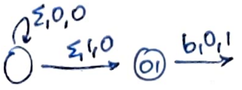
c $\quad \Sigma^* c\, b^*$

$\neg\exists x\, \neg\forall y\ \varphi$

$x<y \wedge \neg Q_b(y)$

$\neg\exists x\,\exists y\,\neg\varphi$

$\neg\exists x\,\exists y\,(Q_a(x) \wedge x<y \wedge \neg Q_b(y))$



$b^* \cup b^*ab^*$

bb‥b