Notes: 'n' represents the dimension of the corresponding problem. 'NI' represents the total number of iterations. 'NF' represents the number of the function evaluation. 'T' represents the running time of the algorithm. '$\|g^\star\|$' represents the gradient value for the solution. The values of 'NI', 'NF', 'T' and '$\|g^\star\|$' are set as 'NaN' if and only if the number of iterations is greater than 2000.

Table 1: Numerical results for Test 1

Problem/n	SHCGM-V1 NI/NF/T/$\|g^\star\|$	SHCGM-V2 NI/NF/T/$\|g^\star\|$	SHCGM-V3 NI/NF/T/$\|g^\star\|$	SHCGM-V4 NI/NF/T/$\|g^\star\|$
dixmaana/3000	28/92/0.088/8.48e-07	23/85/0.059/7.80e-07	18/81/0.054/3.31e-07	46/111/0.077/8.54e-07
dixmaana/30000	21/90/0.491/2.93e-07	17/86/0.471/9.68e-07	18/85/0.458/3.32e-07	21/88/0.481/9.92e-07
dixmaana/600000	23/98/10.742/6.02e-07	16/91/10.048/1.42e-07	18/89/9.703/8.43e-07	29/105/11.500/8.36e-07
dixmaanb/6000	19/85/0.135/9.57e-07	18/81/0.103/6.21e-07	20/80/0.102/6.54e-07	16/77/0.097/3.95e-07
dixmaanb/15000	22/89/0.266/6.84e-07	21/87/0.266/7.97e-07	17/83/0.259/3.80e-07	23/90/0.276/8.44e-07
dixmaanb/60000	20/84/0.863/7.69e-07	19/91/0.926/8.80e-07	19/88/0.894/2.81e-07	22/91/0.957/3.38e-07
dixmaanc/3000	21/86/0.080/1.75e-07	18/80/0.054/9.80e-07	17/80/0.051/3.06e-07	23/88/0.055/8.29e-07
dixmaanc/150000	16/89/2.473/7.63e-07	24/96/2.661/7.21e-07	24/93/2.578/3.22e-07	25/99/2.748/5.17e-07
dixmaand/300000	24/98/5.341/6.98e-07	24/97/5.266/9.10e-07	18/92/4.972/3.90e-07	17/91/4.946/3.47e-07
dixmaane/1500	334/430/0.171/9.78e-07	503/565/0.220/9.36e-07	314/410/0.140/7.47e-07	348/416/0.131/9.80e-07
dixmaane/3000	455/558/0.353/7.89e-07	543/607/0.389/9.99e-07	387/495/0.314/7.90e-07	478/542/0.341/9.53e-07
dixmaane/15000	1153/1431/4.383/9.52e-07	1264/1332/4.065/8.22e-07	945/1167/3.560/9.58e-07	1055/1123/3.432/9.79e-07
dixmaanf/1500	355/464/0.179/5.94e-07	485/549/0.176/9.90e-07	284/385/0.132/8.66e-07	324/391/0.124/9.47e-07
dixmaanf/3000	368/471/0.299/9.69e-07	451/516/0.333/8.75e-07	308/408/0.262/8.90e-07	394/459/0.296/9.47e-07
dixmaanf/30000	1094/1334/7.383/9.93e-07	NaN/NaN/NaN/NaN	724/906/5.016/8.85e-07	1631/1767/9.885/9.98e-07
dixmaang/300	212/291/0.044/5.57e-07	383/443/0.044/9.26e-07	137/202/0.018/8.91e-07	202/262/0.024/9.79e-07
dixmaang/1500	391/509/0.179/7.39e-07	698/771/0.244/9.99e-07	270/357/0.113/6.57e-07	609/698/0.222/9.77e-07
dixmaang/15000	909/1144/3.535/9.68e-07	NaN/NaN/NaN/NaN	1669/2109/6.436/7.24e-07	1440/1569/4.808/8.92e-07
dixmaanh/15000	947/1149/3.598/9.71e-07	1076/1168/3.588/9.05e-07	841/1063/3.284/8.34e-07	1001/1099/3.391/9.85e-07
dixmaanh/21000	828/1035/4.283/9.83e-07	1260/1361/5.592/9.55e-07	772/966/4.017/9.45e-07	872/983/4.043/9.68e-07
dixmaanh/30000	1393/1702/9.561/9.86e-07	1408/1492/8.367/9.97e-07	1031/1282/7.146/7.48e-07	1589/1701/9.621/9.74e-07
dixmaani/30	232/321/0.025/9.95e-07	274/339/0.010/8.21e-07	200/276/0.008/8.95e-07	334/447/0.013/9.52e-07
dixmaani/90	754/961/0.042/9.60e-07	666/730/0.035/8.76e-07	580/740/0.033/9.24e-07	548/624/0.025/8.46e-07
dixmaani/210	1771/2134/0.144/9.03e-07	1194/1264/0.079/9.54e-07	1419/1749/0.105/9.14e-07	1239/1324/0.080/8.94e-07
dixmaanj/6000	NaN/NaN/NaN/NaN	NaN/NaN/NaN/NaN	754/965/1.219/9.68e-07	NaN/NaN/NaN/NaN
dixmaank/1500	NaN/NaN/NaN/NaN	NaN/NaN/NaN/NaN	277/375/0.117/8.31e-07	NaN/NaN/NaN/NaN
dixmaanl/300	1160/1384/0.150/4.93e-07	1375/1480/0.137/9.65e-07	848/1068/0.091/7.34e-07	1302/1429/0.111/8.29e-07
dixmaanl/21000	1800/2133/8.690/9.90e-07	NaN/NaN/NaN/NaN	1120/1422/5.811/9.47e-07	1898/1978/8.156/9.98e-07
dixmaanl/30000	1907/2225/12.356/9.87e-07	NaN/NaN/NaN/NaN	989/1234/6.836/9.37e-07	1961/2033/11.415/9.88e-07
dixon3dq/50	484/646/0.011/9.24e-07	575/635/0.006/9.89e-07	345/450/0.005/8.31e-07	577/729/0.008/8.87e-07
dixon3dq/100	1166/1489/0.017/9.28e-07	1013/1071/0.012/9.47e-07	999/1269/0.014/9.66e-07	1072/1234/0.014/9.61e-07
dixon3dq/150	1583/1998/0.022/8.71e-07	1555/1612/0.017/7.88e-07	1474/1817/0.020/7.25e-07	1545/1716/0.020/9.54e-07
dqdrtic/10000	156/388/0.041/7.16e-07	104/291/0.027/7.98e-07	74/223/0.019/8.25e-07	151/383/0.041/6.43e-07
dqdrtic/50000	121/336/0.122/8.85e-07	102/306/0.099/5.57e-07	68/204/0.071/6.92e-07	178/430/0.154/7.76e-07
dqdrtic/150000	125/308/0.478/8.22e-07	100/238/0.378/7.99e-07	76/214/0.333/9.05e-07	134/382/0.683/9.95e-07
dqrtic/5000	58/166/0.175/7.92e-07	61/173/0.178/3.13e-07	47/154/0.156/9.37e-07	53/161/0.171/7.38e-07
edensch/500	76/269/0.033/6.59e-07	57/170/0.017/7.54e-07	41/108/0.011/8.46e-07	102/444/0.047/9.82e-07
edensch/20000	79/382/1.483/9.48e-07	128/689/2.674/6.39e-07	51/220/0.856/9.77e-07	95/294/1.142/9.37e-07
edensch/120000	95/450/9.647/7.04e-07	146/882/18.813/6.40e-07	67/214/4.633/9.54e-07	140/624/13.368/6.58e-07
eg2/100	664/5753/0.086/7.14e-07	456/3170/0.046/8.74e-07	289/970/0.014/4.42e-07	197/683/0.010/6.86e-07
eg2/200	1167/10779/0.227/1.00e-06	491/3545/0.069/5.73e-07	356/2160/0.043/7.27e-07	547/4051/0.081/8.08e-07
eg2/500	NaN/NaN/NaN/NaN	NaN/NaN/NaN/NaN	1592/14367/0.547/4.70e-07	1212/10283/0.390/8.38e-07
fletchcr/7000	267/2557/0.174/7.79e-07	294/2755/0.155/1.12e-07	168/1533/0.088/7.41e-07	178/1498/0.086/2.56e-07
freuroth/50	301/1200/0.021/9.78e-07	271/1379/0.016/6.23e-07	295/1518/0.017/6.01e-07	399/1790/0.020/9.00e-07
freuroth/500	1429/13579/0.199/5.45e-07	NaN/NaN/NaN/NaN	501/4097/0.060/4.33e-07	NaN/NaN/NaN/NaN
freuroth/5000	NaN/NaN/NaN/NaN	NaN/NaN/NaN/NaN	1286/13047/0.932/7.78e-07	NaN/NaN/NaN/NaN
genrose/10000	352/511/0.073/7.42e-07	419/586/0.085/9.27e-07	259/389/0.046/8.43e-07	348/444/0.054/9.88e-07
himmelbg/1000	2/13/0.005/7.54e-30	2/13/0.001/7.57e-30	2/13/0.001/7.55e-30	2/13/0.001/7.55e-30
himmelbg/70000	2/15/0.036/8.34e-43	2/15/0.030/8.40e-43	2/15/0.030/8.37e-43	2/15/0.030/8.37e-43
himmelbg/600000	3/16/0.259/1.39e-26	3/16/0.256/2.32e-26	3/16/0.257/1.31e-26	3/16/0.256/1.63e-26
liarwhd/10000	141/623/0.067/6.31e-07	132/532/0.052/4.98e-07	128/522/0.039/7.65e-07	135/526/0.038/4.83e-07
liarwhd/30000	211/861/0.186/4.29e-07	178/695/0.154/7.06e-07	107/465/0.101/8.59e-07	191/937/0.205/3.41e-07
liarwhd/120000	138/665/0.414/4.74e-07	258/810/0.520/3.17e-07	144/529/0.348/6.06e-07	165/758/0.477/6.96e-08
penalty1/100	19/109/0.008/9.20e-07	18/108/0.002/8.50e-07	19/109/0.002/9.20e-07	19/109/0.003/8.39e-07
penalty1/10000	20/92/5.340/2.06e-07	20/92/5.312/4.55e-07	20/92/5.202/4.52e-07	20/92/5.231/1.83e-07
quartc/5000	58/166/0.174/7.92e-07	61/173/0.174/3.13e-07	47/154/0.159/9.37e-07	53/161/0.171/7.38e-07
tridia/100	441/573/0.011/9.81e-07	376/443/0.006/9.84e-07	283/375/0.005/9.39e-07	423/508/0.007/9.86e-07
tridia/300	831/1052/0.015/9.66e-07	732/806/0.011/8.52e-07	497/626/0.008/7.81e-07	733/825/0.011/9.98e-07
tridia/1500	NaN/NaN/NaN/NaN	1800/1881/0.034/9.96e-07	1572/1924/0.035/8.29e-07	1557/1681/0.032/9.84e-07
woods/5000	588/1133/0.078/9.44e-07	197/432/0.022/8.78e-07	168/407/0.022/6.03e-07	510/738/0.043/8.69e-07
woods/300000	342/629/1.750/9.36e-07	326/515/1.511/5.21e-07	196/519/1.332/4.89e-07	1182/1776/5.939/8.99e-07
woods/1000000	430/820/7.222/8.37e-07	181/436/3.564/8.50e-07	175/471/3.757/9.24e-07	515/742/7.840/7.81e-07
bdexp/1000	2/11/0.009/2.83e-133	2/11/0.001/3.23e-133	2/11/0.001/2.99e-133	2/11/0.001/2.99e-133
bdexp/10000	2/17/0.014/2.62e-61	2/17/0.010/2.63e-61	2/17/0.010/2.62e-61	2/17/0.010/2.62e-61
bdexp/100000	2/12/0.081/4.56e-106	2/12/0.082/4.57e-106	2/12/0.083/4.57e-106	2/12/0.082/4.56e-106
exdenschnf/10000	54/131/0.037/1.19e-07	39/115/0.012/4.11e-07	32/105/0.010/6.59e-07	59/137/0.015/5.64e-07
exdenschnb/3000	32/92/0.008/7.27e-07	44/107/0.005/6.75e-07	24/81/0.003/8.95e-07	72/140/0.005/6.70e-07
exdenschnb/5000000	36/114/4.470/6.50e-07	28/103/3.789/9.97e-07	26/104/3.645/4.74e-07	53/132/5.562/9.60e-07
genquartic/50000	25/94/0.049/2.43e-07	20/87/0.033/4.80e-07	22/93/0.035/4.56e-07	18/84/0.033/5.77e-07
genquartic/100000	19/88/0.062/2.76e-07	27/98/0.081/5.79e-07	18/87/0.077/4.32e-07	26/97/0.068/9.52e-07
biggsb1/250	1388/1782/0.029/9.86e-07	1312/1383/0.018/9.61e-07	1174/1444/0.018/8.00e-07	1099/1229/0.016/8.62e-07
biggsb1/500	NaN/NaN/NaN/NaN	NaN/NaN/NaN/NaN	1810/2189/0.025/1.00e-06	NaN/NaN/NaN/NaN
fletcbv3/100	1589/3513/0.127/9.83e-07	NaN/NaN/NaN/NaN	1277/3059/0.097/9.88e-07	NaN/NaN/NaN/NaN
nonscomp/10000	240/421/0.052/9.41e-07	NaN/NaN/NaN/NaN	177/359/0.036/7.39e-07	NaN/NaN/NaN/NaN
nonscomp/70000	NaN/NaN/NaN/NaN	NaN/NaN/NaN/NaN	1586/2514/1.504/9.80e-07	NaN/NaN/NaN/NaN
nonscomp/100000	NaN/NaN/NaN/NaN	NaN/NaN/NaN/NaN	1895/2914/2.387/8.57e-07	NaN/NaN/NaN/NaN
power1/100	1501/1932/0.027/9.81e-07	1323/1418/0.016/9.96e-07	1322/1640/0.019/8.18e-07	1093/1289/0.015/9.99e-07
raydan1/300	204/282/0.010/6.93e-07	NaN/NaN/NaN/NaN	159/232/0.005/7.28e-07	390/484/0.010/9.91e-07
raydan2/2000	14/67/0.007/5.34e-07	15/74/0.006/2.78e-07	14/67/0.007/5.34e-07	14/71/0.007/7.00e-07
raydan2/70000	12/72/0.228/2.93e-07	85/416/1.303/4.59e-07	20/100/0.319/3.56e-08	35/144/0.457/2.46e-07
diagonal1/250	646/5287/0.118/9.73e-07	672/4998/0.110/9.84e-07	350/2530/0.056/8.27e-07	620/4883/0.109/8.23e-07
diagonal2/500	226/373/0.014/6.65e-07	174/270/0.010/9.97e-07	160/275/0.012/9.54e-07	231/344/0.016/9.99e-07
diagonal2/5000	642/1000/0.312/9.42e-07	498/735/0.227/8.79e-07	482/747/0.235/7.91e-07	618/894/0.281/9.45e-07
diagonal2/10000	831/1245/0.787/9.96e-07	708/1049/0.667/9.11e-07	732/1121/0.735/9.88e-07	819/1203/0.774/8.62e-07
singx/300	157/413/0.049/5.49e-07	293/720/0.087/6.35e-07	173/445/0.061/4.75e-07	133/369/0.047/6.62e-07
singx/1000	183/482/0.942/9.77e-07	417/1008/1.986/5.26e-07	412/1004/1.964/6.36e-07	139/415/0.796/3.25e-07
singx/3000	185/509/6.566/6.80e-07	279/743/9.753/8.81e-07	175/470/6.164/9.36e-07	175/468/6.080/3.57e-07
lin/30	17/79/0.022/7.20e-07	17/79/0.011/7.21e-07	17/79/0.007/6.97e-07	17/79/0.007/6.58e-07
lin/400	14/72/0.128/7.96e-07	14/75/0.129/8.08e-07	14/72/0.132/7.96e-07	14/72/0.129/7.97e-07
lin/700	18/81/4.463/2.85e-07	18/81/4.447/2.20e-07	18/81/4.412/3.00e-07	18/81/4.358/2.85e-07
osb2/11	534/936/0.038/8.12e-07	506/885/0.031/9.27e-07	435/726/0.026/9.93e-07	578/982/0.036/9.40e-07
pen1/50	57/308/0.011/7.03e-07	90/364/0.006/8.05e-08	60/355/0.006/8.74e-07	61/350/0.007/5.81e-07
pen1/100	66/323/0.009/5.79e-07	112/478/0.014/5.39e-07	112/479/0.013/5.84e-07	121/476/0.012/3.97e-07
pen1/600	182/1275/2.208/3.65e-07	68/306/0.522/7.31e-07	61/327/0.561/5.68e-07	105/644/1.101/8.76e-07
pen2/10	583/1738/0.031/8.16e-07	739/2258/0.032/7.59e-07	831/2348/0.035/9.70e-07	694/2180/0.030/5.49e-07
pen2/50	321/1120/0.025/9.08e-07	288/948/0.020/6.85e-07	265/962/0.020/9.52e-07	382/1347/0.029/9.31e-07
pen2/90	202/740/0.030/9.66e-07	295/1190/0.051/3.89e-07	177/524/0.020/9.77e-07	245/843/0.033/4.94e-07
rosex/300	87/282/0.032/7.96e-07	78/302/0.027/3.12e-07	107/285/0.026/9.52e-07	104/430/0.041/9.50e-07
rosex/1000	114/492/1.424/2.57e-07	102/354/1.048/1.54e-07	116/328/0.960/2.80e-07	75/283/0.824/9.92e-07
trid/400	119/182/0.084/9.15e-07	173/234/0.091/9.65e-07	103/163/0.065/8.06e-07	166/229/0.098/9.44e-07