

Notes: ‘n’ represents the dimension of the corresponding problem. ‘NI’ represents the total number of iterations. ‘NF’ represents the number of the function evaluation. ‘T’ represents the running time of the algorithm. ‘ $\|g^*\|$ ’ represents the gradient value for the solution. The values of ‘NI’, ‘NF’, ‘T’ and ‘ $\|g^*\|$ ’ are set as ‘NaN’ if and only if the number of iterations is greater than 2000.

Table 1: Numerical results for Test 1				
Problem/n	SHCGM-V1 NI/NF/T/ $\ g^*\ $	SHCGM-V2 NI/NF/T/ $\ g^*\ $	SHCGM-V3 NI/NF/T/ $\ g^*\ $	SHCGM-V4 NI/NF/T/ $\ g^*\ $
dixmaana/3000	28/92/0.133/8.48e-07	23/85/0.068/7.80e-07	18/81/0.054/3.31e-07	46/111/0.080/8.54e-07
dixmaana/30000	21/90/0.541/2.93e-07	17/86/0.511/9.68e-07	18/85/0.481/3.32e-07	21/88/0.505/9.92e-07
dixmaana/600000	23/98/11.565/6.02e-07	16/91/19.076/1.42e-07	18/89/14.898/8.43e-07	29/105/12.506/8.36e-07
dixmaanb/6000	19/85/0.239/9.57e-07	18/81/0.110/6.21e-07	20/80/0.112/6.54e-07	16/77/0.103/3.95e-07
dixmaanb/15000	22/89/0.285/6.84e-07	21/87/0.282/7.97e-07	17/83/0.260/3.80e-07	23/90/0.288/8.44e-07
dixmaanb/60000	20/84/0.904/7.69e-07	19/91/1.023/8.80e-07	19/88/0.940/2.81e-07	22/91/1.020/3.38e-07
dixmaanc/3000	21/86/0.088/1.75e-07	18/80/0.055/9.80e-07	17/80/0.053/3.06e-07	23/88/0.059/8.29e-07
dixmaanc/150000	16/89/5.150/7.63e-07	24/96/6.828/7.21e-07	24/93/6.349/3.22e-07	25/99/6.894/5.17e-07
dixmaand/300000	24/98/12.962/6.98e-07	24/97/11.071/9.10e-07	18/92/12.809/3.90e-07	17/91/11.130/3.47e-07
dixmaane/1500	334/430/0.313/9.78e-07	503/565/0.323/9.36e-07	314/410/0.244/7.47e-07	348/416/0.234/9.80e-07
dixmaane/3000	455/558/0.562/7.89e-07	543/607/0.632/9.99e-07	387/495/0.509/7.90e-07	478/542/0.564/9.53e-07
dixmaane/15000	1153/1431/13.475/9.52e-07	1264/1332/13.479/8.22e-07	945/1167/11.879/9.58e-07	1055/1123/11.646/9.79e-07
dixmaanf/1500	355/464/0.364/5.94e-07	485/549/0.306/9.90e-07	284/385/0.219/8.66e-07	324/391/0.216/9.47e-07
dixmaanf/3000	368/471/0.477/9.69e-07	451/516/0.537/8.75e-07	308/408/0.414/8.90e-07	394/459/0.454/9.47e-07
dixmaanf/30000	1094/1334/24.093/9.93e-07	NaN/NaN/NaN/NaN	724/906/15.621/8.85e-07	1631/1767/31.180/9.98e-07
dixmaang/300	212/291/0.142/5.57e-07	383/443/0.100/9.26e-07	137/202/0.034/8.91e-07	202/262/0.057/9.79e-07
dixmaang/1500	391/509/0.303/7.39e-07	698/771/0.435/9.99e-07	270/357/0.205/6.57e-07	609/698/0.396/9.77e-07
dixmaang/15000	909/1144/10.982/9.68e-07	NaN/NaN/NaN/NaN	1669/2109/21.550/7.24e-07	1440/1569/15.405/8.92e-07
dixmaanh/15000	947/1149/9.172/9.71e-07	1076/1168/11.696/9.05e-07	841/1063/10.782/8.34e-07	1001/1099/11.114/9.85e-07
dixmaanh/21000	828/1035/13.512/9.83e-07	1260/1361/14.589/9.55e-07	772/966/12.000/9.45e-07	872/983/11.631/9.68e-07
dixmaanh/30000	1393/1702/28.361/9.86e-07	1408/1492/25.767/9.97e-07	1031/1282/20.861/7.48e-07	1589/1701/27.838/9.74e-07
dixmaani/30	232/321/0.092/9.95e-07	274/339/0.027/8.21e-07	200/276/0.023/8.95e-07	334/447/0.033/9.52e-07
dixmaani/90	754/961/0.109/9.60e-07	666/730/0.067/8.76e-07	580/740/0.067/9.24e-07	548/624/0.060/8.46e-07
dixmaani/210	1771/2134/0.391/9.03e-07	1194/1264/0.180/9.54e-07	1419/1749/0.366/9.14e-07	1239/1324/0.287/8.94e-07
dixmaanj/6000	NaN/NaN/NaN/NaN	NaN/NaN/NaN/NaN	754/965/3.030/9.68e-07	NaN/NaN/NaN/NaN
dixmaank/1500	NaN/NaN/NaN/NaN	NaN/NaN/NaN/NaN	277/375/0.209/8.31e-07	NaN/NaN/NaN/NaN
dixma anl/300	1160/1384/0.296/4.93e-07	1375/1480/0.243/9.65e-07	848/1068/0.215/7.34e-07	1302/1429/0.317/8.29e-07
dixma anl/21000	1800/2133/22.705/9.90e-07	NaN/NaN/NaN/NaN	1120/1422/18.358/9.47e-07	1898/1978/23.803/9.98e-07
dixma anl/30000	1907/2225/33.620/9.87e-07	NaN/NaN/NaN/NaN	989/1234/19.202/9.37e-07	1961/2033/35.018/9.88e-07
dixon3dq/50	484/646/0.086/9.24e-07	575/635/0.035/9.89e-07	345/450/0.016/8.31e-07	577/729/0.044/8.87e-07
dixon3dq/100	1166/1489/0.083/9.28e-07	1013/1071/0.060/9.47e-07	999/1269/0.079/9.66e-07	1072/1234/0.069/9.61e-07
dixon3dq/150	1583/1998/0.114/8.71e-07	1555/1612/0.079/7.88e-07	1474/1817/0.101/7.25e-07	1545/1716/0.078/9.54e-07
dqdrtic/10000	156/388/0.100/7.16e-07	104/291/0.052/7.98e-07	74/223/0.033/8.25e-07	151/383/0.065/6.43e-07
dqdrtic/50000	121/336/0.217/8.85e-07	102/306/0.196/5.57e-07	68/204/0.134/6.92e-07	178/430/0.319/7.76e-07
dqdrtic/150000	125/308/1.107/8.22e-07	100/238/0.812/7.99e-07	76/214/0.693/9.05e-07	134/382/1.268/9.95e-07
dqrtic/5000	58/166/0.327/7.92e-07	61/173/0.282/3.13e-07	47/154/0.261/9.37e-07	53/161/0.243/7.38e-07
edensch/500	76/269/0.065/6.59e-07	57/170/0.053/7.54e-07	41/108/0.041/8.46e-07	102/444/0.117/9.82e-07
edensch/20000	79/382/2.956/9.48e-07	128/689/4.962/6.39e-07	51/220/2.058/9.77e-07	95/294/2.646/9.37e-07
edensch/120000	95/450/22.421/7.04e-07	146/882/43.798/6.40e-07	67/214/11.244/9.54e-07	140/624/30.051/6.58e-07
eg2/100	664/5753/0.324/7.14e-07	456/3170/0.160/8.74e-07	289/970/0.043/4.42e-07	197/683/0.027/6.86e-07
eg2/200	1167/10779/0.661/1.00e-06	491/3545/0.245/5.73e-07	356/2160/0.155/7.27e-07	547/4051/0.318/8.08e-07
eg2/500	NaN/NaN/NaN/NaN	NaN/NaN/NaN/NaN	1592/14367/1.593/4.70e-07	1212/10283/1.200/8.38e-07
fletchcr/7000	267/2557/0.359/7.79e-07	294/2755/0.296/1.12e-07	168/1533/0.165/7.41e-07	178/1498/0.173/2.56e-07
freuroth/50	301/1200/0.062/9.78e-07	271/1379/0.045/6.23e-07	295/1518/0.049/6.01e-07	399/1790/0.057/9.00e-07
freuroth/500	1429/13579/0.825/5.45e-07	NaN/NaN/NaN/NaN	501/4097/0.230/4.33e-07	NaN/NaN/NaN/NaN
freuroth/5000	NaN/NaN/NaN/NaN	NaN/NaN/NaN/NaN	1286/13047/2.183/7.78e-07	NaN/NaN/NaN/NaN
genrose/10000	352/511/0.152/7.42e-07	419/586/0.131/9.27e-07	259/389/0.074/8.43e-07	348/444/0.096/9.88e-07
himmelbg/1000	2/13/0.016/7.54e-30	2/13/0.001/7.57e-30	2/13/0.001/7.55e-30	2/13/0.002/7.55e-30
himmelbg/70000	2/15/0.060/8.34e-43	2/15/0.063/8.40e-43	2/15/0.081/8.37e-43	2/15/0.095/8.37e-43
himmelbg/600000	3/16/0.610/1.39e-26	3/16/0.721/2.32e-26	3/16/0.653/1.31e-26	3/16/0.717/1.63e-26
liarwhd/10000	141/623/0.185/6.31e-07	132/532/0.094/4.98e-07	128/522/0.060/7.65e-07	135/526/0.062/4.83e-07
liarwhd/30000	211/861/0.374/4.29e-07	178/695/0.310/7.06e-07	107/465/0.212/8.59e-07	191/937/0.409/3.41e-07
liarwhd/120000	138/665/0.701/4.74e-07	258/810/0.900/3.17e-07	144/529/0.604/6.06e-07	165/758/0.825/6.96e-08
penalty1/100	19/109/0.021/9.20e-07	18/108/0.006/8.50e-07	19/109/0.006/9.20e-07	19/109/0.006/8.39e-07
penalty1/10000	20/92/8.399/2.06e-07	20/92/8.037/4.55e-07	20/92/8.723/4.52e-07	20/92/8.537/1.83e-07
quartc/5000	58/166/0.297/7.92e-07	61/173/0.284/3.13e-07	47/154/0.262/9.37e-07	53/161/0.245/7.38e-07
tridia/100	441/573/0.044/9.81e-07	376/443/0.018/9.84e-07	283/375/0.014/9.39e-07	423/508/0.017/9.86e-07
tridia/300	831/1052/0.042/9.66e-07	732/806/0.030/8.52e-07	497/626/0.023/7.81e-07	733/825/0.056/9.98e-07
tridia/1500	NaN/NaN/NaN/NaN	1800/1881/0.149/9.96e-07	1572/1924/0.162/8.29e-07	1557/1681/0.106/9.84e-07
woods/5000	588/1133/0.173/9.44e-07	197/432/0.050/8.78e-07	168/407/0.043/6.03e-07	510/738/0.085/8.69e-07
woods/300000	342/629/4.002/9.36e-07	326/515/2.872/5.21e-07	196/519/2.930/4.89e-07	1182/1776/11.421/8.99e-07
woods/1000000	430/820/16.564/8.37e-07	181/436/9.036/8.50e-07	175/471/10.176/9.24e-07	515/742/15.463/7.81e-07
bdexp/1000	2/11/0.047/2.83e-133	2/11/0.003/3.23e-133	2/11/0.002/2.99e-133	2/11/0.003/2.99e-133
bdexp/10000	2/17/0.028/2.62e-61	2/17/0.015/2.63e-61	2/17/0.015/2.62e-61	2/17/0.015/2.62e-61
bdexp/100000	2/12/0.197/4.56e-106	2/12/0.173/4.57e-106	2/12/0.239/4.57e-106	2/12/0.245/4.56e-106
exdenschnf/10000	54/131/0.106/1.19e-07	39/115/0.020/4.11e-07	32/105/0.017/6.59e-07	59/137/0.026/5.64e-07
exdenschnb/3000	32/92/0.020/7.27e-07	44/107/0.008/6.75e-07	24/81/0.005/8.95e-07	72/140/0.014/6.70e-07
exdenschnb/5000000	36/114/8.895/6.50e-07	28/103/7.004/9.97e-07	26/104/7.950/4.74e-07	53/132/11.534/9.60e-07
genquartic/50000	25/94/0.235/2.43e-07	20/87/0.063/4.80e-07	22/93/0.067/4.56e-07	18/84/0.060/5.77e-07
genquartic/100000	19/88/0.112/2.76e-07	27/98/0.134/5.79e-07	18/87/0.108/4.32e-07	26/97/0.106/9.52e-07
biggsb1/250	1388/1782/0.075/9.86e-07	1312/1383/0.033/9.61e-07	1174/1444/0.052/8.00e-07	1099/1229/0.057/8.62e-07
biggsb1/500	NaN/NaN/NaN/NaN	NaN/NaN/NaN/NaN	1810/2189/0.100/1.00e-06	NaN/NaN/NaN/NaN
fletcbv3/100	1589/3513/0.293/9.83e-07	NaN/NaN/NaN/NaN	1277/3059/0.236/9.88e-07	NaN/NaN/NaN/NaN
nonscomp/10000	240/421/0.154/9.41e-07	NaN/NaN/NaN/NaN	177/359/0.059/7.39e-07	NaN/NaN/NaN/NaN
nonscomp/70000	NaN/NaN/NaN/NaN	NaN/NaN/NaN/NaN	1586/2514/3.002/9.80e-07	NaN/NaN/NaN/NaN
nonscomp/100000	NaN/NaN/NaN/NaN	NaN/NaN/NaN/NaN	1895/2914/4.231/8.57e-07	NaN/NaN/NaN/NaN
power1/100	1501/1932/0.082/9.81e-07	1323/1418/0.068/9.96e-07	1322/1640/0.063/8.18e-07	1093/1289/0.062/9.99e-07
raydan1/300	204/282/0.040/6.93e-07	NaN/NaN/NaN/NaN	159/232/0.015/7.28e-07	390/484/0.042/9.91e-07
raydan2/2000	14/67/0.023/5.34e-07	15/74/0.014/2.78e-07	14/67/0.014/5.34e-07	14/71/0.011/7.00e-07
raydan2/70000	12/72/0.449/2.93e-07	85/416/2.762/4.59e-07	20/100/0.875/3.56e-08	35/144/1.125/2.46e-07
diagonal1/250	646/5287/0.298/9.73e-07	672/4998/0.356/9.84e-07	350/2530/0.159/8.27e-07	620/4883/0.286/8.23e-07
diagonal2/500	226/373/0.054/6.65e-07	174/270/0.029/9.97e-07	160/275/0.019/9.54e-07	231/344/0.026/9.99e-07
diagonal2/5000	642/1000/0.471/9.42e-07	498/735/0.363/8.79e-07	482/747/0.336/7.91e-07	618/894/0.424/9.45e-07
diagonal2/10000	831/1245/1.002/9.96e-07	708/1049/0.839/9.11e-07	732/1121/0.886/9.88e-07	819/1203/0.965/8.62e-07
singx/300	157/413/0.150/5.49e-07	293/720/0.168/6.35e-07	173/445/0.110/4.75e-07	133/369/0.072/6.62e-07
singx/1000	183/482/1.580/9.77e-07	417/1008/3.545/5.26e-07	412/1004/5.093/6.36e-07	139/415/1.261/3.25e-07
singx/3000	185/509/13.391/6.80e-07	279/743/19.649/8.81e-07	175/470/12.950/9.36e-07	175/468/12.542/3.57e-07
lin/30	17/79/0.043/7.20e-07	17/79/0.015/7.21e-07	17/79/0.011/6.97e-07	17/79/0.017/6.58e-07
lin/400	14/72/0.235/7.96e-07	14/75/0.168/8.08e-07	14/72/0.171/7.96e-07	14/72/0.171/7.97e-07
lin/700	18/81/12.065/2.85e-07	18/81/9.428/2.20e-07	18/81/10.953/3.00e-07	18/81/9.986/2.85e-07
osb2/11	534/936/0.191/8.12e-07	506/885/0.078/9.27e-07	435/726/0.063/9.93e-07	578/982/0.068/9.40e-07
pen1/50	57/308/0.034/7.03e-07	90/364/0.018/8.05e-08	60/355/0.017/8.74e-07	61/350/0.020/5.81e-07
pen1/100	66/323/0.023/5.79e-07	112/478/0.031/5.39e-07	112/479/0.024/5.84e-07	121/476/0.024/3.97e-07
pen1/600	182/1275/4.694/3.65e-07	68/306/0.831/7.31e-07	61/327/0.877/5.68e-07	105/644/1.736/8.76e-07
pen2/10	583/1738/0.105/8.16e-07	739/2258/0.106/7.59e-07	831/2348/0.096/9.70e-07	694/2180/0.106/5.49e-07
pen2/50	321/1120/0.090/9.08e-07	288/948/0.089/6.85e-07	265/962/0.053/9.52e-07	382/1347/0.125/9.31e-07
pen2/90	202/740/0.094/9.66e-07	295/1190/0.114/3.89e-07	177/524/0.051/9.77e-07	245/843/0.072/4.94e-07
rosex/300	87/282/0.055/7.96e-07	78/302/0.046/3.12e-07	107/285/0.051/9.52e-07	104/430/0.070/9.50e-07
rosex/1000	114/492/2.613/2.57e-07	102/354/1.665/1.54e-07	116/328/1.540/2.80e-07	75/283/1.631/9.92e-07
trid/400	119/182/0.135/9.15e-07	173/234/0.147/9.65e-07	103/163/0.106/8.06e-07	166/229/0.142/9.44e-07