Notes: 'n' represents the dimension of the corresponding problem. 'NI' represents the total number of iterations. 'NF' represents the number of the function evaluation. 'T' represents the running time of the algorithm. '‖$g^\star$‖' represents the gradient value for the solution. The values of 'NI', 'NF', 'T' and '‖$g^\star$‖' are set as 'NaN' if and only if the number of iterations is greater than 3000.

Table 1: Numerical results for Test 3

Problem/n	SHCGM-V3 NI/NF/T/‖$g^\star$‖	CG-Descent NI/NF/T/‖$g^\star$‖	FhDY NI/NF/T/‖$g^\star$‖	FHTTCGM-N NI/NF/T/‖$g^\star$‖	MP+ NI/NF/T/‖$g^\star$‖
dixmaana/3000	18/81/0.077/3.31e-07	24/77/0.054/6.47e-07	19/83/0.057/7.02e-08	24/88/0.060/6.83e-08	25/80/0.056/2.43e-07
dixmaana/30000	18/85/0.468/3.32e-07	31/88/0.469/8.56e-07	19/86/0.462/3.86e-07	20/86/0.467/6.58e-07	28/87/0.467/3.87e-07
dixmaana/600000	18/89/9.628/8.43e-07	24/92/9.951/2.34e-07	17/90/9.744/6.14e-07	17/91/9.869/7.71e-07	22/86/9.346/9.81e-07
dixmaanb/6000	20/80/0.123/6.54e-07	27/83/0.103/5.87e-07	16/82/0.102/2.89e-07	18/84/0.104/1.28e-07	22/78/0.097/6.67e-07
dixmaanb/15000	17/83/0.252/3.80e-07	27/79/0.233/9.37e-07	22/90/0.263/4.55e-09	20/88/0.259/7.29e-08	23/73/0.213/3.05e-07
dixmaanb/60000	19/88/0.886/2.81e-07	27/90/0.908/3.75e-07	23/98/1.015/9.52e-07	19/88/0.883/6.63e-07	25/80/0.807/9.71e-07
dixmaanc/3000	17/80/0.081/3.06e-07	26/82/0.054/3.01e-07	19/84/0.052/5.05e-07	18/83/0.052/6.29e-07	19/75/0.048/5.81e-08
dixmaanc/150000	24/93/2.558/3.22e-07	21/85/2.323/4.33e-07	20/95/2.600/6.35e-07	21/94/2.566/7.80e-07	26/91/2.505/9.73e-07
dixmaand/300000	18/92/4.964/3.90e-07	27/93/4.986/9.15e-07	17/80/4.309/8.83e-07	22/96/5.218/4.28e-07	28/93/5.056/1.66e-07
dixmaane/1500	314/410/0.155/7.47e-07	565/661/0.219/8.38e-07	338/649/0.248/6.50e-07	282/528/0.186/8.03e-07	632/704/0.230/8.56e-07
dixmaane/3000	387/495/0.313/7.90e-07	793/911/0.589/9.52e-07	423/798/0.516/9.58e-07	473/878/0.566/7.49e-07	933/1025/0.672/9.74e-07
dixmaane/15000	945/1167/3.532/9.58e-07	1907/2192/6.482/8.86e-07	880/1701/5.045/9.31e-07	810/1451/4.339/8.93e-07	NaN/NaN/NaN/NaN
dixmaanf/1500	284/385/0.174/8.66e-07	433/486/0.204/9.08e-07	266/535/0.221/9.65e-07	257/478/0.163/7.36e-07	521/583/0.184/9.97e-07
dixmaanf/3000	308/408/0.252/8.90e-07	534/608/0.377/9.03e-07	276/542/0.337/4.42e-07	338/621/0.385/7.47e-07	706/830/0.514/9.74e-07
dixmaanf/30000	724/906/4.978/8.85e-07	1804/1950/10.698/9.93e-07	912/1713/9.272/9.43e-07	947/1681/9.161/9.07e-07	NaN/NaN/NaN/NaN
dixmaang/300	137/202/0.034/8.91e-07	196/248/0.024/7.09e-07	124/254/0.023/6.19e-07	118/222/0.020/9.89e-07	177/233/0.020/7.87e-07
dixmaang/1500	270/357/0.142/6.57e-07	370/430/0.148/9.74e-07	272/521/0.163/6.91e-07	293/553/0.174/6.93e-07	512/573/0.185/9.51e-07
dixmaang/15000	1669/2109/6.397/7.24e-07	1221/1314/3.886/8.31e-07	NaN/NaN/NaN/NaN	NaN/NaN/NaN/NaN	1865/1960/5.990/8.91e-07
dixmaanh/15000	841/1063/3.303/8.34e-07	1040/1167/3.505/9.25e-07	630/1218/3.616/7.63e-07	723/1336/3.999/8.39e-07	1654/1724/5.278/9.90e-07
dixmaanh/21000	772/966/3.906/9.45e-07	1436/1573/6.305/9.79e-07	912/1745/6.956/9.35e-07	754/1451/5.752/6.72e-07	NaN/NaN/NaN/NaN
dixmaanh/30000	1031/1282/7.106/7.48e-07	1432/1542/8.546/9.78e-07	1001/1912/10.434/9.86e-07	912/1692/9.228/8.92e-07	NaN/NaN/NaN/NaN
dixmaani/30	200/276/0.023/8.95e-07	421/508/0.012/7.93e-07	222/425/0.012/6.03e-07	202/363/0.010/9.82e-07	474/535/0.014/9.98e-07
dixmaani/90	580/740/0.035/9.24e-07	839/913/0.045/7.41e-07	513/943/0.045/9.64e-07	570/1002/0.045/7.07e-07	1574/1663/0.072/8.58e-07
dixmaani/210	1419/1749/0.112/9.14e-07	NaN/NaN/NaN/NaN	1416/2727/0.160/8.63e-07	1403/2551/0.150/7.56e-07	NaN/NaN/NaN/NaN
dixmaanj/6000	754/965/1.246/9.68e-07	1481/1670/2.128/7.11e-07	694/1307/1.608/7.85e-07	685/1223/1.540/7.44e-07	NaN/NaN/NaN/NaN
dixmaank/1500	277/375/0.172/8.31e-07	NaN/NaN/NaN/NaN	NaN/NaN/NaN/NaN	1085/1914/0.606/7.35e-07	NaN/NaN/NaN/NaN
dixmaanl/300	848/1068/0.128/7.34e-07	NaN/NaN/NaN/NaN	1004/1896/0.156/9.47e-07	1142/2035/0.163/7.21e-07	NaN/NaN/NaN/NaN
dixmaanl/21000	1120/1422/5.735/9.47e-07	1282/1467/5.875/8.08e-07	1440/2796/11.100/9.16e-07	NaN/NaN/NaN/NaN	1865/1949/7.893/7.55e-07
dixmaanl/30000	989/1234/6.803/9.37e-07	1410/1570/8.598/8.03e-07	859/1635/8.829/8.41e-07	766/1394/7.563/6.61e-07	NaN/NaN/NaN/NaN
dixon3dq/50	345/450/0.022/8.31e-07	776/862/0.008/7.99e-07	616/1166/0.012/8.84e-07	584/1050/0.012/7.86e-07	1782/2074/0.022/9.81e-07
dixon3dq/100	999/1269/0.014/9.66e-07	NaN/NaN/NaN/NaN	821/1576/0.016/9.16e-07	1015/1810/0.016/8.60e-07	NaN/NaN/NaN/NaN
dixon3dq/150	1474/1817/0.019/7.25e-07	NaN/NaN/NaN/NaN	1363/2582/0.026/6.46e-07	1426/2507/0.024/6.75e-07	NaN/NaN/NaN/NaN
dqdrtic/10000	74/223/0.028/8.25e-07	152/258/0.019/3.02e-07	72/223/0.019/2.69e-07	96/250/0.024/3.08e-07	317/474/0.046/9.80e-07
dqdrtic/50000	68/204/0.067/6.92e-07	150/265/0.090/8.12e-07	62/210/0.065/5.14e-07	96/276/0.091/6.43e-07	294/447/0.170/8.98e-07
dqdrtic/150000	76/214/0.293/9.05e-07	149/263/0.338/3.43e-07	72/225/0.263/2.59e-07	120/325/0.438/9.01e-07	267/423/0.668/7.80e-07
dqrtic/5000	47/154/0.174/9.37e-07	62/157/0.158/4.63e-07	41/156/0.156/4.19e-07	39/147/0.170/4.61e-07	189/1248/1.254/9.25e-07
edensch/500	41/108/0.024/8.46e-07	74/399/0.040/9.74e-07	68/474/0.047/3.98e-07	31/95/0.011/4.45e-07	31/84/0.009/9.85e-07
edensch/20000	51/220/0.855/9.77e-07	120/840/3.255/9.79e-07	51/269/1.035/5.85e-07	72/429/1.657/7.25e-07	45/213/0.831/5.72e-07
edensch/120000	67/214/4.614/9.54e-07	111/827/17.787/8.11e-07	125/965/20.582/2.12e-07	49/235/5.059/4.99e-07	58/308/6.606/9.15e-07
eg2/100	289/970/0.032/4.42e-07	1057/6524/0.088/9.94e-07	538/1585/0.022/4.46e-07	476/1401/0.019/7.55e-07	NaN/NaN/NaN/NaN
eg2/200	356/2160/0.044/7.27e-07	NaN/NaN/NaN/NaN	1122/5148/0.101/8.30e-07	NaN/NaN/NaN/NaN	NaN/NaN/NaN/NaN
eg2/500	1592/14367/0.551/4.70e-07	NaN/NaN/NaN/NaN	NaN/NaN/NaN/NaN	NaN/NaN/NaN/NaN	NaN/NaN/NaN/NaN
fletchcr/7000	168/1533/0.122/7.41e-07	179/1292/0.070/9.69e-07	391/3583/0.191/9.27e-07	380/3726/0.203/6.75e-07	255/2295/0.124/7.59e-07
freuroth/50	295/1518/0.036/6.01e-07	383/2205/0.026/8.64e-07	236/1585/0.018/9.83e-07	169/505/0.006/9.28e-07	NaN/NaN/NaN/NaN
freuroth/500	501/4097/0.065/4.33e-07	NaN/NaN/NaN/NaN	NaN/NaN/NaN/NaN	NaN/NaN/NaN/NaN	NaN/NaN/NaN/NaN
freuroth/5000	1286/13047/0.926/7.78e-07	NaN/NaN/NaN/NaN	NaN/NaN/NaN/NaN	NaN/NaN/NaN/NaN	NaN/NaN/NaN/NaN
genrose/10000	259/389/0.066/8.43e-07	302/418/0.052/8.30e-07	225/473/0.059/7.75e-07	218/433/0.045/9.95e-07	845/1051/0.115/8.56e-07
himmelbg/1000	2/13/0.017/7.55e-30	6/24/0.001/2.61e-08	2/13/0.001/7.57e-30	2/13/0.001/6.48e-30	9/17/0.001/1.49e-18
himmelbg/70000	2/15/0.033/8.37e-43	6/26/0.036/5.05e-200	2/15/0.029/8.41e-43	2/15/0.029/6.76e-43	11/42/0.062/0.00e+00
himmelbg/600000	3/16/0.266/1.31e-26	6/37/0.478/1.76e-30	3/16/0.254/8.54e-27	3/16/0.257/8.40e-27	5/17/0.287/4.37e-12
liarwhd/10000	128/522/0.053/7.65e-07	411/823/0.073/3.49e-07	176/628/0.033/5.48e-07	275/833/0.056/9.17e-07	1794/9193/0.551/9.26e-07
liarwhd/30000	107/465/0.088/8.59e-07	459/951/0.211/5.53e-08	201/732/0.136/2.54e-07	177/566/0.114/4.56e-07	NaN/NaN/NaN/NaN
liarwhd/120000	144/529/0.336/6.06e-07	391/836/0.525/5.75e-07	262/966/0.534/1.31e-07	278/824/0.490/7.49e-07	NaN/NaN/NaN/NaN
penalty1/100	19/109/0.018/9.20e-07	14/66/0.001/6.61e-07	18/107/0.002/2.82e-07	20/111/0.003/1.92e-07	14/66/0.001/6.62e-07
penalty1/10000	20/92/3.141/4.52e-07	65/281/9.639/4.56e-07	20/91/3.155/8.17e-07	20/91/3.169/1.61e-07	50/250/8.454/7.08e-07
quartc/5000	47/154/0.162/9.37e-07	62/157/0.154/4.63e-07	41/156/0.154/4.19e-07	39/147/0.154/4.61e-07	189/1248/1.235/9.25e-07
tridia/100	283/375/0.010/9.39e-07	809/940/0.010/9.75e-07	357/664/0.007/8.43e-07	384/689/0.008/7.67e-07	1195/1385/0.015/8.35e-07
tridia/300	497/626/0.008/7.81e-07	1702/1959/0.021/9.71e-07	564/1068/0.011/7.99e-07	543/1000/0.011/6.30e-07	NaN/NaN/NaN/NaN
tridia/1500	1572/1924/0.035/8.29e-07	NaN/NaN/NaN/NaN	1516/2941/0.048/7.80e-07	1575/2844/0.046/6.71e-07	NaN/NaN/NaN/NaN
woods/5000	168/407/0.039/6.03e-07	347/476/0.020/6.25e-07	263/640/0.025/7.03e-07	235/572/0.027/3.04e-07	711/1739/0.084/9.18e-07
woods/300000	196/519/1.441/4.89e-07	461/613/1.758/6.53e-07	376/910/2.352/8.20e-07	292/743/2.023/9.96e-07	932/2508/6.892/9.29e-07
woods/1000000	175/471/4.281/9.24e-07	341/493/4.411/9.18e-07	191/543/4.413/8.42e-07	327/778/7.002/4.65e-07	914/2429/21.798/8.70e-07
bdexp/1000	2/11/0.019/2.99e-133	6/25/0.002/1.76e-16	2/11/0.001/3.25e-133	2/11/0.001/2.09e-136	NaN/NaN/NaN/NaN
bdexp/10000	2/17/0.010/2.62e-61	7/21/0.011/7.59e-161	2/17/0.010/2.63e-61	2/17/0.010/1.36e-62	NaN/NaN/NaN/NaN
bdexp/100000	2/12/0.084/4.57e-106	6/22/0.108/5.22e-44	2/12/0.080/4.57e-106	2/12/0.082/4.51e-106	6/22/0.114/8.46e-38
exdenschnf/10000	32/105/0.033/6.59e-07	25/94/0.008/7.73e-07	25/99/0.011/4.94e-08	37/112/0.012/3.96e-07	27/95/0.009/8.85e-07
exdenschnb/3000	24/81/0.010/8.95e-07	28/82/0.004/6.56e-07	19/84/0.004/8.43e-09	26/89/0.004/4.60e-07	25/76/0.003/2.42e-07
exdenschnb/5000000	26/104/3.894/4.74e-07	34/106/3.501/3.53e-07	32/108/3.581/1.75e-07	31/108/3.680/2.72e-07	22/91/3.056/2.64e-07
genquartic/50000	22/93/0.060/4.56e-07	27/89/0.035/1.78e-08	24/116/0.043/3.82e-07	23/93/0.034/9.08e-07	23/85/0.031/5.06e-07
genquartic/100000	18/87/0.073/4.32e-07	32/95/0.074/9.65e-07	19/96/0.070/3.50e-07	24/92/0.058/5.18e-07	30/96/0.074/5.54e-07
biggsb1/250	1174/1444/0.034/8.00e-07	1912/2019/0.022/7.25e-07	1376/2566/0.026/9.56e-07	1322/2400/0.026/6.72e-07	NaN/NaN/NaN/NaN
biggsb1/500	1810/2189/0.028/1.00e-06	NaN/NaN/NaN/NaN	NaN/NaN/NaN/NaN	NaN/NaN/NaN/NaN	NaN/NaN/NaN/NaN
fletcbv3/100	1277/3059/0.121/9.88e-07	1037/1324/0.047/5.49e-07	1931/3749/0.119/7.50e-07	NaN/NaN/NaN/NaN	NaN/NaN/NaN/NaN
nonscomp/10000	177/359/0.055/7.39e-07	56/122/0.010/8.83e-07	NaN/NaN/NaN/NaN	192/410/0.035/3.32e-07	50/116/0.009/9.56e-07
nonscomp/70000	1586/2514/1.484/9.80e-07	NaN/NaN/NaN/NaN	NaN/NaN/NaN/NaN	NaN/NaN/NaN/NaN	NaN/NaN/NaN/NaN
nonscomp/100000	1895/2914/2.344/8.57e-07	NaN/NaN/NaN/NaN	NaN/NaN/NaN/NaN	NaN/NaN/NaN/NaN	NaN/NaN/NaN/NaN
power1/100	1322/1640/0.033/8.18e-07	NaN/NaN/NaN/NaN	1428/2671/0.030/8.23e-07	1185/2113/0.022/9.42e-07	NaN/NaN/NaN/NaN
raydan1/300	159/232/0.010/7.28e-07	299/370/0.006/4.73e-07	149/273/0.005/6.21e-07	147/276/0.005/9.58e-07	172/224/0.005/6.61e-07
raydan2/2000	14/67/0.008/5.34e-07	14/65/0.003/9.35e-07	14/71/0.005/7.00e-07	14/71/0.007/6.99e-07	14/64/0.006/2.58e-07
raydan2/70000	20/100/0.311/3.56e-08	21/75/0.236/1.28e-07	22/130/0.405/6.99e-07	17/105/0.332/6.01e-07	47/103/0.327/3.02e-07
diagonal1/250	350/2530/0.074/8.27e-07	789/6148/0.134/9.14e-07	449/3517/0.076/9.72e-07	275/1713/0.037/9.16e-07	896/8103/0.176/9.42e-07
diagonal2/500	160/275/0.013/9.54e-07	310/372/0.014/9.53e-07	159/311/0.012/9.06e-07	176/346/0.015/5.28e-07	653/2726/0.099/8.61e-07
diagonal2/5000	482/747/0.233/7.91e-07	1106/1235/0.374/8.59e-07	535/1012/0.302/9.26e-07	518/948/0.285/8.68e-07	NaN/NaN/NaN/NaN
diagonal2/10000	732/1121/0.707/9.88e-07	1750/2011/1.254/9.94e-07	752/1427/0.870/9.87e-07	718/1341/0.846/9.49e-07	NaN/NaN/NaN/NaN
singx/300	173/445/0.055/4.75e-07	725/1059/0.121/8.92e-07	286/671/0.062/5.32e-07	329/764/0.079/7.74e-07	NaN/NaN/NaN/NaN
singx/1000	412/1004/1.922/6.36e-07	895/1307/2.539/7.18e-07	633/1489/2.894/4.88e-07	565/1257/2.444/6.29e-07	NaN/NaN/NaN/NaN
singx/3000	175/470/5.811/9.36e-07	618/912/11.261/9.37e-07	839/2021/25.332/9.04e-07	478/1044/13.178/4.02e-07	NaN/NaN/NaN/NaN
lin/30	17/79/0.033/6.97e-07	19/67/0.012/2.67e-07	17/79/0.006/6.96e-07	17/79/0.006/7.05e-07	19/67/0.005/2.57e-07
lin/400	14/72/0.131/7.96e-07	16/66/0.119/8.77e-07	14/72/0.132/7.97e-07	14/72/0.120/7.97e-07	16/66/0.110/8.77e-07
lin/700	18/81/4.367/3.00e-07	29/90/5.053/1.85e-07	18/81/4.523/2.20e-07	18/81/4.499/3.00e-07	29/90/4.999/6.75e-08
osb2/11	435/726/0.046/9.93e-07	1626/1924/0.065/9.32e-07	581/1123/0.039/7.27e-07	837/1544/0.034/7.61e-07	NaN/NaN/NaN/NaN
pen1/50	60/355/0.013/8.74e-07	145/493/0.007/9.90e-07	111/440/0.006/9.06e-07	174/640/0.011/3.00e-07	1392/7001/0.096/9.45e-07
pen1/100	112/479/0.009/5.84e-07	393/1605/0.027/2.98e-07	195/755/0.011/7.53e-07	168/600/0.009/5.44e-07	656/3112/0.046/9.93e-07
pen1/600	61/327/0.546/5.68e-07	207/756/1.287/5.40e-07	98/574/0.988/3.24e-07	258/1644/2.831/4.00e-07	290/1409/2.391/5.05e-07
pen2/10	831/2348/0.050/9.70e-07	NaN/NaN/NaN/NaN	1807/5105/0.066/8.44e-07	1837/4564/0.052/9.09e-07	NaN/NaN/NaN/NaN
pen2/50	265/962/0.018/9.52e-07	1156/2033/0.037/7.75e-07	424/1318/0.024/7.50e-07	481/1316/0.024/5.79e-07	1142/5971/0.112/9.01e-07
pen2/90	177/524/0.020/9.77e-07	603/1109/0.039/9.05e-07	212/587/0.022/9.21e-07	273/761/0.039/8.70e-07	855/2863/0.134/9.70e-07
rosex/300	107/285/0.040/9.52e-07	184/365/0.024/5.60e-07	95/323/0.028/9.07e-07	144/432/0.033/7.34e-07	574/2486/0.167/9.64e-07
rosex/1000	116/328/0.985/2.80e-07	210/381/1.080/7.20e-07	114/359/1.019/5.02e-07	93/320/0.899/9.20e-07	680/3314/9.655/9.42e-07
trid/400	103/163/0.084/8.06e-07	41/93/0.037/9.71e-07	99/191/0.077/8.01e-07	98/191/0.085/8.34e-07	37/91/0.046/6.92e-07

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