Notes: 'n' represents the dimension of the corresponding problem. 'NI' represents the total number of iterations. 'NF' represents the number of the function evaluation. 'T' represents the running time of the algorithm. '$\|g^\star\|$' represents the gradient value for the solution. The values of 'NI', 'NF', 'T' and '$\|g^\star\|$' are set as 'NaN' if and only if the number of iterations is greater than 2000.

Table 1: Numerical results for Test 1

Problem/n	SHCGM-V1 NI/NF/T/$\|g^\star\|$	SHCGM-V2 NI/NF/T/$\|g^\star\|$	SHCGM-V3 NI/NF/T/$\|g^\star\|$	SHCGM-V4 NI/NF/T/$\|g^\star\|$
dixmaana/3000	19/77/0.139/3.45e-07	28/92/0.095/8.48e-07	18/81/0.067/3.31e-07	46/111/0.090/8.54e-07
dixmaana/30000	21/96/0.576/7.58e-07	21/90/0.487/2.93e-07	18/85/0.461/3.32e-07	21/88/0.489/9.92e-07
dixmaana/600000	15/88/17.799/6.15e-07	23/98/20.264/6.02e-07	18/89/22.968/8.43e-07	29/105/26.199/8.36e-07
dixmaanb/6000	14/80/0.303/3.35e-07	19/85/0.354/9.57e-07	20/80/0.312/6.54e-07	16/77/0.328/3.95e-07
dixmaanb/15000	19/86/0.794/2.83e-07	22/89/0.873/6.84e-07	17/83/0.834/3.80e-07	23/90/0.901/8.44e-07
dixmaanb/60000	23/92/2.436/8.47e-07	20/84/1.978/7.69e-07	19/88/2.462/2.81e-07	22/91/2.343/3.38e-07
dixmaanc/3000	19/82/0.177/2.54e-07	21/86/0.100/1.75e-07	17/80/0.100/3.06e-07	23/88/0.138/8.29e-07
dixmaanc/150000	20/93/6.929/4.67e-07	16/89/6.686/7.63e-07	24/93/3.018/3.22e-07	25/99/3.480/5.17e-07
dixmaand/300000	19/90/11.993/1.45e-07	24/98/13.297/6.98e-07	18/92/12.674/3.90e-07	17/91/12.172/3.47e-07
dixmaane/1500	377/674/0.460/7.50e-07	334/430/0.244/9.78e-07	314/410/0.231/7.47e-07	348/416/0.237/9.80e-07
dixmaane/3000	417/733/0.735/9.90e-07	455/558/0.555/7.89e-07	387/495/0.484/7.90e-07	478/542/0.547/9.53e-07
dixmaane/15000	1499/2725/21.931/3.39e-07	1153/1431/13.066/9.52e-07	945/1167/9.729/9.58e-07	1055/1123/9.204/9.79e-07
dixmaanf/1500	340/613/0.473/6.12e-07	355/464/0.261/5.94e-07	284/385/0.222/8.66e-07	324/391/0.206/9.47e-07
dixmaanf/3000	371/676/0.650/8.92e-07	368/471/0.457/9.69e-07	308/408/0.398/8.90e-07	394/459/0.551/9.47e-07
dixmaanf/30000	1822/3348/55.246/9.33e-07	1094/1334/23.258/9.93e-07	724/906/13.874/8.85e-07	1631/1767/27.439/9.98e-07
dixmaang/300	121/227/0.123/7.06e-07	212/291/0.059/5.57e-07	137/202/0.049/8.91e-07	202/262/0.069/9.79e-07
dixmaang/1500	275/510/0.328/9.76e-07	391/509/0.273/7.39e-07	270/357/0.193/6.57e-07	609/698/0.374/9.77e-07
dixmaang/15000	NaN/NaN/NaN/NaN	909/1144/7.385/9.68e-07	1669/2109/16.524/7.24e-07	1440/1569/11.508/8.92e-07
dixmaanh/15000	1243/2285/20.061/6.81e-07	947/1149/7.386/9.71e-07	841/1063/9.073/8.34e-07	1001/1099/9.492/9.85e-07
dixmaanh/21000	804/1513/18.101/7.97e-07	828/1035/12.705/9.83e-07	772/966/12.238/9.45e-07	872/983/11.796/9.68e-07
dixmaanh/30000	983/1821/26.803/9.75e-07	1393/1702/27.028/9.86e-07	1031/1282/21.896/7.48e-07	1589/1701/25.199/9.74e-07
dixmaani/30	293/540/0.093/5.40e-07	232/321/0.024/9.95e-07	200/276/0.021/8.95e-07	334/447/0.034/9.52e-07
dixmaani/90	864/1557/0.174/9.27e-07	754/961/0.094/9.60e-07	580/740/0.074/9.24e-07	548/624/0.071/8.46e-07
dixmaani/210	1889/3463/0.734/9.69e-07	1771/2134/0.411/9.03e-07	1419/1749/0.316/9.14e-07	1239/1324/0.286/8.94e-07
dixmaanj/6000	1172/2165/8.012/9.79e-07	NaN/NaN/NaN/NaN	754/965/3.305/9.68e-07	NaN/NaN/NaN/NaN
dixmaank/1500	169/324/0.286/7.23e-07	NaN/NaN/NaN/NaN	277/375/0.199/8.31e-07	NaN/NaN/NaN/NaN
dixmaanl/300	NaN/NaN/NaN/NaN	1160/1384/0.295/4.93e-07	848/1068/0.251/7.34e-07	1302/1429/0.420/8.29e-07
dixmaanl/21000	1492/2726/32.756/9.73e-07	1800/2133/26.415/9.90e-07	1120/1422/17.691/9.47e-07	1898/1978/14.899/9.98e-07
dixmaanl/30000	947/1754/18.440/9.16e-07	1907/2225/36.763/9.87e-07	989/1234/19.245/9.37e-07	1961/2033/32.565/9.88e-07
dixon3dq/50	652/1184/0.081/3.60e-07	484/646/0.023/9.24e-07	345/450/0.016/8.31e-07	577/729/0.030/8.87e-07
dixon3dq/100	1447/2697/0.109/8.51e-07	1166/1489/0.062/9.28e-07	999/1269/0.048/9.66e-07	1072/1234/0.051/9.61e-07
dixon3dq/150	NaN/NaN/NaN/NaN	1583/1998/0.104/8.71e-07	1474/1817/0.077/7.25e-07	1545/1716/0.067/9.54e-07
dqdrtic/10000	109/267/0.086/7.80e-07	156/388/0.092/7.16e-07	74/223/0.048/8.25e-07	151/383/0.078/6.43e-07
dqdrtic/50000	100/254/0.186/4.04e-07	121/336/0.235/8.85e-07	68/204/0.145/6.92e-07	178/430/0.331/7.76e-07
dqdrtic/150000	119/295/0.844/8.97e-07	125/308/0.816/8.22e-07	76/214/0.515/9.05e-07	134/382/1.173/9.95e-07
dqrtic/5000	45/152/0.241/3.86e-07	58/166/0.255/7.92e-07	47/154/0.236/9.37e-07	53/161/0.244/7.38e-07
edensch/500	31/103/0.032/9.94e-07	76/269/0.047/6.59e-07	41/108/0.025/8.46e-07	102/444/0.111/9.82e-07
edensch/20000	33/142/1.184/8.30e-07	79/382/3.480/9.48e-07	51/220/2.583/9.77e-07	95/294/2.430/9.37e-07
edensch/120000	115/833/45.673/9.11e-07	95/450/24.666/7.04e-07	67/214/12.662/9.54e-07	140/624/36.979/6.58e-07
eg2/100	NaN/NaN/NaN/NaN	664/5753/0.308/7.14e-07	289/970/0.059/4.42e-07	197/683/0.043/6.86e-07
eg2/200	NaN/NaN/NaN/NaN	1167/10779/0.574/1.00e-06	356/2160/0.131/7.27e-07	547/4051/0.247/8.08e-07
eg2/500	NaN/NaN/NaN/NaN	NaN/NaN/NaN/NaN	1592/14367/1.516/4.70e-07	1212/10283/1.002/8.38e-07
fletchcr/7000	82/439/0.116/4.75e-07	267/2557/0.336/7.79e-07	168/1533/0.189/7.41e-07	178/1498/0.205/2.56e-07
freuroth/50	286/951/0.056/6.02e-07	301/1200/0.040/9.78e-07	295/1518/0.051/6.01e-07	399/1790/0.064/9.00e-07
freuroth/500	NaN/NaN/NaN/NaN	1429/13579/0.861/5.45e-07	501/4097/0.217/4.33e-07	NaN/NaN/NaN/NaN
freuroth/5000	NaN/NaN/NaN/NaN	NaN/NaN/NaN/NaN	1286/13045/2.059/7.78e-07	NaN/NaN/NaN/NaN
genrose/10000	303/570/0.207/3.71e-07	352/511/0.137/7.42e-07	259/389/0.106/8.43e-07	348/444/0.110/9.88e-07
himmelbg/1000	2/13/0.022/7.57e-30	2/13/0.001/7.54e-30	2/13/0.001/7.55e-30	2/13/0.002/7.55e-30
himmelbg/70000	2/15/0.058/8.40e-43	2/15/0.080/8.34e-43	2/15/0.104/8.37e-43	2/15/0.100/8.37e-43
himmelbg/600000	3/16/0.681/2.32e-26	3/16/0.782/1.39e-26	3/16/0.857/1.31e-26	3/16/0.726/1.63e-26
liarwhd/10000	1273/2784/0.671/6.58e-07	141/623/0.093/6.31e-07	128/522/0.073/7.65e-07	135/526/0.076/4.83e-07
liarwhd/30000	1964/4293/2.435/7.13e-07	211/861/0.391/4.29e-07	107/465/0.213/8.59e-07	191/937/0.409/3.41e-07
liarwhd/120000	NaN/NaN/NaN/NaN	138/665/0.714/4.74e-07	144/529/0.606/6.06e-07	165/758/0.826/6.96e-08
penalty1/100	18/108/0.022/8.50e-07	19/109/0.006/9.20e-07	19/109/0.005/9.20e-07	19/109/0.007/8.39e-07
penalty1/10000	20/92/11.472/4.51e-07	20/92/12.098/2.06e-07	20/92/11.128/4.52e-07	20/92/11.806/1.83e-07
quartc/5000	45/152/0.330/3.86e-07	58/166/0.268/7.92e-07	47/154/0.240/9.37e-07	53/161/0.253/7.38e-07
tridia/100	456/820/0.044/7.77e-07	441/573/0.025/9.81e-07	283/375/0.013/9.39e-07	423/508/0.018/9.86e-07
tridia/300	1130/2049/0.077/9.75e-07	831/1052/0.071/9.66e-07	497/626/0.051/7.81e-07	733/825/0.067/9.98e-07
tridia/1500	NaN/NaN/NaN/NaN	NaN/NaN/NaN/NaN	1572/1924/0.171/8.29e-07	1557/1681/0.171/9.84e-07
woods/5000	309/679/0.127/9.91e-07	588/1133/0.136/9.44e-07	168/407/0.041/6.03e-07	510/738/0.088/8.69e-07
woods/300000	495/1006/8.684/9.25e-07	342/629/3.803/9.36e-07	196/519/3.499/4.89e-07	1182/1776/14.779/8.99e-07
woods/1000000	532/1087/19.314/8.96e-07	430/820/14.904/8.37e-07	175/471/7.877/9.24e-07	515/742/14.198/7.81e-07
bdexp/1000	2/11/0.020/3.23e-133	2/11/0.002/2.83e-133	2/11/0.002/2.99e-133	2/11/0.003/2.99e-133
bdexp/10000	2/17/0.027/2.63e-61	2/17/0.017/2.62e-61	2/17/0.016/2.62e-61	2/17/0.016/2.62e-61
bdexp/100000	2/12/0.156/4.57e-106	2/12/0.140/4.56e-106	2/12/0.158/4.57e-106	2/12/0.135/4.56e-106
exdenschnf/10000	29/104/0.077/6.18e-07	54/131/0.025/1.19e-07	32/105/0.019/6.59e-07	59/137/0.030/5.64e-07
exdenschnb/3000	21/82/0.019/5.23e-07	32/92/0.009/7.27e-07	24/81/0.005/8.95e-07	72/140/0.010/6.70e-07
exdenschnb/5000000	28/105/6.727/4.29e-07	36/114/7.740/6.50e-07	26/104/6.820/4.74e-07	53/132/10.442/9.60e-07
genquartic/50000	22/85/0.111/5.72e-07	25/94/0.075/2.43e-07	22/93/0.073/4.56e-07	18/84/0.065/5.77e-07
genquartic/100000	23/92/0.123/5.76e-07	19/88/0.116/2.76e-07	18/87/0.112/4.32e-07	26/97/0.118/9.52e-07
biggsb1/250	NaN/NaN/NaN/NaN	1388/1782/0.073/9.86e-07	1174/1444/0.101/8.00e-07	1099/1229/0.075/8.62e-07
biggsb1/500	NaN/NaN/NaN/NaN	NaN/NaN/NaN/NaN	1810/2189/0.096/1.00e-06	NaN/NaN/NaN/NaN
fletcbv3/100	NaN/NaN/NaN/NaN	1589/3513/0.288/9.83e-07	1277/3059/0.229/9.88e-07	NaN/NaN/NaN/NaN
nonscomp/10000	NaN/NaN/NaN/NaN	240/421/0.074/9.41e-07	177/359/0.112/7.39e-07	NaN/NaN/NaN/NaN
nonscomp/70000	NaN/NaN/NaN/NaN	NaN/NaN/NaN/NaN	1586/2514/2.936/9.80e-07	NaN/NaN/NaN/NaN
nonscomp/100000	NaN/NaN/NaN/NaN	NaN/NaN/NaN/NaN	1895/2914/4.332/8.57e-07	NaN/NaN/NaN/NaN
power1/100	NaN/NaN/NaN/NaN	1501/1932/0.087/9.81e-07	1322/1640/0.083/8.18e-07	1093/1289/0.047/9.99e-07
raydan1/300	157/295/0.023/3.94e-07	204/282/0.014/6.93e-07	159/232/0.012/7.28e-07	390/484/0.032/9.91e-07
raydan2/2000	15/74/0.017/2.78e-07	14/67/0.008/5.34e-07	14/67/0.009/5.34e-07	14/71/0.010/7.00e-07
raydan2/70000	18/95/0.623/6.79e-07	12/72/0.407/2.93e-07	20/100/0.616/3.56e-08	35/144/0.850/2.46e-07
diagonal1/250	1305/10474/0.641/7.75e-07	646/5287/0.364/9.73e-07	350/2530/0.192/8.27e-07	620/4883/0.332/8.23e-07
diagonal2/500	165/301/0.031/4.41e-07	226/373/0.026/6.65e-07	160/275/0.018/9.54e-07	231/344/0.024/9.99e-07
diagonal2/5000	829/1560/0.755/7.75e-07	642/1000/0.502/9.42e-07	482/747/0.344/7.91e-07	618/894/0.418/9.45e-07
diagonal2/10000	1535/2816/2.291/7.17e-07	831/1245/1.033/9.96e-07	732/1121/0.963/9.88e-07	819/1203/1.010/8.62e-07
singx/300	NaN/NaN/NaN/NaN	157/413/0.068/5.49e-07	173/445/0.070/4.75e-07	133/369/0.068/6.62e-07
singx/1000	NaN/NaN/NaN/NaN	183/482/1.474/9.77e-07	412/1004/3.063/6.36e-07	139/415/1.262/3.25e-07
singx/3000	NaN/NaN/NaN/NaN	185/509/13.197/6.80e-07	175/470/12.687/9.36e-07	175/468/13.067/3.57e-07
lin/30	17/79/0.049/6.58e-07	17/79/0.016/7.20e-07	17/79/0.010/6.97e-07	17/79/0.011/6.58e-07
lin/400	14/74/0.190/1.55e-07	14/72/0.180/7.96e-07	14/72/0.182/7.96e-07	14/72/0.180/7.97e-07
lin/700	18/81/13.674/2.85e-07	18/81/16.455/2.85e-07	18/81/15.575/3.00e-07	18/81/17.168/2.85e-07
osb2/11	1332/2463/0.372/8.35e-07	534/936/0.134/8.12e-07	435/726/0.111/9.93e-07	578/982/0.160/9.40e-07
pen1/50	159/477/0.074/1.74e-07	57/308/0.036/7.03e-07	60/355/0.039/8.74e-07	61/350/0.019/5.81e-07
pen1/100	360/955/0.057/8.39e-07	66/323/0.016/5.79e-07	112/479/0.023/5.84e-07	121/476/0.026/3.97e-07
pen1/600	285/1341/8.774/6.63e-07	182/1275/6.900/3.65e-07	61/327/1.749/5.68e-07	105/644/4.198/8.76e-07
pen2/10	NaN/NaN/NaN/NaN	583/1738/0.145/8.16e-07	831/2348/0.206/9.70e-07	694/2180/0.196/5.49e-07
pen2/50	874/1911/0.259/9.14e-07	321/1120/0.134/9.08e-07	265/962/0.116/9.52e-07	382/1347/0.150/9.31e-07
pen2/90	367/1067/0.124/4.75e-07	202/740/0.086/9.66e-07	177/524/0.055/9.77e-07	245/843/0.095/4.94e-07
rosex/300	376/825/0.171/3.19e-07	87/282/0.056/7.96e-07	107/285/0.044/9.52e-07	104/430/0.059/9.50e-07
rosex/1000	354/781/7.447/6.15e-07	114/492/4.836/2.57e-07	116/328/3.270/2.80e-07	75/283/2.277/9.92e-07
trid/400	95/176/0.161/5.23e-07	119/182/0.130/9.15e-07	103/163/0.115/8.06e-07	166/229/0.169/9.44e-07