Notes: 'n' represents the dimension of the corresponding problem. 'NI' represents the total number of iterations. 'NF' represents the number of the function evaluation. 'T' represents the running time of the algorithm. ' $\|g^*\|$ ' represents the gradient value for the solution. The values of 'NI', 'NF', 'T' and ' $\|g^*\|$ ' are set as 'NaN' if and only if the number of iterations is greater than 3000.

Table 1: Numerical results for Test 1
SHCGMcc-MIN SHCGMcc-AVG SHCGMcc-MAX SHCGMcc-GEO Problem/n $\mathrm{NI/NF/T}/\|g^\star\|$ $NI/NF/T/||g^*||$ $NI/NF/T/||g^*||$ $NI/NF/T/||g^*||$ cosine/600 43/100/0.010/7.63e-06 16/66/0.021/4.75e-0632/92/0.008/2.79e-0624/75/0.014/9.69e-0617/68/0.012/9.73e-06 67/134/0.021/9.90e-0619/73/0.012/6.24e-06 $25/78/0.013/9.73\mathrm{e}\text{-}06$ $\cosine/3000$ cosine/1000023/81/0.033/3.89e-0640/101/0.045/8.81e-06 28/84/0.035/9.48e-0629/87/0.034/6.26e-06 ${\rm dixmaana}/6000$ 14/74/0.093/2.17e-0616/74/0.113/4.53e-0617/76/0.093/6.08e-0615/73/0.091/6.24e-06dixmaana/9000 16/76/0.147/4.59e-0621/81/0.155/4.14e-0616/68/0.131/8.38e-0613/70/0.131/4.92e-06dixmaanb/4800 11/66/0.090/7.09e-0619/76/0.076/4.46e-06 16/78/0.077/4.77e-0619/79/0.076/4.19e-06dixmanb/2400016/83/0.366/5.38e-0618/77/0.356/9.35e-06 17/79/0.464/8.12e-0721/84/0.443/2.49e-0613/62/0.039/4.91e-0614/66/0.010/6.45e-0616/68/0.009/1.77e-0610/62/0.007/9.19e-06dixmaanc/270 dixmaanc/2700 22/81/0.057/3.17e-0625/88/0.058/3.85e-0622/78/0.045/6.95e-0629/101/0.061/6.10e-06 dixmaand/1200 20/72/0.043/3.12e-0622/76/0.029/9.51e-06 17/71/0.026/3.54e-0617/74/0.027/2.01e-06dixmaand/9000 14/74/0.182/9.38e-0617/78/0.231/7.23e-06 15/77/0.216/2.24e-0614/71/0.225/8.56e-06440/498/0.384/8.20e-06 dixmaane/2400 439/809/0.666/8.92e-06311/483/0.367/7.73e-06345/536/0.412/9.22e-06390/621/0.936/9.98e-06dixmaane/4800 566/1040/1.484/7.12e-06500/567/0.812/9.08e-06428/650/0.987/7.93e-06dixmaanf/6000 554/1004/2.669/9.75e-06510/571/1.331/8.95e-06527/837/1.815/9.55e-06438/676/2.155/9.12e-061690/1953/18.584/9.98e-061316/2058/19.830/9.03e-06dixmaanf/15000 1589/2872/26.730/6.76e-071383/2112/20.093/9.66e-06438/674/2.865/9.02e-06390/622/2.474/8.05e-06dixmaang/6000 462/845/3.366/9.79e-06728/812/2.775/8.11e-061193/1858/13.808/6.65e-06dixmaang/12000 1721/3062/23.231/7.71e-06 1457/1745/13.547/9.83e-06 1234/1875/12.880/2.80e-06dixmaanh/6000 1176/2113/4.482/8.36e-061167/1476/2.894/8.23e-06969/1494/2.876/5.58e-06979/1484/2.772/5.50e-06858/943/7.734/9.88e-06802/1282/10.784/6.08e-06dixmaanh/15000 1028/1907/14.690/9.86e-06846/1348/11.224/8.52e-06 dixmaani/360 2195/4079/0.785/9.92e-061299/1355/0.289/9.48e-06 2373/3793/0.811/8.28e-06 2378/3733/0.768/9.74e-06dixmaanj/3000 300/538/0.563/7.69e-06929/1051/0.996/8.78e-06 273/426/0.412/8.84e-06258/405/0.382/5.77e-06dixmaanj/15000 435/788/7.409/8.72e-06848/921/8.938/9.64e-06 709/1080/9.530/6.96e-06725/1105/10.858/9.56e-06dixmaank/6000 337/609/1.724/5.95e-06669/825/1.812/8.17e-06 330/526/1.272/7.79e-06346/538/1.207/9.55e-06dixmaank/12000 153/295/2.205/5.08e-061003/1278/10.172/9.94e-06427/686/3.007/6.25e-06660/1023/4.247/7.43e-06 $286/535/0.165/8.20 \mathrm{e}\text{-}06$ 449/703/0.182/9.96e-06dixmaanl/1200 2010/2114/0.600/8.11e-06399/648/0.177/9.82e-06dixmaanl/2400 79/147/0.073/9.73e-06135/228/0.114/9.88e-06NaN/NaN/NaN/NaN 162/271/0.136/9.16e-06 dixon3dq/50 603/946/0.009/9.46e-06 515/935/0.013/9.18e-06 479/528/0.005/9.83e-06 580/891/0.008/8.43e-06 2931/5435/0.049/7.20e-06 dixon3dq/150 1404/1452/0.014/7.95e-062929/4694/0.043/8.71e-061068/1650/0.015/9.73e-06dqdrtic/300 121/263/0.008/9.99e-0672/225/0.003/7.22e-0659/159/0.002/6.85e-0659/148/0.002/8.52e-0652/159/0.157/2.19e-06dqrtic/500041/147/0.150/6.31e-0653/160/0.157/8.64e-06 47/155/0.200/9.56e-07 120/285/0.053/1.39e-06107/336/0.044/5.40e-06dqdrtic/9000 84/217/0.026/8.28e-0689/207/0.028/4.58e-0643/101/0.015/6.31e-06 edensch/500 28/85/0.023/8.86e-0627/83/0.017/5.63e-0632/87/0.026/8.25e-0646/113/0.050/9.91e-06edensch/100033/97/0.049/2.95e-0635/94/0.028/9.63e-0631/92/0.031/6.08e-06 $\rm edensch/4000$ 35/105/0.123/8.38e-06 72/161/0.191/9.83e-06 36/112/0.176/9.36e-0639/111/0.160/7.11e-06eg2/1002620/5449/0.230/9.91e-06177/617/0.033/7.12e-061919/4082/0.218/7.29e-061921/4050/0.192/9.44e-06fletchcr/100 37/89/0.015/4.57e-0647/117/0.005/9.77e-06 36/91/0.004/2.47e-0638/93/0.003/6.63e-06 39/105/0.004/7.13e-0630/90/0.003/2.88e-06 fletchcr/500 81/217/0.009/6.07e-0634/94/0.004/9.85e-0645/139/0.009/3.99e-0665/228/0.013/9.67e-06fletchcr/200068/424/0.027/9.59e-0698/655/0.053/8.23e-06263/865/0.029/8.10e-06291/703/0.023/8.13e-06freuroth/40 295/676/0.035/6.86e-06208/477/0.016/9.94e-06740/845/0.030/8.03e-06 885/1257/0.053/5.59e-06 genrose/100 1432/2371/0.086/5.79e-06945/1369/0.050/9.56e-06 himmelbg/700 2/15/0.001/1.35e-29 2/15/0.009/1.35e-292/15/0.001/1.35e-292/15/0.001/1.35e-292/10/0.002/5.13e-20 himmelbg/2400 2/10/0.002/5.14e-202/10/0.002/5.12e-202/10/0.002/5.13e-20liarwhd/6000 1375/2954/0.373/5.37e-06169/513/0.044/2.20e-06631/1385/0.134/9.83e-06515/1131/0.114/7.42e-06 liarwhd/3000 679/1424/0.111/9.19e-06146/418/0.026/3.41e-06406/901/0.075/8.82e-06265/687/0.061/1.78e-06 $17/101/0.005/2.02\mathrm{e}\text{-}06$ 17/101/0.006/3.08e-06penalty1/100 17/101/0.017/1.61e-06 17/101/0.005/1.51e-0631/199/0.162/7.81e-06 penalty1/1000 27/196/0.143/3.24e-0628/193/0.164/4.58e-07 29/199/0.144/3.28e-06 quartc/400056/157/0.201/2.25e-0656/163/0.243/5.25e-06 53/161/0.247/2.06e-0666/177/0.236/8.54e-06quartc/8000 57/166/0.371/4.31e-06 $62/174/0.395/7.43\mathrm{e}\text{-}06$ 51/165/0.366/5.23e-0649/159/0.344/6.92e-06quartc/50000 93/259/5.912/2.71e-06 89/223/5.181/8.28e-06 107/262/5.868/8.18e-0689/266/5.976/3.57e-06 tridia/300 783/1429/0.102/9.72e-06626/693/0.024/9.68e-06792/1245/0.045/9.87e-06817/1250/0.078/8.84e-06 NaN/NaN/NaN/NaN tridia/2000 NaN/NaN/NaN/NaN 2014/2090/0.175/9.36e-06NaN/NaN/NaN/NaN woods/1500352/733/0.080/6.17e-06150/398/0.019/9.17e-06178/375/0.032/6.81e-06249/546/0.029/9.84e-06woods/2000305/637/0.030/6.51e-06 177/322/0.016/6.08e-06 342/689/0.033/8.20e-06 192/449/0.022/7.66e-06 2/11/0.009/4.39e-1112/11/0.007/4.39e-111bdexp/50002/11/0.024/4.40e-1112/11/0.009/4.39e-111bdexp/500002/16/0.106/3.49e-892/16/0.144/3.49e-892/16/0.146/3.49e-892/16/0.144/3.49e-892/11/0.006/4.39e-111bdexp/50002/11/0.013/4.40e-111 2/11/0.010/4.39e-111 2/11/0.008/4.39e-111exdenschnf/9000 31/99/0.041/7.10e-0646/117/0.019/5.81e-06 29/99/0.014/2.66e-0627/92/0.013/5.29e-06exdenschnf/2800 23/86/0.006/8.75e-0654/123/0.008/8.46e-06 22/88/0.006/3.99e-0631/98/0.007/9.79e-06exdenschnf/6000 23/79/0.010/6.91e-0636/104/0.012/3.09e-0637/107/0.013/2.31e-07 29/94/0.010/4.86e-06 exdenschnb/300018/74/0.012/9.81e-0639/97/0.009/5.06e-06 $18/74/0.005/6.61\mathrm{e}\text{-}06$ 17/70/0.005/9.14e-06exdenschnb/6000 17/74/0.007/3.04e-0617/75/0.006/5.65e-06 18/75/0.006/7.79e-0721/79/0.007/1.17e-06exdenschnb/24000 17/75/0.015/6.09e-0618/76/0.016/1.56e-0621/80/0.017/2.35e-0618/79/0.017/9.65e-0628/88/0.014/4.99e-06 genquartic/9000 15/74/0.028/9.51e-0620/80/0.013/6.98e-0618/76/0.011/2.78e-06genquartic/9000 15/74/0.010/9.51e-0628/88/0.013/4.99e-0620/80/0.012/6.98e-0618/76/0.011/2.78e-0617/79/0.048/9.15e-06genquartic/50000 22/85/0.053/5.72e-0722/87/0.047/7.23e-0616/77/0.040/6.63e-061149/1210/0.043/8.38e-062424/3883/0.188/6.71e-06 biggsb1/300 2049/3755/0.157/8.86e-062906/4539/0.253/9.61e-06sine/500338/750/0.099/9.67e-06228/371/0.030/9.88e-06142/308/0.023/6.95e-0673/159/0.021/6.84e-06sine/100028/82/0.018/9.28e-0658/144/0.034/9.85e-06 38/93/0.022/5.03e-0634/87/0.021/8.73e-06sine/200081/175/0.040/8.65e-06 101/211/0.058/9.54e-07 1742/2524/0.491/6.95e-0667/175/0.055/7.00e-06 fletcbv3/50 596/1265/0.070/9.36e-06584/1111/0.064/9.78e-06NaN/NaN/NaN/NaN NaN/NaN/NaN/NaN ${\rm nonscomp}/5000$ 46/114/0.036/5.75e-06220/310/0.040/9.80e-0647/117/0.012/6.91e-0648/117/0.012/6.57e-06 nonscomp/8000NaN/NaN/NaN/NaN 1966/2599/0.373/7.91e-06 NaN/NaN/NaN/NaN NaN/NaN/NaN/NaN NaN/NaN/NaN/NaN power1/150 NaN/NaN/NaN/NaN 2477/2593/0.113/8.29e-06 NaN/NaN/NaN/NaN 279/346/0.045/9.00e-06182/284/0.035/9.50e-06 218/380/0.031/7.25e-06170/277/0.041/8.77e-06raydan1/500 raydan1/1000 379/450/0.077/8.26e-06 268/427/0.044/5.11e-06 319/587/0.056/8.18e-06296/450/0.040/6.09e-06raydan2/100015/70/0.014/6.53e-0615/70/0.006/6.53e-06 15/70/0.006/6.53e-0615/70/0.008/6.53e-0612/63/0.012/5.09e-06raydan2/2000 12/63/0.010/5.09e-0612/63/0.015/5.09e-06 $12/63/0.015/5.09\mathrm{e}\text{-}06$ 10/71/0.028/1.06e-0610/71/0.020/1.06e-06 10/71/0.019/1.06e-06 raydan2/5000 10/71/0.020/1.06e-06469/2813/0.304/7.48e-06644/4086/0.434/9.64e-061380/13252/1.508/8.79e-06diagonal1/800 1410/12268/1.469/9.23e-06 ${\rm diagonal1/1500}$ 2305/22703/4.308/9.25e-06 731/5398/1.040/9.71e-062701/25074/4.647/6.87e-061213/10506/2.082/9.76e-06diagonal2/5000 758/1437/0.642/8.50e-06420/622/0.276/9.95e-06535/880/0.370/9.16e-06538/858/0.384/9.78e-06910/1441/1.164/8.35e-06 diagonal2/10000 1330/2448/1.935/5.69e-06 584/875/0.804/9.25e-06 917/1475/1.198/9.66e-06 diagonal3/200 129/252/0.022/8.12e-06193/436/0.018/9.48e-06160/599/0.025/6.27e-06209/1177/0.048/9.00e-061143/9001/0.759/8.19e-06 501/3649/0.294/9.42e-06diagonal3/500 208/395/0.023/9.33e-06904/7537/0.536/9.73e-061377/10869/2.023/7.99e-06NaN/NaN/NaN/NaN NaN/NaN/NaN/NaN NaN/NaN/NaN/NaN diagonal3/2000 osb2/111180/2192/0.208/9.56e-06423/752/0.049/8.11e-06 918/1536/0.110/9.63e-06 736/1251/0.101/9.85e-06 ie/15012/46/0.362/9.08e-06 16/52/0.669/8.93e-06 17/53/0.551/3.30e-06 18/55/0.476/3.19e-06ie/50013/52/4.626/8.30e-06 12/45/3.958/7.38e-06 9/41/3.202/4.75e-06 12/50/4.992/7.03e-06singx/5001627/3212/1.557/9.01e-06 $224/571/0.217/7.76\mathrm{e}\text{-}06$ 1829/3670/1.756/9.52e-061126/2298/1.100/7.01e-061228/2414/7.244/9.14e-06singx/10001619/3259/8.995/9.42e-06247/611/1.497/8.39e-06 495/1037/3.409/9.04e-06424/1063/13.889/6.78e-061446/2934/35.144/3.13e-06 819/1685/22.578/4.74e-06singx/20002567/5127/61.154/9.99e-06 lin/5010/57/0.067/2.08e-06 10/57/0.013/2.08e-0610/57/0.010/2.08e-0610/57/0.011/2.08e-0613/63/0.020/8.86e-07 13/63/0.021/8.86e-07 lin/10013/63/0.021/8.86e-07 13/63/0.021/8.86e-07 199/928/0.093/9.07e-0699/402/0.032/8.72e-06127/634/0.049/8.20e-06pen1/200106/532/0.041/4.20e-06 $\mathrm{pen1/1000}$ 144/795/8.027/7.95e-0677/565/6.419/9.04e-06 97/483/4.853/6.41e-06 165/585/6.476/8.74e-06pen2/160NaN/NaN/NaN/NaN 268/635/0.082/9.99e-06 191/795/0.098/5.57e-06 NaN/NaN/NaN/NaN rosex/500 $445/930/1.221/8.52\mathrm{e}\text{-}06$ 69/300/0.310/9.54e-06228/495/0.445/3.10e-06179/406/0.364/8.23e-06rosex/1000427/891/6.950/8.97e-06 95/327/2.795/8.26e-06 100/273/2.457/8.29e-06 204/494/3.933/8.58e-06 trid/40087/165/0.178/1.19e-06 154/209/0.126/7.96e-0685/148/0.093/4.95e-06 79/141/0.091/8.08e-06 $38/92/0.210/5.75\mathrm{e}\text{-}06$ trid/800127/182/0.411/8.68e-0639/93/0.192/6.82e-06 40/93/0.152/4.13e-06