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Introduction to Transmission Lines
Basic Principles and Applications of Quarter-Wavelength Cables and Impedance Matching

Daniel D. Traficante

Departments of Chemistry and Medicinal Chemistry
and NMR Concepts
University of Rhode Island
Kingston, Rhode Island 02881

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The basic electrical properties of transmission lines are reviewed, and it is shown how these properties determine the characteristic impedance of the line. For resonant lines, the formation of standing waves is explained. The variation of the impedance of the line as a function of its length is shown to be caused by the standing waves of voltage and current. Various equations for the standing-wave ratio and the concomitant power losses are derived for systems that are impedance mismatched. Applications in NMR spectroscopy include the use of transmission lines as filters, as impedance-matching devices, and for directing pulses toward the probe and away from the preamplifier. Finally, a simple laboratory experiment demonstrates the properties of open and shorted quarter- and half-wavelength cables.

INTRODUCTION

Transmission lines are an important vehicle for transporting power or signal energy from a source to a destination. They are used extensively in commercial communication — by television, radio, telephones, and satellite — as well as in scientific instrumentation. In many cases, weak signals must be transported with minimum loss. Because NMR signals are notoriously weak, the electrical properties of transmission lines must be studied and understood if the lines are to efficiently transfer signals from a probe to a preamplifier. Moreover, the pulsed radio-frequency (rf) power delivered from the power amplifier to the probe must be of a short duration if the power is to be reasonably flat over the desired spectral width. Again, this power must be transferred efficiently to minimize loss and to ensure that the pulse width remains narrow. This article reviews some basic principles and applications of transmission lines, so that readers can employ them to optimally connect sources to loads.

TRANSMISSION LINES

Generally speaking, a transmission line is a set of conductors that carries electric current or signal energy from one point to another. Familiar examples are the power lines that carry the

120- and 240-volt alternating current (ac) from generating plants to our homes and the telephone lines that carry voice-modulated currents. Much shorter transmission lines connect antennas to radio and television receivers and connect probes to amplifiers in NMR spectrometers.

BASIC LINES

Usually, a transmission line consists of a pair of wires or conductors separated from one another by a nonconducting, or dielectric, material. Figure 1 shows two basic types of transmission lines. The transmission line in Fig. 1A is made up of two parallel conductors; the spacing between them is constant. The conductors are insulated from one another by the dielectric material. This type of transmission line is often referred to as *twin lead*, and it is commonly used to connect an antenna to the input of the UHF section of a television receiver.

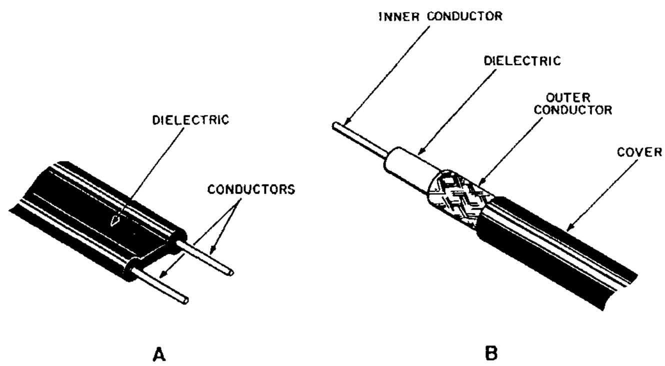


Figure 1. Two transmission lines. (A) Twin-lead line used in commercial UHF television systems and as an antenna for FM radio receivers. (B) Coaxial cable used in commercial UHF/VHF television systems and in NMR spectrometers.

The transmission line in Fig. 1B is a *coaxial cable*. This type of line has a central conductor made of either a single solid wire or of a multifilament stranded wire that is mounted within a surrounding metallic covering, the *shield*. The central wire and the surrounding braided shield form the two conductors of the line. The central conductor is held at the center of the outer conductor by means of a dielectric material, such as nylon or Teflon®. The outer conductor confines the transmitted energy to the inside of the line. It also serves as a shield to prevent pickup of stray electromagnetic fields. As shown in Fig. 1B, a coaxial cable is generally protected by a rubber or plastic cover.

LINE CONSTANTS

Properties of the Line

Depending on their construction and the materials of which they are made, all transmission lines have specific amounts of resistance, capacitance, and inductance. At high frequencies, the distributed inductance and capacitance are most important; for direct current (dc) and low-frequency alternating current (ac), resistance is the important factor.

Resistance

Every conductor has a small amount of resistance per unit length, and all insulating materials have a small amount of conducting ability. The conductor resistance produces a power loss, given by the product I^2R, where I is the current and R is the total resistance of the wire.

Conductance

In addition to the resistance of the wire, a transmission line has a property known as *shunt conductance*, which is a measure of the leakage current through the dielectric. This current exists because no dielectric material is a perfect insulator. The conductivity of the insulating material provides a resistance path between the wires of the line, and thus causes losses due to the shunting of the minute amounts of current carried by the high resistance. Hence, the leakage between conductors can be expressed either as resistance, R', or as conductance, G, where $G = 1/R'$.

Inductance

A changing magnetic field around a wire cuts the wire itself and induces a counter electromotive force (emf) in the wire (*1*). Thus, the conductors of a transmission line have inductance. With ac in the line, the inductance causes a counter emf to be produced along the line. This counter emf opposes the applied voltage so that the voltage between the conductors at the receiving end is somewhat less than that across the terminals of the source.

Skin Effect

The changing magnetic field described above is strongest near the center of the conductor, and it produces a greater counter emf near the center than it does at the conductor's surface. This causes an increase in the effective resistance, known as the *skin effect*. The *skin depth* is the depth below the surface of a conductor at which the current decreases to $1/e$ of its magnitude at the surface.

Capacitance

A capacitor consists of conductors separated by a dielectric (*1*). Thus, there is also capacitance between the conductors that has a shunting effect and permits a small alternating current between the line conductors. Again there is a power loss, because the receiving device has less voltage applied to it and less current delivered to it than it would if the receiver were connected directly to the source terminals.

Distributed Properties

Resistance, conductance, capacitance, and inductance are known as *distributed properties* because they are not concentrated at any one point or group of points; instead, they are distributed uniformly along the length of the line. For convenience, they are indicated schematically as a series of "lumped" components, as shown in Fig. 2. In this figure, the series R and L elements, respectively, represent the resistance and inductance of the two conductors, the capacitors C represent the distributed capacitance, and the shunt conductance is indicated by the resistors R' to show the paths of the leakage current. If the line is considered to be composed of sections of equal length, then all R, L, C, and R' elements are equal for each section.

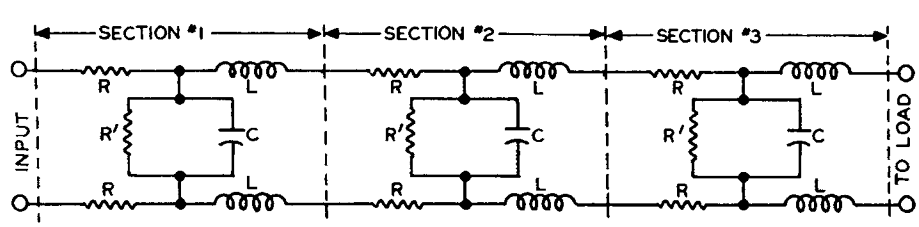


Figure 2. Transmission line schematically represented by lumped components.

It is customary to express leakage between conductors in terms of conductance (G) in micromhos, or resistance (R') in megohms, per foot. The capacitance (C) is expressed in picofarads, the inductance (L) in microhenries, and the resistance (R) in ohms, per foot. In the transmission lines used in NMR spectrometers, R' is large enough that R and G can be neglected, leaving only L and C as the constants to be considered in addition to the load. In many cases, L and C are just as important as is the load itself, and sometimes they are even more important.

CHARACTERISTIC IMPEDANCE

Each line section of Fig. 2 has a definite impedance that depends on the values of R, L, C, and R'. Because of these distributed properties, the transmission line offers impedance to a generator connected at the input terminals. This is the *reflected impedance* of the transmission line and is determined partly by the line's length. If the length is increased, the added series inductance and parallel capacitance of each small segment will have less and less effect on the total reflected impedance, until finally their effect is negligible. The final impedance of an infinitely long line is called the *characteristic impedance* or *surge impedance* (Z_0) of the line.

The value of the characteristic impedance depends on the inductance, resistance, and capacitance characteristics of the line. In turn, these characteristics depend on the size and kind of wire, the nature of the dielectric material that separates the wires, and the distance between the wires. Changes in any of these factors alter the characteristic impedance, which also varies slightly with frequency. Although it is impossible to construct a transmission line of infinite length, it is convenient to rate transmission lines in terms of the impedance (Z_0) that they would have if they were infinitely long. Measurements of a given line can be extrapolated to calculate infinite line impedance, at least to a close approximation (*2*).

In each section of the line represented in Fig. 2, two L and two R elements make up the series impedance Z_1. For any one section, the total series resistance R_T equals $2R$, and the total series inductance L_T equals $2L$. The total series impedance is equal to the vector sum of the resistance and reactance (*1*). Hence,

$$Z_1 = \sqrt{R_T^2 + X_{LT}^2} \qquad [1]$$

Because the shunt impedance, Z_2, consists of the parallel combination of R' and the capactive reactance, X_C, it is equal to the product of R' and X_C divided by their sum. Vector addition is necessary to obtain the sum, so the equation becomes

$$Z_2 = \frac{R'X_C}{\sqrt{(R')^2 + X_C^2}} \qquad [2]$$

In terms of the series and shunt impedances, it will be shown later in this article that the characteristic impedance of the line can be expressed by

$$Z_0 = \sqrt{Z_1 Z_2} \qquad [3]$$

Because R and G are negligible for most practical lines, Z_1 almost equals the inductive reactance, ωL, and because R' is so much greater than X_C at any frequency, Z_2 is nearly equal to the capacitive reactance, $1/(\omega C)$. Substitution of these quantities for Z_1 and Z_2 into Eq. [3] yields

$$Z_0 = \sqrt{L/C} \qquad [4]$$

This last expression assumes that the dielectric material that separates the conductors is air. In solid dielectric lines, the dielectric has the effect of increasing the capacitance per unit length of the line. Therefore, the impedance Z_0 is reduced by the factor $\sqrt{1/k}$, where k is the dielectric constant of the insulating material. Thus, for such transmission lines

$$Z_0 = \sqrt{L/(kC)} \tag{5}$$

WAVELENGTH AND VELOCITY

In Fig. 3, the small shaded circles A and B represent cross sections of parallel conductors. The instantaneous direction of electron flow is assumed to be into the page at conductor A and out of the page at conductor B. The electric and magnetic fields are as indicated by the solid and dashed lines, respectively. The electric lines of force travel from A to B; the magnetic lines move counterclockwise around A and clockwise around B. Thus, at every point where they cross, the electric lines are perpendicular to the magnetic lines, and the planes that contain these lines are perpendicular to the direction of current in the conductors. During succeeding instants, the fields travel along the line, because they are produced by the moving charges or currents in the wires. Thus, the fields follow the path of the current, whether that path is curved or straight.

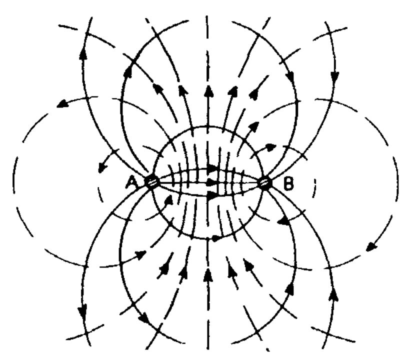


Figure 3. Distribution of electric and magnetic fields around a twin-lead transmission line.

At various points along any one conductor of a transmission line carrying an ac signal, the current is in different directions, as indicated by the arrows drawn in the conductors (A) and (B) shown in Fig. 4A. At points 1, 3, and 5, the currents are in opposite directions in the two conductors. The vertical lines represent the electric field between the conductors, and as the arrows on these lines indicate, at points 1 and 5 the conditions are like those illustrated in Fig. 3. At point 3, the conditions can be illustrated by reversing the direction of the arrows on all of the lines in Fig. 3.

At points 2 and 4, Fig. 4A indicates that the current levels go through zero between direction changes. Because the signal is a sinewave, the currents change constantly, both in magnitude and in polarity. This means that the points that have maximum current will go through zero current one-quarter of a cycle later, and then reach a maximum in the opposite direction another quarter of a cycle later. With a sinewave input, the current and electric field variations will have the shapes shown in Figs. 4B, 4C, and 4D at successive points in time. Note that the wave appears to be moving to the right. The regions of maximum and minimum field strength travel along the transmission line in the direction of energy transmission; from a pulsed-rf amplifier to the probe or from the probe to the receiver amplifier. At the end of a complete cycle of the input, conditions in the line again will be as shown in Fig. 4A; the series of events in this interval is called a *cycle*.

For a sinewave signal input, Fig. 4B indicates the instantaneous amplitude and direction of the conductor currents and the electric fields at successive points along the line in Fig. 4A.

This waveform must be moved to represent the changes that occur as the energy travels along the line. Thus, it is customary to consider the energy as being transmitted in waves along the line. The wavelength is the distance between successive peaks, as indicated at points 1 and 5 in Fig. 4B.

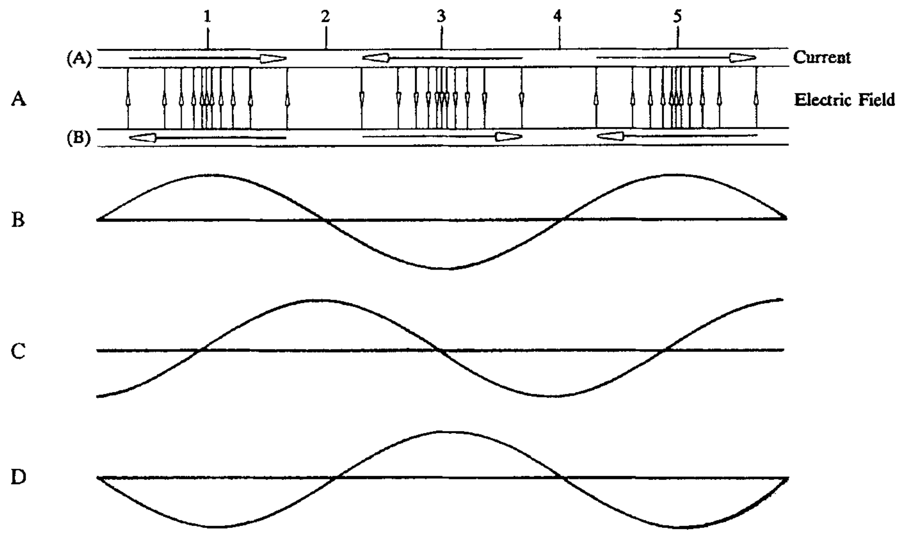


Figure 4. Instantaneous values of the voltage and current in a twin-lead transmission line carrying rf energy.

At the instant shown in Fig. 4A, the arrows in the conductors indicate that the currents have maximum amplitudes at points 1, 3, and 5. Also, as indicated by the concentration of the electric field lines, the difference of potential, or voltage, between the conductors is greatest at these same points. Thus, in the case illustrated, the voltage and current waves are in phase — a desirable condition for most transmission line applications.

When an electromagnetic energy wave travels in free space, its wavelength is given by $\lambda = c/f$, where λ is the wavelength in meters, f is the frequency in hertz, and c is the velocity of light in a vacuum (approximately 3×10^8 m/s). However, in any other medium, such as a transmission line, the velocity, v, of an electromagnetic wave is less than the free-space velocity, c. Thus, for waves of the same frequency, the wavelength λ' in a transmission line is given by $\lambda' = v/f$. These two equations can be combined to give $\lambda' = v\lambda/c$. Because v is a positive number between zero and c, the ratio v/c is always some fraction between 0 and 1. Therefore, the wavelength in the transmission line is always less than it is in free space.

LINE TERMINATION

A transmission line usually connects a source of energy to a load. The load can be a resistance, reactance, or impedance, and it is said to *terminate* the transmission line. However, there are applications in which the line is not terminated in a load of this type. For example, in place of the load a short circuit can be connected across the end of the line. A transmission line terminated this way is a *shorted line*. A line can be terminated in an open circuit; in this case, it is called an *open line*. The voltage, current, and impedance conditions in a transmission line depend on the nature of its termination. The properties and uses of shorted and open lines are explained later in this article.

NONRESONANT LINE

A generator connected to the sending end of a transmission line of infinite length is depicted by Fig. 5. In this arrangement, the characteristic impedance of the line forms the load into which the generator operates. The figure illustrates the instantaneous current (I) and voltage (E) relationships along the line. The waves can be considered to be moving rapidly away from the generator, with the figure showing how the waves would look if they could be stopped at some particular instant. There is, of course, some loss in the line, causing each succeeding peak of current and voltage to have slightly less magnitude than its predecessor. In Fig. 6, the vertical dashed line divides the infinitely long transmission line into two sections; a finite length L_1, and an infinite length L_2. If L_2 is disconnected from L_1, and if the receiving end of L_1 is terminated in a load equal to the characteristic impedance of the line, then, as far as the generator is concerned, the circuit conditions are the same as they would be if the generator were delivering its output to the infinite line. When a transmission line is terminated in a load equal to the characteristic impedance of the line, the current and voltage waves remain in phase for the entire length of the line, just as they do for the infinitely long line depicted in Fig. 5. All of the energy fed into the line by the generator is consumed by the load, and none is returned to the generator. In this respect, the line acts as though it had infinite length. A transmission line operated in this manner — that is, terminated with an impedance equal to Z_0 — is known as a *nonresonant line.*

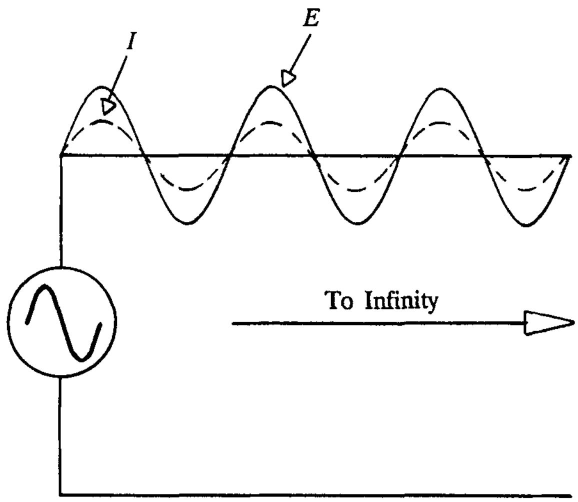


Figure 5. Relationship of the instantaneous voltage and current in an infinitely long transmission line carrying rf energy.

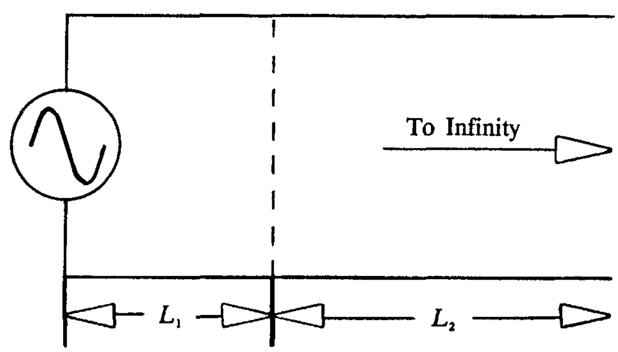


Figure 6. Schematic representation of a generator connected to an infinitely long transmission line.

RESONANT LINE

A transmission line is called a *resonant line* when it is terminated by a load that is unequal to the characteristic impedance of the line. When the waves reach the receiving end of a resonant line, the energy is not completely absorbed by the load, as it is in a nonresonant line. Instead, some energy is reflected back toward the source. At any point along the line, the voltage and current amplitudes are equal to the algebraic sum of the outgoing and reflected waves at that point. Because both waves move with the same speed, but in opposite directions, they reinforce one another at some points along the line and cancel each other at other points. The result is that there are points along the line where the voltage, E, is minimum at all times. Midway between these points, E rises to a maximum, first in one direction and then in the other. Thus, midway between two minimum points, E reaches a maximum in one direction, and 180° away it is at a maximum in the other direction. The same variations occur for the current waveforms. However, in a resonant line the E and I maxima do not occur simultaneously at the same points. Instead, they are out-of-phase with respect to each other. Because these variations of E and I do not move along the resonant line, they are known as *standing waves*.

STANDING WAVES

Figure 7 illustrates the production of a standing-voltage wave. Here, the solid line represents the incident voltage wave, i, which leaves the generator end, indicated by G, and travels toward T, the termination of the transmission line. Wave r, indicated by the dot-dashed line, is relected from the termination end and moves along the line from T toward G. Assume that a voltage indicator is used at various points along the line to measure the voltage between the two conductors that form the line. At any given point, the instrument indicates the algegraic sum of the voltage waves i and r.

Figure 7A shows the two traveling waves at an instant when the positive peaks of one occur at the same points as do the negative peaks of the other. Assuming no losses in the line, the waves i and r have equal amplitude at this instant, and the voltage-indicating instrument reads zero at any point along the line.

In Fig. 7B, the incident wave has traveled a distance of 45° toward the terminating end, T. The reflected wave begins at the point where the incident wave touches T. Because their directions are opposite, the phase relationship between these two waves is a total of 90°. At this instant, the voltage indicator reads zero at points 1 and 3, where the respective waves have equal but opposite values. At other points, the voltage along the line varies, as shown by the dashed curve S. After the wave has traveled another 45°, the incident wave is at its negative peak when it reaches the termination, and the reflected wave begins at this point. The result is that the forward and reflected waves coincide, as shown in Fig. 7C. At this instant, their positive and negative peaks are in-phase and add to produce maximum meter readings, as indicated by the peaks of wave S. Figure 7D shows the conditions 45° later; S has decreased to the same amplitude as in Fig. 7B.

As shown in Fig. 7E, after both waves have moved a total of 180°, they have the same relative phase relationship as shown in Fig. 7A. Notice that Fig. 7E is an inverted copy of Fig. 7A, and the sum of the waves is zero at all points. Similarly, Figs. 7F and 7G are inverted copies of 7B and 7C, respectively. From Fig. 7A to 7G, the waves are shown moving in 45° steps. However, Fig. 7H shows the positions of the waves after they have moved 90° from 7G.

Compare the values of S in the various stages of Fig. 7. In each of these stages, the value is zero at points 1 and 3. In this example, these points are one-quarter and three-quarters of a wavelength, respectively, from the termination of the line at T. The peaks occur at points 2 and 4 and at all other multiples of a half-wavelength from T. As indicated by wave S in Fig. 7C, the sum voltage rises from zero to a positive peak at point 2, falls to a negative peak at point

4, and rises again to zero for all points in Fig. 7E. It then reaches a negative peak at point 2 in Fig. 7G, rises to a positive peak at point 4, and again falls to zero for all points in Fig. 7H.

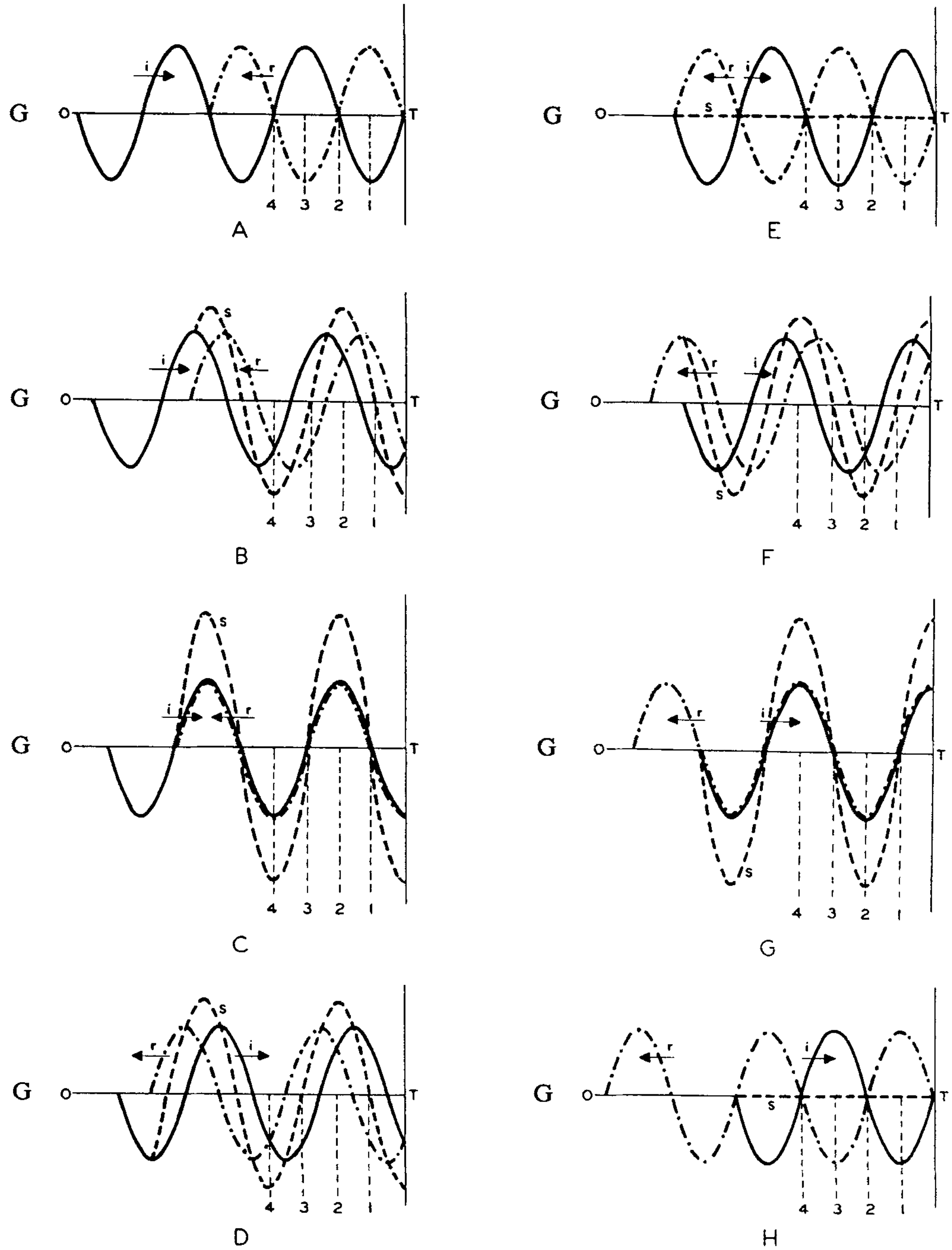


Figure 7. Production of a standing-voltage wave. i is the incident wave leaving the generator G, r is the wave reflected from the termination T, and the standing-voltage wave S is the algebraic sum of i and r.

Another representation of this voltage variation is given in Fig. 8. Here, the numbered waves show a series of instantaneous values of the wave S. The first wave coincides with the axis to represent the zero value for all points on the wave. As this wave increases in amplitude, it is represented successively by 2, 3, 4, and so on, up to the maximum indicated by wave 8. The wave then decreases through the values shown by 7, 6, and 5, and down to zero again. For the second half of its cycle, the wave increases to maximum in the opposite direction, as represented by 9, after which it again falls to zero to complete the cycle. Because the voltage wave S does not travel along the line, but instead remains stationary, it is called a *standing wave.*

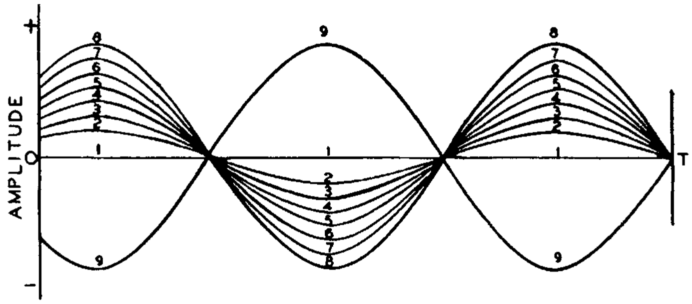


Figure 8. Standing-voltage wave S in Fig. 7, at different points in time.

In addition to a standing wave of voltage, there is also a standing wave of current produced along a resonant line. The phase relationship between the standing waves of voltage and current depends on the termination of the line. For example, if a line is terminated by an open circuit, current cannot flow across the open end, and the standing wave of current is minimum at the termination point, as indicated in Fig. 8. Conversely, because the end is open, the standing wave of voltage must be at a maximum, as indicated in Fig. 7C or Fig. 7G. In this case, the two standing waves are 90° out-of-phase. As another example, when the line is terminated by a short, the current will be high at the shorted end, and the voltage across the short must necessarily be low. Again, the two standing waves are 90° out-of-phase. As described below, other terminations will produce other phase relationships.

EFFECT OF LINE TERMINATION

At any point along a transmission line, the impedance Z is equal to E/I. In the case of a nonresonant line, the ratio of voltage to current is the same at all points, although both E and I decrease with distance from the source. Therefore, E/I is a constant equal to the characteristic impedance Z_0 at all points along the line. A nonresonant line often is called a "flat" line, because the impedance is constant along the line and because its characteristic impedance remains fairly constant over a wide range of frequencies.

However, in a resonant line, the ratio E/I varies from point to point, and therefore the line impedance also varies. When the receiving end of the line is shorted, the standing wave of voltage is minimum, and the standing wave of current is maximum at the shorted end. This situation is indicated in Fig. 9A, where the voltage is represented by the solid line and the current by the dashed line. Thus, when calculated from E/I, Z is minimum at the shorted end. However, at a point located one-quarter wavelength back from the shorted end, E is maximum and I is minimum, as shown in Fig. 9A. At this point on the line, Z is maximum. Because the source is located at this position in the figure, the source "sees" the end of the line as though it were a parallel-tuned circuit, as shown. If the length of the line is doubled, as in Fig. 9B, then at the source end the voltage is minimum and the current is maximum. Now the source "sees" the end of the line as though it were a series-tuned circuit. Because resonant lines can appear to present tuned circuits to a generator, they are sometimes called *tuned lines.*

Conversely, the transmission line in Fig. 10A is open at the receiving end, and the resulting standing waves of voltage and current are as shown. In this case, the current is minimum at the open end, and the voltage is maximum. At this end, then, the impedance is maximum. However, at a point one-quarter wavelength back from the open end, E is minimum and I is maximum. Thus, Z is minimum at this point, and the source "sees" the end of the line as though it were a series-tuned circuit, as shown. Finally, at a distance equal to one-half wavelength, the source "sees" the end of the line as though it were a parallel-tuned circuit, shown in Fig. 10B.

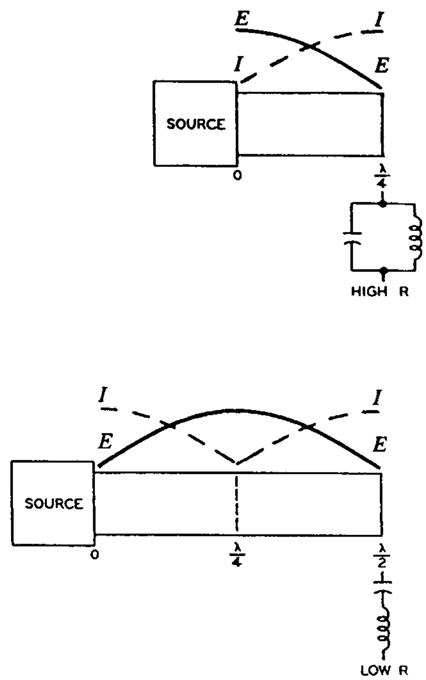


Figure 9. Impedance variation along a resonant line. (A) A shorted quarter-wavelength line, (B) a shorted half-wavelength line.

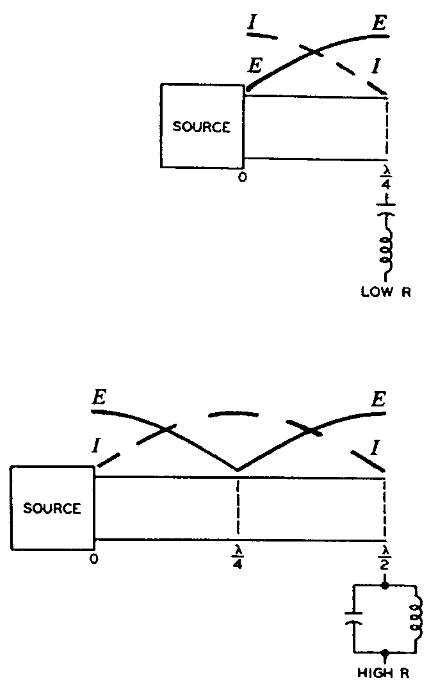


Figure 10. Impedance variation along a resonant line. (A) An open quarter-wavelength line, (B) an open half-wavelength line.

IMPEDANCE VARIATIONS ALONG A LINE

To see how the transmission line's characteristics depend on its length, consider a source that is generating a frequency f_0. In Fig. 11, the source is connected to a two-wire transmission line that is short-circuited at the receiving end. A "shorting bar" is represented by the double-arrowed line, which can be moved along the transmission line to change its length. In this figure, all wavelength measurements are made from the generator end. It has already been explained that if the short is placed at exactly $\lambda/4$, the generator will "see" the line as a very high impedance, represented by the parallel-tuned circuit below the line at $\lambda/4$. At exactly $\lambda/2$, the line appears to be a low-impedance series-tuned circuit.

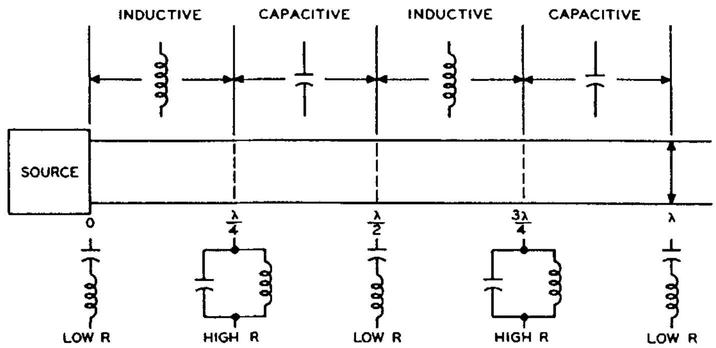


Figure 11. Impedance variation along shorted lines of various lengths.

It has already been shown (*1*) that if a parallel-tuned circuit is placed in a line carrying a frequency other than the tuned frequency, f_0, then the impedance of this circuit will not be maximum, and some of the ac will pass through it. If the frequency is lower than f_0, then X_C increases while X_L decreases. Hence, the current will tend to pass through the inductor rather than through the capacitor, and the circuit appears to be inductive. On the other hand, if the frequency is increased above f_0, then X_C decreases while X_L increases. The current will tend to pass through the capacitor rather than through the inductor, and the circuit appears to be capacitive.

Suppose now that the frequency of the generator is f_0, and that the shorting bar in Fig. 11 is placed at exactly $\lambda/4$. If the frequency of the generator is then decreased to f_1, the new wavelength will be greater than that of f_0, and the line will be shorter than $\lambda/4$ at this new frequency. Because a parallel-tuned circuit appears to be inductive at a low frequency, the transmission line will now also appear to be inductive at this lower frequency. Hence, Fig. 11 shows that the line appears to be inductive at lengths less than $\lambda/4$. If the frequency is increased, the new wavelength will be less than that of f_0, and the line will be greater than $\lambda/4$ at this new frequency. Because a parallel-tuned circuit appears to be capacitive at high frequencies, the transmission line also will appear to be capacitive at the higher frequency. Hence, Fig. 11 shows that the line appears to be capacitive at lengths greater than $\lambda/4$ but less than $\lambda/2$.

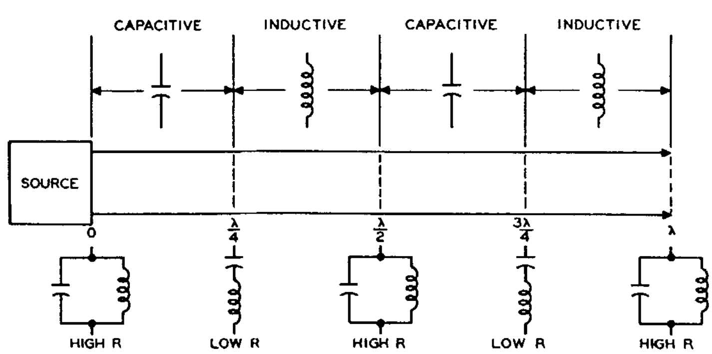


Figure 12. Impedance variation along open lines of various lengths.

Similar reasoning leads to opposite characteristics for an open line, as shown in Fig. 12. For a series-tuned circuit, frequencies other than f_0 are impeded. For a higher frequency, X_C decreases, and the capacitor appears as though it were being replaced by a simple conducting wire. However, X_L increases at higher frequencies, and the inductor seems to be replaced by a larger one. Hence, at frequencies higher or lower than f_0, a series-tuned circuit appears to be inductive or capacitive, respectively.

LINES TERMINATED BY INDUCTORS OR CAPACITORS

Suppose that the shorting bar in Fig. 11 is first set at the $\lambda/4$ point and then is replaced by an inductor. The diagrams above the transmission line in this figure show that the length of the line will then appear to decrease. That is, the right edge of Fig. 9B will appear to move a small distance to the left, and the waveform will move a small distance to the right. Conversely, if the short is replaced by a capacitor, the length of the line will appear to increase, and the waveform will move to the left.

ANOTHER UNIT OF LENGTH FOR TRANSMISSION LINES

It is often convenient to think of the length of a line in terms of degrees, rather than in linear length. In Fig. 13, the solid curves show the magnitude and nature of the impedance of a line as a function of its length expressed in degrees. Above and below the horizontal axis, the reactance is either inductive (X_L) or capacitive (X_C). The horizontal dashed lines represent the characteristic impedance, Z_0, and these lines intersect the solid curves at odd-numbered multiples of 45°.

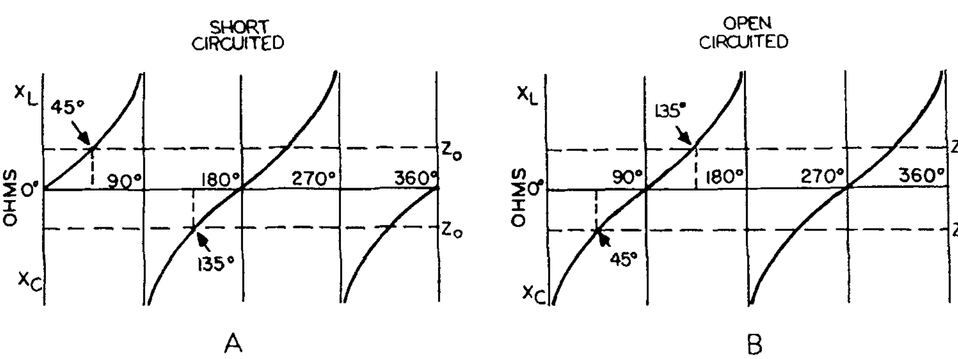


Figure 13. Shorted and open lines represented in degrees, rather than in wavelength, and the variation of reactance as a function of length.

SUMMARY OF TRANSMISSION LINE IMPEDANCES

For various terminations and line lengths, Table 1 lists the type and relative magnitude of the impedance that is presented to a source. The listed terminations consist of opens, shorts, and resistances greater than, equal to, or less than the characteristic impedance of the line, Z_0.

The table shows that a quarter-wavelength section "inverts" the load impedance, so that the opposite magnitude of impedance is presented to the source. A half-wavelength section presents the same impedance as the termination does. When the termination is a resistance that is greater or less than Z_0, the other lengths present an input impedance with a reactive component. When the resistance is equal to Z_0, then the input impedance is Z_0 in all cases. If a line having a length equal to some multiple of a half-wavelength is added to any of the above transmission line sections, then the impedance characteristics at the input terminals are the same as the input of the original section.

TABLE 1
Impedance Seen by a Source

Line Termination	Length of Line $<\lambda/4$	$\lambda/4$	$>\lambda/4, <\lambda/2$	$\lambda/2$
Open	Capacitance	Short, or series-resonant circuit	Inductance	Open, or parallel-resonant circuit
$R > Z_0$	Capacitance and resistance in series	Resistance $< Z_0$	Inductance and resistance in series	Resistance $= R$
$R = Z_0$	Resistance $= Z_0$	Resistance $= Z_0$	Resistance $= Z_0$	Resistance $= Z_0$
$R < Z_0$	Inductance and resistance in series	Resistance $> Z_0$	Capacitance and resistance in series	Resistance $= R$
Short	Inductance	Open, or parallel-resonant circuit	Capacitance	Short, or series-resonant circuit

IMPEDANCE MISMATCH AND STANDING-WAVE RATIO

Standing-wave ratio (*SWR*) is a measure of the mismatch between the transmission line and its load. Commercial instruments are available for measuring *SWR*, and they are generally equipped with a three-position switch labeled FORWARD, REFLECTED, and SWR. It is important to note, however, that *SWR* is not the ratio of the forward to reflected power. Instead, it is the ratio of the maximum to the minimum value of the voltage (or current) of the standing wave:

$$SWR = \frac{E_{max}}{E_{min}} = \frac{I_{max}}{I_{min}} \qquad [6]$$

The maximum voltage of the standing wave occurs when the incident wave (i) and the reflected wave (r) are exactly in-phase, as shown in Fig. 7C. The minimum voltage occurs when these waves are exactly 180° out-of-phase, as shown in Fig. 7A. Hence,

$$SWR = \frac{E_i + E_r}{E_i - E_r} \qquad [7]$$

Using the power rule $P = E^2/R$, we now write

$$SWR = \frac{\sqrt{P_f} + \sqrt{P_r}}{\sqrt{P_f} - \sqrt{P_r}} \qquad [8]$$

P_f and P_r represent the forward and reflected powers, respectively. As an example, if the forward and reflected powers are 10.0 and 0.40 watts, respectively, then *SWR* is 1.5. Equation [8] also shows that under the ideal condition of perfect matching, *SWR* is exactly 1.0, because there is no reflected power.

Another term commonly used for expressing impedance mismatch is the *reflection coefficient*, ρ, which is defined as $\rho = E_r/E_i$. Rearrangement of Eq. [7] gives Eq. [9], which can be further rearranged to give Eq. [10].

$$\rho = \frac{E_r}{E_i} = \frac{SWR - 1}{SWR + 1} \qquad [9]$$

$$SWR = \frac{1 + \rho}{1 - \rho} \qquad [10]$$

Power gains or losses usually are defined in units of decibels, dB, as shown by Eq. [11].

$$\text{dB} = 10\log\frac{P_2}{P_1} \qquad [11]$$

P_1 and P_2 are the input and output powers, respectively. If $P_2 > P_1$, then dB > 0, and there is a power gain; however, if $P_2 < P_1$, then dB < 0, and there is a power loss. If P_f and P_r represent the forward and reflected powers, then $P_1 = P_f$ and $P_2 = P_f - P_r$, leading to

$$\text{dB} = 10\log\frac{P_f - P_r}{P_f} \qquad [12]$$

Rearrangement of Eq. [8] gives

$$SWR(\sqrt{P_f} - \sqrt{P_r}) = \sqrt{P_f} + \sqrt{P_r}$$

$$P_f(SWR - 1)^2 = P_r(SWR + 1)^2$$

$$\frac{P_f - P_r}{P_f} = 1 - \frac{P_r}{P_f} = 1 - \frac{(SWR - 1)^2}{(SWR + 1)^2} = \frac{4SWR}{(SWR + 1)^2} \qquad [13]$$

Combining Eqs. [12] and [13], finally, gives

$$\text{dB} = 10\log\frac{4SWR}{(SWR + 1)^2} \qquad [14]$$

Figure 14 was prepared from Eq. [14]. It shows power loss as a function of SWR. Finally, *return loss* is the logarithmic expression of ρ:

$$\text{dB} = 20\log\rho \qquad [15]$$

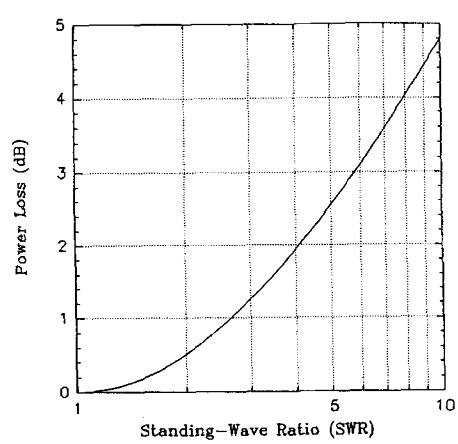


Figure 14. The loss of power, in dB, as a function of *SWR*.

In addition to Eqs. [6], [7], [8], and [10], *SWR* also can be expressed by two more equations, Eqs. [20] and [26]. These equations can be derived with the aid of Fig. 15. In this figure, the

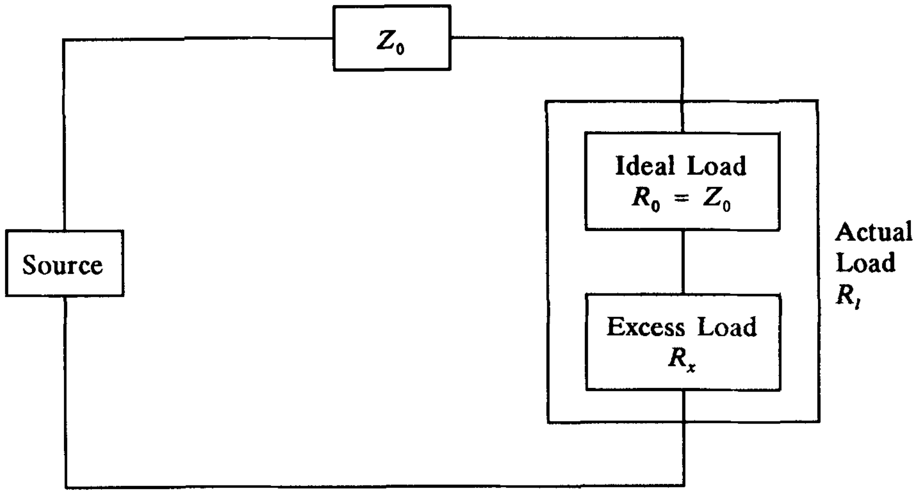


Figure 15. Diagram of an actual load, consisting of an ideal load and an excess load, connected to a source by a transmission line.

actual load is considered to be composed of an ideal load, i.e., one that matches the characteristic impedance of the cable (Z_0), plus an additional, excess resistance (R_x). For example, if the actual load is 75 Ω, and if Z_0 is 50 Ω, then R_x is 75 − 50 = 25 Ω. The current (I) flowing in the circuit is given by

$$I = \frac{E_f}{Z_0 + R_0 + R_x} \qquad [16]$$

E_f is the forward voltage. In this treatment, the reflected voltage shown in Fig. 7 is considered to be due to the presence of R_x. Then the reflected voltage (E_r) is equal to the voltage drop across R_x:

$$E_r = \frac{E_f R_x}{Z_0 + R_0 + R_x} \qquad [17]$$

In Eq. [6], E_{max} is the sum of the forward and reflected voltages, and E_{min} is the difference. Because $R_0 = Z_0$, then substitution in the appropriate locations yields

$$E_{max} = E_f + \frac{E_f R_x}{Z_0 + R_0 + R_x} = E_f\left(1 + \frac{R_x}{2R_0 + R_x}\right) \qquad [18]$$

$$E_{min} = E_f - \frac{E_f R_x}{Z_0 + R_0 + R_x} = E_f\left(1 - \frac{R_x}{2Z_0 + R_x}\right) \qquad [19]$$

$$SWR = \frac{E_{max}}{E_{min}} = \frac{2(R_0 + R_x)(2Z_0 + R_x)}{2Z_0(2R_0 + R_x)} = \frac{R_l}{Z_0} \qquad [20]$$

The combination of Eqs. [9] and [20] leads to

$$\rho = \frac{R_l - Z_0}{R_l + Z_0} \qquad [21]$$

From Eq. [20] and Fig. 14, a 50-Ω cable loaded with 100 Ω will give an *SWR* equal to 100/50, or 2.0, with a 0.5 dB power loss.

If the load impedance is less than the characteristic impedance, then $R_x < 0$ in Fig. 15, and Eqs. [16] to [20] become Eqs. [22] to [26], respectively. Note that R_x can be replaced by $-R_x$ in the denominator, but not in the numerator — power can be a positive quantity only; it cannot be negative.

$$I = \frac{E_f}{Z_0 + R_0 - R_x} \qquad [22]$$

$$E_r = \frac{E_f R_x}{Z_0 + R_0 - R_x} \qquad [23]$$

$$E_{max} = E_f + \frac{E_f R_x}{Z_0 + R_0 - R_x} = E_f\left(1 + \frac{R_x}{2Z_0 - R_x}\right) \qquad [24]$$

$$E_{min} = E_f - \frac{E_f R_x}{Z_0 + R_0 - R_x} = E_f\left(1 - \frac{R_x}{2R_0 - R_x}\right) \qquad [25]$$

$$SWR = \frac{E_{max}}{E_{min}} = \frac{2Z_0(2R_0 - R_x)}{2(R_0 - R_x)(2Z_0 - R_x)} = \frac{Z_0}{R_l} \qquad [26]$$

The results from Eqs. [20] and [26] show that when the cable is not impedance matched by the load, then the *SWR* can be calculated simply by taking the ratio of R_l and Z_0 in whatever form is required to obtain an $SWR > 1.0$, the ideal value. For additional details on this subject and related topics, see References 2 and 3.

TRANSMISSION LINES AS IMPEDANCE-MATCHING DEVICES

To achieve maximum power transfer, when a source must be connected to a load whose input impedance is not equal to the output impedance of the source, an impedance-transforming device must be inserted between the source and the load. Some common devices employed to accomplish this are standard transformers, resistor networks, and resonant circuits. Sections of transmission lines also can be used.

In Fig. 16A, a matching network is used to match the output impedance of the source to the input impedance of the load. If all of the impedances are real, then these three components can be schematically represented as shown in Fig. 16B, where Z_s, Z_l, and Z_0, respectively, represent the impedances of the source, the load, and the transmission line, and where E and V represent the voltage of the source and the voltage drop across Z_0, respectively. Here, the matching network acts as a load for the source at the left and as a source for the load at the right. When it acts as a load, current I_1 from the voltage source flows through the series combination of Z_s and Z_0, producing the voltage drop V across Z_0. This voltage drop, in turn, acts as a source from which current I_2 then flows through the series combination of Z_0 and Z_l. The power consumed by the load, P_l, can then be calculated from

$$E = I_1(Z_s + Z_0)$$

$$V = I_1 Z_0 = \frac{EZ_0}{Z_s + Z_0} = I_2(Z_l + Z_0)$$

$$P_l = I_2^2 Z_l = \frac{E^2 Z_0^2 Z_l}{(Z_s + Z_0)^2 (Z_l + Z_0)^2} \quad [27]$$

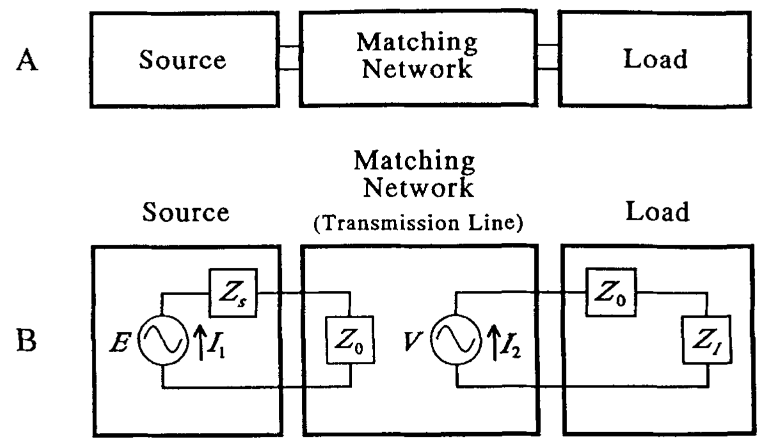


Figure 16. Diagram of a transmission line used to match the impedance of a load to that of a source.

To determine the optimum value of Z_0 for maximum power transfer, the derivative of P_l with respect to Z_0 is set equal to zero, and the equation is solved for Z_0 in terms of Z_s and Z_l. The following substitutions are made here for simplicity: $a = Z_s$, $b = Z_l$, and $x = Z_0$. Then,

$$P_l = \frac{E^2 b x^2}{(a + x)^2 (b + x)^2}$$

$$\frac{dP_l}{dx} = \frac{2E^2 bx}{(a + x)^2 (b + x)^2} - \frac{2E^2 bx^2 (a + x)(b + x)[(a + x) + (b + x)]}{(a + x)^4 (b + x)^4} = 0 \quad [28]$$

Solving for x gives

$$x = \sqrt{ab}$$

$$Z_0 = \sqrt{Z_s Z_l} \quad [29]$$

In the section entitled "Characteristic Impedance" it was stated that Z_0 is equal to the square root of the product of the shunt impedance, Z_1, multiplied by the series impedance, Z_2 (see Eq. [3]). For an infinitely long line that is not connected to a source or a load, then the source impedance (Z_S) of the matching network in Fig. 16 is nonexistent. Hence, the impedance Z_0 on the left side of the matching network can be replaced by the shunt impedance, Z_1. Similarly, the load impedance (Z_l) does not exist, and Z_0 on the right side can be replaced by the series impedance, Z_2. In other words, Z_1 and Z_2 can be considered the source and load impedances, respectively. Equation [29] then becomes equal to Eq. [3].

Figures 17 and 18 are provided here to illustrate the equations derived above. Figure 17 was prepared from Eq. [27] and is a graph of the normalized power delivered to a 200-Ω load as a function of the characteristic impedance of a cable used to connect this load to a 50-Ω source. Although transmission lines can be used to match two different impedances, maximum power transfer is achieved when a line connects two equal impedances. This can be shown mathematically, and Question 7 in the problem set appended to this article is a useful exercise. The optimum combination is obtained when all three networks have the same impedance; i.e., when the load has the same impedance as the source and when they are connected by a transmission line that has the same characteristic impedance. Then Eq. [29] becomes $Z_0 = Z_s = Z_l$. This can also be seen by making reference to Fig. 18, which was prepared from Eq. [27].

It is a graph of the normalized power delivered to a load as a function of the impedance of the load and the characteristic impedance of a cable that is connected to a 50-Ω source.

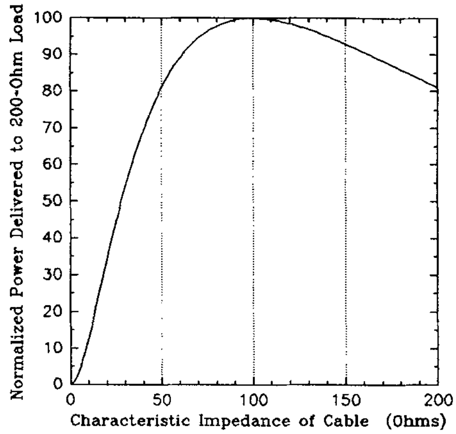


Figure 17. **Normalized power delivered to a 200-Ω load as a function of the characteristic impedance of a cable used to connect this load to a 50-Ω source.**

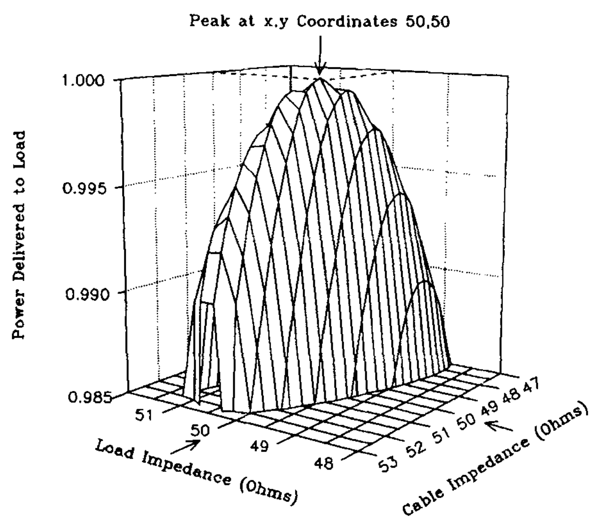


Figure 18. Normalized power delivered to a load as a function of the impedance of the load and the characteristic impedance of a cable when it is used to connect the load to a 50-Ω source.

If the impedance of either the source or the load is complex, then Eq. [29] becomes more complicated by the incorporation of another term (*4*). For one real impedance R_1, and one complex impedance $(R_2 + iX)$,

$$Z_0 = \sqrt{R_1 R_2} \cdot \sqrt{(1 - X^2)/[R_2(R_1 - R_2)]} \qquad [30]$$

OTHER RELATED DEVICES

There are several other devices that operate on the same basic principles as do coaxial transmission lines. Brief descriptions of three are given here. More detailed descriptions are beyond the scope of this article, but can be found in References 3-5.

Microstrip Transmission Line

For transmitter circuitry operating above 100 MHz, matching circuits can be made by constructing "microstrip-line" networks on a printed circuit board. As shown in Fig. 19 (*4*), this transmission line consists of a wide conductor on one side of the board and a ground plane made by leaving all the copper on the other side of the board. The characteristic impedance of the line will depend on the width (W) of the upper conductor and the thickness (T) and dielectric constant of the circuit board.

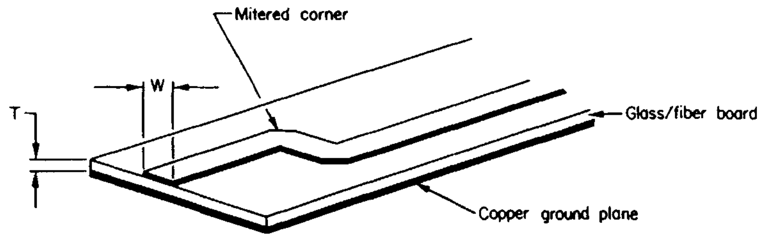
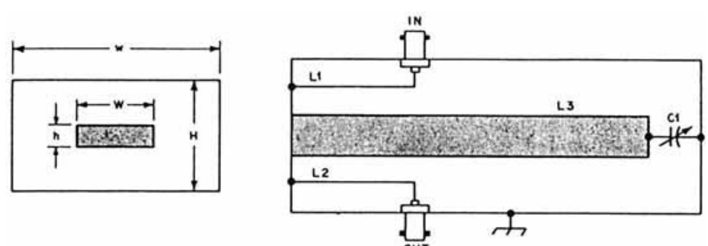


Figure 19. Microstrip transmission line formed on a double-sided printed circuit board. When corners are necessary, they should be mitered as shown.

Strip Transmission Line

Strip transmission lines differ from microstrips in that a second ground plane is placed above the conductor strip, as shown in Fig. 20 (*3, 5*). From 50 to 220 MHz, a quarter-wavelength stripline filter is suitable, but at higher frequencies it is difficult to construct a practical filter unless a half-wavelength format is adopted. One problem with the quarter-wavelength style above 220 MHz is that the box enclosure itself tends to become a resonant cavity, and an unwanted resonance can fall in the desired filter pass-band. Furthermore, by using the dimensions for a half-wavelength the effects of undesirable stray capacitance within the filter are less pronounced.



$\frac{h}{H} = 0.25$

$\frac{W}{H} = 0.4$

$w \geqslant 2H$

$L3 = \frac{2808}{f_{MHz}} \times 0.65$ (inches)

L1, L2 = 0.25 X L3 (inches)

Figure 20. Design details for a quarter-wavelength transmission line (stripline) filter.

Helical Resonator

In physical terms, a helical resonator is a high-Q (*1*), single-layer, solenoid coil contained in a cylindrical or rectangular shield compartment, as shown in Fig. 21 (*5*). Electrically, these resonators are quarter-wavelength devices, but they offer the distinct advantage of being much smaller than are the more common coaxial-line resonators. In addition, for a given physical volume, helical resonators have a Q of approximately twice that of a lumped-element circuit. Therefore, the application of two or more helical resonators in a filter configuration is practical. Values of Q for several hundred to approximately 1000 can be achieved.

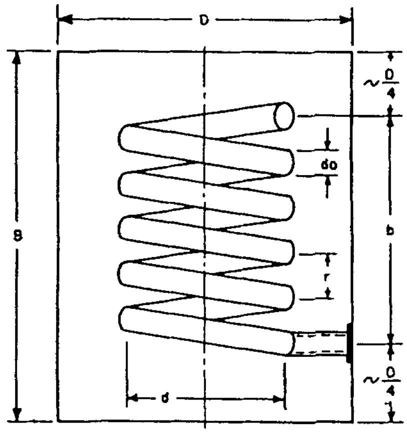


Figure 21. Details of helical-resonator design.

APPLICATIONS

Because of their electrical equivalence to various circuit elements, transmission line sections are often used instead of ordinary lumped elements. They find extensive use in high-frequency applications, such as in VHF, UHF, and SHF equipment; in low-frequency circuits the use of transmission lines would be impractical because of their excessive lengths.

Delay Lines in Commercial Communications Equipment

For any type of transmission line, it takes a short, but finite amount of time for an applied signal to travel from one end of the line to the other. This property is used in applications where it is necessary to delay the signal by some interval. A transmission network used for this purpose is known as a *delay line*. For example, in a color TV receiver, the color video signal is delayed as it passes through a filter circuit. Hence, it is necessary to insert a delay line in the path of the black-and-white video signal to delay it by the same amount of time. Delay lines also are used in pulse-generating circuits for delaying pulses and as a means of obtaining the desired pulse waveform. In these applications, the delay time is approximately 10 μs or less.

Applications in NMR Spectroscopy

Filters

During the observation of carbon signals with strong proton decoupling, weak carbon signals as well as relatively high power at the proton frequency travel along the coaxial cable that connects the probe to the preamplifier. If the proton frequency is not separated from the carbon signals, then the high power will saturate the first stage of preamplification, resulting in a poor signal-to-noise (S/N) ratio for the carbon signals. Conventional filters, quarter-wavelength cables (*6, 7*), and a combination of these, are all useful for separating the two frequencies. Figure 22 shows one method of accomplishing this filtering by using a shorted cable that is a quarter wavelength at the carbon frequency. Because $\omega_H = 4\omega_C$, then $\lambda_H = \lambda_C/4$. This means that the same cable is approximately a full wavelength at ω_H. The line acts like a high impedance for the carbon signals, but acts like a short to ground at ω_H. Hence, the high power at the proton frequency is shunted, or "dumped" to ground and does not enter the input of the preamplifier.

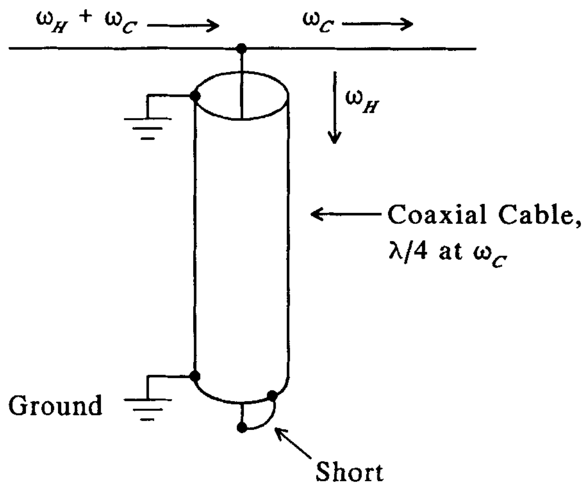


Figure 22. A shorted quarter-wavelength cable for shunting the proton decoupling power and separating it from the carbon signal.

Pulse-Reflecting Circuits

In the early stages of NMR spectroscopy, most probes were of the "cross-coil" variety. In these systems, the transmitter and receiver were connected to separate coils within the probe. These coils were wound on the x and y axes, respectively; hence the term "cross-coil." The cross-coil configuration minimizes the cross-talk, or coupling, between the two channels because the coil axes are orthogonal to one another. This arrangement reduces the amount of the rf pulse that leaks into the receiver while the transmitter is on. If the leakage is too great in continuous-wave (cw) systems, it can saturate the preamplifier and reduce the S/N ratio. In Fourier transform systems, the preamplifier can be damaged by the high power of the rf pulse.

"Single-coil" probes were used in later designs. In these probes, the transmitter and receiver are connected to one coil, which is frequently called a "transceiver coil." This arrangement has many advantages, the most important of which is probably that the B_1 field at the sample is higher than it is with the cross-coil arrangement, resulting in shorter 90° pulse widths. However, if proper precautions are not taken, the strong rf pulse can damage the preamplifier because its input is connected directly to the output of the transmitter. Protection often is provided by a quarter-wavelength cable arranged as shown in Fig. 23.

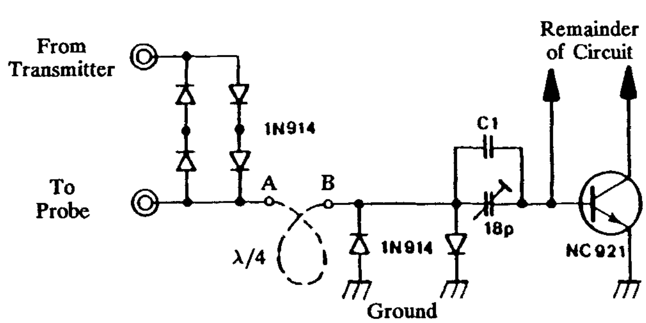


Figure 23. Portion of a schematic diagram for a commercial NMR preamplifier. The quarter-wavelength cable connecting points A and B directs the pulsed rf from the transmitter to the probe, but passes the NMR signal from the probe to the first stage of amplification provided by the NC921 transistor.

In this figure, the rf pulse from the transmitter turns on the bank of four 1N914 diodes (*1*). At point A, there are two possible paths for the rf signal. The first is to the left toward the probe. The second is to the right toward the quarter-wavelength cable, indicated by the dashed loop. In fact, both paths are taken, but the distribution of power to the two sides is unequal. As soon as enough voltage develops at point B to forward bias the two diodes connected to ground, they will conduct, and point B will be shorted to ground. Only a few tenths of a volt is required to do this. It is common for schematics to bear some resemblance to the actual physical construction of the device, but frequently it seems that the esthetics of the drawing are more important. In this diagram, the two ends of the bank of four diodes are connected directly to point A, and the ends of the other two diodes to ground are connected directly to point B. Hence, the wavelength from A to ground is exactly $\lambda/4$, and this shorted cable presents a high impedance to the rf pulse at point A. Thus, only a small amount of current can flow through the cable to ground; in fact, only enough to barely keep the two diodes turned on. If the current were to drop below this value, the diodes would open, the cable would not offer a high impedance to the pulse, current would then flow again, and the diodes would turn on again.

Because of the high impedance at point A, the pulse is directed to the left, toward the probe, along another coaxial cable. Assuming that the probe is properly tuned and matched, generally to 50 Ω, the cable will be terminated in its characteristic impedance. Hence, the cable will be a nonresonant line, and its length will not be important unless it is excessively long. Essentially all of the pulsed rf is then delivered to the probe.

After the pulse, the NMR signal from the probe travels along the same cable that carried the pulse to the probe. When it reaches point A, its voltage will be too low to turn on the bank of four diodes, and none of the signal will travel toward the transmitter output. Instead, the signal will proceed through the $\lambda/4$ cable. At point B, its voltage again will be too low to turn on the two diodes connected to ground, and none of the signal will be lost by being shunted to ground. It must then continue through the coupling capacitors to the input of the first stage of amplification, transistor NC921. Note that the total length of the path from the probe to the input of NC921 is now very much longer than $\lambda/4$. However, the input impedance of the preamplifier is also 50 Ω, and again the length of this path is not important. For an earlier form of this circuit, see Reference 8.

An interesting and clever adaptation of the above principles has been used by J. L. Engle and J. Sorge (personal communication) for probes that are frequently used for two or three fixed frequencies. In Fig. 24, three series-tuned circuits are connected to the output of the transmitter. Each circuit is tuned to pass one of the desired operating frequencies, f_A, f_{AB}, or

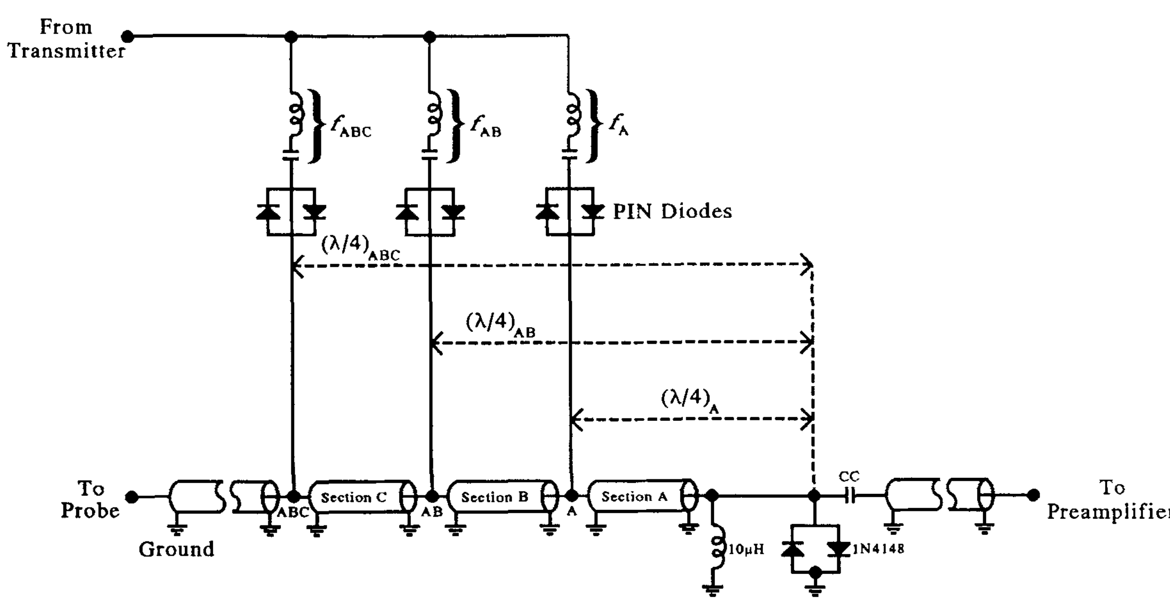


Figure 24. Variation of the use of quarter-wavelength cables as shown in Fig. 23. Here, the fixed cables direct three different pulsed frequencies from the transmitter to the probe.

f_{ABC}. Three sections of 50-Ω cable are connected in series. The length of section A is $\lambda/4$ at f_A. The length of section B is such that its length plus section A is $\lambda/4$ at f_{AB}. Finally, the overall length of the three sections is $\lambda/4$ at f_{ABC}.

Suppose that the desired operating frequency is f_{AB}. An rf pulse of this frequency will pass through only the series circuit tuned for f_{AB}, and it will turn on the two attached PIN diodes. Some of the rf current will pass through sections A and B, and turn on the two 1N4148 diodes, shorting the cable to ground at this point and presenting a high impedance to rf at point AB. Hence, most of the rf pulse is directed toward the probe, traveling through section C and through the cable that connects point ABC to the probe. However, the length of the path is not critical because this line is terminated by the probe, which should be tuned and matched to 50 Ω. After the pulse, the NMR signal voltage will be too low to turn on any of the diodes, and the signal will travel from the probe to the preamplifier. Again, the length of this path is not critical because it is terminated by the preamplifier, whose input impedance is 50 Ω. The 10 μH *nonferrite* choke provides for quicker discharge of any forward bias of the 1N4148 diodes after the pulse. The coupling capacitor CC will pass the NMR signal, but will block any dc bias voltages in the preamplifier from shorting to ground through one of the 1N4148 diodes, or through the choke.

LABORATORY EXPERIMENT

The following experiment is an instructive demonstration of the properties of resonant and nonresonant lines and of how shorted and open quarter- and half-wavelength cables can filter signals.

Equipment Needed

A. An ordinary oscilloscope ("scope").
B. A frequency synthesizer or other source of a variable frequency.
C. A BNC "T" connector.
D. A 50-Ω load with a BNC connector.
E. A double-female ("straight-through") BNC connector.
F. Two 50-Ω coaxial cables, 0.5 to 1.0 m long, each with BNC connectors on both ends.
G. A 50-Ω coaxial cable, 0.5 to 1.0 m long, with a BNC connector on one end. The other end should be prepared as follows: After cleanly cutting the end, strip back approximately 1 cm of the rubber covering to expose the braided shield. Unwind the braid and twist its separated wires into a "pigtail." Finally, strip away approximately 5 mm of the dielectric to expose the center conductor. When properly prepared, the end of the cable should look like the illustration (Fig. 25).

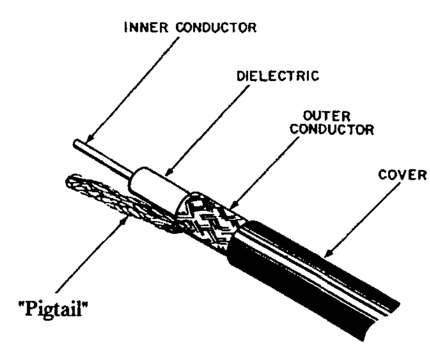


Figure 25. Coaxial cable prepared as described in the text.

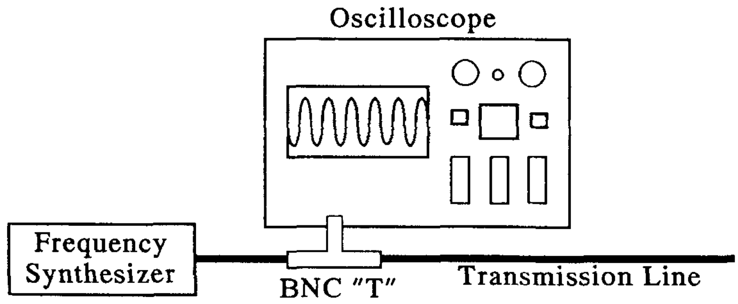


Figure 26. Arrangement of equipment used in the experiment for determining the properties of open and shorted quarter-wavelength cables.

Procedure (Refer to Fig. 26)

A. Resonant and nonresonant lines

1. Connect the center port of the "T" to the input of the scope.
2. Using one of the cables that has connectors at both ends, connect the output of the frequency synthesizer to one side of the "T."
3. To the other side of the "T," connect the other cable that has connectors at both ends.
4. Preparing for step 7, attach the double-female connector to the other end of the cable.
5. Set the frequency to approximately 50 MHz, and adjust the scope so that a clean, stationary waveform is obtained.
6. Vary the frequency between 10 and 90 MHZ, and note that the amplitude does not remain constant as the frequency changes.
7. Attach the 50-Ω load to the double-female connector.
8. Repeat step 6 and note that the amplitude now remains constant, or "flat."

B. Shorted and open quarter-wavelength cable

1. Measure the length of the cable with the prepared end, and calculate the frequency f_0 at which it is one-quarter of a wavelength.
2. Set the frequency to f_0.
3. Short the end by holding the pigtail braid to the center conductor.
4. Readjust the frequency to maximize the amplitude on the scope. Note the peak-to-peak amplitude.
5. Release the pigtail and separate it from the center conductor.
6. Note the new amplitude on the scope, and calculate the attenuation in decibels, using the equation dB = 20 log (E_2/E_1). A typical value is 20-25 dB.

C. Shorted and open half-wavelength cables

1. Change the synthesizer to $2f_0$. The $\lambda/4$ cable now becomes a $\lambda/2$ cable.
2. Repeat the above procedures (B, steps 3-6) used for the quarter-wavelength cable, and note that the properties for the half-wavelength cable are reversed.

SUMMARY

The electrical properties of coaxial cables are defined in terms of resistance, conductance, inductance, and capacitance. Together, these are called distributed properties. They determine the impedance of the line because they act as though they were a series combination of lumped components.

When a line is infinitely long or is terminated by a load whose impedance is equal to the characteristic impedance of the line, then the impedance along the line is relatively constant. The line is then called a flat or nonresonant line. When rf energy is transmitted along a nonresonant line, the waves of current and voltage will be in-phase. Under these conditions, all of the energy fed into the line by the generator is consumed by the load, and none is returned to the generator.

A transmission line of finite length that is not terminated by its characteristic impedance is called a resonant line. Radio-frequency energy propagated along the line will be reflected by the load, and standing waves of current and voltage will form along the line. The impedance varies along a resonant line, and the impedance seen by the generator is a function of the impedance of the load and the physical length of the line. The ratio of the maximum to the minimum values of the standing waves of current or voltage is called the standing-wave ratio — a measure of the mismatch between the line and its load. Other expressions of the mismatch are reflection coefficient and return loss.

Sections of transmission lines can act as impedance-matching devices and can be used to maximize the transfer of power from a source to a load when their impedances are unequal. They also can act as tuned filters when the sections are a quarter- or half-wavelength long and terminated by an open end or by a short. A shorted quarter-wavelength cable presents a high impedance to its source, whereas an open end presents a low impedance. Half-wavelength cables have the opposite properties.

Shorted quarter-wavelength cables are commonly used in NMR instrumentation to direct the high-power rf pulse from the pulse amplifier toward the probe and away from the preamplifier. This is accomplished by attaching diodes to the end of the cable, so that it will be shorted when the pulsed rf is present but open at other times. With this configuration, the shorted cable presents a high impedance to the power amplifier and the rf is directed toward the probe. After the pulse, the cable is no longer shorted to ground. Instead, it is connected to the preamplifier, whose input impedance matches the characteristic impedance of the cable. The NMR signal returning from the probe is then efficiently transferred to the preamplifier.

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APPENDIX

Mathematical Expressions

I . . . . Current
R . . . . Resistance
R' . . . . Leakage between conductors in a transmission line
G . . . . Conductance
L . . . . Inductance

C .... Capacitance
Z_0 .... Characteristic impedance, or surge impedance of a transmission line
Z_1 .... Series impedance of a transmission line
R_T .... Total resistance
L_T .... Total inductance
Z_2 .... Shunt impedance of a transmission line
X_C .... Capacitive reactance
k .... Dielectric constant of the insulating material separating the conductors in a transmission line
λ .... Wavelength of an electromagnetic wave
c .... Velocity of an electromagnetic wave in a vacuum
f .... Frequency of an electromagnetic wave
v .... Velocity of an electromagnetic wave in a transmission line
E .... Voltage
X_L .... Inductive reactance
SWR .... Standing-wave ratio
E_{max} .... Maximum value of the voltage in a standing wave
E_{min} .... Minimum value of the voltage in a standing wave
I_{max} .... Maximum value of the current in a standing wave
I_{min} .... Minimum value of the current in a standing wave
E_i .... Voltage of the incident wave in a transmission line
E_r .... Voltage of the reflected wave in a transmission line
P_f .... Forward power along a transmission line
P_r .... Reflected power along a transmission line
ρ .... Reflection coefficient
P_1 .... Input power
P_2 .... Output power
R_l .... Load resistance
R_x .... The portion of the actual resistance of a load that is in excess of the ideal resistance
V_r .... Reflected voltage due to the presence of R_x
P_l .... Power consumed by a load
Z_s .... Impedance of a source
Z_l .... Impedance of a load

Important Equations

Standing Wave Ratio (SWR)

$$SWR = \frac{E_{max}}{E_{min}} = \frac{I_{max}}{I_{min}} \qquad [6]$$

$$SWR = \frac{E_i + E_r}{E_i - E_r} \qquad [7]$$

$$SWR = \frac{\sqrt{P_f} + \sqrt{P_r}}{\sqrt{P_f} - \sqrt{P_r}} \qquad [8]$$

$$SWR = \frac{1+\rho}{1-\rho} \qquad [10]$$

$$SWR = \frac{E_{max}}{E_{min}} = \frac{2(R_0+R_x)(2Z_0+R_x)}{2Z_0(2R_0+R_x)} = \frac{R_l}{Z_0} \qquad [20]$$

$$SWR = \frac{E_{max}}{E_{min}} = \frac{2Z_0(2R_0-R_x)}{2(R_0-R_x)(2Z_0-R_x)} = \frac{Z_0}{R_l} \qquad [26]$$

SWR Power Loss

$$\mathrm{dB} = 10\log\frac{4SWR}{(SWR+1)^2} \qquad [14]$$

Reflection Coefficient

$$\rho = \frac{E_r}{E_i} = \frac{SWR-1}{SWR+1} \qquad [9]$$

Return Loss

$$\mathrm{dB} = 20\log\rho \qquad [11]$$

Impedance-Matching Transmission Line

$$Z_0 = \sqrt{Z_s Z_l} \qquad [29]$$

$$Z_0 = \sqrt{R_1 R_2}\cdot\sqrt{(1-X^2)/[R_2(R_1-R_2)]} \qquad [30]$$

Glossary

Characteristic impedance . . . The input impedance of an infinitely long transmission line.

Distributed properties . . . . . The resistance, conductance, capacitance, and inductance distributed throughout the length of a transmission line.

Flat line . . . . A nonresonant line, as defined below.

Impedance transformation . . The process of changing an impendance to a different value.

Line termination . . . . The load connected to the end of a transmission line. The load can be a resistance, reactance, or impedance; it can also have a shorted or open end.

Nonresonant line . . . . A transmission line that is not resonant at any length. It is terminated by an impedance that is equal to the characteristic impedance of the line.

Quarter-wavelength cable . . A transmission line whose length is equal to one-quarter of a wavelength of the electromagnetic wave being transmitted along the line.

Reflection coefficient . . . . . A measure of the degree of impedance mismatch; equal to the reflected voltage divided by the incident voltage.

Resonant line . . . . A transmission line that is not terminated in its characteristic impedance. An example is one whose length is exactly equal to an integer number of quarter-wavelengths of the electromagnetic wave transmitted along a line whose end is open or shorted.

Return loss . . . . A measure of the degree of impedance mismatch; equal to 20 times the logarithm of the reflection coefficient.

Standing waves . . . . Stationary variation in voltage or current along a resonant transmission line.

Standing-wave ratio . . . . . . The ratio of the maximum to the minimum values of the standing waves of voltage or current along a transmission line. A measure of the degree of mismatch between the load and the characteristic impedance of the line.

Tuned line . . . . A resonant transmission line tuned to the desired frequency by varying its electrical length.

PROBLEM SET

1. What are the four distributed properties of a line that determine its characteristic impedance?

2. In Fig. 7H, the positions of the waves are shown after they have moved 90° from their positions shown in 7G. Draw the positions of the waves after they have moved only 45°. Show the incident and reflected waves and their instantaneous sum at different points in time.

3. A flat line is

   A. A flat twin-conductor as shown in Fig. 1A.
   B. A line terminated by its characteristic impedance.
   C. A line terminated by a short.
   D. An open line.

4. Which of the following is NOT correct?

   A. An open $\lambda/4$ line presents a low impedance to its source.
   B. A shorted $\lambda/4$ line presents a high impedance to its source.
   C. An open line that is slightly shorter than $\lambda/2$ presents an inductance to its source.
   D. An open line that is slightly longer than $\lambda/4$ presents a capacitor to its source.

5. At 100 MHz, a shorted transmission line 75 cm long presents to its source which of the following:

   A. Inductance.
   B. Capacitance.
   C. Low impedance.
   D. High impedance.

6. Suppose ^{31}P is to be decoupled from ^{15}N on a 7.046 T system. Assuming ideal conditions, what approximate length of cable should be used if it is to be used as shown in Fig. 22? Should the cable be open or shorted?

7. Show mathematically that although transmission lines can be used to optimally match two different impedances, maximum power transfer is achieved when a line connects two impedances that are equal to the characteristic impedance of the line.

8. Suppose (1) that an NMR probe is connected by a 50-Ω cable to a pulsed-power amplifier that has a 50-Ω output impedance, and (2) that there is a malfunction inside the probe so that it can be tuned and matched only to 200 Ω. What will be the loss of power under this condition of impedance mismatch?

9. Under the conditions stated in Question 8, what will be the 90° pulse width if it is normally 10 μs when the probe can be tuned and matched to 50 Ω?

10. Assume that time constraints require that an NMR experiment be run under the conditions stated in Question 8. What should be the characteristic impedance of a cable that is to be used to optimally match the amplifier to the probe? What will be the 90° pulse width if this cable is used?

The answers to these questions appear on page 88-89. If you disagree with the published answers, please submit your counterarguments for publication.

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