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Wilson's Algorithm for Randomized Linear Algebra

Yusuf Yiğit Pilavcı

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Introduction

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- In large scale, faster algorithms might be necessary
- Randomized numerical linear algebra (RNLA) aims to facilitate them via Monte Carlo methods [Martinsson and Tropp 2020].
- The algorithms in this branch run under a scheme called sample-and-solve.

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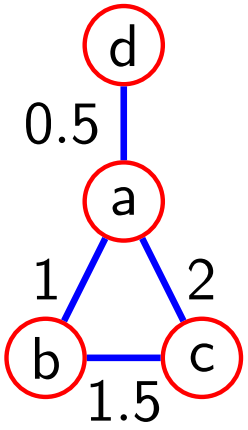
  - Graph visualization, clustering, sparsification ...
- We leverage the rich theoretical links between the graph Laplacians and a special random process over graphs, called Random Spanning Forests.

Graphs and Matrices



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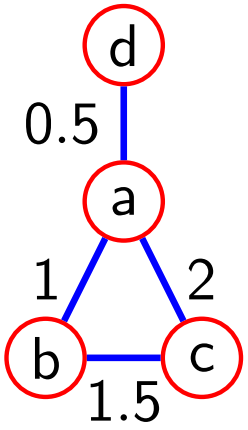

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Adjacency matrix W

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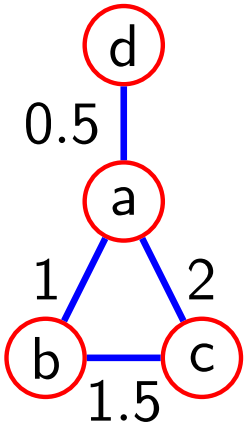

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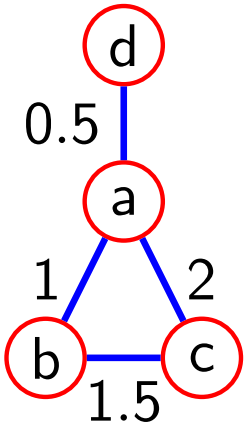
Degree matrix D

Graphs and Matrices



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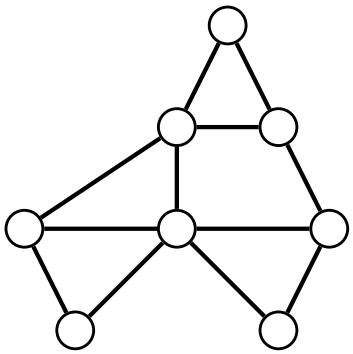
Laplacian matrix $\mathrm{L} = \mathrm{D} - \mathrm{W}$

Spanning Forests, Roots and Partitions



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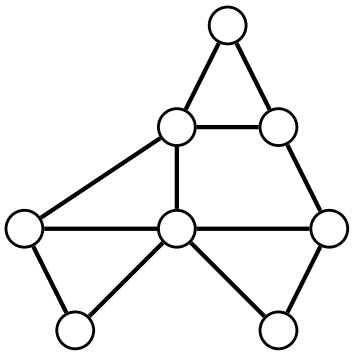

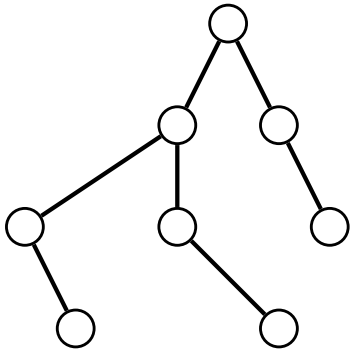

Spanning Tree

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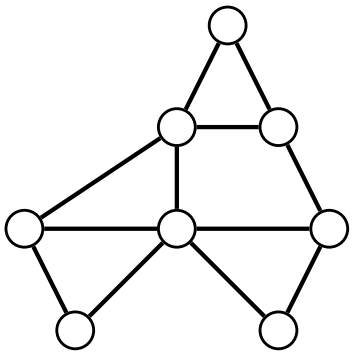

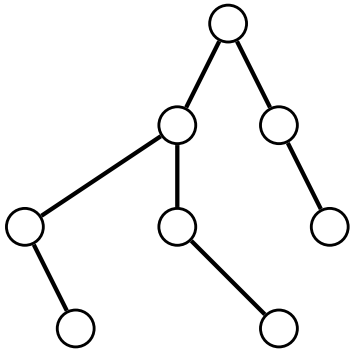

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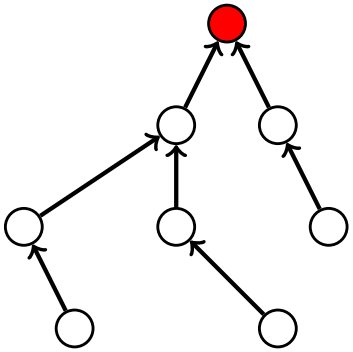

Rooted Spanning Tree

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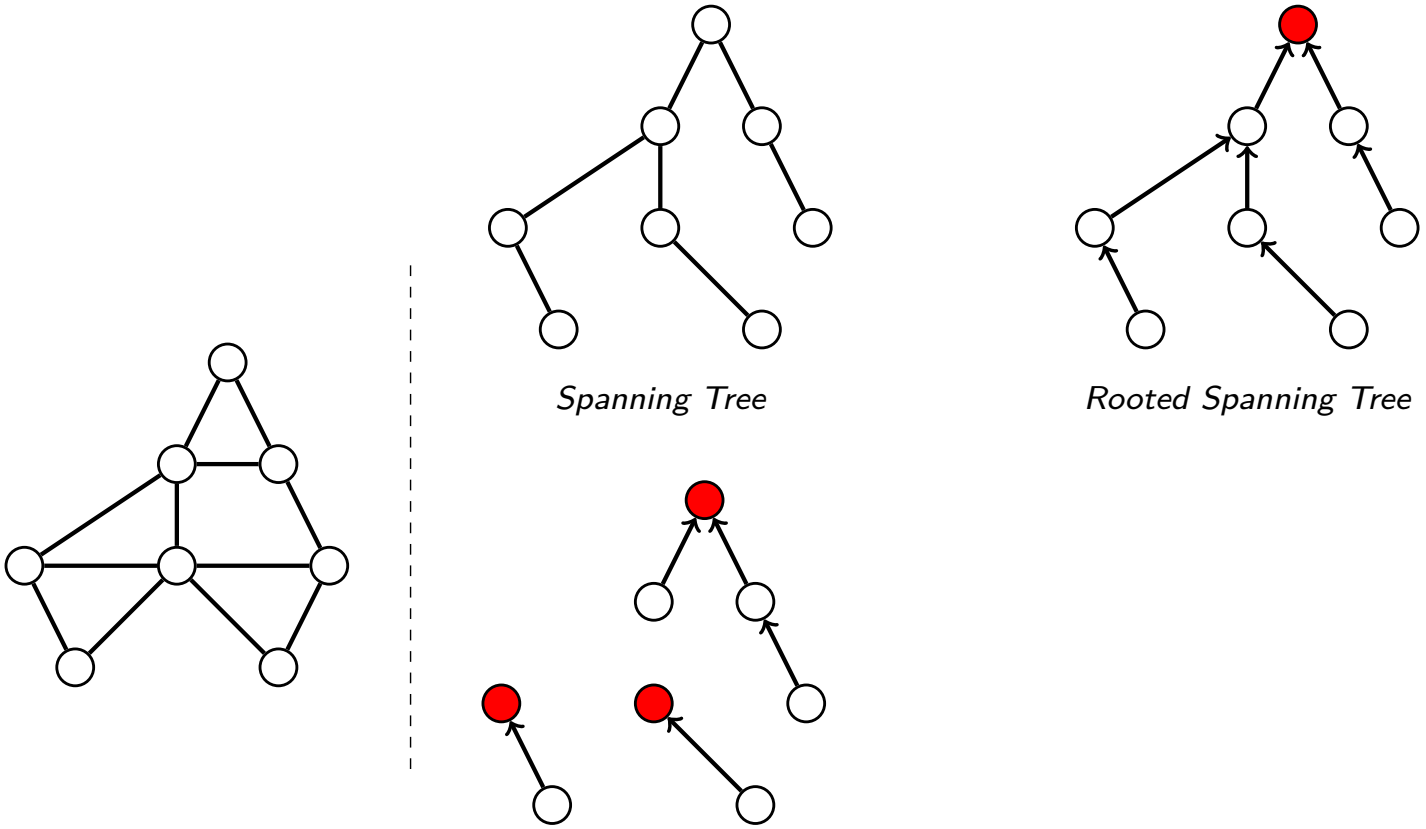

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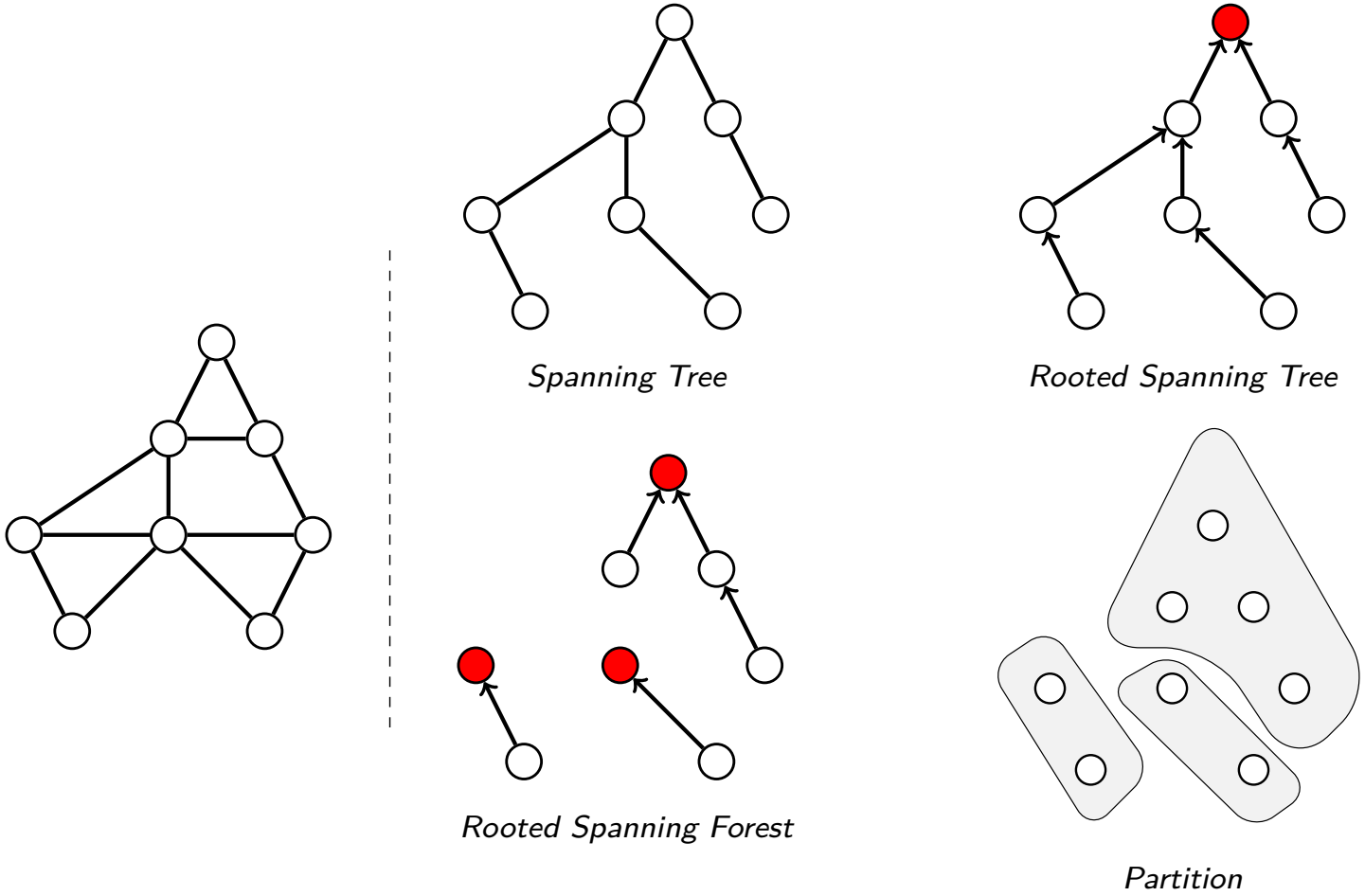

A few more notation

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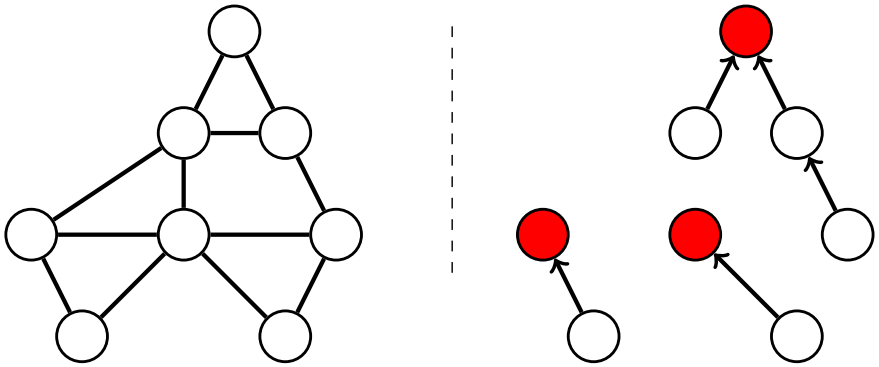

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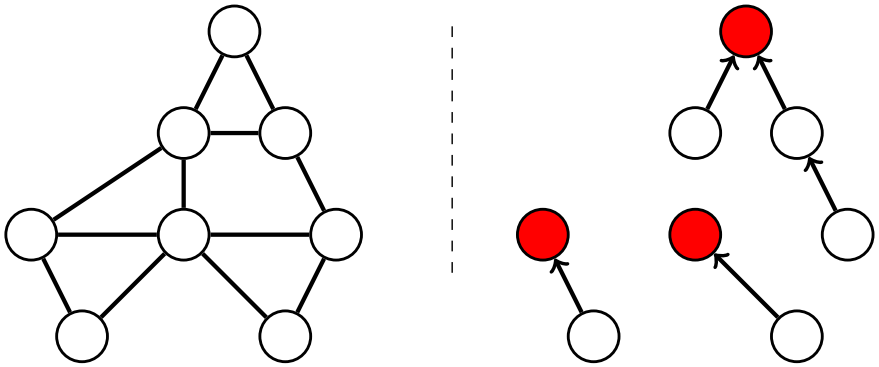

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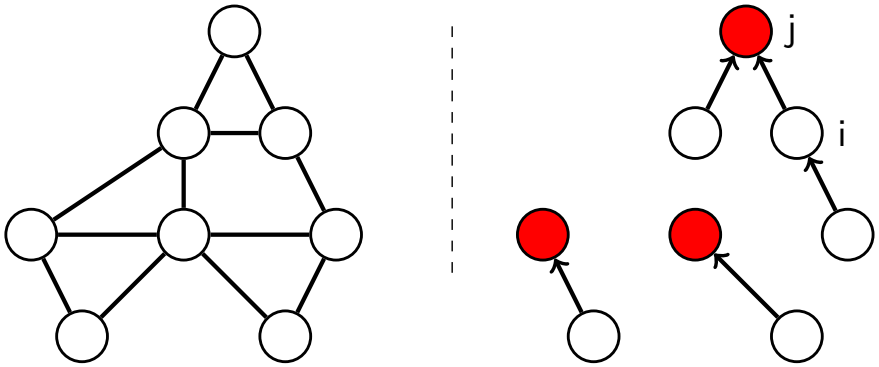

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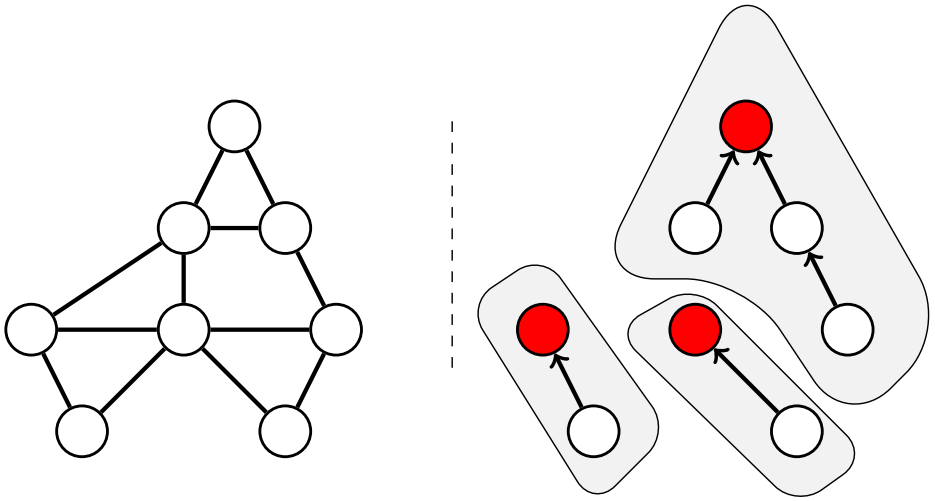

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Random Spanning Forests: **What**, Why and How?

Definition (RSF)

A random spanning forest Φ_q on a graph $\mathcal{G}$ is spanning forest selected over all spanning forests of $\mathcal{G}$ according to the following distribution:

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- $q > 0$ changes the expected number of roots.

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- Many other properties of USTs and more also extend to the RSFs.
- Moreover, we have the following identity:

$$\forall i, j \in \mathcal{V}, \quad \mathbb{P}(r_{\Phi_q}(i) = j) = \mathrm{K}_{i,j}, \text{ with } \mathrm{K} = q(\mathrm{L} + q\mathrm{I})^{-1}.$$

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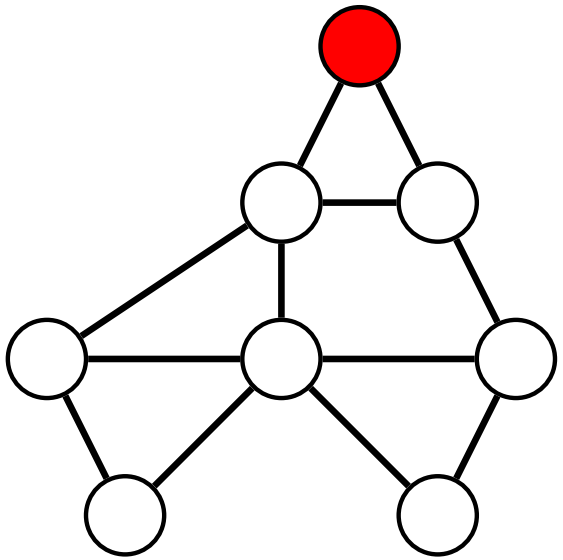

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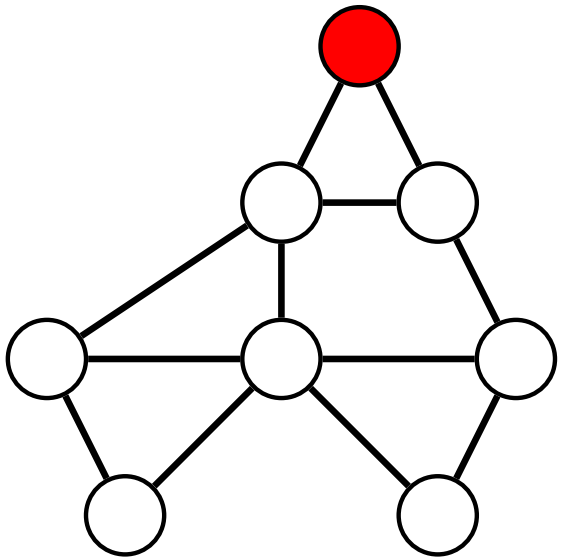

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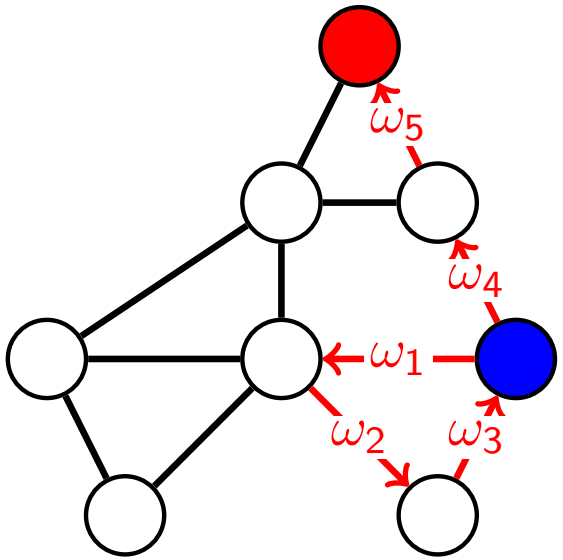

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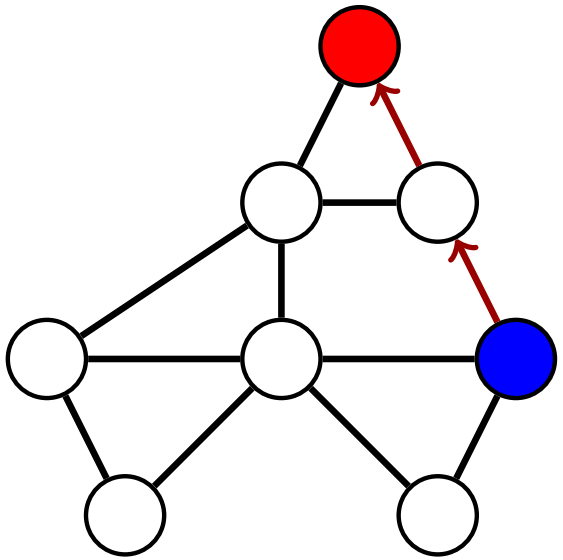

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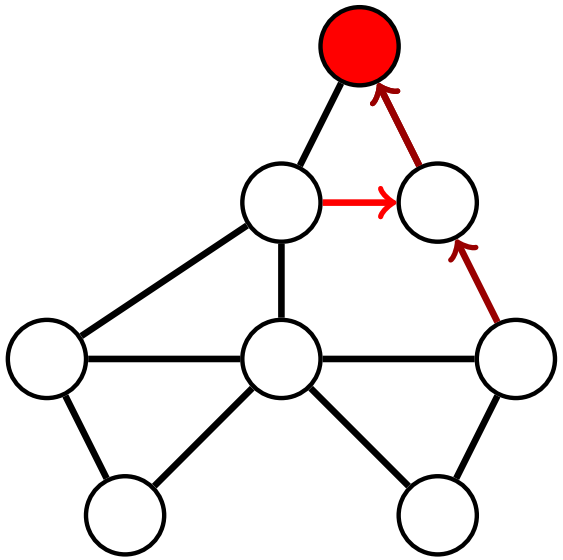

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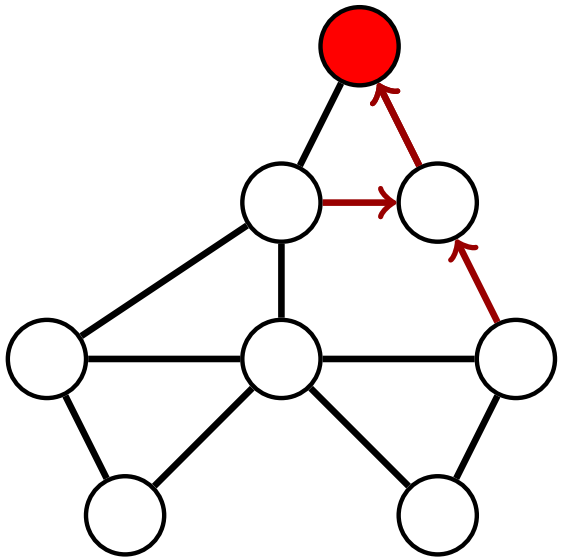

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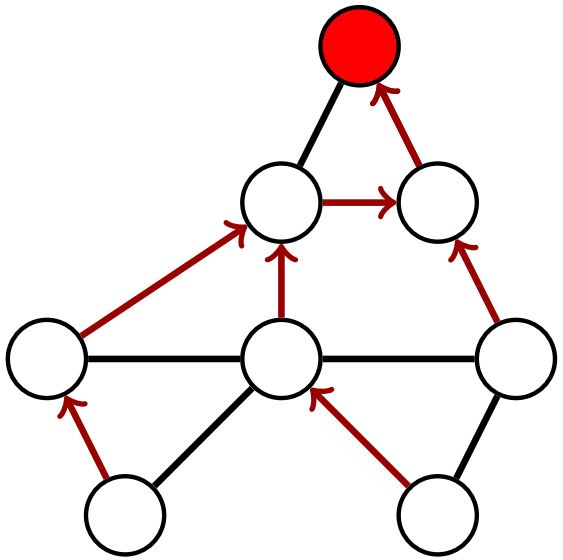

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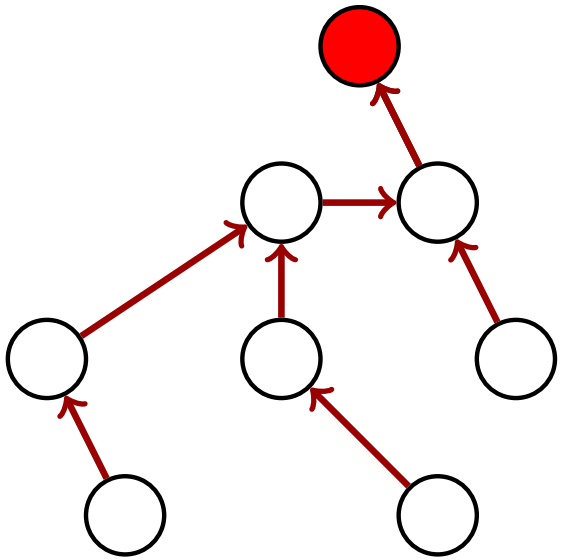

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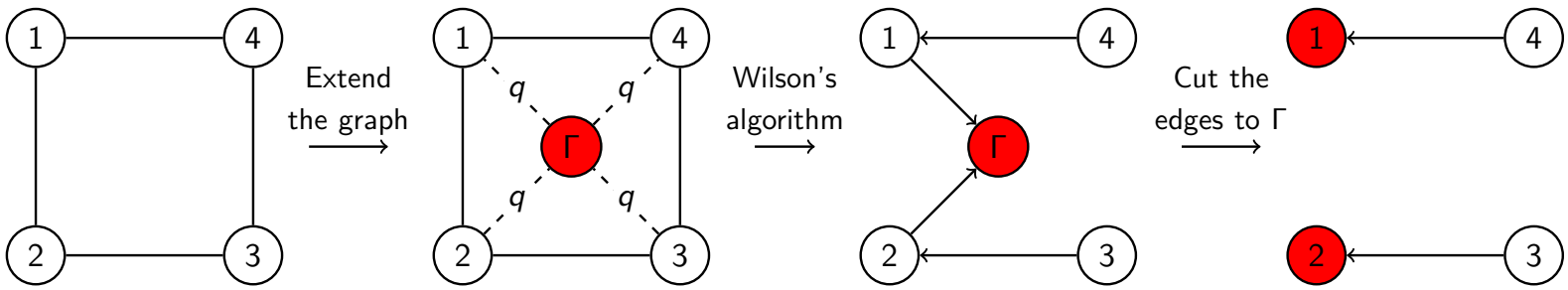

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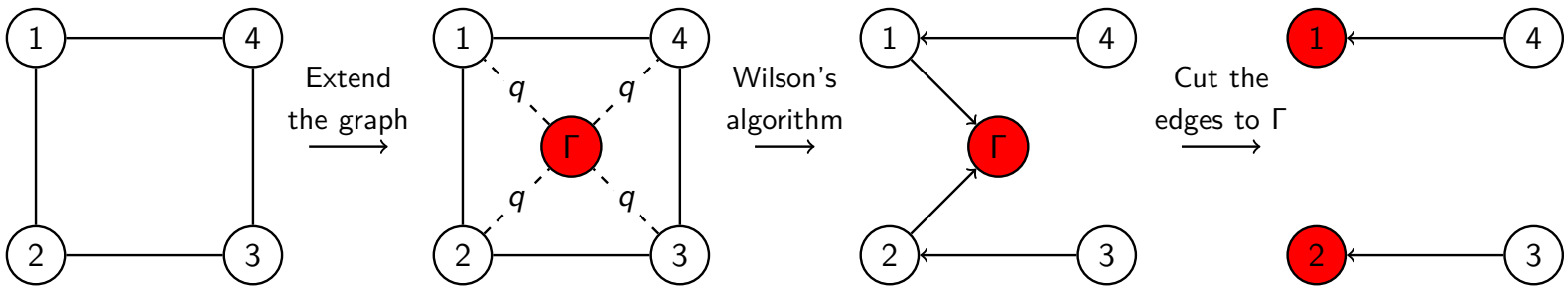

- The time complexity is $\mathcal{O}(\frac{|\mathcal{E}|}{q})$.

Problems

- Graph signal smoothing.
- Trace estimation.

Graph Signal Smoothing

Given a graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, w)$,

$$\hat{\mathbf{x}} = \arg\min_{\mathbf{z}\in\mathbb{R}^n} q \underbrace{||\mathbf{y} - \mathbf{z}||^2}_{\text{Fidelity}} + \underbrace{\mathbf{z}^T \mathrm{L} \mathbf{z}}_{\text{Regularization}} \quad , \quad q > 0$$

where L is the graph Laplacian and $\mathbf{z}^T \mathrm{L} \mathbf{z} = \sum_{(i,j)\in\mathcal{E}} w(i,j)(z_i - z_j)^2$.

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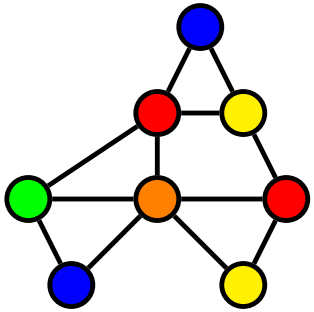
- Direct computation of K requires $\mathcal{O}(n^3)$ elementary operations due to the inverse.
- For large n, iterative methods and polynomial approximations are state-of-the-art. Both compute $\hat{\mathbf{x}}$ in linear time in the number of edges $|\mathcal{E}|$.

Forest Estimator



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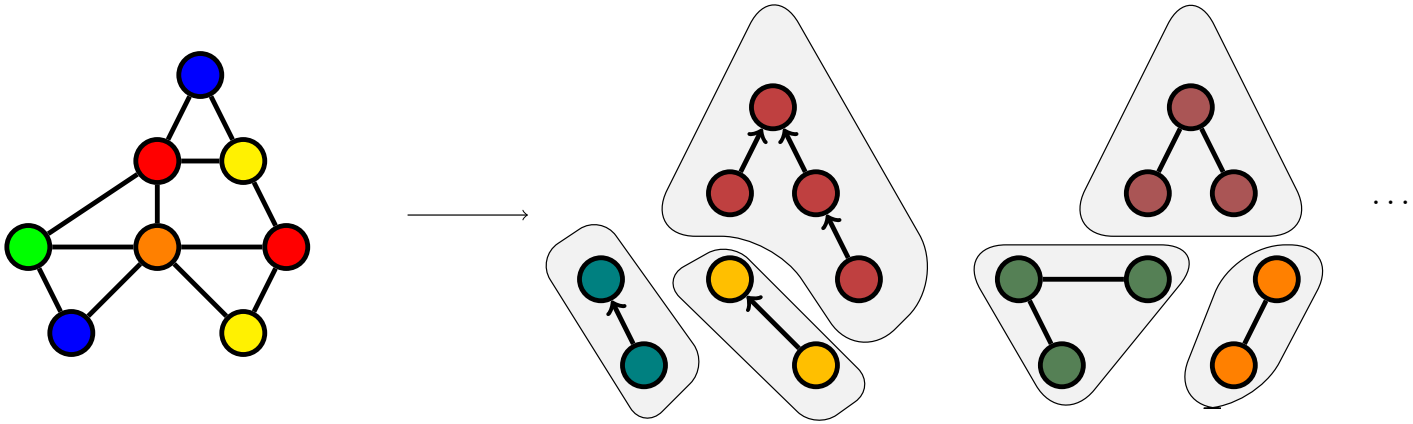

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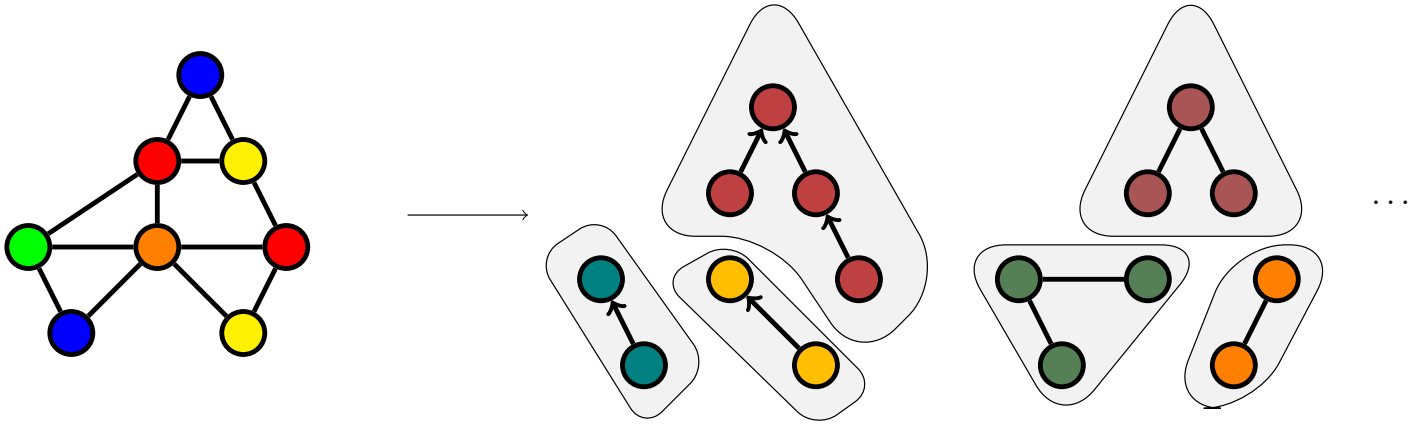

- Random partitions are sampled via random spanning forests.
- This yields an unbiased estimator $\bar{\mathbf{x}}$.

Comparison with State of the art (SOTA)

- We compare $\bar{\mathbf{x}}$ with:
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- We compare $\bar{\mathbf{x}}$ with:
  - Direct computation via Cholesky decomposition,
  - Polynomial approximation,
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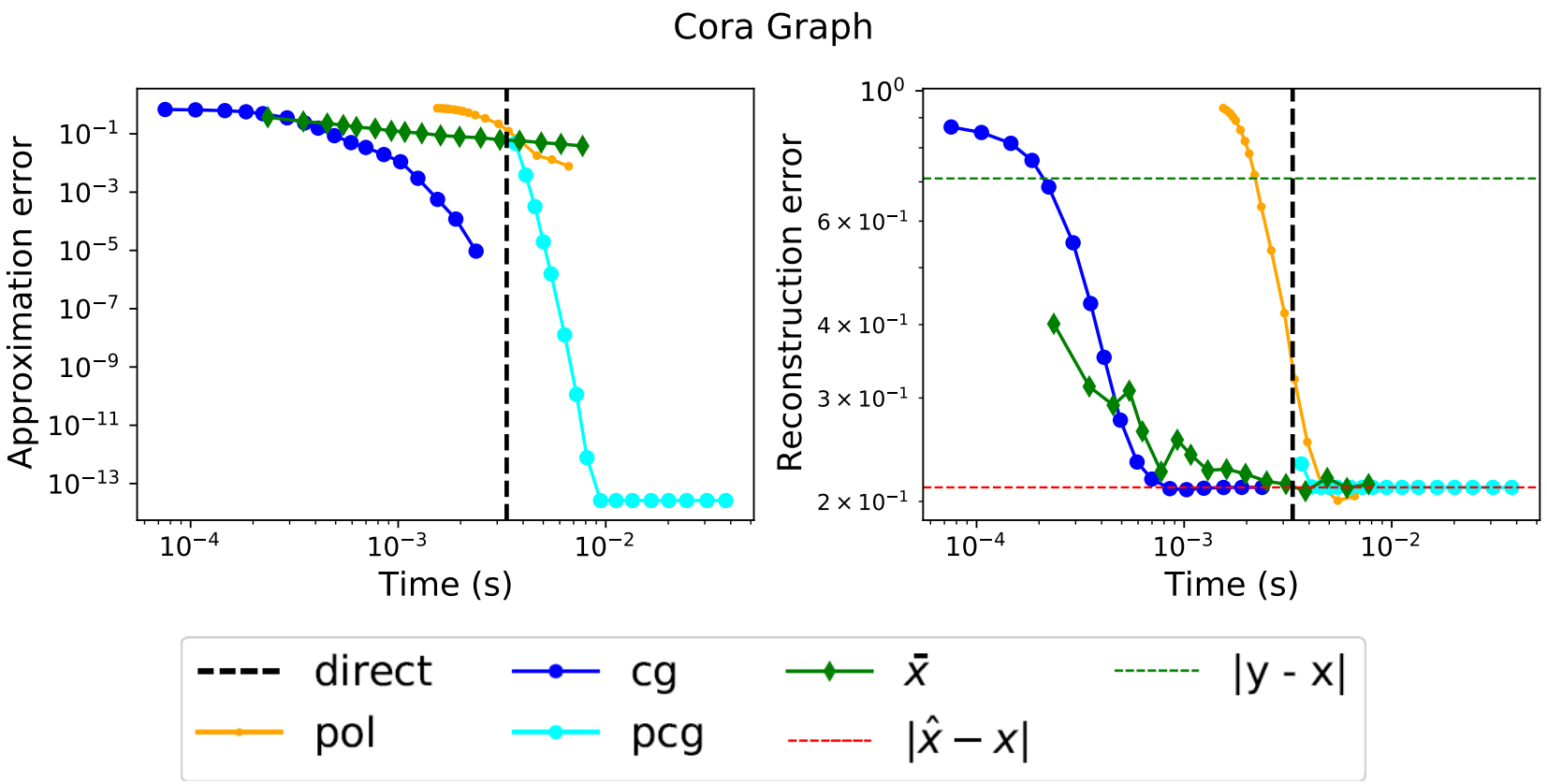
- We assume a smooth original signal $\mathbf{x}$ and noisy measurements $\mathbf{y} = \mathbf{x} + \epsilon$ with $\epsilon \in \mathcal{N}(0, \sigma)$.

Comparison with State of the art (SOTA)

- We compare all algorithms in approximation error (error respect to $\hat{\mathbf{x}}$) and reconstruction error (error respect to $\mathbf{x}$)



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- One technique is applicable to our case is called *control variate*.
- Control variate method deploys an additional random quantity which
  - is correlated with the estimated one,
  - has a known expectation

  to reduce the variance.

Gradient Descent Update as Control Variate

- The solution $\hat{\mathbf{x}}$ also minimizes:

$$F(\mathbf{x}) = \frac{1}{2}\mathbf{x}^\top \mathrm{K}^{-1}\mathbf{x} - \mathbf{x}^\top \mathbf{y}.$$

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  where $\alpha \in \mathbb{R}$ and $\nabla F(\mathbf{x}_k) = \mathrm{K}^{-1}\mathbf{x}_k - \mathbf{y}$.
- We propose to apply the gradient descent update on the previous estimator $\bar{\mathbf{x}}$:

$$\bar{\mathbf{z}} := \bar{\mathbf{x}} - \alpha(\mathrm{K}^{-1}\bar{\mathbf{x}} - \mathbf{y})$$

Properties of this estimator

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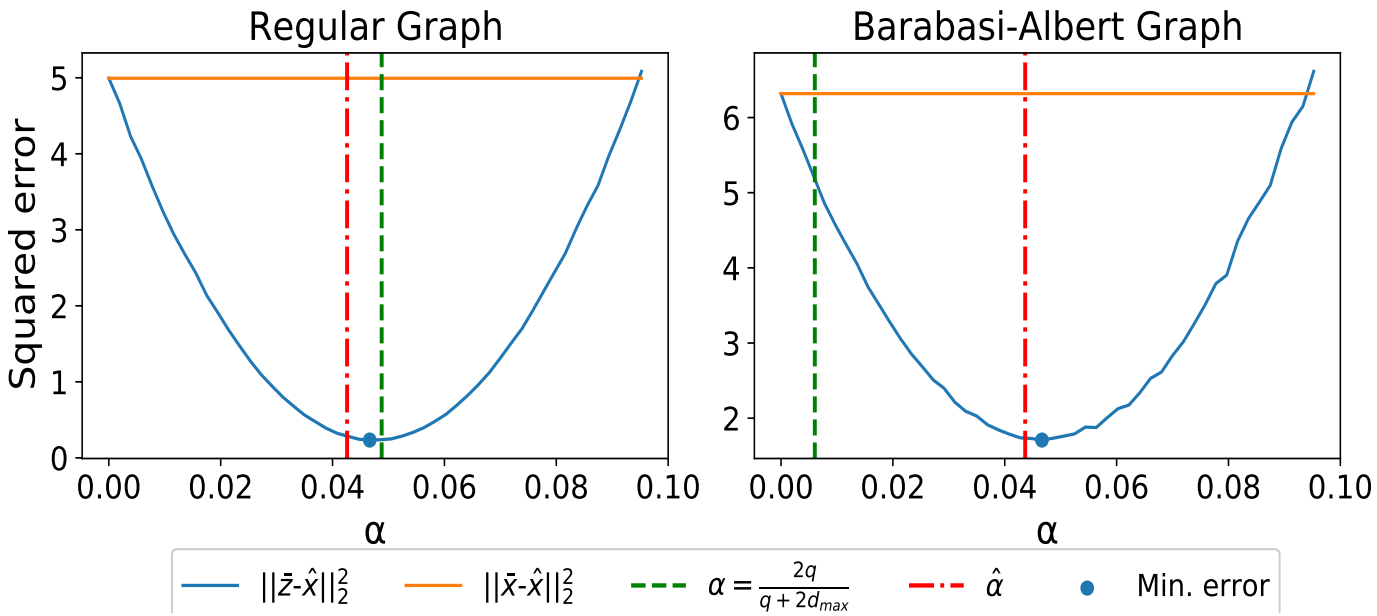
- One can either choose a value for α from the safe range (*e.g.* $\alpha = \frac{2q}{q+2d_{max}}$) or estimate from the samples:

$$\hat{\alpha} = \frac{\mathrm{tr}(\widehat{\mathrm{Cov}}(\mathrm{K}^{-1}\bar{\mathbf{x}}, \bar{\mathbf{x}}))}{\mathrm{tr}(\widehat{\mathrm{Var}}(\mathrm{K}^{-1}\bar{\mathbf{x}}))}.$$

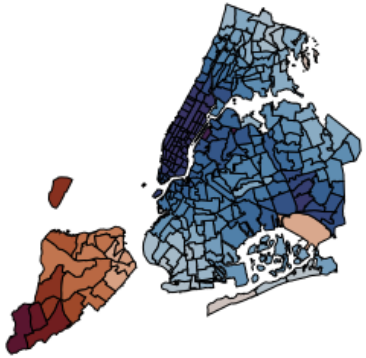
Range of α

- We empirically compare these options of α over a regular and irregular graph:

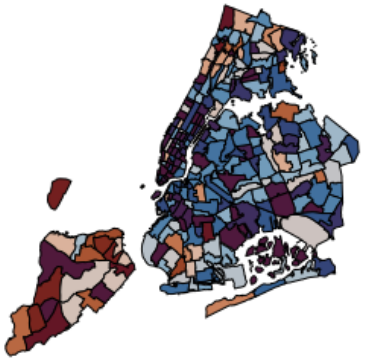


An Illustration

$\mathbf{x}$:



$\mathbf{y}$:



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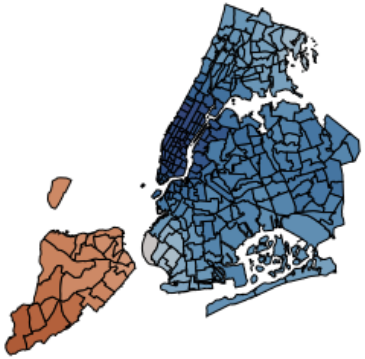

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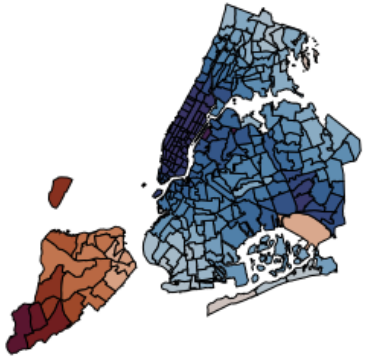

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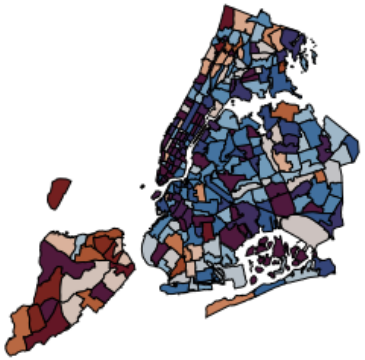

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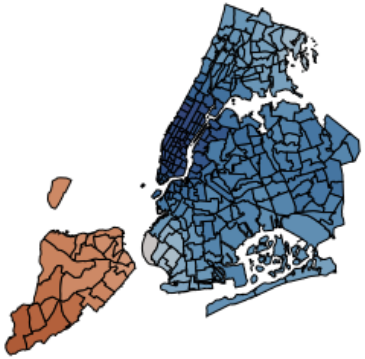

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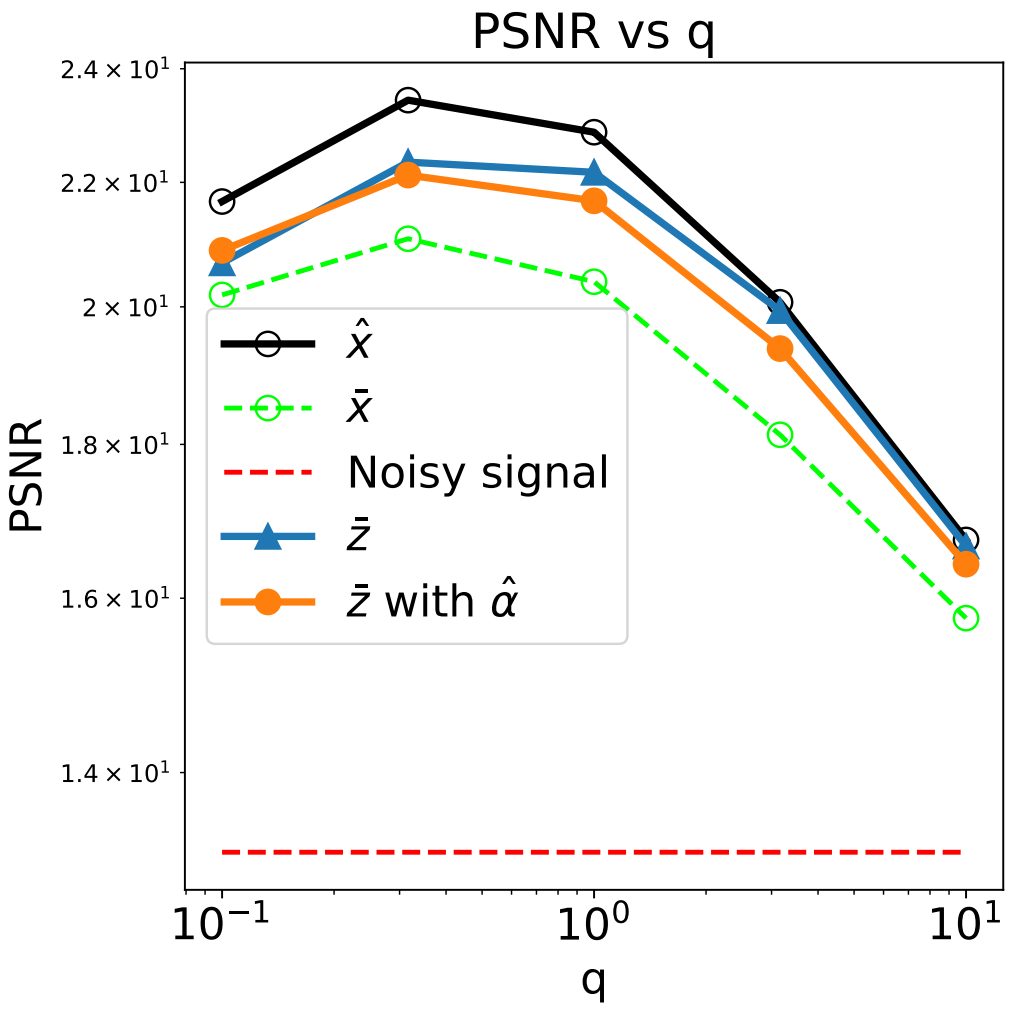

Figure: PSNR vs q, N=2

Inverse Trace Estimation: A Motivation

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- There are several methods such as Akaike's or Bayesian information criterion, generalized cross validation or Stein's unbiased risk estimator.
- Each uses a quantity called the effective degree of freedom which is equal to tr(K).

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- For some samples of a random vector $\mathbf{a}$, it needs to calculate $\mathbf{a}^T \mathrm{K}\mathbf{a}$.
- Solving for $\mathrm{K}\mathbf{a}$ can be done via:
  - Direct computation via Cholesky decomposition
  - (Preconditioned) Iterative solvers
  - Algebraic Multigrid solvers
  - ...

Forest based Trace Estimator

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- One can use this estimator in case of symmetric diagonally dominant matrices instead of the graph Laplacians.

Variance Reduction via Control Variates

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- A safe value of α is $\frac{2q}{q+d_{max}}$. We also observe that $\frac{q}{q+d_{avg}}$ is usually a good estimate of $\alpha^\star$.

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- Stratified sampling can substantially decrease approximation error when applicable.
- Thanks to rich theory of RSFs, we can apply stratified sampling on s.

Comparison with SOTA

- We compare the time needed by the estimators for reaching a certain accuracy.

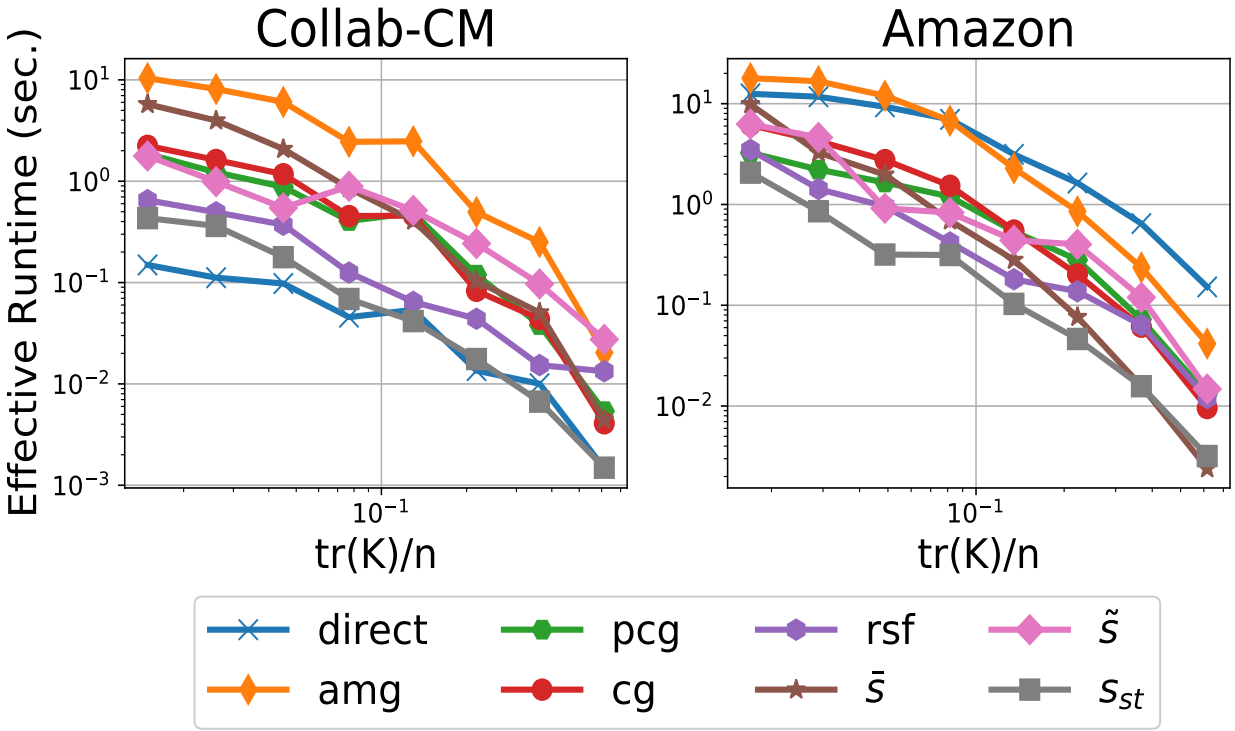

Figure: Effective Runtime vs $\mathrm{tr}(K)/n$.

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- This perspective leverages fascinating links between the graph Laplacians and RSFs, yielding applications in graph signal smoothing and inverse trace estimation.
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- Estimating effective resistances.

$$\mathrm{R}_{i,j} = \mathrm{L}^{\dagger}_{i,i} + \mathrm{L}^{\dagger}_{j,j} - \mathrm{L}^{\dagger}_{i,j} - \mathrm{L}^{\dagger}_{j,i}.$$

Questions

Thanks! Questions?

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