ECE 581K Computer Project 4 (Due Dec 2th 11:59pm 2021)

The problems are from the reference book listed in the syllabus: "Intuitive Probability and Random Processes using MATLAB" by Steven Kay.

1. Problem 1

16.7 (⌣̈) (c,f) A *biased* random walk process is defined as $X[n] = \sum_{i=0}^{n} U[i]$, where $U[i]$ is a Bernoulli random process with

$$p_U[k] = \begin{cases} \frac{1}{4} & k = -1 \\ \frac{3}{4} & k = 1. \end{cases}$$

What is $E[X[n]]$ and $\text{var}(X[n])$ as a function of n? Next, simulate on a computer a realization of this random process. What happens as $n \to \infty$ and why?

Please also try to tune the PMF of the Bernoulli random variable, P[K], and try to explain the results.

2. Problem 2

puter simulation. Specifically, generate $M = 10,000$ realizations of the AR random process $X[n] = 0.95X[n-1] + U[n]$ for $n = 0, 1, \ldots, 49$, where $U[n]$ is WGN with $\sigma_U^2 = 1$. Do so two ways: for the first set of realizations let $X[-1] = 0$ and for the second set of realizations let $X[-1] \sim \mathcal{N}(0, \sigma_U^2/(1-a^2))$, using a different random variable for each realization. Now estimate the variance for each sample time n, which is $r_X[0]$, by averaging $X^2[n]$ down the ensemble of realizations. Do you obtain the theoretical result of $r_X[0] = \sigma_U^2/(1-a^2)$?

3. Problem 3

18.5 (f,c) A discrete-time WSS random process $X[n]$ is defined by the difference equation $X[n] = aX[n-1] + U[n] - bU[n-1]$, where $U[n]$ is a discrete-time white noise random process with variance $\sigma_U^2 = 1$. Plot the PSD of $X[n]$ if $a = 0.9, b = 0.2$ and also if $a = 0.2, b = 0.9$ and explain your results.

4. Problem 4

18.13 (⌣̈) (f,c) A zero mean signal with PSD $P_S(f) = 2 - 2\cos(2\pi f)$ is embedded in white noise with variance $\sigma_W^2 = 1$. Plot the frequency response of the optimal Wiener smoother. Also, compute the minimum MSE. Hint: For the MSE use a "sum" approximation to the integral (see Problem 1.14).

5. Problem 5

18.14 (c) In this problem we simulate the Wiener smoother. First generate $N = 50$ samples of a signal $S[n]$, which is an AR random process (assumes that $U[n]$ is white Gaussian noise) with $a = 0.25$ and $\sigma_U^2 = 0.5$. Remember to set the initial condition $S[-1] \sim \mathcal{N}(0, \sigma_U^2/(1-a^2))$. Next add white Gaussian noise $W[n]$ with $\sigma_W^2 = 1$ to the AR random process realization. Finally, use the MATLAB code in the chapter to smooth the noise-corrupted signal. Plot the true signal and the smoothed signal. How well does the smoother perform?

6. Problem 6

20.30 (⌣̈) **(w)** It is desired to generate a realization of a WSS Gaussian random process by filtering WGN with an LSI filter. If the desired PSD is $P_X(f) = 2 - 2\cos(2\pi f)$, explain how to do this.

20.31 (⌣̈) **(c)** Using the results of Problem 20.30, generate a realization of $X[n]$. To verify that your data generation appears correct, estimate the ACS for $k = 0, 1, \ldots, 9$ and compare it to the theoretical ACS.