ECE 581K Computer Project 2 (Due Oct 11th 10:30am Beijing time 2021)

The problems are from the reference book listed in the syllabus: "Intuitive Probability and Random Processes using MATLAB" by Steven Kay.

1. Problem 1:

	$j=0$	$j=1$
$i=0$	$\frac{1}{8}$	$\frac{1}{8}$
$i=1$	$\frac{1}{4}$	$\frac{1}{2}$

Table 7.7: Joint PMF values for Example 7.15.

(⌣̈) **(c)** Using the joint PMF shown in Table 7.7 generate realizations of the random vector (X, Y) and estimate its joint and marginal PMFs. Compare your estimated results to the true values.

2. Problem 2

(⌣̈) **(c)** For the joint PMF shown in Table 7.7 determine the correlation coefficient. Next use a computer simulation to generate realizations of the random vector (X, Y) and estimate the correlation coefficient as

$$\hat{\rho}_{X,Y} = \frac{\frac{1}{M}\sum_{m=1}^{M} x_m y_m - \bar{x}\bar{y}}{\sqrt{\left(\frac{1}{M}\sum_{m=1}^{M} x_m^2 - \bar{x}^2\right)\left(\frac{1}{M}\sum_{m=1}^{M} y_m^2 - \bar{y}^2\right)}}$$

where

$$\begin{aligned}\bar{x} &= \frac{1}{M}\sum_{m=1}^{M} x_m \\ \bar{y} &= \frac{1}{M}\sum_{m=1}^{M} y_m\end{aligned}$$

and (x_m, y_m) is the mth realization.

3. Problem 3

(w,c) If $X \sim \text{geom}(p)$, $Y \sim \text{geom}(p)$, and X and Y are independent, show that the PMF of $Z = X + Y$ is given by

$$p_Z[k] = p^2(k-1)(1-p)^{k-2} \qquad k = 2, 3, \ldots.$$

To avoid errors use the discrete unit step sequence. Next, for $p = 1/2$ generate realizations of Z by first generating realizations of X, then generating realizations of Y and adding each pair of realizations together. Estimate the PMF of Z and compare it to the true PMF.

4. Problem 4

(w,c) Consider the nonlinear transformation

$$\begin{aligned} W &= X^2 + 5Y^2 \\ Z &= -5X^2 + Y^2. \end{aligned}$$

Write a computer program to plot in the x-y plane the points (x_i, y_j) for $x_i = 0.95 + (i-1)/100$ for $i = 1, 2, \ldots, 11$ and $y_j = 1.95 + (j-1)/100$ for $j = 1, 2, \ldots, 11$. Next transform all these points into the w-z plane using the given nonlinear transformation. What kind of figure do you see? Next calculate the area of the figure (you can use a rough approximation based on the computer generated figure output) and finally take the ratio of the areas of the figures in the two planes. Does this ratio agree with the Jacobian factor

$$\left| \det\left(\frac{\partial(w,z)}{\partial(x,y)} \right) \right|$$

when evaluated at $x = 1$, $y = 2$?

5. Problem 5

(c) If $X_i \sim \mathcal{N}(1,1)$ for $i = 1, 2 \ldots, N$ are IID random variables, plot a realization of the sample mean random variable versus N. Should the realization converge and if so to what value?

6. Problem 6

(w,c) Let $X_{1_i} \sim \mathcal{U}(0,2)$ for $i = 1, 2 \ldots, N$ be IID random variables and let $X_{2_i} \sim \mathcal{N}(1,4)$ for $i = 1, 2 \ldots, N$ be another set of IID random variables. If the sample mean random variable is formed for each set of IID random variables, which one should converge faster? Implement a computer simulation to check your results.

7. Problem 7

14.12 (⌣̈) **(w,c)** An $N \times 1$ random vector $\mathbf{X}$ has $E_{X_i}[X_i] = \mu$ and $\text{var}(X_i) = i\sigma^2$ for $i = 1, 2, \ldots, N$. The components of $\mathbf{X}$ are independent. Does the sample mean random variable converge to μ as N becomes large? Carry out a computer simulation for this problem and explain your results.