SDDiP_with_EnhancedCut

Generation Expansion

$\min \ \sum_{t=1}^T (c_t^1 x_t + c_t^2 y_t + p s_t)$

$\text{ s.t. } \sum_{s=1}^t x_s \le \bar{u},$

$\mathbf{1}^\top y_t + s_t \ge d_t^\omega,$

$h_t N(S_t + S_0) \ge y_t,$

$S_t = \sum_{s=1}^t x_s,$

$x_t \in \mathbb{Z}_d^+, y_t \in \mathbb{R}_d^+.$

Let x_t be a vector representing number of different types of generators to be built in stage t, y_t be a vector of the amount of electricity produced by each type of generator per hour in stage t, and s_t be a scalar slack variable in stage t to ensure the relatively complete recourse with corresponding penalty p. In the formulation, c_t^1 and c_t^2 are investment and generation cost with the operation and maintenance (OM) at stage t, respectively.

The matrix N Contains maximum rating and maximum capacity information of generators, $\bar{u}$ is a pre-determined construction limits on each type of generators due to resource and regulatory constraints, h_t is the number of hours in stage t and d_t^ω is the electricity demand at stage t, where only $\{d_t^\omega\}_{t=1,...,T}$ are subject to uncertainty.

To specify the data, according to paper `Jin2011`, we have the following data:

There are total 6 kind of generators, and the parameter d stands for the category of generators, hence $d = 6$.

- From table 4, we can obtain the build cost c_g For each type.
- From table 7, $\bar{u}$.
- From the table 6, we can obatin the following data:
  - The number of already existed generators S_0;
  - Install capacity m_g;
  - Generator rating N.
- Based m_g and c_g, we can compute the investment for type i at stage t by $c_g[i] * m_g[i]/(1+r)^t$ where $r = 0.008$ is the annualized interest rate.
- From the table 5, we can compute the generation cost c_2 for each type at stage t with OM costs by

$$\text{FuelCost} \times 1.02^t \times 10^{-6} \times \text{HeatRate} \times \text{Efficiency} + \text{OM cost} \times 1.03^t \ .$$

- And the penalty is $p = 1e5$.
- $h_t = 8760$ for all t from table 2.

> @article{Jin2011ModelingAS, title={Modeling and solving a large-scale generation expansion planning problem under uncertainty}, author={Shan Jin and Sarah M. Ryan and Jean-Paul Watson and David L. Woodruff}, journal={Energy Systems}, year={2011}, volume={2}, pages={209-242} }

SDDiP Formulation

In order to make the above model conform to the conditions of the sddip algorithm, i.e., the stage variable is binary and the problem is multi-stage, we do the following transformation:

Note that $S_t \leq (4, 10, 10, 1, 45, 4)^\top$ from the data; to binarize the variables, we introduce the binary variables for each compoent.

$$S_t = \begin{bmatrix} l_1 + 2l_2 + 4l_3 \\ l_1 + 2l_2 + 4l_3 + 8l_4 \\ l_1 + 2l_2 + 4l_3 + 8l_4 \\ l_1 \\ l_1 + 2l_2 + 4l_3 + 8l_4 + 16l_5 + 32l_6 \\ l_1 + 2l_2 + 4l_3 \end{bmatrix} = AL_t,$$

Where A Is a coefficient matrix and L_t is a vector with 21 components. ($n = 21$)

$$\begin{aligned}
&\min \ \textstyle\sum_{t=1}^{T} (c_t^1 x_t + c_t^2 y_t + p s_t) \\
&\text{ s.t. } S_t \leq \bar{u}, \\
&\mathbf{1}^\top y_t + s_t \geq d_t^\omega, \\
&h_t N(S_t + S_0) \geq y_t, \\
&S_t = \textstyle\sum_{s=1}^{t} x_s, \\
&S_t = AL_t, \\
&L_t \in \{0, 1\}^n, x_t \in \mathbb{Z}_d^+, y_t \in \mathbb{R}_d^+.
\end{aligned}$$

For stage t, given the previous decision L_{t-1}, we can formulate it by following:

$$\begin{aligned}
&\min \ c_t^1 x_t + c_t^2 y_t + p s_t + \theta_t \\
&\text{ s.t. } AL_{t-1} + x_t \leq \bar{u},
\end{aligned}$$

$\mathbf{1}^\top y_t + s_t \geq d_t^\omega,$

$hN(AL_{t-1} + x_t + S_0) \geq y_t,$

$AL_t = AL_{t-1} + x_t,$

$L_t \in \{0,1\}^n, x_t \in \mathbb{Z}_d^+, y_t \in \mathbb{R}_d^+.$

We introduce a local binary copy L_c for L_{t-1}, then we have: (Stage variable is L_t)

$\min \; c_t^1 x_t + c_t^2 y_t + p s_t + \theta_t$

$\text{ s.t. } AL_c + x_t \leq \bar{u},$

$\mathbf{1}^\top y_t + s_t \geq d_t^\omega,$

$L_c = L_{t-1},$

$hN(AL_c + x_t + S_0) \geq y_t,$

$AL_t = AL_c + x_t,$

$L_t \in \{0,1\}^n, L_c \in [0,1]^n, y_t \in \mathbb{R}_d^+.$

Enhanced Cut Formulation

- Forward Step

$\min \; c_t^1 x_t + c_t^2 y_t + p s_t + \theta_t$

$\text{ s.t. } AL_c + x_t \leq \bar{u},$

$\mathbf{1}^\top y_t + s_t \geq d_t^\omega,$

$L_c = L_{t-1},$

$hN(AL_c + x_t + S_0) \geq y_t,$

$AL_t = AL_c + x_t,$

$L_t \in \{0,1\}^n, \; L_c \in [0,1]^n, \; x_t \in \mathbb{Z}_d^+, \; y_t \in \mathbb{R}_d^+,$

$\theta_t \geq \sum_{\omega \in \Omega_t} q^\omega (v_l^\omega + (\pi_l^\omega)^\top (L_t - \tilde{L})), \; \forall l = 1, \ldots, i-1.$

- Backward Step

$\max \; F(\pi) + \pi^\top (\tilde{L} - \hat{L}_{t-1})$

$$\text{s.t. } F(\pi) \geq (1-\epsilon) f^*$$

Where $F(\pi)$ is the following optimization problem (Backward_F)

$$\min\ c_t^1 x_t + c_t^2 y_t + p s_t + \theta_t + \pi^\top (\hat{L}_{t-1} - L_c)$$

$$\text{s.t. } AL_c + x_t \leq \bar{u},$$

$$\mathbf{1}^\top y_t + s_t \geq d_t^\omega,$$

$$hN(AL_c + x_t + S_0) \geq y_t,$$

$$AL_t = AL_c + x_t,$$

$$L_t \in \{0,1\}^n,\ L_c \in [0,1]^n,\ y_t \in \mathbb{R}_d^+,$$

$$\theta_t \geq \sum_{\omega \in \Omega_t} q^\omega (v_l^\omega + (\pi_l^\omega)^\top (L_t - \tilde{L})),\ \forall l = 1, \ldots, i-1.$$

Where f^* the optimal value of the forward optimization problem.

Level-set Method