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Strategic network expansion of urban rapid transit systems: A bi-objective programming model

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Abstract
With the development of urbanization and the extension of city boundaries, the expansion of rapid transit systems based on the existing lines becomes an essential issue in urban transportation systems. In this study, the network expansion problem is formulated as a bi-objective programming model to minimize the construction cost and maximize the total travel demand covered by the newly introduced transit lines. To solve the bi-objective mixed-integer linear program, an approach called minimum distance to the utopia point is applied. Thus, the specific trade-off is suggested to the decision makers instead of a series of optimal solutions. A real-world case study based on the metro network in Wuxi, China, is conducted, and the results demonstrate the effectiveness and efficiency of the proposed model and solution method. It is found that the utopia method can not only provide a reasonable connecting pattern of the network expansion problem but also identify the corridors with high priority under the limited budget condition.

1 | INTRODUCTION

Mass rapid transit as the most important public transportation system has been promoted by many governments worldwide for several reasons: (1) it is a large-capacity transport mode whose capacity is five to 10 times that of a bus, (2) it is reliable and environmentally friendly, and (3) it reduces road traffic congestion as it attracts tremendous travel demand from private transport. As a result, building an effective rapid transit system is a vital task for governments to promote public transport, especially for megacities, such as Beijing and London. Because of the progressive nature of the city development, the rapid transit system is commonly developed phase by phase, which can be seen from the construction history of some urban transit systems, such as Shanghai Metro (Table 1). Thus, how to design new rapid transit lines based on the existing transit network while respecting the construction budget is a critical challenge. This problem can become extremely difficult when the network size is large; thus, decision tools are necessary.

The traditional rapid transit network design problem (RTNDP) has been investigated in the literature, and some effective models have been proposed to determine the optimal station location and line routing. However, several unique issues arise in the network expansion problem compared to the traditional network design problem:

- The transfer connection between the existing lines and the new planning lines is an inevitable constraint;
- The uncovered travel demand in the city shows more urgency than the covered one, which has been fulfilled by the existing system. The traditional RTNDP assumes that all preassigned corridors lay the lines, but the network expansion problem calls for corridor selection when the budget is limited. Under the limited budget circumstances, those areas with urgent and enormous travel demand should be given higher priority for developing rapid transit system. Seeking the balance between the number of lines and cost is asked for in the network expansion problem.

TABLE 1 The construction plan of Shanghai Metro

Years	Construction plans
1996–2000	Line 1
2000–2005	Line 2 (part), Line 3
2005–2007	Line 4 (part)
2007–2009	Line 1, Line 6, Line 4 (part)
2009–2010	Line 7, Line 11
2010–2012	Line 10
2012–2013	Line 13
2013	Line 12, Line 16

The rapid transit system design and operation problem are commonly tackled in a sequential manner that involves the following decision subproblems: network design, frequency settings, capacity settings, vehicle scheduling, and crew scheduling and rostering. Among them, the network design is considered the most important subproblem because it belongs to the strategic level and is difficult to change once constructed. This article studies transit line planning and station location problem based on an existing network at the strategic level, which is called the *transit network expansion problem*.

For transit network design and optimization, one most frequently used objective is to cover the travel demand nodes as many as possible. With a given set of demand nodes scattering among the study area, the network design problem is to determine a line in such a way that as many demand nodes lie on or close to the line as possible. Another common way is to take the origin–destination (OD) trips into account, and to maximize the satisfied OD trips by the constructed transit network. Considering that accurate OD trip data are not available in the strategic planning stage, we choose maximizing the covered travel demand and minimizing the budget as two objectives, and propose a bi-objective model to determine the optimal transit network expansion decisions.

Bi-objective optimizing model usually offers a series of solutions on the Pareto front, and it leaves decision makers at a loss for selecting a particular solution. In this article, we introduce an approach based on the criteria of minimal distance to the utopia point to obtain a balanced trade-off between the construction cost and the amount of travel demand covered. The contributions of this article are as follows:

- Proposal of a bi-objective programming model to tackle the strategic transit network expansion problem to provide the locations of stations and connecting patterns of lines.
- Application of the approach based on the minimal distance to the utopia point into the RTNDP to suggest a trade-off as a specific plan, which is commonly presented as the Pareto front in previous studies.
- Demonstration of the effectiveness of the proposed model and solution method and obtain managerial insights using a real-world case study based on the rapid transit network in Wuxi, China.

The remainder of this article is organized as follows. The literature review is presented in Section 2. In Section 3, the bi-objective model is developed; then, the solution method for the bi-objective model is presented. A case study based on Wuxi's transit network system planning is conducted in Section 4, and Section 5 concludes the article.

2 | LITERATURE REVIEW

In this section, we review the existing studies on rapid transit network design, network extension, and solution methods on bi-objective models.

2.1 | Network design problem

Guihaire and Hao (2008), Laporte, Marín, Mesa, and Perea (2011a), Farahani, Miandoabchi, Szeto, and Rashidi (2013), and Kepaptsoglou and Karlaftis (2009) reviewed the transit network design literature, and a few mathematical programs were available for the strategic network design problem. Hamacher, Liebers, Schöbel, Wagner, and Wagner (2001) found the effects of adding new stations in the railway network in two stages. Then, Laporte, Mesa, and Ortega (2002) proposed an effective model to solve the problem of locating stations on a given alignment by maximizing the covered population. Laporte, Marín, Mesa, and Ortega (2007) extended the previous models by incorporating the four-stage process and optimization method to settle the locations of stations and lines. The objective is to maximize the covered demand based on the OD demand while satisfying the budget constraint. Marín (2007) studied a network that contained a free but bounded number of lines to select the OD locations of lines. Escudero and Muñoz (2009) developed a modification of the transit network design problem to enable the definition of circular lines and presented a two-stage approach for solving this new problem. Daganzo (2010) idealized the urban regions as squares with uniform demand and described the network shapes as grids and hub-and-spoke structures. Then, Estrada, Roca-Riu, Badia, Robusté, and Daganzo (2011) developed Daganzo's idea to combine the agency's cost and user level of service in the optimization. Daganzo (2012) proposed the model that guided the government to optimize the performance of the urban public infrastructure including the cover demand, cost, and price. Then, Laporte et al. (2011a) proposed a network design model to provide a near-optimal network both when an arc was inoperative and when no failure occurred. Gutiérrez-Jarpa, Obreque, Laporte, and Marianov (2013) first proposed a model to find the exact configuration (locations of stations and segments) of a set of lines (more than one), with the objectives of minimizing the construction cost and maximizing the OD demand. The authors introduced the concept of corridors to address the shape of lines in advance. The travel speed in the corridors can be obtained from the speed prediction model (see Tajalli & Hajbabaie, 2018; Yao et al., 2017). Laporte and Pascoal (2015) proposed a path-based algorithm for the metro network design with the objectives of maximizing the covered demand and minimizing the construction cost. Gutiérrez-Jarpa, Laporte, Marianov, and Moccia (2017) improved the model proposed in 2013 to compare the travel time difference between taking the metro and driving cars and developed a multi-objective model, which included satisfied travel demand, saving travel time, and construction cost. Gutiérrez-Jarpa, Laporte, and Marianov (2018) then treated the corridors themselves as the lines instead of computing the exact shapes of the line in the corridors and the main contribution of the article is to present a new method to generate the corridors by an algorithm instead of manually determining the corridors according to experience. Jin, Tang, Sun, and Lee (2014) integrated the bus service into the metro network to increase the metro network resilience. Hossein Rashidi, Rey, Jian, and Waller (2016) proposed a bi-objective model that could find the best stations choices on a given route of transit line and considered the elastic demand. Szeto, Jaber, and O'Mahony (2010) proposed a single-objective discrete network design framework considering the land-use transport interaction over time. Moreover, Wang and Szeto (2017) developed the bi-objective design problem into a multi-objective problem by considering the environmental sustainability, where the three-dimensional Pareto front is used to list all optimal solutions.

One of the most common objectives in the network design problem is maximizing the covered population as suggested by Escudero and Muñoz (2009), Laporte, Mesa, Ortega, and Perea (2011b), and Marín (2007). Some other works focus on the covered trip based on the OD matrix as suggested by Sun, Jin, Lee, Axhausen, and Erath (2014). In conclusion, some models can suggest the precise shape of the transit network. Gutiérrez-Jarpa et al. (2013) and Gutiérrez-Jarpa et al. (2017) defined the network based on the preassigned corridors, and Gutiérrez-Jarpa et al. (2018) treated the corridors as lines and found an algorithm to define corridors. However, these models and cases assume that once a corridor is defined, the embedded line in it must exist. Besides, the terminal stations (i.e., ending stations) have to lie in the predetermined *extreme set* (see Gutiérrez-Jarpa et al., 2013). Aiming at relaxing the assumptions, we propose a model that is capable of selecting corridors with higher priority among a given corridor set, and at the same time locating the terminal stations anywhere along the corridors.

2.2 | Network expansion problem

The network expansion possesses different characteristics with the network design problem. One of the most distinct points is adjoint of new lines and existing network. Matisziw, Murray, and Kim (2006) introduced an integrated method of geographic information systems and spatial optimization model to solve the existing transit network. The article proposed a bi-objective model that maximized the cover demand while minimizing the extension distance. It derived a suboptimal solution by the weighting method, which involved specifying a weight that reflected the relative desirability of each objective. Based on the single-line solution, Matisziw et al. (2006) solved two lines separately and enumerated and combined them based on different weights. Therefore, it is a line extension instead of a network expansion plan, and we would like to develop the line extension problem to the network level. In addition to the design problem, the network extension problem is also highly related to the phase split and construction management. Adeli and Karim (1997) proposed a general mathematical formulation to schedule construction projects, which can be applied in the transit network construction. Then, Adeli and Karim (2001) and Karim and Adeli (1999a, 1999b) improved the object-oriented model to determine the optimized construction cost.

2.3 | Solution to the bi-objective problem

The most popular method to solve the bi-objective problem is to obtain the Pareto front (see Pan, He, Tian, Su, & Zhang, 2017; Sousa, Alçada-Almeida, & Coutinho-Rodrigues, 2017). Gutiérrez-Jarpa et al. (2013), Gutiérrez-Jarpa et al. (2017), and Wang and Szeto (2017) and most network design problems use the trade-off to illustrate a series of the optimal solutions. Chakroborty (2003) identified some features of the RTNDP that made it difficult to solve; then, they proposed genetic algorithms, which were an evolutionary optimization mechanism to solve it. Fan and Machemehl (2008) testified that the Tabu search-based heuristic methods showed better performance than the genetic algorithms when solving the nonlinear bus transit route network design problem. Belotti, Soylu, and Wiecek (2016) reviewed the algorithm to solve the bi-objective and provided a quick solution using the extended Branch and Bound to obtain the Pareto front. Rostami, Neri, and Epitropakis (2017) proposed a novel algorithm by employing a progressive preference sector approach to conduct the decision. The preference sector approach (also referred to as the utopia point method) has been successfully applied in several fields such as the electric power network problem (see Aghaei, Baharvandi, Rabiee, & Akbari, 2015; Deng, Liu, Ouyang, Lin, & Xie, 2017; Ning & You, 2018), energy harvesting problem (see Wang, Liu, Yuan, & Chen, 2018), and shortest path problem (Granat & Guerriero, 2003). However, this approach has not been applied in the network design problem.

As discussed above, the mathematical model for network expansion problem has not been fully investigated. The model is accomplished by grouping the candidate points, which represent the travel demand, into several corridors and selecting the corridors where lines should be constructed. In this article, we propose a bi-objective model and apply the approach of minimum distance to the utopia point to obtain the best trade-off.

3 | MODEL FORMULATIONS

In this section, the formal description of the problem is presented. Section 3.1 defines the transit network expansion problem. Section 3.2 emphasizes the assumption in the model. Section 3.3 lists all variables and parameters in the problem, and the bi-objective mixed integer linear program is presented in Section 3.4. Section 3.5 introduces a solution approach called minimum distance to the utopia point method.

3.1 | Problem definition

A transit network consists of several lines defined by the nodes and the arcs that connect them. Therefore, the key task of the RTNDP is to determine the locations of the stations and the connecting patterns. In the case of the network expansion problem, the stations that must be rebuilt as transfer stations between existing and new lines should also be identified. With the distribution of unsatisfied travel demand and the candidates of rebuilt stations, the underlying network can be defined.

The strategic expansion of rapid transit network is defined on an undirected graph $G(N_0, A)$, where $N_0 = \{1, \ldots, n\}$ includes all nodes and $A = \{(i, j) : i, j \in N, i < j\}$ contains all arcs. The node set consists of three subsets. The first subset N_t contains the stations on the existing lines, and they are the candidate transfer stations that connect the existing and new lines. The second subset N contains candidate stations whose travel demand is to be satisfied by the new lines. The third subset N_e is the set of dummy end nodes. Connecting arcs between every two different nodes are defined as the arc set A. However, in consideration of the minimum–maximum distance rules of adjacent stations, only those arcs satisfying the range constraint are considered. The arc between any node and the dummy node is defined regardless of the range constraint. We define a line segment as a series of connecting arcs, and a corridor as an area where the metro line segment should be located. In the network expansion planning problem, the most distinct difference with the new planning one is that there are already several operational lines. The continuity between the existing lines and the new constructing lines is primarily considered. Therefore, a single segment should go through the candidate transfer nodes and the new candidate stations to ensure the continuity of existing and new network.

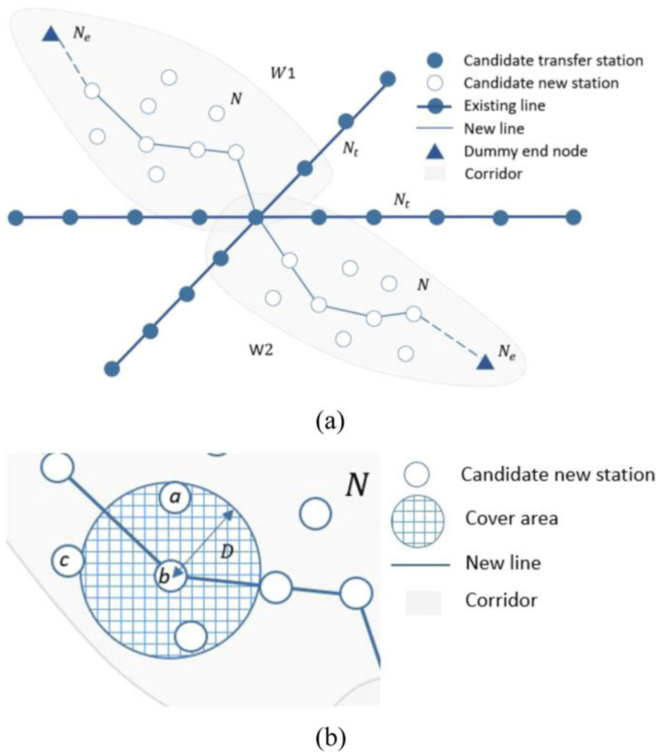

FIGURE 1 Illustration of the urban rail transit network: (a) A simple example of the corridors; (b) Conditions of travel demand being covered

As an example, Figure 1a depicts the elements of the model. In the example, there are two existing lines, made up of the set N_t (solid circles), which represent the candidate transfer stations. The candidate nodes (set N) are shown as hollow circles, whose travel demand has not been satisfied by the existing transit network. The demand will be fulfilled under two conditions. First, the demand of a node is satisfied when it is constructed as a station (e.g., Node b in Figure 1b). Second, the sufficiently close node is selected to be a station (e.g., the travel demand of Node a in Figure 1b can be satisfied as Node b is a station). The dummy end node set N_e (solid triangle) is sufficiently far from all other nodes. N_t, N, and N_e jointly form a corridor set (the shaded area), and there are two corridors in the example. A segment in a corridor extends from the candidate transfer station, passes through the candidate nodes, and ends at the dummy end node. The arcs between the candidate node and the dummy end node are included only for modeling purpose. The entire network consists of several lines and corridors.

3.2 | Assumptions

We make some realistic assumptions regarding different factors that affect network expansion problem to maintain the problem computationally tractable.

1. The number of construction lines is limited.

According to the experiences of construction of large-scale transit networks worldwide, the urban transit network is designed and constructed phase by phase; in each phase, only several line segments are completed. The number of constructing lines in a period is limited because of the high expenditures and significant inconvenience on the daily trip in the process of construction.

2. The corridors are nonoverlapping.

The purpose of the network expansion is to cover as much travel demand as possible with limited resources. The overlapping corridors will cause the repeated coverages of one travel demand. Therefore, the nonoverlapping corridors are necessary.

3.3 | Parameters and variables

Sets:

- N_t set of the candidate transfer stations (nodes on existing lines)
- N set of candidate stations (nodes on new lines)
- N_e set of dummy end stations
- W set of corridors
- N_0 set of nodes in the network, where $N_0 = N_t \cup N \cup N_e$
- A set of candidate arcs
- Ω_j^w set of consecutive arcs in corridor w that connect with node j and cannot be simultaneously constructed
- $G(N_0, A)$ graph where N_0 represents the nodes and A represents the arcs

Parameters:

- d_{ij}^w the distance between i and j in corridor w
- r_i^w the travel demand generated by node i in corridor w
- f_i^w the construction cost of node i in corridor w
- g_t^w the construction cost of transfer node t in corridor w
- h_{ij}^w the construction cost of the arc (i, j) in corridor w
- S the limitation of the transfer lines at a single station
- D the cover range of the station
- L the largest number of new lines in a period

Decision variable:

- z_t^w equal to 1 if node t in corridor w is selected to be the transfer station; 0 otherwise
- x_i^w equal to 1 if candidate station i in corridor w is selected to be the new station; 0 otherwise
- y_i^w equal to 1 if the demand of point i in corridor w is satisfied; 0 otherwise
- λ_{ij}^w equal to 1 if there is a path between node i and j in corridor w; 0 otherwise
- ρ_{ij}^w equal to 1 if candidate node i is in the covered area of node j in corridor w; 0 otherwise

3.4 | Mathematical program

The bi-objective mixed integer linear program is as follows:

Objective:

$$\max z_d = \sum_{w\in W}\sum_{i\in N} r_i^w y_i^w \tag{1}$$

$$\min z_c = \sum_{w\in W}\sum_{i,j\in N} d_{ij}^w \lambda_{ij}^w + \sum_{w\in W}\sum_{i\in N} f_i^w x_i^w + \sum_{w\in W}\sum_{t\in T} g_t^w z_t^w \tag{2}$$

Subject to:

$$\sum_{j\in N\cup N_e} \lambda_{tj}^w = z_t^w \quad \forall\, t \in N_t, w \in W \tag{3}$$

$$\sum_{j\in N} \lambda_{tj}^w \le S \;\forall\, t \in N_t, w \in W \tag{4}$$

$$\sum_{i\in N\cup N_t} \lambda_{ie}^w = x_e^w \quad \forall e \in N_e, w \in W \tag{5}$$

$$\sum_{i\in N, i\ne j} \lambda_{ij}^w - \sum_{l\in N, l\ne j} \lambda_{jl}^w = 0 \;\forall j,\; j \ne t, e, w \in W \tag{6}$$

$$x_i^w \le \sum_{j|(i,j)\in A} \lambda_{ij}^w \;\forall i \in N, w \in W \tag{7}$$

$$x_i^w \le \sum_{j|(i,j)\in A} \lambda_{ji}^w \;\forall i \in N, w \in W \tag{8}$$

$$\lambda_{ij}^w \le x_i^w \;\forall i \in N, w \in W \tag{9}$$

$$\lambda_{ij}^w \le x_j^w \;\forall j \in \mathrm{N}, w \in W \tag{10}$$

$$\mathrm{d}_{ij}^w \rho_{ij}^w \le D \;\forall i, j \in N, w \in W \tag{11}$$

$$x_j^w + \sum_{i\in N, i\ne j} x_i^w \rho_{ij}^w \ge y_j^w \;\forall j \in N, w \in W \tag{12}$$

$$\sum_{w \in W} \sum_{t \in T} z_t^w - \sum_{w \in W} \lambda_{te}^w \leq L \; t \in N_t, \; e \in N_e \tag{13}$$

$$\sum_{\forall (i,j) \in A} \lambda_{ij}^w - \sum_{i \in N} x_i + 1 \leq 0, w \in W \tag{14}$$

$$\lambda_{ij}^w + \lambda_{jk}^w \leq 1, \; \forall w \in W, \forall (i,j), (j,k) \in \Omega_j^w \tag{15}$$

$$z_t^w \in \{0, 1\} \tag{16}$$

$$x_i^w \in \{0, 1\} \tag{17}$$

$$y_i^w \in \{0, 1\} \tag{18}$$

$$\lambda_{ij}^w \in \{0, 1\} \tag{19}$$

$$\rho_{ij}^w \in \{0, 1\} \tag{20}$$

Objective (1) maximizes the satisfied travel demand. Objective (2) minimizes the construction cost, which includes the cost of rebuilding the transfer nodes, constructing new lines and stations. Constraint (3) guarantees the continuity between the existing line and the new line at the transfer station. Constraint (4) ensures a maximum number of interchange lines crossing at each transfer station. This constraint can be useful in the condition that the massive transfer passenger flow is a critical concern and should be kept within a safety level. Constraint (5) represents all segments that end at dummy nodes. Constraint (6) is the flow conservation restriction to ensure the balance of inflow and outflow at each node. Constraints (7) and (8) indicate that if a node is selected to be constructed, at least one line must start and end at the node. Constraints (9) and (10) guarantee that both ends of a selected arc should be constructed. Constraint (11) determines the value of ρ_{ij}^w. Constraint (12) determines the condition when the demand of node j can be fulfilled, that is, node j or the neighboring nodes that can cover the demand of node j are selected (as shown in Figure 1b). Constraint (13) restricts the number of planning lines in a construction period. The amount of transfer stations excludes the number of arcs that directly connect the existing stations, and the dummy end node represents the number of effective segments.

Constraint (14) guarantees that there are no sub-tours in the corridors. Without the constraint, there may be circular lines, which are neither reasonable in the real plan nor logical in the model. The shapes of the lines are controlled by the direction of the corridors, and thus the next connecting station only locates in the forward direction. For example, in Figure 2, the nodes in the shaded area are candidate stations to directly connect with Node 7. Another way of avoiding sub-tours is to

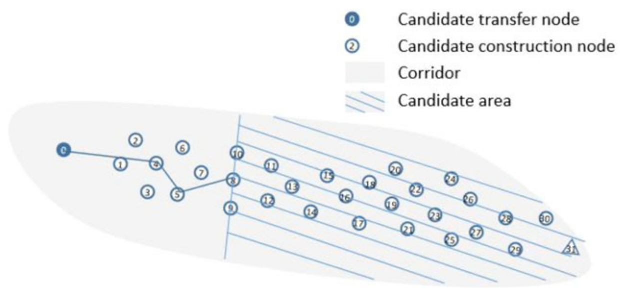

FIGURE 2 Illustration of the ordinal labeling method

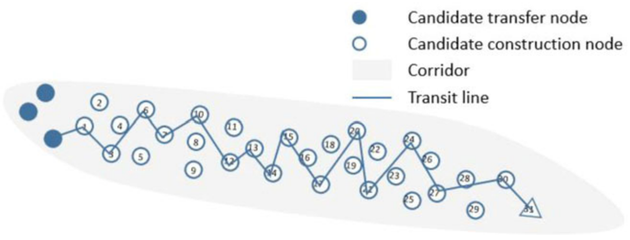

FIGURE 3 Illustration of the squiggly line

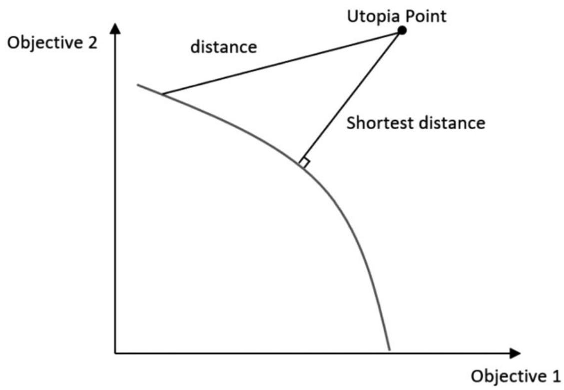

FIGURE 4 Illustration of the utopia point

employ the ordinal labeling method, in which nodes in a corridor are labeled in an increasing manner from the candidate transfer station to the dummy end node. Therefore, for any station on the line, it will connect with a small-label station and a large-label station in both directions to avoid the circular line.

Constraint (15) eliminates the unreasonable connection patterns such as the squiggly line in Figure 3. For example, for Node 21, the simultaneous connection with Nodes 20 and 24 is unreasonable. Set Ω_j^w is introduced in the constraint, which restricts that two arcs connected to node j cannot be simultaneously constructed.

3.5 | Solution to the bi-objective model

The most popularmethod to solve a bi-objective problem is to transfer the problem into a single-objective model using the ϵ-constraint method. We can change the weights of two objectives to generate the Pareto front. As the two objectives require maximum and minimum values, we present the objective as

FIGURE 5 Travel demand in Wuxi

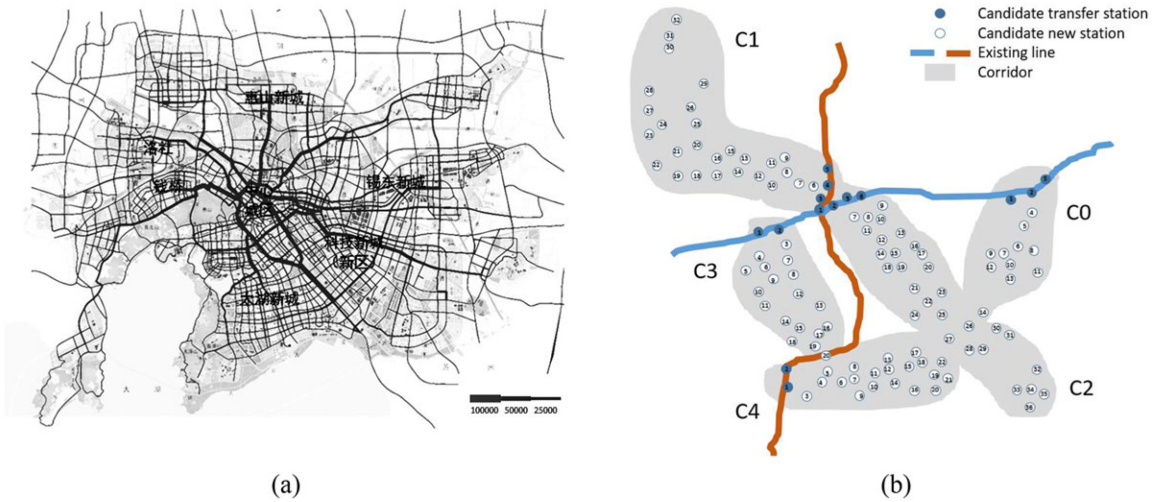

FIGURE 6 (a) Illustration of the travel corridor; (b) Illustration of the network graph

TABLE 2 Parameters in the model and index in the solution

Model parameters			Solution indexes	
Parameter	**Value**	**Unit of measure**	**Index**	**Value**
Travel demand	10	Thousand passengers per day	Step	0.1
Line cost	1	Billion RMB/km	ξ_d	0.05
Station cost	5	Billion RMB per station	ξ_s	0.001
Transfer station cost	30	Billion RMB per station	Initial weight	(0.1,0.9)
D	1	km		
L	5	Number of lines		

Note: RMB indicates China Yuan Renminbi.

TABLE 3 Solution of the network design

Corridor	Connecting station (z_i)	Candidate station (x_i)	Covered demand (y_i)	Total demand in corridor
0	1	4,7,14	70	110
1	4	7,8,13,16,24,28,30,32	200	270
2	2	8,13,16,20,22,26,29,32,36	260	300
3	1	4,6,14,16,20	150	180
4	1	3,4,5,7,12,15,18,22,27	190	210

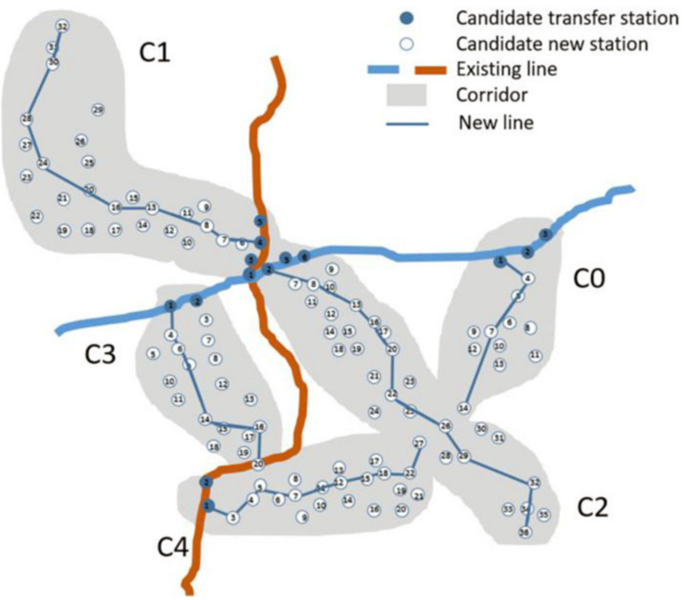

FIGURE 7 Explicit solution of the case

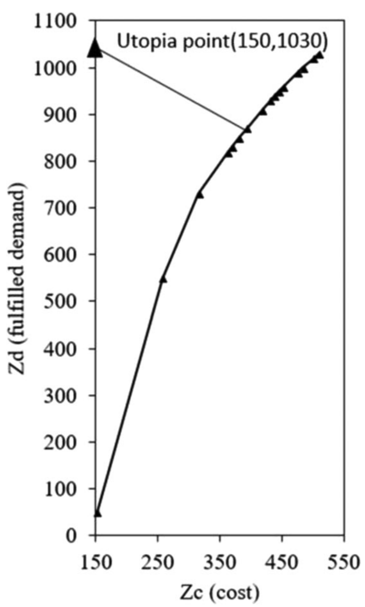

FIGURE 8 Trade-off curve, z_d vs. z_c

follows:

$$\text{objective } z_\alpha = \max\left(\alpha_1{}^* z_d - \alpha_2{}^* z_c\right), \quad \alpha_1 + \alpha_2 = 1 \tag{21}$$

Objective z_α shows the comprehensive value of z_d and z_c, where α_1 and α_2 are the weights of z_d and z_c, respectively. In the rectangular coordinate system, the two objective values project to the x-axis and y-axis, and the curve presents the Pareto front, which is a series of optimal solutions. However, it does not provide the decision maker a specific plan, which finds a balance between two or more objectives. By applying the utopia point, we can find the point with the shortest distance to the utopia point and seek a balance between demand and budget.

The utopia point is computed in the ideal condition, where one of the objectives is entirely considered while the other is completely relaxed. The utopia point is an ideal solution that cannot be reached in real cases. The dimension of the utopia point is decided by the number of objective functions. The distance between the point on the Pareto front and the utopia point is calculated using the Pythagorean theorem. When the distance is minimal, the connection between two points is vertical to the Pareto front as shown in Figure 4. This solution is considered the balanced one among multiple objectives.

The optimal solution with the shortest distance to the utopia point can be obtained by the following procedure:

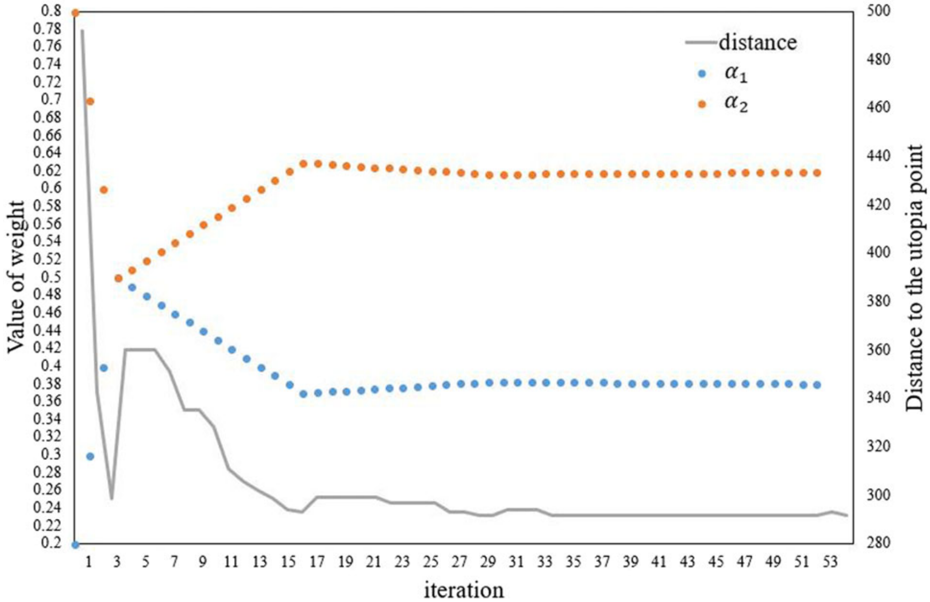

FIGURE 9 Distances to the utopia point, values and weights of two objectives of each iteration

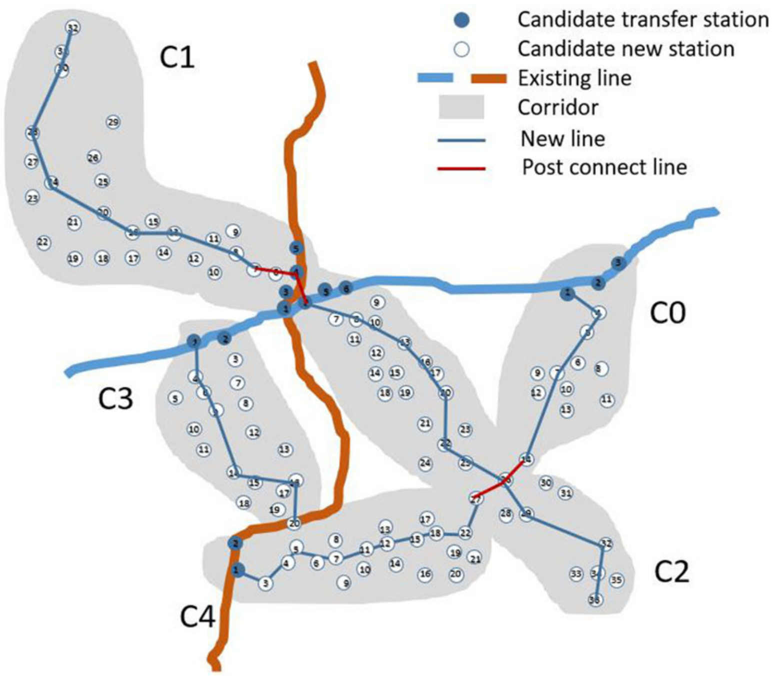

FIGURE 10 Post-optimization of the solution

Step 1. Find the utopia point and initialize parameters. Let α_1 be equal to 1 (α_2 equal to 0) to find the optimal satisfied travel demand z_d^*. Similarly, let α_1 be equal to 0 (α_2 equal to 1) to find the optimal construction cost z_c^*. Thus, the position of the utopia point (z_c^*, z_d^*) can be obtained. Then, initialize weight parameter α_1 and step size β.

Step 2. Calculate current active solution. Given the weight parameter α_1, we solve the weighted single objective model by the solver CPLEX. Calculate the construction cost and satisfied travel demand (z_c^1, z_d^1) associated with the optimal solution, and calculate the distance between the solution and the utopia point D_1.

Step 3. Obtain the neighboring solution. Set $\alpha_1 \leftarrow \alpha_1 + \beta$, compute the corresponding solution (z_c^2, z_d^2) and its distance to the utopia point D_2.

Step 4. Update solution and parameters. If $D_2 < D_1$, update the current active solution as the neighboring one and set $\alpha_1 \leftarrow \alpha_1 + \beta$. Else update $\beta \leftarrow -0.1\beta$.

Step 5. Check stopping criteria. If the gap $\frac{|D_2 - D_1|}{D_1} \leq \xi_d$ or the step size $|\beta| \leq \xi_s$, stop. Else go to Step 2.

TABLE 4 Parameters in case 2

Parameter	Value	Unit of measure	Parameter	Value	Unit of measure
Travel demand	10	Thousand passengers per day	Transfer station cost	30	Billion RMB per station
Line cost	10	Billion RMB/km	Cover range	1	km
Station cost	20	Billion RMB per station	L	5	Number of lines

Note: RMB indicates China Yuan Renminbi.

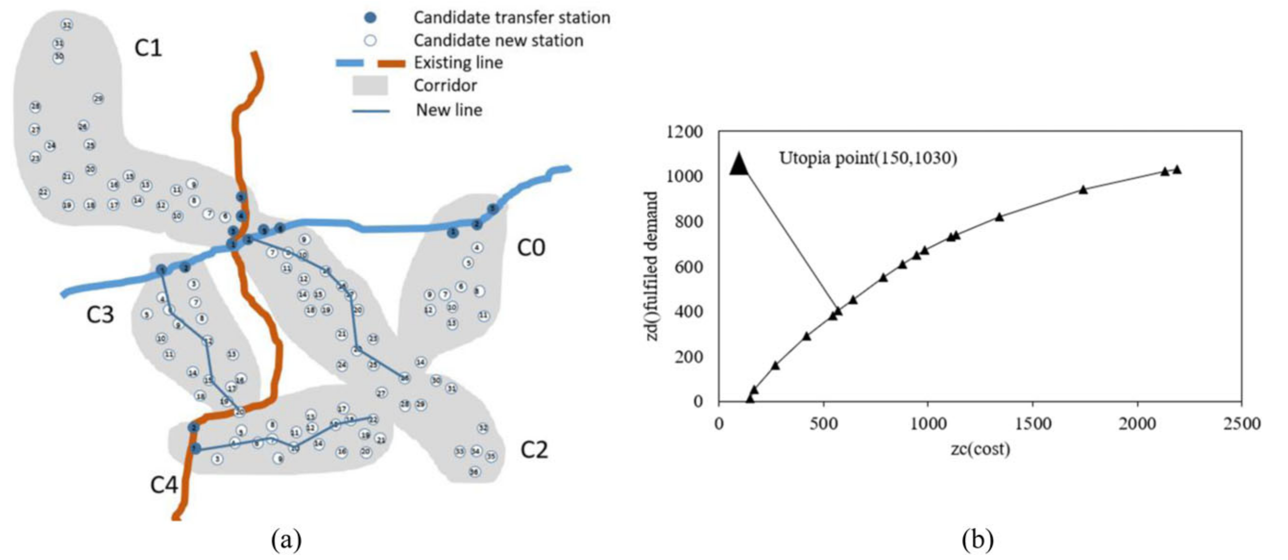

FIGURE 11 Solution of the corridor choice case: (a) Corridor choice solution; (b) Trade-off curve of z_d vs. z_c

4 | CASE STUDY

A real-world case study is conducted to validate the performance of the network expansion model. We first present the data set and parameters in Section 4.1. The computational results are presented in Section 4.2. A post-optimization is conducted in Section 4.3. The corridor selection is discussed in Section 4.4. The utopia point is further discussed in Section 4.5.

4.1 | Data and parameters

The data set is based on the transit network and travel demand in Wuxi City, China. The two thick full lines in Figure 5 represent two existing subway lines in Wuxi and the solid circles represent the travel demand zone. First, the study area is partitioned into a set of zones considering the land-use patterns and population size. Subsequently, each zone is represented by an individual node (see Figure 5) serving as an aggregation of all the actual travel origins and destinations in the zone. The exact locations of the nodes should be determined considering the road network and popular origins/destinations within the zone. For example, it should be put in close proximity to road junctions and commercial blocks. Lastly, the travel demand of each zone is estimated based on the land-use pattern and population characteristics. Methods for transportation planning, such as household travel survey, traffic counts, could be employed for estimating the travel demand. Considering that each node serves as a station candidate, the zoning step needs to be adjusted repeatedly to avoid significant variation of the travel demand among different zones. In this study, we adjust the zoning and set the travel demand of each zone roughly the same. Accordingly, the nodes unevenly distribute in the study area as the central city owns more travel demand than the suburb. Figure 6a shows a map of the road network of the city of Wuxi, where roads are represented by lines. The thickness of the lines indicates the traffic volume on the roads. This map is very helpful for determining the corridors of the rapid transit systems, as the orientation of the major roads indicated by thicker lines reflects the overall travel demand pattern of the entire Wuxi City. In this study, we identify five corridors accordingly, as are shown in Figure 6b. In total, more than 150 travel demand zones in the entire city have not been satisfied by the existing lines, and the five corridors may cover most of the travel demand in the central city. The solid and hollow circles represent existing and candidate stations, respectively.

A significant parameter is the cover range of the candidate station. With the development of the bike-sharing system, a significant number of people are willing to ride a bike to the metro station. An investigation on the willingness to take the metro shows that most people think that less than 12 min of walking or less than 8 min of riding appears to be acceptable to take the subway for commuting. Therefore, one candidate node can cover the travel demand within the 1 km range.

Parameters in the model computation are listed on the left of Table 2. The average costs are obtained from the public transport agency and government statistic report. The equivalent construction cost of the transfer station is composed of the construction cost and equivalent economic loss in the period. The index in the computational experiment is presented on the right of Table 2.

4.2 | Computational results

The computational experiment is solved by CPLEX 12.8.1 with JAVA on a PC Intel Core i7 at 3.4 GHz with 8GB RAM. The optimal solution with the minimum distance to the utopia point is shown in Table 3 and Figure 7. The searching procedure of solving the bi-objective programming model ran 55 iterations until the optimal solution was obtained. The utopia point was computed to be 150 for z_c and 1030 for z_d. The trade-off of the two objectives is shown in Figure 8, where the vertical axis shows the value of the fulfilled demand and the horizontal axis shows the construction cost. The distances to the utopia point, objective values, and weights of two objectives of each iteration are listed in Figure 9; the weight of z_d converges to 0.38081. More than 80% of the travel demand can be covered by the new transit lines. The computational time is 22.86 s, which is acceptable considering of the strategic planning nature of the transit network expansion problem. We remark that the optimal solution obtained from our proposed model is highly similar to the manually designed development plan of the transport agency. The cover demands in the solution and real plan vary by less than 20%. Moreover, 28 stations of 34 total are constructed in real life, and four of five transfer stations are fit to the real plan. Therefore, the proposed model could be helpful for decision making in the transit network expansion.

In some scenarios, the solution does not change with the weight, for example, iterations 16–20 in Figure 9. Two stop conditions were set to prevent the program from stopping when the optimal solution has not been reached. The first one is to restrict that the error of distance is small enough. The second one is to restrict that the precision of the weight should satisfy the demand.

4.3 | Post-optimization

A post-adjustment procedure for the corridors' connection is necessary if the suggested lines are not well-linked together. Based on the optimal solution, adding connections among the corridors promotes the connectivity of the entire network. In the intersection of two corridors, we manually connect the corridor as shown in Figure 10. Five segments in different corridors can be well-integrated into three lines. The covered demand and station construction cost will not increase due to the additional connection.

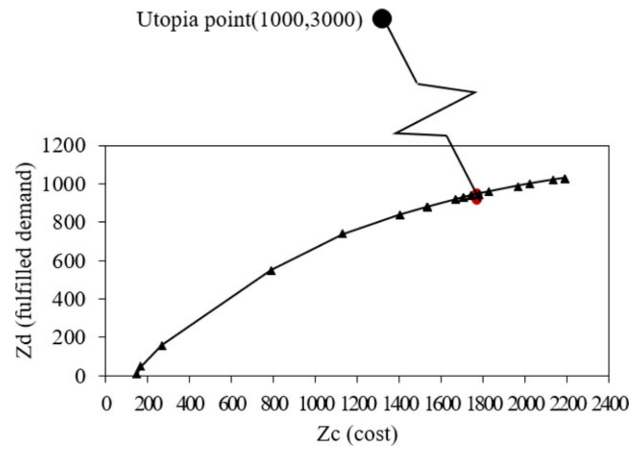

FIGURE 12 Searching process of the proposed bi-objective model solution approach and its optimal solution

4.4 | Corridor choice

The proposed model is also capable of identifying corridors with high priority and suggesting those to be constructed first in the case of limited budget. We further run an experiment in which the construction cost is very high (as shown in Table 4). The optimal solution is shown in Figure 11a, and the trade-off is illustrated in Figure 11b. When the weight of z_d is 0.58689, the optimal solution is found within 15.07 s. Given a limited budget and high cost, there is no line lying in Corridor 0 and Corridor 1, and the segments in Corridors 2, 3, and 4 are shorter than those in the first case, and only 40 travel demands are covered. The model and solution algorithm can well obtain the optimal corridor choice under different budget scenarios and show the capability of finding the priority of the corridors.

4.5 | Discussion of the utopia point method

As discussed in Section 3.5, the utopia point is determined by relaxing one of the objective functions and combining the two unreachable goals together. Another way is for decision makers to set the utopia point manually, according to their goals. Figure 12 shows an illustrative example, in which the utopia point is set at (1000, 3000). As a result, the optimal solution is updated to a point that is closest to the new utopia point after 39 iterations.

As for the effectiveness, the utopia point method shows much higher efficiency than the Pareto front. As Figure 13 illustrates, the trade-off curve of the case in the Section 4.4 is generated by 100 iterations for the accuracy of 0.01 for the weight. In comparison, for the accuracy of 0.0001 in the utopia point method, the average number of iterations needed based on 10 runs is only 47. This is because the

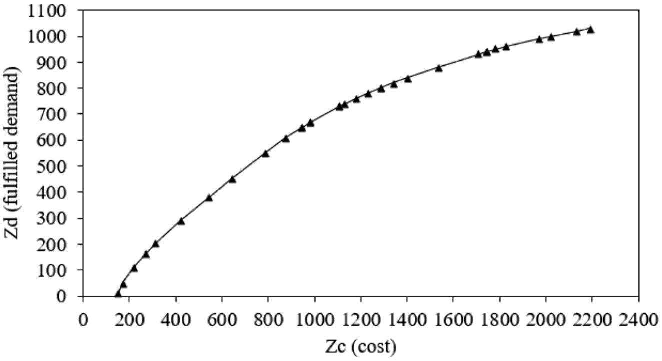

FIGURE 13 Pareto front of optimal solutions

search procedure mainly focuses on the segment when the solution approaches the optimal point. Therefore, the utopia point method is very computationally efficient, thanks to its capability in finding the superior section of the Pareto front.

5 | CONCLUSIONS

In this article, we studied the network expansion problem for urban rapid transit systems, and developed a bi-objective mixed integer linear programming model with the objective of minimizing the construction cost and maximizing the covering demand. The model presents an exact shape of the construction plan of new lines, which includes the decisions of station locations and line segments based on the predesigned corridors. It is capable of selecting corridors with higher priority among a given corridor set, and at the same time locating the terminal stations anywhere along the corridors. The solution method based on the criteria of minimum distance to the utopia point is introduced to solve the bi-objective model. The method does not require the generation of all properly Pareto-optimal points along the front, but it smartly provides points closest to the decision maker's goal with relatively short computational time. A post-optimization procedure adjusts the solution and provides a reasonable connection pattern. A real-world case study based on the metro system in Wuxi, China, is conducted, and the effectiveness and computational efficiency are verified.

For further research, there are two possible extensions to this work: combining the post-optimization with model formulation and considering the OD demand. Additionally, with the strategic network expansion plan, the tactical and operational decision problems can also be considered, including frequency setting, vehicle and crew scheduling.

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REFERENCES

Adeli, H., & Karim, A. (1997). Scheduling/cost optimization and neural dynamics model for construction. *Journal of Construction Engineering and Management*, *123*(4), 450–458.

Adeli, H., & Karim, A. (2001). *Construction scheduling, cost optimization, and management: A new model based on neurocomputing and object technologies*. London, England: Spon Press.

Aghaei, J., Baharvandi, A., Rabiee, A., & Akbari, M. A. (2015). Probabilistic PMU placement in electric power networks: An MILP-based multiobjective model. *IEEE Transactions on Industrial Informatics*, *11*(2), 332–341.

Belotti, P., Soylu, B., & Wiecek, M. M. (2016). Fathoming rules for biobjective mixed integer linear programs: Review and extensions. *Discrete Optimization*, *22*, 341–363.

Chakroborty, P. (2003). Genetic algorithms for optimal urban transit network design. *Computer-Aided Civil and Infrastructure Engineering*, *18*(3), 184–200.

Daganzo, C. F. (2010). Structure of competitive transit networks. *Transportation Research Part B: Methodological*, *44*(4), 434–446.

Daganzo, C. F. (2012). On the design of public infrastructure systems with elastic demand. *Transportation Research Part B: Methodological*, *46*(9), 1288–1293.

Deng, Z., Liu, M., Ouyang, Y., Lin, S., & Xie, M. (2017). Multi-objective mixed-integer dynamic optimization method applied to optimal allocation of dynamic var sources of power systems. *IEEE Transactions on Power Systems*, *33*(2), 1683–1697.

Escudero, L. F., & Muñoz, S. (2009). An approach for solving a modification of the extended rapid transit network design problem. *TOP*, *17*(2), 320–334.

Estrada, M., Roca-Riu, M., Badia, H., Robusté, F., & Daganzo, C. F. (2011). Design and implementation of efficient transit networks: Procedure, case study and validity test. *Procedia-Social and Behavioral Sciences*, *17*, 113–135.

Fan, W., & Machemehl, R. B. (2008). Tabu search strategies for the public transportation network optimizations with variable transit demand. *Computer-Aided Civil and Infrastructure Engineering*, *23*(7), 502–520.

Farahani, R. Z., Miandoabchi, E., Szeto, W. Y., & Rashidi, H. (2013). A review of urban transportation network design problems. *European Journal of Operational Research*, *229*(2), 281–302.

Granat, J., & Guerriero, F. (2003). The interactive analysis of the multicriteria shortest path problem by the reference point method. *European Journal of Operational Research*, *151*(1), 103–118.

Guihaire, V., & Hao, J. K. (2008). Transit network design and scheduling: A global review. *Transportation Research Part A: Policy & Practice*, *42*(10), 1251–1273.

Gutiérrez-Jarpa, G., Laporte, G., & Marianov, V. (2018). Corridor-based metro network design with travel flow capture. *Computers & Operations Research*, *89*, 58–67.

Gutiérrez-Jarpa, G., Laporte, G., Marianov, V., & Moccia, L. (2017). Multi-objective rapid transit network design with modal competition:

The case of Concepción, Chile. *Computers & Operations Research*, *78*, 27–43.

Gutiérrez-Jarpa, G., Obreque, C., Laporte, G., & Marianov, V. (2013). Rapid transit network design for optimal cost and origin–destination demand capture. *Computers & Operations Research*, *40*(12), 3000–3009.

Hamacher, H. W., Liebers, A., Schöbel, A., Wagner, D., & Wagner, F. (2001). Locating new stops in a railway network. *Electronic Notes in Theoretical Computer Science*, *50*(1), 13–23.

Hossein Rashidi, T., Rey, D., Jian, S., & Waller, T. (2016). A clustering algorithm for bi-criteria stop location design with elastic demand. *Computer-Aided Civil and Infrastructure Engineering*, *31*(2), 117–131.

Jin, J. G., Tang, L. C., Sun, L., & Lee, D. H. (2014). Enhancing metro network resilience via localized integration with bus services. *Transportation Research Part E: Logistics and Transportation Review*, *63*, 17–30.

Karim, A., & Adeli, H. (1999a). OO information model for construction project management. *Journal of Construction Engineering and Management*, *125*(5), 361–367.

Karim, A., & Adeli, H. (1999b). CONSCOM: An OO construction scheduling and change management system. *Journal of Construction Engineering and Management*, *125*(5), 368–376.

Kepaptsoglou, K., & Karlaftis, M. (2009). Transit route network design problem: Review. *Journal of Transportation Engineering*, *135*(8), 491–505.

Laporte, G., Marín, Á., Mesa, J. A., & Ortega, F. A. (2007). An integrated methodology for the rapid transit network design problem. In F. Geraets, L. Kroon, A. Schoebel, D. Wagner, C. D. Zaroliagis (Eds.), *Algorithmic methods for railway optimization* (pp. 187–199). Berlin Heidelberg, Germany: Springer.

Laporte, G., Marín, A., Mesa, J. A., & Perea, F. (2011a). Designing robust rapid transit networks with alternative routes. *Journal of Advanced Transportation*, *45*(1), 54–65.

Laporte, G., Mesa, J. A., & Ortega, F. A. (2002). Locating stations on rapid transit lines. *Computers & Operations Research*, *29*(6), 741–759.

Laporte, G., Mesa, J. A., Ortega, F. A., & Perea, F. (2011b). Planning rapid transit networks. *Socio-Economic Planning Sciences*, *45*(3), 95–104.

Laporte, G., & Pascoal, M. M. (2015). Path based algorithms for metro network design. *Computers & Operations Research*, *62*, 78–94.

Marín, Á. (2007). An extension to rapid transit network design problem. *TOP*, *15*(2), 231–241.

Matisziw, T. C., Murray, A. T., & Kim, C. (2006). Strategic route extension in transit networks. *European Journal of Operational Research*, *171*(2), 661–673.

Ning, C., & You, F. (2018). Adaptive robust optimization with minimax regret criterion: Multiobjective optimization framework and computational algorithm for planning and scheduling under uncertainty. *Computers & Chemical Engineering*, *108*, 425–447.

Pan, L., He, C., Tian, Y., Su, Y., & Zhang, X. (2017). A region division based diversity maintaining approach for many-objective optimization. *Integrated Computer-Aided Engineering*, *24*(3), 279–296.

Rostami, S., Neri, F., & Epitropakis, M. (2017). Progressive preference articulation for decision making in multi-objective optimisation problems. *Integrated Computer-Aided Engineering*, *24*(4), 315–335.

Sousa, N., Alçada-Almeida, L., & Coutinho-Rodrigues, J. (2017). Bi-objective modeling approach for repairing multiple feature infrastructure systems. *Computer-Aided Civil and Infrastructure Engineering*, *32*(3), 213–226.

Sun, L., Jin, J. G., Lee, D. H., Axhausen, K. W., & Erath, A. (2014). Demand-driven timetable design for metro services. *Transportation Research Part C: Emerging Technologies*, *46*, 284–299.

Szeto, W. Y., Jaber, X., & O'Mahony, M. (2010). Time-dependent discrete network design frameworks considering land use. *Computer-Aided Civil and Infrastructure Engineering*, *25*(6), 411–426.

Tajalli, M., & Hajbabaie, A. (2018). Dynamic speed harmonization in connected urban street networks. *Computer-Aided Civil and Infrastructure Engineering*, *33*(6), 510–523.

Wang, Q., Liu, H. L., Yuan, J., & Chen, L. (2018). Optimizing the energy-spectrum efficiency of cellular systems by evolutionary multi-objective algorithm. *Integrated Computer-Aided Engineering*, 1–14. https://doi.org/10.3233/ICA-180575

Wang, Y., & Szeto, W. Y. (2017). Multiobjective environmentally sustainable road network design using Pareto optimization. *Computer-Aided Civil and Infrastructure Engineering*, *32*(11), 964–987.

Yao, B., Chen, C., Cao, Q., Jin, L., Zhang, M., Zhu, H., & Yu, B. (2017). Short-term traffic speed prediction for an urban corridor. *Computer-Aided Civil and Infrastructure Engineering*, *32*(2), 154–169.

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