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Optimizing Passenger Flow Control and Bus-Bridging Service for Commuting Metro Lines

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Abstract: *Oversaturated conditions are often observed during peak-hour periods, especially for commuting metro lines serving as a corridor connecting suburb and urban areas due to its unidirectional passenger flow pattern. System operators are concerned about the amount of passengers accumulated inside station and at platform when train service cannot meet the travel demand. In this article, we tackle the metro system congestion issue and develop a compound strategy integrating passenger flow control and bus-bridging service, to mitigate overcrowded situation. A two-stage mathematical modeling procedure is proposed. Stage 1 determines the stations and time periods for taking passenger flow control strategy. Stage 2 identifies the optimal bus-bridging services. Mixed integer linear programming models are developed to find the demand-responsive flow control pattern and bus-bridging services. The proposed passenger flow control and bus-bridging strategy is applied to a commuting metro line in Shanghai. The results show that the proposed strategy is effective in reducing the number of stranded passengers, releasing the overcrowding pressure, and improving passengers' satisfaction.*

1 INTRODUCTION

With increasing amount of urban mobility needs and rapid growth of urban resident population, metro systems have been developed as a key solution for enhancing mobility and accessibility in metropolis because of its larger capacity, higher reliability, and better efficiency. It is reported that 25 cities in China have built metro systems (Fan et al., 2016). Taking Shanghai as an example, a metropolis with 14 metro lines and 366 stations, the average daily ridership was 9.56 million in 2015 and the highest ridership was 10.83 million recorded on December 31, 2015.

Such a high travel demand has brought a heavy pressure to metro systems. In many megacities in China, the population growth rate is far more than the expansion of metro systems. During daily commuting period or under unexpected extreme large flow condition, a large number of passengers will aggregate in metro stations or platforms. This brings a high risk to operational safety of metro systems. Strategies adopted by metro operators for improving service quality and reducing passenger waiting time can be divided into two categories:

1. Increasing the metro system's capacity, such as optimizing metro timetable, running additional train services, and adjusting running speed. All of those measures aim to improve metro service capacity.
2. Deploying alternative transportation services, such as running bus services in parallel with metro lines and between different lines. Such measures can help to balance passenger demand over the metro network and shift overcrowded demand indirectly.

The first category focuses on taking various measures to improve the capacity of the metro system itself. For instance, running additional train services provides additional capacity of the metro line. Optimizing metro service timetable is one effective solution in reducing passenger waiting time, since a well-designed timetable matches the metro service supply with

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passengers' travel demand. Those measures all focus on the operations of the metro system itself. Strategies concentrating on passengers like flow control operation of metro network have not been widely studied. For the latter category, measures like bus-bridging service are implemented as alternative choices for meeting excessive travel demand. In this article, we focus on optimizing the passenger flow control strategy together with bus-bridging service for metro lines under oversaturated situation (e.g., morning and evening peak periods). Compared to other strategies, passenger flow control (e.g., reducing the amount of gates opened) can effectively resolve the overcrowded problem inside stations and guarantee the operational safety. However, certain amount of passengers will be stranded outside the metro station due to the flow control operation implemented. Thus, we subsequently determine optimal bus-bridging services as a complementary measure to the passenger flow control. With bus-bridging services, stranded passengers can take them and fulfill commuting needs. In other words, we introduce flow control measure for certain stations to mitigate the overcrowded condition during peak hours, and at the same time deploy bus-bridging services accordingly for those affected commuters. In this article, we model the passenger flow control and bus-bridging services separately instead of formulating the whole problem as a single optimization model. This is due to the fact that in many cities (e.g., Shanghai and Beijing), the metro transit agency, as the key player in charge of the flow control operations, commonly has no or very limited bus fleet to implement the bus-bridging services by itself. Based on the determined flow control operations, the metro operator turns to bus companies for running bus-bridging services accordingly. For this reason, a hierarchical two-stage mathematical modeling procedure is more suitable for both metro transit agency and bus companies to make optimal decisions. The contribution of this article lies in the following aspects:

- Propose a compound strategy integrating passenger flow control and bus bridging for resolving the congestion issue of metro systems during peak periods.
- Develop a two-stage mathematical modeling framework for optimizing the operations of metro systems under oversaturated condition. The strategy aims to avoid overcrowding, improve passenger's satisfaction, and ensure the operational safety of metro stations.
- Demonstrate the significance of the proposed strategy and obtain managerial insights for a suburban-to-urban commuting metro line operation. A case study based on the Shanghai metro line No. 5 shows that the flow control strategy can reduce the number of passengers stranded on platform, and the bus-bridging service can help to shorten the waiting time of affected commuters.

The remainder of this article is organized as follows. Literature review is presented in Section 2. In Section 3, a two-stage modeling framework is developed. A case study based on Shanghai metro line No. 5 is conducted in Section 4, and Section 5 concludes the article.

2 LITERATURE REVIEW

Various operations research and optimization methods for tackling challenges related to metro systems have been developed in past decades. Readers can refer to Bussieck et al. (1997) and Odoni et al. (1994) for more details. Although for metro systems, operators should better understand passenger demand pattern and develop demand-sensitive strategy to meet oversaturated demand with limited train service capacity. Since this study focuses on two measures for mitigating the metro system congestion problem, passenger flow control and bus-bridging service deployment, the literature review is conducted accordingly.

To improve service quality and reliability of metro systems, recent researches demonstrate an increasing interest in developing various operational strategies. Among them, metro network flow modeling and train timetable design are two main streams. A comprehensive description about public metro operations and corresponding modeling was conducted by Ceder (2007). For earlier research, Yang and Hayashi (2002) employed microsimulation analysis method to analyze railway's origin/destination route choice behavior in big cities. Li et al. (2015) proposed the resource allocation model for multiclass services in multipath networks with the objective of utility maximization. Si et al. (2016) focused on metro passenger assignment problem, and developed a multiclass transit assignment model for Beijing metro system to estimate transit passenger flows. Based on automated fare collection data, Zhu et al. (2014) proposed a methodology based on genetic algorithm for calibrating urban rail transit assignment. Path candidates for each origin–destination pair are first generated, and the proposed calibration method subsequently assigns the detailed passenger flow over the metro network. To validate existing metro flow assignment models, Zhu et al. (2015) further developed a methodology based on automatic fare collection data and a cluster analysis technique. It is found that the accuracy of flow assignment depends on whether travel cost and transferring cost can be determined accurately.

Lin and Ku (2014) used genetic algorithms to optimize stopping patterns in passenger railway systems with the objective of maximizing the profit of the railway company. Xu et al. (2015) compared three strategies (e.g., increasing train frequency, improving train capacity, and applying extra trains), which aim to relieve traffic pressure in a metro system. Sun et al. (2014) studied the train timetable design problem, and formulated three optimization models to obtain demand-driven timetable for metro service. A real-world case study based on the Singapore metro system demonstrated that demand-driven timetable is beneficial for reducing passenger's waiting time. Castillo et al. (2014) proposed a linear programming mathematical model with double–single track (ADST) lines as an alternative to double-track lines, which aims to decide the optimal sequence of single and double tracks and optimize the railway timetable. To reduce the complexity of the problem, Castillo et al. (2016) proposed a time partitioning technique, which was successfully applied in line planning and timetable optimization. Similar research has been conducted by Niu and Zhou (2013), which focused on metro service optimization with oversaturated demand during peak hours. The primary objective is to minimize the passenger waiting time after satisfying the high demand by metro service with limited capacity. The existing literature on demand predict and delay estimation for air traffic management is abundant, whereas studies on delay management in the context of railway traffic management are quite few. Recently, delay management for railway systems has been receiving increasing attention. A first integer programming model for this problem was proposed by Schöbel (2001). Dollevoet et al. (2012) assumed that passengers always take the fastest route to destination and developed an integer programming model with consideration of rerouting of passengers. The results showed that delay can be reduced significantly. Dollevoet and Huisman (2014) developed several heuristic methods to deal with a large scale problem. And it was further extended in Dollevoet et al. (2014), in which station capacity is specifically taken into consideration.

Besides optimizing the metro service during normal operating condition, a few researchers have also studied the disruption management for metro systems. Disruptions (e.g., track intrusions, medical emergencies, weather extremes, and track and rolling stock failures) can be critical, since even small ones could result in significant travel delay and widespread confusion among affected passengers, not to mention large disruptions (Pender et al., 2013). Cacchiani et al. (2014) analyzed methods for real-time railway rescheduling under disruption condition. Jin et al. (2016) developed a planning procedure to supplement a degraded urban mass rapid transit network through intelligent introduction of shuttle bus services in the disrupted area. Results showed that the approach can be applied well to routine and scheduled metro service. Jin et al. (2015) focused on disruption situation caused by intentional attacks, and introduced a trilevel defender–attacker-user game-theoretic model for optimally allocating protection resources. A Markovian–Bayesian network model, which takes into account human error, was presented by Castillo et al. (2015) to evaluate the probability of accident on high speed and conventional railways. Furthermore, Castillo et al. (2015) developed a Bayesian model, which takes into account terrain, light signals, rolling stock, and even driver behavior, to analyze the probabilistic safety for railway lines.

The integration of bus services with metro system has received much attention in urban transit optimization. For instance, feeder bus lines linked to metro lines can expand public transit service to other areas. The feeder-bus network design problem given to a metro system was first defined by Kuah and Perl (1989). In more recent studies, Li et al. (2009) proposed an analytical model for optimizing a bus-rail system with feeder bus service under different market regimes. Cheng and Tseng (2016) explored the effect of perceived values, free bus transfer, and penalties on intermodal metro–bus transfer user's intension. Jin et al. (2014) addressed the resilience enhancement for metro systems by leveraging on public bus service. Results showed that the metro system's resilience can be enhanced from the integration with public bus service. Taking passenger demand into consideration, Codina et al. (2013) developed a mathematical programming model and a heuristically derived solution method for planning a bus-bridging system. The model presented took into account the operation under congested condition (i.e., the peak morning traffic hour). By using automatic fare collection data, Song et al. (2012) developed a method for improving the integration of Beijing's metro and bus network to obtain the optimal plan for adjusting bus services. The results showed that the proposed approach works well in practice and helps planning for multimodal corridors in Chinese cities. Xiong et al. (2015) investigated the last-mile issue, and proposed an optimization method for timetable synchronization between community shuttles and metro service. It aims to minimize passengers' delay cost and transfer cost.

As discussed above, most studies have been devoted to traditional planning and operation problems of metro systems (e.g., passenger flow prediction, distribution modeling, and timetable design) under normal operating condition, whereas the metro system operation under oversaturated condition is not very well investigated. Furthermore, in contrast to previous works by Jin et al. in 2014, 2015, and 2016, which focus on metro transit disruption management and metro–bus network integration, this study deals with the passenger flow control from the demand management perspective. This article particularly focuses on the morning peak period of a metro line, and investigates how to optimize the flow control measure under the oversaturated condition. To the best of our knowledge, very few studies have been conducted for optimizing the passenger flow control for metro lines, and bus bridging has not been considered when developing flow control operations in the previous literature. Hence, this article aims to optimize simultaneously the passenger flow control and bus-bridging service for a suburban-to-urban commuting metro line, and develops a two-stage mathematical modeling procedure to find the best flow control plan integrated with bus-bridging service.

3 PASSENGER FLOW CONTROL PROBLEM

In this section, a detailed description of passenger flow control problem is first presented. Then, mathematical optimization models are developed for identifying optimal passenger flow control manner (i.e., stations and time slots) and determining bus-bridging services, respectively.

3.1 Problem description

Oversaturated condition arises when the travel demand exceeds the train service capacity, especially during morning and evening peak periods. This problem may become even worse for metro commuting lines, which connect urban and suburban areas. Figure 1a shows the demand and headway variation on the No. 5 line in Shanghai during a weekday from 5:00 to 23:00. It can be seen that in peak and off-peak periods, passenger demand exhibits significant variation. In this case, passengers suffer from great waiting time because of unbalanced demand. Large demand flow into station within a short time can cause overcrowding problems. Passengers waiting on platform inside the station may cause safety issue. Figure 1b shows the demand unbalance among stations on the No. 5 line during peak period. In this case, some passengers in the downstream station may be left behind if a coming train is full. One more characteristic for metro lines like No. 5 in Shanghai, which connect suburban and urban areas is that most passengers are heading toward the final station of the line. So the key for solving the overcrowding problem is to control the passenger inflow in certain stations from the system perspective, and to provide additional bus-bridging services for passengers affected by flow control operation.

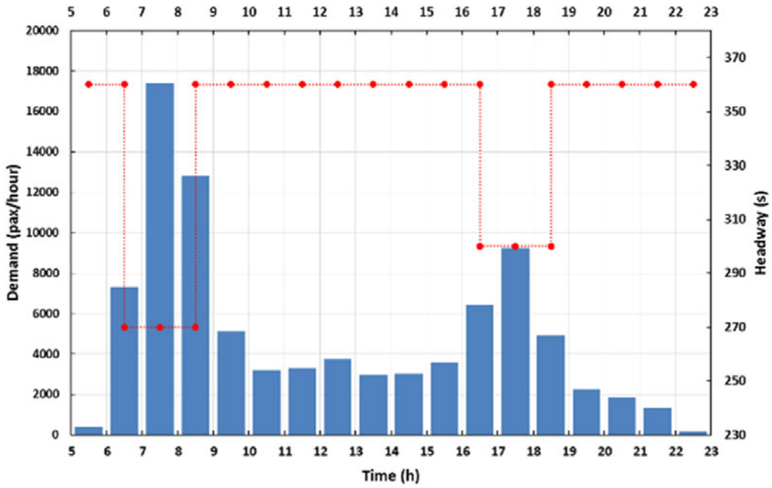
The bus-bridging services refer to those temporary shuttle bus services provided as a complementary measure when the metro service cannot meet the passenger demand. As such, the bus-bridging service provides additional service capacity satisfying those unfulfilled travel demands of the metro system. As illustrated in Figure 2, there are two types of bus-bridging services: (1) parallel bus-bridging services running in parallel with metro lines and (2) interline bus-bridging services connecting different metro lines.

The compound flow control and bus-bridging strategy aims to determine: (1) the stations, the inflow rates, and time interval for implementing flow control operation and (2) the bus-bridging services to run in such a way that the overall system service performance can be maximized. In previous study, train trajectories are often used as an instrument in describing demand patterns and modeling on service operations (Sun et al., 2014). Here, Figure 3a shows the way in which train trajectories are represented by a straight line in spatial–temporal diagram. For modeling purposes, we model train departure times as discrete values instead of continuous ones. The time horizon is discretized into a set of time intervals, as shown in Figure 3b, such as 10 seconds, 1 minute, and 2 minutes. Note that we use the equivalent time concept proposed by Sun et al. (2014). The concept of equivalent time is employed to synchronize train operation and passenger demand collectively. For any station in the metro line, its equivalent time is defined as the exact time minus its operation offset. For example, in Figure 3a, train departs the station at the end of the time $t = \{1, 3, 8, 14, 18, \ldots\}$ intervals. With the redefined equivalent time interval, Figure 3b groups the passenger demand as a two-dimensional matrix, P_t^s, on behalf of the number of passengers arriving at station s in time interval t. The travel demand can be obtained with the help of smart card-based automated fare collection (AFC) system, which records full information of each trip including entering station, entering time, exit station, and exit time.

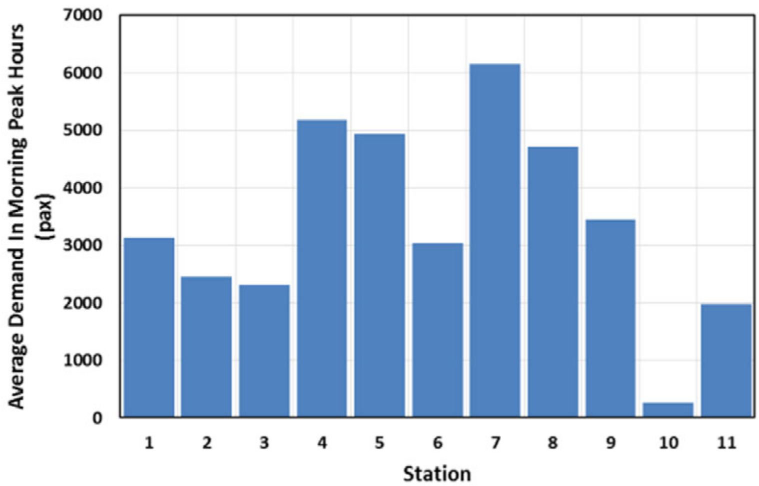
Before presenting mathematical programming models, we first introduce notations as follows:

Sets

$\boldsymbol{S}$:	Set of metro stations, $S = \{1, 2, 3, \ldots, S_n\}$
$\boldsymbol{T}$:	Set of time intervals, $T = \{1, 2, 3, \ldots, T_n\}$
$\boldsymbol{N}$:	Set of station turnstiles, $N = \{1, 2, 3, \ldots, N_s\}$
$\boldsymbol{B}$:	Set of bus stations in parallel bus route, $B = \{1, 2, \ldots, B_n\}$
$\boldsymbol{A}_{ij}$:	Set of arcs connect bus station i and j, $i, j \in B$ and $i \neq j$



(a) Demand variation of time from 5:00 to 23:00



(b) Demand variation of stations

Fig. 1. Demand variation of time and station in No. 5 line in Shanghai. (a) Demand variation of time from 5:00 to 23:00. (b) Demand variation of stations.

Parameters (Stage 1)

S_n:	Number of stations in a specific metro line
T_n:	Number of time intervals in research period
N_s:	Number of turnstiles in station s
CAP:	Train capacity (number of passengers in train never exceed CAP)
$vol_{\max}$:	Maximum inflow volume for each turnstile in interval t
P_t^s:	Passenger demand at station s in time interval t
B_t^s:	Real passenger demand (include passengers left behind in last time interval)
A_t^s:	Number of alighting passengers at station s in time interval t
$W_{\max}$:	Maximum waiting time for passengers outside the station
W_{ave}:	Average waiting time for passengers on platform
M:	Constant parameter with a large value
μ:	The minimum percentage of passengers who can get in the station s during the control period t
α_s:	Penalty cost for passengers waiting outside the station s due to the flow control
β_s:	Penalty cost for passengers waiting in the station s (left behind by first arriving train)

Parameters (Stage 2)

B_n:	Number of bus stations in parallel bus route, $B_n = S_n$

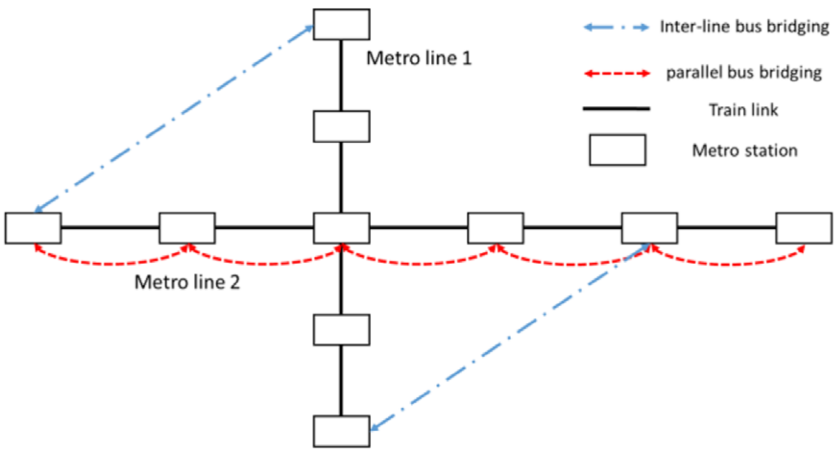


Fig. 2. Parallel and interline bus-bridging services.

$\boldsymbol{L}$:	Number of buses available for running bus-bridging services
$\boldsymbol{CAP_b}$:	Bus capacity (number of passengers should not exceed CAP_b)
$\boldsymbol{v}$:	Bus running velocity
$\boldsymbol{d_{ij}}$:	Distance between bus station i and j
$\boldsymbol{h_{\min}}$:	Minimum service headway
$\boldsymbol{q_{ij}}$:	Waiting passengers outside the metro station
$\boldsymbol{t_1^s}$:	Begin time of flow control operation at metro station s
$\boldsymbol{t_2^s}$:	End time of flow control operation at metro station s
Decision variables (Stage 1)	
$\boldsymbol{u_t^s}$:	Passenger inflow volume at station s in time interval t
$\boldsymbol{O_t^s}$:	Number of passengers who can board the train at station s in time interval t
$\boldsymbol{R_t^s}$:	Number of passengers remaining in train after the train departs from station s in time interval t
$\boldsymbol{Wout_t^s}$:	Passengers waiting outside the station s for flow control in time interval t
$\boldsymbol{Win_t^s}$:	Passengers waiting on platform at station s in time interval t
$\boldsymbol{b_s}$:	$\in \{0,\ 1\}\ \forall s \in S$. 0 if stations make the flow control operation, 1 otherwise
$\boldsymbol{x_t^s}$:	$\in \{0,\ 1\}\ \forall s \in S\ t \in T$. 1 if at station s and in time interval t make flow control strategy, 0 otherwise
$\boldsymbol{y1_{t'}^s}$:	$\in \{0,\ 1\}\ \forall s \in S\ t' \in T$. 1 if station s begin to do flow control operation in time interval t'
$\boldsymbol{y2_{t''}^s}$:	$\in \{0,\ 1\}\ \forall s \in S\ t'' \in T$. 1 if station s stop to do flow control operation in time interval t''
Decision variables (Stage 2)	
$\boldsymbol{b_{ij}}$:	Quantity of buses running in arc $(i,\ j)$
$\boldsymbol{z_{ij}}$:	$\in \{0,\ 1\}\ \forall (i,\ j) \in B$. 1 if arc $(i,\ j)$ has been selected for service, 0 otherwise

3.2 Stage 1: Optimizing passenger flow control

First, the following assumptions are made:

1. All passengers prefer to board trains as soon as possible. During morning and afternoon peak periods, commuters account for a large percentage of all passengers, and have higher request for punctuality. In case the demand exceeds the metro service capacity, certain amount of passengers have to wait outside of the station. This incurs significant dissatisfaction among affected passengers. The critical task for metro operator is to minimize the number of passengers affected by the flow control operation, and reduce their travel delay.
2. Trains hold the same speed and dwell time, which means they have identical trajectories. It aims to simplify the flow control operation process. In fact, metro services are not always punctual to designed timetable, and various disturbances often appear in daily operations. This assumption is reasonable since trains can be operated regularly in disruption-free scenarios (Sun et al., 2014).

Then, the objective function of passenger flow control optimization problem can be formulated as follows:

$$\min \sum_{s\in S}\sum_{t\in T} Wout_t^s \cdot W_{\max} \cdot \alpha_s + \sum_{s\in S}\sum_{t\in T} Win_t^s \cdot W_{\mathrm{ave}} \cdot \beta_s \tag{1}$$

Subject to

$$u_t^s \le \min\left\{P_t^s,\ N_s \cdot vol_{\max}\right\} \tag{2}$$

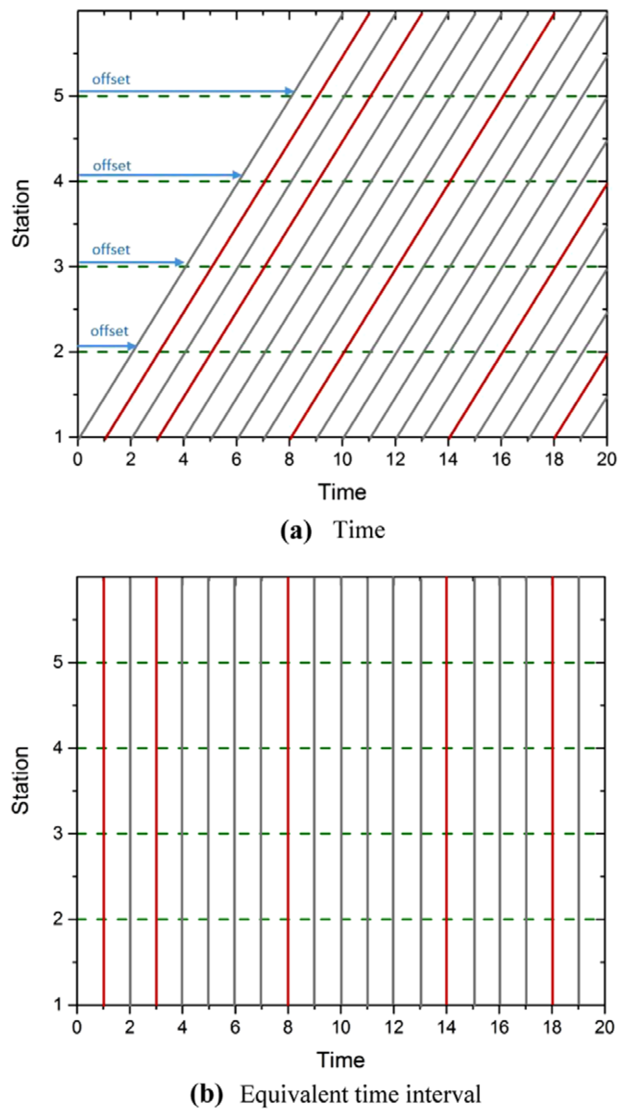


Fig. 3. Illustration of time discretization and equivalent time concept. (a) Time. (b) Equivalent time interval.

$$R_t^s = \begin{cases} R_t^{s-1} - A_t^s + O_t^s & \forall s \in S\ t \in T \backslash \{1\} \\ O_t^s & \forall s \in S\ t = 1 \end{cases} \tag{3}$$

$$R_t^s \le CAP \quad \forall s \in S\ t \in T \tag{4}$$

$$Wout_t^s = P_t^s - u_t^s \quad \forall s \in S\ t \in T \tag{5}$$

$$Win_t^s = u_t^s - O_t^s \quad \forall s \in S\ t \in T \tag{6}$$

$$B_t^s = \begin{cases} P_t^s - Wout_{t-1}^s + Win_{t-1}^s & \forall s \in S\ t \in T \backslash \{1\} \\ P_t^s & \forall s \in S\ t = 1 \end{cases} \tag{7}$$

$$Wout_t^s \le (1-\mu) \cdot P_t^s \quad \forall s \in S\ t \in T \tag{8}$$

$$(u_t^s + 1) - P_t^s \le M \cdot (1 - x_t^s) \quad \forall s \in S\ t \in T \tag{9}$$

$$P_t^s - u_t^s \le M \cdot x_t^s \quad \forall s \in S\ t \in T \tag{10}$$

$$\sum_{t' \in T} y1_{t'}^s + b_s = 1 \quad \forall s \in S\ t \in T \tag{11}$$

$$\sum_{t'' \in T} y2_{t''}^s + b_s = 1 \quad \forall s \in S\ t \in T \tag{12}$$

$$x_t^s = \sum_{t' \le t} y1_{t'}^s - \sum_{t'' \le t} y2_{t''}^s \quad \forall s \in S\ t \in T \tag{13}$$

$$x_t^s \in \{0,\ 1\} \quad \forall s \in S\ t \in T \tag{14}$$

$$y1_{t'}^s \in \{0,\ 1\} \quad \forall s \in S\ t \in T \tag{15}$$

$$y2_{t''}^s \in \{0,\ 1\} \quad \forall s \in S\ t \in T \tag{16}$$

$$b_s \in \{0,\ 1\} \quad \forall s \in S\ t \in T \tag{17}$$

Objective function (1) minimizes the sum of (a) the total waiting time of passengers outside the station and (b) the total waiting time of passengers on platform. The parameters α_s and β_s represent penalty cost for passengers waiting outside and inside station s, respectively. Constraint (2) ensures that the optimum inflow volume should be no more than the demand and maximum allowed inflow volume. More specifically, once the flow control is implemented, part of passengers who arrive at station s in time interval t must wait outside the station. The number of those passengers can be calculated by $P_t^s - u_t^s$. On the other hand, the maximum loading volume of station s can be defined as the product of N_s and $vol_{\max}$. Constraint (3) calculates the train occupancy at each time period for each station, whereas Constraint (4) imposes the capacity restriction for the occupancy, ensuring that the occupancy does not exceed capacity. Constraints (5) and (6) define the number of passengers waiting outside the stations and on platform, respectively. Constraint (7) guarantees the real passenger demand in time interval t of each station. In the first time interval, the number of left behind passengers equals to 0, which indicates the numerical equality of B_t^s and P_t^s. Otherwise, the real passenger demand in time interval t is defined as the sum of passenger demand generated in time interval t and passengers left behind in time interval $t-1$, minus the total passengers waiting outside the station. Constraint (8) ensures that at least μ% of total passengers can get in the station during the flow control period. The large constant M used in Constraints (9) and (10) provides a way for binary variables to turn constraints on and off. Those two constraints ensure that if and only if passenger demand (P_t^s) is larger

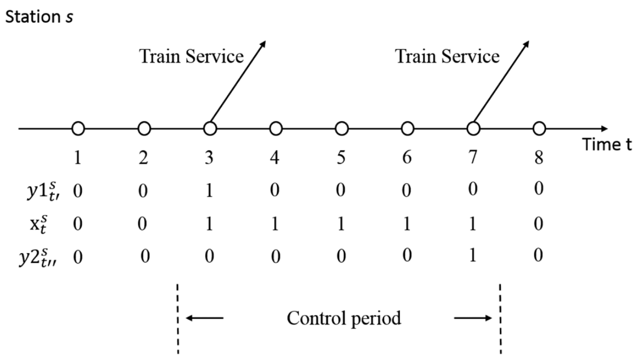


Fig. 4. Illustration of the control period by binary variable x^s_t and $y1^s_{t'}$, $y2^s_{t''}$.

than optimum inflow volume (u^s_t), the flow control operation should be taken. The value of M here needs to be chosen carefully. If M is smaller than $[(u^s_t+1)-P^s_t]$ and $(P^s_t-u^s_t)$, valid solutions may be cut off. However, by setting M to be an extremely large value, the linear relaxation of the model becomes weaker and this in turn slows down the solution process. In the actual case, for operation convenience, the flow control should be implemented for consecutive time periods. Thus, the binary decision variables x^s_t take value of 1 within the flow control period. As illustrated in Figure 4, the binary variable x^s_t equals to 1 during control period (3, 7). Constraints (11)–(13) are imposed to define the relationship between x^s_t and $y1^s_{t'}$, $y2^s_{t''}$. Finally, the domains of the decision variables are defined by Constraints (14)–(17).

3.3 Stage 2: Bus-bridging problem

Once the flow control operation has been activated, a certain amount of passengers may have to wait outside the station. Therefore, bus-bridging services should be implemented efficiently to provide alternative choices for those affected passengers.

Here we develop an optimization model to determine the optimal bus-bridging routes and the corresponding bus fleet size for each route with the objective of maximizing total amount of covered passengers.

Note that through solving the passenger flow control problem, the input variable q_{ij} can be obtained. Then, the bus-bridging problem can be formulated as follows:

$$\max \sum_{j\in B}\sum_{i\in B} q_{ij}z_{ij} \tag{18}$$

Subject to

$$b_{ij} \le M z_{ij} \quad \forall i, j \in B \text{ and } i \ne j \tag{19}$$

$$1-b_{ij} \le M(1-z_{ij}) \quad \forall i, j \in B \text{ and } i \ne j \tag{20}$$

$$q_{ij}z_{ij} \le CAP_b b_{ij} \quad \forall i, j \in B \text{ and } i \ne j \tag{21}$$

$$\sum_{j\in B}\sum_{i\in B} b_{ij} \le L \quad \forall i, j \in B \text{ and } i \ne j \tag{22}$$

$$h_{\min}(b_{ij}-1) \le t^i_2 - t^i_1 \quad \forall i, j \in B \text{ and } i \ne j \tag{23}$$

$$z_{ij} \in \{0,1\} \quad \forall i, j \in B \text{ and } i \ne j \tag{24}$$

Objective function (18) maximizes the number of waiting passengers who are served by bus-bridging services during the flow control operation period. As mentioned in the explanation of Constraints (9) and (10) in Stage 1, the large constants M used in (19) and (20) provide a way for binary variables to turn constraints on and off. Once the arc (i, j) has been selected, at least one bus should be assigned to the corresponding bus-bridging service. Otherwise, there is no need to implement bus-bridging service on it. Besides, the value of constant M should always be greater than b_{ij}. Constraint (21) ensures enough buses are employed for each route. Constraint (22) imposes the limit of the buses available. Constraint (23) guarantees that the service headway should be larger than its minimum value (t^i_1 and t^i_2 are the starting and ending time intervals in station i for flow control operation). Finally, the domain of the decision variable is defined by Constraint (24).

The two optimization models developed here help to balance passenger demand along the whole metro line. Meanwhile, the employed bus-bridging services can reduce passengers' travel delay caused by flow control operation.

4 CASE STUDY

Up to now, a two-stage modeling framework has been formulated to determine the optimal passenger flow control pattern as well as the complementary bus-bridging services. In this section, we conduct a case study based on one commuting line (No. 5) in Shanghai and apply the proposed optimization framework. Computational experiments are conducted to validate the performance of the developed flow control strategy with bus-bridging service.

4.1 Description of the case

Figure 5 shows the location of the commuting line No. 5 in the entire Shanghai metro network. As is shown, the commuting line consists of 11 stations, and serves as a key transport corridor carrying a large amount of

Fig. 5. Line No. 5 in Shanghai metro network.

passengers from the suburban to the city area. Most of the passengers alight at the terminal station (i.e., Xinzhuang) of the line and transfer to metro line No. 1 toward their final destination.

According to the trip data recorded by AFC system, during 8:00 to 9:00, in the direction to the urban area, the total inflow demand for all 11 stations in line No. 5 is 11,492, whereas the amount of passengers transferring to metro line No. 1 is 11,174, indicating that nearly 97% of all passengers alight at the terminal station Xinzhuang. If train capacity is not considered, the occupancy rates at the end of time interval t ($t = 20$) at the station Beiqiao, Zhuanqiao, Yindu Road, and Chunshen Road are 113.9%, 135.5%, 145.2%, and 156.4%, respectively. Besides, the average number of passengers left behind at those four stations per hour in the morning peak period (06:00–09:00) are 93, 660, 433, and 530. Currently, the metro operator has taken flow control operation for two stations with the greatest amount of passenger demand (i.e., Dongchuan Road and Zhuanqiao) during morning and evening peak periods. In this case, we focus on a period from 6:00 to 9:00 in a typical working day. According to the train timetable, the headway in morning peak hours is fixed to 240 seconds. So the total number of time intervals in the study period is 45. Figure 6 shows the demand variation during the study period of two typical stations of the line (No. 1 and No. 5 stations). As can be seen, the largest travel demand appears during 7:00–8:30, whereas the demand is relatively lower during other time periods. Moreover, Dongchuan Road station has the largest average hourly passenger demand. And the demand in Huaning Road station is smallest. Table 1 lists the detailed parameter settings of the case study.

Table 1
Main parameters of the case study

Parameters	*Value*	*Parameters*	*Value*
Stage 1: Flow control			
S_n	10	$vol_{\max}$	100 pax/interval
T_n	45 (06:00–09:00)	$W_{\max}$	12 minutes
N_s	5	W_{ave}	6 minutes
CAP	986 pax/service	M	10,000
Stage 2: Bus-bridging			
B_n	10 (equal to S_n)	v	20/25/30 km/h
L	30	$h_{\min}$	120 seconds
CAP_b	150 pax/service	M	10,000

Table 2
Computational time of the solution approach

	Passenger flow control	*Bus bridging*
Running time (seconds)	3.8	0.7

All computational experiments are conducted on a personal computer with Intel Core i5 2.4 GHz with 8G RAM. Mixed integer linear programming (MILP) models are directly solved by IBM ILOG Cplex solver 12.6.1. Table 2 shows the computational time for solving

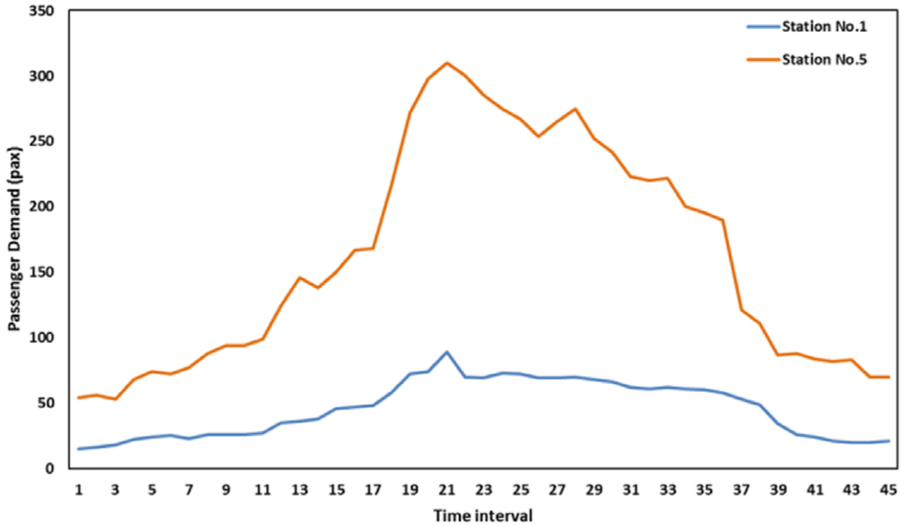


Fig. 6. Demand variation of two stations of line No. 5.

the passenger flow control and bus-bridging models. As can be seen, the developed models can be solved very efficiently within a few seconds.

4.2 Optimizing flow control operation

We evaluate the performance of the passenger flow control strategy proposed in previous section. Table 3 shows the optimal flow control pattern generated by the proposed optimization models. Compared to the currently employed flow control operation (only Dongchuan Road and Zhuanqiao stations from 7:20 to 8:20) adopted by the metro operator, our proposed approach suggests six stations for taking the flow control measure, and each of them has its own flow control time period. We remark that the optimal flow control pattern (i.e., stations and the corresponding flow control time periods) generated by the proposed model is demand driven, and can be automatically updated once the travel demand changes (e.g., working day and public holiday). Besides, the detailed flow control decisions, including stations and corresponding time periods, help metro operators implement the flow control operation in a more convenient way.

To verify the effectiveness of the optimized flow control operation, we look into two indicators: number of waiting passengers and boarding rate. Figure 7 shows the distribution of the two indicators of each station before and after the flow control measure was taken. As can be seen from Figure 7a, without flow control measure, passengers of upstream stations can always board the train, whereas waiting passengers concentrate at the downstream four stations. In addition, almost all passengers at Yindu Road and Chunshen Road cannot board the first train and have to wait for the second one. In contrast, our proposed flow control operation helps to decrease the number of waiting passengers significantly at the downstream four stations, whereas the number of waiting passengers at Dongchuan Road increases substantially due to train capacity. We remark

Table 3
Computational results of the flow control problem

Station name	*Minhang DA*	*Wenjin Road*	*Huaning Road*	*Jinping Road*	*Dongchuan Road*
Offset (second)	0	120	240	420	540
b_s	1	1	0	0	0
$x_t^s = 1$	–	–	20–27	19–34	13–36
Station name	*Jianchuan Road*	*Beiqiao*	*Zhuanqiao*	*Yindu Road*	*Chunshen Road*
Offset (second)	660	840	1,080	1,260	1,380
b_s	0	0	0	1	1
$x_t^s = 1$	22–28	17–36	17–36	–	–

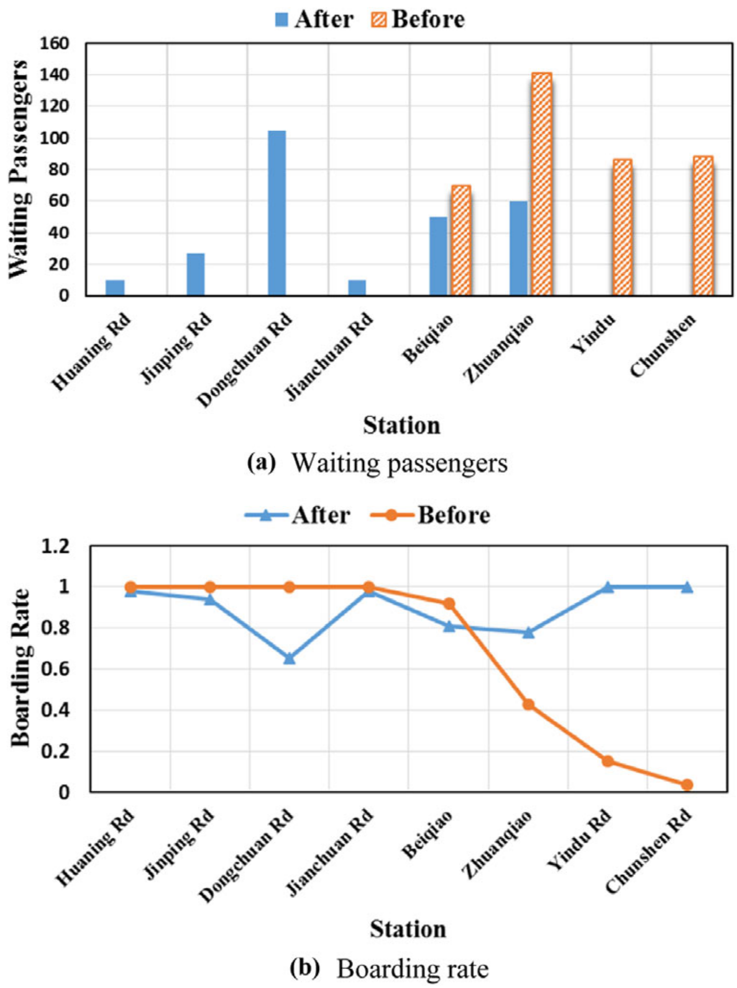


Fig. 7. Comparing number of waiting passengers and boarding rate before and after the flow control operation. (a) Waiting passengers. (b) Boarding rate.

that our proposed flow control pattern achieves a more equitable condition for passengers among all stations. This is because the boarding rate is relatively stable once the optimized flow control measure was taken, whereas it decreases substantially for the downstream four stations in the case of no flow control operation, as is shown in Figure 7b. What's more, although the number of total waiting passengers has increased from 5,141 to 5,362, the value of total waiting time has reduced from 61,692 to 32,172. Thus, the overcrowding pressure of the downstream stations can be mitigated and the overall dissatisfaction of passengers can be minimized.

We further investigate the train occupancy for each station. Owing to the train capacity restriction, the occupancy of each train (R_t^s) is always less than or equal to 986. It is observed that, from 20th to 29th time interval, the occupancy reaches full train load at Chunshen Road, which is the last boarding station before the terminal station. Besides, the optimized flow control operation guarantees that the train services reserve space for boarding passengers for each station. Figure 8 reports the number of boarding passengers at each station for situations with and without flow control operation. As can be seen, before the flow control operation was taken, the number of boarding passengers shows a significant variation along the metro line, especially in Dongchuan Road and Beiqiao. And no passengers at the last two stations can board the train; passengers who get on at Yindu Road and Chunshen Road may have to wait a long time to board a train although their destination is only one or two stations ahead. The passenger demand between stations is extremely unbalanced. In contrast, the optimized flow control pattern balances the number of boarding passengers along the line. Some upstream stations with large passenger demand will take flow control operation measures, which guides

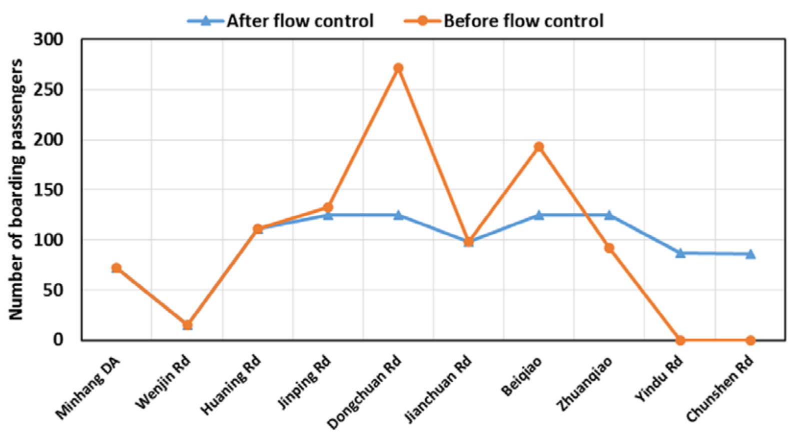


Fig. 8. Number of boarding passengers for each station.

passengers to take bus-bridging services. At the same time, it guarantees that passengers at the last two stations can still board the train. It further demonstrates the effectiveness of the flow control measure in achieving balanced equity among all stations of the metro line.

4.3 Optimizing bus-bridging service

The flow control operation precludes certain passengers at the selected station from entering the station, and those affected passengers may have to find alternative ways to their destination. With the number of affected passengers $Wout_t^s$ known from the flow control stage, the bus-bridging optimization stage runs bus-bridging services aiming at minimizing the dissatisfaction of those affected passengers. As mentioned previously, about 97% of total passengers head toward the terminal station of the metro line and transfer to another line during peak hours. With this regard, we fix the terminal station of deployed bus services to be exactly the terminal station of the metro line. Table 4 shows the computational results obtained from the bus-bridging optimization model, and the optimal bus-bridging routes are shown in Figure 9. As can be seen, three bus-bridging service routes are employed and a total number of 30 buses are allocated correspondingly: Jinping–Xinzhuang (3 buses), Dongchuan–Xinzhuang (18 buses), and Zhuanqiao–Xinzhuang (9 buses). During the flow control period, there are total 5,362 passengers who have to wait outside the station, whereas 4,208 of them are able to turn to the bus-bridging services to continue their journey. So with the help of bus-bridging services with 30 buses, about 78% of waiting passengers can reach the terminal station with minimal delay. Further computational experiments show that with an additional nine buses, all the passengers affected by the flow control operation can be covered. In summary, running bus-bridging service is an effective way for complementing metro services, especially during peak periods.

Table 4
Computational results of the bus-bridging service problem

Parameters	*Value*
b_{ij} for $j = 10$	{0, 0, 0, 3, 18, 0, 0, 9, 0, 0}
z_{ij} for $j = 10$	{0, 0, 0, 1, 1, 0, 0, 1, 0, 0}

4.4 Efficiency of bus-bridging services

The efficiency of taking bus-bridging services is related to the transferring time from metro to bus, bus waiting time, as well as the bus riding time. The transferring time from metro to bus depends on the distance from the metro station to the bus stop. It is common that bus stations are located quite near to metro stations, and thus the transferring time can be low. The bus waiting time is affected by the frequency of the bus services, and the average waiting time can be estimated to be half of the service interval. The bus riding time is affected by road traffic condition and can be quite uncertain. For the case study, we compare the bus riding time under three circumstances: (1) stable flow: the bus can be running at an average speed of 30 km/h; (2) a little bit crowded: the bus can be running at an average speed of 25 km/h; (3) congestion: the average velocity of bus can only reach 20 km/h. Figure 10 compares the bus ridging time from three origin stations to the terminal station with the travel time of the metro system. As can be seen, the direct bus services can be faster than the

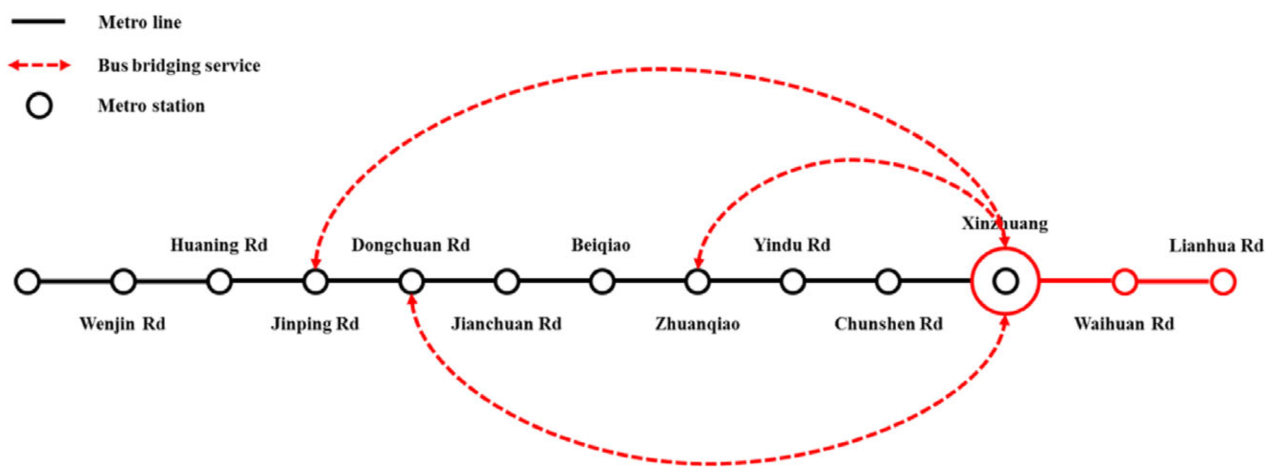


Fig. 9. Optimal bus-bridging route in case study.

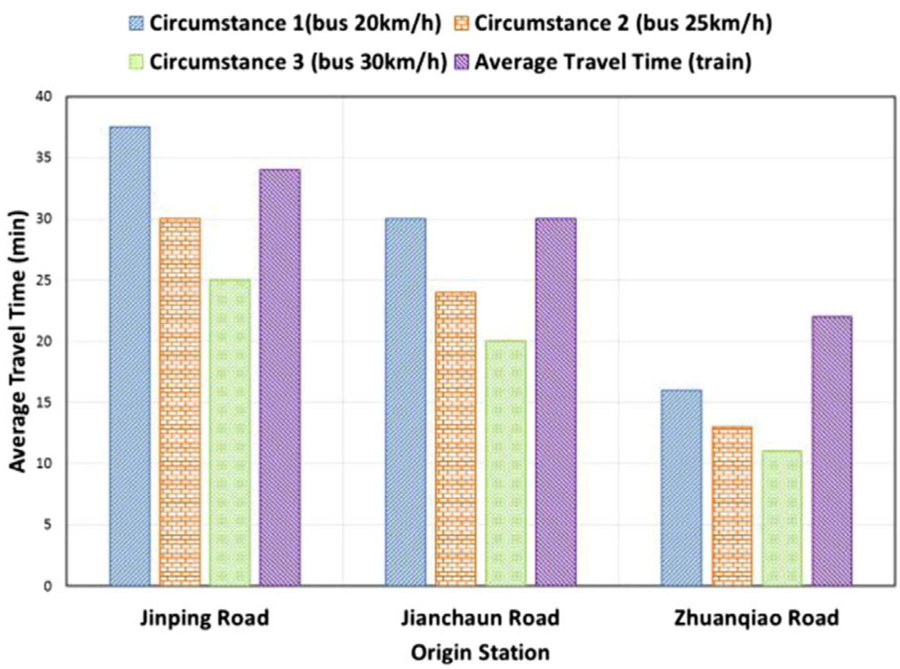


Fig. 10. Bus riding time under different traffic conditions.

metro service if the traffic condition is good. Even if taking the transferring time and waiting time into consideration, running direct bus-bridging services is indeed an effective strategy for complementing metro services, especially during peak periods on a commuting metro line.

4.5 Sensitivity of penalty coefficients

To investigate the impact of penalty coefficients on the performance of the proposed modeling framework, we run series of experiments with different values of the penalty coefficients α_s and β_s, which serve as the weighting parameters for the passengers waiting outside and inside stations, respectively. The proportion of α_s and β_s reflects the extent to which the system operator is concerned with the passengers waiting outside stations over those waiting inside stations. If the value of α_s is larger than that of β_s, it indicates that the system operator is concerned more about passengers waiting outside the stations. In this article, we run the computational experiments with the following four parameter settings for

Table 5
Sensitivity analysis of penalty coefficients

$\frac{\alpha_s}{\beta_s}$		*Minhang DA*	*Wenjin Road*	*Huaning Road*	*Jinping Road*	*Dongchuan Road*	*Jianchuan Road*	*Beiqiao*	*Zhuanqiao*	*Yindu Road*	*Chunshen Road*
0.5	$\sum_t Wout_t^s$	0	0	83	576	2,602	118	1,130	1,203	0	0
	b_s	1	1	0	0	0	0	0	0	1	1
	$x_t^s = 1$	–	–	20–28	19–34	13–36	22–28	17–36	17–36	–	–
1	$\sum_t Wout_t^s$	0	0	83	576	2,602	118	1,130	1,203	0	0
	b_s	1	1	0	0	0	0	0	0	1	1
	$x_t^s = 1$	–	–	20–28	19–34	13–36	22–28	17–36	17–36	–	–
2	$\sum_t Wout_t^s$	0	0	83	576	2,602	118	1,130	1,203	0	0
	b_s	1	1	0	0	0	0	0	0	1	1
	$x_t^s = 1$	–	–	20–28	19–34	13–36	22–28	17–36	17–36	–	–
4	$\sum_t Wout_t^s$	0	0	82	435	2,531	68	1,004	1,194	0	0
	$\sum_t Win_t^s$	–	–	–	–	216	173	–	–	–	–
	b_s	1	1	0	0	0	0	0	0	1	1
	$x_t^s = 1$	–	–	20–27	19–34	13–36	22–28	17–36	17–35	–	–

α_s/β_s: 0.5, 1, 2, 4. Table 5 compares the results given different parameter settings, including number of waiting passengers, stations selected for conducting flow control operation, and their corresponding flow control time window.

As can be seen, the optimal flow control operation (i.e., the flow control stations and their corresponding time window) remains the same for $\alpha_s/\beta_s = 0.5, 1$, and 2. For α_s/β_s, the flow control stations keep the same, whereas the flow control time window for two stations (Huaning and Zhuanqiao) changes slightly. In addition, there is no change in the total amount of waiting passengers. However, the number of passengers waiting outside the station decreases slightly at the cost of the increased number of passengers waiting on the platform (Dongchuan Road and Jianchuan Road). With larger α_s/β_s, the model tends to hold the passengers to wait for next trains inside stations rather than taking bus-bridging services. The penalty coefficients should be calibrated case by case so that the total number of passengers taking bus-bridging services can be obtained accurately from the model. In summary, the proposed flow control optimization model yields quite robust solutions without much sensitivity to the penalty coefficients.

4.6 Applicability of bus-bridging services

The applicability of bus-bridging services during the flow control period depends very much on the efficiency of the bus services as well as their attractiveness to passengers. In our case study, an arterial road runs in parallel with the metro line, and there is one dedicated bus lane available for running the bus-bridging services. This ensures high efficiency of transferring affected passengers by bus services. Note that running bus-bridging services is subject to the road condition, especially when the road traffic is not good during peak hours. It can be a good choice if there is an expressway or dedicated bus lane available. It is also important to run direct bus-bridging services with few stops, since direct bus services ensure shorter riding time as compared to those stopping at every station along the metro line.

The attractiveness of bus-bridging services to passengers is another key consideration. To encourage passengers to use the bus-bridging services during the flow control period, metro operators need to take certain measures, such as offering free bus services, and providing clear instruction in stations to guide affected passengers to take bus services.

5 CONCLUSION

This article addressed the passenger flow control optimization problem arising from metro systems under oversaturated condition. Compared to the naive standard response of rescheduling train timetable or running additional train services, the passenger flow control strategy is an emerging operational challenge in practice and has not been well investigated. In this article, we develop a compound modeling framework to tackle the metro system congestion issue including (1) the flow control optimization model, which optimally determines the stations for taking flow control operations and their corresponding time windows and (2) the bus-bridging optimization model, which identifies the optimal bus-bridging routes and allocation. Mixed integer linear programming models are developed to find the demand-responsive flow control pattern and bus-bridging services. The developed optimization models can support metro operators to determine the exact stations and time periods for taking flow control measures, and how the bus-bridging services are implemented. A real-world case study based on one commuting line in Shanghai demonstrates that the proposed approach is effective to reduce the number of stranded passengers, release the overcrowding pressure, and improve passengers' satisfaction. Besides, it is found that the bus-bridging services should be implemented when the flow control operation is taken, and the two operations should be well coordinated according to the travel demand. This requires effective collaboration between metro and bus operators. For some metro operators who also run bus services (e.g., SMRT in Singapore), it may not be a challenge. However, for the case of Shanghai where metro and bus systems are run by different operators, the proposed flow control and bus-bridging measure works only if there exists a partnership between metro and bus operators.

For future research, we are interested in extending the model from single line situation to a metro network with multiple lines. Another promising research topic is to design efficient solution algorithm for tackling larger scale problems. At last, we would like to simultaneously consider the flow control and bus-bridging decisions and develop an integrated optimization model.

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