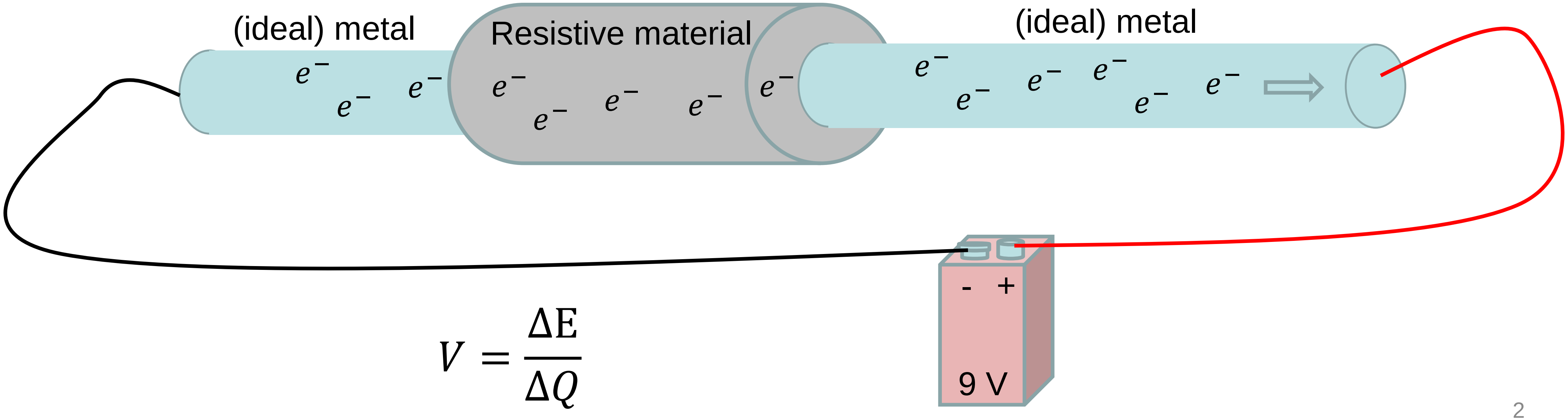
Lecture 3: Measuring and Modeling Circuits

- Ammeter
- Voltmeter
- Current-Voltage (IV) plots
  - Ohm’s Law
  - Cylindrical Conductors/Resistors
  - IV for linear circuit models

Voltage

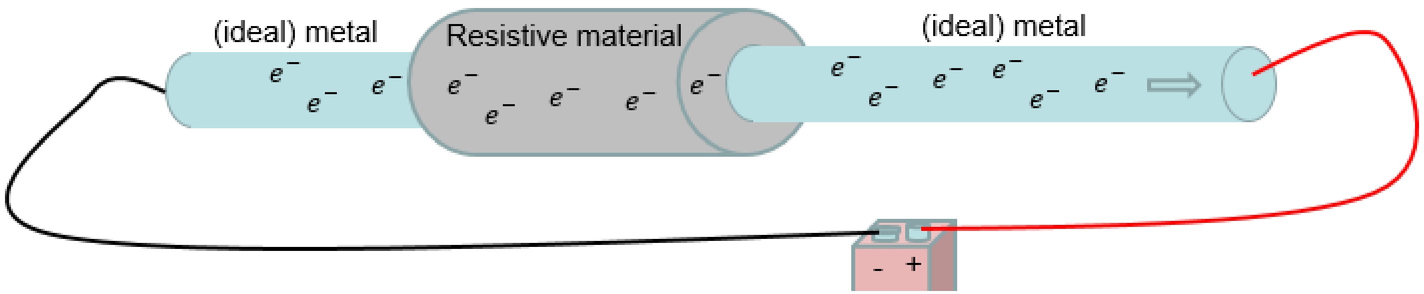
- Remember that voltage is *differential* (measured between two points) and not absolute (cannot be measured at a single point without a reference)
- It would take no energy (0 volts) to move charge through an *ideal conductor* (zero-resistance)
- A 9-volt battery delivers 9 Joules of energy to each Coulomb of charge it moves

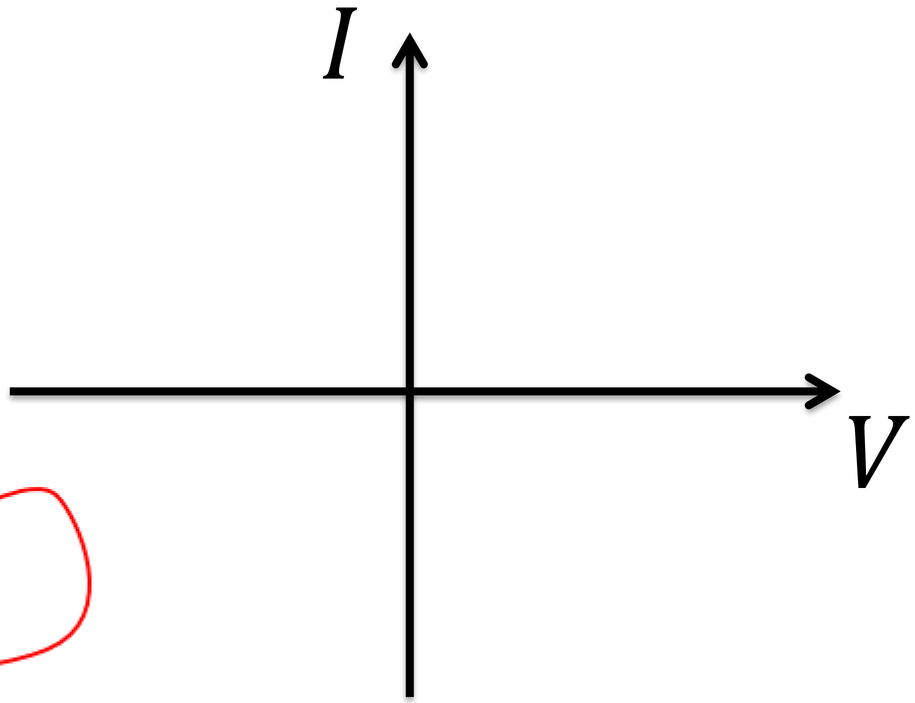


$$V = \frac{\Delta E}{\Delta Q}$$

Current vs Voltage Measurements

- Current-vs-Voltage plots
- IV characteristics
- Ohm's Law





Ohm's law models the current and voltage relationship in conductors

Motivated by applications of long-distance telegraphy, Georg Ohm (~1825) conducted careful experimentation to find this widely-used approximate mathematical model:

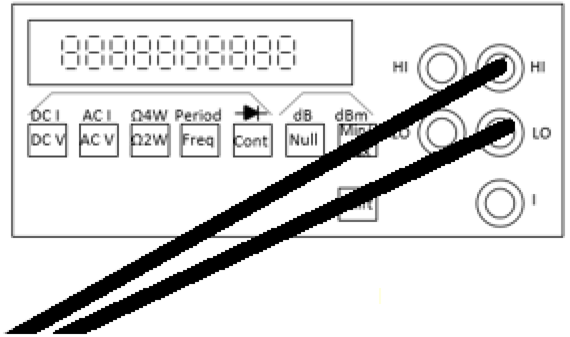
$$I = \frac{V}{R}$$

where $R = \rho \frac{l}{A}$ is resistance of a *conductor* (e.g. wire)

with length, l, and area A, and where ρ is *resistivity* - a material parameter

Resistors also known as Conductors

$$I = \frac{V}{R} \qquad R = \rho \frac{l}{A}$$



Question: Find the diameter of one mile of Cu ($\rho = 1.7 \times 10^{-8}\ \Omega\, m$) wire when $R\ =\ 10\ \Omega$.

$$R = \rho \frac{l}{\pi r^2}$$

one mile = 1607 meters

Diameter = $2r$

A. $1.7\ \mu m$
B. $\boxed{1.9\ mm}$
C. $1\ cm$
D. $19\ cm$
E. $1.7\ m$

Q: If the resistance of one wire is $10\ \Omega$, what is the resistance of two such wires in parallel?

A. $2.5\ \Omega$
B. $\boxed{5\ \Omega}$
C. $10\ \Omega$
D. $20\ \Omega$
E. $40\ \Omega$

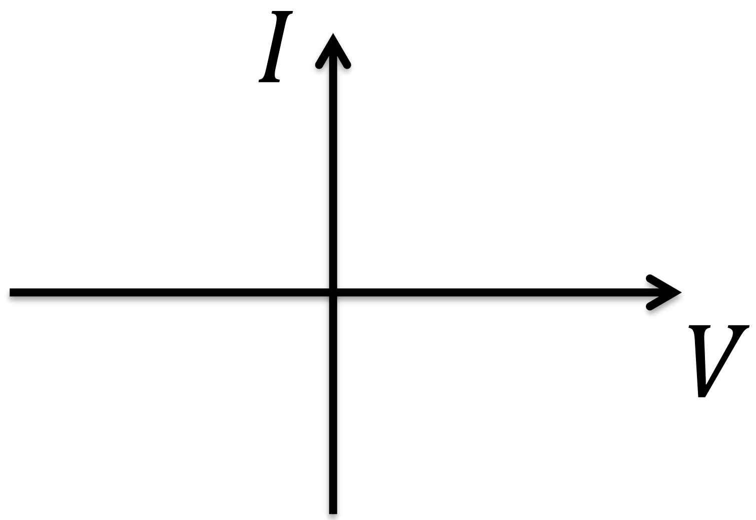
Our ohmmeter uses the same connections but different settings than the voltmeter! Polarity doesn't matter for Ohms. Why?

The Relationship between Current and Voltage is very revealing for many devices

Devices composed of voltage sources, current sources, and resistors have "IV" relationships described by a simple line:

$$I \approx mV + b$$

where m is the slope and b is the intercept of this line on the I (current) axis.

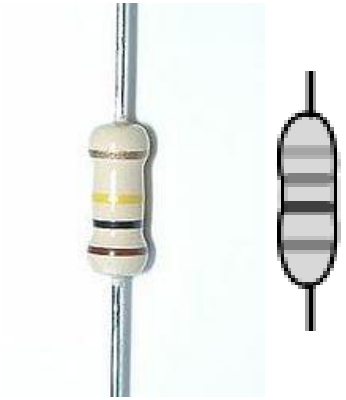


Linear IV Characteristics

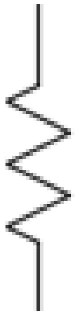
$$I \approx mV + b$$

Example: For a "**resistor**", zero voltage means zero current and the intercept is at the origin ($b = 0$).

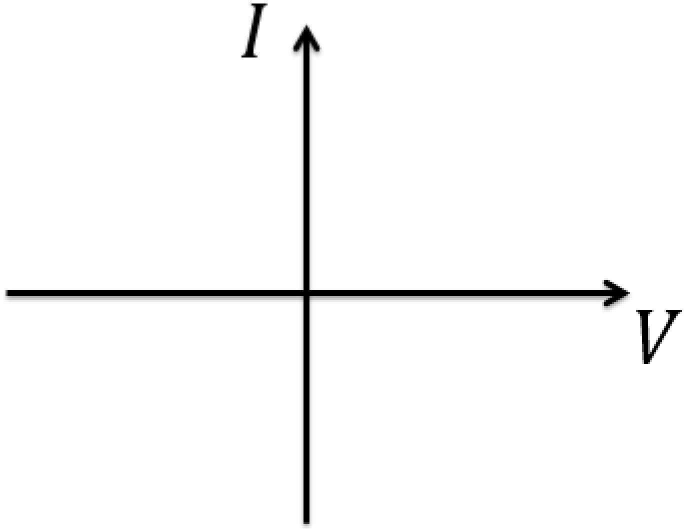
Physical



Circuit schematic



$$\boxed{I = \frac{V}{R}}$$



Linear IV Characteristics

$$I \approx mV + b$$

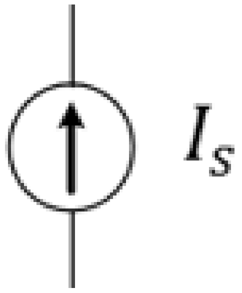
Example: For an **ideal current source**, $m = 0$ such that $I = b$ independent of V (the voltage across the current source).

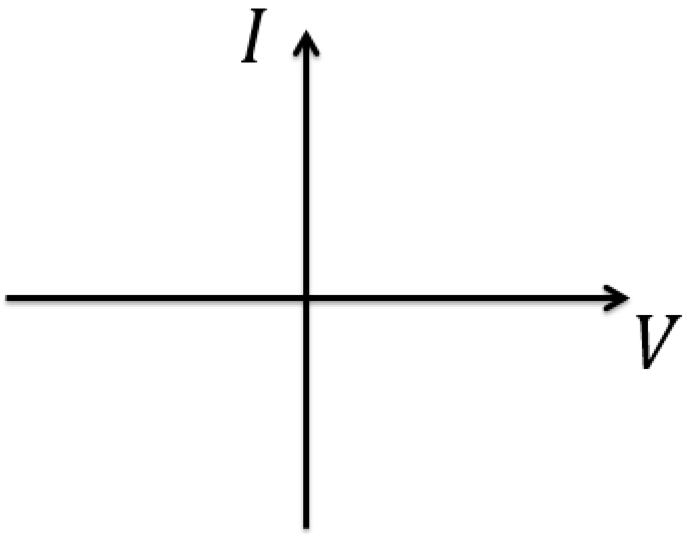
Physical

?

(later…)

Circuit schematic



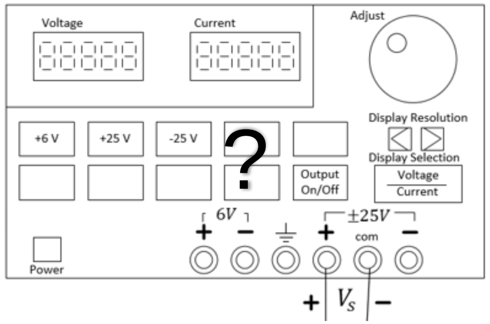


Linear IV Characteristics

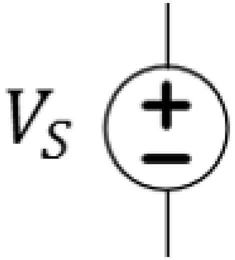
$$I \approx mV + b$$

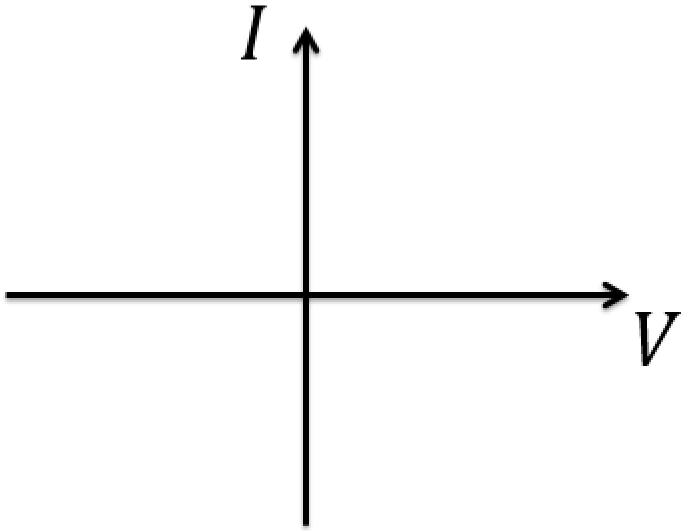
Example: For an **ideal voltage source**, $V = V_S$ and the current through the source is unconstrained (the limit as $m \to \infty, b \to -\infty$).

Physical



Circuit schematic



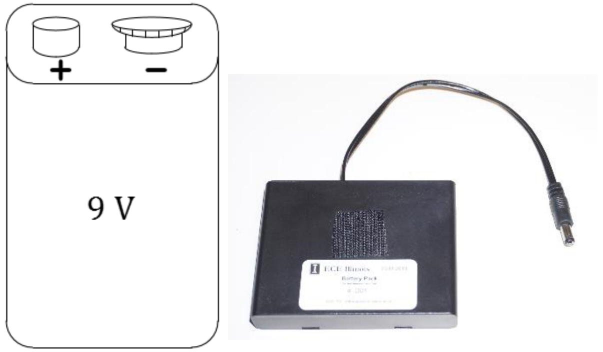


Linear IV Characteristics

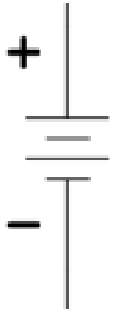
$$I \approx mV + b$$

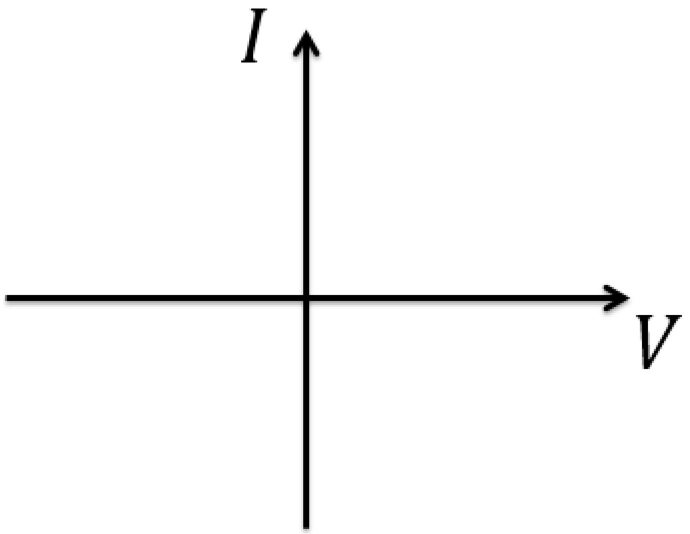
Example: What happens with a non-ideal voltage source, for example, a **battery**?

Physical



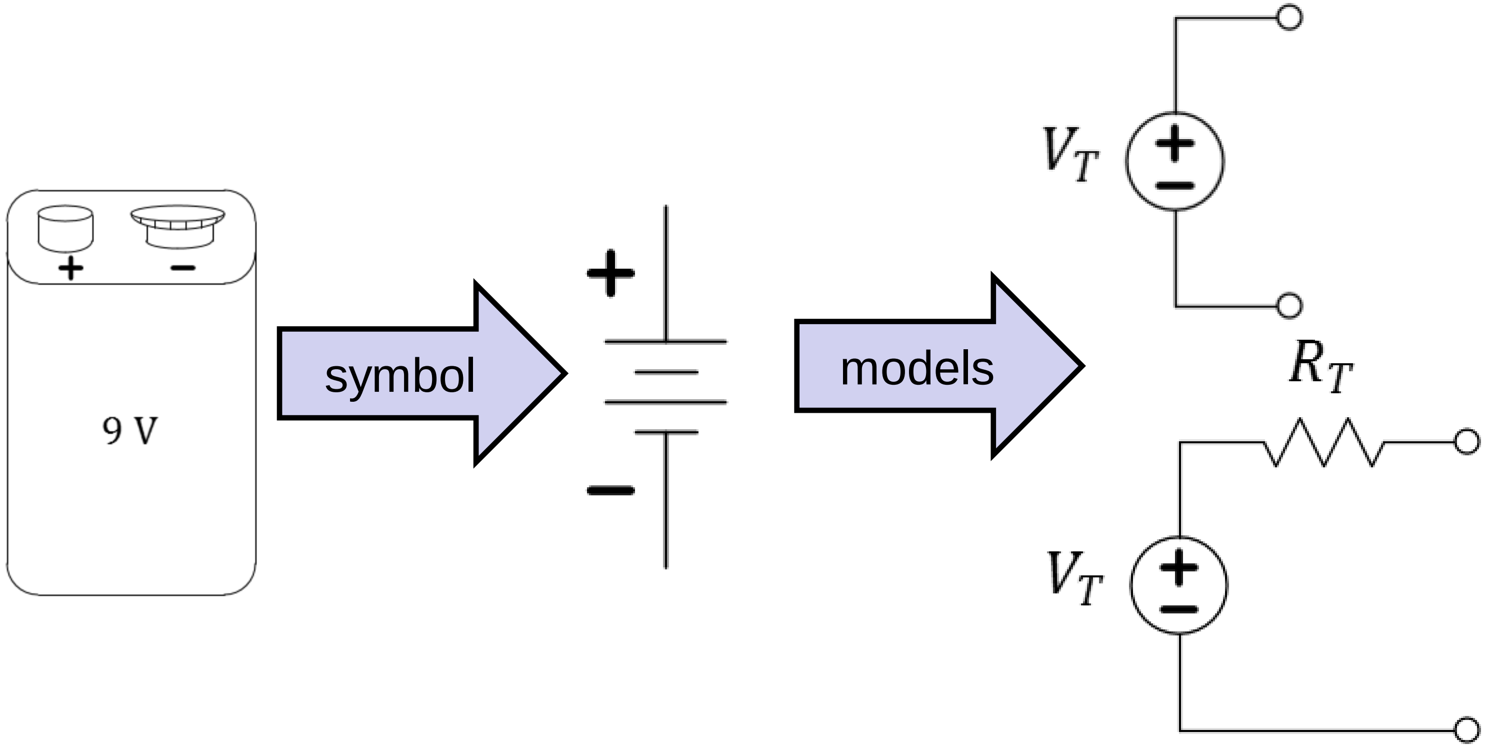
Circuit schematic





Resistance can be used in device <u>models</u>

- Lengths of wire
- Incandescent bulbs
- Heating elements
- Battery terminals
- Stalled motors
- Fuses, etc.

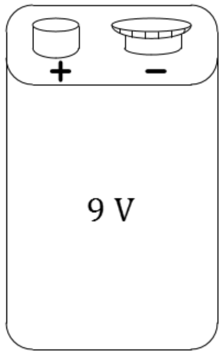


Q: If a 9 V battery provides (at maximum) a current of 2 A, what is its modelled “internal” resistance, R_T?

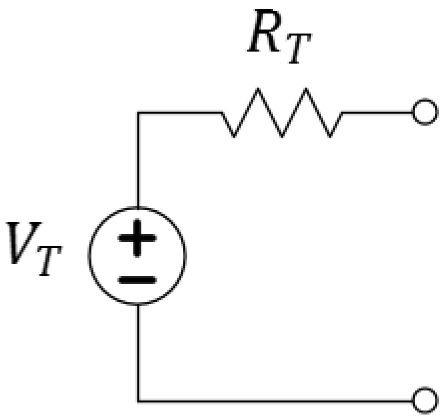
A. $0\ \Omega$
B. $2\ \Omega$
C. $4.5\ \Omega$
D. $18\ \Omega$
E. $\infty\ \Omega$

Linear IV Characteristics

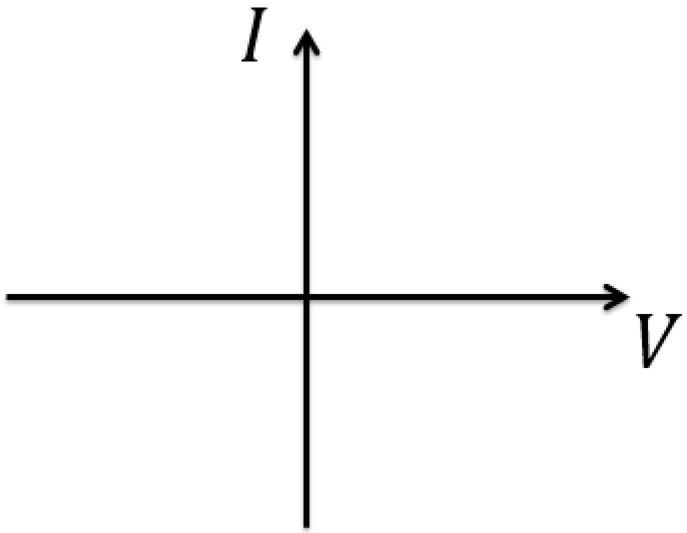
Physical



Circuit schematic



IV plot



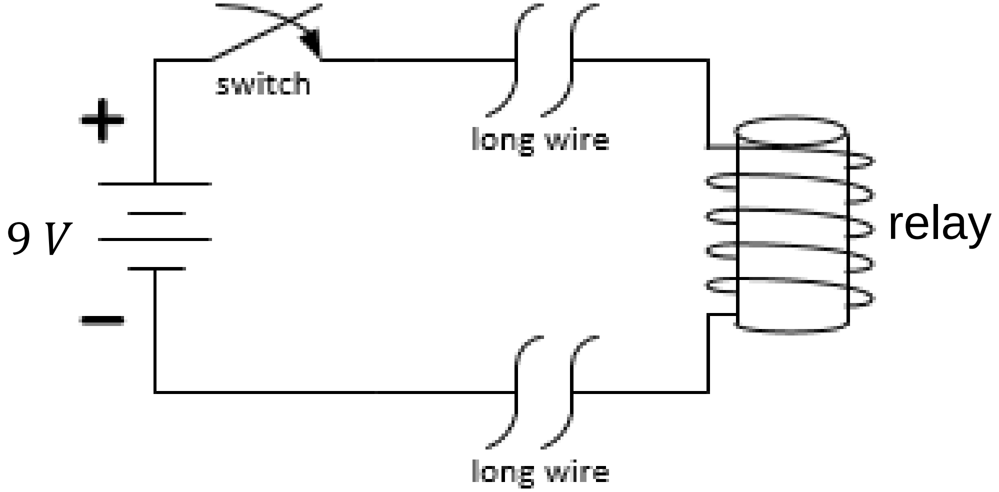
IV equation

$$I = mV + b$$

Q: For what region of the empirical data might we want the model to best fit?

A. Near the intersection with the I-axis.
B. Near the intersection with the V-axis.
C. Halfway between the two axis.
D. Minimize the average error between the equation’s prediction and all data.
E. Minimize the maximum error between the equation’s prediction and all data.

Circuit Model For a Telegraph Loop



(This wire is sometimes replaced by earth)

Q: If a 9 V battery with 4 Ω contact resistance is used and the relay has 80 Ω and the wire has 10 Ω/mile, what is the maximum telegraph distance which will result in a 50 mA current through the relay circuit loop?

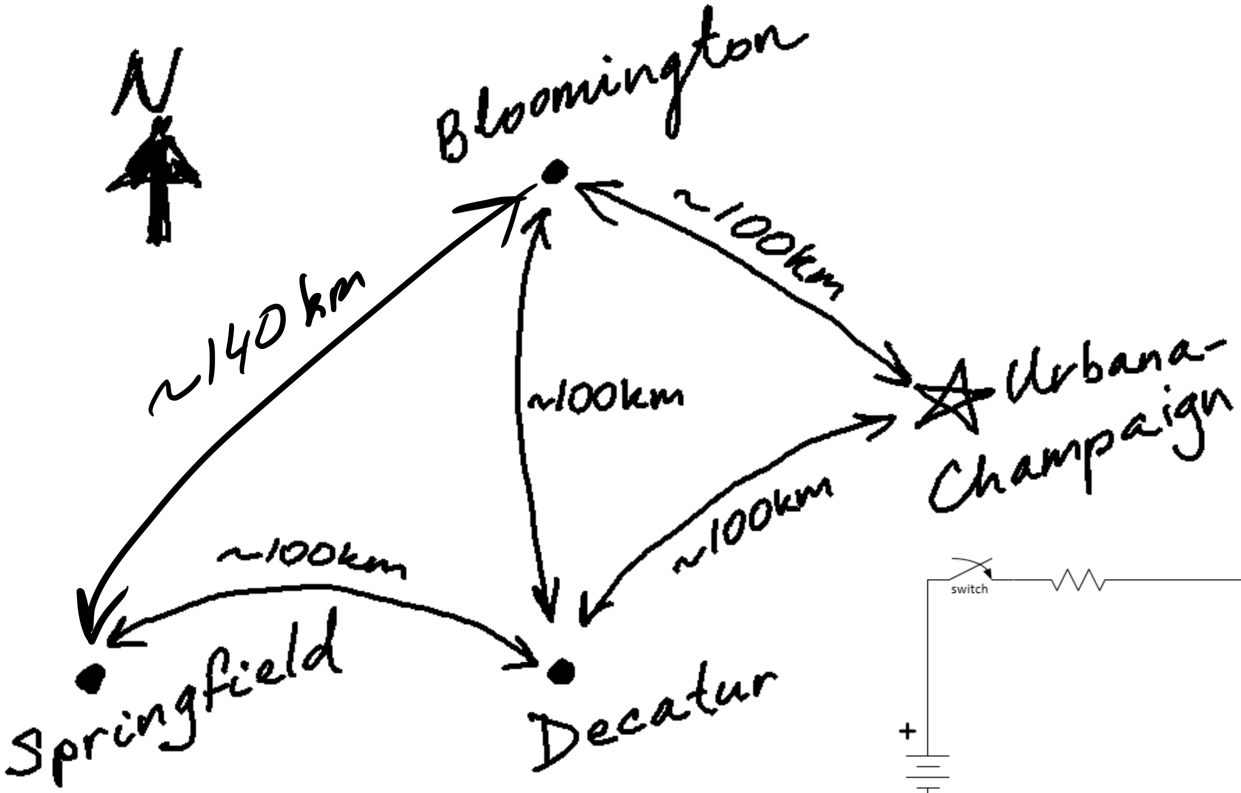
A. 0.5 miles
B. $\boxed{\text{5 miles}}$
C. 10 miles
D. 100 miles
E. 500 miles

total resistance = 9V/50mA = 180 Ohm

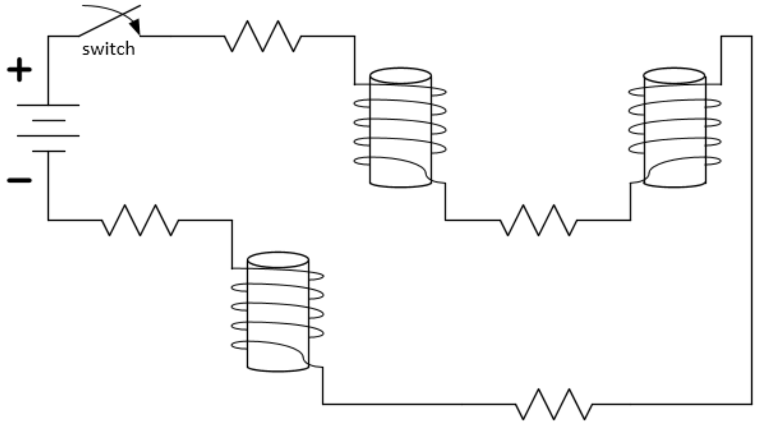
$$180\Omega - 4\Omega - 80\Omega = 96\Omega$$

$$\frac{96\Omega}{10\Omega/\text{mile}} = 9.6\ miles \qquad \frac{9.6 miles}{2} = 4.8\ miles$$

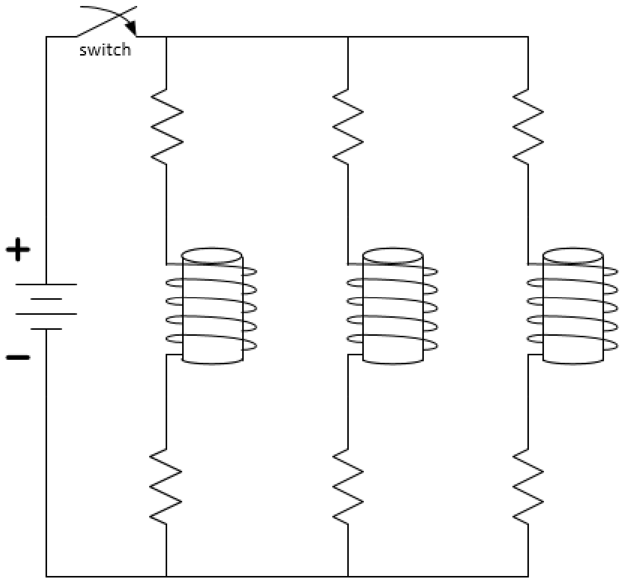
Broadcasting: multiple ways to wire relays



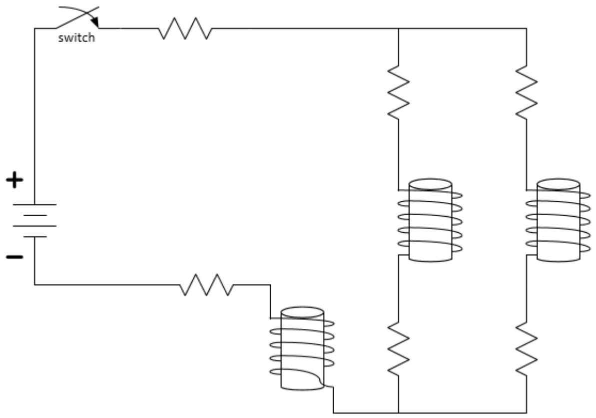
A.



B.



C.

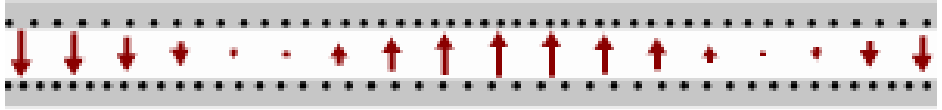


i>clicker Q:
Which method (A, B, or C) is a parallel combination of towns?

L3 Learning Objectives

a. Relate voltage and current for an "Ohmic" conductor.
b. Compute resistance of a cylindrical conductor given dimensions.
c. Use Ohm's Law to model the internal resistance of a physical battery.
d. Draw source and resistor circuits to model real-world problems

Explore More!



ECE 329 Fields and Waves I
ECE 350 Fields and Waves II

A wave traveling rightward along a lossless transmission line.
Black dots represent electrons, and arrows show the electric field.

Source: https://en.wikipedia.org/wiki/Transmission_line