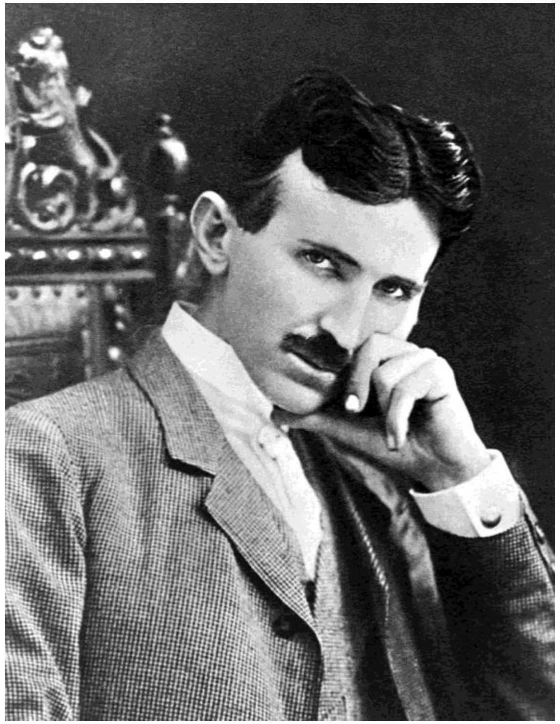
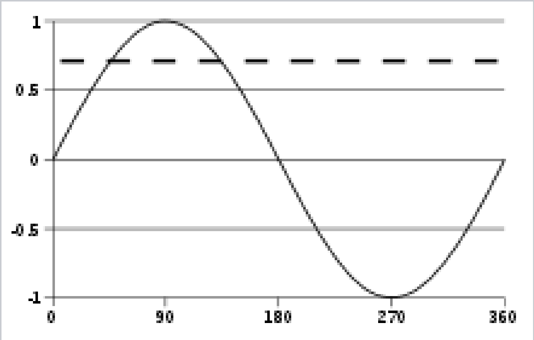
Lecture 9: AC and Time-average Power

- AC and DC
- Time-average Power
- Root-Means-Square (RMS) Voltage

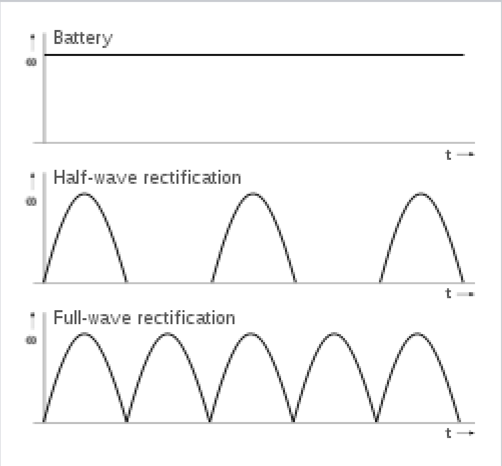
Alternating vs. Direct Current



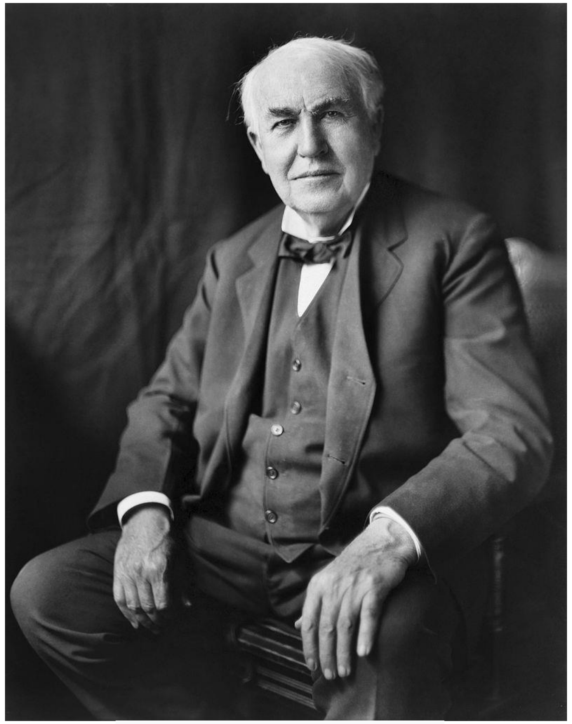
AC/DC

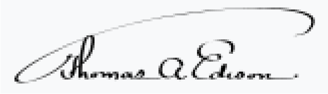


A sine wave, over one cycle (360°). The dashed line represents the root mean square (RMS) value at about 0.707



Types of direct current





Explore More!…

Search for “Current War.” An interesting read!

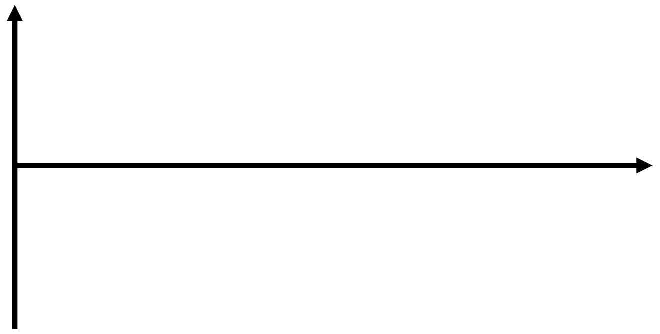
Power

For time-varying signals, power is a time-varying signal.

$$p(t) = i(t)v(t)$$

The **time-average power** is often of interest. Time average is computed by the equation

$$P_{avg} = \frac{\int_{-\infty}^{\infty} p(t)dt}{\int_{-\infty}^{\infty} dt}$$



Power Calculation: periodic

$$P_{avg} = \frac{\int_{-\infty}^{\infty} p(t)dt}{\int_{-\infty}^{\infty} dt}$$

If v(t) and i(t) are periodic, then $p(t)$ is periodic with period T

$$\boxed{P_{avg} = \frac{\int_T p(t)dt}{T}}$$

$$= area\ under\ p(t)\ divided\ by\ T$$

$$= Energy\ in\ one\ period\ divided\ by\ T$$

Power Calculation: DC

If v(t) and i(t) are constant (DC), then $p(t)$ is constant

$$\boxed{P_{avg} \equiv P = IV}$$

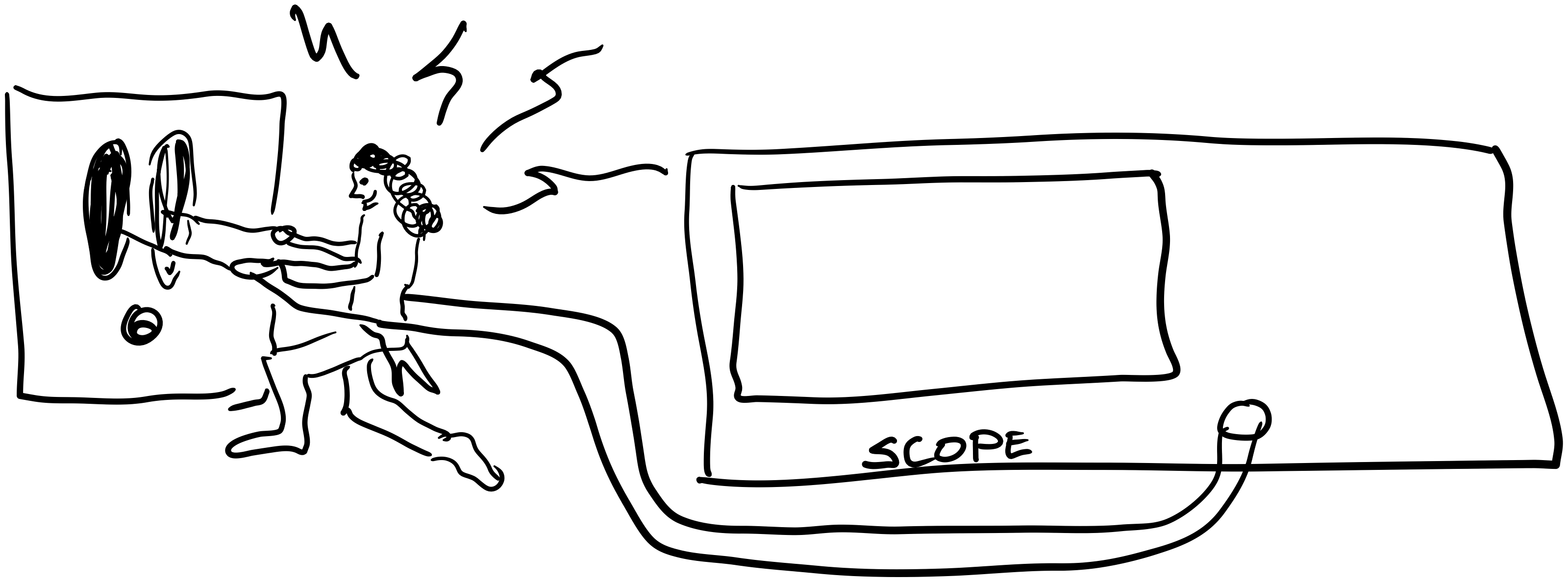
Power Calculation: Noise

For non-periodic signals (e.g. constant white noise)

$$P_{avg} \approx \frac{\int_T p(t)dt}{T}$$

Where T is a sufficiently-long observation time

Voltage from the wall plug is *sinusoidal*



Q: What is the peak instantaneous power absorbed by a 250Ω light bulb powered by the MAINS?

A. 1 W
B. 10 W
C. $\boxed{\text{100 W}}$
D. 1 kW
E. 10 kW

In History…

In the 1880's and 1890's, **Nikola Tesla** played a large role in improving DC motors, developing AC motors and generators, and developing many high-frequency/high-voltage experiments including many in the area of remote control and wireless telephony. **Marconi**'s 1901 cross-Atlantic wireless transmission likely infringed upon a few of Tesla's nearly 300 patents.

We want Time Average Power, so What's This Stuff About RMS??

$$P_{avg} = \frac{\int_{-\infty}^{\infty} p(t)dt}{\int_{-\infty}^{\infty} dt}$$

$$= \frac{\int_{-\infty}^{\infty} v(t)i(t)dt}{\int_{-\infty}^{\infty} dt}$$

$$= \frac{\int_{-\infty}^{\infty} \frac{v^2(t)}{R} dt}{\int_{-\infty}^{\infty} dt}$$ (for a resistor)

$$= \frac{1}{R} \frac{\int_{-\infty}^{\infty} v^2(t)dt}{\int_{-\infty}^{\infty} dt}$$

$$= \frac{1}{R}\, avg\{v^2(t)\}$$

Define $V_{rms} \stackrel{\text{def}}{=} \sqrt{\frac{\int_{-\infty}^{\infty} v^2(t)dt}{\int_{-\infty}^{\infty} dt}}$ so that $P_{avg} = \frac{V_{rms}^2}{R}$

(for a resistor)

Important Comment: RMS voltage helps us find time-averaged power. We don't want RMS power…what does that even mean??

Important Comment #2: for things that are not resistors, we may need to look at $p(t)$ directly as V_{rms} doesn't tell the whole story.

Important Comment #3: You can use both V_{rms}, I_{rms}, and something called a power factor in more-advanced circuit courses. Eg. ECE342

Root-Mean-Square averages

RMS is meaningful when interested in power production/dissipation in AC.

$$V_{RMS} = \sqrt{Average[v^2(t)]}$$

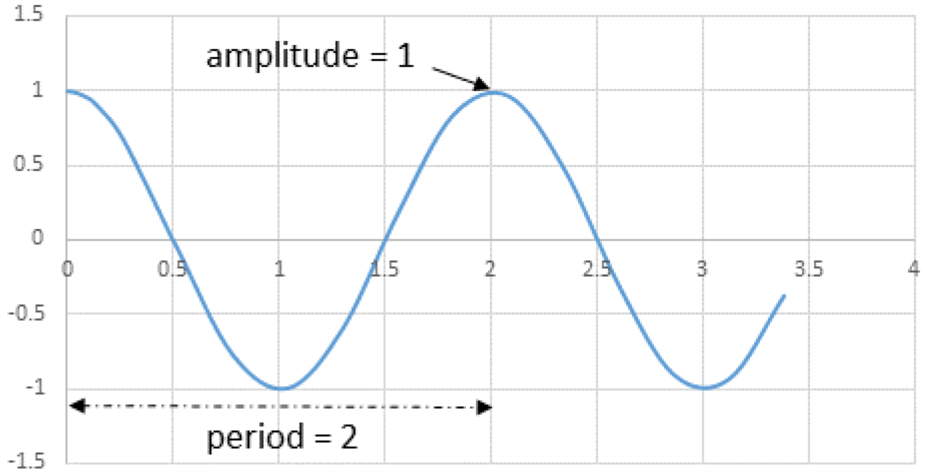
1. Sketch $v^2(t)$
2. Compute $Average[v^2(t)]$
3. Take $\sqrt{}$ of the value found in part 2.

Calculating P_{avg} and V_{rms}

$$Trig\ identity: \cos(\alpha)\cos(\beta) = \frac{1}{2}[\cos(\alpha-\beta)+\cos(\alpha+\beta)]$$

$$y(t) = A\cos\left(\frac{2\pi}{T}t\right)$$

$$A = Amplitude$$
$$T = period$$

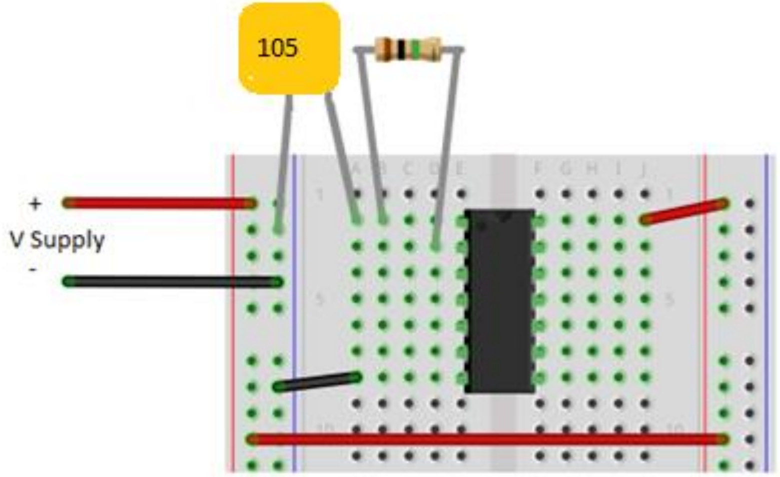


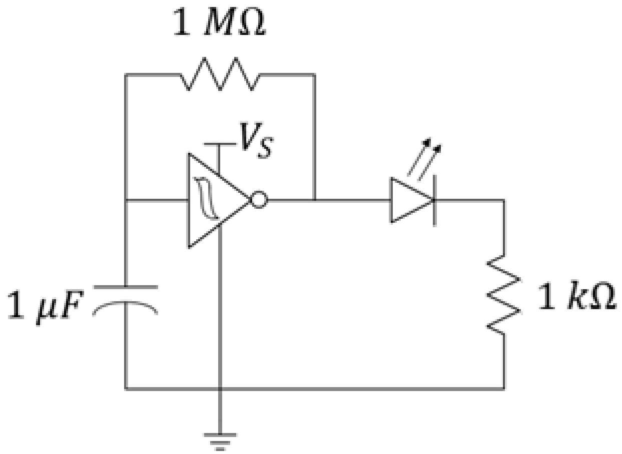
USA "Mains voltage"

Q: What is the <u>average</u> power absorbed by a 250Ω light bulb if A = 170V?

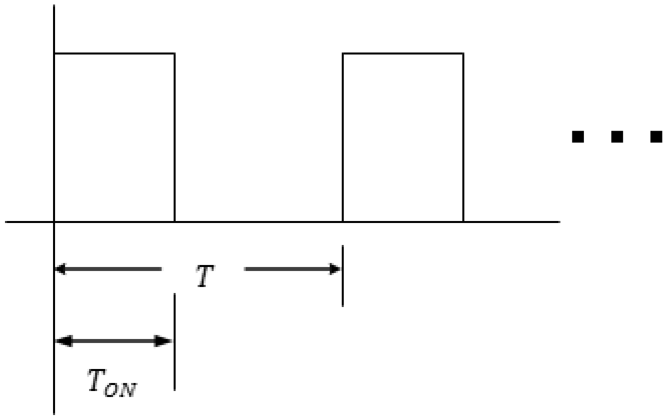
In Practice: Time-varying signals

In lab, we use the output of the invertor to change the input in a feedback loop. A "high" output drives the input high and a low input drives the input low. The invertor's function causes "oscillation" to occur and the LED to flash. Note how the capacitor allows for changing input voltage.





Calculating P_{avg} and V_{rms}



$$Duty\ Cycle\ Definition: \quad \frac{T_{ON}}{T}$$

Q: What happens to power and V_{rms} when T_{ON} is halved while T is unchanged?

L9 Learning Objectives

a. Compute the time-average power from $p(t)$ plots
b. Explain the meaning of V_{rms} and its relationship to P_{avg}
c. Compute the rms-voltage from $v(t)$ plots