Date ______
Page ______

Chapter 1
Mathematical Logic.

* Meaning of statement : In a logic that is Mathematical in its method manipulating symbols according to definite and explicit (स्पष्ट) rules of derivation.
In a logic a statement is either

a) A meaningful declarative sentence ie either True or False.

OR

b) That which a true or False declarative sentence asserts (States Fact).

A mathematical sentence is a sentence that states a fact or content a complete idea.
A sentence that can be judged to be true or false is called a statement or a closed sentence.
A statement or preposition is a declarative sentence that is either true or false but not both.
Is either true denoted by "T" or "1"
and If either false denoted by "F" or "0"

eg: 1) The sun rises in the west

2) $128 = 2^6$

3) $x = 2^6$

4) is $(2^{10} - 1)$ is a even Integer.

5) Maths is Fun

* Primitive and Compound Statement

* Primitive Statement: A statement represented by a single statement variable (without any connective) is called a simple or primitive statement.

* Compound Statement:
A statement represented by some combination of statement variables and connectives is called a compound statement.

eg: P : The clock is slow
Q : The time is correct.

$P \vee Q$ - The clock is slow or the time is correct

P = You are absent.
Q = You have a makeup assignment to complete.

p→q ~~P∨Q~~ = If you are absent then you have a makeup assignment to complete.

* Truth value of Statement:

The truth value of Perepositión is true (T) if it is true preposition and False (F) if it is a False Preposition
eg: P : The year 1973 was a leap year.
Q/: Is a preposition readily decideable as a false.

NOTE: The use of the label 'P ....', so that overall statement is a read P is a statement : "The year 1973 was a leap year") so we use p, q, r, s & t to represent the statement and there letters are called statements variables i.e variable that can be replaced by statement

* # Truth Table :-

A truth table display the relationship between the truth values of the statement
Truth tables are specially valuable in the determination of truth values of statements constructed ~~of~~ for simplier statement.
A table that gives the truth values of compound statement in terms of its component part is called a truth table.

Since each variable can take only two values a statement with n variable require a ~~table~~ ~~variable~~ table with 2^n rows using the letter p, q, r etc.

P
T
F

(1 variable)

p	q
T	T
T	F
F	T
F	F

(2 variable)

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* Connectives :

*) Most mathematical statement are combination of simple statement form through some chose of the word NOT, AND, OR, if-then, if-only if.
This are called logical connectives or simply connective and are denoted by following symbols

$\wedge$	AND
$\vee$	OR
$\sim$ or $\neg$	NOT
$\Rightarrow$ or $\rightarrow$	If ~~and~~ then
$\Leftrightarrow$ or $\leftrightarrow$	If And only IF

* Logical Operators :
There are five logical operators

1) AND Conjunction
2) OR Disjunction
3) NOT Negation
4) if_them Implication / conditional
5) if_only if Double Implication / Biconditional

The first logical operator, which we will discuss is "AND" ~~OR~~ or conjunction operator.
It is a binary operator (a binary operator is defined as an operator that takes two operants & not binary in the sense of binary number system).

If p and q are statement variable the conjunction of p and q "p and q" denoted by $(p \wedge q)$

The compound statement of p and q are true if both p and q are true otherwise it is false.

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

This will be very useful for us when we get to boolean Algebra there we will use 1 in place of T and 0 in place of F. and the "AND" operator will be used to "mask bits"

Date
Page

2) OR : The OR or disjunction operator is also a binary operator and is Traditionally represented using the symbol "v".

If p and q are statement variables the disjunction of p and q is "p or q" denoted p v q

The compound statement p and q is true if atleast one of statement is true. If it is False when both statement are False.

p	q	p v q
T	T	T
T	F	T
F	T	T
F	F	F

3) NOT
The NOT or negation or inversion.
Operator is a unary operator It takes just one offerand. like the urany minus in Arithmatic ie −x

NOT is traditionally represented using either (~) tilde or "¬"

If p is a statement variable the negation of p is "not p") OR "It is not the case that p") and is denoted ~p
It has opposite truth value for p

p	~p
T	F
F	T

The notation for inequality involve "AND" "OR" statement

Let a, b, c be particular real numbers $a \leq b$ means $a < b$ or $a = b$
$a < b < c$ means $a < b$ and $b < c$

~ is a unary operator while OR and AND are binary operators

4) Conditional operator (→) : In a logic conditional statement is a compound sentence that is usually expressed with a keyword "if — then —" using the variable p and q to represent two simple sentence
The conditional if p then q. such statements are called conditional statements.

and are denoted by "$p \to q$"
The conditional $p \to q$ is frequent read as p implies q or p only if q

Other common statement is of the form p ~~and~~ if and only if q such that ~~are~~ statements are called biconditional statement and is denoted by $p \leftrightarrow q$

p	q	$p \to q$	$p \leftrightarrow q$
T	T	T	T
T	F	F	F
F	T	T	F
F	F	T	T

The truth value of $p \to q$ and $p \leftrightarrow q$ are ~~denoted~~ b defined by the table
Observe that

1) The conditional $p \to q$ is a false only when p is true and q is False.
Accordingly when p is false the conditional $p \to q$ is true regardless of the truth value of
T

The biconditional $p \leftrightarrow q$ is true whenever p and q have the same truth values and false otherwise.

1) $\sim p \vee q$

p	q	$\sim p$	$\sim p \vee q$
T	T	F	T
T	F	F	F
F	T	T	T
F	F	T	T

2) Construct a truth table for a statement form $(p \wedge q) \vee \sim r$

p q r

Set up column lable p, q, r $\sim r$, $p \wedge q$ and $(p \wedge q) \vee \sim r$

p q r $\sim r$ $p \wedge q$

Fill in the p, q, r column with all the logically possible combination of T and F.

3) Use the truth table for the negation and $\wedge$ to fill in the $\sim r$ and $(p \wedge q)$ column with the appropriate truth value.

4) Finally p and q or $\sim r$ column by considering truth values for ~~p an~~ $p \wedge q$ and $\sim r$

p	q	r	$\sim r$	$p \wedge q$	$(p \wedge q) \vee \sim r$
T	T	T	F	T	T
T	T	F	T	T	T
T	F	T	F	F	F
T	F	F	T	F	T
F	T	T	F	F	F
F	T	F	T	F	T
F	F	T	F	F	F
F	F	F	T	F	T

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1) $(\sim p \vee q) \wedge \sim r$

p	q	r	a = $\sim r$	$\sim p$	b $\sim p \vee q$	$a \wedge b$
T	T	T	F	F	T	F
T	T	F	T	F	T	T
T	F	T	F	F	F	F
T	F	F	T	F	F	F
F	T	T	F	T	T	F
F	T	F	T	T	T	T
F	F	T	F	T	T	F
F	F	F	T	T	T	T

2) $(\sim p \wedge \sim r) \vee (r \wedge q)$

$\sim p \wedge \sim r = a$
$r \wedge q = b$

p	q	r	$\sim p$	$\sim r$	$\sim p \wedge \sim r$	$r \wedge q$	$a \vee b$
T	T	T	F	F	F	T	T
T	T	F	F	T	F	F	F
T	F	T	F	F	F	F	F
T	F	F	F	T	F	F	F
F	T	T	T	F	F	T	T
F	T	F	T	T	T	F	T
F	F	T	T	F	F	F	F
F	F	F	T	T	T	F	T

3) ~~p~~ $\overset{b}{q} \wedge \sim(\underbrace{\sim p \vee r}_{a=})$

p	q	r	~p	~p ∨ r	~a	b ∧ ~a
T	T	T	F	T	F	F
T	T	F	F	F	T	F
T	F	T	F	T	F	F
T	F	F	F	F	T	F
F	T	T	T	T	F	F
F	T	F	T	T	F	F
F	F	T	T	T	F	F
F	F	F	T	T	F	F

4) $p \vee (q \wedge \underbrace{(p \vee r)}_{a})$

p	q	r	p∨r	q∧a	p∨(q∧a)
T	T	T	T	T	~~F~~ T
T	T	F	T	T	~~T~~ T
T	F	T	T	F	~~F~~ T
T	F	F	T	F	~~F~~ T
F	T	T	T	T	~~F~~ T
F	T	F	F	F	~~F~~ F
F	F	T	T	F	~~F~~ F
F	F	F	F	F	~~F~~ F

1) $\sim q \rightarrow \sim p$

p	q	$\sim p$	$\sim q$	$\sim q \rightarrow \sim p$
T	T	F	F	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

2) $\underbrace{(p \rightarrow q) \wedge \sim p)}_{a} \rightarrow \sim q$

$\sim p$	$\sim q$	p	q	$p \rightarrow q$	$p \rightarrow q \wedge \sim p$	$a \rightarrow \sim q$
F	F	T	T	T	F	T
F	T	T	F	F	F	T
T	F	F	T	T	T	F
T	T	F	F	T	T	T

classmate
Date
Page

* Tautologies and Contradiction

Some prepositions P (p, q……) content only T in the last column of there truth table or in the other words, they are true for any truth value of these variables such preposition are called tautology.
Analogifly the prepositions P (p, q……) is called the contradiction if it content only F in the last column of truth table or in other words if it is false for any truth values of its variable.

Example :

p∧~p
The prepositions "P or not P" ie $P \vee \sim P$ is a tautology and the prepositions "$P \wedge \sim P$" ie $P \wedge \sim P$ is the contradiction
p∨~p
This is verify by looking at these truth tables.
(The truth tables have only 2 rows seems is preposition has only the one variable P)

p	$\sim p$	$p \vee \sim p$
T	F	T
F	T	T

p	$\sim p$	$p \wedge \sim p$
T	F	F
F	T	F

NOTE : The Negation of the tatology is the contradiction seens it is always false and the negation of the contradiction is tautology is always true.

* Logical Equivalence
To propositions $P(p, q, \ldots\ldots)$ and $Q(p, q, \ldots\ldots)$ are said to be logically equivalence or simply equivalence or equal denoted by $P(p, q, \ldots\ldots) \equiv Q(p, q, \ldots\ldots)$ If they have identical truth table

Consider Example : The truth table of :

$\sim(p \wedge q)$

$\sim p \vee \sim q$.

observed that both truth table are the same ie propositions are false in the first case &

true in the other three cases.
Accordingly we can write.

$\sim(p \wedge q) \equiv \sim p \vee \sim q$

In other words the prepositions is logically equivalunce.

Consider the statement "It is not the case that roses are red violets are blue"

this statement can written in the form $\sim(p \vee q)$

where p is = "Roses are Red" and
q is = "Violets are blue.

However as noted above
$\sim(p \wedge q) \equiv \sim p \vee \sim q$

Thus the statement "Roses are not red or voilets are not blue" as the same meaning as the given statement

1)

p	q	$p \wedge q$	$\sim(p \wedge q)$
T	T	T	F
T	F	F	T
F	T	F	T
F	F	F	T

2)

p	q	$\sim p$	$\sim q$	$\sim p \vee \sim q$
T	T	F	F	F
T	F	F	T	T
F	T	T	F	T
F	F	T	T	T

In table ① & ② $\sim(p \wedge q) \equiv \sim p \vee \sim q$ is logically equivalence.

① Verify that the prepositions
1) $p \vee \sim(p \wedge q)$ is tautology
2) Show that the preposition $\sim(p \wedge q)$ and $\sim p \vee \sim q$.

1)

p	q	$p \wedge q$	$\sim(p \wedge q)$	$p \vee \sim(p \wedge q)$
T	T	T	F	T
T	F	F	T	T
F	T	F	T	T
F	F	F	T	T

3) Verify that prepositions $(p \wedge q) \wedge \sim(p \vee q)$ is a contradiction.

p	q	ⓐ $p \wedge q$	$p \vee q$	ⓑ $\sim p \vee q$	$a \wedge b$
T	T	T	T	F	F
T	F	F	T	F	F
F	T	F	T	F	F
F	F	F	F	T	F

$\vdash$ = Logical Arguement

Arguments : An arguement is an assertion that a given set of proposition $P_1, P_2 \ldots\ldots P_n$ called premises.
An another proposition Q called concuclsion. Such an arguement is denoted by $P_1, P_2 \ldots\ldots\ldots P_n \vdash Q$
The notations of logical arguement or valid arguement

An arguement $P_1, P_2 \ldots\ldots P_n \vdash Q$ is said to be valid if Q is true whenever all the premises $P_1, P_2 \ldots P_n$ are true.
An arguement which is not valid is called "fallancy".

eg: The following arguement is valid

(a) $P, P \to q \vdash q$

The proof of this rule follows from the truth table bracicat specifically p and $p \to q$ are true simultaneously only in case row1 and in this case q is true.

The following arguement is a fallancy

$p \to q, q \vdash p$

For $p \to q$ and q are both true in case row 3 in the truth table but in this case p is false.

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

Now the proposition $P_1, P_2 \ldots P_n$ are true simultaneously if and only if the propositions $P_1 \wedge P_2 \wedge \ldots\ldots P_n$ is True. Thus the argument $P_1, P_2 \ldots P_n \vdash Q$ is valid if and only if Q is true.

Whenever $P_1 \wedge P_2 \wedge \ldots\ldots P_n$ is true or equivalently $(P_1 \wedge P_2 \wedge \ldots\ldots \wedge P_n) \vdash Q$ is a tautology.

Date
Page

* A fundamental principal of logical reasoning states

if "($p \rightarrow q$) p implies q and q implies r, then p implies r" ie the following arguement is valid.

$$(p \rightarrow q \wedge q \rightarrow r) \vdash (p \rightarrow r)$$

This fact is very fine by the truth table which show that the following proposition is tautology.

$$(p \rightarrow q \wedge q \rightarrow r) \rightarrow (\overset{c}{p \rightarrow r})$$

p	q	r	$\overset{a}{p \rightarrow q}$	$\overset{b}{q \rightarrow r}$	$a \wedge b$	c	$a \wedge b \rightarrow c$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	F	T	T
T	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	F	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

From the Last coloum it is verified that this arguement is valid and $(p \rightarrow q \wedge q \rightarrow r) \rightarrow (p \rightarrow r)$ is a tautology.

$\vdash$ $\rightarrow$

* Show that the following argu... are fallancy.

$p \rightarrow q, \sim p \vdash \sim q$

Construct a truth table $p \rightarrow q$

$p \rightarrow q \wedge \sim p \rightarrow \sim q$

Since the preposition $(p \rightarrow q \wedge \sim p) \rightarrow \sim q$ is not a tautology the argument is a fallancy.

Equivalently the arguement is a fallancy since in a third line of the truth table $(p \rightarrow q \wedge \sim p)$ are true but $\sim q$ is a false

p	q	a = $p \rightarrow q$	b = $\sim p$	$\sim q$	c = $a \wedge b$	$c \rightarrow \sim q$
T	T	T	F	F	F	T
T	F	F	F	T	F	T
F	T	T	T	F	T	F
F	F	T	T	T	T	T

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Prove that Following arguement is valid

* $p \to \sim q,\ r \to q,\ r \vdash \sim p$

$((p \to \sim q) \land (r \to q) \land r) \to \sim p$

a = b = c =

p	q	r	$\sim q$	$\sim p$	$p \to \sim q$	$r \to q$	$a \land b \land r$	$c \to \sim p$
T	T	T	F	F	F	T	F	T
T	T	F	F	F	F	T	F	T
T	F	T	T	F	T	F	F	T
T	F	F	T	F	T	T	F	T
F	T	T	F	T	T	T	T	T
F	T	F	F	T	T	T	F	T
F	F	T	T	T	T	F	F	T
F	F	F	T	T	T	T	F	T