JEE Mains PCM Formula Book
Advanced Formulas

Compiled for JEE Preparation

Important Constants and Values

Fundamental Constants

Constant	Symbol	Value
Speed of light	c	3×10^{8} m/s
Gravitational constant	G	6.674×10^{-11} N m❑^{2}/kg ❑^{2}
Planck's constant	h	6.626×10^{-34} J s
Boltzmann constant	k_B	1.381×10^{-23} J/K
Avogadro's number	N_A	6.022×10^{23} mol❑$^{-1}$
Electronic charge	e	1.602×10^{-19} C
Permittivity of free space	ϵ_0	8.854×10^{-12} C❑^{2}/N m ❑^{2}
Permeability of free space	μ_0	$4\pi \times 10^{-7}$ T m/A
Gas constant	R	8.314 J/mol·K
Faraday constant	F	96485 C/mol

Advanced Physics Formulas

Mechanics - Advanced

Variable Mass System: $F = m\frac{dv}{dt} + v_{rel}\frac{dm}{dt}$

Definition: Rocket equation accounting for changing mass.
Elastic Collision in 2D:

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} + \frac{2m_2}{m_1 + m_2} v_{2i} \cos\phi$$

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i} \cos\phi + \frac{m_2 - m_1}{m_1 + m_2} v_{2i}$$

Definition: Velocities after oblique elastic collision.
Moment of Inertia (Advanced):

$$\text{Thin rod about end: } I=\frac{1}{3}ML^2$$

$$\text{Solid sphere: } I=\frac{2}{5}MR^2$$

$$\text{Hollow sphere: } I=\frac{2}{3}MR^2$$

$$\text{Circular disc: } I=\frac{1}{2}MR^2$$

Definition: Common moments of inertia for standard shapes.
Radius of Gyration: $I=Mk^2$
Definition: Distance where entire mass can be concentrated for same moment of inertia.
Elastic Potential Energy: $U=\frac{1}{2}kx^2$
Definition: Energy stored in deformed elastic material.
Critical Velocity: $v_c=\sqrt{\frac{2GM}{R}}$
Definition: Minimum velocity for circular orbit.
Time Period of Satellite: $T=2\pi\sqrt{\frac{r^3}{GM}}$
Definition: Kepler's third law for artificial satellites.

Electrodynamics - Advanced

Capacitors with Dielectric: $C=\kappa C_0$
Definition: Capacitance increases by dielectric constant κ.
Energy Density in Electric Field: $u_E=\frac{1}{2}\epsilon_0 E^2$
Definition: Energy per unit volume in electric field.
Energy Density in Magnetic Field: $u_B=\frac{1}{2\mu_0}B^2$
Definition: Energy per unit volume in magnetic field.
Self-Inductance: $L=\frac{\mu_0 N^2 A}{l}$
Definition: For solenoid of length l, area A, and N turns.
Growth of Current in LR Circuit: $I=I_0\left(1-e^{-t/\tau}\right)$
Definition: Exponential growth with time constant $\tau=L/R$.
Decay of Current in LR Circuit: $I=I_0e^{-t/\tau}$
Definition: Exponential decay in LR circuit.
Charging of Capacitor: $q=Q\left(1-e^{-t/RC}\right)$
Definition: Exponential charging in RC circuit.

Discharging of Capacitor: $q = Q\,e^{-t/RC}$
Definition: Exponential discharging in RC circuit.

Optics - Advanced

Lens Maker's Formula: $\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$
Definition: Relates focal length to radii of curvature and refractive index.
Newton's Formula: $f^2 = x_1 x_2$
Definition: Product of object and image distances from focal points equals f^2.
Resolving Power of Telescope: $R = \frac{D}{1.22\,\lambda}$
Definition: Ability to distinguish close objects.
Resolving Power of Microscope: $R = \frac{2\,\mu \sin\theta}{\lambda}$
Definition: Depends on wavelength and numerical aperture.
Brewster's Angle: $\tan i_p = \mu$
Definition: Angle for complete polarization of reflected light.

Modern Physics - Advanced

Compton Effect: $\Delta\lambda = \frac{h}{m_0 c}(1 - \cos\theta)$
Definition: Wavelength shift in photon-electron scattering.
Bragg's Law: $2\,d \sin\theta = n\,\lambda$
Definition: Condition for constructive interference in crystal diffraction.
Moseley's Law: $\sqrt{\nu} = a(Z - b)$
Definition: Relationship between X-ray frequency and atomic number.
Radioactive Decay Law: $N = N_0 e^{-\lambda t}$
Definition: Exponential decay of radioactive substances.
Half-life: $T_{1/2} = \frac{\ln 2}{\lambda}$
Definition: Time for half the atoms to decay.
Mean life: $\tau = \frac{1}{\lambda}$
Definition: Average lifetime of radioactive atoms.

Thermodynamics - Advanced

Work Done in Various Processes:

$$\text{Isothermal: } W = n R T \ln \frac{V_f}{V_i}$$
$$\text{Adiabatic: } W = \frac{P_i V_i - P_f V_f}{\gamma - 1}$$
$$\text{Isobaric: } W = P\left(V_f - V_i\right)$$
$$\text{Isochoric: } W = 0$$

Definition: Work calculations for different thermodynamic processes.

Efficiency of Heat Engine: $\eta = 1 - \frac{T_2}{T_1}$

Definition: Carnot efficiency for ideal heat engine.

Coefficient of Performance: $COP = \frac{Q_2}{W}$

Definition: For refrigerator, ratio of heat extracted to work done.

Advanced Mathematics Formulas

Calculus - Advanced

Leibnitz Rule: $\frac{d^n}{dx^n}(uv) = \sum_{r=0}^{n} \binom{n}{r} u^{n-r} v^r$

Definition: nth derivative of product of two functions.

Rolle's Theorem: If $f(a) = f(b)$ and continuous, then $f'(c) = 0$ for some $c \in (a, b)$

Definition: Special case of Mean Value Theorem.

Lagrange's Mean Value Theorem: $f'(c) = \frac{f(b) - f(a)}{b - a}$

Definition: There exists c where instantaneous rate equals average rate.

Cauchy's Mean Value Theorem: $\frac{f'(c)}{g'(c)} = \frac{f(b) - f(a)}{g(b) - g(a)}$

Definition: Generalized mean value theorem.

Integration by Parts: $\int u\,dv = uv - \int v\,du$

Definition: For product of functions.

Reduction Formulas:

$$\int \sin^n x\,dx = -\frac{1}{n}\sin^{n-1} x \cos x + \frac{n-1}{n}\int \sin^{n-2} x\,dx$$
$$\int \cos^n x\,dx = \frac{1}{n}\cos^{n-1} x \sin x + \frac{n-1}{n}\int \cos^{n-2} x\,dx$$

Definition: Recursive formulas for trigonometric integrals.

Coordinate Geometry - Advanced

Equation of Chord: $T = S_1$
Definition: Chord of contact from external point.
Director Circle: $x^2 + y^2 = a^2 + b^2$ (for ellipse)
Definition: Locus of points from which perpendicular tangents can be drawn.
Parametric Coordinates:

$$\text{Circle: } x = a\cos\theta,\ y = a\sin\theta$$
$$\text{Ellipse: } x = a\cos\theta,\ y = b\sin\theta$$
$$\text{Hyperbola: } x = a\,sec\,\theta,\ y = b\tan\theta$$

Definition: Parametric representations of conic sections.
Polar Coordinates: $x = r\cos\theta,\ y = r\sin\theta$
Definition: Alternative coordinate system.

Vector Algebra - Advanced

Scalar Triple Product: $\left[\vec{a}\,\vec{b}\,\vec{c}\right] = \vec{a}\cdot\left(\vec{b}\times\vec{c}\right)$
Definition: Volume of parallelepiped formed by vectors.
Vector Triple Product: $\vec{a}\times\left(\vec{b}\times\vec{c}\right) = \left(\vec{a}\cdot\vec{c}\right)\vec{b} - \left(\vec{a}\cdot\vec{b}\right)\vec{c}$
Definition: Expansion formula for vector triple product.
Equation of Plane:

$$\text{Vector: } \vec{r} = \vec{a} + \lambda\vec{b} + \mu\vec{c}$$
$$\text{Cartesian: } a\,x + b\,y + c\,z = d$$
$$\text{Intercept: } \frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$

Definition: Different forms of plane equation in 3D.

Probability - Advanced

Binomial Distribution: $P(X = r) = \binom{n}{r} p^r (1-p)^{n-r}$
Definition: Probability of r successes in n independent trials.
Mean and Variance:

$$\text{Mean: } \mu = n\,p$$
$$\text{Variance: } \sigma^2 = n\,p\,(1-p)$$

Definition: For binomial distribution.
Bayes' Theorem: $P(A \vee B) = \dfrac{P(B \vee A)\,P(A)}{P(B)}$
Definition: Conditional probability relationship.

Advanced Chemistry Formulas

Physical Chemistry - Advanced

Arrhenius Equation: $k = A e^{-E_a/RT}$
Definition: Temperature dependence of reaction rate constant.
Integrated Rate Laws:

$$\text{Zero order: } [A] = [A]_0 - kt$$
$$\text{First order: } \ln[A] = \ln[A]_0 - kt$$
$$\text{Second order: } \frac{1}{[A]} = \frac{1}{[A]_0} + kt$$

Definition: Concentration-time relationships for different orders.
Van't Hoff Equation: $\frac{d \ln K}{dT} = \frac{\Delta H}{RT^2}$
Definition: Temperature dependence of equilibrium constant.
Gibbs-Helmholtz Equation: $\Delta G = \Delta H - T\Delta S$
Definition: Relationship between free energy, enthalpy and entropy.
Debye-Hückel Limiting Law: $\log \gamma = -A z^2 \sqrt{I}$
Definition: Activity coefficient in electrolyte solutions.

Organic Chemistry - Advanced Reactions

Hofmann Elimination:
Definition: Anti-elimination giving less substituted alkene (Hofmann product).
Saytzeff Rule: More substituted alkene is major product
Definition: In elimination reactions, hydrogen is preferentially removed from carbon with fewer hydrogens.
Markovnikov's Rule: Hydrogen adds to carbon with more hydrogens
Definition: In addition to unsymmetrical alkenes.
Anti-Markovnikov Addition: (with peroxide)
Definition: Peroxide effect reverses Markovnikov addition for HBr.
Diels-Alder Reaction:
Definition: [4+2] cycloaddition forming six-membered rings.

Coordination Chemistry - Advanced

Crystal Field Splitting Energy:

$$\text{Octahedral: } \Delta_o = \frac{5}{3}\Delta_t$$
$$\text{Tetrahedral: } \Delta_t = \frac{4}{9}\Delta_o$$

Definition: Energy difference between split d-orbitals.
Spectrochemical Series:

Definition: Ligands arranged by increasing crystal field splitting.
Magnetic Moment: $\mu = \sqrt{n(n+2)}$ BM
Definition: For n unpaired electrons, predicts paramagnetism.

Electrochemistry - Advanced

Kohlrausch's Law: $\Lambda_m^\infty = \lambda_{+¿^\infty + \lambda_-^\infty ¿}$
Definition: Limiting molar conductivity equals sum of ionic conductivities.
Debye-Hückel-Onsager Equation: $\Lambda_m = \Lambda_m^\infty - A\sqrt{c}$
Definition: Conductivity dependence on concentration for strong electrolytes.

Solid State - Advanced

Bragg's Equation: $n\lambda = 2d\sin\theta$
Definition: X-ray diffraction by crystals.
Density of Crystal: $\rho = \frac{Z \times M}{N_A \times a^3}$
Definition: For cubic crystal with edge length a.
Atomic Packing Factor:

FCC: 0.74
BCC: 0.68
Simple cubic: 0.52

Definition: Fraction of volume occupied by atoms in unit cell.

Important Shortcuts and Tricks

Physics Shortcuts

Time of Flight (Projectile): $T = \frac{2u\sin\theta}{g}$
Range Maximum: $R_{max} = \frac{u^2}{g}$ at $\theta = 45^\circ$
Maximum Height: $H = \frac{u^2\sin^2\theta}{2g}$
Lens Combination: $\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$
Power: $P = P_1 + P_2$
Resistors in Series: $R_s = R_1 + R_2 + R_3$
Resistors in Parallel: $\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$
Capacitors in Series: $\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$
Capacitors in Parallel: $C_p = C_1 + C_2 + C_3$

Mathematics Shortcuts

Quadratic Roots: $\alpha+\beta=-\frac{b}{a}, \alpha\beta=\frac{c}{a}$

Cubic Roots: $\alpha+\beta+\gamma=-\frac{b}{a}, \alpha\beta+\beta\gamma+\gamma\alpha=\frac{c}{a}$

Binomial Expansion: $(1+x)^n=1+nx+\frac{n(n-1)}{2!}x^2+\cdots$

For small x: $(1+x)^n\approx 1+nx$

Trigonometric Values:

$$\sin 15^\circ=\frac{\sqrt{3}-1}{2\sqrt{2}}, \cos 15^\circ=\frac{\sqrt{3}+1}{2\sqrt{2}}$$

$$\sin 75^\circ=\frac{\sqrt{3}+1}{2\sqrt{2}}, \cos 75^\circ=\frac{\sqrt{3}-1}{2\sqrt{2}}$$

Chemistry Shortcuts

Mole Fraction: $x_i=\frac{n_i}{\sum n_i}$

Molality: $m=\frac{\text{moles of solute}}{\text{kg of solvent}}$

pH calculation: $pH=-\log ¿$

For weak acid: $pH=\frac{1}{2}pK_a-\frac{1}{2}\log C$

Henderson-Hasselbalch: $pH=pK_a+\log\frac{[A^-)}{[HA)}$

For buffer solutions