FINAL JEE–MAIN EXAMINATION – APRIL, 2024

(Held On Friday 05th April, 2024) **TIME : 3 : 00 PM to 6 : 00 PM**

MATHEMATICS — TEST PAPER WITH SOLUTION

SECTION-A

1. Let f: $[-1, 2] \to R$ be given by $f(x) = 2x^2 + x + [x^2] - [x]$, where [t] denotes the greatest integer less than or equal to t. The number of points, where f is not continuous, is :

(1) 6 (2) 3

(3) 4 (4) 5

Ans. (3)

Sol. Doubtful points : $-1, 0, 1, \sqrt{2}, \sqrt{3}, 2$

at $x = \sqrt{2}, \sqrt{3}$

$$f(x) = \underbrace{\left(2x^2 + x - [x]\right)}_{\text{Cont.}} + \underbrace{\left[x^2\right]}_{\text{Cont.}} = \text{Discount}$$

at $x = -1$:

$$\left.\begin{aligned} \text{RHL} \Rightarrow f(x) &= (2 - 1 - (-1)) + 0 = 2 \\ f(-1) &= 2 - 1 - (-1) + 1 = 3 \end{aligned}\right\} \text{Dis.}$$

at $x = 2$:

$$\left.\begin{aligned} \text{LHL} \Rightarrow f(x) &= 8 + 2 - 1 + 3 = 12 \\ f(2) &= 8 + 2 - 2 + 4 = 12 \end{aligned}\right\} \text{Cont.}$$

at $x = 0$:

$$\left.\begin{aligned} \text{LHL} \Rightarrow 0 + 0 - (-1) + 0 &= 1 \\ f(0) &= 0 \end{aligned}\right\} \text{Dis.}$$

at $x = 1$

$$\left.\begin{aligned} \text{LHL} \Rightarrow 2 + 1 - 0 + 0 &= 3 \\ f(1) &= 3 - 1 + 1 = 3 \\ \text{RHL} \Rightarrow 2 + 1 - 1 + 1 &= 3 \end{aligned}\right\} \text{Cont.}$$

2. The differential equation of the family of circles passing the origin and having center at the line $y = x$ is :

(1) $(x^2 - y^2 + 2xy)dx = (x^2 - y^2 + 2xy)dy$

(2) $(x^2 + y^2 + 2xy)dx = (x^2 + y^2 - 2xy)dy$

(3) $(x^2 - y^2 + 2xy)dx = (x^2 - y^2 - 2xy)dy$

(4) $(x^2 + y^2 - 2xy)dx = (x^2 + y^2 + 2xy)dy$

Ans. (3)

Sol. $C \equiv x^2 + y^2 + gx + gy = 0$ .....(1)

$2x + 2yy' + g + gy' = 0$

$$g = -\left(\frac{2x + 2yy'}{1 + y'}\right)$$

Put in (1)

$$x^2 + y^2 - \left(\frac{2x + 2yy'}{1 + y'}\right)(x + y) = 0$$

$(x^2 - y^2 - 2xy)y' = x^2 - y^2 + 2xy$

3. Let $S_1 = \{z \in C : |z| \le 5\}$,

$$S_2 = \left\{z \in C : \text{Im}\left(\frac{z + 1 - \sqrt{3}\,i}{1 - \sqrt{3}\,i}\right) \ge 0\right\} \text{ and}$$

$S_3 = \{z \in C : \text{Re}(z) \ge 0\}$. Then

(1) $\frac{125\pi}{6}$ (2) $\frac{125\pi}{24}$

(3) $\frac{125\pi}{4}$ (4) $\frac{125\pi}{12}$

Ans. (4)

Sol. $S_1 : x^2 + y^2 \le 25$ .....(1)

$$S_2 : \text{lm of } \frac{z + \left(1 - \sqrt{3}\,i\right)}{\left(1 - \sqrt{3}\,i\right)} \ge 0$$

$$\text{lm of } \left(\frac{x + iy}{1 - \sqrt{3}\,i} + 1\right) \ge 0$$

$$\text{lm of } \left(\frac{(x + iy)\left(1 + \sqrt{3}\,i\right)}{4}\right) \ge 0$$

$\Rightarrow \sqrt{3}x + y \ge 0$ .....(2)

$S_3 : x \ge 0$ .....(3)

$$\text{Area} = \frac{5}{12}\left(\pi(5)^2\right)$$

4. The area enclosed between the curves y = x|x| and y = x – |x| is :

(1) $\frac{8}{3}$ (2) $\frac{2}{3}$

(3) 1 (4) $\frac{4}{3}$

Ans. (4)

Sol.

$$A = \int_{-2}^{0} -x^2 - 2x \ = \frac{4}{3}$$

5. 60 words can be made using all the letters of the word BHBJO, with or without meaning. If these words are written as in a dictionary, then the 50th word is :

(1) OBBHJ (2) HBBJO

(3) OBBJH (4) JBBOH

Ans. (3)

Sol. B B H J O

$\boxed{B}$_____ $4! = 24$

$\boxed{H}$_____ $\frac{4!}{2!} = 12$

$\boxed{J}$_____ $\frac{4!}{2!} = 12$

O B B H J

O B B J H $\rightarrow$ 50th rank

6. Let $\vec{a} = 2\hat{i} + 5\hat{j} - \hat{k}$, $\vec{b} = 2\hat{i} - 2\hat{j} + 2\hat{k}$ and $\vec{c}$ be three vectors such that $\left(\vec{c} + \hat{i}\right) \times \left(\vec{a} + \vec{b} + \hat{i}\right) = \vec{a} \times \left(\vec{c} + \hat{i}\right)$. $\vec{a}.\vec{c} = -29$, then $\vec{c}.\left(-2\hat{i} + \hat{j} + \hat{k}\right)$ is equal to :

(1) 10 (2) 5

(3) 15 (4) 12

Ans. (2)

Sol. Let's assume $\vec{v} = \vec{a} + \vec{b} + \hat{i}$

$$= 5\hat{i} + 3\hat{j} + \hat{k}$$

and $\vec{c} + \hat{i} = \vec{p}$

So,

$\vec{p} \times \vec{v} = \vec{a} \times \vec{p}$

$\vec{p} \times \vec{v} + \vec{p} \times \vec{a} = \vec{0}$

$\vec{p} \times \left(\vec{v} + \vec{a}\right) = \vec{0}$

$\Rightarrow \vec{p} = \lambda\left(\vec{v} + \vec{a}\right)$

$\vec{c} + i = \lambda\left(7\hat{i} + 8\hat{j}\right)$

$\vec{a}.\vec{c} + \vec{a}.\hat{i} = \lambda\vec{a}.\left(7\hat{i} + 8\hat{j}\right)$

$-29 + 2 = \lambda\left(14 + 40\right)$

$\lambda = -\frac{1}{2}$

$\vec{c}.\left(-2\hat{i} + \hat{j} + \hat{k}\right) + \hat{i}.\left(-2\hat{i} + \hat{j} + \hat{k}\right) = \lambda\left(7\hat{i} + 8\hat{j}\right).\left(-2\hat{i} + \hat{j} + \hat{k}\right)$

$= -\frac{1}{2}\left(-14 + 8\right) + 2 = 5$

7. Consider three vectors $\vec{a}, \vec{b}, \vec{c}$. Let $\left|\vec{a}\right| = 2, \left|\vec{b}\right| = 3$ and $\vec{a} = \vec{b} \times \vec{c}$. If $\alpha \in \left[0, \frac{\pi}{3}\right]$ is the angle between the vectors $\vec{b}$ and $\vec{c}$, then the minimum value of $27\left|\vec{c} - \vec{a}\right|^2$ is equal to :

(1) 110 (2) 105

(3) 124 (4) 121

Ans. (3)

Sol. $|\vec{c}-\vec{a}| = |\vec{c}|^2 + |\vec{a}|^2 - 2\vec{a}.\vec{c}$

$= |\vec{c}|^2 + 4 - 0$

$\because\ \vec{a} = \vec{b}\times\vec{c}$

$|\vec{a}| = |\vec{b}\times\vec{c}|$

$2 = 3|\vec{c}|\sin\alpha$

$|\vec{c}| = \frac{2}{3}\operatorname{cosec}\alpha \qquad \alpha \in \left[0, \frac{\pi}{3}\right]$

$|\vec{c}|_{min} = \frac{2}{3}\times\frac{2}{\sqrt{3}} \qquad \operatorname{cosec}\alpha \in \left[\frac{2}{\sqrt{3}}, \infty\right)$

$\Rightarrow 27|\vec{c}-\vec{a}|^2_{min} = 27\left(\frac{16}{27}+4\right) = 124$

8. Let A(–1, 1) and B(2, 3) be two points and P be a variable point above the line AB such that the area of ΔPAB is 10. If the locus of P is ax + by = 15, then 5a + 2b is :

(1) $-\frac{12}{5}$ (2) $-\frac{6}{5}$

(3) 4 (4) 6

Ans. (1)

Sol.

$$\frac{1}{2}\begin{vmatrix} h & k & 1 \\ -1 & 1 & 1 \\ 2 & 3 & 1 \end{vmatrix} = 10$$

$-2x + 3y = 25$

$-\frac{6}{5}x + \frac{9}{5}y = 15$

$a = -\frac{6}{5},\ b = \frac{9}{5}$

$5a = -6,\ 2b = \frac{18}{5}$

9. Let (α, β, γ) be the point (8, 5, 7) in the line $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-2}{5}$. Then α + β + γ is equal to

(1) 16 (2) 18

(3) 14 (4) 20

Ans. (3)

Sol.

$\overrightarrow{AM}.\left(2\hat{i}+3\hat{j}+5\hat{k}\right) = 0$

(2λ – 7)(2) + (3λ – 6)(3) + (5λ – 5)(5) = 0

38λ = 57

$\lambda = \frac{3}{2}$

$M\left(4, \frac{7}{2}, \frac{19}{2}\right)$

A'(0,2,12)

10. If the constant term in the expansion of $\left(\frac{\sqrt[5]{3}}{x} + \frac{2x}{\sqrt[3]{5}}\right)^{12}$, x ≠ 0, is $\alpha\times 2^8\times\sqrt[5]{3}$, then 25α is equal to :

(1) 639 (2) 724

(3) 693 (4) 742

Ans. (3)

Sol. $T_{r+1} = {}^{12}C_r\left(\frac{3^{1/5}}{x}\right)^{12-r}\left(\frac{2x}{5^{1/3}}\right)^r$

$T_{r+1} = \frac{{}^{12}C_r(3)^{\frac{12-r}{5}}(2)^r(x)^{2r-12}}{(5)^{r/3}}$

r = 6

$T_7 = \frac{{}^{12}C_6(3)^{6/5}(2)^6}{5^2} = \left(\frac{9\times11\times7}{25}\right)2^8.3^{1/5}$

25α = 693

11. Let f, g : R $\rightarrow$ R be defined as : $f(x) = |x - 1|$ and $g(x) = \begin{cases} e^x, & x \geq 0 \\ x+1, & x \leq 0 \end{cases}$. Then the function $f(g(x))$ is

(1) neither one-one nor onto.

(2) one-one but not onto.

(3) both one-one and onto.

(4) onto but not one-one.

Ans. (1)

Sol. $f(g(x)) = |g(x) - 1|$

$$fog\begin{cases} |e^x - 1| & x \geq 0 \\ |x+1-1| & x \leq 0 \end{cases}$$

$$fog\begin{cases} e^x - 1 & x \geq 0 \\ -x & x \leq 0 \end{cases}$$

12. Let the circle $C_1 : x^2 + y^2 - 2(x + y) + 1 = 0$ and C_2 be a circle having centre at (–1, 0) and radius 2. If the line of the common chord of C_1 and C_2 intersects the y-axis at the point P, then the square of the distance of P from the centre of C_1 is :

(1) 2 (2) 1

(3) 6 (4) 4

Ans. (1)

Sol. $S_1 : x^2 + y^2 - 2x - 2y + 1 = 0$

$S_2 : x^2 + y^2 + 2x - 3 = 0$

Common chord = $S_1 - S_2 = 0$

$-4x - 2y + 4 = 0$

$2x + y = 2 \Rightarrow P(0, 2)$

$d^2_{(c,p)} = (1 - 0)^2 + (2 - 1)^2 = 2$

13. Let the set S = {2, 4, 8, 16, ..., 512} be partitioned into 3 sets A, B, C with equal number of elements such that $A \cup B \cup C = S$ and $A \cap B = B \cap C = A \cap C = \phi$. The maximum number of such possible partitions of S is equal to :

(1) 1680 (2) 1520

(3) 1710 (4) 1640

Ans. (1)

Sol.

$$\frac{9!}{(3!3!3!)3!} \times 3!$$

14. The values of m, n, for which the system of equations

$x + y + z = 4$,

$2x + 5y + 5z = 17$,

$x + 2y + mz = n$

has infinitely many solutions, satisfy the equation :

(1) $m^2 + n^2 - m - n = 46$

(2) $m^2 + n^2 + m + n = 64$

(3) $m^2 + n^2 + mn = 68$

(4) $m^2 + n^2 - mn = 39$

Ans. (4)

Sol. $D = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 5 & 5 \\ 1 & 2 & m \end{vmatrix} = 0 \Rightarrow m = 2$

$D_3 = \begin{vmatrix} 1 & 1 & 4 \\ 2 & 5 & 17 \\ 1 & 2 & n \end{vmatrix} = 0 \Rightarrow n = 7$

15. The coefficients a, b, c in the quadratic equation $ax^2 + bx + c = 0$ are from the set {1, 2, 3, 4, 5, 6}. If the probability of this equation having one real root bigger than the other is p, then 216p equals :

(1) 57 (2) 38

(3) 19 (4) 76

Ans. (2)

Sol. $D > 0$

$b^2 > 4ac$

$b = 3 : (a, c) = (1, 1)(1, 2)(2,1)$

$b = 4 : (a, c) = (1, 1)(1, 2)(2,1)(1,3)(3,1)$

$b = 5 : (a, c) = (1,1)(1,2)(2,1)\ (1,3)(3,1)(1,4)(4,1)$
$(1,5)(5,1)(1,6)(6,1)(2,3)(3,2)(2,2)$

$b = 6 : (a, c) = (1,1)(1,2)(2,1)\ (1,3)(3,1)(1,4)(4,1)$
$(1,5)(5,1)(1,6)(6,1)(2,3)(3,2)(2,4)(4,2)(2,2)$

fav. cases = 38

Prob. : $\dfrac{38}{6\times6\times6}$

16. Let ABCD and AEFG be squares of side 4 and 2 units, respectively. The point E is on the line segment AB and the point F is on the diagonal AC. Then the radius r of the circle passing through the point F and touching the line segments BC and CD satisfies :

(1) $r = 1$ (2) $r^2 - 8r + 8 = 0$

(3) $2r^2 - 4r + 1 = 0$ (4) $2r^2 - 8r + 7 = 0$

Ans. (2)

Sol.

$OF^2 = r^2$

$(2 - r)^2 + (2 - r)^2 = r^2$

$r^2 - 8r + 8 = 0$

17. Let $\beta(m, n) = \int_0^1 x^{m-1}(1-x)^{n-1}dx$, m, n > 0. If $\int_0^1 (1-x^{10})^{20}dx = a\times\beta(b,c)$, then $100(a + b + x)$ equals _____.

(1) 1021 (2) 1120

(3) 2012 (4) 2120

Ans. (4)

Sol. $I = \int_0^1 1.\left(1-x^{10}\right)^{20} dx$

$x^{10} = t$

$x = t^{1/10}$

$dx = \frac{1}{10}(t)^{-9/10}\,dt$

$I = \int_0^1 (1-t)^{20}\frac{1}{10}(t)^{-9/10}\,dt$

$I = \frac{1}{10}\int_0^1 t^{-9/10}(1-t)^{20}\,dt$

$a = \frac{1}{10}$ $b = \frac{1}{10}$ $c = 21$

18. Let $\alpha\beta \neq 0$ and $A = \begin{bmatrix} \beta & \alpha & 3 \\ \alpha & \alpha & \beta \\ -\beta & \alpha & 2\alpha \end{bmatrix}$.

If $B = \begin{bmatrix} 3\alpha & -9 & 3\alpha \\ -\alpha & 7 & -2\alpha \\ -2\alpha & 5 & -2\beta \end{bmatrix}$ is the matrix of cofactors of the elements of A, then det(AB) is equal to :

(1) 343 (2) 125

(3) 64 (4) 216

Ans. (4)

Sol. Equating co-factor fo A_{21}

$(2\alpha^2 - 3\alpha) = \alpha$

$\alpha = 0, 2$ (accept)

Now, $2\alpha^2 - \alpha\beta = 3\alpha$

$\alpha = 2$ $\beta = 1$

$|AB| = |A \text{ cof } (A)| = |A|^3$

$$A = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 2 & 1 \\ -1 & 2 & 4 \end{vmatrix} = 6 - 2(9) + 3(6) = 6$$

19. If $y(\theta)=\dfrac{2\cos\theta+\cos 2\theta}{\cos 3\theta+4\cos 2\theta+5\cos\theta+2}$,

then at $\theta=\dfrac{\pi}{2}$, y'' + y' + y is equal to:

(1) $\dfrac{3}{2}$ (2) 1

(3) $\dfrac{1}{2}$ (4) 2

Ans. (4)

Sol. $y=\dfrac{2\cos\theta+2\cos^2\theta-1}{4\cos^3\theta-3\cos\theta+8\cos^2\theta-4+5\cos\theta+2}$

$$y=\frac{\left(2\cos^2\theta+2\cos\theta-1\right)}{\left(2\cos^2\theta+2\cos\theta-1\right)\left(2\cos\theta+2\right)}$$

$$y=\frac{1}{2}\left(\frac{1}{1+\cos\theta}\right)$$

$\Rightarrow \theta=\dfrac{\pi}{2}$ $\quad y=\dfrac{1}{2}$

$$y'=\frac{1}{2}\left(\frac{-1}{\left(1+\cos\theta\right)^2}\times\left(-\sin\theta\right)\right)$$

$\Rightarrow \theta=\dfrac{\pi}{2}$ $\quad y=\dfrac{1}{2}$

$$y''=\frac{1}{2}\left[\frac{\cos\theta\left(1+\cos\theta\right)^2-\sin\theta(2)(1+\cos\theta)(-\sin\theta)}{(1+\cos\theta)^4}\right]$$

$\Rightarrow \theta=\dfrac{\pi}{2}$ $\quad y=1$

20. For $x\ge 0$, the least value of K, for which $4^{1+x}+4^{1-x}$, $\dfrac{K}{2}$, 16^x+16^{-x} are three consecutive terms of an A.P. is equal to :

(1) 10 (2) 4

(3) 8 (4) 16

Ans. (1)

Sol. $k=\underbrace{4\left(4^x+\dfrac{1}{4^x}\right)}_{\ge 2}+\underbrace{\left(4^{2x}+\dfrac{1}{4^{2x}}\right)}_{\ge 2}$

$k\ge 10$

SECTION-B

21. Let the mean and the standard deviation of the probability distribution

X	α	1	0	–3
P(X)	$\frac{1}{3}$	K	$\frac{1}{6}$	$\frac{1}{4}$

be μ and σ, respectively. If $\sigma-\mu=2$, then $\sigma+\mu$ is equal to _____.

Ans. (5)

Sol. $\dfrac{1}{3}+k+\dfrac{1}{6}+\dfrac{1}{4}=1$ $\quad\Rightarrow k=\dfrac{1}{4}$

$$\mu=\frac{\alpha}{3}+\frac{1}{4}-\frac{3}{4}$$

$$\boxed{\mu=\frac{\alpha}{3}-\frac{1}{2}}$$

$$\sigma=\sqrt{\left(\alpha^2\frac{1}{3}+\frac{1}{4}+9\frac{1}{4}\right)-\left(\frac{\alpha}{3}-\frac{1}{2}\right)^2}$$

$$\sigma=\sqrt{\frac{2\alpha^2}{9}+\frac{\alpha}{3}+\frac{9}{4}}$$

$\sigma=\mu+2$

$$\sigma^2=\left(\mu+2\right)^2\Rightarrow\frac{2\alpha^2}{9}+\frac{\alpha}{3}+\frac{9}{4}=\frac{\alpha^2}{9}+\frac{9}{4}+\alpha$$

$$\frac{\alpha^2}{9}-\frac{2\alpha}{3}=0$$

$\alpha=0$, (reject) or $\alpha=6$

($\because$ $x=0$ is already given)

$\Rightarrow \sigma+\mu=2\mu+2$

$=5$

22. Let y = y(x) be the solution of the differential equation $\dfrac{dy}{dx}+\dfrac{2x}{\left(1+x^2\right)^2}y=xe^{\frac{1}{\left(1+x^2\right)}}$; y(0) = 0.

Then the area enclosed by the curve $f(x)=y(x)e^{-\frac{1}{\left(1+x^2\right)}}$ and the line y – x = 4 is ____.

Ans. (18)

Sol. $IF = e^{\int \frac{2x}{(1+x^2)^2}dx} = e^{\frac{-1}{1+x^2}}$

$$y \cdot e^{\frac{-1}{1+x^2}} = \int x \cdot e^{\frac{1}{1+x^2}} \cdot e^{\frac{-1}{1+x^2}} dx$$

$$y \cdot e^{\frac{-1}{1+x^2}} = \frac{x^2}{2} + c$$

$(0, 0) \Rightarrow \boxed{C = 0}$

$$y(x) = \frac{x^2}{2} e^{\frac{1}{1+x^2}}$$

$$f(x) = \frac{x^2}{2}$$

$$A = \int_{-2}^{4} (x+4) - \frac{x^2}{2} dx = 18$$

23. The number of solutions of $\sin^2 x + (2 + 2x - x^2)\sin x - 3(x - 1)^2 = 0$, where $-\pi \le x \le \pi$, is

Ans. (2)

Sol. $\sin^2 x - (x^2 - 2x - 2)\sin x - 3(x - 1)^2 = 0$

$\sin^2 x - (x - 1)^2)\sin x - 3(x-1)^2 = 0$

roots :

$\sin x = -3$ (reject) or $(x - 1)^2$

$\sin x = (x - 1)^2$

24. Let the point $(-1, \alpha, \beta)$ lie on the line of the shortest distance between the lines $\frac{x+2}{-3} = \frac{y-2}{4} = \frac{z-5}{2}$ and $\frac{x+2}{-1} = \frac{y+6}{2} = \frac{z-1}{0}$.

Then $(\alpha - \beta)^2$ is equal to _____.

Ans. (25)

Sol.

$P(-3\lambda - 2, 4\lambda + 2, 2\lambda + 5)$

$Q(-\mu - 2, 2\mu - 6, 1)$

DRS of PQ = $(3\lambda - \mu, 2\mu - 4\lambda - 8, -2\lambda - 4)$

$$\text{DRS of PQ} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 2 & 0 \\ -3 & 4 & 2 \end{vmatrix}$$

$= \left(4\hat{i} + 2\hat{j} + 2\hat{k}\right)$

OR

(2, 1, 1)

$$\frac{3\lambda - \mu}{2} = \frac{2\mu - 4\lambda - 8}{1} = \frac{-2\lambda - 4}{1}$$

$\Rightarrow \mu = \lambda + 2$ & $7\lambda = \mu - 8$

$\boxed{\lambda = -1}$ $\boxed{\mu = 1}$

Q : (–3, –4, 1)

$$L_{PQ} = \frac{x+3}{2} = \frac{y+4}{1} = \frac{z-1}{1}$$

$$(-1, \alpha, \beta) \Rightarrow 1 = \frac{\alpha + 4}{1} = \frac{\beta - 1}{1}$$

$\Rightarrow \alpha = -3, \beta = 2$

$(\alpha - \beta)^2 = 25$

25. If

$$1+\frac{\sqrt{3}-\sqrt{2}}{2\sqrt{3}}+\frac{5-2\sqrt{6}}{18}+\frac{9\sqrt{3}-11\sqrt{2}}{36\sqrt{3}}+\frac{49-20\sqrt{6}}{180}+....$$

upto ∞ = $2\left(\sqrt{\frac{b}{a}}+1\right)\log_e\left(\frac{a}{b}\right)$, where a and b are integers with gcd(a, b) = 1, then 11a + 18b is equal to _____.

Ans. (76)

Sol. $S=1+\frac{x}{2\sqrt{3}}+\frac{x^2}{18}+\frac{x^3}{36\sqrt{3}}+\frac{x^4}{180}+....\infty$

Put $\frac{x}{\sqrt{3}}=t$, where $x=\sqrt{3}-\sqrt{2}$

$$S=1+\frac{t}{2}+\frac{t^2}{6}+\frac{t^3}{12}+\frac{t^4}{20}+....$$

$$S=1+t\left(1-\frac{1}{2}\right)+t^2\left(\frac{1}{2}-\frac{1}{3}\right)+t^3\left(\frac{1}{3}-\frac{1}{4}\right)+t^4\left(\frac{1}{4}-\frac{1}{5}\right)$$

$$S=\left(1+t+\frac{t^2}{2}+\frac{t^3}{3}+\frac{t^3}{4}+...\right)-\left(\frac{t}{2}+\frac{t^2}{3}+\frac{t^3}{4}+\frac{t^4}{5}+...\right)$$

$$S=\left(t+\frac{t^2}{2}+...\right)-\frac{1}{t}\left(t+\frac{t^2}{2}+\frac{t^3}{3}+...\right)+2$$

$$S=2+\left(1-\frac{1}{t}\right)(-\log(1-t))=\left(\frac{1}{t}-1\right)\log(1-t)+2$$

$$S=2+\left(\frac{\sqrt{3}}{\sqrt{3}-\sqrt{2}}-1\right)\log\left(1-\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}}\right)$$

$$S=2+\left(\frac{\sqrt{2}}{\sqrt{3}-\sqrt{2}}\right)\log e\frac{\sqrt{2}}{\sqrt{3}}$$

$$S=2+\frac{(\sqrt{6}+2)}{2}\log e\frac{2}{3}=2+\left(\sqrt{\frac{3}{2}}+1\right)\log e\frac{2}{3}$$

a = 2, b = 3

11a + 18b = 11 × 2 +18 × 3 = 76

26. Let a > 0 be a root of the equation $2x^2 + x - 2 = 0$.

If $\lim\limits_{x\to\frac{1}{a}}\frac{16\left(1-\cos(2+x-2x^2)\right)}{(1-ax^2)}=\alpha+\beta\sqrt{17}$, where $\alpha, \beta \in Z$ then $\alpha + \beta$ is equal to _____.

Ans. (170)

Sol. $2x^2 + x - 2 = 0 \begin{cases} a \\ b \end{cases}$

$2x^2 - x - 2 = 0 \begin{cases} \frac{1}{a} \\ \frac{1}{b} \end{cases}$

$$\lim_{x\to\frac{1}{a}}16\cdot\frac{\left(1-\cos 2\left(x-\frac{1}{a}\right)\left(x-\frac{1}{b}\right)\right)}{4\left(x-\frac{1}{b}\right)^2}\times\frac{4\left(x-\frac{1}{b}\right)^2}{a^2\left(x-\frac{1}{a}\right)^2}$$

$$=16\times\frac{2}{a^2}\left(\frac{1}{a}-\frac{1}{b}\right)^2$$

$$=\frac{32}{a^2}\left(\frac{17}{4}\right)=\frac{17.8}{a^2}=\frac{17\times8\times16}{\left(-1+\sqrt{117}\right)^2}$$

$$=\frac{136.16}{18.2\sqrt{7}}\times\frac{18+2\sqrt{7}}{18+2\sqrt{7}}$$

$$=\frac{136}{256}\left(18+2\sqrt{7}\right)\cdot16$$

$$=153+17\sqrt{17}=\alpha+\beta\sqrt{17}$$

$\alpha + \beta = 153 + 17 = 170$

27. If $f(t)=\int_0^{\pi}\frac{2xdx}{1-\cos^2 t\sin^2 x}$, $0 < t < \pi$, then the value of $\int_0^{\frac{\pi}{2}}\frac{\pi^2 dt}{f(t)}$ equals _____.

Ans. (1)

Sol. $f(t)=\int_0^{\pi}\frac{2x}{1-\cos^2 t\sin^2 x}dx$ .....(1)

$$= 2\int_0^{\pi} \frac{(\pi - x)dx}{1 - \cos^2 t \sin^2 x} \quad ....(2)$$

$$2f(t) = 2\int_0^{\pi} \frac{\pi}{1 - \cos^2 t \sin^2 x} dx$$

$$f(t) = \int_0^{\pi} \frac{\pi}{1 - \cos^2 t \sin^2 x} dx$$

divide & by $\cos^2 x$

$$f(t) = \pi\int_0^{\pi} \frac{\sec^2 x dx}{\sec^2 x - \cos^2 t \tan^2 x}$$

$$f(t) = 2\pi\int_0^{\pi/2} \frac{\sec^2 x dx}{\sec^2 x - \cos^2 t \tan^2 x}$$

$\tan x = z$

$\sec^2 x dx = dz$

$$f(t) = 2\pi\int_0^{\infty} \frac{dz}{1 + \sin^2 t \cdot z^2}$$

$$= \frac{\pi^2}{\sin t}$$

Then $\int_0^{\pi/2} \frac{\pi^2}{f(t)} dt$

$$= \int_0^{\pi/2} \sin t \, dt$$

$= 1$

28. Let the maximum and minimum values of $\left(\sqrt{8x - x^2 - 12} - 4\right)^2 + (x - 7)^2$, $x \in R$ be M and m respectively. Then $M^2 - m^2$ is equal to _____.

Ans. (1600)

Sol. $(x - 7)^2 + (y - 4)^2$

$$y = \sqrt{8x - x^2 - 12}$$

$$y^2 = -(x - 4)^2 + 16 - 12$$

$$(x - 4)^2 + y^2 = 4$$

$m = 9$

$M = 41$

$M^2 - m^2 = 41^2 - 9^2 = 1600$

29. Let a line perpendicular to the line $2x - y = 10$ touch the parabola $y^2 = 4(x - 9)$ at the point P. The distance of the point P from the centre of the circle $x^2 + y^2 - 14x - 8y + 56 = 0$ is _____.

Ans. (10)

Sol. $y^2 = 4(x - 9)$

slope of tangent $= \frac{-1}{2}$

Point of contact $P\left(9 + \frac{1}{\left(-\frac{1}{2}\right)^2}, \frac{2 \times 1}{\frac{-1}{2}}\right)$

$P(13, -4)$

center of circle $C(7, 4)$

distance $CP = \sqrt{(13 - 7)^2 + (-4 - 4)^2}$

$= 10$

30. The number of real solutions of the equation $x|x + 5| + 2|x + 7| - 2 = 0$ is _____.

Ans. (3)

30. The number of real solutions of the equation $x|x + 5| + 2|x + 7| - 2 = 0$ is _____.

Allen Ans. (3)

Sol. **Case I :** $x \geq -5$

$x^2 + 5x + 2x + 12 = 0$

$x^2 + 7x + 12 = 0$

$x = -3, -4$

Case II : $-7 < x < -5$

$-x^2 - 5x + 2x + 14 - 2 = 0$

$-x^2 - 3x + 12 = 0$

$$x = \frac{-3 \pm \sqrt{9+48}}{2}$$

$$= \frac{-3 \pm \sqrt{57}}{2}$$

$$x = \frac{-3-\sqrt{57}}{2}, \frac{-3+\sqrt{57}}{2} \text{(rejected)}$$

Case III : $x \le -7$

$-x^2 - 5x - 2x - 14 - 2 = 0$

$x^2 + 7x + 16 = 0$

$D = 49 - 64 < 0$

No solutions

No. of solutions = 3

SECTION-A

31. Given below are two statements :

Statement I : When the white light passed through a prism, the red light bends lesser than yellow and violet.

Statement II : The refractive indices are different for different wavelengths in dispersive medium. In the light of the above statements, choose the correct answer from the options given below :

(1) Both Statement I and Statement II are true.

(2) Statement I is true but Statement II is false.

(3) Both Statement I and Statement II are false.

(4) Statement I is false but Statement II is true.

Ans. **(1)**

Sol. As $\lambda_{red} > \lambda_{yellow} > \lambda_{violet}$

Light ray with longer wavelength bends less.

32. Which of the following statement is not true about stopping potential (V_0)?

(1) It depends on the nature of emitter material.

(2) It depends upon frequency of the incident light.

(3) It increases with increase in intensity of the incident light.

(4) It is 1/e times the maximum kinetic energy of electrons emitted.

Ans. **(3)**

Sol. $KE_{max} = h\nu - \phi_0 = eV$

33. The angular momentum of an electron in a hydrogen atom is proportional to : (Where r is the radius of orbit of electron)

(1) $\sqrt{r}$ (2) $\frac{1}{r}$

(3) r (4) $\frac{1}{\sqrt{r}}$

Ans. **(1)**

Sol. $F_C = \frac{mv^2}{r}$

$$\frac{Kq_1q_2}{r^2} = \frac{mv^2}{r}$$

$$mv^2r^2 = Kq_1q_2r$$

$$\frac{L^2}{m} = Kq_1q_2r$$

$$L \propto \sqrt{r}$$

34. A galvanometer of resistance 100 Ω when connected in series with 400 Ω measures a voltage of upto 10 V. The value of resistance required to convert the galvanometer into ammeter to read upto 10 A is $x \times 10^{-2}$ Ω. The value of x is :

(1) 2 (2) 800

(3) 20 (4) 200

Ans. **(3)**

Sol. $i_g = \frac{10}{400+100} = 20\times10^{-3}\,A$

For ammeter

Let shunt resistance = S

$i_gR = (i - i_g)S$

$20 \times 10^{-3} \times 100 = 10\ S$

$S = 20 \times 10^{-2}\ \Omega$

35. The vehicles carrying inflammable fluids usually have metallic chains touching the ground :

(1) To conduct excess charge due to air friction to ground and prevent sparking.

(2) To alert other vehicles.

(3) To protect tyres from catching dirt from ground.

(4) It is a custom.

Ans. **(1)**

Sol. Static charge is developed due to air friction. This can result in combustion. So, metallic chains is used to discharge excess charge.

36. If n is the number density and d is the diameter of the molecule, then the average distance covered by a molecule between two successive collisions (i.e. mean free path) is represented by :

(1) $\frac{1}{\sqrt{2n\pi d^2}}$ (2) $\sqrt{2}n\pi d^2$

(3) $\frac{1}{\sqrt{2}n\pi d^2}$ (4) $\frac{1}{\sqrt{2}n^2\pi^2 d^2}$

Ans. (3)

Sol. n = number of molecule per unit volume

d = diameter of the molecule

$\lambda = \frac{1}{\sqrt{2}\pi d^2 n}$ (By Theory)

37. A particle moves in x-y plane under the influence of a force $\vec{F}$ such that its linear momentum is $\vec{P}(t) = \hat{i}\cos(kt) - \hat{j}\sin(kt)$. If k is constant, the angle between $\vec{F}$ and $\vec{P}$ will be :

(1) $\frac{\pi}{2}$ (2) $\frac{\pi}{6}$

(3) $\frac{\pi}{4}$ (4) $\frac{\pi}{3}$

Ans. (1)

Sol. $\vec{P} = \cos(kt)\hat{i} - \sin(kt)\hat{j}$; $|\vec{P}| = 1$

$\because \vec{P} = m\vec{v}$

$\therefore \hat{P} = \hat{v}$

$\Rightarrow \hat{v} = \cos(kt)\hat{i} - \sin(kt)\hat{j}$

$\hat{a} = \frac{-k\sin(kt)\hat{i} - k\cos(kt)\hat{j}}{k}$

$\Rightarrow \hat{a} = -\sin kt\hat{i} - \cos kt\hat{j}$

$\because \hat{F} = \hat{a} = -\sin kt\,\hat{i} - \cos kt\,\hat{j}$

$\cos\theta = \frac{\hat{F}.\hat{P}}{|\hat{F}||\hat{P}|} = -\frac{\sin kt\cos t + \sin kt\cos t}{1\times 1} = 0$

$\Rightarrow \theta = \frac{\pi}{2}$

38. The electrostatic force $(\vec{F}_1)$ and magnetic force $(\vec{F}_2)$ acting on a charge q moving with velocity v can be written :

(1) $\vec{F}_1 = q\vec{V}.\vec{E}$, $\vec{F}_2 = q(\vec{B}.\vec{V})$

(2) $\vec{F}_1 = q\vec{B}$, $\vec{F}_2 = q(\vec{B}\times\vec{V})$

(3) $\vec{F}_1 = q\vec{E}$, $\vec{F}_2 = q(\vec{V}\times\vec{B})$

(4) $\vec{F}_1 = q\vec{E}$, $\vec{F}_2 = q(\vec{B}\times\vec{V})$

Ans. (3)

Sol. $\vec{F}_1 = q\vec{E}$ (Theory)

$\vec{F}_2 = q(\vec{V}\times\vec{B})$

39. A man carrying a monkey on his shoulder does cycling smoothly on a circular track of radius 9m and completes 120 revolutions in 3 minutes. The magnitude of centripetal acceleration of monkey is (in m/s^2) :

(1) zero (2) $16\pi^2\ ms^{-2}$

(3) $4\pi^2\ ms^{-2}$ (4) $57600\ \pi^2\ ms^{-2}$

Ans. (2)

Sol. Given : R = 9m,

120 revolution in 3 min

$\omega = \frac{120\,\text{Rev.}}{3\,\text{min.}} = \frac{120\times 2\pi\ \text{rad}}{3\times 60\ \text{sec}} = \frac{4\pi}{3}\ \text{rad/s}$

$a_{centripetal} = \omega^2 R = \left(\frac{4\pi}{3}\right)^2 \times 9 = 16\pi^2\ m/s^2$

40. A series LCR circuit is subjected to an AC signal of 200 V, 50 Hz. If the voltage across the inductor (L = 10 mH) is 31.4 V, then the current in this circuit is _____ :

(1) 68 A (2) 63 A

(3) 10 A (4) 10 mA

Ans. (3)

Sol. Voltage across inductor $V_L = IX_L$

$31.4 = I[L\omega]$

$31.4 = I[L(2\pi f)]$

$31.4 = I[10 \times 10^{-3}(2 \times 3.14) \times 50$

$\Rightarrow I = 10$ A

41. What is the dimensional formula of ab^{-1} in the equation $\left(P+\frac{a}{V^2}\right)(V - b) = RT$, where letters have their usual meaning.

(1) $[M^0L^3T^{-2}]$ (2) $[ML^2T^{-2}]$

(3) $[M^{-1}L^5T^3]$ (4) $[M^6L^7T^4]$

Ans. (2)

Sol. $\because$ [V] = [b]

$\therefore$ Dimension of b = $[L^3]$

$$\& \ [P]=\left[\frac{a}{V^2}\right]$$

$[a] = [PV^2] = [ML^{-1}T^{-2}][L^6]$

Dimension of a = $[ML^5T^{-2}]$

$$\therefore \ ab^{-1}=\frac{[ML^5T^{-2}]}{[L^3]}=[ML^2T^{-2}]$$

42. The output (Y) of logic circuit given below is 0 only when :

(1) A = 1, B = 0 (2) A = 0, B = 0

(3) A = 1, B = 1 (4) A = 0, B = 1

Ans. (2)

Sol.

43. A body is moving unidirectionally under the influence of a constant power source. Its displacement in time t is proportional to :

(1) t^2 (2) $t^{2/3}$

(3) $t^{3/2}$ (4) t

Ans. (3)

Sol. P = costant $\Rightarrow$ FV = constant

$$\Rightarrow m\frac{dV}{dt}V = \text{constant}$$

$$\int_0^V VdV = (C)\int_0^t dt$$

$$\left(\frac{V^2}{2}\right) = Ct$$

$V \propto t^{1/2}$

$$\frac{ds}{dt} \propto t^{1/2}$$

$$\int_0^S ds = K\int_0^t t^{1/2}dt$$

$$S = K \times \frac{2}{3}t^{3/2}$$

$S \propto t^{3/2}$

$\therefore$ displacement is proportional to $(t)^{3/2}$

44. Match **List-I** with **List-II** :-

	List-I EM-Wave		**List-II Wavelength Range**
(A)	Infra-red	(I)	$< 10^{-3}$ nm
(B)	Ultraviolet	(II)	400 nm to 1 nm
(C)	X-rays	(III)	1 mm to 700 nm
(D)	Gamma rays	(IV)	1 nm to 10^{-3} nm

Choose the correct answer from the options given below :

(1) (A)-(II), (B)-(I), (C)-(IV), (D)-(III)

(2) (A)-(III), (B)-(II), (C)-(IV), (D)-(I)

(3) (A)-(IV), (B)-(III), (C)-(II), (D)-(I)

(4) (A)-(I), (B)-(III), (C)-(II), (D)-(IV)

Ans. (2)

Sol. Infrared is the least energetic thus having biggest wavelength (λ) & gamma rays are most energetic thus having smallest wavelength (λ).

45. During an adiabatic process, if the pressure of a gas is found to be proportional to the cube of its absolute temperature, then the ratio of $\frac{C_P}{C_V}$ for the gas is :

(1) $\frac{5}{3}$ (2) $\frac{9}{7}$

(3) $\frac{3}{2}$ (4) $\frac{7}{5}$

Ans. (3)

Sol. $P \propto T^3$

PT^{-3} = constant

$\because \frac{PV}{T} = nR$ = constant from ideal gas equation

(P) $(PV)^{-3}$ = constant

$P^{-2} V^{-3}$ = cosntant ...(1)

$\because$ Process equation for adiabatic process is

PV^y = constant ...(2)

Comparing equation (1) and (2)

$$\frac{C_P}{C_V} = y = \frac{3}{2}$$

46. Match **List-I** with **List-II** :

	List-I		**List-II**
(A)	A force that restores an elastic body of unit area to its original state	(I)	Bulk modulus
(B)	Two equal and opposite forces parallel to opposite faces	(II)	Young's modulus
(C)	Forces perpendicular everywhere to the surface per unit area same everywhere	(III)	Stress
(D)	Two equal and opposite forces perpendicular to opposite faces	(IV)	Shear modulus

Choose the correct answer from the options given below :

(1) (A)-(II), (B)-(IV), (C)-(I), (D)-(III)

(2) (A)-(IV), (B)-(II), (C)-(III), (D)-(I)

(3) (A)-(III), (B)-(IV), (C)-(I), (D)-(II)

(4) (A)-(III), (B)-(I), (C)-(II), (D)-(IV)

Ans. (3)

Sol. **(A)** $\text{stress} = \frac{F_{restoring}}{A}$

If A = 1

Stress = $F_{restoring}$

(A)-(III)

(B)

$\Rightarrow$ Shear stress

(B)-(IV)

(C)

$\Rightarrow$ Volumetric stress

(C)-(I)

(D)

$\Rightarrow$ Longitudinal stress

(D)-(II)

47. A vernier callipers has 20 divisions on the vernier scale, which coincides with 19th division on the main scale. The least count of the instrument is 0.1 mm. One main scale division is equal to ____mm.

(1) 1 (2) 0.5

(3) 2 (4) 5

Ans. (3)

Sol. 20 VSD = 19 MSD

$$1\text{VSD} = \frac{19}{20}\text{MSD}$$

L.C. = 1 MSD – 1 VSD

$$0.1 \text{ mm} = 1\text{MSD} - \frac{19}{20}\text{MSD}$$

$$0.1 = \frac{1}{20}\text{MSD}$$

1 MSD = 2mm

48. A heavy box of mass 50 kg is moving on a horizontal surface. If co-efficient of kinetic friction between the box and horizontal surface is 0.3 then force of kinetic friction is :

(1) 14.7 N

(2) 147 N

(3) 1.47 N

(4) 1470 N

Ans. (2)

Sol.

$F_k = \mu_k N = 0.3 \times 50 \times 9.8 = 147$ N

49. A satellite revolving around a planet in stationary orbit has time period 6 hours. The mass of planet is one-fourth the mass of earth. The radius orbit of planet is : (Given = Radius of geo-stationary orbit for earth is 4.2×10^4 km)

(1) 1.4×10^4 km

(2) 8.4×10^4 km

(3) 1.68×10^5 km

(4) 1.05×10^4 km

Ans. (4)

Sol. $T = \dfrac{2\pi r^{3/2}}{\sqrt{GM}}$

$$\frac{T_1}{T_2} = \left(\frac{r_1}{r_2}\right)^{3/2}\left(\frac{M_2}{M_1}\right)^{1/2}$$

$$\frac{6}{24} = \frac{(r_1)^{3/2}}{(4.2\times10^4)^{3/2}}\left(\frac{M}{M/4}\right)^{1/2}$$

$r_1 = 1.05 \times 10^4$ km

50. The ratio of heat dissipated per second through the resistance 5 Ω and 10 Ω in the circuit given below is :

(1) 1 : 2 (2) 2 : 1

(3) 4 : 1 (4) 1 : 1

Ans. (2)

Sol.

$$\frac{i_1}{i_2} = \frac{10}{5} = \frac{2}{1}$$

$$\frac{P_1}{P_2} = \frac{i_1^2R_1}{i_2^2R_2} = \left(\frac{2}{1}\right)^2 \times \frac{5}{10} = \frac{2}{1}$$

SECTION-B

51. A solenoid of length 0.5 m has a radius of 1 cm and is made up of 'm' number of turns. It carries a current of 5A. If the magnitude of the magnetic field inside the solenoid is 6.28×10^{-3} T, then the value of m is :

Ans. (500)

Sol. $\mu_0 ni = B$ n = number of turns per unit length

$$\mu_0\left(\frac{m}{\ell}\right)i = B$$

$$m = \frac{B.\ell}{\mu_0 i} = \frac{6.28\times10^{-3}\times0.5}{12.56\times10^{-7}\times5}$$

m = 500

52. The shortest wavelength of the spectral lines in the Lyman series of hydrogen spectrum is 915 Å. The longest wavelength of spectral lines in the Balmer series will be _____ Å.

Ans. **(6588)**

Sol. Lyman Series

Shortest, $\frac{hc}{\lambda} = -13.6\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

$\lambda\downarrow$ E$\uparrow$; $\frac{hc}{\lambda_0} = -13.6(1)$

Balmer Series :

———— n = 3

———— n = 2

$$\frac{hc}{\lambda_1} = -13.6\left(\frac{1}{2^2} - \frac{1}{3^2}\right)$$

$$\frac{hc}{\lambda_1} = -13.6\left(\frac{1}{4} - \frac{1}{9}\right)$$

$$\frac{hc}{\lambda_1} = -13.6 \times \left(\frac{5}{36}\right)$$

$$\Rightarrow \frac{-13.6\lambda_0}{\lambda_1} = -13.6 \times \frac{5}{36}$$

$$\lambda_1 = \frac{\lambda_0 \times 36}{5} = \frac{915 \times 36}{5} = 6588$$

53. In a single slit experiment, a parallel beam of green light of wavelength 550 nm passes through a slit of width 0.20 mm. The transmitted light is collected on a screen 100 cm away. The distance of first order minima from the central maximum will be $x \times 10^{-5}$ m. The value of x is :

Ans. **(275)**

Sol.

$$y = \frac{\lambda D}{d} = \frac{550 \times 10^{-9} \times 100 \times 10^{-2}}{0.2 \times 10^{-3}} = 275$$

54. A sonometer wire of resonating length 90 cm has a fundamental frequency of 400 Hz when kept under some tension. The resonating length of the wire with fundamental frequency of 600 Hz under same tension _____ cm.

Ans. **(60)**

Sol.

$f_0 = 400$ Hz ; $v = \sqrt{\frac{T}{\mu}}$ = constant

$\frac{\lambda}{2} = L$; $v = f_0\lambda$

$\frac{v}{2f_0} = L \Rightarrow v = 2Lf_0$

$L' = \frac{v}{2f'} = \frac{2Lf_0}{2f'}$

$= \frac{Lf_0}{f'} = \frac{90 \times 400}{600} = 60$

55. A hollow sphere is rolling on a plane surface about its axis of symmetry. The ratio of rotational kinetic energy to its total kinetic energy is $\frac{x}{5}$. The value of x is _____.

Ans. **(2)**

Sol. $$\frac{\frac{1}{2}I\omega^2}{\frac{1}{2}I\omega^2 + \frac{1}{2}mv^2} = \frac{\left(\frac{1}{2}\right)\left(\frac{2}{3}mR^2\right)\omega^2}{\left(\frac{1}{2}\right)\left(\frac{2}{3}mR^2\right)\omega^2 + \frac{1}{2}m(R\omega)^2}$$

$$= \frac{\frac{2}{3}}{\frac{2}{3}+1} = \frac{2}{5}$$

x = 2

56. A hydraulic press containing water has two arms with diameters as mentioned in the figure. A force of 10 N is applied on the surface of water in the thinner arm. The force required to be applied on the surface of water in the thicker arm to maintain equilibrium of water is _____ N.

Ans. (1000 N)

Sol. $\frac{F_1}{A_1} = \frac{F_2}{A_2}$

$\frac{F_1}{\pi(7)^2} = \frac{10}{\pi \times (0.7)^2}$

$F_1 = 1000$ N

57. The electric field at point p due to an electric dipole is E. The electric field at point R on equitorial line will be $\frac{E}{x}$. The value of x :

Ans. (16)

Sol. $E_P = \frac{2KP}{r^3} = E$

$E_R = \frac{KP}{(2r)^3} = \frac{E}{16}$

x = 16

58. The maximum height reached by a projectile is 64 m. If the initial velocity is halved, the new maximum height of the projectile is _____ m.

Ans. (16)

Sol. $H_{max} = \frac{u^2 \sin^2\theta}{2g}$

$\frac{H_{1max}}{H_{2max}} = \frac{u_1^2}{u_2^2}$

$\frac{64}{H_{2max}} = \frac{u^2}{(u/2)^2}$

$H_{2max} = 16$ m

59. A wire of resistance 20 Ω is divided into 10 equal parts. A combination of two parts are connected in parallel and so on. Now resulting pairs of parallel combination are connected in series. The equivalent resistance of final combination is _____ Ω.

Ans. (5)

Sol.

⇒ 10 equal part

Each part has resistance = 2Ω

2 parts are connected in parallel so, R = 1Ω

Now, there will be 5 parts each of resistance 1Ω, they are connected in series.

$R_{eq} = 5R$, $R_{eq} = 5\Omega$

60. The current in an inductor is given by I = (3t + 8) where t is in second. The magnitude of induced emf produced in the inductor is 12 mV. The self-inductance of the inductor _____ mH.

Ans. (4)

Sol. I = 3t + 8

ε = 12 mV

$|\varepsilon| = L\left|\frac{dI}{dt}\right|$

12 = L × 3

L = 4 mH

SECTION-A

61. Match **List - I** with **List - II**.

List - I	**List - II**
(A) ICI	(I) T -Shape
(B) ICI_3	(II) Square pyramidal
(C) ClF_5	(III) Pentagonal bipyramidal
(D) IF_7	(IV) Linear

Choose the **correct** answer from the options given below:

(1) (A)–(I), (B)–(IV), C–(III), D–(II)
(2) (A)–(I), (B)–(III), C–(II), D–(IV)
(3) (A)–(IV), (B)–(I), C–(II), D–(III)
(4) (A)–(IV), (B)–(III), C–(II), D–(I)

Ans. **(3)**

Sol. A. I – Cl (iv) linear

B.

(I) T-shape

C.

(II) Square pyramidal

D.

(III) Pentagonal bipyramidal

62. While preparing crystals of Mohr's salt, dil. H_2SO_4 is added to a mixture of ferrous sulphate and ammonium sulphate, before dissolving this mixture in water, dil. H_2SO_4 is added here to:

(1) prevent the hydrolysis of ferrous sulphate
(2) prevent the hydrolysis of ammonium sulphate
(3) make the medium strongly acidic
(4) increase the rate of formation of crystals

Ans. **(1)**

Sol. Fe^{+2} ions undergoes hydrolysis, therefore while preparing aqueous solution of ferrous sulphate and ammonium sulphate in water dilute sulphuric acid is added to prevent hydrolysis of ferrous sulphate.

63. Identify the major product in the following reaction.

(1)

(2)

(3)

(4)

Ans. **(3)**

Sol.

64. The correct nomenclature for the following compound is:

(1) 2–carboxy–4–hydroxyhept–6–enal
(2) 2–carboxy–4–hydroxyhept–7–enal
(3) 2–formyl–4–hydroxyhept–6–enoic acid
(4) 2–formyl–4–hydroxyhept–7–enoic acid

Ans. **(3)**

Sol.

2-formly-4-hydroxyhept-6-enoic acid

65. Given below are two statements : one is labelled as **Assertion (A)** and the other is labelled as **Reason (R)**.

Assertion (A) : NH_3 and NF_3 molecule have pyramidal shape with a lone pair of electrons on nitrogen atom. The resultant dipole moment of NH_3 is greater than that of NF_3.

Reason (R) : In NH_3, the orbital dipole due to lone pair is in the same direction as the resultant dipole moment of the N–H bonds. F is the most electronegative element.

In the light of the above statements, choose the **correct** answer from the options given below:

(1) Both **(A)** and **(R)** are true and **(R)** is the correct explanation of **(A)**

(2) **(A)** is false but **(R)** is true

(3) **(A)** is true but **(R)** is false

(4) Both **(A)** and **(R)** are true but **(R)** is NOT the correct explanation of **(A)**

Ans. (1)

Sol.

Resultant dipole moment = 0.80×10^{-30} Cm

Resultant dipole moment = 4.90×10^{-30} cm

66. Given below are two statements:

Statement I : On passing $HCl_{(g)}$ through a saturated solution of $BaCl_2$, at room temperature white turbidity appears.

Statement II : When HCl gas is passed through a saturated solution of NaCl, sodium chloride is precipitated due to common ion effect.

In the light of the above statements, choose the **most appropriate** answer from the options given below:

(1) **Statement I** is correct but **Statement II** is incorrect

(2) Both **Statement I** and **Statement II** are incorrect

(3) **Statement I** is incorrect but **Statement II** is correct

(4) Both **Statement I** and **Statement II** are correct

Ans. (1)

Sol. $BaCl_2$, NaCl are soluble but on adding HCl(g) to $BaCl_2$, NaCl solutions, Sodium or Barium chlorides may precipitate out, as a consequence of the law of mass action.

67. The metal atom present in the complex MABXL (where A, B, X and L are unidentate ligands and M is metal) involves sp^3 hybridization. The number of geometrical isomers exhibited by the complex is:

(1) 4 (2) 0

(3) 2 (4) 3

Ans. (2)

Sol. Tetrahedral complex does not show geometrical isomerism.

68. Match **List - I** with **List - II**.

	List - I (Pair of Compounds)		**List - II (Isomerism)**
(A)	n-propanol and Isopropanol	(I)	Metamerism
(B)	Methoxypropane and ethoxyethane	(II)	Chain Isomerism
(C)	Propanone and propanal	(III)	Position Isomerism
(D)	Neopentane and Isopentane	(IV)	Functional Isomerism

(1) (A)–(II), (B)–(I), (C)–(IV), (D)–(III)

(2) (A)–(III), (B)–(I), (C)–(II), (D)–(IV)

(3) (A)–(I), (B)–(III), (C)–(IV), (D)–(II)

(4) (A)–(III), (B)–(I), (C)–(IV), (D)–(II)

Ans. (4)

Sol.

69. The quantity of silver deposited when one coulomb charge is passed through $AgNO_3$ solution:

(1) 0.1 g atom of silver

(2) 1 chemical equivalent of silver

(3) 1 g of silver

(4) 1 electrochemical equivalent of silver

Ans. (4)

Sol. W = ZIt

W = ZQ

$Q = \frac{W}{Z}$

W = ZQ = (electrochemical equivalent)

70. Which one of the following reactions is NOT possible?

Ans. (2)

Sol.

71. Given below are two statements :

Statement I : The metallic radius of Na is 1.86 A° and the ionic radius of Na^+ is lesser than 1.86 A°.

Statement II : Ions are always smaller in size than the corresponding elements.

In the light of the above statements, choose the **correct** answer from the options given below :

(1) **Statement I** is correct but **Statement II** is false

(2) Both **Statement I** and **Statement II** are true

(3) Both **Statement I** and **Statement II** are false

(4) **Statement I** is incorrect but **Statement II** is true

Ans. (1)

Sol. $r_{Na} > r_{Na^+}$

So, Statement (I) is correct but size of anions are greater than size of neutral atoms.

So statement (II) is incorrect.

72. $CH_3CH_2–OH \xrightarrow[\text{(ii) } KMnO_4 \text{ (iii)NaOH, CaO,}\Delta]{\text{(i) Jone's Reagent}} P$

Consider the above reaction sequence and identify the major product P.

(1) Methane (2) Methanal

(3) Methoxymethane (4) Methanoic acid

Ans. (1)

Sol.

73. Consider the given chemical reaction :

Product "A" is :

(1) picric acid (2) oxalic acid

(3) acetic acid (4) adipic acid

Ans. (4)

Sol.

74. For the electro chemical cell

$M|M^{2+}||X|X^{2-}$

If $E^0_{\left(M^{2+}/M\right)} = 0.46\,V$ and $E^0_{\left(X/X^{2-}\right)} = 0.34V$.

Which of the following is **correct** ?

(1) $E_{cell} = -0.80$ V

(2) $M + X \rightarrow M^2 + X^{2-}$ is a spontaneous reaction

(3) $M^{2+} + X^{2-} \rightarrow M + X$ is a spontaneous reaction

(4) $E_{cell} = 0.80$ V

Ans. **(3)**

Sol. $M \mid M^{+2} \parallel X / X^{2-}$

$E^o_{cell} = E^o_{M/M^{+2}} + E^o_{X/X^{-2}}$

$= -0.46 + 0.34 = -0.12V$

As E^o_{cell} is negative so anode becomes cathode and cathode become anode. Spontaneous reaction will be

$M^{+2} + X^{2-} \longrightarrow M + X$

75. The number of moles of methane required to produce 11g CO_2(g) after complete combustion is:

(Given molar mass of methane in g mol^{-1} : 16)

(1) 0.75 (2) 0.25

(3) 0.35 (4) 0.5

Ans. **(2)**

Sol. $C_nH_{2n+2} + \frac{3n+1}{2}O_2 \longrightarrow nCO_2 + (n+1)H_2O$

$CH_4 + 2O_2 \longrightarrow CO_2 + 2H_2O$

4gm 11gm

0.25 mole 0.25 mole

0.25 mole CH_4 gives 0.25 mole (or 11gm) CO_2

76. The number of complexes from the following with no electrons in the t_2 orbital is _____.

$TiCl_4$, $[MnO_4]^-$, $[FeO_4]^{2-}$, $[FeCl_4]^-$, $[CoCl_4]^{2-}$

(1) 3 (2) 1

(3) 4 (4) 2

Ans. **(1)**

Sol. $TiCl_4 \Rightarrow Ti^{+4}$ $e^0t_2^0$

$MnO_4^- \Rightarrow Mn^{+7}$ $e^0t_2^0$

$FeO_4^{2-} \Rightarrow Fe^{+6}$ $e^2t_2^0$

$FeCl_4^{2-} \Rightarrow Fe^{+2}$ $e^3t_2^3$

$CoCl_4^{2-} \Rightarrow Co^{+2}$ $e^4t_2^3$

77. The number of ions from the following that have the ability to liberate hydrogen from a dilute acid is _____. Ti^{2+}, Cr^{2+} and V^{2+}

(1) 0 (2) 2

(3) 3 (4) 1

Ans. **(3)**

Sol. The ions Ti^{+2}, V^{+2} Cr^{+2} are strong reducing agents and will liberate hydrogen from a dilute acid, eg.

$2Cr^{+2}_{(aq.)} + 2H^+_{(aq.)} \longrightarrow 2Cr^{+3}_{(aq.)} + H_2(g)$

78. Identify A and B in the given chemical reaction sequence : -

(1) A -

, B -

(2) A -

, B -

(3) A -
, B -

(4) A -
, B -

Ans. **(2)**

Sol.

(A) (B)

79. The correct statements from the following are :

(A) The decreasing order of atomic radii of group 13 elements is Tl > In > Ga > Al > B.

(B) Down the group 13 electronegativity decreases from top to bottom.

(C) Al dissolves in dil. HCl and liberate H_2 but conc. HNO_3 renders Al passive by forming a protective oxide layer on the surface.

(D) All elements of group 13 exhibits highly stable +1 oxidation state.

(E) Hybridisation of Al in $[Al(H_2O)_6]^{3+}$ ion is sp^3d^2.

Choose the **correct** answer from the options given below :

(1) (C) and (E) only

(2) (A), (C) and (E) only

(3) (A), (B), (C) and (E) only

(4) (A) and (C) only

Ans. **(1)**

Sol. A. size order Tℓ > In > Al > Ga > B

B. Electronegativity order B > Al < Ga < In < Tℓ

D. B, Al are more stable in +3 oxidation state

So, only C, E statements are correct.

80. Coagulation of egg, on heating is because of :

(1) Denaturation of protein occurs

(2) The secondary structure of protein remains unchanged

(3) Breaking of the peptide linkage in the primary structure of protein occurs

(4) Biological property of protein remains unchanged

Ans. **(1)**

Sol. Coagulation of egg give primary structure of protein, which is known as denaturation of protein

SECTION-B

81. Combustion of 1 mole of benzene is expressed at

$$C_6H_6(1) + \frac{15}{2}O_2(g) \rightarrow CO_2(g) + 3H_2O(1).$$

The standard enthalpy of combustion of 2 mol of benzene is – 'x' kJ.

x = _____.

(1) standard Enthalpy of formation of 1 mol of $C_6H_6(1)$, for the reaction
$6C(graphite) + 3H_2(g) \rightarrow C_6H_6(1)$ is 48.5 kJ mol^{-1}.

(2) Standard Enthalpy of formation of 1 mol of $CO_2(g)$, for the reaction
$C(graphite) + O_{2(g)} \rightarrow CO_2(g)$ is –393.5 kJ mol^{-1}.

(3) Standard and Enthalpy of formation of 1 mol of $H_2O(1)$, for the reaction
$H_2(g) + \frac{1}{2}O_2(g) \rightarrow H_2O(1)$ is –286 kJ mol^{-1}.

Ans. **(6535)**

Sol. $6C(graphite)+3H_2(g) \rightarrow C_6H_6(\ell)$; $\Delta H = 48.5$ kJ/mol

$C(graphite)+O_2(g) \rightarrow CO_2(g)$; $\Delta H = -393.5$ kJ/mol

$H_2^{(g)} + \frac{1}{2}(g) \longrightarrow H_2O(\ell)$; $\Delta H = -286$ kJ/mol

equation –(1) × 1 + (2) × 6 + (3) × 3

– 48.5 –6 × 393.5 – 3 × 286

= – 3267.5 kJ for 1 mol

= – 6535 kJ for 2 mol

Ans. 6535 kJ

82. The fusion of chromite ore with sodium carbonate in the presence of air leads to the formation of products A and B along with the evolution of CO_2. The sum of spin-only magnetic moment values of A and B is ___ B.M. (Nearest integer)

(Given atomic number : C : 6, Na : 11, O : 8, Fe : 26, Cr : 24]

Ans. **(6)**

Sol. $4FeCr_2O_4 + 8Na_2CO_3 + 7O_2 \rightarrow$

$$\underset{A}{8Na_2CrO_4} + \underset{B}{2Fe_2O_3} + 8CO_2$$

Spin only magnetic moment

For Na_2CrO_4 $\mu_B = 0$

For Fe_2O_3 $\mu_B = 5.9$

sum = 5.9

83. X of enthanamine was subjected to reaction with $NaNO_2/HCl$ followed by hydrolysis to liberate N_2 and HCl. The HCl generated was completely neutralised by 0.2 moles of NaOH. X is ____ g.

Ans. **(9)**

Sol.

84. In an atom, total number of electrons having quantum numbers n = 4, $|m_l| = 1$ and $m_s = -\frac{1}{2}$ is

Ans. **(6)**

Sol. n = 4

ℓ	m_ℓ
0	0
1	–1, 0, +1
2	–2, –1, 0, +1, +2, +3

So number of orbital associated with

n = 4, $|m_\ell| = 1$ are 6

Now each orbital contain one e^- with $m_s = -\frac{1}{2}$

85. Using the given figure, the ratio of R_f values of sample A and sample C is x × 10^{-2}. Value of x is _____.

Fig : Paper chromatography of Samples

Ans. **(50)**

Sol. R_f of A = $\frac{5}{12.5}$ R_f of C = $\frac{10}{12.5}$

Ratio = $\frac{R_{f(A)}}{R_{f(C)}} = \frac{1}{2}$ = 0.5 or 50 × 10^{-2}

86. In the Claisen-Schmidt reaction to prepare 351 g of dibenzalacetone using 87 g of acetone, the amount of benzaldehyde required is _____g. (Nearest integer)

Ans. **(318)**

Sol. Claisen Schmidt reaction

mw of benzaldehyde = 106

106 × 3 = 318 gm. Benzaldehyde is required to give 1.5 mole (or 351 gm) product

87. Consider the following single step reaction in gas phase at constant temperature.

$2A_{(g)} + B_{(g)} \rightarrow C_{(g)}$

The initial rate of the reaction is recorded as r_1 when the reaction starts with 1.5 atm pressure of A and 0.7 atm pressure of B. After some time, the rate r_2 is recorded when the pressure of C becomes 0.5 atm. The ratio $r_1 : r_2$ is _____ × 10^{-1}. (Nearest integer)

Ans. **(315)**

Sol. 2A(g) + B(g) ⟶ C(g)

r_1	1.5 atm	0.7 atm	
r_2	0.5 atm	0.2 atm	0.5 atm

$\because r = K [P_A]^2[P_B]$

$r_1 = K [1.5]^2[0.7]$

$r_2 = K [0.5]^2[0.2]$

$\frac{r_1}{r_2} = 9 \times \frac{7}{2} = 31.5 = 315 \times 10^{-1}$

Ans. 315

88. The product © in the following sequence of reactions has _____ π bonds.

Ans. (4)

Sol. A =

B =

C =

π bonds = 4

89. Considering acetic acid dissociates in water, its dissociation constant is 6.25×10^{-5}. If 5 mL of acetic acid is dissolved in 1 litre water, the solution will freeze at $-x \times 10^{-2}$ °C, provided pure water freezes at 0 °C.

x = _____. (Nearest integer)

Given : $(K_f)_{water} = 1.86$ K kg mol^{-1}.

density of acetic acid is 1.2 g mol^{-1}

molar mass of water = 18 g mol^{-1}.

molar mass of acetic acid = 60 g mol^{-1}.

density of water = 1 g cm^{-3}

Acetic acid dissociates as

$$CH_3COOH \rightleftharpoons CH_3COO^{\ominus} + H^{\oplus}$$

Ans. (19)

Sol. Mass of CH_3COOH = V × d

= 5 ml × 1.2 g/ml

= 6 gm

$$n_{CH_3COOH} = \frac{6}{60} = 0.1\text{mol}$$

$$m_{CH_3COOH} \approx M_{CH_3COOH} = \frac{0.1}{1} = 0.1\text{M}$$

$CH_3COOH \rightleftharpoons CH_3COO^- + H^+$

C

C – Cα Cα Cα

$$K_a = \frac{C\alpha^2}{1-\alpha}$$

$1 - \alpha \approx 1 \Rightarrow K_a = C\alpha^2$

$$\alpha = \sqrt{\frac{Ka}{C}} = \sqrt{\frac{6.25\times10^{-5}}{0.1}} = 25 \times 10^{-3}$$

V.f. (i) = 1 + α(n – 1) = 1 + α(2 – 1) = 1 + α

$= 1 + 25 \times 10^{-3} = 1.025$

$\Delta T_f = iK_f m$

= (1.025)(1.86)(0.1)

= 0.19

$= 19 \times 10^{-2}$

90. Number of compounds from the following with zero dipole moment is _____.

HF, H_2, H_2S, CO_2, NH_3, BF_3, CH_4, $CHCl_3$, SiF_4, H_2O, BeF_2

Ans. (6)

Sol. H_2, CO_2, BF_3, CH_4, SiF_4 , BeF_2

are symm. molecule so dipole moment is zero

FINAL JEE–MAIN EXAMINATION – APRIL, 2024

(Held On Friday 05th April, 2024) **TIME : 9 : 00 AM to 12 : 00 NOON**

MATHEMATICS — TEST PAPER WITH SOLUTION

SECTION-A

1. Let d be the distance of the point of intersection of the lines $\frac{x+6}{3}=\frac{y}{2}=\frac{z+1}{1}$ and $\frac{x-7}{4}=\frac{y-9}{3}=\frac{z-4}{2}$ from the point (7, 8, 9). Then $d^2 + 6$ is equal to :

(1) 72 (2) 69

(3) 75 (4) 78

Ans. (3)

Sol. $\frac{x+6}{3}=\frac{y}{2}=\frac{z+1}{1}=\lambda$ …(1)

$x = 3\lambda - 6,\ y = 2\lambda,\ z = \lambda - 1$

$\frac{x-7}{4}=\frac{y-9}{3}=\frac{z-4}{2}=\mu$ …(2)

$x = 4\mu + 7,\ y = 3\mu + 9,\ z = 2\mu + 4$

$3\lambda - 6 = 4\mu + 7 \Rightarrow 3\lambda - 4\mu = 13$ …(3) × 2

$2\lambda = 3\mu + 9 \Rightarrow 2\lambda - 3\mu = 9$ …(4) × 3

$6\lambda - 8\mu = 26$

$6\lambda - 9\mu = 27$

$- \quad + \quad -$

$\mu = -1$

$\Rightarrow 3\lambda - 4(-1) = 13$

$3\lambda = 9$

$\lambda = 3$

int. point (3, 6, 2) ; (7, 8, 9)

$d^2 = 16 + 4 + 49 = 69$

Ans. $d^2 + 6 = 69 + 6 = 75$

2. Let a rectangle ABCD of sides 2 and 4 be inscribed in another rectangle PQRS such that the vertices of the rectangle ABCD lie on the sides of the rectangle PQRS. Let a and b be the sides of the rectangle PQRS when its area is maximum. Then $(a + b)^2$ is equal to :

(1) 72 (2) 60

(3) 80 (4) 64

Ans. (1)

Sol.

Area = $(4\cos\theta + 2\sin\theta)(2\cos\theta + 4\sin\theta)$

$= 8\cos^2\theta + 16\sin\theta\cos\theta + 4\sin\theta\cos\theta + 8\sin^2\theta$

$= 8 + 20\sin\theta\cos\theta$

$= 8 + 10\sin 2\theta$

Max Area = 8 + 10 = 18 ($\sin 2\theta = 1$) $\theta = 45°$

$(a + b)^2 = (4\cos\theta + 2\sin\theta + 2\cos\theta + 4\sin\theta)^2$

$= (6\cos\theta + 6\sin\theta)^2$

$= 36(\sin\theta + \cos\theta)^2$

$= 36(\sqrt{2})^2$

$= 72$

3. Let two straight lines drawn from the origin O intersect the line $3x + 4y = 12$ at the points P and Q such that ΔOPQ is an isosceles triangle and $\angle POQ = 90°$. If $l = OP^2 + PQ^2 + QO^2$, then the greatest integer less than or equal to l is :

(1) 44 (2) 48

(3) 46 (4) 42

Ans. (3)

Sol.

$3x + 4y = 12$

$3(r\cos\theta) + 4(r\sin\theta) = 12$

$r(3\cos\theta + 4\sin\theta) = 12$ ...(1)

$3(-r\sin\theta) + 4(r\cos\theta) = 12$

$r(-3\sin\theta + 4\cos\theta) = 12$ ...(2)

$$\left(\frac{12}{r}\right)^2 + \left(\frac{12}{r}\right)^2 = (3\cos\theta + 4\sin\theta)^2 + (-3\sin\theta + 4\cos\theta)^2$$

$$2\left(\frac{12}{r}\right)^2 = 9 + 16$$

$$\frac{2 \times 144}{r^2} = 25 \Rightarrow 288 = 25r^2$$

$$\Rightarrow \frac{288}{25} = r^2$$

$$\Rightarrow \sqrt{2}\left(\frac{12}{5}\right) = r$$

$\ell = OP^2 + PQ^2 + QO^2$

$\ell = r^2 + r^2 \quad + r^2(\cos\theta + \sin\theta)^2 + r^2(\sin\theta + \cos\theta)^2$

$= 2r^2 + r^2(1 + \sin 2\theta + 1 - 2\sin 2\theta)$

$= 2r^2 + 2r^2$

$= 4r^2$

$$= 4\left(\frac{288}{25}\right) = \frac{1152}{25} = 46.08$$

$[\ell] = 46$

4. If $y = y(x)$ is the solution of the differential equation $\frac{dy}{dx} + 2y = \sin(2x)$, $y(0) = \frac{3}{4}$, then $y\left(\frac{\pi}{8}\right)$ is equal to :

(1) $e^{-\pi/8}$ (2) $e^{-\pi/4}$

(3) $e^{\pi/4}$ (4) $e^{\pi/8}$

Ans. (2)

Sol. $\frac{dy}{dx} + 2y = \sin 2x$, $y(0) = \frac{3}{4}$

$I.F = e^{\int 2dx} = e^{2x}$

$y.e^{2x} = \int e^{2x} \sin 2x\, dx$

$$y.e^{2x} = \frac{e^{2x}(2\sin 2x - 2\cos 2x)}{4+4} + C$$

$$x = 0, y = \frac{3}{4} \Rightarrow \frac{3}{4}.1 = \frac{1(0-2)}{8} + C$$

$$\frac{3}{4} = -\frac{1}{4} + C$$

$1 = C$

$$y = \frac{2\sin 2x - 2\cos 2x}{8} + 1.e^{-2x}$$

$$x = \frac{\pi}{8}, \quad y = \frac{1}{8}\left(2\sin\frac{\pi}{4} - 2\cos\frac{\pi}{4}\right) + e^{-2\left(\frac{\pi}{8}\right)}$$

$$y = 0 + e^{-\frac{\pi}{4}}$$

5. For the function

$f(x) = \sin x + 3x - \frac{2}{\pi}(x^2 + x)$, where $x \in \left[0, \frac{\pi}{2}\right]$,

consider the following two statements :

(I) f is increasing in $\left(0, \frac{\pi}{2}\right)$.

(II) f′ is decreasing in $\left(0, \frac{\pi}{2}\right)$.

Between the above two statements,

(1) only (I) is true.

(2) only (II) is true.

(3) neither (I) nor (II) is true.

(4) both (I) and (II) are true.

Ans. (4)

Sol. $f(x) = \sin x + 3x - \frac{2}{\pi}(x^2 + x) \quad x \in \left[0, \frac{\pi}{2}\right]$

$f'(x) = \cos x + 3 - \frac{2}{\pi}(2x + 1) > 0 \quad f(x) \uparrow$

$f'(x) = -\sin x + 0 - \frac{\pi}{2}(2)$

$= -\sin x - \frac{4}{\pi} < 0 \quad f'(x) \downarrow$

$0 < x < \frac{\pi}{2}$

$\Rightarrow -\frac{2}{\pi}\left(\underset{+1}{0} < \underset{+1}{2x} < \underset{+1}{\pi}\right)$

$-\underset{+3}{\frac{2}{\pi}} > \underset{+3}{\frac{-2}{\pi}(2x+1)} > -\underset{+3}{\frac{2}{\pi}(\pi+1)}$

$3 - \underset{(+ve)}{\frac{2}{\pi}} > 3 - \frac{2}{\pi}(2x+1) > 3 - \underset{(+ve)}{\frac{2}{\pi}(\pi+1)}$

6. If the system of equations

$11x + y + \lambda z = -5$

$2x + 3y + 5z = 3$

$8x - 19y - 39z = \mu$

has infinitely many solutions, then $\lambda^4 - \mu$ is equal to :

(1) 49 (2) 45

(3) 47 (4) 51

Ans. (3)

Sol. $11x + y + \lambda z = -5$

$2x + 3y + 5z = 3$

$8x - 19y - 39z = \mu$

for infinite sol.

$$D = \begin{vmatrix} 11 & 1 & \lambda \\ 2 & 3 & 5 \\ 8 & -19 & -39 \end{vmatrix} = 0$$

$\Rightarrow 11(-117 + 95) - 1(-78 - 40) + \lambda(-38 - 24)$

$\Rightarrow 11(-22) + 118 - \lambda(62) = 0$

$\Rightarrow 62\lambda = 118 - 242$

$\Rightarrow \lambda = \frac{-124}{62} = -2$

$$D_1 = \begin{vmatrix} -5 & 1 & -2 \\ 3 & 3 & 5 \\ \mu & -19 & -39 \end{vmatrix} = 0$$

$\Rightarrow -5(-117 + 95) - 1(-117 - 5\mu) - 2(-57 - 3\mu) = 0$

$\Rightarrow -5(-22) + 117 + 5\mu + 114 + 6\mu = 0$

$\Rightarrow 11\mu = -110 - 231 = -341$

$\Rightarrow \mu = -31$

$\lambda^4 - \mu = (-2)^4 - (-31) = 16 + 31 = 47$

7. Let A = {1, 3, 7, 9, 11} and B = {2, 4, 5, 7, 8, 10, 12}. Then the total number of one-one maps $f : A \to B$, such that f (1) + f(3) = 14, is :

(1) 180 (2) 120

(3) 480 (4) 240

Ans. (4)

Sol.

A = {1, 3, 7, 9, 11}

B = {2, 4, 5, 7, 8, 10, 12}

f(1) + f(3) = 14

(i) 2 + 12

(ii) 4 + 10

$2 \times (2 \times 5 \times 4 \times 3) = 240$

8. If the function $f(x) = \frac{\sin 3x + \alpha \sin x - \beta \cos 3x}{x^3}$, $x \in R$, is continuous at x = 0, then f(0) is equal to :

(1) 2 (2) –2

(3) 4 (4) –4

Ans. (4)

Sol. $f(x) = \frac{\sin 3x + \alpha \sin x - \beta \cos 3x}{x^3}$

is continuous at x = 0

$$\lim_{x \to 0} = \frac{3x - \frac{(3x)^3}{\lfloor 3} + \ldots + \alpha\left(x - \frac{x^3}{\lfloor 3} \ldots\right) - \beta\left(1 - \frac{(3x)^2}{\lfloor 2} \ldots\right)}{x^3} = f(0)$$

$$\lim_{x\to 0} = \frac{-\beta + x(3+\alpha) + \frac{9\beta x^2}{\lfloor 2} + \left(\frac{-27}{\lfloor 3} - \frac{\alpha}{\lfloor 3}\right)x^3 \ldots}{x^3} = f(0)$$

for exist

$\beta = 0,\ 3+\alpha = 0,\ -\frac{27}{\lfloor 3} - \frac{\alpha}{\lfloor 3} = f(0)$

$\alpha = -3,\ -\frac{27}{6} - \frac{(-3)}{6} = f(0)$

$f(0) = \frac{-27+3}{6} = -4$

9. The integral $\int_0^{\frac{\pi}{4}} \frac{136\sin x}{3\sin x + 5\cos x}dx$ is equal to :

(1) $3\pi - 50\log_e 2 + 20\log_e 5$

(2) $3\pi - 25\log_e 2 + 10\log_e 5$

(3) $3\pi - 10\log_e\left(2\sqrt{2}\right) + 10\log_e 5$

(4) $3\pi - 30\log_e 2 + 20\log_e 5$

Ans. (1)

Sol. $I = \int_0^{\pi/4} \frac{136\sin x}{3\sin x + 5\cos x}dx$

$136\sin x = A(3\sin x + 5\cos x) + B(3\cos x - 5\sin x)$

$136 = 3A - 5B$ …(1)

$0 = 5A + 3B$ …(2)

$3B = -5A \Rightarrow B = -\frac{5}{3}A$

$136 = 3A - 5\left(-\frac{5}{3}A\right)$

$136 = 3A + \frac{25}{3}A$

$136 = \frac{34A}{3}$

$\Rightarrow A = \frac{136\times 3}{34} = 12$

$B = \frac{-5}{3}(12) = -20$

$I = \int_0^{\pi/4} \frac{A(3\sin x + 5\cos x)}{3\sin x + 5\cos x} + \int_0^{\pi/4} \frac{B(3\cos x - 5\sin x)}{3\sin x + 5\cos x}$

$= A(x)_0^{\pi/4} + B\left[\ell n(3\sin x + 5\cos x)\right]_0^{\pi/4}$

$= 12\left(\frac{\pi}{4}\right) - 20\ell n\left(\frac{3}{\sqrt{2}} + \frac{5}{\sqrt{2}}\right) - \ell n(0+5)$

$= 3\pi - 20\ell n 4\sqrt{2} + 20\ell n 5$

$= 3\pi - 20\times\frac{5}{2}\ell n 2 + 20\ell n 5$

$= 3\pi - 50\ell n 2 + 20\ell n 5$

10. The coefficients a, b, c in the quadratic equation $ax^2 + bx + c = 0$ are chosen from the set {1, 2, 3, 4, 5, 6, 7, 8}. The probability of this equation having repeated roots is :

(1) $\frac{3}{256}$ (2) $\frac{1}{128}$

(3) $\frac{1}{64}$ (4) $\frac{3}{128}$

Ans. (3)

Sol. $ax^2 + bx + c = 0$

$a, b, c \in \{1, 2, 3, 4, 5, 6, 7, 8\}$

Repeated roots D = 0

$\Rightarrow b^2 - 4ac = 0 \Rightarrow b^2 = 4ac$

Prob = $\frac{8}{8\times 8\times 8} = \frac{1}{64}$

$\Rightarrow$ (a, b, c)

(1, 2, 1) ; (2, 4, 2) ; (1, 4, 4) ; (4, 4, 1) ; (3, 6, 3) ; (2, 8, 8) ; (8, 8, 2) ; (4, 8, 4)

8 case

11. Let A and B be two square matrices of order 3 such that $|A| = 3$ and $|B| = 2$.

Then $|A^T A(\text{adj}(2A))^{-1} (\text{adj}(4B))(\text{adj}(AB))^{-1}AA^T|$ is equal to :

(1) 64 (2) 81

(3) 32 (4) 108

Ans. (1)

Sol. $|A| = 3, |B| = 2$

$|A^TA(\text{adj}(2A))^{-1} (\text{adj}(4B)) (\text{adj}(AB))^{-1}AA^T|$

$= 3\times3\times|(\text{adj}(2A)^{-1}| \times |\text{adj}(4B)| \times |(\text{adj}(AB))^{-1}|\times3\times3$

$$\downarrow \qquad \downarrow \qquad \downarrow$$

$$\frac{1}{|\text{adj}(2A)|} \qquad 2^{12}\times 2^2 \qquad \frac{1}{|\text{adj}(AB)|}$$

$$= \frac{1}{2^6\,|\text{adj}A|} \qquad = \frac{1}{|\text{adjB}\cdot\text{adjA}|}$$

$$= \frac{1}{2^6\cdot3^2} \qquad = \frac{1}{2^2\cdot3^2}$$

$$= 3^4\cdot\frac{1}{2^6\cdot3^2}\cdot2^{12}\cdot2^2\cdot\frac{1}{2^2\cdot3^2} = 64$$

12. Let a circle C of radius 1 and closer to the origin be such that the lines passing through the point (3, 2) and parallel to the coordinate axes touch it. Then the shortest distance of the circle C from the point (5, 5) is :

(1) $2\sqrt{2}$ (2) 5

(3) $4\sqrt{2}$ (4) 4

Ans. (4)

Sol.

Coordinates of the centre will be (2, 1)

Equation of circle will be

$(x - 2)^2 + (y - 1)^2 = 1$

$QC = \sqrt{(5-2)^2 + (5-1)^2}$

$QC = 5$

shortest distance

$= RQ = CQ - CR$

$= 5 - 1$

$= 4$

13. Let the line $2x + 3y - k = 0$, $k > 0$, intersect the x-axis and y-axis at the points A and B, respectively. If the equation of the circle having the line segment AB as a diameter is $x^2 + y^2 - 3x - 2y = 0$ and the length of the latus rectum of the ellipse $x^2 + 9y^2 = k^2$ is $\frac{m}{n}$, where m and n are coprime, then $2m + n$ is equal to

(1) 10 (2) 11

(3) 13 (4) 12

Ans. (2)

Sol. Centre of the circle $= \left(\frac{3}{2}, 1\right)$

Equation of diameter $= 2x + 3y - k = 0$

$$2\left(\frac{3}{2}\right) + 3(1) - k = 0$$

$\Rightarrow k = 6$

Now, Equation of ellipse becomes

$x^2 + 9y^2 = 36$

$$\frac{x^2}{6^2} + \frac{y^2}{2^2} = 1$$

length of LR $= \frac{2b^2}{a} = \frac{2.2^2}{6} = \frac{8}{6} = \frac{4}{3} = \frac{m}{n}$

$\therefore$ 2m + n = 2(4) + 3 = 11

14. Consider the following two statements :

Statement I : For any two non-zero complex numbers z_1, z_2

$\left(|z_1|+|z_2|\right)\left|\frac{z_1}{|z_1|}+\frac{z_2}{|z_2|}\right| \le 2\left(|z_1|+|z_2|\right)$ and

Statement II : If x, y, z are three distinct complex numbers and a, b, c are three positive real numbers such that $\frac{a}{|y-z|} = \frac{b}{|z-x|} = \frac{c}{|x-y|}$, then

$\frac{a^2}{y-z} + \frac{b^2}{z-x} + \frac{c^2}{x-y} = 1$.

Between the above two statements,

(1) both Statement I and Statement II are incorrect.

(2) Statement I is incorrect but Statement II is correct.

(3) Statement I is correct but Statement II is incorrect.

(4) both Statement I and Statement II are correct.

Ans. (3)

Sol. **Statement I :**

$\left(|z_1|+|z_2|\right)\left|\frac{z_1}{|z_1|}+\frac{z_2}{|z_2|}\right|$

Since $\left|\frac{z_1}{|z_1|}+\frac{z_2}{|z_2|}\right| \le \left|\frac{z_1}{|z_1|}\right| + \left|\frac{z_2}{|z_2|}\right|$

$\left|\frac{z_1}{|z_1|}+\frac{z_2}{|z_2|}\right| \le \frac{|z_1|}{|z_1|} + \frac{|z_2|}{|z_2|}$

$\left|\frac{z_1}{|z_1|}+\frac{z_2}{|z_2|}\right| \le 2$

$\left(|z_1|+|z_2|\right)\left(\left|\frac{z_1}{|z_1|}+\frac{z_2}{|z_2|}\right|\right) \le 2\left(|z_1|+|z_2|\right)$

$\therefore$ statement I is correct

For **Statement II :**

$\frac{a}{|y-z|} = \frac{b}{|z-x|} = \frac{c}{|x-y|}$

$\frac{a^2}{|y-z|^2} = \frac{b^2}{|z-x|^2} = \frac{c^2}{|x-y|^2} = \lambda$

$a^2 = \lambda(|y-z|^2) = \lambda(y-z)\left(\bar{y}-\bar{z}\right)$

$b^2 = \lambda(z-x)\left(\bar{z}-\bar{x}\right)$ and $c^2 = \lambda(x-y)\left(\bar{x}-\bar{y}\right)$

$\frac{a^2}{y-z} + \frac{b^2}{z-x} + \frac{c^2}{x-y} = \lambda\left(\bar{y}-\bar{z}+\bar{z}-\bar{x}+\bar{x}-\bar{y}\right) = 0$

Statement II is false

15. Suppose $\theta \in \left[0, \frac{\pi}{4}\right]$ is a solution of $4\cos\theta - 3\sin\theta = 1$.

Then $\cos\theta$ is equal to :

(1) $\frac{4}{\left(3\sqrt{6}-2\right)}$ (2) $\frac{6-\sqrt{6}}{\left(3\sqrt{6}-2\right)}$

(3) $\frac{6+\sqrt{6}}{\left(3\sqrt{6}+2\right)}$ (4) $\frac{4}{\left(3\sqrt{6}+2\right)}$

Ans. (1)

Sol. $4\left(\frac{1-\tan^2\theta/2}{1+\tan^2\theta/2}\right) - 3\left(\frac{2\tan\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}}\right) = 1$

let $\tan\frac{\theta}{2} = t$

$\frac{4-4t^2-6t}{1+t^2} = 1$

$4 - 4t^2 - 6t = 1 + t^2$

$\Rightarrow 5t^2 + 6t - 3 = 0$

$$\Rightarrow t = \frac{-6 \pm \sqrt{36 - 4(5)(-3)}}{2(5)}$$

$$= \frac{-6 \pm \sqrt{96}}{10}$$

$$= \frac{-6 \pm 4\sqrt{6}}{10}$$

$$t = \frac{-3 + 2\sqrt{6}}{5}$$

$$\cos\theta = \frac{1-t^2}{1+t^2} = \frac{1-\left(\frac{2\sqrt{6}-3}{5}\right)^2}{1+\left(\frac{2\sqrt{6}-3}{5}\right)^2} = \frac{1-\left(\frac{24+9-12\sqrt{6}}{25}\right)}{1+\left(\frac{24+9-12\sqrt{6}}{25}\right)}$$

$$= \frac{25-33+12\sqrt{6}}{25+33-12\sqrt{6}} = \frac{12\sqrt{6}-8}{58-12\sqrt{6}} = \frac{6\sqrt{6}-4}{29-6\sqrt{6}} \times \frac{29+6\sqrt{6}}{29+6\sqrt{6}}$$

$$= \frac{100+150\sqrt{6}}{625} = \frac{4+6\sqrt{6}}{25} \times \frac{4-6\sqrt{6}}{4-6\sqrt{6}}$$

$$= \frac{-200}{25\left(4-6\sqrt{6}\right)} = \frac{-8}{4-6\sqrt{6}} = \frac{4}{3\sqrt{6}-2}$$

16. If $\frac{1}{\sqrt{1}+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \ldots + \frac{1}{\sqrt{99}+\sqrt{100}} = m$ and $\frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \ldots + \frac{1}{99\cdot 100} = n$, then the point (m, n) lies on the line

(1) $11(x - 1) - 100(y - 2) = 0$

(2) $11(x - 2) - 100(y - 1) = 0$

(3) $11(x - 1) - 100y = 0$

(4) $11x - 100y = 0$

Ans. (4)

Sol. $\frac{1}{\sqrt{1}+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \ldots + \frac{1}{\sqrt{99}+\sqrt{100}} = m$

$$\frac{\sqrt{1}-\sqrt{2}}{-1} + \frac{\sqrt{2}-\sqrt{3}}{-1} \ldots \frac{\sqrt{99}-\sqrt{100}}{-1} = m$$

$\sqrt{100} - 1 = m \Rightarrow m = 9$

$$\frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \ldots \frac{1}{99\cdot 100} = n$$

$$\frac{1}{1} - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} \ldots \frac{1}{99} - \frac{1}{100} = n$$

$$1 - \frac{1}{100} = n$$

$$\frac{99}{100} = n$$

$$(m, n) = \left(9, \frac{99}{100}\right)$$

$$\Rightarrow 11(9) - 100\left(\frac{99}{100}\right)$$

$= 99 - 99 = 0$

Ans. option (4) $11x - 100y = 0$

17. Let $f(x) = x^5 + 2x^3 + 3x + 1$, $x \in R$, and g(x) be a function such that $g(f(x)) = x$ for all $x \in R$. Then $\frac{g(7)}{g'(7)}$ is equal to :

(1) 7 (2) 42

(3) 1 (4) 14

Ans. (4)

Sol. $f(x) = x^5 + 2x^3 + 3x + 1$

$f'(x) = 5x^4 + 6x^2 + 3$

$f'(1) = 5 + 6 + 3 = 14$

$g(f(x)) = x$

$g'(f(x))f'(x) = 1$

for $f(x) = 7$

$\Rightarrow x^5 + 2x^3 + 3x + 1 = 7$

$\Rightarrow x = 1$

$$g'(7)\, f'(1) = 1 \Rightarrow g'(7) = \frac{1}{f'(1)} = \frac{1}{14}$$

$x = 1, f(x) = 7 \Rightarrow g(7) = 1$

$$\frac{g(7)}{g'(7)} = \frac{1}{1/14} = 14$$

18. If A(l, –1, 2), B(5, 7, –6), C(3, 4, –10) and D(–l, –4, –2) are the vertices of a quadrilateral ABCD, then its area is :

(1) $12\sqrt{29}$ (2) $24\sqrt{29}$

(3) $24\sqrt{7}$ (4) $48\sqrt{7}$

Ans. (1)

Sol. A(1, –1, 2)

B(5, 7, –6)

C(3, 4, –10)

D(–1, –4, –2)

Area = $\frac{1}{2}\left|\overrightarrow{AC}\times\overrightarrow{BD}\right| = \frac{1}{2}\left|\left(2\hat{i}+5\hat{j}-12\hat{k}\right)\times\left(6\hat{i}+11\hat{j}-4\hat{k}\right)\right|$

$$= \frac{1}{2}\left|112\hat{i}-64\hat{j}-8\hat{k}\right|$$

$$= 4\left|14\hat{i}-8\hat{j}-\hat{k}\right|$$

$$= 4\sqrt{196+64+1}$$

$$= 4\sqrt{261}$$

$$= 12\sqrt{29}$$

19. The value of $\int_{-\pi}^{\pi}\frac{2y(1+\sin y)}{1+\cos^2 y}dy$ is :

(1) π^2 (2) $\frac{\pi^2}{2}$

(3) $\frac{\pi}{2}$ (4) $2\pi^2$

Ans. (1)

Sol. $\int_{-\pi}^{\pi}\frac{2y(1+\sin y)}{1+\cos^2 y}dy$

$$= \underbrace{\int_{-\pi}^{\pi}\frac{2y}{1+\cos^2 y}dy}_{\text{(Odd)}} + \underbrace{\int_{-\pi}^{\pi}\frac{2y\sin y}{1+\cos^2 y}dy}_{\text{(Even)}}$$

$$= 0 + 2.2\int_{0}^{\pi} y\left(\frac{\sin y}{1+\cos^2 y}\right)dy$$

$$I = 4\int_{0}^{\pi}\frac{y\sin y}{1+\cos^2 y}dy$$

$$I = 4\int_{0}^{\pi}\frac{(\pi-y)\sin y}{1+\cos^2 y}dy$$

$$2I = 4\int_{0}^{\pi}\frac{\pi\sin y}{1+\cos^2 y}dy$$

$$I = 2\pi\int_{0}^{\pi}\frac{\sin y}{1+\cos^2 y}dy$$

$$= 2\pi\left(-\tan^{-1}(\cos y)\right)_0^{\pi}$$

$$= -2\pi\left[\left(-\frac{\pi}{4}\right)-\left(\frac{\pi}{4}\right)\right]$$

$$= -2\pi\left[-\frac{2\pi}{4}\right] = \pi^2$$

20. If the line $\frac{2-x}{3} = \frac{3y-2}{4\lambda+1} = 4-z$ makes a right angle with the line $\frac{x+3}{3\mu} = \frac{1-2y}{6} = \frac{5-z}{7}$, then $4\lambda + 9\mu$ is equal to :

(1) 13 (2) 4

(3) 5 (4) 6

Ans. (4)

Sol. $\frac{2-x}{3} = \frac{3y-2}{4\lambda+1} = 4-z$ …(1)

$$\frac{x-2}{(-3)} = \frac{y-\frac{2}{3}}{\left(\frac{4\lambda+1}{3}\right)} = \frac{z-4}{(-1)}$$

$$\frac{x+3}{3\mu}=\frac{1-2y}{6}=\frac{5-z}{7} \quad \ldots(2)$$

$$\frac{x+3}{3\mu}=\frac{y-\frac{1}{2}}{(-3)}=\frac{z-5}{(-7)}$$

Right angle $\Rightarrow (-3)(3\mu)+\left(\frac{4\lambda+1}{3}\right)(-3)+(-1)(-7)=0$

$-9\mu - 4\lambda - 1 + 7 = 0$

$4\lambda + 9\mu = 6$

SECTION-B

21. From a lot of 10 items, which include 3 defective items, a sample of 5 items is drawn at random. Let the random variable X denote the number of defective items in the sample. If the variance of X is σ^2, then $96\sigma^2$ is equal to_____.

Ans. (56)

Sol. X = denotes number of defective

x	0	1	2	3
P(x)	$\frac{7}{15}$	$\frac{5}{12}$	$\frac{5}{12}$	$\frac{1}{12}$
x_1^2	0	1	4	9
$P_i x_1^2$	0	$\frac{5}{12}$	$\frac{20}{12}$	$\frac{9}{12}$
$p_i x_i$	0	$\frac{5}{12}$	$\frac{10}{12}$	$\frac{3}{12}$

$$\mu=\Sigma p_i x_i=\frac{18}{12}$$

$$\Sigma p_i x_1^2=\frac{34}{12}$$

$$\sigma^2=\Sigma p_i x_1^2-(\mu)^2$$

$$=\frac{34}{12}-\left(\frac{18}{12}\right)^2=\frac{17}{6}-\frac{9}{4}$$

$$\frac{34-27}{12}=\frac{7}{12}$$

$$96\sigma^2=96\times\frac{7}{12}=56$$

22. If the constant term in the expansion of $(1+2x-3x^3)\left(\frac{3}{2}x^2-\frac{1}{3x}\right)^9$ is p, then 108p is equal to

Ans. (54)

Sol. $\left(1+2x-3x^3\right)\left(\frac{3}{2}x^2-\frac{1}{3x}\right)^9$

General term m $\left(\frac{3}{2}x^2-\frac{1}{3x}\right)^9$

$$={}^9C_r\cdot\frac{3^{9-2r}}{2^{9-r}}(-1)^r\cdot x^{18-3r}$$

Put r = 6 to get coeff. of $x^0={}^9C_6\cdot\frac{1}{6^3}\cdot x^0=\frac{7}{18}x^0$

Put r = 7 to get coeff. of $x^{-3}={}^9C_r\cdot\frac{3^{-5}}{2^2}(-1)^7\cdot x^{-3}$

$$=-{}^9C_7\cdot\frac{1}{3^5\cdot 2^2}\cdot x^{-3}=\frac{-1}{27}x^{-3}$$

$$\left(1+2x-3x^3\right)\left(\frac{7}{18}x^0-\frac{1}{27}x^{-3}\right)$$

$$\frac{7}{18}+\frac{3}{27}=\frac{7}{18}+\frac{1}{9}=\frac{7+2}{18}=\frac{9}{18}=\frac{1}{2}$$

$$\therefore\ 108\cdot\frac{1}{2}=54$$

23. The area of the region enclosed by the parabolas $y = x^2 - 5x$ and $y = 7x - x^2$ is _____.

Ans. (72)

NTA Ans. (198)

Sol. $y = x^2 - 5x$ and $y = 7x - x^2$

$\int_0^6 (g(x) - f(x))dx$

$\int_0^6 \left((7x - x^2) - (x^2 - 5x)\right)dx$

$\int_0^6 \left(12x - 2x^2\right)dx = \left[12\frac{x^2}{2} - \frac{2x^3}{3}\right]_0^6$

$\Rightarrow 6(6)^2 - \frac{2}{3}(6)^3$

$= 216 - 144 = 72$ unit^2

24. The number of ways of getting a sum 16 on throwing a dice four times is _____.

Ans. (125)

Sol. $(x^1 + x^2 .... + x^6)^4$

$x^4 \cdot \left(\frac{1 - x^6}{1 - x}\right)^4$

$x^4 \cdot (1 - x^6)^4 \cdot (1 - x)^{-4}$

$x^4[1 - 4x^6 + 6x^{12}....]\ [(1 - x)^{-4}]$

$(x^4 - 4x^{10} + 6x^{16}....)\ (1 - x)^{-4}$

$(x^4 - 4x^{10} + 6x^{16})\ (1 + {}^{15}C_{12}\, x^{12} + {}^{9}C_6 x^6 ....)$

$({}^{15}C_{12} - 4 \cdot {}^{9}C_6 + 6)x^{16}$

$({}^{15}C_3 - 4 \cdot {}^{9}C_6 + 6)$

$= 35 \times 13 - 6 \times 8 \times 7 + 6$

$= 455 - 336 + 6$

$= 125$

25. If $S = \{a \in R : |2a - 1| = 3[a] + 2\{a\}\}$, where $[t]$ denotes the greatest integer less than or equal to t and $\{t\}$ represents the fractional part of t, then $72\sum_{a \in S} a$ is equal to _____.

Ans. (18)

Sol. $|2a - 1| = 3[a] + 2\{a\}$

$|2a - 1| = [a] + 2a$

Case-1 : $a > \frac{1}{2}$

$2a - 1 = [a] + 2a$

$[a] = -1 \quad \therefore a \in [-1, 0)$ Reject

Case-2 : $a < \frac{1}{2}$

$-2a + 1 = [a] + 2a$

$a = I + f$

$-2(I + f) + 1 = I + 2I + 2f$

$I = 0,\ f = \frac{1}{4} \quad \therefore\ a = \frac{1}{4}$

Hence $a = \frac{1}{4}$

$72\sum_{a \in S} a = 72 \times \frac{1}{4} = 18$

26. Let f be a differentiable function in the interval $(0, \infty)$ such that $f(1) = 1$ and $\lim_{t \to x} \frac{t^2 f(x) - x^2 f(t)}{t - x} = 1$ for each $x > 0$. Then $2f(2) + 3f(3)$ is equal to _____.

Ans. (24)

Sol. $\lim_{t \to x} \frac{t^2 f(x) - x^2 f(t)}{t - x} = 1$

$\lim_{t \to x} \frac{2t.f(x) - x^2 f'(x)}{1} = 1$

$2x.f(x) - x2f'(x) = 1$

$\frac{dy}{dx} - \frac{2}{x} \cdot y = \frac{-1}{x^2}$

$\text{I.f.} = e^{\int -\frac{2}{x}dx} = \frac{1}{x^2}$

$\therefore\ \frac{y}{x^2} = \int -\frac{1}{x^4}dx + C$

$\frac{y}{x^2} = \frac{1}{3x^3} + C$

Put $f(1) = 1$

$C = \frac{2}{3}$

$y = \frac{1}{3x} + \frac{2x^2}{3}$

$y = \frac{2x^3 + 1}{3x}$

$f(2) = \frac{17}{6}$

$f(3) = \frac{55}{9}$

$2f(2) + 3f(3) = \frac{17}{3} + \frac{55}{3} = \frac{72}{3} = 24$

27. Let $a_1, a_2, a_3, \ldots$ be in an arithmetic progression of positive terms.

Let $A_k = a_1^2 - a_2^2 + a_3^2 - a_4^2 + \ldots + a_{2k-1}^2 - a_{2k}^2$.

If $A_3 = -153$, $A_5 = -435$ and $a_1^2 + a_2^2 + a_3^2 = 66$, then $a_{17} - A_7$ is equal to _____.

Ans. (910)

Sol. $d \rightarrow$ common diff.

$A_k = -kd[2a + (2k - 1)d]$

$A_3 = -153$

$\Rightarrow 153 = 13d[2a + 5d]$

$51 = d[2a + 5d]$ …(1)

$A_5 = -435$

$435 = 5d[2a + 9d]$

$87 = d[2a + 9d]$

(2) – (1)

$36 = 4d^2$

$d = 3, a = 1$

$a_{17} - A_7 = 49 - [-7.3[2 + 39]] = 910$

28. Let $\vec{a} = \hat{i} - 3\hat{j} + 7\hat{k}$, $\vec{b} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{c}$ be a vector such that $(\vec{a} + 2\vec{b}) \times \vec{c} = 3(\vec{c} \times \vec{a})$. If $\vec{a} \cdot \vec{c} = 130$, then $\vec{b} \cdot \vec{c}$ is equal to _____.

Ans. (30)

Sol. $(\vec{a} + 2\vec{b}) \times \vec{c} = 3(\vec{c} \times \vec{a})$

$(2\vec{b} + 4\vec{a}) \times \vec{c} = 0$

$\vec{c} = \lambda(4\vec{a} + 2\vec{b}) = \lambda(8\hat{i} - 14\hat{j} + 30\hat{k})$

$\vec{a} \cdot \vec{c} = 130$

$8\lambda + 42\lambda + 210\lambda = 130$

$\lambda = \frac{1}{2}$

$\vec{c} = 4\hat{i} - 7\hat{j} + 15\hat{k}$

$\vec{b} \cdot \vec{c} = 8 + 7 + 15 = 30$

29. The number of distinct real roots of the equation $|x|\,|x + 2| - 5|x + 1| - 1 = 0$ is _____.

Ans. (3)

Sol.

Case-1

$x \geq 0$

$x^2 + 2x - 5x - 5 - 1 = 0$

$x^2 - 3x - 6 = 0$

$x = \frac{3 \pm \sqrt{9 + 24}}{2} = \frac{3 \pm \sqrt{33}}{2}$

One positive root

Case-2

$-1 \leq x < 0$

$-x^2 - 2x - 5x - 5 - 1 = 0$

$x^2 + 7x + 6 = 0$

$(x + 6)(x + 1) = 0$

$x = -1$

one root in range

Case-3

$-2 \leq x < -1$

$x^2 - 2x + 5x + 5 - 1 = 0$

$x^2 - 3x - 4 = 0$

$(x - 4)(x + 1) = 0$

No root in range

Case-4

$x < -2$

$x^2 + 7x + 4 = 0$

$$x = \frac{-7 \pm \sqrt{49-16}}{2} = \frac{7 \pm \sqrt{33}}{2}$$

one root in range

Total number of distinct roots are 3

30. Suppose AB is a focal chord of the parabola $y^2 = 12x$ of length l and slope $m < \sqrt{3}$. If the distance of the chord AB from the origin is d, then ld^2 is equal to _____.

Ans. (108)

Sol.

$\ell = 4a \operatorname{cosec}^2\theta$

$$\ell = 12 \times \frac{9}{d^2}$$

$\ell d^2 = 108$

SECTION-A

31. Light emerges out of a convex lens when a source of light kept at its focus. The shape of wavefront of the light is:

(1) Both spherical and cylindrical

(2) Cylindrical

(3) Spherical

(4) Plane

Ans. (4)

Sol. Light emerges parallel

$\therefore$ planor wavefront

32. Following gates section is connected in a complete suitable circuit.

For which of the following combination, bulb will glow (ON):

(1) A = 0, B = 1, C = 1, D = 1

(2) A = 1, B = 0, C = 0, D = 0

(3) A = 0, B = 0, C = 0, D = 1

(4) A = 1, B = 1, C = 1, D = 0

Ans. (2)

Sol. Bulb will glow if bulb have potential drop on it. One end of bulb must be at high (1) and other must be at low (0).

Option (2) satisfy this condition

33. If G be the gravitational constant and u be the energy density then which of the following quantity have the dimension as that the $\sqrt{uG}$:

(1) Pressure gradient per unit mass

(2) Force per unit mass

(3) Gravitational potential

(4) Energy per unit mass

Ans. (2)

Sol. $[uG] = [(M^1L^{-1}T^{-2})\ (M^{-1}L^3T^{-2})]$

$[uG] = [M^0L^2T^{-4}]$

$[\sqrt{uG}] = [L^1T^{-2}]$

Option (2) is correct

34. Given below are two statements :

Statement-I: When a capillary tube is dipped into a liquid, the liquid neither rises nor falls in the capillary. The contact angle may be 0°.

Statement-II: The contact angle between a solid and a liquid is a property of the material of the solid and liquid as well :

In the light of above statement, choose the **correct** answer from the options given below.

(1) **Statement-I** is false but **Statement-II** is true.

(2) Both **Statement-I** and **Statement-II** are true.

(3) Both **Statement-I** and **Statement-II** are false.

(4) **Statement-I** is true and **Statement-II** is false.

Ans. (1)

Sol. Capillary rise

$$h = \frac{2T\cos\theta}{\rho g r};$$

If $\theta = 0°$ then rise is non-zero

$\therefore$ Statement-1 is incorrect.

Option(1) is correct

35. Given below are two statements:

Statement-I: Figure shows the variation of stopping potential with frequency (ν) for the two photosensitive materials M_1 and M_2. The slope gives value of $\frac{h}{e}$, where h is Planck's constant, e is the charge of electron.

Statement-II: M_2 will emit photoelectrons of greater kinetic energy for the incident radiation having same frequency.

In the light of the above statements, choose the most appropriate answer from the options given below.

(1) **Statement-I** is correct and **Statement-II** is incorrect.

(2) **Statement-I** is incorrect but **Statement-II** is correct.

(3) Both **Statement-I** and **Statement-II** are incorrect.

(4) Both **Statement-I** and **Statement-II** are correct.

Ans. (1)

Sol. $eV_0 = hv - \phi$

$$V_0 = \frac{h}{e}v - \frac{\phi}{e}$$

M_2 material has higher work function, so statement-(II) is incorrect.

Option (1) is correct.

36. The angle between vector $\vec{Q}$ and the resultant of $(2\vec{Q}+2\vec{P})$ and $(2\vec{Q}-2\vec{P})$ is:

(1) 0°

(2) $\tan^{-1}\frac{(2\vec{Q}-2\vec{P})}{2\vec{Q}+2\vec{P}}$

(3) $\tan^{-1}\left(\frac{P}{Q}\right)$

(4) $\tan^{-1}\left(\frac{2Q}{P}\right)$

Ans. (1)

Sol. $\vec{R} = (2\vec{Q}+2\vec{P}) + (2\vec{Q}-2\vec{P})$

$\vec{R} = 4\vec{Q}$

Angle between $\vec{Q}$ and $\vec{R}$ is zero

Option (1) is correct

37. In hydrogen like system the ratio of coulombian force and gravitational force between an electron and a proton is in the order of:

(1) 10^{39} (2) 10^{19}

(3) 10^{29} (4) 10^{36}

Ans. (1)

Sol. $F_e = \frac{kQ_1Q_2}{r^2} = \frac{9\times10^9\times1.6\times10^{-19}\times1.6\times10^{-19}}{r^2}$

$$F_g = \frac{Gm_1m_2}{r^2} = \frac{6.67\times10^{-11}\times9.1\times10^{-31}\times1.6\times10^{-27}}{r^2}$$

$$\frac{F_e}{F_g} \cong 0.23\times10^{40} \cong 2.3\times10^{39}$$

Option (1)

38. In a co-axial straight cable, the central conductor and the outer conductor carry equal currents in opposite directions. The magnetic field is zero.

(1) inside the outer conductor

(2) in between the two conductors

(3) outside the cable

(4) inside the inner conductor

Ans. (3)

Sol.

$$\oint \vec{B}.d\vec{\ell} = \mu_0 i_{enc} = 0$$

$\therefore$ B = 0 outside the cable

39. An electron rotates in a circle around a nucleus having positive charge Ze. Correct relation between total energy (E) of electron to its potential energy (U) is:

(1) E = 2U (2) 2E = 3U

(3) E = U (4) 2E = U

Ans. (4)

Sol. $F = \frac{k(Ze)(e)}{r^2} = \frac{mv^2}{r}$

$$KE = \frac{1}{2}mv^2 = \frac{1}{2}\frac{K(Ze)(e)}{r}$$

$$PE = -\frac{K(Ze)(e)}{r}$$

$$TE = \frac{K(Ze)(e)}{2r} - \frac{K(Ze)(e)}{r} = \frac{-K(Ze)(e)}{2r}$$

$$TE = \frac{PE}{2}$$

2TE = PE

Option (4)

40. If the collision frequency of hydrogen molecules in a closed chamber at 27°C is Z, then the collision frequency of the same system at 127° C is :

(1) $\frac{\sqrt{3}}{2}Z$ (2) $\frac{4}{3}Z$

(3) $\frac{2}{\sqrt{3}}Z$ (4) $\frac{3}{4}Z$

Ans. (3)

Sol. Assuming mean free path constant.

$$f \propto v \propto \sqrt{T}$$

$$\frac{f_1}{f_2} = \sqrt{\frac{T_1}{T_2}} = \sqrt{\frac{300}{400}}$$

$$f_2 = \sqrt{\frac{4}{3}} = f_1 = \frac{2}{\sqrt{3}}Z$$

41. Ratio of radius of gyration of a hollow sphere to that of a solid cylinder of equal mass, for moment of Inertia about their diameter axis AB as shown in figure is $\sqrt{\frac{8}{x}}$. The value of x is:

(1) 34 (2) 17

(3) 67 (4) 51

Ans. (3)

Sol. $I_{sphere} = \frac{2}{3}MR^2 = Mk_1^2$

$$I_{cylinder} = \frac{1}{12}M(4R^2) + \frac{1}{4}MR^2 + M(2R)^2$$

$$= \frac{67}{12}MR^2 = Mk_2^2$$

$$\frac{k_1}{k_2} = \sqrt{\frac{2}{3}.\frac{12}{67}} = \sqrt{\frac{8}{67}}$$

42. Two conducting circular loops A and B are placed in the same plane with their centres coinciding as shown in figure. The mutual inductance between them is:

(1) $\frac{\mu_0 \pi a^2}{2b}$ (2) $\frac{\mu_0}{2\pi} \cdot \frac{b^2}{a}$

(3) $\frac{\mu_0 \pi b^2}{2a}$ (4) $\frac{\mu_0}{2\pi} \cdot \frac{a^2}{b}$

Ans. (1)

Sol. $\phi = Mi = BA$

$$\Rightarrow Mi = \frac{\mu_0 i}{2b} \pi a^2$$

$$\therefore M = \frac{\mu_0 \pi a^2}{2b}$$

43. Match **list-I** with **list-II**:

	List-I		**List-II**
(A)	Kinetic energy of planet	(I)	$-\frac{GMm}{a}$
(B)	Gravitation Potential energy of Sun-planet system.	(II)	$\frac{GMm}{2a}$
(C)	Total mechanical energy of planet	(III)	$\frac{Gm}{r}$
(D)	Escape energy at the surface of planet for unit mass object	(IV)	$-\frac{GMm}{2a}$

(Where a = radius of planet orbit, r = radius of planet, M = mass of Sun, m = mass of planet)

Choose the correct answer from the options given below:

(1) (A) – II, (B) – I, (C) – IV, (D) – III

(2) (A) – III, (B) – IV, (C) – I, (D) – II

(3) (A) – I, (B) – IV, (C) – II, (D) – III

(4) (A) – I, (B) – II, (C) – III, (D) – IV

Ans. (1)

Sol. $KE = \frac{1}{2}mv^2 = \frac{GMm}{2a}$

PE = –2KE

TE = –KE

44. A wooden block of mass 5kg rests on soft horizontal floor. When an iron cylinder of mass 25 kg is placed on the top of the block, the floor yields and the block and the cylinder together go down with an acceleration of 0.1 ms^{-2}. The action force of the system on the floor is equal to:

(1) 297 N (2) 294 N

(3) 291 N (4) 196 N

Ans. (3)

Sol. Taking g = 9.8 m/s^2

294 – N = 30 × 0.1

N = 291

45. A simple pendulum doing small oscillations at a place R height above earth surface has time period of T_1 = 4 s. T_2 would be it's time period if it is brought to a point which is at a height 2R from earth surface. Choose the correct relation [R = radius of Earth]:

(1) $T_1 = T_2$ (2) $2T_1 = 3T_2$

(3) $3T_1 = 2T_2$ (4) $2T_1 = T_2$

Ans. (3)

Sol. $T_1 = 2\pi\sqrt{\frac{\ell}{GM}(2R)^2}$

$$T_2 = 2\pi\sqrt{\frac{\ell}{GM}(3R)^2}$$

$$\therefore \frac{T_1}{T_2} = \frac{2}{3}$$

46. A body of mass 50 kg is lifted to a height of 20 m from the ground in the two different ways as shown in the figures. The ratio of work done against the gravity in both the respective cases, will be:

Case-1: Pulled straight up

Case-2: Along the ramp

(1) 1 : 1 (2) 2 : 1

(3) $\sqrt{3}:2$ (4) 1 : 2

Ans. (1)

Sol. Work done by gravity is independent of path. It depends only on vertical displacement so work done in both cases will be same.

Option (1) is correct

47. Time periods of oscillation of the same simple pendulum measured using four different measuring clocks were recorded as 4.62 s, 4.632 s, 4.6 s and 4.64 s. The arithmetic mean of these reading in correct significant figure is.

(1) 4.623 s (2) 4.62 s

(3) 4.6 s (4) 5 s

Ans. (3)

Sol. Sum of number by considering significant digit

sum = 4.6 + 4.6 + 4.6 + 4.6 = 18.4

Arithmetic Mean = $\frac{\text{sum}}{4} = \frac{18.4}{4} = 4.6$

48. The heat absorbed by a system in going through the given cyclic process is :

(1) 61.6 J (2) 431.2 J

(3) 616 J (4) 19.6 J

Ans. (1)

Sol. $\Delta U = 0$ (Cyclic process)

$\Delta Q = W$ = area of P-V curve.

$= \pi \times (140 \times 10^3 \text{ Pa}) \times (140 \times 10^{-6} \text{ m}^3)$

$\Delta Q = 61.6$ J

49. In the given figure $R_1 = 10\Omega$, $R_2 = 8\Omega$, $R_3 = 4\Omega$ and $R_4 = 8\Omega$. Battery is ideal with emf 12V. Equivalent resistant of the circuit and current supplied by battery are respectively.

(1) 12 Ω and 11.4 A (2) 10.5 Ω and 1.14 A

(3) 10.5 Ω and 1 A (4) 12 Ω and 1 A

Ans. (4)

Sol. Here R_2, R_3, R_4 are in parallel

$$\frac{1}{R_{234}} = \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}$$

$R_{234} = 2\Omega$

R_{234} is in series with R_1 so

$R_{eq} = R_{234} + R_1 = 2 + 10 = 12\Omega$

$$i = \frac{12}{12} = 1\text{Amp}$$

50. An alternating voltage of amplitude 40 V and frequency 4 kHz is applied directly across the capacitor of 12 μF. The maximum displacement current between the plates of the capacitor is nearly:

(1) 13 A (2) 8 A

(3) 10 A (4) 12 A

Ans. (4)

Sol. Displacement current is same as conduction current in capacitor.

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$

$$= \frac{1}{2\pi \times 4 \times 10^3 \times 12 \times 10^{-6}} = 3.317\Omega$$

$$I = \frac{V}{X_C} = \frac{40}{3.317} = 12A$$

SECTION-B

51. In Young's double slit experiment, carried out with light of wavelength 5000Å, the distance between the slits is 0.3 mm and the screen is at 200 cm from the slits. The central maximum is at x = 0 cm. The value of x for third maxima is ..... mm.

Ans. (10)

Sol. $\beta = \frac{\lambda D}{d} = \frac{5\times10^{-7}\times2}{3\times10^{-4}} = \frac{10\times10^{-3}}{3}$ m

For 3rd maxima $y_3 = 3\beta = 10\times10^{-3}$ m = 10 mm

52. A 2A current carrying straight metal wire of resistance 1 Ω, resistivity 2×10^{-6} Ωm, area of cross-section 10 mm^2 and mass 500 g is suspended horizontally in mid air by applying a uniform magnetic field $\vec{B}$. The magnitude of B is ..... $\times10^{-1}$ T (given, g = 10 m/s^2)

Ans. (5)

Sol. $R = \frac{\rho\ell}{A} \Rightarrow \frac{2\times10^{-6}\times\ell}{10^{-5}} = 1 \Rightarrow \ell = 5$

$mg = Bi\ell$

$B = \frac{mg}{i\ell} = \frac{5}{2\times5} = 0.5 = 5\times10^{-1}$ Tesla

53. The electric field between the two parallel plates of a capacitor of 1.5 μF capacitance drops to one third of its initial value in 6.6 μs when the plates are connected by a thin wire. The resistance of this wire is ..... Ω. (Given, log 3 = 1.1)

Ans. (4)

Sol. $E = \frac{E_0}{3} \Rightarrow V = \frac{V_0}{3}$

$\frac{V_0}{3} = V_0 e^{-\frac{t}{\tau}}$

$t = \tau\ell n 3$

$6.6\times10^{-6} = R\,(1.5\times10^{-6})(1.1)$

$R = \frac{6}{1.5} = 4\Omega$

54. Three blocks M_1, M_2, M_3 having masses 4 kg, 6 kg and 10 kg respectively are hanging from a smooth pully using rope 1, 2 and 3 as shown in figure. The tension in the rope 1, T_1 when they are moving upward with acceleration of $2ms^{-2}$ is ..... N (if g = 10 m/s^2)

Ans. (240)

Sol. FBD of M_1 :

$T_1 - 200 = (4 + 6 + 10)\times2$

$\therefore T_1 = 240$

55. The density and breaking stress of a wire are 6×10^4 kg /m^3 and 1.2×10^8 N/m^2 respectively. The wire is suspended from a rigid support on a planet where acceleration due to gravity is $\frac{1^{rd}}{3}$ of the value on the surface of earth. The maximum length of the wire with breaking is ..... m (take, g = 10 m/s^2)

Ans. (600)

Sol.

$T = mg$

$$\sigma = \frac{T}{A} = \frac{mg}{A}$$

$$\frac{(\sigma A \ell)g}{A}$$

$$\Rightarrow \ell = \frac{\sigma}{\rho g} = \frac{1.2\times10^8\times3}{6\times10^4\times10} = 600$$

56. A body moves on a frictionless plane starting from rest. If S_n is distance moved between t = n – 1 and t = n and S_{n-1} is distance moved between t = n –2 and t = n – 1, then the ratio $\frac{S_{n-1}}{S_n}$ is $\left(1-\frac{2}{x}\right)$ for n = 10. The value of x is .....

Ans. (19)

Sol. $S_n = \frac{1}{2}a(2n-1) = \frac{19a}{2}$

$$S_{n-1} = \frac{1}{2}a(2n-3) = \frac{17a}{2}$$

$$\frac{S_{n-1}}{S_n} = \frac{17}{19} = 1-\frac{2}{x} \Rightarrow x = 19$$

57. If three helium nuclei combine to form a carbon nucleus then the energy released in this reaction is ..... × 10^{-2} MeV. (Given 1 u = 931 MeV/c^2, atomic mass of helium = 4.002603 u)

Ans. (727)

Sol. Reaction :

$$3\,{}^4_2He \longrightarrow {}^{12}_6C + \gamma \text{ rays}$$

Mass defect = $\Delta m = (3m_{He} - m_C)$

= (3 × 4.002603 – 12) = 0.007809 u

Energy released

= 931 Δm MeV

= 7.27 MeV = 727 × 10^{-2} MeV

58. An ac source is connected in given series LCR circuit. The rms potential difference across the capacitor of 20 μF is ..... V.

Ans. (50)

Sol. $X_L = \omega L = 100 \times 1 = 100\Omega$

$$X_C = \frac{1}{\omega C} = \frac{1}{100\times20\times10^{-6}} = 500\Omega$$

$$Z = \sqrt{(X_L - X_C)^2 + R^2}$$

$$\sqrt{(100-500)^2 + 300^2}$$

$Z = 500\Omega$

$$i_{rms} = \frac{V_{rms}}{Z} = \frac{50}{500} = 0.1A$$

rms voltage across capacitor

$V_{rms} = X_C\, i_{rms}$

=500 × 0.1 = 50V

59. In the experiment to determine the galvanometer resistance by half-deflection method, the plot of $\frac{1}{\theta}$ vs the resistance (R) of the resistance box is shown in the figure. The figure of merit of the galvanometer is ..... ×10^{-1} A/division. [The source has emf 2V]

Ans. (5)

Sol. $i = K\theta$

$$\frac{2}{G+R} = K\theta$$

$$\Rightarrow \frac{1}{\theta} = \frac{(G+R)K}{2} = R\left(\frac{K}{2}\right) + \frac{KG}{2}$$

$$\text{Slope} = \frac{K}{2} = \frac{1}{4} \Rightarrow K = 0.5 = 5\times10^{-1}\,A$$

60. Three capacitors of capacitances 25 μF, 30 μF and 45 μF are connected in parallel to a supply of 100 V. Energy stored in the above combination is E. When these capacitors are connected in series to the same supply, the stored energy is $\frac{9}{x}E$. The value of x is .....

Ans. (86)

Sol. In parallel combination : Potential difference is same across all

$$\text{Energy} = \frac{1}{2}(C_1 + C_2 + C_3)V^2$$

$$= \frac{1}{2}(25+30+45)\times(100)^2\times10^{-6} = 0.5 = E$$

In series combination: Charge is same on all.

$$\frac{1}{C_{equ}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = \frac{1}{25} + \frac{1}{30} + \frac{1}{45}$$

$$\frac{1}{C_{equ}} = \frac{(18+15+10)}{450} = \frac{43}{450} \Rightarrow C_{equ} = \frac{450}{43}$$

$$\text{Energy} = \frac{Q^2}{2C_1} + \frac{Q^2}{2C_2} + \frac{Q^2}{2C_3}$$

$$= \frac{Q^2}{2}\left[\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right]$$

$$\frac{(V\times C_{equ})^2}{2}\times\frac{1}{C_{equ}} = \frac{V^2C_{equ}}{2}$$

$$\frac{(100)^2}{2}\times\frac{450}{43}\times10^{-6}$$

$$\Rightarrow \frac{4.5}{86} = \frac{9}{x}E = \frac{9}{x}\times0.5 \Rightarrow x = 86$$

SECTION-A

61. The **incorrect** postulates of the Dalton's atomic theory are :

(A) Atoms of different elements differ in mass.

(B) Matter consists of divisible atoms.

(C) Compounds are formed when atoms of different element combine in a fixed ratio.

(D) All the atoms of given element have different properties including mass.

(E) Chemical reactions involve reorganisation of atoms.

Choose the **correct** answer from the options given below :

(1) (B), (D), (E) only

(2) (A), (B), (D) only

(3) (C), (D), (E) only

(4) (B), (D) only

Ans. (4)

Sol. B, D

62. The following reaction occurs in the Blast furnance where iron ore is reduced to iron metal

$$Fe_2O_{3(s)} + 3CO_{(g)} \rightleftharpoons Fe_{(l)} + 3CO_{2(g)}$$

Using the Le-chatelier's principle, predict which one of the following will not disturb the equilibrium.

(1) Addition of Fe_2O_3

(2) Addition of CO_2

(3) Removal of CO

(4) Removal of CO_2

Ans. (1)

Sol. When solid added no effect on equilibrium.

63. Identify compound (Z) in the following reaction sequence.

(1)

(2)

(3)

(4)

Ans. (3)

Sol.

64. Given below are two statements : One is labelled as **Assertion (A)** and the other is labelled as **Reason (R)**

Assertion (A): Enthalpy of neutralisation of strong monobasic acid with strong monoacidic base is always –57 kJ mol^{-1}

Reason (R): Enthalpy of neutralisation is the amount of heat liberated when one mole of H^+ ions furnished by acid combine with one mole of ^-OH ions furnished by base to form one mole of water.

In the light of the above statements, choose the **correct** answer from the options given below.

(1) **(A)** is true but **(R)** is false

(2) Both **(A)** and **(R)** are true and **(R)** is the correct explanation of **(A)**

(3) **(A)** is false but **(R)** is true

(4) Both **(A)** and **(R)** are true but **(R)** is **not** the correct explanation of **(A)**

Ans. (2)

Sol. Enthalpy of neutralization of SA & SB is always –57 kJ / mol because strong monoacid gives one mole of H^+ and strong mono base gives one mole of OH^- which form one mole of water.

65. The statement(s) that are **correct** about the species O^{2-}, F^-, Na^+ and Mg^{2+}.

(A) All are isoelectronic

(B) All have the same nuclear charge

(C) O^{2-} has the largest ionic radii

(D) Mg^{2+} has the smallest ionic radii

Choose the **most appropriate** answer from the options given below :

(1) (B), (C) and (D) only

(2) (A), (B), (C) and (D)

(3) (C) and (D) only

(4) (A), (C) and (D) only

Ans. (4)

Sol.

	O^{-2}	F^-	Na^+	Mg^{+2}
(No. of e^-)	10	10	10	10

(Ionic radius) $O^{-2} > F^- > Na^+ > Mg^{+2}$

Zeff $O^{-2} < F^- < Na^+ < Mg^{+2}$

66. For the compounds:

(A) $H_3C–CH_2–O–CH_2–CH_2–CH_3$

(B) $H_3C–CH_2–CH_2–CH_2–CH_3$

(C) $CH_3–CH_2–\underset{\underset{O}{\|}}{C}–CH_2–CH_3$

(D) $H_3C–\underset{\underset{OH}{|}}{CH}– CH_2– CH_2–CH_3$

The increasing order of boiling point is :

Choose the **correct** answer from the options given below :

(1) (A) < (B) < (C) < (D)

(2) (B) < (A) < (C) < (D)

(3) (D) < (C) < (A) < (B)

(4) (B) < (A) < (D) < (C)

Ans. (2)

Sol. Compounds having same number of carbon atoms follow the boiling point order as:

$(\text{Boiling point})_{\text{Hydrogen bonding}} > (\text{Boiling point})_{\text{high polarity}} > (\text{Boiling point})_{\text{low polarity}} > (\text{Boiling point})_{\text{non polar}}$

67. Given below are two statements :

Statement I: In group 13, the stability of +1 oxidation state increases down the group.

Statement II: The atomic size of gallium is greater than that of aluminium.

In the light of the above statements, choose the **most appropriate** answer from the options given below:

(1) **Statement I** is incorrect but **Statement II** is correct

(2) Both **Statement I** and **Statement II** are correct

(3) Both **Statement I** and **Statement II** are incorrect

(4) **Statement I** is correct but **Statement II** is incorrect

Ans. (4)

Sol. **Statement I :** Number of d & f electrons, increases down the group and due to poor shielding of d & f e^-, stability of lower oxidation states increases down the group

Statement II : The atomic size of aluminium is greater than that of gallium.

68. Number of σ and π bonds present in ethylene molecule is respectively :

(1) 3 and 1 (2) 5 and 2

(3) 4 and 1 (4) 5 and 1

Ans. (4)

Sol. ethylene is , it has 5σ bonds and 1π bond.

69. Identify 'A' in the following reaction :

(1)

(2)

(3)

(4)

Ans. (2)

Sol.

– Wolf kishner reduction.

70. The reaction at cathode in the cells commonly used in clocks involves.

(1) reduction of Mn from +4 to +3
(2) oxidation of Mn from +3 to +4
(3) reduction of Mn from + 7 to +2
(4) oxidation of Mn from + 2 to +7

Ans. (1)

Sol. In the cathode reaction manganese (Mn) is reduced from the +4 oxidation state to the +3 state.

71. Which one of the following complexes will exhibit the least paramagnetic behaviour ?

[Atomic number, Cr = 24, Mn = 25, Fe = 26, Co = 27]

(1) $[Co(H_2O)_6]^{2+}$ (2) $[Fe(H_2O)_6]^{2+}$
(3) $[Mn(H_2O)_6]^{2+}$ (4) $[Cr(H_2O)_6]^{2+}$

Ans. (1)

Sol.

	Number of unpaired e^-	$\mu = \sqrt{n(n+2)}$ B.M.
$[Co(H_2O)_6]^{2+}$	3	3.87
$[Fe(H_2O)_6]^{2+}$	4	4.89
$[Mn(H_2O)_6]^{2+}$	5	5.92
$[Cr(H_2O)_6]^{2+}$	4	4.89

Least paramagnetic behaviour = $[Co(H_2O)_6]^{2+}$

72. Given below are two statements : one is labelled as **Assertion (A)** and the other is labelled as **Reason (R).**

Assertion (A): Cis form of alkene is found to be more polar than the trans form

Reason (R): Dipole moment of trans isomer of 2-butene is zero.

In the light of the above statements, choose the **correct** answer from the options given below :

(1) Both **(A)** and **(R)** are true but **(R)** is **NOT** the correct explanation of **(A)**
(2) **(A)** is true but **(R)** is false
(3) Both **(A)** and **(R)** are true and **(R)** is the correct explanation of **(A)**
(4) **(A)** is false but **(R)** is true

Ans. (3)

Sol. Dipole moment is a vector quantity and for compound net dipole moment is the vector sum of all dipoles hence dipole moment of cis form is greater than trans form.

73. Given below are two statements :

Statement I: Nitration of benzene involves the following step –

$$H-\overset{H}{\underset{\bullet\bullet}{\overset{|}{O}}}{}^{\oplus}-NO_2 \rightleftharpoons H_2O + \overset{\oplus}{N}O_2$$

Statement II: Use of Lewis base promotes the electrophilic substitution of benzene.

In the light of the above statements, choose the **most appropriate** answer from the options given below :

(1) Both **Statement I** and **Statement II** are incorrect
(2) **Statement I** is correct but **Statement II** is incorrect
(3) Both **Statement I** and **Statement II** are correct
(4) **Statement I** is incorrect but **Statement II** is correct

Ans. (2)

Sol. In nitration of benzene concentrated H_2SO_4 and HNO_3 is used as reagent which generates electrophile $N\overset{\oplus}{O}_2$ in following steps:

$$H_2SO_4 + HNO_3 \rightleftharpoons HSO_4^{\ominus} + H-\overset{H}{\underset{\oplus}{\overset{|}{O}}}-NO_2$$

$$\Updownarrow$$

$$HSO_4^{\ominus} + H_2O + NO_2^{\oplus}$$

Lewis acids can promote the formation of electrophiles not Lewis base

74. The correct order of ligands arranged in increasing field strength.

(1) $Cl^- < {}^-OH < Br^- < CN^-$

(2) $F^- < Br^- < I^- < NH_3$

(3) $Br^- < F^- < H_2O < NH_3$

(4) $H_2O < {}^-OH < CN^- < NH_3$

Ans. (3)

Sol. Experimental order $Br^- < F^- < H_2O < NH_3$

75. Which of the following gives a positive test with ninhydrin ?

(1) Cellulose (2) Starch

(3) Polyvinyl chloride (4) Egg albumin

Ans. (4)

Sol. Ninhydrin test is a test of amino acids. Egg albumin contains protein which is a natural polymer of amino acids which will show positive ninhydrin test

76. The metal that shows highest and maximum number of oxidation state is:

(1) Fe (2) Mn

(3) Ti (4) Co

Ans. (2)

Sol. Mn shows highest oxidation state (Mn^{+7}) in 3d series metals.

77. Ail organic compound has 42.1% carbon, 6.4% hydrogen and remainder is oxygen. If its molecular weight is 342, then its molecular formula is :

(1) $C_{11}H_{18}O_{12}$ (2) $C_{12}H_{20}O_{12}$

(3) $C_{14}H_{20}O_{10}$ (4) $C_{12}H_{22}O_{11}$

Ans. (4)

Sol. only $C_{12}H_{22}O_{11}$ has 42.1% carbon, 6.4% hydrogen & 51.5 percent oxygen.

78. Given below are two statement :

Statement I : Bromination of phenol in solvent with low polarity such as $CHCl_3$ or CS_2 requires Lewis acid catalyst.

Statement II : The lewis acid catalyst polarises the bromine to generate Br^+.

In the light of the above statements, choose the **correct** answer from the options given below :

(1) Statement I is true but Statement II is false.

(2) Both Statement I and Statement II are true

(3) Both Statement I and Statement II are false.

(4) Statement I is false but Statement II is true.

Ans. (4)

Sol. Phenol is a highly activated compound which can undergo bromination directly with Bromine without any lewis acid.

79. Molar ionic conductivities of divalent cation and anion are 57 S cm^2 mol^{-1} and 73 S cm^2 mol^{-1} respectively. The molar conductivity of solution of an electrolyte with the above cation and anion will be :

(1) 65 S cm^2 mol^{-1} (2) 130 S cm^2 mol^{-1}

(3) 187 S cm^2 mol^{-1} (4) 260 S cm^2 mol^{-1}

Ans. (2)

Sol. $\Lambda_C^{+2} = 57\,S\,cm^2 mol^{-1}$

$\Lambda_A^{+2} = 73\,S\,cm^2 mol^{-1}$

$\Lambda_{Solution} = \lambda_C^{+2} + \Lambda_A^{-2}$

= 57 + 73 = 130

80. The number of neutrons present in the more abundant isotope of boron is 'x'. Amorphous boron upon heating with air forms a product, in which the oxidation state of boron is 'y'. The value of x + y is …

(1) 4 (2) 6

(3) 3 (4) 9

Ans. (4)

Sol. More abundant isotope = B^{11}

[Number of neutrons = 6]

x = 6

$B + O_2 \rightarrow B_2O_3$

Oxidation state of B in B_2O_3 = +3

So, y = 3

Hence x + y = 9

SECTION-B

81. The value of Rydberg constant (R_H) is 2.18×10^{-18} J. The velocity of electron having mass 9.1×10^{-31} kg in Bohr's first orbit of hydrogen atom = …… $\times10^5$ ms^{-1} (nearest integer)

Ans. (22)

Sol. $V = 2.18 \times 10^6 \times \frac{Z}{n}$

$= 21.8 \times 10^5 \times \frac{1}{1} \approx 22 \times 10^5$ (nearest)

82.

In a borax bead test under hot condition, a metal salt (one from the given) is heated at point B of the flame, resulted in green colour salt bead. The spin-only magnetic moment value of the salt is ……. BM (Nearest integer)

[Given atomic number of Cu = 29, Ni = 28, Mn = 25, Fe = 26]

Ans. **(6)**

Sol. Fe^{+3} will give green coloured bead when heated at point B.

Number of unpaired e^- in Fe^{+3} = 5

$\mu = 5.92$

Nearest integer = 6

83. The heat of combustion of solid benzoic acid at constant volume is –321.30 kJ at 27°C. The heat of combustion at constant pressure is (–321.30 – xR) kJ, the value of x is …….

Ans. **(150)**

Sol. $C_6H_5COOH(S) + \frac{15}{2}O_2(g) \rightarrow 7CO_2(g) + 3H_2O(\ell)$

$\Delta H = \Delta U + \Delta n_g RT$

$= -321.30 - \frac{1}{2}\frac{R}{100} \times 300$

= (–321.30 – 150R) kJ

84. Consider the given chemical reaction sequence :

Total sum of oxygen atoms in Product A and Product B are ……

Ans. **(14)**

Sol. Picric acid is prepared by treating phenol first with concentrated sulphuric acid which converts it to phenol-2,4-disulphonic acid and then with concentrated nitric acid to get 2, 4, 6 trinitrophenol.

85. The spin only magnetic moment value of the ion among Ti^{2+}, V^{2+}, Co^{3+} and Cr^{2+}, that acts as strong oxidising agent in aqueous solution is …….. BM (Near integer).

(Given atomic numbers : Ti : 22, V : 23, Cr : 24, Co : 27)

Ans. **(5)**

Sol. Strong oxidising agent = Co^{+3}

No. of unpaired e^- in $Co^{+3}[3d^6]$ = 4

Hence $\mu = \sqrt{n(n+2)} = \sqrt{24}$ BM

Nearest integer = 5

86. During Kinetic study of reaction 2A + B → C + D, the following results were obtained :

	A[M]	B[M]	initial rate of formation of D
I	0.1	0.1	6.0×10^{-3}
II	0.3	0.2	7.2×10^{-2}
III	0.3	0.4	2.88×10^{-1}
IV	0.4	0.1	2.40×10^{-2}

Based on above data, overall order of the reaction is …….

Ans. **(3)**

Sol. $r = K[A]^x[B]^y$

(I) $6 \times 10^{-3} = K[0.1]^x[0.1]^y$

(IV) $2.4 \times 10^{-2} = K[0.4]^x[0.1]^y$

(IV)/(I)

$4 = (4)^x$

x = 1

$r = K[A]^x[B]^y$

(III) $2.88 \times 10^{-1} = K[0.3]^x[0.4]^y$

(II) $7.2 \times 10^{-2} = K[0.3]^x[0.2]^y$

(III)/(II)

$4 = 2^y$

y = 2

Overall order = x + y = 1 + 2 = 3

87. An artificial cell is made by encapsulating 0.2 M glucose solution within a semipermeable membrane. The osmotic pressure developed when the artificial cell is placed within a 0.05 M solution of NaCl at 300 K is _____ $\times 10^{-1}$ bar. (Nearest Integer)

[Given : R = 0.083 L bar mol^{-1} K^{-1}]

Assume complete dissociation of NaCl

Ans. **(25)**

Sol.

$NaCl \longrightarrow Na^+ + Cl^-$

0.05M 0.05M 0.05M

Total $C_1 = 0.05 + 0.05 = 0.1$ M (NaCl)

$C_2 = 0.2$ M (glucose)

$\pi = (C_2 - C_1)\,RT$

$= (0.2 - 0.1) \times 0.083 \times 300$

$= 2.49$ bar

$= 24.9 \times 10^{-1}$ bar

88. The number of halobenzenes from the following that can be prepared by Sandmeyer's reaction is .....

I **II** **III** **IV** **V**

Ans. **(2)**

Sol. In Sandmayer reaction only bromobenzene & chlorobenzene are prepared

89. In the lewis dot structure for NO_2^-, total number of valence electrons around nitrogen is …….

Ans. **(8)**

Sol.

Number of valence e^- around N-atom = 8

90. 9.3 g of pure aniline is treated with bromine water at room temperature to give a white precipitate of the product 'P'. The mass of product 'P' obtained is 26.4 g. The percentage yield is …… %.

Ans. **(80)**

Sol.

93 g of aniline produces 330 g of 2, 4, 6-tribromoaniline. Hence 9.3 g of aniline should produce 33g of 2, 4, 6-tribromoaniline. Hence percentage yield $\frac{26.4 \times 100}{33} = 80\%$

FINAL JEE–MAIN EXAMINATION – APRIL, 2024

(Held On Thursday 04th April, 2024) **TIME : 3 : 00 PM to 6 : 00 PM**

MATHEMATICS — TEST PAPER WITH SOLUTION

SECTION-A

1. If the function $f(x)=\begin{cases}\dfrac{72^x-9^x-8^x+1}{\sqrt{2}-\sqrt{1+\cos x}} & , \quad x\neq 0\\ a\log_e 2\log_e 3 & , \quad x=0\end{cases}$

is continuous at x = 0, then the value of a^2 is equal to

(1) 968 (2) 1152

(3) 746 (4) 1250

Ans. (2)

Sol. $\lim\limits_{x\to 0} f(x)=a\ell n2\ell n3$

$$\lim_{n\to 0}\frac{72^x-9^x-8^x+1}{\sqrt{2}-\sqrt{1+\cos x}}=\lim_{x\to 0}\frac{(8^x-1)(9^x-1)}{\sqrt{2}-\sqrt{1+\cos x}}$$

$$\lim_{n\to 0}\left(\frac{8^x-1}{x}\right)\left(\frac{9^x-1}{x}\right)\left(\frac{x^2}{1-\cos x}\right)\left(\sqrt{2}+\sqrt{1+\cos x}\right)$$

$\therefore\ \ell n8\times\ell n9\times 2\times 2\sqrt{2}=24\sqrt{2}\ell n2\ell n3$

$\therefore$ a = $24\sqrt{2}$, $a^2 = 576 \times 2 = 1152$

2. If $\lambda > 0$, let θ be the angle between the vectors $\vec{a}=\hat{i}+\lambda\hat{j}-3\hat{k}$ and $\vec{b}=3\hat{i}-\hat{j}+2\hat{k}$. If the vectors $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$ are mutually perpendicular, then the value of $(14\cos\theta)^2$ is equal to

(1) 25 (2) 20

(3) 50 (4) 40

Ans. (1)

Sol. $(\vec{a}+\vec{b}).(\vec{a}-\vec{b})=0$, $\lambda>0$

$|\vec{a}|^2-|\vec{b}|^2=0 \rightarrow 1+\lambda^2+9=9+1+4$

$\therefore\ \lambda=2,\ \cos\theta=\dfrac{\vec{a}-\vec{b}}{|\vec{a}|.|\vec{b}|}=\dfrac{3-\lambda-6}{\sqrt{14}.\sqrt{14}}$

$14\cos\theta = 3 - 8 = -5$

$\therefore\ (14\cos\theta)^2 = 25$

3. Let C be a circle with radius $\sqrt{10}$ units and centre at the origin. Let the line x + y = 2 intersects the circle C at the points P and Q. Let MN be a chord of C of length 2 unit and slope –1. Then, a distance (in units) between the chord PQ and the chord MN is

(1) $2-\sqrt{3}$ (2) $3-\sqrt{2}$

(3) $\sqrt{2}-1$ (4) $\sqrt{2}+1$

Ans. (2)

$C : x^2 + y^2 = 10$

$AN = \dfrac{MN}{2}=1$

$\therefore$ In $\Delta OAN \rightarrow (ON)^2 = (OA)^2 + (AN)^2$

$10 = (OA)^2 + 1 \rightarrow OA = 3$

Perpendicular distance of center from

$PQ=\dfrac{|0+0-2|}{\sqrt{2}}=\sqrt{2}$

Perpendicular distance between MN and

$PQ=OA+\sqrt{2}$ or $|OA-\sqrt{2}|$

$=3+\sqrt{2}$ or $3-\sqrt{2}$

4. Let a relation R on $\mathbb{N}\times\mathbb{N}$ be defined as :

$(x_1,y_1)\ R(x_2,y_2)$ if and only if $x_1 \le x_2$ or $y_1 \le y_2$

Consider the two statements :

(I) R is reflexive but not symmetric.

(II) R is transitive

Then which one of the following is true ?

(1) Only (II) is correct.

(2) Only (I) is correct.

(3) Both (I) and (II) are correct.

(4) Neither (I) nor (II) is correct.

Ans. (2)

Sol. All $((x_1y_1), (x_1,y_1))$ are in R where

$x_1, y_1 \in N \therefore$ R is reflexive

$((1,1), (2,3)) \in R$ but $((2,3), (1,1)) \notin R$

$\therefore$ R is not symmetric

$((2,4), (3,3)) \in R$ and $((3,3), (1,3)) \in R$ but $((2,4), (1,3)) \notin R$

$\therefore$ R is not transitive

5. Let three real numbers a,b,c be in arithmetic progression and a + 1, b, c + 3 be in geometric progression. If a > 10 and the arithmetic mean of a,b and c is 8, then the cube of the geometric mean of a,b and c is

(1) 120 (2) 312

(3) 316 (4) 128

Ans. (1)

Sol. $2b = a + c$, $b^2 = (a + 1)(c + 3)$,

$$\frac{a+b+c}{3} = 8 \rightarrow b = 8, a + c = 16$$

$64 = (a + 1)(19 - a) = 19 + 18a - a^2$

$a^2 - 18a - 45 = 0 \rightarrow (a - 15)(a + 3) = 0, (a > 10)$

$a = 15, c = 1, b = 8$

$((abc)^{1/3})^3 = abc = 120$

6. Let $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$ and $B = I + adj(A) + (adj\ A)^2 + \ldots + (adj\ A)^{10}$. Then, the sum of all the elements of the matrix B is :

(1) –110 (2) 22

(3) –88 (4) –124

Ans. (3)

Sol. $Adj(A) = \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix}$

$(AdjA)^2 = \begin{bmatrix} 1 & -4 \\ 0 & 1 \end{bmatrix}$

$\vdots$

$(AdjA)^{10} = \begin{bmatrix} 1 & -20 \\ 0 & 1 \end{bmatrix}$

$$B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & -4 \\ 0 & 1 \end{bmatrix} + \ldots + \begin{bmatrix} 1 & -20 \\ 0 & 1 \end{bmatrix}$$

$B = \begin{bmatrix} 11 & -110 \\ 0 & 11 \end{bmatrix} \Rightarrow$ sum of elements of B

$= -88$

7. The value of $\dfrac{1\times 2^2 + 2\times 3^2 + \ldots + 100\times(101)^2}{1^2\times 2 + 2^2\times 3 + \ldots + 100^2\times 101}$ is

(1) $\frac{306}{305}$ (2) $\frac{305}{301}$

(3) $\frac{32}{31}$ (4) $\frac{31}{30}$

Ans. (2)

Sol. $$\frac{1\times 2^2 + 2\times 3^2 + \ldots + 100\times(101)^2}{1^2\times 2 + 2^2\times 3 + \ldots + 100^2\times 101} = \frac{\sum_{r=1}^{100} r(r+1)^2}{\sum_{r=1}^{100} r^2(r+1)}$$

$$= \frac{\sum_{r=1}^{100}(r^3 + 2r^2 + r)}{\sum_{r=1}^{100}(r^3 + r^2)} = \frac{\left(\frac{n(n+1)^2}{2}\right) + \frac{2.n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}}{\left(\frac{n(n+1)}{2}\right)^2 + \frac{n(n+1)(2n+1)}{6}}$$

$$= \frac{\frac{n(n+1)}{2}\left[\frac{n(n+1)}{2} + \frac{2}{3}.(2n+1) + 1\right]}{\frac{n(n+1)}{2}\left[\frac{n(n+1)}{2} + \frac{(2n+1)}{3}\right]} \text{;Put } n = 100$$

$$= \frac{\frac{100(101)}{2} + \frac{2}{3}(201) + 1}{\frac{100\times 101}{2} + \frac{201}{3}} = \frac{5185}{5117} = \frac{305}{301}$$

8. Let $f(x)=\int_0^x\left(t+\sin\left(1-e^t\right)\right)dt, x\in\mathbb{R}$.

Then $\lim_{x\to 0}\frac{f(x)}{x^3}$ is equal to

(1) $\frac{1}{6}$ (2) $-\frac{1}{6}$

(3) $-\frac{2}{3}$ (4) $\frac{2}{3}$

Ans. (2)

Sol. $\lim_{x\to 0}\frac{f(x)}{x^3}$

Using L Hopital Rule.

$\lim_{x\to 0}\frac{f'(x)}{3x^2}=\lim_{x\to 0}\frac{x+\sin\left(1-e^x\right)}{3x^2}$ (Again L Hopital)

Using L.H. Rule

$=\lim_{x\to 0}\frac{-\left[\sin\left(1-e^x\right)\left(-e^x\right).e^x+\cos\left(1-e^x\right).e^x\right]}{6}$

$=-\frac{1}{6}$

9. The area (in sq. units) of the region described by $\{(x,y) : y^2\le 2x, \text{ and } y\ge 4x-1\}$ is

(1) $\frac{11}{32}$ (2) $\frac{8}{9}$

(3) $\frac{11}{12}$ (4) $\frac{9}{32}$

Ans. (4)

Sol.

Shaded area $=\int_{-\frac{1}{2}}^{1}\left(x_{Right}-x_{Left}\right)dy$

$y^2=2x$
$y=4x-1$ Solve
$y=1, y=-\frac{1}{2}$

Shaded area $=\int_{-\frac{1}{2}}^{1}\left(\frac{y+1}{4}-\frac{y^2}{2}\right)dy$

$=\left(\frac{1}{4}\left(\frac{y^2}{2}+y\right)-\frac{y^3}{6}\right)_{-\frac{1}{2}}^{1}=\frac{9}{32}$

10. The area (in sq. units) of the region $S=\left\{z\in\mathbb{C};|z-1|\le 2;\left(z+\bar{z}\right)+i\left(z-\bar{z}\right)\le 2, \operatorname{Im}(z)\ge 0\right\}$ is

(1) $\frac{7\pi}{3}$ (2) $\frac{3\pi}{2}$

(3) $\frac{17\pi}{8}$ (4) $\frac{7\pi}{4}$

Ans. (2)

Sol. Put z = x + iy

$|z-1|\le 2\Rightarrow(x-1)^2+y^2\le 4$ …(1)

$\left(z+\bar{z}\right)+i\left(z-\bar{z}\right)\le 2\Rightarrow 2x+i(2iy)\le 2$

$\Rightarrow x-y\le 1$ …(2)

$\operatorname{Im}(z)\ge 0\Rightarrow y\ge 0$ …(3)

Required area

= Area of semi-circle – area of sector A

$\frac{1}{2}\pi(2)^2-\frac{\pi}{2}$

$=\frac{3\pi}{2}$

11. If the value of the integral $\int_{-1}^{1} \frac{\cos \alpha x}{1+3^x} dx$ is $\frac{2}{\pi}$. Then, a value of α is

(1) $\frac{\pi}{6}$ (2) $\frac{\pi}{2}$

(3) $\frac{\pi}{3}$ (4) $\frac{\pi}{4}$

Ans. (2)

Sol. Let $I = \int_{-1}^{+1} \frac{\cos \alpha x}{1+3^x} dx$ ...(I)

$I = \int_{-1}^{+1} \frac{\cos \alpha x}{1+3^{-x}} dx$

$\left(\text{using} \int_a^b f(x) dx = \int_a^b f(a+b-x) dx \right)$ ...(II)

Add (1) and (II)

$2I = \int_{-1}^{+1} \cos(\alpha x) dx = 2\int_0^1 \cos(\alpha x) dx$

$I = \frac{\sin \alpha}{\alpha} = \frac{2}{\pi} (\text{given})$

$\therefore \alpha = \frac{\pi}{2}$

12. Let $f(x) = 3\sqrt{x-2} + \sqrt{4-x}$ be a real valued function. If α and β are respectively the minimum and the maximum values of f, then $\alpha^2 + 2\beta^2$ is equal to

(1) 44 (2) 42

(3) 24 (4) 38

Ans. (2)

Sol. $f(x) = 3\sqrt{x-2} + \sqrt{4-x}$

$x - 2 \geq 0$ & $4 - x \geq 0$

$\therefore x \in [2, 4]$

Let $x = 2\sin^2\theta + 4\cos^2\theta$

$\therefore f(x) = 3\sqrt{2}|\cos\theta| + \sqrt{2}|\sin\theta|$

$\therefore \sqrt{2} \leq 3\sqrt{2}|\cos\theta| + \sqrt{2}|\sin\theta| \leq \sqrt{9 \times 2 + 2}$

$\sqrt{2} \leq 3\sqrt{2}|\cos\theta| + \sqrt{2}|\sin\theta| \leq \sqrt{20}$

$\therefore \alpha = \sqrt{2}$ $\beta = \sqrt{20}$

$\alpha^2 + 2\beta^2 = 2 + 40 = 42$

13. If the coefficients of x^4, x^5 and x^6 in the expansion of $(1 + x)^n$ are in the arithmetic progression, then the maximum value of n is :

(1) 14 (2) 21

(3) 28 (4) 7

Ans. (1)

Sol. Coeff. of $x^4 = {}^nC_4$

Coeff. of $x^5 = {}^nC_5$

Coeff. of $x^6 = {}^nC_6$

${}^nC_4, {}^nC_5, {}^nC_6$ .... AP

$2.{}^nC_5 = {}^nC_4 + {}^nC_6$

$2 = \frac{{}^nC_4}{{}^nC_5} + \frac{{}^nC_6}{{}^nC_5}$ $\left\{ \frac{{}^nC_r}{{}^nC_{r-1}} = \frac{n-r+1}{r} \right\}$

$2 = \frac{5}{n-4} + \frac{n-5}{6}$

$12(n - 4) = 30 + n^2 - 9n + 20$

$n^2 - 21n + 98 = 0$

$(n - 14)(n - 7) = 0$

$n_{max} = 14$ $n_{min} = 7$

14. Consider a hyperbola H having centre at the origin and foci and the x-axis. Let C_1 be the circle touching the hyperbola H and having the centre at the origin. Let C_2 be the circle touching the hyperbola H at its vertex and having the centre at one of its foci. If areas (in sq. units) of C_1 and C_2 are 36π and 4π, respectively, then the length (in units) of latus rectum of H is

(1) $\frac{28}{3}$ (2) $\frac{14}{3}$

(3) $\frac{10}{3}$ (4) $\frac{11}{3}$

Ans. (1)

Sol. Let H : $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ $\quad (b^2 = a^2(e^2 - 1))$

$\therefore$ eq^n of $C_1 = x^2 + y^2 = a^2$

Ar. = 36π

$\pi a^2 = 36\pi$

$a = 6$

Now radius of C_2 can be $a(e - 1)$ or $a(e + 1)$

for $r = a(e - 1)$	for $r = a(e + 1)$
Ar. = 4π	$\pi r^2 = 4\pi$
$\pi a^2(e - 1)^2 = 4\pi$	$a^2(e + 1)^2 = 4$
$36\pi(e - 1)^2 = 4\pi$	$36(e + 1)^2 = 4$
$e - 1 = \frac{1}{3}$	$e + 1 = \frac{1}{3}$
$e = \frac{4}{3}$	$-\frac{2}{3}$
	Not possible

$\therefore b^2 = 36\left(\frac{16}{9} - 1\right) = 28$

$\therefore LR = \frac{2b^2}{a} = \frac{2 \times 28}{6} = \frac{28}{3}$

15. If the mean of the following probability distribution of a random variable X;

X	0	2	4	6	8
P(X)	a	2a	a + b	2b	3b

is $\frac{46}{9}$, then the variance of the distribution is

(1) $\frac{581}{81}$ (2) $\frac{566}{81}$

(3) $\frac{173}{27}$ (4) $\frac{151}{27}$

Ans. (2)

Sol. $\sum P_i = 1$

$a + 2a + a + b + 2b + 3b = 1$

$4a + 6b = 1$ …. (I)

$E(x) = \text{mean} = \frac{46}{9}$

$\sum P_i X_i = \frac{46}{9} \Rightarrow 4a + 4a + 4b + 12b + 24b = \frac{46}{9}$

$8a + 40b = \frac{46}{9}$

$4a + 20b = \frac{23}{9}$ …. (II)

Subtract (I) from (II) we get

$b = \frac{1}{9}$ & $a = \frac{1}{12}$

Variance = $E(x_i^2) - E(x_i)^2$

$E(x_i^2) = 0^2 \times 9^2 + 2^2 \times 2a + 4^2(a + b) + 6^2(2b) + 8^2(3b)$

$= 24a + 280b$

Put $a = \frac{1}{12}$ $\quad b = \frac{1}{9}$

$E(x_i^2) = 2 + \frac{280}{9} = \frac{298}{9}$

$\therefore \sigma^2 = E(x_i^2) - E(x_i)^2$

$= \frac{298}{9} - \left(\frac{46}{9}\right)^2$

$\sigma^2 = \frac{298}{9} - \frac{2116}{81}$

$= \frac{566}{81}$

16. Let PQ be a chord of the parabola $y^2 = 12x$ and the midpoint of PQ be at (4,1). Then, which of the following point lies on the line passing through the points P and Q ?

(1) (3,–3) (2) $\left(\frac{3}{2}, -16\right)$

(3) (2,–9) (4) $\left(\frac{1}{2}, -20\right)$

Ans. (4)

Sol.

$T = S_1$

$y - 6(x + 4)$

$= 1 - 48$

$6x - y = 23$

Option 4 $\left(\frac{1}{2}, -20\right)$ will satisfy

17. Given the inverse trigonometric function assumes principal values only. Let x, y be any two real numbers in [–1,1] such that

$\cos^{-1}x - \sin^{-1} y = \alpha, \frac{-\pi}{2} \le \alpha \le \pi$.

Then, the minimum value of $x^2 + y^2 + 2xy \sin\alpha$ is

(1) –1 (2) 0

(3) $\frac{-1}{2}$ (4) $\frac{1}{2}$

Ans. (2)

Sol. $\cos^{-1}x - \left(\frac{\pi}{2} - \cos^{-1} y\right) = \alpha$

$\cos^{-1}x + \cos^{-1}y = \frac{\pi}{2} + \alpha$

$\alpha \in \left[-\frac{\pi}{2}, \pi\right], \frac{\pi}{2} + \alpha \in \left[0, \frac{3\pi}{2}\right]$

$\cos^{-1}\left(xy - \sqrt{1-x^2}\sqrt{1-y^2}\right) = \frac{\pi}{2} + \alpha$

$xy - \sqrt{1-x^2}\sqrt{1-y^2} = -\sin\alpha$

$(xy + \sin\alpha) = (1 - x^2)(1 - y^2)$

$x^2y^2 + 2xy\sin a + \sin^2 a = 1 - x^2 - y^2 + x^2y^2$

$x^2 + y^2 + 2xy \sin\alpha = 1 - \sin^2\alpha$

$x^2 + y^2 + 2xy\sin\alpha = \cos^2\alpha$

Min. value of $\cos^2\alpha = 0$

At $\boxed{\alpha = \frac{\pi}{2}}$

Option (2) is correct

18. Let y = y(x) be the solution of the differential equation

$(x^2 + 4)^2dy + (2x^3y + 8xy - 2)dx = 0$. If y(0) = 0, then y(2) is equal to

(1) $\frac{\pi}{8}$ (2) $\frac{\pi}{16}$

(3) 2π (4) $\frac{\pi}{32}$

Ans. (4)

Sol. $\frac{dy}{dx} + y\left(\frac{2x^3 + 8x}{\left(x^2+4\right)^2}\right) = \frac{2}{\left(x^2+4\right)^2}$

$\frac{dy}{dx} + y\left(\frac{2x}{x^2+4}\right) = \frac{2}{\left(x^2+4\right)^2}$

$IF = e^{\int \frac{2x}{x^2+4} dx}$

$IF = x^2 + 4$

$y \times (x^2 + 4) = \int \frac{2}{\left(x^2+4\right)^2} \times \left(x^2+4\right)$

$y(x^2 + 4) = 2\int \frac{dx}{x^2 + 2^2}$

$y\left(x^2+4\right) = \frac{2}{2}\tan^{-1}\left(\frac{x}{2}\right) + c$

$0 = 0 + c = c = 0$

$y(x^2 + 4) = \tan^{-1}\left(\frac{x}{2}\right)$

y at x = 2

$y(4 + 4) = \tan^{-1}(1)$

$\boxed{y(2) = \frac{\pi}{32}}$

Option (4) is correct

19. Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{b} = 2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\vec{c} = x\hat{i} + 2\hat{j} + 3\hat{k}, x \in \mathbb{R}$. If $\vec{d}$ is the unit vector in the direction of $\vec{b} + \vec{c}$ such that $\vec{a}.\vec{d} = 1$, then $\left(\vec{a} \times \vec{b}\right).\vec{c}$ is equal to

(1) 9 (2) 6

(3) 3 (4) 11

Ans. (4)

Sol. $\vec{d} = \lambda\left(\vec{b} + \vec{c}\right)$

$\vec{a}.\vec{d} = \lambda\left(\vec{b}.\vec{a} + \vec{c}.\vec{a}\right)$

$1 = \lambda(1 + x + 5)$

$1 = \lambda(x + 6)$ ….(1)

$\left|\vec{d}\right| = 1$ $\boxed{\frac{1}{\lambda} = x + 6}$

$\left|\lambda\left(\vec{b} + \vec{c}\right)\right| = 1$

$\left|\lambda\left((x+2)\hat{i} + 6\hat{j} - 2\hat{k}\right)\right| = 1$

$\lambda^2((x + 2)^2 + 6^2 + 2^2) = 1$

$x^2 + 4x + 4 + 36 + 4 = (x + 6)^2$

$x^2 + 4x + 44 = x^2 + 12x + 36$

$8x = 8, x = 1$

$$\begin{vmatrix} 1 & 1 & 1 \\ 2 & 4 & -5 \\ x & 2 & 3 \end{vmatrix} = \left(\vec{a} \times \vec{b}\right).\vec{c}$$

$$\begin{vmatrix} 0 & 0 & 1 \\ -2 & 9 & -4 \\ x-2 & -1 & 3 \end{vmatrix} = 2 - 9(x - 2)$$

$= 20 - 9x$

at $x = 1$

$20 - 9 = 11$

Option 4 is correct

20. Let P the point of intersection of the lines $\frac{x-2}{1} = \frac{y-4}{5} = \frac{z-2}{1}$ and $\frac{x-3}{2} = \frac{y-2}{3} = \frac{z-3}{2}$.

Then, the shortest distance of P from the line

$4x = 2y = z$ is

(1) $\frac{5\sqrt{14}}{7}$ (2) $\frac{\sqrt{14}}{7}$

(3) $\frac{3\sqrt{14}}{7}$ (4) $\frac{6\sqrt{14}}{7}$

Ans. (3)

$L_1 \equiv \frac{x-2}{1} = \frac{y-4}{5} = \frac{z-2}{1} = \lambda$

$P\left(\lambda + 2, 5\lambda + 4, \lambda + 2\right)$

$L_2 \equiv \frac{x-3}{2} = \frac{y-2}{3} = \frac{z-3}{2}$

$P\left(2\mu + 3, 3\mu + 2, 2\mu + 3\right)$

$\lambda + 2 = 2\mu + 3$ $3\mu + 2 = 5\lambda + 4$

$\lambda = 2\mu + 1$ $3\mu = 5\lambda + 2$

$3\mu = 5(2\mu + 1) + 2$

$3\mu = 10\mu + 7$

$\mu = -1$ $\lambda = -1$

Both satisfies (P)

P(1,–1,1)

$L_3 \equiv \frac{x}{1/4} = \frac{y}{1/2} = \frac{z}{1}$

$L_3 = \frac{x}{1} = \frac{y}{2} = \frac{z}{4} = k$

Coordinates of Q(k,2k,4k)

DR's of PQ = <k–1, 2k + 1, 4k – 1>

PQ $\perp$ to L_3

$(k - 1) + 2(2k + 1) + 4(4k - 1) = 0$

$k - 1 + 4k + 2 + 16k - 4 = 0$

$k = \frac{1}{7}$

$Q\left(\frac{1}{7}, \frac{2}{7}, \frac{4}{7}\right)$

$PQ = \sqrt{\left(1 - \frac{1}{7}\right)^2 + \left(-1 - \frac{2}{7}\right)^2 + \left(1 - \frac{4}{7}\right)^2}$

$= \sqrt{\frac{36}{49} + \frac{81}{49} + \frac{9}{49}} = \frac{\sqrt{126}}{7}$

$PQ = \frac{3\sqrt{14}}{7}$

Option-3 will satisfy

SECTION-B

21. Let S = {$\sin^2 2\theta$: $(\sin^4\theta + \cos^4\theta)x^2 + (\sin 2\theta)x + (\sin^6\theta + \cos^6\theta) = 0$ has real roots}. If α and β be the smallest and largest elements of the set S, respectively, then $3((\alpha-2)^2 + (\beta-1)^2)$ equals…..

Ans. (4)

Sol. $D = (\sin 2\theta)^2 - 4\left(1 - \frac{\sin^2 2\theta}{2}\right)\left(1 - \frac{3}{4}\sin^2 2\theta\right)$

$= (\sin 2\theta)^2 - 4\left(1 - \frac{5}{4}\sin^2 2\theta + \frac{3}{8}\sin^4 2\theta\right)$

$D = -\frac{3}{2}\sin^4 2\theta + 6\sin^2 2\theta - 4 > 0$

$3\sin^4 2\theta - 12\sin^2 2\theta + 8 < 0$

$\sin^2 2\theta = \frac{12 \pm \sqrt{12^2 - 12.8}}{6} = \frac{12 \pm 4\sqrt{3}}{6} = \frac{6 \pm 2\sqrt{3}}{3}$

$\sin^2 2\theta = 2 \pm \frac{2}{\sqrt{3}}$, but $\sin^2 2\theta \in [0, 1]$

$\therefore\ \alpha = 2 - \frac{2}{\sqrt{3}}, \beta = 1 \rightarrow (\alpha-2)^2 = \frac{4}{3}, (\beta-1)^2 = 0$

$\boxed{3(\alpha-2)^2 + (\beta-1)^2 = 4}$

22. If $\int \operatorname{cosec}^5 x\,dx = \alpha \cot x \operatorname{cosec} x\left(\operatorname{cosec}^2 x + \frac{3}{2}\right) + \beta \log_e\left|\tan\frac{x}{2}\right| + C$

where $\alpha, \beta \in \mathbb{R}$ and C is constant of integration , then the value of $8(\alpha + \beta)$ equals …..

Ans. (1)

Sol. $\int \operatorname{cosec}^3 x \cdot \operatorname{cosec}^2 x\,dx = I$

By applying integration by parts

$I = -\cot x \operatorname{cosec}^3 x + \int \cot x(-3\operatorname{cosec}^2 x \cot x \operatorname{cosec} x)dx$

$I = -\cot x \operatorname{cosec}^3 x - 3\int \operatorname{cosec}^3 x(\operatorname{cosec}^2 x - 1)dx$

$I = -\cot x \operatorname{cosec}^3 x - 3I + 3\int \operatorname{cosec}^3 x\,dx$

let

$I_1 = \int \operatorname{cosec}^3 x\,dx = -\operatorname{cosec} x \cot x - \int \cot^2 x \operatorname{cosec} x\,dx$

$I_1 = -\operatorname{cosec} x \cot x - \int(\operatorname{cosec}^2 x - 1)\operatorname{cosec} x\,dx$

$2I_1 = -\operatorname{cosec} x \cot x + \ell n\left|\tan\frac{x}{2}\right|$

$I_1 = -\frac{1}{2}\operatorname{cosec} x \cot x + \frac{1}{2}\ell n\left|\tan\frac{x}{2}\right|$

$4I = -\cot x \operatorname{cosec}^3 x - \frac{3}{2}\operatorname{cosec} x \cot x + \frac{3}{2}\ell n\left|\tan\frac{x}{2}\right| + 4c$

$I = -\frac{1}{4}\operatorname{cosec} x \cot x\left(\operatorname{cosec}^2 x + \frac{3}{2}\right) + \frac{3}{8}\ell n\left|\tan\frac{x}{2}\right| + c$

$\therefore\ \alpha = \frac{-1}{4}, \beta = \frac{3}{8} \rightarrow \boxed{8(\alpha+\beta) = 1}$

23. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a thrice differentiable function such that f(0) = 0, f(1) = 1, f(2) = –1, f(3) = 2 and f(4) = –2. Then, the minimum number of zeros of (3f' f'' + ff''') (x) is …..

Ans. (5)

Sol. $(3f'f'' + ff''')(x) = \left(\left(ff'' + (f')^2\right)(x)\right)'$

$\left(ff'' + (f')^2\right)(x) = ((ff')(x))'$

$\therefore\ (3f'f'' + f''')(x) = (f(x) \cdot f'(x))''$

min. roots of f(x) $\rightarrow$ 4

$\therefore$ min. roots of f '(x) $\rightarrow$ 3

$\therefore$ min. roots of $(f(x) \cdot f'(x)) \rightarrow 7$

$\boxed{\therefore\ \text{min. roots of } (f(x) \cdot f'(x))'' \rightarrow 5}$

24. Consider the function $f : \mathbb{R} \rightarrow \mathbb{R}$ defined by

$f(x) = \frac{2x}{\sqrt{1+9x^2}}$. If the composition of

$f, \underbrace{(f \circ f \circ f \circ ... \circ f)}_{10\text{ times}}(x) = \frac{2^{10}x}{\sqrt{1+9\alpha x^2}}$, then the value of $\sqrt{3\alpha+1}$ is equal to …..

Ans. (1024)

Sol. $f(f(x)) = \dfrac{2f(x)}{\sqrt{1+9f^2(x)}} = \dfrac{4x}{\sqrt{1+9x^2+9.2^2x^2}}$

$f(f(f(x))) = \dfrac{2^3x/\sqrt{1+9x^2}}{\sqrt{1+9(1+2^2)\dfrac{2^2x^2}{1+9x^2}}} = \dfrac{2^3x}{\sqrt{1+9x^2(1+2^2+2^4)}}$

$\therefore$ By observation

$\alpha = 1+2^2+2^4+....+2^{18} = 1\left(\dfrac{(2^2)^{10}-1}{2^2-1}\right) = \dfrac{2^{20}-1}{3}$

$3\alpha+1 = 2^{20} \rightarrow \sqrt{3\alpha+1} = 2^{10} = \boxed{1024}$

25. Let A be a 2 × 2 symmetric matrix such that $A\begin{bmatrix}1\\1\end{bmatrix} = \begin{bmatrix}3\\7\end{bmatrix}$ and the determinant of A be 1.

If $A^{-1} = \alpha A + \beta I$, where I is an identity matrix of order 2 × 2, then $\alpha + \beta$ equals …..

Ans. (5)

Sol. Let $A = \begin{bmatrix}a & b\\b & d\end{bmatrix}$

$\begin{bmatrix}a & b\\b & d\end{bmatrix}\begin{bmatrix}1\\1\end{bmatrix} = \begin{bmatrix}3\\7\end{bmatrix}, ad - b^2 = 1$

$a + b = 3$, $b + d = 7$, $(3 - b)(7 - b) - b^2 = 1$

$21 - 10b = 1 \rightarrow b = 2, a = 1, d = 5$

$A = \begin{bmatrix}1 & 2\\2 & 5\end{bmatrix}$, $A^{-1} = \begin{bmatrix}5 & -2\\-2 & 1\end{bmatrix}$

$A^{-1} = \alpha A + \beta I$

$\begin{bmatrix}5 & -2\\-2 & 1\end{bmatrix} = \begin{bmatrix}\alpha+\beta & 2\alpha\\2\alpha & 5\alpha+\beta\end{bmatrix}$

$\alpha = -1, \beta = 6 \rightarrow \boxed{\alpha+\beta = 5}$

26. There are 4 men and 5 women in Group A, and 5 men and 4 women in Group B. If 4 persons are selected from each group, then the number of ways of selecting 4 men and 4 women is …..

Ans. (5626)

Sol.

From Group A	From Group B	Ways of selection
4M	4W	${}^4C_4\,{}^4C_4 = 1$
3M 1W	1M 3W	${}^4C_3\,{}^5C_1\,{}^5C_1\,{}^4C_3 = 400$
2M 2W	2M 2W	${}^4C_2\,{}^5C_2\,{}^5C_2\,{}^4C_2 = 3600$
1M 3W	3M 1W	${}^4C_1\,{}^5C_3\,{}^5C_3\,{}^4C_1 = 1600$
4W	4M	${}^5C_4\,{}^5C_4 = 25$
Total		5626

Ans. 5626

27. In a tournament, a team plays 10 matches with probabilities of winning and losing each match as $\frac{1}{3}$ and $\frac{2}{3}$ respectively. Let x be the number of matches that the team wins, and y be the number of matches that team loses. If the probability $P(|x - y| \le 2)$ is p, then 3^9p equals……

Ans. (8288)

Sol. $P(W) = \dfrac{1}{3}$ $\qquad$ $P(L) = \dfrac{2}{3}$

x = number of matches that team wins

y = number of matches that team loses

$|x-y| \le 2$ and $x + y = 10$

$|x-y| = 0,1,2 \quad x, y \in N$

Case-I : $|x-y| = 0 \Rightarrow x = y$

$\because\ x+y = 10 \Rightarrow x = 5 = y$

$P(|x-y| = 0) = {}^{10}C_5\left(\dfrac{1}{3}\right)^5\left(\dfrac{2}{3}\right)^5$

Case-II : $|x-y| = 1 \Rightarrow x - y = \pm 1$

x = y + 1	x = y –1
$\because\ x+y=10$	$\because\ x+y=10$
2y = 9	2y = 11
Not possible	Not possible

Case-III : $|x-y|=2 \Rightarrow x-y=\pm 2$

$x-y=2$ OR $x-y=-2$

$\because\ x+y=10$ $\quad\quad$ $\because\ x+y=10$

$x=6, y=4$ $\quad\quad$ $x=4, y=6$

$$P(|x-y|=2)={}^{10}C_6\left(\frac{1}{3}\right)^6\left(\frac{2}{3}\right)^4+{}^{10}C_4\left(\frac{1}{3}\right)^4\left(\frac{2}{3}\right)^6$$

$$p={}^{10}C_5\frac{2^5}{3^{10}}+{}^{10}C_6\frac{2^4}{3^{10}}+{}^{10}C_4\frac{2^6}{3^{10}}$$

$$3^9p=\frac{1}{3}\left({}^{10}C_5 2^5+{}^{10}C_6 2^4+{}^{10}C_4 2^6\right)$$

$= 8288$

28. Consider a triangle ABC having the vertices A(1,2), B(α,β) and C(γ,δ) and angles $\angle ABC=\frac{\pi}{6}$ and $\angle BAC=\frac{2\pi}{3}$. If the points B and C lie on the line y = x + 4, then $\alpha^2+\gamma^2$ is equal to …..

Ans. (14)

Sol.

Equation of line passes through point A(1, 2) which makes angle $\frac{\pi}{6}$ from y = x + 4 is

$$y-2=\frac{1\pm\tan\frac{\pi}{6}}{1\mp\tan\frac{\pi}{6}}(x-1)$$

$$y-2=\frac{\sqrt{3}\pm 1}{\sqrt{3}\mp 1}(x-1)$$

$\oplus$	$\ominus$
$y-2=(2+\sqrt{3})(x-1)$	$y-2=(2-\sqrt{3})(x-1)$
solve with $y=x+4$	solve with $y=x+4$
$x+2=(2+\sqrt{3})x-2-\sqrt{3}$	$x+2=(2-\sqrt{3})x-2+\sqrt{3}$
$x=\frac{4+\sqrt{3}}{1+\sqrt{3}}$	$x=\frac{4-\sqrt{3}}{1-\sqrt{3}}$

$$\alpha^2+\gamma^2=\left(\frac{4+\sqrt{3}}{1+\sqrt{3}}\right)^2+\left(\frac{4-\sqrt{3}}{1-\sqrt{3}}\right)^2$$

$\alpha^2+\gamma^2=14$

29. Consider a line L passing through the points P(1,2,1) and Q(2,1,–1). If the mirror image of the point A(2,2,2) in the line L is (α,β,γ), then α + β + 6γ is equal to …..

Ans. (6)

DR's of Line L ≡ –1 : 1 : 2

DR's of AB ≡ α – 2 : β – 2 : γ – 2

AB $\perp_{ar}$ L ⇒ 2 – α + β – 2 + 2 γ – 4 = 0

2γ + β – α = 4 ...(1)

Let C is mid-point of AB

$$C\left(\frac{\alpha+2}{2},\frac{\beta+2}{2},\frac{\gamma+2}{2}\right)$$

DR's of PC = $\frac{\alpha}{2}:\frac{\beta-2}{2}:\frac{\gamma}{2}$

line L || PC $\Rightarrow \frac{-\alpha}{2}=\frac{\beta-2}{2}=\frac{\gamma}{4}=K(\text{let})$

α = – 2K

β = 2K + 2

γ = 4K

use in (1) $\Rightarrow K=\frac{1}{6}$

value of α + β + 6γ = 24K + 2 = 6

30. Let y = y(x) be the solution of the differential equation $(x + y + 2)^2\, dx = dy$, $y(0) = -2$. Let the maximum and minimum values of the function $y = y(x)$ in $\left[0, \frac{\pi}{3}\right]$ be α and β, respectively. If $(3\alpha + \pi)^2 + \beta^2 = \gamma + \delta\sqrt{3}, \gamma, \delta \in \mathbb{Z}$, then $\gamma + \delta$ equals …..

Ans. (31)

Sol. $\frac{dy}{dx} = (x + y + 2)^2$ ...(1), $y(0) = -2$

Let $x + y + 2 = v$

$1 + \frac{dy}{dx} = \frac{dv}{dx}$

from (1) $\frac{dv}{dx} = 1 + v^2$

$\int \frac{dv}{1 + v^2} = \int dx$

$\tan^{-1}(v) = x + C$

$\tan^{-1}(x + y + 2) = x + C$

at $x = 0$ $y = -2$ $\Rightarrow C = 0$

$\Rightarrow \tan^{-1}(x + y + 2) = x$

$y = \tan x - x - 2$

$f(x) = \tan x - x - 2,\ x \in \left[0, \frac{\pi}{3}\right]$

$f'(x) = \sec^2 x - 1 > 0 \Rightarrow f(x) \uparrow$

$f_{min} = f(0) = -2 = \beta$

$f_{max} = f\left(\frac{\pi}{3}\right) = \sqrt{3} - \frac{\pi}{3} - 2 = \alpha$

now $(3\alpha + \pi)^2 + \beta^2 = \gamma + \delta\sqrt{3}$

$\Rightarrow (3\alpha + \pi)^2 + \beta^2 = (3\sqrt{3} - 6)^2 + 4$

$\gamma + \delta\sqrt{3} = 67 - 36\sqrt{3}$

$\Rightarrow \gamma = 67$ and $\delta = -36 \Rightarrow \gamma + \delta = 31$

SECTION-A

31. The translational degrees of freedom (f_t) and rotational degrees of freedom (f_r) of CH_4 molecule are :

(1) $f_t = 2$ and $f_r = 2$ (2) $f_t = 3$ and $f_r = 3$
(3) $f_t = 3$ and $f_r = 2$ (4) $f_t = 2$ and $f_r = 3$

Ans. (2)

Sol. Since CH_4 is polyatomic Non-Linear
D.O.F of CH_4
T. DOF = 3
R DOF = 3

32. A cyclist starts from the point P of a circular ground of radius 2 km and travels along its circumference to the point S. The displacement of a cyclist is :

(1) 6 km (2) $\sqrt{8}$ km
(3) 4 km (4) 8 km

Ans. (2)

Sol.

$\therefore$ Displacement = $R\sqrt{2} = 2\sqrt{2} = \sqrt{8}$ km

33. The magnetic moment of a bar magnet is 0.5 Am^2. It is suspended in a uniform magnetic field of 8×10^{-2} T. The work done in rotating it from its most stable to most unstable position is :

(1) 16×10^{-2} J (2) 8×10^{-2} J
(3) 4×10^{-2} J (4) Zero

Ans. (2)

Sol. At stable equilibrium
$U = -mB \cos 0° = -mB$
At unstable equilibrium
$U = -mB \cos 180° = +mB$
$W = \Delta U$
W.D. = 2 mB
$= 2\,(0.5)\,8 \times 10^{-2} = 8 \times 10^{-2}$ J

34. Which of the diode circuit shows correct biasing used for the measurement of dynamic resistance of p-n junction diode :

(1)

(2)

(3)

(4)

Ans. (2)

Sol. Diode should be in forward biased to calculate dynamic resistance

Hence correct answer would be 2.

35. Arrange the following in the ascending order of wavelength :

(A) Gamma rays (λ_1) (B) x-ray (λ_2)

(C) Infrared waves (λ_3) (D) Microwaves (λ_4)

Choose the most appropriate answer from the options given below :

(1) $\lambda_4 < \lambda_3 < \lambda_1 < \lambda_2$ (2) $\lambda_4 < \lambda_3 < \lambda_2 < \lambda_1$

(3) $\lambda_1 < \lambda_2 < \lambda_3 < \lambda_4$ (4) $\lambda_2 < \lambda_1 < \lambda_4 < \lambda_3$

Ans. **(3)**

Sol. $\lambda_1 < \lambda_2 < \lambda_3 < \lambda_4$

36. Identify the logic gate given in the circuit :

(1) NAND - gate (2) OR - gate

(3) AND gate (4) NOR gate

Ans. **(2)**

Sol. $Y = \overline{\bar{A}.\bar{B}}$

By De-Morgan Law

$Y = \overline{\overline{A+B}}$

Y = A + B

Hence OR gate

37. The width of one of the two slits in a Young's double slit experiment is 4 times that of the other slit. The ratio of the maximum of the minimum intensity in the interference pattern is :

(1) 9 :1 (2) 16 : 1

(3) 1 : 1 (4) 4 : 1

Ans. **(1)**

Sol. Since, Intensity $\propto$ width of slit (ω)

so, $I_1 = I$, $I_2 = 4I$

$$I_{min} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2 = I$$

$$I_{max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2 = 9I$$

$$\frac{I_{max}}{I_{min}} = \frac{9I}{I} = \frac{9}{1}$$

38. Correct formula for height of a satellite from earths surface is :

(1) $\left(\frac{T^2R^2g}{4\pi}\right)^{1/2} - R$ (2) $\left(\frac{T^2R^2g}{4\pi^2}\right)^{1/3} - R$

(3) $\left(\frac{T^2R^2}{4\pi^2g}\right)^{1/3} - R$ (4) $\left(\frac{T^2R^2}{4\pi^2}\right)^{-1/3} + R$

Ans. **(2)**

Sol.

$$\Rightarrow \frac{GMm}{(R+h)^2} = \frac{mv^2}{(R+h)}$$

$$\Rightarrow \frac{GM}{(R+h)} = v^2 \ldots.(1)$$

$\Rightarrow v = (R + h)\omega$

$$\Rightarrow v = (R+h)\frac{2\pi}{T} \ldots\ldots.(2)$$

$$\Rightarrow \frac{GM}{R^2} = g$$

$\Rightarrow GM = gR^2 \ldots\ldots..(3)$

Put value from (2) & (3) in eq. (1)

$$\Rightarrow \frac{gR^2}{(R+h)} = (R+h)^2\left(\frac{2\pi}{T}\right)^2$$

$$\Rightarrow \frac{T^2R^2g}{(2\pi)^2} = (R+h)^3$$

$$\Rightarrow \left[\frac{T^2R^2g}{(2\pi)^2}\right]^{1/3} - R = h$$

39. Match List I with List II

	List–I		List–II
A.	Purely capacitive circuit	I.	
B.	Purely inductive circuit	II.	
C.	LCR series at resonance	III.	
D.	LCR series circuit	IV.	

Choose the correct answer from the options given below :

(1) A-I, B-IV, C-III, D-II

(2) A-IV, B-I, C-III, D-II

(3) A-IV, B-I, C-II, D-III

(4) A-I, B-IV, C-II, D-III

Ans. (4)

Sol. A – V lags by 90° from I hence option (I) is correct.

B – V lead by 90° from I hence option (IV) is correct

C – In LCR resonance $X_L = X_C$. Hence circuit is purely resistive so option (II) is correct

D – In LCR series V is at some angle from I hence (III) is correct

Hence option (4) is correct.

40. Given below are two statements :

Statement I : The contact angle between a solid and a liquid is a property of the material of the solid and liquid as well.

Statement II : The rise of a liquid in a capillary tube does not depend on the inner radius of the tube.

In the light of the above statements, choose the correct answer from the options given below :

(1) Both Statement I and Statement II are false

(2) Statement I is false but Statement II is true.

(3) Statement I is true but Statement II is false.

(4) Both Statement I and Statement II are true.

Ans. (3)

Sol. Statement I is correct as we know contact angle depends on cohesine and adhesive forces.

Statement II is incorrect because height of liquid is given by $h = = \frac{2T\cos\theta_C}{\rho g r}$ where r is radius of Tube (assuming length of capillary is sufficient)

Hence option (3) is correct.

41. A body of m kg slides from rest along the curve of vertical circle from point A to B in friction less path. The velocity of the body at B is :

(given, R = 14 m, g = 10 m/s^2 and $\sqrt{2}$ = 1.4)

(1) 19.8 m/s (2) 21.9 m/s

(3) 16.7 m/s (4) 10.6 m/s

Ans. (2)

Sol.

Apply W.E.T. from A to B

$\Rightarrow W_{mg} = K_B - K_A$

$\Rightarrow mg \times \left(\frac{R}{\sqrt{2}} + R\right) = \frac{1}{2}mv_B^2 - 0 \quad \{v_A = 0 \text{ rest}\}$

$\Rightarrow mgR\frac{\left(\sqrt{2}+1\right)}{\sqrt{2}} = \frac{1}{2}mv_B^2$

$\Rightarrow \sqrt{gR\frac{2\left(\sqrt{2}+1\right)}{\sqrt{2}}} = v_B$

$\Rightarrow \sqrt{\frac{10 \times 14 \times 2(2.4)}{1.4}} = v_B$

$\Rightarrow 21.9 = v_B$

Hence option (2) is correct

42. An electric bulb rated 50 W – 200 V is connected across a 100 V supply. The power dissipation of the bulb is :

(1) 12.5 W (2) 25 W

(3) 50 W (4) 100 W

Ans. (1)

Sol. Rated power & voltage gives resistance

$$R = \frac{V^2}{P} = \frac{(200)^2}{50} = \frac{40000}{50}$$

R = 800

$$P = \frac{\left(V_{applied}\right)^2}{R} = \frac{(100)^2}{800}$$

P = 12.5 watt

Hence option 1 is correct.

43. A 2 kg brick begins to slide over a surface which is inclined at an angle of 45° with respect to horizontal axis. The co-efficient of static friction between their surfaces is :

(1) 1 (2) $\frac{1}{\sqrt{3}}$

(3) 0.5 (4) 1.7

Ans. (1)

Sol.

mg sin 45 = f_L

mg cos 45 = N

$f_L = \mu_s N$

μ_s = tan 45 = 1

or

tan θ = μ_s (θ is angle of repose)

tan 45 = μ_s = 1

correct option (1)

44. In simple harmonic motion, the total mechanical energy of given system is E. If mass of oscillating particle P is doubled then the new energy of the system for same amplitude is :

(1) $\frac{E}{\sqrt{2}}$ (2) E

(3) $E\sqrt{2}$ (4) 2E

Ans. (2)

Sol. T.E. = $\frac{1}{2}kA^2$

since A is same T.E. will be same

correct option (2)

45. Given below are two statements : one is labelled as **Assertion A** and the other is labelled as **Reason R**.

Assertion A : Number of photons increases with increase in frequency of light.

Reason R : Maximum kinetic energy of emitted electrons increases with the frequency of incident radiation.

In the light of the above statements, choose the **most appropriate** answer from the options given below :

(1) Both **A** and **R** are correct and **R** is **NOT** the correct explanation of **A**.

(2) **A** is correct but **R** is not correct.

(3) Both **A** and **R** are correct and **R** is the correct explanation of **A**.

(4) **A** is not correct but **R** is correct.

Ans. (4)

Sol. Intensity of light $I = \frac{nh\nu}{A}$

Here n is no. of photons per unit time.

$n = \frac{IA}{h\nu}$ so on increasing frequency ν, n decreases taking intensity constant.

$k_{max} = h\nu - \phi$

So on increasing ν, kinetic energy increases.

46. According to Bohr's theory, the moment of momentum of an electron revolving in 4th orbit of hydrogen atom is :

(1) $8\frac{h}{\pi}$ (2) $\frac{h}{\pi}$

(3) $2\frac{h}{\pi}$ (4) $\frac{h}{2\pi}$

Ans. (3)

Sol. Moment of momentum is $\vec{r} \times \vec{P}$

$\vec{L} = \vec{r} \times m\vec{v}$

$L = mvr = \frac{nh}{2\pi} = \frac{4h}{2\pi} = \frac{2h}{\pi}$

47. A sample of gas at temperature T is adiabatically expanded to double its volume. Adiabatic constant for the gas is γ = 3/2. The work done by the gas in the process is : (μ = 1 mole)

(1) $RT\left[\sqrt{2}-2\right]$ (2) $RT\left[1-2\sqrt{2}\right]$

(3) $RT\left[2\sqrt{2}-1\right]$ (4) $RT\left[2-\sqrt{2}\right]$

Ans. (4)

Sol. $W = \frac{nR\Delta T}{1-\gamma}$

$TV^{\gamma-1} = \text{constant} = T_f\left(2V\right)^{\gamma-1}$

$T_f = T\left(\frac{1}{2}\right)^{1/2} = \frac{T}{\sqrt{2}}$

$W = \frac{R\left(\frac{T}{\sqrt{2}} - T\right)}{1-\frac{3}{2}} = 2RT\frac{\left(\sqrt{2}-1\right)}{\sqrt{2}}$

$= RT\left(2-\sqrt{2}\right)$

48. A charge q is placed at the center of one of the surface of a cube. The flux linked with the cube is :-

(1) $\frac{q}{4\epsilon_0}$ (2) $\frac{q}{2\epsilon_0}$

(3) $\frac{q}{8\epsilon_0}$ (4) Zero

Ans. (2)

Sol. From

$2\phi = \frac{q}{\epsilon_0}$

$\phi = \frac{q}{2\epsilon_0}$

49. Applying the principle of homogeneity of dimensions, determine which one is correct.

where T is time period, G is gravitational constant, M is mass, r is radius of orbit.

(1) $T^2 = \frac{4\pi^2 r}{GM^2}$ (2) $T^2 = 4\pi^2 r^3$

(3) $T^2 = \frac{4\pi^2 r^3}{GM}$ (4) $T^2 = \frac{4\pi^2 r^2}{GM}$

Ans. (3)

Sol. According to principle of homogeneity dimension of LHS should be equal to dimensions of RHS so option (3) is correct.

$$T^2 = \frac{4\pi^2 r^3}{GM}$$

$$\left[T^2\right] = \frac{\left[L^3\right]}{\left[M^{-1}L^3T^{-2}\right]\left[M\right]}$$

(Dimension of G is $\left[M^{-1}L^3T^{-2}\right]$)

$$\left[T^2\right] = \frac{\left[L^3\right]}{\left[L^3T^{-2}\right]} = \left[T^2\right]$$

50. A 90 kg body placed at 2R distance from surface of earth experiences gravitational pull of :

(R = Radius of earth, g = 10 ms^{-2})

(1) 300 N (2) 225 N

(3) 120 N (4) 100 N

Ans. (4)

Sol. Value of g = $g_s\left(1+\frac{h}{R}\right)^{-2}$

$= g_s(1+2)^{-2} = \frac{g_s}{9}$

Here g_s = gravitational acceleration at surface

Force = mg = 90 × $\frac{g_s}{9}$ = 100 N

SECTION-B

51. The displacement of a particle executing SHM is given by x = 10 sin $\left(\omega t + \frac{\pi}{3}\right)$m . The time period of motion is 3.14 s. The velocity of the particle at t = 0 is _____ m/s.

Ans. (10)

Sol. Given,

$T = 3.14 = \frac{2\pi}{\omega}$

$\omega = 2$ rad/s

$x = 10\sin\left(\omega t + \frac{\pi}{3}\right)$

$v = \frac{dx}{dt} = 10\omega\cos\left(\omega t + \frac{\pi}{3}\right)$

at t = 0

$v = 10\omega\cos\left(\frac{\pi}{3}\right) = 10\times 2\times\frac{1}{2}$ [using $\omega = 2$ rad/s]

v = 10 m/s

52. A bus moving along a straight highway with speed of 72 km/h is brought to halt within 4s after applying the brakes. The distance travelled by the bus during this time (Assume the retardation is uniform) is _____m.

Ans. (40)

Sol. Initial velocity = u = 72 km/h = 20 m/s

v = u + at

$\Rightarrow 0 = 20 + a \times 4$

$a = -5$ m/s^2

$v^2 - u^2 = 2as$

$\Rightarrow 0^2 - 20^2 = 2(-5).s$

s = 40 m

53. A parallel plate capacitor of capacitance 12.5 pF is charged by a battery connected between its plates to potential difference of 12.0 V. The battery is now disconnected and a dielectric slab (ϵ_r = 6) is inserted between the plates. The change in its potential energy after inserting the dielectric slab is _____× 10^{-12} J.

Ans. (750)

Sol. Before inserting dielectric capacitance is given $C_0 = 12.5$ pF and charge on the capacitor $Q = C_0V$

After inserting dielectric capacitance will become $\in_r C_0$.

Change in potential energy of the capacitor $= E_i - E_f$

$$= \frac{Q^2}{2C_i} - \frac{Q^2}{2C_f} = \frac{Q^2}{2C_0}\left[1 - \frac{1}{\in_r}\right]$$

$$= \frac{(C_0V)^2}{2C_0}\left[1 - \frac{1}{\in_r}\right] = \frac{1}{2}C_0V^2\left[1 - \frac{1}{\in_r}\right]$$

Using $C_0 = 12.5$ pF, $V = 12$ V, $\in_r = 6$

$$= \frac{1}{2}(12.5)\times 12^2\left[1 - \frac{1}{6}\right] = \frac{1}{2}(12.5)\times 12^2 \times \frac{5}{6}$$

$= 750$ pJ $= 750 \times 10^{-12}$J

54. In a system two particles of masses $m_1 = 3$kg and $m_2 = 2$kg are placed at certain distance from each other. The particle of mass m_1 is moved towards the center of mass of the system through a distance 2cm. In order to keep the center of mass of the system at the original position, the particle of mass m_2 should move towards the center of mass by the distance ____ cm.

Ans. **(3)**

Sol. m_1=3kg $\quad$ m_2=2kg

$\bullet\!\!\rightarrow$ 2cm $\quad$ $\leftarrow\!\!\bullet$ x

$$\Delta X_{C.O.M.} = \frac{m_1\Delta x_1 + m_2\Delta x_2}{m_1 + m_2}$$

$$\Rightarrow 0 = \frac{3\times 2 + 2(-x)}{3+2}$$

$\Rightarrow x = 3$ cm

55. The disintegration energy Q for the nuclear fission of $^{235}U \rightarrow ^{140}Ce + ^{94}Zr + n$ is ____MeV.

Given atomic masses of

$^{235}U : 235.0439u; ^{140}Ce; 139.9054u$,

$^{94}Zr : 93.9063u; n : 1.0086u$,

Value of $c^2 = 931$ MeV/u.

Ans. **(208)**

Sol. $^{235}U \rightarrow ^{140}Ce + ^{94}Zr + n$

Disintegration energy

$Q = (m_R - m_P).c^2$

$m_R = 235.0439$ u

$m_P = 139.9054u + 93.9063u + 1.0086$ u

$= 234.8203u$

$Q = (235.0439u - 234.8203u)c^2$

$= 0.2236\ c^2$

$= 0.2236 \times 931$

$Q = 208.1716$

56. A light ray is incident on a glass slab of thickness $4\sqrt{3}$ cm and refractive index $\sqrt{2}$. The angle of incidence is equal to the critical angle for the glass slab with air. The lateral displacement of ray after passing through glass slab is ____cm.

(Given sin 15° = 0.25)

Ans. **(2)**

Sol.

$i = \theta_C$

$$\Rightarrow i = \sin^{-1}\left(\frac{1}{\mu}\right)$$

$\Rightarrow i = 45^o$

and according to snell's law

$1 \sin 45° = \sqrt{2}\sin r$

$\Rightarrow r = 30°$

Lateral displacement $\Delta = \dfrac{t\sin(i-r)}{\cos r}$

$$\Rightarrow \Delta = \frac{4\sqrt{3}\times\sin 15°}{\cos 30°}$$

$\Rightarrow \Delta = 2$cm

57. A rod of length 60 cm rotates with a uniform angular velocity 20 rad s^{-1} about its perpendicular bisector, in a uniform magnetic field 0.5 T. The direction of magnetic field is parallel to the axis of rotation. The potential difference between the two ends of the rod is ____V.

Ans. (0)

Sol.

$$\because V_0 - V_A = \frac{B\omega\ell^2}{2}$$

$$V_0 - V_B = \frac{B\omega\ell^2}{2}$$

$$\therefore\ V_A = V_B \therefore\ V_A - V_B = 0$$

58. Two wires A and B are made up of the same material and have the same mass. Wire A has radius of 2.0 mm and wire B has radius of 4.0 mm. The resistance of wire B is 2Ω. The resistance of wire A is _____Ω.

Ans. (32)

Sol. $\because R = \frac{\rho\ell}{A} = \frac{\rho V}{A^2}$

$$\therefore \frac{R_A}{R_B} = \frac{A_B^2}{A_A^2} = \frac{r_B^4}{r_A^4}$$

$$\Rightarrow \frac{R_A}{2} = \left[\frac{4\times10^{-3}}{2\times10^{-3}}\right]^4$$

$\Rightarrow R_A = 32\ \Omega.$

59. Two parallel long current carrying wire separated by a distance 2r are shown in the figure. The ratio of magnetic field at A to the magnetic field produced at C is $\frac{x}{7}$. The value of x is ___.

Ans. (5)

Sol. $B_A = \frac{\mu_0 i}{2\pi r} + \frac{\mu_0(2i)}{2\pi(3r)} = \frac{5\mu_0 i}{6\pi r}$

$$B_C = \frac{\mu_0(2i)}{2\pi r} + \frac{\mu_0 i}{2\pi(3r)} = \frac{7\mu_0 i}{6\pi r}$$

$$\therefore \frac{B_A}{B_C} = \frac{5}{7}$$

$\therefore x = 5$

60. Mercury is filled in a tube of radius 2 cm up to a height of 30 cm. The force exerted by mercury on the bottom of the tube is ___N.
(Given, atmospheric pressure = 10^5 Nm^{-2}, density of mercury = 1.36×10^4 kg m^{-3}, g = 10 ms^{-2}, $\pi = \frac{22}{7}$)

Ans. (177)

Sol. $F = P_0 A + \rho_m ghA$

$$= 10^5 \times \frac{22}{7}\times\left(2\times10^{-2}\right)^2$$

$$+1.36\times10^4\times10\times\left(30\times10^{-2}\right)\left(\frac{22}{7}\times\left(2\times10^{-2}\right)^2\right)$$

F = 51.29 + 125.71 = 177 N

SECTION-A

61. The equilibrium constant for the reaction

$$SO_3(g) \rightleftharpoons SO_2(g) + \frac{1}{2}O_2(g)$$

is $K_C = 4.9 \times 10^{-2}$. The value of K_C for the reaction given below is

$$2SO_2(g) + O_2(g) \rightleftharpoons 2SO_3(g) \text{ is}$$

(1) 4.9 (2) 41.6

(3) 49 (4) 416

Ans. (4)

Sol. $K'_C = \left(\frac{1}{K_C}\right)^2 = \left(\frac{1}{4.9\times10^{-2}}\right)^2$

$K'_C = 416.49$

62. Find out the major product formed from the following reaction. [Me: $-CH_3$]

(1)

(2)

(3)

(4)

Ans. (2)

Sol.

The above mechanism valid for both cis and trans isomers. So the products are same for both cis and trans isomers.

63. When MnO_2 and H_2SO_4 is added to a salt (A), the greenish yellow gas liberated as salt (A) is :

(1) NaBr (2) CaI_2

(3) KNO_3 (4) NH_4Cl

Ans. (4)

Sol. $2NH_4Cl + MnO_2 + 2H_2SO_4 \xrightarrow{\Delta} MnSO_4 + (NH_4)_2SO_4 + 2H_2O + \underset{\text{greenish yellow solution}}{Cl_2} \uparrow$

64. The correct statement/s about Hydrogen bonding is/are :

A. Hydrogen bonding exists when H is covalently bonded to the highly electro negative atom.

B. Intermolecular H bonding is present in o-nitro phenol

C. Intramolecular H bonding is present in HF.

D. The magnitude of H bonding depends on the physical state of the compound.

E. H-bonding has powerful effect on the structure and properties of compounds.

Choose the **correct** answer from the options given below :

(1) A only (2) A, D, E only

(3) A, B, D only (4) A, B, C only

Ans. (2)

Sol. (A) Generally hydrogen bonding exists when H is covalently bonded to the highly electronegative atom like F, O, N.

(B) Intramolecular H bonding is present in

(C) Intermolecular Hydrogen bonding is present in HF

(D) The magnitude has Hydrogen bonding in solid state is greater than liquid state.

(E) Hydrogen bonding has powerfull effect on the structure & properties of compound like melting point, boiling point, density etc.

65.

In the above chemical reaction sequence "A'' and "B" respectively are :

(1) O_3, Zn/H_2O and $NaOH_{(alc.)}$ / I_2

(2) H_2O, H^+ and $NaOH_{(alc.)}$ / I_2

(3) H_2O, H^+ and $KMnO_4$

(4) O_3, Zn/H_2O and $KMnO_4$

Ans. **(1)**

Sol.

66. Common name of Benzene-1, 2-diol is

(1) quinol (2) resorcinol

(3) catechol (4) o-cresol

Ans. **(3)**

Sol.

IUPAC name : Benzene-1,2-diol

Common name : catechol

67. $CH_3-CH_2-CH_2-Br + NaOH \xrightarrow{C_2H_5OH}$ Product 'A'

Product A $\xrightarrow{H_2O,\ H^+}$ Product "B"

Product A $\xrightarrow[H_2O/H_2O_2/^-OH]{\text{Diborane}}$ Product "C"

Consider the above reactions, identify product B and product C.

(1) B = C = 2-Propanol

(2) B = 2-Propanol C = 1-Propanol

(3) B = 1-Propanol C = 2-Propanol

(4) B = C = 1-Propanol

Ans. **(2)**

Sol.

68. The adsorbent used in adsorption chromatography is/are

A. silica gel B. alumina

C. quick lime D. magnesia

Choose the **most appropriate** answer from the options given below :

(1) B only (2) C and D only

(3) A and B only (4) A only

Ans. **(3)**

Sol. The most common polar and acidic support used is adsorption chromatography is silica. The surface silanol groups on their supported to adsorb polar compound and work particularly well for basic substances. Alumina is the example of polar and basic adsorbent that is used in adsorption chromatography.

69.

major product "P"

Product P is

(1)

(2)

(3)

(4)

Ans. **(2)**

Sol.

(Major)

70. Correct order of stability of carbanion is

(1) c > b > d > a (2) a > b > c > d
(3) d > a > c > b (4) d > c > b > a

Ans. **(4)**

Sol. As we know compound (d) is aromatic and the compound (a) is anti-aromatic. Hence compound (d) is most stable and compound (a) is least stable among these in compound (b) and (c) carbon atom of that positive charge is sp^3 hybridised they on the basis of angle strain theory compound (c) is more stable than compound (b).

71. The correct order of the first ionization enthalpy is
(1) Al > Ga > Tl (2) Ga > Al > B
(3) B > Al > Ga (4) Tl > Ga > Al

Ans. **(4)**

Sol. (i) due to lanthanide contraction Tℓ has more I.E. as compared to Ga and Aℓ

(ii) due to scandide contraction Ga has more I.E. as compared to Aℓ

72. If an iron (III) complex with the formula $\left[Fe(NH_3)_x(CN)_y\right]^-$ has no electron in its e_g orbital, then the value of x + y is
(1) 5 (2) 6
(3) 3 (4) 4

Ans. **(2)**

Sol. Complex is $[\overset{III}{Fe}(NH_3)_2(CN)_4]^{\ominus}$
x = 2
y = 4
so x + y = 6

73. Fuel cell, using hydrogen and oxygen as fuels,
A. has been used in spaceship
B. has as efficiency of 40% to produce electricity
C. uses aluminium as catalysts
D. is eco-friendly
E. is actually a type of Galvanic cell only
(1) A,B,C only (2) A,B,D only
(3) A,B,D,E only (4) A,D,E only

Ans. **(4)**

Sol. Fuel cell is used in spaceship and it is type of galvanic cell.

74. Choose the **Incorrect** Statement about Dalton's Atomic Theory
(1) Compounds are formed when atoms of different elements combine in any ratio
(2) All the atoms of a given element have identical properties including identical mass
(3) Matter consists of indivisible atoms
(4) Chemical reactions involve recorganization of atoms

Ans. **(1)**

Sol. In compound atoms of different elements combine in fixed ratio by mass.

75. Match List I with List II

	LIST I		LIST II
A.	α- Glucose and α-Galactose	I.	Functional isomers
B.	α- Glucose and β-Glucose	II.	Homologous
C.	α- Glucose and α-Fructose	III.	Anomers
D.	α- Glucose and α-Ribose	IV.	Epimers

Choose the **correct** answer from the options given below:

(1) A-III, B-IV, C-II, D-I

(2) A-III, B-IV, C-I, D-II

(3) A-IV, B-III, C-I, D-II

(4) A-IV, B-III, C-II, D-I

Ans. (3)

Sol. Based on biomolecules theory and structure of these named compounds –

(A) α-Glucose and α-Galactose (IV) Epimers.

(B) α-Glucose and β-Glucose (III) Anomers

(C) α-Glucose and α-Fructose (I) Functional isomers

(D) α-Glucose and α-Ribose (II) Homologous

76. Given below are two statements:

Statement I : The correct order of first ionization enthalpy values of Li, Na, F and Cl is Na < Li < Cl < F.

Statement II : The correct order of negative electron gain enthalpy values of Li, Na, F and Cl is Na < Li < F < Cl

In the light of the above statements, choose the **correct** answer from the options given below :

(1) Both Statement I and Statement II are true

(2) Both Statement I and Statement II are false

(3) Statement I is false but Statement II is true

(4) Statement I is true but Statement II is false

Ans. (1)

Sol.. (i) Na < Li < Cl < F

$I.E._1$ in kJ/mol: Na ↓ 496, Li ↓ 520, Cl ↓ 1256, F ↓ 1681

(ii) Na < Li < F < Cl

$\Delta_{eg}H$ in kJ/mol: Na ↓ –53, Li ↓ –60, F ↓ –328, Cl ↓ –349

77. For a strong electrolyte, a plot of molar conductivity against $(\text{concentration})^{1/2}$ is a straight line, with a negative slope, the correct unit for the slope is

(1) $S\ cm^2\ mol^{-3/2}\ L^{1/2}$ (2) $S\ cm^2\ mol^{-1}\ L^{1/2}$

(3) $S\ cm^2\ mol^{-3/2}\ L$ (4) $S\ cm^2\ mol^{-3/2}\ L^{-1/2}$

Ans. (1)

Sol. $\Lambda_m = \Lambda_m^o - A\sqrt{C}$

Units of $A\sqrt{C}$ = $S\ cm^2\ mole^{-1}$

Uits of A = $S\ cm^2\ mole^{-3/2}\ L^{1/2}$

78. A first row transition metal in its +2 oxidation state has a spin-only magnetic moment value of 3.86 BM. The atomic number of the metal is

(1) 25 (2) 26

(3) 22 (4) 23

Ans. (4)

Sol. $_{22}Ti^{+2} \Rightarrow [Ar]3d^2$

$_{23}V^{+2} \Rightarrow [Ar]3d^3$

$_{25}Mn^{+2} \Rightarrow [Ar]3d^5$

$_{26}Fe^{+2} \Rightarrow [Ar]3d^6$

79. The number of unpaired d-electrons in $[Co(H_2O)_6]^{3+}$ is_____

(1) 4 (2) 2

(3) 0 (4) 1

Ans. (3)

Sol. $\Rightarrow [Co(H_2O)_6]^{+3}$

No unpaired electrons

80. The number of species from the following that have pyramidal geometry around the central atom is_____

$S_2O_3^{2-}, SO_4^{2-}, SO_3^{2-}, S_2O_7^{2-}$

(1) 4 (2) 3

(3) 1 (4) 2

Ans. **(3)**

Sol.

SECTION-B

81. The maximum number of orbitals which can be identified with n = 4 and $m_l = 0$ is_____

Ans. **(4)**

Sol.

	4s	4p	4d	4f
n = 4 ,	□	□□□	□□□□□	□□□□□□□
$m_\ell \Rightarrow$	1	1	1	1

So answer is 4.

82. Number of compounds/species from the following with non-zero dipole moment is_____

$BeCl_2$, BCl_3, NF_3, XeF_4, CCl_4, H_2O H_2S, HBr, CO_2, H_2, HCl

Ans. **(5)**

Sol. Polar molecule : $NF_3, H_2O, H_2S, HBr, HCl$
($\mu \neq 0$)

Non Polar molecule : $BeCl_2, BCl_3, XeF_4, CCl_4, CO_2, H_2$
($\mu = 0$)

So answer is 5.

83. Three moles of an ideal gas are compressed isothermally from 60 L to 20 L using constant pressure of 5 atm. Heat exchange Q for the compression is – ____ Lit. atm.

Ans. **(200)**

Sol. As isothermal $\Delta U = 0$

and process is irreversible

$Q = -W = -[-P_{ext}(V_2 - V_1)]$

$Q = 5(20 - 60) = -200$ atm-L

84. From 6.55 g of aniline, the maximum amount of acetanilide that can be prepared will be___$\times 10^{-1}$ g.

Ans. **(95)**

Sol.

93 g aniline form 135 gm acetanlide

so 6.55 g anilne form $\frac{135}{93} \times 6.55 = 9.5$

95×10^{-1}

85. Consider the following reaction, the rate expression of which is given below

$A + B \rightarrow C$

rate = $k[A]^{1/2}[B]^{1/2}$

The reaction is initiated by taking 1M concentration A and B each. If the rate constant (k) is 4.6×10^{-2} s^{-1}, then the time taken for A to become 0.1 M is_____sec. (nearest integer)

Ans. **(50)**

Sol. $K = \frac{2.303}{t}\log\frac{1}{0.1}$

$4.6 \times 10^{-2} = \frac{2.303}{t}$

t = 50 sec.

86. Phthalimide is made to undergo following sequence of reactions.

$$\text{Phthalimide} \xrightarrow[\text{(ii)Benzylchloride}]{\text{(i)KOH}} \text{'P'}$$

Total number of π bonds present in product 'P' is/are

Ans. **(8)**

Sol.

Total number of π-bonds present in product P is 8

87. The total number of 'sigma' and 'Pi' bonds in 2-oxohex-4-ynoic acid is____.

Ans. **(18)**

Sol.

2-Oxohex-4-ynoic acid

Number of σ-bonds = 14

Number of π-bonds = 4

= 18

88. A first row transition metal with highest enthalpy of atomisation, upon reaction with oxygen at high temperature forms oxides of formula M_2O_n (where n = 3,4,5). The 'spin-only' magnetic moment value of the amphoteric oxide from the above oxides is___ BM (near integer)

(Given atomic number : Sc : 21, Ti : 22, V : 23, Cr : 24, Mn : 25, Fe : 26, Co : 27, Ni : 28 ,Cu : 29, Zn : 30)

Ans. **(0)**

Sol. 'V' has highest enthalpy of atomisation (515 kJ/mol) among first row transition elements.

V_2O_5

Here 'V' is in +5 oxidation state

$V^{+5} \Rightarrow 1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6$ (no unpaired electrons)

89. 2.7 Kg of each of water and acetic acid are mixed, The freezing point of the solution will be –x °C. Consider the acetic acid does not dimerise in water, nor dissociates in water x = _____(nearest integer)

[Given : Molar mass of water = 18 g mol^{-1}, acetic acid = 60 g mol^{-1}]

K_f H_2O : 1.86 K kg mol^{-1}

K_f acetic acid : 3.90 K kg mol^{-1}

freezing point : H_2O = 273 K, acetic acid = 290 K]

Ans. **(31)**

Sol. As moles of water > moles of CH_3COOH

water is solvent.

$T°_F - (T_F)_S = K_F \times M$

$$0 - (T_F)_S = 1.86 \times \frac{2700/60}{2700/1000}$$

$(T_F)_S = -31°C$.

90. Vanillin compound obtained from vanilla beans, has total sum of oxygen atoms and π electrons is__

Ans. **(11)**

Sol. Vanillin compound is an organic compound molecular formula $C_8H_8O_3$. It is a phenolic aldehyde. Its functional compounds include aldehyde, hydroxyl and ether. It is the primary component of the extract of the vanilla beans.

Total sum of oxygen atoms and π-electrons is 3 + 8 = 11

Total number of oxygen atoms = 3

Total number of π-bonds = 4

∴ Total number of π-electrons = 8

FINAL JEE–MAIN EXAMINATION – APRIL, 2024

(Held On Thursday 04th April, 2024) **TIME : 9 : 00 AM to 12 : 00 NOON**

MATHEMATICS — TEST PAPER WITH SOLUTION

SECTION-A

1. Let $f : R \to R$ be a function given by

$$f(x)=\begin{cases}\dfrac{1-\cos 2x}{x^2} & , \ x<0\\ \alpha & , \ x=0\\ \dfrac{\beta\sqrt{1-\cos x}}{x} & , \ x>0\end{cases}$$

, where $\alpha, \beta \in R$. If f is continuous at x = 0, then $\alpha^2 + \beta^2$ is equal to :

(1) 48 (2) 12

(3) 3 (4) 6

Ans. (2)

Sol. $f(0^-) = \lim\limits_{x\to 0^-} \dfrac{2\sin^2 x}{x^2} = 2 = \alpha$

$$f(0^+) = \lim_{x\to 0^+} \beta\times\sqrt{2}\,\frac{\sin\frac{x}{2}}{2\frac{x}{2}} = \frac{\beta}{\sqrt{2}} = 2$$

$\Rightarrow \beta = 2\sqrt{2}$

$\alpha^2 + \beta^2 = 4 + 8 = 12$

2. Three urns A, B and C contain 7 red, 5 black; 5 red, 7 black and 6 red, 6 black balls, respectively. One of the urn is selected at random and a ball is drawn from it. If the ball drawn is black, then the probability that it is drawn from urn A is :

(1) $\dfrac{4}{17}$ (2) $\dfrac{5}{18}$

(3) $\dfrac{7}{18}$ (4) $\dfrac{5}{16}$

Ans. (2)

Sol.

A	B	C
7R, 5B	5R, 7B	6R, 6B

$$P(B) = \frac{1}{3}.\frac{5}{12}+\frac{1}{3}.\frac{7}{12}+\frac{1}{3}.\frac{6}{12}$$

$$\text{required probability} = \frac{\frac{1}{3}.\frac{5}{12}}{\frac{1}{3}.\left[\frac{5}{12}+\frac{7}{12}+\frac{6}{12}\right]} = \frac{5}{18}$$

3. The vertices of a triangle are A(–1, 3), B(–2, 2) and C(3, –1). A new triangle is formed by shifting the sides of the triangle by one unit inwards. Then the equation of the side of the new triangle nearest to origin is :

(1) $x - y - \left(2+\sqrt{2}\right) = 0$

(2) $-x + y - \left(2-\sqrt{2}\right) = 0$

(3) $x + y - \left(2-\sqrt{2}\right) = 0$

(4) $x + y + \left(2-\sqrt{2}\right) = 0$

Ans. (3)

Sol.

equation of AC $\to x + y = 2$

equation of line parallel to AC $x + y = d$

$$\left|\frac{d-2}{\sqrt{2}}\right| = 1$$

$d = 2-\sqrt{2}$

eq^n of new required line

$$\boxed{x+y=2-\sqrt{2}}$$

4. If the solution y = y(x) of the differential equation $(x^4 + 2x^3 + 3x^2 + 2x + 2)dy - (2x^2 + 2x + 3)dx = 0$ satisfies $y(-1) = -\dfrac{\pi}{4}$, then y(0) is equal to :

(1) $-\dfrac{\pi}{12}$ (2) 0

(3) $\dfrac{\pi}{4}$ (4) $\dfrac{\pi}{2}$

Ans. (3)

Sol. $\int dy = \int \frac{(2x^2+2x+3)}{x^4+2x^3+3x^2+2x+2}dx$

$y = \int \frac{(2x^2+2x+3)}{(x^2+1)(x^2+2x+2)}dx$

$y = \int \frac{dx}{x^2+2x+2} + \int \frac{dx}{x^2+1}$

$y = \tan^{-1}(x+1) + \tan^{-1}x + C$

$y(-1) = \frac{-\pi}{4}$

$\frac{-\pi}{4} = 0 - \frac{\pi}{4} + C \Rightarrow C = 0$

$\Rightarrow y = \tan^{-1}(x+1) + \tan^{-1}x$

$y(0) = \tan^{-1}1 = \frac{\pi}{4}$

5. Let the sum of the maximum and the minimum values of the function $f(x) = \frac{2x^2-3x+8}{2x^2+3x+8}$ be $\frac{m}{n}$, where gcd(m, n) = 1. Then m + n is equal to :

(1) 182 (2) 217

(3) 195 (4) 201

Ans. (4)

Sol. $y = \frac{2x^2-3x+8}{2x^2+3x+8}$

$x^2(2y-2) + x(3y+3) + 8y - 8 = 0$

use $D \geq 0$

$(3y+3)^2 - 4(2y-2)(8y-8) \geq 0$

$(11y-5)(5y-11) \leq 0$

$\Rightarrow y \in \left[\frac{5}{11}, \frac{11}{5}\right]$

y = 1 is also included

6. One of the points of intersection of the curves $y = 1 + 3x - 2x^2$ and $y = \frac{1}{x}$ is $\left(\frac{1}{2}, 2\right)$. Let the area of the region enclosed by these curves be $\frac{1}{24}\left(\ell\sqrt{5}+m\right) - n\log_e\left(1+\sqrt{5}\right)$, where ℓ, m, n $\in$ N. Then ℓ + m + n is equal to

(1) 32 (2) 30

(3) 29 (4) 31

Ans. (2)

Sol.

$A = \int_{\frac{1}{2}}^{\frac{1+\sqrt{5}}{2}} \left(1 + 3x - 2x^2 - \frac{1}{x}\right)dx$

$A = \left[x + \frac{3x^2}{2} - \frac{2x^3}{3} - \ell n\, x\right]_{\frac{1}{2}}^{\frac{1+\sqrt{5}}{2}}$

$A = \frac{1+\sqrt{5}}{2} + \frac{3}{2}\left(\frac{1+\sqrt{5}}{2}\right)^2 - \frac{2}{3}\left(\frac{1+\sqrt{5}}{2}\right)^3 - \ell n\left(\frac{1+\sqrt{5}}{2}\right)$

$-\frac{1}{2} - \frac{3}{2}\left(\frac{1}{4}\right) + \frac{2}{3}\left(\frac{1}{8}\right) + \ell n\left(\frac{1}{2}\right)$

$A = \frac{1}{2} + \frac{\sqrt{5}}{2} + \frac{3}{8} + \frac{3}{4}\sqrt{5} + \frac{15}{8} - \frac{4}{3} - \frac{2}{3}\sqrt{5}$

$-\frac{1}{2} - \frac{3}{8} + \frac{1}{12} - \ell n\left(1+\sqrt{5}\right)$

$= \sqrt{5}\left(\frac{1}{2} + \frac{3}{4} - \frac{2}{3}\right) + \frac{15}{8} - \frac{4}{3} + \frac{1}{12} - \ell n\left(1+\sqrt{5}\right)$

$= \frac{14}{24}\sqrt{5} + \frac{15}{24} - \ell n\left(1+\sqrt{5}\right)$

7. If the system of equations

$x + \left(\sqrt{2}\sin\alpha\right)y + \left(\sqrt{2}\cos\alpha\right)z = 0$

$x + (\cos\alpha)y + (\sin\alpha)z = 0$

$x + (\sin\alpha)y - (\cos\alpha)z = 0$

has a non-trivial solution, then $\alpha \in \left(0, \frac{\pi}{2}\right)$ is equal to :

(1) $\frac{3\pi}{4}$ (2) $\frac{7\pi}{24}$

(3) $\frac{5\pi}{24}$ (4) $\frac{11\pi}{24}$

Ans. (3)

Sol. $\begin{vmatrix} 1 & \sqrt{2}\sin\alpha & \sqrt{2}\cos\alpha \\ 1 & \sin\alpha & -\cos\alpha \\ 1 & \cos\alpha & \sin\alpha \end{vmatrix} = 0$

$\Rightarrow 1-\sqrt{2}\sin\alpha(\sin\alpha+\cos\alpha)+\sqrt{2}\cos\alpha(\cos\alpha-\sin\alpha)=0$

$\Rightarrow 1+\sqrt{2}\cos 2\alpha-\sqrt{2}\sin 2\alpha=0$

$\cos 2\alpha-\sin 2\alpha=-\frac{1}{\sqrt{2}}$

$\cos\left(2\alpha+\frac{\pi}{4}\right)=-\frac{1}{2}$

$2\alpha+\frac{\pi}{4}=2n\pi\pm\frac{2\pi}{3}$

$\alpha+\frac{\pi}{8}=n\pi\pm\frac{\pi}{3}$

n = 0,

$x=\frac{\pi}{3}-\frac{\pi}{8}=\frac{5\pi}{24}$

8. There are 5 points P_1, P_2, P_3, P_4, P_5 on the side AB, excluding A and B, of a triangle ABC. Similarly there are 6 points P_6, P_7, …, P_{11} on the side BC and 7 points P_{12}, P_{13}, …, P_{18} on the side CA of the triangle. The number of triangles, that can be formed using the points P_1, P_2, …, P_{18} as vertices, is :

(1) 776 (2) 751

(3) 796 (4) 771

Ans. (2)

Sol. ${}^{18}C_3 - {}^{5}C_3 - {}^{6}C_3 - {}^{7}C_3$

= 751

9. Let $f(x)=\begin{cases} -2, & -2\le x\le 0 \\ x-2, & 0<x\le 2 \end{cases}$ and h(x) = f(|x|) + |f(x)|.

Then $\int_{-2}^{2} h(x)dx$ is equal to :

(1) 2 (2) 4

(3) 1 (4) 6

Ans. (1)

Sol.

f(|x|) → |f(x)|

$h(x)=\begin{cases} x-2+2-x=0, & 0\le x\le 2 \\ -x-2+2=-x & -2\le x<0 \end{cases}$

$\Rightarrow \int_0^2 h(x)dx=0$ and $\int_{-2}^{0} h(x)dx=2$

10. The sum of all rational terms in the expansion of $\left(2^{\frac{1}{5}}+5^{\frac{1}{3}}\right)^{15}$ is equal to :

(1) 3133 (2) 633

(3) 931 (4) 6131

Ans. (1)

Sol. $T_{r+1}={}^{15}C_r\left(5^{\frac{1}{3}}\right)^r\left(2^{\frac{1}{5}}\right)^{15-r}$

$={}^{15}C_r\,5^{\frac{r}{3}}.2^{\frac{15-r}{5}}$

R = 3λ, 15μ

⇒ r = 0, 15

2 rational terms

$\Rightarrow {}^{15}C_0 2^3+{}^{15}C_{15}(5)^5$

= 8 + 3125 = 3133

11. Let a unit vector which makes an angle of 60° with $2\hat{i}+2\hat{j}-\hat{k}$ and an angle of 45° with $\hat{i}-\hat{k}$ be $\vec{C}$. Then $\vec{C}+\left(-\frac{1}{2}\hat{i}+\frac{1}{3\sqrt{2}}\hat{j}-\frac{\sqrt{2}}{3}\hat{k}\right)$ is :

(1) $-\frac{\sqrt{2}}{3}\hat{i}+\frac{\sqrt{2}}{3}\hat{j}+\left(\frac{1}{2}+\frac{2\sqrt{2}}{3}\right)\hat{k}$

(2) $\frac{\sqrt{2}}{3}\hat{i}+\frac{1}{3\sqrt{2}}\hat{j}-\frac{1}{2}\hat{k}$

(3) $\left(\frac{1}{\sqrt{3}}+\frac{1}{2}\right)\hat{i}+\left(\frac{1}{\sqrt{3}}-\frac{1}{3\sqrt{2}}\right)\hat{j}+\left(\frac{1}{\sqrt{3}}+\frac{\sqrt{2}}{3}\right)\hat{k}$

(4) $\frac{\sqrt{2}}{3}\hat{i}-\frac{1}{2}\hat{k}$

Ans. (4)

Sol. $\vec{C}=C_1\hat{i}+C_2\hat{j}+C_3\hat{k}$

$C_1^2+C_2^2+C_3^2=1$

$\vec{C}.\left(2\hat{i}+2\hat{j}-\hat{k}\right)=|C|\sqrt{9}\cos 60°$

$2C_1+2C_2-C_3=\frac{3}{2}$

$C_1-C_3=1$

$C_1+2C_2=\frac{1}{2}$

$C_1=\frac{\sqrt{2}}{3}+\frac{1}{2}$

$C_2=\frac{-1}{3\sqrt{2}}$

$C_3=\frac{\sqrt{2}}{3}-\frac{1}{2}$

12. Let the first three terms 2, p and q, with $q \neq 2$, of a G.P. be respectively the 7th, 8th and 13th terms of an A.P. If the 5th term of the G.P. is the nth term of the A.P., then n is equal to

(1) 151 (2) 169

(3) 177 (4) 163

Ans. (4)

Sol. $p^2 = 2q$

$2 = a + 6d$ ...(i)

$p = a + 7d$ ...(ii)

$q = a + 12d$ ...(iii)

$p - 2 = d$ ((ii) – (i))

$q - p = 5d$ ((iii) – (ii))

$q - p = 5(p - 2)$

$q = 6p - 10$

$p^2 = 2(6p - 10)$

$p^2 - 12p + 20 = 0$

$p = 10, 2$

$p = 10$; $q = 50$

$d = 8$

$a = -46$

2, 10, 50, 250, 1250

$ar^4 = a + (n - 1)d$

$1250 = -46 + (n - 1)8$

$n = 163$

13. Let a, b $\in$ R. Let the mean and the variance of 6 observations –3, 4, 7, –6, a, b be 2 and 23, respectively. The mean deviation about the mean of these 6 observations is :

(1) $\frac{13}{3}$ (2) $\frac{16}{3}$

(3) $\frac{11}{3}$ (4) $\frac{14}{3}$

Ans. (1)

Sol. $\frac{\sum x_i}{6}=2$ and $\frac{\sum x_i^2}{N}-\mu^2=23$

$\alpha + \beta = 10$

$\alpha^2 + \beta^2 = 52$

solving we get $\alpha = 4$, $\beta = 6$

$\frac{\sum|x_i-\bar{x}|}{6}=\frac{5+2+5+8+2+4}{6}=\frac{13}{3}$

14. If 2 and 6 are the roots of the equation $ax^2 + bx + 1 = 0$, then the quadratic equation, whose roots are $\frac{1}{2a+b}$ and $\frac{1}{6a+b}$, is :

(1) $2x^2 + 11x + 12 = 0$ (2) $4x^2 + 14x + 12 = 0$

(3) $x^2 + 10x + 16 = 0$ (4) $x^2 + 8x + 12 = 0$

Ans. (4)

Sol. Sum $= 8 = -\frac{b}{a}$

Product $= 12 = \frac{1}{a}$ $\Rightarrow a = \frac{1}{12}$

$b = -\frac{2}{3}$

$2a + b = \frac{2}{12} - \frac{2}{3} = -\frac{1}{2}$

$6a + b = \frac{6}{12} - \frac{2}{3} = -\frac{1}{6}$

sum = –8

P = 12

$x^2 + 8x + 12 = 0$

15. Let α and β be the sum and the product of all the non-zero solutions of the equation $(\bar{z})^2 + |z| = 0$, $z \in C$. Then $4(\alpha^2 + \beta^2)$ is equal to :

(1) 6 (2) 4

(3) 8 (4) 2

Ans. (2)

Sol. $z = x + iy$

$\bar{z} = x - iy$

$\bar{z}^2 = x^2 - y^2 - 2ixy$

$\Rightarrow x^2 - y^2 - 2ixy + \sqrt{x^2 + y^2} = 0$

$\Rightarrow x = 0$ or $y = 0$

$-y^2 + |y| = 0$ $x^2 + |x| = 0$

$|y| = |y|^2$ $\Rightarrow x = 0$

$y = 0, \pm 1$

$\Rightarrow i, -i$ $\Rightarrow \alpha = i - i = 0$

are roots $\beta = i(-i) = 1$

$4(0 + 1) = 4$

16. Let the point, on the line passing through the points P(1, –2, 3) and Q(5, –4, 7), farther from the origin and at a distance of 9 units from the point P, be (α, β, γ). Then $\alpha^2 + \beta^2 + \gamma^2$ is equal to :

(1) 155 (2) 150

(3) 160 (4) 165

Ans. (1)

Sol. PQ line

$\frac{x-1}{4} = \frac{y+2}{-2} = \frac{z-3}{4}$

pt $(4t + 1, -2t - 2, 4t + 3)$

distance$^2 = 16t^2 + 4t^2 + 16t^2 = 81$

$t = \pm\frac{3}{2}$

pt (7, –5, 9)

$\alpha^2 + \beta^2 + \gamma^2 = 155$

option (1)

17. A square is inscribed in the circle $x^2 + y^2 - 10x - 6y + 30 = 0$. One side of this square is parallel to $y = x + 3$. If (x_i, y_i) are the vertices of the square, then $\sum(x_i^2 + y_i^2)$ is equal to :

(1) 148 (2) 156

(3) 160 (4) 152

Ans. (4)

Sol.

$y = x + c$ & $x + y + d = 0$

$\left|\frac{5-3+c}{\sqrt{2}}\right| = \sqrt{2}$ $\left|\frac{8+d}{\sqrt{2}}\right| = \sqrt{2}$

$|c + 2| = 2$ $8 + d = \pm 2$

$c = 0, -4$ $d = -10, -6$

pts (5, 5), (3, 3), (7, 3), (5, 1)

$\sum(x_i^2 + y_1^2) = 25 + 25 + 9 + 9 + 49 + 9 + 25 + 1$

$= 152$

Option (4)

18. If the domain of the function $\sin^{-1}\left(\frac{3x-22}{2x-19}\right)+\log_e\left(\frac{3x^2-8x+5}{x^2-3x-10}\right)$ is $(\alpha, \beta]$, then $3\alpha + 10\beta$ is equal to :

(1) 97 (2) 100

(3) 95 (4) 98

Ans. (1)

Sol. $-1 \le \frac{3x-22}{2x-19} \le 1$ $\frac{3x^2-8x+5}{x^2-3x-10} > 0$

$x \in \left(5, \frac{41}{5}\right]$

$3\alpha + 10\beta = 97$

Option (1)

19. Let $f(x) = x^5 + 2e^{x/4}$ for all $x \in R$. Consider a function g(x) such that (gof) (x) = x for all $x \in R$. Then the value of $8g'(2)$ is :

(1) 16 (2) 4

(3) 8 (4) 2

Ans. (1)

Sol. f(x) = 2

when x = 0

$\because g'(f(x))\, f'(x) = 1$

$g'(2) = \frac{1}{f'(0)}$

$\because f'(x) = 5x^4 + \frac{2}{4}e^{x/4}$

$g'(2) = 2$

Ans = 16

Option (1)

20. Let $\alpha \in (0, \infty)$ and $A = \begin{bmatrix} 1 & 2 & \alpha \\ 1 & 0 & 1 \\ 0 & 1 & 2 \end{bmatrix}$.

If $\det(\text{adj}(2A - A^T).\text{adj}(A - 2A^T)) = 2^8$, then $(\det(A))^2$ is equal to :

(1) 1 (2) 49

(3) 16 (4) 36

Ans. (3)

Sol. $|\text{adj}(A - 2A^T)\,(2A - A^T)| = 28$

$|(A - 2A^T)\,(2A - A^T)| = 24$

$|A - 2A^T|\,|2A - A^T| = \pm 16$

$(A - 2A^T)^T = A^T - 2A$

$|A - 2A^T| = |A^T - 2A|$

$\Rightarrow |A - 2A^T|^2 = 16$

$|A - 2A^T| = \pm 4$

$$\begin{bmatrix} 1 & 2 & \alpha \\ 1 & 0 & 1 \\ 0 & 1 & 2 \end{bmatrix} - \begin{bmatrix} 2 & 2 & 0 \\ 4 & 0 & 2 \\ 2\alpha & 2 & 4 \end{bmatrix}$$

$$\begin{vmatrix} -1 & 0 & \alpha \\ -3 & 0 & -1 \\ -2\alpha & -1 & -2 \end{vmatrix}$$

$1 + 3\alpha = 4$

$3\alpha = 3$

$\alpha = 1$

$$|A| = \begin{vmatrix} 1 & 2 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 2 \end{vmatrix} = -1 - 3 = -4$$

$|A|^2 = 16$

SECTION-B

21. If $\lim_{x\to 1} \frac{(5x+1)^{1/3} - (x+5)^{1/3}}{(2x+3)^{1/2} - (x+4)^{1/2}} = \frac{m\sqrt{5}}{n(2n)^{2/3}}$, where gcd(m, n) = 1, then 8m + 12n is equal to _____

Ans. (100)

Sol. $$\lim_{x\to 1} \frac{\frac{1}{3}(5x+1)^{-2/3}5 - \frac{1}{3}(x+5)^{-2/3}}{\frac{1}{2}(2x+3)^{-1/2}.2 - \frac{1}{2}(x+4)^{-1/2}}$$

$$= \frac{8}{3}\frac{\sqrt{5}}{6^{2/3}} \quad \begin{matrix} m = 8 \\ n = 3 \end{matrix}$$

8m + 12n = 100

22. In a survey of 220 students of a higher secondary school, it was found that at least 125 and at most 130 students studied Mathematics; at least 85 and at most 95 studied Physics; at least 75 and at most 90 studied Chemistry; 30 studied both Physics and Chemistry; 50 studied both Chemistry and Mathematics; 40 studied both Mathematics and Physics and 10 studied none of these subjects. Let m and n respectively be the least and the most number of students who studied all the three subjects. Then m + n is equal to _____

Ans. (45)

Sol.

$125 \le m + 90 - x \le 130$

$85 \le P + 70 - x \le 95$

$75 \le C + 80 - x \le 90$

$m + P + C + 120 - 2x = 210$

$\Rightarrow 15 \le x \le 45$ & $30 - x \ge 0$

$\Rightarrow 15 \le x \le 30$

$30 + 15 = 45$

23. Let the solution y = y(x) of the differential equation $\frac{dy}{dx} - y = 1 + 4\sin x$ satisfy $y(\pi) = 1$. Then $y\left(\frac{\pi}{2}\right) + 10$ is equal to _____

Ans. (7)

Sol. $ye^{-x} = \int\left(e^{-x} + 4e^{-x}\sin x\right)dx$

$ye^{-x} = -e^{-x} - 2(e^{-x}\sin x \;\; e^{-x}\cos x) + C$

$y = -1 - 2(\sin x + \cos x) + ce^{x}$

$\because y(\pi) = 1 \Rightarrow c = 0$

$y(\pi/2) = -1 - 2 = -3$

Ans = 10 – 3 = 7

24. If the shortest distance between the lines $\frac{x+2}{2} = \frac{y+3}{3} = \frac{z-5}{4}$ and $\frac{x-3}{1} = \frac{y-2}{-3} = \frac{z+4}{2}$ is $\frac{38}{3\sqrt{5}}k$ and $\int_0^k [x^2]dx = \alpha - \sqrt{\alpha}$, where [x] denotes the greatest integer function, then $6\alpha^3$ is equal to _____

Ans. (48)

Sol. $\frac{38}{3\sqrt{5}}\hat{k} = \frac{(5\hat{i} + 5\hat{j} - 9\hat{k})}{\sqrt{5}} \cdot \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 1 & -3 & 2 \end{vmatrix}$

$\frac{38}{3\sqrt{5}}\hat{k} = \frac{19}{\sqrt{5}}$

$k = \frac{19}{\sqrt{5}}$

$k = \frac{3}{2}$

$\int_0^{3/2}[x^2] = \int_0^1 0 + \int_1^{\sqrt{2}} 1 + \int_{\sqrt{2}}^{3/2} 2$

$= \sqrt{2} - 1 + 2\left(\frac{3}{2} - \sqrt{2}\right)$

$= 2 - \sqrt{2}$

$\alpha = 2$

$\Rightarrow 6\alpha^3 = 48$

25. Let A be a square matrix of order 2 such that |A| = 2 and the sum of its diagonal elements is –3. If the points (x, y) satisfying $A^2 + xA + yI = 0$ lie on a hyperbola, whose transverse axis is parallel to the x-axis, eccentricity is e and the length of the latus rectum is ℓ, then $e^4 + \ell^4$ is equal to _____

Ans. (Bouns)

NTA Ans. (25)

Sol. Given |A| = 2

trace A = –3

and $A^2 + xA + yI = 0$

$\Rightarrow x = 3, y = 2$

so, information is incomplete to determine eccentricity of hyperbola (e) and length of latus rectum of hyperbola (ℓ)

26. Let $a = 1 + \frac{{}^2C_2}{3!} + \frac{{}^3C_2}{4!} + \frac{{}^4C_2}{5!} + ...,$

$$b = 1 + \frac{{}^1C_0 + {}^1C_1}{1!} + \frac{{}^2C_0 + {}^2C_1 + {}^2C_2}{2!} + \frac{{}^3C_0 + {}^3C_1 + {}^3C_2 + {}^3C_3}{3!} + ...$$

Then $\frac{2b}{a^2}$ is equal to _____

Ans. (8)

Sol. $f(x) = 1 + \frac{(1+x)}{1!} + \frac{(1+x)^2}{2!} + \frac{(1+x)^3}{3!} + ...$

$$\frac{e^{(1+x)}}{1+x} = \frac{1}{1+x} + 1 + \frac{(1+x)}{2!} + \frac{(1+x)^2}{3!} + \frac{(1+x)^2}{4!}$$

coef x^2 in RHS : $1 + \frac{{}^2C_2}{3} + \frac{{}^3C_2}{4} + ... = a$

coeff. x^2 in L.H.S.

$$e\left(1 + x + \frac{x^2}{2!}\right)....\left(1 - x + \frac{x^2}{2!}.....\right)$$

is $e - e + \frac{e}{2!} = a$

$$b = 1 + \frac{2}{1!} + \frac{2^2}{2!} + \frac{2^3}{3!} + ... = e^2$$

$$\frac{2b}{a^2} = 8$$

27. Let A be a 3 × 3 matrix of non-negative real elements such that $A\begin{bmatrix}1\\1\\1\end{bmatrix} = 3\begin{bmatrix}1\\1\\1\end{bmatrix}$. Then the maximum value of det(A) is _____

Ans. (27)

Sol. Let $A = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$

$$A\begin{bmatrix}1\\1\\1\end{bmatrix} = 3\begin{bmatrix}1\\1\\1\end{bmatrix}$$

$\Rightarrow a_1 + a_2 + a_3 = 3$ .....(1)

$\Rightarrow b_1 + b_2 + b_3 = 3$ .....(2)

$\Rightarrow c_1 + ca_2 + c_3 = 3$ .....(3)

Now,

$|A| = (a_1b_2c_3 + a_2b_3c_1 + a_3b_1c_2)$

$- (a_3b_2c_1 + a_2b_1c_3 + a_1b_3c_2)$

$\therefore$ From above in formation, clearly $|A|_{max} = 27$, when $a_1 = 3, b_2 = 3, c_3 = 3$

28. Let the length of the focal chord PQ of the parabola $y^2 = 12x$ be 15 units. If the distance of PQ from the origin is p, then $10p^2$ is equal to ____

Ans. (72)

Sol.

length of focal chord = $4a \operatorname{cosec}^2\theta = 15$

$12\operatorname{cosec}^2\theta = 15$

$\sin^2\theta = \frac{4}{5}$

$\tan^2\theta = 4$

$\tan\theta = 2$

equation $\frac{y-0}{x-3} = 2$

$y = 2x - 6$

$2x - y - 6 = 0$

$P = \frac{6}{\sqrt{5}}$

$10p^2 = 10.\frac{36}{5} = 72$

29. Let ABC be a triangle of area $15\sqrt{2}$ and the vectors $\overrightarrow{AB} = \hat{i} + 2\hat{j} - 7\hat{k}$, $\overrightarrow{BC} = a\hat{i} + b\hat{j} + c\hat{k}$ and $\overrightarrow{AC} = 6\hat{i} + d\hat{j} - 2\hat{k}$, d > 0. Then the square of the length of the largest side of the triangle ABC is

Ans. (54)

Sol.

$$\text{Area} = \frac{1}{2}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -7 \\ 6 & d & -2 \end{vmatrix} = 15\sqrt{2}$$

$(-4 + 7d)\,\hat{i} - \hat{j}\,(-2 + 42) + \hat{k}\,(d - 12)$

$(7d - 4)^2 + (40)^2 + (d - 12)^2 = 1800$

$50d^2 - 80d - 40 = 0$

$5d^2 - 8d - 4 = 0$

$5d^2 - 10d - 2d - 4$

$5d(d - 2) + 2(d - 2) = 0$

$d = 2$ or $d = -\frac{2}{5}$

$\because d > 0, d = 2$

$(a + 1)\,\hat{i} + (b + 2)\,\hat{j} + (c - 7)\,\hat{k} = 6\,\hat{i} + 2\,\hat{j} - 2\,\hat{k}$

$a + 1 = 6$ & $b + 2 = 2$, $c - 7 = -2$

$a = 5 \quad b = 0 \quad c = 5$

$|AB| = \sqrt{1+4+49} = \sqrt{54}$

$|BC| = \sqrt{25+25} = \sqrt{50}$

$|AC| = \sqrt{86+4+4} = \sqrt{44}$

Ans. 54

30. If $\int_0^{\frac{\pi}{4}} \frac{\sin^2 x}{1+\sin x\cos x}\,dx = \frac{1}{a}\log_e\left(\frac{a}{3}\right) + \frac{\pi}{b\sqrt{3}}$, where a, $b \in N$, then a + b is equal to _____

Ans. (8)

Sol. $\int_0^{\frac{\pi}{2}} \frac{\sin^2 x}{1+\frac{1}{2}\sin 2x}\,dx = \int_0^{\frac{\pi}{4}} \frac{1-\cos 2x}{2+\sin 2x}\,dx$

$\int \frac{1}{2+\sin 2x} - \int \frac{\cos 2x}{2+\sin 2x}$

$(I_1) \quad - \quad (I_2)$

$(I_1) = \int \frac{dx}{2 + \frac{2\tan x}{1+\tan^2 x}}$

$\int_0^{\frac{\pi}{4}} \frac{\sec^2 x\,dx}{2\tan^2 x + 2\tan x + 2}$

$\tan x = t$

$\frac{1}{2}\int_0^1 \frac{dt}{\left(t+\frac{1}{2}\right)^2 + \frac{3}{4}} = \frac{\pi}{6\sqrt{3}}$

$I_2 = \int_0^{\pi/4} \frac{\cos 2x}{2+\sin 2x}\,dx = \frac{1}{2}\left(\ell n\frac{3}{2}\right)$

$I_1 - I_2 = \frac{1}{\sqrt{3}}\frac{\pi}{6} + \frac{1}{2}\ell n\frac{2}{3}$

$\Rightarrow a = 2, b = 6$

Ans. 8

SECTION-A

31. An electron is projected with uniform velocity along the axis inside a current carrying long solenoid. Then :

(1) the electron will be accelerated along the axis.

(2) the electron will continue to move with uniform velocity along the axis of the solenoid.

(3) the electron path will be circular about the axis.

(4) the electron will experience a force at 45° to the axis and execute a helical path.

Ans. **(2)**

Sol.

Since $\vec{v} \parallel \vec{B}$ so force on electron due to magnetic field is zero. So it will move along axis with uniform velocity.

32. The electric field in an electromagnetic wave is given by $\vec{E} = \hat{i}40\cos\omega\left(t - \frac{z}{c}\right)NC^{-1}$. The magnetic field induction of this wave is (in SI unit):

(1) $\vec{B} = \hat{i}\frac{40}{c}\cos\omega\left(t - \frac{z}{c}\right)$

(2) $\vec{B} = \hat{j}40\cos\omega\left(t - \frac{z}{c}\right)$

(3) $\vec{B} = \hat{k}\frac{40}{c}\cos\omega\left(t - \frac{z}{c}\right)$

(4) $\vec{B} = \hat{j}\frac{40}{c}\cos\omega\left(t - \frac{z}{c}\right)$

Ans. **(4)**

Sol. $\vec{E} = \hat{i}40\cos\omega\left(t - \frac{z}{c}\right)$

$\vec{E}$ is along +x direction

$\vec{v}$ is along +z direction

So direction of $\vec{B}$ will be along +y and magnitude of B will be $\frac{E}{c}$

So answer is $\frac{40}{c}\cos\omega\left(t - \frac{z}{c}\right)\hat{j}$

33. Which of the following nuclear fragments corresponding to nuclear fission between neutron $\left({}^{1}_{0}n\right)$ and uranium isotope $\left({}^{235}_{92}U\right)$ is correct:

(1) ${}^{144}_{56}Ba + {}^{89}_{36}Kr + 4{}^{1}_{0}n$ (2) ${}^{140}_{56}Xe + {}^{94}_{38}Sr + 3{}^{1}_{0}n$

(3) ${}^{153}_{51}Sb + {}^{99}_{41}Nb + 3{}^{1}_{0}n$ (4) ${}^{144}_{56}Ba + {}^{89}_{36}Kr + 3{}^{1}_{0}n$

Ans. **(4)**

Sol. Balancing mass number and atomic number

${}^{235}_{92}U + {}^{1}_{0}n \rightarrow {}^{144}_{56}Ba + {}^{89}_{36}Kr + 3{}^{1}_{0}n$

34. In an experiment to measure focal length (f) of convex lens, the least counts of the measuring scales for the position of object (u) and for the position of image (v) are Δu and Δv, respectively. The error in the measurement of the focal length of the convex lens will be :

(1) $\frac{\Delta u}{u} + \frac{\Delta v}{v}$ (2) $f^2\left[\frac{\Delta u}{u^2} + \frac{\Delta v}{v^2}\right]$

(3) $2f\left[\frac{\Delta u}{u} + \frac{\Delta v}{v}\right]$ (4) $f\left[\frac{\Delta u}{u} + \frac{\Delta v}{v}\right]$

Ans. **(2)**

Sol. $f^{-1} = v^{-1} - u^{-1}$

$-f^{-2}\,df = -v^{-2}dv - u^{-2}du$

$\frac{df}{f^2} = \frac{dv}{v^2} + \frac{du}{u^2}$

$df = f^2\left[\frac{dv}{v^2} + \frac{du}{u^2}\right]$

35. Given below are two statements :

Statement I : When speed of liquid is zero everywhere, pressure difference at any two points depends on equation $P_1 - P_2 = \rho g\,(h_2 - h_1)$

Statement II : In ventury tube shown $2gh = \upsilon_1^2 - \upsilon_2^2$

In the light of the above statements, choose the most appropriate answer from the options given below.

(1) Both Statement I and Statement II are correct.

(2) Statement I is incorrect but Statement II is correct.

(3) Both Statement I and Statement II are incorrect.

(4) Statement I is correct but Statement II is incorrect.

Ans. **(4)**

Sol.

Applying Bernoulli's equation

$$P_1 + \rho g h_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g h_2 + \frac{1}{2}\rho v_2^2$$

[h_1 & h_2 are height of point from any reference level]

Given $V_1 = V_2 = 0$ (for statement-1)

$\therefore P_1 - P_2 = \rho g(h_2 - h_2)$

For statement-2

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

$P_1 - P_2 = \rho gh$

$$P_1 - P_2 = \frac{1}{2}\rho v_2^2 - \frac{1}{2}\rho v_1^2$$

$$\rho gh = \frac{1}{2}\rho v_2^2 - \frac{1}{2}\rho v_1^2$$

$$2gh = v_2^2 - v_1^2$$

Hence answer (4)

36. The resistances of the platinum wire of a platinum resistance thermometer at the ice point and steam point are 8 Ω and 10 Ω respectively. After inserting in a hot bath of temperature 400°C, the resistance of platinum wire is :

(1) 2Ω (2) 16 Ω (3) 8 Ω (4) 10 Ω

Ans. **(2)**

Sol. Given $R_0 = 8\Omega$, $R_{100} = 10\Omega$

$\therefore R_{100} = R_0\,(1 + \alpha\Delta T)$

Also, $R_{400} = R_0\,(1 + \alpha\Delta T^1)$

$\therefore 10 = 8\,(1 + \alpha \times 100) \Rightarrow 100\alpha = \frac{1}{4}$

$\therefore R_{400} = 8\,(1 + 400\alpha) = 8\,(1 + 1) = 16\Omega$

Hence option (2)

37. A metal wire of uniform mass density having length L and mass M is bent to form a semicircular arc and a particle of mass m is placed at the centre of the arc. The gravitational force on the particle by the wire is:

(1) $\frac{GMm\pi}{2L^2}$ (2) 0

(3) $\frac{GmM\pi^2}{L^2}$ (4) $\frac{2GmM\pi}{L^2}$

Ans. **(4)**

Sol.

We have $R = \frac{L}{\pi}$

$$g_0 = \frac{2G\frac{M}{L}}{R} = \frac{2GM\pi}{L^2}$$

$$\therefore F_m = mg_0 = \frac{2GM\pi m}{L^2}$$

Hence option (4)

38. On celcius scale the temperature of body increases by 40°C. The increase in temperature on Fahrenheit scale is:

(1) 70°F

(2) 68°F

(3) 72°F

(4) 75°F

Ans. **(3)**

Sol. We know that per °C change is equivalent to 1.8° change in °F.

∴ 40° change on celcius scale will corresponds to 72° change on Fahrenheit scale.

Hence option (3)

39. An effective power of a combination of 5 identical convex lenses which are kept in contact along the principal axis is 25 D. Focal length of each of the convex lens is :

(1) 20 cm

(2) 50 cm

(3) 500 cm

(4) 25 cm

Ans. **(1)**

Sol. We know that $P_{eq} = \Sigma P_i$

∴ given all lenses are identical

∴ 5P = 25D

∴ P = 5D

$$\therefore \frac{1}{f} = 5 \Rightarrow f = \frac{1}{5}m = 20cm$$

Hence option (1)

40. Which figure shows the correct variation of applied potential difference (V) with photoelectric current (I) at two different intensities of light ($I_1 < I_2$) of same wavelengths :

(1)

(2)

(3)

(4)

Ans. **(3)**

Sol. Given lights are of same wavelength.

Hence stopping potential will remain same.

Since $I_2 > I_1$, hence saturation current corresponding to I_2 will be greater than that corresponding to I_1.

Hence option (3)

41. A wooden block, initially at rest on the ground, is pushed by a force which increases linearly with time t. Which of the following curve best describes acceleration of the block with time :

(1)

(2)

(3)

(4)

Ans. **(2)**

Sol.

$$F = ma \Rightarrow a = \frac{F}{m} = \frac{kt}{m}$$

a vs t will be straight line passing through origin.
Since option (2).

42. If a rubber ball falls from a height h and rebounds upto the height of h/2. The percentage loss of total energy of the initial system as well as velocity ball before it strikes the ground, respectively, are :

(1) 50%, $\sqrt{\frac{gh}{2}}$ (2) 50%, $\sqrt{gh}$

(3) 40%, $\sqrt{2gh}$ (4) 50%, $\sqrt{2gh}$

Ans. **(4)**

Sol. Velocity just before collision = $\sqrt{2gh}$

Velocity just after collision = $\sqrt{2g\left(\frac{h}{2}\right)}$

$$\therefore \Delta KE = \frac{1}{2}m(2gh) - \frac{1}{2}mgh$$

$$= \frac{1}{2}mgh$$

$\therefore$ % loss in energy

$$= \frac{\Delta KE}{KE_i} \times 100 = \frac{\frac{1}{2}mgh}{\frac{1}{2}mg2h} \times 100 = 50\%$$

Hence option (4)

43. The equation of stationary wave is :

$$y = 2a\sin\left(\frac{2\pi nt}{\lambda}\right)\cos\left(\frac{2\pi x}{\lambda}\right)$$

Which of the following is NOT correct

(1) The dimensions of nt is [L]

(2) The dimensions of n is $[LT^{-1}]$

(3) The dimensions of n/λ is [T]

(4) The dimensions of x is [L]

Ans. **(3)**

Sol. Comparing the given equation with standard equation of standing $\frac{2\pi n}{\lambda} = \omega$ & $\frac{2\pi}{\lambda} = k$

$$\left[\frac{n}{\lambda}\right] = [\omega] = T^{-1}$$

$[nt] = [\lambda] = L$

$[n] = [\lambda\omega] = LT^{-1}$

$[x] = [\lambda] = L$

Hence option (3)

44. A body travels 102.5 m in n^{th} second and 115.0 m in $(n + 2)^{th}$ second. The acceleration is :

(1) 9 m/s^2 (2) 6.25 m/s^2

(3) 12.5 m/s^2 (4) 5 m/s^2

Ans. **(2)**

Sol. Given, $102.5 = u + \frac{a}{2}(2n-1)$ &

$115 = u + \frac{a}{2}(2n+3)$

$\Rightarrow 102.5 = u + an - \frac{a}{2}$ &

$115 = u + an + \frac{3a}{2}$

$12.5 = 2a \Rightarrow a = 6.25\ m/s^2$

Hence option (2)

45. To measure the internal resistance of a battery, potentiometer is used. For R = 10 Ω, the balance point is observed at ℓ = 500 cm and for R = 1 Ω the balance point is observed at ℓ = 400 cm. The internal resistance of the battery is approximately :

(1) 0.2 Ω (2) 0.4 Ω

(3) 0.1 Ω (4) 0.3 Ω

Ans. **(4)**

Sol. Let potential gradient be λ.

$\therefore i \times 10 = \lambda \times 500 = \varepsilon - ir_s$

$\Rightarrow 500\lambda = \varepsilon - 50\lambda r_s$

Also,

$i' \times 1 = \lambda \times 400 = \varepsilon - i'r_s$

$\Rightarrow 400\lambda = \varepsilon - 400\ \lambda r_s$

$\therefore 100\lambda = 350\lambda\ r_s \Rightarrow r_s = \frac{10}{35} \approx 0.3\Omega$

Hence option (4)

46. An infinitely long positively charged straight thread has a linear charge density λ Cm^{-1}. An electron revolves along a circular path having axis along the length of the wire. The graph that correctly represents the variation of the kinetic energy of electron as a function of radius of circular path from the wire is :

(1)

(2)

(3)

(4)

Ans. **(2)**

Sol.

Electric field E at a distance r due to infinite long wire is $E = \frac{2k\lambda}{r}$

Force of electron $\Rightarrow F = eE$

$$F = e\left(\frac{2k\lambda}{r}\right)$$

$$F = \frac{2k\lambda e}{r}$$

This force will provide required centripetal force

$$F = \frac{mv^2}{r} = \frac{2k\lambda e}{r}$$

$$v = \sqrt{\frac{2k\lambda e}{m}}$$

$$KE = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{2k\lambda e}{m}\right)$$

$$= k\lambda e$$

This is constant so option (2) is correct.

47. The value of net resistance of the network as shown in the given figure is :

(1) $\left(\frac{5}{2}\right)\Omega$ (2) $\left(\frac{15}{4}\right)\Omega$

(3) 6Ω (4) $\left(\frac{30}{11}\right)\Omega$

Ans. **(3)**

Sol.

Diode 2 is in reverse bias

So current will not flow in branch of 2nd diode, So we can assume it to be broken wire.

Diode 1 is in forward bias

So it will behave like conducting wire. So new circuit will be

$$R_{eq} = \frac{15\times10}{15+10} = \frac{15\times10}{25} = 6\Omega$$

Correct answer (3)

48. P-T diagram of an ideal gas having three different densities ρ_1, ρ_2, ρ_3 (in three different cases) is shown in the figure. Which of the following is correct :

(1) $\rho_2 < \rho_3$ (2) $\rho_1 > \rho_2$

(3) $\rho_1 < \rho_2$ (4) $\rho_1 = \rho_2 = \rho_3$

Ans. **(2)**

Sol. For ideal gas

$PV = nRT$

$$PV = \frac{m}{M}RT$$

$$P = \left(\frac{M}{V}\right)\frac{RT}{M}$$

$$P = \frac{\rho RT}{M}$$

(Where m is mass of gas and M is molecular mass of gas)

for same temperature $P_1 > P_2 > P_3$

So $\rho_1 > \rho_2 > \rho_3$

So correct answer is (2)

49. The co-ordinates of a particle moving in x-y plane are given by :

$x = 2 + 4t$, $y = 3t + 8t^2$.

The motion of the particle is :

(1) non-uniformly accelerated.

(2) uniformly accelerated having motion along a straight line.

(3) uniform motion along a straight line.

(4) uniformly accelerated having motion along a parabolic path.

Ans. **(4)**

Sol. $x = 2 + 4t$

$$\frac{dx}{dt} = v_x = 4$$

$$\frac{dv_x}{dt} = a_x = 0$$

$y = 3t + 8t^2$

$$\frac{dy}{dt} = v_y = 3 + 16t$$

$$\frac{dv_y}{dt} = a_y = 16$$

the motion will be uniformly accelerated motion.

For path

$x = 2 + 4t$

$$\frac{(x-2)}{4} = t$$

Put this value of t is equation of y

$$y = 3\left(\frac{x-2}{4}\right) + 8\left(\frac{x-2}{4}\right)^2$$

this is a quadratic equation so path will be parabola.

Correct answer (4)

50. In an ac circuit, the instantaneous current is zero, when the instantaneous voltage is maximum. In this case, the source may be connected to :

A. pure inductor.

B. pure capacitor.

C. pure resistor.

D. combination of an inductor and capacitor.

Choose the correct answer from the options given below :

(1) A, B and C only (2) B, C and D only

(3) A and B only (4) A, B and D only

Ans. **(4)**

Sol. This is possible when phase difference is $\frac{\pi}{2}$ between current and voltage so correct answer will be (4)

SECTION-B

51. An infinite plane sheet of charge having uniform surface charge density $+\sigma_s$ C/m^2 is placed on x-y plane. Another infinitely long line charge having uniform linear charge density $+\lambda_e$ C/m is placed at z = 4m plane and parallel to y-axis. If the magnitude values $|\sigma_s| = 2\,|\lambda_e|$ then at point (0, 0, 2), the ratio of magnitudes of electric field values due to sheet charge to that of line charge is $\pi\sqrt{n} : 1$. The value of n is_____.

Ans. **(16)**

Sol.

$$\frac{E_S}{E_\ell} = \frac{\sigma}{2\epsilon_0} \times \frac{2\pi\epsilon_0 r}{\lambda}$$

$$= \frac{\pi \times \sigma r}{\lambda}$$

$$= \frac{\pi \times 2\lambda \times 2}{\lambda} = \frac{4\pi}{1}$$

$\therefore$ n = 16

52. A hydrogen atom changes its state from n = 3 to n = 2. Due to recoil, the percentage change in the wave length of emitted light is approximately 1×10^{-n}. The value of n is_____.

[Given Rhc = 13.6 eV, hc = 1242 eV nm, $h = 6.6 \times 10^{-34}$ J s, mass of the hydrogen atom $= 1.6 \times 10^{-27}$ kg]

Ans. **(7)**

Sol. $\Delta E = 13.6\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = 1.9$ eV

$$\Delta E = \frac{hc}{\lambda}$$

$$\lambda = \frac{hc}{\Delta E}$$

$P_i = P_f$

$$0 = -mv + \frac{h}{\lambda'}$$

$$\Rightarrow v = \frac{h}{m\lambda'}$$

$$\Delta E = \frac{1}{2}mv^2 + \frac{hc}{\lambda'}$$

$$= \frac{1}{2}m\left(\frac{h}{m\lambda'}\right)^2 + \frac{hc}{\lambda'}$$

Now

$$\Delta E = \frac{h^2}{2m\lambda'^2} + \frac{hc}{\lambda'}$$

$$\lambda'^2 \Delta E - hc\lambda' - \frac{h^2}{2m} = 0$$

$$\lambda' = \frac{hc \pm \sqrt{h^2c^2 + \frac{4\Delta E h^2}{2m}}}{2\Delta E}$$

$$\lambda' = \frac{hc \pm hc\sqrt{1 + \frac{2\Delta E}{mc^2}}}{2\Delta E}$$

$$\frac{\lambda'}{\lambda} = \frac{1 + \left(1 + \frac{2\Delta E}{mc^2}\right)^{\frac{1}{2}}}{2} = \frac{1 + 1 + \frac{\Delta E}{mc^2}}{2}$$

$$\frac{\lambda'}{\lambda} = 1 + \frac{\Delta E}{2mc^2}$$

$$\frac{\lambda' - \lambda}{\lambda} = \frac{\Delta E}{2mc^2} = \frac{1.9 \times 1.6 \times 10^{-19}}{2 \times 1.67 \times 10^{-27} \times 9 \times 10^{16}} = 10^{-9}$$

$\therefore$ % change $\approx 10^{-7}$

Correct answer 7

53. The magnetic field existing in a region is given by $\vec{B} = 0.2(1 + 2x)\hat{k}T$. A square loop of edge 50 cm carrying 0.5 A current is placed in x-y plane with its edges parallel to the x-y axes, as shown in figure. The magnitude of the net magnetic force experienced by the loop is_____ mN.

Ans. **(50)**

Sol. Force on segment parallel to x-axis will cancel each other. Hence F_{net} will be due to portion parallel to y-axis.

$F = 0.5 \times 0.5 \times 6 \times 0.2 - 0.5 \times 0.5 \times 0.2 \times 5$

$= 0.5 \times 0.5 \times 0.2$

$= 0.25 \times 0.2$

$= 50 \times 10^{-3}$ N

$= 50$ mN

54. A alternating current at any instant is given by $i = \left[6 + \sqrt{56}\sin\left(100\pi t + \frac{\pi}{3}\right)\right]$ A. The rms value of the current is_____ A.

Ans. **(8)**

Sol. $I_{rms} = \sqrt{\frac{\int i^2 dt}{\int dt}}$

$I_{rms} = \sqrt{(6)^2 + \frac{(\sqrt{56})^2}{2}}$

$= \sqrt{36 + 28}$

$= \sqrt{64}$

$= 8A$

55. Twelve wires each having resistance 2Ω are joined to form a cube. A battery of 6 V emf is joined across point a and c. The voltage difference between e and f is_____ V.

Ans. **(1)**

Sol.

From symmetry, current through e-b & g-d = 0

$\therefore R_{eq} = \frac{3}{4} \times R = \frac{3}{2}\Omega$

$\therefore$ Current through battery $= \frac{6 \times 2}{3} = 4A$

$i_2 = \frac{4}{8} \times 2 = 1A$

$\therefore \Delta V$ across e-f $= \frac{i_2}{2} \times R = \frac{1}{2} \times 2 = 1V$

56. A soap bubble is blown to a diameter of 7 cm. 36960 erg of work is done in blowing it further. If surface tension of soap solution is 40 dyne/cm then the new radius is _____ cm. Take : $\left(\pi = \frac{22}{7}\right)$.

Ans. **(7)**

Sol. $\omega = \Delta U = S\Delta A$

$36960 \text{ erg} = \frac{40\text{dyne}}{\text{cm}} 8\pi\left[(r)^2 - \left(\frac{7}{2}\right)^2\right]\text{cm}^2$

r = 7 cm

57. Two wavelengths λ_1 and λ_2 are used in Young's double slit experiment λ_1 = 450 nm and λ_2 = 650 nm. The minimum order of fringe produced by λ_2 which overlaps with the fringe produced by λ_1 is n. The value of n is ____.

Ans. **(9)**

Sol. $n_2\lambda_2 = n_1\lambda_1$

$\frac{n_2}{n_1} = \frac{\lambda_1}{\lambda_2} = \frac{450}{650} = \frac{9}{13}$

$n_2 = 9$

58. An elastic spring under tension of 3 N has a length a. Its length is b under tension 2 N. For its length (3a – 2b), the value of tension will be_____ N.

Ans. **(5)**

Sol. $3 = K(a - \ell)$

$2 = K(b - \ell)$

$T = K(3a - 2b - \ell)$

$T = K(3(a - \ell) - 2(b - \ell)$

$= K\left[3\left(\frac{3}{K}\right) - 2\left(\frac{2}{K}\right)\right]$

$= 9 - 4$

$= 5$ N

59. Two forces $\vec{F}_1$ and $\vec{F}_2$ are acting on a body. One force has magnitude thrice that of the other force and the resultant of the two forces is equal to the force of larger magnitude. The angle between $\vec{F}_1$ and $\vec{F}_2$ is $\cos^{-1}\left(\frac{1}{n}\right)$. The value of |n| is ____.

Ans. **(6)**

Sol. $|\vec{F}_1| = F$

$|\vec{F}_R| = |\vec{F}_2| = 3F$

$F_R^2 = F_1^2 + F_2^2 + 2F_1F_2\cos\theta$

$9F^2 = F^2 + 9F^2 + 6F^2\cos\theta$

$\cos\theta = -\frac{1}{6}$

$\theta = \cos^{-1}\left(\frac{1}{-6}\right)$

n = –6

|n| = 6

60. A solid sphere and a hollow cylinder roll up without slipping on same inclined plane with same initial speed v. The sphere and the cylinder reaches upto maximum heights h_1 and h_2, respectively, above the initial level. The ratio $h_1 : h_2$ is $\frac{n}{10}$. The value of n is_____.

Ans. **(7)**

Sol Gain in P.E. = Loss in K.E.

$$mgh = \frac{1}{2}mv^2\left(1+\frac{K^2}{R^2}\right)$$

$$h \propto 1+\frac{K^2}{R^2}$$

$$\frac{h_1}{h_2} = \frac{1+\frac{2}{5}}{1+1} = \frac{7}{5\times 2} = \frac{7}{10}$$

n = 7

SECTION-A

61. What pressure (bar) of H_2 would be required to make emf of hydrogen electrode zero in pure water at 25ºC ?

(1) 10^{-14} (2) 10^{-7} (3) 1 (4) 0.5

NTA Ans. (3)

Sol. $2e^- + 2H^+(aq) \rightarrow H_2(g)$

$$E = E^o - \frac{0.059}{n}\log\frac{P_{H_2}}{[H^+]^2}$$

$$0 = 0 - \frac{0.059}{2}\log\frac{P_{H_2}}{(10^{-7})^2}$$

$$\log\frac{P_{H_2}}{(10^{-7})^2} = 0$$

$$\frac{P_{H_2}}{10^{-14}} = 1$$

$$P_{H_2} = 10^{-14}\,\text{bar}$$

62. The correct sequence of ligands in the order of decreasing field strength is :

(1) $CO > H_2O > F^- > S^{2-}$

(2) $^-OH > F^- > NH_3 > CN^-$

(3) $NCS^- > EDTA^{4-} > CN^- > CO$

(4) $S^{2-} > {}^-OH > EDTA^{4-} > CO$

Ans. (1)

Sol. According to spectrochemical series ligand field strength is $CO > H_2O > F^- > S^{2-}$

63. Match List -I with List II:

List - I Mechanism steps		List - II Effect	
(A)		(I)	– E effect
(B)		(II)	– R effect
(C)		(III)	+ E effect
(D)		(IV)	+ R effect

Choose the **correct** answer from the options given below :

(1) (A) – (IV), (B) – (III), (C) – (I), (D) – (II)

(2) (A) – (III), (B) – (I), (C) – (II), (D) – (IV)

(3) (A) – (II), (B) – (IV), (C) – (III), (D) – (I)

(4) (A) – (I), (B) – (II), (C) – (IV), (D) – (III)

Ans. (1)

Sol.

+ R effect ($\ddot{N}H_2$ electron donating)

– R effect (NO_2 electron withdrawing)

+E Effect H^+ electron deficient species)

– E Effect ($\bar{C}N$ electron deficient species)

64. What will be the decreasing order of basic strength of the following conjugate bases ?

$^-OH, R\bar{O}, CH_3CO\bar{O}, C\bar{l}$

(1) $C\bar{l} > OH > R\bar{O} > CH_3CO\bar{O}$

(2) $R\bar{O} > ^-OH > CH_3CO\bar{O} > C\bar{l}$

(3) $^-OH > R\bar{O} > CH_3CO\bar{O} > C\bar{l}$

(4) $C\bar{l} > R\bar{O} > ^-OH > CH_3CO\bar{O}$

Ans. (2)

Sol. Strong acid have weak conjugate base

Acidic strength :

$H–Cl > CH_3COOH > H_2O > R–OH$

Conjugate base strength :

$Cl^- < CH_3COO^- < \bar{O}H < RO^-$

65. In the precipitation of the iron group (III) in qualitative analysis, ammonium chloride is added before adding ammonium hydroxide to :

(1) prevent interference by phosphate ions

(2) decrease concentration of ^-OH ions

(3) increase concentration of Cl^- ions

(4) increase concentration of NH_4^+ ions

Ans. (2)

Sol. $NH_4OH \rightleftharpoons NH_4^+ + OH^-$

$NH_4Cl \rightarrow NH_4^+ + Cl$

Due to common ion effect of NH_4^+,

$[OH^-]$ decreases in such extent that only group-III cation can be precipitated , due to their very low K_{sp} in the range of 10^{-38}.

66.

Identify Ⓑ and Ⓒ and how are Ⓐ and Ⓒ related ?

	(B)	(C)	
(1)			functional group isomers
(2)			Derivative
(3)			position isomers
(4)			chain isomers

Ans. (3)

Sol.

A and C are position isomer.

67. One of the commonly used electrode is calomel electrode. Under which of the following categories calomel electrode comes ?

(1) Metal – Insoluble Salt – Anion electrodes

(2) Oxidation – Reduction electrodes

(3) Gas – Ion electrodes

(4) Metal ion – Metal electrodes

Ans. **(1)**

Sol. Theory based

68. Number of complexes from the following with even number of unpaired "d" electrons is____.

$[V(H_2O)_6]^{3+}$, $[Cr(H_2O)_6]^{2+}$, $[Fe(H_2O)_6]^{3+}$, $[Ni(H_2O)_6]^{3+}$, $[Cu(H_2O)_6]^{2+}$

[Given atomic numbers : V = 23, Cr = 24, Fe = 26, Ni = 28, Cu = 29]

(1) 2 (2) 4

(3) 5 (4) 1

Ans. **(1)**

Sol. $[V(H_2O)_6]^{3+} \rightarrow d^2sp^3$

$_{23}V$:- $[Ar]3d^34s^2$

V^{+3} :- $[Ar]3d^2$, n = 2 (even number of unpaired e^-)

$[Cr(H_2O)_6]^{2+} \rightarrow sp^3d^2$

$_{24}Cr$:- $[Ar]3d^54s^1$

Cr^{+2} :- $[Ar]3d^4$, n = 4 (even number of unpaired e^-)

e_g | 1 | |

t_{2g} | 1 | 1 | 1 |

$[Fe(H_2O)_6]^{3+} \rightarrow sp^3d^2$

Fe^{3+} :– $[Ar]3d^54s^0$

n = 5 (odd number of unpaired e^-)

$[Ni(H_2O)_6]^{3+} \rightarrow sp^3d^2$

Ni :- $[Ar]3d^84s^2$

Ni^{+3} :- $[Ar]3d^7$, n = 3 (odd number of unpaired e^-)

$[Cu(H_2O)_6]^{2+} \rightarrow sp^3d^2$

Cu :- $[Ar]3d^94s^0$

n = 1 (odd number of unpaired e^-)

69. Which one of the following molecules has maximum dipole moment ?

(1) NF_3 (2) CH_4

(3) NH_3 (4) PF_5

Ans. **(3)**

Sol. CH_4 & PF_5 , $\mu_{net} = 0$ (non polar)

μ_{NH_3} > μ_{NF_3}

Vector addition of bond moment & lone pair moment > Vector subtraction of bond moment & lone pair moment

70. Number of molecules/ions from the following in which the central atom is involved in sp^3 hybridization is _____.

NO_3^-, BCl_3, ClO_2^-, ClO_3

(1) 2 (2) 4

(3) 3 (4) 1

Ans. **(1)**

Sol.

71. Which among the following is **incorrect** statement?

(1) Electromeric effect dominates over inductive effect

(2) The electromeric effect is, temporary effect

(3) The organic compound shows electromeric effect in the presence of the reagent only

(4) Hydrogen ion (H^+) shows negative electromeric effect

Ans. **(4)**

Sol. Hydrogen ion (H^+) shows positive electromeric effect.

72. Given below are two statements :

Statement I : Acidity of α-hydrogens of aldehydes and ketones is responsible for Aldol reaction.

Statement II : Reaction between benzaldehyde and ethanal will NOT give Cross – Aldol product.

In the light of above statements, choose the **most appropriate** answer from the options given below.

(1) Both **Statement I** and **Statement II** are correct.

(2) Both **Statement I** and **Statement II** are incorrect.

(3) **Statement I** is incorrect but **Statement II** is correct.

(4) **Statement I** is correct but **Statement II** is incorrect.

Ans. (4)

Sol. Aldehyde and ketones having acidic α-hydrogen show aldol reaction

73. Which of the following nitrogen containing compound does not give Lassaigne's test ?

(1) Phenyl hydrazine (2) Glycene

(3) Urea (4) Hydrazine

Ans. (4)

Sol. Hydrazine (NH_2–NH_2) have no carbon so does not show Lassaigne's test.

74. Which of the following is the correct structure of L-Glucose ?

(1)

(2)

(3)

(4)

Ans. (1)

Sol. Structure of L-Glucose is

75. The element which shows only one oxidation state other than its elemental form is :

(1) Cobalt (2) Scandium

(3) Titanium (4) Nickel

Ans. (2)

Sol. Co, Ti, Ni can show +2, +3 and +4 oxidation state, But 'Sc' only shows +3 stable oxidation state.

76. Identify the product in the following reaction :

(1)

(2)

(3)

(4)

Ans. (4)

Sol.

77. Number of elements from the following that CANNOT form compounds with valencies which match with their respective group valencies is _____.

B, C, N, S, O, F, P, Al, Si

(1) 7 (2) 5 (3) 6 (4) 3

Ans. **(4)**

Sol. N,O, F can't extend their valencies upto their group number due to the non-availability of vacant 2d like orbital.

78. The Molarity (M) of an aqueous solution containing 5.85 g of NaCl in 500 mL water is :

(Given : Molar Mass Na : 23 and Cl : 35.5 $gmol^{-1}$)

(1) 20 (2) 0.2

(3) 2 (4) 4

Ans. **(2)**

Sol. $M = \frac{n_{NaCl}}{V_{sol}\,(\text{in L})}$

$M = \frac{\frac{5.85}{58.5}}{0.5} = 0.2\,M$

79. Identify the correct set of reagents or reaction conditions 'X' and 'Y' in the following set of transformation.

$$CH_3-CH_2-CH_2-Br \xrightarrow{\text{'X'}} \text{Product} \xrightarrow{\text{'Y'}} CH_3-\underset{\underset{Br}{|}}{CH}-CH_3$$

(1) X = conc.alc. NaOH, 80°C, Y = $Br_2/CHCl_3$

(2) X = dil.aq. NaOH, 20°C, Y = HBr/acetic acid

(3) X = conc.alc. NaOH, 80°C, Y = HBr/acetic acid

(4) X = dil.aq. NaOH, 20°C, Y = $Br_2/CHCl_3$

Ans. **(3)**

Sol. $CH_3–CH_2–CH_2–Br \xrightarrow[80\,^oC]{X=\text{conc.alc.NaOH}}$

$CH_3–CH=CH_2 \xrightarrow{Y\equiv HBr/\text{Acetic acid}} CH_3–CHBr – CH_3$

80. The correct order of first ionization enthalpy values of the following elements is :

(A) O (B) N

(C) Be (D) F

(E) B

Choose the correct answer from the options given below :

(1) B < D < C < E < A (2) E < C < A < B < D

(3) C < E < A < B < D (4) A < B < D < C < E

Ans. **(2)**

Sol. Correct order of I^{st} IE

Li < B < Be < C < O < N < F < Ne

↓ ↓ ↓ ↓ ↓

E < C < A < B < D

SECTION-B

81. The enthalpy of formation of ethane (C_2H_6) from ethylene by addition of hydrogen where the bond-energies of C – H, C – C, H – H are 414 kJ, 347 kJ, 615 kJ and 435 kJ respectively is - _____ kJ.

Ans. **(125)**

Sol. $C_2H_4(g) + H_2(g) \rightarrow C_2H_6(g)$

ΔH = BE(C = C) + 4BE (C – H) + BE (H – H) – BE(C – C) – 6BE (C – H)

ΔH = BE(C = C) + BE(H – H) – BE(C – C) – 2BE(C – H)

= 615 + 435 – 347 – 2 × 414

= – 125 kJ

82. The number of correct reaction(s) among the following is _____.

(A)

(B)

(C)

(D)

Ans. **(1)**

Sol.

(A)

(Incorrect)

(B)

(Incorrect)

(C)

(Correct)

(D)

(Incorrect)

83. X g of ethylamine is subjected to reaction with $NaNO_2$/HCl followed by water; evolved dinitrogen gas which occupied 2.24 L volume at STP.

X is _____ $\times 10^{-1}$ g.

Ans. (45)

Sol.

$$\underset{\text{Mol.wt.45g}}{CH_3CH_2NH_2} \xrightarrow{NaNO_2+HCl} \xrightarrow{H_2O} CH_3CH_2\text{–}OH + \underset{14g}{N_2}$$

given : N_2 evolved is 2.24 L i.e. 0.1 mole.

i.e. $CH_3CH_2NH_2$ (ethyl amine) will be 4.5 g (=0.1 mole)

Hence the answer = 45×10^{-1} g

84. The de-Broglie's wavelength of an electron in the 4th orbit is _____ πa_0. (a_0 = Bohr's radius)

Ans. (8)

Sol. $2\pi r_n = n\lambda_d$

$$2\pi a_0 \frac{n^2}{Z} = n\lambda_d$$

$$2\pi a_0 \frac{4^2}{1} = 4\lambda_d$$

$$\lambda_d = 8\pi a_0$$

85. Only 2 mL of $KMnO_4$ solution of unknown molarity is required to reach the end point of a titration of 20 mL of oxalic acid (2 M) in acidic medium. The molarity of $KMnO_4$ solution should be _____ M.

NTA Ans. (50)

Sol. eq.($KMnO_4$) = eq.($H_2C_2O_4$)

$M \times 2 \times 5 = 2 \times 20 \times 2$

M = 8M

86. Consider the following reaction

$MnO_2 + KOH + O_2 \rightarrow A + H_2O$.

Product 'A' in neutral or acidic medium disproportionate to give products 'B' and 'C' along with water. The sum of spin-only magnetic moment values of B and C is _____ BM. (nearest integer)

(Given atomic number of Mn is 25)

Ans. (4)

Sol. $MnO_2 + KOH + O_2 \rightarrow K_2MnO_4 + H_2O$

(A)

$$K_2MnO_4 \xrightarrow{\text{Neutral/acidic solution}} KMnO_4 + MnO_2$$

Mn^{+4} :- $[Ar]3d^3$

$n = 3, \mu = \sqrt{3(3+2)} = 3.87$ B.M.

Nearest integer is (4)

87. Consider the following transformation involving first order elementary reaction in each step at constant temperature as shown below.

$$A + B \underset{\text{Step 3}}{\overset{\text{Step 1}}{\rightleftharpoons}} C \xrightarrow{\text{Step 2}} P$$

Some details of the above reaction are listed below.

Step	Rate constant (sec^{-1})	Activation energy ($kJ\ mol^{-1}$)
1	k_1	300
2	k_2	200
3	k_3	Ea_3

If the overall rate constant of the above transformation (k) is given as $k = \frac{k_1 k_2}{k_3}$ and the overall activation energy (E_a) is 400 kJ mol^{-1}, then the value of Ea_3 is _____ kJ mol^{-1} (nearest integer)

Ans. **(100)**

Sol. $K = \frac{K_1 K_2}{K_3}$

$$Ae^{\frac{-E_a}{RT}} = \frac{A_1 e^{\frac{-E_{a_1}}{RT}} A_2 e^{\frac{-E_{a_2}}{RT}}}{A_3 e^{\frac{-E_{a_3}}{RT}}}$$

$$Ae^{\frac{-E_a}{RT}} = \frac{A_1 A_2}{A_3} e^{\frac{-(E_{a_1}+E_{a_2}-E_{a_3})}{RT}}$$

$$E_a = E_{a_1} + E_{a_2} - E_{a_3}$$

$400 = 300 + 200 - E_{a_3}$

$E_{a_3} = 100$ kJ/mole

88. 2.5 g of a non-volatile, non-electrolyte is dissolved in 100 g of water at 25°C. The solution showed a boiling point elevation by 2°C. Assuming the solute concentration in negligible with respect to the solvent concentration, the vapour pressure of the resulting aqueous solution is _____ mm of Hg (nearest integer)

[Given : Molal boiling point elevation constant of water (K_b) = 0.52 K. kg mol^{-1},

1 atm pressure = 760 mm of Hg, molar mass of water = 18 g mol^{-1}]

Ans. **(707)**

Sol. $2 = 0.52 \times m$

$m = \frac{2}{0.52}$

According to question, solution is much diluted

so $\frac{\Delta P}{P^o} = \frac{n_{solute}}{n_{solvent}}$

$$\frac{\Delta P}{P^o} = \frac{m}{1000} \times M_{solvent}$$

$$\Delta P = P^o \times \frac{m}{1000} \times M_{solvent}$$

$$= 760 \times \frac{\frac{2}{0.52}}{1000} \times 18 = 52.615$$

$P_5 = 760 - 52.615 = 707.385$ mm of Hg

89. The number of different chain isomers for C_7H_{16} is _____.

Ans. **(9)**

Sol.

90. Number of molecules/species from the following having one unpaired electron is _____.

$O_2, O_2^{-1}, NO, CN^{-1}, O_2^{2-}$

Ans. **(2)**

Sol. According to M.O.T.

$O_2 \rightarrow$ no. of unpaired electrons = 2

$O_2^- \rightarrow$ no. of unpaired electron = 1

$NO \rightarrow$ no. of unpaired electron = 1

$CN^- \rightarrow$ no. of unpaired electron = 0

$O_2^{2-} \rightarrow$ no. of unpaired electron = 0

FINAL JEE–MAIN EXAMINATION – JANUARY, 2024

(Held On Monday 29th January, 2024) **TIME : 3 : 00 PM to 6 : 00 PM**

MATHEMATICS — TEST PAPER WITH SOLUTION

SECTION-A

1. Let $A = \begin{bmatrix} 2 & 1 & 2 \\ 6 & 2 & 11 \\ 3 & 3 & 2 \end{bmatrix}$ and $P = \begin{bmatrix} 1 & 2 & 0 \\ 5 & 0 & 2 \\ 7 & 1 & 5 \end{bmatrix}$. The sum of the prime factors of $\left|P^{-1}AP - 2I\right|$ is equal to

(1) 26 (2) 27 (3) 66 (4) 23

Ans. **(1)**

Sol. $\left|P^{-1}AP - 2I\right| = \left|P^{-1}AP - 2P^{-1}P\right|$

$= \left|P^{-1}(A - 2I)P\right|$

$= \left|P^{-1}\right|\left|A - 2I\right|\left|P\right|$

$= |A-2I|$

$= \begin{vmatrix} 0 & 1 & 2 \\ 6 & 0 & 11 \\ 3 & 3 & 0 \end{vmatrix} = 69$

So, Prime factor of 69 is 3 & 23

So, sum = 26

2. Number of ways of arranging 8 identical books into 4 identical shelves where any number of shelves may remain empty is equal to

(1) 18 (2) 16 (3) 12 (4) 15

Ans. **(4)**

Sol. 3 Shelf empty : (8, 0, 0, 0) → 1way

2 shelf empty : $\left.\begin{matrix} (7,1,0,0) \\ (6,2,0,0) \\ (5,3,0,0) \\ (4,4,0,0) \end{matrix}\right] \rightarrow 4\text{ways}$

1 shelf empty : $\left.\begin{matrix} (6,1,1,0) & (3,3,2,0) \\ (5,2,1,0) & (4,2,2,0) \\ (4,3,1,0) & \end{matrix}\right] \rightarrow 5\text{ways}$

0 Shelf empty : $\left.\begin{matrix} (1,2,3,2) & (5,1,1,1) \\ (2,2,2,2) & \\ (3,3,1,1) & \\ (4,2,1,1) & \end{matrix}\right] \rightarrow 5\text{ways}$

Total = 15 ways

3. Let P(3, 2, 3), Q (4, 6, 2) and R (7, 3, 2) be the vertices of ΔPQR. Then, the angle ∠QPR is

(1) $\frac{\pi}{6}$ (2) $\cos^{-1}\left(\frac{7}{18}\right)$

(3) $\cos^{-1}\left(\frac{1}{18}\right)$ (4) $\frac{\pi}{3}$

Ans. **(4)**

Sol.

Direction ratio of PR = (4, 1, -1)

Direction ratio of PQ = (1, 4, -1)

Now, $\cos\theta = \left|\frac{4+4+1}{\sqrt{18}\cdot\sqrt{18}}\right|$

$\theta = \frac{\pi}{3}$

4. If the mean and variance of five observations are $\frac{24}{5}$ and $\frac{194}{25}$ respectively and the mean of first four observations is $\frac{7}{2}$, then the variance of the first four observations in equal to

(1) $\frac{4}{5}$ (2) $\frac{77}{12}$ (3) $\frac{5}{4}$ (4) $\frac{105}{4}$

Ans. **(3)**

Sol. $\bar{X} = \frac{24}{5}; \sigma^2 = \frac{194}{25}$

Let first four observation be x_1, x_2, x_3, x_4

Here, $\frac{x_1 + x_2 + x_3 + x_4 + x_5}{5} = \frac{24}{5}$.....(1)

Also, $\frac{x_1 + x_2 + x_3 + x_4}{4} = \frac{7}{2}$

$\Rightarrow \boxed{x_1 + x_2 + x_3 + x_4 = 14}$

Now from eqn -1

$x_5 = 10$

Now, $\sigma^2 = \frac{194}{25}$

$$\frac{x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2}{5} - \frac{576}{25} = \frac{194}{25}$$

$$\Rightarrow x_1^2 + x_2^2 + x_3^2 + x_4^2 = 54$$

Now, variance of first 4 observations

$$\text{Var} = \frac{\sum_{i=1}^{4} x_i^2}{4} - \left(\frac{\sum_{i=1}^{4} x_i}{4}\right)^2$$

$$= \frac{54}{4} - \frac{49}{4} = \frac{5}{4}$$

5. The function $f(x) = 2x + 3(x)^{\frac{2}{3}}, x \in \mathbb{R}$, has

(1) exactly one point of local minima and no point of local maxima

(2) exactly one point of local maxima and no point of local minima

(3) exactly one point of local maxima and exactly one point of local minima

(4) exactly two points of local maxima and exactly one point of local minima

Ans. (3)

Sol. $f(x) = 2x + 3(x)^{\frac{2}{3}}$

$$f'(x) = 2 + 2x^{\frac{-1}{3}}$$

$$= 2\left(1 + \frac{1}{x^{\frac{1}{3}}}\right)$$

$$= 2\left(\frac{x^{\frac{1}{3}} + 1}{x^{\frac{1}{3}}}\right)$$

So, maxima (M) at x = -1 & minima(m) at x = 0

6. Let r and θ respectively be the modulus and amplitude of the complex number $z = 2 - i\left(2\tan\frac{5\pi}{8}\right)$, then (r, θ) is equal to

(1) $\left(2\sec\frac{3\pi}{8}, \frac{3\pi}{8}\right)$

(2) $\left(2\sec\frac{3\pi}{8}, \frac{5\pi}{8}\right)$

(3) $\left(2\sec\frac{5\pi}{8}, \frac{3\pi}{8}\right)$

(4) $\left(2\sec\frac{11\pi}{8}, \frac{11\pi}{8}\right)$

Ans. (1)

Sol. $z = 2 - i\left(2\tan\frac{5\pi}{8}\right) = x + iy$ (let)

$$r = \sqrt{x^2 + y^2} \quad \& \quad \theta = \tan^{-1}\frac{y}{x}$$

$$r = \sqrt{(2)^2 + \left(2\tan\frac{5\pi}{8}\right)^2}$$

$$= \left|2\sec\frac{5\pi}{8}\right| = \left|2\sec\left(\pi - \frac{3\pi}{8}\right)\right|$$

$$= 2\sec\frac{3\pi}{8}$$

$$\& \quad \theta = \tan^{-1}\left(\frac{-2\tan\frac{5\pi}{8}}{2}\right)$$

$$= \tan^{-1}\left(\tan\left(\pi - \frac{5\pi}{8}\right)\right)$$

$$= \frac{3\pi}{8}$$

7. The sum of the solutions $x \in \mathbb{R}$ of the equation $\frac{3\cos 2x + \cos^3 2x}{\cos^6 x - \sin^6 x} = x^3 - x^2 + 6$ is

(1) 0 (2) 1

(3) –1 (4) 3

Ans. (3)

Sol. $\dfrac{3\cos 2x+\cos^3 2x}{\cos^6 x-\sin^6 x}=x^3-x^2+6$

$\Rightarrow \dfrac{\cos 2x\,(3+\cos^2 2x)}{\cos 2x\,(1-\sin^2 x\cos^2 x)}=x^3-x^2+6$

$\Rightarrow \dfrac{4(3+\cos^2 2x)}{(4-\sin^2 2x)}=x^3-x^2+6$

$\Rightarrow \dfrac{4(3+\cos^2 2x)}{(3+\cos^2 2x)}=x^3-x^2+6$

$x^3-x^2+2=0 \Rightarrow (x+1)(x^2-2x+2)=0$

so, sum of real solutions = –1

8. Let $\overrightarrow{OA}=\vec{a}, \overrightarrow{OB}=12\vec{a}+4\vec{b}$ and $\overrightarrow{OC}=\vec{b}$, where O is the origin. If S is the parallelogram with adjacent sides OA and OC, then

$\dfrac{\text{area of the quadrilateral OABC}}{\text{area of S}}$ is equal to ___

(1) 6 (2) 10

(3) 7 (4) 8

Ans. (4)

Sol.

Area of parallelogram, $S=|\vec{a}\times\vec{b}|$

Area of quadrilateral =Area(ΔOAB)+Area (ΔOBC)

$=\dfrac{1}{2}\left\{|\vec{a}\times(12\vec{a}+4\vec{b})|+|\vec{b}\times(12\vec{a}+4\vec{b})|\right\}$

$=8|(\vec{a}\times\vec{b})|$

Ratio $=\dfrac{8|(\vec{a}\times\vec{b})|}{|(\vec{a}\times\vec{b})|}=8$

9. If $\log_e a$, $\log_e b$, $\log_e c$ are in an A.P. and $\log_e a-\log_e 2b$, $\log_e 2b-\log_e 3c$, $\log_e 3c-\log_e a$ are also in an A.P, then a : b : c is equal to

(1) 9 : 6 : 4 (2) 16 : 4 : 1

(3) 25 : 10 : 4 (4) 6 : 3 : 2

Ans. (1)

Sol. $\log_e a$, $\log_e b$, $\log_e c$ are in A.P.

$\therefore b^2=ac$ .....(i)

Also

$\log_e\left(\dfrac{a}{2b}\right), \log_e\left(\dfrac{2b}{3c}\right), \log_e\left(\dfrac{3c}{a}\right)$ are in A.P.

$\left(\dfrac{2b}{3c}\right)^2=\dfrac{a}{2b}\times\dfrac{3c}{a}$

$\dfrac{b}{c}=\dfrac{3}{2}$

Putting in eq. (i) $b^2=a\times\dfrac{2b}{3}$

$\dfrac{a}{b}=\dfrac{3}{2}$

a : b : c = 9 : 6 : 4

10. If

$\int\dfrac{\sin^{\frac{3}{2}}x+\cos^{\frac{3}{2}}x}{\sqrt{\sin^3 x\cos^3 x\,\sin(x-\theta)}}dx=A\sqrt{\cos\theta\tan x-\sin\theta}+B\sqrt{\cos\theta-\sin\theta\cot x}+C,$

where C is the integration constant, then AB is equal to

(1) $4\operatorname{cosec}(2\theta)$ (2) $4\sec\theta$

(3) $2\sec\theta$ (4) $8\operatorname{cosec}(2\theta)$

Ans. (4)

Sol. $\int\dfrac{\sin^{\frac{3}{2}}x+\cos^{\frac{3}{2}}x}{\sqrt{\sin^3 x\cos^3 x\,\sin(x-\theta)}}dx$

$I=\int\dfrac{\sin^{\frac{3}{2}}x+\cos^{\frac{3}{2}}x}{\sqrt{\sin^3 x\cos^3 x\,(\sin x\cos\theta-\cos x\sin\theta)}}dx$

$=\int\dfrac{\sin^{\frac{3}{2}}x}{\sin^{\frac{3}{2}}x\cos^2 x\sqrt{\tan x\cos\theta-\sin\theta}}dx+\int\dfrac{\cos^{\frac{3}{2}}x}{\sin^2 x\cos^{\frac{3}{2}}x\sqrt{\cos\theta-\cot x\sin\theta}}dx=$

$\int\dfrac{\sec^2 x}{\sqrt{\tan x\cos\theta-\sin\theta}}dx+\int\dfrac{\operatorname{cosec}^2 x}{\sqrt{\cos\theta-\cot x\sin\theta}}dx$

$I=I_1+I_2$ ..... {Let}

For I_1, let $\tan x\cos\theta-\sin\theta=t^2$

$\sec^2 x\,dx=\dfrac{2t\,dt}{\cos\theta}$

For I_2, let $\cos\theta-\cot x\sin\theta=z^2$

$\operatorname{cosec}^2 x\,dx=\dfrac{2z\,dz}{\sin\theta}$

$I = I_1 + I_2$

$$= \int \frac{2t\ dt}{\cos\theta\ t} + \int \frac{2z\ dz}{\sin\theta\ z}$$

$$= \frac{2t}{\cos\theta} + \frac{2z}{\sin\theta}$$

$$= 2\sec\theta\sqrt{\tan x\cos\theta - \sin\theta} + 2\operatorname{cosec}\theta\sqrt{\cos\theta - \cot x\sin\theta}$$

Comparing

$AB = 8\ \operatorname{cosec}2\theta$

11. The distance of the point (2, 3) from the line $2x - 3y + 28 = 0$, measured parallel to the line $\sqrt{3}x - y + 1 = 0$, is equal to

(1) $4\sqrt{2}$ (2) $6\sqrt{3}$

(3) $3 + 4\sqrt{2}$ (4) $4 + 6\sqrt{3}$

Ans. (4)

Sol.

Writing P in terms of parametric co-ordinates $2 + r\cos\theta$, $3 + r\sin\theta$ as $\tan\theta = \sqrt{3}$

$$P(2 + \frac{r}{2}, 3 + \frac{\sqrt{3}r}{2})$$

P must satisfy $2x - 3y + 28 = 0$

So, $2(2 + \frac{r}{2}) - 3(3 + \frac{\sqrt{3}r}{2}) + 28 = 0$

We find $r = 4 + 6\sqrt{3}$

12. If $\sin\left(\frac{y}{x}\right) = \log_e|x| + \frac{\alpha}{2}$ is the solution of the differential equation $x\cos\left(\frac{y}{x}\right)\frac{dy}{dx} = y\cos\left(\frac{y}{x}\right) + x$ and $y(1) = \frac{\pi}{3}$, then α^2 is equal to

(1) 3 (2) 12

(3) 4 (4) 9

Ans. (1)

Sol. Differential equation :–

$$x\cos\frac{y}{x}\frac{dy}{dx} = y\cos\frac{y}{x} + x$$

$$\cos\frac{y}{x}\left[x\frac{dy}{dx} - y\right] = x$$

Divide both sides by x^2

$$\cos\frac{y}{x}\left(\frac{x\frac{dy}{dx} - y}{x^2}\right) = \frac{1}{x}$$

Let $\frac{y}{x} = t$

$$\cos t\left(\frac{dt}{dx}\right) = \frac{1}{x}$$

$$\cos t\ dt = \frac{1}{x}dx$$

Integrating both sides

$\sin t = \ln|x| + c$

$$\sin\frac{y}{x} = \ln|x| + c$$

Using $y(1) = \frac{\pi}{3}$, we get $c = \frac{\sqrt{3}}{2}$

So, $\alpha = \sqrt{3} \Rightarrow \alpha^2 = 3$

13. If each term of a geometric progression $a_1, a_2, a_3, \ldots$ with $a_1 = \frac{1}{8}$ and $a_2 \neq a_1$, is the arithmetic mean of the next two terms and $S_n = a_1 + a_2 + \ldots + a_n$, then $S_{20} - S_{18}$ is equal to

(1) 2^{15} (2) -2^{18}

(3) 2^{18} (4) -2^{15}

Ans. (4)

Sol. Let r'th term of the GP be ar^{n-1}. Given,

$2a_r = a_{r+1} + a_{r+2}$

$2ar^{n-1} = ar^n + ar^{n+1}$

$$\frac{2}{r} = 1 + r$$

$r^2 + r - 2 = 0$

Hence, we get, $r = -2$ (as $r \neq 1$)

So, $S_{20} - S_{18}$ = (Sum upto 20 terms) – (Sum upto 18 terms) = $T_{19} + T_{20}$

$T_{19} + T_{20} = ar^{18}(1 + r)$

Putting the values $a = \frac{1}{8}$ and $r = -2$;

we get $T_{19} + T_{20} = -2^{15}$

14. Let A be the point of intersection of the lines $3x + 2y = 14$, $5x - y = 6$ and B be the point of intersection of the lines $4x + 3y = 8$, $6x + y = 5$. The distance of the point P(5, –2) from the line AB is

(1) $\frac{13}{2}$ (2) 8 (3) $\frac{5}{2}$ (4) 6

Ans. **(4)**

Sol. Solving lines L_1 ($3x + 2y = 14$) and L_2 ($5x - y = 6$) to get A(2, 4) and solving lines L_3 ($4x + 3y = 8$) and L_4 ($6x + y = 5$) to get $B\left(\frac{1}{2}, 2\right)$.

Finding Eqn. of AB : $4x - 3y + 4 = 0$

Calculate distance PM

$$\Rightarrow \left|\frac{4(5)-3(-2)+4}{5}\right| = 6$$

15. Let $x = \frac{m}{n}$ (m, n are co-prime natural numbers) be a solution of the equation $\cos\left(2\sin^{-1}x\right) = \frac{1}{9}$ and let $\alpha, \beta (\alpha > \beta)$ be the roots of the equation $mx^2 - nx - m + n = 0$. Then the point (α, β) lies on the line

(1) $3x + 2y = 2$ (2) $5x - 8y = -9$

(3) $3x - 2y = -2$ (4) $5x + 8y = 9$

Ans. **(4)**

Sol. Assume $\sin^{-1}x = \theta$

$$\cos(2\theta) = \frac{1}{9}$$

$$\sin\theta = \pm\frac{2}{3}$$

as m and n are co-prime natural numbers,

$$x = \frac{2}{3}$$

i.e. m = 2, n = 3

So, the quadratic equation becomes $2x^2 - 3x + 1 = 0$ whose roots are $\alpha = 1, \beta = \frac{1}{2}$

$\left(1, \frac{1}{2}\right)$ lies on $5x + 8y = 9$

16. The function $f(x) = \frac{x}{x^2 - 6x - 16}, x \in \mathbb{R} - \{-2, 8\}$

(1) decreases in (–2, 8) and increases in $(-\infty, -2) \cup (8, \infty)$

(2) decreases in $(-\infty, -2) \cup (-2, 8) \cup (8, \infty)$

(3) decreases in $(-\infty, -2)$ and increases in $(8, \infty)$

(4) increases in $(-\infty, -2) \cup (-2, 8) \cup (8, \infty)$

Ans. **(2)**

Sol. $f(x) = \frac{x}{x^2 - 6x - 16}$

Now,

$$f'(x) = \frac{-(x^2 + 16)}{(x^2 - 6x - 16)^2}$$

$$f'(x) < 0$$

Thus f(x) is decreasing in

$(-\infty, -2) \cup (-2, 8) \cup (8, \infty)$

17. Let $y = \log_e\left(\frac{1 - x^2}{1 + x^2}\right)$, $-1 < x < 1$. Then at $x = \frac{1}{2}$, the value of $225(y' - y'')$ is equal to

(1) 732 (2) 746

(3) 742 (4) 736

Ans. **(4)**

Sol. $y = \log_e\left(\frac{1 - x^2}{1 + x^2}\right)$

$$\frac{dy}{dx} = y' = \frac{-4x}{1 - x^4}$$

Again,

$$\frac{d^2y}{dx^2} = y'' = \frac{-4(1 + 3x^4)}{(1 - x^4)^2}$$

Again

$$y' - y'' = \frac{-4x}{1 - x^4} + \frac{4(1 + 3x^4)}{(1 - x^4)^2}$$

at $x = \frac{1}{2}$,

$y' - y'' = \frac{736}{225}$

Thus $225(y' - y'') = 225 \times \frac{736}{225} = 736$

18. If R is the smallest equivalence relation on the set {1, 2, 3, 4} such that {(1,2), (1,3)} $\subset$ R, then the number of elements in R is _____

(1) 10 (2) 12

(3) 8 (4) 15

Ans. **(1)**

Sol. Given set {1, 2, 3, 4}

Minimum order pairs are

(1, 1), (2, 2), (3, 3), (4, 4), (3, 1),(2, 1), (2, 3),(3,2), (1, 3), (1, 2)

Thus no. of elements = 10

19. An integer is chosen at random from the integers 1, 2, 3, …, 50. The probability that the chosen integer is a multiple of atleast one of 4, 6 and 7 is

(1) $\frac{8}{25}$ (2) $\frac{21}{50}$

(3) $\frac{9}{50}$ (4) $\frac{14}{25}$

Ans. **(2)**

Sol. Given set = {1, 2, 3, …….. 50}

P(A) = Probability that number is multiple of 4

P(B) = Probability that number is multiple of 6

P(C) = Probability that number is multiple of 7

Now,

$P(A) = \frac{12}{50},\ P(B) = \frac{8}{50},\ P(C) = \frac{7}{50}$

again

$P(A \cap B) = \frac{4}{50}, P(B \cap C) = \frac{1}{50}, P(A \cap C) = \frac{1}{50}$

$P(A \cap B \cap C) = 0$

Thus

$P(A \cup B \cup C) = \frac{12}{50} + \frac{8}{50} + \frac{7}{50} - \frac{4}{50} - \frac{1}{50} - \frac{1}{50} + 0$

$= \frac{21}{50}$

20. Let a unit vector $\hat{u} = x\hat{i} + y\hat{j} + z\hat{k}$ make angles $\frac{\pi}{2}, \frac{\pi}{3}$ and $\frac{2\pi}{3}$ with the vectors $\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{k}$, $\frac{1}{\sqrt{2}}\hat{j} + \frac{1}{\sqrt{2}}\hat{k}$ and $\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j}$ respectively. If $\vec{v} = \frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j} + \frac{1}{\sqrt{2}}\hat{k}$, then $|\hat{u} - \vec{v}|^2$ is equal to

(1) $\frac{11}{2}$ (2) $\frac{5}{2}$

(3) 9 (4) 7

Ans. **(2)**

Sol. Unit vector $\hat{u} = x\hat{i} + y\hat{j} + z\hat{k}$

$\vec{p}_1 = \frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{k}, \vec{p}_2 = \frac{1}{\sqrt{2}}\hat{j} + \frac{1}{\sqrt{2}}\hat{k}$

$\vec{p}_3 = \frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j}$

Now angle between $\hat{u}$ and $\vec{p}_1 = \frac{\pi}{2}$

$\hat{u} \cdot \vec{p}_1 = 0 \Rightarrow \frac{x}{\sqrt{2}} + \frac{z}{\sqrt{2}} = 0$

$\Rightarrow x + z = 0$ ….(i)

Angle between $\hat{u}$ and $\vec{p}_2 = \frac{\pi}{3}$

$\hat{u} \cdot \vec{p}_2 = |\hat{u}| \cdot |\vec{p}_2| \cos\frac{\pi}{3}$

$\Rightarrow \frac{y}{\sqrt{2}} + \frac{z}{\sqrt{2}} = \frac{1}{2} \Rightarrow y + z = \frac{1}{\sqrt{2}}$ ….(ii)

Angle between $\hat{u}$ and $\vec{p}_3 = \frac{2\pi}{3}$

$\hat{u} \cdot \vec{p}_3 = |\hat{u}| \cdot |\vec{p}_3| \cos\frac{2\pi}{3}$

$\Rightarrow \frac{x}{\sqrt{2}} + \frac{4}{\sqrt{2}} = \frac{-1}{2} \Rightarrow x + y = \frac{-1}{\sqrt{2}}$ …. (iii)

from equation (i), (ii) and (iii) we get

$x = \frac{-1}{\sqrt{2}}$ $y = 0$ $z = \frac{1}{\sqrt{2}}$

Thus $\hat{u}-\vec{v}=\frac{-1}{\sqrt{2}}\hat{i}+\frac{1}{\sqrt{2}}\hat{k}-\frac{1}{\sqrt{2}}\hat{i}-\frac{1}{\sqrt{2}}\hat{j}-\frac{1}{\sqrt{2}}\hat{k}$

$\hat{u}-\vec{v}=\frac{-2}{\sqrt{2}}\hat{i}-\frac{1}{\sqrt{2}}\hat{j}$

$\therefore |\hat{u}-\vec{v}|^2=\left(\sqrt{\frac{4}{2}+\frac{1}{2}}\right)^2=\frac{5}{2}$

SECTION-B

21. Let α, β be the roots of the equation $x^2-\sqrt{6}x+3=0$ such that Im (α) > Im (β). Let a, b be integers not divisible by 3 and n be a natural number such that $\frac{\alpha^{99}}{\beta}+\alpha^{98}=3^n(a+ib), i=\sqrt{-1}$. Then n + a + b is equal to _____.

Ans. **49**

Sol. $x^2-\sqrt{6}x+6=0 \begin{smallmatrix}\nearrow\alpha\\ \searrow\beta\end{smallmatrix}$

$x=\frac{\sqrt{6}\pm i\sqrt{6}}{2}=\frac{\sqrt{6}}{2}(1\pm i)$

$\alpha=\sqrt{3}(e^{i\frac{\pi}{4}}),\ \beta=\sqrt{3}(e^{-i\frac{\pi}{4}})$

$\therefore \frac{\alpha^{99}}{\beta}+\alpha^{98}=\alpha^{98}\left(\frac{\alpha}{\beta}+1\right)$

$=\frac{\alpha^{98}(\alpha+\beta)}{\beta}=3^{49}\left(e^{i99\frac{\pi}{4}}\right)\times\sqrt{2}$

$=3^{49}(-1+i)$

$=3^n(a+ib)$

$\therefore$ n = 49, a = -1, b = 1

$\therefore$ n + a + b = 49 – 1 + 1 = 49

22. Let for any three distinct consecutive terms a, b, c of an A.P, the lines ax + by + c = 0 be concurrent at the point P and Q (α,β) be a point such that the system of equations

x + y + z = 6,

2x + 5y + αz = β and

x + 2y + 3z = 4, has infinitely many solutions. Then $(PQ)^2$ is equal to _____.

Ans. **113**

Sol. $\because$ a, b, c and in A.P

$\Rightarrow 2b = a + c \Rightarrow a - 2b + c = 0$

$\therefore$ ax + by + c passes through fixed point (1, -2)

$\therefore$ P = (1, - 2)

For infinite solution,

$D = D_1 = D_2 = D_3 = 0$

$D:\begin{vmatrix}1&1&1\\2&5&\alpha\\1&2&3\end{vmatrix}=0$

$\Rightarrow \alpha = 8$

$D_1:\begin{vmatrix}6&1&1\\\beta&5&\alpha\\4&2&3\end{vmatrix}=0 \Rightarrow \beta = 6$

$\therefore$ Q = (8,6)

$\therefore PQ^2 = 113$

23. Let P(α,β) be a point on the parabola $y^2 = 4x$. If P also lies on the chord of the parabola $x^2 = 8y$ whose mid point is $\left(1,\frac{5}{4}\right)$. Then (α-28) (β-8) is equal to _____.

Ans. **192**

Sol. Parabola is $x^2 = 8y$

Chord with mid point (x_1,y_1) is $T = S_1$

$\therefore xx_1 - 4(y+y_1) = x_1^2-8y_1$

$\therefore (x_1, y_1) = \left(1,\frac{5}{4}\right)$

$\Rightarrow x-4\left(y+\frac{5}{4}\right)=1-8\times\frac{5}{4}=-9$

$\therefore$ x-4y + 4 = 0 ..... (i)

(α,β) lies on (i) & also on $y^2 = 4x$

$\therefore$ α - 4β + 4 = 0 ..... (ii)

& $\beta^2 = 4\alpha$ ..... (iii)

Solving (ii) & (iii)

$\beta^2 = 4(4\beta-4) \Rightarrow \beta^2-16\beta + 16 = 0$

$\therefore\ \beta = 8 \pm 4\sqrt{3}$ and $\alpha = 4\beta - 4 = 28 \pm 16\sqrt{3}$

$\therefore (\alpha, \beta) = (28+16\sqrt{3}, 8+4\sqrt{3})$ & $\left(28-16\sqrt{3}, 8-4\sqrt{3}\right)$

$\therefore (\alpha-28)(\beta-8) = \left(\pm 16\sqrt{3}\right)\left(\pm 4\sqrt{3}\right)$

$= 192$

24. If $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sqrt{1-\sin 2x}\,dx = \alpha + \beta\sqrt{2} + \gamma\sqrt{3}$, where α, β and γ are rational numbers, then $3\alpha+4\beta-\gamma$ is equal to _____.

Ans. **6**

Sol. $= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sqrt{1-\sin 2x}\ dx$

$= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} |\sin x - \cos x|\,dx$

$= \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} (\cos x - \sin x)\,dx + \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} (\sin x - \cos x)\,dx$

$= -1 + 2\sqrt{2} - \sqrt{3}$

$= \alpha + \beta\sqrt{2} + \gamma\sqrt{3}$

$\alpha = -1, \beta = 2, \gamma = -1$

$3\alpha + 4\beta - \gamma = 6$

25. Let the area of the region $\{(x, y): 0 \le x \le 3, 0 \le y \le \min\{x^2 + 2, 2x + 2\}\}$ be A. Then 12A is equal to _____.

Ans. **164**

Sol.

$A = \int_0^2 (x^2+2)dx + \int_2^3 (2x+2)dx$

$A = \frac{41}{3}$

$12A = 41 \times 4 = 164$

26. Let O be the origin, and M and N be the points on the lines $\frac{x-5}{4} = \frac{y-4}{1} = \frac{z-5}{3}$ and $\frac{x+8}{12} = \frac{y+2}{5} = \frac{z+11}{9}$ respectively such that MN is the shortest distance between the given lines. Then $\overrightarrow{OM}\cdot\overrightarrow{ON}$ is equal to _____.

Ans. **9**

Sol. $L_1 : \frac{x-5}{4} = \frac{y-4}{1} = \frac{z-5}{3} = \lambda$ drs $(4,1,3) = b_1$

M $(4\lambda+5, \lambda + 4, 3\lambda + 5)$

$L_2 : \frac{x+8}{12} = \frac{y+2}{5} = \frac{z+11}{9} = \mu$

$N(12\mu - 8, 5\mu - 2, 9\mu - 11)$

$\overrightarrow{MN} = (4\lambda - 12\mu + 13, \lambda - 5\mu + 6, 3\lambda - 9\mu + 16)$ ..(1)

Now

$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 1 & 3 \\ 12 & 5 & 9 \end{vmatrix} = -6\hat{i} + 8\hat{k}$ …(2)

Equation (1) and (2)

$\therefore\ \frac{4\lambda - 12\mu + 13}{-6} = \frac{\lambda - 5\mu + 6}{0} = \frac{3\lambda - 9\mu + 16}{8}$

I and II

$\lambda - 5\mu + 6 = 0$ ….(3)

I and III

$\lambda - 3\mu + 4 = 0$ ….(4)

Solve (3) and (4) we get

$\lambda = -1, \mu = 1$

$\therefore$ M (1, 3, 2)

N (4, 3, –2)

$\therefore$ $\overrightarrow{OM}\cdot\overrightarrow{ON} = 4 + 9 - 4 = 9$

27. Let $f(x) = \sqrt{\lim_{r\to x}\left\{\frac{2r^2\left[(f(r))^2 - f(x)f(r)\right]}{r^2 - x^2} - r^3 e^{\frac{f(r)}{r}}\right\}}$

be differentiable in $(-\infty,0)\cup(0,\infty)$ and f(1) = 1. Then the value of ea, such that f(a) = 0, is equal to _____.

Ans. **2**

Sol. f(1)=1, f(a) = 0

$$f^2(x) = \lim_{r\to x}\left(\frac{2r^2(f^2(r) - f(x)f(r))}{r^2 - x^2} - r^3 e^{\frac{f(r)}{r}}\right)$$

$$= \lim_{r\to x}\left(\frac{2r^2 f(r)}{r+x}\frac{(f(r) - f(x))}{r - x} - r^3 e^{\frac{f(r)}{r}}\right)$$

$$f^2(x) = \frac{2x^2 f(x)}{2x} f'(x) - x^3 e^{\frac{f(x)}{x}}$$

$$y^2 = xy\frac{dy}{dx} - x^3 e^{\frac{y}{x}}$$

$$\frac{y}{x} = \frac{dy}{dx} - \frac{x^2}{y} e^{\frac{y}{x}}$$

Put $\boxed{y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}}$

$$v = v + x\frac{dv}{dx} - \frac{x}{v}e^v$$

$$\frac{dv}{dx} = \frac{e^v}{v} \Rightarrow e^{-v}\, v dv = dx$$

Integrating both side

$e^v (x + c) + 1 + v = 0$

$f(1) = 1 \Rightarrow x = 1, y = 1$

$$\Rightarrow c = -1 - \frac{2}{e}$$

$$e^v\left(-1 - \frac{2}{e} + x\right) + 1 + v = 0$$

$$e^{\frac{y}{x}}\left(-1 - \frac{2}{e} + x\right) + 1 + \frac{y}{x} = 0$$

$x = a, y = 0 \Rightarrow a = \frac{2}{e}$

ae = 2

28. Remainder when $64^{32^{32}}$ is divided by 9 is equal to _____.

Ans. **1**

Sol. Let $32^{32} = t$

$64^{32^{32}} = 64^t = 8^{2t} = (9 - 1)^{2t}$

$= 9k + 1$

Hence remainder = 1

29. Let the set $C = \{(x, y) \mid x^2 - 2^y = 2023, x, y \in \mathbb{N}\}$. Then $\sum_{(x,y)\in C}(x + y)$ is equal to _____.

Ans. **46**

Sol. $x^2 - 2^y = 2023$

$\Rightarrow \boxed{x = 45, y = 1}$

$$\sum_{(x,y)\in C}(x + y) = 46.$$

30. Let the slope of the line 45x + 5y + 3 = 0 be $27r_1 + \frac{9r_2}{2}$ for some r_1, $r_2 \in R$. Then

$\lim_{x\to 3}\left(\int_3^x \frac{8t^2}{\frac{3r_2 x}{2} - r_2 x^2 - r_1 x^3 - 3x} dt\right)$ is equal to ____.

Ans. **12**

Sol. According to the question ,

$$27r_1 + \frac{9r_2}{2} = -9$$

$$\lim_{x\to 3} \frac{\int_3^x 8t^2 dt}{\frac{3r_2 x}{2} - r_2 x^2 - r_1 x^3 - 3x}$$

$$= \lim_{x\to 3} \frac{8x^2}{\frac{3r_2^2}{2} - 2r_2 x - 3r_1 x^2 - 3} \text{ (using LH' Rule)}$$

$$= \frac{72}{\frac{3r_2}{2} - 6r_2 - 27r_1 - 3}$$

$$= \frac{72}{-\frac{9r_2}{2} - 27r_1 - 3}$$

$$= \frac{72}{9-3} = 12$$

SECTION-A

31. Two sources of light emit with a power of 200 W. The ratio of number of photons of visible light emitted by each source having wavelengths 300 nm and 500 nm respectively, will be :

(1) 1 : 5 (2) 1 : 3
(3) 5 : 3 (4) 3 : 5

Ans. (4)

Sol. $n_1 \times \frac{hc}{\lambda_1} = 200$

$n_2 \times \frac{hc}{\lambda_2} = 200$

$\frac{n_1}{n_2} = \frac{\lambda_1}{\lambda_2} = \frac{300}{500}$

$\frac{n_1}{n_2} = \frac{3}{5}$

32. The truth table for this given circuit is :

(1)

A	B	Y
0	0	1
0	1	1
1	0	1
1	1	0

(2)

A	B	Y
0	0	0
0	1	1
1	0	0
1	1	1

(3)

A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

(4)

A	B	Y
0	0	1
0	1	0
1	0	1
1	1	0

Ans. (2)

Sol.

$Y = A.B + \overline{A}.B$

$= \left(A + \overline{A}\right).B$

$Y = 1.B$

$Y = B$

33. A physical quantity Q is found to depend on quantities a, b, c by the relation $Q = \frac{a^4b^3}{c^2}$. The percentage error in a, b and c are 3%, 4% and 5% respectively. Then, the percentage error in Q is :

(1) 66% (2) 43%
(3) 34% (4) 14%

Ans. (3)

Sol. $Q = \frac{a^4b^3}{c^2}$

$\frac{\Delta Q}{Q} = 4\frac{\Delta a}{a} + 3\frac{\Delta b}{b} + 2\frac{\Delta c}{c}$

$\frac{\Delta Q}{Q} \times 100 = 4\left(\frac{\Delta a}{a} \times 100\right) + 3\left(\frac{\Delta b}{b} \times 100\right) + 2\left(\frac{\Delta c}{c} \times 100\right)$

% error in Q = 4 × 3% + 3 × 4% + 2 × 5%

= 12% + 12% + 10%

= 34%

34. In an a.c. circuit, voltage and current are given by :
V = 100 sin (100 t) V and

I = 100 sin (100 t + $\frac{\pi}{3}$) mA respectively.

The average power dissipated in one cycle is :

(1) 5 W (2) 10 W
(3) 2.5 W (4) 25 W

Ans. (3)

Sol. $P_{avg} = V_{rms}\ I_{rms}\ \cos(\Delta\phi)$

$$= \frac{100}{\sqrt{2}} \times \frac{100 \times 10^{-3}}{\sqrt{2}} \times \cos\left(\frac{\pi}{3}\right)$$

$$= \frac{10^4}{2} \times \frac{1}{2} \times 10^{-3}$$

$$= \frac{10}{4} = 2.5\,W$$

35. The temperature of a gas having 2.0×10^{25} molecules per cubic meter at 1.38 atm (Given, k = 1.38×10^{-23} JK^{-1}) is :

(1) 500 K (2) 200 K

(3) 100 K (4) 300 K

Ans. **(1)**

Sol. PV = nRT

$$PV = \frac{N}{N_A} RT$$

N = Total no. of molecules

$$P = \frac{N}{V} kT$$

$1.38 \times 1.01 \times 10^5 = 2 \times 10^{25} \times 1.38 \times 10^{-23} \times T$

$1.01 \times 10^5 = 2 \times 10^2 \times T$

$$T = \frac{1.01 \times 10^3}{2} \approx 500\ K$$

36. A stone of mass 900g is tied to a string and moved in a vertical circle of radius 1m making 10 rpm. The tension in the string, when the stone is at the lowest point is (if $\pi^2 = 9.8$ and $g = 9.8\ m/s^2$)

(1) 97 N (2) 9.8 N

(3) 8.82 N (4) 17.8 N

Ans. **(2)**

Sol. Given that

$$m = 900\ gm = \frac{900}{1000} kg = \frac{9}{10} kg$$

r = 1m

$$\omega = \frac{2\pi N}{60} = \frac{2\pi(10)}{60} = \frac{\pi}{3} rad/sec$$

$T - mg = mr\omega^2$

$T = mg + mr\omega^2$

$$= \frac{9}{10} \times 9.8 + \frac{9}{10} \times 1\left(\frac{\pi}{3}\right)^2$$

$$= 8.82 + \frac{9}{10} \times \frac{\pi^2}{9}$$

= 8.82 + 0.98

= 9.80 N

37. The bob of a pendulum was released from a horizontal position. The length of the pendulum is 10m. If it dissipates 10% of its initial energy against air resistance, the speed with which the bob arrives at the lowest point is : [Use, g : 10 ms^{-2}]

(1) $6\sqrt{5}\ ms^{-1}$ (2) $5\sqrt{6}\ ms^{-1}$

(3) $5\sqrt{5}\ ms^{-1}$ (4) $2\sqrt{5}\ ms^{-1}$

Ans. **(1)**

Sol. $\ell = 10$ m,

Initial energy = $mg\ell$

So, $\frac{9}{10} mg\ell = \frac{1}{2} mv^2$

$$\Rightarrow \frac{9}{10} \times 10 \times 10 = \frac{1}{2} v^2$$

$v^2 = 180$

$$v = \sqrt{180} = 6\sqrt{5}\ m/s$$

38. If the distance between object and its two times magnified virtual image produced by a curved mirror is 15 cm, the focal length of the mirror must be :

(1) 15 cm (2) –12 cm

(3) –10 cm (4) 10/3 cm

Ans. **(3)**

Sol.

$m = 2 = \frac{-v}{u}$

$2 = \frac{-(15-u)}{-u}$

$2u = 15 - u$

$3u = 15 \Rightarrow u = 5$ cm

$v = 15 - u = 15 - 5 = 10$ cm

$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$

$= \frac{1}{10} + \frac{1}{(-5)} = \frac{1-2}{10} = \frac{-1}{10}$

$f = -10$ cm

39. Two particles X and Y having equal charges are being accelerated through the same potential difference. Thereafter they enter normally in a region of uniform magnetic field and describes circular paths of radii R_1 and R_2 respectively. The mass ratio of X and Y is :

(1) $\left(\frac{R_2}{R_1}\right)^2$ (2) $\left(\frac{R_1}{R_2}\right)^2$

(3) $\left(\frac{R_1}{R_2}\right)$ (4) $\left(\frac{R_2}{R_1}\right)$

Ans. (2)

Sol. $R = \frac{mv}{qB} = \frac{p}{qB} = \frac{\sqrt{2m(KE)}}{qB} = \frac{\sqrt{2mqV}}{qB}$

$R \propto \sqrt{m}$

$m \propto R^2$

$\frac{m_1}{m_2} = \left(\frac{R_1}{R_2}\right)^2$

40. In Young's double slit experiment, light from two identical sources are superimposing on a screen. The path difference between the two lights reaching at a point on the screen is $\frac{7\lambda}{4}$. The ratio of intensity of fringe at this point with respect to the maximum intensity of the fringe is :

(1) 1/2 (2) 3/4 (3) 1/3 (4) 1/4

Ans. (1)

Sol. $\Delta x = \frac{7\lambda}{4}$

$\phi = \frac{2\pi}{\lambda}\Delta x = \frac{2\pi}{\lambda} \times \frac{7\lambda}{4} = \frac{7\pi}{2}$

$I = I_{max}\cos^2\left(\frac{\phi}{2}\right)$

$\frac{I}{I_{max}} = \cos^2\left(\frac{\phi}{2}\right) = \cos^2\left(\frac{7\pi}{2\times 2}\right) = \cos^2\left(\frac{7\pi}{4}\right)$

$= \cos^2\left(2\pi - \frac{\pi}{4}\right)$

$= \cos^2\frac{\pi}{4}$

$= \frac{1}{2}$

41. A small liquid drop of radius R is divided into 27 identical liquid drops. If the surface tension is T, then the work done in the process will be :

(1) $8\pi R^2T$ (2) $3\pi R^2T$

(3) $\frac{1}{8}\pi R^2T$ (4) $4\pi R^2T$

Ans. (1)

Sol. Volume constant

$\frac{4}{3}\pi R^3 = 27 \times \frac{4}{3} \times \pi r^3$

$R^3 = 27r^3$

$R = 3r$

$r = \frac{R}{3}$

$r^2 = \frac{R^2}{9}$

Work done $= T.\Delta A$

$= 27\,T(4\pi r^2) - T\,4\pi R^2$

$= 27T4\pi\frac{R^2}{9} - 4\pi R^2T$

$= 8\pi R^2T$

42. A bob of mass 'm' is suspended by a light string of length 'L'. It is imparted a minimum horizontal velocity at the lowest point A such that it just completes half circle reaching the top most position B. The ratio of kinetic energies $\frac{(K.E.)_A}{(K.E.)_B}$ is :

(1) 3 : 2 (2) 5 : 1

(3) 2 : 5 (4) 1 : 5

Ans. (2)

Sol. Apply energy conservation between A & B

$$\frac{1}{2}mV_L^2 = \frac{1}{2}mV_H^2 + mg(2L)$$

$$\because V_L = \sqrt{5gL}$$

So, $V_H = \sqrt{gL}$

$$\frac{(K.E)_A}{(K.E)_B} = \frac{\frac{1}{2}m\left(\sqrt{5gL}\right)^2}{\frac{1}{2}m\left(\sqrt{gL}\right)^2} = \frac{5}{1}$$

43. A wire of length L and radius r is clamped at one end. If its other end is pulled by a force F, its length increases by *l*. If the radius of the wire and the applied force both are reduced to half of their original values keeping original length constant, the increase in length will become.

(1) 3 times (2) 3/2 times

(3) 4 times (4) 2 times

Ans. (4)

Sol. $Y = \frac{\text{stress}}{\text{strain}}$

$$Y = \frac{\frac{F}{\pi r^2}}{\frac{\ell}{L}}$$

$$F = Y\pi r^2 \times \frac{\ell}{L} \qquad \text{....(i)}$$

$$Y = \frac{\frac{F/2}{\pi r^2/4}}{\frac{\Delta\ell}{L}}$$

$$F = Y\frac{\Delta\ell}{L} \times 2 \times \frac{\pi r^2}{4}$$

From (i)

$$Y\pi r^2 \frac{\ell}{L} = Y\frac{\Delta\ell}{L}\frac{\pi r^2}{2}$$

$$\Delta\ell = 2\ell$$

44. A planet takes 200 days to complete one revolution around the Sun. If the distance of the planet from Sun is reduced to one fourth of the original distance, how many days will it take to complete one revolution ?

(1) 25 (2) 50

(3) 100 (4) 20

Ans. (1)

Sol. $T^2 \propto r^3$

$$\frac{T_1^2}{r_1^3} = \frac{T_2^2}{r_2^3}$$

$$\frac{(200)^2}{r^3} = \frac{T_2^2}{\left(\frac{r}{4}\right)^3}$$

$$\frac{200 \times 200}{4 \times 4 \times 4} = T_2^2$$

$$T_2 = \frac{200}{4 \times 2}$$

$T_2 = 25$ days

45. A plane electromagnetic wave of frequency 35 MHz travels in free space along the X-direction. At a particular point (in space and time) $\vec{E} = 9.6\hat{j}\,V/m$. The value of magnetic field at this point is :

(1) $3.2\times10^{-8}\hat{k}T$ (2) $3.2\times10^{-8}\hat{i}T$

(3) $9.6\hat{j}T$ (4) $9.6\times10^{-8}\hat{k}T$

Ans. (1)

Sol. $\frac{E}{B} = C$

$\frac{E}{B} = 3\times10^{8}$

$B = \frac{E}{3\times10^{8}} = \frac{9.6}{3\times10^{8}}$

$B = 3.2 \times 10^{-8}$ T

$\hat{B} = \hat{v}\times\hat{E}$

$= \hat{i}\times\hat{j} = \hat{k}$

So,

$\vec{B} = 3.2\times10^{-8}\hat{k}$ T

46. In the given circuit, the current in resistance R_3 is :

(1) 1 A (2) 1.5 A

(3) 2 A (4) 2.5 A

Ans. (1)

Sol.

$R_{eq} = 2\Omega + 2\,\Omega + 1\,\Omega = 5\,\Omega$

$i = \frac{V}{R_{eq}} = \frac{10}{5} = 2A$

Current in resistance $R_3 = 2\times\left(\frac{4}{4+4}\right)$

$= 2\times\frac{4}{8}$

$= 1A$

47. A particle is moving in a straight line. The variation of position 'x' as a function of time 't' is given as $x = (t^3 - 6t^2 + 20t + 15)$ m. The velocity of the body when its acceleration becomes zero is :

(1) 4 m/s (2) 8 m/s

(3) 10 m/s (4) 6 m/s

Ans. (2)

Sol. $x = t^3 - 6t^2 + 20t + 15$

$\frac{dx}{dt} = v = 3t^2 - 12t + 20$

$\frac{dv}{dt} = a = 6t - 12$

When a = 0

$6t - 12 = 0$; $t = 2$ sec

At t = 2 sec

$v = 3(2)^2 - 12(2) + 20$

$v = 8$ m/s

48. N moles of a polyatomic gas (f = 6) must be mixed with two moles of a monoatomic gas so that the mixture behaves as a diatomic gas. The value of N is :

(1) 6 (2) 3 (3) 4 (4) 2

Ans. (3)

Sol. $f_{eq} = \frac{n_1f_1 + n_2f_2}{n_1 + n_2}$

For diatomic gas $f_{eq} = 5$

$5 = \frac{(N)(6) + (2)(3)}{N+2}$

$5N + 10 = 6N + 6$

$N = 4$

49. Given below are two statements :

Statement I : Most of the mass of the atom and all its positive charge are concentrated in a tiny nucleus and the electrons revolve around it, is Rutherford's model.

Statement II : An atom is a spherical cloud of positive charges with electrons embedded in it, is a special case of Rutherford's model.

In the light of the above statements, choose the most appropriate from the options given below.

(1) Both statement I and statement II are false
(2) Statement I is false but statement II is true
(3) Statement I is true but statement II is false
(4) Both statement I and statement II are true

Ans. (3)

Sol. According to Rutherford atomic model, most of mass of atom and all its positive charge is concentrated in tiny nucleus & electron revolve around it.

According to Thomson atomic model, atom is spherical cloud of positive charge with electron embedded in it.

Hence,

Statement I is true but statement II false.

50. An electric field is given by $(6\hat{i}+5\hat{j}+3\hat{k})$ N/C. The electric flux through a surface area $30\hat{i}\ m^2$ lying in YZ-plane (in SI unit) is :

(1) 90 (2) 150
(3) 180 (4) 60

Ans. (3)

Sol. $\vec{E}=6\hat{i}+5\hat{j}+3\hat{k}$

$\vec{A}=30\hat{i}$

$\phi=\vec{E}.\vec{A}$

$\phi=(6\hat{i}+5\hat{j}+3\hat{k}).(30\hat{i})$

$\phi=6\times30=180$

SECTION-B

51. Two metallic wires P and Q have same volume and are made up of same material. If their area of cross sections are in the ratio 4 : 1 and force F_1 is applied to P, an extension of Δl is produced. The force which is required to produce same extension in Q is F_2.

The value of $\frac{F_1}{F_2}$ is_____ .

Ans. (16)

Sol. $Y=\frac{\text{Stress}}{\text{Strain}}=\frac{F/A}{\Delta\ell/\ell}=\frac{F\ell}{A\Delta\ell}$

$\Delta\ell=\frac{F\ell}{AY}$

$V=A\ell\Rightarrow\ell=\frac{V}{A}$

$\Delta\ell=\frac{FV}{A^2Y}$

Y & V is same for both the wires

$\Delta\ell\propto\frac{F}{A^2}$

$\frac{\Delta\ell_1}{\Delta\ell_2}=\frac{F_1}{A_1^2}\times\frac{A_2^2}{F_2}$

$\Delta\ell_1=\Delta\ell_2$

$F_1A_2^2=F_2A_1^2$

$\frac{F_1}{F_2}=\frac{A_1^2}{A_2^2}=\left(\frac{4}{1}\right)^2=16$

52. A horizontal straight wire 5 m long extending from east to west falling freely at right angle to horizontal component of earth's magnetic field 0.60×10^{-4} Wbm^{-2}. The instantaneous value of emf induced in the wire when its velocity is 10 ms^{-1} is _____ $\times10^{-3}$ V.

Ans. (3)

Sol. $B_H=0.60\times10^{-4}$ Wb/m^2

Induced emf $e=B_Hv\ell$

$=0.60\times10^{-4}\times10\times5$

$=3\times10^{-3}$ V

53. Hydrogen atom is bombarded with electrons accelerated through a potential different of V, which causes excitation of hydrogen atoms. If the experiment is being formed at T = 0 K. The minimum potential difference needed to observe any Balmer series lines in the emission spectra will be $\frac{\alpha}{10}$ V , where α = _____.

Ans. (121)

Sol. For minimum potential difference electron has to make transition from n = 3 to n = 2 state but first electron has to reach to n = 3 state from ground state. So, energy of bombarding electron should be equal to energy difference of n = 3 and n = 1 state.

$$\Delta E = 13.6\left[1-\frac{1}{3^2}\right]e = eV$$

$$\frac{13.6\times 8}{9} = V$$

$$V = 12.09\ V \approx 12.1\ V$$

So, $\alpha = 121$

54. A charge of 4.0 μC is moving with a velocity of $4.0 \times 10^6\ ms^{-1}$ along the positive y-axis under a magnetic field $\vec{B}$ of strength $\left(2\hat{k}\right)$ T. The force acting on the charge is $x\hat{i}\,N$. The value of x is __.

Ans. (32)

Sol. $q = 4\ \mu C,\ \vec{v} = 4\times 10^6\hat{j}$ m/s

$\vec{B} = 2\hat{k}T$

$\vec{F} = q\left(\vec{v}\times\vec{B}\right)$

$= 4\times 10^{-6}\left(4\times 10^6\hat{j}\times 2\hat{k}\right)$

$= 4\times 10^{-6}\times 8\times 10^6\hat{i}$

$\vec{F} = 32\hat{i}$ N

x = 32

55. A simple harmonic oscillator has an amplitude A and time period 6π second. Assuming the oscillation starts from its mean position, the time required by it to travel from x = A to $x = \frac{\sqrt{3}}{2}A$ will be $\frac{\pi}{x}$ s, where x = _____ :

Ans. (2)

Sol.

From phasor diagram particle has to move from P to Q in a circle of radius equal to amplitude of SHM.

$$\cos\phi = \frac{\frac{\sqrt{3}A}{2}}{A} = \frac{\sqrt{3}}{2}$$

$$\phi = \frac{\pi}{6}$$

Now, $\frac{\pi}{6} = \omega t$

$$\frac{\pi}{6} = \frac{2\pi}{T}t$$

$$\frac{\pi}{6} = \frac{2\pi}{6\pi}t$$

$$t = \frac{\pi}{2}$$

So, x = 2

56. In the given figure, the charge stored in 6μF capacitor, when points A and B are joined by a connecting wire is _____μC.

Ans. (36)

Sol. At steady state, capacitor behaves as an open circuit and current flows in circuit as shown in the diagram.

$R_{eq} = 9\ \Omega$

$i = \frac{9\ V}{9\ \Omega} = 1A$

$\Delta V_{6\Omega} = 1 \times 6 = 6\ V$

$V_A = 3\ V$

So, potential difference across 6μF is 6 V.

Hence $Q = C\Delta V$

$= 6 \times 6 \times 10^{-6}\ C$

$= 36\ \mu C$

57. In a single slit diffraction pattern, a light of wavelength 6000 Å is used. The distance between the first and third minima in the diffraction pattern is found to be 3 mm when the screen in placed 50 cm away from slits. The width of the slit is _____ $\times 10^{-4}$ m.

Ans. **(2)**

Sol. For n^{th} minima

$b\sin\theta = n\lambda$

(λ is small so $\sin\theta$ is small, hence $\sin\theta \simeq \tan\theta$)

$b\tan\theta = n\lambda$

$b\frac{y}{D} = n\lambda$

$\Rightarrow y_n = \frac{n\lambda D}{b}$ (Position of nth minima)

$B \rightarrow 1^{st}$ minima, $A \rightarrow 3^{rd}$ minima

$y_3 = \frac{3\lambda D}{b}$, $y_1 = \frac{\lambda D}{b}$

$\Delta y = y_3 - y_1 = \frac{2\lambda D}{b}$

$3 \times 10^{-3} = \frac{2 \times 6000 \times 10^{-10} \times 0.5}{b}$

$b = \frac{2 \times 6000 \times 10^{-10} \times 0.5}{3 \times 10^{-3}}$

$b = 2 \times 10^{-4}$ m

$x = 2$

58. In the given circuit, the current flowing through the resistance 20Ω is 0.3 A, while the ammeter reads 0.9 A. The value of R_1 is _____ Ω.

Ans. **(30)**

Sol.

Given, $i_1 = 0.3$ A, $i_1 + i_2 + i_3 = 0.9$ A

So, $V_{AB} = i_1 \times 20\Omega = 20 \times 0.3\ V = 6\ V$

$$i_2 = \frac{6V}{15\Omega} = \frac{2}{5}A$$

$$i_1 + i_2 + i_3 = \frac{9}{10}A$$

$$\frac{3}{10} + \frac{2}{5} + i_3 = \frac{9}{10}$$

$$\frac{7}{10} + i_3 = \frac{9}{10}$$

$i_3 = 0.2$ A

So, $i_3 \times R_1 = 6$ V

$(0.2)R_1 = 6$

$$R_1 = \frac{6}{0.2} = 30\ \Omega$$

59. A particle is moving in a circle of radius 50 cm in such a way that at any instant the normal and tangential components of its acceleration are equal. If its speed at t = 0 is 4 m/s, the time taken to complete the first revolution will be $\frac{1}{\alpha}\left[1 - e^{-2\pi}\right]$s, where α = _____.

Ans. **(8)**

Sol. $|\vec{a}_C| = |\vec{a}_t|$

$$\frac{v^2}{r} = \frac{dv}{dt}$$

$$\Rightarrow \int_4^v \frac{dv}{v^2} = \int_0^t \frac{dt}{r}$$

$$\Rightarrow \left[\frac{-1}{v}\right]_4^v = \frac{t}{r}$$

$$\Rightarrow \frac{-1}{v} + \frac{1}{4} = 2t$$

$$\Rightarrow v = \frac{4}{1-8t} = \frac{ds}{dt}$$

$$4\int_0^t \frac{dt}{1-8t} = \int_0^s ds$$

(r = 0.5 m

s = 2πr = π)

$$4 \times \frac{\left[\ell n(1-8t)\right]_0^t}{-8} = \pi$$

$$\ell n(1-8t) = -2\pi$$

$$1 - 8t = e^{-2\pi}$$

$$t = \left(1 - e^{-2\pi}\right)\frac{1}{8}\ s$$

So, $\alpha = 8$

60. A body of mass 5 kg moving with a uniform speed $3\sqrt{2}\ ms^{-1}$ in X – Y plane along the line y = x + 4. The angular momentum of the particle about the origin will be _____ kg m^2s^{-1}.

Ans. **(60)**

Sol. y – x – 4 = 0

d_1 is perpendicular distance of given line from origin.

$$d_1 = \left|\frac{-4}{\sqrt{1^2 + 1^2}}\right| \Rightarrow 2\sqrt{2}\ m$$

So, $|\vec{L}| = mvd_1 = 5 \times 3\sqrt{2} \times 2\sqrt{2}$ kg m^2/s

= 60 kg m^2/s

SECTION-A

61. The ascending acidity order of the following H atoms is

(1) C < D < B < A

(2) A < B < C < D

(3) A < B < D < C

(4) D < C < B < A

Ans. (1)

Sol. $CH \equiv C^{\ominus} > CH_2 = \overset{\ominus}{C}H > H_3C - \overset{\ominus}{C}H_2 >$

Stability of conjugate base α acidic strength

C < D < B < A

62. Match List I with List II

List I (Bio Polymer)		List II (Monomer)	
A.	Starch	I.	nucleotide
B.	Cellulose	II.	α-glucose
C.	Nucleic acid	III.	β-glucose
D.	Protein	IV.	α-amino acid

Choose the correct answer from the options given below :-

(1) A-II, B-I, C-III, D-IV

(2) A-IV, B-II, C-I, D-III

(3) A-I, B-III, C-IV, D-II

(4) A-II, B-III, C-I, D-IV

Ans. (4)

Sol. A-II, B-III, C-I, D-IV

Fact based.

63. Match List I with List II

List I (Compound)		List II (pK_a value)	
A.	Ethanol	I.	10.0
B.	Phenol	II.	15.9
C.	m-Nitrophenol	III.	7.1
D.	p-Nitrophenol	IV.	8.3

Choose the correct answer from the options given below :-

(1) A-I, B-II, C-III, D-IV

(2) A-IV, B-I, C-II, D-III

(3) A-III, B-IV, C-I, D-II

(4) A-II, B-I, C-IV, D-III

Ans. (4)

Sol. Ethanol → 15.9

Phenol → 10

M-Nitrophenol → 8.3

P-Nitrophenol → 7.1

64. Which of the following reaction is correct ?

(1) $CH_3CH_2CH_2NH_2 \xrightarrow[H_2O]{HNO_2, 0°C} CH_3CH_2OH + N_2 + HCl$

(2)

(3)

(4) $C_2H_5CONH_2 + Br_2 + NaOH$
$\rightarrow C_2H_5CH_2NH_2 + Na_2CO_3 + NaBr + H_2O$

Ans. (2)

Sol.

Markovnikov addition

65. According to IUPAC system, the compound is named as

(1) Cyclohex-1-en-2-ol (2) 1-Hydroxyhex-2-ene
(3) Cyclohex-1-en-3-ol (4) Cyclohex-2-en-1-ol

Ans. (4)
Cyclohex-2-en-1-ol

66. The correct IUPAC name of K_2MnO_4 is
(1) Potassium tetraoxopermanganate (VI)
(2) Potassium tetraoxidomanganate (VI)
(3) Dipotassium tetraoxidomanganate (VII)
(4) Potassium tetraoxidomanganese (VI)

Ans. (2)

Sol. K_2MnO_4
$2 + x - 8 = 0$
$\Rightarrow x = +6$
O.S. of Mn = +6
IUPAC Name =
Potassium tetraoxidomanganate(VI)

67. A reagent which gives brilliant red precipitate with Nickel ions in basic medium is
(1) sodium nitroprusside
(2) neutral $FeCl_3$
(3) meta-dinitrobenzene
(4) dimethyl glyoxime

Ans. (4)

Sol. $Ni^{2+} + 2dmg^- \rightarrow [Ni(dmg)_2]$
Rosy red/Bright Red precipitate

68. Phenol treated with chloroform in presence of sodium hydroxide, which further hydrolysed in presence of an acid results
(1) Salicyclic acid
(2) Benzene-1,2-diol
(3) Benzene-1, 3-diol
(4) 2-Hydroxybenzaldehyde

Ans. (4)

Sol.

It is Reimer Tiemann Reaction

69. Match List I with List II

List I (Spectral Series for Hydrogen)		**List II (Spectral Region/Higher Energy State)**	
A.	Lyman	I.	Infrared region
B.	Balmer	II.	UV region
C.	Paschen	III.	Infrared region
D.	Pfund	IV.	Visible region

Choose the correct answer from the options given below :-
(1) A-II, B-III, C-I, D-IV
(2) A-I, B-III, C-II, D-IV
(3) A-II, B-IV, C-III, D-I
(4) A-I, B-II, C-III, D-IV

Ans. (3)

Sol. A – II, B – IV, C – III, D – I
Fact based.

70. On passing a gas, 'X', through Nessler's reagent, a brown precipitate is obtained. The gas 'X' is
(1) H_2S (2) CO_2
(3) NH_3 (4) Cl_2

Ans. (3)

Sol. Nessler's Reagent Reaction :

$$\underset{(\text{Nessler's Reagent})}{2K_2HgI_4} + NH_3 + 3KOH \rightarrow \underset{\left(\substack{\text{Iodine of Millon's base}\\ \text{Brown precipitate}}\right)}{HgO.\,Hg(NH_2)I} + 7KI + 2H_2O$$

71. The product A formed in the following reaction is:

(1)

(2)

(3)

(4)

Ans. (3)

Sol.

72. Identify the reagents used for the following conversion

(1) A = $LiAlH_4$, B = $NaOH_{(aq)}$, C = NH_2–NH_2/KOH, ethylene glycol

(2) A = $LiAlH_4$, B = $NaOH_{(alc)}$, C =Zn/HCl

(3) A = DIBAL-H, B= $NaOH_{(aq)}$, C = NH_2–NH_2/KOH, ethylene glycol

(4) A = DIBAL-H, B = $NaOH_{(alc)}$, C = Zn/HCl

Ans. (4)

Sol.

73. Which of the following acts as a strong reducing agent? (Atomic number : Ce = 58, Eu = 63, Gd = 64, Lu = 71)

(1) Lu^{3+} (2) Gd^{3+}

(3) Eu^{2+} (4) Ce^{4+}

Ans. (3)

Sol. $Eu^{+2} \longrightarrow Eu^{+3} + 1e^-$

$[Xe]4f^7 6s^0$ $[Xe]\ 4f^6 6s^0$

74. Chromatographic technique/s based on the principle of differential adsorption is/are

A. Column chromatography

B. Thin layer chromatography

C. Paper chromatography

Choose the most appropriate answer from the options given below:

(1) B only (2) A only

(3) A & B only (4) C only

Ans. (3)

Sol. Memory Based

75. Which of the following statements are correct about Zn, Cd and Hg?

A. They exhibit high enthalpy of atomization as the d-subshell is full.

B. Zn and Cd do not show variable oxidation state while Hg shows +I and +II.

C. Compounds of Zn, Cd and Hg are paramagnetic in nature.

D. Zn, Cd and Hg are called soft metals.

Choose the ***most appropriate*** from the options given below:

(1) B, D only (2) B, C only

(3) A, D only (4) C, D only

Sol. Ans. (1)

(A) Zn, Cd, Hg exhibit lowest enthalpy of atomization in respective transition series.

(C) Compounds of Zn, Cd and Hg are diamagnetic in nature.

76. The element having the highest first ionization enthalpy is

(1) Si (2) Al

(3) N (4) C

Ans. (3)

Sol. Al < Si < C < N ; IE_1 order.

77. Alkyl halide is converted into alkyl isocyanide by reaction with

(1) NaCN (2) NH_4CN

(3) KCN (4) AgCN

Ans. (4)

Sol. Covalent character of AgCN.

78. Which one of the following will show geometrical isomerism?

(1)

(2)

(3)

(4)

Ans. (3)

Sol. Due to unsymmetrical.

79. Given below are two statements:
Statement I: Fluorine has most negative electron gain enthalpy in its group.
Statement II: Oxygen has least negative electron gain enthalpy in its group.
In the light of the above statements, choose the most appropriate from the options given below.
(1) Both Statement I and Statement II are true
(2) Statement I is true but Statement II is false
(3) Both Statement I and Statement II are false
(4) Statement I is false but Statement II is true

Ans. (4)

Sol. Statement-1 is false because chlorine has most negative electron gain enthalpy in its group.

80. Anomalous behaviour of oxygen is due to its
(1) Large size and high electronegativity
(2) Small size and low electronegativity
(3) Small size and high electronegativity
(4) Large size and low electronegativity

Ans. (3)

Sol. Fact Based.

SECTION-B

81. The total number of anti bonding molecular orbitals, formed from 2s and 2p atomic orbitals in a diatomic molecule is _____.

Ans. (4)

Sol. Antibonding molecular orbital from 2s = 1
Antibonding molecular orbital from 2p = 3
Total = 4

82. The oxidation number of iron in the compound formed during brown ring test for NO_3^- ion is _____.

Ans. (1)

Sol. $[Fe(H_2O)_5(NO)]^{2+}$,
Oxidation no. of Fe = +1

83. The following concentrations were observed at 500 K for the formation of NH_3 from N_2 and H_2. At equilibrium :$[N_2] = 2 \times 10^{-2}$ M, $[H_2] = 3 \times 10^{-2}$ M and $[NH_3] = 1.5 \times 10^{-2}$M. Equilibrium constant for the reaction is _____.

Ans. (417)

Sol. $$K_C = \frac{[NH_3]^2}{[N_2][H_2]^3}$$

$$K_C = \frac{(1.5\times10^{-2})^2}{(2\times10^{-2})\times(3\times10^{-2})^3}$$

$K_C = 417$

84. Molality of 0.8 M H_2SO_4 solution (density 1.06 g cm^{-3}) is _____$\times10^{-3}$ m.

Ans. (815)

Sol. $$m = \frac{M\times1000}{d_{sol}\times1000 - M\times \text{Molar mass}_{solute}}$$

815×10^{-3} m

85. If 50 mL of 0.5 M oxalic acid is required to neutralise 25 mL of NaOH solution, the amount of NaOH in 50 mL of given NaOH solution is _____g.

Ans. (4)

Sol. Equivalent of Oxalic acid = Equivalents of NaOH
$50 \times 0.5 \times 2 = 25 \times M \times 1$
$M_{NaOH} = 2M$
W_{NaOH} in 50ml = $2 \times 50 \times 40 \times 10^{-3}$ g = 4g

86. The total number of 'Sigma' and Pi bonds in 2-formylhex-4-enoic acid is ____.

Ans. (22)

Sol.

22 bonds

87. The half-life of radioisotopic bromine - 82 is 36 hours. The fraction which remains after one day is _____ $\times10^{-2}$.
(Given antilog 0.2006 = 1.587)

Ans. (63)

Sol. Half life of bromine – 82 = 36 hours

$$t_{1/2} = \frac{0.693}{K}$$

$$K = \frac{0.693}{36} = 0.01925\ hr^{-1}$$

1st order rxn kinetic equation

$$t = \frac{2.303}{K}\log\frac{a}{a-x}$$

$$\log\frac{a}{a-x} = \frac{t\times K}{2.303} \quad (t = 1day = 24hr)$$

$$\log\frac{a}{a-x} = \frac{24hr \times 0.01925hr^{-1}}{2.303}$$

$$\log\frac{a}{a-x} = 0.2006$$

$$\frac{a}{a-x} = \text{anti}\log(0.2006)$$

$$\frac{a}{a-x} = 1.587$$

If a = 1

$\frac{1}{1-x} = 1.587 \Rightarrow 1 - x = 0.6301$ = Fraction remain after one day

88. Standard enthalpy of vapourisation for CCl_4 is 30.5 kJ mol^{-1}. Heat required for vapourisation of 284g of CCl_4 at constant temperature is ____kJ.
(Given molar mass in g mol^{-1} ; C = 12, Cl = 35.5)

Ans. (56)

Sol. ΔH^0_{vap} CCl_4 = 30.5 kJ / mol

Mass of CCl_4 = 284 gm

Molar mass of CCl_4 = 154 g/mol

Moles of CCl_4 = $\frac{284}{154}$ = 1.844 mol

$\Delta H_{vap}°$ for 1 mole = 30.5 kJ/mol

$\Delta H_{vap}°$ for 1.844 mol = 30.5 × 1.844

= 56.242 kJ

89. A constant current was passed through a solution of $AuCl_4^-$ ion between gold electrodes. After a period of 10.0 minutes, the increase in mass of cathode was 1.314 g. The total charge passed through the solution is _____ × 10^{-2} F.
(Given atomic mass of Au = 197)

Ans. (2)

Sol. $\frac{W}{E} = \frac{charge}{1F}$

$\frac{1.314}{\frac{197}{3}} = \frac{Q}{1F}$

Q = 2 × 10^{-2} F

90. The total number of molecules with zero dipole moment among CH_4, BF_3, H_2O, HF, NH_3, CO_2 and SO_2 is _____.

Ans. (3)

Sol. Molecules with zero dipole moment = CO_2, CH_4, BF_3

FINAL JEE–MAIN EXAMINATION – JANUARY, 2024

(Held On Monday 29th January, 2024) **TIME : 9 : 00 AM to 12 : 00 NOON**

MATHEMATICS — TEST PAPER WITH SOLUTION

SECTION-A

1. If in a G.P. of 64 terms, the sum of all the terms is 7 times the sum of the odd terms of the G.P, then the common ratio of the G.P. is equal to

(1) 7 (2) 4

(3) 5 (4) 6

Ans. (4)

Sol. $a + ar + ar^2 + ar^3 + ... + ar^{63}$

$= 7(a + ar^2 + ar^4 ..... + ar^{62})$

$\Rightarrow \dfrac{a(1-r^{64})}{1-r} = \dfrac{7a(1-r^{64})}{1-r^2}$

$r = 6$

2. In an A.P., the sixth terms $a_6 = 2$. If the $a_1a_4a_5$ is the greatest, then the common difference of the A.P., is equal to

(1) $\frac{3}{2}$ (2) $\frac{8}{5}$ (3) $\frac{2}{3}$ (4) $\frac{5}{8}$

Ans. (2)

Sol. $a_6 = 2 \Rightarrow a + 5d = 2$

$a_1a_4a_5 = a(a+3d)(a+4d)$

$= (2-5d)(2-2d)(2-d)$

$f(d) = 8 - 32d + 34d^2 - 20d + 30d^2 - 10d^3$

$f'(d) = -2(5d-8)(3d-2)$

$d = \dfrac{8}{5}$

3. If $f(x) = \begin{cases} 2+2x, & -1 \le x < 0 \\ 1-\frac{x}{3}, & 0 \le x \le 3 \end{cases}$; $g(x) = \begin{cases} -x, & -3 \le x \le 0 \\ x, & 0 < x \le 1 \end{cases}$,

then range of (fog(x)) is

(1) (0, 1] (2) [0, 3)

(3) [0, 1] (4) [0, 1)

Ans. (3)

Sol. $f(g(x)) = \begin{cases} 2+2g(x) & , \quad -1 \le g(x) < 0 \quad .....(1) \\ 1-\dfrac{g(x)}{3} & , \quad 0 \le g(x) \le 3 \quad .....(2) \end{cases}$

By (1) $x \in \phi$

And by (2) $x \in [-3, 0]$ and $x \in [0, 1]$

Range of $f(g(x))$ is [0, 1]

4. A fair die is thrown until 2 appears. Then the probability, that 2 appears in even number of throws , is

(1) $\frac{5}{6}$ (2) $\frac{1}{6}$ (3) $\frac{5}{11}$ (4) $\frac{6}{11}$

Ans. (3)

Sol. Required probability =

$\dfrac{5}{6} \times \dfrac{1}{6} + \left(\dfrac{5}{6}\right)^3 \times \dfrac{1}{6} + \left(\dfrac{5}{6}\right)^5 \times \dfrac{1}{6} + ...$

$= \dfrac{1}{6} \times \dfrac{\frac{5}{6}}{1 - \frac{25}{36}} = \dfrac{5}{11}$

5. If $z=\frac{1}{2}-2i$, is such that

$|z+1|=\alpha z+\beta(1+i), i=\sqrt{-1}$ and $\alpha,\beta\in R$, then $\alpha+\beta$ is equal to

(1) –4 (2) 3

(3) 2 (4) –1

Ans. (2)

Sol. $z=\frac{1}{2}-2i$

$|z+1|=\alpha z+\beta(1+i)$

$\left|\frac{3}{2}-2i\right|=\frac{\alpha}{2}-2\alpha i+\beta+\beta i$

$\left|\frac{3}{2}-2i\right|=\left(\frac{\alpha}{2}+\beta\right)+(\beta-2\alpha)i$

$\beta=2\alpha$ and $\frac{\alpha}{2}+\beta=\sqrt{\frac{9}{4}+4}$

$\alpha+\beta=3$

6. $\lim\limits_{x\to\frac{\pi}{2}}\left(\frac{1}{\left(x-\frac{\pi}{2}\right)^2}\int_{x^3}^{\left(\frac{\pi}{2}\right)^3}\cos\left(\frac{1}{t^3}\right)dt\right)$ is equal to

(1) $\frac{3\pi}{8}$ (2) $\frac{3\pi^2}{4}$

(3) $\frac{3\pi^2}{8}$ (4) $\frac{3\pi}{4}$

Ans. (3)

Sol. Using L'hopital rule

$$=\lim_{x\to\frac{\pi}{2}^-}\frac{0-\cos x\times 3x^2}{2\left(x-\frac{\pi}{2}\right)}$$

$$=\lim_{x\to\frac{\pi}{2}^-}\frac{\sin\left(x-\frac{\pi}{2}\right)}{2\left(x-\frac{\pi}{2}\right)}\times\frac{3\pi^2}{4}$$

$$=\frac{3\pi^2}{8}$$

7. In a ΔABC, suppose y = x is the equation of the bisector of the angle B and the equation of the side AC is 2x –y =2. If 2AB = BC and the point A and B are respectively (4, 6) and (α,β), then $\alpha+2\beta$ is equal to

(1) 42 (2) 39

(3) 48 (4) 45

Ans. (1)

Sol.

AD : DC = 1 : 2

$\frac{4-\alpha}{6-\alpha}=\frac{10}{8}$

$\alpha=\beta$

$\alpha=14$ and $\beta=14$

8. Let $\vec{a},\vec{b}$ and $\vec{c}$ be three non-zero vectors such that $\vec{b}$ and $\vec{c}$ are non-collinear .if $\vec{a}+5\vec{b}$ is collinear with $\vec{c},\vec{b}+6\vec{c}$ is collinear with $\vec{a}$ and $\vec{a}+\alpha\vec{b}+\beta\vec{c}=\vec{0}$, then $\alpha+\beta$ is equal to

(1) 35 (2) 30

(3) – 30 (4)–25

Ans. (1)

Sol. $\vec{a}+5\vec{b}=\lambda\vec{c}$

$\vec{b}+6\vec{c}=\mu\vec{a}$

Eliminating $\vec{a}$

$\lambda\vec{c}-5\vec{b}=\frac{6}{\mu}\vec{c}+\frac{1}{\mu}\vec{b}$

$\therefore\mu=\frac{-1}{5},\lambda=-30$

$\alpha=5,\beta=30$

9. Let $\left(5,\frac{a}{4}\right)$, be the circumcenter of a triangle with vertices $A(a,-2), B(a,6)$ and $C\left(\frac{a}{4},-2\right)$. Let α denote the circumradius, β denote the area and γ denote the perimeter of the triangle. Then $\alpha+\beta+\gamma$ is

(1) 60 (2) 53

(3) 62 (4) 30

Ans. (2)

Sol. $A(a,-2), B(a,6), C\left(\frac{a}{4},-2\right)$, $O\left(5,\frac{a}{4}\right)$

AO = BO

$$(a-5)^2+\left(\frac{a}{4}+2\right)^2=(a-5)^2+\left(\frac{a}{4}-6\right)^2$$

a = 8

AB = 8, AC = 6, BC = 10

$\alpha=5,\beta=24,\gamma=24$

10. For $x\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$, if

$y(x)=\int\frac{\operatorname{cosec}x+\sin x}{\operatorname{cosec}x\sec x+\tan x\sin^2 x}dx$ and

$\lim\limits_{x\to\left(\frac{\pi}{2}\right)^-}y(x)=0$ then $y\left(\frac{\pi}{4}\right)$ is equal to

(1) $\tan^{-1}\left(\frac{1}{\sqrt{2}}\right)$ (2) $\frac{1}{2}\tan^{-1}\left(\frac{1}{\sqrt{2}}\right)$

(3) $-\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{1}{\sqrt{2}}\right)$ (4) $\frac{1}{\sqrt{2}}\tan^{-1}\left(-\frac{1}{2}\right)$

Ans. (4)

Sol. $y(x)=\int\frac{(1+\sin^2 x)\cos x}{1+\sin^4 x}dx$

Put sinx = t

$$=\int\frac{1+t^2}{t^4+1}dt=\frac{1}{\sqrt{2}}\tan^{-1}\frac{\left(t-\frac{1}{t}\right)}{\sqrt{2}}+C$$

$x=\frac{\pi}{2},t=1$ $\therefore C=0$

$$y\left(\frac{\pi}{4}\right)=\frac{1}{\sqrt{2}}\tan^{-1}\left(-\frac{1}{2}\right)$$

11. If $\alpha, -\frac{\pi}{2}<\alpha<\frac{\pi}{2}$ is the solution of $4\cos\theta+5\sin\theta=1$, then the value of $\tan\alpha$ is

(1) $\frac{10-\sqrt{10}}{6}$ (2) $\frac{10-\sqrt{10}}{12}$

(3) $\frac{\sqrt{10}-10}{12}$ (4) $\frac{\sqrt{10}-10}{6}$

Ans. (3)

Sol. $4+5\tan\theta=\sec\theta$

Squaring : $24\tan^2\theta+40\tan\theta+15=0$

$$\tan\theta=\frac{-10\pm\sqrt{10}}{12}$$

and $\tan\theta=-\left(\frac{10+\sqrt{10}}{12}\right)$ is Rejected.

(3) is correct.

12. A function y = f(x) satisfies

$f(x)\sin 2x+\sin x-(1+\cos^2 x)f'(x)=0$ with condition

f(0) = 0 . Then $f\left(\frac{\pi}{2}\right)$ is equal to

(1) 1 (2) 0 (3) –1 (4) 2

Ans. (1)

Sol. $\frac{dy}{dx}-\left(\frac{\sin 2x}{1+\cos^2 x}\right)y=\sin x$

I.F. = $1+\cos^2 x$

$$y\cdot(1+\cos^2 x)=\int(\sin x)dx$$

= – cosx + C

x = 0, C = 1

$$y\left(\frac{\pi}{2}\right)=1$$

13. Let O be the origin and the position vector of A and B be $2\hat{i}+2\hat{j}+\hat{k}$ and $2\hat{i}+4\hat{j}+4\hat{k}$ respectively. If the internal bisector of $\angle AOB$ meets the line AB at C, then the length of OC is

(1) $\frac{2}{3}\sqrt{31}$ (2) $\frac{2}{3}\sqrt{34}$

(3) $\frac{3}{4}\sqrt{34}$ (4) $\frac{3}{2}\sqrt{31}$

Ans. (2)

Sol.

length of $OC = \frac{\sqrt{136}}{3} = \frac{2\sqrt{34}}{3}$

14. Consider the function $f:\left[\frac{1}{2},1\right] \to R$ defined by $f(x) = 4\sqrt{2}x^3 - 3\sqrt{2}x - 1$. Consider the statements

(I) The curve y = f(x) intersects the x-axis exactly at one point

(II) The curve y = f(x) intersects the x-axis at $x = \cos\frac{\pi}{12}$

Then

(1) Only (II) is correct

(2) Both (I) and (II) are incorrect

(3) Only (I) is correct

(4) Both (I) and (II) are correct

Ans. (4)

Sol. $f'(x) = 12\sqrt{2}x^2 - 3\sqrt{2} \geq 0$ for $\left[\frac{1}{2},1\right]$

$f\left(\frac{1}{2}\right) < 0$

f(1) > 0 ⇒ (A) is correct.

$f(x) = \sqrt{2}\left(4x^3 - 3x\right) - 1 = 0$

Let cosα = x,

cos 3α = cos $\frac{\pi}{4} \Rightarrow \alpha = \frac{\pi}{12}$

$x = \cos\frac{\pi}{12}$

(4) is correct.

15. Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \alpha & \beta \\ 0 & \beta & \alpha \end{bmatrix}$ and $|2A|^3 = 2^{21}$ where $\alpha, \beta \in Z$,

Then a value of α is

(1) 3 (2) 5

(3) 17 (4) 9

Ans. (2)

Sol. $|A| = \alpha^2 - \beta^2$

$|2A|^3 = 2^{21} \Rightarrow |A| = 2^4$

$\alpha^2 - \beta^2 = 16$

$(\alpha+\beta)(\alpha-\beta) = 16 \Rightarrow \alpha = 4$ or 5

16. Let PQR be a triangle with $R(-1, 4, 2)$. Suppose M(2, 1, 2) is the mid point of PQ. The distance of the centroid of ΔPQR from the point of intersection of the line $\frac{x-2}{0} = \frac{y}{2} = \frac{z+3}{-1}$ and $\frac{x-1}{1} = \frac{y+3}{-3} = \frac{z+1}{1}$ is

(1) 69 (2) 9

(3) $\sqrt{69}$ (4) $\sqrt{99}$

Ans. (3)

Sol. Centroid G divides MR in 1 : 2

G(1, 2, 2)

Point of intersection A of given lines is (2,–6, 0)

$AG = \sqrt{69}$

17. Let R be a relation on Z × Z defined by

(a, b)R(c, d) if and only if ad – bc is divisible by 5.

Then R is

(1) Reflexive and symmetric but not transitive

(2) Reflexive but neither symmetric not transitive

(3) Reflexive, symmetric and transitive

(4) Reflexive and transitive but not symmetric

Ans. (1)

Sol. (a, b)R(a, b) as ab – ab = 0

Therefore reflexive

Let (a,b)R(c,d)⇒ad – bc is divisible by 5

⇒ bc – ad is divisible by 5 ⇒(c,d)R(a,b)

Therefore symmetric

Relation not transitive as (3,1)R(10,5) and (10,5)R(1,1) but (3,1) is not related to (1,1)

18. If the value of the integral

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\left(\frac{x^2\cos x}{1+\pi^x}+\frac{1+\sin^2 x}{1+e^{\sin x^{2023}}}\right)dx=\frac{\pi}{4}(\pi+a)-2,$$

then the value of a is

(1) 3 (2) $-\frac{3}{2}$ (3) 2 (4) $\frac{3}{2}$

Ans. (1)

Sol. $$I=\int_{-\pi/2}^{\pi/2}\left(\frac{x^2\cos x}{1+\pi^x}+\frac{1+\sin^2 x}{1+e^{\sin x^{2023}}}\right)dx$$

$$I=\int_{-\pi/2}^{\pi/2}\left(\frac{x^2\cos x}{1+\pi^{-x}}+\frac{1+\sin^2 x}{1+e^{\sin(-x)^{2023}}}\right)dx$$

On Adding, we get

$$2I=\int_{-\pi/2}^{\pi/2}\left(x^2\cos x+1+\sin^2 x\right)dx$$

On solving

$$I=\frac{\pi^2}{4}+\frac{3\pi}{4}-2$$

a = 3

19. Suppose

$$f(x)=\frac{\left(2^x+2^{-x}\right)\tan x\sqrt{\tan^{-1}\left(x^2-x+1\right)}}{\left(7x^2+3x+1\right)^3},$$

Then the value of f '(0) is equal to

(1) π (2) 0

(3) $\sqrt{\pi}$ (4) $\frac{\pi}{2}$

Ans. (3)

Sol. $$f'(0)=\lim_{h\to 0}\frac{f(h)-f(0)}{h}$$

$$=\lim_{h\to 0}\frac{(2^h+2^{-h})\tan h\ \sqrt{\tan^{-1}(h^2-h+1)}-0}{(7h^2+3h+1)^3 h}$$

$$=\sqrt{\pi}$$

20. Let A be a square matrix such that $AA^T=I$. Then $\frac{1}{2}A\left[\left(A+A^T\right)^2+\left(A-A^T\right)^2\right]$ is equal to

(1) A^2+I (2) A^3+I

(3) A^2+A^T (4) A^3+A^T

Ans. (4)

Sol. $AA^T=I=A^TA$

On solving given expression, we get

$$\frac{1}{2}A\left[A^2+(A^T)^2+2AA^T+A^2+(A^T)^2-2AA^T\right]$$

$$=A[A^2+(A^T)^2]=A^3+A^T$$

SECTION-B

21. Equation of two diameters of a circle are $2x-3y=5$ and $3x-4y=7$. The line joining the points $\left(-\frac{22}{7},-4\right)$ and $\left(-\frac{1}{7},3\right)$ intersects the circle at only one point $P(\alpha,\beta)$. Then $17\beta-\alpha$ is equal to

Ans. (2)

Sol. Centre of circle is (1, –1)

Equation of AB is 7x – 3y + 10 = 0 …(i)

Equation of CP is 3x + 7y + 4 = 0 …(ii)

Solving (i) and (ii)

$$\alpha=\frac{-41}{29},\beta=\frac{1}{29} \qquad \therefore 17\beta-\alpha=2$$

22. All the letters of the word "GTWENTY" are written in all possible ways with or without meaning and these words are written as in a dictionary. The serial number of the word "GTWENTY" IS

Ans. (553)

Sol. Words starting with E = 360

Words starting with GE = 60

Words starting with GN = 60

Words starting with GTE = 24

Words starting with GTN = 24

Words starting with GTT = 24

GTWENTY = 1

Total = 553

23. Let α,β be the roots of the equation $x^2-x+2=0$ with $\text{Im}(\alpha)>\text{Im}(\beta)$. Then $\alpha^6+\alpha^4+\beta^4-5\alpha^2$ is equal to

Ans. (13)

Sol. $\alpha^6+\alpha^4+\beta^4-5\alpha^2$

$= \alpha^4(\alpha-2)+\alpha^4-5\alpha^2+(\beta-2)^2$

$= \alpha^5-\alpha^4-5\alpha^2+\beta^2-4\beta+4$

$= \alpha^3(\alpha-2)-\alpha^4-5\alpha^2+\beta-2-4\beta+4$

$= -2\alpha^3-5\alpha^2-3\beta+2$

$= -2\alpha(\alpha-2)-5\alpha^2-3\beta+2$

$= -7\alpha^2+4\alpha-3\beta+2$

$= -7(\alpha-2)+4\alpha-3\beta+2$

$= -3\alpha-3\beta+16=-3(1)+16=13$

24. Let $f(x)=2^x-x^2, x\in R$. If m and n are respectively the number of points at which the curves y = f(x) and y = f '(x) intersects the x-axis, then the value of m + n is

Ans. (5)

Sol.

$\therefore$ m = 3

$f'(x)=2^x\ln 2-2x=0$

$2^x\ln 2=2x$

$\therefore$ n = 2

$\Rightarrow$ m + n = 5

25. If the points of intersection of two distinct conics $x^2+y^2=4b$ and $\frac{x^2}{16}+\frac{y^2}{b^2}=1$ lie on the curve $y^2=3x^2$, then $3\sqrt{3}$ times the area of the rectangle formed by the intersection points is __

Ans. (432)

Sol. Putting $y^2=3x^2$ in both the conics

We get $x^2=b$ and $\frac{b}{16}+\frac{3}{b}=1$

$\Rightarrow b=4,12$ (b = 4 is rejected because curves coincide)

$\therefore b=12$

Hence points of intersection are

$\left(\pm\sqrt{12},\pm 6\right) \Rightarrow$ area of rectangle = 432

26. If the solution curve $y = y\ x$ of the differential equation $(1+y^2)(1+\log_e x)dx + x\,dy = 0$, $x > 0$ passes through the point (1, 1) and $y(e) = \dfrac{\alpha - \tan\left(\frac{3}{2}\right)}{\beta + \tan\left(\frac{3}{2}\right)}$, then $\alpha + 2\beta$ is

Ans. (3)

Sol. $\int\left(\dfrac{1}{x} + \dfrac{\ln x}{x}\right)dx + \int \dfrac{dy}{1+y^2} = 0$

$\ln x + \dfrac{(\ln x)^2}{2} + \tan^{-1} y = C$

Put x = y = 1

$\therefore C = \dfrac{\pi}{4}$

$\Rightarrow \ln x + \dfrac{(\ln x)^2}{2} + \tan^{-1} y = \dfrac{\pi}{4}$

Put x = e

$\Rightarrow y = \tan\left(\dfrac{\pi}{4} - \dfrac{3}{2}\right) = \dfrac{1 - \tan\frac{3}{2}}{1 + \tan\frac{3}{2}}$

$\therefore \alpha = 1, \beta = 1$

$\Rightarrow \alpha + 2\beta = 3$

27. If the mean and variance of the data 65, 68, 58, 44, 48, 45, 60, $\alpha, \beta, 60$ where $\alpha > \beta$ are 56 and 66.2 respectively, then $\alpha^2 + \beta^2$ is equal to

Ans. (6344)

Sol. $\bar{x} = 56$

$\sigma^2 = 66.2$

$\Rightarrow \dfrac{\alpha^2 + \beta^2 + 25678}{10} - (56)^2 = 66.2$

$\therefore \alpha^2 + \beta^2 = 6344$

28. The area (in sq. units) of the part of circle $x^2 + y^2 = 169$ which is below the line $5x - y = 13$ is $\dfrac{\pi\alpha}{2\beta} - \dfrac{65}{2} + \dfrac{\alpha}{\beta}\sin^{-1}\left(\dfrac{12}{13}\right)$ where α, β are coprime numbers. Then $\alpha + \beta$ is equal to

Ans. (171)

Sol.

Area = $\displaystyle\int_{-13}^{12} \sqrt{169 - y^2}\,dy - \frac{1}{2} \times 25 \times 5$

$= \dfrac{\pi}{2} \times \dfrac{169}{2} - \dfrac{65}{2} + \dfrac{169}{2}\sin^{-1}\dfrac{12}{13}$

$\therefore \alpha + \beta = 171$

29. If $\dfrac{{}^{11}C_1}{2} + \dfrac{{}^{11}C_2}{3} + \ldots + \dfrac{{}^{11}C_9}{10} = \dfrac{n}{m}$ with gcd(n, m) = 1, then n +m is equal to

Ans. (2041)

Sol. $\displaystyle\sum_{r=1}^{9} \frac{{}^{11}C_r}{r+1}$

$= \dfrac{1}{12}\displaystyle\sum_{r=1}^{9} {}^{12}C_{r+1}$

$= \dfrac{1}{12}\left[2^{12} - 26\right] = \dfrac{2035}{6}$

$\therefore m + n = 2041$

30. A line with direction ratios 2, 1, 2 meets the lines x = y +2 = z and x + 2 = 2y = 2z respectively at the point P and Q. if the length of the perpendicular from the point (1, 2, 12) to the line PQ is l, then l^2 is

Ans. (65)

Sol. Let $P(t, t-2, t)$ and $Q(2s-2, s, s)$

D.R's of PQ are 2, 1, 2

$$\frac{2s-2-t}{2} = \frac{s-t+2}{1} = \frac{s-t}{2}$$

$\Rightarrow t = 6$ and $s = 2$

$\Rightarrow P(6,4,6)$ and $Q(2,2,2)$

$$PQ : \frac{x-2}{2} = \frac{y-2}{1} = \frac{z-2}{2} = \lambda$$

Let $F(2\lambda+2, \lambda+2, 2\lambda+2)$

$A(1,2,12)$

$\overrightarrow{AF} \cdot \overrightarrow{PQ} = 0$

$\therefore \lambda = 2$

So F(6,4, 6) and $AF = \sqrt{65}$

SECTION-A

31. In the given circuit, the breakdown voltage of the Zener diode is 3.0 V. What is the value of I_z?

(1) 3.3 mA (2) 5.5 mA
(3) 10 mA (4) 7 mA

Ans. (2)

Sol.

$V_z = 3V$

Let potential at B = 0 V

Potential at $E(V_E) = 10$ V

$V_C = V_A = 3$ V

$I_z + I_1 = I$

$$I = \frac{10-3}{1000} = \frac{7}{1000} A$$

$$I_1 = \frac{3}{2000} A$$

Therefore $I_z = \frac{7-1.5}{1000} = 5.5mA$

32. The electric current through a wire varies with time as $I = I_0 + \beta t$. where $I_0 = 20$ A and $\beta = 3$ A/s. The amount of electric charge crossed through a section of the wire in 20 s is :

(1) 80 C (2) 1000 C
(3) 800 C (4) 1600 C

Ans. (2)

Sol. Given that

Current $I = I_0 + \beta t$

$I_0 = 20A$

$\beta = 3A/s$

$I = 20 + 3t$

$$\frac{dq}{dt} = 20 + 3t$$

$$\int_0^q dq = \int_0^{20} (20+3t)dt$$

$$q = \int_0^{20} 20dt + \int_0^{20} 3tdt$$

$$q = \left[20t + \frac{3t^2}{2}\right]_0^{20} = 1000 \text{ C}$$

33. Given below are two statements:

Statement I : If a capillary tube is immersed first in cold water and then in hot water, the height of capillary rise will be smaller in hot water.

Statement II : If a capillary tube is immersed first in cold water and then in hot water, the height of capillary rise will be smaller in cold water.

In the light of the above statements, choose the ***most appropriate*** from the options given below

(1) Both **Statement I** and **Statement II** are true
(2) Both **Statement I** and **Statement II** are false
(3) **Statement I** is true but **Statement II** is false
(4) **Statement I** is false but **Statement II** is true

Ans. (3)

Sol. Surface tension will be less as temperature increases

$$h = \frac{2T\cos\theta}{\rho gr}$$

Height of capillary rise will be smaller in hot water and larger in cold water.

34. A convex mirror of radius of curvature 30 cm forms an image that is half the size of the object. The object distance is :

(1) –15 cm (2) 45 cm

(3) –45cm (4) 15 cm

Ans. **(1)**

Sol.

Given R = 30 cm

f = R/2 = +15 cm

Magnification (m) $= \pm\frac{1}{2}$

For convex mirror, virtual image is formed for real object.

Therefore, m is +ve

$$\frac{1}{2} = \frac{f}{f-u}$$

u = – 15 cm

35. Two charges of 5Q and –2Q are situated at the points (3a, 0) and (–5a, 0) respectively. The electric flux through a sphere of radius '4a' having center at origin is :

(1) $\frac{2Q}{\varepsilon_0}$ (2) $\frac{5Q}{\varepsilon_0}$

(3) $\frac{7Q}{\varepsilon_0}$ (4) $\frac{3Q}{\varepsilon_0}$

Ans. **(2)**

Sol.

5Q charge is inside the spherical region

flux through sphere = $\frac{5Q}{\varepsilon_0}$

36. A body starts moving from rest with constant acceleration covers displacement S_1 in first (p – 1) seconds and S_2 in first p seconds. The displacement $S_1 + S_2$ will be made in time :

(1) $(2p+1)$s

(2) $\sqrt{(2p^2-2p+1)}$s

(3) $(2p-1)$s

(4) $(2p^2-2p+1)$s

Ans. **(2)**

Sol. S_1 in first (p – 1) sec

S_2 in first p sec

$$S_1 = \frac{1}{2}a(p-1)^2$$

$$S_2 = \frac{1}{2}a(p)^2$$

$$S_1 + S_2 = \frac{1}{2}at^2$$

$(p-1)^2 + p^2 = t^2$

$$t = \sqrt{2p^2+1-2p}$$

37. The potential energy function (in J) of a particle in a region of space is given as $U = (2x^2 + 3y^3 + 2z)$. Here x, y and z are in meter. The magnitude of x - component of force (in N) acting on the particle at point P (1, 2, 3) m is :

(1) 2 (2) 6

(3) 4 (4) 8

Ans. **(3)**

Sol. Given $U = 2x^2 + 3y^3 + 2z$

$$F_x = -\frac{\partial U}{\partial x} = -4x$$

At x = 1 magnitude of F_x is 4N

38. The resistance $R = \frac{V}{I}$ where $V = (200 \pm 5)V$ and $I = (20 \pm 0.2)A$, the percentage error in the measurement of R is :

(1) 3.5%

(2) 7%

(3) 3%

(4) 5.5%

Ans. (1)

Sol. $R = \frac{V}{I}$

According to error analysis

$$\frac{dR}{R} = \frac{dV}{V} + \frac{dI}{I}$$

$$\frac{dR}{R} = \frac{5}{200} + \frac{0.2}{20}$$

$$\frac{dR}{R} = \frac{7}{200}$$

% error $\frac{dR}{R} \times 100 = \frac{7}{200} \times 100 = 3.5\%$

39. A block of mass 100 kg slides over a distance of 10 m on a horizontal surface. If the co-efficient of friction between the surfaces is 0.4, then the work done against friction (in J) is :

(1) 4200

(2) 3900

(3) 4000

(4) 4500

Ans. (3)

Sol. Given m = 100 kg

s = 10 m

$\mu = 0.4$

As $f = \mu mg = 0.4 \times 100 \times 10 = 400$ N

Now $W = f.s = 400 \times 10 = 4000$ J

40. Match List I with List II

	List I		List II
A.	$\oint \vec{B}.\vec{dl} = \mu_0 i_c + \mu_0 \varepsilon_0 \frac{d\phi_E}{dt}$	I.	Gauss' law for electricity
B.	$\oint \vec{E}.\vec{dl} = \frac{d\phi_B}{dt}$	II.	Gauss' law for magnetism
C.	$\oint \vec{E}.\vec{dA} = \frac{Q}{\varepsilon_0}$	III.	Faraday law
D.	$\oint \vec{B}.\vec{dA} = 0$	IV.	Ampere – Maxwell law

Chose the correct answer from the options given below

(1) A-IV, B-I, C-III, D-II

(2) A-II, B-III, C-I, D-IV

(3) A-IV, B-III, C-I, D-II

(4) A-I, B-II, C-III, D-IV

Ans. (3)

Sol. Ampere – Maxwell law

$$\rightarrow \oint \vec{B}.\vec{dl} = \mu_0 i_c + \mu_0 \varepsilon_0 \frac{d\phi_E}{dt}$$

Faraday law $\rightarrow \oint \vec{E}.\vec{dl} = \frac{d\phi_B}{dt}$

Gauss' law for electricity $\rightarrow \oint \vec{E}.\vec{dA} = \frac{Q}{\varepsilon_0}$

Gauss ' law for magnetism $\rightarrow \oint \vec{B}.\vec{dA} = 0$

41. If the radius of curvature of the path of two particles of same mass are in the ratio 3:4, then in order to have constant centripetal force, their velocities will be in the ratio of:

(1) $\sqrt{3} : 2$ (2) $1 : \sqrt{3}$

(3) $\sqrt{3} : 1$ (4) $2 : \sqrt{3}$

Ans. (1)

Sol. Given $m_1 = m_2$

and $\frac{r_1}{r_2} = \frac{3}{4}$

As centripetal force $F = \frac{mv^2}{r}$

In order to have constant (same in this question) centripetal force

$F_1 = F_2$

$$\frac{m_1 v_1^2}{r_1} = \frac{m_2 v_2^2}{r_2}$$

$$\Rightarrow \frac{v_1}{v_2} = \sqrt{\frac{r_1}{r_2}} = \frac{\sqrt{3}}{2}$$

42. A galvanometer having coil resistance 10 Ω shows a full scale deflection for a current of 3mA. For it to measure a current of 8A, the value of the shunt should be:

(1) $3 \times 10^{-3}\Omega$ (2) $4.85 \times 10^{-3}\Omega$

(3) $3.75 \times 10^{-3}\Omega$ (4) $2.75 \times 10^{-3}\Omega$

Ans. **(3)**

Sol. Given G = 10 Ω

$I_g = 3mA$

I = 8A

In case of conversion of galvanometer into ammeter.

We have $I_g G = (I - I_g)S$

$$S = \frac{I_g G}{I - I_g}$$

$$S = \frac{(3\times10^{-3})10}{8 - 0.003} = 3.75 \times 10^{-3}\Omega$$

43. The de-Broglie wavelength of an electron is the same as that of a photon. If velocity of electron is 25% of the velocity of light, then the ratio of K.E. of electron and K.E. of photon will be:

(1) $\frac{1}{1}$ (2) $\frac{1}{8}$

(3) $\frac{8}{1}$ (4) $\frac{1}{4}$

Ans. **(2)**

Sol. For photon

$$E_P = \frac{hc}{\lambda_p} \Rightarrow \lambda_p = \frac{hc}{E_P}$$

For electron

$$\lambda_e = \frac{h}{m_e v_e} = \frac{hv_e}{2K_e}$$

Given $v_e = 0.25$ c

$$\lambda_e = \frac{h \times 0.25c}{2K_e} = \frac{hc}{8K_e}$$

Also $\lambda_p = \lambda_e$

$$\frac{hc}{E_p} = \frac{hc}{8K_e}$$

$$\frac{K_e}{E_p} = \frac{1}{8}$$

44. The deflection in moving coil galvanometer falls from 25 divisions to 5 division when a shunt of 24Ω is applied. The resistance of galvanometer coil will be :

(1) 12Ω (2) 96Ω

(3) 48Ω (4) 100Ω

Ans. **(2)**

Sol. Let x = current/division

After applying shunt

Now 5x × G = 20x × 24

G = 4 × 24

G = 96Ω

45. A biconvex lens of refractive index 1.5 has a focal length of 20 cm in air. Its focal length when immersed in a liquid of refractive index 1.6 will be:

(1) – 16 cm

(2) – 160 cm

(3) + 160 cm

(4) + 16 cm

Ans. (2)

Sol. $\mu_l = 1.5$

$\mu_m = 1.6$

$f_a = 20$ cm

As $\frac{f_m}{f_a} = \frac{(\mu_l - 1)\mu_m}{(\mu_l - \mu_m)}$

$\frac{f_m}{20} = \frac{(1.5-1)1.6}{(1.5-1.6)}$

$f_m = -160$ cm

46. A thermodynamic system is taken from an original state A to an intermediate state B by a linear process as shown in the figure. It's volume is then reduced to the original value from B to C by an isobaric process. The total work done by the gas from A to B and B to C would be :

(1) 33800 J (2) 2200 J

(3) 600 J (4) 1200 J

Ans. (BONUS)

Sol.

Work done AB = $\frac{1}{2}$ (8000 + 6000) Dyne/cm^2 × 4m^3 = (6000Dyne/cm^2) × 4m^3

Work done BC = –(4000 Dyne/cm^2) × 4m^3

Total work done = 2000 Dyne/cm^2 × 4m^3

$$= 2\times10^3 \times \frac{1}{10^5}\frac{N}{cm^2}\times 4m^3$$

$$= 2\times10^{-2} \times \frac{N}{10^{-4}m^2}\times 4m^3$$

$= 2 \times 10^2 \times 4$ Nm = 800 J

47. At what distance above and below the surface of the earth a body will have same weight, (take radius of earth as R.)

(1) $\sqrt{5}R - R$ (2) $\frac{\sqrt{3}R - R}{2}$

(3) $\frac{R}{2}$ (4) $\frac{\sqrt{5}R - R}{2}$

Ans. (4)

Sol. $g_p = \frac{gR^2}{(R+h)^2}$

$g_q = g\left(1 - \frac{h}{R}\right)$

$g_p = g_q$

$$\frac{g}{\left(1+\frac{h}{R}\right)^2} = g\left(1-\frac{h}{R}\right)$$

$$\left(1-\frac{h^2}{R^2}\right)\left(1+\frac{h}{R}\right) = 1$$

Take $\frac{h}{R} = x$

So

$x^3 - x + x^2 = 0$

$x = \frac{\sqrt{5}-1}{2}$

$h = \frac{R}{2}\left(\sqrt{5}-1\right)$

48. A capacitor of capacitance 100 μF is charged to a potential of 12 V and connected to a 6.4 mH inductor to produce oscillations. The maximum current in the circuit would be :

(1) 3.2 A (2) 1.5 A
(3) 2.0 A (4) 1.2 A

Ans. (2)

Sol. By energy conservation

$$\frac{1}{2}CV^2 = \frac{1}{2}LI_{max}^2$$

$$I_{max} = \sqrt{\frac{C}{L}}\,V$$

$$= \sqrt{\frac{100\times10^{-6}}{6.4\times10^{-3}}}\times12$$

$$= \frac{12}{8} = \frac{3}{2} = 1.5\text{ A}$$

49. The explosive in a Hydrogen bomb is a mixture of ${}_1H^2$, ${}_1H^3$ and ${}_3Li^6$ in some condensed form. The chain reaction is given by

${}_3Li^6 + {}_0n^1 \rightarrow {}_2He^4 + {}_1H^3$

${}_1H^2 + {}_1H^3 \rightarrow {}_2He^4 + {}_0n^1$

During the explosion the energy released is approximately

[Given : M(Li) = 6.01690 amu. M (${}_1H^2$) = 2.01471 amu. M (${}_2He^4$) = 4.00388amu, and 1 amu = 931.5 MeV]

(1) 28.12 MeV (2) 12.64 MeV
(3) 16.48 MeV (4) 22.22 MeV

Ans. (4)

Sol. ${}_3Li^6 + {}_0n^1 \rightarrow {}_2He^4 + {}_1H^3$

${}_1H^2 + {}_1H^3 \rightarrow {}_2He^4 + {}_0n^1$

$${}_3Li^6 + {}_1H^2 \rightarrow 2\left({}_2He^4\right)$$

Energy released in process

$Q = \Delta mc^2$

$Q = [M(Li) + M({}_1H^2) - 2 \times M({}_2He^4)] \times 931.5$ MeV

$Q = [6.01690 + 2.01471 - 2 \times 4.00388] \times 931.5$ MeV

$Q = 22.216$ MeV

$Q = 22.22$ MeV

50. Two vessels A and B are of the same size and are at same temperature. A contains 1g of hydrogen and B contains 1g of oxygen. P_A and P_B are the pressures of the gases in A and B respectively, then $\frac{P_A}{P_B}$ is :

(1) 16 (2) 8 (3) 4 (4) 32

Ans. (1)

Sol. $$\frac{P_AV_A}{P_BV_B} = \frac{n_ART_A}{n_BRT_B}$$

Given $V_A = V_B$

And $T_A = T_B$

$$\frac{P_A}{P_B} = \frac{n_A}{n_B}$$

$$\frac{P_A}{P_B} = \frac{1/2}{1/32} = 16$$

SECTION-B

51. When a hydrogen atom going from n = 2 to n = 1 emits a photon, its recoil speed is $\frac{x}{5}$ *m/s*. Where x = _____ . (Use : mass of hydrogen atom = 1.6×10^{-27} kg)

Ans. (17)

Sol.

ΔE = 10.2 eV

Recoil speed(v) = $\frac{\Delta E}{mc}$

$$= \frac{10.2\text{eV}}{1.6\times10^{-27}\times3\times10^{8}}$$

$$= \frac{10.2\times1.6\times10^{-19}}{1.6\times10^{-27}\times3\times10^{8}}$$

$$v = 3.4\text{ m/s} = \frac{17}{5}\text{ m/s}$$

Therefore, x = 17

52. A ball rolls off the top of a stairway with horizontal velocity u. The steps are 0.1 m high and 0.1 m wide. The minimum velocity u with which that ball just hits the step 5 of the stairway will be $\sqrt{x}$ ms^{-1} where x = _____[use g = 10 m/s^2].

Ans. (2)

Sol.

The ball needs to just cross 4 steps to just hit 5th step

Therefore, horizontal range (R) = 0.4 m

R = u.t

Similarly, in vertical direction

$$h = \frac{1}{2}gt^2$$

$$0.4 = \frac{1}{2}gt^2$$

$$0.4 = \frac{1}{2}g\left(\frac{0.4}{u}\right)^2$$

$u^2 = 2$

$u = \sqrt{2}$ m/s

Therefore, x = 2

53. A square loop of side 10 cm and resistance 0.7Ω is placed vertically in east-west plane. A uniform magnetic field of 0.20 T is set up across the plane in north east direction. The magnetic field is decreased to zero in 1 s at a steady rate. Then, magnitude of induced emf is $\sqrt{x} \times 10^{-3}$V. The value of x is _____.

Ans. (2)

Sol.

$$\vec{A} = (0.1)^2\,\hat{j}$$

$$\vec{B} = \frac{0.2}{\sqrt{2}}\hat{i} + \frac{0.2}{\sqrt{2}}\hat{j}$$

Magnitude of induced emf

$$e = \frac{\Delta\phi}{\Delta t} = \frac{\vec{B}\cdot\vec{A} - 0}{1} = \sqrt{2}\times 10^{-3}\,V$$

54. A cylinder is rolling down on an inclined plane of inclination 60°. It's acceleration during rolling down will be $\frac{x}{\sqrt{3}}$ *m/s*^{2}, where x = _____. (use g = 10 m/s^2).

Ans. (10)

Sol.

For rolling motion, $a = \dfrac{g\sin\theta}{1 + \dfrac{I_{cm}}{MR^2}}$

$$a = \frac{g\sin\theta}{1 + \frac{1}{2}}$$

$$= \frac{2\times 10\times\frac{\sqrt{3}}{2}}{3}$$

$$= \frac{10}{\sqrt{3}}$$

Therefore x = 10

55. The magnetic potential due to a magnetic dipole at a point on its axis situated at a distance of 20 cm from its center is 1.5×10^{-5}Tm. The magnetic moment of the dipole is_____Am^2.

(Given : $\frac{\mu_0}{4\pi} = 10^{-7}TmA^{-1}$)

Ans. (6)

Sol. $V = \frac{\mu_0}{4\pi}\frac{M}{r^2}$

$$\Rightarrow 1.5\times10^{-5} = 10^{-7}\times\frac{M}{\left(20\times10^{-2}\right)^2}$$

$$\Rightarrow M = \frac{1.5\times10^{-5}\times20\times20\times10^{-4}}{10^{-7}}$$

$M = 1.5 \times 4 = 6$

56. In a double slit experiment shown in figure, when light of wavelength 400 nm is used, dark fringe is observed at P. If D = 0.2 m. the minimum distance between the slits S_1 and S_2 is _____ mm.

Ans. (0.20)

Sol. Path difference for minima at P

$$2\sqrt{D^2+d^2} - 2D = \frac{\lambda}{2}$$

$$\therefore \sqrt{D^2+d^2} - D = \frac{\lambda}{4}$$

$$\therefore \sqrt{D^2+d^2} = \frac{\lambda}{4} + D$$

$$\Rightarrow D^2 + d^2 = D^2 + \frac{\lambda^2}{16} + \frac{D\lambda}{2}$$

$$\Rightarrow d^2 = \frac{D\lambda}{2} + \frac{\lambda^2}{16}$$

$$\Rightarrow d^2 = \frac{0.2\times400\times10^{-9}}{2} + \frac{4\times10^{-14}}{4}$$

$$\Rightarrow d^2 \approx 400\times10^{-10}$$

$$\therefore d = 20\times10^{-5}$$

$$\Rightarrow d = 0.20\,mm$$

57. A 16Ω wire is bend to form a square loop. A 9V battery with internal resistance 1Ω is connected across one of its sides. If a 4μF capacitor is connected across one of its diagonals, the energy stored by the capacitor will be $\frac{x}{2}\mu J$. where x = _____.

Ans. (81)

Sol.

$$I = \frac{V}{R_{eq}} \quad I = \frac{V}{R_{eq}} = \frac{9}{1+\frac{12\times4}{12+4}} = \frac{9}{4}$$

$$I_1 = \frac{9}{4}\times\frac{4}{16} = \frac{9}{16}$$

$$V_A - V_B = I_1\times8 = \frac{9}{16}\times8 = \frac{9}{2}\ \mathbf{V}$$

$$\therefore U = \frac{1}{2}\times4\times\frac{81}{4}\mu J$$

$$\therefore U = \frac{81}{2}\mu J$$

$$\therefore x = 81$$

58. When the displacement of a simple harmonic oscillator is one third of its amplitude, the ratio of total energy to the kinetic energy is $\frac{x}{8}$, where x = _____.

Ans. (9)

Sol. Let total energy = E = $\frac{1}{2}KA^2$

$$U = \frac{1}{2}K\left(\frac{A}{3}\right)^2 = \frac{KA^2}{2\times9} = \frac{E}{9}$$

$$KE = E - \frac{E}{9} = \frac{8E}{9}$$

$$\text{Ratio } \frac{\text{Total}}{KE} = \frac{E}{\frac{8E}{9}} = \frac{9}{8}$$

x = 9

59. An electron is moving under the influence of the electric field of a uniformly charged infinite plane sheet S having surface charge density $+\sigma$. The electron at t = 0 is at a distance of 1 m from S and has a speed of 1 m/s. The maximum value of σ if the electron strikes S at t = 1 s is $\alpha\left[\frac{m\in_0}{e}\right]\frac{C}{m^2}$ the value of α is

Ans. **(8)**

Sol. $u = 1\ m/s;\ a = -\frac{\sigma e}{2\varepsilon_0 m}$

$t = 1\ s$

$S = -1\ m$

Using $S = ut + \frac{1}{2}at^2$

$-1 = 1\times 1 - \frac{1}{2}\times\frac{\sigma e}{2\varepsilon_0 m}\times(1)^2$

$\therefore \sigma = 8\frac{\varepsilon_0 m}{e}$

$\therefore \alpha = 8$

60. In a test experiment on a model aeroplane in wind tunnel, the flow speeds on the upper and lower surfaces of the wings are 70 ms^{-1} and 65 ms^{-1} respectively. If the wing area is 2 m^2 the lift of the wing is _____ N.

(Given density of air = 1.2 kg m^{-3})

Ans. **(810)**

Sol. $F = \frac{1}{2}\rho\left(v_1^2 - v_2^2\right)A$

$F = \frac{1}{2}\times 1.2\times\left(70^2 - 65^2\right)\times 2$

$= 810\ N$

SECTION-A

61. Given below are two statements : one is labelled as **Assertion A** and the other is labelled as **Reason R**:

Assertion A: The first ionisation enthalpy decreases across a period.

Reason R: The increasing nuclear charge outweighs the shielding across the period.

In the light of the above statements, choose the most appropriate from the options given below:

(1) Both A and R are true and R is the correct explanation of A

(2) A is true but R is false

(3) A is false but R is true

(4) Both A and R are true but R is NOT the correct explanation of A

Ans. (3)

Sol. First ionisation energy **increases** along the period. Along the period Z increases which outweighs the shielding effect

62. Match List I with List II

LIST-I (Substances)	**LIST-II (Element Present)**
A.Ziegler catalyst	I.Rhodium
B.Blood Pigment	II. Cobalt
C.Wilkinson catalyst	III.Iron
D.Vitamin B_{12}	IV.Titanium

Choose the correct answer from the options given below:

(1) A-II, B-IV, C-I, D-III

(2) A-II, B-III, C-IV, D-I

(3) A-III, B-II, C-IV, D-I

(4) A-IV, B-III, C-I, D-II

Ans. (4)

Sol. Ziegler catalyst $\rightarrow$ Titanium

Blood pigment $\rightarrow$ Iron

Wilkinson catalyst $\rightarrow$ Rhodium

Vitamin $B_{12} \rightarrow$ Cobalt

63. In chromyl chloride test for confirmation of Cl^- ion, a yellow solution is obtained. Acidification of the solution and addition of amyl alcohol and 10% H_2O_2 turns organic layer blue indicating formation of chromium pentoxide. The oxidation state of chromium in that is

(1)+6 (2)+5

(3)+10 (4)+3

Ans. (1)

Sol.

$$Cl^- + K_2Cr_2O_7 + H_2SO_4 \rightarrow CrO_2Cl_2 \xrightarrow{\text{Basic medium}} \underset{\text{yellow solution}}{CrO_4^{2-}} + Cl^-$$

$$\underset{\text{yellow solution}}{CrO_4^{2-}} \xrightarrow[3.\,10\%\ H_2O_2]{\substack{1.\text{Acidification} \\ 2.\text{Amyl alcohol}}} \underset{\text{blue compound}}{CrO_5}$$

64. The difference in energy between the actual structure and the lowest energy resonance structure for the given compound is

(1) electromeric energy

(2) resonance energy

(3) ionization energy

(4) hyperconjugation energy

Ans. (2)

Sol. The difference in energy between the actual structure and the lowest energy resonance structure for the given compound is known as resonance energy.

65. Given below are two statements :

Statement I : The electronegativity of group 14 elements from Si to Pb gradually decreases.

Statement II : Group 14 contains non-metallic, metallic, as well as metalloid elements.

In the light of the above statements, choose the most appropriate from the options given below :

(1) Statement I is false but Statement II is true

(2) Statement I is true but Statement II is false

(3) Both Statement I and Statement II are true

(4) Both Statement I and Statement II are false

Ans. (1)

Sol.

Gr-14	EN
C	2.5
Si	1.8
Ge	1.8
Sn	1.8
Pb	1.9

The electronegativity values for elements from Si to Pb are almost same. So Statement I is false.

66. The correct set of four quantum numbers for the valence electron of rubidium atom (Z = 37) is:

(1) $5, 0, 0, +\frac{1}{2}$ (2) $5, 0, 1, +\frac{1}{2}$

(3) $5, 1, 0, +\frac{1}{2}$ (4) $5, 1, 1, +\frac{1}{2}$

Ans. (1)

Sol. Rb = $[Kr]5s^1$

n = 5

$l = 0$

m = 0

s = +½ or –½

67. The major product(P) in the following reaction is

(1)

(2)

(3)

(4)

Ans. (4)

Sol.

68. The arenium ion which is not involved in the bromination of Aniline is .

(1)

(2)

(3)

(4)

Ans. (3)

Sol. Since $-\ddot{N}H_2$ group is o/p directing hence arenium ion will not be formed by attack at meta position i.e.

Hence Answer is (3)

69. Appearance of blood red colour, on treatment of the sodium fusion extract of an organic compound with $FeSO_4$ in presence of concentrated H_2SO_4 indicates the presence of element/s

(1) Br (2) N

(3) N and S (4) S

Ans. (3)

Sol. $Fe^{2+} \xrightarrow[Conc.H_2SO_4]{H^+} Fe^{+3}$

$Fe^{+3} \xrightarrow{^-SCN} Fe(SCN)_3$ (blood red colour)

Appearance of blood red colour indicates presence of both nitrogen and sulphur.

70. Given below are two statements : one is labelled as Assertion A and the other is labelled as Reason R :

Assertion A : Aryl halides cannot be prepared by replacement of hydroxyl group of phenol by halogen atom.

Reason R : Phenols react with halogen acids violently.

In the light of the above statements, choose the most appropriate from the options given below:

(1) Both A and R are true but R is NOT the correct explanation of A

(2) A is false but R is true

(3) A is true but R is false

(4) Both A and R are true and R is the correct explanation of A

Ans. (3)

Sol. Assertion (A): Given statement is correct because in phenol hydroxyl group cannot be replaced by halogen atom.

Reason (R) :

Given reason is false

Hence Assertion (A) is correct but Reason (R) is false

71. Identify product A and product B :

(1) A :

(2) A :

(3) A :

(4) A :

Ans. (4)

Sol.

Hence correct Ans. (4)

72. Identify the incorrect pair from the following :

(1) Fluorspar- BF_3

(2) Cryolite-Na_3AlF_6

(3) Fluoroapatite-$3Ca_3(PO_4)_2.CaF_2$

(4) Carnallite-$KCl.MgCl_2.6H_2O$

Ans. (1)

Sol. (1) Fluorspar is CaF_2

73. The interaction between π bond and lone pair of electrons present on an adjacent atom is responsible for

(1) Hyperconjugation

(2) Inductive effect

(3) Electromeric effect

(4) Resonance effect

Ans. (4)

Sol. It is a type of conjugation responsible for resonance.

74. $KMnO_4$ decomposes on heating at 513K to form O_2 along with

(1) MnO_2 & K_2O_2

(2) K_2MnO_4 & Mn

(3) Mn & KO_2

(4) K_2MnO_4 & MnO_2

Ans. (4)

Sol. $KMnO_4 \xrightarrow{\Delta} K_2MnO_4 + MnO_2 + O_2$

75. In which one of the following metal carbonyls, CO forms a bridge between metal atoms?

(1) $[Co_2(CO)_8]$ (2) $[Mn_2(CO)_{10}]$

(3) $[Os_3(CO)_{12}]$ (4) $[Ru_3(CO)_{12}]$

Ans. (1)

Sol. (1)

(2)

(3)

(4)

76. Type of amino acids obtained by hydrolysis of proteins is :

(1) β (2) α

(3) δ (4) γ

Ans. (2)

Sol. Proteins are natural polymers composed of α-amino acids which are connected by peptide linkages.

Hence proteins upon acidic hydrolysis produce α-amino acids.

77. The final product A formed in the following multistep reaction sequence is

(1)

(2)

(3)

(4)

Ans. (1)

Sol.

78. Which of the following is **not** correct?

(1) ΔG is negative for a spontaneous reaction

(2) ΔG is positive for a spontaneous reaction

(3) ΔG is zero for a reversible reaction

(4) ΔG is positive for a non-spontaneous reaction

Ans. (2)

Sol. $(\Delta G)_{P,T} = (+)ve$ for non-spontaneous process

79. Chlorine undergoes disproportionation in alkaline medium as shown below :

a $Cl_2(g)$ + b $OH^-(aq)$ → c $ClO^-(aq)$ + d $Cl^-(aq)$ + e $H_2O(l)$

The values of a, b, c and d in a balanced redox reaction are respectively :

(1) 1, 2, 1 and 1 (2) 2, 2, 1 and 3

(3) 3, 4, 4 and 2 (4) 2, 4, 1 and 3

Ans. (1)

Sol.

$$\Rightarrow Cl_2 + 2\bar{O}H \longrightarrow Cl^- + ClO^- + H_2O$$

80. In alkaline medium. MnO_4^- oxidises I^- to

(1) IO_4^- (2) IO^-

(3) I_2 (4) IO_3^-

Ans. (4)

Sol. $2MnO_4^- + H_2O + I^- \xrightarrow{\text{alkaline medium}} 2MnO_2 + 2OH^- + IO_3^-$

SECTION-B

81. Number of compounds with one lone pair of electrons on central atom amongst following is _

O_3, H_2O, SF_4, ClF_3, NH_3, BrF_5, XeF_4

Ans. (4)

Sol.

82. The mass of zinc produced by the electrolysis of zinc sulphate solution with a steady current of 0.015 A for 15 minutes is____ $\times 10^{-4}$ g.

(Atomic mass of zinc = 65.4 amu)

Ans. (45.75) or (46)

Sol. $Zn^{+2} + 2e^- \longrightarrow Zn$

$W = Z \times i \times t$

$$= \frac{65.4}{2 \times 96500} \times 0.015 \times 15 \times 60$$

$$= 45 \cdot 75 \times 10^{-4} \text{gm}$$

83. For a reaction taking place in three steps at same temperature, overall rate constant $K = \frac{K_1K_2}{K_3}$. If Ea_1, Ea_2 and Ea_3 are 40, 50 and 60 kJ/mol respectively, the overall Ea is____kJ/mol.

Ans. (30)

Sol. $$K = \frac{K_1 \cdot K_2}{K_3} = \frac{A_1 \cdot A_2}{A_3} \cdot e^{-\frac{(E_{a_1} + E_{a_2} - E_{a_3})}{RT}}$$

$$A \cdot e^{-E_a/RT} = \frac{A_1A_2}{A_3} \cdot e^{-\frac{(E_{a_1} + E_{a_2} - E_{a_3})}{RT}}$$

$$E_a = E_{a_1} + E_{a_2} - E_{a_3} = 40 + 50 - 60 = 30 \text{ kJ / mole.}$$

84. For the reaction $N_2O_4(g) \rightleftharpoons 2NO_2(g)$, K_p= 0.492 atm at 300K. K_c for the reaction at same temperature is___ $\times 10^{-2}$.

(Given : R = 0.082 L atm mol^{-1} K^{-1})

Ans. (2)

Sol. $K_P = K_C \cdot (RT)^{\Delta n_g}$

$\Delta n_g = 1$

$$\Rightarrow K_c = \frac{K_P}{RT} = \frac{0.492}{0.082 \times 300} = 2 \times 10^{-2}$$

85. A solution of H_2SO_4 is 31.4% H_2SO_4 by mass and has a density of 1.25g/mL.The molarity of the H_2SO_4 solution is____M (nearest integer)

[Given molar mass of H_2SO_4 = 98g mol^{-1}]

Ans. (4)

Sol. $$M = \frac{n_{solute}}{V} \times 1000$$

$$= \frac{\left(\frac{31.4}{98}\right)}{\left(\frac{100}{1.25}\right)} \times 1000$$

$$= 4.005 \approx 4$$

86. The osmotic pressure of a dilute solution is 7×10^5 Pa at 273K. Osmotic pressure of the same solution at 283K is____$\times 10^4$ Nm^{-2}.

Ans. (72.56) or (73)

Sol. $\pi = CRT$

$$\Rightarrow \frac{\pi_1}{\pi_2} = \frac{T_1}{T_2}$$

$$\Rightarrow \pi_2 = \frac{\pi_1 T_2}{T_1} = \frac{7 \times 10^5 \times 283}{273}$$

$$= 72.56 \times 10^4 \text{ Nm}^{-2}$$

87. Number of compounds among the following which contain sulphur as heteroatom is____.

Furan,Thiophene, Pyridine, Pyrrole, Cysteine, Tyrosine

Ans. (2)

Sol.

Thiophene , Cysteine

88. The number of species from the following which are paramagnetic and with bond order equal to one is____.

$H_2, He_2^+, O_2^+, N_2^{2-}, O_2^{2-}, F_2, Ne_2^+, B_2$

Ans. (1)

Sol.

	Magnetic behaviour	Bond order
H_2	Diamagnetic	1
He_2^+	Paramagnetic	0.5
O_2^+	Paramagnetic	2.5
N_2^{2-}	Paramagnetic	2
O_2^{2-}	Diamagnetic	1
F_2	Diamagnetic	1
Ne_2^+	Paramagnetic	0.5
B_2	Paramagnetic	1

89. From the compounds given below, number of compounds which give positive Fehling's test is____.

Benzaldehyde, Acetaldehyde, Acetone,

Acetophenone,Methanal, 4-nitrobenzaldehyde, cyclohexane carbaldehyde.

Ans. (3)

Sol. Acetaldehyde (CH_3CHO),Methanal(HCHO), and cyclohexane carbaldehyde

.

90.

Consider the given reaction. The total number of oxygen atoms present per molecule of the product (P) is____.

Ans. (1)

Sol.

Hence total number of oxygen atom present per molecule

is 1.

FINAL JEE–MAIN EXAMINATION – JANUARY, 2024

(Held On Saturday 27th January, 2024) **TIME : 3 : 00 PM to 6 : 00 PM**

MATHEMATICS — TEST PAPER WITH SOLUTION

SECTION-A

1. Considering only the principal values of inverse trigonometric functions, the number of positive real values of x satisfying $\tan^{-1}(x)+\tan^{-1}(2x)=\frac{\pi}{4}$ is :

(1) More than 2

(2) 1

(3) 2

(4) 0

Ans. (2)

Sol. $\tan^{-1}x+\tan^{-1}2x=\frac{\pi}{4}$; $x>0$

$\Rightarrow \tan^{-1}2x=\frac{\pi}{4}-\tan^{-1}x$

Taking tan both sides

$\Rightarrow 2x=\frac{1-x}{1+x}$

$\Rightarrow 2x^2+3x-1=0$

$x=\frac{-3\pm\sqrt{9+8}}{8}=\frac{-3\pm\sqrt{17}}{8}$

Only possible $x=\frac{-3+\sqrt{17}}{8}$

2. Consider the function $f:(0,2)\to R$ defined by $f(x)=\frac{x}{2}+\frac{2}{x}$ and the function g(x) defined by

$$g(x)=\begin{cases}\min\{f(t)\}, & 0<t\le x \text{ and } 0<x\le 1\\ \frac{3}{2}+x, & 1<x<2\end{cases}$$

. Then

(1) g is continuous but not differentiable at x = 1

(2) g is not continuous for all $x\in(0,2)$

(3) g is neither continuous nor differentiable at x = 1

(4) g is continuous and differentiable for all $x\in(0,2)$

Ans. (1)

Sol. $f:(0,2)\to R$; $f(x)=\frac{x}{2}+\frac{2}{x}$

$f'(x)=\frac{1}{2}-\frac{2}{x^2}$

$\therefore f(x)$ is decreasing in domain.

$$g(x)=\begin{cases}\frac{x}{2}+\frac{2}{x} & 0<x\le 1\\ \frac{3}{2}+x & 1<x<2\end{cases}$$

3. Let the image of the point (1, 0, 7) in the line $\frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{3}$ be the point (α, β, γ). Then which one of the following points lies on the line passing through (α, β, γ) and making angles $\frac{2\pi}{3}$ and $\frac{3\pi}{4}$ with y-axis and z-axis respectively and an acute angle with x-axis ?

(1) $\left(1,-2,1+\sqrt{2}\right)$

(2) $\left(1,2,1-\sqrt{2}\right)$

(3) $\left(3,4,3-2\sqrt{2}\right)$

(4) $\left(3,-4,3+2\sqrt{2}\right)$

Ans. (3)

Sol. $L_1 = \frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3} = \lambda$

$M(\lambda, 1+2\lambda, 2+3\lambda)$

$\overrightarrow{PM} = (\lambda-1)\hat{i} + (1+2\lambda)\hat{j} + (3\lambda-5)\hat{k}$

$\overrightarrow{PM}$ is perpendicular to line L_1

$\overrightarrow{PM}.\vec{b} = 0 \quad (\vec{b} = \hat{i} + 2\hat{j} + 3\hat{k})$

$\Rightarrow \lambda - 1 + 4\lambda + 2 + 9\lambda - 15 = 0$

$14\lambda = 14 \Rightarrow \lambda = 1$

$\therefore M = (1,3,5)$

$\vec{Q} = 2\vec{M} - \vec{P}$ [M is midpoint of $\vec{P}$ & $\vec{Q}$]

$\vec{Q} = 2\hat{i} + 6\hat{j} + 10\hat{k} - \hat{i} - 7\hat{k}$

$\vec{Q} = \hat{i} + 6\hat{j} + 3\hat{k}$

$\therefore (\alpha, \beta, \gamma) = (1,6,3)$

Required line having direction cosine (l, m, n)

$l^2 + m^2 + n^2 = 1$

$\Rightarrow l^2 + \left(-\frac{1}{2}\right)^2 + \left(-\frac{1}{\sqrt{2}}\right)^2 = 1$

$l^2 = \frac{1}{4}$

$\therefore l = \frac{1}{2}$ [Line make acute angle with x-axis]

Equation of line passing through (1, 6, 3) will be

$\vec{r} = (\hat{i} + 6\hat{j} + 3\hat{k}) + \mu\left(\frac{1}{2}\hat{i} - \frac{1}{2}\hat{j} - \frac{1}{\sqrt{2}}\hat{k}\right)$

Option (3) satisfying for $\mu = 4$

4. Let R be the interior region between the lines $3x - y + 1 = 0$ and $x + 2y - 5 = 0$ containing the origin. The set of all values of a, for which the points $(a^2, a + 1)$ lie in R, is :

(1) $(-3,-1) \cup \left(-\frac{1}{3}, 1\right)$

(2) $(-3,0) \cup \left(\frac{1}{3}, 1\right)$

(3) $(-3,0) \cup \left(\frac{2}{3}, 1\right)$

(4) $(-3,-1) \cup \left(\frac{1}{3}, 1\right)$

Ans. **(2)**

Sol. $P(a^2, a + 1)$

$L_1 = 3x - y + 1 = 0$

Origin and P lies same side w.r.t. L_1

$\Rightarrow L_1(0) \,.\, L_1(P) > 0$

$\therefore\ 3(a^2) - (a + 1) + 1 > 0$

$\Rightarrow 3a^2 - a > 0$

$a \in (-\infty, 0) \cup \left(\frac{1}{3}, \infty\right)$ …….(1)

Let $L_2 : x + 2y - 5 = 0$

Origin and P lies same side w.r.t. L_2

$\Rightarrow L_2(0).L_2(P) > 0$

$\Rightarrow a^2 + 2(a+1) - 5 < 0$

$\Rightarrow a^2 + 2a - 3 < 0$

$\Rightarrow (a+3)(a-1) < 0$

$\therefore a \in (-3, 1)$ …….(2)

Intersection of (1) and (2)

$a \in (-3, 0) \cup \left(\frac{1}{3}, 1\right)$

5. The 20th term from the end of the progression $20, 19\frac{1}{4}, 18\frac{1}{2}, 17\frac{3}{4}, ..., -129\frac{1}{4}$ is :-

(1) -118

(2) -110

(3) -115

(4) -100

Ans. (3)

Sol. $20, 19\frac{1}{4}, 18\frac{1}{2}, 17\frac{3}{4}, ..., -129\frac{1}{4}$

This is A.P. with common difference

$d_1 = -1 + \frac{1}{4} = -\frac{3}{4}$

$-129\frac{1}{4}, ..., 19\frac{1}{4}, 20$

This is also A.P. $a = -129\frac{1}{4}$ and $d = \frac{3}{4}$

Required term =

$-129\frac{1}{4} + (20-1)\left(\frac{3}{4}\right)$

$= -129 - \frac{1}{4} + 15 - \frac{3}{4} = -115$

6. Let $f : R - \left\{\frac{-1}{2}\right\} \to R$ and $g : R - \left\{\frac{-5}{2}\right\} \to R$ be defined as $f(x) = \frac{2x+3}{2x+1}$ and $g(x) = \frac{|x|+1}{2x+5}$. Then the domain of the function fog is :

(1) $R - \left\{-\frac{5}{2}\right\}$

(2) R

(3) $R - \left\{-\frac{7}{4}\right\}$

(4) $R - \left\{-\frac{5}{2}, -\frac{7}{4}\right\}$

Ans. (1)

Sol. $f(x) = \frac{2x+3}{2x+1}; x \neq -\frac{1}{2}$

$g(x) = \frac{|x|+1}{2x+5}, x \neq -\frac{5}{2}$

Domain of f(g(x))

$f(g(x)) = \frac{2g(x)+3}{2g(x)+1}$

$x \neq -\frac{5}{2}$ and $\frac{|x|+1}{2x+5} \neq -\frac{1}{2}$

$x \in R - \left\{-\frac{5}{2}\right\}$ and $x \in R$

$\therefore$ Domain will be $R - \left\{-\frac{5}{2}\right\}$

7. For $0 < a < 1$, the value of the integral $\int_0^{\pi} \frac{dx}{1 - 2a\cos x + a^2}$ is :

(1) $\frac{\pi^2}{\pi + a^2}$

(2) $\frac{\pi^2}{\pi - a^2}$

(3) $\frac{\pi}{1 - a^2}$

(4) $\frac{\pi}{1 + a^2}$

Ans. (3)

Sol. $I = \int_0^{\pi} \frac{dx}{1 - 2a\cos x + a^2}; 0 < a < 1$

$I = \int_0^{\pi} \frac{dx}{1 + 2a\cos x + a^2}$

$2I = 2\int_0^{\pi/2} \frac{2(1+a^2)}{(1+a^2)^2 - 4a^2\cos^2 x} dx$

$\Rightarrow I = \int_0^{\pi/2} \frac{2(1+a^2).\sec^2 x}{(1+a^2)^2.\sec^2 x - 4a^2} dx$

$\Rightarrow I = \int_0^{\pi/2} \frac{2.(1+a^2).\sec^2 x}{(1+a^2)^2.\tan^2 x + (1-a^2)^2} dx$

$$\Rightarrow I=\int_0^{\pi/2}\frac{\frac{2.\sec^2 x}{1+a^2}.dx}{\tan^2 x+\left(\frac{1-a^2}{1+a^2}\right)^2}$$

$$\Rightarrow I=\frac{2}{(1-a^2)}\left[\frac{\pi}{2}-0\right]$$

$$I=\frac{\pi}{1-a^2}$$

8. Let $g(x)=3f\left(\frac{x}{3}\right)+f(3-x)$ and $f''(x)>0$ for all $x\in(0,3)$. If g is decreasing in (0, α) and increasing in (α, 3), then 8α is

(1) 24

(2) 0

(3) 18

(4) 20

Ans. (3)

Sol. $g(x)=3f\left(\frac{x}{3}\right)+f(3-x)$ and $f''(x)>0\ \forall\ x\in(0,3)$

$\Rightarrow f'(x)$ is increasing function

$$g'(x)=3\times\frac{1}{3}.f'\left(\frac{x}{3}\right)-f'(3-x)$$

$$=f'\left(\frac{x}{3}\right)-f'(3-x)$$

If g is decreasing in (0, α)

$g'(x)<0$

$$f'\left(\frac{x}{3}\right)-f'(3-x)<0$$

$$f'\left(\frac{x}{3}\right)<f'(3-x)$$

$$\Rightarrow\frac{x}{3}<3-x$$

$$\Rightarrow x<\frac{9}{4}$$

Therefore $\alpha=\frac{9}{4}$

Then $8\alpha=8\times\frac{9}{4}=18$

9. If $\lim\limits_{x\to0}\frac{3+\alpha\sin x+\beta\cos x+\log_e(1-x)}{3\tan^2 x}=\frac{1}{3}$, then 2α–β is equal to :

(1) 2

(2) 7

(3) 5

(4) 1

Ans. (3)

Sol. $\lim\limits_{x\to0}\frac{3+\alpha\sin x+\beta\cos x+\log_e(1-x)}{3\tan^2 x}=\frac{1}{3}$

$$\Rightarrow\lim_{x\to0}\frac{3+\alpha\left[x-\frac{x^3}{3!}+....\right]+\beta\left[1-\frac{x^2}{2!}+\frac{x^4}{4!}....\right]+\left(-x-\frac{x^2}{2}-\frac{x^3}{3}...\right)}{3\tan^2 x}=\frac{1}{3}$$

$$\Rightarrow\lim_{x\to0}\frac{(3+\beta)+(\alpha-1)x+\left(-\frac{1}{2}-\frac{\beta}{2}\right)x^2+....}{3x^2}\times\frac{x^2}{\tan^2 x}=\frac{1}{3}$$

$$\Rightarrow\beta+3=0,\ \alpha-1=0\text{ and }\frac{-\frac{1}{2}-\frac{\beta}{2}}{3}=\frac{1}{3}$$

$$\Rightarrow\beta=-3,\alpha=1$$

$$\Rightarrow2\alpha-\beta=2+3=5$$

10. If α, β are the roots of the equation, $x^2-x-1=0$ and $S_n=2023\alpha^n+2024\beta^n$, then

(1) $2S_{12}=S_{11}+S_{10}$

(2) $S_{12}=S_{11}+S_{10}$

(3) $2S_{11}=S_{12}+S_{10}$

(4) $S_{11}=S_{10}+S_{12}$

Ans. (2)

Sol. $x^2-x-1=0$

$S_n=2023\alpha^n+2024\beta^n$

$S_{n-1}+S_{n-2}=2023\alpha^{n-1}+2024\beta^{n-1}+2023\alpha^{n-2}+2024\beta^{n-2}$

$=2023\alpha^{n-2}[1+\alpha]+2024\beta^{n-2}[1+\beta]$

$=2023\alpha^{n-2}[\alpha^2]+2024\beta^{n-2}[\beta^2]$

$=2023\alpha^n+2024\beta^n$

$S_{n-1}+S_{n-2}=S_n$

Put n = 12

$S_{11}+S_{10}=S_{12}$

11. Let A and B be two finite sets with m and n elements respectively. The total number of subsets of the set A is 56 more than the total number of subsets of B. Then the distance of the point P(m, n) from the point Q(–2, –3) is

(1) 10

(2) 6

(3) 4

(4) 8

Ans. (1)

Sol. $2^m - 2^n = 56$

$2^n(2^{m-n} - 1) = 2^3 \times 7$

$2^n = 2^3$ and $2^{m-n} - 1 = 7$

$\Rightarrow n = 3$ and $2^{m-n} = 8$

$\Rightarrow n = 3$ and $m - n = 3$

$\Rightarrow n = 3$ and $m = 6$

P(6,3) and Q(–2, –3)

$PQ = \sqrt{8^2 + 6^2} = \sqrt{100} = 10$

Hence option (1) is correct

12. The values of α, for which

$$\begin{vmatrix} 1 & \frac{3}{2} & \alpha + \frac{3}{2} \\ 1 & \frac{1}{3} & \alpha + \frac{1}{3} \\ 2\alpha + 3 & 3\alpha + 1 & 0 \end{vmatrix} = 0$$, lie in the interval

(1) (–2, 1)

(2) (–3, 0)

(3) $\left(-\frac{3}{2}, \frac{3}{2}\right)$

(4) (0, 3)

Ans. (2)

Sol. $$\begin{vmatrix} 1 & \frac{3}{2} & \alpha + \frac{3}{2} \\ 1 & \frac{1}{3} & \alpha + \frac{1}{3} \\ 2\alpha + 3 & 3\alpha + 1 & 0 \end{vmatrix} = 0$$

$\Rightarrow (2\alpha + 3)\left\{\frac{7\alpha}{6}\right\} - (3\alpha + 1)\left\{\frac{-7}{6}\right\} = 0$

$\Rightarrow (2\alpha + 3).\frac{7\alpha}{6} + (3\alpha + 1).\frac{7}{6} = 0$

$\Rightarrow 2\alpha^2 + 3\alpha + 3\alpha + 1 = 0$

$\Rightarrow 2\alpha^2 + 6\alpha + 1 = 0$

$\Rightarrow \alpha = \frac{-3 + \sqrt{7}}{2}, \frac{-3 - \sqrt{7}}{2}$

Hence option (2) is correct.

13. An urn contains 6 white and 9 black balls. Two successive draws of 4 balls are made without replacement. The probability, that the first draw gives all white balls and the second draw gives all black balls, is :

(1) $\frac{5}{256}$ (2) $\frac{5}{715}$

(3) $\frac{3}{715}$ (4) $\frac{3}{256}$

Ans. (3)

Sol. $\frac{{}^6C_4}{{}^{15}C_4} \times \frac{{}^9C_4}{{}^{11}C_4} = \frac{3}{715}$

Hence option (3) is correct.

14. The integral $\int \frac{(x^8 - x^2)dx}{(x^{12} + 3x^6 + 1)\tan^{-1}\left(x^3 + \frac{1}{x^3}\right)}$ is equal to :

(1) $\log_e\left(\left|\tan^{-1}\left(x^3 + \frac{1}{x^3}\right)\right|\right)^{1/3} + C$

(2) $\log_e\left(\left|\tan^{-1}\left(x^3 + \frac{1}{x^3}\right)\right|\right)^{1/2} + C$

(3) $\log_e\left(\left|\tan^{-1}\left(x^3 + \frac{1}{x^3}\right)\right|\right) + C$

(4) $\log_e\left(\left|\tan^{-1}\left(x^3 + \frac{1}{x^3}\right)\right|\right)^{3} + C$

Ans. (1)

Sol. $I=\int\frac{x^8-x^2}{(x^{12}+3x^6+1)\tan^{-1}\left(x^3+\frac{1}{x^3}\right)}dx$

Let $\tan^{-1}\left(x^3+\frac{1}{x^3}\right)=t$

$\Rightarrow\frac{1}{1+\left(x^3+\frac{1}{x^3}\right)^2}.\left(3x^2-\frac{3}{x^4}\right)dx=dt$

$\Rightarrow\frac{x^6}{x^{12}+3x^6+1}.\frac{3x^6-3}{x^4}dx=dt$

$I=\frac{1}{3}\int\frac{dt}{t}=\frac{1}{3}\ln|t|+C$

$I=\frac{1}{3}\ln\left|\tan^{-1}\left(x^3+\frac{1}{x^3}\right)\right|+C$

$I=\ln\left|\tan^{-1}\left(x^3+\frac{1}{x^3}\right)\right|^{1/3}+C$

Hence option (1) is correct

15. If $2\tan^2\theta-5\sec\theta=1$ has exactly 7 solutions in the interval $\left[0,\frac{n\pi}{2}\right]$, for the least value of $n\in N$ then $\sum_{k=1}^{n}\frac{k}{2^k}$ is equal to :

(1) $\frac{1}{2^{15}}(2^{14}-14)$

(2) $\frac{1}{2^{14}}(2^{15}-15)$

(3) $1-\frac{15}{2^{13}}$

(4) $\frac{1}{2^{13}}(2^{14}-15)$

Ans. **(4)**

Sol. $2\tan^2\theta-5\sec\theta-1=0$

$\Rightarrow 2\sec^2\theta-5\sec\theta-3=0$

$\Rightarrow(2\sec\theta+1)(\sec\theta-3)=0$

$\Rightarrow\sec\theta=-\frac{1}{2},3$

$\Rightarrow\cos\theta=-2,\frac{1}{3}$

$\Rightarrow\cos\theta=\frac{1}{3}$

For 7 solutions n = 13

So, $\sum_{k=1}^{13}\frac{k}{2^k}=S$ (say)

$S=\frac{1}{2}+\frac{2}{2^2}+\frac{3}{2^3}+....+\frac{13}{2^{13}}$

$\frac{1}{2}S=\frac{1}{2^2}+\frac{1}{2^3}+.....+\frac{12}{2^{13}}+\frac{13}{2^{14}}$

$\Rightarrow\frac{S}{2}=\frac{1}{2}.\frac{1-\frac{1}{2^{13}}}{1-\frac{1}{2}}-\frac{13}{2^{14}}\Rightarrow S=2.\left(\frac{2^{13}-1}{2^{13}}\right)-\frac{13}{2^{13}}$

16. The position vectors of the vertices A, B and C of a triangle are $2\hat{i}-3\hat{j}+3\hat{k}$, $2\hat{i}+2\hat{j}+3\hat{k}$ and $-\hat{i}+\hat{j}+3\hat{k}$ respectively. Let l denotes the length of the angle bisector AD of $\angle BAC$ where D is on the line segment BC, then $2l^2$ equals :

(1) 49

(2) 42

(3) 50

(4) 45

Ans. **(4)**

Sol. AB = 5

AC = 5

$\therefore$ D is midpoint of BC

$D\left(\frac{1}{2},\frac{3}{2},3\right)$

$\therefore l=\sqrt{\left(2-\frac{1}{2}\right)^2+\left(-3-\frac{3}{2}\right)^2+(3-3)^2}$

$l=\sqrt{\frac{45}{2}}$

$\therefore 2l^2=45$

17. If y = y(x) is the solution curve of the differential equation $(x^2-4)dy-(y^2-3y)dx=0$, $x>2, y(4)=\frac{3}{2}$ and the slope of the curve is never zero, then the value of y(10) equals :

(1) $\frac{3}{1+(8)^{1/4}}$

(2) $\frac{3}{1+2\sqrt{2}}$

(3) $\frac{3}{1-2\sqrt{2}}$

(4) $\frac{3}{1-(8)^{1/4}}$

Ans. (1)

Sol. $(x^2-4)dy-(y^2-3y)dx=0$

$\Rightarrow \int \frac{dy}{y^2-3y}=\int \frac{dx}{x^2-4}$

$\Rightarrow \frac{1}{3}\int \frac{y-(y-3)}{y(y-3)}dy=\int \frac{dx}{x^2-4}$

$\Rightarrow \frac{1}{3}(\ln|y-3|-\ln|y|)=\frac{1}{4}\ln\left|\frac{x-2}{x+2}\right|+C$

$\Rightarrow \frac{1}{3}\ln\left|\frac{y-3}{y}\right|=\frac{1}{4}\ln\left|\frac{x-2}{x+2}\right|+C$

At $x=4,\ y=\frac{3}{2}$

$\therefore C=\frac{1}{4}\ln 3$

$\therefore \frac{1}{3}\ln\left|\frac{y-3}{y}\right|=\frac{1}{4}\ln\left|\frac{x-2}{x+2}\right|+\frac{1}{4}\ln(3)$

At x = 10

$\frac{1}{3}\ln\left|\frac{y-3}{y}\right|=\frac{1}{4}\ln\left|\frac{2}{3}\right|+\frac{1}{4}\ln(3)$

$\ln\left|\frac{y-3}{y}\right|=\ln 2^{3/4},\ \forall x>2, \frac{dy}{dx}<0$

as $y(4)=\frac{3}{2}\Rightarrow y\in(0,3)$

$-y+3=8^{1/4}.y$

$y=\frac{3}{1+8^{1/4}}$

18. Let e_1 be the eccentricity of the hyperbola $\frac{x^2}{16}-\frac{y^2}{9}=1$ and e_2 be the eccentricity of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, a > b, which passes through the foci of the hyperbola. If $e_1e_2 = 1$, then the length of the chord of the ellipse parallel to the x-axis and passing through (0, 2) is :

(1) $4\sqrt{5}$

(2) $\frac{8\sqrt{5}}{3}$

(3) $\frac{10\sqrt{5}}{3}$

(4) $3\sqrt{5}$

Ans. (3)

Sol. $H:\frac{x^2}{16}-\frac{y^2}{9}=1$ $\qquad e_1=\frac{5}{4}$

$\therefore e_1e_2=1\Rightarrow e_2=\frac{4}{5}$

Also, ellipse is passing through $(\pm 5,0)$

$\therefore$ a = 5 and b = 3

$E:\frac{x^2}{25}+\frac{y^2}{9}=1$

End point of chord are $\left(\pm\frac{5\sqrt{5}}{3},2\right)$

$\therefore L_{PQ}=\frac{10\sqrt{5}}{3}$

19. Let $\alpha=\frac{(4!)!}{(4!)^{3!}}$ and $\beta=\frac{(5!)!}{(5!)^{4!}}$. Then :

(1) $\alpha\in N$ and $\beta\notin N$

(2) $\alpha\notin N$ and $\beta\in N$

(3) $\alpha\in N$ and $\beta\in N$

(4) $\alpha\notin N$ and $\beta\notin N$

Ans. (3)

Sol. $\alpha = \frac{(4!)!}{(4!)^{3!}}, \beta = \frac{(5!)!}{(5!)^{4!}}$

$\alpha = \frac{(24)!}{(4!)^{6}}, \beta = \frac{(120)!}{(5!)^{24}}$

Let 24 distinct objects are divided into 6 groups of 4 objects in each group.

No. of ways of formation of group = $\frac{24!}{(4!)^6.6!} \in N$

Similarly,

Let 120 distinct objects are divided into 24 groups of 5 objects in each group.

No. of ways of formation of groups

$= \frac{(120)!}{(5!)^{24}.24!} \in N$

20. Let the position vectors of the vertices A, B and C of a triangle be $2\hat{i}+2\hat{j}+\hat{k}$, $\hat{i}+2\hat{j}+2\hat{k}$ and $2\hat{i}+\hat{j}+2\hat{k}$ respectively. Let l_1, l_2 and l_3 be the lengths of perpendiculars drawn from the ortho center of the triangle on the sides AB, BC and CA respectively, then $l_1^2+l_2^2+l_3^2$ equals :

(1) $\frac{1}{5}$ (2) $\frac{1}{2}$

(3) $\frac{1}{4}$ (4) $\frac{1}{3}$

Ans. (2)

Sol. ΔABC is equilateral

Orthocentre and centroid will be same

$G\left(\frac{5}{3},\frac{5}{3},\frac{5}{3}\right)$

Mid-point of AB is $D\left(\frac{3}{2},2,\frac{3}{2}\right)$

$\therefore \ell_1 = \sqrt{\frac{1}{36}+\frac{1}{9}+\frac{1}{36}}$

$\ell_1 = \sqrt{\frac{1}{6}} = \ell_2 = \ell_3$

$\therefore \ell_1^2+\ell_2^2+\ell_3^2 = \frac{1}{2}$

SECTION-B

21. The mean and standard deviation of 15 observations were found to be 12 and 3 respectively. On rechecking it was found that an observation was read as 10 in place of 12. If μ and σ^2 denote the mean and variance of the correct observations respectively, then $15(\mu+\mu^2+\sigma^2)$ is equal to ……..

Ans. (2521)

Sol. Let the incorrect mean be μ' and standard deviation be σ'

We have

$\mu' = \frac{\Sigma x_i}{15} = 12 \Rightarrow \Sigma x_i = 180$

As per given information correct Σx_i = 180–10+12

$\Rightarrow \mu$ (correct mean) = $\frac{182}{15}$

Also

$\sigma' = \sqrt{\frac{\Sigma x_i^2}{15} - 144} = 3 \Rightarrow \Sigma x_i^2 = 2295$

Correct Σx_i^2 = 2295 – 100 + 144 = 2339

σ^2 (correct variance) = $\frac{2339}{15} - \frac{182\times182}{15\times15}$

Required value

$= 15(\mu+\mu^2+\sigma^2)$

$= 15\left(\frac{182}{15}+\frac{182\times182}{15\times15}+\frac{2339}{15}-\frac{182\times182}{15\times15}\right)$

$= 15\left(\frac{182}{15}+\frac{2339}{15}\right)$

= 2521

22. If the area of the region

$\{(x,y): 0 \le y \le \min\{2x, 6x - x^2\}\}$ is A, then 12 A

is equal to……

Ans. **(304)**

Sol. We have

$$A = \frac{1}{2} \times 4 \times 8 + \int_4^6 (6x - x^2)dx$$

$$A = \frac{76}{3}$$

12A = 304

23. Let A be a 2 × 2 real matrix and I be the identity matrix of order 2. If the roots of the equation $|A - xI| = 0$ be –1 and 3, then the sum of the diagonal elements of the matrix A^2 is……

Ans. **(10)**

Sol. $|A - xI| = 0$

Roots are –1 and 3

Sum of roots = tr(A) = 2

Product of roots = |A| = –3

Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

We have a + d = 2

ad – bc = –3

$$A^2 = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \times \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a^2 + bc & ab + bd \\ ac + cd & bc + d^2 \end{bmatrix}$$

We need $a^2 + bc + bc + d^2$

$= a^2 + 2bc + d^2$

$= (a + d)^2 - 2ad + 2bc$

$= 4 - 2(ad - bc)$

$= 4 - 2(-3)$

$= 4 + 6$

$= 10$

24. If the sum of squares of all real values of α, for which the lines 2x – y + 3 = 0, 6x + 3y + 1 = 0 and αx + 2y –2 = 0 do not form a triangle is p, then the greatest integer less than or equal to p is ……

Ans. **(32)**

Sol. $2x - y + 3 = 0$

$6x + 3y + 1 = 0$

$\alpha x + 2y - 2 = 0$

Will not form a Δ if $\alpha x + 2y - 2 = 0$ is concurrent with $2x - y + 3 = 0$ and $6x + 3y + 1 = 0$ or parallel to either of them so

Case-1: Concurrent lines

$$\begin{vmatrix} 2 & -1 & 3 \\ 6 & 3 & 1 \\ \alpha & 2 & -2 \end{vmatrix} = 0 \Rightarrow \alpha = \frac{4}{5}$$

Case-2 : Parallel lines

$$-\frac{\alpha}{2} = \frac{-6}{3} \text{ or } -\frac{\alpha}{2} = 2$$

$$\Rightarrow \alpha = 4 \text{ or } \alpha = -4$$

$$P = 16 + 16 + \frac{16}{25}$$

$$[P] = \left[32 + \frac{16}{25}\right] = 32$$

25. The coefficient of x^{2012} in the expansion of $(1 - x)^{2008}(1 + x + x^2)^{2007}$ is equal to

Ans. **(0)**

Sol. $(1-x)(1-x)^{2007}(1+x+x^2)^{2007}$

$(1-x)(1-x^3)^{2007}$

$(1-x)({}^{2007}C_0 - {}^{2007}C_1(x^3)+.....)$

General term

$(1-x)((-1)^r\,{}^{2007}C_r x^{3r})$

$(-1)^r\,{}^{2007}C_r x^{3r} - (-1)^r\,{}^{2007}C_r x^{3r+1}$

$3r = 2012$

$r \neq \frac{2012}{3}$

$3r + 1 = 2012$

$3r = 2011$

$r \neq \frac{2011}{3}$

Hence there is no term containing x^{2012}.

So coefficient of $x^{2012} = 0$

26. If the solution curve, of the differential equation $\frac{dy}{dx} = \frac{x+y-2}{x-y}$ passing through the point (2, 1) is

$$\tan^{-1}\left(\frac{y-1}{x-1}\right) - \frac{1}{\beta}\log_e\left(\alpha + \left(\frac{y-1}{x-1}\right)^2\right) = \log_e|x-1|,$$

then $5\beta+\alpha$ is equal to

Ans. (11)

Sol. $\frac{dy}{dx} = \frac{x+y-2}{x-y}$

$x = X+h, y = Y+k$

$\frac{dY}{dX} = \frac{X+Y}{X-Y}$

$\left.\begin{array}{l} h+k-2=0 \\ h-k=0 \end{array}\right\} h = k = 1$

$Y = vX$

$v + \frac{dv}{dX} = \frac{1+v}{1-v} \Rightarrow X - \frac{dv}{dX} = \frac{1+v^2}{1-v}$

$\frac{1-v}{1+v^2}dv = \frac{dX}{X}$

$\tan^{-1}v - \frac{1}{2}\ln(1+v^2) = \ln|X| + C$

As curve is passing through (2, 1)

$$\tan^{-1}\left(\frac{y-1}{x-1}\right) - \frac{1}{2}\ln\left(1+\left(\frac{y-1}{x-1}\right)^2\right) = \ln|x-1|$$

$\therefore \alpha = 1$ and $\beta = 2$

$\Rightarrow 5\beta + \alpha = 11$

27. Let $f(x) = \int_0^x g(t)\log_e\left(\frac{1-t}{1+t}\right)dt$, where g is a continuous odd function.

If $\int_{-\pi/2}^{\pi/2}\left(f(x) + \frac{x^2\cos x}{1+e^x}\right)dx = \left(\frac{\pi}{\alpha}\right)^2 - \alpha$, then α is equal to……

Ans. (2)

Sol. $f(x) = \int_0^x g(t)\ln\left(\frac{1-t}{1+t}\right)dt$

$f(-x) = \int_0^{-x} g(t)\ln\left(\frac{1-t}{1+t}\right)dt$

$f(-x) = -\int_0^x g(-y)\ln\left(\frac{1+y}{1-y}\right)dy$

$= -\int_0^x g(y)\ln\left(\frac{1-y}{1+y}\right)dy$ (g is odd)

$f(-x) = -f(x) \Rightarrow f$ is also odd

Now,

$$I = \int_{-\pi/2}^{\pi/2}\left(f(x) + \frac{x^2\cos x}{1+e^x}\right)dx \qquad .....(1)$$

$$I = \int_{-\pi/2}^{\pi/2}\left(f(-x) + \frac{x^2e^x\cos x}{1+e^x}\right)dx \qquad .....(2)$$

$$2I = \int_{-\pi/2}^{\pi/2} x^2\cos x\,dx = 2\int_0^{\pi/2} x^2\cos x\,dx$$

$$I = \left(x^2 \sin x\right)_0^{\pi/2} - \int_0^{\pi/2} 2x \sin x dx$$

$$= \frac{\pi^2}{4} - 2\left(-x\cos x + \int \cos x dx\right)_0^{\pi/2}$$

$$= \frac{\pi^2}{4} - 2(0+1) = \frac{\pi^2}{4} - 2 \Rightarrow \left(\frac{\pi}{2}\right)^2 - 2$$

$$\therefore \alpha = 2$$

28. Consider a circle $(x-\alpha)^2 + (y-\beta)^2 = 50$, where $\alpha, \beta > 0$. If the circle touches the line y + x = 0 at the point P, whose distance from the origin is $4\sqrt{2}$, then $(\alpha+\beta)^2$ is equal to……

Ans. (100)

Sol.

$$S: (x-\alpha)^2 + (y-\beta)^2 = 50$$

CP = r

$$\left|\frac{\alpha+\beta}{\sqrt{2}}\right| = 5\sqrt{2}$$

$$\Rightarrow (\alpha+\beta)^2 = 100$$

29. The lines $\frac{x-2}{2} = \frac{y}{-2} = \frac{z-7}{16}$ and $\frac{x+3}{4} = \frac{y+2}{3} = \frac{z+2}{1}$ intersect at the point P. If the distance of P from the line $\frac{x+1}{2} = \frac{y-1}{3} = \frac{z-1}{1}$ is l, then $14l^2$ is equal to……..

Ans. (108)

Sol. $\frac{x-2}{1} = \frac{y}{-1} = \frac{z-7}{8} = \lambda$

$$\frac{x+3}{4} = \frac{y+2}{3} = \frac{z+2}{1} = k$$

$$\Rightarrow \lambda + 2 = 4k - 3$$

$$-\lambda = 3k - 2$$

$$\Rightarrow k = 1, \lambda = -1$$

$$8\lambda + 7 = k - 2$$

$$\therefore P = (1,1,-1)$$

Projection of $2\hat{i} - 2\hat{k}$ on $2\hat{i} + 3\hat{j} + \hat{k}$ is

$$= \frac{4-2}{\sqrt{4+9+1}} = \frac{2}{\sqrt{14}}$$

$$\therefore l^2 = 8 - \frac{4}{14} = \frac{108}{14}$$

$$\Rightarrow 14l^2 = 108$$

30. Let the complex numbers α and $\frac{1}{\bar{\alpha}}$ lie on the circles $|z - z_0|^2 = 4$ and $|z - z_0|^2 = 16$ respectively, where $z_0 = 1 + i$. Then, the value of $100|\alpha|^2$ is……

Ans. (20)

Sol. $|z - z_0|^2 = 4$

$$\Rightarrow (\alpha - z_0)(\bar{\alpha} - \bar{z}_0) = 4$$

$$\Rightarrow \alpha\bar{\alpha} - \alpha\bar{z}_0 - z_0\bar{\alpha} + |z_0|^2 = 4$$

$$\Rightarrow |\alpha|^2 - \alpha\bar{z}_0 - z_0\bar{\alpha} = 2 \text{ ……(1)}$$

$$|z - z_0|^2 = 16$$

$$\Rightarrow \left(\frac{1}{\bar{\alpha}} - z_0\right)\left(\frac{1}{\alpha} - \bar{z}_0\right) = 16$$

$$\Rightarrow (1 - \bar{\alpha} z_0)(1 - \alpha \bar{z}_0) = 16|\alpha|^2$$

$$\Rightarrow 1 - \bar{\alpha} z_0 - \alpha \bar{z}_0 + |\alpha|^2 |z_0|^2 = 16|\alpha|^2$$

$$\Rightarrow 1 - \bar{\alpha} z_0 - \alpha \bar{z}_0 = 14|\alpha|^2 \ldots\ldots\ldots\ldots(2)$$

From (1) and (2)

$$\Rightarrow 5|\alpha|^2 = 1$$

$$\Rightarrow 100|\alpha|^2 = 20$$

SECTION-A

31. The equation of state of a real gas is given by $\left(P+\frac{a}{V^2}\right)(V-b)=RT$, where P, V and T are pressure. volume and temperature respectively and R is the universal gas constant. The dimensions of $\frac{a}{b^2}$ is similar to that of :

(1) PV

(2) P

(3) RT

(4) R

Ans. **(2)**

Sol. $[P]=\left[\frac{a}{V^2}\right]\Rightarrow[a]=\left[PV^2\right]$

And [V] = [b]

$$\frac{[a]}{\left[b^2\right]}=\frac{\left[PV^2\right]}{\left[V^2\right]}=[P]$$

32. Wheatstone bridge principle is used to measure the specific resistance (S_1) of given wire, having length L, radius r. If X is the resistance of wire, then specific resistance is : $S_1=X\left(\frac{\pi r^2}{L}\right)$. If the length of the wire gets doubled then the value of specific resistance will be :

(1) $\frac{S_1}{4}$

(2) $2S_1$

(3) $\frac{S_1}{2}$

(4) S_1

Ans. **(4)**

Sol. As specific resistance does not depends on dimension of wire so, it will not change.

33. Given below are two statements : one is labelled as Assertion (A) and the other is labelled as Reason (R).

Assertion (A) : The angular speed of the moon in its orbit about the earth is more than the angular speed of the earth in its orbit about the sun.

Reason (R) : The moon takes less time to move around the earth than the time taken by the earth to move around the sun.

In the light of the above statements, choose the most appropriate answer from the options given below :

(1) (A) is correct but (R) is not correct

(2) Both (A) and (R) are correct and (R) is the correct explanation of (A)

(3) Both (A) and (R) are correct but (R) is not the correct explanation of (A)

(4) (A) is not correct but (R) is correct

Ans. **(2)**

Sol. $\omega=\frac{2\pi}{T}\Rightarrow\omega\propto\frac{1}{T}$

$T_{moon}=27$ days

T_{earth} = 365 days 4 hour

$\Rightarrow\omega_{moon}>\omega_{earth}$

34. Given below are two statements :

Statement (I) : The limiting force of static friction depends on the area of contact and independent of materials.

Statement (II) : The limiting force of kinetic friction is independent of the area of contact and depends on materials.

In the light of the above statements, choose the most appropriate answer from the options given below :

(1) Statement I is correct but Statement II is incorrect

(2) Statement I is incorrect but Statement II is correct

(3) Both Statement I and Statement II are incorrect

(4) Both Statement I and Statement II are correct

Ans. **(2)**

Sol. Co-efficient of friction depends on surface in contact So, depends on material of object.

35. The truth table of the given circuit diagram is :

(1)

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	1

(2)

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

(3)

A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

(4)

A	B	Y
0	0	1
0	1	1
1	0	1
1	1	0

Ans. (2)

Sol.

$Y = A \cdot \bar{B} + \bar{A} \cdot B$

This is XOR GATE

36. A current of 200 μA deflects the coil of a moving coil galvanometer through 60^0. The current to cause deflection through $\frac{\pi}{10}$ radian is :

(1) 30 μA (2) 120 μA

(3) 60 μA (4) 180 μA

Ans. (3)

Sol. $i \propto \theta$ (angle of deflection)

$$\therefore \frac{i_2}{i_1} = \frac{\theta_2}{\theta_1} \Rightarrow \frac{i_2}{200\ \mu A} = \frac{\pi/10}{\pi/3} = \frac{3}{10}$$

$$\Rightarrow i_2 = 60\ \mu A$$

37. The atomic mass of ${}_6C^{12}$ is 12.000000 u and that of ${}_6C^{13}$ is 13.003354 u. The required energy to remove a neutron from ${}_6C^{13}$, if mass of neutron is 1.008665 u, will be :

(1) 62. 5 MeV (2) 6.25 MeV

(3) 4.95 MeV (4) 49.5 MeV

Ans. (3)

Sol. ${}_6C^{13} + \text{Energy} \rightarrow {}_6C^{12} + {}_0n^1$

$\Delta m =$ (12.000000 + 1.008665) – 13.003354

= – 0.00531 u

∴ Energy required = 0.00531 × 931.5 MeV

= 4.95 MeV

38. A ball suspended by a thread swings in a vertical plane so that its magnitude of acceleration in the extreme position and lowest position are equal. The angle (θ) of thread deflection in the extreme position will be :

(1) $\tan^{-1}\left(\sqrt{2}\right)$ (2) $2\tan^{-1}\left(\frac{1}{2}\right)$

(3) $\tan^{-1}\left(\frac{1}{2}\right)$ (4) $2\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)$

Ans. (2)

Sol.

Loss in kinetic energy = Gain in potential energy

$$\Rightarrow \frac{1}{2}mv^2 = mg\ell(1-\cos\theta)$$

$$\Rightarrow \frac{v^2}{\ell} = 2g(1-\cos\theta)$$

Acceleration at lowest point = $\frac{v^2}{\ell}$

Acceleration at extreme point = gsinθ

Hence, $\frac{v^2}{\ell} = g\sin\theta$

$$\therefore \sin\theta = 2(1-\cos\theta)$$

$$\Rightarrow \tan\frac{\theta}{2} = \frac{1}{2} \Rightarrow \theta = 2\tan^{-1}\left(\frac{1}{2}\right)$$

39. Three voltmeters, all having different internal resistances are joined as shown in figure. When some potential difference is applied across A and B, their readings are V_1, V_2 and V_3. Choose the correct option.

(1) $V_1 = V_2$ (2) $V_1 \neq V_3 - V_2$

(3) $V_1 + V_2 > V_3$ (4) $V_1 + V_2 = V_3$

Ans. (4)

Sol. From KVL,

$\mathbf{V_1 + V_2 - V_3 = 0} \Rightarrow V_1 + V_2 = V_3$

40. The total kinetic energy of 1 mole of oxygen at 27°C is :

[Use universal gas constant (R)= 8.31 J/mole K]

(1) 6845.5 J (2) 5942.0 J

(3) 6232.5 J (4) 5670.5J

Ans. (3)

Sol. Kinetic energy = $\frac{f}{2}nRT$

$= \frac{5}{2} \times 1 \times 8.31 \times 300$ J

= 6232.5 J

41. Given below are two statements : one is labelled as Assertion(A) and the other is labelled as Reason (R).

Assertion (A) : In Vernier calliper if positive zero error exists, then while taking measurements, the reading taken will be more than the actual reading.

Reason (R) : The zero error in Vernier Calliper might have happened due to manufacturing defect or due to rough handling.

In the light of the above statements, choose the correct answer from the options given below :

(1) Both (A) and (R) are correct and (R) is the correct explanation of (A)

(2) Both (A) and (R) are correct but (R) is not the correct explanation of (A)

(3) (A) is true but (R) is false

(4) (A) is false but (R) is true

Ans. (2)

Sol. Assertion & Reason both are correct

Theory

42. Primary side of a transformer is connected to 230 V, 50 Hz supply. Turns ratio of primary to secondary winding is 10 : 1. Load resistance connected to secondary side is 46 Ω. The power consumed in it is :

(1) 12.5 W (2) 10.0 W

(3) 11.5 W (4) 12.0 W

Ans. (3)

Sol. $\frac{V_1}{V_2} = \frac{N_1}{N_2}$

$\frac{230}{V_2} = \frac{10}{1}$

$V_2 = 23V$

Power consumed = $\frac{V_2^2}{R}$

$= \frac{23 \times 23}{46} = 11.5$ W

43. During an adiabatic process, the pressure of a gas is found to be proportional to the cube of its absolute temperature. The ratio of $\frac{C_p}{C_v}$ for the gas is :

(1) $\frac{5}{3}$ (2) $\frac{3}{2}$

(3) $\frac{7}{5}$ (4) $\frac{9}{7}$

Ans. (2)

Sol. $P \propto T^3 \Rightarrow PT^{-3} = \text{constant}$

$PV^{\gamma} = \text{const}$

$P\left(\frac{nRT}{P}\right)^{\gamma} = \text{const}$

$P^{1-\gamma}T^{\gamma} = \text{const}$

$PT^{\frac{\gamma}{1-\gamma}} = \text{const}$

$\frac{\gamma}{1-\gamma} = -3$

$\gamma = -3 + 3\gamma$

$3 = 2\gamma$

$\gamma = \frac{3}{2}$

44. The threshold frequency of a metal with work function 6.63 eV is :

(1) 16×10^{15} Hz

(2) 16×10^{12} Hz

(3) 1.6×10^{12} Hz

(4) 1.6×10^{15} Hz

Ans. (4)

Sol. $\phi_0 = h\nu_0$

$6.63\times1.6\times10^{-19} = 6.63\times10^{-34}\nu_0$

$\nu_0 = \frac{1.6\times10^{-19}}{10^{-34}}$

$\nu_0 = 1.6\times10^{15}\text{Hz}$

45. Given below are two statements : one is labelled as Assertion (A) and the other is labelled as Reason (R)

Assertion (A) : The property of body, by virtue of which it tends to regain its original shape when the external force is removed, is Elasticity.

Reason (R) : The restoring force depends upon the bonded inter atomic and inter molecular force of solid.

In the light of the above statements, choose the correct answer from the options given below :

(1) (A) is false but (R) is true

(2) (A) is true but (R) is false

(3) Both (A) and (R) are true and (R) is the correct explanation (A)

(4) Both (A) and (R) are true but (R) is not the correct explanation of (A)

Ans. (3 or 4)

Sol. Theory

46. When a polaroid sheet is rotated between two crossed polaroids then the transmitted light intensity will be maximum for a rotation of :

(1) 60° (2) 30°

(3) 90° (4) 45°

Ans. (4)

Sol. Let I_0 be intensity of unpolarised light incident on first polaroid.

I_1 = Intensity of light transmitted from 1st polaroid

$= \frac{I_0}{2}$

θ be the angle between 1st and 2nd polaroid

ϕ be the angle between 2nd and 3rd polaroid

$\theta + \phi = 90^0$ (as 1st and 3rd polaroid are crossed)

$\phi = 90^0 - \theta$

I_2 = Intensity from 2nd polaroid

$I_2 = I_1\cos^2\theta = \frac{I_0}{2}\cos^2\theta$

I_3 = Intensity from 3rd polaroid

$I_3 = I_2\cos^2\phi$

$I_3 = I_1\cos^2\theta\cos^2\phi$

$I_3 = \frac{I_0}{2}\cos^2\theta\cos^2\phi$

$\phi = 90 - \theta$

$I_3 = \frac{I_0}{2}\cos^2\theta\sin^2\theta$

$I_3 = \frac{I_0}{2}\left[\frac{2\sin\theta\cos\theta}{2}\right]^2$

$I_3 = \frac{I_0}{8}\sin^2 2\theta$

I_3 will be maximum when $\sin 2\theta = 1$

$2\theta = 90°$

$\theta = 45°$

47. An object is placed in a medium of refractive index 3. An electromagnetic wave of intensity 6×10^8 W/m^2 falls normally on the object and it is absorbed completely. The radiation pressure on the object would be (speed of light in free space $= 3 \times 10^8$ m/s) :

(1) 36 Nm^{-2} (2) 18 Nm^{-2}

(3) 6 Nm^{-2} (4) 2 Nm^{-2}

Ans. (3)

Sol. Radiation pressure $= \frac{I}{v}$

$= \frac{I\cdot\mu}{c}$

$= \frac{6\times10^8\times3}{3\times10^8}$

$= 6$ N/m^2

48. Given below are two statements : one is labelled a Assertion (A) and the other is labelled as Reason(R)

Assertion (A) : Work done by electric field on moving a positive charge on an equipotential surface is always zero.

Reason (R) : Electric lines of forces are always perpendicular to equipotential surfaces.

In the light of the above statements, choose the most appropriate answer from the options given below :

(1) Both (A) and (R) are correct but (R) is not the correct explanation of (A)

(2) ((A) is correct but (R) is not correct

(3) (A) is not correct but (R) is correct

(4) Both (A) and (R) are correct and (R) is the correct explanation of (A)

Ans. (4)

Sol. Electric line of force are always perpendicular to equipotential surface so angle between farce and displacement will always be 90°. So work done equal to 0.

49. A heavy iron bar of weight 12 kg is having its one end on the ground and the other on the shoulder of a man. The rod makes an angle 60° with the horizontal, the weight experienced by the man is :

(1) 6 kg

(2) 12 kg

(3) 3 kg

(4) $6\sqrt{3}$ kg

Ans. (3)

Sol.

Torque about O = 0

$$120\left(\frac{L}{2}\cos 60°\right) - N_2L = 0$$

$N_2 = 30$ N

50. A bullet is fired into a fixed target looses one third of its velocity after travelling 4 cm. It penetrates further D × 10^{-3} m before coming to rest. The value of D is :

(1) 2

(2) 5

(3) 3

(4) 4

Ans. (Bonus)

Sol. $v^2 - u^2 = 2aS$

$$\left(\frac{2u}{3}\right)^2 = u^2 + 2(-a)\left(4\times10^{-2}\right)$$

$$\frac{4u^2}{9} = u^2 - 2a\left(4\times10^{-2}\right)$$

$$-\frac{5u^2}{9} = -2a\left(4\times10^{-2}\right) \quad \ldots(1)$$

$$0 = \left(\frac{2u}{3}\right)^2 + 2(-a)(x)$$

$$-\frac{4u^2}{9} = -2ax \quad \ldots(2)$$

(1) **/(2)**

$$\frac{5}{4} = \frac{4\times10^{-2}}{x}$$

$$x = \frac{16}{5}\times10^{-2}$$

$$x = 3\cdot2\times10^{-2}\,m$$

$$x = 32\times10^{-3}\,m$$

Note : Since no option is matching, Question should be bonus.

SECTION-B

51. The magnetic field at the centre of a wire loop formed by two semicircular wires of radii $R_1 = 2\pi$ m and $R_2 = 4\pi$ m carrying current I = 4A as per figure given below is $\alpha \times 10^{-7}$ T. The value of α is _____. (Centre O is common for all segments)

Ans. **(3.00)**

Sol.

$$\frac{\mu_0 i}{2R_2}\left(\frac{\pi}{2\pi}\right)\otimes + \frac{\mu_0 i}{2R_1}\left(\frac{\pi}{2\pi}\right)\otimes$$

$$\left(\frac{\mu_0 i}{4R_2} + \frac{\mu_0 i}{4R_1}\right)\otimes$$

$$\frac{4\pi\times10^{-7}\times4}{4\times4\pi} + \frac{4\pi\times10^{-7}\times4}{4\times2\pi}$$

$$= 3\times10^{-7} = \alpha\times10^{-7}$$

$$\alpha = 3$$

52. Two charges of –4 μC and +4 μC are placed at the points A(1, 0, 4)m and B(2, –1, 5) m located in an electric field $\vec{E} = 0.20\,\hat{i}$ V / cm. The magnitude of the torque acting on the dipole is $8\sqrt{\alpha}\times10^{-5}$ Nm, Where α = _____.

Ans. **(2.00)**

Sol.

$$\vec{\tau} = \vec{p}\times\vec{E}$$

$$\vec{p} = q\vec{\ell}$$

$$\vec{E} = 0.2\frac{V}{cm} = 20\frac{V}{m}$$

$$\vec{p} = 4\times\left(\hat{i}-\hat{j}+\hat{k}\right)$$

$$= \left(4\hat{i}-4\hat{j}+4\hat{k}\right)\mu C - m$$

$$\vec{\tau} = \left(4\hat{i}-4\hat{j}+4\hat{k}\right)\times\left(20\hat{i}\right)\times10^{-6}\text{ Nm}$$

$$= \left(8\hat{k}+8\hat{j}\right)\times10^{-5} = 8\sqrt{2}\times10^{-5}$$

$$\alpha = 2$$

53. A closed organ pipe 150 cm long gives 7 beats per second with an open organ pipe of length 350 cm, both vibrating in fundamental mode. The velocity of sound is _____ m/s.

Ans. **(294.00)**

Sol.

$$f_c = \frac{v}{4\ell_1} \qquad f_o = \frac{v}{2\ell_2}$$

$$|f_c - f_0| = 7$$

$$\frac{v}{4\times150} - \frac{v}{2\times350} = 7$$

$$\frac{v}{600\text{ cm}} - \frac{v}{700\text{ cm}} = 7$$

$$\frac{v}{6m} - \frac{v}{7m} = 7$$

$$v\left(\frac{1}{42}\right) = 7$$

$$v = 42\times7$$

=294 m/s

54. A body falling under gravity covers two points A and B separated by 80 m in 2s. The distance of upper point A from the starting point is _____ m (use g = 10 ms^{-2})

Ans. **(45.00)**

Sol.

From $A \to B$

$$-80 = -v_1 t - \frac{1}{2} \times 10t^2$$

$$-80 = -2v_1 - \frac{1}{2} \times 10 \times 2^2$$

$$-80 = -2v_1 - 20$$

$$-60 = -2v_1$$

$v_1 = 30$ m/s

From O to A

$$v^2 = u^2 + 2gS$$

$$30^2 = 0 + 2 \times (-10)(-S)$$

900 = 20 S

S = 45 m

55. The reading of pressure metre attached with a closed pipe is 4.5 × 10^4 N/m^2. On opening the valve, water starts flowing and the reading of pressure metre falls to 2.0 × 10^4 N/m^2. The velocity of water is found to be $\sqrt{V}$ m/s. The value of V is _____

Ans. **(50)**

Sol. Change in pressure $= \frac{1}{2}\rho v^2$

$$4.5 \times 10^4 - 2.0 \times 10^4 = \frac{1}{2} \times 10^3 \times v^2$$

$$2.5 \times 10^4 = \frac{1}{2} \times 10^3 \times v^2$$

$v^2 = 50$

$v = \sqrt{50}$

Velocity of water = $\sqrt{V} = \sqrt{50}$

= V = 50

56. A ring and a solid sphere roll down the same inclined plane without slipping. They start from rest. The radii of both bodies are identical and the ratio of their kinetic energies is $\frac{7}{x}$ where x is _____.

Ans. **(7.00)**

Sol. In pure rolling work done by friction is zero. Hence potential energy is converted into kinetic energy. Since initially the ring and the sphere have same potential energy, finally they will have same kinetic energy too.

∴ Ratio of kinetic energies = 1

$$\Rightarrow \frac{7}{x} = 1 \Rightarrow x = 7$$

57. A parallel beam of monochromatic light of wavelength 5000 Å is incident normally on a single narrow slit of width 0.001 mm. The light is focused by convex lens on screen, placed on its focal plane. The first minima will be formed for the angle of diffraction of _____ (degree).

Ans. **(30.00)**

Sol. For first minima

$$a \sin\theta = \lambda$$

$$\Rightarrow \sin\theta = \frac{\lambda}{a} = \frac{5000 \times 10^{-10}}{1 \times 10^{-6}} = \frac{1}{2}$$

$$\Rightarrow \theta = 30°$$

58. The electric potential at the surface of an atomic nucleus (z = 50) of radius 9×10^{-13} cm is _____ $\times 10^{6}$ V.

Ans. **(8.00)**

Sol. Potential $= \frac{kQ}{R} = \frac{k.Ze}{R}$

$$= \frac{9\times10^{9}\times50\times1.6\times10^{-19}}{9\times10^{-13}\times10^{-2}}$$

$$= 8\times10^{6}\,V$$

59. If Rydberg's constant is R, the longest wavelength of radiation in Paschen series will be $\frac{\alpha}{7R}$, where $\alpha =$ _____.

Ans. **(144.00)**

Sol. Longest wavelength corresponds to transition between n = 3 and n = 4

$$\frac{1}{\lambda} = RZ^2\left(\frac{1}{3^2} - \frac{1}{4^2}\right) = RZ^2\left(\frac{1}{9} - \frac{1}{16}\right)$$

$$= \frac{7RZ^2}{9\times16}$$

$$\Rightarrow \lambda = \frac{144}{7R} \text{ for } Z = 1 \qquad \therefore \alpha = 144$$

60. A series LCR circuit with $L = \frac{100}{\pi}$ mH, $C = \frac{10^{-3}}{\pi}$ F and R = 10 Ω, is connected across an ac source of 220 V, 50 Hz supply. The power factor of the circuit would be _____.

Ans. **(1.00)**

Sol. $X_c = \frac{1}{\omega C} = \frac{\pi}{2\pi\times50\times10^{-3}} = 10\Omega$

$$X_L = \omega L = 2\pi\times50\times\frac{100}{\pi}\times10^{-3}$$

$$= 10\Omega$$

$\because X_C = X_L$,Hence, circuit is in resonance

$\therefore$ power factor $= \frac{R}{Z} = \frac{R}{R} = 1$

SECTION-A

61. The order of relative stability of the contributing structure is:

Choose the **correct** answer from the options given below:

(1) I > II > III

(2) II > I > III

(3) I = II = III

(4) III > II > I

Ans. **(1)**

Sol. I > II > III, since neutral resonating structures are more stable than charged resonating structure. II > III, since stability of structure with –ve charge on more electronegative atom is higher.

62. Which among the following halide/s will not show S_N1 reaction:

(A) $H_2C = CH - CH_2Cl$

(B) $CH_3 - CH = CH - Cl$

(C)

(D)

Choose the **most appropriate** answer from the options given below:

(1) (A), (B) and (D) only
(2) (A) and (B) only
(3) (B) and (C) only
(4) (B) only

Ans. **(4)**

Sol. Since $CH_3 - CH = \overset{+}{C}H$ is very unstable, CH_3–CH = CH – Cl cannot give S_{N1} reaction.

63. Which of the following statements is not correct about rusting of iron?

(1) Coating of iron surface by tin prevents rusting, even if the tin coating is peeling off.
(2) When pH lies above 9 or 10, rusting of iron does not take place.
(3) Dissolved acidic oxides SO_2, NO_2 in water act as catalyst in the process of rusting.
(4) Rusting of iron is envisaged as setting up of electrochemical cell on the surface of iron object.

Ans. **(1)**

Sol. As tin coating is peeled off, then iron is exposed to atmosphere.

64. Given below are two statements:

Statement (I) : In the Lanthanoids, the formation of Ce^{+4} is favoured by its noble gas configuration.

Statement (II) : Ce^{+4} is a strong oxidant reverting to the common +3 state.

In the light of the above statements, choose the most appropriate answer from the options given below:

(1) Statement I is false but Statement II is true
(2) Both Statement I and Statement II are true
(3) Statement I is true but Statement II is false
(4) Both Statement I and Statement II are false

Ans. **(2)**

Sol. Statement (1) is true, Ce^{+4} has noble gas electronic configuration.

Statement (2) is also true due to high reduction potential for Ce^{4+}/Ce^{3+} (+1.74V), and stability of Ce^{3+}, Ce^{4+} acts as strong oxidizing agent.

65. Choose the correct option having all the elements with d^{10} electronic configuration from the following:

(1) ^{27}Co, ^{28}Ni, ^{26}Fe, ^{24}Cr
(2) ^{29}Cu, ^{30}Zn, ^{48}Cd, ^{47}Ag
(3) ^{46}Pd, ^{28}Ni, ^{26}Fe, ^{24}Cr
(4) ^{28}Ni, ^{24}Cr, ^{26}Fe, ^{29}Cu

Ans. **(2)**

Sol. $[Cr] = [Ar]4s^1 3d^5$
$[Cd] = [Kr]5s^2 4d^{10}$
$[Cu] = [Ar]4s^1 3d^{10}$
$[Ag] = [Kr]5s^1 4d^{10}$
$[Zn] = [Ar]4s^2 3d^{10}$

66. Phenolic group can be identified by a positive:
(1) Phthalein dye test
(2) Lucas test
(3) Tollen's test
(4) Carbylamine test

Ans. **(1)**

Sol. Carbylamine Test-Identification of primary amines
Lucas Test - Differentiation between 1°, 2° and 3° alcohols
Tollen's Test - Identification of Aldehydes
Phthalein Dye Test - Identification of phenols

67. The molecular formula of second homologue in the homologous series of mono carboxylic acids is _____.
(1) $C_3H_6O_2$
(2) $C_2H_4O_2$
(3) CH_2O
(4) $C_2H_2O_2$

Ans. **(2)**

Sol. $HCOOH, \underset{\substack{\uparrow \\ \text{Second homologue}}}{CH_3COOH}$

68. The technique used for purification of steam volatile water immiscible substance is:
(1) Fractional distillation
(2) Fractional distillation under reduced pressure
(3) Distillation
(4) Steam distillation

Ans. **(4)**

Sol. Steam distillation is used for those liquids which are insoluble in water, containing non-volatile impurities and are steam volatile.

69. The final product A, formed in the following reaction sequence is:

$$Ph{-}CH{=}CH_2 \xrightarrow[\text{(iii) HBr; (iv) Mg, ether, then } HCHO/H_3O^+]{\text{(i) } BH_3;\ \text{(ii) } H_2O_2,\ ^{\ominus}OH} A$$

(1) $Ph-CH_2-CH_2-CH_3$
(2) $Ph-\underset{\underset{CH_3}{|}}{CH}-CH_3$
(3) $Ph-\underset{\underset{CH_2OH}{|}}{CH}-CH_3$
(4) $Ph-CH_2-CH_2-CH_2-OH$

Ans. **(4)**

Sol. $PhCH=CH_2 \xrightarrow{B_2H_6/H_2O_2,OH^-} PhCH_2CH_2OH$

$PhCH_2CH_2OH + HBr \longrightarrow PhCH_2CH_2Br + H_2O\ (SN^{NGP})$

$PhCH_2CH_2Br \xrightarrow{Mg/dry\ ether} PhCH_2CH_2MgBr$

$PhCH_2CH_2MgBr \xrightarrow[\text{(II) } H_3O^+]{\text{(I) } H-\overset{\overset{O}{||}}{C}-H} PhCH_2CH_2CH_2OH$

70. Match List-I with List-II.

List – I (Reaction)	**List – II (Reagent(s))**
(A)	(I) $Na_2Cr_2O_7$, H_2SO_4
(B)	(II) (i) NaOH (ii) CH_3Cl
(C)	(III) (i) NaOH, $CHCl_3$ (ii) NaOH (iii) HCl
(D)	(IV) (i) NaOH (ii) CO_2 (iii) HCl

Choose the correct answer from the options given below:
(1) (A)-(IV), (B)-(I), (C)-(III), (D)-(II)
(2) (A)-(II), (B)-(III), (C)-(I), (D)-(IV)
(3) (A)-(II), (B)-(I), (C)-(III), (D)-(IV)
(4) (A)-(IV), (B)-(III), (C)-(I), (D)-(II)

Ans. **(4)**

Sol. (A) → Kolbe Schmidt Reaction
(B) → Reimer Tiemann Reaction
(C) → Oxidation of phenol to p-benzoquinone
(D) → $PhOH + NaOH \rightarrow H_2O + PhO^-$

$PhO^- + CH_3-Cl \longrightarrow PhOCH_3 + Cl^-$

71. Major product formed in the following reaction is a mixture of:

(1) and $(CH_3)_3CI$
(2) and $(CH_3)_3COH$
(3) and $(CH_3)_3COH$
(4) and $CH_3-\underset{\underset{CH_3}{|}}{\overset{\overset{CH_3}{|}}{C}}-I$

Ans. **(4)**

Sol.

72. Bond line formula of $HOCH(CN)_2$ is:

(1)

(2)

(3)

(4)

Ans. (4)

Sol. $CH(OH)(CN)_2$ is or

73. Given below are two statements:

Statement (I) : Oxygen being the first member of group 16 exhibits only –2 oxidation state.

Statement (II) : Down the group 16 stability of +4 oxidation state decreases and +6 oxidation state increases.

In the light of the above statements, choose the **most appropriate** answer from the options given below:

(1) Statement I is correct but Statement II is incorrect
(2) Both Statement I and Statement II are correct
(3) Both Statement I and Statement II are incorrect
(4) Statement I is incorrect but Statement II is correct

Ans. (3)

Sol. Statement-I: Oxygen can have oxidation state from –2 to +2, so statement I is incorrect

Statement- II: On moving down the group stability of +4 oxidation state increases whereas stability of +6 oxidation state decreases down the group, according to inert pair effect.

So both statements are wrong.

74. Identify from the following species in which d^2sp^3 hybridization is shown by central atom:

(1) $[Co(NH_3)_6]^{3+}$
(2) BrF_5
(3) $[Pt(Cl)_4]^{2-}$
(4) SF_6

Ans. (1)

Sol. $[Co(NH_3)_6]^{+3}$ – d^2sp^3 hybridization

BrF_5 – sp^3d^2 hybridization

$[PtCl_4]^{-2}$ – dsp^2 hybridization

SF_6 – sp^3d^2 hybridization

75. Identify B formed in the reaction.

$$Cl-(CH_2)_4-Cl \xrightarrow{\text{excess } NH_3} A \xrightarrow{NaOH} B + H_2O + NaCl$$

(1)

(2) $H_2N-(CH_2)_4-NH_2$

(3) $Cl\overset{+}{N}H_3-(CH_2)_4-\overset{+}{N}H_3Cl^-$

(4)

Ans. (2)

Sol.

76. The quantity which changes with temperature is:

(1) Molarity
(2) Mass percentage
(3) Molality
(4) Mole fraction

Ans. (1)

Sol. $$\text{Molarity} = \frac{\text{Moles of solute}}{\text{Volume of solution}}$$

Since volume depends on temperature, molarity will change upon change in temperature.

77. Which structure of protein remains intact after coagulation of egg white on boiling?

(1) Primary

(2) Tertiary

(3) Secondary

(4) Quaternary

Ans. (1)

Sol. Boiling an egg causes denaturation of its protein resulting in loss of its quarternary, tertiary and secondary structures.

78. Which of the following cannot function as an oxidising agent?

(1) N^{3-}

(2) SO_4^{2-}

(3) BrO_3^-

(4) MnO_4^-

Ans. (1)

Sol. In N^{3-} ion 'N' is present in its lowest possible oxidation state, hence it cannot be reduced further because of which it cannot act as an oxidizing agent.

79. The incorrect statement regarding conformations of ethane is:

(1) Ethane has infinite number of conformations

(2) The dihedral angle in staggered conformation is 60°

(3) Eclipsed conformation is the most stable conformation.

(4) The conformations of ethane are inter-convertible to one-another.

Ans. (3)

Sol. Eclipsed conformation is the least stable conformation of ethane.

80. Identity the incorrect pair from the following:

(1) Photography - AgBr

(2) Polythene preparation – $TiCl_4$, $Al(CH_3)_3$

(3) Haber process - Iron

(4) Wacker process – Pt Cl_2

Ans. (4)

Sol. The catalyst used in Wacker's process is $PdCl_2$

SECTION-B

81. Total number of ions from the following with noble gas configuration is _____.

Sr^{2+} (Z = 38), Cs^+ (Z = 55), La^{2+} (Z = 57) Pb^{2+} (Z = 82), Yb^{2+} (Z = 70) and Fe^{2+} (Z = 26)

Ans. (2)

Sol. Noble gas configuration = $ns^2\ np^6$

$[Sr^{2+}]$ = [Kr]

$[Cs^+]$ = [Xe]

$[Yb^{2+}]$ = [Xe] $4f^{14}$

$[La^{2+}]$ = [Xe] $5d^1$

$[Pb^{2+}]$ = [Xe] $4f^{14}\ 5d^{10}\ 6s^2$

$[Fe^{2+}]$ = [Ar] $3d^6$

82. The number of non-polar molecules from the following is _____

HF, H_2O, SO_2, H_2, CO_2, CH_4, NH_3, HCl, $CHCl_3$, BF_3

Ans. (4)

Sol. The non-polar molecules are CO_2, H_2, CH_4 and BF_3

83. Time required for completion of 99.9% of a First order reaction is _____ times of half life ($t_{1/2}$) of the reaction.

Ans. (10)

Sol.

$$\frac{t_{99.9\%}}{t_{1/2}} = \frac{\frac{2.303}{k}\left(\frac{a}{a-x}\right)}{\frac{2.303}{k}\log 2} = \frac{\log\left(\frac{100}{100-99.9}\right)}{\log 2} = \frac{\log 10^3}{\log 2} = \frac{3}{0.3} = 10$$

84. The Spin only magnetic moment value of square planar complex $[Pt(NH_3)_2Cl(NH_2CH_3)]Cl$ is _____ B.M. (Nearest integer)

(Given atomic number for Pt = 78)

Ans. (0)

Sol. Pt^{2+} (d^8)

$Pt^{2+} \rightarrow dsp^2$ hybridization and have no unpaired e^-s.

$\therefore$ Magnetic moment = 0

85. For a certain thermochemical reaction $M \rightarrow N$ at T = 400 K, $\Delta H^{\ominus} = 77.2$ kJ mol^{-1}, ΔS = 122 JK^{-1}, log equilibrium constant (logK) is –_____ × 10^{-1}.

Ans. (37)

Sol.

$$\Delta G^\circ = \Delta H^\circ - T\Delta S^\circ$$

$$= 77.2 \times 10^3 - 400 \times 122 = 28400\,J$$

$$\Delta G^\circ = -2.303\,RT\log K$$

$$\Rightarrow 28400 = -2.303 \times 8.314 \times 400\ \log K$$

$$\Rightarrow \log K = -3.708 = -37.08 \times 10^{-1}$$

86. Volume of 3 M NaOH (formula weight 40 g mol^{-1}) which can be prepared from 84 g of NaOH is _____ $\times 10^{-1}$ dm^3.

Ans. **(7)**

Sol. $M = \frac{n_{NaOH}}{V_{sol}(\text{in L})} \Rightarrow 3 = \frac{(84/40)}{V} \Rightarrow V = 0.7L = 7\times10^{-1}L$

87. 1 mole of PbS is oxidised by "X" moles of O_3 to get "Y" moles of O_2. X + Y = _____

Ans. **(8)**

Sol. $PbS + 4O_3 \rightarrow PbSO_4 + 4O_2$

x = 4, y = 4

88. The hydrogen electrode is dipped in a solution of pH = 3 at 25°C. The potential of the electrode will be –_____ $\times 10^{-2}$ V.

$\left(\frac{2.303RT}{F} = 0.059V\right)$

Ans. **(18)**

Sol. $2H^+_{(aq.)} + 2e^- \rightarrow H_2(g)$

$$E_{cell} = E^0_{cell} - \frac{0.059}{2}\log\frac{P_{H_2}}{[H^+]^2}$$

= 0- 0.059 × 3 = - 0.177 volts. = -17.7 × 10^{-2} V.

89. 9.3 g of aniline is subjected to reaction with excess of acetic anhydride to prepare acetanilide. The mass of acetanilide produced if the reaction is 100% completed is _____ $\times 10^{-1}$ g.

(Given molar mass in g mol^{-1} N : 14, O : 16, C : 12, H : 1)

Ans. **(135)**

Sol. $\underset{(\text{Aniline MM}=93)}{C_6H_5NH_2} + CH_3-\overset{O}{\overset{||}{C}}-O-\overset{O}{\overset{||}{C}}-CH_3 \rightarrow$

$\underset{(\text{Acetanilide MM}=135)}{C_6H_5NH-\overset{O}{\overset{||}{C}}-CH_3} + CH_3COOH$

$n_{Acetanilide} = n_{Aniline}$

$\Rightarrow \frac{m}{135} = \frac{9.3}{93}$

$\Rightarrow m = 13.5g$

90. Total number of compounds with Chiral carbon atoms from following is _____.

$CH_3-CH_2-CH(NO_2)-COOH$

$CH_3-CH_2-CHBr-CH_2-CH_3$

$CH_3-CH(I)-CH_2-NO_2$

$CH_3-CH_2-CH(OH)-CH_2OH$

$CH_3-\underset{\underset{I}{|}}{CH}-CH(I)-C_2H_5$

Ans. **(5)**

Sol. Chiral carbons are marked by.

FINAL JEE–MAIN EXAMINATION – JANUARY, 2024

(Held On Saturday 27th January, 2024) **TIME : 9 : 00 AM to 12 : 00 NOON**

MATHEMATICS — TEST PAPER WITH SOLUTION

SECTION-A

1. ${}^{n-1}C_r = \left(k^2-8\right){}^{n}C_{r+1}$ if and only if :

(1) $2\sqrt{2} < k \le 3$ (2) $2\sqrt{3} < k \le 3\sqrt{2}$

(3) $2\sqrt{3} < k < 3\sqrt{3}$ (4) $2\sqrt{2} < k < 2\sqrt{3}$

Ans. (1)

Sol. ${}^{n-1}C_r = (k^2 - 8)\,{}^{n}C_{r+1}$

$$\underbrace{r+1 \ge 0,\quad r \ge 0}_{r\ge 0}$$

$$\frac{{}^{n-1}C_r}{{}^{n}C_{r+1}} = k^2 - 8$$

$$\frac{r+1}{n} = k^2 - 8$$

$$\Rightarrow k^2 - 8 > 0$$

$$\left(k-2\sqrt{2}\right)\left(k+2\sqrt{2}\right) > 0$$

$$k \in \left(-\infty, -2\sqrt{2}\right) \cup \left(2\sqrt{2}, \infty\right) \qquad \ldots(I)$$

$$\therefore\ n \ge r+1,\ \frac{r+1}{n} \le 1$$

$$\Rightarrow k^2 - 8 \le 1$$

$$k^2 - 9 \le 0$$

$$-3 \le k \le 3 \qquad \ldots.(II)$$

From equation (I) and (II) we get

$$k \in \left[-3, -2\sqrt{2}\right) \cup \left(2\sqrt{2}, 3\right]$$

2. The distance, of the point (7, –2, 11) from the line $\frac{x-6}{1} = \frac{y-4}{0} = \frac{z-8}{3}$ along the line $\frac{x-5}{2} = \frac{y-1}{-3} = \frac{z-5}{6}$, is :

(1) 12 (2) 14

(3) 18 (4) 21

Ans. (2)

Sol. B = (2λ +7, –3λ – 2, 6λ + 11)

Point B lies on $\frac{x-6}{1} = \frac{y-4}{0} = \frac{z-8}{3}$

$$\frac{2\lambda+7-6}{1} = \frac{-3\lambda-2-4}{0} = \frac{6\lambda+11-8}{3}$$

$-3\lambda - 6 = 0$

$\lambda = -2$

B ⇒ (3, 4, –1)

$$AB = \sqrt{(7-3)^2 + (4+2)^2 + (11+1)^2}$$

$$= \sqrt{16+36+144}$$

$$= \sqrt{196} = 14$$

3. Let x = x(t) and y = y(t) be solutions of the differential equations $\frac{dx}{dt} + ax = 0$ and $\frac{dy}{dt} + by = 0$ respectively, a, b ∈ R. Given that x(0) = 2; y(0) = 1 and 3y(1) = 2x(1), the value of t, for which x(t) = y(t), is :

(1) $\log_{\frac{2}{3}} 2$ (2) $\log_4 3$

(3) $\log_3 4$ (4) $\log_{\frac{4}{3}} 2$

Ans. (4)

Sol. $\frac{dx}{dt} + ax = 0$

$\frac{dx}{x} = -adt$

$\int \frac{dx}{x} = -a\int dt$

$\ln|x| = -at + c$

at t = 0, x = 2

$\ln 2 = 0 + c$

$\ln x = -at + \ln 2$

$\frac{x}{2} = e^{-at}$

$x = 2e^{-at}$ ….(i)

$\frac{dy}{dt} + by = 0$

$\frac{dy}{y} = -bdt$

$\ln|y| = -bt + \lambda$

t = 0, y = 1

$0 = 0 + \lambda$

$y = e^{-bt}$ ….(ii)

According to question

$3y(1) = 2x(1)$

$3e^{-b} = 2(2\,e^{-a})$

$e^{a-b} = \frac{4}{3}$

For x(t) = y(t)

$\Rightarrow\ 2e^{-at} = e^{-bt}$

$2 = e^{(a-b)t}$

$2 = \left(\frac{4}{3}\right)^t$

$\log_{\frac{4}{3}} 2 = t$

4. If (a, b) be the orthocentre of the triangle whose vertices are (1, 2), (2, 3) and (3, 1), and $I_1 = \int_a^b x\sin\left(4x - x^2\right)dx$, $I_2 = \int_a^b \sin\left(4x - x^2\right)dx$, then $36\frac{I_1}{I_2}$ is equal to :

(1) 72 (2) 88

(3) 80 (4) 66

Ans. (1)

Sol. Equation of CE

$y - 1 = -(x - 3)$

$x + y = 4$

orthocentre lies on the line x + y = 4

so, a + b = 4

$I_1 = \int_a^b x\sin\left(x(4-x)\right)dx$ …(i)

Using king rule

$I_1 = \int_a^b (4-x)\sin\left(x(4-x)\right)dx$ …(ii)

(i) + (ii)

$2I_1 = \int_a^b 4\sin\left(x(4-x)\right)dx$

$2I_1 = 4I_2$

$I_1 = 2I_2$

$\frac{I_1}{I_2} = 2$

$\frac{36I_1}{I_2} = 72$

5. If A denotes the sum of all the coefficients in the expansion of $(1 - 3x + 10x^2)^n$ and B denotes the sum of all the coefficients in the expansion of $(1 + x^2)^n$, then :

(1) $A = B^3$ (2) 3A = B

(3) $B = A^3$ (4) A = 3B

Ans. (1)

Sol. Sum of coefficients in the expansion of

$(1 - 3x + 10x^2)^n = A$

then $A = (1 - 3 + 10)^n = 8^n$ (put x = 1)

and sum of coefficients in the expansion of

$(1 + x^2)^n = B$

then $B = (1 + 1)^n = 2^n$

$A = B^3$

6. The number of common terms in the progressions 4, 9, 14, 19, ... , up to 25th term and 3, 6, 9, 12, ..., up to 37th term is :

(1) 9 (2) 5

(3) 7 (4) 8

Ans. (3)

Sol. 4, 9, 14, 19, …., up to 25th term

$T_{25} = 4 + (25 - 1)\,5 = 4 + 120 = 124$

3, 6, 9, 12, …, up to 37th term

$T_{37} = 3 + (37 - 1)3 = 3 + 108 = 111$

Common difference of I^{st} series $d_1 = 5$

Common difference of IInd series $d_2 = 3$

First common term = 9, and

their common difference = 15 (LCM of d_1 and d_2)

then common terms are

9, 24, 39, 54, 69, 84, 99

7. If the shortest distance of the parabola $y^2 = 4x$ from the centre of the circle $x^2 + y^2 - 4x - 16y + 64 = 0$ is d, then d^2 is equal to :

(1) 16 (2) 24

(3) 20 (4) 36

Ans. (3)

Sol. Equation of normal to parabola

$y = mx - 2m - m^3$

this normal passing through center of circle (2, 8)

$8 = 2m - 2m - m^3$

$m = -2$

So point P on parabola $\Rightarrow (am^2, -2am) = (4, 4)$

And C = (2, 8)

$PC = \sqrt{4+16} = \sqrt{20}$

$d^2 = 20$

8. If the shortest distance between the lines $\frac{x-4}{1} = \frac{y+1}{2} = \frac{z}{-3}$ and $\frac{x-\lambda}{2} = \frac{y+1}{4} = \frac{z-2}{-5}$ is $\frac{6}{\sqrt{5}}$, then the sum of all possible values of λ is :

(1) 5 (2) 8

(3) 7 (4) 10

Ans. (2)

Sol. $\frac{x-4}{1} = \frac{y+1}{2} = \frac{z}{-3}$

$\frac{x-\lambda}{2} = \frac{y+1}{4} = \frac{z-2}{-5}$

the shortest distance between the lines

$$= \left|\frac{\left(\vec{a}-\vec{b}\right)\cdot\left(\vec{d_1}\times\vec{d_2}\right)}{\left|\vec{d_1}\times\vec{d_2}\right|}\right|$$

$$= \left|\frac{\begin{vmatrix}\lambda-4 & 0 & 2\\ 1 & 2 & -3\\ 2 & 4 & -5\end{vmatrix}}{\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 2 & -3\\ 2 & 4 & -5\end{vmatrix}}\right|$$

$$= \left|\frac{(\lambda-4)(-10+12)-0+2(4-4)}{\left|2\hat{i}-1\hat{j}+0\hat{k}\right|}\right|$$

$$\frac{6}{\sqrt{5}} = \left|\frac{2(\lambda-4)}{\sqrt{5}}\right|$$

$3 = |\lambda - 4|$

$\lambda - 4 = \pm 3$

$\lambda = 7, 1$

Sum of all possible values of λ is = 8

9. If $\int_0^1 \frac{1}{\sqrt{3+x}+\sqrt{1+x}}dx = a + b\sqrt{2} + c\sqrt{3}$, where a, b, c are rational numbers, then 2a + 3b – 4c is equal to :

(1) 4 (2) 10

(3) 7 (4) 8

Ans. (4)

Sol. $\int_0^1 \frac{1}{\sqrt{3+x}+\sqrt{1+x}}dx = \int_0^1 \frac{\sqrt{3+x}-\sqrt{1+x}}{(3+x)-(1+x)}dx$

$$\frac{1}{2}\left[\int_0^1 \sqrt{3+x}\,dx - \int_0^1 \left(\sqrt{1+x}\right)dx\right]$$

$$\frac{1}{2}\left[2\frac{(3+x)^{\frac{3}{2}}}{3}-\frac{2(1+x)^{\frac{3}{2}}}{3}\right]_0^1$$

$$\frac{1}{2}\left[\frac{2}{3}\left(8-3\sqrt{3}\right)-\frac{2}{3}\left(2^{\frac{3}{2}}-1\right)\right]$$

$$\frac{1}{3}\left[8-3\sqrt{3}-2\sqrt{2}+1\right]$$

$$=3-\sqrt{3}-\frac{2}{3}\sqrt{2}=a+b\sqrt{2}+c\sqrt{3}$$

$$a=3,\ b=-\frac{2}{3},\ c=-1$$

$2a + 3b - 4c = 6 - 2 + 4 = 8$

10. Let S = {l, 2, 3, ... , 10}. Suppose M is the set of all the subsets of S, then the relation

$R = \{(A, B): A \cap B \neq \phi;\ A, B \in M\}$ is :

(1) symmetric and reflexive only

(2) reflexive only

(3) symmetric and transitive only

(4) symmetric only

Ans. (4)

Sol. Let S = {1, 2, 3, …, 10}

$R = \{(A, B): A \cap B \neq \phi;\ A, B \in M\}$

For Reflexive,

M is subset of 'S'

So $\phi \in M$

for $\phi \cap \phi = \phi$

$\Rightarrow$ but relation is $A \cap B \neq \phi$

So it is not reflexive.

For symmetric,

ARB $\quad A \cap B \neq \phi$,

$\Rightarrow$ BRA $\quad \Rightarrow B \cap A \neq \phi$,

So it is symmetric.

For transitive,

If $A = \{(1, 2), (2, 3)\}$

$B = \{(2, 3), (3, 4)\}$

$C = \{(3, 4), (5, 6)\}$

ARB & BRC but A does not relate to C

So it not transitive

11. If $S = \{z \in C : |z - i| = |z + i| = |z-1|\}$, then, n(S) is:

(1) 1 (2) 0

(3) 3 (4) 2

Ans. (1)

Sol. $|z - i| = |z + i| = |z - 1|$

ABC is a triangle. Hence its circum-centre will be the only point whose distance from A, B, C will be same.

So n(S) = 1

12. Four distinct points (2k, 3k), (1, 0), (0, 1) and (0, 0) lie on a circle for k equal to :

(1) $\frac{2}{13}$ (2) $\frac{3}{13}$

(3) $\frac{5}{13}$ (4) $\frac{1}{13}$

Ans. (3)

Sol. (2k, 3k) will lie on circle whose diameter is AB.

$(x - 1)(x) + (y - 1)(y) = 0$

$x^2 + y^2 - x - y = 0$ …(i)

Satisfy (2k, 3k) in (i)

$(2k)^2 + (3k)^2 - 2k - 3k = 0$

$13k^2 - 5k = 0$

$k = 0,\ k = \frac{5}{13}$

hence $k = \frac{5}{13}$

13. Consider the function.

$$f(x)=\begin{cases}\dfrac{a\left(7x-12-x^2\right)}{b\left|x^2-7x+12\right|} & , \quad x<3\\ 2^{\frac{\sin(x-3)}{x-[x]}} & , \quad x>3\\ b & , \quad x=3\end{cases}$$

Where [x] denotes the greatest integer less than or equal to x. If S denotes the set of all ordered pairs (a, b) such that f(x) is continuous at x = 3, then the number of elements in S is :

(1) 2 (2) Infinitely many

(3) 4 (4) 1

Ans. (4)

Sol. $f\left(3^-\right)=\dfrac{a}{b}\dfrac{\left(7x-12-x^2\right)}{\left|x^2-7x+12\right|}$ (for f(x) to be cont.)

$\Rightarrow f(3^-)=\dfrac{-a}{b}\dfrac{(x-3)(x-4)}{(x-3)(x-4)};x<3 \Rightarrow \dfrac{-a}{b}$

Hence $f\left(3^-\right)=\dfrac{-a}{b}$

Then $f\left(3^+\right)=2^{\lim\limits_{x\to 3^+}\left(\frac{\sin(x-3)}{x-3}\right)}=2$ and

f(3) = b.

Hence $f(3) = f(3^+) = f(3^-)$

$\Rightarrow\ b=2=-\dfrac{a}{b}$

b = 2, a = –4

Hence only 1 ordered pair (–4, 2).

14. Let $a_1, a_2, \ldots\ldots a_{10}$ be 10 observations such that $\sum\limits_{k=1}^{10}a_k=50$ and $\sum\limits_{\forall k<j}a_k\cdot a_j=1100$. Then the standard deviation of $a_1, a_2, .., a_{10}$ is equal to :

(1) 5 (2) $\sqrt{5}$

(3) 10 (4) $\sqrt{115}$

Ans. (2)

Sol. $\sum\limits_{k=1}^{10}a_k=50$

$a_1 + a_2 + \ldots + a_{10} = 50$ ….(i)

$\sum\limits_{\forall k<j}a_ka_j=1100$ ....(ii)

If $a_1 + a_2 + \ldots + a_{10} = 50$.

$(a_1 + a_2 + \ldots + a_{10})^2 = 2500$

$\Rightarrow\sum\limits_{i=1}^{10}a_i^2+2\sum\limits_{k<j}a_ka_j=2500$

$\Rightarrow\sum\limits_{i=1}^{10}a_i^2=2500-2(1100)$

$\sum\limits_{i=1}^{10}a_i^2=300$, Standard deviation 'σ'

$$=\sqrt{\frac{\sum a_i^2}{10}-\left(\frac{\sum a_i}{10}\right)^2}=\sqrt{\frac{300}{10}-\left(\frac{50}{10}\right)^2}$$

$=\sqrt{30-25}=\sqrt{5}$

15. The length of the chord of the ellipse $\dfrac{x^2}{25}+\dfrac{y^2}{16}=1$, whose mid point is $\left(1,\dfrac{2}{5}\right)$, is equal to :

(1) $\dfrac{\sqrt{1691}}{5}$ (2) $\dfrac{\sqrt{2009}}{5}$

(3) $\dfrac{\sqrt{1741}}{5}$ (4) $\dfrac{\sqrt{1541}}{5}$

Ans. (1)

Sol. Equation of chord with given middle point.

$T = S_1$

$\dfrac{x}{25}+\dfrac{y}{40}=\dfrac{1}{25}+\dfrac{1}{100}$

$\dfrac{8x+5y}{200}=\dfrac{8+2}{200}$

$y=\dfrac{10-8x}{5}$ …(i)

$$\frac{x^2}{25}+\frac{(10-8x)^2}{400}=1 \qquad \text{(put in original equation)}$$

$$\frac{16x^2+100+64x^2-160x}{400}=1$$

$4x^2 - 8x - 15 = 0$

$$x=\frac{8\pm\sqrt{304}}{8}$$

$$x_1=\frac{8+\sqrt{304}}{8};\ x_2=\frac{8-\sqrt{304}}{8}$$

Similarly, $y=\frac{10-18\pm\sqrt{304}}{5}=\frac{2\pm\sqrt{304}}{5}$

$$y_1=\frac{2-\sqrt{304}}{5};\ y_2=\frac{2+\sqrt{304}}{5}$$

Distance $=\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$

$$=\sqrt{\frac{4\times304}{64}+\frac{4\times304}{25}}=\frac{\sqrt{1691}}{5}$$

16. The portion of the line 4x + 5y = 20 in the first quadrant is trisected by the lines L_1 and L_2 passing through the origin. The tangent of an angle between the lines L_1 and L_2 is :

(1) $\frac{8}{5}$ (2) $\frac{25}{41}$

(3) $\frac{2}{5}$ (4) $\frac{30}{41}$

Ans. (4)

Sol. Co-ordinates of $A=\left(\frac{5}{3},\frac{8}{3}\right)$

Co-ordinates of $B=\left(\frac{10}{3},\frac{4}{3}\right)$

Slope of OA = $m_1=\frac{8}{5}$

Slope of OB = $m_2=\frac{2}{5}$

$$\tan\theta=\left|\frac{m_1-m_2}{1+m_1m_2}\right|$$

$$\tan\theta=\frac{\frac{6}{5}}{1+\frac{16}{25}}=\frac{30}{41}$$

$$\tan\theta=\frac{30}{41}$$

17. Let $\vec{a}=\hat{i}+2\hat{j}+k$, $\vec{b}=3(\hat{i}-\hat{j}+k)$. Let $\vec{c}$ be the vector such that $\vec{a}\times\vec{c}=\vec{b}$ and $\vec{a}\cdot\vec{c}=3$. Then $\vec{a}\cdot\left((\vec{c}\times\vec{b})-\vec{b}-\vec{c}\right)$ is equal to :

(1) 32 (2) 24

(3) 20 (4) 36

Ans. (2)

Sol. $\vec{a}\cdot\left[(\vec{c}\times\vec{b})-\vec{b}-\vec{c}\right]$

$\vec{a}\cdot(\vec{c}\times\vec{b})-\vec{a}\cdot\vec{b}-\vec{a}\cdot\vec{c}$ ….(i)

given $\vec{a}\times\vec{c}=\vec{b}$

$\Rightarrow\ (\vec{a}\times\vec{c})\cdot\vec{b}=\vec{b}\cdot\vec{b}=|\vec{b}|^2=27$

$\Rightarrow\ \vec{a}\cdot(\vec{c}\times\vec{b})=[\vec{a}\ \vec{c}\ \vec{b}]=(\vec{a}\times\vec{c})\cdot\vec{b}=27$ …(ii)

Now $\vec{a}\cdot\vec{b}=3-6+3=0$ …(iii)

$\vec{a}\cdot\vec{c}=3$ …(iv) (given)

By (i), (ii), (iii) & (iv)

27 – 0 – 3 = 24

18. If $a = \lim_{x\to 0} \frac{\sqrt{1+\sqrt{1+x^4}} - \sqrt{2}}{x^4}$ and

$b = \lim_{x\to 0} \frac{\sin^2 x}{\sqrt{2} - \sqrt{1+\cos x}}$, then the value of ab^3 is :

(1) 36 (2) 32 (3) 25 (4) 30

Ans. (2)

Sol. $a = \lim_{x\to 0} \frac{\sqrt{1+\sqrt{1+x^4}} - \sqrt{2}}{x^4}$

$= \lim_{x\to 0} \frac{\sqrt{1+x^4} - 1}{x^4\left(\sqrt{1+\sqrt{1+x^4}} + \sqrt{2}\right)}$

$= \lim_{x\to 0} \frac{x^4}{x^4\left(\sqrt{1+\sqrt{1+x^4}} + \sqrt{2}\right)\left(\sqrt{1+x^4} + 1\right)}$

Applying limit $a = \frac{1}{4\sqrt{2}}$

$b = \lim_{x\to 0} \frac{\sin^2 x}{\sqrt{2} - \sqrt{1+\cos x}}$

$= \lim_{x\to 0} \frac{\left(1-\cos^2 x\right)\left(\sqrt{2} + \sqrt{1+\cos x}\right)}{2-\left(1+\cos x\right)}$

$b = \lim_{x\to 0} \left(1+\cos x\right)\left(\sqrt{2} + \sqrt{1+\cos x}\right)$

Applying limits $b = 2\left(\sqrt{2} + \sqrt{2}\right) = 4\sqrt{2}$

Now, $ab^3 = \frac{1}{4\sqrt{2}} \times \left(4\sqrt{2}\right)^3 = 32$

19. Consider the matrix $f(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

Given below are two statements :

Statement I: f(–x) is the inverse of the matrix f(x).

Statement II: f(x) f(y) = f(x + y).

In the light of the above statements, choose the correct answer from the options given below

(1) Statement I is false but Statement II is true

(2) Both Statement I and Statement II are false

(3) Statement I is true but Statement II is false

(4) Both Statement I and Statement II are true

Ans. (4)

Sol. $f(-x) = \begin{bmatrix} \cos x & \sin x & 0 \\ -\sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$f(x)\cdot f(-x) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$

Hence statement- I is correct

Now, checking statement II

$f(y) = \begin{bmatrix} \cos y & -\sin y & 0 \\ \sin y & \cos y & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$f(x)\cdot f(y) = \begin{bmatrix} \cos(x+y) & -\sin(x+y) & 0 \\ \sin(x+y) & \cos(x+y) & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\Rightarrow f(x)\cdot f(y) = f(x+y)$

Hence statement-II is also correct.

20. The function f : N – {1} → N; defined by f(n) = the highest prime factor of n, is :

(1) both one-one and onto

(2) one-one only

(3) onto only

(4) neither one-one nor onto

Ans. (4)

Sol. f : N – {1} → N

f(n) = The highest prime factor of n.

f(2) = 2

f(4) = 2

⇒ many one

4 is not image of any element

⇒ into

Hence many one and into

Neither one-one nor onto.

SECTION-B

21. The least positive integral value of α, for which the angle between the vectors $\alpha\hat{i}-2\hat{j}+2k$ and $\alpha\hat{i}+2\alpha\hat{j}-2k$ is acute, is _____.

Ans. (5)

Sol. $\cos\theta = \dfrac{\left(\alpha\hat{i}-2\hat{j}+2\hat{k}\right)\cdot\left(\alpha\hat{i}+2\alpha\hat{j}-2\hat{k}\right)}{\sqrt{\alpha^2+4+4}\ \sqrt{\alpha^2+4\alpha^2+4}}$

$\cos\theta = \dfrac{\alpha^2-4\alpha-4}{\sqrt{\alpha^2+8}\ \sqrt{5\alpha^2+4}}$

$\Rightarrow \alpha^2 - 4\alpha - 4 > 0$

$\Rightarrow \alpha^2 - 4\alpha + 4 > 8 \qquad \Rightarrow (\alpha - 2)^2 > 8$

$\Rightarrow \alpha-2>2\sqrt{2}$ or $\alpha-2<-2\sqrt{2}$

$\alpha>2+2\sqrt{2}$ or $\alpha<2-2\sqrt{2}$

$\alpha\in(-\infty, -0.82) \cup (4.82, \infty)$

Least positive integral value of $\alpha \Rightarrow 5$

22. Let for a differentiable function $f:(0,\infty)\to R$, $f(x)-f(y)\ge \log_e\left(\dfrac{x}{y}\right)+x-y$, $\forall$ x, y $\in$ (0, ∞).

Then $\sum_{n=1}^{20} f'\left(\dfrac{1}{n^2}\right)$ is equal to _____.

Ans. (2890)

Sol. $f(x) - f(y) \ge \ln x - \ln y + x - y$

$\dfrac{f(x)-f(y)}{x-y} \ge \dfrac{\ln x-\ln y}{x-y}+1$

Let $x > y$

$\lim_{y\to x} f'(x^-) \ge \dfrac{1}{x}+1$ .....(1)

Let $x < y$

$\lim_{y\to x} f'(x^+) \le \dfrac{1}{x}+1$ ..... (2)

$f^{l}(x^-) = f^{l}(x^+)$

$f^{l}(x) = \dfrac{1}{x}+1$

$f'\left(\dfrac{1}{x^2}\right) = x^2+1$

$\sum_{x=1}^{20}\left(x^2+1\right) = \sum_{x-1}^{20} x^2 + 20$

$= \dfrac{20\times21\times41}{6}+20$

$= 2890$

23. If the solution of the differential equation $(2x + 3y - 2)\,dx + (4x + 6y - 7)\,dy = 0$, $y(0) = 3$, is $\alpha x + \beta y + 3\log_e|2x + 3y - \gamma| = 6$, then $\alpha + 2\beta + 3\gamma$ is equal to _____.

Ans. (29)

Sol. $2x + 3y - 2 = t \qquad 4x + 6y - 4 = 2t$

$2+3\dfrac{dy}{dx} = \dfrac{dt}{dx} \qquad 4x + 6y - 7 = 2t - 3$

$\dfrac{dy}{dx} = \dfrac{-(2x+3y-2)}{4x+6y-7}$

$\dfrac{dt}{dx} = \dfrac{-3t+4t-6}{2t-3} = \dfrac{t-6}{2t-3}$

$\int \dfrac{2t-3}{t-6}\,dt = \int dx$

$\int\left(\dfrac{2t-12}{t-6}+\dfrac{9}{t-6}\right)\cdot dt = x$

$2t + 9\ln(t - 6) = x + c$

$2(2x + 3y - 2) + 9\ln(2x + 3y - 8) = x + c$

$x = 0,\ y = 3$

$c = 14$

$4x + 6y - 4 + 9\ln(2x + 3y - 8) = x+14$

$x + 2y + 3\ln(2x + 3y - 8) = 6$

$\alpha = 1,\ \beta = 2,\ \gamma = 8$

$\alpha + 2\beta + 3\gamma = 1 + 4 + 24 = 29$

24. Let the area of the region $\{(x, y) :\ x - 2y + 4 \ge 0,\ x + 2y^2 \ge 0,\ x + 4y^2 \le 8,\ y \ge 0\}$ be $\dfrac{m}{n}$, where m and n are coprime numbers. Then m + n is equal to _____.

Ans. (119)

Sol.

$A = \int_0^1\left[\left(8-4y^2\right)-\left(-2y^2\right)\right]dy +$

$\int_1^{3/2}\left[\left(8-4y^2\right)-\left(2y-4\right)\right]dy$

$= \left[8y-\dfrac{2y^3}{3}\right]_0^1 + \left[12y-y^2-\dfrac{4y^3}{3}\right]_1^{3/2} = \dfrac{107}{12} = \dfrac{m}{n}$

$\therefore$ m + n = 119

25. If

$$8=3+\frac{1}{4}(3+p)+\frac{1}{4^2}(3+2p)+\frac{1}{4^3}(3+3p)+....\infty,$$

then the value of p is _____.

Ans. (9)

Sol. $8=\frac{3}{1-\frac{1}{4}}+\frac{p\cdot\frac{1}{4}}{\left(1-\frac{1}{4}\right)^2}$

(sum of infinite terms of A.G.P = $\frac{a}{1-r}+\frac{dr}{(1-r)^2}$)

$\Rightarrow \frac{4p}{9}=4 \Rightarrow p=9$

26. A fair die is tossed repeatedly until a six is obtained. Let X denote the number of tosses required and let a = P(X = 3), b = P(X ≥ 3) and c = P(X ≥ 6 |X > 3). Then $\frac{b+c}{a}$ is equal to _____.

Ans. (12)

Sol. $a=P(X=3)=\frac{5}{6}\times\frac{5}{6}\times\frac{1}{6}=\frac{25}{216}$

$b=P(X\geq 3)=\frac{5}{6}\times\frac{5}{6}\times\frac{1}{6}+\left(\frac{5}{6}\right)^3\cdot\frac{1}{6}+\left(\frac{5}{6}\right)^4\cdot\frac{1}{6}+.....$

$=\frac{\frac{25}{216}}{1-\frac{5}{6}}=\frac{25}{216}\times\frac{6}{1}=\frac{25}{36}$

$P(X\geq 6)=\left(\frac{5}{6}\right)^5\cdot\frac{1}{6}+\left(\frac{5}{6}\right)^6\cdot\frac{1}{6}+.....$

$=\frac{\left(\frac{5}{6}\right)^5\cdot\frac{1}{6}}{1-\frac{5}{6}}=\left(\frac{5}{6}\right)^5$

$c=\frac{\left(\frac{5}{6}\right)^5}{\left(\frac{5}{6}\right)^3}=\frac{25}{36}$

$\frac{b+c}{a}=\frac{\left(\frac{5}{6}\right)^2+\left(\frac{5}{6}\right)^2}{\left(\frac{5}{6}\right)^2\cdot\frac{1}{6}}=12$

27. Let the set of all a ∈ R such that the equation $\cos 2x+a\sin x=2a-7$ has a solution be [p, q] and $r=\tan 9°-\tan 27°-\frac{1}{\cot 63°}+\tan 81°$, then pqr is equal to _____.

Ans. (48)

Sol. cos2x + a·sinx = 2a – 7

$a(\sin x-2)=2(\sin x-2)(\sin x+2)$

$\sin x=2,\ \ a=2(\sin x+2)$

$\Rightarrow a\in[2, 6]$

p = 2 q = 6

r = tan 9° + cot 9° – tan27 – cot27

$r=\frac{1}{\sin 9\cdot\cos 9}-\frac{1}{\sin 27\cdot\cos 27}$

$=2\left[\frac{4}{\sqrt{5}-1}-\frac{4}{\sqrt{5}+1}\right]$

r = 4

p . q . r = 2 × 6 × 4 = 48

28. Let $f(x)=x^3+x^2f'(1)+xf''(2)+f'''(3)$, x ∈ R. Then $f'(10)$ is equal to _____.

Ans. (202)

Sol. $f(x) = x^3 + x^2 \cdot f'(1) + x \cdot f''(2) + f'''(3)$

$f'(x)=3x^2+2xf'(1)+f''(2)$

$f''(x)=6x+2f'(1)$

$f'''(x)=6$

$f'(1) = -5,\ f''(2) = 2,\ f'''(3) = 6$

$f(x) = x^3 + x^2 \cdot (-5) + x \cdot (2) + 6$

$f'(x) = 3x^2 - 10x + 2$

$f'(10) = 300 - 100 + 2 = 202$

29. Let $A = \begin{bmatrix} 2 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$, $B = [B_1, B_2, B_3]$, where B_1, B_2, B_3 are column matrices, and $AB_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$,

$AB_2 = \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix}$, $AB_3 = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$

If $\alpha = |B|$ and β is the sum of all the diagonal elements of B, then $\alpha^3 + \beta^3$ is equal to _____.

Ans. (28)

Sol. $A = \begin{bmatrix} 2 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$ $\quad B = [B_1, B_2, B_3]$

$B_1 = \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix}$, $B_2 = \begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix}$, $B_3 = \begin{bmatrix} x_3 \\ y_3 \\ z_3 \end{bmatrix}$

$$AB_1 = \begin{bmatrix} 2 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$x_1 = 1, y_1 = -1, z_1 = -1$

$$AB_2 = \begin{bmatrix} 2 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix}$$

$x_2 = 2, y_2 = 1, z_2 = -2$

$$AB_3 = \begin{bmatrix} 2 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_3 \\ y_3 \\ z_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$$

$x_3 = 2, y_3 = 0, z_3 = -1$

$$B = \begin{bmatrix} 1 & 2 & 2 \\ -1 & 1 & 0 \\ -1 & -2 & -1 \end{bmatrix}$$

$\alpha = |B| = 3$

$\beta = 1$

$\alpha^3 + \beta^3 = 27 + 1 = 28$

30. If α satisfies the equation $x^2 + x + 1 = 0$ and $(1 + \alpha)^7 = A + B\alpha + C\alpha^2$, $A, B, C \geq 0$, then $5(3A - 2B - C)$ is equal to _____.

Ans. (5)

Sol. $x^2 + x + 1 = 0 \Rightarrow x = \omega, \omega^2 = \alpha$

Let $\alpha = \omega$

Now $(1 + \alpha)^7 = -\omega^{14} = -\omega^2 = 1 + \omega$

$A = 1, B = 1, C = 0$

$\therefore 5(3A - 2B - C) = 5(3 - 2 - 0) = 5$

SECTION-A

31. Position of an ant (S in metres) moving in Y-Z plane is given by $S = 2t^2\hat{j} + 5\hat{k}$ (where t is in second). The magnitude and direction of velocity of the ant at t = 1 s will be :

(1) 16 m/s in y-direction

(2) 4 m/s in x-direction

(3) 9 m/s in z-direction

(4) 4 m/s in y-direction

Ans. (4)

Sol. $\vec{v} = \frac{d\vec{s}}{dt} = 4t\hat{j}$

At t = 1 sec $\vec{v} = 4\hat{j}$

32. Given below are two statements :

Statement (I) : Viscosity of gases is greater than that of liquids.

Statement (II) : Surface tension of a liquid decreases due to the presence of insoluble impurities.

In the light of the above statements, choose the most appropriate answer from the options given below :

(1) Statement I is correct but statement II is incorrect

(2) Statement I is incorrect but Statement II is correct

(3) Both Statement I and Statement II are incorrect

(4) Both Statement I and Statement II are correct

Ans. (2)

Sol. Gases have less viscosity.

Due to insoluble impurities like detergent surface tension decreases

33. If the refractive index of the material of a prism is $\cot\left(\frac{A}{2}\right)$, where A is the angle of prism then the angle of minimum deviation will be

(1) $\pi - 2A$ (2) $\frac{\pi}{2} - 2A$

(3) $\pi - A$ (4) $\frac{\pi}{2} - A$

Ans. (1)

Sol. $\cot\frac{A}{2} = \frac{\sin\left(\frac{A+\delta_{min}}{2}\right)}{\sin\frac{A}{2}}$

$\Rightarrow \cos\frac{A}{2} = \sin\left(\frac{A+\delta_{min}}{2}\right)$

$\frac{A+\delta_{min}}{2} = \frac{\pi}{2} - \frac{A}{2}$

$\delta_{min} = \pi - 2A$

34. A proton moving with a constant velocity passes through a region of space without any change in its velocity. If $\vec{E}$ and $\vec{B}$ represent the electric and magnetic fields respectively, then the region of space may have :

(A) $E = 0, B = 0$ (B) $E = 0, B \neq 0$

(C) $E \neq 0, B = 0$ (D) $E \neq 0, B \neq 0$

Choose the most appropriate answer from the options given below :

(1)(A), (B) and (C) only

(2) (A), (C) and (D) only

(3) (A), (B) and (D) only

(4) (B), (C) and (D) only

Ans. (3)

Sol. Net force on particle must be zero i.e. $q\vec{E} + q\vec{V} \times \vec{B} = 0$

Possible cases are

(i) $\vec{E} \,\&\, \vec{B} = 0$

(ii) $\vec{V} \times \vec{B} = 0, \vec{E} = 0$

(iii) $q\vec{E} = -q\vec{V} \times \vec{B}$

$\vec{E} \neq 0 \,\&\, \vec{B} \neq 0$

35. The acceleration due to gravity on the surface of earth is g. If the diameter of earth reduces to half of its original value and mass remains constant, then acceleration due to gravity on the surface of earth would be :

(1) g/4 (2) 2g
(3) g/2 (4) 4g

Ans. (4)

Sol. $g = \frac{GM}{R^2} \Rightarrow g \propto \frac{1}{R^2}$

$\frac{g_2}{g_1} = \frac{R_1^2}{R_2^2}$

$g_2 = 4g_1 \left(R_2 = \frac{R_1}{2} \right)$

36. A train is moving with a speed of 12 m/s on rails which are 1.5 m apart. To negotiate a curve radius 400 m, the height by which the outer rail should be raised with respect to the inner rail is (Given, g = 10 m/s^2) :

(1) 6.0 cm (2) 5.4 cm
(3) 4.8 cm (4) 4.2 cm

Ans. (2)

Sol. $\tan\theta = \frac{v^2}{Rg} = \frac{12 \times 12}{10 \times 400}$

$\tan\theta = \frac{h}{1.5}$

$\Rightarrow \frac{h}{1.5} = \frac{144}{4000}$

h = 5.4 cm

37. Which of the following circuits is reverse - biased ?

(1)

(2)

(3)

(4)

Ans. (4)

Sol. P end should be at higher potential for forward biasing.

38. Identify the physical quantity that cannot be measured using spherometer :

(1) Radius of curvature of concave surface
(2) Specific rotation of liquids
(3) Thickness of thin plates
(4) Radius of curvature of convex surface

Ans. (2)

Sol. Spherometer can be used to measure curvature of surface.

39. Two bodies of mass 4 g and 25 g are moving with equal kinetic energies. The ratio of magnitude of their linear momentum is :

(1) 3 : 5 (2) 5 : 4
(3) 2 : 5 (4) 4 : 5

Ans. (3)

Sol. $\frac{P_1^2}{2m_1} = \frac{P_2^2}{2m_2}$

$\frac{P_1}{P_2} = \sqrt{\frac{m_1}{m_2}} = \frac{2}{5}$

40. 0.08 kg air is heated at constant volume through 5°C. The specific heat of air at constant volume is 0.17 kcal/kg°C and J = 4.18 joule/cal. The change in its internal energy is approximately.

(1) 318 J (2) 298 J
(3) 284 J (4) 142 J

Ans. (3)

Sol. Q = ΔU as work done is zero [constant volume]

$\Delta U = ms\Delta T$

$= 0.08 \times (170 \times 4.18) \times 5$

$\simeq 284$ J

41. The radius of third stationary orbit of electron for Bohr's atom is R. The radius of fourth stationary orbit will be:

(1) $\frac{4}{3}R$ (2) $\frac{16}{9}R$
(3) $\frac{3}{4}R$ (4) $\frac{9}{16}R$

Ans. (2)

Sol. $r \propto \frac{n^2}{Z}$

$\frac{r_4}{r_3} = \frac{4^2}{3^2}$

$r_4 = \frac{16}{9}R$

42. A rectangular loop of length 2.5 m and width 2 m is placed at 60° to a magnetic field of 4 T. The loop is removed from the field in 10 sec. The average emf induced in the loop during this time is

(1) – 2V (2) + 2V

(3) + 1V (4) – 1V

Ans. (3)

Sol. Average emf= $\frac{\text{Change in flux}}{\text{Time}} = -\frac{\Delta\phi}{\Delta t}$

$= -\frac{0-(4\times(2.5\times2)\cos 60°)}{10}$

$= +1V$

43. An electric charge $10^{-6}\mu C$ is placed at origin (0, 0) m of X –Y co-ordinate system. Two points P and Q are situated at $(\sqrt{3},\sqrt{3})$m and $(\sqrt{6},0)$m respectively. The potential difference between the points P and Q will be :

(1) $\sqrt{3}$V

(2) $\sqrt{6}$V

(3) 0 V

(4) 3 V

Ans. (3)

Sol. Potential difference $= \frac{KQ}{r_1} - \frac{KQ}{r_2}$

$r_1 = \sqrt{(\sqrt{3})^2 + (\sqrt{3})^2}$

$r_2 = \sqrt{(\sqrt{6})^2 + 0}$

As $r_1 = r_2 = \sqrt{6}$m

So potential difference = 0

44. A convex lens of focal length 40 cm forms an image of an extended source of light on a photo-electric cell. A current I is produced. The lens is replaced by another convex lens having the same diameter but focal length 20 cm. The photoelectric current now is :

(1) $\frac{I}{2}$ (2) 4 I

(3) 2 I (4) I

Ans. (4)

Sol. As amount of energy incident on cell is same so current will remain same.

45. A body of mass 1000 kg is moving horizontally with a velocity 6 m/s. If 200 kg extra mass is added, the final velocity (in m/s) is:

(1) 6 (2) 2

(3) 3 (4) 5

Ans. (4)

Sol. Momentum will remain conserve

$1000 \times 6 = 1200 \times v$

$v = 5$ m/s

46. A plane electromagnetic wave propagating in x-direction is described by

$E_y = (200\ Vm^{-1}) \sin[1.5 \times 10^7 t - 0.05\ x]$;

The intensity of the wave is :

(Use $\epsilon_0 = 8.85 \times 10^{-12}\ C^2N^{-1}m^{-2}$)

(1) 35.4 Wm^{-2} (2) 53.1 Wm^{-2}

(3) 26.6 Wm^{-2} (4) 106.2 Wm^{-2}

Ans. (2)

Sol. $I = \frac{1}{2}\varepsilon_0 E_0^2 \times c$

$I = \frac{1}{2}\times 8.85\times10^{-12}\times4\times10^4\times3\times10^8$

$I = 53.1\ W/m^2$

47. Given below are two statements :

Statement (I) : Planck's constant and angular momentum have same dimensions.

Statement (II) : Linear momentum and moment of force have same dimensions.

In the light of the above statements, choose the correct answer from the options given below :

(1) Statement I is true but Statement II is false

(2) Both Statement I and Statement II are false

(3) Both Statement I and Statement II are true

(4) Statement I is false but Statement II is true

Ans. (1)

Sol. $[h] = ML^2T^{-1}$

$[L] = ML^2T^{-1}$

$[P] = MLT^{-1}$

$[\tau] = ML^2T^{-2}$

(Here h is Planck's constant, L is angular momentum, P is linear momentum and τ is moment of force)

48. A wire of length 10 cm and radius $\sqrt{7}\times10^{-4}$ m connected across the right gap of a meter bridge. When a resistance of 4.5 Ω is connected on the left gap by using a resistance box, the balance length is found to be at 60 cm from the left end. If the resistivity of the wire is R × 10^{-7}Ωm, then value of R is :

(1) 63 (2) 70

(3) 66 (4) 35

Ans. (3)

Sol. For null point,

$$\frac{4.5}{60}=\frac{R}{40}$$

Also, $R=\frac{\rho\ell}{A}=\frac{\rho\ell}{\pi r^2}$

$$4.5\times40=\rho\times\frac{0.1}{\pi\times7\times10^{-8}}\times60$$

$$\rho=66\times10^{-7}\Omega\times m$$

49. A wire of resistance R and length L is cut into 5 equal parts. If these parts are joined parallely, then resultant resistance will be :

(1) $\frac{1}{25}R$ (2) $\frac{1}{5}R$

(3) 25 R (4) 5 R

Ans. (1)

Sol. Resistance of each part = $\frac{R}{5}$

Total resistance = $\frac{1}{5}\times\frac{R}{5}=\frac{R}{25}$

50. The average kinetic energy of a monatomic molecule is 0.414 eV at temperature :

(Use K_B = 1.38 × 10^{-23} J/mol-K)

(1) 3000 K

(2) 3200 K

(3) 1600 K

(4) 1500 K

Ans. (2)

Sol. For monoatomic molecule degree of freedom = 3.

$$\therefore\ K_{avg}=\frac{3}{2}K_BT$$

$$T=\frac{0.414\times1.6\times10^{-19}\times2}{3\times1.38\times10^{-23}}$$

= 3200 K

SECTION-B

51. A particle starts from origin at t = 0 with a velocity $5\hat{i}$ m / s and moves in x-y plane under action of a force which produces a constant acceleration of $(3\hat{i}+2\hat{j})m/s^2$. If the x-coordinate of the particle at that instant is 84 m, then the speed of the particle at this time is $\sqrt{\alpha}$ m / s. The value of α is _____.

Ans. (673)

Sol u_x = 5 m/s a_x = 3 m/s^2 x = 84 m

$$v_x^2-u_x^2=2ax$$

$$v_x^2-25=2(3)(84)$$

$$V_x=23\text{ m/s}$$

$$v_x-u_x=a_xt$$

$$t=\frac{23-5}{3}=6s$$

$$v_y=0+a_yt=0+2\times(6)=12\text{ m/s}$$

$$v^2=v_x^2+v_y^2=23^2+12^2=673$$

$$v=\sqrt{673}\text{ m/s}$$

52. A thin metallic wire having cross sectional area of 10^{-4} m^2 is used to make a ring of radius 30 cm. A positive charge of 2π C is uniformly distributed over the ring, while another positive charge of 30 pC is kept at the centre of the ring. The tension in the ring is _____ N ; provided that the ring does not get deformed (neglect the influence of gravity).

(given, $\frac{1}{4\pi\in_0}=9\times10^9$ SI units)

Ans. (3)

Sol.

$$2T\sin\frac{d\theta}{2}=\frac{kq_0}{R^2}\cdot\lambda Rd\theta$$

$$\left[\lambda=\frac{Q}{2\pi R}\right]$$

$$\Rightarrow T = \frac{Kq_0Q}{(R^2)\times 2\pi}$$

$$= \frac{(9\times10^9)(2\pi\times30\times10^{-12})}{(0.30)^2\times2\pi}$$

$$= \frac{9\times10^{-3}\times30}{9\times10^{-2}} = 3N$$

53. Two coils have mutual inductance 0.002 H. The current changes in the first coil according to the relation $i = i_0 \sin \omega t$, where $i_0 = 5A$ and $\omega = 50\pi$ rad/s. The maximum value of emf in the second coil is $\frac{\pi}{\alpha}$ V. The value of α is ____.

Ans. **(2)**

Sol. $\phi = Mi = Mi_0 \sin\omega t$

$$EMF = -M\frac{di}{dt} = -0.002(i_0\omega\cos\omega t)$$

$$EMF_{max} = i_0\omega\ (0.002) = (5)(50\pi)(0.002)$$

$$EMF_{max} = \frac{\pi}{2}\ V$$

54. Two immiscible liquids of refractive indices $\frac{8}{5}$ and $\frac{3}{2}$ respectively are put in a beaker as shown in the figure. The height of each column is 6 cm. A coin is placed at the bottom of the beaker. For near normal vision, the apparent depth of the coin is $\frac{\alpha}{4}$ cm. The value of α is_____.

Ans. **(31)**

Sol. $h_{app} = \frac{h_1}{\mu_1} + \frac{h_2}{\mu_2} = \frac{6}{3/2} + \frac{6}{8/5} = 4 + \frac{15}{4} = \frac{31}{4}$ cm

55. In a nuclear fission process, a high mass nuclide $(A \approx 236)$ with binding energy 7.6 MeV/Nucleon dissociated into middle mass nuclides $(A \approx 118)$, having binding energy of 8.6 MeV/Nucleon. The energy released in the process would be ____ MeV.

Ans. **(236)**

Sol. $Q = BE_{Product} - BE_{Rectant}$

$= 2(118)(8.6) - 236(7.6)$

$= 236 \times 1 = 236$ MeV

56. Four particles each of mass 1 kg are placed at four corners of a square of side 2 m. Moment of inertia of system about an axis perpendicular to its plane and passing through one of its vertex is _____ kgm^2.

Ans. **(16)**

Sol.

$$I = ma^2 + ma^2 + m(\sqrt{2}a)^2$$

$= 4ma^2$

$= 4 \times 1 \times (2)^2 = 16$

57. A particle executes simple harmonic motion with an amplitude of 4 cm. At the mean position, velocity of the particle is 10 cm/s. The distance of the particle from the mean position when its speed becomes 5 cm/s is $\sqrt{\alpha}$ cm, where α = _____.

Ans. **(12)**

Sol. $V_{at\ mean\ position} = A\omega \Rightarrow 10 = 4\omega$

$$\omega = \frac{5}{2}$$

$$v = \omega\sqrt{A^2 - x^2}$$

$$5 = \frac{5}{2}\sqrt{4^2 - x^2} \Rightarrow x^2 = 16 - 4$$

$$x = \sqrt{12}\ cm$$

58. Two long, straight wires carry equal currents in opposite directions as shown in figure. The separation between the wires is 5.0 cm. The magnitude of the magnetic field at a point P midway between the wires is ____ μT

(Given : $\mu_0 = 4\pi \times 10^{-7}$ TmA^{-1})

Ans. (160)

Sol. $B = \left(\frac{\mu_0 i}{2\pi a}\right) \times 2 = \frac{4\pi \times 10^{-7} \times 10}{\pi \times \left(\frac{5}{2} \times 10^{-2}\right)}$

$= 16 \times 10^{-5} = 160\mu T$

59. The charge accumulated on the capacitor connected in the following circuit is ____ μC

(Given C = 150 μF)

Ans. (400)

Sol.

$V_A + \frac{10}{3}(1) - 6(1) = V_B$

$V_A - V_B = 6 - \frac{10}{3} = \frac{8}{3}$ volt

$Q = C(V_A - V_B)$

$= 150 \times \frac{8}{3} = 400\mu C$

60. If average depth of an ocean is 4000 m and the bulk modulus of water is 2×10^9 Nm^{-2}, then fractional compression $\frac{\Delta V}{V}$ of water at the bottom of ocean is $\alpha \times 10^{-2}$. The value of α is _____

(Given, g = 10 ms^{-2}, ρ = 1000 kg m^{-3})

Ans. (2)

Sol. $B = -\frac{\Delta P}{\left(\frac{\Delta V}{V}\right)}$

$-\left(\frac{\Delta V}{V}\right) = \frac{\rho g h}{B} = \frac{1000 \times 10 \times 4000}{2 \times 10^9}$

$= 2 \times 10^{-2}$ [–ve sign represent compression]

SECTION-A

61. Two nucleotides are joined together by a linkage known as :

(1) Phosphodiester linkage
(2) Glycosidic linkage
(3) Disulphide linkage
(4) Peptide linkage

Ans. (1)

Sol. Phosphodiester linkage

62. Highest enol content will be shown by :

(1)

(2)

(3)

(4)

Ans. (2)

Sol.

Aromatic

63. Element not showing variable oxidation state is :

(1)Bromine (2)Iodine
(3)Chlorine (4)Fluorine

Ans. (4)

Sol. Fluorine does not show variable oxidation state.

64. Which of the following is strongest Bronsted base?

(1)

(2)

(3)

(4)

Ans. (4)

Sol.

65. Which of the following electronic configuration would be associated with the highest magnetic moment ?

(1) [Ar] $3d^7$ (2) [Ar] $3d^8$
(3) [Ar] $3d^3$ (4) [Ar] $3d^6$

Ans. (4)

Sol.

	$3d^7$	$3d^8$	$3d^3$	$3d^6$
No.of. unpaired e^-	3	2	3	4
Spin only Magnetic moment	$\sqrt{15}$ BM	$\sqrt{8}$ BM	$\sqrt{15}$ BM	$\sqrt{24}$ BM

66. Which of the following has highly acidic hydrogen?

(1)

(2)

(3)

(4)

Ans. (4)

Sol.

Conjugate base is more stable due to more resonance of negative charge.

67. A solution of two miscible liquids showing negative deviation from Raoult's law will have :

(1) increased vapour pressure, increased boiling point

(2) increased vapour pressure, decreased boiling point

(3) decreased vapour pressure, decreased boiling point

(4) decreased vapour pressure, increased boiling point

Ans. (4)

Sol. Solution with negative deviation has

$P_T < P_A^0X_A + P_B^0X_B$

$P_A < P_A^0X_A$

$P_B < P_B^0X_B$

If vapour pressure decreases so boiling point increases.

68. Consider the following complex ions

$P = [FeF_6]^{3-}$

$Q = [V(H_2O)_6]^{2+}$

$R = [Fe(H_2O)_6]^{2+}$

The correct order of the complex ions, according to their spin only magnetic moment values (in B.M.) is :

(1) R < Q < P (2) R < P< Q

(3) Q < R < P (4) Q < P < R

Ans. (3)

Sol. $[FeF_6]^{3-}$: Fe^{+3} : [Ar] $3d^5$

F : Weak field Ligand

↑	↑	↑	↑	↑

No. of unpaired electron's = 5

$\mu = \sqrt{5(5+2)}$

$\mu = \sqrt{35}$ BM

$[V(H_2O)_6]^{+2}$: V^{+2} : $3d^3$

↑	↑	↑		

No. of unpaired electron's = 3

$\mu = \sqrt{3(3+2)}$

$\mu = \sqrt{15}$ BM

$[Fe(H_2O)_6]^{+2}$: Fe^{+2} : $3d^6$

H_2O : Weak field Ligand

↑↓	↑	↑	↑	↑

No. of unpaired electron's = 4

$\mu = \sqrt{4(4+2)}$

$\mu = \sqrt{24}$ BM

69. Choose the polar molecule from the following :

(1) CCl_4 (2) CO_2

(3) $CH_2 = CH_2$ (4) $CHCl_3$

Ans. (4)

Sol.

$\mu \neq 0$

$CHCl_3$ is polar molecule and rest all molecules are non-polar.

70. Given below are two statements :

Statement (I) : The 4f and 5f - series of elements are placed separately in the Periodic table to preserve the principle of classification.

Statement (II) :S-block elements can be found in pure form in nature. In the light of the above statements, choose the most appropriate answer from the options given below :

(1) Statement I is false but Statement II is true

(2) Both Statement I and Statement II are true

(3) Statement I is true but Statement II is false

(4) Both Statement I and Statement II are false

Ans. (3)

Sol. s-block elements are highly reactive and found in combined state.

71. Given below are two statements :

Statement (I) : p-nitrophenol is more acidic than m-nitrophenol and o-nitrophenol.

Statement (II) : Ethanol will give immediate turbidity with Lucas reagent.

In the light of the above statements, choose the correct answer from the options given below :

(1) Statement I is true but Statement II is false

(2) Both Statement I and Statement II are true

(3) Both Statement I and Statement II are false

(4) Statement I is false but Statement II is true

Ans. (1)

Sol. Acidic strength

Ethanol give lucas test after long time

Statement (I)→correct

Statement (II) → incorrect

72. The ascending order of acidity of –OH group in the following compounds is :

(A) Bu – OH

(B)

(C)

(D)

(E)

Choose the correct answer from the options given below :

(1) (A) < (D) < (C) < (B) < (E)

(2) (C) < (A) < (D) < (B) < (E)

(3) (C) < (D) < (B) < (A) < (E)

(4) (A) < (C) < (D) < (B) < (E)

Ans. (4)

Sol.

73. Given below are two statements : one is labelled as Assertion (A) and the other is labelled as Reason (R).

Assertion (A) : Melting point of Boron (2453 K) is unusually high in group 13 elements.

Reason (R) : Solid Boron has very strong crystalline lattice.

In the light of the above statements, choose the most appropriate answer from the options given below ;

(1) Both (A) and (R) are correct but (R) Is not the correct explanation of (A)

(2) Both (A) and (R) are correct and (R) is the correct explanation of (A)

(3) (A) is true but (R) is false

(4) (A) is false but (R) is true

Ans. (2)

Sol. Solid Boron has very strong crystalline lattice so its melting point unusually high in group 13 elements

74. Cyclohexene is _____ type of an organic compound.

(1) Benzenoid aromatic
(2) Benzenoid non-aromatic
(3) Acyclic
(4) Alicyclic

Ans. (4)

Sol. is Alicyclic

75. Yellow compound of lead chromate gets dissolved on treatment with hot NaOH solution. The product of lead formed is a :

(1) Tetraanionic complex with coordination number six
(2) Neutral complex with coordination number four
(3) Dianionic complex with coordination number six
(4) Dianionic complex with coordination number four

Ans. (4)

Sol. $PbCrO_4$ + NaOH (hot excess) $\rightarrow [Pb(OH)_4]^{-2}$ + Na_2CrO_4

Dianionic complex with coordination number four

76. Given below are two statements :

Statement (I) : Aqueous solution of ammonium carbonate is basic.

Statement (II) : Acidic/basic nature of salt solution of a salt of weak acid and weak base depends on K_a and K_b value of acid and the base forming it.

In the light of the above statements, choose the most appropriate answer from the options given below :

(1) Both Statement I and Statement II are correct
(2) Statement I is correct but Statement II is incorrect
(3) Both Statement I and Statement II are incorrect
(4) Statement I is incorrect but Statement II is correct

Ans. (1)

Sol. Aqueous solution of $(NH_4)_2CO_3$is Basic

pH of salt of weak acid and weak base depends on Ka and Kb value of acid and the base forming it

77. IUPAC name of following compound (P) is :

(P)

(1) l-Ethyl-5, 5-dimethylcyclohexane
(2) 3-Ethyl-1,1-dimethylcyclohexane
(3) l-Ethyl-3, 3-dimethylcyclohexane
(4) l,l-Dimethyl-3-ethylcyclohexane

Ans. (2)

Sol.

3-ethy 1, 1 -dimethylcyclohexane

78. NaCl reacts with conc. H_2SO_4 and $K_2Cr_2O_7$ to give reddish fumes (B), which react with NaOH to give yellow solution (C). (B) and (C) respectively are ;

(1) CrO_2Cl_2, Na_2CrO_4 (2) Na_2CrO_4, CrO_2Cl_2
(3) CrO_2Cl_2, $KHSO_4$ (4) CrO_2Cl_2, $Na_2Cr_2O_7$

Ans. (1)

Sol. NaCl + conc. H_2SO_4 + $K_2Cr_2O_7$

$\rightarrow CrO_2Cl_2$ + $KHSO_4$ + $NaHSO_4$ + H_2O

(B)

Reddish brown

CrO_2Cl_2 + NaOH $\rightarrow$ Na_2CrO_4 + NaCl + H_2O

(C)

Yellow colour

79. The correct statement regarding nucleophilic substitution reaction in a chiral alkyl halide is ;

(1) Retention occurs in S_Nl reaction and inversion occurs in S_N2 reaction.
(2) Racemisation occurs in S_Nl reaction and retention occurs in S_N2 reaction.
(3) Racemisation occurs in both S_N1 and S_N2 reactions.
(4) Racemisation occurs in S_N1 reaction and inversion occurs in S_N2 reaction.

Ans. (4)

Sol. SN^1 – Racemisation

SN^2 – Inversion

80. The electronic configuration for Neodymium is:

[Atomic Number for Neodymium 60]

(1)[Xe] $4f^4\ 6s^2$ (2) [Xe] $5f^4 7s^2$

(3) [Xe] $4f^6\ 6s^2$ (4) [Xe] $4f^1 5d^1 6s^2$

Ans. (1)

Sol. Electronic configuration of Nd(Z = 60) is;

[Xe] $4f^4\ 6s^2$

SECTION-B

81. The mass of silver (Molar mass of Ag : 108 $gmol^{-1}$) displaced by a quantity of electricity which displaces 5600 mL of O_2 at S.T.P. will be _____ g.

Ans. 107 gm or 108

Sol. Eq. of Ag = Eq. of O_2

Let x gm silver displaced,

$$\frac{x \times 1}{108} = \frac{5.6}{22.7} \times 4$$

(Molar volume of gas at STP = 22.7 lit)

x = 106.57 gm

Ans. 107

OR,

as per old STP data, molar volume=22.4 lit

$$\frac{x \times 1}{108} = \frac{5.6}{22.4} \times 4$$, x= 108 gm.

Ans. 108

82. Consider the following data for the given reaction

$2HI_{(g)} \rightarrow H_{2(g)} + I_{2(g)}$

	1	2	3
HI (mol L^{-1})	0.005	0.01	0.02
Rate (mol $L^{-1}s$–1)	7.5×10^{-4}	3.0×10^{-3}	1.2×10^{-2}

The order of the reaction is _____.

Ans. (2)

Sol. Let, $R = k[HI]^n$

using any two of given data,

$$\frac{3 \times 10^{-3}}{7.5 \times 10^{-4}} = \left(\frac{0.01}{0.005}\right)^n$$

n = 2

83. Mass of methane required to produce 22 g of CO_2 after complete combustion is _____g.

(Given Molar mass in g mol–1 C = 12.0

H = 1.0

O = 16.0)

Ans. (8)

Sol. $CH_4 + 2O_2 \rightarrow CO_2 + 2H_2O$

Moles of $CO_2 = \frac{22}{44} = 0.5$

So, required moles of CH_4 = 0.5

Mass = 0.5 × 16 = 8gm

84. If three moles of an ideal gas at 300 K expand isotherrnally from 30 dm^3 to 45 dm^3 against a constant opposing pressure of 80 kPa, then the amount of heat transferred is_____ J.

Ans. (1200)

Sol. Using, first law of thermodynamics,

$\Delta U = Q + W$,

$\Delta U = 0$: Process is isothermal

$Q = -W$

$W = -P_{ext}\Delta V$: Irreversible

$= -80 \times 10^3\ (45 - 30) \times 10^{-3}$

$= -1200$ J

85. 3-Methylhex-2-ene on reaction with HBr in presence of peroxide forms an addition product (A). The number of possible stereoisomers for 'A' is_____.

Ans. (4)

Sol.

2chiral centres

No. of stereoisomers = 4

86. Among the given organic compounds, the total number of aromatic compounds is

(A)

(B)

(C)

(D)

Ans. **(3)**

Sol. B,C and D are Aromatic

87. Among the following, total number of meta directing functional groups is (Integer based)

$-OCH_3$, $-NO_2$, $-CN$, $-CH_3$ $-NHCOCH_3$,

$-COR$, $-OH$, $-COOH$, $-Cl$

Ans. **(4)**

Sol. $-NO_2$, $-C\equiv N$, $-COR$, $-COOH$

are meta directing.

88. The number of electrons present in all the completely filled subshells having n=4 and $s=+\frac{1}{2}$ is _____.

(Where n = principal quantum number and s = spin quantum number)

Ans. **(16)**

Sol. n = 4 can have,

	4s	**4p**	**4d**	**4f**
Total e^-	2	6	10	14
Total e^- with $S=+\frac{1}{2}$	1	3	5	7

So, Ans.16

89. Sum of bond order of CO and NO^+ is _____.

Ans. **(6)**

Sol. $CO \Rightarrow \quad \bar{C}\equiv\overset{+}{O}$: BO = 3

$NO^+ \Rightarrow \quad N\equiv O^+$: BO = 3

90. From the given list, the number of compounds with + 4 oxidation state of Sulphur _____.

SO_3, H_2SO_3, $SOCl_2$, SF_4, $BaSO_4$, $H_2S_2O_7$

Ans. **(3)**

Sol.

Compounds	SO_3	H_2SO_3	$SOCl_2$	SF_4	$BaSO_4$	$H_2S_2O_7$
O.S.of Sulphur:	+6	+4	+4	+4	+6	+6