FINAL JEE–MAIN EXAMINATION – JULY, 2022

(Held On Tuesday 26th July, 2022) **TIME : 3 : 00 PM to 6 : 00 PM**

PHYSICS

TEST PAPER WITH SOLUTION

SECTION-A

1. Two projectiles are thrown with same initial velocity making an angle of 45° and 30° with the horizontal respectively. The ratio of their respective ranges will be

(A) $1:\sqrt{2}$ (B) $\sqrt{2}:1$

(C) $2:\sqrt{3}$ (D) $\sqrt{3}:2$

Official Ans. by NTA (C)

Sol. Let projection speed is u

$$R_1=\frac{u^2\text{Sin}(90^\circ)}{g};R_2=\frac{u^2\sin(60^\circ)}{g}$$

$$\frac{R_1}{R_2}=\frac{2}{\sqrt{3}}$$

2. In a Vernier Calipers. 10 divisions of Vernier scale is equal to the 9 divisions of main scale. When both jaws of Vernier calipers touch each other, the zero of the Vernier scale is shifted to the left of zero of the main scale and 4th Vernier scale division exactly coincides with the main scale reading. One main scale division is equal to 1 mm. While measuring diameter of a spherical body, the body is held between two jaws. It is now observed that zero of the Vernier scale lies between 30 and 31 divisions of main scale reading and 6th Vernier scale division exactly. coincides with the main scale reading. The diameter of the spherical body will be :

(A) 3.02 cm (B) 3.06 cm

(C) 3.10 cm (D) 3.20 cm

Official Ans. by NTA (C)

Sol. 1 M.S.D = 1mm

9 M.S.D = 10 V.S.D

1 V.S.D = 0.9 M.S.D = 0.9 mm

L.C of vernier caliper = 1–0.9 = 0.1 mm = 0.01 cm

zero error =–(10–4)×0.1mm = – 0.6 mm

Reading = M.S.R + V.S.R – Zero error

= 3cm + 6 × 0.01 – [–0.06]

= 3 + 0.06 + 0.06

= 3.12 cm

Nearest given answer in the options is 3.10

3. A ball of mass 0.15 kg hits the wall with its initial speed of 12 ms^{-1} and bounces back without changing its initial speed. If the force applied by the wall on the ball during the contact is 100 N. calculate the time duration of the contact of ball with the wall.

(A) 0.018 s (B) 0.036 s

(C) 0.009 s (D) 0.072 s

Official Ans. by NTA (B)

Sol. $\vec{P}_i=0.15\times12\left(\hat{i}\right)$

$\vec{P}_f=0.15\times12\left(-\hat{i}\right)$

$\left|\overrightarrow{\Delta P}\right|=3.6\text{ kg}-\text{m/s}$

$3.6=F\Delta t$

$3.6=100\ \Delta t$

$\Delta t=0.036$ sec

4. A body of mass 8 kg and another of mass 2 kg are moving with equal kinetic energy. The ratio of their respective momenta will be :

(A) 1:1 (B) 2:1 (C) 1:4 (D) 4:1

Official Ans. by NTA (B)

Sol. $K.E=\frac{P^2}{2m}$

$K_1=\frac{P_1^2}{2(8)}\ ;\ K_2=\frac{P_2^2}{2(2)}$

$K_1=K_2$

So,

$4P_2^2=P_1^2$

$\frac{P_1}{P_2}=2$

5. Two uniformly charged spherical conductors A and B of radii 5 mm and 10 mm are separated by a distance of 2 cm. If the spheres are connected by a conducting wire, then in equilibrium condition, the ratio of the magnitudes of the electric fields at the surface of the sphere A and B will be :

(A) 1 : 2 (B) 2 : 1 (C) 1 : 1 (D) 1 : 4

Official Ans. by NTA (B)

Sol. $V_A = V_B$

$$\frac{KQ_A}{R_A} = \frac{KQ_B}{R_B}$$

$$\frac{Q_A}{Q_B} = \frac{R_A}{R_B} = \frac{1}{2}$$

$$E_A = \frac{KQ_A}{R_A^2}; E_B = \frac{KQ_B}{R_B^2}$$

$$\frac{E_A}{E_B} = \frac{Q_A}{Q_B} \times \frac{R_B^2}{R_A^2} = \frac{R_B}{R_A} = \frac{2}{1}$$

6. The oscillating magnetic field in a plane electromagnetic wave is given by $B_y = 5 \times 10^{-6}$ sin 1000π $(5x - 4 \times 10^8 t)$T. The amplitude of electric field will be :

(A) 15×10^2 Vm^{-1} (B) 5×10^{-6} Vm^{-1}

(C) 16×10^{12} Vm^{-1} (D) 4×10^2 Vm^{-1}

Official Ans. by NTA (D)

Sol. $B_0 = 5 \times 10^{-6}$

$$v = \text{Speed of wave} = \frac{4 \times 10^8}{5} = 8 \times 10^7 \quad \left[\therefore v = \frac{w}{k}\right]$$

$$E_0 = vB_0 = 40 \times 10^1$$

$$= 4 \times 10^2 \text{ V/m}$$

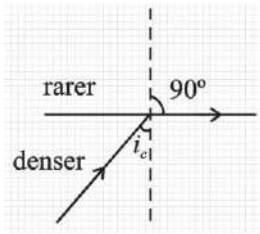
7. Light travels in two media M_1 and M_2 with speeds 1.5×10^8 ms^{-1} and 2.0×10^8 ms^{-1} respectively. The critical angle between them is:

(A) $\tan^{-1}\left(\frac{3}{\sqrt{7}}\right)$ (B) $\tan^{-1}\left(\frac{2}{3}\right)$

(C) $\cos^{-1}\left(\frac{3}{4}\right)$ (D) $\sin^{-1}\left(\frac{2}{3}\right)$

Official Ans. by NTA (A)

Sol.



$$v = \frac{c}{n}$$

$$n_d \sin i_c = n_r \sin 90^\circ$$

$$\sin i_c = \frac{n_r}{n_d} = \frac{v_d}{v_r}$$

$$\sin i_c = \frac{1.5 \times 10^8}{2 \times 10^8} = \frac{1.5}{2}$$

$$\sin i_c = \frac{3}{4}$$

$$\tan i_c = \frac{3}{\sqrt{4^2 - 3^2}} \Rightarrow \frac{3}{\sqrt{7}}$$

$$i_c = \tan^{-1}\left(\frac{3}{\sqrt{7}}\right)$$

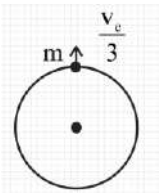
8. A body is projected vertically upwards from the surface of earth with a velocity equal to one third of escape velocity. The maximum height attained by the body will be:
(Take radius of earth = 6400 km and g=10 ms^{-2})

(A) 800 km (B) 1600 km

(C) 2133 km (D) 4800 km

Official Ans. by NTA (A)

Sol.



$$v_e = \sqrt{\frac{2GM}{R}}$$

$$\frac{-GMm}{R} + \frac{1}{2}m\frac{V_e^2}{9} = -\frac{GMm}{R+h}$$

$$\frac{GM}{R+h} = \frac{GM}{R} - \frac{V_e^2}{18}$$

$$\frac{GM}{R+h} = \frac{GM}{R} - \frac{GM}{9R}$$

$$\frac{GM}{R+h} = \frac{8GM}{9R}$$

$$\frac{1}{R+h} = \frac{8}{9R}$$

$$9R = 8R + 8h$$

$$h = \frac{R}{8} \Rightarrow \frac{6400}{8} \Rightarrow 800\text{km}$$

9. The maximum and minimum voltage of an amplitude modulated signal are 60 V and 20 V respectively. The percentage modulation index will be :

(A) 0.5% (B) 50% (C) 2% (D) 30%

Official Ans. by NTA (B)

Sol. $V_{max} = 60$

$V_{min} = 20$

% modulation =

$$\left(\frac{V_{max}-V_{min}}{V_{max}+V_{min}}\right)100 \Rightarrow \left(\frac{60-20}{60+20}\right)100 \Rightarrow \left(\frac{40}{80}\right)100$$

$\Rightarrow$ 50%

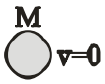
10. A nucleus of mass M at rest splits into two parts having masses $\frac{M'}{3}$ and $\frac{2M'}{3}\left(M' < M\right)$. The ratio of de Broglie wavelength of two parts will be :

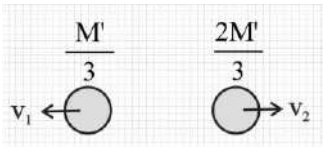
(A) 1 : 2 (B) 2 : 1

(C) 1 : 1 (D) 2 : 3

Official Ans. by NTA (C)

Sol.





$\left|\vec{P}_1\right| = \left|\vec{P}_2\right|$

Here $\vec{P}$ is momentum

So $\lambda = \frac{h}{P}$

Hence both will have same de broglie wavelength.

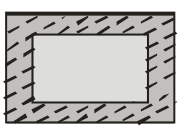
11. An ice cube of dimensions 60 cm × 50 cm × 20 cm is placed in an insulation box of wall thickness 1 cm. The box keeping the ice cube at 0°C of temperature is brought to a room of temperature 40°C. The rate of melting of ice is approximately: (Latent heat of fusion of ice is 3.4×10^5 J kg^{-1} and thermal conducting of insulation wall is 0.05 $Wm^{-1}{}^{o}C^{-1}$)

(A) 61×10^{-1} kg s^{-1} (B) 61×10^{-5} kg s^{-1}

(C) 208 kg s^{-1} (D) 30×10^{-5} kg s^{-1}

Official Ans. by NTA (B)

Sol.



$$\frac{dQ}{dt} = \frac{KA\Delta T}{\ell}$$

A = 2 (0.6 × 0.5 + 0.5 × 0.2 + 0.2 × 0.6)

= 2(0.3 + 0.1 + 0.12)

= 2(0.4 + 0.12)

= 2(0.52)

= 1.04 m^2

$$R_{th} = \frac{\ell}{KA} \Rightarrow \frac{1\times10^{-2}}{0.05\times1.04} \Rightarrow \frac{10^{-2}}{0.052}$$

$$\frac{dQ}{dt} = \frac{\Delta T}{R_{th}} \Rightarrow \frac{40\times0.052}{10^{-2}} \Rightarrow 2.08\times10^2 \text{ J/s}$$

$2.08 \times 10^2 = m \times 3.4 \times 10^5$

$$m = \frac{2.08}{3.4\times10^3} \Rightarrow 0.61\times10^{-3} \text{ kg/s}$$

$= 61 \times 10^{-5}$ Kg/s

12. A gas has n degrees of freedom. The ratio of specific heat of gas at constant volume to the specific heat of gas at constant pressure will be :

(A) $\frac{n}{n+2}$ (B) $\frac{n+2}{n}$

(C) $\frac{n}{2n+2}$ (D) $\frac{n}{n-2}$

Official Ans. by NTA (A)

Sol. $C_v = \frac{nR}{2}$ $C_p = \frac{(n+2)R}{2}$

$$\frac{C_v}{C_p} = \frac{n}{n+2}$$

13. A transverse wave is represented by y = 2sin $(\omega t - kx)$ cm. The value of wavelength (in cm) for which the wave velocity becomes equal to the maximum particle velocity, will be ;

(A) 4π (B) 2π

(C) π (D) 2

Official Ans. by NTA (A)

Sol. y = 2 sin (ωt – kx)

Maximum particle velocity = A ω

Wave velocity = $\frac{\omega}{k}$

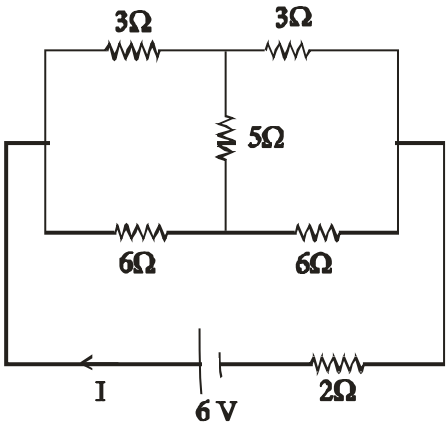
$\frac{\omega}{k}$ = A ω

$k = \frac{1}{A} = \frac{2\pi}{\lambda}$

λ = 2πA

= 4 π cm

14. A battery of 6 V is connected to the circuit as shown below. The current I drawn from the battery is :

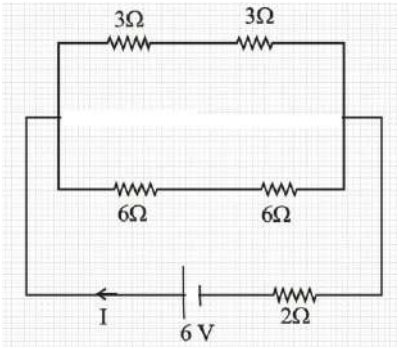


(A) 1A (B) 2A

(C) $\frac{6}{11}$A (D) $\frac{4}{3}$A

Official Ans. by NTA (A)

Sol. Balanced wheat stone bridge in circuit so there is no current in 5 Ω resistor so it can be removed from the circuit.



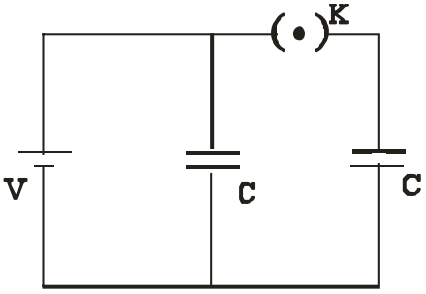
$$R_{eq} = \frac{6\times12}{6+12} + 2$$

$$= \frac{6\times12}{18} + 2$$

$$R_{eq} = 6\Omega$$

$$I = \frac{V}{R_{eq}} = \frac{6}{6} = 1\ \text{Amp.}$$

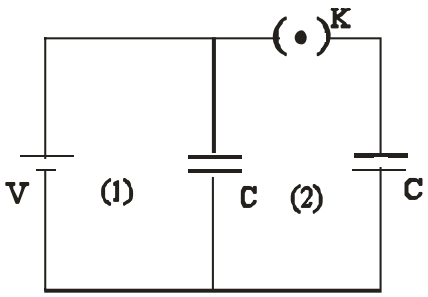
15. A source of potential difference V is connected to the combination of two identical capacitors as shown in the figure. When key 'K' is closed, the total energy stored across the combination is E_1. Now key 'K' is opened and dielectric of dielectric constant 5 is introduced between the plates of the capacitors. The total energy stored across the combination is now E_2. The ratio E_1/E_2 will be :



(A) $\frac{1}{10}$ (B) $\frac{2}{5}$

(C) $\frac{5}{13}$ (D) $\frac{5}{26}$

Official Ans. by NTA (C)



Sol.

(1) Switch is closed

$C_{eq} = 2C$

Energy $E_1 = \frac{1}{2}C_{eq}V^2$

$= \frac{1}{2}2C\times V^2$

$E_1 = CV^2$

(ii) When switch is opened charge on right capacitor remain CV while potential on left capacitor remain same

Dielectric K = 5

$C' = KC$

$C' = 5C$

$$E_2 = \frac{1}{2}(5C)V^2 + \frac{(CV)^2}{2(5C)}$$

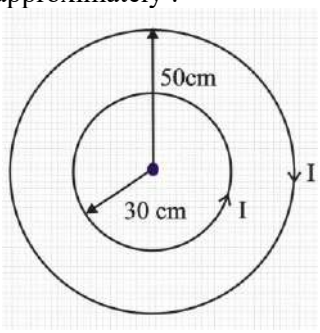
$$E_2 = \frac{5CV^2}{2} + \frac{CV^2}{10}$$

$$E_2 = \frac{13CV^2}{5}$$

$$\frac{E_1}{E_2} = \frac{CV^2}{\frac{13CV^2}{5}} = \frac{5}{13}$$

$$\frac{E_1}{E_2} = \frac{5}{13}$$

16. Two concentric circular loops of radii r_1=30 cm and r_2=50 cm are placed in X-Y plane as shown in the figure. A current I = 7A is flowing through them in the direction as shown in figure. The net magnetic moment of this system of two circular loops is approximately :

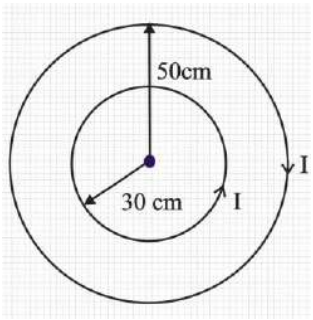


(A) $\frac{7}{2}\hat{k}$ Am^2 (B) $-\frac{7}{2}\hat{k}$ Am^2

(C) $7\hat{k}$ Am^2 (D) $-7\hat{k}$ Am^2

Official Ans. by NTA (B)

Sol.



Magnetic moment

$$\vec{M} = -i\pi(0.5)^2\hat{k} + i\pi(0.3)^2\hat{k}$$

$$\vec{M} = -7\times\frac{22}{7}\left(\frac{25}{100} - \frac{9}{100}\right)\hat{k}$$

$$= -22\left(\frac{16}{100}\right)\hat{k}$$

$$\vec{M} = -3.52\hat{k}\ \text{Am}^2$$

$$= -\frac{7}{2}\hat{k}\ \text{Am}^2$$

17. A velocity selector consists of electric field $\vec{E} = E\hat{k}$ and magnetic field $\vec{B} = B\hat{j}$ with B=12 mT. The value E required for an electron of energy 728 eV moving along the positive x-axis to pass undeflected is :
(Given, mass of electron = 9.1×10^{-31}kg)

(A) 192 kVm^{-1} (B) 192 m Vm^{-1}

(C) 9600 kVm^{-1} (D) 16 kVm^{-1}

Official Ans. by NTA (A)

Sol. $\vec{E} = E\hat{k}$ B = 12 mT

$\vec{B} = B\hat{j}$ Energy = 728 eV

$$\text{Energy} = \frac{1}{2}mv^2$$

$$728\text{eV} = \frac{1}{2}\times9.1\times10^{-31}\times v^2$$

$$728\times1.6\times10^{-19} = \frac{1}{2}\times9.1\times10^{-31}\times v^2$$

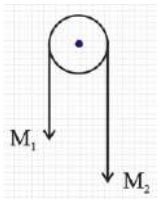
$v = 16 \times 10^6$ m/s

$E = vB$

$E = 16\times10^6 \times 12 \times 10^{-3}$

$E = 192 \times 10^3$ V/m

18. Two masses M_1 and .M_2 are tied together at the two ends of a light inextensible string that passes over a frictionless pulley. When the mass M_2 is twice that of M_1. the acceleration of the system is a_1. When the mass M_2 is thrice that of M_1. The acceleration of The system is a_2. The ratio $\frac{a_1}{a_2}$ will be:



(A) $\frac{1}{3}$ (B) $\frac{2}{3}$

(C) $\frac{3}{2}$ (D) $\frac{1}{2}$

Official Ans. by NTA (B)

Sol. $$a = \frac{m_2g - m_1g}{m_1 + m_2}$$

Case 1 $M_2 = 2m_1$

$$a_1 = \frac{2m_1g - m_1g}{3m_1}$$

$$a_1 = g/3$$

Case -2

$M_2 = 3m_1$

$a_2 = \frac{3m_1g - m_1g}{4m_1}$

$a_2 = \frac{g}{2}$

$\frac{a_1}{a_2} = \frac{\frac{g}{3}}{\frac{g}{2}} = \frac{2}{3}$

19. Mass numbers of two nuclei are in the ratio of 4:3. Their nuclear densities will be in the ratio of

(A) 4:3 (B) $\left(\frac{3}{4}\right)^{\frac{1}{3}}$

(C) 1 : 1 (D) $\left(\frac{4}{3}\right)^{\frac{1}{3}}$

Official Ans. by NTA (C)

Sol. Radius of nucleus $R = R_0A^{\frac{1}{3}}$

Density of nucleus = $\frac{\text{Mass of nucleus}}{\text{volume of nucleus}}$

$\rho = \frac{m \times A}{\frac{4}{3}\pi R^3}$ Where m : mass of proton or neutron

$\rho = \frac{m \times A}{\frac{4}{3}\pi R_0{}^3 A}$

$\rho \propto A^0$

Hence density of nucleus is independent of mass number

20. The area of cross section of the rope used to lift a load by a crane is $2.5 \times 10^{-4}m^2$. The maximum lifting capacity of the crane is 10 metric tons. To increase the lifting capacity of the crane to 25 metric tons, the required area of cross section of the rope should be :
(take g =10 ms^{-2})
(A) $6.25 \times 10^{-4}m^2$
(B) $10 \times 10^{-4}m^2$
(C) $1 \times 10^{-4}m^2$
(D) $1.67 \times 10^{-4}m^2$

Official Ans. by NTA (A)

Sol. Since breaking stress (Maximum lifting capacity) is the property of material so it will remain same.

breaking stress = $\frac{\text{Maximum lifting capacity}}{\text{Area of cross section of rope}}$

$\frac{10}{2.5\times10^{-4}} = \frac{25}{A}$

$A = 625 \times 10^{-6}$

$= 6.25 \times 10^{-4}\ m^2$

SECTION-B

1. If $\vec{A} = \left(2\hat{i} + 3\hat{j} - \hat{k}\right)$ m and $\vec{B} = \left(\hat{i} + 2\hat{j} + 2\hat{k}\right)$ m. The magnitude of component of vector $\vec{A}$ along vector $\vec{B}$ will be_____ m.

Official Ans. by NTA (2)

Sol. $\vec{A} = \left(2\hat{i} + 3\hat{j} - \hat{k}\right)$m and $\vec{B} = \left(\hat{i} + 2\hat{j} + 2\hat{k}\right)$m

Component of $\vec{A}$ along $\vec{B} = \vec{A}\cdot\hat{B}$

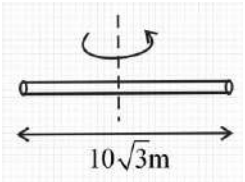
$= \frac{\vec{A}.\vec{B}}{|\vec{B}|} = \frac{2+6-2}{\sqrt{1^2+2^2+2^2}}$

$= \frac{6}{3} = 2$

2. The radius of gyration of a cylindrical rod about an axis of rotation perpendicular to its length and passing through the center will be_____ m. Given, the length of the rod is $10\sqrt{3}$ m.

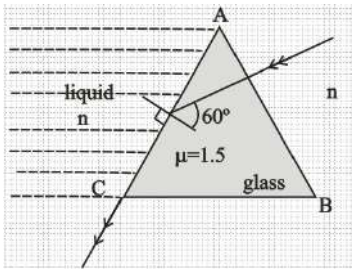
Official Ans. by NTA (5)

Sol.



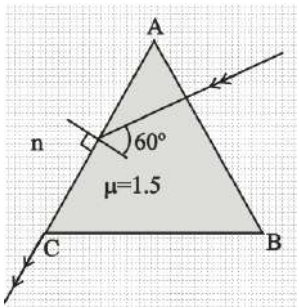
$I = \frac{m\ell^2}{12} = mk^2 \Rightarrow k^2 = \frac{\ell^2}{12} \Rightarrow k = \frac{\ell}{\sqrt{12}} = \frac{\ell}{2\sqrt{3}} = \frac{10\sqrt{3}}{2\sqrt{3}} = 5$

3. In the given figure, the face AC of the equilateral prism is immersed in a liquid of refractive index 'n'. For incident angle 60° at the side AC, the refracted light beam just grazes along face AC. The refractive index of the liquid $n=\frac{\sqrt{x}}{4}$. The value of x is_____ .
(Given refractive index of glass = 1.5)



Official Ans. by NTA (27)

Sol.



Using snell's law at face AC

$1.5 \sin 60^\circ = n \times \sin 90^\circ$

$1.5\times \frac{\sqrt{3}}{2} = n = \frac{\sqrt{x}}{4}$

$3\sqrt{3} = \sqrt{x}$

$x = 27$

4. Two lighter nuclei combine to form a comparatively heavier nucleus by the relation given below:
${}^{2}_{1}X + {}^{2}_{1}X = {}^{4}_{2}Y$
The binding energies per nucleon ${}^{2}_{1}X$ and ${}^{4}_{2}Y$ are 1.1 MeV and 7.6 MeV respectively. The energy released in this process is_____ . MeV.

Official Ans. by NTA (26)

Sol. Energy released in the given process = Binding energy of product – Binding energy of reactants

$= 7.6 \times 4 - (1.1 \times 2) \times 2$

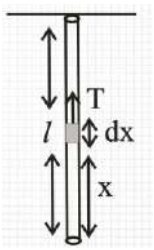
$= 30.4 - 4.4$

$=26$ MeV

5. A uniform heavy rod of mass 20 kg. Cross sectional area 0.4 m^2 and length 20 m is hanging from a fixed support. Neglecting the lateral contraction, the elongation in the rod due to its own weight is $x \times 10^{-9}$ m. The value of x is_____.
:(Given. Young's modulus $Y=2 \times 10^{11}$ Nm^{-2} and $g=10$ ms^{-2})

Official Ans. by NTA (25)

Sol.



$Y = \frac{T}{A}\frac{dx}{dy}$

$m = 20$ kg

A = 0.4 m^2

$l = 20$m

let extension is dy in length dx

$Y = \frac{\text{stress}}{\text{strain}}$

$Y = \frac{\frac{T}{A}}{\frac{dy}{dx}} = \frac{T}{A}.\frac{dx}{dy}$

$dy = \frac{Tdx}{AY}$

Tension at a distance x from lower end = $\frac{mg}{\ell}x$

So. $\int_0^{\Delta l} dy = \int_0^{\ell} \frac{mg}{\ell} x \frac{dx}{AY}$

$\Delta \ell = \frac{mg}{\ell AY}\left[\frac{x^2}{2}\right]_0^{\ell}$

$\Delta \ell = \frac{mg\ell}{2AY}$

$\Delta \ell = \frac{20\times 10\times 20}{2\times 0.4\times 2\times 10^{11}}$

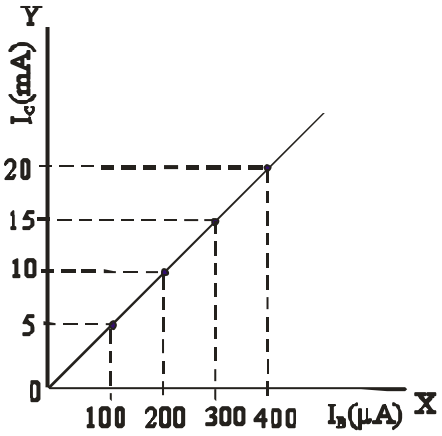
2500×10^{-11}

$\Delta \ell = 25\times 10^{-9}$

$= x \times 10^{-9}$

$x = 25$

6. The typical transfer characteristic of a transistor in CE configuration is shown in figure. A load resistor of 2 kΩ is connected in the collector branch of the circuit used. The input resistance of the transistor is 0.50 kΩ. The voltage gain of the transistor is



Official Ans. by NTA (200)

Sol. Current gain in C-E configuration

$$\Rightarrow \beta = \frac{\Delta I_C}{\Delta I_B}$$

$R_C = 2k\Omega$, R_B=0.50 kΩ

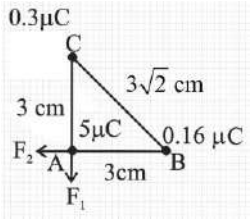
$$\text{Voltage gain} = \frac{\Delta I_C R_C}{\Delta I_B R_B} = \frac{5\times10^{-3}}{100\times10^{-6}} \times \frac{2}{0.5}$$

$$= \frac{10^{-2}}{5\times10^{-5}} = \frac{1000}{5} = 200$$

7. Three point charges of magnitude 5μC, 0.16μC and 0.3μC are located at the vertices A, B, C of a right angled triangle whose sides are AB = 3cm, BC = $3\sqrt{2}$ cm and CA=3 cm and point A is the right angle corner. Charge at point A experiences _____ N of electrostatic force due to the other two charges.

Official Ans. by NTA (17)

Sol.



$$F_1 = \frac{k\times5\times0.3\times10^{-12}}{9\times10^{-4}}$$

$$= \frac{9\times10^9\times5\times0.3\times10^{-12}}{9\times10^{-4}}$$

$$= 1.5\times10 = 15\ \text{N}$$

$$F_2 = \frac{9\times10^9\times5\times0.16\times10^{-12}}{9\times10^{-4}} = 8\text{N}$$

force experienced by charge at $A = \sqrt{F_1^2 + F_2^2}$

$$= \sqrt{15^2 + 8^2}$$

$$= \sqrt{289} = 17\ \text{N}$$

8. In a coil of resistance 8Ω, the magnetic flux due to an external magnetic field varies with time as $\phi = \frac{2}{3}\left(9 - t^2\right)$. The value of total heat produced in the coil, till the flux becomes zero, will be_____J.

Official Ans. by NTA (2)

Sol. $\phi = \frac{2}{3}(9 - t^2) = 0$

$t = 3$ sec

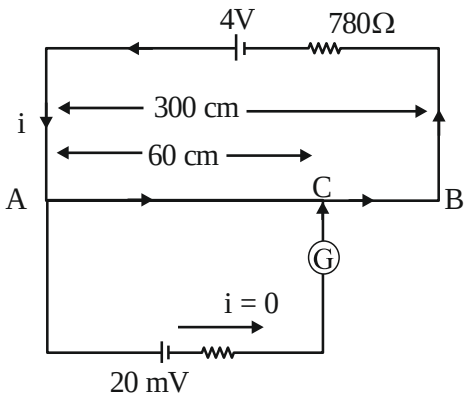
$$e = \frac{-d\phi}{dt} = -\frac{2}{3}(0 - 2t) = \frac{4t}{3}$$

$$\text{Heat produced in 3 sec} = \int \frac{e^2}{r}dt = \int_0^3 \frac{16t^2}{9\times8}dt = 2\text{J}$$

9. A potentiometer wire of length 300 cm is connected in series with a resistance 780 Ω and a standard cell of emf 4V. A constant current flows through potentiometer wire. The length of the null point for cell of emf 20 mV is found to be 60 cm. The resistance of the potentiometer wire is____Ω.

Official Ans. by NTA (20)

Sol.



Let resistance of potentiometers wire is R

$$i = \frac{4}{R+780}$$

Potential difference across AB

$$= \frac{4R}{R+780}$$

Potential difference across AC

$$= \frac{4R \times 60}{(R+780)\times 300} = \frac{4R}{5(R+780)}$$

This should be equal to 20 mV

$$\frac{4R}{5(R+780)} = 20\times10^{-3} = 2\times10^{-2}$$

$4R = 10^{-1}(R + 780)$

$$4R = \frac{R}{10} + 78$$

$$4R - \frac{R}{10} = 78$$

$$\frac{39R}{10} = 78$$

$$\boxed{R = 20\Omega}$$

10. As per given figures, two springs of spring constants K and 2K are connected to mass m. If the period of oscillation in figure (a) is 3s, then the period of oscillation in figure (b) will be $\sqrt{x}$ s. The value of x is_____.

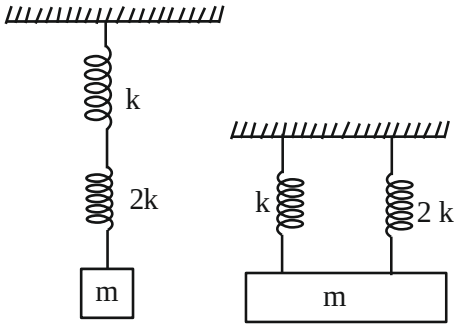


figure (a) figure (b)

Official Ans. by NTA (2)

Sol.

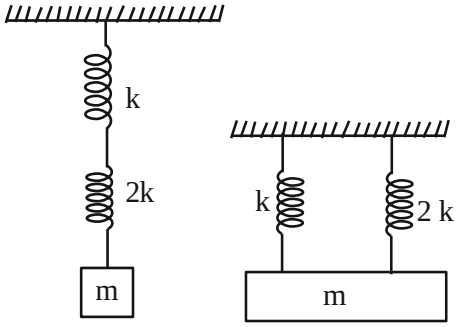


figure (a) figure (b)

For figure (a) :

$$K_{eq} = \frac{K\times 2K}{K+2K} = \frac{2K}{3}$$

$$T = 2\pi\sqrt{\frac{m}{K_{eq}}} = 2\pi\sqrt{\frac{m}{2K/3}} = 2\pi\sqrt{\frac{3m}{2K}}$$

For figure (b):

$$K_{eq} = 3K,\ T' = 2\pi\sqrt{\frac{m}{3K}}$$

$$\frac{T'}{T} = \sqrt{\frac{m\times 2K}{3K\times 3m}} = \frac{\sqrt{2}}{3}$$

$T' = \sqrt{2}$

$x = 2$

FINAL JEE–MAIN EXAMINATION – JULY, 2022

(Held On Tuesday 26th July, 2022) **TIME : 3 : 00 PM to 6 : 00 PM**

CHEMISTRY

TEST PAPER WITH SOLUTION

SECTION-A

1. Hemoglobin contains 0.34% of iron by mass. The number of Fe atoms in 3.3 g of hemoglobin is : (Given : Atomic mass of Fe is 56 u, N_A in 6.022×10^{23} mol^{-1})

(A) 1.21×10^5 (B) 12.0×10^{16}

(C) 1.21×10^{20} (D) 3.4×10^{22}

Official Ans. by NTA (C)

Sol. No. of Fe atoms $= \frac{0.34}{100} \times \frac{3.3}{56} \times 6.022 \times 10^{23}$

$= 1.206 \times 10^{20}$

2. Arrange the following in increasing order of their covalent character.

(A) CaF_2 (B) $CaCl_2$

(C) $CaBr_2$ (D) CaI_2

Choose the correct answer from the options given below.

(A) B < A < C < D (B) A < B < C < D

(C) A < B < D < C (D) A < C < B < D

Official Ans. by NTA (B)

Sol. According to Fajan's rule,

Covalent character $\propto$ size of Anion

3. Class XII students were asked to prepare one litre of buffer solution of pH 8.26 by their chemistry teacher. The amount of ammonium chloride to be dissolved by the student in 0.2 M ammonia solution to make one litre of the buffer is (Given pK_b (NH_3) = 4.74; Molar mass of NH_3 = 17 g mol^{-1}; Molar mass of NH_4Cl = 53.5 g mol^{-1})

(A) 53.5 g (B) 72.3 g

(C) 107.0 g (D) 126.0 g

Official Ans. by NTA (C)

Sol. POH = 14 – 8.26

$$= pK_b + \log\frac{[NH_4^+]}{[NH_3]}$$

$$= 5.74 = 4.74 + \log\frac{[NH_4^+]}{0.2} \Rightarrow [NH_4^+] = 2$$

Hence

$NH_4Cl = 2 \times 53.5 = 107$ g

4. At 30°C, the half life for the decomposition of AB_2 is 200 s and is independent of the initial concentration of AB_2. The time required for 80% of the AB_2 to decompose is (Given: log 2 = 0.30; log 3 = 0.48)

(A) 200 s (B) 323 s

(C) 467 s (D) 532 s

Official Ans. by NTA (C)

Sol. $T_{1/2}$ = 200 s and 1st order reaction

$$K = \frac{2.303\log 2}{200} = \frac{2.303}{t}\log\frac{A_0}{0.2A_0}$$

$$\frac{\log 2}{200} = \frac{1}{t}\log 5$$

$$t = \frac{7}{3} \times 200 = 466.67s = 467 \text{ s}$$

5. Given below are two statements : one is labelled as Assertion A and the other is labelled as Reason R.

Assertion A : Finest gold is red in colour, as the size of the particles increases, it appears purple then blue and finally gold.

Assertion R : The colour of the colloidal solution depends on the wavelength of light scattered by the dispersed particles.

In the light of the above statements, choose the most appropriate answer from the options given below;

(A) Both A and R are true and R is the correct explanation of A

(B) Both A and R are true but R is NOT the correct explanation of A

(C) A is true but R is false

(D) A is false but R is true

Official Ans. by NTA (A)

6. The metal that has very low melting point and its periodic position is closer to a metalloid is :

(A) Al (B) Ga

(C) Se (D) In

Official Ans. by NTA (B)

Sol.

	Melting point
Al →	933 K
Ga →	303 K
In →	430 K
Se →	490 K

7. The metal that is not extracted from its sulphide ore is :

(A) Aluminium (B) Iron

(C) Lead (D) Zinc

Official Ans. by NTA (A)

Sol. Al is extracted from $Al_2O_3 \cdot 2H_2O$ i.e., Bauxite ore

8. The products obtained from a reaction of hydrogen peroxide and acidified potassium permanganate are

(A) Mn^{4+}, H_2O only (B) Mn^{2+}, H_2O only

(C) Mn^{4+}, H_2O, O_2 only (D) Mn^{2+}, H_2O, O_2 only

Official Ans. by NTA (D)

Sol. $6H^+ + 2MnO_4^- + 5H_2O_2 \longrightarrow 2Mn^{+2} + 8H_2O + 5O_2$

9. Given below are two statements : one is labelled as Assertion A and the other is labelled as Reason R.

Assertion A : LiF is sparingly soluble in water.

Reason R : The ionic radius of Li^+ ion is smallest among its group members, hence has least hydration enthalpy.

In the light of the above statements, choose the most appropriate answer from the options given below .

(A) Both A and R are true and R is the correct explanation of A

(B) Both A and R are true but R is NOT the correct explanation of A

(C) A is true but R is false

(D) A is false but R is true

Official Ans. by NTA (C)

Sol. Due to high lattice energy LiF is sparingly soluble in water. Li^+ has high hydration energy among its group members due to smallest size.

10. Given below are two statements : one is labelled as Assertion A and the other is labelled as Reason R.

Assertion A : Boric acid is a weak acid

Reason R : Boric acid is not able to release H^+ ion on its own. It receives OH^- ion from water and releases H^+ ion.

In the light of the above statements, choose the most appropriate answer from the options given below.

(A) Both A and R are correct and R is the correct explanation of A

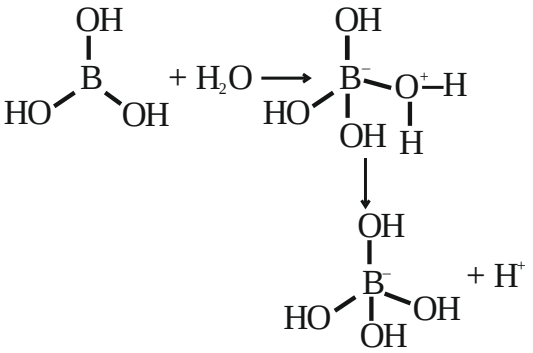
(B) Both A and R are correct but R is NOT the correct explanation of A

(C) A is correct but R is not correct

(D) A is not correct but R is correct

Official Ans. by NTA (A)

Sol.



11. The metal complex that is diamagnetic is (Atomic number : Fe, 26; Cu, 29)

(A) $K_3[Cu(CN)_4]$

(B) $K_2[Cu(CN)_4]$

(C) $K_3[Fe(CN)_4]$

(D) $K_4[FeCl_6]$

Official Ans. by NTA (A)

Sol. $K_3[Cu(CN)_4]$

O.N. of copper is Cu^{+1}

$Cu^{+1} = [Ar]3d^{10} \Rightarrow$ Diamagnetic

12. Match List I with List II

List I **Pollutant**	**List II** **Source**
A. Microorganisms	I. Strip mining
B. Plant nutrients	II. Domestic sewage
C. Toxic heavy metals	III. Chemical fertilizer
D. Sediment	IV. Chemical factory

Choose the correct answer from the options given below :

(A) A-II, B-III, C-IV, D-I

(B) A-II, B-I, C-IV, D-III

(C) A-I, B-IV, C-II, D-III

(D) A-I, B-IV, C-III, D-II

Official Ans. by NTA (A)

Sol.

List I Pollutant	List II Source
A. Microorganisms	Domestic sewage
B. Plant nutrients	Chemical fertilizer
C. Toxic heavy metals	Chemical factory
D. Sediment	Strip mining

13. The correct decreasing order of priority of functional groups in naming an organic compound as per IUPAC system of nomenclature is :

(A) —COOH > —$CONH_2$ > —COCl > —CHO

(B) —SO_3H > —COCl > —$CONH_2$ > —CN

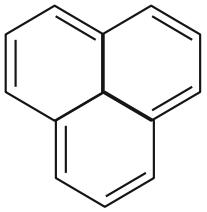
(C) —COOR > —COCl > —NH_2 > $\rangle C = o$

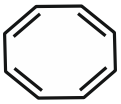
(D) —COOH > —COOR > —$CONH_2$ > —COCl

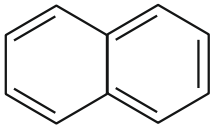
Official Ans. by NTA (B)

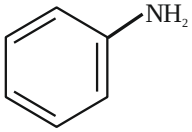
Sol. —SO_3H > —COCl > —$CONH_2$ > —CN

14. Which of the following is not an example of benzenoid compound ?

(A) 

(B) 

(C) 

(D) 

Official Ans. by NTA (B)

15. Hydrolysis of which compound will give carbolic acid ?

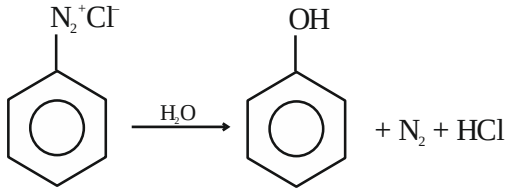
(A) Cumene

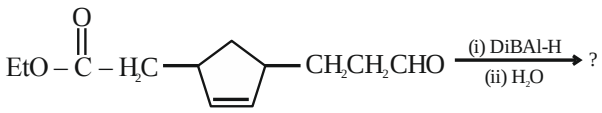
(B) Benzenediazonium chloride

(C) Benzal chloride

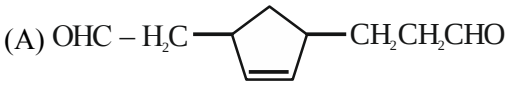
(D) Ethylene glycol ketal

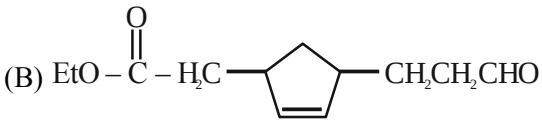
Official Ans. by NTA (B)

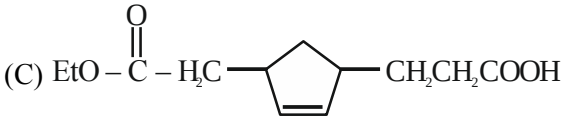
Sol. 

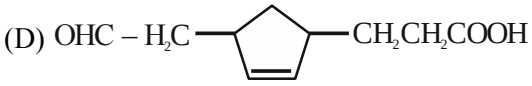
16. 

Consider the above reaction and predict the major product.

(A) 

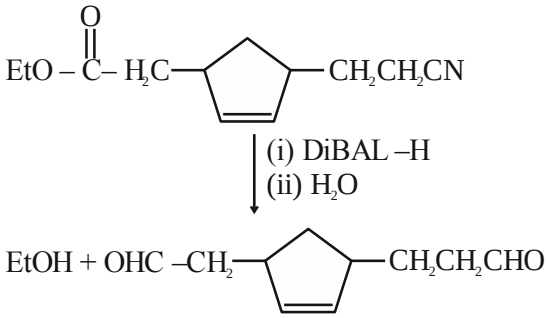
(B) 

(C) 

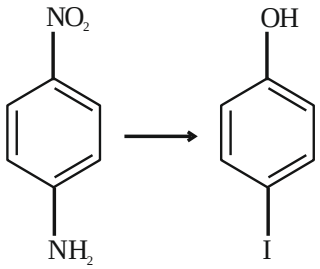
(D) 

Official Ans. by NTA (A)

Sol.



17. The correct sequential order of the reagents for the given reaction is :



(A) HNO_2, Fe/H^+, HNO_2, KI, H_2O/H^+

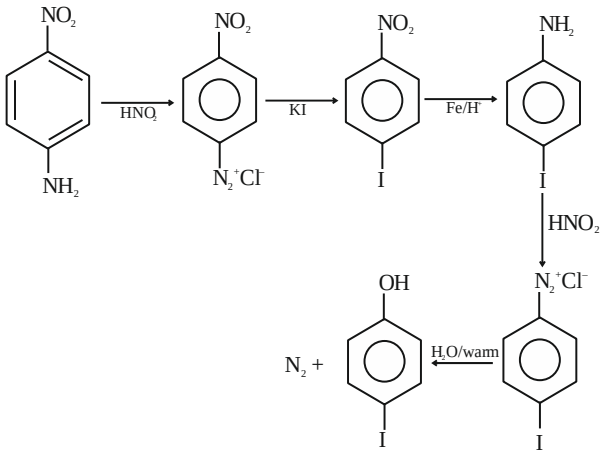
(B) HNO_2, KI, Fe/H^+, HNO_2, H_2O/warm

(C) HNO_2, KI, HNO_2, Fe/H^+, H_2O/H^+

(D) HNO_2, Fe/H^+, KI, HNO_2, H_2O/warm

Official Ans. by NTA (B)

Sol.



18. Vulcanization of rubber is carried out by heating a mixture of :

(A) isoprene and styrene

(B) neoprene and sulphur

(C) isoprene and sulphur

(D) neoprene and styrene

Official Ans. by NTA (C)

Sol. Vulcanization of rubber is carried out by heating a mixture of isoprene & sulphur

19. Animal starch is the other name of :

(A) amylose (B) maltose

(C) glycogen (D) amylopectin

Official Ans. by NTA (C)

Sol. Glycogen

20. Given below are two statements : one is labelled as Assertion A and the other is labelled as Reason R.

Assertion A : Phenolphthalein is a pH dependent indicator, remains colourless in acidic solution and gives pink colour in basic medium

Reason R : Phenolphthalein is a weak acid. It doesn't dissociate in basic medium.

In the light of the above statements, choose the most appropriate answer from the options given below :

(A) Both A and R are true and R is the correct explanation of A

(B) Both A and R are true but R is NOT the correct explanation of A.

(C) A is true but R is false

(D) A is false but R is true

Official Ans. by NTA (C)

Sol. Phenolphthalein dissociate in basic medium

$HPh(aq) \rightleftharpoons H^+ + Ph^-$

(colourless) (Pink)

SECTION-B

1. A 10 g mixture of hydrogen and helium is contained in a vessel of capacity 0.0125 m^3 at 6 bar and 27°C. The mass of helium in the mixture is _____ g. (nearest integer)

Given : R = 8.3 $JK^{-1}mol^{-1}$ (Atomic masses of H and He are 1u and 4u, respectively)

Official Ans. by NTA (8)

Sol. $PV = n_{mix}RT$

$$n_{mix} = \frac{6 \times 12.5}{0.083 \times 300} \approx 3$$

Let mole of He = x

Mole of $H_2 = 3 - x$

$4x + 2(3 - x) = 10$

$\boxed{x = 2mol}$

Mass of He = 8g

2. Consider an imaginary ion $^{48}_{22}X^{3-}$. The nucleus contains 'a'% more neutrons than the number of electrons in the ion. The value of 'a' is ____. [nearest integer]

Official Ans. by NTA (4)

Sol. $^{48}_{22}X^{3-}$

No. of neutrons = 26

No. of electrons = 25

% of extra neutrons

than electrons = $\frac{26-25}{25}\times 100 = 4$

3. For the reaction

$H_2F_2(g) \rightarrow H_2(g) + F_2(g)$

$\Delta U = -59.6$ kJ mol^{-1} at 27°C.

The enthalpy change for the above reaction is (–) ___ kJ mol^{-1} [nearest integer] Given : R = 8.314 JK^{-1} mol^{-1}.

Official Ans. by NTA (57)

Sol. $\Delta H = \Delta U + \Delta n_g RT$

$\Delta H = -59.6 + 1 \times 8.314 \times 300 \times 10^{-3} = -57.10$

4. The elevation in boiling point for 1 molal solution of non-volatile solute A is 3K. The depression in freezing point for 2 molal solution of A in the same solvent is 6 K. The ratio of K_b and K_f i.e., K_b/K_f is 1 : X. The value of X is [nearest integer]

Official Ans. by NTA (1)

Sol. $\Delta T_b = iK_bm_1$ $\Delta T_f = iK_fm_2$

$$\frac{\Delta T_b}{\Delta T_f} = \frac{K_b \times 1}{K_f \times 2} \Rightarrow \frac{3}{6} = \frac{1}{2} = \frac{K_b}{K_f} \times \frac{1}{2}$$

$$\frac{K_b}{K_f} = \frac{1}{1} \Rightarrow x = 1$$

5. 20 mL of 0.02 M hypo solution is used for the titration of 10 mL of copper sulphate solution, in the presence of excess of KI using starch as an indicator. The molarity of Cu^{2+} is found to be _____ $\times 10^{-2}$ M [nearest integer]

Given : $2Cu^{2+} + 4I^- \rightarrow Cu_2I_2 + I_2$

$I_2 + 2S_2O_3^{2-} \rightarrow 2I^- + S_4O_6^{2-}$

Official Ans. by NTA (4)

Sol. $n_{eq.}$ of I_2 = n_{eq} of $Na_2S_2O_3 = 20 \times 0.002 \times 1$

$2 \times n_{mol}$ of $I_2 = 0.4$

n_{mol} of $I_2 = 0.2$ m mol

n_{mol} of $Cu^{+2} = 0.2 \times 2 \times 10^{-3}$

$$[Cu^{+2}] = \frac{0.4\times10^{-3}}{10\times10^{-3}} = 0.04 = 4\times10^{-2}$$

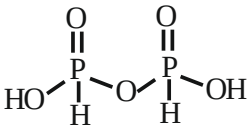
6. The number of non-ionisable protons present in the product B obtained from the following reaction is _____. $C_2H_5OH + PCl_3 \rightarrow C_2H_5Cl + A$

$A + PCl_3 \rightarrow B$

Official Ans. by NTA (2)

Sol. $C_2H_5OH + PCl_3 \longrightarrow C_2H_5Cl + H_3PO_3$

$H_3PO_3 + PCl_3 \longrightarrow H_4P_2O_5 + HCl$



7. The spin-only magnetic moment value of the compound with strongest oxidizing ability among MnF_4, MnF_3 and MnF_2 is _____ B.M. [nearest integer]

Official Ans. by NTA (5)

Sol.

MnF_4	MnF_3	MnF_2
+4	+3	+2
E.C = [Ar] $3d^3$	[Ar] $3d^4$	[Ar] $3d^5$

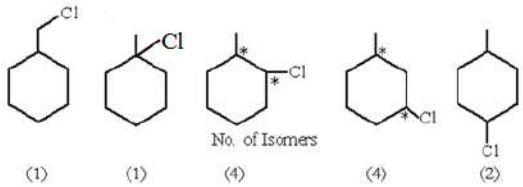
Hence $MnF_3 \Rightarrow$ strongest O.A

$\mu = \sqrt{4(4+2)} = \sqrt{24} = 4.89 = 5$

8. Total number of isomers (including stereoisomers) obtain on monochlorination of methylcyclohexane is _____.

Official Ans. by NTA (12)

Sol.



9. A 100 mL solution of CH_3CH_2MgBr on treatment with methanol produces 2.24 mL of a gas at STP. The weight of gas produced is _____ mg. [nearest integer]

Official Ans. by NTA (3)

Sol. $CH_3–CH_2–MgBr + CH_3OH \longrightarrow$

$$CH_3–CH_3 + Mg\begin{matrix}OCH_3\\ Br\end{matrix}$$

$$n = \frac{2.24\times10^{-3}}{22.4} = 10^{-4}$$

$W = n \times M$

$= 10^{-4} \times 30 = 3$ mg

10. How many of the following drugs is/are example(s) of broad spectrum antibiotic ? Ofloxacin, Penicillin G, Terpineol, Salvarsan

Official Ans. by NTA (1)

Sol. Ofloxacin

FINAL JEE–MAIN EXAMINATION – JULY, 2022

(Held On Tuesday 26th July, 2022) **TIME : 3 : 00 PM to 6 : 00 PM**

MATHEMATICS

TEST PAPER WITH SOLUTION

SECTION-A

1. The minimum value of the sum of the squares of the roots of $x^2+(3-a)x+1=2a$ is:

(A) 4 (B) 5

(C) 6 (D) 8

Official Ans. by NTA (C)

Sol. $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

let $f(a) = (3 - a)^2 - 2(1 - 2a)$

$f(a) = a^2 - 2a + 7$

$f(a) = (a - 1)^2 + 6$

$f(a)\big)_{min.} = 6$

2. If $z = x + iy$ satisfies $|z| - 2 = 0$ and $|z-i|-|z+5i|=0$, then

(A) $x + 2y - 4 = 0$ (B) $x^2 + y - 4 = 0$

(C) $x + 2y + 4 = 0$ (D) $x^2 - y + 3 = 0$

Official Ans. by NTA (C)

Sol. $|z-i|-|z+5i|=0$

$\Rightarrow |x + (y - 1)i| = |x + (y + 5)i|$

$x^2 + (y - 1)^2 = x^2 + (y + 5)^2$

$(y - 1)^2 - (y + 5)^2 = 0$

$(2y + 4)(-6) = 0$

$y = -2$

$\therefore x^2 + (-2)^2 = 4$

$x = 0$

$Z \equiv (0, -2)$, check options

3. Let $A = \begin{bmatrix}1\\1\\1\end{bmatrix}$ and $B = \begin{bmatrix}9^2 & -10^2 & 11^2\\ 12^2 & 13^2 & -14^2\\ -15^2 & 16^2 & 17^2\end{bmatrix}$, then the value of $A'BA$ is:

(A) 1224 (B) 1042 (C) 540 (D) 539

Official Ans. by NTA (D)

Sol. $A'BA = [1\ \ 1\ \ 1]\begin{bmatrix}9^2 & -10^2 & 11^2\\ 12^2 & 13^2 & -14^2\\ -15^2 & 16^2 & 17^2\end{bmatrix}\begin{bmatrix}1\\1\\1\end{bmatrix}$

$=[9^2+12^2-15^2 \quad -10^2+13^2+16^2 \quad 11^2-14^2+17^2]\begin{bmatrix}1\\1\\1\end{bmatrix}$

$= [9^2+12^2-15^2 -10^2+13^2+16^2 + 11^2-14^2+17^2]$

$= [539]$

4. $\sum\limits_{\substack{i,j=0\\ i\neq j}}^{n} {}^nC_i\ {}^nC_j$ is equal to

(A) $2^{2n} - {}^{2n}C_n$ (B) $2^{2n-1} - {}^{2n-1}C_{n-1}$

(C) $2^{2n} - \frac{1}{2}{}^{2n}C_n$ (D) $2^{n-1} + {}^{2n-1}C_n$

Official Ans. by NTA (B)

Sol. $\sum\limits_{\substack{i,j=0\\ i\neq j}}^{n} {}^nC_i\ {}^nC_j$

$= \sum\limits_{i=0}^{n} {}^nC_i \cdot \sum\limits_{j=0}^{n} {}^nC_j - \sum\limits_{i=j=0}^{n} \left({}^nC_i\right)^2$

$= (2^n)(2^n) - {}^{2n}C_n$

$= 2^{2n} - {}^{2n}C_n$

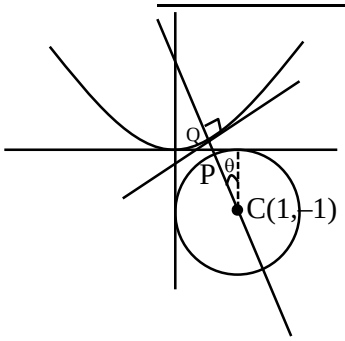
5. Let P and Q be any points on the curves $(x-1)^2+(y+1)^2=1$ and $y = x^2$, respectively. The distance between P and Q is minimum for some value of the abscissa of P in the interval

(A) $\left(0, \frac{1}{4}\right)$ (B) $\left(\frac{1}{2}, \frac{3}{4}\right)$

(C) $\left(\frac{1}{4}, \frac{1}{2}\right)$ (D) $\left(\frac{3}{4}, 1\right)$

Official Ans. by NTA (C)

Sol.



$Q=\left(t,t^2\right)$

$m_{CQ}=m_{normal}$

$\frac{t^2+1}{t-1}=-\frac{1}{2t}$

Let $f\left(t\right)=2t^3+3t-1$

$f\left(\frac{1}{4}\right)f\left(\frac{1}{3}\right)<0\Rightarrow t\in\left(\frac{1}{4},\frac{1}{3}\right)$

$P\equiv\left(1+\cos\left(90+\theta\right),-1+\sin\left(90+\theta\right)\right)$

$P=\left(1-\sin\theta,-1+\cos\theta\right)$

$m_{normal}=m_{CP}\Rightarrow-\frac{1}{2t}=\frac{\cos\theta}{-\sin\theta}\Rightarrow\tan\theta=2t$

$x=1-\sin\theta=1-\frac{2t}{\sqrt{1+4t^2}}=g\left(t\right)\ \ \left(let\right)$

$\Rightarrow g'\left(t\right)<0$

$g\left(t\right)\downarrow function$

$t\in\left(\frac{1}{4},\frac{1}{3}\right)$

$\Rightarrow g\left(t\right)\in\left(0.44,0.485\right)\in\left(\frac{1}{4},\frac{1}{2}\right)$

6. If the maximum value of a, for which the function $f_a(x)=\tan^{-1}2x-3ax+7$ is non-decreasing in $\left(-\frac{\pi}{6},\frac{\pi}{6}\right)$, is $\bar{a}$, then $f_{\bar{a}}\left(\frac{\pi}{8}\right)$ is equal to

(A) $8-\frac{9\pi}{4\left(9+\pi^2\right)}$ (B) $8-\frac{4\pi}{9\left(4+\pi^2\right)}$

(C) $8\left(\frac{1+\pi^2}{9+\pi^2}\right)$ (D) $8-\frac{\pi}{4}$

Official Ans. by NTA (A)

Sol. $f_a(x)=\tan^{-1}2x-3ax+7$

$f_a'(x)=\frac{2}{1+4x^2}-3a\geq0$

$a\leq\left(\frac{2}{3(1+4x^2)}\right)_{min.}$ at $x=\pm\frac{\pi}{6}$

$a_{max}=\bar{a}=\frac{6}{9+\pi^2}$

$f_{\bar{a}}(\frac{\pi}{8})=\tan^{-1}\frac{\pi}{4}-3\frac{6}{9+\pi^2}\frac{\pi}{8}+7=\tan^{-1}\frac{\pi}{4}-\frac{9\pi}{4\left(\pi^2+9\right)}+7$

7. Let $\beta=\lim_{x\to0}\frac{\alpha x-\left(e^{3x}-1\right)}{\alpha x\left(e^{3x}-1\right)}$ for some $\alpha\in\mathbb{R}$. Then the value of $\alpha+\beta$ is :

(A) $\frac{14}{5}$ (B) $\frac{3}{2}$ (C) $\frac{5}{2}$ (D) $\frac{7}{2}$

Official Ans. by NTA (C)

Sol. $\beta=\lim_{x\to0}\frac{\alpha x-(e^{3x}-1)}{\alpha x(e^{3x}-1)}$

$\beta=\lim_{x\to0}\frac{1+\alpha x-\left[1+3x+\frac{9x^2}{2!}+.....\right]}{(\alpha x)\frac{(e^{3x}-1)}{3x}3x}$

$\beta=\lim_{x\to0}\frac{(\alpha x-3x)-\frac{9x^2}{2!}-.....}{3\alpha x^2}$

For existence of limit $\alpha-3=0$

$\alpha=3$

Limit $\beta=\frac{-3}{2\alpha}$

$\beta=-\frac{1}{2}$

Now,

$\alpha+\beta=\frac{5}{2}$

8. The value of $\log_e 2\frac{d}{dx}(\log_{\cos x}\operatorname{cosec}x)$ at $x=\frac{\pi}{4}$ is

(A) $-2\sqrt{2}$ (B) $2\sqrt{2}$ (C) -4 (D) 4

Official Ans. by NTA (D)

Sol. $\log_e 2\dfrac{d}{dx}(\log_{\cos x}\operatorname{cosec}x)$

Let,

$y=\log_{\cos x}\operatorname{cosec}x$

$y=-\dfrac{\ln(\sin x)}{\ln(\cos x)}$

$\dfrac{dy}{dx}=-\dfrac{[\cot x\cdot\ln(\cos x)+\tan x\cdot\ln(\sin x)]}{(\ln(\cos x))^2}$

$\left.\dfrac{dy}{dx}\right)_{x=\frac{\pi}{4}}=\dfrac{4}{\ln 2}$

Now,

$\Rightarrow \log_e 2\cdot\dfrac{4}{\ln 2}=4$

9. $\int_0^{20\pi}(|\sin x|+|\cos x|)^2\,dx$ is equal to :-

(A) $10(\pi+4)$ (B) $10(\pi+2)$

(C) $20(\pi-2)$ (D) $20(\pi+2)$

Official Ans. by NTA (D)

Sol. $I=\int_0^{20\pi}(|\sin x|+|\cos x|)^2\,dx$; (Jack property)

$I=40\int_0^{\pi/2}(\sin x+\cos x)^2\,dx$

$I=40\int_0^{\pi/2}(1+\sin 2x)\,dx$

$I=20[\pi+2]$

10. Let the solution curve y = f(x) of the differential equation $\dfrac{dy}{dx}+\dfrac{xy}{x^2-1}=\dfrac{x^4+2x}{\sqrt{1-x^2}}, x\in(-1,1)$ pass through the origin. Then $\int_{-\frac{\sqrt3}{2}}^{\frac{\sqrt3}{2}} f(x)dx$ is equal to

(A) $\dfrac{\pi}{3}-\dfrac{1}{4}$ (B) $\dfrac{\pi}{3}-\dfrac{\sqrt3}{4}$

(C) $\dfrac{\pi}{6}-\dfrac{\sqrt3}{4}$ (D) $\dfrac{\pi}{6}-\dfrac{\sqrt3}{2}$

Official Ans. by NTA (B)

Sol. $\dfrac{dy}{dx}+\dfrac{xy}{x^2-1}=\dfrac{x^4+2x}{\sqrt{1-x^2}}$

I.F = $e^{\int\frac{x}{x^2-1}dx}$

I.F = $\sqrt{1-x^2}$

Solution of D.E.

$y\cdot\sqrt{1-x^2}=\int\dfrac{x^4+2x}{\sqrt{1-x^2}}\cdot\sqrt{1-x^2}\,dx$

$y\cdot\sqrt{1-x^2}=\int(x^4+2x)\,dx$

$y\cdot\sqrt{1-x^2}=\dfrac{x^5}{5}+x^2+C$

At x = 0, y = 0, get C = 0

$y=\dfrac{x^5}{5\sqrt{1-x^2}}+\dfrac{x^2}{\sqrt{1-x^2}}$

Now,

$\int_{-\frac{\sqrt3}{2}}^{\frac{\sqrt3}{2}} f(x)dx=\int_{-\frac{\sqrt3}{2}}^{\frac{\sqrt3}{2}}\dfrac{x^5}{5\sqrt{1-x^2}}dx+\int_{-\frac{\sqrt3}{2}}^{\frac{\sqrt3}{2}}\dfrac{x^2}{\sqrt{1-x^2}}dx$

$\int_{-\frac{\sqrt3}{2}}^{\frac{\sqrt3}{2}} f(x)dx=0+2\int_0^{\frac{\sqrt3}{2}}\dfrac{x^2}{\sqrt{1-x^2}}dx$

$\int_{-\frac{\sqrt3}{2}}^{\frac{\sqrt3}{2}} f(x)dx=\dfrac{\pi}{3}-\dfrac{\sqrt3}{4}$

11. The acute angle between the pair of tangents drawn to the ellipse $2x^2+3y^2=5$ from the point (1,3) is

(A) $\tan^{-1}\left(\dfrac{16}{7\sqrt5}\right)$ (B) $\tan^{-1}\left(\dfrac{24}{7\sqrt5}\right)$

(C) $\tan^{-1}\left(\dfrac{32}{7\sqrt5}\right)$ (D) $\tan^{-1}\left(\dfrac{3+8\sqrt5}{35}\right)$

Official Ans. by NTA (B)

Sol. Equation of tangent to the ellipse $2x^2+3y^2=5$ is

$y=mx\pm\sqrt{\dfrac{5}{2}m^2+\dfrac{5}{3}}$

It pass through (1, 3)

$$3 = m \pm \sqrt{\frac{5}{2}m^2 + \frac{5}{3}}$$

$$3m^2 + 12m - \frac{44}{3} = 0$$

Let θ be the angle between the tangents

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

$$\tan\theta = \left|\frac{3\sqrt{320}}{-35}\right|$$

$$\theta = \tan^{-1}\left(\frac{24}{7\sqrt{5}}\right)$$

12. The equation of a common tangent to the parabolas $y = x^2$ and $y = -(x-2)^2$ is

(A) $y = 4(x-2)$ (B) $y = 4(x-1)$

(C) $y = 4(x+1)$ (D) $y = 4(x+2)$

Official Ans. by NTA (B)

Sol. Equation of tangent of $y = x^2$ be

$tx = y + at^2$ .....(1)

$$y = tx - \frac{t^2}{4}$$

Solve with $y = -(x-2)^2$

$$tx - \frac{t^2}{4} = -(x-2)^2$$

$$x^2 + x(t-4) - \frac{t^2}{4} + 4 = 0$$

$D = 0$

$$(t-4)^2 - 4\cdot\left(4 - \frac{t^2}{4}\right) = 0$$

$t^2 - 4t = 0$

$t = 0$ or $t = 4$

From eq. (1), required common tangent is

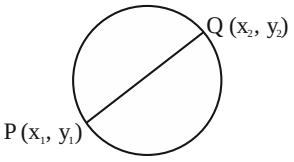
$y = 4(x-1)$

13. Let the abscissae of the two points P and Q on a circle be the roots of $x^2 - 4x - 6 = 0$ and the ordinates of P and Q be the roots of $y^2 + 2y - 7 = 0$. If PQ is a diameter of the circle $x^2 + y^2 + 2ax + 2by + c = 0$, then the value of (a+b-c) is

(A) 12 (B) 13 (C) 14 (D) 16

Official Ans. by NTA (A)

Sol.



Equation of circle diameter form

$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$

(where x_1 , x_2 are the roots of $x^2 - 4x - 6 = 0$ and y_1, y_2 are the roots of $y^2 + 2y - 7 = 0$)

$x^2 + y^2 - 4x + 2y - 13 = 0$

Now,

Compare it with the given equation, we get

$a = -2, b = 1, c = -13$

Now

$a + b - c = 12$

14. If the line x-1 = 0, is a directrix of the hyperbola $kx^2 - y^2 = 6$, then the hyperbola passes through the point

(A) $\left(-2\sqrt{5}, 6\right)$ (B) $\left(-\sqrt{5}, 3\right)$

(C) $\left(\sqrt{5}, -2\right)$ (D) $\left(2\sqrt{5}, 3\sqrt{6}\right)$

Official Ans. by NTA (C)

Sol. $$\frac{x^2}{6/k} - \frac{y^2}{6} = 1 \quad \text{.....(1)}$$

$$e^2 = 1 + \frac{6}{6/k}$$

$$e = \sqrt{1+k}$$

$$a = \sqrt{\frac{6}{k}}$$

Eq. of directrix $x = \frac{a}{e} \Rightarrow x = \sqrt{\frac{6}{k(k+1)}}$

$$\frac{6}{k(k+1)} = 1$$

$k = 2$

From eq. (1) ,we get $2x^2 - y^2 = 6$

Check options

15. A vector $\vec{a}$ is parallel to the line of intersection of the plane determined by the vectors $\hat{i}, \hat{i}+\hat{j}$ and the plane determined by the vectors $\hat{i}-\hat{j}, \hat{i}+\hat{k}$. The obtuse angle between $\vec{a}$ and the vector $\vec{b}=\hat{i}-2\hat{j}+2\hat{k}$ is

(A) $\frac{3\pi}{4}$ (B) $\frac{2\pi}{3}$

(C) $\frac{4\pi}{5}$ (D) $\frac{5\pi}{6}$

Official Ans. by NTA (A)

Sol. $\vec{n}_1=\hat{i}\times(\hat{i}+\hat{j})=\hat{k}$

$\vec{n}_2=(\hat{i}+\hat{k})\times(\hat{i}-\hat{j})$

$=\hat{i}+\hat{j}-\hat{k}$

Line of intersection along $\vec{n}_1\times\vec{n}_2$

$=\hat{k}\times(\hat{i}+\hat{j}-\hat{k})=-\hat{i}+\hat{j}$

D.R of $\vec{a}=-\hat{i}+\hat{j}$

D.R of $\vec{b}=\hat{i}-2\hat{j}+2\hat{k}$

$\vec{a}\cdot\vec{b}=-3$ and $(\vec{a}\wedge\vec{b})=\theta$

$\cos\theta=\frac{-3}{\sqrt{2}\times3}$

$\theta=\frac{3\pi}{4}$

16. If $0<x<\frac{1}{\sqrt{2}}$ and $\frac{\sin^{-1}x}{\alpha}=\frac{\cos^{-1}x}{\beta}$, then a value of $\sin\left(\frac{2\pi\alpha}{\alpha+\beta}\right)$ is

(A) $4\sqrt{(1-x^2)}(1-2x^2)$

(B) $4x\sqrt{(1-x^2)}(1-2x^2)$

(C) $2x\sqrt{(1-x^2)}(1-4x^2)$

(D) $4\sqrt{(1-x^2)}(1-4x^2)$

Official Ans. by NTA (B)

Sol. $\frac{\sin^{-1}x}{\alpha}=\frac{\cos^{-1}x}{\beta}=k$

$\sin^{-1}x=k\alpha$

$\cos^{-1}x=k\beta$

$k=\frac{\pi}{2(\alpha+\beta)}$ ....(i)

$\sin\left(\frac{2\pi\alpha}{\alpha+\beta}\right)=\sin(4\sin^{-1}x)$

$=2\sin(2\sin^{-1}x)\cos(2\sin^{-1}x)$

$=4x\sqrt{1-x^2}(1-2x^2)$

17. Negation of the Boolean expression $p\Leftrightarrow(q\Rightarrow p)$ is

(A) $(\sim p)\wedge q$ (B) $p\wedge(\sim q)$

(C) $(\sim p)\vee(\sim q)$ (D) $(\sim p)\wedge(\sim q)$

Official Ans. by NTA (D)

Sol. $\sim(p\leftrightarrow(q\rightarrow p))$

$\sim(p\leftrightarrow q)=(p\wedge\sim q)\vee(q\wedge\sim p)$

$\sim(p\leftrightarrow(q\rightarrow p))=(p\wedge\sim(q\rightarrow p))\vee((q\rightarrow p)\wedge\sim p)$

$(p\wedge\sim(q\rightarrow p))=p\wedge(q\wedge\sim p)=(p\wedge\sim p)\wedge q=c$

$(q\rightarrow p)\wedge\sim p=(\sim q\vee p)\wedge\sim p=\sim p\wedge(\sim q\vee p)$

$=(\sim p\wedge\sim q)\vee(\sim p\wedge p)=\sim p\wedge\sim q$

$\sim(p\leftrightarrow(q\rightarrow p))=c\vee(\sim p\wedge\sim q)=\sim p\wedge\sim q$

18. Let X be a binomially distributed random variable with mean 4 and variance $\frac{4}{3}$. Then 54 P(X≤ 2) is equal to

(A) $\frac{73}{27}$ (B) $\frac{146}{27}$

(C) $\frac{146}{81}$ (D) $\frac{126}{81}$

Official Ans. by NTA (B)

Sol. np = 4

npq = 4/3

n = 6, p = 2/3, q = 1/3

54(P(X = 2) + P (X = 1) + P(X = 0))

$54\left({}^6C_2\left(\frac{2}{3}\right)^2\left(\frac{1}{3}\right)^4+{}^6C_1\left(\frac{2}{3}\right)^1\left(\frac{1}{3}\right)^5+{}^6C_0\left(\frac{2}{3}\right)^0\left(\frac{1}{3}\right)^6\right)$

$=\frac{146}{27}$

19. The integral $\int \frac{\left(1-\frac{1}{\sqrt{3}}\right)(\cos x-\sin x)}{\left(1+\frac{2}{\sqrt{3}}\sin 2x\right)}dx$ is equal to

(A) $\frac{1}{2}\log_e\left|\frac{\tan\left(\frac{x}{2}+\frac{\pi}{12}\right)}{\left(\frac{x}{2}+\frac{\pi}{6}\right)}\right|+C$

(B) $\frac{1}{2}\log_e\left|\frac{\tan\left(\frac{x}{2}+\frac{\pi}{6}\right)}{\left(\frac{x}{2}+\frac{\pi}{3}\right)}\right|+C$

(C) $\log_e\left|\frac{\tan\left(\frac{x}{2}+\frac{\pi}{6}\right)}{\tan\left(\frac{x}{2}+\frac{\pi}{12}\right)}\right|+C$

(D) $\frac{1}{2}\log_e\left|\frac{\tan\left(\frac{x}{2}-\frac{\pi}{12}\right)}{\tan\left(\frac{x}{2}-\frac{\pi}{6}\right)}\right|+C$

Official Ans. by NTA (A)

Sol. $I=\int \frac{\left(1-\frac{1}{\sqrt{3}}\right)(\cos x-\sin x)}{\left(1+\frac{2}{\sqrt{3}}\sin 2x\right)}dx$

$\frac{\sqrt{3}}{2}\int \frac{\left(1-\frac{1}{\sqrt{3}}\right)(\cos x-\sin x)}{\left(\frac{\sqrt{3}}{2}+\sin 2x\right)}dx$

$\int \frac{\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\right)(\cos x-\sin x)}{\sin 60^\circ+\sin 2x}dx$

$\int \frac{\left(\frac{\sqrt{3}}{2}\cos x-\frac{1}{2}\cos x-\frac{\sqrt{3}}{2}\sin x+\frac{1}{2}\sin x\right)}{2\sin\left(x+\frac{\pi}{6}\right)\cos\left(x-\frac{\pi}{6}\right)}dx$

$\int \frac{\left(\cos\left(x-\frac{\pi}{6}\right)-\sin\left(x+\frac{\pi}{6}\right)\right)}{2\sin\left(x+\frac{\pi}{6}\right)\cos\left(x-\frac{\pi}{6}\right)}dx$

$\frac{1}{2}\left(\int \frac{dx}{\sin\left(x+\frac{\pi}{6}\right)}-\int \frac{dx}{\cos\left(x-\frac{\pi}{6}\right)}\right)$

$\frac{1}{2}\ln\left|\frac{\tan\left(\frac{x}{2}+\frac{\pi}{12}\right)}{\tan\left(\frac{x}{2}+\frac{\pi}{6}\right)}\right|$

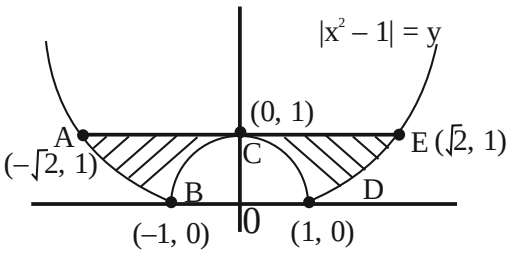
20. The area bounded by the curves $y = |x^2-1|$ and $y = 1$ is

(A) $\frac{2}{3}\left(\sqrt{2}+1\right)$ (B) $\frac{4}{3}\left(\sqrt{2}-1\right)$

(C) $2\left(\sqrt{2}-1\right)$ (D) $\frac{8}{3}\left(\sqrt{2}-1\right)$

Official Ans. by NTA (D)

Sol. $y = |x^2 - 1|$



Area = ABCDEA

$=2\left(\int_0^1\left(1-(1-x^2)\right)dx+\int_1^{\sqrt{2}}\left(1-(x^2-1)\right)dx\right)$

$=\frac{8}{3}(\sqrt{2}-1)$

SECTION-B

1. Let A = {1,2,3,4,5,6,7} and B = {3,6,7,9}. Then the number of elements in the set $\{C\subseteq A : C\cap B\neq\phi\}$ is_____

Official Ans. by NTA (112)

Sol. A = {1,2,3,4,5,6,7} and

B = {3,6,7,9}

Total subset of A = 2^7 = 128

$C\cap B=\phi$ when set C contains the element 1, 2, 4, 5

$\therefore S = \{C \subseteq A; C \cap B \neq \phi\}$

= Total – $(C \cap B = \phi)$

= $128 - 2^4 = 112$

2. The largest value of a, for which the perpendicular distance of the plane containing the lines $\vec{r} = (\hat{i} + \hat{j}) + \lambda(\hat{i} + a\hat{j} - \hat{k})$ and $\vec{r} = (\hat{i} + \hat{j}) + \mu(-\hat{i} + \hat{j} - a\hat{k})$ from the point (2,1,4) is $\sqrt{3}$, is_____.

Official Ans. by NTA (20)

Sol. $\vec{r} = (\hat{i} + \hat{j}) + \lambda(\hat{i} + a\hat{j} - \hat{k})$

$\vec{r} = (\hat{i} + \hat{j}) + \mu(-\hat{i} + \hat{j} - a\hat{k})$

D.R's of plane containing these lines is

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & a & -1 \\ -1 & 1 & -a \end{vmatrix} = \hat{i}(1-a^2) - \hat{j}(-a-1) + \hat{k}(1+a)$$

$\vec{n} = (1-a)\hat{i} + \hat{j} + \hat{k}$

One point in plane : (1, 1, 0)

$\therefore$ equation of plane is

$(1 - a)(x - 1) + (y - 1) + (z - 0) = 0$

$(1 - a)x + y + z + a - 2 = 0$

$$\therefore D = \frac{|(1-a)2 + 1 + 4 + a - 2|}{\sqrt{(1-a)^2 + 1 + 1}}$$

$\Rightarrow |5 - a| = \sqrt{3}.\sqrt{a^2 - 2a + 3}$

$\Rightarrow a^2 + 2a - 8 = 0$

$\Rightarrow a = 2, -4$

$\therefore$ largest value of a = 2

3. Numbers are to be formed between 1000 and 3000, which are divisible by 4, using the digits 1,2,3,4,5 and 6 without repetition of digits. Then the total number of such numbers is _____.

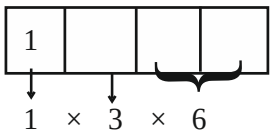
Official Ans. by NTA (30)

Sol. Here 1st digit is 1 or 2 only

Case-I

If first digit is 1

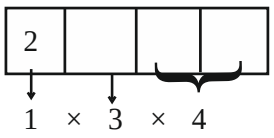
Then last two digits can be 24, 32, 36, 52, 56, 64



= 18 ways

Case – II

If first digit is 2 then last two digit can be 16, 36, 56, 64



= 12 ways

Total ways = 12 + 18 = 30 ways

4. If $\sum_{k=1}^{10} \frac{k}{k^4 + k^2 + 1} = \frac{m}{n}$, where m and n are co-prime, then m + n is equal to

Official Ans. by NTA (166)

Sol. $\sum_{k=1}^{10} \frac{k}{k^4 + k^2 + 1} = \frac{m}{n}$

$$\Rightarrow \frac{1}{2}\sum_{k=1}^{10} \frac{(k^2 + k + 1) - (k^2 - k + 1)}{(k^2 + k + 1)(k^2 - k + 1)}$$

$$\Rightarrow \frac{1}{2}\left(\sum_{k=1}^{10}\left(\frac{1}{(k^2 - k + 1)} - \frac{1}{k^2 + k + 1}\right)\right)$$

$$\Rightarrow \frac{55}{111} = \frac{m}{n}$$

m + n = 166

5. If the sum of solutions of the system of equations $2\sin^2\theta - \cos 2\theta = 0$ and $2\cos^2\theta + 3\sin\theta = 0$ in the interval $[0, 2\pi]$ is $k\pi$, then k is equal to _____.

Official Ans. by NTA (3)

Sol. $2\sin^2\theta - \cos 2\theta = 0$

$2\sin^2\theta - (1 - 2\sin^2\theta) = 0$

$$\Rightarrow \sin^2\theta = \left(\frac{1}{2}\right)^2$$

$\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

$2\cos^2\theta + 3\sin\theta = 0$

$\Rightarrow 2\sin^2\theta - 3\sin\theta - 2 = 0$

$\therefore \sin\theta = -\frac{1}{2}$

$\theta = \frac{7\pi}{6}, \frac{11\pi}{6}$

So, the common solution is

$\theta = \frac{7\pi}{6}, \frac{11\pi}{6}$

Sum = $\frac{7\pi + 11\pi}{6} = 3\pi = k\pi$

K = 3

6. The mean and standard deviation of 40 observations are 30 and 5 respectively. It was noticed that two of these observations 12 and 10 were wrongly recorded. If σ is the standard deviation of the data after omitting the two wrong observations from the data, then $38\sigma^2$ is equal to_____.

Official Ans. by NTA (238)

Sol. Wrong mean = $\mu_1 = 30$

Wrong S.D = $\sigma_1 = 5$

$\frac{\sum x_i}{40} = 30$

$\Rightarrow \sum x_i = 1200$

$\sigma_1^2 = 25$

$\Rightarrow \frac{\sum x_i^2}{40} - 30^2 = 25$

$\Rightarrow \sum x_i^2 = 925 \times 40 = 37000$

New sum = $\sum x_i' = 1200 - 10 - 12 = 1178$

New mean = $\mu_1' = \frac{1178}{38} = 31$

New $\sum x_i^2 = 37000 - (10)^2 - (12)^2 = 36756$

New S.D, $\sigma_1' = \sqrt{\frac{36756}{38} - (31)^2} = \sigma$

$36756 - (31)^2 \times 38 = 38\sigma^2$

$\Rightarrow 38\sigma^2 = 238$

7. The plane passing through the line L: ℓ x-y+3(1- ℓ) z = 1, x+2y - z = 2 and perpendicular to the plane 3x+2y+z = 6 is 3x-8y+7z=4. If θ is the acute angle between the line L and the y-axis, then 415 $\cos^2\theta$ is equal to_____.

Official Ans. by NTA (125)

Sol. $\vec{n}_1 = \ell\hat{i} - \hat{j} + 3(1-\ell)\hat{k}$

$\vec{n}_2 = \hat{i} + 2\hat{j} - \hat{k}$

Direction ratio of line = $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \ell & -1 & 3(1-\ell) \\ 1 & 2 & -1 \end{vmatrix}$

$= (6\ell - 5)\hat{i} + (3 - 2\ell)\hat{j} + (2\ell + 1)\hat{k}$

3x – 8y + 7z = 4 will contain the line $(6\ell - 5)\hat{i} + (3 - 2\ell)\hat{j} + (2\ell + 1)\hat{k}$

Normal of 3x – 8y + 7z = 4 will be perpendicular to the line

$= 3(6\ell - 5) + (3 - 2\ell)(-8) + 7(2\ell + 1) = 0$

$\Rightarrow \ell = \frac{2}{3}$

$\therefore$ direction ratio of line $\left(-1, \frac{5}{3}, \frac{7}{3}\right)$

Angle with y axis

$\cos\theta = \frac{5/3}{\sqrt{1 + \frac{25}{9} + \frac{49}{9}}}$

$\cos\theta = \frac{5}{\sqrt{83}}$

$\therefore 415\cos^2\theta = \frac{25}{83} \times 415 = 125$

8. Suppose y = y(x) be the solution curve to the differential equation $\frac{dy}{dx} - y = 2 - e^{-x}$ such that $\lim_{x\to\infty} y(x)$ is finite. If a and b are respectively the x- and y- intercepts of the tangent to the curve at x=0, then the value of a-4b is equal to _____.

Official Ans. by NTA (3)

Sol. $\frac{dy}{dx} - y = 2 - e^{-x}$

I.F. $= e^{-\int dx} = e^{-x}$

$\therefore$ solution of D.E

$y \cdot e^{-x} = \int \left(2e^{-x} - e^{-2x}\right) dx$

$\Rightarrow y = -2 + \frac{e^{-x}}{2} + C.e^{x}$

$\because \lim_{x\to\infty} y$ is finite

$\therefore \lim_{x\to\infty} \left(-2 + \frac{e^{-x}}{2} + C.e^{x}\right) \to$ finite

This is possible only when C = 0

$\therefore y = y(x) = -2 + \frac{e^{-x}}{2}$

$\frac{dy}{dx} = -\frac{1}{2}e^{-x}$

$\left.\frac{dy}{dx}\right|_{x=0} = -\frac{1}{2} = m$, $y(0) = -2 + \frac{1}{2} = \frac{-3}{2}$

$\therefore$ equation of tangent

$y + \frac{3}{2} = -\frac{1}{2}(x - 0)$

$\Rightarrow x + 2y = -3$

$a = -3$, $b = \frac{-3}{2}$

$a - 4b = -3 + 6 = 3$

9. Different A.P.'s are constructed with the first term 100, the last term 199, And integral common differences. The sum of the common differences of all such, A.P's having at least 3 terms and at most 33 terms is.

Official Ans. by NTA (53)

Sol. 1st term = 100 = a

Last term = 199 = ℓ

If 3 term

a, a + d, a + 2d

$a_n = \ell = a + (n - 1)d$

$d_i = \frac{\ell - a}{n - 1}$

$n \to$ number of terms

n=3, $d_1 = \frac{199 - 100}{2}$

$= \frac{99}{2} \notin I$

n = 4, $d_2 = \frac{99}{3} = 33 \in I$

n = 10, $d_3 = \frac{99}{9} = 11 \in I$

n = 12, $d_4 = \frac{99}{11} = 9 \in I$

$\therefore \sum d_i = 33 + 11 + 9 = 53$

10. The number of matrices $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, where a,b,c,d $\in$ {-1,0,1,2,3,……,10}, such that $A = A^{-1}$, is_____.

Official Ans. by NTA (50)

Sol. $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

Given $A = A^{-1}$

$\therefore A^2 = A \cdot A^{-1} = I$

$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} a^2 + bc & ab + bd \\ ac + cd & bc + d^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\therefore a^2 + bc = 1$ ….(1)

$ab + bd = 0$ …..(2)

$ac + cd = 0$ …(3)

$bc + d^2 = 1$ …….(4)

(1) – (4) gives

$a^2 - d^2 = 0$

$\Rightarrow (a + d) = 0$ or $a - d = 0$

Case – I

$a + d = 0 \Rightarrow (a, d) = (-1, 1), (0, 0), (1, -1)$

(a) $(a, d) = (-1, 1)$

$\therefore$ from equation (1)

$1 + bc = 1 \Rightarrow bc = 0$

$b = 0$ C = 12 possibilities

$c = 0$ b = 12 possibilities

but (0, 0) is repeated

$\therefore 2 \times 12 = 24$

24 – 1 (repeated) = 23 pairs

(b) $(a, d) = (1, -1) \Rightarrow bc = 0 \rightarrow$ 23 pairs

(c) $(a, d) = (0, 0) \Rightarrow bc = 1$

$\Rightarrow$ (b, c) = (1, 1) & (–1, –1), 2 pairs

Case – II

$a = d$

from (2) and (3)

$a \neq 0$ then $b = c = 0$

$a^2 = 1$

$a = \pm 1 = d$

$(a, d) = (1, 1), (-1, -1) \rightarrow$ 2 pairs

$\therefore$ Total = 23 + 23 + 2 + 2

= 50 pairs

FINAL JEE–MAIN EXAMINATION – JULY, 2022

(Held On Tuesday 26th July, 2022) **TIME : 9 : 00 AM to 12 : 00 NOON**

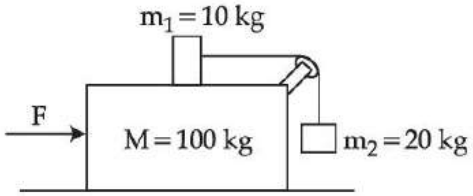
PHYSICS

TEST PAPER WITH SOLUTION

SECTION-A

1. Three masses M = 100 kg, m_1 = 10 kg and m_2=20 kg are arranged in a system as shown in figure. All the surfaces are frictionless and strings are inextensible and weightless. The pulleys are also weightless and frictionless. A force F is applied on the system so that the mass m_2 moves upward with an acceleration of 2 ms^{-2}. The value of F is :

(Take g = 10 ms^{-2})



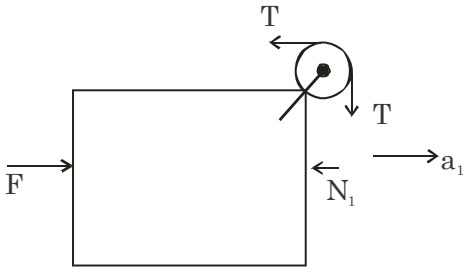
(A) 3360 N (B) 3380 N

(C) 3120 N (D) 3240 N

Official Ans. by NTA (C)

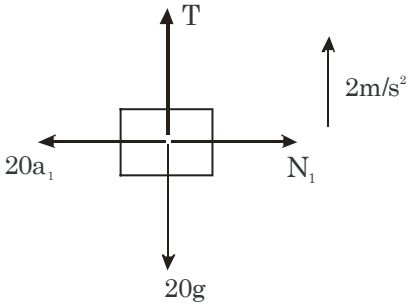
Sol. Let acceleration of 100 kg block = a_1

FBD of 100 kg block w.r.t ground



$F - T - N_1 = 100\ a_1$ ……..(i)

FBD of 20 block wrt 100kg

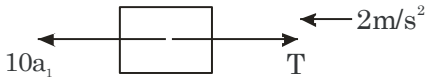


$T - 20g = 20(2)$

$T = 240$ ….. (ii)

$N_1 = 20a_1$ ….. (iii)

FBD of 10 kg block wrt 100 kg



$10a_1 - 240 = 10(2)$

$a_1 = 26\,m/s^2$

$F - 240 - 20(26) = 100 \times 26$

$\Rightarrow F = 3360$ N

2. A radio can tune to any station in 6 MHz to 10 MHz band. The value of corresponding wavelength bandwidth will be :

(A)4 m (B) 20 m

(C) 30 m (D) 50 m

Official Ans. by NTA (B)

Sol. Given: Frequency f_1 = 6MHz

Frequency f_2 = 10MHz

$$\lambda_1 = \frac{c}{f_1}$$

$$\lambda_2 = \frac{c}{f_2}$$

Wavelength bandwidth = λ_2-λ_1 = 20 m

3. The disintegration rate of a certain radioactive sample at any instant is 4250 disintegrations per minute. 10 minutes later, the rate becomes 2250 disintegrations per minute. The approximate decay constant is :

(Take $\log_{10}1.88 = 0.274$)

(A)0.02 min^{-1} (B) 2.7 min^{-1}

(C) 0.063 min^{-1} (D) 6.3 min^{-1}

Official Ans. by NTA (C)

Sol. At t=0 disintegration rate = 4250 dpm

At t=10 disintegration rate = 2250 dpm

$A = A_o e^{-\lambda t}$

$2250 = 4250\, e^{-\lambda(10)}$

$\Rightarrow \lambda(10) = \ln\left(\frac{4250}{2250}\right)$

$\Rightarrow \lambda = 0.063 \text{min}^{-1}$

4. A parallel beam of light of wavelength 900 nm and intensity 100 Wm^{-2} is incident on a surface perpendicular to the beam. Tire number of photons crossing 1 cm^2 area perpendicular to the beam in one second is :

(A) 3×10^{16} (B) 4.5×10^{16}

(C) 4.5×10^{17} (D) 4.5×10^{20}

Official Ans. by NTA (B)

Sol. Wavelength of incident beam $\lambda = 900\times10^{-9}$ m

Intensity of incident beam =I= 100 W/m^2

No. of photons crossing per unit sec

$= n = \frac{E_{net}}{E_{single\ photon}} = \frac{IA\lambda}{hc}$

$= \frac{(100)(1\times10^{-4})(900\times10^{-9})}{6.62\times10^{-34}\times3\times10^{8}} = 4.5\times10^{16}$

5. In young's double slit experiment, the fringe width is 12mm. If the entire arrangement is placed in water of refractive index $\frac{4}{3}$, then the fringe width becomes (in mm)

(A) 16 (B) 9

(C)48 (D) 12

Official Ans. by NTA (B)

Sol. For a given light wavelength corresponding a medium of refractive index μ

$\lambda_{med} = \frac{\lambda_{vacuum}}{\mu}$

and we know that fringe width $\beta = \frac{\lambda D}{d}$

Therefore, $\beta_{med} = \frac{\beta_{vacuum}}{\mu} = \frac{12}{\frac{4}{3}} = 9$ mm

6. The magnetic field of a plane electromagnetic wave is given by

$\vec{B} = 2\times10^{-8}\sin\left(0.5\times10^{3}x + 1.5\times10^{11}t\right)\hat{j}T$

The amplitude of the electric field would be

(A) $6Vm^{-1}$ along x-axis

(B) $3Vm^{-1}$ along z-axis

(C) $6Vm^{-1}$ along z-axis

(D) $2\times10^{-8}Vm^{-1}$ along z-axis

Official Ans. by NTA (C)

Sol. $c = \frac{E_0}{B_0} \Rightarrow E_0 = cB_0$

$E_0 = (3\times10^{8})(2\times10^{-8})$

$E_0 = 6Vm^{-1}$

As, $\vec{B}$ = along y-axis

$\vec{v}$ = along negative x-axis

hence $\vec{E}_0$ = along z-axis

7. In a series LR circuit $X_L = R$ and power factor of the circuit is P_1. When capacitor with capacitance C such that $X_L = X_C$ is put in series, the power factor becomes P_2. The ratio $\frac{P_1}{P_2}$ is

(A) $\frac{1}{2}$ (B) $\frac{1}{\sqrt{2}}$

(C) $\frac{\sqrt{3}}{\sqrt{2}}$ (D)2 : 1

Official Ans. by NTA (B)

Sol. In case of L-R circuit

$Z=\sqrt{X_L^2+R^2}$ & power factor

$P_1=\cos\phi=\dfrac{R}{Z}$

As $X_L=R$

$\Rightarrow Z=\sqrt{2}R$

$\Rightarrow P_1=\dfrac{R}{\sqrt{2}R}\Rightarrow P_1=\dfrac{1}{\sqrt{2}}$

In case of L-C-R circuit

$Z=\sqrt{R^2+\left(X_L-X_C\right)^2}$

As $X_L=X_C$

$\Rightarrow Z=R$

$\Rightarrow P_2=\cos\phi=\dfrac{R}{R}=1$

$\Rightarrow \dfrac{P_1}{P_2}=\dfrac{1}{\sqrt{2}}$

8. A charge particle is moving in a uniform magnetic field $\left(2\hat{i}+3\hat{j}\right)T$. If it has an acceleration of $\left(\alpha\hat{i}-4\hat{j}\right)m/s^2$, then the value of α will be

(A) 3 (B) 6 (C) 12 (D) 2

Official Ans. by NTA (B)

Sol. As $\vec{F}=q\left(\vec{v}\times\vec{B}\right)$

$\vec{a}=\dfrac{q}{m}\left(\vec{v}\times\vec{B}\right)$

So, $\vec{a}\,\&\,\vec{B}$ are $\perp$ to each other

Hence, $\vec{a}.\vec{B}=0$

$\left(\alpha\hat{i}-4\hat{j}\right).\left(2\hat{i}+3\hat{j}\right)=0$

$\alpha(2)+(-4)(3)=0$

$\alpha=\dfrac{12}{2}\Rightarrow\alpha=6$

9. B_X and B_Y are the magnetic field at the centre of two coils of two coils X and Y respectively, each carrying equal current. If coil X has 200 turns and 20 cm radius and coil Y has 400 turns and 20 cm radius, the ratio of B_X and B_Y is

(A)1 : 1 (B) 1 : 2

(C)2 : 1 (D) 4 : 1

Official Ans. by NTA (B)

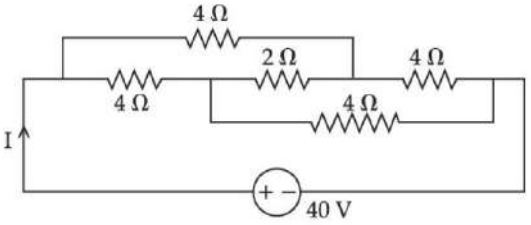
Sol. At centre $B=N\left(\dfrac{\mu_0 i}{2R}\right)$

$B_x=200\left(\dfrac{\mu_0 i}{2\times 20cm}\right)$

$B_y=400\left(\dfrac{\mu_0 i}{2\times 20cm}\right)$

$\dfrac{B_x}{B_y}=\dfrac{1}{2}$

10. The current I in the given circuit will be :

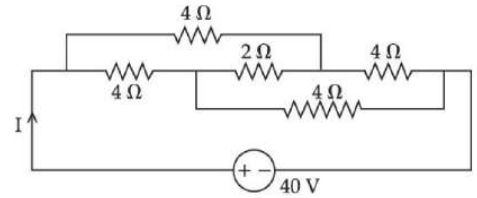


(A) 10A (B) 20 A

(C) 4A (D) 40A

Official Ans. by NTA (A)

Sol.



Given circuit is balanced wheat stone bridge

Hence 2Ω can be neglected

$R_{net}=4\Omega$

$I=\dfrac{40}{4}$

I = 10A

11. The total charge on the system of capacitance $C_1 = 1\mu F,\ C_2 = 2\mu F, C_3 = 4\mu F$ and $C_4 = 3\mu F$ connected in parallel is

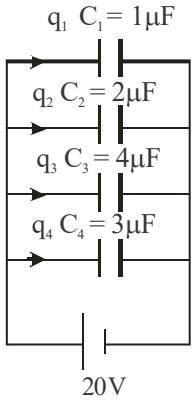
(Assume a battery of 20V is connected to the combination)

(A) $200\mu C$ (B) $200C$

(C) $10\mu C$ (D) $10C$

Official Ans. by NTA (A)

Sol.



Total charge = $q_1 + q_2 + q_2 + q_4$

$= 1\times 20 + 2\times 20 + 4\times 20 + 3\times 20 = 200\mu C$

12. When a particle executes simple Harmonic motion, the nature of graph of velocity as function of displacement will be :

(A)Circular (B)Ellipitical

(C)Sinusoidal (D) Straight line

Official Ans. by NTA (B)

Sol. For a particle in SHM, its speed depends on position as

$v = \omega\sqrt{A^2 - x^2}$

Where ω is angular frequency and A is amplitude

Now $v^2 = \omega^2 A^2 - \omega^2 x^2$

So, $\dfrac{v^2}{(\omega A)^2} + \dfrac{x^2}{(A)^2} = 1$

So graph between v and x is elliptical

13. 7 mole of certain monoatomic ideal gas undergoes a temperature increase of 40K at constant pressure. The increase in the internal energy of the gas in this process is

(Given $R = 8.3\ JK^{-1}mol^{-1}$)

(A) 5810 J (B) 3486 J

(C)11620J (D) 6972 J

Official Ans. by NTA (B)

Sol. For a quasi-static process the change in internal energy of an ideal gas is

$\Delta U = nC_V\,\Delta T$

$= n\times\dfrac{3R}{2}\times\Delta T$

[molar heat capacity at constant volume for mono atomic gas = $\dfrac{3R}{2}$]

$\Delta U = 7\times\dfrac{3}{2}\times 8.3\times 40 = 3486J$

14. A monoatomic gas at pressure P and volume V is suddenly compressed to one eighth of its original volume. The final pressure at constant entropy will be:

(A) P (B) 8P (C) 32P (D) 64 P

Official Ans. by NTA (C)

Sol. Constant entropy means process is adiabatic

$PV^{\gamma} = \text{constant}$

$V_2 = \dfrac{V_1}{8}$

$P_1V_1^{\gamma} = P_2V_2^{\gamma}$

$P_1V_1^{\gamma} = P_2\left(\dfrac{V_1}{8}\right)^{5/3}$

$P_1V_1^{5/3} = \dfrac{P_2V_1^{5/3}}{32}$

$\mathbf{P_2} = 32P_1$

15. A water drop of radius 1cm is broken into 729 equal droplets. If surface tension of water is 75 dyne/cm, then the gain in surface energy upto first decimal place will be :

[Given $\pi = 3.14$]

(A) 8.5×10^{-4} J (B) 8.2×10^{-4} J

(C) 7.5×10^{-4} J (D) 5.3×10^{-4} J

Official Ans. by NTA (C)

Sol. Initial surface energy = TA

Where T is surface tension and A is surface area

$$U_i = \left(\frac{75\times10^{-5}}{10^{-2}}\frac{N}{m}\right)\times\left[4\pi\left(1\times10^{-2}\right)^2\right]$$

$$= 75\times10^{-3}\times4\pi\times10^{-4} = 942\times10^{-7}\ J$$

To get final radius of drops by volume conservation

$$\frac{4}{3}\pi R^3 = 729\left(\frac{4}{3}\pi r^3\right)$$

R = Initial radius

r = final radius

$$r = \frac{R}{(729)^{1/3}} = \frac{R}{9} = \frac{1}{9}\text{cm}$$

Final surface energy

$$U_f = 729[TA]$$

$$= 729\left[\frac{75\times10^{-5}}{10^{-2}}\frac{N}{m}\right]\times\left[4\pi\left(\frac{1}{9}\times10^{-2}\right)^2\right]$$

$$= 729\left[75\times10^{-3}\times\frac{4\pi\times10^{-4}}{81}\right]$$

$$= 9\left[942\times10^{-7}\ J\right]$$

Gain in surface energy

$$\Delta U = 9\times942\times10^{-7} - 942\times10^{-7}$$

$$= 8\times942\times10^{-7}\ J = 7536\times10^{-7}\ J$$

$$= 7.5\times10^{-4}\ J$$

16. The percentage decrease in the weight of a rocket, when taken to a height of 32 km above the surface of earth will, be :

(Radius of earth = 6400km)

(A) 1 % (B) 3%

(C) 4% (D) 0.5%

Official Ans. by NTA (A)

Sol. Acceleration due to gravity at a height h<< R is

$$g' = g\left(1-\frac{2h}{R}\right)$$

$$\therefore \frac{\Delta g}{g} = \frac{2h}{R}$$

$$\Rightarrow \frac{\Delta g}{g}\times100 = \frac{2h}{R}\times100$$

$$= 2\times\frac{32}{6400}\times100 = 1\%$$

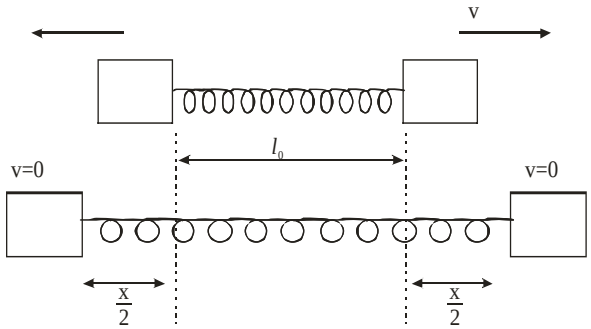
17. As per the given figure, two blocks each of mass 250g are connected to a spring of spring constant 2Nm^{-1}. If both are given velocity v in opposite directions, then maximum elongation of the spring is :

(A) $\frac{v}{2\sqrt{2}}$ (B) $\frac{v}{2}$

(C) $\frac{v}{4}$ (D) $\frac{v}{\sqrt{2}}$

Official Ans. by NTA (B)

Sol.



using energy conservation

$$\frac{1}{2}mv^2\times2 = \frac{1}{2}kx^2$$

$$\Rightarrow \frac{1}{4}v^2 = \frac{1}{2}\times2\times x^2$$

$$\therefore x = \frac{v}{2}$$

18. A monkey of mass 50kg climbs on a rope which can withstand the tension (T) of 350N. If monkey initially climbs down with an acceleration of $4m/s^2$ and then climbs up with an acceleration of $5m/s^2$. Choose the correct option $\left(g = 10m/s^2\right)$

(A) T = 700N while climbing upward

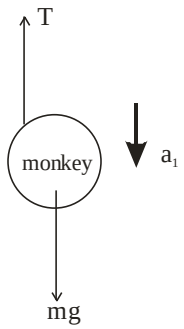
(B) T = 350 N while going downward

(C) Rope will break while climbing upward

(D) Rope will break while going downward

Official Ans. by NTA (C)

Sol. F.B.D of monkey while moving downward

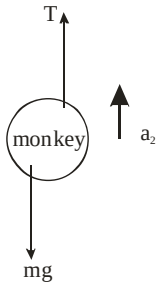


Using Newton's second law

$mg - T = ma_1$

$\therefore$ $500 - T = 50 \times 4 \Rightarrow T = 300$ N

F.B.D of monkey while moving up



Using Newton's second law of motion

$T - mg = ma_2$

$\Rightarrow$ $T - 500 = 50 \times 5$

$\Rightarrow$ T = 750 N

Breaking strength of string = 350 N

$\therefore$ String will break while monkey is moving upward

19. Two projectile thrown at 30^0 and 45^0 with the horizontal respectively, reach the maximum height in same time. The ratio of their initial velocities is

(A) $1:\sqrt{2}$ (B) $2:1$

(C) $\sqrt{2}:1$ (D) $1:2$

Official Ans. by NTA (C)

Sol. **Time taken to reach maximum height**

$$t = \frac{u\sin\theta}{g}$$

$$\therefore \frac{u_1\sin\theta_1}{g} = \frac{u_2\sin\theta_2}{g}$$

$$\Rightarrow u_1\sin 30 = u_2\sin 45$$

$$\Rightarrow \frac{u_1}{u_2} = \frac{1/\sqrt{2}}{1/2} = \frac{\sqrt{2}}{1}$$

20. A screw gauge of pitch 0.5mm is used to measure the diameter of uniform wire of length 6.8cm, the main scale reading is 1.5 mm and circular scale reading is 7. The calculated curved surface area of wire to appropriate significant figures is :

[Screw gauge has 50 divisions on the circular scale]

(A) $6.8cm^2$ (B) $3.4cm^2$

(C) $3.9cm^2$ (D) $2.4cm^2$

Official Ans. by NTA (B)

Sol. $L.C. = \frac{P}{N} = \frac{0.5mm}{50} = 0.01$ mm

Length of wire = 6.8 cm

Diameter of wire = 1.5 mm + 7 × L.C

= 1.5 mm + 7 × .01 = 1.57 mm

Curved surface area = $\pi D\ell$

$= 3.14 \times 6.8 \times 1.57 \times 10^{-1}\ cm^2$

$= 3.352\ cm^2 = 3.4\ cm^2$

SECTION-B

1. If the initial velocity in horizontal direction of a projectile is unit vector $\hat{i}$ and the equation of trajectory is $y = 5x(1-x)$. The y component vector of the initial velocity is _____ $\hat{j}$

(Take $g = 10 m/s^2$)

Official Ans. by NTA (5)

Sol. $u_x = 1$

$y = 5x(1-x)$

$$\frac{dy}{dt} = 5\frac{dx}{dt} - 10x\frac{dx}{dt}$$

For initial y-component of velocity

$$u_y = \left(\frac{dy}{dt}\right)_{x=0} \Rightarrow 5(1) = 5$$

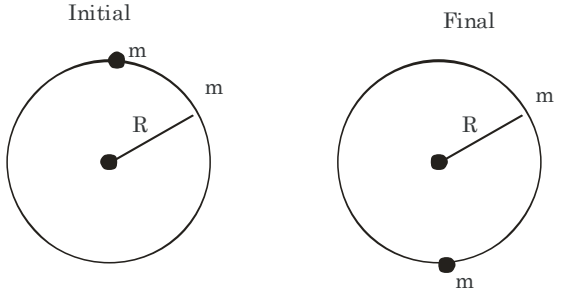
$$\vec{u}_y = 5\hat{j}$$

2. A disc of mass 1 kg and radius R is free of rotate about a horizontal axis passing through its centre and perpendicular to the plane of disc. A body of same mass as that of disc is fixed at the highest point of the disc. Now the system is released, when the body comes to the lowest position, its angular speed will be $4\sqrt{\frac{x}{3R}}\ rad\,s^{-1}$ where x = _____

$\left(g = 10 ms^{-2}\right)$

Official Ans. by NTA (5)

Sol.



using conservation of mechanical energy

$$mg2R = \frac{1}{2}I_{disc}\ \omega^2 + \frac{1}{2}I_{particle}\omega^2$$

$$mg2R = \frac{\omega^2}{2}\left[\frac{mR^2}{2} + mR^2\right]$$

$$mg2R = \frac{\omega^2}{2}\frac{3}{2}mR^2$$

$$\frac{3}{4}\omega^2 = \frac{2g}{R}$$

$$\omega^2 = \frac{8g}{3R}$$

$$\omega = \sqrt{\frac{80}{3R}}$$

Given $\omega = 4\sqrt{\frac{x}{3R}}$

$$16\frac{x}{3R} = \frac{80}{3R}$$

$x = 5$

3. In an experiment of determine the Young's modulus of wire of a length exactly 1m, the extension in the length of the wire is measured as 0.4mm with an uncertainty of ± 0.02 mm when a load of 1kg is applied. The diameter of the wire is measured as 0.4mm with an uncertainty of ± 0.01 mm. The error in the measurement of Young's modulus (ΔY) is found to be $x \times 10^{10}\ Nm^{-2}$. The value of x is _____

[Take $g = 10 m/s^2$]

Official Ans. by NTA (2)

Sol. $L = 1m$

$\Delta L = 0.4 \times 10^{-3}$ m

$m = 1kg$

$d = 0.4 \times 10^{-3} m$

$$\frac{F}{A} = Y\frac{\Delta L}{L}$$

$$Y=\frac{FL}{A\Delta L}=\frac{(mg).(1)}{\left(\frac{\pi d^2}{4}\right)0.4\times10^{-3}}$$

$$\Rightarrow\frac{10\times4}{\pi\left(0.4\times10^{-3}\right)^2\times0.4\times10^{-3}}$$

$$Y=\frac{40}{\pi\left(0.4\times10^{-3}\right)^3}$$

$$Y=\frac{40\times7}{22\times64\times10^{-3}\times10^{-9}}$$

$Y = 0.199 \times 10^{-12}\ N/m^2$

$$\frac{\Delta Y}{Y}=\frac{\Delta F}{F}+\frac{\Delta L}{L}+\frac{\Delta A}{A}+\frac{\Delta(\Delta L)}{(\Delta L)}$$

$$=\frac{0.02}{0.4}+2\frac{\Delta d}{d}=\frac{0.2}{4}+2\times\frac{0.01}{0.4}$$

$$=\frac{0.1}{2}+\frac{0.1}{2}=0.1$$

$$\Rightarrow \Delta Y=0.1\times Y$$

$= 0.199 \times 10^{11} = 1.99 \times 10^{10}$

4. When a car is approaching the observer, the frequency of horn is 100Hz. After passing the observer, it is 50Hz. If the observer moves with the car, the frequency will be $\frac{x}{3}$ Hz where x = ___

Official Ans. by NTA (200)

Sol. $$f_1=100=f_0\left(\frac{C}{C-V_s}\right)$$

C = speed of sound

V_S = speed of source

$$f_2=50=f_0\left(\frac{C}{C+V_s}\right)$$

$$\frac{f_1}{f_2}=2=\frac{C+V_s}{C-V_s}$$

$2C - 2V_s = C + V_s$

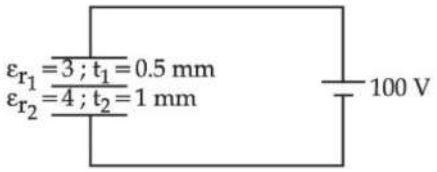
$3V_s = C$

$$V_S=\frac{C}{3}$$

$$100=f_0\frac{C}{\frac{2C}{3}}=\frac{3}{2}f_0$$

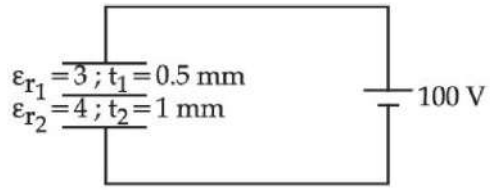
$$f_0=\frac{200}{3}$$

5. A composite parallel plate capacitor is made up of two different dielectric materials with different thickness $(t_1 \text{ and } t_2)$ as shown in figure. The two different dielectric material are separated by a conducting foil F. The voltage of the conducting foil is ____V.



Official Ans. by NTA (60)

Sol.



Capacitance of each capacitor

$$C_1=\frac{A3\in_0}{\frac{1}{2}}=6A\in_0$$

$$C_2=A4\in_0=4A\in_0$$

Equivalent capacitance

$$C_{eq}=\frac{C_1C_2}{C_1+C_2}\Rightarrow\frac{24}{10}A\in_0$$

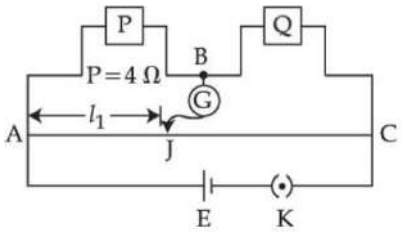
$$q_{net}=C_{eq}(\Delta V)\Rightarrow240\,A\in_0$$

$$\Delta V_2=\frac{240\ A\in_0}{4A\in_0}=60V$$

$(\Delta V_2 = \text{Potential drop across } C_2)$

$V_{foil} = 60V$

6. Resistance are connected in a meter bridge circuit as shown in the figure. The balancing length l_1 is 40cm. Now an unknown resistance x is connected in series with P and new balancing length is found to be 80cm measured from the same end. Then the value of x will be _____Ω



Official Ans. by NTA (20)

Sol. Initially, $\frac{P}{Q}=\frac{40cm}{60cm}=\frac{2}{3}$ ...(1)

Finally, $\frac{P+x}{Q}=\frac{80cm}{20cm}=\frac{4}{1}$ ...(2)

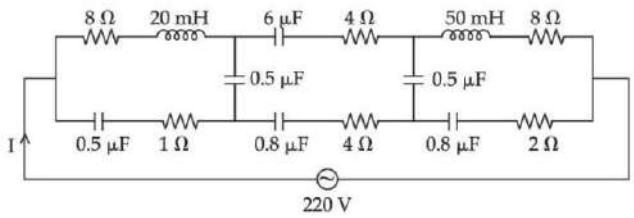
Divide (2) by (1)

$$\frac{P+x}{P}=4\times\frac{3}{2}=6$$

$$\Rightarrow \quad 1+\frac{x}{P}=6\Rightarrow\frac{x}{P}=5$$

$\therefore$ $x = 5P = 5 \times 4 = 20\Omega$

7. The effective current I in the given circuit at very high frequencies will be ____A



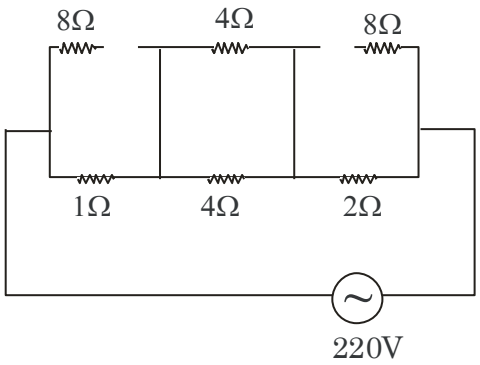
Official Ans. by NTA (44)

Sol. At very high frequencies,

$$X_C=\frac{1}{\omega C}\approx 0$$

Also $X_L=\omega L\approx\infty$

Thus, equivalent circuit can be redrawn as

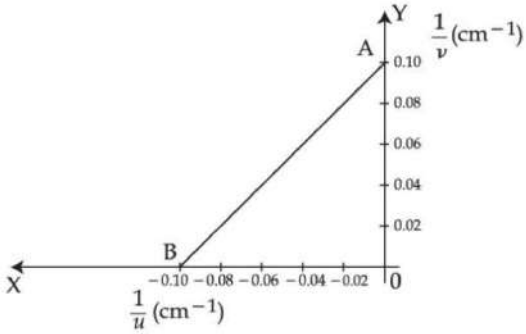


$Z = 1 + 2 + 2 = 5\Omega$

$$I=\frac{220V}{5\Omega}=44A$$

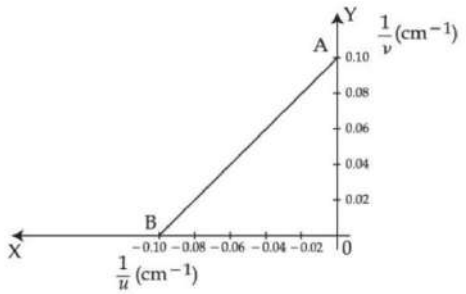
8. The graph between $\frac{1}{u}$ and $\frac{1}{v}$ for a thin convex lens in order to determine its focal length is plotted as shown in the figure. The refractive index of length is 1.5 and its both the surfaces have same radius of curvatures R. The value of R will be ____cm.

(Where u = object distance , v = image distance)



Official Ans. by NTA (10)

Sol.



For point B, $\frac{1}{u} = -0.10\,cm^{-1}, \frac{1}{v} = 0$

$\therefore$ Thus, u = –10cm, v = ∞

i.e. f = 10 cm

$$\Rightarrow \quad \frac{1}{10cm} = (1.5-1)\left(\frac{2}{R}\right) = \frac{1}{R} \Rightarrow R = 10cm$$

9. In a hydrogen spectrum ,λ be the wavelength of first transition line of Lyman series. The wavelength difference will be "aλ" between the wavelength of 3rd transition line of Paschen series and that of 2nd transition line of Balmer Series where a = _____

Official Ans. by NTA (5)

Sol. **For first line of Lyman**

$$\frac{1}{\lambda} = R\left(1-\frac{1}{4}\right) = R\left(\frac{3}{4}\right)$$

$$\Rightarrow \quad \lambda = \frac{4}{3R} \quad \ldots(1)$$

3rd line(Paschen)

$$\frac{1}{\lambda_3} = R\left(\frac{1}{3^2}-\frac{1}{6^2}\right) = \frac{R}{9}\times\frac{3}{4}$$

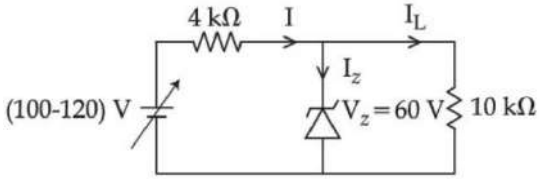
2nd line(Balmer)

$$\frac{1}{\lambda_2} = R\left(\frac{1}{2^2}-\frac{1}{4^2}\right) = \frac{R}{4}\times\frac{3}{4}$$

Thus $a\lambda = \lambda_3 - \lambda_2 = \frac{12}{R} - \frac{16}{3R} = \frac{20}{3R}$ putting (1)

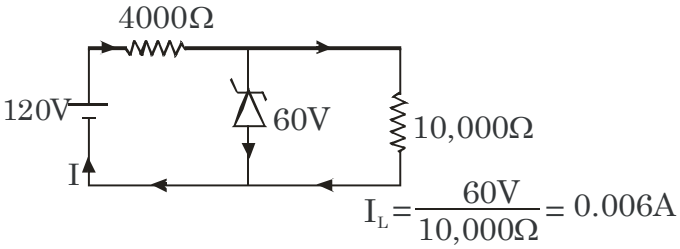
$$a\left(\frac{4}{3R}\right) = \frac{20}{3R} \Rightarrow a = 5$$

10. In the circuit shown below, maximum zener diode current will be ____mA



Official Ans. by NTA (9)

Sol. Consider input 120V



$$I = \frac{(120-60)\,V}{4000\Omega} = 0.015A$$

Thus $I_2 = I - I_L$

= 0.015 – 0.006 = 0.009 A = 9 mA

FINAL JEE–MAIN EXAMINATION – JULY, 2022

(Held On Tuesday 26th July, 2022) **TIME : 9 : 00 AM to 12 : 00 NOON**

CHEMISTRY

TEST PAPER WITH SOLUTION

SECTION-A

1. Match **List - I** with **List - II**.

List - I (Compound)	**List – II (Shape)**
(A) BrF_5	(I) bent
(B) $[CrF_6]^{3-}$	(II) square pyramidal
(C) O_3	(III) trigonal bipyramidal
(D) PCl_5	(IV) octahedral

Choose the **correct** answer from the options given below :

(A) (A) – (I), (B) - (II), (C) - (III), (D) - (IV)

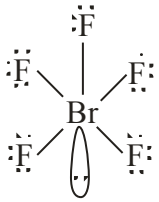
(B) (A) - (IV), (B) - (III), (C) - (II), (D) - (I)

(C) (A) - (II), (B) - (IV), (C) - (I), (D) - (III)

(D) (A) - (III), (B) - (IV), (C) - (II), (D) - (I)

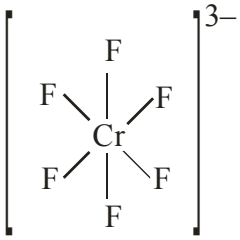
Official Ans. by NTA (C)

Sol.



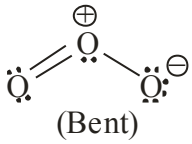
(Square pyramidal)

$[CrF_6]^{3-}$:



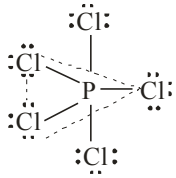
(Octahedral)

O_3 :



(Bent)

PCl_5 :



(Trigonal bipyramidal)

2. Match **List - I** with **List - II.**

List –I (Processes/Reactions)	**List – II (Catalyst)**
(A) $2SO_2(g)+O_2(g)\rightarrow 2SO_3(g)$	(I) Fe(s)
(B) $4NH_3(g)+5O_2(g)\rightarrow 4NO(g)+6H_2O(g)$	(II) Pt(s)-Rh(s)
(C) $N_2(g)+3H_2(g)\rightarrow 2NH_3(g)$	(III) V_2O_5
(D) Vegetable oil(l)+$H_2\rightarrow$Vegetable ghee(s)	(IV) Ni(s)

Choose the correct answer from the options given below :

(A) (A) - (III), (B) - (I), (C) - (II), (D) - (IV)

(B) (A) - (III), (B) - (II), (C) - (I), (D) - (IV)

(C) (A) - (IV), (B) - (III), (C) - (I), (D) - (II)

(D) (A) - (IV), (B) - (II), (C) - (III), (D) - (I)

Official Ans. by NTA (B)

Sol. $2SO_2(g)+O_2(g)\xrightarrow{V_2O_5}2SO_3(g)$:

contact process

$4NH_3(g)+5O_2(g)\xrightarrow{Pt(s)-Rh(s)}4NO(g)+6H_2O(g)$:

Ostwald's process

$N_2(g)+3H_2(g)\xrightarrow{Fe(s)}2NH_3(g)$; Haber's process

$\text{Vegetable oil}\ (l)+H_2(g)\xrightarrow{Ni(s)}$ vegetable ghee

: Hydrogenation

3. Given two statements below :

Statement I : In Cl_2 molecule the covalent radius is double of the atomic radius of chlorine.

Statement II : Radius of anionic species is always greater than their parent atomic radius.

Choose the **most appropriate** answer from options given below :

(A) Both Statement I and Statement II are correct.

(B) Both Statement I and Statement II are incorrect.

(C) Statement I is correct but Statement II is incorrect.

(D) Statement I is incorrect but Statement II is correct.

Official Ans. by NTA (D)

Sol. In Cl_2 molecule, the covalent radius is half of the internuclear distance, so statement(I) is false.

For the same element, anion has lower effective nuclear charge than atom $\Rightarrow$ so anion is larger than atom. $\Rightarrow$ statement (II) is correct.

4. Refining using liquation method is the most suitable for metals with :

(A) Low melting point

(B) High boiling point

(C) High electrical conductivity

(D) Less tendency to be soluble in melts than impurities

Official Ans. by NTA (A)

Sol. Liquation is used to purify metals having lower melting point than impurities present in them.

5. Which of the following can be used to prevent the decomposition of H_2O_2?

(A) Urea

(B) Formaldehyde

(C) Formic acid

(D) Ethanol

Official Ans. by NTA (A)

Sol. Urea acts as stabiliser for H_2O_2.

6. Reaction of $BeCl_2$ with $LiAlH_4$ gives :

(A) $AlCl_3$

(B) BeH_2

(C) LiH

(D) LiCl

(E) $BeAlH_4$

Choose the **correct** answer from options given below :

(A) (A), (D) and (E)

(B) (A) , (B) and (D)

(C) (D) and (E)

(D) (B) , (C) and (D)

Official Ans. by NTA (B)

Sol. $2BeCl_2 + LiAlH_4 \rightarrow 2BeH_2 + LiCl + AlCl_3$

7. Borazine, also known as inorganic benzene, can be prepared by the reaction of 3-equivalents of "X" with 6-equivalents of "Y". "X" and "Y", respectively are :

(A) $B(OH)_3$ and NH_3 (B) B_2H_6 and NH_3

(C) B_2H_6 and HN_3 (D) NH_3 and B_2O_3

Official Ans. by NTA (B)

Sol. $3B_2H_6 + 6NH_3 \xrightarrow{\Delta} 2B_3N_3H_6 + 12\ H_2$

8. Which of the given reactions is not an example of disproportionation reaction ?

(A) $2H_2O_2 \rightarrow 2H_2O + O_2$

(B) $2NO_2 + H_2O \rightarrow HNO_3 + HNO_2$

(C) $MnO_4^- + 4H^+ + 3e^- \rightarrow MnO_2 + 2H_2O$

(D) $3MnO_4^{2-} + 4H^+ \rightarrow 2MnO_4^- + MnO_2 + 2H_2O$

Official Ans. by NTA (C)

Sol. $\overset{-1}{H_2O_2} \longrightarrow \overset{2-}{H_2O} + \overset{0}{O_2}$: Disproportionation

$\overset{+4}{2NO_2} + H_2O \rightarrow \overset{+5}{HNO_3} + \overset{+3}{HNO_2}$: Disproportionation

$MnO_4^- + 4H^+ + 3e^- \rightarrow MnO_2 + 2H_2O$: reduction

$\overset{+6}{3MnO_4^{2-}} + 4H^+ \rightarrow \overset{+7}{2MnO_4^-} + \overset{+4}{MnO_2} + 2H_2O$: Disproportionation

9. The dark purple colour of $KMnO_4$ disappears in the titration with oxalic acid in acidic medium.

The overall change in the oxidation number of manganese in the reaction is :

(A) 5 (B) 1

(C) 7 (D) 2

Official Ans. by NTA (A)

Sol. In acidic medium,

$\overset{+7}{MnO_4^-} \rightarrow Mn^{+2}$

change in ox. no. = 5

10. $\dot{C}l + CH_4 \rightarrow A + B$

A and B in the above atmospheric reaction step are

(A) C_2H_6 and Cl_2 (B) $\dot{C}HCl_2$ and H_2

(C) $\dot{C}H_3$ and HCl (D) C_2H_6 and HCl

Official Ans. by NTA (C)

Sol. $\dot{C}l + CH_4 \longrightarrow \dot{C}H_3 + HCl$

11. Which technique among the following, is most appropriate in separation of a mixture of 100 mg of p-nitrophenol and picric acid ?

(A) Steam distillation

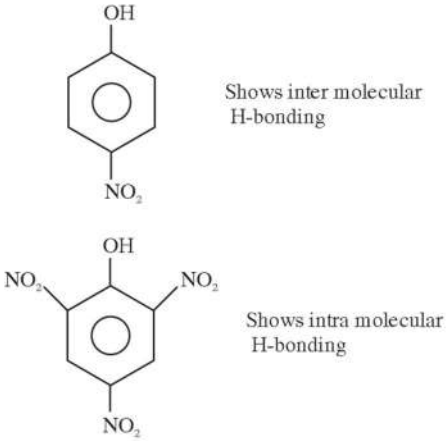
(B) 2-5 ft long column of silica gel

(C) Sublimation

(D) Preparative TLC (Thin Layer Chromatography)

Official Ans. by NTA (D)

Sol.



Solvent polarity has been related to R_f value of nitrocompounds.

100 mg p-nitrophenol and picric acid have different R_f value on silica gel plate

$\therefore$ Preparative TLC is best to separate 100 mg of para nitrophenol and picric acid

12. The difference in the reaction of phenol with bromine in chloroform and bromine in water medium is due to :

(A) Hyperconjugation in substrate

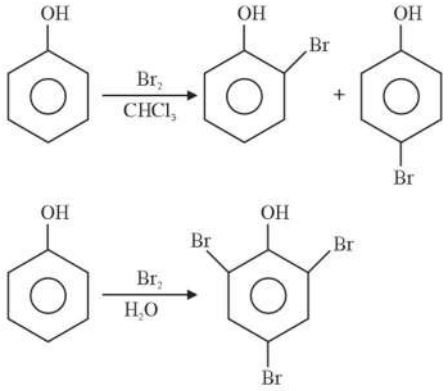
(B) Polarity of solvent

(C) Free radical formation

(D) Electromeric effect of the substrate

Official Ans. by NTA (B)

Sol.

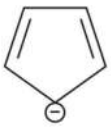


Difference in reactions is observed due to solvent polarity, which

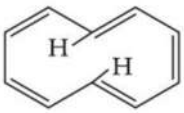
(i) Ionizes phenol to make more reactive phenoxide ion

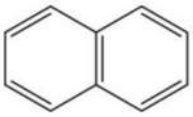
(ii) Increases electrophilicity of bromine.

13. Which of the following compounds is **not** aromatic?

(A) 

(B) 

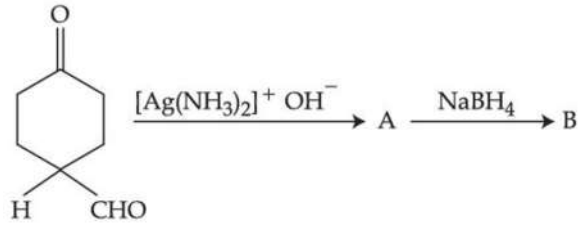
(C) 

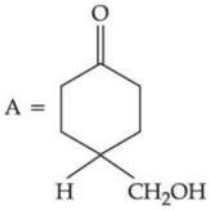
(D) 

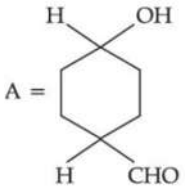
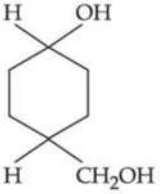
Official Ans. by NTA (C)

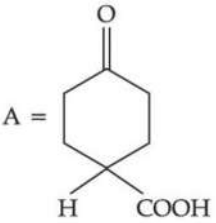
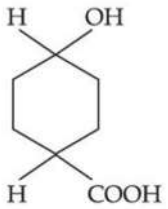
Sol. [10] Annulene, although follow $(4n + 2)\pi$ electron rule, but it is non-aromatic due to its non planar nature. It is nonplanar due to repulsion of C – H bonds present inside the ring.

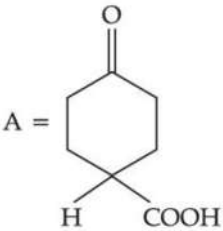
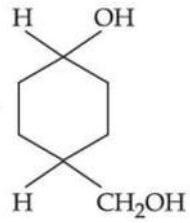
14. The products formed in the following reaction, A and B are



(A) A =  , B = 

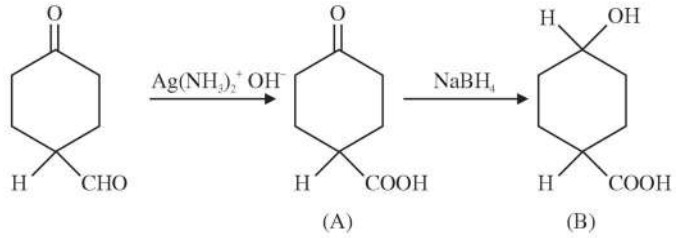
(B) A =  , B = 

(C) A =  , B = 

(D) A =  , B = 

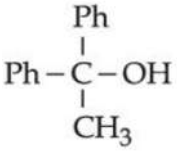
Official Ans. by NTA (C)

Sol.



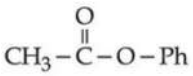
$NaBH_4$ does not reduce carboxylic acid.

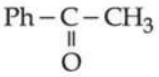
15. Which reactant will give the following alcohol on reaction with one mole of phenyl magnesium bromide (PhMgBr) followed by acidic hydrolysis ?



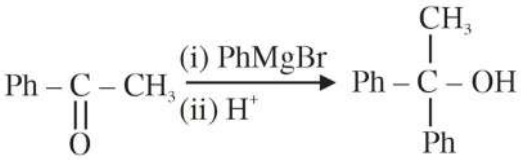
(A) $CH_3 - C \equiv N$

(B) $Ph - C \equiv N$

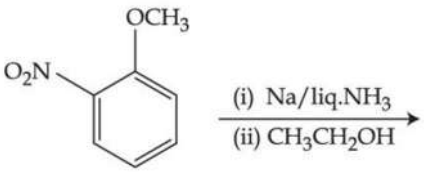
(C) 

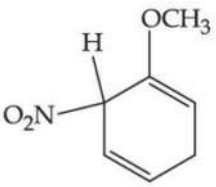
(D) 

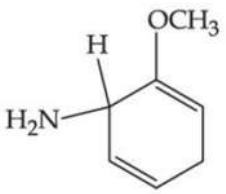
Official Ans. by NTA (D)

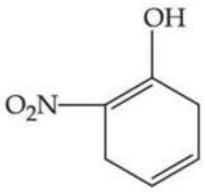
Sol. 

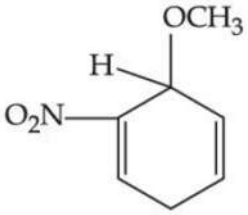
16. The major product of the following reaction is



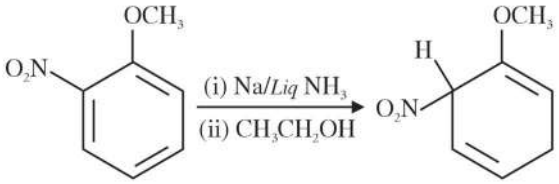
(A) 

(B) 

(C) 

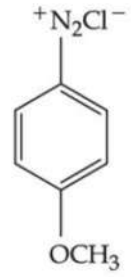
(D) 

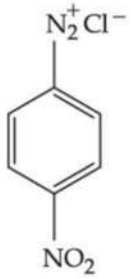
Official Ans. by NTA (A)

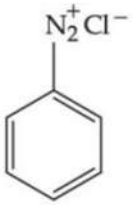
Sol. 

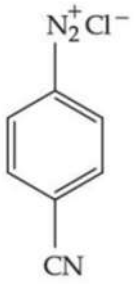
Given reaction is an example of birch reduction.

17. The correct stability order of the following diazonium salt is

(A) 

(B) 

(C) 

(D) 

(A) (A) > (B) > (C) > (D)

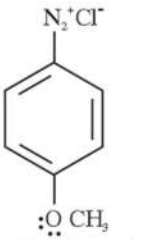
(B) (A) > (C) > (D) > (B)

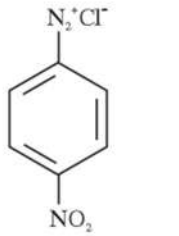
(C) (C) > (A) > (D) > (B)

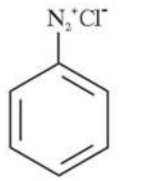
(D) (C) > (D) > (B) > (A)

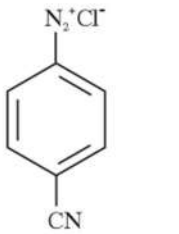
Official Ans. by NTA (B)

Sol.

(A) 
(+ M group)

(B) 
(– M & – I group)

(C) 

(D) 
(– M & – I group)

Since diazonium ion is a cation hence it is stabilized by electron donating groups and destabilized by electron withdrawing group.

Hence Stability order should be A > C > D > B.

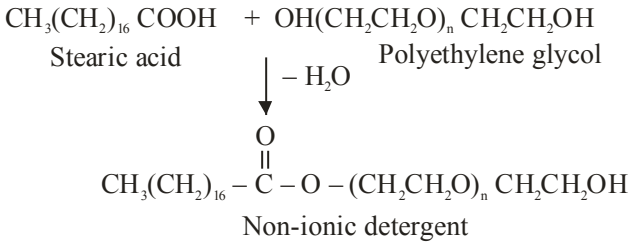
18. Stearic acid and polyethylene glycol react to form which one of the following soap/s detergents ?

(A) Cationic detergent (B) Soap

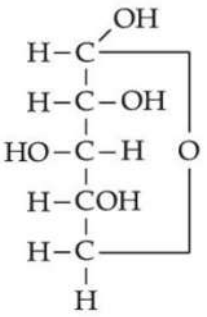
(C) Anionic detergent (D) Non-ionic detergent

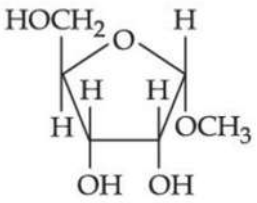
Official Ans. by NTA (D)

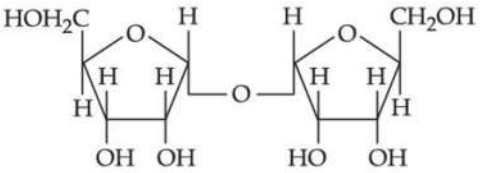
Sol.

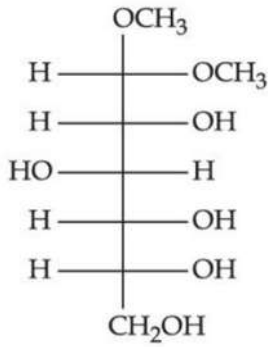


19. Which of the following is reducing sugar?

(A) 

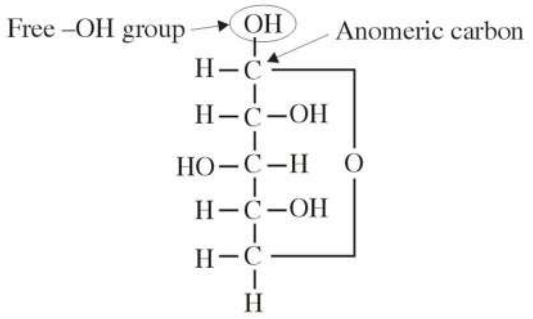
(B) 

(C) 

(D) 

Official Ans. by NTA (A)

Sol. If any sugar is having free –OH group at anomeric carbon then it will be a reducing sugar



20. Given below are two statements : one is labelled as **Assertion (A)** and the other is labelled as **Reason (R).**

Assertion (A) : Experimental reaction of CH_3Cl with aniline and anhydrous $AlCl_3$ does **not** give o and p-methylaniline.

Reason (R) : The — NH_2 group of aniline becomes deactivating because of salt formation with anhydrous $AlCl_3$ and hence yields *m*-methyl aniline as the product.

In the light of the above statements, choose the most appropriate answer from the options given below :

(A) Both (A) and (R) are true and (R) is the correct explanation of (A).

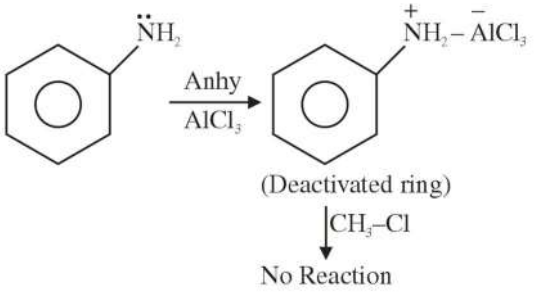
(B) Both (A) and (R) are true but (R) is not the correct explanation of (A).

(C) (A) is true, but (R) is false.

(D) (A) is false, but (R) is true.

Official Ans. by NTA (C)

Sol.



Friedel Craft Alkylation does not occur on this deactivated ring.

SECTION-B

1. Chlorophyll extracted from the crushed green leaves was dissolved in water to make 2 L solution of Mg of concentration 48 ppm. The number of atoms of Mg in this solution is $x \times 10^{20}$ atoms. The value of x is_____. (Nearest Integer)

(Given : Atomic mass of Mg is 24 g mol^{-1}, $N_A = 6.02 \times 10^{23}$ mol^{-1})

Official Ans. by NTA (24)

Sol. $$ppm = \frac{W_{Mg}}{V_{soln}} \times 10^6 = 48$$

$$\Rightarrow W_{Mg} = \frac{48 \times 2 \times 1000}{10^6}$$

$$= 48 \times 2 \times 10^{-3}\ g$$

$$n_{Mg} = \frac{W_{Mg}}{24} = \frac{48 \times 2 \times 10^{-3}}{24}$$

$$= 4 \times 10^{-3}$$

Number of Mg atoms $= 4 \times 10^{-3} \times 6.02 \times 10^{23}$

$= 4 \times 6.02 \times 10^{20}$

$= 24.08 \times 10^{20}$

$\therefore \quad x = 24.08$

2. A mixture of hydrogen and oxygen contains 40% hydrogen by mass when the pressure is 2.2 bar. The partial pressure of hydrogen is bar.

(Nearest Integer)

Official Ans. by NTA (2)

Sol. Let $W_{H_2} = 40\ g \Rightarrow n_{H_2} = \frac{40}{2} = 20$

$$W_{O_2} = 60g \Rightarrow n_{O_2} = \frac{60}{32} = \frac{15}{8}$$

$$P_{H_2} = \left(\frac{20}{20 + \frac{15}{8}}\right) \times 2.2$$

$$= \frac{20}{20 + 1.875} \times 2.2$$

$$= \frac{20}{21.875} \times 2.2$$

$$= 2.0114$$

$$\simeq 2.01 \text{ bar}$$

3. The wavelength of an electron and a neutron will become equal when the velocity of the electron is x times the velocity of neutron. The value of x is_____. (Nearest Integer)

(Mass of electron is 9.1×10^{-31} kg and mass of neutron is 1.6×10^{-27} kg)

Official Ans. by NTA (1758)

Sol. $v_e = x\, v_N$

$\lambda_e = \lambda_N$

$$\Rightarrow \frac{h}{m_e v_e} = \frac{h}{m_N v_N}$$

$$v_e = \frac{m_N}{m_e}.v_N$$

$$= \frac{1.6\times10^{-27}}{9.1\times10^{-31}} v_N$$

$v_e = 1758.24 \times v_N$

$\therefore$ x = 1758.24

4. 2.4 g coal is burnt in a bomb calorimeter in excess of oxygen at 298 K and 1 atm pressure.

The temperature of the calorimeter rises from 298 K to 300 K. The enthalpy change during the combustion of coal is – x kJ mol^{-1}. The value of x is_____. (Nearest Integer)

(Given : Heat capacity of bomb calorimeter 20.0 kJ K^{-1} . Assume coal to be pure carbon)

Official Ans. by NTA (200)

Sol. $C(s) + O_2(g) \rightarrow CO_2(g)$; ΔH = -x kJ/mole

$Q = C\Delta T = 20$ kJ × 2

40 kJ heat is released for 2.4 g of C

For 1 mole 'C' : $Q = \frac{40}{2.4}\times12$

$$= \frac{400}{24}\times12 = 200 \text{ kJ/mole}$$

$Q = \Delta E = \Delta H = 200$ kJ ($\because \Delta n_g = 0$)

x = 200

5. When 800 mL of 0.5 M nitric acid is heated in a beaker, its volume is reduced to half and 11.5 g of nitric acid is evaporated. The molarity of the remaining nitric acid solution is x × 10^{-2} M. (Nearest Integer)

(Molar mass of nitric acid is 63 g mol^{-1})

Official Ans. by NTA (54)

Sol. $n_{HNO_3} = 0.5\times0.8$

= 0.4 mole

$$(n_{HNO_3})_{remains} = 0.4 - \frac{11.5}{63}$$

= 0.4 – 0.1825

= 0.2175

$$\text{Molarity} = \frac{0.2175}{400}\times1000$$

$$= \frac{0.2175}{0.4}$$

= 0.5437 mole/lit.

$\simeq$ 0.54 mole/lit.

= 54 × 10^{-2} mol/lit.

6. At 298 K, the equilibrium constant is 2 × 10^{15} for the reaction :

$Cu(s) + 2Ag^+(aq) \rightleftharpoons Cu^{2+}(aq) + 2Ag(s)$

The equilibrium constant for the reaction

$$\frac{1}{2}Cu^{2+}(aq) + Ag(s) \rightleftharpoons \frac{1}{2}Cu(s) + Ag^+(aq)$$

is x × 10^{-8}. The value of x is_____.

(Nearest Integer)

Official Ans. by NTA (2)

Sol. $$K'_{eq} = \frac{1}{\sqrt{K_{eq}}} = \frac{1}{\sqrt{2\times10^{15}}} = x\times10^{-8}$$

$$\Rightarrow \frac{1}{\sqrt{20}}\times\frac{1}{10^7} = x\times10^{-8}$$

$$\Rightarrow \frac{1}{\sqrt{20}}\times10^{-7} = x\times10^{-8}$$

$$\frac{10}{\sqrt{20}} = x$$

$$\Rightarrow x = \frac{\sqrt{10}}{\sqrt{2}} = \sqrt{5} = 2.236$$

$\simeq$ 2.24

7. The amount of charge in F (Faraday) required to obtain one mole of iron from Fe_3O_4 is _____. (Nearest Integer)

Official Ans. by NTA (8)

Sol. $Fe_3O_4 \xrightarrow{+8e^-} 3Fe$

Charge for 1 mole Fe = 8/3 F

= 2.67 F

8. For a reaction A $\rightarrow$ 2B + C the half lives are 100 s and 50 s when the concentration of reactant A is 0.5 and 1.0 mol L^{-1} respectively. The order of the reaction is___ _____. (Nearest Integer)

Official Ans. by NTA (2)

Sol. $t_{\frac{1}{2}} \propto \frac{1}{[A_0]^{n-1}}$

$[100] \propto \frac{1}{(0.5)^{n-1}}$

$(50) \propto \frac{1}{(1)^{n-1}}$

$[2]^1 = \left[\frac{1}{0.5}\right]^{n-1}$

$[2]^1 = [2]^{n-1}$

$n - 1 = 1$

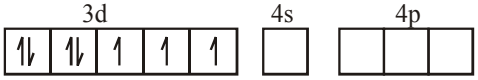
$n = 2$

order = 2

9. The difference between spin only magnetic moment values of $[Co(H_2O)_6]Cl_2$ and $[Cr(H_2O)_6]Cl_3$ is_____.

Official Ans. by NTA (0)

Sol. $[Co(H_2O)_6]^{2+}$

Co^{+2} : 

number of unpaired e^- = 3

$\mu = \sqrt{15}BM$

$[Cr(H_2O)_6]^{3+}$

Cr^{+3} : 

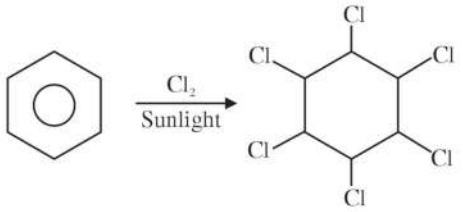
number of unpaired e^- = 3

$\mu = \sqrt{15}BM$

Difference in spin only magnetic moment = 0

10. In the presence of sunlight, benzene reacts with Cl_2 to give product, X. The number of hydrogens in X is_____.

Official Ans. by NTA (6)

Sol. 

FINAL JEE–MAIN EXAMINATION – JULY, 2022

(Held On Tuesday 26th July, 2022) **TIME : 9 : 00 AM to 12 : 00 NOON**

MATHEMATICS

TEST PAPER WITH SOLUTION

SECTION-A

1. Let f: R $\to$ R be a continuous function such that f (3x) – f (x) = x. If f (8) = 7, then f (14) is equal to :

(A) 4 (B) 10

(C) 11 (D) 16

Official Ans. by NTA (B)

Sol. f(x) – f(x/3) = x/3

f(x/3) – f(x/3²) = x/3²

.... on adding

$$f(x)-\lim_{n\to\infty} f\left(\frac{x}{3^n}\right)=x\left(\frac{1}{3}+\frac{1}{3^2}....\infty\right)$$

$$f(x)-f(0)=\frac{x}{2}$$

f(8) = 7 ; f(0) = 3

f(x) = x/2 + 3

f(14) = 10

2. Let O be the origin and A be the point $z_1 = 1 + 2i$. If B is the point z_2, $Re(z_2) < 0$, such that OAB is a right angled isosceles triangle with OB as hypotenuse, then which of the following is NOT true ?

(A) $\arg z_2 = \pi - \tan^{-1} 3$

(B) $\arg (z_1 - 2z_2) = -\tan^{-1} \frac{4}{3}$

(C) $|z_2| = \sqrt{10}$

(D) $|2z_1 - z_2| = 5$

Official Ans. by NTA (D)

Sol. $AB = AO.\, z^{-i\pi/2} = -2 + i$

So OB = (–2 + *i*) + (1 + 2*i*)

$z_2 = -1 + 3i$

$\therefore |2z_1 - z_2| = \sqrt{10}$

3. If the system of linear equations.

8x + y + 4z = –2

x + y + z = 0

λx – 3y = μ

has infinitely many solutions, then the distance of the point $\left(\lambda, \mu, -\frac{1}{2}\right)$ from the plane 8x + y + 4z + 2 = 0 is :

(A) $3\sqrt{5}$ (B) 4

(C) $\frac{26}{9}$ (D) $\frac{10}{3}$

Official Ans. by NTA (D)

Sol. $D = \begin{vmatrix} 8 & 1 & 4 \\ 1 & 1 & 1 \\ \lambda & -3 & 0 \end{vmatrix} = 0 \Rightarrow \lambda = 4$

Also $D_1 = D_2 = D_3 = 0$

So μ = –2

Point $\left(4, -2, -\frac{1}{2}\right)$

Distance from plane = $\frac{10}{3}$

4. Let A be a 2 × 2 matrix with det (A) = –1 and det ((A + I) (Adj (A) + I)) = 4. Then the sum of the diagonal elements of A can be :

(A) –1 (B) 2

(C) 1 (D) $-\sqrt{2}$

Official Ans. by NTA (B)

Sol. Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$; ad – bc = –1

$|A + I||adj\ A + I| = 4$

$\Rightarrow$ ad – bc + a + d + 1 = 2 or –2

a + d = 2 or –2

5. The odd natural number a, such that the area of the region bounded by y= 1, y = 3, x = 0, x = y^a is $\frac{364}{3}$, equal to :

(A) 3 (B) 5

(C) 7 (D) 9

Official Ans. by NTA (B)

Sol. $A=\int_1^3 y^a.dy=\left.\frac{y^{a+1}}{a+1}\right|_1^3=\frac{364}{3}$

$\Rightarrow a=5$

6. Consider two G.Ps. 2, 2^2, 2^3, ... and 4, 4^2, 4^3,.... of 60 and n terms respectively. If the geometric mean of all the 60 + n terms is $(2)^{\frac{225}{8}}$, then $\sum_{k=1}^{n} k(n-k)$ is equal to :

(A) 560 (B) 1540

(C) 1330 (D) 2600

Official Ans. by NTA (C)

Sol. $\left(\left(2^1 2^2.....2^{60}\right)\left(4^1.4^2.....4^n\right)\right)^{\frac{1}{60+n}}=2^{\frac{225}{8}}$

$\left(2^{30\times 61}\,4^{\frac{n(n+1)}{2}}\right)^{\frac{1}{60+n}}=2^{\frac{225}{8}}$

$2^{1830+n^2+n}=2^{\frac{(225)(60+n)}{8}}$

$= 8n^2-217n+1140=0$

$n=20,\ \frac{57}{8}$

$\sum_{k=1}^{n} nk-k^2=\frac{n^2(n+1)}{2}-\frac{n(n+1)(2n+1)}{6}$

$= 1330$

7. If the function

$$f(x)=\begin{cases}\frac{\log_e(1-x+x^2)+\log_e(1+x+x^2)}{\sec x-\cos x}, & x\in\left(\frac{-\pi}{2},\frac{\pi}{2}\right)-\{0\}\\ k, & x=0\end{cases}$$

is continuous at x = 0, then k is equal to :

(A) 1 (B) –1

(C) e (D) 0

Official Ans. by NTA (A)

Sol. $\lim_{x\to 0}\frac{\left(\ln\left(1+x^2+x^4\right)\right)\cos x}{1-\cos^2 x}$

$\lim_{x\to 0}\frac{\left(\frac{\ln\left(1+x^2+x^4\right)}{x^2+x^4}\right)x^2\left(1+x^2\right)\cos x}{\left(\frac{\sin^2 x}{x^2}\right)x^2}=1$

$\therefore k=1$

8. If $f(x)=\begin{cases}x+a, & x\le 0\\ |x-4|, & x>0\end{cases}$ and

$g(x)=\begin{cases}x+1, & x<0\\ (x-4)^2+b, & x\ge 0\end{cases}$

are continuous on R, then (gof) (2) + (fog) (–2) is equal to :

(A) –10 (B) 10

(C) 8 (D) –8

Official Ans. by NTA (D)

Sol. $f(x)=\begin{cases}x+a & ;x\le 0\\ |x-4| & ;x>0\end{cases}$; $g(x)=\begin{cases}x+1 & ;x<0\\ (x-4)^2+b & ;x\ge 0\end{cases}$

For continuity a = 4 and b = –15

$g(f(2))+f(g(-2))$

$=g(2)+f(-1)=-8$

9. Let $f(x) = \begin{cases} x^3 - x^2 + 10x - 7, & x \le 1 \\ -2x + \log_2(b^2 - 4), & x > 1 \end{cases}$

Then the set of all values of b, for which f(x) has maximum value at x = 1, is :

(A) (–6, –2)

(B) (2, 6)

(C) $[-6, -2) \cup (2, 6]$

(D) $[-\sqrt{6}, -2) \cup (2, \sqrt{6}]$

Official Ans. by NTA (C)

Sol. $f(1) = 3$

For $x < 1$, $f'(x) = 3x^2 - 2x + 10 > 0$

$\Rightarrow f(x)$ is increasing

For $x > 1$, $f'(x) < 0$

$\Rightarrow$ function is decreasing.

$\lim\limits_{x \to 1^+} f(x) = -2 + \log_2(b^2 - 4)$

For maximum value at x = 1

$3 \ge -2 + \log_2(b^2 - 4)$

$32 \ge b^2 - 4 > 0$

$b \in [-6, -2) \cup (2, 6]$

10. If $a = \lim\limits_{n \to \infty} \sum\limits_{k=1}^{n} \frac{2n}{n^2 + k^2}$ and $f(x) = \sqrt{\frac{1 - \cos x}{1 + \cos x}}, x \in (0,1)$, then :

(A) $2\sqrt{2} f\left(\frac{a}{2}\right) = f'\left(\frac{a}{2}\right)$

(B) $f\left(\frac{a}{2}\right) f'\left(\frac{a}{2}\right) = \sqrt{2}$

(C) $\sqrt{2} f\left(\frac{a}{2}\right) = f'\left(\frac{a}{2}\right)$

(D) $f\left(\frac{a}{2}\right) = \sqrt{2} f'\left(\frac{a}{2}\right)$

Official Ans. by NTA (C)

Sol. $a = \frac{1}{n} \sum\limits_{k=1}^{n} \frac{2}{1 + \left(\frac{k}{n}\right)^2} = \int_0^1 \frac{2}{1 + x^2} dx = \frac{\pi}{2}$

$f(x) = \tan\left(\frac{x}{2}\right); x \in (0, 1)$

$f\left(\frac{\pi}{4}\right) = \sqrt{2} - 1$

$f'\left(\frac{\pi}{4}\right) = \frac{1}{2} \sec^2\left(\frac{\pi}{8}\right) = \frac{\sqrt{2}}{\sqrt{2} + 1}$

$f'\left(\frac{\pi}{4}\right) = \sqrt{2} f\left(\frac{\pi}{4}\right)$

11. If $\frac{dy}{dx} + 2y \tan x = \sin x$, $0 < x < \frac{\pi}{2}$ and $y\left(\frac{\pi}{3}\right) = 0$, then the maximum value of y(x) is

(A) $\frac{1}{8}$ (B) $\frac{3}{4}$

(C) $\frac{1}{4}$ (D) $\frac{3}{8}$

Official Ans. by NTA (A)

Sol. $\frac{dy}{dx} + 2y \tan x = \sin x$

I.F $= e^{\int 2 \tan x \, dx} = e^{\ln(\sec x)^2} = \sec^2 x$

$y(\sec^2 x) = \int \sin x \sec^2 x \, dx + C$

$y \cdot \sec^2 x = \sec x + C$

Put $x = \frac{\pi}{3}, y = 0$

$y = \cos x - 2\cos^2 x$

$= \frac{1}{8} - 2\left(\cos x - \frac{1}{4}\right)^2$

$\therefore y_{max} = \frac{1}{8}$

12. A point P moves so that the sum of squares of its distances from the points (1, 2) and (–2, 1) is 14. Let f(x, y) = 0 be the locus of P, which intersects the x-axis at the points A, B and the y-axis at the point C, D . Then the area of the quadrilateral ACBD is equal to

(A) $\frac{9}{2}$ (B) $\frac{3\sqrt{17}}{2}$

(C) $\frac{3\sqrt{17}}{4}$ (D)9

Official Ans. by NTA (B)

Sol. $(x-1)^2+(y-2)^2+(x+2)^2+(y-1)^2=14$

$\Rightarrow x^2+y^2+x-3y-2=0$

Put x = 0

$\Rightarrow y^2-3y-2=0$

$\Rightarrow y=\frac{3\pm\sqrt{17}}{2}$

Put y = 0

$\Rightarrow x^2+x-2=0$

$(x+2)(x-1)=0$

$\therefore A(-2,0), B(1,0), C\left(0,\frac{3+\sqrt{17}}{2}\right), D\left(0,\frac{3-\sqrt{17}}{2}\right)$

Area = $\frac{1}{2}.3.\sqrt{17}=\frac{3\sqrt{17}}{2}$

13. Let the tangent drawn to the parabola $y^2=24x$ at the point (α,β) is perpendicular to the line 2x +2y = 5. Then the normal to the hyperbola $\frac{x^2}{\alpha^2}-\frac{y^2}{\beta^2}=1$ at the point $(\alpha+4,\beta+4)$ does NOT pass through the point :

(A) (25, 10) (B) (20, 12)

(C) (30, 8) (D) (15, 13)

Official Ans. by NTA (D)

Sol. Tangent at (α,β) has slope 1

$\beta^2=24\alpha$

Equation of tangent $y\beta=12(x+\alpha), \frac{12}{\beta}=1$

$\Rightarrow \alpha=6, \beta=12$

$\therefore(\alpha+4, \beta+4)=(10,16)$

Normal at (10, 16) to $\frac{x^2}{36}-\frac{y^2}{144}=1$ is

2x + 5y = 100

14. The length of the perpendicular from the point (1, –2, 5) on the line passing through (1, 2, 4) and parallel to the line x + y – z = 0 = x – 2y + 3z – 5 is :

(A) $\sqrt{\frac{21}{2}}$ (B) $\sqrt{\frac{9}{2}}$

(C) $\sqrt{\frac{73}{2}}$ (D)1

Official Ans. by NTA (A)

Sol. d.r's of the line = $\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\1&1&-1\\1&-2&3\end{vmatrix}=\hat{i}-4\hat{j}-3\hat{k}$

$\therefore$ equation of line is

$\vec{r}=\hat{i}+2\hat{j}+4\hat{k}+\lambda(\hat{i}-4\hat{j}-3\hat{k})$

Let A(1, 2, 4) and P be $(1+\lambda, 2-4\lambda, 4-3\lambda)$

$\therefore \overrightarrow{PA}.(\hat{i}-4\hat{j}-3\hat{k})=0$

$\lambda=\frac{1}{2}$

$\Rightarrow P\left(\frac{1}{2},2,\frac{-5}{2}\right)$

$|AP|=\sqrt{\frac{21}{2}}$

15. Let $\vec{a}=\alpha\hat{i}+\hat{j}-\hat{k}$ and $\vec{b}=2\hat{i}+\hat{j}-\alpha\hat{k}, \alpha>0$. If the projection of $\vec{a}\times\vec{b}$ on the vector $-\hat{i}+2\hat{j}-2\hat{k}$ is 30, then α is equal to

(A) $\frac{15}{2}$ (B) 8

(C) $\frac{13}{2}$ (D)7

Official Ans. by NTA (D)

Sol. $\vec{a}\times\vec{b}=(1-\alpha)\hat{i}+(\alpha^2-2)\hat{j}+(\alpha-2)\hat{k}$

Projection of $\vec{a}\times\vec{b}$ on $-\hat{i}+2\hat{j}-2\hat{k}$

$$=\frac{(\vec{a}\times\vec{b}).(-\hat{i}+2\hat{j}-2\hat{k})}{3}=30$$

$$\Rightarrow 2\alpha^2-\alpha-91=0$$

$$\Rightarrow \alpha=7,-\frac{13}{2}$$

16. The mean and variance of a binomial distribution are α and $\frac{\alpha}{3}$ respectively. If $P(X=1)=\frac{4}{243}$, then P(X = 4 or 5) is equal to :

(A) $\frac{5}{9}$ (B) $\frac{64}{81}$

(C) $\frac{16}{27}$ (D) $\frac{145}{243}$

Official Ans. by NTA (C)

Sol. $np = \alpha$ .....(1)

$npq = \alpha/3$ .....(2)

From (1) & (2)

$q = 1/3$ & $p = 2/3$

$${}^nC_1\, q^{n-1}p^1=\frac{4}{243}$$

$$\frac{n}{3^n}=\frac{2}{243}$$

$n = 6$

$$P(4 \text{ or } 5) = {}^6C_4\left(\frac{2}{3}\right)^4\left(\frac{1}{3}\right)^2+{}^6C_5\left(\frac{2}{3}\right)^5\cdot\left(\frac{1}{3}\right)^0$$

$$=\frac{16}{27}$$

17. Let E_1, E_2, E_3 be three mutually exclusive events such that $P(E_1)=\frac{2+3p}{6}$, $P(E_2)=\frac{2-p}{8}$ and $P(E_3)=\frac{1-p}{2}$. If the maximum and minimum values of p are p_1 and p_2, then $(p_1 + p_2)$ is equal to :

(A) $\frac{2}{3}$ (B) $\frac{5}{3}$

(C) $\frac{5}{4}$ (D) 1

Official Ans. by NTA (D)

Sol. $0 \le P(E_i) \le 1$ for i = 1, 2, 3

$\Rightarrow -2/3 \le p \le 1$

E_1 & E_2 & E_3 are mutually exclusive

$P(E_1) + P(E_2) + P(E_3) \le 1$

$\Rightarrow 2/3 \le p \le 1$

$p_1 = 1$, $p_2 = 2/3$

$p_1 + p_2 = 5/3$

18. Let

$S=\{\theta\in[0,2\pi]: 8^{2\sin^2\theta}+8^{2\cos^2\theta}=16\}$. Then

$n(S) + \sum_{\theta\in S}\left(\sec\left(\frac{\pi}{4}+2\theta\right)\text{cosec}\left(\frac{\pi}{4}+2\theta\right)\right)$ is equal to :

(A) 0 (B) –2

(C) –4 (D) 12

Official Ans. by NTA (C)

Sol. $8^{2\sin^2\theta}+8^{2-2\sin^2\theta}=16$

$$y+\frac{64}{y}=16$$

$\Rightarrow y = 8$

$\Rightarrow \sin^2\theta = ½$

$$n(S)+\sum_{\theta\in S}\frac{1}{\cos(\pi/4+2\theta)\sin(\pi/4+2\theta)}$$

$= 4 + (–2) \times 4 = –4$

19. $\tan\left(2\tan^{-1}\frac{1}{5}+\sec^{-1}\frac{\sqrt{5}}{2}+2\tan^{-1}\frac{1}{8}\right)$ is equal to:

(A) 1 (B) 2

(C) $\frac{1}{4}$ (D) $\frac{5}{4}$

Official Ans. by NTA (B)

Sol. $\tan\left(2(\tan^{-1}\frac{1}{5}+\tan^{-1}\frac{1}{8})+\tan^{-1}\left(\frac{1}{2}\right)\right)$

$= \tan\left[2\tan^{-1}\left(\frac{1}{3}\right)+\tan^{-1}\left(\frac{1}{2}\right)\right]$

$= 2$

20. The statement $(\sim(p \Leftrightarrow \sim q)) \wedge q$ is :

(A) a tautology
(B) a contradiction

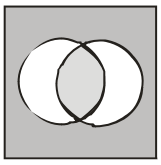
(C) equivalent to $(p \Rightarrow q) \wedge q$

(D) equivalent to $(p \Rightarrow q) \wedge p$

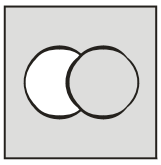
Official Ans. by NTA (D)

Sol. $(\sim(p \Leftrightarrow \sim q)) \wedge q \equiv (p \Leftrightarrow q) \wedge q$

$(p \Leftrightarrow q) \wedge q \equiv p \wedge q$



$p \Leftrightarrow q$



$p \rightarrow q$

SECTION-B

1. If for some p, q, r $\in$ R, not all have same sign, one of the roots of the equation $(p^2 + q^2)x^2 - 2q(p + r)x + q^2 + r^2 = 0$ is also a root of the equation $x^2 + 2x - 8 = 0$, then $\frac{q^2+r^2}{p^2}$ is equal to-

Official Ans. by NTA (272)

Sol. $(px - q)^2 + (qx - r)^2 = 0$

$\Rightarrow x = \frac{q}{p} = \frac{r}{q} = -4$

$\Rightarrow \frac{q^2+r^2}{p^2} = 272$

2. The number of 5-digit natural numbers, such that the product of their digits is 36, is

Official Ans. by NTA (180)

Sol. $3\times\frac{5!}{2!2!}+\frac{5!}{3!\times 2!}+\frac{5!}{2!}+\frac{5!}{3!}=180$

3. The series of positive multiples of 3 is divided into sets : {3}, {6, 9,12}, {15, 18, 21, 24, 27},... Then the sum of the elements in the 11^{th} set is equal to_____,

Official Ans. by NTA (6993)

Sol. $S_{11} = 3[101+102+.....+121]$

$=\frac{3}{2}(222)\times 21 = 6993$

4. The number of distinct real roots of the equation $x^5(x^3 - x^2 - x + 1) + x(3x^3 - 4x^2 - 2x + 4) - 1 = 0$ is

Official Ans. by NTA (3)

Sol. $x^5(x^3 - x^2 - x + 1) + x(3x^3 - 4x^2 - 2x + 4) - 1 = 0$

$\Rightarrow (x-1)^2 (x + 1)(x^5 + 3x - 1) = 0$

Let $f(x) = x^5 + 3x - 1$

$f'(x) > 0 \ \forall \ x \in R$

Hence 3 real distinct roots.

5. If the coefficients of x and x^2 in the expansion of $(1 + x)^p(1 - x)^q$, p, q $\leq$ 15, are –3 and–5 respectively, then the coefficient of x^3 is equal to_____.

Official Ans. by NTA (23)

Sol. Since coefficient of x is –3

$\Rightarrow {}^pC_1 - {}^qC_1 = -3$

$\Rightarrow p - q = -3$ ....(1)

Comparing coefficients of x^2

$-{}^pC_1\,{}^qC_1 + {}^pC_2 + {}^qC_2 = -5$

$-pq + \frac{p(p-1)}{2} + \frac{q(q-1)}{2} = -5$ ....(2)

Solving (1) and (2)

p = 8, q = 11

Coefficient of x^3 is

$-{}^{q}C_3 + {}^{p}C_3 + {}^{p}C_1\,{}^{q}C_2 - {}^{p}C_2\,{}^{q}C_1$

$= -{}^{11}C_3 + {}^{8}C_3 + {}^{8}C_1\,{}^{11}C_2 - {}^{8}C_2\,{}^{11}C_1$

$= 23$

6. If

$n(2n+1)\int_0^1 (1-x^n)^{2n}\,dx = 1177\int_0^1 (1-x^n)^{2n+1}dx$, then

$n \in N$ is equal to _____

Official Ans. by NTA (24)

Sol. Let $I_1 = \int_0^1 (1-x^n)^{2n}\,dx$, $I_2 = \int_0^1 (1-x^n)^{2n+1}\,dx$

$I_2 = \int_0^1 (1-x^n)^{2n+1}.1\,dx$

$= (1-x^n)^{2n+1}.x\Big|_0^1 - \int_0^1 (2n+1)(1-x^n)^{2n}(-nx^{n-1})\,x\,dx$

$I_2 = -n(2n+1)\{I_2 - I_1\}$

$(2n^2+n+1)I_2 = n(2n+1)I_1$

$\frac{I_1}{I_2} = \frac{2n^2+n+1}{n(2n+1)} = \frac{1177}{n(2n+1)}$

$\Rightarrow 2n^2 + n - 1176 = 0 \Rightarrow n = 24$

7. Let a curve $y = y(x)$ pass through the point (3, 3) and the area of the region under this curve, above the x-axis and between the abscissae 3 and x(>3) be $\left(\frac{y}{x}\right)^3$. If this curve also passes through the point $\left(\alpha, 6\sqrt{10}\right)$ in the first quadrant, then α is equal to _____

Official Ans. by NTA (6)

Sol. $x^4 = 3yx.y' - 3y^2$

$\Rightarrow 3xy\frac{dy}{dx} = 3y^2 + x^4$

Put $y^2 = t, y\frac{dy}{dx} = \frac{1}{2}\frac{dt}{dx}$

$\frac{dt}{dx} - \frac{2}{x}t = \frac{2}{3}x^3$

$\therefore \frac{t}{x^2} = \frac{x^2}{3} + C$

$\Rightarrow \frac{y^2}{x^2} = \frac{x^2}{3} - 2$

Put (3, 3) , C = –2

$\therefore \frac{y^2}{x^2} = \frac{x^2}{3} - 2$

$3y^2 = x^4 - 6x^2$

$x^4 - 6x^2 = 1080$

$\therefore x = 6$

8. The equations of the sides AB, BC and CA of a triangle ABC are 2x + y = 0, x + py = 15a and x – y = 3 respectively. If its orthocentre is (2, a), $-\frac{1}{2} < a < 2$, then p is equal to

Official Ans. by NTA (3)

Sol. Coordinates of A(1, –2), $B\left(\frac{15a}{1-2p}, \frac{-30a}{1-2p}\right)$ and orthocentre H(2, a)

Slope of AH = p

a + 2 = p .....(1)

Slope of BH = –1

31a – 2ab = 15a + 4p–2 .....(2)

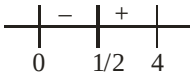
From (1) and (2)

a = 1 & p = 3

9. Let the function $f(x) = 2x^2 - \log_e x$, $x > 0$, be decreasing in (0, a) and increasing in (a, 4). A tangent to the parabola $y^2 = 4ax$ at a point P on it passes through the point (8a, 8a – 1) but does not pass through the point $\left(-\frac{1}{a}, 0\right)$. If the equation of the normal at P is $\frac{x}{\alpha}+\frac{y}{\beta}=1$, then $\alpha + \beta$ is equal to-

Official Ans. by NTA (45)

Sol. $f'(x) = 4x - \frac{1}{x}$



$a = \frac{1}{2}$

Let $P(x_1, y_1)$ be any point on $y^2 = 4ax$

$\frac{1}{y_1}=\frac{3-y_1}{4-x_1} \Rightarrow y_1^2-6y_1+8=0$

$y_1 = 2, 4$

$\Rightarrow$ P(8, 4) as P(2, 2) rejected

Equation of normal at P.

$y - 4 = -4(x - 8)$

$\frac{x}{9}+\frac{y}{36}=1$

$\alpha = 9, \beta = 36$

$\alpha + \beta = 45$

10. Let Q and R be two points on the line $\frac{x+1}{2}=\frac{y+2}{3}=\frac{z-1}{2}$ at a distance $\sqrt{26}$ from the point P(4, 2, 7). Then the square of the area of the triangle PQR is_____.

Official Ans. by NTA (153)

Sol. Let $(2\lambda - 1, 3\lambda - 2, 2\lambda + 1)$ be any point on the line

$(2\lambda - 5)^2 + (3\lambda - 4)^2 + (2\lambda - 6)^2 = 26$

$\lambda = 1, 3$

Q (1, 1, 3) ; R (5, 7, 7) ; P (4, 2, 7)

Area of triangle PQR = ½ $\left|\vec{PQ}\times\vec{PR}\right|$

$=\sqrt{153}$

FINAL JEE–MAIN EXAMINATION – JULY, 2022

(Held On Monday 25th July, 2022) **TIME : 3 : 00 PM to 6 : 00 PM**

PHYSICS

TEST PAPER WITH SOLUTION

SECTION-A

1. In AM modulation, a signal is modulated on a carrier wave such that maximum and minimum amplitude are found to be 6V and 2V respectively. The modulation index is

(A) 100% (B) 80%

(C) 60% (D) 50%

Official Ans. by NTA (D)

Sol. $\text{modulation index} = \frac{V_{max} - V_{min.}}{V_{max} + V_{min.}} \times 100\%$

$= \frac{6-2}{6+2} \times 100\% = 50\%$

2. The electric current in a circular coil of 2 turns produces a magnetic induction B_1 at its centre. The coil is unwound and is rewound into a circular coil of 5 turns and the same current produces a magnetic induction B_2 at its centre.

The ratio of $\frac{B_2}{B_1}$ is :

(A) $\frac{5}{2}$ (B) $\frac{25}{4}$

(C) $\frac{5}{4}$ (D) $\frac{25}{2}$

Official Ans. by NTA (B)

Sol. $B = \frac{N\mu_0 i}{2R}$

$B_1 = \frac{N_1\mu_0 i}{2R_1}$

For $N_2 = 5$

Radius of coil $= R_2 = \frac{N_1 \times R_1}{N_2}$

$B_2 = \frac{N_2\mu_0 i}{R_2}$

$\frac{B_2}{B_1} = \frac{N_2}{N_1}\frac{R_1}{R_2} = \frac{N_2}{N_1} \times \frac{N_2}{N_1}$; $\frac{B_2}{B_1} = \frac{25}{4}$

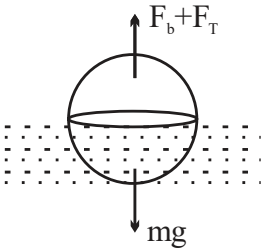
3. A drop of liquid of density ρ is floating half immersed in a liquid of density σ and surface tension 7.5×10^{-4} Ncm^{-1}. The radius of drop in cm will be : (Take : g = 10 m/s^2)

(A) $\frac{15}{\sqrt{2\rho-\sigma}}$ (B) $\frac{15}{\sqrt{\rho-\sigma}}$

(C) $\frac{3}{2\sqrt{\rho-\sigma}}$ (D) $\frac{3}{20\sqrt{2\rho-\sigma}}$

Official Ans. by NTA (A)

Sol.



Boyant force + surace tension = mg

$\sigma\frac{V}{2}g + 2\pi RT = \rho Vg$

$2\pi RT = \frac{(2\rho-\sigma)}{2}\cdot\frac{4}{3}\pi R^3 g; \left[V = \frac{4}{3}\pi R^3\right]$

$R^3 = \frac{3T}{(2\rho-\sigma)g} \Rightarrow R = \sqrt{\frac{3\times 7.5\times 10^{-2}\,N-m^{-1}}{(2\rho-\sigma)\times 10}}$

$R = \frac{3}{20\sqrt{(2\rho-\sigma)}}\,m = \frac{15}{\sqrt{2\rho-\sigma}}\,cm$

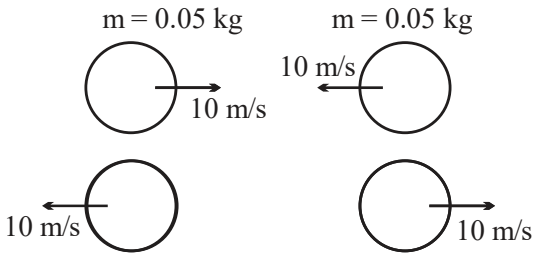
4. Two billiard balls of mass 0.05 kg each moving in opposite directions with 10 ms^{-1} collide and rebound with the same speed. If the time duration of contact is t = 0.005 s, then what is the force exerted on the ball due to each other?

(A) 100 N (B) 200 N

(C) 300 N (D) 400 N

Official Ans. by NTA (B)

Sol.



Change in momentum of any one ball

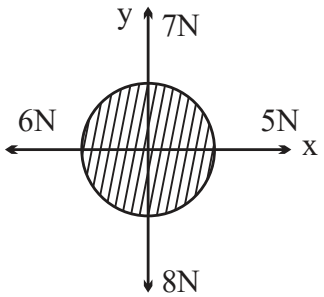
$$|\Delta\vec{P}| = 2\times 0.05\times 10$$

$$|\Delta\vec{P}| = 1$$

$$|\vec{F}_{av}| = \frac{|\Delta\vec{P}|}{\Delta t}$$

$F_{av.}$ = 200 N

5. For a free body diagram shown in the figure, the four forces are applied in the 'x' and 'y' directions. What additional force must be applied and at what angle with positive x-axis so that the net acceleration of body is zero?



(A) $\sqrt{2}$ N, 45° (B) $\sqrt{2}$ N, 135°

(C) $\frac{2}{\sqrt{3}}$ N, 30° (D) 2 N, 45°

Official Ans. by NTA (A)

Sol. Let addition force required is = $\vec{F}$

$$\vec{F} + 5\hat{i} - 6\hat{i} + 7\hat{j} - 8\hat{j} = 0$$

$$\vec{F} = \hat{i} + \hat{j}, |\vec{F}| = \sqrt{2}$$

Angle with x-axis: $\tan\theta = \frac{\text{y component}}{\text{x component}} = \frac{1}{1}$

$\theta = 45°$

6. Capacitance of an isolated conducting sphere of radius R_1 becomes n times when it is enclosed by a concentric conducting sphere of radius R_2 connected to earth. The ratio of their radii $\left(\frac{R_2}{R_1}\right)$ is:

(A) $\frac{n}{n-1}$ (B) $\frac{2n}{2n+1}$

(C) $\frac{n+1}{n}$ (D) $\frac{2n+1}{n}$

Official Ans. by NTA (A)

Sol. Capacitance of isolated Conducting sphere = $4\pi\varepsilon_0 R_1$

By enclosing inside another sphere of radius R_2, new capacitance = $\frac{4\pi\varepsilon_0 R_1 R_2}{(R_2 - R_1)}$

Given: $\frac{4\pi\varepsilon_0 R_1 R_2}{(R_2 - R_1)} = n\times 4\pi\varepsilon_0 R_1$

$$\Rightarrow \frac{R_2}{(R_2 - R_1)} = n \Rightarrow \frac{\frac{R_2}{R_1}}{\left(\frac{R_2}{R_1} - 1\right)} = n$$

$$\Rightarrow \frac{R_2}{R_1} = n\frac{R_2}{R_1} - n \Rightarrow \frac{R_2}{R_1} = \frac{n}{(n-1)}$$

7. The ratio of wavelengths of proton and deuteron accelerated by potential V_p and V_d is $1:\sqrt{2}$. Then, the ratio of V_p to V_d will be

(A) 1 : 1 (B) $\sqrt{2}:1$

(C) 2 : 1 (D) 4 : 1

Official Ans. by NTA (D)

Sol. Kinetic energy gained by a charged particle accelerated by a potential V is qV

KE = qV

$$\Rightarrow \frac{p^2}{2m} = qV \Rightarrow p = \sqrt{2mqV}$$

$$p = \frac{h}{\lambda}, \text{ thus } \lambda = \frac{h}{\sqrt{2mqV}}$$

$$\text{now } \frac{\lambda_p}{\lambda_d} = \sqrt{\frac{m_d V_d}{m_p V_p}}$$

$$\Rightarrow \frac{1}{\sqrt{2}} = \sqrt{\frac{2V_d}{V_p}} \Rightarrow \frac{V_p}{V_d} = 4$$

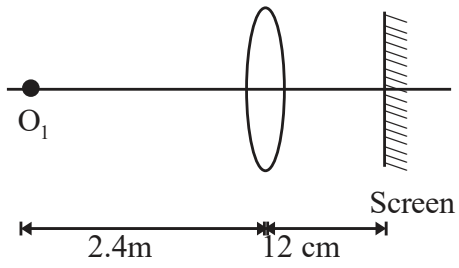
8. For an object placed at a distance 2.4 m from a lens, a sharp focused image is observed on a screen placed at a distance 12 cm from the lens. A glass plate of refractive index 1.5 and thickness 1 cm is introduced between lens and screen such that the glass plate plane faces parallel to the screen. By what distance should the object be shifted so that a sharp focused image is observed again on the screen?

(A) 0.8 m (B) 3.2 m

(C) 1.2 m (D) 5.6 m

Official Ans. by NTA (B)

Sol.



Applying lens formula

$$\frac{1}{0.12} + \frac{1}{2.4} = \frac{1}{f} \Rightarrow \frac{1}{f} = \frac{210}{24}$$

Upon putting the glass slab, shift of image is

$$\Delta x = t\left(1 - \frac{1}{\mu}\right) = \frac{1}{3}\text{cm}$$

$$\text{Now } v = 12 - \frac{1}{3} = \frac{35}{3}\text{cm}$$

Again apply lens formula

$$\frac{1}{0.12} + \frac{1}{u} = \frac{1}{f} = \frac{210}{24}$$

Solving u = – 5.6 m

Thus shift of object is

5.6 – 2.4 = 3.2 m

9. Light wave traveling in air along x-direction is given by $E_y = 540 \sin \pi \times 10^4(x - ct)\ Vm^{-1}$. Then, the peak value of magnetic field of wave will be (Given $c = 3 \times 10^8\ ms^{-1}$)

(A) 18×10^{-7} T (B) 54×10^{-7} T

(C) 54×10^{-8} T (D) 18×10^{-8} T

Official Ans. by NTA (A)

Sol. $E_y = 540 \sin \pi \times 10^4(x - ct)\ Vm^{-1}$

$E_0 = 540\ Vm^{-1}$

$$B_0 = \frac{E_0}{C} = \frac{540}{3\times10^8} = 18\times10^{-7}\text{T}$$

10. When you walk through a metal detector carrying a metal object in your pocket, it raises an alarm. This phenomenon works on

(A) Electromagnetic induction

(B) Resonance in ac circuits

(C) Mutual induction in ac circuits

(D) interference of electromagnetic waves

Official Ans. by NTA (B)

)

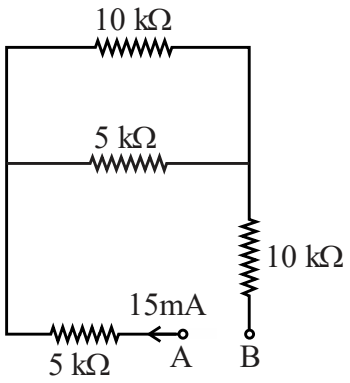
Sol. Metal detector works on the principle of transmitting an electromagnetic signal and analyses a return signal from the target. So it works on the principle of resonance in AC circuit.

11. An electron with energy 0.1 keV moves at right angle to the earth's magnetic field of 1×10^{-4} Wbm^{-2}. The frequency of revolution of the electron will be
(Take mass of electron = 9.0×10^{-31} kg)
(A) 1.6×10^5 Hz (B) 5.6×10^5 Hz
(C) 2.8×10^6 Hz (D) 1.8×10^6 Hz
Official Ans. by NTA (C)

Sol. $f = \frac{1}{T} = \frac{eB}{2\pi m}$

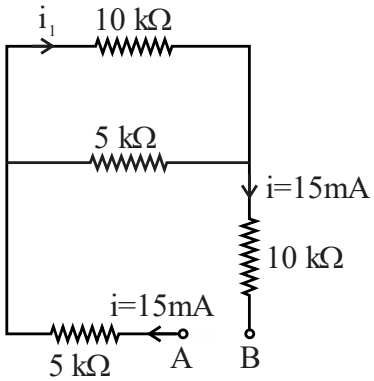
$= \frac{1.6\times10^{-19}\times10^{-4}}{2\pi\times9\times10^{-31}} = 2.8\times10^{6}\,Hz$

12. A current of 15 mA flows in the circuit as shown in figure. The value of potential difference between the points A and B will be



(A) 50V (B) 75V
(C) 150V (D) 275V
Official Ans. by NTA (D)

Sol.



$i_1 = \frac{5}{10+5}\times15mA = 5mA$

$V_A - 5i - 10i_1 - 10i = V_B$

$V_A - V_B = 75 + 50 + 150 = 275$ V

13. The length of a seconds pendulum at a height h = 2R from earth surface will be:
(Given: R=Radius of earth and acceleration due to gravity at the surface of earth $g = \pi^2 m/s^{-2}$)

(A) $\frac{2}{9}$m (B) $\frac{4}{9}$m

(C) $\frac{8}{9}$m (D) $\frac{1}{9}$m

Official Ans. by NTA (D)

Sol. $T = 2\pi\sqrt{\frac{L}{g}}$, $g' = \frac{GM}{9R^2} = \frac{g}{9} = \frac{\pi^2}{9}$

$2 = 2\pi\sqrt{\frac{L}{\pi^2}\times9}$

$\Rightarrow 1 = \pi\sqrt{L}\times\frac{3}{\pi} \Rightarrow L = \frac{1}{9}$m

14. Sound travels in a mixture of two moles of helium and n moles of hydrogen. If rms speed of gas molecules in the mixture is $\sqrt{2}$ times the speed of sound, then the value of n will be
(A) 1 (B) 2
(C) 3 (D) 4
Official Ans. by NTA (B)

Sol. $v_s = \sqrt{\frac{\gamma RT}{M}}$

$v_{rms} = \sqrt{\frac{3RT}{M}}$

$\frac{v_s}{v_{rms}} = \sqrt{\frac{\gamma}{3}} = \frac{1}{\sqrt{2}} \Rightarrow \frac{\gamma}{3} = \frac{1}{2} \Rightarrow \gamma = \frac{3}{2}$

$\gamma = 1 + \frac{2}{f_{mix.}}$

$f_{mix.} = \frac{2\times3+n\times5}{n+2} = \frac{6+n\times5}{(n+2)}$

$\gamma = 1 + \frac{2(n+2)}{6+n\times5} = \frac{6+5n+2n+4}{6+5n}$

$\gamma = \frac{7n+10}{6+5n} = \frac{3}{2}$

$14n + 20 = 18 + 15n$

$n = 2$

15. Let η_1 is the efficiency of an engine at $T_1 = 447°C$ and $T_2 = 147°C$ while η_2 is the efficiency at $T_1 = 947°C$ and $T_2 = 47°C$. The ratio $\frac{\eta_1}{\eta_2}$ will be :

(A) 0.41 (B) 0.56
(C) 0.73 (D) 0.70

Official Ans. by NTA (B)

Sol. Efficiency $\eta = 1-\frac{T_L}{T_H}$

$$\eta_1 = 1-\frac{147+273}{447+273} = 1-\frac{420}{720}$$

$$\eta_1 = \frac{300}{720}$$

$$\eta_2 = 1-\frac{47+273}{947+273} = 1-\frac{320}{1220}$$

$$\eta_2 = \frac{900}{1220}$$

$$\frac{\eta_1}{\eta_2} = \frac{300}{720}\times\frac{1220}{900} = \frac{122}{72\times 3}$$

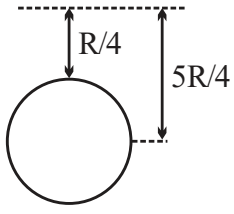
$$\frac{\eta_1}{\eta_2} = 0.56$$

16. An object is taken to a height above the surface of earth at a distance $\frac{5}{4}R$ from the centre of the earth. Where radius of earth, R = 6400 km. The percentage decrease in the weight of the object will be

(A) 36% (B) 50%
(C) 64% (D) 25%

Official Ans. by NTA (A)

Sol.



$$g_{eff} = \frac{g}{\left(1+\frac{h}{R}\right)^2};\ g_{eff} = \frac{g}{\left(1+\frac{1}{4}\right)^2} = \frac{16g}{25}$$

$$\text{change} = \frac{g_{eff}-g}{g}\times 100 = \frac{\frac{16}{25}-1}{1}\times 100$$

$$= \frac{-9}{25}\times 100 = -36\%$$

Hence % decrease in the weight = 36%

17. A bag of sand of mass 9.8 kg is suspended by a rope. A bullet of 200 g travelling with speed 10 ms^{-1} gets embedded in it, then loss of kinetic energy will be

(A) 4.9 J (B) 9.8 J
(C) 14.7 (D) 19.6 J

Official Ans. by NTA (B)

Sol. $P_i = P_f$ (no any external force)

$0.2 \times 10 = 10 \times v$

$v = 0.2$ m/sec

$$\text{Loss in K.E.} = \frac{1}{2}\times(0.2)\times 10^2 - \frac{1}{2}\times 10(0.2)^2$$

$$= \frac{1}{2}\times 10\times(0.2)[10-0.2]$$

$= 9.8$ J

18. A ball is projected from the ground with a speed 15 ms^{-1} at an angle θ with horizontal so that its range and maximum height are equal, then'tan θ' will be equal to

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$
(C) 2 (D) 4

Official Ans. by NTA (D)

Sol. R = H

$$\frac{2v_x\times v_y}{g} = \frac{v_y^2}{2g}$$

$$v_x = \frac{v_y}{4};\ u\cos\theta = \frac{u\sin\theta}{4}$$

$\tan\theta = 4$

19. The maximum error in the measurement of resistance, current and time for which current flows in an electrical circuit are 1%, 2% and 3% respectively. The maximum percentage error in the detection of the dissipated heat will be:

(A) 2 (B) 4

(C) 6 (D) 8

Official Ans. by NTA (D)

Sol. $E_H = I^2R \times t$

$$\frac{\Delta E}{E}\times 100 = \frac{2\Delta I}{I}\times 100 + \frac{\Delta R}{R}\times 100 + \frac{\Delta T}{T}\times 100$$

$$= 2 \times 2 + 1 + 3 = 8$$

20. Hydrogen atom from excited state comes to the ground by emitting a photon of wavelength λ. The value of principal quantum number 'n' of the excited state will be :

(R : Rydberg constant)

(A) $\sqrt{\frac{\lambda R}{\lambda - 1}}$ (B) $\sqrt{\frac{\lambda R}{\lambda R - 1}}$

(C) $\sqrt{\frac{\lambda}{\lambda R - 1}}$ (D) $\sqrt{\frac{\lambda R^2}{\lambda R - 1}}$

Official Ans. by NTA (B)

Sol.

$$E_n = \frac{-Rch}{n^2}(1)$$

n=n

$E_{photon} = E_n - E_1$

n=1

$$E_1 = \frac{-Rch}{(1)^2}(1)$$

$$\frac{-Rch}{(n)^2} + \frac{Rch}{1} = \frac{hc}{\lambda}$$

$$\frac{-R}{n^2} + R = \frac{1}{\lambda}$$

$$R - \frac{1}{\lambda} = \frac{R}{n^2}$$

$$\frac{\lambda R - 1}{\lambda} = \frac{R}{n^2}$$

$$n^2 = \frac{\lambda R}{\lambda R - 1} \Rightarrow n = \sqrt{\frac{\lambda R}{\lambda R - 1}}$$

SECTION-B

1. A particle is moving in a straight line such that its velocity is increasing at 5 ms^{-1} per meter. The acceleration of the particle is _____ ms^{-2} at a point where its velocity is 20 ms^{-1}.

Official Ans. by NTA (100)

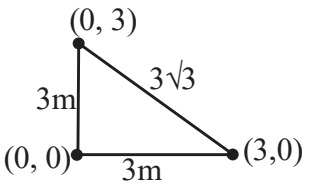
Sol. $\frac{dv}{ds} = 5$

$$a = v\frac{dv}{ds} = 20 \times 5 = 100 \text{ m/sec}^2$$

2. Three identical spheres each of mass M are placed at the corners of a right angled triangle with mutually perpendicular sides equal to 3 m each. Taking point of intersection of mutually perpendicular sides as origin, the magnitude of position vector of centre of mass of the system will be $\sqrt{x}$ m. The value of x is

Official Ans. by NTA (2)

Sol.



$$\vec{r}_{com} = \frac{M(0\hat{i} + 0\hat{j}) + M(3\hat{i}) + M(3\hat{j})}{3M}$$

$$\vec{r}_{com} = \hat{i} + \hat{j}$$

$$|\vec{r}_{com}| = \sqrt{2} = \sqrt{x}$$

$$x = 2$$

3. A block of ice of mass 120 g at temperature 0°C is put in 300 gm of water at 25°C. The xg of ice melts as the temperature of the water reaches 0°C. The value of x is

[Use: Specific heat capacity of water = 4200 $Jkg^{-1}K^{-1}$, Latent heat of ice = 3.5×10^5 Jkg^{-1}]

Official Ans. by NTA (90)

Sol. Energy released by water

$= 0.3 \times 25 \times 4200 = 31500$ J

let m kg ice melts

$m \times 3.5 \times 10^5 = 31500$

$$m = \frac{31500 \times 10^{-5}}{3.5} = 9000 \times 10^{-5}$$

$m = 0.09$ kg $= 90$ gm

$x = 90$

4. $\frac{x}{x+4}$ is the ratio of energies of photons produced due to transition of an electron of hydrogen atom from its

(i) third permitted energy level to the second level and

(ii) the highest permitted energy level to the second permitted level.

The value of x will be

Official Ans. by NTA (5)

Sol. $$\frac{13.6\left(\frac{1}{2^2}-\frac{1}{3^2}\right)}{13.6\left(\frac{1}{2^2}-0\right)} = \frac{x}{x+4}; \quad \frac{\frac{1}{4}-\frac{1}{9}}{\frac{1}{4}} = \frac{x}{x+4}$$

$$\frac{5}{9} = \frac{x}{x+4}$$

$5x + 20 = 9x$

$4x = 20$

$x = 5$

5. In a potentiometer arrangement, a cell of emf 1.20 V gives a balance point at 36 cm length of wire. This cell is now replaced by another cell of emf 1.80 V. The difference in balancing length of potentiometer wire in above conditions will be _____ cm.

Official Ans. by NTA (18)

Sol. 1.2 = (Potential Gradient) × 36

1.8 = (Potential Gradient) × x

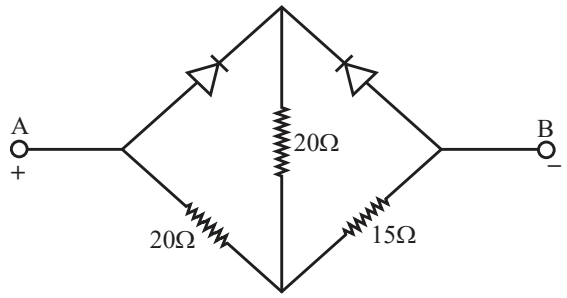
On dividing, we get

$$\frac{2}{3} = \frac{36}{x}$$

$x = 18 \times 3 = 54$ cm

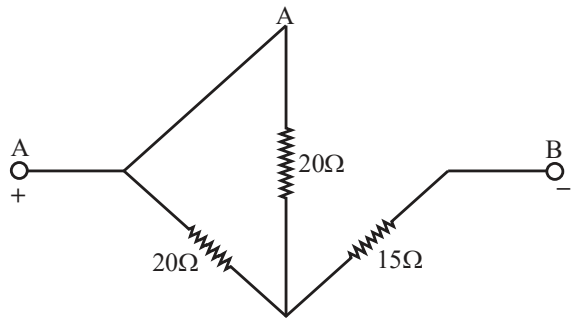
Hence difference = 54 – 36 = 18 cm

6. Two ideal diodes are connected in the network as shown in figure. The equivalent resistance between A and B is _____Ω.

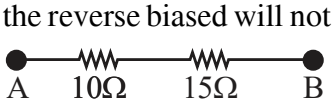


Official Ans. by NTA (25)

Sol.



The forward biased diode will conduct while the reverse biased will not



∴ Equivalent resistance = 10 + 15 = 25Ω

7. Two waves executing simple harmonic motion travelling in the same direction with same amplitude and frequency are superimposed. The resultant amplitude is equal to the $\sqrt{3}$ times of amplitude of individual motions. The phase difference between the two motions is _____ (degree)

Official Ans. by NTA (60)

Sol. $A_{resultant} = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\phi}$

$\Rightarrow \sqrt{3}A = \sqrt{A^2 + A^2 + 2A^2\cos\phi}$

$\Rightarrow 3A^2 = 2A^2 + 2A^2\cos\phi$

$\Rightarrow \cos\phi = \frac{1}{2}$

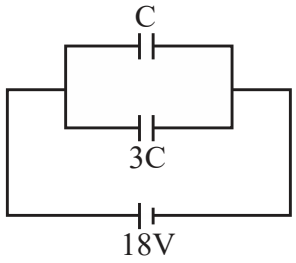
$\therefore \phi = 60°$

$\therefore$ Phase difference = 60 degree

8. Two parallel plate capacitors of capacity C and 3C are connected in parallel combination and charged to a potential difference 18V. The battery is then disconnected and the space between the plates of the capacitor of capacity C is completely filled with a material of dielectric constant 9. The final potential difference across the combination of capacitors will be _____ V

Official Ans. by NTA (6)

Sol.



Initial charge on C = 18 CV

initial charge on 3C = 54 CV

Let final common potential difference = V'

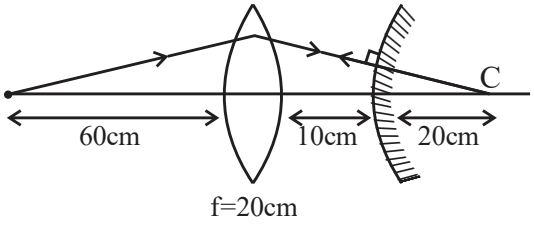
9CV' + 3CV' = 18CV + 54CV

$\Rightarrow$ 12CV' = 72 CV $\Rightarrow$ V' = 6 V

9. A convex lens of focal length 20 cm is placed in front of convex mirror with principal axis coinciding each other. The distance between the lens and mirror is 10 cm. A point object is placed on principal axis at a distance of 60 cm from the convex lens. The image formed by combination coincides the object itself. The focal length of the convex mirror is _____ cm.

Official Ans. by NTA (10)

Sol.



For lens

$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$\Rightarrow \frac{1}{v} - \frac{1}{(-60)} = \frac{1}{20} \Rightarrow \frac{1}{v} + \frac{1}{60} = \frac{1}{20}$

v = 30 cm

For final image to be formed on the object itself, after refraction from lens the ray should meet the mirror perpendicularly and the image by lens should be on the centre of curvature of mirror

R = 30 – 10 = 20 cm

Focal length of mirror = R/2 = 10 cm

10. Magnetic flux (in weber) in a closed circuit of resistance 20 Ω varies with time t(s) as $\phi = 8t^2 - 9t + 5$. The magnitude of the induced current at t = 0.25 s will be _____ mA

Official Ans. by NTA (250)

Sol. $\phi = 8t^2 - 9t + 5$

$\text{emf} = -\frac{d\phi}{dt} = -(16t - 9)$

At t = 0.25 s

Emf = –[(16 × 0.25)–9] = 5V

$\text{Current} = \frac{\text{Emf}}{\text{Resistance}} = \frac{5V}{20\Omega}$

$= \frac{1}{4}A = \frac{1000}{4}mA = 250mA$

FINAL JEE–MAIN EXAMINATION – JULY, 2022

(Held On Monday 25th July, 2022) **TIME : 3 : 00 PM to 06 : 00 PM**

CHEMISTRY

TEST PAPER WITH SOLUTION

SECTION-A

1. Match **List I** with **List II** :

List-I (molecule)	List-II (hybridization; shape)
A. XeO_3	I. sp^3d ; linear
B. XeF_2	II. sp^3; pyramidal
C. $XeOF_4$	III. sp^3d^3; distorted octahedral
D. XeF_6	IV. sp^3d^2 ;square pyramidal

Choose the correct answer from the options given below:

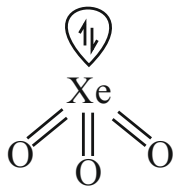
(A) A-II, B-I, C-IV, D-III

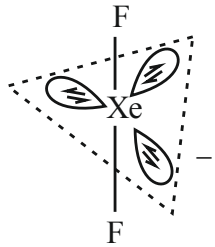
(B) A-II, B-IV, C-III, D-I

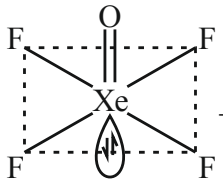
(C) A-IV, B-II, C-III, D-I

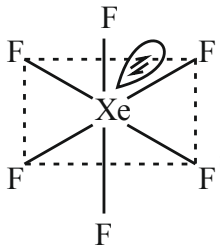
(D) A-IV, B-II, C-I, D-III

Official Ans. by NTA (A)

Sol. (A)  – sp^3 – pyramidal

(B)  – sp^3d – linear

(C)  –sp^3d^2–square pyramidal

(D)  sp^3d^3–distorted octahedral

2. Two solutions A and B are prepared by dissolving 1 g of non-volatile solutes X and Y. respectively in 1 kg of water. The ratio of depression in freezing points for A and B is found to be 1 : 4. The ratio of molar masses of X and Y is :

(A) 1 : 4

(B) 1 : 0.25

(C) 1 : 0.20

(D) 1 : 5

Official Ans. by NTA (B)

Sol. $$\frac{\Delta T_{fx}}{\Delta T_{fy}}=\frac{k_f\cdot m_x}{k_f\cdot m_y}=\frac{\frac{1}{M_x}}{\frac{1}{M_y}}$$

$$\Rightarrow \frac{1}{4}=\frac{M_y}{M_x}$$

$$\Rightarrow M_x : M_y = 1 : 0.25$$

3. Ka_1, Ka_2 and Ka_3 are the respective ionization constants for the following reactions (a),(b), and (c).

(a) $H_2C_2O_4 \rightleftharpoons H^+ + HC_2O_4^-$

(b) $HC_2O_4^- \rightleftharpoons H^+ + HC_2O_4^{2-}$

(c) $H_2C_2O_4 \rightleftharpoons 2H^+ + C_2O_4^{2-}$

The relationship between K_{a_1}, K_{a_2} and K_{a_3} is given as

(A) $K_{a_3}=K_{a_1}+K_{a_2}$ (B) $K_{a_3}=K_{a_1}-K_{a_2}$

(C) $K_{a_3}=K_{a_1}/K_{a_2}$ (D) $K_{a_3}=K_{a_1}\times K_{a_2}$

Official Ans. by NTA (D)

Sol. $H_2C_2O_4 \rightleftharpoons H^+ + HC_2O_4^-$ K_{a_1}

$H_2C_2O_4^- \rightleftharpoons H^+ + C_2O_4^{2-}$ K_{a_2}

$H_2C_2O_4 \rightleftharpoons 2H^+ + C_2O_4^{2-}$ $K_{a_3}=K_{a_1}\times K_{a_2}$

4. The molar conductivity of a conductivity cell filled with 10 moles of 20 mL NaCl solution is Λ_{m1} and that of 20 moles another identical cell heaving 80 mL NaCl solution is Λ_{m2}, The conductivities exhibited by these two cells are same.
The relationship between Λ_{m2} and Λ_{m1} is

(A) $\Lambda_{m2} = 2\Lambda_{m1}$ (B) $\Lambda_{m2} = \Lambda_{m1}/2$

(C) $\Lambda_{m2} = \Lambda_{m1}$ (D) $\Lambda_{m2} = 4\Lambda_{m1}$

Official Ans. by NTA (A)

Sol. $\Lambda_m = \kappa \times \frac{1000}{M}$

$\Rightarrow \Lambda_m \propto \frac{1}{M}$

$\frac{\Lambda_{m_1}}{\Lambda_{m_2}} = \frac{M_2}{M_1} = \frac{\frac{20}{80}}{\frac{10}{20}} = \frac{1}{4} \times \frac{2}{1} = \frac{1}{2}$

$\Rightarrow \Lambda_{m_2} = 2\Lambda_{m_1}$

5. For micelle formation, which of the following statements are correct?

(A) Micelle formation is an exothermic process.

(B) Micelle formation is an endothermic process.

(C) The entropy change is positive.

(D) The entropy change is negative.

(A) A and D only (B) A and C only

(C) B and C only (D) B and D only

Official Ans. by NTA (A)

Sol. For micelle formation, $\Delta S > 0$ (hydrophobic effect) This is possible because, the decrease in entropy due to clustering is offset by increase in entropy due to desolvation of the surfactant, Also $\Delta H > 0$

6. The first ionization enthalpies of Be, B, N and O follow the order

(A) O < N < B < Be (B) Be < B < N < O

(C) B < Be < N < O (D) B < Be < O < N

Official Ans. by NTA (D)

Sol. **1st I.E.** $\underset{(2p^3)}{N} > \underset{(2p^4)}{O} > \underset{(2s^2)}{Be} > \underset{(2p^1)}{B}$

7. Given below are two statements.

Statement I : Pig iron is obtained by heating cast iron with scrap iron.

Statement II: Pig iron has a relatively lower carbon content than that of cast iron. In the light of the above statements, choose the correct answer from the options given below.

(A) Both Statement I and Statement II are correct.

(B) Both Statement I and Statement II are not correct.

(C) Statement I is correct but Statement II is not correct

(D) Statement I is not correct but Statement II is correct.

Official Ans. by NTA (B)

Sol. Statement –I is incorrect because cast iron is obtained by heating pig iron with scrap iron
Statement–II is also incorrect because pig iron has more carbon content (~4%) than cast iron (~3%)

8. High purity (>99.95%) dihydrogen is obtained by

(A) reaction of zinc with aqueous alkali.

(B) electrolysis of acidified water using platinum electrodes.

(C) electrolysis of warm aqueous barium hydroxide solution between nickel electrodes.

(D) reaction of zinc with dilute acid.

Official Ans. by NTA (C)

Sol. High purity (>99.95%) dihydrogen is obtained by electrolysis of warm aqueous $Ba(OH)_2$ solution between Ni-electrodes

9. The correct order of density is

(A) Be > Mg > Ca > Sr

(B) Sr > Ca > Mg > Be

(C) Sr > Be > Mg > Ca

(D) Be > Sr > Mg > Ca

Official Ans. by NTA (C)

Sol. In II'A' group density decreases down the group till Ca and after that it increases.
Correct order of density is
Sr > Be > Mg > Ca

10. The total number of acidic oxides from the following list is: NO, N_2O, B_2O_3, N_2O_5, CO, SO_3, P_4O_{10}

(A) 3 (B) 4

(C) 5 (D) 6

Official Ans. by NTA (B)

Sol. Neutral Oxides — N_2O, NO, CO

Acidic Oxides — B_2O_3, N_2O_5, SO_3, P_4O_{10}

11. The correct order of energy of absorption for the following metal complexes is

A: $[Ni(en)_3]^{2+}$, B: $[Ni(NH_3)_6]^{2+}$, C: $[Ni(H_2O)_6]^{2+}$

(A) C < B < A

(B) B < C < A

(C) C < A < B

(D) A < C < B

Official Ans. by NTA (A)

Sol. Stronger the ligand, larger the splitting & higher the energy of absorption.

$$\underset{(A)}{[Ni(en)_3]^{+2}} > \underset{(B)}{[Ni(NH_3)_6]^{+2}} > \underset{(C)}{[Ni(H_2O)_6]^{+2}}$$

12. **Match List I with List II.**

List–I		List–II	
A.	Sulphate	I.	Pesticide
B.	Fluoride	II.	Bending of bones
C.	Nicotine	III.	Laxative effect
D.	Sodium arsinite	IV.	Herbicide

Choose the correct answer from the options given below:

(A) A-II, B-III. C-IV, D-I

(B) A-IV, B-III, C-II, D-I

(C) A-III, B-II, C-I, D-IV

(D) A-III, B-II, C-IV, D-I

Official Ans. by NTA (C)

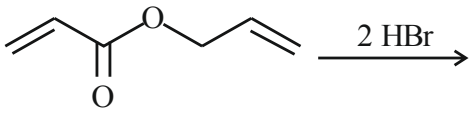
Sol. **A-Sulphate – III (Laxative effect)**

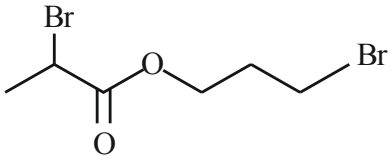
B-Fluoride – II (Bending of bones)

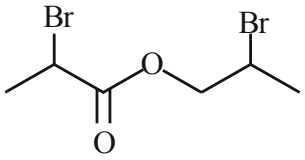
C-Nictoine – I (pesticides)

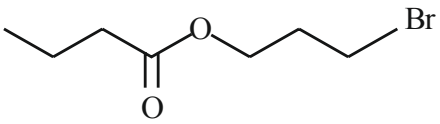
D-Sodium Arsinite – IV (herbicide)

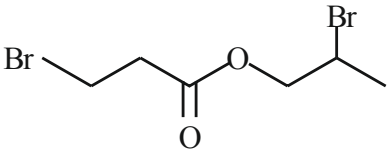
13. Major product of the following reaction is



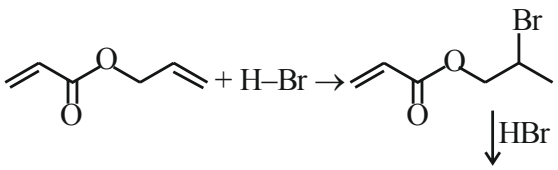
(A)


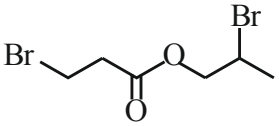
(B)


(C)


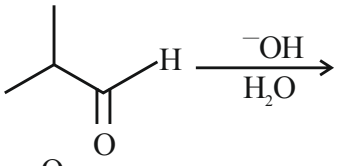
(D)


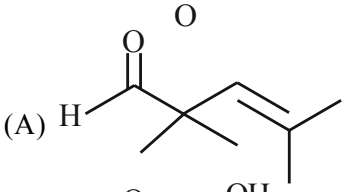
Official Ans. by NTA (D)

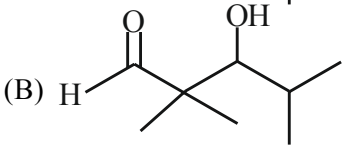
Sol.


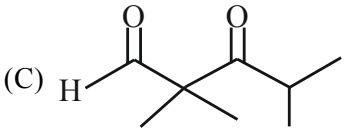


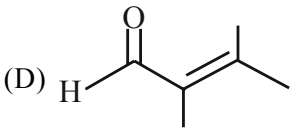
14. What is the major product of the following reaction?



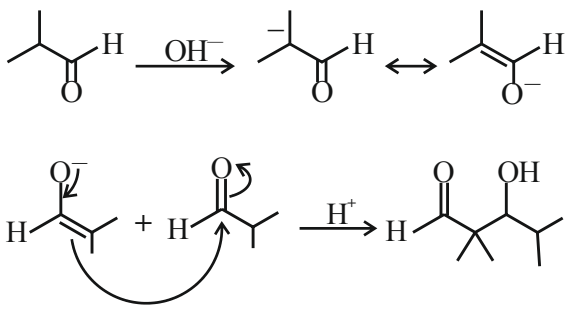
(A) 

(B) 

(C) 

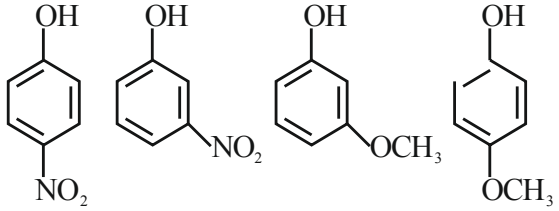
(D) 

Official Ans. by NTA (B)

Sol. 

Aldol formation takes place.

15. Arrange the following in decreasing acidic strength.



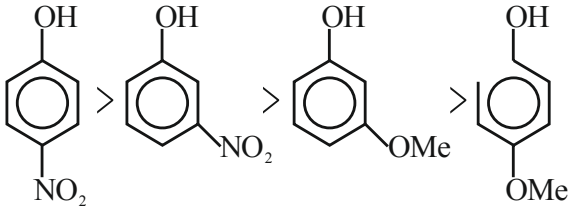
(A) (B) (C) (D)

(A) A > B > C > D (B) B > A > C > D

(C) D > C > A > B (D) D > C > B > A

Official Ans. by NTA (A)

Sol. The correct order of acid strength is



16. $CH_3-CH_2-CN \xrightarrow[Ether]{CH_3MgBr} A \xrightarrow{H_3O^+} B \xrightarrow[HCl]{Zn-Hg} C$

The correct structure of C is

(A) $CH_3—CH_2—CH_2—CH_3$

(B) $CH_3-CH_2-\overset{\overset{O}{\|}}{C}-CH_3$

(C) $CH_3-CH_2-\overset{\overset{OH}{|}}{CH}-CH_3$

(D) $CH_3—CH_2—CH=CH_2$

Official Ans. by NTA (A)

Sol.

$$CH_3CH_2-C\equiv N \xrightarrow{CH_3MgBr} CH_3CH_2-\overset{\overset{N\,MgBr}{\|}}{C}-CH_3$$

$$\downarrow H_3O^+$$

$$CH_3CH_2CH_2CH_3 \xleftarrow[HCl]{Zn/Hg} CH_3CH_2-\overset{\overset{O}{\|}}{C}-CH_3$$

(Clemmensen Reduction)

17. **Match List I with List II :**

List-I Polymer	List-II used for items
A. Nylon 6,6	I. Buckets
B. Low density polythene	II. Non-stick utensils
C. High density polythene	III. Bristles of brushes
D. Teflon	IV. Toys

Choose the correct answer from the options given below:

(A) A–III, B-I, C-IV, D-II

(B) A–III, B-IV, C-I, D-II

(C) A–II, B-I, C-IV, D-III

(D) A–II, B-IV, C-I, D-III

Official Ans. by NTA (B)

Sol. **LDPE → Toys**

HDPE → Buckets **(As per NCERT)**

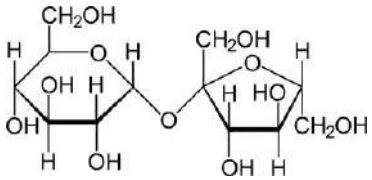
18. Glycosidic linkage between C_1 of α-glucose and C_2 of β-fructose is found in

(A) maltose (B) sucrose

(C) lactose (D) amylose

Official Ans. by NTA (B)

Sol. Theoretical



19. Some drugs bind to a site other than, the active site of an enzyme. This site is known as

(A) non-active site (B) allosteric site

(C) competitive site (D) therapeutic site

Official Ans. by NTA (B)

Sol. Theoretical

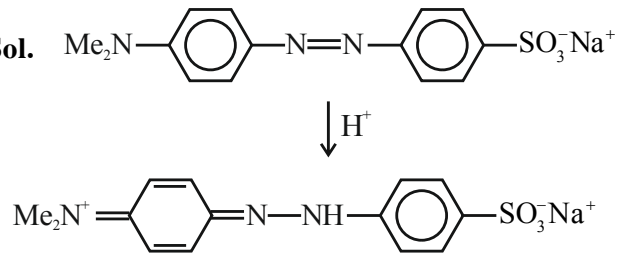
20. In base vs. Acid titration, at the end point methyl orange is present as

(A) quinonoid form (B) heterocyclic form

(C) phenolic form (D) benzenoid form

Official Ans. by NTA (A)

Sol.



(QUINONOID FORM)

SECTION-B

1. 56.0 L of nitrogen gas is mixed with excess of hydrogen gas and it is found that 20 L of ammonia gas is produced. The volume of unused nitrogen gas is found to be____ L.

Official Ans. by NTA (46)

Sol.

N_2	+	$3H_2$	→	$2NH_3$
56L		excess		O
−10L		−30L		+20L
46L				20L

2. A sealed flask with a capacity of 2 dm^3 contains 11 g of propane gas. The flask is so weak that it will burst if the pressure becomes 2 MPa. The minimum temperature at which the flask will burst is _____ °C. [Nearest integer]

(Given: R = 8.3 J K^{-1} mol^{-1}. Atomic masses of C and H are 12u and 1u respectively.) (Assume that propane behaves as an ideal gas.)

Official Ans. by NTA (1655)

Sol. Moles of $C_3H_8 = \frac{11}{44} = 0.25$ moles

$PV = nRT$

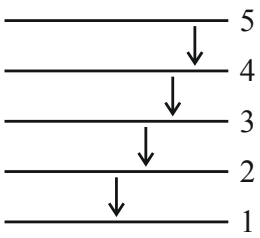
$\Rightarrow 2 \times 10^6 \times 2 \times 10^{-3} = 0.25 \times 8.3 \times T$

$\Rightarrow T = 1927.710\ K = 1654.56°C$

3. When the excited electron of a H atom from n = 5 drops to the ground state, the maximum number of emission lines observed are _____

Official Ans. by NTA (10)

Sol. Since only a single H atom is present, maximum number of spectral lines = 4



4. While performing a thermodynamics experiment, a student made the following observations,

$HCl + NaOH \rightarrow NaCl + H_2O\ \Delta H = -57.3$ kJ mol^{-1}

$CH_3COOH + NaOH \rightarrow CH_3COONa + H_2O$

$\Delta H = -55.3$ kJ mol^{-1}.

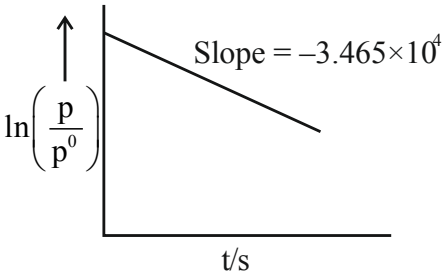
The enthalpy of ionization of CH_3COOH as calculated by the student is _____ kJ mol^{-1}. (nearest integer)

Official Ans. by NTA (2)

Sol. $\Delta H_{ionisation}$ of $CH_3COOH = |-57.3 - (-55.3)|$

$= 2$ KJ/mol

5. For the decomposition of azomethane.
$CH_3N_2CH_3(g) \rightarrow CH_3CH_3(g)+N_2(g)$ a first order reaction, the variation in partial pressure with time at 600 K is given as



The half life of the reaction is _____ × 10^{-5}s.
[Nearest integer]

Official Ans. by NTA (2)

Sol. For first order reaction

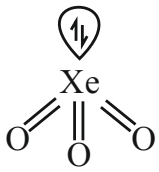
$$k = \frac{1}{t}\ln\left(\frac{P_0}{P}\right)$$

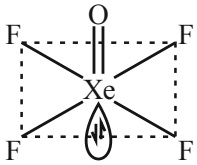
$$\ln\left(\frac{P_0}{P}\right) = kt$$

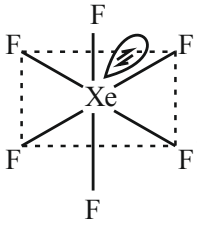
$$t_{1/2} = \frac{\ln 2}{k} = \frac{0.693}{3.465\times10^4} = 2\times10^{-5}$$

6. The sum of number of lone pairs of electrons present on the central atoms of XeO_3, $XeOF_4$ and XeF_6 is _____

Official Ans. by NTA (3)

Sol.  - one lone pair on central atom

 - one lone pair on central atom

 - one lone pair on central atom

7. The spin-only magnetic moment value of M^{3+} ion (in gaseous state) from the pairs Cr^{3+}/Cr^{2+}, Mn^{3+}/Mn^{2}, Fe^{3+}/Fe^{2+} and Co^{3+}/Co^{2+} that has negative standard electrode potential, is B.M.

[Nearest integer]

Official Ans. by NTA (4)

Sol. $E^0_{Cr^{+3}|Cr^{+2}} = -0.41V$

$[Cr^{+3}] = 4s^0 3d^3$

$\mu = \sqrt{n(n+2)}$ B.M

$= \sqrt{15}$ B.M ~ 4 B.M

8. A sample of 4.5 mg of an unknown monohydric alcohol, R–OH was added to methylmagnesium iodide. A gas is evolved and is collected and its volume measured to be 3.1 mL. The molecular weight of the unknown alcohol is ____ g/mol.
[Nearest integer]

Official Ans. by NTA (33)

Sol. $ROH + CH_3MgI \rightarrow ROMgI + CH_4(g)$

moles of CH_4 = moles of ROH

$$\Rightarrow \frac{V}{22400} = \frac{m}{M.M} \text{ (Assuming NTP Condition)}$$

$$\Rightarrow \frac{3.1}{22400} = \frac{4.5\times10^{-3}}{M.M}$$

$\Rightarrow$ MM = 32.51

Nearest Integer = 33

9. The separation of two coloured substances was done by paper chromatography. The distances travelled by solvent front, substance A and substance B from the base line are 3.25 cm. 2.08 cm and 1.05 cm. respectively. The ratio of R_f values of A to B is _____

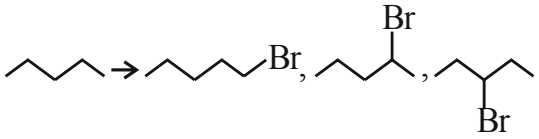
Official Ans. by NTA (2)

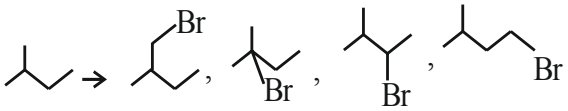
Sol. $$\frac{R_{F_A}}{R_{F_B}} = \frac{\frac{2.08}{3.25}}{\frac{1.05}{3.25}} = \frac{2.08}{1.05} \simeq 2$$

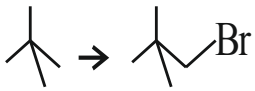
10. The total number of monobromo derivatives formed by the alkanes with molecular formula C_5H_{12} is (excluding stereo isomers)_____

Official Ans. by NTA (8)

Sol. The Alkanes and their monobromodervative are

1. 

2. 

3. 

FINAL JEE–MAIN EXAMINATION – JULY, 2022

(Held On Monday 25th July, 2022) **TIME : 3 : 00 PM to 6 : 00 PM**

MATHEMATICS

TEST PAPER WITH ANSWER

SECTION-A

1. For $z \in \mathbb{C}$ if the minimum value of $\left(|z-3\sqrt{2}|+|z-p\sqrt{2}i|\right)$ is $5\sqrt{2}$, then a value of p is _____

(A) 3 (B) $\frac{7}{2}$

(C) 4 (D) $\frac{9}{2}$

Official Ans. by NTA (C)

2. The number of real values λ, such that the system of linear equations

$2x - 3y + 5z = 9$

$x + 3y - z = -18$

$3x - y + (\lambda^2 - |\lambda|)z = 16$

has no solution, is :-

(A) 0 (B) 1

(C) 2 (D) 4

Official Ans. by NTA (C)

3. The number of bijective functions $f : \{1, 3, 5, 7,.....99\} \to \{2, 4, 6, 8,....., 100\}$, such that $f(3) \geq f(9) \geq f(15) \geq f(21) \geq ..... \geq f(99)$, is _____

(A) ${}^{50}P_{17}$ (B) ${}^{50}P_{33}$

(C) $33! \times 17!$ (D) $\frac{50!}{2}$

Official Ans. by NTA (B)

4. The remainder when $(11)^{1011} + (1011)^{11}$ is divided by 9 is

(A) 1 (B) 4

(C) 6 (D) 8

Official Ans. by NTA (D)

5. The sum $\sum_{n=1}^{21} \frac{3}{(4n-1)(4n+3)}$ is equal to

(A) $\frac{7}{87}$ (B) $\frac{7}{29}$

(C) $\frac{14}{87}$ (D) $\frac{21}{29}$

Official Ans. by NTA (B)

6. $\lim\limits_{x \to \frac{\pi}{4}} \frac{8\sqrt{2} - (\cos x + \sin x)^7}{\sqrt{2} - \sqrt{2}\sin 2x}$ is equal to

(A) 14 (B) 7

(C) $14\sqrt{2}$ (D) $7\sqrt{2}$

Official Ans. by NTA (A)

7. $\lim\limits_{n \to \infty} \frac{1}{2^n}\left(\frac{1}{\sqrt{1-\frac{1}{2^n}}} + \frac{1}{\sqrt{1-\frac{2}{2^n}}} + \frac{1}{\sqrt{1-\frac{3}{2^n}}} + ... + \frac{1}{\sqrt{1-\frac{2^n-1}{2^n}}}\right)$

is equal to

(A) $\frac{1}{2}$ (B) 1

(C) 2 (D) –2

Official Ans. by NTA (C)

8. If A and B are two events such that $P(A) = \frac{1}{3}$, $P(B) = \frac{1}{5}$ and $P(A \cup B) = \frac{1}{2}$, then $P(A \mid B') + P(B \mid A')$ is equal to

(A) $\frac{3}{4}$ (B) $\frac{5}{8}$

(C) $\frac{5}{4}$ (D) $\frac{7}{8}$

Official Ans. by NTA (B)

9. Let [t] denote the greatest integer less than or equal to t. Then the value of the integral $\int_{-3}^{101}([\sin(\pi x)]+e^{[\cos(2\pi x)]})dx$ is equal to

(A) $\frac{52(1-e)}{e}$ (B) $\frac{52}{e}$

(C) $\frac{52(2+e)}{e}$ (D) $\frac{104}{e}$

Official Ans. by NTA (B)

10. Let the point P (α, β) be at a unit distance from each of the two lines $L_1 : 3x - 4y + 12 = 0$, and $L_2 : 8x + 6y + 11 = 0$. If P lies below L_1 and above L_2, then $100(\alpha + \beta)$ is equal to

(A) –14 (B) 42

(C) –22 (D) 14

Official Ans. by NTA (D)

11. Let a smooth curve y = f(x) be such that the slope of the tangent at any point (x, y) on it is directly proportional to $\left(\frac{-y}{x}\right)$. If the curve passes through the point (1, 2) and (8, 1), then $\left|y\left(\frac{1}{8}\right)\right|$ is equal to

(A) $2\log_e 2$ (B) 4

(C) 1 (D) $4\log_e 2$

Official Ans. by NTA (B)

12. If the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ meets the line $\frac{x}{7}+\frac{y}{2\sqrt{6}}=1$ on the x-axis and the line $\frac{x}{7}-\frac{y}{2\sqrt{6}}=1$ on the y-axis, then the eccentricity of the ellipse is

(A) $\frac{5}{7}$ (B) $\frac{2\sqrt{6}}{7}$

(C) $\frac{3}{7}$ (D) $\frac{2\sqrt{5}}{7}$

Official Ans. by NTA (A)

13. The tangents at the point A(1, 3) and B(1, –1) on the parabola $y^2 - 2x - 2y = 1$ meet at the point P. Then the area (in unit^2) of the triangle PAB is :-

(A) 4 (B) 6

(C) 7 (D) 8

Official Ans. by NTA (D)

14. Let the foci of the ellipse $\frac{x^2}{16}+\frac{y^2}{7}=1$ and the hyperbola $\frac{x^2}{144}-\frac{y^2}{\alpha}=\frac{1}{25}$ coincide. Then the length of the latus rectum of the hyperbola is:-

(A) $\frac{32}{9}$ (B) $\frac{18}{5}$

(C) $\frac{27}{4}$ (D) $\frac{27}{10}$

Official Ans. by NTA (D)

15. A plane E is perpendicular to the two planes $2x - 2y + z = 0$ and $x - y + 2z = 4$, and passes through the point P(1, –1, 1). If the distance of the plane E from the point Q(a, a, 2) is $3\sqrt{2}$, then $(PQ)^2$ is equal to

(A) 9 (B) 12

(C) 21 (D) 33

Official Ans. by NTA (C)

16. The shortest distance between the lines $\frac{x+7}{-6}=\frac{y-6}{7}=z$ and $\frac{7-x}{2}=y-2=z-6$ is

(A) $2\sqrt{29}$ (B) 1

(C) $\sqrt{\frac{37}{29}}$ (D) $\frac{\sqrt{29}}{2}$

Official Ans. by NTA (A)

17. Let $\vec{a}=\hat{i}-\hat{j}+2\hat{k}$ and $\vec{b}$ be a vector such that $\vec{a}\times\vec{b}=2\hat{i}-\hat{k}$ and $\vec{a}.\vec{b}=3$. Then the projection of $\vec{b}$ on the vector $\vec{a}-\vec{b}$ is :-

(A) $\frac{2}{\sqrt{21}}$ (B) $2\sqrt{\frac{3}{7}}$

(C) $\frac{2}{3}\sqrt{\frac{7}{3}}$ (D) $\frac{2}{3}$

Official Ans. by NTA (A)

18. If the mean deviation about median for the number 3, 5, 7, 2k, 12, 16, 21, 24 arranged in the ascending order, is 6 then the median is

(A) 11.5 (B) 10.5

(C) 12 (D) 11

Official Ans. by NTA (D)

19. $2\sin\left(\frac{\pi}{22}\right)\sin\left(\frac{3\pi}{22}\right)\sin\left(\frac{5\pi}{22}\right)\sin\left(\frac{7\pi}{22}\right)\sin\left(\frac{9\pi}{22}\right)$ is equal to

(A) $\frac{3}{16}$ (B) $\frac{1}{16}$

(C) $\frac{1}{32}$ (D) $\frac{9}{32}$

Official Ans. by NTA (B)

20. Consider the following statements :

P : Ramu is intelligent

Q : Ramu is rich

R : Ramu is not honest

The negation of the statement "Ramu is intelligent and honest if and only if Ramu is not rich" can be expressed as :

(A) $((P\wedge(\sim R))\wedge Q)\wedge((\sim Q)\wedge((\sim P)\vee R))$

(B) $((P\wedge R)\wedge Q)\vee((\sim Q)\wedge((\sim P)\vee(\sim R)))$

(C) $((P\wedge R)\wedge Q)\wedge((\sim Q)\wedge((\sim P)\vee(\sim R)))$

(D) $((P\wedge(\sim R))\wedge Q)\vee((\sim Q)\wedge((\sim P)\vee R))$

Official Ans. by NTA (D)

SECTION-B

1. Let A : {1, 2, 3, 4, 5, 6, 7}. Define B = {T $\subseteq$ A : either $1\notin T$ or $2\in T$} and C = T $\subseteq$ A : T the sum of all the elements of T is a prime number}. Then the number of elements in the set $B\cup C$ is _____

Official Ans. by NTA (107)

2. Let f(x) be a quadratic polynomial with leading coefficient 1 such that f(0) = p, $p\neq 0$ and $f(1)=\frac{1}{3}$. If the equation f(x) = 0 and fofofof(x) = 0 have a common real root, then f(–3) is equal to.....

Official Ans. by NTA (25)

3. Let $A=\begin{bmatrix}1 & a & a\\ 0 & 1 & b\\ 0 & 0 & 1\end{bmatrix}, a,b\in\mathbb{R}$. If for some $n\in N$, $A^n=\begin{bmatrix}1 & 48 & 2160\\ 0 & 1 & 96\\ 0 & 0 & 1\end{bmatrix}$ then n + a + b is equal to _____

Official Ans. by NTA (24)

4. The sum of the maximum and minimum values of the function f(x) = |5x – 7| + [x^2 + 2x] is the interval $\left[\frac{5}{4},2\right]$, where [t] is the greatest integer $\leq t$ is _____

Official Ans. by NTA (15)

5. Let y = y(x) be the solution of the differential equation $\frac{dy}{dx}=\frac{4y^3+2yx^2}{3xy^2+x^3}, y(1)=1$. If for some $n\in N, y(2)\in[n-1,n)$, then n is equal to _____

Official Ans. by NTA (3)

6. Let f be a twice differentiable function on R. If $f'(0)=4$ and

$$f(x)+\int_0^x (x-t)f'(t)dt=(e^{2x}+e^{-2x})\cos 2x+\frac{2}{a}x,$$

then $(2a+1)^5 a^2$ is equal to _____

Official Ans. by NTA (8)

7. Let $a_n=\int_{-1}^{n}\left(1+\frac{x}{2}+\frac{x^2}{2}+\frac{x^3}{3}+.....+\frac{x^{n-1}}{n}\right)dx$ for $n\in N$. Then the sum of all the elements of the set $\{n\in N: a_n\in(2,30)\}$ is _____

Official Ans. by NTA (5)

8. If the circles $x^2+y^2+6x+8y+16=0$ and $x^2+y^2+2(3-\sqrt{3})x+x+2(4-\sqrt{6})y=k+6\sqrt{3}+8\sqrt{6}$, $k>0$, touch internally at the point $P(\alpha, \beta)$, then $(\alpha+\sqrt{3})^2+(\beta+\sqrt{6})^2$ is equal to _____

Official Ans. by NTA (25)

9. Let the area enclosed by the x-axis, and the tangent and normal drawn to the curve $4x^3-3xy^2+6x^2-5xy-8y^2+9x+14=0$ at the point (–2, 3) be A. Then 8A is equal to _____

Official Ans. by NTA (170)

10. Let $x=\sin(2\tan^{-1}\alpha)$ and $y=\sin\left(\frac{1}{2}\tan^{-1}\frac{4}{3}\right)$. If $S=\{\alpha\in R: y^2=1-x\}$, then $\sum_{\alpha\in S}16\alpha^3$ is equal to _____

Official Ans. by NTA (130)

FINAL JEE–MAIN EXAMINATION – JULY, 2022

(Held On Monday 25th July, 2022) **TIME : 9 : 00 AM to 12 : 00 NOON**

PHYSICS

TEST PAPER WITH SOLUTION

SECTION-A

1. If momentum [P], area [A] and time [T] are taken as fundamental quantities, then the dimensional formula for coefficient of viscosity is :

(A) $[P\ A^{-1}\ T^{0}]$ (B) $[P\ A\ T^{-1}]$

(C) $[P\ A^{-1}\ T]$ (D) $[P\ A^{-1}\ T^{-1}]$

Official Ans. by NTA (A)

Sol. Viscosity = pascal.second

$P^x\ A^y\ T^z = [M^1\ L^{-1}\ T^{-1}]$

$[M^1\ L^{+1}\ T^{-1}]^x\ [L^2]^y[T^1]^z = M^1L^{-1}\ T^{-1}$

$M^xL^{+x+2y}\ T^{-x+z} = M^1\ L^{-1}\ T^{-1}$

$x = 1 \quad x + 2y = -1 \quad -x + z = -1$

$y = -1$

$z = 0$

Viscosity $= P^1\ A^{-1}\ T^0$

2. Which of the following physical quantities have the same dimensions ?

(A) Electric displacement ($\vec{D}$) and surface charge density

(B) Displacement current and electric field

(C) Current density and surface charge density

(D) Electric potential and energy

Official Ans. by NTA (A)

Sol. Electric displacement

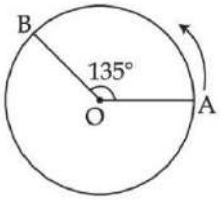
$\vec{D} = \epsilon_0\ \vec{E}$

$$[D] = [\epsilon_0\ E] = \left[\epsilon_0\ \frac{\sigma}{\epsilon_0}\right]$$

$[D] = [\sigma]$

$\rightarrow$ Surface change density = σ.

3. A person moved from A to B on a circular path as shown in figure. If the distance travelled by him is 60 m, then the magnitude of displacement would be : (Given cos135° = - 0.7)



(A) 42 m (B) 47 m

(C) 19 m (D) 40 m

Official Ans. by NTA (B)

Sol. $d = R\theta$

$$60 = R\left(\frac{3\pi}{4}\right)$$

$$R = \frac{60\times 4}{3\pi} = \frac{80}{\pi}\text{m}$$

$$\text{Displacement} = \sqrt{R^2 + R^2 - 2R^2\cos 135}$$

$$\Rightarrow \sqrt{2R^2 - 2R^2(-0.7)}$$

$$\Rightarrow \sqrt{3.4R^2} = \sqrt{3.4\left(\frac{80}{\pi}\right)^2}$$

≈ 47 m

4. A body of mass 0.5 kg travels on straight line path with velocity $v = (3x^2 + 4)$m/s. The net workdone by the force during its displacement from x = 0 to x = 2 m is :

(A) 64 J (B) 60 J

(C) 120 J (D) 128 J

Official Ans. by NTA (B)

Sol. $v_i = 3(0^2) + 4 = 4 \quad \cong \quad x = 0$

$v_F = 3(2)^2 + 4 \quad \cong \quad x = 2$

$= 16$

$$W = \Delta K = \frac{1}{2}m\left(16^2 - 4^2\right)$$

$$= \frac{1}{2}\times\frac{1}{2}(256 - 16)$$

$$= \frac{240}{4} = 60\text{J}$$

5. A solid cylinder and a solid sphere, having same mass M and radius R, roll down the same inclined plane from top without slipping. They start from rest. The ratio of velocity of the solid cylinder to that of the solid sphere, with which they reach the ground, will be :

(A) $\sqrt{\frac{5}{3}}$ (B) $\sqrt{\frac{4}{5}}$

(C) $\sqrt{\frac{3}{5}}$ (D) $\sqrt{\frac{14}{15}}$

Official Ans. by NTA (D)

Sol. $V = \sqrt{\frac{2gH}{1+k^2/R^2}}$

$$\frac{V_{cylinder}}{V_{sphere}} = \sqrt{\frac{(1+k^2/R^2)_{sphere}}{(1+k^2/R^2)_{cylinder}}}$$

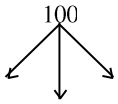
$$= \sqrt{\frac{1+2/5}{1+1/2}} = \sqrt{\frac{7}{5}\times\frac{2}{3}} = \sqrt{\frac{14}{15}}$$

6. Three identical particle A, B and C of mass 100 kg each are placed in a straight line with AB = BC = 13 m. The gravitational force on a fourth particle P of the same mass is F, when placed at a distance 13 m from the particle B on the perpendicular bisector of the line AC. The value of F will be approximately :

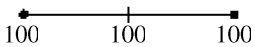
(A) 21 G (B) 100 G

(C) 59 G (D) 42 G

Official Ans. by NTA (B)



Sol.



$$F = \frac{GMM}{r^2} + \sqrt{2}\frac{GMM}{(\sqrt{2}r)^2}$$

$$= \frac{GMM}{r^2}\left(1+\frac{1}{\sqrt{2}}\right)$$

$$= \frac{G\times10^4}{13^2}\left(1+\frac{1}{\sqrt{2}}\right)$$

$$F \simeq 100G$$

7. A certain amount of gas of volume V at 27°C temperature and pressure 2×10^7 Nm^{-2} expands isothermally until its volume gets doubled. Later it expands adiabatically until its volume gets redoubled. The final pressure of the gas will be (Use $\gamma = 1.5$)

(A) 3.536×10^5 Pa (B) 3.536×10^6 Pa

(C) 1.25×10^6 Pa (D) 1.25×10^5 Pa

Official Ans. by NTA (B)

Sol. $P_1 = 2\times10^7$ Pa

$P_1V_1 = P_2V_2$

Since $V_2 = 2V_1$ Hence $P_2 = P_1/2$ (isothermal expansion)

$P_2 = 1\times10^7$ Pa

$P_2(V_2)^\gamma = P_3(2V_2)^\gamma$

$$P_3 = \frac{1\times10^7}{2^{1.5}} = 3.536\times10^6$$

8. Following statements are given :

(1) The average kinetic energy of a gas molecule decreases when the temperature is reduced.

(2) The average kinetic energy of a gas molecule increases with increase in pressure at constant temperature.

(3) The average kinetic energy of a gas molecule decreases with increases in volume.

(4) Pressure of a gas increases with increase in temperature at constant pressure.

(5) The volume of gas decreases with increase in temperature.

Choose the correct answer from the options given below :

(A) (1) and (4) only (B) (1), (2) and (4) only

(C) (2) and (4) only (D) (1), (2) and (5) only

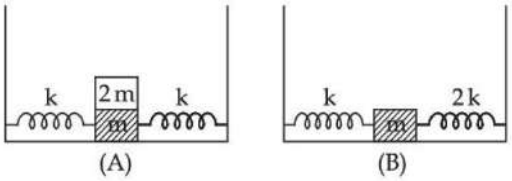
Official Ans. by NTA (A)

Sol. $KE_{avg} = \frac{3}{2}KT$

$$P = \frac{1}{3}\rho V_{rms}^2$$

Note : Statement (4) is correct only if we consider it at constant volume and not constant pressure. Ideally, this question must be bonus but most appropriate answer is option (A)

9. In figure (A), mass '2 m' is fixed on mass 'm' which is attached to two springs of spring constant k. In figure (B), mass 'm' is attached to two spring of spring constant 'k' and '2k'. If mass 'm' in (A) and (B) are displaced by distance 'x' horizontally and then released, then time period T_1 and T_2 corresponding to (A) and (B) respectively follow the relation.



(A) $\frac{T_1}{T_2}=\frac{3}{\sqrt{2}}$ (B) $\frac{T_1}{T_2}=\sqrt{\frac{3}{2}}$

(C) $\frac{T_1}{T_2}=\sqrt{\frac{2}{3}}$ (D) $\frac{T_1}{T_2}=\frac{\sqrt{2}}{3}$

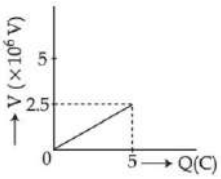
Official Ans. by NTA (A)

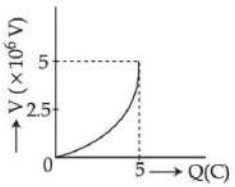
Sol. $T_1=2\pi\sqrt{\frac{3m}{2k}}$

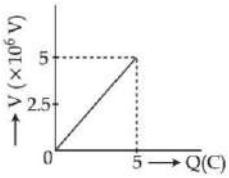
$T_2=2\pi\sqrt{\frac{m}{3k}}$

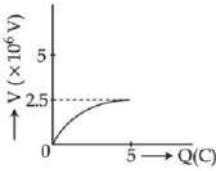
$\frac{T_1}{T_2}=\frac{2\pi\sqrt{\frac{3m}{2k}}}{2\pi\sqrt{\frac{m}{3k}}}=\frac{3}{\sqrt{2}}$

10. A condenser of 2 μF capacitance is charged steadily from 0 to 5C. Which of the following graph represents correctly the variation of potential difference (V) across it's plates with respect to the charge (Q) on the condenser ?

(A) 

(B) 

(C) 

(D) 

Official Ans. by NTA (A)

Sol. $Q=CV$

$V=\frac{1}{C}Q$

Straight line with slope $=\frac{1}{C}$

Slope $=\frac{1}{C}=\frac{1}{2\times10^{-6}}=5\times10^5$

11. Two charged particles, having same kinetic energy, are allowed to pass through a uniform magnetic field perpendicular to the direction of motion. If the ratio of radii of their circular paths is 6 : 5 and their respective masses ratio is 9 : 4. Then, the ratio of their charges will be :

(A) 8 : 5 (B) 5 : 4

(C) 5 : 3 (D) 8 : 7

Official Ans. by NTA (B)

Sol. Radius of circular path $R=\frac{\sqrt{2mk}}{qB}$

$q=\frac{\sqrt{2mk}}{RB}$

$\frac{q_1}{q_2}=\sqrt{\frac{m_1}{m_2}}\times\frac{R_2}{R_1}=\sqrt{\frac{9}{4}}\times\frac{5}{6}=\frac{5}{4}$

12. To increase the resonant frequency in series LCR circuit,

(A) Source frequency should be increased

(B) Another resistance should be added in series with the first resistance.

(C) Another capacitor should be added in series with the first capacitor

(D) The source frequency should be decreased

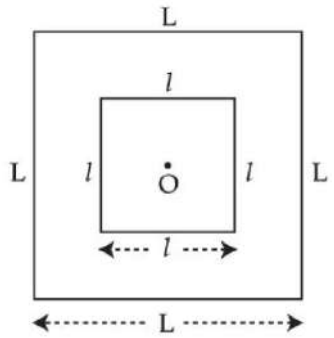
Official Ans. by NTA (C)

Sol. $f=\frac{1}{2\pi\sqrt{LC}}$

To increase the resonating frequency product of L and C should decrease.

By joining capacitor in series, capacitor will decrease

13. A small square loop of wire of side l is placed inside a large square loop of wire L (L >> l). Both loops are coplanar and their centres coincide at point O as shown in figure. The mutual inductance of the system is :

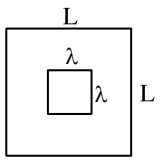


(A) $\frac{2\sqrt{2}\mu_0 L^2}{\pi\ell}$ (B) $\frac{\mu_0\ell^2}{2\sqrt{2}\pi L}$

(C) $\frac{2\sqrt{2}\mu_0\ell^2}{\pi L}$ (D) $\frac{\mu_0 L^2}{2\sqrt{2}\pi\ell}$

Official Ans. by NTA (C)

Sol. Assuming current I in outer loop magnetic field at centre $= 4\times\frac{\mu_0 i}{4\pi\times\frac{L}{2}}\times(2\sin 45°) = \frac{2\sqrt{2}\mu_0 i}{\pi L}$



$$M = \frac{\text{Flux through inner loop}}{i}$$

$$M = \frac{2\sqrt{2}\mu_0\ell^2}{\pi L}$$

14. The rms value of conduction current in a parallel plate capacitor is 6.9 μA. The capacity of this capacitor, if it is connected to 230 V ac supply with an angular frequency of 600 rad/s, will be :

(A) 5 pF (B) 50 pF

(C) 100 pF (D) 200 pF

Official Ans. by NTA (B)

Sol. Current in capacitor $I = \frac{V}{X_C}$

$$I = (V)\times(\omega C)$$

$$C = \frac{I}{V\omega} = \frac{6.9\times10^{-6}}{230\times600} = 50\text{pF}$$

15. Which of the following statement is correct ?

(A) In primary rainbow, observer sees red colour on the top and violet on the bottom

(B) In primary rainbow, observer sees violet colour on the top and red on the bottom

(C) In primary rainbow, light wave suffers total internal reflection twice before coming out of water drops

(D) Primary rainbow is less bright than secondary rainbow.

Official Ans. by NTA (A)

Sol. In primary rainbow, red colour is at top and violet is at bottom.

Intensity of secondary rainbow is less in comparison to primary rainbow.

16. Time taken by light to travel in two different materials A and B of refractive indices μ_A and μ_B of same thickness is t_1 and t_2 respectively. If $t_2 - t_1 = 5\times10^{-10}$ s and the ratio of μ_A to μ_B is 1 : 2. Then the thickness of material, in meter is : (Given v_A and v_B are velocities of light in A and B materials respectively).

(A) $5\times10^{-10}\ v_a$ m (B) 5×10^{-10} m

(C) 1.5×10^{-10} m (D) $5\times10^{-10}\ v_B$ m

Official Ans. by NTA (A)

Sol. $\frac{\mu_A}{\mu_B} = \frac{c/V_A}{c/V_B} = \frac{V_B}{V_A} = \frac{1}{2}$

Let the thickness is d

$$\frac{d}{v_B} - \frac{d}{v_A} = 5\times10^{-10}$$

$$d = \frac{5\times10^{-10}\times v_A v_B}{v_A - v_B}$$

As $v_A = 2v_B \Rightarrow d = 5\times10^{-10}\times 2v_B$

Or $d = 5\times10^{-10}\times v_A$

17. A metal exposed to light of wavelength 800 nm and emits photoelectrons with a certain kinetic energy. The maximum kinetic energy of photo-electron doubles when light of wavelength 500 nm is used. The work function of the metal is (Take hc = 1230 eV-nm).

(A) 1.537 eV (B) 2.46 eV

(C) 0.615 eV (D) 1.23 eV

Official Ans. by NTA (C)

Sol. $k_1 = \frac{1230}{800} - \phi$ ...(1)

$k_2 = 2k_1 = \frac{1230}{500} - \phi$ ...(2)

Eliminating k_1 from (1) and (2) we get

$0 = \frac{1230}{500} - \frac{1230}{400} + \phi$

$\phi = 0.615 eV$

18. The momentum of an electron revolving in n^{th} orbit is given by : (Symbols have their usual meanings)

(A) $\frac{nh}{2\pi r}$ (B) $\frac{nh}{2r}$

(C) $\frac{nh}{2\pi}$ (D) $\frac{2\pi r}{nh}$

Official Ans. by NTA (A)

Sol. Angular momentum is integral multiple of $\frac{h}{2\pi}$

$mvr = \frac{nh}{2\pi}$

So momentum $mv = \frac{nh}{2\pi r}$

19. The magnetic moment of an electron (e) revolving in an orbit around nucleus with an orbital angular momentum is given by :

(A) $\vec{\mu}_L = \frac{e\vec{L}}{2m}$ (B) $\vec{\mu}_L = -\frac{e\vec{L}}{2m}$

(C) $\vec{\mu}_l = -\frac{e\vec{L}}{m}$ (D) $\vec{\mu}_l = \frac{2e\vec{L}}{m}$

Official Ans. by NTA (B)

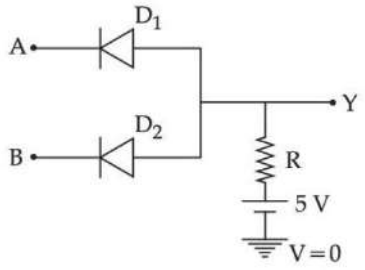
Sol. Ratio of magnetic moment and angular momentum

$\frac{\vec{\mu}}{\vec{L}} = \frac{q}{2m}$

For e^-

$\vec{\mu} = -\frac{e}{2m}\vec{L}$

20. In the circuit, the logical value of A = 1 or B = 1 when potential at A or B is 5V and the logical value of A = 0 or B = 0 when potential at A or B is 0 V.



The truth table of the given circuit will be :

	A	B	Y
	0	0	0
(A)	1	0	0
	0	1	0
	1	1	1

	A	B	Y
	0	0	0
(B)	1	0	1
	0	1	1
	1	1	1

	A	B	Y
	0	0	0
(C)	1	0	0
	0	1	0
	1	1	0

	A	B	Y
	0	0	0
(D)	1	0	1
	0	1	1
	1	1	0

Official Ans. by NTA (A)

Sol. When both A and B have logical value '1' both diode are reverse bias and current will flow in resistor hence output will be 5 volt i.e. logical value '1'.

In all other case conduction will take place, hence output will be zero volt i.e. logical value '0'.

So truth table is

A	B	Y	
0	0	0	
0	1	0	(AND gate)
1	0	0	
1	1	1	

SECTION-B

1. A car is moving with speed of 150 km/h and after applying the brake it will move 27 m before it stops. If the same car is moving with a speed of one third the reported speed then it will stop after travelling _____ m distance.

Official Ans. by NTA (3)

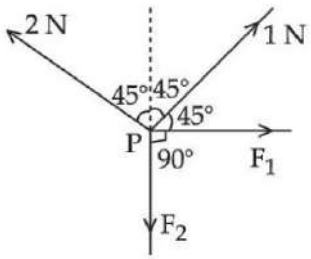
Sol. Stopping distance $=\frac{v^2}{2a}=d$

If speed is made $\frac{1}{3}$rd

$d'=\frac{1}{9}d.\qquad d'=\frac{27}{9}=3.$

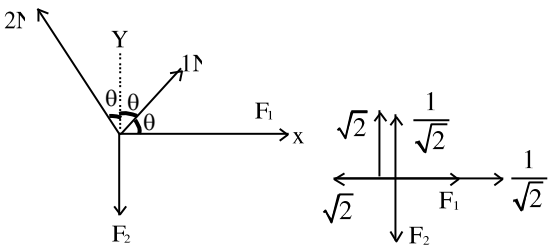
Braking acceleration remains same

2. Four forces are acting at a point P in equilibrium as shown in figure. The ratio of force F_1 to F_2 is 1 : x where x = _____ .



Official Ans. by NTA (3)

Sol.



$\theta=45°$

Taking components along x & y

$F_1=\sqrt{2}-\frac{1}{\sqrt{2}}=\frac{2-1}{\sqrt{2}}=\frac{1}{\sqrt{2}}$

$F_2=\sqrt{2}+\frac{1}{\sqrt{2}}=\frac{2+1}{\sqrt{2}}=\frac{3}{\sqrt{2}}$

$F_1:F_2=1:3$

x = 3

3. A wire of length L and radius r is clamped rigidly at one end. When the other end of the wire is pulled by a force F, its length increases by 5 cm. Another wire of the same material of length 4L and radius 4r is pulled by a force 4F under same conditions. The increase in length of this wire is _____ cm.

Official Ans. by NTA (5)

Sol. $\Delta\ell_1=\frac{F\ell}{AY}=\frac{F\ell}{\pi r^2Y}=5cm$

$\Delta\ell_2=\frac{4F4\ell}{\pi 16r^2Y}=\frac{F\ell}{\pi r^2Y}=5cm$

4. A unit scale is to be prepared whose length does not change with temperature and remains 20 cm, using a bimetallic strip made of brass and iron each of different length. The length of both components would change in such a way that difference between their lengths remains constant. If length of brass is 40 cm and length of iron will be _____cm.

($\alpha_{iron} = 1.2 \times 10^{-5}\ K^{-1}$ and $\alpha_{brass} = 1.8 \times 10^{-5}\ K^{-1}$).

Official Ans. by NTA (60)

Sol. $\ell_B(1+\alpha_B\Delta T)-\ell_i(1+\alpha_i\Delta T)=\ell_B-\ell_i$

$\alpha_B\ell_B=\ell_i\alpha_i$

$1.8\times10^{-5}\times40=\ell_i\times1.2\times10^{-5}$

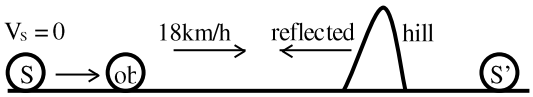
$\ell_i=\frac{1.8\times10^{-5}\times40}{1.2\times10^{-5}}=\frac{3\times40}{2}=60$

$\ell_i=60cm$

5. An observer is riding on a bicycle and moving towards a hill at 18 kmh^{-1}. He hears a sound from a source at some distance behind him directly as well as after its reflection from the hill. If the original frequency of the sound as emitted by source is 640 Hz and velocity of the sound in air is 320 m/s, the beat frequency between the two sounds heard by observer will be _____ Hz.

Official Ans. by NTA (20)

Sol.



$V_S=0,\ V_{ob}=5m/s$

$f_{direct}=\left(\frac{320-5}{320}\right)640=630Hz$

$f_{reflected}=\left(\frac{320+5}{320}\right)640=650Hz$

$f_{beat}=650-630=20Hz$

6. The volume charge density of a sphere of radius 6 m is 2 μC cm^{-3}. The number of lines of force per unit surface area coming out from the surface of the sphere is _____ × 10^{10} NC^{-1}.

[Given : Permittivity of vacuum $\epsilon_0 = 8.85 \times 10^{-12}\ C^2\ N^{-1} - m^{-2}$]

Official Ans. by NTA (45)

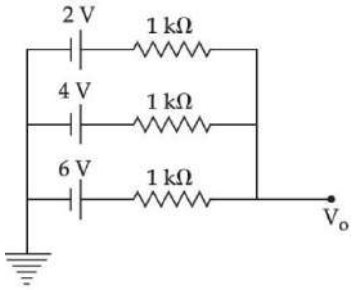
Sol. No. of electric field lines per unit area = electric field.

$$E = \frac{\rho r}{3\epsilon_0}, \text{ for } r = R$$

$$E = \frac{\rho R}{3\epsilon_0} = \frac{2\times 6}{3\times 8.85\times 10^{-12}} = 0.45\times 10^{12}\ NC^{-1}$$

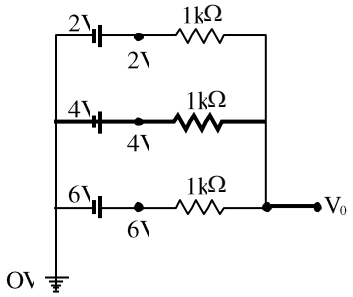
$$= 45\times 10^{10}\ N/C$$

7. In the given figure, the value of V_0 will be ____ V.



Official Ans. by NTA (4)

Sol.



By nodal analysis $\frac{V_0-2}{1k\Omega}+\frac{V_0-4}{1k\Omega}+\frac{V_0-6}{1k\Omega}=0$

$$3V_0 - 12 = 0$$

$$V_0 = 4$$

8. Eight copper wire of length *l* and diameter d are joined in parallel to form a single composite conductor of resistance R. If a single copper wire of length 2*l* have the same resistance (R) then its diameter will be _____ d.

Official Ans. by NTA (4)

Sol. Each wire has resistance $= \rho\frac{4\ell}{\pi d^2} = r$

Eight wire in parallel, then equivalent resistance is

$$\frac{r}{8} = \frac{\rho\ell}{2\pi d^2}$$

Single copper wire of length 2*l* has resistance

$$R = \rho\frac{2\ell\times 4}{\pi d_1^2} = \frac{\rho\ell}{2\pi d^2}$$

$$\Rightarrow d_1 = 4d$$

9. The energy band gap of semiconducting material to produce violet (wavelength = 4000 Å) LED is _____ eV. (Round off to the nearest integer).

Official Ans. by NTA (3)

Sol. $E_g = \frac{hc}{\lambda} = \frac{1242}{\lambda(nm)} = \frac{1242}{400} = 3.105$

Answer rounded to 3 eV

10. The required height of a TV tower which can cover the population of 6.03 lakh is h. If the average population density is 100 per square km and the radius of earth is 6400 km, then the value of h will be _____ m.

Official Ans. by NTA (150)

Sol. $d = \sqrt{2Rh}$

$d = \sqrt{2\times 6400\times h\times 10^{-3}}$ (h in m)

Area = πd^2

= $(\pi \times 2 \times 6400 \times h \times 10^{-3})\ km^2$

$6.03 \times 100000 = 100 \times \pi \times 2 \times 6400 \times 10^{-3}\ h$

$$h = \frac{6.03\times 10^5}{10\times\pi\times 128}$$

h = 150 m

FINAL JEE–MAIN EXAMINATION – JULY, 2022

(Held On Monday 25th July, 2022) **TIME : 9 : 00 AM to 12 : 00 NOON**

CHEMISTRY — TEST PAPER WITH SOLUTION

SECTION-A

1. SO_2Cl_2 on reaction with excess of water results into acidic mixture

$SO_2Cl_2 + 2H_2O \rightarrow H_2SO_4 + 2HCl$

16 moles of NaOH is required for the complete neutralisation of the resultant acidic mixture. The number of moles of SO_2Cl_2 used is :

(A) 16 (B) 8

(C) 4 (D) 2

Official Ans. by NTA (C)

Sol. Let $n(SO_2Cl_2) = x$ moles

$\therefore n(H_2SO_4) = x, n(HCl) = 2x$

$\Rightarrow n(H^+) = 4x$

For Neutralisation

$\Rightarrow n(H^+) = n(OH^-)$

$\Rightarrow 4x = 16$

$\Rightarrow \mathbf{x = 4}$

2. Which of the following sets of quantum numbers is not allowed ?

(A) $n = 3, l = 2, m_l = 0, s = +\frac{1}{2}$

(B) $n = 3, l = 2, m_l = -2, s = +\frac{1}{2}$

(C) $n = 3, l = 3, m_l = -3, s = -\frac{1}{2}$

(D) $n = 3, l = 0, m_l = 0, s = -\frac{1}{2}$

Official Ans. by NTA (C)

Sol. $l = 0, 1, 2 \ldots\ldots (n-1)$

$\therefore$ for $n = 3$

$l = 0, 1, 2$

$\Rightarrow l = 3,$

not possible for $n = 3$

3. The depression in freezing point observed for a formic acid solution of concentration 0.5 mL L^{-1} is 0.0405°C. Density of formic acid is 1.05 g mL^{-1}. The Van't Hoff factor of the formic acid solution is nearly : (Given for water $k_f = 1.86$ K kg mol^{-1})

(A) 0.8 (B) 1.1

(C) 1.9 (D) 2.4

Official Ans. by NTA (C)

Sol. $[HCOOH] = 0.5 \text{ ml } l^{-1}$

$\Rightarrow (0.5 \text{ ml} \times 1.05 \text{ g ml}^{-1})$ HCOOH in 1L

$\Rightarrow 0.525$ g HCOOH in 1L

$$m = \frac{(0.525/46)}{1\text{kg}} \text{mol} \quad \text{[Assuming dilute solution]}$$

$$\therefore \Delta T_f = iK_f m \Rightarrow i = \frac{\Delta T_f}{k_f m} = \frac{0.0405 \times 46}{1.86 \times 0.525} = 1.9$$

4. 20 mL of 0.1 M NH_4OH is mixed with 40 mL of 0.05 M HCl. The pH of the mixture is nearest to:

(Given: $K_b(NH_4OH) = 1 \times 10^{-5}$, log 2 = 0.30, log 3 = 0.48, log 5 = 0.69, log 7 = 0.84, log 11 =1.04)

(A) 3.2 (B) 4.2

(C) 5.2 (D) 6.2

Official Ans. by NTA (C)

Sol. $NH_4OH + HCl \rightarrow NH_4Cl + H_2O$

mmole 2 2

\- - 2 mmole

$$\left[NH_4^+\right] = \frac{2\text{mmole}}{60\,\text{ml}} = \frac{1}{30}\,\text{M}$$

$$pH = \frac{pK_w - pK_b - \log C}{2} = \frac{14 - 5 + 1.48}{2} = 5.24$$

5.

Match **List - I** with **List - II**

	List - I		List - II
(A)	$N_2(g)+3H_2(g) \rightarrow 2NH_3(g)$	(I)	Cu
(B)	$CO(g)+3H_2(g) \rightarrow CH_4(g)+H_2O(g)$	(II)	$Cu/ZnO-Cr_2O_3$
(C)	$CO(g)+H_2(g) \rightarrow HCHO(g)$	(III)	$Fe_xO_y+K_2O+Al_2O_3$
(D)	$CO(g)+2H_2(g) \rightarrow CH_3OH(g)$	(IV)	Ni

Choose the correct answer from the options given below :

(A) (A) - (II), (B) - (IV), (C) - (I), (D) - (III)

(B) (A) - (II), (B) - (I), (C) - (IV), (D) - (III)

(C) (A) - (III), (B) - (IV), (C) - (I), (D) - (II)

(D) (A) - (III), (B) - (I), (C) - (IV), (D) - (II)

Official Ans. by NTA (C)

Sol. **Factual**

6. The IUPAC nomenclature of an element with electronic configuration $[Rn]5f^{14}6d^{1}7s^{2}$ is :

(A) Unnilbium (B) Unnilunium

(C) Unnilquadium (D) Unniltrium

Official Ans. by NTA (D)

Sol. Atomic Number 103

7. The compound(s) that is(are) removed as slag during the extraction of copper is :

(1) CaO (2) FeO

(3) Al_2O_3 (4) ZnO

(5) NiO

Choose the correct answer from the options given below :

(A) (3) (4) Only (B) (1), (2), (5) Only

(C) (1), (2) Only (D) (2) Only

Official Ans. by NTA (D)

Sol. $FeO + SiO_2 \rightarrow FeSiO_3$

8. The reaction of H_2O_2 with potassium permanganate in acidic medium leads to the formation of mainly:

(A) Mn^{2+} (B) Mn^{4+}

(C) Mn^{3+} (D) Mn^{6+}

Official Ans. by NTA (A)

Sol. $H_2O_2 + MnO_4^- \rightarrow Mn^{2+} + O_2$ (unbalanced)

9. Choose the correct order of density of the alkali metals :

(A) Li < K < Na < Rb < Cs

(B) Li < Na < K < Rb < Cs

(C) Cs < Rb < K < Na < Li

(D) Li < Na < K < Cs < Rb

Official Ans. by NTA (A)

Sol. **Factual**

10. The geometry around boron in the product 'B' formed from the following reaction is

$$BF_3 + NaH \xrightarrow{450K} A + NaF$$

$$A + NMe_3 \rightarrow B$$

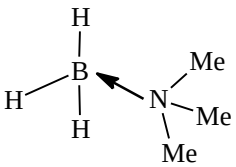
(A) trigonal planar (B) tetrahedral

(C) pyramidal (D) square planar

Official Ans. by NTA (B)

Sol. $BF_3 + NaH \xrightarrow{450K} \underset{\text{(diborane)}}{B_2H_6} + NaF$

$$B_2H_6 + NMe_3 \longrightarrow \underset{\text{symmetrical cleavage}}{2[BH_3 \leftarrow NMe_3]}$$



11. The interhalogen compound formed from the reaction of bromine with excess of fluorine is a :

(A) hypohalite (B) halate

(C) perhalate (D) halite

Official Ans. by NTA (B)

Sol. $Br_2 + 5\underset{\text{(excess)}}{F_2} \longrightarrow 2\overset{+5}{Br}F_5 \xrightarrow{H_2O} HBrO_3$ (Forms bromate)

12. The photochemical smog does not generally contain :

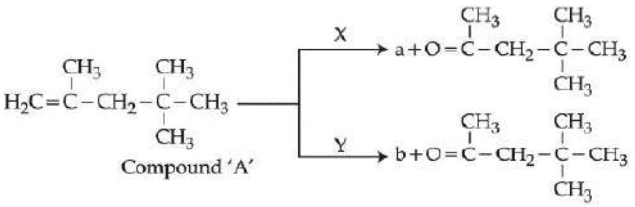
(A) NO (B) NO_2

(C) SO_2 (D) HCHO

Official Ans. by NTA (C)

Sol. **Factual**

13. A compound 'A' on reaction with 'X' and 'Y produces the same major product but different by product 'a' and 'b'. Oxidation of 'a' gives a substance produced by ants.



'X' and 'Y' respectively are :

(A) $KMnO_4/H^+$ and dil. $KMnO_4$, 273 K

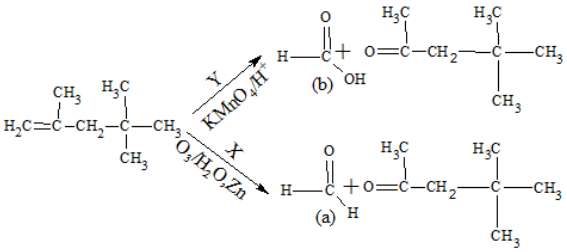
(B) $KMnO_4$,(dilute), 273 K and $KMnO_4/H^+$

(C) $KMnO_4/H^+$ and O_3, H_2O/Zn

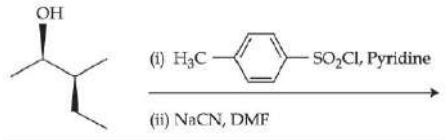
(D) O_3, H_2O/Zn and $KMnO_4/H^+$

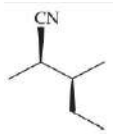
Official Ans. by NTA (D)

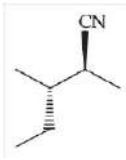
Sol.

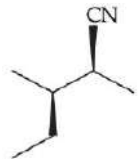


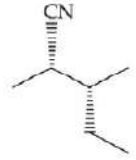
14. Most stable product of the following reaction is:



(A) 

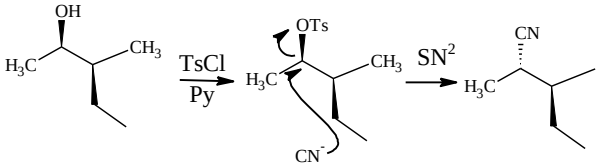
(B) 

(C) 

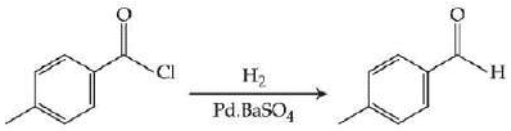
(D) 

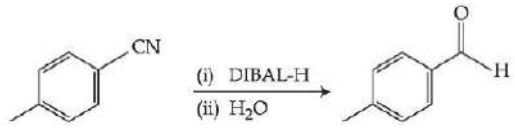
Official Ans. by NTA (B)

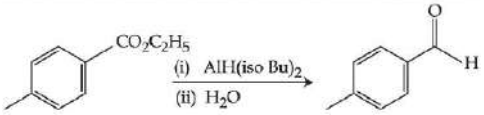
Sol.

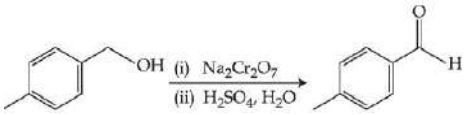


15. Which one of the following reactions does not represent correct combination of substrate and product under the given conditions ?

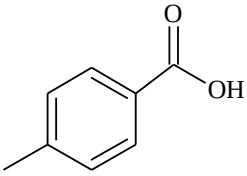
(A) 

(B) 

(C) 

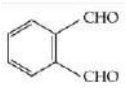
(D) 

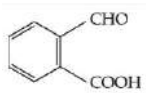
Official Ans. by NTA (D)

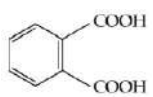
Sol. 

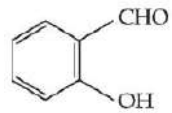
is formed

16. An organic compound 'A' on reaction with NH_3 followed by heating gives compound B. Which on further strong heating gives compound C ($C_8H_5NO_2$). Compound C on sequential reaction with ethanolic KOH, alkyl chloride and hydrolysis with alkali gives a primary amine. The compound A is :

(A) 

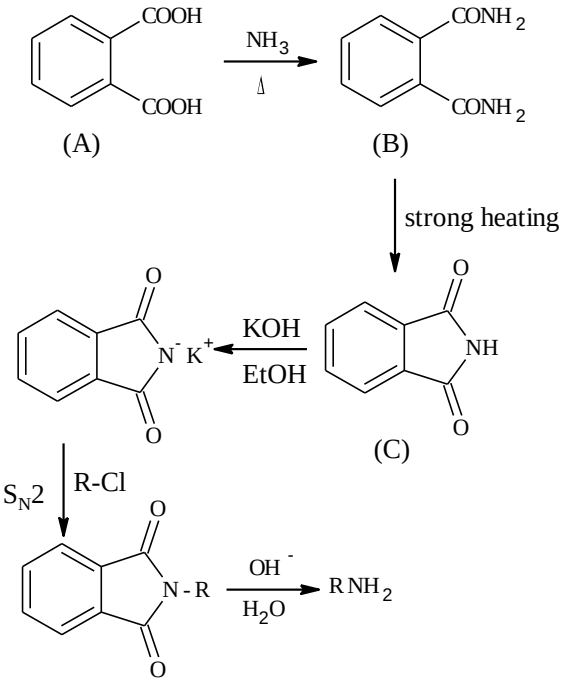
(B) 

(C) 

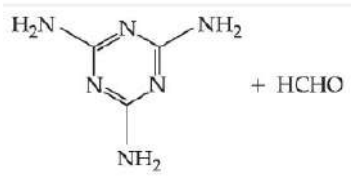
(D) 

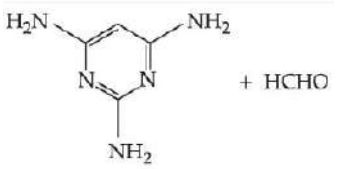
Official Ans. by NTA (C)

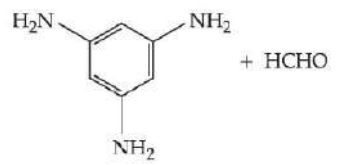
Sol. Gabriel Pthalimide reaction

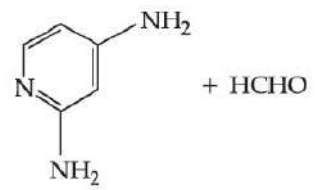


17. Melamine polymer is formed by the condensation of :

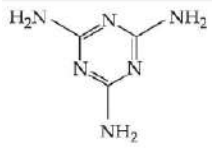
(A) 

(B) 

(C) 

(D) 

Official Ans. by NTA (A)

Sol. Melamine : 

Formaldehyde HCHO

Melamine formaldehyde Resin is melamine polymer

18. During the denaturation of proteins, which of these structures will remain intact ?

(A) Primary

(B) Secondary

(C) Tertiary

(D) Quaternary

Official Ans. by NTA (A)

Sol. Primary structure remains intact during denaturation of proteins

19. Drugs used to bind to receptors, inhibiting its natural function and blocking a message are called :

(A) Agonists

(B) Antagonists

(C) Allosterists

(D) Anti histaminists

Official Ans. by NTA (B)

Sol. Factual

20. Given below are two statements :

Statement I : On heating with $KHSO_4$, glycerol is dehydrated and acrolein is formed.

Statement II : Acrolein has fruity odour and can be used to test glycerol's presence.

Choose the correct option.

(A) Both Statement I and Statement II are correct.

(B) Both Statement I and Statement II are incorrect

(C) Statement I is correct but Statement II is incorrect.

(D) Statement I is incorrect but Statement II is correct.

Official Ans. by NTA (B)

Sol. Acrolein has a pungent, suffocating odour.

Acrolein is used to detect presence of glycerol

SECTION-B

1. Among the following species

$N_2, N_2^+, N_2^-, N_2^{2-}, O_2, O_2^+, O_2^-, O_2^{2-}$

the number of species showing diamagnetism is

Official Ans. by NTA (2)

Sol. Diamagnetic species are: N_2, O_2^{2-}

2. The enthalpy of combustion of propane, graphite and dihydrogen at 298 K are: –2220.0 kJ mol^{-1}, –393.5 kJ mol^{-1} and –285.8 kJ mol^{-1} respectively. The magnitude enthalpy of formation of propane (C_3H_8) is……kJ mol^{-1}. (Nearest integer)

Official Ans. by NTA (104)

Sol. $3C_{(gr)} + 4H_{2(g)} \rightarrow C_3H_{8(g)}$

$= -103.7$ kJ mol^{-1}

3. The pressure of a moist gas at 27°C is 4 atm. The volume of the container is doubled at the same temperature. The new pressure of the moist gas is …. $\times 10^{-1}$ atm. (Nearest integer)

(Given : The vapour pressure of water at 27°C is 0.4 atm)

Official Ans. by NTA (22)

Sol. $\left[P_{gas}\right]_0 + V.P. = 4$

$\left[P_{gas}\right]_0 = 4 - 0.4 = 3.6$

As volume is doubled, $\left[P_{gas}\right]_{new} = 1.8$ atm

New Total Pressure = 1.8 + 0.4 = 2.2 atm

4. The cell potential for $Zn|Zn^{2+}(aq)||Sn^{x+}|Sn$ is 0.801 V at 298 K. The reaction quotient for the above reaction is 10^{-2}. The number of electrons involved in the given electrochemical cell reaction is. . . . .

(Given $E^0_{Zn^{2+}|Zn} = -0.763V$, $E^0_{Sn^{x+}|Sn} = +0.008V$ and $\frac{2.303RT}{F} = 0.06V$**)**

Official Ans. by

Sol. $E = E^0 - \frac{2.303\,RT}{nF}\log Q$

Here, $E = +0.801V$, $E^0 = 0.008 - (-0.763)$

$= +0.771$ V

$\therefore 0.801 = +0.771 - \frac{0.06}{n}\log 10^{-2}$

$\Rightarrow n = 4$

5. The half life for the decomposition of gaseous compound A is 240 s when the gaseous pressure was 500 Torr initially. When the pressure was 250 Torr, the half life was found to be 4.0 min. The order of the reaction is……. (Nearest integer)

Official Ans. by NTA (1)

Sol. $(t_{1/2})_{500\ torr} = 240$ sec = 4 min.

$(t_{1/2})_{250\ torr} =$ 4 min.

$t_{1/2} \propto a^{1-n}$

As $t_{1/2}$ is independent of initial pressure. Hence, order is 1st order.

6. Consider the following metal complexes :

$[Co(NH_3)]^{3+}$

$[CoCl(NH_3)_5]^{2+}$

$[Co(CN)_6]^{3-}$

$[Co(NH_3)_5(H_2O)]^{3+}$

The spin-only magnetic moment value of the complex that absorbs light with shortest wavelength is B.M. (Nearest integer)

Official Ans. by NTA (0)

Sol. $\Delta_0 \propto \frac{1}{\lambda}$

Here, CN^- being SFL will have maximum CFSE

So, $[Co(CN)_6]^{3-}$ will be d^2sp^3, $\mu = 0$

7. Among Co^{3+}, Ti^{2+}, V^{2+} and Cr^{2+} ions, one if used as a reagent cannot liberate H_2 from dilute mineral acid solution, its spin-only magnetic moment in gaseous state is ……B.M. (Nearest integer)

Official Ans. by NTA (5)

Sol. Co^{3+} can't liberate H_2.

It has d^6 configuration,

Number of unpaired electrons = 4

$\mu = \sqrt{4 \times 6} = 4.92$ B.M.

8. While estimating the nitrogen present in an organic compound by Kjeldahl's method, the ammonia evolved from 0.25 g of the compound neutralized 2.5 mL of 2 M H_2SO_4. The percentage of nitrogen present in organic compound is ..... .

Official Ans. by NTA (56)

Sol. $$\%N = \frac{1.4(N_1V_1)}{\text{mass of organic compound}}$$

$$\%N = \frac{1.4(2.5\times2\times2)}{0.25} = 56$$

9. The number of sp^3 hybridised carbons in an acyclic neutral compound with molecular formula C_4H_5N is :

Official Ans. by NTA (1)

Sol. $$DU = 4+1-\left(\frac{5-1}{2}\right) = 3$$

$H_3C-CH=CH-C\equiv N$

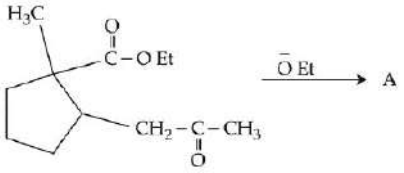
(sp^3)

or

$CH_2=C=CH=CH=NH$

(Zero sp^3 carbon)

10. In the given reaction

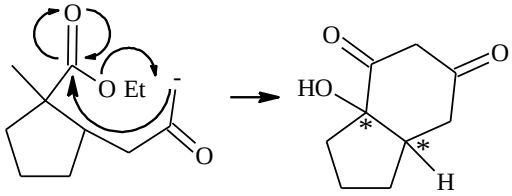


(Where Et is $-C_2H_5$)

The number of chiral carbon/s in product A is

Official Ans. by NTA (2)

Sol.



2 chiral carbons

FINAL JEE–MAIN EXAMINATION – JULY, 2022

(Held On Monday 25th July, 2022) **TIME : 9 : 00 AM to 12 : 00 NOON**

MATHEMATICS — TEST PAPER WITH SOLUTION

SECTION-A

1. The total number of functions,

$f : \{1, 2, 3, 4\} \bullet \{1, 2, 3, 4, 5, 6\}$

such that $f(1) + f(2) = f(3)$, is equal to :

(A) 60 (B) 90

(C) 108 (D) 126

Official Ans. by NTA (B)

Sol. $A = \{1, 2, 3, 4\}$

$B = \{1, 2, 3, 4, 5, 6\}$

Here f(3) can be 2, 3, 4, 5, 6

$f(3) = 2, (f(1), f(2)) \rightarrow (1,1) \rightarrow$ 6 cases

$f(3) = 3, (f(1), f(2)) \rightarrow (1,2), (2,1)$

$\rightarrow 2 \times 6 = 12$ cases

$f(3) = 4, (f(1), f(2)) \rightarrow (1,3), (3,1), (2,2)$

$\rightarrow 3 \times 6 = 18$ cases

$f(3) = 5, (f(1), f(2)) \rightarrow (1,4), (4,1), (2,3), (3,2)$

$\rightarrow 4 \times 6 = 24$ cases

$f(3) = 6, (f(1), f(2)) \rightarrow (1,5),(5,1),(2,4),(4,2),(3,3)$

$\rightarrow 5 \times 6 = 30$ cases

Total number of cases = 6+ 12 + 18 + 24 + 30 = 90

2. If $\alpha,\beta, \gamma, \delta$ are the roots of the equation $x^4 + x^3 + x^2 + x + 1 = 0$, then

$\alpha^{2021} + \beta^{2021} + \gamma^{2021} + \delta^{2021}$ is equal to .

(A) –4 (B) –1

(C) 1 (D) 4

Official Ans. by NTA (B)

Sol. $\alpha,\beta, \gamma, \delta$ root of the equation

$x^4 + x^3 + x^2 + x + 1 = 0$

Which are 5^{th} roots of unity except 1.

then $\alpha^{2021} + \beta^{2021} + \gamma^{2021} + \delta^{2021} =$

$\alpha + \beta + \gamma + \delta = -1$

3. For $n \in N$, let $S_n = \left\{z \in C : |z - 3 + 2i| = \frac{n}{4}\right\}$ and

$T_n = \left\{z \in C : |z - 2 + 3i| = \frac{1}{n}\right\}$.

Then the number of elements in the set

$\{n \in N : S_n \cap T_n = \phi\}$ is :

(A) 0 (B) 2 (C) 3 (D) 4

Official Ans. by NTA (D)

Sol. $S_n : |z - (3 - 2i)| = \frac{n}{4}$ is a circle center $C_1(3,-2)$

and radius n/4

$T_n : |z - (2 - 3i)| = \frac{1}{n}$ is a circle center $C_2(2,-3)$

and radius 1/n

Here $S_n \cap T_n = \phi$

Both circles do not intersect each other

Case-1 : $C_1C_2 > n/4 + 1/n$

$\sqrt{2} > \frac{n}{4} + \frac{1}{n}$

then n = 1, 2, 3, 4

Case-2 : $C_1C_2 < \left|\frac{n}{4} - \frac{1}{n}\right|$

$\Rightarrow \sqrt{2} < \left|\frac{n^2 - 4}{4n}\right|$

$\Rightarrow$ n has infinite solutions for $n \in \mathbf{N}$

4. The number of $\theta \in (0, 4\pi)$ for which the system of linear equations

$3 (\sin 3\theta) x - y + z = 2$

$3 (\cos 2\theta) x + 4y + 3z = 3$

$6x + 7y + 7z = 9$

has no solution is :

(A) 6 (B) 7 (C) 8 (D) 9

Official Ans. by NTA (B)

Sol. The system of equation has no solution.

$$D=\begin{vmatrix}3\sin 3\theta & -1 & 1\\ 3\cos 2\theta & 4 & 3\\ 6 & 7 & 7\end{vmatrix}=0$$

$21\sin 3\theta + 42\cos 2\theta - 42 = 0$

$\sin 3\theta + 2\cos 2\theta - 2 = 0$

Number of solution is 7 in $(0,4\pi)$

5. If $\lim\limits_{n\to\infty}\left(\sqrt{n^2-n-1}+n\alpha+\beta\right)=0$ then $8(\alpha+\beta)$ is equal to :

(A) 4 (B) –8

(C) –4 (D) 8

Official Ans. by NTA (C)

Sol. $\lim\limits_{n\to\infty} n\left(1-\dfrac{n+1}{n^2}\right)^{\frac{1}{2}}+\alpha n+\beta=0$

$$\lim_{n\to\infty} n\left\{1-\frac{1}{2}\left(\frac{n+1}{n^2}\right)+\frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2!}\left(\frac{n+1}{n^2}\right)^2+....\right\}+\alpha n+\beta=0$$

$$\lim_{n\to\infty} n-\frac{1}{2}+\frac{1}{n}+....+n\alpha+\beta=0$$

$$\alpha=-1,\beta=\frac{1}{2}$$

$8(\alpha+\beta) = -4$

6. If the absolute maximum value of the function f(x) $= (x^2 - 2x + 7)\, e^{(4x^3-12x^2-180x+31)}$ in the interval [–3, 0] is $f(\alpha)$, then :

(A) $\alpha = 0$ (B) $\alpha = -3$

(C) $\alpha \in (-1,0)$ (D) $\alpha \in (-3,-1)$

Official Ans. by NTA (B)

Sol. $f'(x)=e^{(4x^3-12x^2-180x+31)}\left(12(x^2-2x+7)(x+3)(x-5)+2(x-1)\right)$

for $x \in [-3,0]$

$\Rightarrow f'(x) < 0$

f(x) is decreasing function on [–3,0]

The absolute maximum value of the function f(x) is at $x = -3$

$\Rightarrow \alpha = -3$

7. The curve $y(x) = ax^3 + bx^2 + cx + 5$ touches the x-axis at the point P(–2, 0) and cuts the y-axis at the point Q, where y' is equal to 3. Then the local maximum value of y(x) is :

(A) $\frac{27}{4}$ (B) $\frac{29}{4}$ (C) $\frac{37}{4}$ (D) $\frac{9}{2}$

Official Ans. by NTA (A)

Sol. $y(x) = ax^3 + bx^2 + cx + 5$ is passing through (-2,0) then $8a - 4b + 2c = 5$……(1)

$y'(x) = 3ax^2 + 2bx + c$ touches x-axis at (-2,0)

$12a - 4b + c = 0$ ……(2)

again, for $x = 0$, $y'(x) = 3$

$c = 3$ ……(3)

Solving eq. (1), (2) & (3) $a=-\frac{1}{2},\ b=-\frac{3}{4}$

$$y'(x)=-\frac{3}{2}x^2-\frac{3}{2}x+3$$

y(x) has local maxima at x = 1

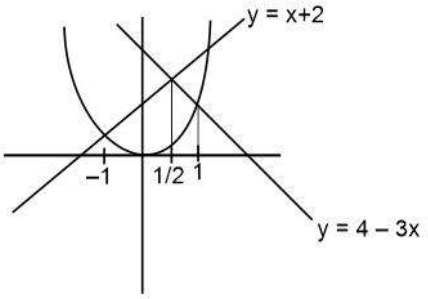
$$y(1)=\frac{27}{4}$$

8. The area of the region given by $A=\{(x, y) : x^2 \le y \le \min\{x + 2, 4 - 3x\}\}$ is :

(A) $\frac{31}{8}$ (B) $\frac{17}{6}$ (C) $\frac{19}{6}$ (D) $\frac{27}{8}$

Official Ans. by NTA (B)

Sol.



$$A=\int_{-1}^{\frac{1}{2}}(x+2-x^2)dx+\int_{\frac{1}{2}}^{1}(4-3x-x^2)dx=\frac{17}{6}$$

9. For any real number x, let [x] denote the largest integer less than equal to x. Let f be a real valued function defined on the interval [–10,10] by $f(x)=\begin{cases} x-[x], & \text{if } x \text{ is odd} \\ 1+[x]-x & \text{if } x \text{ is even} \end{cases}$

Then the value of $\frac{\pi^2}{10}\int_{-10}^{10} f(x)\cos\pi x\,dx$ is :

(A) 4 (B) 2

(C) 1 (D) 0

Official Ans. by NTA (A)

Sol. f(x) is periodic function whose period is 2

$$\frac{\pi^2}{10}\int_{-10}^{10} f(x)\cos\pi x\,dx=\frac{\pi^2}{10}\times 10\int_0^2 f(x)\cos\pi x\,dx$$

$$=\pi^2\left(\int_0^1 (1-x)\cos\pi x\,dx+\int_1^2 (x-1)\cos\pi x\,dx\right)$$

Using by parts

$$=\pi^2\times\frac{4}{\pi^2}=4$$

10. The slope of the tangent to a curve C : y =y(x) at any point [x, y) on it is $\frac{2e^{2x}-6e^{-x}+9}{2+9e^{-2x}}$. If C passes through the points $\left(0,\frac{1}{2}+\frac{\pi}{2\sqrt{2}}\right)$ and $\left(\alpha,\frac{1}{2}e^{2\alpha}\right)$ then e^{α} is equal to :

(A) $\frac{3+\sqrt{2}}{3-\sqrt{2}}$

(B) $\frac{3}{\sqrt{2}}\left(\frac{3+\sqrt{2}}{3-\sqrt{2}}\right)$

(C) $\frac{1}{\sqrt{2}}\left(\frac{\sqrt{2}+1}{\sqrt{2}-1}\right)$

(D) $\frac{\sqrt{2}+1}{\sqrt{2}-1}$

Official Ans. by NTA (B)

Sol. $\frac{dy}{dx}=\frac{2e^{2x}-6e^{-x}+9}{2+9e^{-2x}}$

$$\frac{dy}{dx}=e^{2x}-\frac{6e^{x}}{2e^{2x}+9}$$

$$y=\frac{e^{2x}}{2}-\tan^{-1}\left(\frac{\sqrt{2}e^{x}}{3}\right)+c$$

If C passes through the point $\left(0,\frac{1}{2}+\frac{\pi}{2\sqrt{2}}\right)$

$$c=-\frac{\pi}{4}-\tan^{-1}\frac{\sqrt{2}}{3}$$

Again C passes through the point $\left(\alpha,\frac{1}{2}e^{2\alpha}\right)$

then $e^{\alpha}=\frac{3}{\sqrt{2}}\left(\frac{3+\sqrt{2}}{3-\sqrt{2}}\right)$

11. The general solution of the differential equation $(x-y^2)dx+y(5x+y^2)dy=0$ is :

(A) $(y^2+x)^4=C|(y^2+2x)^3|$

(B) $(y^2+2x)^4=C|(y^2+x)^3|$

(C) $|(y^2+x)^3|=C(2y^2+x)^4$

(D) $|(y^2+2x)^3|=C(2y^2+x)^4$

Official Ans. by NTA (A)

Sol. $(x-y^2)dx+y(5x+y^2)dy=0$

$$\frac{dy}{dx}=\frac{y^2-x}{y(5x+y^2)}.\ \text{Let } y^2=v$$

$$\frac{2ydy}{dx}=2\left(\frac{y^2-x}{5x+y^2}\right)$$

$$\frac{dv}{dx}=2\left(\frac{v-x}{5x+v}\right) \qquad v=kx$$

$$k+x\frac{dk}{dx}=2\left(\frac{kx-x}{5x+kx}\right)$$

$$x\frac{dk}{dx}=-\frac{(k^2+3k+2)}{k+5}$$

$$\int\frac{(5+k)}{(k+1)(k+2)}dk=\int-\frac{dx}{x}$$

$$\int\left(\frac{4}{k+1}-\frac{3}{k+2}\right)dk=-\int\frac{dx}{x}$$

$$4\ln(k+1)-3\ln(k+2)=-\ln x+\ln c$$

$$\frac{(k+1)^4}{(k+2)^3}=-\ln x+\ln c$$

$$c(y^2+2x)^3=(y^2+x)^4$$

12. A line, with the slope greater than one, passes through the point A(4, 3) and intersects the line $x - y - 2 = 0$ at the point B. If the length of the line segment AB is $\frac{\sqrt{29}}{3}$, then B also lies on the line :

(A) $2x + y = 9$ (B) $3x - 2y = 7$

(C) $x + 2y = 6$ (D) $2x - 3y = 7$

Official Ans. by NTA (C)

Sol. Let $B(x_1, x_1 - 2)$

$$\sqrt{(x_1-4)^2+(x_1-2-3)^2} = \frac{\sqrt{29}}{3}$$

Squaring on both side

$18x_1^2 - 162x_1 + 340 = 0$

$x_1 = \frac{51}{9}$ or $x_1 = \frac{10}{3}$

$y_1 = \frac{33}{9}$ or $y_1 = \frac{4}{3}$

Option (C) will satisfy $\left(\frac{10}{3}, \frac{4}{3}\right)$

13. Let the locus of the centre (α, β), $\beta > 0$, of the circle which touches the circle $x^2 + (y - 1)^2 = 1$ externally and also touches the x-axis be L. Then the area bounded by L and the line $y = 4$ is :

(A) $\frac{32\sqrt{2}}{3}$ (B) $\frac{40\sqrt{2}}{3}$ (C) $\frac{64}{3}$ (D) $\frac{32}{3}$

Official Ans. by NTA (C)

Sol. $(\alpha - 0)^2 + (\beta - 1)^2 = (\beta + 1)^2$

$\alpha^2 = 4\beta$

$x^2 = 4y$

$$A = 2\int_0^4 \left(4 - \frac{x^2}{4}\right) dx = \frac{64}{3}$$

14. Let P be the plane containing the straight line $\frac{x-3}{9} = \frac{y+4}{-1} = \frac{z-7}{-5}$ and perpendicular to the plane containing the straight lines $\frac{x}{2} = \frac{y}{3} = \frac{z}{5}$ and $\frac{x}{3} = \frac{y}{7} = \frac{z}{8}$. If d is the distance of P from the point $(2, -5, 11)$, then d^2 is equal to :

(A) $\frac{147}{2}$ (B) 96 (C) $\frac{32}{3}$ (D) 54

Official Ans. by NTA (D)

Sol. $a(x - 3) + b(y + 4) + c(z - 7) = 0$

P : $9a - b - 5c = 0$

$-11a - b + 5c = 0$

After solving DR's $\propto (1, -1, 2)$

Equation of plane

$x - y + 2z = 21$

$d = \frac{8}{\sqrt{6}}$

$d^2 = \frac{32}{3}$

15. Let ABC be a triangle such that $\overrightarrow{BC} = \vec{a}$, $\overrightarrow{CA} = \vec{b}$, $\overrightarrow{AB} = \vec{c}$, $|\vec{a}| = 6\sqrt{2}$, $|\vec{b}| = 2\sqrt{3}$ and $\vec{b} \cdot \vec{c} = 12$

Consider the statements :

(S1) : $\left|\left(\vec{a} \times \vec{b}\right) + \left(\vec{c} \times \vec{b}\right)\right| - |\vec{c}| = 6(2\sqrt{2} - 1)$

(S2) : $\angle ABC = \cos^{-1}\left(\sqrt{\frac{2}{3}}\right)$. Then

(A) both (S1) and (S2) are true

(B) only (S1) is true

(C) only (S2) is true

(D) both (S1) and (S2) are false

Official Ans. by NTA (D)

Sol. $\vec{a} + \vec{b} + \vec{c} = 0$

$\vec{b} + \vec{c} = -\vec{a}$

$|\vec{b}|^2 + |\vec{c}|^2 + 2\vec{b} \cdot \vec{c} = |\vec{a}|^2$

$|\vec{c}|^2 = 36$

$|\vec{c}| = 6$

S1 : $|\vec{a} \times \vec{b} + \vec{c} \times \vec{b}| - |\vec{c}|$

$\left|\left(\vec{a} + \vec{c}\right) \times \vec{b}\right| - |\vec{c}|$

$|-\vec{b} \times \vec{b}| - |\vec{c}|$

$0 - 6 = -6$

S2 : $\vec{a} + \vec{b} + \vec{c} = 0$

$\vec{b} + \vec{c} = -\vec{a}$

$|\vec{a}|^2 + |\vec{b}|^2 - 2|\vec{a}||\vec{b}|\cos(\angle ACB) = |\vec{c}|^2$

$\cos(\angle ACB) = \sqrt{\frac{2}{3}}$

16. If the sum and the product of mean and variance of a binomial distribution are 24 and 128 respectively, then the probability of one or two successes is :

(A) $\frac{33}{2^{32}}$ (B) $\frac{33}{2^{29}}$ (C) $\frac{33}{2^{28}}$ (D) $\frac{33}{2^{27}}$

Official Ans. by NTA (C)

Sol. $np + npq = 24$ …(1)

$np \cdot npq = 128$ …(2)

Solving (1) and (2) :

We get $p = \frac{1}{2}$, $q = \frac{1}{2}$, $n = 32$.

Now,

$P(X = 1) + P(X = 2)$

$= {}^{32}C_1\, pq^{31} + {}^{32}C_2\, p^2 q^{30}$

$= \frac{33}{2^{28}}$

17. If the numbers appeared on the two throws of a fair six faced die are α and β, then the probability that $x^2 + \alpha x + \beta > 0$, for all $x \in R$, is :

(A) $\frac{17}{36}$ (B) $\frac{4}{9}$ (C) $\frac{1}{2}$ (D) $\frac{19}{36}$

Official Ans. by NTA (A)

Sol. $x^2 + \alpha x + \beta > 0, \forall\, x \in R$

$D = \alpha^2 - 4\beta < 0$

$\alpha^2 < 4\beta$

Total cases $= 6 \times 6 = 36$

Fav. cases $= \beta = 1, \alpha = 1$

$\beta = 2, \alpha = 1, 2$

$\beta = 3, \alpha = 1, 2, 3$

$\beta = 4, \alpha = 1, 2, 3$

$\beta = 5, \alpha = 1, 2, 3, 4$

$\beta = 6, \alpha = 1, 2, 3, 4$

Total favourable cases = 17

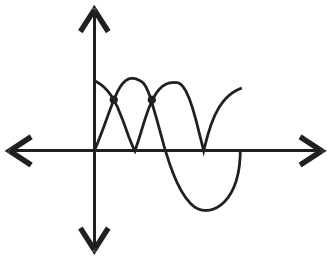
$P(x) = \frac{17}{36}$

18. The number of solutions of $|\cos x| = \sin x$, such that $-4\pi \le x \le 4\pi$ is :

(A) 4 (B) 6 (C) 8 (D) 12

Official Ans. by NTA (C)

Sol.



2 solutions in $(0, 2\pi)$

So, 8 solutions in $[-4\pi, 4\pi]$

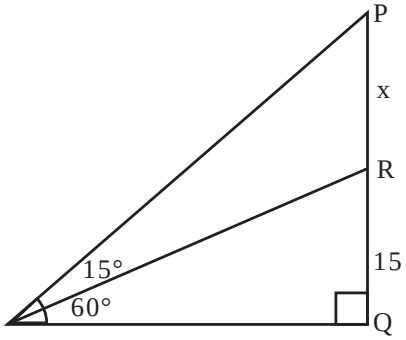
19. A tower PQ stands on a horizontal ground with base Q on the ground. The point R divides the tower in two parts such that QR = 15 m. If from a point A on the ground the angle of elevation of R is 60° and the part PR of the tower subtends an angle of 15° at A, then the height of the tower is :

(A) $5(2\sqrt{3}+3)$ m (B) $5(\sqrt{3}+3)$ m

(C) $10(\sqrt{3}+1)$ m (D) $10(2\sqrt{3}+1)$ m

Official Ans. by NTA (A)

Sol.



$\frac{15}{AQ} = \tan 60°$ …(1)

$\frac{15+x}{AQ} = \tan 75°$ …(2)

$\frac{(1)}{(2)} \Rightarrow x = 10\sqrt{3}$

So, $PQ = 5(2\sqrt{3}+3)$ m

20. Which of the following statements is a tautology ?

(A) $((\sim p) \vee q) \Rightarrow p$ (B) $p \Rightarrow ((\sim p) \vee q)$

(C) $((\sim p) \vee q) \Rightarrow q$ (D) $q \Rightarrow ((\sim p) \vee q)$

Official Ans. by NTA (D)

Sol.

p	q	~p	~q	~pvq
T	T	F	F	T
T	F	F	T	F
F	T	T	F	T
F	F	T	T	T

options	1	2	3	4
	T	T	T	T
	T	F	T	T
	F	T	T	T
	F	T	F	T

SECTION-B

1. Let $A = \begin{bmatrix} 2 & -1 & -1 \\ 1 & 0 & -1 \\ 1 & -1 & 0 \end{bmatrix}$ and $B = A - I$. If $\omega = \frac{\sqrt{3}i - 1}{2}$, then the number of elements in the set $\{n \in \{1, 2, \ldots, 100\} : A^n + (\omega B)^n = A + B\}$ is equal to _____.

Official Ans. by NTA (17)

Sol. $A = \begin{bmatrix} 2 & -1 & -1 \\ 1 & 0 & -1 \\ 1 & -1 & 0 \end{bmatrix} \Rightarrow A^2 = A \Rightarrow A^n = A.$

$\forall\ n \in \{1, 2, \ldots, 100\}$

Now, $B = A - I = \begin{bmatrix} 1 & -1 & -1 \\ 1 & -1 & -1 \\ 1 & -1 & -1 \end{bmatrix}$

$B^2 = -B$

$\Rightarrow B^3 = -B^2 = B$

$\Rightarrow B^5 = B$

$\Rightarrow B^{99} = B$

Also, $\omega^{3k} = 1$

So, n = common of $\{1, 3, 5, \ldots, 99\}$ and $\{3, 6, 9, \ldots, 99\} = 17$

2. The letters of the word 'MANKIND' are written in all possible orders and arranged in serial order as in an English dictionary. Then the serial number of the word 'MANKIND' is _____.

Official Ans. by NTA (1492)

Sol.

M	A	N	K	I	N	D

$$\left(\frac{4\times 6!}{2!}\right) + (5!\times 0) + \left(\frac{4!\times 3}{2!}\right) + (3!\times 2) + (2!\times 1) + (1!\times 1) + (0!\times 0) + 1 = 1492$$

3. If the maximum value of the term independent of t in the expansion of $\left(t^2 x^{\frac{1}{5}} + \frac{(1-x)^{\frac{1}{10}}}{t}\right)^{15}$, $x \geq 0$, is K, then 8K is equal to _____.

Official Ans. by NTA (6006)

Sol. $\left(t^2 x^{\frac{1}{5}} + \frac{(1-x)^{\frac{1}{10}}}{t}\right)^{15}$

$T_{r+1} = {}^{15}C_r \left(t^2 x^{\frac{1}{5}}\right)^{15-r} \cdot \frac{(1-x)^{\frac{r}{10}}}{t^r}$

For independent of t,

$30 - 2r - r = 0$

$\Rightarrow r = 10$

So, Maximum value of ${}^{15}C_{10}\ x(1 - x)$ will be at $x = \frac{1}{2}$

i.e. 6006

4. Let a, b be two non-zero real numbers. If p and r are the roots of the equation $x^2 - 8ax + 2a = 0$ and q and s are the roots of the equation $x^2 + 12bx + 6b = 0$, such that $\frac{1}{p}, \frac{1}{q}, \frac{1}{r}, \frac{1}{s}$ are in A.P., then $a^{-1} - b^{-1}$ is equal to _____.

Official Ans. by NTA (38)

Sol. $x^2 - 8ax + 2a = 0$ $\quad$ $x^2 + 12bx + 6b = 0$

$p + r = 8a$ $\quad$ $q + s = -12b$

$pr = 2a$ $\quad$ $qs = 6b$

$\frac{1}{p}+\frac{1}{r} = 4$ $\quad$ $\frac{1}{q}+\frac{1}{s} = -2$

$\frac{2}{q} = 4$ $\quad$ $\frac{2}{r} = -2$

$q = \frac{1}{2}$ $\quad$ $r = -1$

$p = \frac{1}{5}$ $\quad$ $s = \frac{-1}{4}$

Now, $\frac{1}{a}-\frac{1}{b}=\frac{2}{pr}-\frac{6}{qs} = 38$

5. Let $a_1 = b_1 = 1$, $a_n = a_{n-1} + 2$ and $b_n = a_n + b_{n-1}$ for every natural number $n \geq 2$. Then $\sum_{n=1}^{15} a_n \cdot b_n$ is equal to _____.

Official Ans. by NTA (27560)

Sol. $a_1 = b_1 = 1$

$a_2 = a_1 + 2 = 3$

$a_3 = a_2 + 2 = 5$

$a_4 = a_2 + 2 = 7$

$\Rightarrow a_n = 2n - 1$

$b_2 = a_1 + b_1 = 4$

$b_3 = a_3 + b_2 = 9$

$b_4 = a_4 + b_3 = 16$

$b_n = n^2$

$\sum_{n=1}^{15} a_n b_n$

$\sum_{n=1}^{15} (2n-1)n^2$

$\sum_{n=1}^{15} (2n^3 - n^2)$

$= 2\frac{n^2(n+1)^2}{4} - \frac{n(n+1)(2n+1)}{6}$

Put $n = 15$

$= \frac{2\times 225\times 16\times 16}{4} - \frac{15\times 16\times 31}{6}$

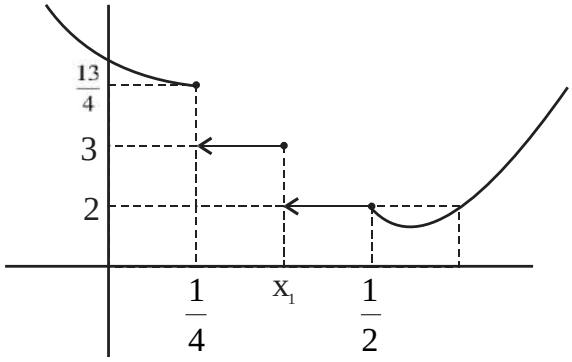
$= 27560$

6. Let $f(x) = \begin{cases} |4x^2 - 8x + 5|, \text{ if } 8x^2 - 6x + 1 \geq 0 \\ [4x^2 - 8x + 5], \text{ if } 8x^2 - 6x + 1 < 0 \end{cases}$,

where $[\alpha]$ denotes the greatest integer less than or equal to α. Then the number of points in R where f is not differentiable is _____.

Official Ans. by NTA (3)

Sol.



ND at $\frac{1}{4}, x_1, \frac{1}{2}$

7. If $\lim_{n\to\infty} \frac{(n+1)^{k-1}}{n^{k+1}}$ $[(nk + 1) + (nk + 2) + \ldots + (nk + n)] = 33$. $\lim_{n\to\infty} \frac{1}{n^{k+1}} \cdot [1^k + 2^k + 3^k + \ldots + n^k]$, then the integral value of k is equal to _____.

Official Ans. by NTA (5)

Sol. LHS

$\lim_{n\to\infty} \frac{(n+1)^{k-1}}{n^{k+1}} [nk \cdot n + 1 + 2 + \ldots + n]$

$= \lim_{n\to\infty} \frac{(n+1)^{k-1}}{n^{k+1}} \cdot \left[n^2 k + \frac{n(n+1)}{2}\right]$

$= \lim_{n\to\infty} \frac{(n+1)^{k-1} \cdot n^2 \left(k + \frac{\left(1+\frac{1}{n}\right)}{2}\right)}{n^{k+1}}$

$\Rightarrow \lim_{n\to\infty} \left(1+\frac{1}{n}\right)\left(k + \frac{\left(1+\frac{1}{n}\right)}{2}\right)$

$\Rightarrow \left(k + \frac{1}{2}\right)$

RHS

$\Rightarrow \lim\limits_{n\to\infty}\frac{1}{n^{k+1}}(1^k+2^k+...+n^k)=\frac{1}{k+1}$

LHS = RHS

$\Rightarrow k+\frac{1}{2}=33\cdot\frac{1}{k+1}$

$\Rightarrow (2k+1)(k+1)=66$

$\Rightarrow (k-5)(2k+13)=0$

$\Rightarrow k=5 \text{ or } -\frac{13}{2}$

8. Let the equation of two diameters of a circle $x^2+y^2-2x+2fy+1=0$ be $2px-y=1$ and $2x+py=4p$. Then the slope $m \in (0, \infty)$ of the tangent to the hyperbola $3x^2-y^2=3$ passing through the centre of the circle is equal to _____.

Official Ans. by NTA (2)

Sol. $2p+f-1=0 \quad ...(1)$

$2-pf-4p=0 \quad ...(2)$

$2=p(f+4)$

$p=\frac{2}{f+4}$

$2p=1-f$

$\frac{4}{f+4}=1-f$

$f^2+3f=0$

$f=0 \text{ or } -3$

Hyperbola $3x^2-y^2=3$, $x^2-\frac{y^2}{3}=1$

$y=mx\pm\sqrt{m^2-3}$

It passes (1, 0)

$o=m\pm\sqrt{m^2-3}$

m tends ∞

It passes (1, 3)

$3=m\pm\sqrt{m^2-3}$

$(3-m)^2=m^2-3$

$m=2$

9. The sum of diameters of the circles that touch (i) the parabola $75x^2=64(5y-3)$ at the point $\left(\frac{8}{5},\frac{6}{5}\right)$ and (ii) the y-axis, is equal to _____.

Official Ans. by NTA (10)

Sol. $x^2=\frac{64.5}{75}\left(y-\frac{3}{5}\right)$

equation of tangent at $\left(\frac{8}{5},\frac{6}{5}\right)$

$x\,.\,\frac{8}{5}=\frac{64}{15}\left(\frac{y+\frac{6}{5}}{2}-\frac{3}{5}\right)$

$3x-4y=0$

equation of family of circle is

$\left(x-\frac{8}{5}\right)^2+\left(y-\frac{6}{5}\right)^2+\lambda(3x-4y)=0$

It touches y axis so $f^2=c$

$x^2+y^2+x\left(3\lambda-\frac{16}{5}\right)+y\left(-4\lambda-\frac{12}{5}\right)+4=0$

$\frac{\left(4\lambda+\frac{12}{5}\right)^2}{4}=4$

$\lambda=\frac{2}{5} \text{ or } \lambda=-\frac{8}{5}$

$\lambda=\frac{2}{5}, \quad r=1$

$\lambda=-\frac{8}{5}, \quad r=4$

$d_1+d_2=10$

10. The line of shortest distance between the lines $\frac{x-2}{0}=\frac{y-1}{1}=\frac{z}{1}$ and $\frac{x-3}{2}=\frac{y-5}{2}=\frac{z-1}{1}$ makes an angle of $\cos^{-1}\left(\sqrt{\frac{2}{27}}\right)$ with the plane P : $ax-y-z=0$, $(a>0)$. If the image of the point $(1, 1, -5)$ in the plane P is (α, β, γ), then $\alpha+\beta-\gamma$ is equal to _____.

Official Ans. by NTA (3)

Sol. DR's of line of shortest distance

$\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\0&1&1\\2&2&1\end{vmatrix}=-\hat{i}+2\hat{j}-2\hat{k}$

angle between line and plane is $\cos^{-1}\sqrt{\frac{2}{27}}=\alpha$

$\cos\alpha=\sqrt{\frac{2}{27}}$, $\sin\alpha=\frac{5}{3\sqrt{3}}$

DR's normal to plane $(1, -1, -1)$

$\sin\alpha=\left|\frac{-a-2+2}{\sqrt{4+4+1}\sqrt{a^2+1+1}}\right|=\frac{5}{3\sqrt{3}}$

$\sqrt{3}\,|a|=5\sqrt{a^2+2}$

$3a^2=25a^2+50$

No value of (a)

FINAL JEE–MAIN EXAMINATION – JUNE, 2022

(Held On Saturday 25th June, 2022) **TIME : 3:00 PM to 6:00 PM**

PHYSICS

TEST PAPER WITH SOLUTION

SECTION-A

1. Given below are two statements. One is labelled as **Assertion A** and the other is labelled as **Reason R.**

Assertion A : Two identical balls A and B thrown with same velocity 'u' at two different angles with horizontal attained the same range R. If A and B reached the maximum height h_1 and h_2 respectively, then $R = 4\sqrt{h_1h_2}$

Reason R: Product of said heights.

$$h_1h_2 = \left(\frac{u^2\sin^2\theta}{2g}\right).\left(\frac{u^2\cos^2\theta}{2g}\right)$$

Choose the CORRECT answer :

(A) Both A and R are true and R is the correct explanation of A.

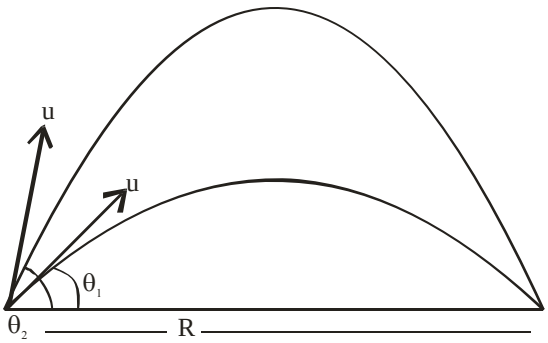
(B) Both A and R are true but R is NOT the correct explanation of A.

(C) A is true but R is false

(D) A is false but R is true

Official Ans. by NTA (A)

Sol. For same range $\theta_1 + \theta_2 = 90^0$



$$h_1 = \frac{u^2\sin^2\theta_1}{2g} \qquad h_2 = \frac{u^2\sin^2\theta_2}{2g}$$

$$h_1h_2 = \frac{u^2\sin^2\theta_1}{2g} \times \frac{u^2\sin^2\theta_2}{2g}$$

$$\theta_2 = 90 - \theta_1$$

$$h_1h_2 = \frac{u^2\sin^2\theta_1}{2g}.\frac{u^2\cos^2\theta_1}{2g}$$

$$= \left[\frac{u^2\sin\theta_1\cos\theta_1}{2g}\right]^2$$

$$= \left[\frac{u^2\sin\theta_1\cos\theta_1}{2g} \times \frac{2}{2}\right]^2 = \frac{R^2}{16}$$

$$R = 4\sqrt{h_1h_2}$$

So R is correct explanation of A

2. Two buses P and Q start from a point at the same time and move in a straight line and their positions are represented by $X_P(t) = \alpha t + \beta t^2$ and $X_Q(t) = ft - t^2$. At what time, both the buses have same velocity ?

(A) $\frac{\alpha - f}{1+\beta}$ (B) $\frac{\alpha + f}{2(\beta - 1)}$

(C) $\frac{\alpha + f}{2(1+\beta)}$ (D) $\frac{f - \alpha}{2(1+\beta)}$

Official Ans. by NTA (D)

Sol. $X_P(t) = \alpha t + \beta t^2$ $X_Q = ft - t^2$

$V_P(t) = \alpha + 2\beta t$ $V_Q = f - 2t$

$V_P = V_Q$

$\alpha + 2\beta t = f - 2t$

$$t = \frac{f - \alpha}{2\beta + 2}$$

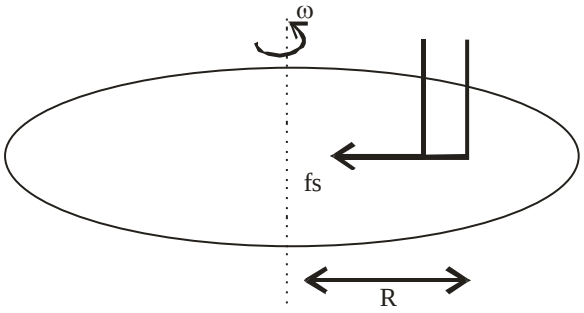
3. A disc with a flat small bottom beaker placed on it at a distance R from its center is revolving about an axis passing through the center and perpendicular to its plane with an angular velocity ω. The coefficient of static friction between the bottom of the beaker and the surface of the disc is μ. The beaker will revolve with the disc if :

(A) $R \le \frac{\mu g}{2\omega^2}$ (B) $R \le \frac{\mu g}{\omega^2}$

(C) $R \ge \frac{\mu g}{2\omega^2}$ (D) $R \ge \frac{\mu g}{\omega^2}$

Official Ans. by NTA (B)

Sol. For beaker to move with disc



$f_s = m\omega^2 R$

We know that $f_S \le f_{s\max}$

$m\omega^2 R \le \mu mg$

$R \le \frac{\mu g}{\omega^2}$

4. A solid metallic cube having total surface area 24 m^2 is uniformly heated. If its temperature is increased by 10°C, calculate the increase in volume of the cube $\left(\text{Given} : \alpha = 5.0\times10^{-4}\,^\circ C^{-1}\right)$

(A) 2.4×10^6 cm^3

(B) 1.2×10^5 cm^3

(C) 6.0×10^4 cm^3

(D) 4.8×10^5 cm^3

Official Ans. by NTA (B)

Sol. Increase in volume $\Delta V = \gamma V_0 \Delta T$

$\gamma = 3\alpha$

So $\Delta V = (3\alpha) V_0 \Delta T$

Total surface area = $6a^2$, where a is side length

$24 = 6a^2$ a= 2m

Volume $V_0 = (2)^3 = 8m^3$

$\Delta V = \left(3\times5\times10^{-4}\right)(8)\times10$

$= 1.2\times10^5 cm^3$

5. A copper block of mass 5.0 kg is heated to a temperature of 500°C and is placed on a large ice block. What is the maximum amount of ice that can melt? [Specific heat of copper: $0.39\,J\,g^{-1}\,^\circ C^{-1}$ and latent heat of fusion of water : 335 J g^{-1}]

(A) 1.5 kg (B) 5.8 kg

(C) 2.9 kg (D) 3.8 kg

Official Ans. by NTA (C)

Sol. Heat given by block to get 0ºC temperature

$\Delta Q_1 = 5\times\left(0.39\times10^3\right)\times(500-0)$

$= 975\times10^3 J$

Heat absorbed by ice to melt m mass

$\Delta Q_2 = m\times\left(335\times10^3\right)J$

$\Delta Q_1 = \Delta Q_2$

$m\times\left(335\times10^3\right) = 975\times10^3$

$m = \frac{975}{335} = 2.910\,kg$

6. The ratio of specific heats $\left(\frac{C_P}{C_V}\right)$ in terms of degree of freedom (f) is given by:

(A) $\left(1+\frac{f}{3}\right)$ (B) $\left(1+\frac{2}{f}\right)$

(C) $\left(1+\frac{f}{2}\right)$ (D) $\left(1+\frac{1}{f}\right)$

Official Ans. by NTA (B)

Sol. Molar heat capacity at constant volume $C_V = \frac{fR}{2}$

where f is degree of freedom.

Molar heat capacity at constant pressure can be written as $C_P = R + C_V = R + \frac{fR}{2} = \left(1 + \frac{f}{2}\right)R$

So $\frac{C_P}{C_v} = 1 + \frac{2}{f}$

7. For a particle in uniform circular motion, the acceleration $\vec{a}$ at any point P(R,θ) on the circular path of radius R is (when θ is measured from the positive x –axis and v is uniform speed) :

(A) $-\frac{v^2}{R}\sin\theta\hat{i} + \frac{v^2}{R}\cos\theta\hat{j}$

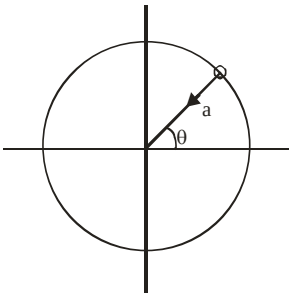
(B) $-\frac{v^2}{R}\cos\theta\hat{i} + \frac{v^2}{R}\sin\theta\hat{j}$

(C) $-\frac{v^2}{R}\cos\theta\hat{i} - \frac{v^2}{R}\sin\theta\hat{j}$

(D) $-\frac{v^2}{R}\hat{i} + \frac{v^2}{R}\hat{j}$

Official Ans. by NTA (C)

Sol. $a = |\vec{a}| = \frac{V^2}{R}$



$\vec{a} = -a\cos\theta\hat{i} - a\sin\theta\hat{j}$

$= -\frac{V^2}{R}\cos\theta\hat{i} - \frac{V^2}{R}\sin\theta\hat{j}$

8. Two metallic plates form a parallel plate capacitor. The distance between the plates is 'd'. A metal sheet of thickness $\frac{d}{2}$ and of area equal to area of each plate is introduced between the plates. What will be the ratio of the new capacitance to the original capacitance of the capacitor ?

(A) 2:1 (B) 1:2

(C) 1:4 (D) 4:1

Official Ans. by NTA (A)

Sol. $C_1 = \frac{\epsilon_0 A}{d}$

$C_2 = \frac{\epsilon_0 A}{\frac{d}{2} + \frac{d/2}{\infty}} = \frac{2\epsilon_0 A}{d}$

$\frac{C_2}{C_1} = \frac{2}{1}$

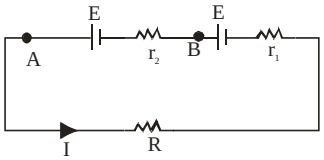
9. Two cells of same emf but different internal resistances r_1 and r_2 are connected in series with a resistance R. The value of resistance R, for which the potential difference across second cell is zero, is

(A) $r_2 - r_1$ (B) $r_1 - r_2$

(C) r_1 (D) r_2

Official Ans. by NTA (A)

Sol. $I = \frac{2E}{R + r_1 + r_2}$ **.....(i)**



But $V_A - V_B = E - Ir_2 = 0$

$\Rightarrow I = \frac{E}{r_2}$ ....(ii)

Comparing values of I from (i) and (ii)

$\frac{E}{r_2} = \frac{2E}{R + r_1 + r_2}$

$\Rightarrow R = r_2 - r_1$

10. Given below are two statements:

Statement – I : Susceptibilities of paramagnetic and ferromagnetic substances increase with decrease in temperature.

Statement – II: Diamagnetism is a result of orbital motions of electrons developing magnetic moments opposite to the applied magnetic field.

Choose the **CORRECT** answer from the options given below : -

(A) Both statement – I and statement -II are true.

(B) Both statement – I and Statement – II are false.

(C) Statement – I is true but statement – II is false.

(D) Statement-I is false but Statement-II is true.

Official Ans. by NTA (A)

Sol. According to curie's law, magnetic susceptibility is inversely proportional to temperature for a fixed value of external magnetic field i.e. $\chi = \frac{C}{T}$.

The same is applicable for ferromagnet & the relation is given as $\chi = \frac{C}{T - T_C}$ (T_C is curie temperature)

Diamagnetism is due to non-cooperative behaviour of orbiting electrons when exposed to external magnetic field.

Hence option (A).

11. A long solenoid carrying a current produces a magnetic field B along its axis. If the current is doubled and the number of turns per cm is halved, the new value of magnetic field will be equal to

(A) B (B) 2 B

(C) 4 B (D) $\frac{B}{2}$

Official Ans. by NTA (A)

Sol. $B_1 = \mu_0 nI$

$B_2 = \mu_0\left(\frac{n}{2}\right)(2I)$

$\Rightarrow B_1 = B_2$

12. A sinusoidal voltage V(t) = 210 sin 3000t volt is applied to a series LCR circuit in which L = 10 mH, C = 25 μF and R = 100Ω. The phase difference (Φ) between the applied voltage and resultant current will be :

(A) $\tan^{-1}(0.17)$ (B) $\tan^{-1}(9.46)$

(C) $\tan^{-1}(0.30)$ (D) $\tan^{-1}(13.33)$

Official Ans. by NTA (A)

Sol. $X_L = 10^{-2} \times 3000 = 30\,\Omega$

$X_C = \frac{1}{3000 \times 25 \times 10^{-6}} = \frac{40}{3}\Omega$

$X = X_L - X_C$

$= 30 - \frac{40}{3} = \frac{50}{3}$

$\tan\delta = \frac{X}{R} = \frac{50}{3 \times 100} = \frac{1}{6}$

$\delta = \tan^{-1}\left(\frac{1}{6}\right) = \tan^{-1}(0.17)$

13. The electromagnetic waves travel in a medium at a speed of 2.0×10^8 m/s.The relative permeability of the medium is 1.0. The relative permittivity of the medium will be:

(A) 2.25 (B) 4.25

(C) 6.25 (D) 8.25

Official Ans. by NTA (A)

Sol. $V = 2 \times 10^8 \text{ m/s}$

$C = 3 \times 10^8 \text{ m/s}$

$\frac{C}{V} = \sqrt{\mu_r \epsilon_r}$

$\frac{9}{4} = 1 \times \epsilon_r$

$\epsilon_r = \frac{9}{4} = 2.25$

14. The interference pattern is obtained with two coherent light sources of intensity ratio 4 :1. And the ratio $\frac{I_{max}+I_{min}}{I_{max}-I_{min}}$ is $\frac{5}{x}$. Then, the value of x will be equal to :

(A) 3 (B) 4

(C) 2 (D) 1

Official Ans. by NTA (B)

Sol. $\frac{I_1}{I_2}=4$

$$\frac{I_{max}}{I_{min}}=\left[\frac{\sqrt{I_1}+\sqrt{I_2}}{\sqrt{I_1}-\sqrt{I_2}}\right]^2$$

$$\frac{I_{max}}{I_{min}}=\left[\frac{2\sqrt{I_2}+\sqrt{I_2}}{2\sqrt{I_2}-\sqrt{I_2}}\right]^2$$

$$\frac{I_{max}}{I_{min}}=9$$

$$\frac{I_{max}+I_{min}}{I_{max}-I_{min}}=\frac{10}{8}$$

$$\frac{5}{x}=\frac{10}{8}$$

x = 4

15. A light whose electric field vectors are completely removed by using a good Polaroid, allowed to incident on the surface of the prism at Brewster's angle. Choose the most suitable option for the phenomenon related to the prism.

(A) Reflected and refracted rays will be perpendicular to each other

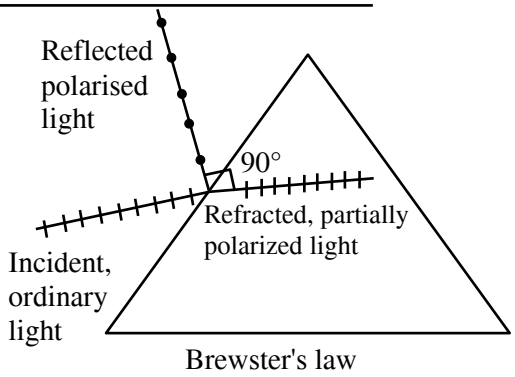
(B) Wave will propagate along the surface of prism

(C) No refraction, and there will be total reflection of light.

(D) No reflection and there will be total transmission of light.

Official Ans. by NTA (D)

Sol.



Brewster's law

But as the incident light electric field vectors are completely removed so there will be no reflection and there will be total transmission of light, explained by an experiment in NCERT.

[Reference NCERT Part-2 Pg-380, (A special case of total transmission)]

Note : Since direction of polarization is not mentioned hence most suitable option (D) corresponding to case in which electric field is absent perpendicular to plane consisting incident and normal.

16. A proton, a neutron, an electron and an α-particle have same energy. If $\lambda_p,\lambda_n,\lambda_e$ and λ_α are the de Broglie's wavelengths of proton, neutron, electron and α particle respectively, then choose the correct relation from the following :

(A) $\lambda_p=\lambda_n>\lambda_e>\lambda_\alpha$

(B) $\lambda_\alpha<\lambda_n<\lambda_p<\lambda_e$

(C) $\lambda_e<\lambda_p=\lambda_n>\lambda_\alpha$

(D) $\lambda_e=\lambda_p=\lambda_n=\lambda_\alpha$

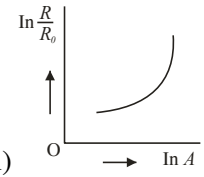
Official Ans. by NTA (B)

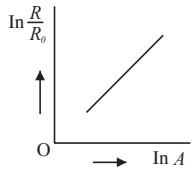
Sol. $\lambda=\frac{h}{\sqrt{2Em}}$

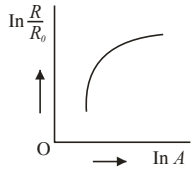
$$\lambda\propto\frac{1}{\sqrt{m}}$$

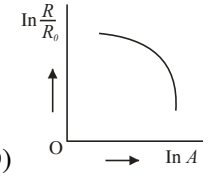
$\therefore$ $\lambda_e>\lambda_p>\lambda_n>\lambda_\alpha$

17. Which of the following figure represents the variation of $\ln\left(\frac{R}{R_0}\right)$ with ln A(If R = radius of a nucleus and A = its mass number)

(A) 

(B) 

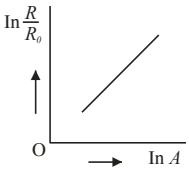
(C) 

(D) 

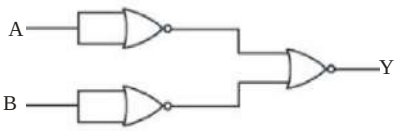
Official Ans. by NTA (B)

Sol. $R = R_0 A^{\frac{1}{3}}$

$$\ln\frac{R}{R_0} = \frac{1}{3}\ln A$$



18. Identify the logic operation performed by the given circuit :



(A) AND gate (B) OR gate

(C) NOR gate (D) NAND gate

Official Ans. by NTA (A)

Sol. $= \left[\overline{\left[\overline{A+A}\right]+\left[\overline{B+B}\right]}\right]$

$Y = \overline{\overline{A}+\overline{B}}$ (D' MORGAN LAW)

$Y = AB$

19. Match List I with List II

List -I		List – II	
A	Facsimile	I.	Static Document Image
B.	Guided media Channel	II.	Local Broadcast Radio
C.	Frequency Modulation	III.	Rectangular wave
D.	Digital Signal	IV.	Optical Fiber

Choose the correct answer from the following options :

(A) A –IV, B-III, C-II, D-I

(B)A-I, B-IV, C-II, D-III

(C) A –IV, B-II, C-III, D-I

(D) A-I, B-II, C-III, D-IV

Official Ans. by NTA (B)

Sol. Question based on the theory given in NCERT.

20. If n represents the actual number of deflections in a converted galvanometer of resistance G and shunt resistance S. Then the total current I when its figure of merit is K will be :

(A) $\frac{KS}{(S+G)}$ (B) $\frac{(G+S)}{nKS}$

(C) $\frac{nKS}{(G+S)}$ (D) $\frac{nK(G+S)}{S}$

Official Ans. by NTA (D)

Sol.

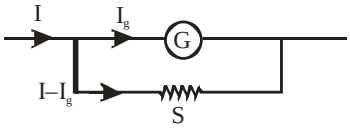


Figure of merit $\frac{I_g}{\theta}=K$

$I_g = Kn$

$I=\frac{I_g}{s}(G+S)$

$I=\frac{nK}{S}(G+S)$

SECTION-B

1. For $z=a^2x^3y^{\frac{1}{2}}$, where 'a' is a constant. If percentage error in measurement of 'x' and 'y' are 4% and 12%, respectively, then the percentage error for 'z' will be %.

Official Ans. by NTA (18)

Sol. $z=a^2x^3y^{1/2}$

$\frac{\Delta z}{z}=\frac{2\Delta a}{a}+\frac{3\Delta x}{x}+\frac{1}{2}\frac{\Delta y}{y}$

a is constant

$\frac{\Delta z}{z}\times 100=3(4\%)+\frac{1}{2}(12\%)=18\%$

2. A curved in a level road has a radius 75m. The maximum speed of a car turning this curved road can be 30 m/s without skidding. If radius of curved road is changed to 48 m and the coefficient of friction between the tyres and the road remains same, then maximum allowed speed would be__ m/s.

Official Ans. by NTA (24)

Sol. $f_{s\max}=\frac{mv^2}{R}$

$\mu mg=\frac{mv^2}{R}$

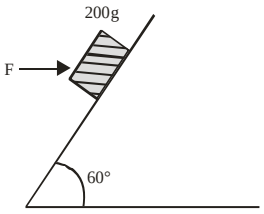
$v=\sqrt{\mu Rg}$

$\frac{v_2}{v_1}=\sqrt{\frac{R_2}{R_1}}$

$\frac{v_2}{30}=\sqrt{\frac{48}{75}}$

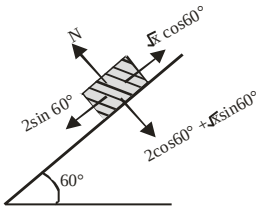
$v_2=24\,m/s$

3. A block of mass 200 g is kept stationary on a smooth inclined plane by applying a minimum horizontal force $F=\sqrt{x}N$ as shown in figure. The value of x = _____.



Official Ans. by NTA (12)

Sol. mg = 2N



$\sqrt{x}\frac{1}{2}=\frac{2\sqrt{3}}{2}$

x = 12

4. Moment of Inertia (M.I.) of four bodies having same mass 'M' and radius '2R' are as follows:

I_1= M.I. of solid sphere about its diameter

I_2 = M.I. of solid cylinder about its axis

I_3 = M.I. of solid circular disc about its diameter

I_4 = M.I. of thin circular ring about its diameter

If $2(I_2 + I_3) + I_4 = x.\ I_1$ then the value of x will be ____

Official Ans. by NTA (5)

Sol. $I_1 = \frac{2}{5}M(2R)^2 = \frac{8}{5}MR^2$

$I_1 = \frac{1}{2}M(2R)^2 = 2MR^2$

$I_3 = \frac{M(2R)^2}{4} = MR^2$

$I_4 = \frac{M(2R)^2}{2} = 2MR^2$

$2(I_2 + I_3) + I_4 = x\ I_1$

$8MR^2 = x\frac{8}{5}MR^2$

$x = 5$

5. Two satellites S_1 and S_2 are revolving in circular orbits around a planet with radius R_1 = 3200 km and R_2 = 800 km respectively. The ratio of speed of satellite S_1 to the speed of satellite S_2 in their respective orbits would be $\frac{1}{x}$ where x =

Official Ans. by NTA (2)

Sol. $V = \frac{GM}{r} \Rightarrow \frac{V_1}{V_2} = \sqrt{\frac{800}{3200}} = \frac{1}{2}$

6. When a gas filled in a closed vessel is heated by raising the temperature by 1°C, its pressure increase by 0.4%. The initial temperature of the gas is_____ K.

Official Ans. by NTA (250)

Sol. $pV = nRT$

$\Delta P.V = nR\Delta T$

$\Rightarrow \frac{\Delta P}{P} = \frac{\Delta T}{T} = \frac{0.4}{100}$

$\Rightarrow T = \frac{100 \times 1}{0.4} = 250K$

7. 27 identical drops are charged at 22V each. They combine to form a bigger drop. The potential of the bigger drop will be ____ V.

Official Ans. by NTA (198)

Sol. $q \rightarrow nq$

$n\frac{4}{3}\pi r^3 = \frac{4}{3}\pi(r')^3$

$\Rightarrow r' = n^{\frac{1}{3}}r$

$V = \frac{kq}{r} \propto \frac{n}{n^{1/3}} \propto n^{2/3} \propto 27^{2/3} \Rightarrow v' = 9V = 9 \times 22 = 198$

8. The length of a given cylindrical wire is increased to double of its original length. The percentage increase in the resistance of the wire will be ____%.

Official Ans. by NTA (300)

Sol. $V' = V$

$\ell' A = \ell A$

$2\ell A' = \ell A$

$A' = \frac{A}{2}$

$R = \rho\frac{\ell}{A}$ … (i)

$\ell' = 2\ell$

$A' = \frac{A}{2}$

$R' = \frac{\rho\ell'}{A'} = \frac{\rho 2\ell}{\frac{A}{2}}$

$R' = \frac{4\rho\ell}{A}$

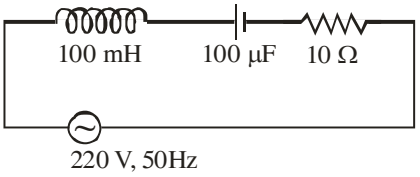
R' = 4R from equation (i)

% increase in resistance

$= \frac{R' - R}{R} \times 100 = \frac{4R - R}{R} \times 100$

= 300 %

9. In a series LCR circuit, the inductance, capacitance and resistance are L = 100mH, C = 100μF and R = 10Ω respectively. They are connected to an AC source of voltage 220V and frequency of 50 Hz. The approximate value of current in the circuit will be____ A.



Official Ans. by NTA (22)

Sol. $X_L = \omega L = 2\pi \times 50 \times 10^{-1} = 10\pi$

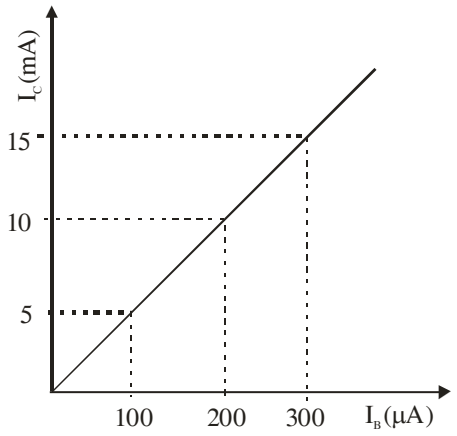
$$X_X = \frac{1}{\omega C} = \frac{1}{2\pi \times 50} \times 10^4 = \frac{100}{\pi}$$

$$R = 10\Omega$$

$$Z = \sqrt{\left(10\pi - \frac{100}{\pi}\right)^2 + 10^2} \simeq 10\Omega$$

$$i = \frac{E}{2} \simeq \frac{220}{10} \simeq 22\text{Amp}$$

10. In an experiment of CE configuration of n-p-n transistor, the transfer characteristics are observed as given in figure.



If the input resistance is 200Ω and output resistance is 60 Ω the voltage gain in this experiment will be _____

Official Ans. by NTA (15)

Sol. $\text{Voltage Gain} = \frac{I_C}{I_B} \times \frac{R_0}{R_I} = \frac{10 \times 10^{-3}}{200 \times 10^{-6}} \times \frac{60}{200} = 15$

FINAL JEE–MAIN EXAMINATION – JUNE, 2022

(Held On Saturday 25th June, 2022) **TIME : 3 : 00 PM to 6 : 00 PM**

CHEMISTRY | TEST PAPER WITH SOLUTION

SECTION-A

1. The minimum energy that must be possessed by photons in order to produce the photoelectric effect with platinum metal is:

[Given: The threshold frequency of platinum is $1.3 \times 10^{15}\ s^{-1}$ and $h = 6.6 \times 10^{-34}$ J s.**]**

(A) 3.21×10^{-14} J (B) 6.24×10^{-16} J

(C) 8.58×10^{-19} J (D) 9.76×10^{-20} J

Official Ans. by NTA (C)

Sol. $W = h\nu$

$= 6.6 \times 10^{-34} \times 1.3 \times 10^{15}$

$= 8.58 \times 10^{-19}$ J

2. At 25°C and 1 atm pressure, the enthalpy of combustion of benzene (1) and acetylene (g) are –3268 kJ mol^{-1} and –1300 kJ mol^{-1}, respectively. The change in enthalpy for the reaction $3\ C_2H_2(g) \rightarrow C_6H_6(l)$, is

(A) + 324 kJ mol^{-1} (B) +632 kJ mol^{-1}

(C) – 632 kJ mol^{-1} (D) – 732 kJ mol^{-1}

Official Ans. by NTA (C)

Sol. $\Delta H = \sum \Delta H_{Combustion}$ (Reactant) - $\sum \Delta H_{Combustion}$ (Product)

$= 3 \times (-1300) - [-3268]$

$= -632$ kJ mol^{-1}

3. Solute A associates in water. When 0.7 g of solute A is dissolved in 42.0 g of water, it depresses the freezing point by 0.2°C. The percentage association of solute A in water, is

[Given : Molar mass of A = 93 g mol^{-1}. Molal depression constant of water is 1.86 K kg mol^{-1}]

(A) 50 % (B) 60 %

(C) 70 % (D) 80 %

Official Ans. by NTA (D)

Sol. $\Delta T = i\ k_f \times m$

$$0.2 = i \times 1.86 \times \frac{0.7}{93} \times \frac{1000}{42}$$

$$i = \frac{0.2 \times 93 \times 6}{1.86 \times 100}$$

$i = 0.60$

$2A \rightleftharpoons A_2$

$1 - \alpha \qquad \frac{\alpha}{2}$

$$i = 1 - \alpha + \frac{\alpha}{2}$$

$$i = 1 - \frac{\alpha}{2}$$

$$1 - \frac{\alpha}{2} = 0.60$$

$$1 - 0.60 = \frac{\alpha}{2}$$

$\alpha = 0.80$

4. The K_{sp} for bismuth sulphide (Bi_2S_3) is 1.08×10^{-73}. The solubility of Bi_2S_3 in mol L^{-1} at 298 K is

(A) 1.0×10^{-15} (B) 2.7×10^{-12}

(C) 3.2×10^{-10} (D) 4.2×10^{-8}

Official Ans. by NTA (A)

Sol. $Bi_2S_3 \rightleftharpoons 2Bi^{3+} + 3S^{2-}$

$\qquad\qquad 2s \qquad 3s$

$k_{sp} = (2s)^2\ (3s)^3$

$= 4s^2 \times 27\ (s)^3$

$= 108\ (s)^5$

$$(s)^5 = \frac{1.08 \times 10^{-73}}{108}$$

$\Rightarrow s = 10^{-15}$

5. Match List I with List II.

List I	List II
A. Zymase	I. Stomach
B. Diastase	II. Yeast
C. Urease	III. Malt
D. Pepsin	IV. Soyabean

Choose the correct answer from the options given below:

(A) A-II, B-III, C-I, D-IV

(B) A-II, B-III, C-IV, D-I

(C) A-III, B-II, C-IV, D-I

(D) A-III, B-II, C-I, D-IV

Official Ans. by NTA (B)

Sol. Zymase naturally occurs in yeast.

Diastase is found in malt.

Urease is found in soyabean

Pepsin is found in stomach

6. The correct order of electron gain enthalpies of Cl, F, Te and Po is

(A) F < Cl < Te < Po (B) Po < Te < F < Cl

(C) Te < Po < Cl < F (D) Cl < F < Te < Po

Official Ans. by NTA (D)

Sol. As Cl has maximum electron affinity among all elements.

Element	**$\Delta_{eg}H$ (kJ/mol)**
F	–328
Cl	–349
Te	–190
Po	–174

7. Given below are two statements.

Statement I: During electrolytic refining, blister copper deposits precious metals

Statement II: In the process of obtaining pure copper by electrolysis method, copper blister is used to make the anode.

In the light of the above statements, choose the correct answer from the options given below.

(A) Both Statement I and Statement II are true.

(B) Both Statement I and Statement II are false.

(C) Statement I is true but Statement II is false.

(D) Statement I is false but Statement II is true.

Official Ans. by NTA (A)

Sol. In the electro-refining, impure metal (here blister copper) is used as an anode while precious metal like Au, Pt get deposited as anode mud.

8. Given below are two statements one is labelled as **Assertion A** and the other is labelled as **Reason R:**

Assertion A : The amphoteric nature of water is explained by using Lewis acid/base concept.

Reason R : Water acts as an acid with NH_3 and as a base with H_2S.

In the light of the above statements choose the correct answer from the options given below :

(A) Both A and R are true and R is the correct explanation of A.

(B) Both A and R are true but R is NOT the correct explanation of A.

(C) A is true but R is false.

(D) A is false but R is true.

Official Ans. by NTA (D)

Sol. $\underset{\text{Acid}}{H_2S} + \underset{\text{Base}}{H_2O} \rightleftharpoons H_3O^+ + HS^-$

$\underset{\text{Acid}}{H_2O} + \underset{\text{Base}}{NH_3} \rightleftharpoons NH_4OH$

9. The correct order of reduction potentials of the following pairs is

A. Cl_2/Cl^-

B. I_2/I^-

C. Ag^+/Ag

D. Na^+/Na

E. Li^+/Li

Choose the correct answer from the options given below.

(A) A > C > B > D > E

(B) A > B > C > D > E

(C) A > C > B > E > D

(D) A > B > C > E > D

Official Ans. by NTA (A)

Sol. $E^{\circ}_{Cl_2/Cl^-} = +1.36\,V$

$E^{\circ}_{I_2/I^-} = +0.54\,V$

$E^{\circ}_{Ag^+/Ag} = +0.80\,V$

$E^{\circ}_{Na^+/Na} = -2.71\,V$

$E^{\circ}_{Li^+/Li} = -3.05\,V$

10. The number of bridged oxygen atoms present in compound B formed from the following reactions is

$Pb(NO_3)_2 \xrightarrow{673\ K} A + PbO + O_2$

$A \xrightarrow{Dimerise} B$

(A) 0 (B) 1

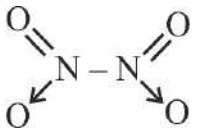
(C) 2 (D) 3

Official Ans. by NTA (A)

Sol. $Pb(NO_3)_2 \xrightarrow[673\ K]{\Delta} PbO + NO_2 + O_2$

(A)

$2\underset{A}{NO_2} \xrightarrow{Dimerise} \underset{B}{N_2O_4}$



(no bridged oxygen)

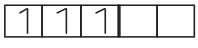
11. The metal ion (in gaseous state) with lowest spin-only magnetic moment value is

(A) V^{2+} (B) Ni^{2+}

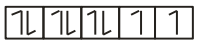
(C) Cr^{2+} (D) Fe^{2+}

Official Ans. by NTA (B)

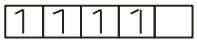
Sol. $V^{2+} : 1s^2\,2s^2\,2p^6\,3s^2\,3p^6\,3d^3$

 (3d) (unpaired e^- = 3)

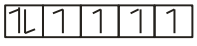
$Ni^{2+} : 1s^2\,2s^2\,2p^6\,3s^2\,3p^6\,3d^8$

 (3d) (unpaired e^- = 2)

$Cr^{2+} : 1s^2\,2s^2\,2p^6\,3s^2\,3p^6\,3d^4$

 (3d) (unpaired e^- = 4)

$Fe^{2+} : 1s^2\,2s^2\,2p^6\,3s^2\,3p^6\,3d^6$

 (3d) (unpaired e^- = 4)

12. Given below are two statements: one is labelled as **Assertion A** and the other is labelled as **Reason R**

Assertion A: Polluted water may have a value of BOD of the order of 17 ppm.

Reason R: BOD is a measure of oxygen required to oxidise both the biodegradable and non-biodegradable organic material in water.

In the light of the above statements, choose the most appropriate answer from the options given below.

(A) Both A and R are correct and R is the correct explanation of A.

(B) Both A and R are correct but R is NOT the correct explanation of A.

(C) A is correct but R is not correct.

(D) A is not correct but R is correct.

Official Ans. by NTA (C)

Sol. Clean water have BOD less than 5 ppm while highly polluted water has BOD greater or equal to 17 ppm. So, assertion is correct.

BOD is measure of oxygen required to oxidise only bio-degradable organic matter. So, reason is false.

13. Given below are two statements: one is labelled as **Assertion A** and the other is labelled as **Reason R.**

Assertion A: A mixture contains benzoic acid and napthalene. The pure benzoic acid can be separated out by the use of benzene.

Reason R: Benzoic acid is soluble in hot water.

In the light of the above statements, choose the most appropriate answer from the options given below.

(A) Both A and R are true and R is the correct explanation of A.

(B) Both A and R are true but R is NOT the correct explanation of A.

(C) A is true but R is false.

(D) A is false but R is true.

Official Ans. by NTA (D)

Sol. Benzoic acid and Napthalene can be effectively separated by crystallization. Benzoic acid is soluble in hot water whereas Napthalene is insoluble.

Hence assertion is incorrect but reason is correct

14. During halogen test, sodium fusion extract is boiled with concentrated HNO_3 to

(A) remove unreacted sodium

(B) decompose cyanide or sulphide of sodium

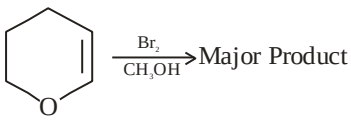
(C) extract halogen from organic compound

(D) maintain the pH of extract

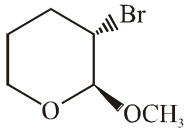
Official Ans. by NTA (B)

Sol. Sodium fusion extract is boiled with concentrated HNO_3 to remove sodium cyanide and sodium sulphide

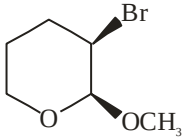
15. Amongst the following, the major product of the given chemical reaction is



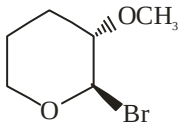
(A)



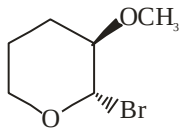
(B)



(C)

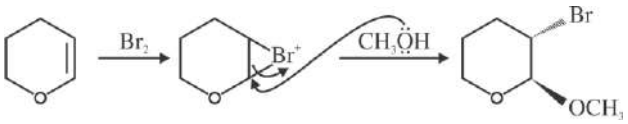


(D)

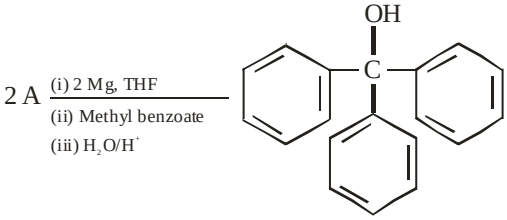


Official Ans. by NTA (A)

Sol.



16. In the given reaction



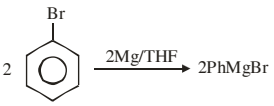
'A' can be

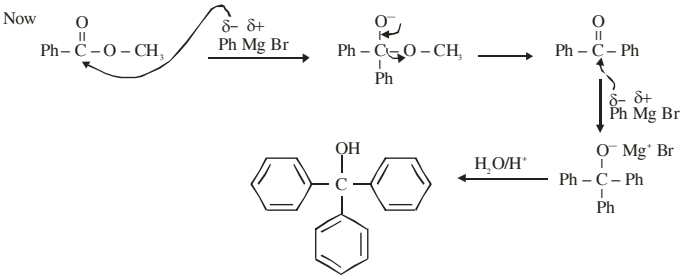
(A) benzyl bromide (B) bromobenzene

(C) cyclohexyl bromide (D) methyl bromide

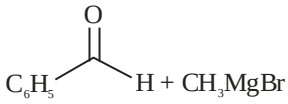
Official Ans. by NTA (B)

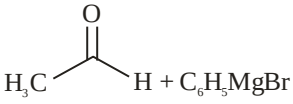
Sol.

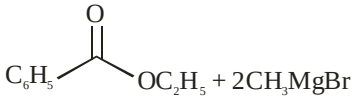


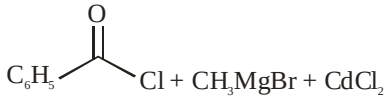


17. Which of the following conditions or reaction sequence will NOT give acetophenone as the major product ?

(A) (a)  (b) $Na_2Cr_2O_7$, H^+

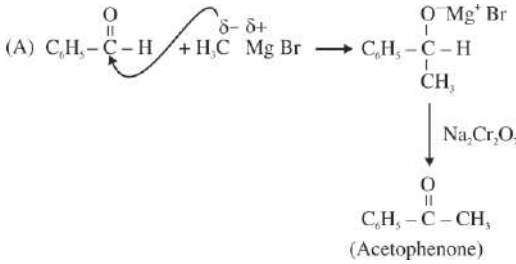
(B) (a)  (b) PCC, DCM

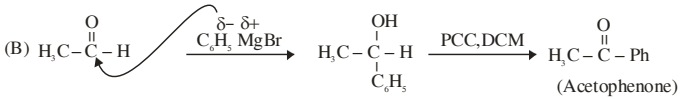
(C) 

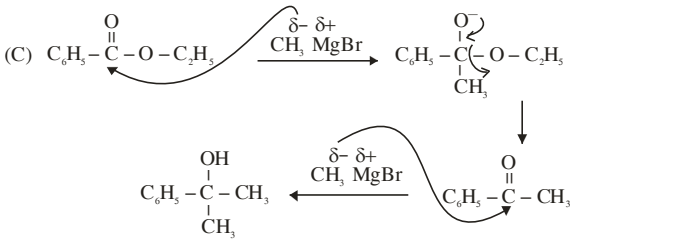
(D) 

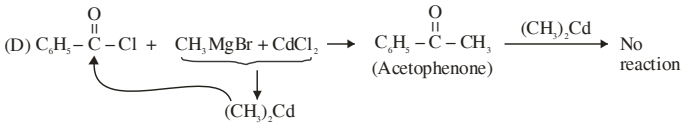
Official Ans. by NTA (C)

Sol.

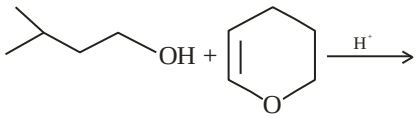


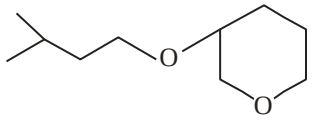


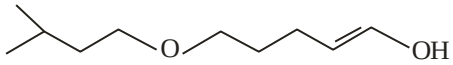


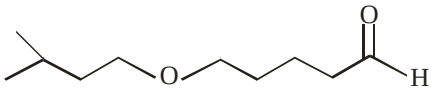


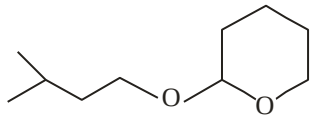
18. The major product formed in the following reaction, is



(A) 

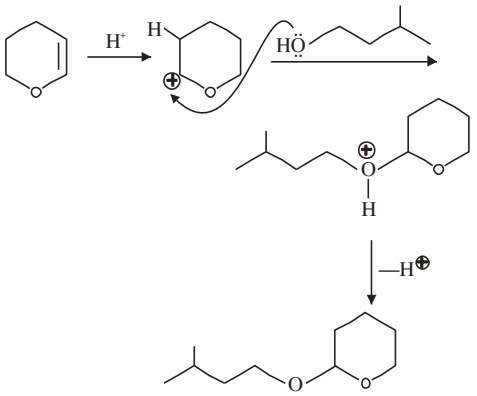
(B) 

(C) 

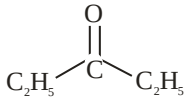
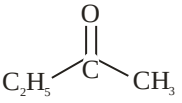
(D) 

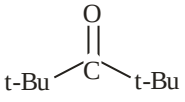
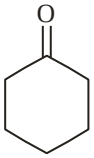
Official Ans. by NTA (D)

Sol.



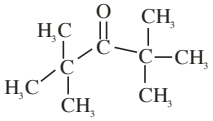
19. Which of the following ketone will NOT give enamine on treatment with secondary amines? [where t-Bu is $-C(CH_3)_3$]

(A)  (B) 

(C)  (D) 

Official Ans. by NTA (C)

Sol. Enamine formation is an example of nucleophilic addition elimination reaction

Since in ketone  Carbonyl

Group is highly sterically hindered hence attack of nucleophile will not be possible.

20. An antiseptic dettol is a mixture of two compounds 'A' and 'B' where A has 6π electrons and B has 2π electrons. What is 'B'?

(A) Bithionol

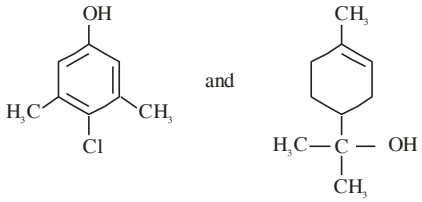
(B) Terpineol

(C) Chloroxylenol

(D) Chloramphenicol

Official Ans. by NTA (B)

Sol. Dettol is mixture of



Chloroxylenol (Compound A) It has $6\pi e^-$

Terpineol (Compound B) It has $2\pi e^-$

Hence compound 'B' is Terpineol.

SECTION-B

1. A protein 'A' contains 0.30% of glycine (molecular weight 75). The minimum molar mass of the protein 'A' is_____ × 10^3 g mol^{-1} [nearest integer]

Official Ans. by NTA (25)

Sol. 0.30 % glycine is equal to 75

$$1\ \% \longrightarrow \frac{75}{0.30}$$

$$100\ \% \longrightarrow \frac{75}{0.30}\times 100$$

$$= 25000\ g$$

2. A rigid nitrogen tank stored inside a laboratory has a pressure of 30 atm at 06:00 am when the temperature is 27 °C. At 03:00 pm, when the temperature is 45°C, the pressure in the tank will be _____atm. [nearest integer]

Official Ans. by NTA (32)

Sol. $\frac{P_1}{T_1} = \frac{P_2}{T_2}$

$$\frac{30}{300} = \frac{P_2}{318}$$

$$P_2 = \frac{30}{300}\times 318$$

$$= \frac{1}{10}\times 318$$

$$= 32$$

3. Amongst BeF_2, BF_3, H_2O, NH_3, CCl_4 and HCl, the number of molecules with non-zero net dipole moment is _____.

Official Ans. by NTA (3)

Sol. BeF_2, BF_3 and $CCl_4 \Rightarrow \mu_{net} = 0$

H_2O, NH_3 and HCl $\Rightarrow \mu_{net} \neq 0$

4. At 345 K, the half life for the decomposition of a sample of a gaseous compound initially at 55.5 kPa was 340 s. When the pressure was 27.8 kPa, the half life was fund to be 170 s. The order of the reaction is_____. [integer answer]

Official Ans. by NTA (0)

Sol. $t_{1/2} \times \frac{1}{[P_0]^{n-1}}$

$$\frac{t_1}{t_2} = \frac{(P_2)^{n-1}}{(P_1)^{n-1}}$$

$$\frac{340}{170} = \left(\frac{27.8}{55.5}\right)^{n-1}$$

$$\Rightarrow\ 2 = \frac{1}{(2)^{n-1}}$$

$$n = 0$$

5. A solution of $Fe_2(SO_4)_3$ is electrolyzed for 'x' min with a current of 1.5 A to deposit 0.3482 g of Fe. The value of x is_____. [nearest integer]

Given : 1 F = 96500 C mol^{-1}

Atomic mass of Fe = 56 g mol^{-1}

Official Ans. by NTA (20)

Sol. $Fe^{3+} + 3e^- \longrightarrow Fe$

$3F \longrightarrow 1$ mole Fe is deposited

For 56 g $\longrightarrow 3 \times 96500$ (required charge)

For 1g $\longrightarrow \frac{3\times96500}{56}$ (required charge)

For 0.3482 g $\longrightarrow \frac{3\times96500}{56}\times0.3482$

$= 1800.06$

$Q = it$

$1800.06 = 1.5\ t$

$t = 20$ min

6. Consider the following reactions :

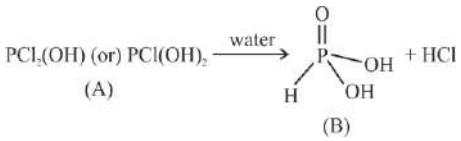
$PCl_3 + H_2O \longrightarrow A + HCl$

$A + H_2O \longrightarrow B + HCl$

number of ionisable protons present in the product B_____.

Official Ans. by NTA (2)

Sol. $PCl_3+H_2O \xrightarrow[\text{hydrolysis}]{\text{Partial}} PCl_2(OH)$ (or) $PCl(OH)_2 +$ HCl



no. of ionisable protons in B = 2

7. Amongst $FeCl_3.3H_2O$, $K_3[Fe(CN)_6]$ and $[Co(NH_3)_6]Cl_3$, the spin-only magnetic moment value of the inner-orbital complex that absorbs light at shortest wavelength is_____ B.M. [nearest integer]

Official Ans. by NTA (2)

Sol. $[Fe(H_2O)_3Cl_3]$, $\underbrace{K_3[Fe(CN)_6], [Co(NH_3)_6]Cl_3}_{\text{inner orbital complexes}}$

$K_3[Fe(CN)_6]$ has more value of Δ_0 than that of $[Co(NH_3)_6]Cl_3$; as $\bar{C}N$ is stronger ligand.

More $\Delta_0 \Rightarrow$ smaller value of absorbed λ

$K_3[Fe(CN)_6]$: Fe^{3+} : $3d^5$ $4s^0$ $4p^0$

1 unpaired e^- d^2sp^3

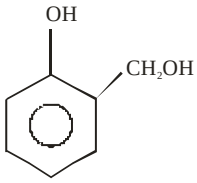
Spin only magnetic moment (μ) = $\sqrt{3}$ BM

$= 1.732$ BM

Rounding off $\Rightarrow 2$

8. The Novolac polymer has mass of 963 g. The number of monomer units present in it are

Official Ans. by NTA (9)

Sol. Monomer unit of Novolac is 

its molecular mass is 124 amu.

Upon considering molecular weight of polymer as 963 amu (In question its given as 963 gram)

Now if during formation of Novolac, (n–1) unit of water are removed then

$n\times124 = 963 + \left[18\times(n-1)\right]$

$n = 9$

9. How many of the given compounds will give a positive Biuret test _____ ? Glycine, Glycylalanine, Tripeptide, Biuret

Official Ans. by NTA (2)

Sol. Biuret test is given by all proteins and peptides having atleast two peptide linkages.

Hence positive test must be given by tripeptide and Biuret.

10. The neutralization occurs when 10 mL of 0.1 M acid 'A' is allowed to react with 30 mL of 0.05 M base $M(OH)_2$. The basicity of the acid 'A' is_____. [M is a metal]

Official Ans. by NTA (3)

Sol. Acid + Base $\longrightarrow$ Salt + H_2O

0.1 M $M(OH)_2$

10ml 0.05 M

30 ml

at equivalence point

equivalent of acid = equivalent of base

$0.1 \times 10 \times n = 30 \times 0.05 \times 2$

$n = 3$

FINAL JEE–MAIN EXAMINATION – JUNE, 2022

(Held On Saturday 25th June, 2022) **TIME : 3 : 00 PM to 6 : 00 PM**

MATHEMATICS

TEST PAPER WITH SOLUTION

SECTION-A

1. Let $A=\{x\in R:|x+1|<2\}$ and $B=\{x\in R:|x-1|\geq 2\}$. Then which one of the following statements is **NOT** true ?

(A) $A-B=(-1,1)$ (B) $B-A=R-(-3,1)$

(C) $A\cap B=(-3,-1]$ (D) $A\cup B=R-[1,3)$

Official Ans. by NTA (B)

Sol. $A: x\in(-3, 1)$ $B: x\in(-\infty, -1]\cup[3,\infty)$

$B-A=(-\infty, -3]\cup[3,\infty)$ = R–(–3, 3)

2. Let $a,b\in R$ be such that the equation $ax^2-2bx+15=0$ has a repeated root α. If α and β are the roots of the equation $x^2-2bx+21=0$, then $\alpha^2+\beta^2$ is equal to:

(A) 37 (B) 58

(C) 68 (D) 92

Official Ans. by NTA (B)

Sol. $ax^2-2bx+15=0$

$2\alpha=\frac{2b}{a}, \alpha^2=\frac{15}{a}$

$\frac{\alpha}{2}=\frac{15}{2b}$

$\alpha=\frac{15}{b}$

$x^2-2bx+21=0$

$\left(\frac{15}{b}\right)^2-2b\left(\frac{15}{b}\right)+21=0$

$b^2=25$

$\alpha+\beta=2b,\ \alpha\beta=21$

$\alpha^2+\beta^2=4b^2-42$

= 58

3. Let z_1 and z_2 be two complex numbers such that $\overline{z}_1=i\overline{z}_2$ and $\arg\left(\frac{z_1}{\overline{z}_2}\right)=\pi$. Then

(A) $\arg z_2=\frac{\pi}{4}$ (B) $\arg z_2=-\frac{3\pi}{4}$

(C) $\arg z_1=\frac{\pi}{4}$ (D) $\arg z_1=-\frac{3\pi}{4}$

Official Ans. by NTA (C)

Sol. $\overline{z}_1=i\overline{z}_2$

$z_1=-iz_2$

$\arg\left(\frac{z_1}{\overline{z}_2}\right)=\pi$

$\arg\left(-i\frac{z_2}{\overline{z}_2}\right)=\pi$ $\arg(z_2)=\theta$

$-\frac{\pi}{2}+\theta+\theta=\pi$

$2\theta=\frac{3\pi}{2}$

$\arg(z_2)=\theta=\frac{3\pi}{4}$, $\arg z_1=\frac{\pi}{4}$

4. The system of equations

$-kx+3y-14z=25$

$-15x+4y-kz=3$

$-4x+y+3z=4$

is consistent for all k in the set

(A) R (B) $R-\{-11,13\}$

(C) $R-\{13\}$ (D) $R-\{-11,11\}$

Official Ans. by NTA (D)

Sol. $\Delta = \begin{vmatrix} -k & 3 & -14 \\ -15 & 4 & -k \\ -4 & 1 & 3 \end{vmatrix} = 121 - k^2$

$\Delta \neq 0 \qquad k \in R - \{11, -11\}$ (Unique sol.)

If $k = 11$

$$\Delta_z = \begin{vmatrix} -11 & 3 & 25 \\ -15 & 4 & 3 \\ -4 & 1 & 4 \end{vmatrix} \neq 0$$

No solution

If $k = -11$

$$\Delta_z = \begin{vmatrix} 11 & 3 & 25 \\ -15 & 4 & 3 \\ -4 & 1 & 4 \end{vmatrix} \neq 0$$

No solution

5. $\lim\limits_{x\to\frac{\pi}{2}}\left(\tan^2 x\left(\left(2\sin^2 x+3\sin x+4\right)^{\frac{1}{2}}-\left(\sin^2 x+6\sin x+2\right)^{\frac{1}{2}}\right)\right)$

is equal to

(A) $\frac{1}{12}$ (B) $-\frac{1}{18}$

(C) $-\frac{1}{12}$ (D) $-\frac{1}{6}$

Official Ans. by NTA (A)

Sol.

$$\lim_{x\to\frac{\pi}{2}} \tan^2 x\left[\sqrt{2\sin^2 x+3\sin x+4}-\sqrt{\sin^2 x+6\sin x+2}\right] =$$

$$\lim_{x\to\frac{\pi}{2}} \frac{\tan^2 x\,[\sin^2 x-3\sin x+2]}{\sqrt{9}+\sqrt{9}}$$

$$= \lim_{x\to\frac{\pi}{2}} \frac{\tan^2 x(\sin x-1)(\sin x-2)}{6}$$

$$= \frac{1}{6}\lim_{x\to\frac{\pi}{2}} \tan^2 x(1-\sin x)$$

$$= \frac{1}{6}\lim_{x\to\frac{\pi}{2}} \frac{\sin^2 x\,(1-\sin x)}{(1-\sin x)(1+\sin x)} = \frac{1}{12}$$

6. The area of the region enclosed between the parabolas $y^2 = 2x - 1$ and $y^2 = 4x - 3$ is

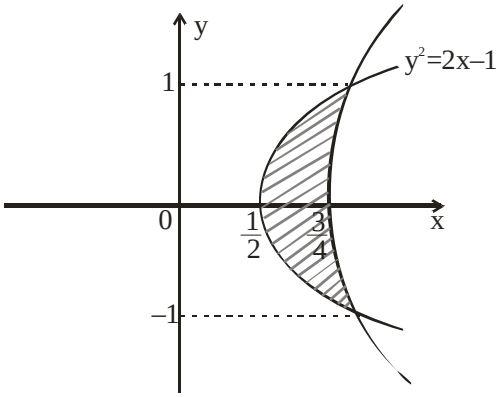
(A) $\frac{1}{3}$ (B) $\frac{1}{6}$

(C) $\frac{2}{3}$ (D) $\frac{3}{4}$

Official Ans. by NTA (A)

Sol. Required area $= 2\int_0^1 \left(\frac{y^2+3}{4}-\frac{y^2+1}{2}\right)dy$

$$= 2\int_0^1 \frac{1-y^2}{4}dy = \frac{1}{2}\left|y-\frac{y^3}{3}\right|_0^1 = \frac{1}{3}$$



7. The coefficient of x^{101} in the expression

$(5+x)^{500} + x(5+x)^{499} + x^2(5+x)^{498} + ...x^{500}$,

$x > 0$, is

(A) ${}^{501}C_{101}(5)^{399}$ (B) ${}^{501}C_{101}(5)^{400}$

(C) ${}^{501}C_{100}(5)^{400}$ (D) ${}^{500}C_{101}(5)^{399}$

Official Ans. by NTA (A)

Sol. $(5+x)^{500} + x(5+x)^{499} + x^2(5+x)^{498} + ... + x^{500}$

$$= \frac{(5+x)^{501}-x^{501}}{(5+x)-x} = \frac{(5+x)^{501}-x^{501}}{5}$$

$\Rightarrow$ coefficient x^{101} in given expression

$$= \frac{{}^{501}C_{101}5^{400}}{5} = {}^{501}C_{101}5^{399}$$

8. The sum $1+2\cdot3+3\cdot3^2+.....+10\cdot3^9$ is equal to

(A) $\frac{2\cdot3^{12}+10}{4}$ (B) $\frac{19\cdot3^{10}+1}{4}$

(C) $5\cdot3^{10}-2$ (D) $\frac{9\cdot3^{10}+1}{2}$

Official Ans. by NTA (B)

Sol. $S=1\cdot3^0+2\cdot3^1+3\cdot3^2+.....+10.3^9$

$3S=1\cdot3^1+2.3^2.....+9\times3^9+10\times3^{10}$

$-2S=(1\cdot3^0+3^1+3^2.....3^9)-10.3^{10}$

$S=5\times3^{10}-\left(\frac{3^{10}-1}{4}\right)$

$S=\frac{20.3^{10}-3^{10}+1}{4}=\frac{19.3^{10}+1}{4}$

9. Let P be the plane passing through the intersection of the planes $\vec{r}\cdot\left(\hat{i}+3\hat{j}-\hat{k}\right)=5$ and $\vec{r}\cdot\left(2\hat{i}-\hat{j}+\hat{k}\right)=3$, and the point $(2,1,-2)$. Let the position vectors of the points X and Y be $\hat{i}-2\hat{j}+4\hat{k}$ and $5\hat{i}-\hat{j}+2\hat{k}$ respectively. Then the points

(A) X and X + Y are on the same side of P

(B) Y and Y − X are on the opposite sides of P

(C) X and Y are on the opposite sides of P

(D) X + Y and X − Y are on the same side of P

Official Ans. by NTA (C)

Sol. $P_1+\lambda P_2=0$

$\Rightarrow(x+3y-z-5)+\lambda(2x-y+z-3)=0$

$(2,1,-2)$ lies on this plane

$\therefore\lambda=1\Rightarrow$ plane is $3x+2y-8=0$

10. A circle touches both the y-axis and the line x + y = 0. Then the locus of its center is

(A) $y=\sqrt{2}x$ (B) $x=\sqrt{2}y$

(C) $y^2-x^2=2xy$ (D) $x^2-y^2=2xy$

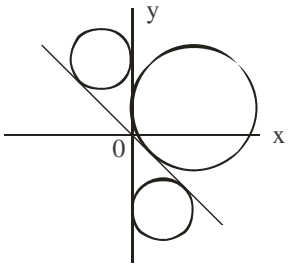
Official Ans. by NTA (D)

Sol. Let (h, k) is centre of circle

$\left|\frac{h-k}{\sqrt{2}}\right|=|h|$

$k^2-h^2+2hk=0$

$\therefore$ Equation of locus is $y^2-x^2+2xy=0$



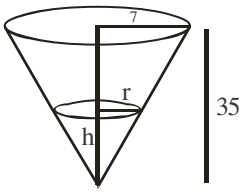
11. Water is being filled at the rate of $1\,cm^3/sec$ in a right circular conical vessel (vertex downwards) of height 35 cm and diameter 14 cm. When the height of the water level is 10 cm, the rate (in cm^2/sec) at which the wet conical surface area of the vessel increases is

(A) 5 (B) $\frac{\sqrt{21}}{5}$

(C) $\frac{\sqrt{26}}{5}$ (D) $\frac{\sqrt{26}}{10}$

Official Ans. by NTA (C)

Sol. From figure $\frac{r}{h}=\frac{7}{35}\Rightarrow h=5r$



Given $\frac{dV}{dt}=1\Rightarrow\frac{d}{dt}\left(\frac{\pi r^2h}{3}\right)=1$

$\Rightarrow\frac{d}{dt}\left(\frac{5\pi}{3}r^3\right)=1\Rightarrow r^2\frac{dr}{dt}=\frac{1}{5\pi}$

Let wet conical surface area = S

$=\pi r\ell=\pi r\sqrt{h^2+r^2}$

$=\sqrt{26}\pi r^2\Rightarrow\frac{dS}{dt}=2\sqrt{26}\pi r\frac{dr}{dt}$

When h = 10 then r = 2 $\Rightarrow\frac{dS}{dt}=\frac{2\sqrt{26}}{10}$

12. If $b_n = \int_0^{\frac{\pi}{2}} \frac{\cos^2 nx}{\sin x} dx$, $n \in N$, then

(A) $b_3 - b_2, b_4 - b_3, b_5 - b_4$ are in an A.P. with common difference –2

(B) $\frac{1}{b_3 - b_2}, \frac{1}{b_4 - b_3}, \frac{1}{b_5 - b_4}$ are in an A.P. with common difference 2

(C) $b_3 - b_2, b_4 - b_3, b_5 - b_4$ are in a G.P.

(D) $\frac{1}{b_3 - b_2}, \frac{1}{b_4 - b_3}, \frac{1}{b_5 - b_4}$ are in an A.P. with common difference –2

Official Ans. by NTA (D)

Sol. $b_n = \int_0^{\pi/2} \frac{1+\cos 2nx}{\sin x} dx$

$b_{n+1} - b_n = \int_0^{\pi/2} \frac{\cos^2(n+1)x - \cos^2 nx}{\sin x} dx$

$= \int_0^{\pi/2} \frac{-\sin(2n+1)x \sin x}{\sin x} dx$

$= \left(\frac{\cos(2n+1)x}{2n+1} \right)_0^{\pi/2} = \frac{-1}{2n+1}$

$\frac{1}{b_3 - b_2}, \frac{1}{b_4 - b_3}, \frac{1}{b_5 - b_4}$ are in A.P. with c.d.= –2

13. If y = y(x) is the solution of the differential equation $2x^2 \frac{dy}{dx} - 2xy + 3y^2 = 0$ such that $y(e) = \frac{e}{3}$, then y(1) is equal to

(A) $\frac{1}{3}$ (B) $\frac{2}{3}$

(C) $\frac{3}{2}$ (D) 3

Official Ans. by NTA (B)

Sol. $\frac{dy}{dx} - \frac{y}{x} = -\frac{3}{2}\left(\frac{y}{x}\right)^2$ $y = vx$

$\frac{dv}{v^2} = -\frac{3dx}{2x}$

$-\frac{1}{v} = -\frac{3}{2} \ell n|x| + C$

$-\frac{x}{y} = \frac{-3}{2} \ell n|x| + C$

$x = e,\ y = \frac{e}{3}$

$C = -\frac{3}{2}$

When $x = 1,\ y = \frac{2}{3}$

14. If the angle made by the tangent at the point (x_0, y_0) on the curve $x = 12(t + \sin t \cos t)$, $y = 12(1+\sin t)^2, 0 < t < \frac{\pi}{2}$, with the positive x-axis is $\frac{\pi}{3}$, then y_0 is equal to

(A) $6\left(3+2\sqrt{2}\right)$ (B) $3\left(7+4\sqrt{3}\right)$

(C) 27 (D) 48

Official Ans. by NTA (C)

Sol. $\frac{dy}{dx} = \frac{2(1+\sin t) \times \cos t}{1+\cos 2t}$

$\Rightarrow \frac{2(1+\sin t)\cos t}{2\cos^2 t} = \sqrt{3}$

$\Rightarrow t = \frac{\pi}{6}, y_0 = 27$

15. The value of $2\sin(12°) - \sin(72°)$ is :

(A) $\frac{\sqrt{5}\left(1-\sqrt{3}\right)}{4}$ (B) $\frac{1-\sqrt{5}}{8}$

(C) $\frac{\sqrt{3}\left(1-\sqrt{5}\right)}{2}$ (D) $\frac{\sqrt{3}\left(1-\sqrt{5}\right)}{4}$

Official Ans. by NTA (D)

Sol. $\sin 12^\circ + \sin 12^\circ - \sin 72^\circ$

$= \sin 12^\circ - 2\cos 42^\circ \sin 30^\circ$

$= \sin 12^\circ - \sin 48^\circ$

$= -2\cos 30^\circ \sin 18^\circ$

$= -2 \times \frac{\sqrt{3}}{2} \times \frac{\sqrt{5}-1}{4}$

$= \frac{\sqrt{3}}{4}(1-\sqrt{5})$

16. A biased die is marked with numbers 2,4, 8, 16, 32, 32 on its faces and the probability of getting a face with mark n is $\frac{1}{n}$. If the die is thrown thrice, then the probability, that the sum of the numbers obtained is 48, is

(A) $\frac{7}{2^{11}}$ (B) $\frac{7}{2^{12}}$

(C) $\frac{3}{2^{10}}$ (D) $\frac{13}{2^{12}}$

Official Ans. by NTA (D)

Sol. $P(n) = \frac{1}{n}$

$P(2) = \frac{1}{2}$ $P(8) = \frac{1}{8}$

$P(4) = \frac{1}{4}$ $P(16) = \frac{1}{16}$

$P(32) = \frac{2}{32}$

Possible cases

16, 16, 16 and 32, 8, 8

Probability $= \frac{1}{16^3} + \frac{2}{32} \times \frac{1}{8} \times \frac{1}{8} \times 3 = \frac{13}{16^3}$

17. The negation of the Boolean expression $((\sim q) \wedge p) \Rightarrow ((\sim p) \vee q)$ is logically equivalent to

(A) $p \Rightarrow q$ (B) $q \Rightarrow p$

(C) $\sim(p \Rightarrow q)$ (D) $\sim(q \Rightarrow p)$

Official Ans. by NTA (C)

Sol. $\sim p \vee q \equiv p \to q$

$\sim q \wedge p \equiv \sim (p \to q)$

Negation of $\sim(p \to q) \to (p \to q)$

is $\sim(p \to q) \wedge (\sim(p \to q))$ i.e. $\sim(p \to q)$

18. If the line $y = 4 + kx$, $k > 0$, is the tangent to the parabola $y = x - x^2$ at the point P and V is the vertex of the parabola, then the slope of the line through P and V is :

(A) $\frac{3}{2}$ (B) $\frac{26}{9}$

(C) $\frac{5}{2}$ (D) $\frac{23}{6}$

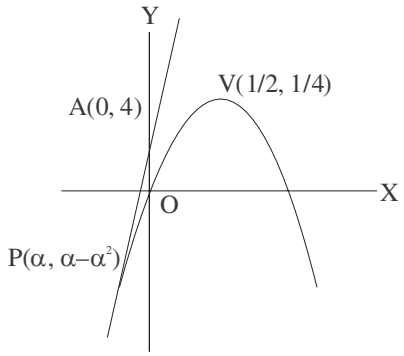
Official Ans. by NTA (C)

Sol. Slope of tangent at P = Slope of line AP

$y'|_P = 1 - 2\alpha = \frac{\alpha - \alpha^2 - 4}{\alpha}$

Solving $\alpha = -2 \Rightarrow P(-2, -6)$

Slope of PV = $\frac{5}{2}$



19. The value of $\tan^{-1}\left(\frac{\cos\left(\frac{15\pi}{4}\right)-1}{\sin\left(\frac{\pi}{4}\right)}\right)$ is equal to

(A) $-\frac{\pi}{4}$ (B) $-\frac{\pi}{8}$

(C) $-\frac{5\pi}{12}$ (D) $-\frac{4\pi}{9}$

Official Ans. by NTA (B)

Sol. $\tan^{-1}\left[\dfrac{\cos\left(4\pi-\frac{\pi}{4}\right)-1}{\sin\frac{\pi}{4}}\right] \Rightarrow \tan^{-1}\left(\dfrac{\cos\frac{\pi}{4}-1}{\sin\frac{\pi}{4}}\right)$

$\tan^{-1}\left(\dfrac{1-\sqrt{2}}{1}\right)=-\dfrac{\pi}{8}$

20. The line $y=x+1$ meets the ellipse $\dfrac{x^2}{4}+\dfrac{y^2}{2}=1$ at two points P and Q. If r is the radius of the circle with PQ as diameter then $(3r)^2$ is equal to

(A) 20 (B) 12

(C) 11 (D) 8

Official Ans. by NTA (A)

Sol. Ellipse $x^2+2y^2=4$

Line $y = x + 1$

Point of intersection

$x^2+2(x+1)^2=4$

$3x^2+4x-2=0$

$|x_1-x_2|=\dfrac{\sqrt{40}}{3}$

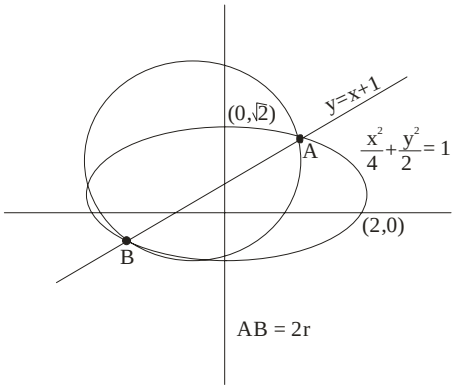
$AB=2r=|x_1-x_2|\sqrt{1+m^2}$,

m is slope of given line

$AB=\dfrac{\sqrt{40}}{3}\sqrt{1+1}$

$2r=\dfrac{\sqrt{80}}{3}\Rightarrow r=\dfrac{\sqrt{80}}{6}$

$(3r)^2=\left(3\times\dfrac{\sqrt{80}}{6}\right)^2=\dfrac{80}{4}=20$



SECTION-B

1. Let $A=\begin{pmatrix}2 & -2\\ 1 & -1\end{pmatrix}$ and $B=\begin{pmatrix}-1 & 2\\ -1 & 2\end{pmatrix}$. Then the number of elements in the set

$\{(n,m): n,m\in\{1,2,.....,10\}$ and $nA^n+mB^m=I\}$ is ____

Official Ans. by NTA (1)

Sol. $A^2=A$ and $B^2=B$

Therefore equation $nA^n+mB^m=I$ becomes

$nA + mB = I$, which gives $m = n = 1$

Only one set possible

2. Let $f(x)=[2x^2+1]$ and $g(x)=\begin{cases}2x-3, & x<0\\ 2x+3, & x\geq 0\end{cases}$, where [t] is the greatest integer $\leq t$. Then, in the open interval (–1, 1), the number of points where fog is discontinuous is equal to _____

Official Ans. by NTA (62)

Sol. $f\left(g(x)\right)=\left[2g^2(x)\right]+1$

$=\begin{cases}\left[2(2x-3)^2\right]+1; & x<0\\ \left[2(2x+3)^2\right]+1; & x\geq 0\end{cases}$

$\therefore$ fog is discontinuous whenever $2(2x-3)^2$ or $2(2x+3)^2$ belongs to integer except $x = 0$.

$\therefore$ 62 points of discontinuity.

3. The value of $b > 3$ for which

$12\int_3^b \dfrac{1}{(x^2-1)(x^2-4)}dx=\log_e\left(\dfrac{49}{40}\right)$, is equal to

Official Ans. by NTA (6)

Sol. $\frac{12}{3}\left[\int_3^b\left(\frac{1}{x^2-4}-\frac{1}{x^2-1}\right)dx\right]=\log\frac{49}{40}$

$$\frac{12}{3}\cdot\left[\frac{1}{4}\ell n\left|\frac{x-2}{x+2}\right|-\frac{1}{2}\ell n\left|\frac{x-1}{x+1}\right|\right]_3^b=\log\frac{49}{40}$$

$$\ell n\frac{(b-2)(b+1)^2}{(b+2)(b-1)^2}=\ell n\frac{49}{50}$$

b = 6

4. If the sum of the coefficients of all the positive even powers of x in the binomial expansion of $\left(2x^3+\frac{3}{x}\right)^{10}$ is $5^{10}-\beta\cdot3^9$, then β is equal to _____

Official Ans. by NTA (83)

Sol. $T_{r+1}={}^{10}C_r(2x^3)^{10-r}\left(\frac{3}{x}\right)^r$

$={}^{10}C_r\,2^{10-r}3^r x^{30-4r}$

Put r = 0, 1, 2, ... 7 and we get β = 83

5. If the mean deviation about the mean of the numbers 1, 2, 3, ..., n, where n is odd, is $\frac{5(n+1)}{n}$, then n is equal to _____

Official Ans. by NTA (21)

Sol. Mean deviation about mean of first n natural numbers is $\frac{n^2-1}{4n}$

$\therefore n=21$

6. Let $\vec{b}=\hat{i}+\hat{j}+\lambda\hat{k},\lambda\in R$. If $\vec{a}$ is a vector such that $\vec{a}\times\vec{b}=13\hat{i}-\hat{j}-4\hat{k}$ and $\vec{a}\cdot\vec{b}+21=0$, then $\left(\vec{b}-\vec{a}\right)\cdot\left(\hat{k}-\hat{j}\right)+\left(\vec{b}+\vec{a}\right)\cdot\left(\hat{i}-\hat{k}\right)$ is equal to

Official Ans. by NTA (14)

Sol. $\left(\vec{a}\times\vec{b}\right).\vec{b}=0$

$\Rightarrow 13-1-4\lambda=0\Rightarrow\lambda=3$

$\Rightarrow\vec{b}=\hat{i}+\hat{j}+3\hat{k}\ \Rightarrow\vec{a}\times\vec{b}=13\hat{i}-\hat{j}-4\hat{k}$

$\Rightarrow\left(\vec{a}\times\vec{b}\right)\times\vec{b}=\left(13\hat{i}-\hat{j}-4\hat{k}\right)\times\left(\hat{i}+\hat{j}+3\hat{k}\right)$

$\Rightarrow-21\vec{b}-11\vec{a}=\hat{i}-43\hat{j}+14\hat{k}$

$\Rightarrow\vec{a}=-2\hat{i}+2\hat{j}-7\hat{k}$

Now $\left(\vec{b}-\vec{a}\right).\left(\hat{k}-\hat{j}\right)+\left(\vec{b}+\vec{a}\right).\left(\hat{i}-\hat{k}\right)=14$

7. The total number of three-digit numbers, with one digit repeated exactly two times, is

Official Ans. by NTA (243)

Sol. If 0 taken twice then ways = 9

If 0 taken once then ${}^9C_1\times2=18$

If 0 not taken then ${}^9C_1\,{}^8C_1.3=216$

Total = 243

8. Let $f(x)=\left|(x-1)(x^2-2x-3)\right|+x-3,\ x\in R$. If m and M are respectively the number of points of local minimum and local maximum of f in the interval (0, 4), then m + M is equal to ____

Official Ans. by NTA (3)

Sol. $f(x)=\begin{cases}\left(x^2-1\right)(x-3)+(x-3), x\in(0,1]\cup[3,4)\\ -\left(x^2-1\right)(x-3)+(x-3), x\in[1,3]\end{cases}$

$$\Rightarrow f'(x)=\begin{cases}3x^2-6x,\ x\in(0,1)\cup(3,4)\\ -3x^2+6x+2,\ x\in(1,3)\end{cases}$$

f(x) is non-derivable at x = 1 and x = 3

also f'(x) = 0 at $x=1+\sqrt{\frac{5}{3}}\ \Rightarrow$ m + M = 3

9. Let the eccentricity of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ be $\frac{5}{4}$. If the equation of the normal at the point $\left(\frac{8}{\sqrt{5}},\frac{12}{5}\right)$ on the hyperbola is $8\sqrt{5}x+\beta y=\lambda$, then $\lambda-\beta$ is equal to

Official Ans. by NTA (85)

Sol. $e^2=1+\frac{b^2}{a^2}=\frac{25}{16}\Rightarrow\frac{b^2}{a^2}=\frac{9}{16}$ **.....(1)**

$A\left(\frac{8}{\sqrt{5}},\frac{12}{5}\right)$ satisfies $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

$\Rightarrow\frac{64}{5a^2}-\frac{144}{25b^2}=1$ **.....(2)**

Solving (1) & (2) $b=\frac{6}{5}$ $a=\frac{8}{5}$

Normal at A is $\frac{\sqrt{5}a^2x}{8}+\frac{5b^2y}{12}=a^2+b^2$

Comparing it $8\sqrt{5}x+\beta y=\lambda$

Gives $\lambda=100,\ \beta=15$

$\lambda-\beta=85$

10. Let l_1 be the line in xy-plane with x and y intercepts $\frac{1}{8}$ and $\frac{1}{4\sqrt{2}}$ respectively, and l_2 be the line in zx-plane with x and z intercepts $-\frac{1}{8}$ and $-\frac{1}{6\sqrt{3}}$ respectively. If d is the shortest distance between the line l_1 and l_2, then d^{-2} is equal to

Official Ans. by NTA (51)

Sol. $8x+4\sqrt{2}y=1,\ z=0$

$\Rightarrow\frac{x-\frac{1}{8}}{1}=\frac{y-0}{-\sqrt{2}}=\frac{z-0}{0}=\lambda$

$-8x-6\sqrt{3}z=1,y=0$

$\Rightarrow\frac{x+\frac{1}{8}}{3\sqrt{3}}=\frac{y-0}{0}=\frac{z-0}{-4}$

$$\begin{vmatrix}\frac{1}{4} & 0 & 0\\ 1 & -\sqrt{2} & 0\\ 3\sqrt{3} & 0 & -4\end{vmatrix}=\sqrt{2}$$

$d=\frac{1}{\sqrt{51}}$

$\frac{1}{d^2}=51$

FINAL JEE–MAIN EXAMINATION – JUNE, 2022

(Held On Saturday 25th June, 2022) **TIME : 9 : 00 AM to 12 : 00 PM**

PHYSICS

TEST PAPER WITH SOLUTION

SECTION-A

1. If $Z=\frac{A^2B^3}{C^4}$, then the relative error in Z will be :

(A) $\frac{\Delta A}{A}+\frac{\Delta B}{B}+\frac{\Delta C}{C}$

(B) $\frac{2\Delta A}{A}+\frac{3\Delta B}{B}-\frac{4\Delta C}{C}$

(C) $\frac{2\Delta A}{A}+\frac{3\Delta B}{B}+\frac{4\Delta C}{C}$

(D) $\frac{\Delta A}{A}+\frac{\Delta B}{B}-\frac{\Delta C}{C}$

Official Ans. by NTA (C)

Sol. $Z=\frac{A^2B^3}{C^4}$

In case of error

$$\frac{dZ}{Z}=\frac{2dA}{A}+\frac{3dB}{B}+\frac{4dC}{C}$$

$$\boxed{\frac{\Delta Z}{Z}=\frac{2\Delta A}{A}+\frac{3\Delta B}{B}+\frac{4\Delta C}{C}}$$

2. $\vec{A}$ is a vector quantity such that $|\vec{A}|=$ non-zero constant. Which of the following expressions is true for $\vec{A}$?

(A) $\vec{A}.\vec{A}=0$

(B) $\vec{A}\times\vec{A}<0$

(C) $\vec{A}\times\vec{A}=0$

(D) $\vec{A}\times\vec{A}>0$

Official Ans. by NTA (C)

Sol. $|\vec{A}|\neq 0$

$$\vec{A}\times\vec{A}=|\vec{A}||\vec{A}|\sin 0°\hat{n}=0$$

3. Which of the following relations is true for two unit vectors $\hat{A}$ and $\hat{B}$ making an angle θ to each other?

(A) $|\hat{A}+\hat{B}|=|\hat{A}-\hat{B}|\tan\frac{\theta}{2}$

(B) $|\hat{A}-\hat{B}|=|\hat{A}+\hat{B}|\tan\frac{\theta}{2}$

(C) $|\hat{A}+\hat{B}|=|\hat{A}-\hat{B}|\cos\frac{\theta}{2}$

(D) $|\hat{A}-\hat{B}|=|\hat{A}+\hat{B}|\cos\frac{\theta}{2}$

Official Ans. by NTA (B)

Sol. $|\hat{A}+\hat{B}|=\sqrt{|\hat{A}|^2+|\hat{B}|^2+2|\hat{A}||\hat{B}|\cos\theta}$

$=\sqrt{1+1+2\cos\theta}$

$=\sqrt{2(1+\cos\theta)}$

$=\sqrt{2\times 2\cos^2\frac{\theta}{2}}$

$=2\cos\frac{\theta}{2}$

$|\hat{A}-\hat{B}|=\sqrt{|\hat{A}|^2+|\hat{B}|^2-2|\hat{A}||\hat{B}|\cos\theta}$

$=\sqrt{2-2\cos\theta}$

$=2\sin\frac{\theta}{2}$

$\frac{|\hat{A}+\hat{B}|}{|\hat{A}-\hat{B}|}=\cot\frac{\theta}{2}$

4. If force $\vec{F} = 3\hat{i} + 4\hat{j} - 2\hat{k}$ acts on a particle having position vector $2\hat{i} + \hat{j} + 2\hat{k}$ then, the torque about the origin will be :-

(A) $3\hat{i} + 4\hat{j} - 2\hat{k}$

(B) $-10\hat{i} + 10\hat{j} + 5\hat{k}$

(C) $10\hat{i} + 5\hat{j} - 10\hat{k}$

(D) $10\hat{i} + \hat{j} - 5\hat{k}$

Official Ans. by NTA (B)

Sol. $\vec{\tau} = \vec{r} \times \vec{F}$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & 2 \\ 3 & 4 & -2 \end{vmatrix}$$

$$= \hat{i}(-2-8) - \hat{j}(-4-6) + \hat{k}(8-3)$$

$$= -10\hat{i} + 10\hat{j} + 5\hat{k}$$

5. The height of any point P above the surface of earth is equal to diameter of earth. The value of acceleration due to gravity at point P will be : (Given g = acceleration due to gravity at the surface of earth)

(A) g/2

(B) g/4

(C) g/3

(D) g/9

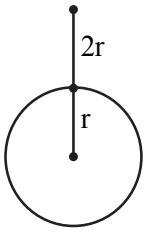
Official Ans. by NTA (D)

Sol. $g = \frac{Gm}{r^2}$

$g' = \frac{Gm}{(3r)^2}$

$g' = \frac{Gm}{9r^2}$

$g' = \frac{g}{9}$



6. The terminal velocity (v_t) of the spherical rain drop depends on the radius (r) of the spherical rain drop as:-

(A) $r^{1/2}$ (B) r

(C) r^2 (D) r^3

Official Ans. by NTA (C)

Sol. $v_t = \frac{2}{9}\frac{gr^2(\rho_p - \rho_l)}{\eta}$; $v_t \propto r^2$

7. The relation between root mean square speed (v_{rms}) and most probable speed (v_p) for the molar mass M of oxygen gas molecule at the temperature of 300 K will be :-

(A) $v_{rms} = \sqrt{\frac{2}{3}} v_p$ (B) $v_{rms} = \sqrt{\frac{3}{2}} v_p$

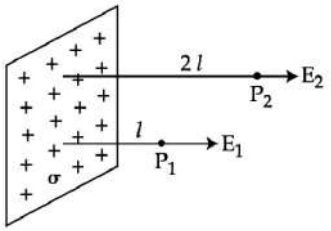
(C) $v_{rms} = v_p$ (D) $v_{rms} = \sqrt{\frac{1}{3}} v_p$

Official Ans. by NTA (B)

Sol. $v_{rms} = \sqrt{\frac{3RT}{M}}$ and $v_{mp} = \sqrt{\frac{2RT}{M}}$

Thus $v_{rms} = \sqrt{\frac{3}{2}} v_{mp}$

8. In the figure, a very large plane sheet of positive charge is shown. P_1 and P_2 are two points at distance l and $2l$ from the charge distribution. If σ is the surface charge density, then the magnitude of electric fields E_1 and E_2 at P_1 and P_2 respectively are :



(A) $E_1 = \sigma/\varepsilon_0$, $E_2 = \sigma/2\varepsilon_0$

(B) $E_1 = 2\sigma/\varepsilon_0$, $E_2 = \sigma/\varepsilon_0$

(C) $E_1 = E_2 = \sigma/2\varepsilon_0$

(D) $E_1 = E_2 = \sigma/\varepsilon_0$

Official Ans. by NTA (C)

Sol. As the sheet is very large $\vec{E}$ is independent of distance from it.

Thus $E_1 = E_2 = \frac{\sigma}{2\varepsilon_0}$

9. Match List-I with List-II

List-I	List-II
(A) AC generator	(I) Detects the presence of current in the circuit
(B) Galvanometer	(II) Converts mechanical energy into electrical energy
(C) Transformer	(III) Works on the principle of resonance in AC circuit
(D) Metal detector	(IV) Changes an alternating voltage for smaller or greater value

Choose the **correct answer** from the options given below :-

(A) (A)–(II), B–(I), (C)–(IV), (D)–(III)

(B) (A)–(II), B–(I), (C)–(III), (D)–(IV)

(C) (A)–(III), B–(IV), (C)–(II), (D)–(I)

(D) (A)–(III), B–(I), (C)–(II), (D)–(IV)

Official Ans. by NTA (A)

Sol. AC generator converts mechanical energy into electrical energy. Galvanometer shows deflection when current passes through it so it is used to show presence of current in any wire. Transformer is used to step up or step down the voltage. Metals detectors contain inductor coils and use principle of induction and resonance in AC circuit.

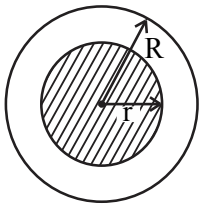
10. A long straight wire with a circular cross-section having radius R, is carrying a steady current I. The current I is uniformly distributed across this cross-section. Then the variation of magnetic field due to current I with distance r (r < R) from its centre will be :-

(A) $B \propto r^2$ (B) $B \propto r$

(C) $B \propto \frac{1}{r^2}$ (D) $B \propto \frac{1}{r}$

Official Ans. by NTA (B)

Sol. Use Ampere's law



$B.2\pi r = \mu_0 . \frac{I}{\pi R^2} . \pi r^2$

Thus $B \propto r$

11. If wattless current flows in the AC circuit, then the circuit is

(A) Purely Resistive circuit

(B) Purely Inductive circuit

(C) LCR series circuit

(D) RC series circuit only

Official Ans. by NTA (B)

Sol. Purely Inductive circuit

$\theta = \frac{\pi}{2}$

$\cos\frac{\pi}{2} = 0$

Average power = 0

12. The electric field in an electromagnetic wave is given by $E = 56.5 \sin\omega(t - x/c)\, NC^{-1}$. Find the intensity of the wave if it is propagating along x-axis in the free space. (Given $\varepsilon_0 = 8.85 \times 10^{-12} C^2 N^{-1} m^{-2}$)

(A) 5.65 Wm^{-2} (B) 4.24 Wm^{-2}

(C) 1.9×10^{-7} Wm^{-2} (D) 56.5 Wm^{-2}

Official Ans. by NTA (B)

Sol. $I = \frac{1}{2}\varepsilon_0 E_0^2 c$

$I = \frac{1}{2} \times (8.85 \times 10^{-12})(56.5)^2 \times (3 \times 10^8)$

= 4.24 Wm^{-2}.

13. The two light beams having intensities I and 9I interfere to produce a fringe pattern on a screen. The phase difference between the beams is $\frac{\pi}{2}$ at point P and π at point Q. Then the difference between the resultant intensities at P and Q will be :

(A) 2 I (B) 6 I

(C) 5 I (D) 7 I

Official Ans. by NTA (B)

Sol. $I_P = I + 9I + 2\sqrt{I \times 9I}\cos\frac{\pi}{2}$

$I_P = 10\ I$

$I_Q = I + 9I + 2\sqrt{I \times 9I}\cos\pi$

$= 10\ I - 6I = 4I$

$\therefore I_P - I_Q = 10I - 4I = 6I$

14. A light wave travelling linearly in a medium of dielectric constant 4, incident on the horizontal interface separating medium with air. The angle of incidence for which the total intensity of incident wave will be reflected back into the same medium will be (Given : relative permeability of medium $\mu_r = 1$)

(A) 10° (B) 20°

(C) 30° (D) 60°

Official Ans. by NTA (D)

Sol. For total internal reflection, $i > \theta_C$

$\Rightarrow \sin i > \sin\theta_C$

$\Rightarrow \sin i > \frac{\mu_R}{\mu_D}$ .....(1)

Also $\mu = \sqrt{\mu_r \in_r}$

$\frac{\mu_R}{\mu_D} = \frac{\sqrt{1\times1}}{\sqrt{4\times1}} = \frac{1}{2}$

From (1), $\sin i > \frac{1}{2} \Rightarrow i > 30°$, i = 60°

15. Given below are two statements :-

Statement I : Davisson-Germer experiment establishes the wave nature of electrons.

Statement II : If electrons have wave nature, they can interfere and show diffraction.

In the light of the above statements choose the **correct answer** from the options given below:-

(A) Both **Statement I** and **Statement II** are true

(B) Both **Statement I** and **Statement II** are false

(C) **Statement I** is true but **Statement II** is false

(D) **Statement I** is false but **Statement II** is true

Official Ans. by NTA (A)

Sol. In Davisson-Germer experiment the electrons exhibit diffraction there by proving that electrons have wave nature. Hence both statement are correct.

Sol. Both the options are correct by concept.

16. The ratio for the speed of the electron in the 3rd orbit of He^+ to the speed of the electron in the 3rd orbit of hydrogen atom will be :-

(A) 1 : 1 (B) 1 : 2

(C) 4 : 1 (D) 2 : 1

Official Ans. by NTA (D)

Sol. $v \propto \frac{Z}{n} \propto Z$ (n = constant)

$\Rightarrow \frac{v_{He^+}}{v_H} = \frac{Z_{He^+}}{Z_H} = \frac{2}{1}$

17. The photodiode is used to detect the optiocal signals. These diodes are preferably operated in reverse biased mode because.

(A) fractional change in majority carriers produce higher forward bias current

(B) fractional change in majority carriers produce higher reverse bias current

(C) fractional change in minority carriers produce higher forward bias current

(D) fractional change in minority carriers produce higher reverse bias current

Official Ans. by NTA (D)

Sol. Very small change in minority charge carriers produces high value of reverse bias current.

18. A signal of 100 THz frequency can be transmitted with maximum efficiency by :

(A) Coaxial cable

(B) Optical fibre

(C) Twisted pair of copper wires

(D) Water

Official Ans. by NTA (B)

Sol. Optical fibre frequency range is 1 THz to 1000 THz.

19. The difference of speed of light in the two media A and B ($v_A - v_B$) is 2.6×10^7 m/s. If the refractive index of medium B is 1.47, then the ratio of refractive index of medium B to medium A is : (Given : speed of light in vacuum $c = 3 \times 10^8$ ms^{-1})

(A) 1.303

(B) 1.318

(C) 1.13

(D) 0.12

Official Ans. by NTA (C)

Sol. $v = \frac{c}{\mu}$

$$\Rightarrow v_B = \frac{3\times10^8}{1.47} = 2.04\times10^8 = 20.4\times10^7\,m/s$$

$$\because v_A - v_B = 2.6\times10^7\,m/s$$

$$\therefore v_A = (20.4+2.6)\times10^7 = 23\times10^7\,m/s$$

$$\therefore \frac{\mu_B}{\mu_A} = \frac{v_A}{v_B} = \frac{23\times10^7}{20.4\times10^7} = 1.13$$

20. A teacher in his physics laboratory allotted an experiment to determine the resistance (G) of a galvanometer. Students took the observations for $\frac{1}{3}$ deflection in the galvanometer. Which of the below is **true** for measuring value of G?

(A) $\frac{1}{3}$ deflection method cannot be used for determining the resistance of the galvanometer.

(B) $\frac{1}{3}$ deflection method can be used and in this case the G equals to twice the value of shunt resistance(s).

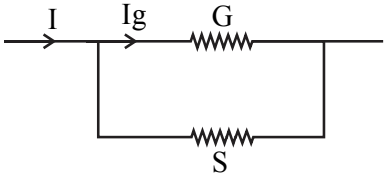
(C) $\frac{1}{3}$ deflection method can be used and in this case, the G equals to three times the value of shunt resistance(s)

(D) $\frac{1}{3}$ deflection method can be used and in this case the G value equals to the shunt resistance(s).

Official Ans. by NTA (B)

Sol. In galvanometer

$$\Rightarrow (I - I_g)S = I_g G$$



$$\frac{I_g}{I} = \frac{S}{S+G}$$

$$\Rightarrow \frac{1}{3} = \frac{S}{S+G} \Rightarrow S+G = 3S \Rightarrow G = 2S$$

SECTION-B

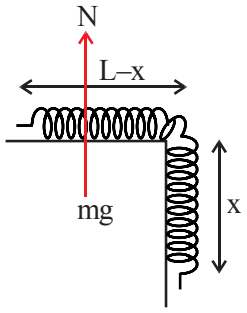
1. A uniform chain of 6 m length is placed on a table such that a part of its length is hanging over the edge of the table. The system is at rest. The co-efficient of static friction between the chain and the surface of the table is 0.5, the maximum length of the chain hanging from the table is _____m.

Official Ans. by NTA 2

Sol. Mass per unit length = λ

$$N = mg = \lambda(L-x)g$$

$$fs_{max} = \mu_s N$$



$$fs_{max} = (0.5)(\lambda)(L-x)g$$

And also $fs_{max} = m_x g$

$$0.5\lambda(L-x)g = \lambda x g$$

$$\frac{L-x}{2} = x$$

$$\frac{L}{2} = \frac{3x}{2} \Rightarrow x = \frac{L}{3} = \frac{6}{3} = 2m$$

2. A 0.5 kg block moving at a speed of 12 ms^{-1} compresses a spring through a distance 30 cm when its speed is halved. The spring constant of the spring will be _____ Nm^{-1}.

Official Ans. by NTA 600

Sol. $U_i + K_i = U_f + K_f$

$$\Rightarrow 0 + \frac{1}{2}m(12)^2 = \frac{1}{2}K(0.3)^2 + \frac{1}{2}m(6)^2$$

$$\Rightarrow 0.5(12^2 - 6^2) = K(0.3)^2$$

K = 600 N/m

3. The velocity of upper layer of water in a river is 36 kmh^{-1}. Shearing stress between horizontal layers of water is 10^{-3} Nm^{-2}. Depth of the river is _____m. (Co-efficiency of viscosity of water is 10^{-2} Pa.s)

Official Ans. by NTA 100

Sol. $$F = \eta A \frac{\Delta v_x}{\Delta y}$$

$$\frac{F}{A} = \eta \frac{\Delta v_x}{\Delta y}$$

$$\Rightarrow 10^{-3} = 10^{-2} \times \frac{36 \times 1000}{h \times 3600}$$

$$\Rightarrow h = 10^{-2} \times \frac{36 \times 1000}{10^{-3} \times 3600} = 100 \text{ m}$$

4. A steam engine intakes 50g of steam at 100°C per minute and cools it down to 20°C. If latent heat of vaporization of steam is 540 cal g^{-1}, then the heat rejected by the steam engine per minute is _____ $\times 10^3$ cal.

Official Ans. by NTA 31

Sol. Heat rejected = $mL_f + mS\Delta T$

= (50 × 540) + 50 (1) (100 – 20)

= 31000 Cal

= 31 × 10^3 Cal

5. The first overtone frequency of an open organ pipe is equal to the fundamental frequency of a closed organ pipe. If the length of the closed organ pipe is 20 cm. The length of the open organ pipe is _____ cm.

Official Ans. by NTA 80

Sol. $$f_1 = \frac{2v}{2l_1}$$

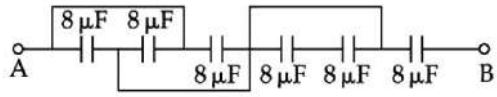
$$f_2 = \frac{v}{4l_2}$$

$f_1 = f_2$

$$= \frac{2v}{2l_1} = \frac{v}{4l_2}$$

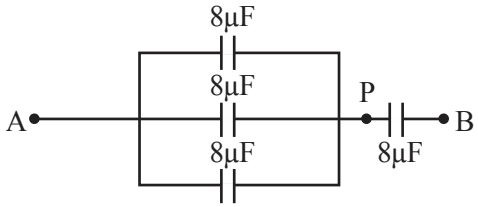
$l_1 = 4l_2 = 80$ cm

6. The equivalent capacitance between points A and B in below shown figure will be _____μF.



Official Ans. by NTA 6

Sol. Two capacitors are short circuited



Finally equivalent capacitance

$$=\frac{24\times8}{24+8}=\frac{24\times8}{32}=6\mu F$$

7. A resistor develops 300 J of thermal energy in 15s, when a current of 2A is passed through it. If the current increases to 3A, the energy developed in 10s is _____ J.

Official Ans. by NTA 450

Sol. $H = i^2Rt$

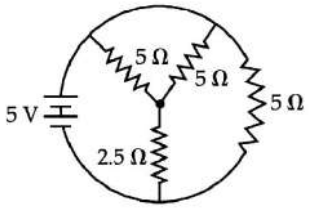
$300 = 2^2 \times R \times 15$

$$\Rightarrow R=\frac{300}{60}=5\Omega$$

Now, for i = 3A, t = 10s, R = 5Ω

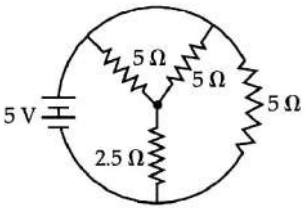
$H = 3^2 \times 5 \times 10 = 450$ J

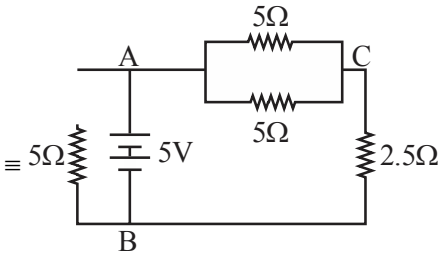
8. The total current supplied to the circuit as shown in figure by the 5V battery is _____A

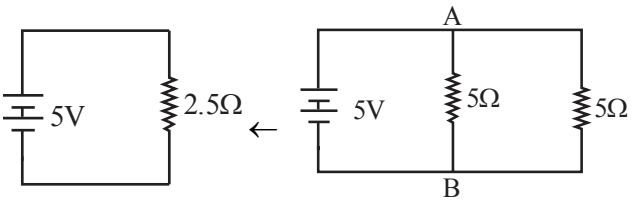


Official Ans. by NTA 2

Sol.







Current supplied by 5V battery

$$=\frac{5V}{2.5\Omega}=2A$$

9. The current in a coil of self inductance 2.0 H is increasing according to $I = 2\sin(t^2)$A. The amount of energy spent during the period when current changes from 0 to 2A is _____ J.

Official Ans. by NTA 4

Sol. $I=2\sin(t^2)\Rightarrow dI=4t\sin(t^2)dt$

If $I=0\Rightarrow t=0$

and $I=2\Rightarrow 2=2\sin t^2$

$$\Rightarrow t=\sqrt{\frac{\pi}{2}}$$

$$E=\int LI\,dI$$

$$=\int 2\times2\sin(t^2)\times4t\cos(t^2)dt$$

$$=8\int_0^{\sqrt{\pi/2}} t\sin(2t^2)dt$$

$$=2\left[-\cos(2t^2)\right]_0^{\sqrt{\pi/2}}$$

$$=2\left[-\cos\pi+\cos0\right]=4$$

10. A force on an object of mass 100g is $(10\hat{i}+5\hat{j})$ N. The position of that object at t = 2s is $(a\hat{i}+b\hat{j})$ m after starting from rest. The value of $\frac{a}{b}$ will be _____

Official Ans. by NTA 2

Sol. $\vec{F}=10\hat{i}+5\hat{j}$

m = 100 g = 0.1 kg

$\vec{a}=\frac{\vec{F}}{m}=100\hat{i}+50\hat{j}$

$\vec{S}=\vec{u}t+\frac{1}{2}\vec{a}t^2=\frac{1}{2}\vec{a}t^2(\text{as } \vec{u}=0)$

$=\frac{1}{2}(100\hat{i}+50\hat{j})2^2$

$=200\hat{i}+100\hat{j}$

$=a\hat{i}+b\hat{j}$

$a=200,\ b=100$

$\therefore \frac{a}{b}=2$

FINAL JEE–MAIN EXAMINATION – JUNE, 2022

(Held On Saturday 25th June, 2022) **TIME : 9 : 00 AM to 12 : 00 PM**

CHEMISTRY

TEST PAPER WITH SOLUTION

SECTION-A

1. Bonding in which of the following diatomic molecule(s) become(s) stronger, on the basis of MO Theory, by removal of an electron ?

(A) NO (B) N_2

(C) O_2 (D) C_2

(E) B_2

Choose the most appropriate answer from the options given below :-

(A) (A), (B), (C) only (B) (B), (C), (E) only

(C) (A), (C) only (D) (D) only

Official Ans. by NTA (C)

Sol. Bond strength $\propto$ Bond order

removal of electron from antibonding MO increases B.O.

NO & O_2 has valence e⁻ in π^*orbital.

2. Incorrect statement for Tyndall effect is :-

(A) The refractive indices of the dispersed phase and the dispersion medium differ greatly in magnitude.

(B) The diameter of the dispersed particles is much smaller than the wavelength of the light used.

(C) During projection of movies in the cinemas hall, Tyndall effect is noticed.

(D) It is used to distinguish a true solution from a colloidal solution.

Official Ans. by NTA (B)

Sol. The diameter of dispersed particle should be somewhat below or near the wavelength of light.

3. The pair, in which ions are isoelectronic with Al^{3+} is :-

(A) Br^- and Be^{2+} (B) Cl^- and Li^+

(C) S^{2-} and K^+ (D) O^{2-} and Mg^{2+}

Official Ans. by NTA (D)

Sol. Isoelectronic species have same no. of electrons

Al^{+3}, O^{2-}, Mg^{+2} all have 10 electrons.

4. Leaching of gold with dilute aqueous solution of NaCN in presence of oxygen gives complex [A], which on reaction with zinc forms the elemental gold and another complex [B]. [A] and [B], respectively are :-

(A) $[Au(CN)_4]^-$ and $[Zn(CN)_2(OH)_2]^{2-}$

(B) $[Au(CN)_2]^-$ and $[Zn(OH)_4]^{2-}$

(C) $[Au(CN)_2]^-$ and $[Zn(CN)_4]^{2-}$

(D) $[Au(CN)_4]^{2-}$ and $[Zn(CN)_6]^{4-}$

Official Ans. by NTA (C)

Sol. $Au + NaCN \rightarrow Na[Au(CN)_2]$

$Zn + Na[Au(CN)_2] \rightarrow Na_2[Zn(CN)_4] + Au$

5. Number of electron deficient molecules among the following

PH_3, B_2H_6, CCl_4, NH_3, LiH and BCl_3 is

(A) 0 (B) 1

(C) 2 (D) 3

Official Ans. by NTA (C)

Sol. Electron deficient species have less than 8 electrons (or two electrons for H) in their valence (incomplete octet)

B_2H_6, BCl_3 have incomplete octet.

6. Which one of the following alkaline earth metal ions has the highest ionic mobility in its aqueous solution?

(A) Be^{2+} (B) Mg^{2+}

(C) Ca^{2+} (D) Sr^{2+}

Official Ans. by NTA (D)

Sol. Highest ionic mobility corresponds to lowest extent of hydration and highest size of gaseous ion.

Hence Sr^{2+} has the highest ionic mobility in its aqueous solution

7. White precipitate of AgCl dissolves in aqueous ammonia solution due to formation of :

(A) $[Ag(NH_3)_4]Cl_2$ (B) $[Ag(Cl)_2(NH_3)_2]$

(C) $[Ag(NH_3)_2]Cl$ (D) $[Ag(NH_3)Cl]Cl$

Official Ans. by NTA (C)

Sol. $AgCl + 2NH_3 \rightarrow [Ag(NH_3)_2]^+Cl^-$

soluble

8. Cerium (IV) has a noble gas configuration. Which of the following is correct statement about it?

(A) It will not prefer to undergo redox reactions.

(B) It will prefer to gain electron and act as an oxidizing agent

(C) It will prefer to give away an electron and behave as reducing agent

(D) It acts as both, oxidizing and reducing agent.

Official Ans. by NTA (B)

Sol. Cerium exists in two different oxidation state +3, +4

$Ce^{+4} + e^- \rightarrow Ce^{3+}$ $E^0 = +1.61$ V

$Ce^{+3} + 3e^- \rightarrow Ce$ $E^0 = -2.336$ V

It shows Ce^{+4} acts as a strong oxidising agent & accepts electron.

9. Among the following, which is the strongest oxidizing agent ?

(A) Mn^{3+} (B) Fe^{3+}

(C) Ti^{3+} (D) Cr^{3+}

Official Ans. by NTA (A)

Sol. Strongest oxidising agent have highest reduction potential value

$E^0_{Mn^{+3}/Mn^{+2}} = 1.51V$ (highest)

10. The eutrophication of water body results in :

(A) loss of Biodiversity

(B) breakdown of organic matter

(C) increase in biodiversity

(D) decrease in BOD.

Official Ans. by NTA (A)

Sol. Eutrophication of water body results in loss of Biodiversity.

11. Phenol on reaction with dilute nitric acid, gives two products. Which method will be most effective for large scale separation ?

(A) Chromatographic separation

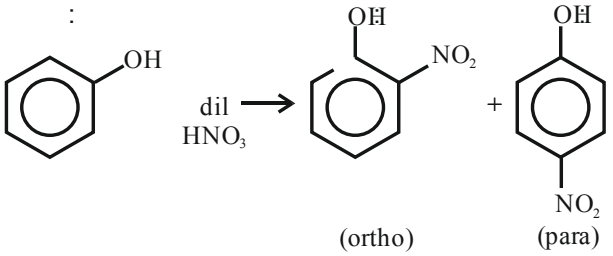
(B) Fractional Crystallisation

(C) Steam distillation

(D) Sublimation

Official Ans. by NTA (C)

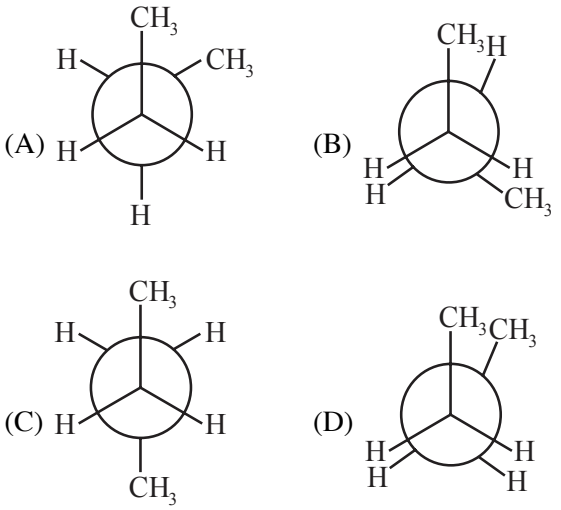
Sol.



Para product has higher boiling point than ortho as intermolecular H-bond is possible in former, where as intramolecular H-bond is possible in ortho product.

Steam distillation can separate them as ortho product is steam volatile.

12. In the following structures, which one is having staggered conformation with maximum dihedral angle?

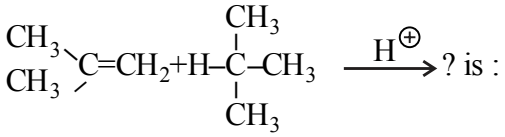


Official Ans. by NTA (C)

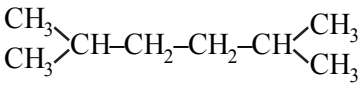
Sol. Dihedral angle : It's the angle b/w 2 specified groups ($-CH_3$ here)

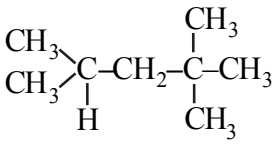
Staggered form is Given in option (C) & the angle is 180^0

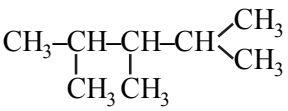
13. The products formed in the following reaction.

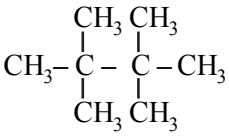


? is :

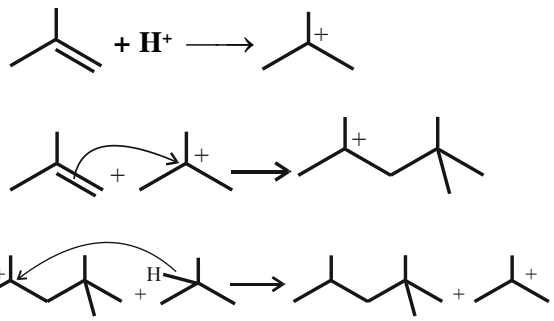
(A) 

(B) 

(C) 

(D) 

Official Ans. by NTA (B)

Sol. 

14. The IUPAC name of ethylidene chloride is :-

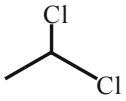
(A) 1-Chloroethene

(B) 1-Chloroethyne

(C) 1,2-Dichloroethane

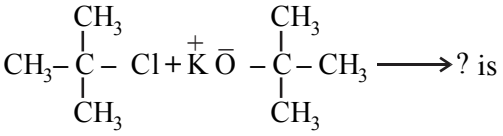
(D) 1,1-Dichloroethane

Official Ans. by NTA (D)

Sol. 

"1, 1-Dichloroethane is Ethylidene chloride"

15. The major product in the reaction



? is

(A) t-Butyl ethyl ether

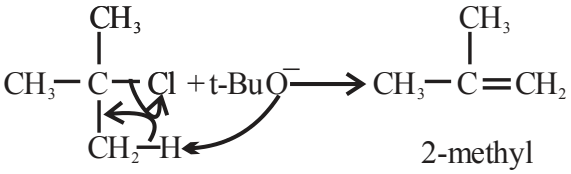
(B) 2,2-Dimethyl butane

(C) 2-Methyl pent-1-ene

(D) 2-Methyl prop-1-ene

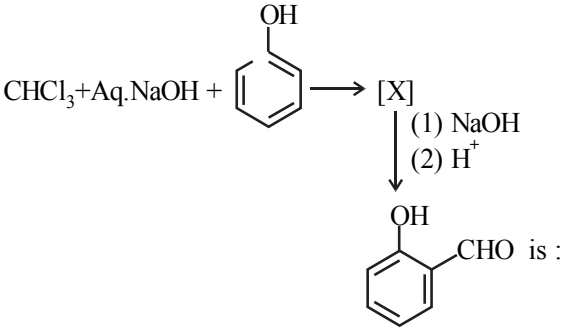
Official Ans. by NTA (D)

Sol. We have been given a bulky base, hence elimination will take place & not substitution.

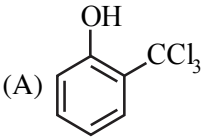


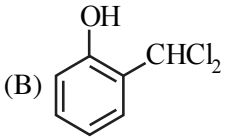
2-methyl prop-1-ene

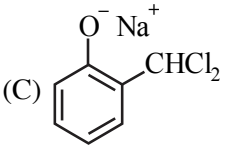
16. The intermediate X, in the reaction

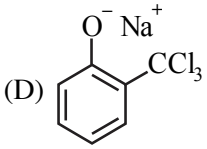


? is :

(A) 

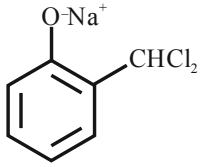
(B) 

(C) 

(D) 

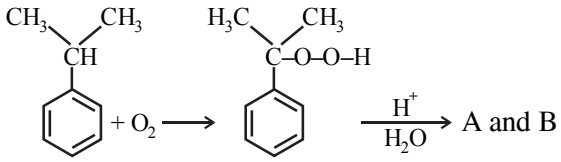
Official Ans. by NTA (C)

Sol. It's a classic Reimer-Tiemann reaction.

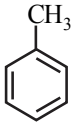


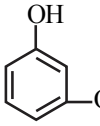
Will be the intermediate formed.

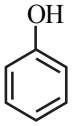
17. In the following reaction :

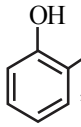


The compounds A and B respectively are :-

(A)  , CH_3COOH

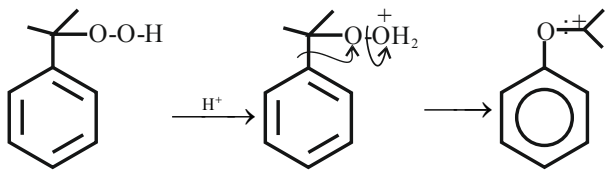
(B)  , CH_3COOH

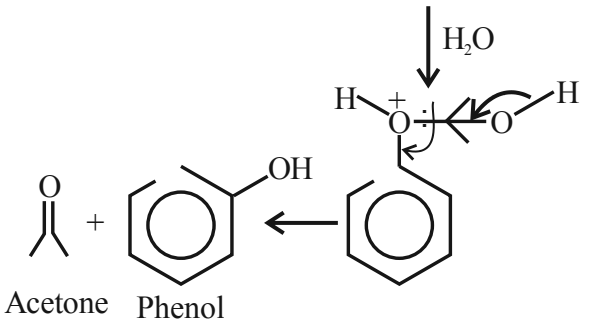
(C)  , CH_3COCH_3

(D)  , CH_3COCH_3

Official Ans. by NTA (C)

Sol. Given reaction is cumene-Peroxide method for the preparation of phenol.
In this reaction





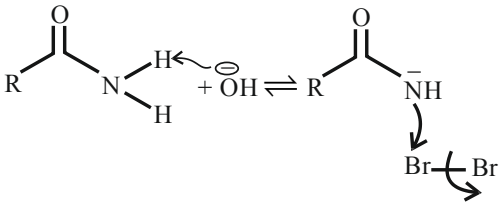
18. The reaction of $R-\underset{\underset{O}{\|}}{C}-NH_2$ with bromine and KOH gives RNH_2 as the end product. Which one of the following is the intermediate product formed in this reaction ?

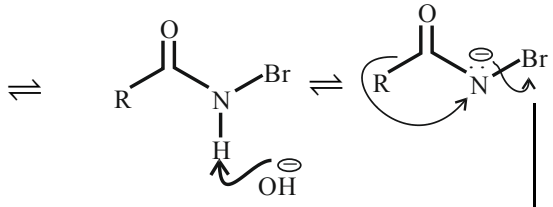
(A) $R-\underset{\underset{O}{\|}}{C}-NH-Br$ (B) $R-NH-Br$

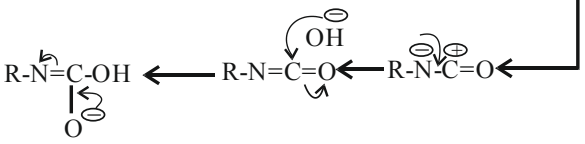
(C) $R-N=C=O$ (D) $R-\underset{\underset{O}{\|}}{C}-NBr_2$

Official Ans. by NTA (C)

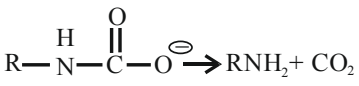
Sol. The given reaction is Hoffmann-Bromide degradation method.







Proton transfer



19. Using very little soap while washing clothes, does not serve the purpose of cleaning of clothes because

(A) soap particles remain floating in water as ions

(B) the hydrophobic part of soap is not able to take away grease

(C) the micelles are not formed due to concentration of soap, below its CMC value

(D) colloidal structure of soap in water is completely disturbed.

Official Ans. by NTA (C)

Sol. Micelle formation only takes place above CMC.

20. Which one of the following is an example of artificial sweetner ?

(A) Bithional (B) Alitame

(C) Salvarsan (D) Lactose

Official Ans. by NTA (B)

Sol. Alitame is a second generation dipeptide sweetner that is 200 times sweeter than sucrose.

SECTION-B

1. The number of N atoms is 681 g of $C_7H_5N_3O_6$ is $x \times 10^{21}$. The value of x is _____ ($N_A = 6.02 \times 10^{23}$ mol^{-1}) (Nearest Integer)

Official Ans. by NTA (5418)

Sol. M.M. of $C_7H_5N_3O_6$ is 84 + 5 + 42 + 96 = 227

$$n_{C_7H_5N_3O_6} = \frac{681}{227} = 3$$

$$n_N = \frac{681}{227} \times 3 = 9 \text{ mol}$$

no. of N atoms = $9 \times 6.02 \times 10^{23}$

$= 5418 \times 10^{21}$

$\therefore$ The answer is 5418.

2. The distance between Na^+ and Cl^- ions in solid NaCl of density 43.1 g cm^{-3} is _____ $\times 10^{-10}$m. (Nearest Integer)

(Given : $N_A = 6.02 \times 10^{23}$ mol^{-1})

Official Ans. by NTA (1)

Sol. Unit cell formula – Na_4Cl_4

$$\text{Mass per unit cell} = \frac{Z \times M.M.}{N_A} g$$

$$= \frac{4 \times 58.5}{N_A} g$$

$$d_{\text{unit cell}} = \frac{m}{V} = \frac{m}{a^3}$$

$$\Rightarrow \frac{4 \times 58.5}{N_A \cdot a^3} = 43.1$$

$$\Rightarrow a^3 = 9.02 \times 10^{-24} cm^3$$

$$\Rightarrow a = 2.08 \times 10^{-8} cm$$

$$\Rightarrow a = 2.08 \times 10^{-10} m$$

Also $a = 2\left(r_{Na^+} + r_{Cl^-}\right)$

$$\Rightarrow r_{Na^+} + r_{Cl^-} = 1.04 \times 10^{-10} m$$

$\therefore$ The answer is 1

3. The longest wavelength of light that can be used for the ionisation of lithium atom (Li) in its ground state is $x \times 10^{-8}$ m. The value of x is _____. (Nearest Integer)

(Given : Energy of the electron in the first shell of the hydrogen atom is -2.2×10^{-18} J; $h = 6.63 \times 10^{-34}$ Js and $c = 3 \times 10^8$ ms^{-1})

Official Ans. by NTA (4)

Sol. We can not calculate I.E. of lithium atom.

4. The standard entropy change for the reaction $4Fe(s) + 3O_2(g) \rightarrow 2Fe_2O_3(s)$ is -550 JK^{-1} at 298 K.

[Given : The standard enthalpy change for the reaction is -165 kJ mol^{-1}]. The temperature in K at which the reaction attains equilibrium is _____. (Nearest Integer)

Official Ans. by NTA (300)

Sol. $\Delta G = \Delta H - T\Delta S = 0$ at equilibrium

$\Rightarrow -165\times10^3 - T\times(-505) = 0$

$\Rightarrow T = 300K$

The answer is 300

5. 1 L aqueous solution of H_2SO_4 contains 0.02 m mol H_2SO_4. 50% of this solution is diluted with deionized water to give 1 L solution (A). In solution (A), 0.01 m mol of H_2SO_4 are added. Total m mols of H_2SO_4 in the final solution is _____ $\times 10^3$ m mols.

Official Ans. by NTA (0)

Sol. $n_{H_2SO_4}$ in Sol^n A = 50% of original solution

= 0.01 m mol.

$n_{H_2SO_4}$ in Final solution = 0.01 + 0.01

= 0.02 mmol

= 0.00002 $\times$ 10^3 mmol

The answer 0

6. The standard free energy change ($\Delta G°$) for 50% dissociation of N_2O_4 into NO_2 at 27°C and 1 atm pressure is –x J mol^{-1}. The value of x is _____. (Nearest Integer)

[Given : R = 8.31 J K^{-1} mol^{-1}, log 1.33 = 0.1239 ln 10 = 2.3]

Official Ans. by NTA (710)

Sol.

	N_2O_4	$\rightleftharpoons$	$2NO_2$
t = 0	1 mol		
t = t	(1-0.5) mol		0.5 × 2 mol
	= 0.5 mol		1 mol

$$k_P = \frac{\left(\frac{1}{1.5}\times1\right)^2}{\left(\frac{0.5}{1.5}\times1\right)} = \frac{1}{0.75} = \frac{100}{75}$$

= 1.33

$\Delta G^0 = -RT\ell nk_P$

$= -8.31\times300\times\ell n(1.33) = -710.45$ J / mol

= –710 J/mol.

7. In a cell, the following reactions take place

$Fe^{2+} \rightarrow Fe^{3+}e^-$ $\quad E^o_{Fe^{3+}/Fe^{2+}} = 0.77$ V

$2I^- \rightarrow I_2 + 2e^-$ $\quad E^o_{I_2/I^-} = 0.54$ V

The standard electrode potential for the spontaneous reaction in the cell is x × 10^{-2}V 298 K. The value of x is _____ (Nearest Integer)

Official Ans. by NTA (23)

Sol. $\underset{\text{Cathode}}{Fe^{+3}} + \underset{\text{anode}}{I^-} \longrightarrow I_2 + Fe^{+2}$

$E^0_{Cell} = E^0_{cathode} - E^0_{anode}$

=0.77–0.54

=0.23

$= 23\times10^{-2}$ V

8. For a given chemical reaction

$$\gamma_1A + \gamma_2B \rightarrow \gamma_3C + \gamma_4D$$

Concentration of C changes from 10 mmol dm^{-3} to 20 mmol dm^{-3} in 10 seconds. Rate of appearance of D is 1.5 times the rate of disappearance of B which is twice the rate of disappearance A. The rate of appearance of D has been experimentally determined to be 9 mmol dm^{-3} s^{-1}. Therefore the rate of reaction is _____mmol dm^{-3} s^{-1}. (Nearest Integer)

Official Ans. by NTA (1)

Sol. $\gamma_1A + \gamma_2B \longrightarrow \gamma_3C + \gamma_4D$

$$\text{Given}: +\frac{d[D]}{dt} = \frac{-3}{2}\frac{d[B]}{dt}$$

$$\Rightarrow \frac{-1}{2}\frac{d[B]}{dt} = \frac{+1}{3}\frac{d[D]}{dt}$$

$$-\frac{d[B]}{dt}=-2\frac{d[A]}{dt}\Rightarrow-\frac{1}{2}\frac{d[B]}{dt}=\frac{-d(A)}{dt}$$

$$+\frac{d[B]}{dt}=9\ \text{mmol}\,\text{dm}^{-3}\text{s}^{-1}$$

$$\frac{+d[C]}{dt}=\frac{20-10}{10}=1\,\text{mmol}\,\text{dm}^{-3}\text{s}^{-1}$$

$$\frac{+d[C]}{dt}=\frac{1}{9}\times\frac{+d[D]}{dt}$$

$$1A+2B\longrightarrow\frac{1}{3}C+3D$$

$$\Rightarrow 3A+6B\longrightarrow C+9D$$

$$\text{Rate of reaction}=\frac{+d[C]}{dt}=1\ \text{mmol dm}^{-3}\ \text{s}^{-1}$$

9. If $[Cu(H_2O)_4]^{2+}$ absorbs a light of wavelength 600 nm for d–d transition, then the value of octahedral crystal field splitting energy for $[Cu(H_2O)_6]^{2+}$ will be_____× 10^{-21} J. (Nearest Integer)

(Given : h = 6.63 × 10^{-34}Js

and c = 3.08 × 10^8 ms^{-1})

Official Ans. by NTA (746)

Sol. $$\Delta_t=\frac{hc}{\lambda}=\frac{6.63\times10^{-34}\times3.08\times10^{8}}{600\times10^{-9}}$$

$$=\frac{6.63\times3.08\times10^{-17}}{600}$$

$$=0.034034\times10^{-17}$$

$$=340.34\times10^{-21}\,\text{J}$$

$$\Delta_0=\frac{9}{4}\Delta_t$$

$$=\frac{9}{4}\times340.34\times10^{-21}$$

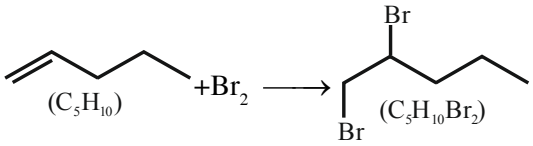
$$=765.765\times10^{-21}\,\text{J}$$

$$\approx766\times10^{-21}\,\text{J}$$

Answer = 766

10. Number of grams of bromine that will completely react with 5.0g of pent-1-ene is _____ × 10^{-2}g. (Atomic mass of Br = 80 g/mol) [Nearest Integer)

Official Ans. by NTA (1143)

Sol. 

moles of Br_2 = moles of C_5H_{10}

$$\Rightarrow\frac{w}{160}=\frac{5}{70}$$

$$\Rightarrow w=\frac{5\times160}{70}\text{g}$$

$$=11.428\ \text{g}$$

$$=1142.8\times10^{-2}\,\text{g}\approx1143\times10^{-2}\,\text{g}$$

FINAL JEE–MAIN EXAMINATION – JUNE, 2022

(Held On Saturday 25th June, 2022) **TIME : 3 : 00 PM to 6 : 00 PM**

MATHEMATICS

TEST PAPER WITH SOLUTION

SECTION-A

1. Let a circle C touch the lines $L_1 : 4x - 3y + K_1 = 0$ and $L_2 : 4x - 3y + K_2 = 0$, $K_1, K_2 \in R$. If a line passing through the centre of the circle C intersects L_1 at (–1, 2) and L_2 at (3, – 6), then the equation of the circle C is

(A) $(x - 1)^2 + (y - 2)^2 = 4$

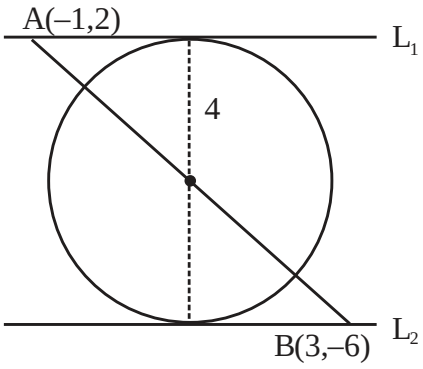
(B) $(x + 1)^2 + (y - 2)^2 = 4$

(C) $(x - 1)^2 + (y + 2)^2 = 16$

(D) $(x - 1)^2 + (y - 2)^2 = 16$

Official Ans. by NTA (C)

Sol.



$L_1 : 4x - 3y + K_1 = 0$

$L_2 : 4x - 3y + K_2 = 0$

now

$-4 - 6 + K_1 = 0 \Rightarrow K_1 = 10$

$12 + 18 + K_2 = 0 \Rightarrow K_2 = -30$

$\Rightarrow$ Tangent to the circle are

$4x - 3y + 10 = 0$

$4x - 3y - 30 = 0$

Length of diameter $2r = \dfrac{|10+30|}{5} = 8$

$\Rightarrow r = 4$

Now centre is mid point of A & B

$x = 1, y = -2$

Equation of circle

$(x - 1)^2 + (y + 2)^2 = 16$ **Ans.**

2. The value of $\displaystyle\int_0^{\pi} \frac{e^{\cos x}\sin x}{(1+\cos^2 x)(e^{\cos x}+e^{-\cos x})}dx$ is equal to

(A) $\dfrac{\pi^2}{4}$ (B) $\dfrac{\pi^2}{2}$

(C) $\dfrac{\pi}{4}$ (D) $\dfrac{\pi}{2}$

Official Ans. by NTA (C)

Sol. $\displaystyle\int_0^{\pi} \frac{e^{\cos x}\sin x}{(1+\cos^2 x)(e^{\cos x}+e^{-\cos x})}dx$ ....(1)

Use King's property

$$I = \int_0^{\pi} \frac{e^{-\cos x}\sin x}{(1+\cos^2 x)(e^{-\cos x}+e^{\cos x})}dx \quad ....(2)$$

On adding equation (1) and (2), we get

$$2I = \int_0^{\pi} \frac{\sin x}{1+\cos^2 x}dx = 2\int_0^{\pi/2} \frac{\sin x}{1+\cos^2 x}dx$$

On putting cos x = t, we get

$$I = \int_0^1 \frac{dt}{1+t^2} = \left(\tan^{-1} t\right)_0^1 = \frac{\pi}{4}$$

3. Let a, b and c be the length of sides of a triangle ABC such that $\dfrac{a+b}{7} = \dfrac{b+c}{8} = \dfrac{c+a}{9}$. If r and R are the radius of incircle and radius of circumcircle of the triangle ABC, respectively, then the value of $\dfrac{R}{r}$ is equal to

(A) $\dfrac{5}{2}$ (B) 2

(C) $\dfrac{3}{2}$ (D) 1

Official Ans. by NTA (A)

Sol. $\frac{a+b}{7}=\frac{b+c}{8}=\frac{c+a}{9}=\lambda$

$a + b = 7\lambda$, $b + c = 8\lambda$, $a + c = 9\lambda$

$\Rightarrow \quad a + b + c = 12\lambda$

Now $a = 4\lambda$, $b = 3\lambda$, $c = 5\lambda$

$\because \quad c^2 = b^2 + a^2$

$\angle C = 90°$

$$\Delta = \frac{1}{2}ab\sin C = \frac{1}{2}ab$$

$$\frac{R}{r} = \frac{c}{2\sin C}\times\frac{s}{\Delta} = \frac{c}{2}\times\frac{6\lambda}{\frac{1}{2}ab} = \frac{c}{ab}\times 6\lambda = \frac{5}{2}$$

4. Let $f : N \to R$ be a function such that $f(x+y)=2f(x)f(y)$ for natural numbers x and y. If $f(1) = 2$, then the value of α for which

$$\sum_{k=1}^{10} f(\alpha+k)=\frac{512}{3}(2^{20}-1)$$

holds, is

(A) 2 (B) 3

(C) 4 (D) 6

Official Ans. by NTA (C)

Sol. $f : N \to R$, $f(x + y) = 2 f(x) f(y)$ ....(1)

$f(1) = 2$,

$$\sum_{k=1}^{10} f(\alpha+k) = 2f(\alpha)\sum_{k=1}^{10} f(k)$$

$$= 2f(\alpha)\left(f(1)+f(2)+.....+f(10)\right) \quad ....(2)$$

From (1)

$f(2) = 2 f^2(1) = 2^3$

$f(3) = 2 f(2) f(1) = 2^5$

$\vdots \qquad \vdots$

$f(10) = 2^9 f^{10}(1) = 2^{19}$

$f(\alpha) = 2^{2\alpha - 1}$; $\alpha \in N$

from (2)

$$\sum_{k=1}^{10} f(\alpha+k) = 2\left(2^{2\alpha-1}\right)\left(2+2^3+2^5+....+2^{19}\right)$$

$$\frac{512}{3}(2^{20}-1) = 2^{2\alpha}\left(2\frac{(2^{20}-1)}{3}\right)$$

Hence $\alpha = 4$

5. Let A be a 3×3 real matrix such that $A\begin{pmatrix}1\\1\\0\end{pmatrix}=\begin{pmatrix}1\\1\\0\end{pmatrix}; A\begin{pmatrix}1\\0\\1\end{pmatrix}=\begin{pmatrix}-1\\0\\1\end{pmatrix}$ and $A\begin{pmatrix}0\\0\\1\end{pmatrix}=\begin{pmatrix}1\\1\\2\end{pmatrix}$.

If $X = (x_1, x_2, x_3)^T$ and I is an identity matrix of order 3, then the system $(A - 2I)X = \begin{pmatrix}4\\1\\1\end{pmatrix}$ has

(A) no solution

(B) infinitely many solutions

(C) unique solution

(D) exactly two solutions

Official Ans. by NTA (B)

Sol. $A = \begin{bmatrix}a_1 & b_1 & c_1\\ a_2 & b_2 & c_2\\ a_3 & b_3 & c_3\end{bmatrix}$

$$A\begin{bmatrix}0\\0\\1\end{bmatrix}=\begin{bmatrix}c_1\\c_2\\c_3\end{bmatrix}=\begin{bmatrix}1\\1\\2\end{bmatrix}$$

$\Rightarrow \quad c_1 = 1, c_2 = 1, c_3 = 2$

$$A\begin{bmatrix}1\\0\\1\end{bmatrix}=\begin{bmatrix}c_1+a_1\\c_2+a_2\\c_3+a_3\end{bmatrix}=\begin{bmatrix}-1\\0\\1\end{bmatrix}$$

$\Rightarrow \quad a_1 = -2, a_2 = -1, a_3 = -1$

$$A\begin{bmatrix}1\\1\\0\end{bmatrix}=\begin{bmatrix}a_1+b_1\\a_2+b_2\\a_3+b_3\end{bmatrix}=\begin{bmatrix}1\\1\\0\end{bmatrix}$$

$\Rightarrow \quad b_1 = 3, b_2 = 2, b_3 = 1$

$$\Rightarrow \quad A = \begin{bmatrix}-2 & 3 & 1\\ -1 & 2 & 1\\ -1 & 1 & 2\end{bmatrix}$$

$$\Rightarrow \quad A - 2I = \begin{bmatrix}-4 & 3 & 1\\ -1 & 0 & 1\\ -1 & 1 & 0\end{bmatrix}$$

$|A - 2I| = 0$

Now, $\begin{bmatrix}-4 & 3 & 1\\ -1 & 0 & 1\\ -1 & 1 & 0\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}4\\1\\1\end{bmatrix}$

$-4x_1 + 3x_2 + x_3 = 4$ ....(1)

$-x_1 + x_3 = 1$ .....(2)

$-x_1 + x_2 = 1$ ....(3)

(1) – [(2) + 3(3)]

$0 = 0 \quad \Rightarrow$ infinite solutions

6. Let $f : R \to R$ be defined as $f(x) = x^3 + x - 5$. If $g(x)$ is a function such that $f(g(x)) = x$, $\forall\ x \in R$, then $g'(63)$ is equal to _____.

(A) $\frac{1}{49}$ (B) $\frac{3}{49}$

(C) $\frac{43}{49}$ (D) $\frac{91}{49}$

Official Ans. by NTA (A)

Sol. $f(x) = x^3 + x - 5$

$\Rightarrow$ $f'(x) = 3x^2 + 1$ $\Rightarrow$ increasing function

$\Rightarrow$ invertible

$\Rightarrow$ $g(x)$ is inverse of $f(x)$

$\Rightarrow$ $g(f(x)) = x$

$\Rightarrow$ $g'(f(x))f'(x) = 1$

$f(x) = 63$

$\Rightarrow$ $x^3 + x - 5 = 63$

$\Rightarrow$ $x = 4$

put $x = 4$

$g'(f(4))f'(4) = 1$

$g'(63) \times 49 = 1$ $\{f'(4) = 49\}$

$g'(63) = \frac{1}{49}$

7. Consider the following two propositions:

P1 : $\sim(p \to \sim q)$

P2 : $(p \wedge \sim q) \wedge ((\sim p) \vee q)$

If the proposition $p \to ((\sim p) \vee q)$ is evaluated as FALSE, then:

(A) P1 is TRUE and P2 is FALSE

(B) P1 is FALSE and P2 is TRUE

(C) Both P1 and P2 are FALSE

(D) Both P1 and P2 are TRUE

Official Ans. by NTA (C)

Sol.

p	q	~p	~q	~p ∨ q	p → (~p ∨ q)	p → ~q	~(p → ~q)	p ∧ ~q	p_2
T	T	F	F	T	T	F	T	F	F
T	F	F	T	F	F	T	F	T	F
F	T	T	F	T	T	T	F	F	F
F	F	T	T	T	T	T	F	F	F

$p \to (\sim p \vee q)$ is F when p is true q is false

From table

P1 & P2 both are false

8. If $\frac{1}{2\cdot 3^{10}} + \frac{1}{2^2\cdot 3^9} + \ldots \frac{1}{2^{10}\cdot 3} = \frac{K}{2^{10}\cdot 3^{10}}$, then the remainder when K is divided by 6 is

(A) 1 (B) 2

(C) 3 (D) 5

Official Ans. by NTA (D)

Sol. $\frac{1}{2\cdot 3^{10}} + \frac{1}{2^2\cdot 3^9} + \frac{1}{2^3\cdot 3^8} + \ldots\ldots + \frac{1}{2^{10}\cdot 3} = \frac{K}{2^{10}\cdot 3^{10}}$

$K = 2^9 + 2^8 \cdot 3 + 2^7 \cdot 3^2 + \ldots\ldots + 3^9$

$= \frac{2^9\left(\left(\frac{3}{2}\right)^{10} - 1\right)}{\frac{3}{2} - 1} = 3^{10} - 2^{10}$

Now, $3^{10} - 2^{10} = (3^5 - 2^5)(3^5 + 2^5)$

$= (211)(275)$

$= (35 \times 6 + 1)(45 \times 6 + 5)$

$= 6\lambda + 5$

Remainder is 5.

9. Let $f(x)$ be a polynomial function such that $f(x) + f'(x) + f''(x) = x^5 + 64$. Then, the value of $\lim\limits_{x\to 1} \frac{f(x)}{x-1}$

(A) – 15 (B) – 60

(C) 60 (D) 15

Official Ans. by NTA (A)

Sol. $\lim\limits_{x\to 1} \frac{f(x)}{x-1} = f'(1)\left(\text{and } f(1) = 0\right)$

$f(x) + f'(x) + t''(x) = x^5 + 64$

$f'(x) + f''(x) + f'''(x) = 5x^4$

$f''(x) + f'''(x) + f^{iv}(x) = 20x^3$

$f'''(x) + f^{iv}(x) + f^{v}(x) = 60x^2$

$\therefore\ f^{v}(x) - f''(x) = 60x^2 - 20x^3$

$\Rightarrow 120 - f''(1) = 40 \Rightarrow f''(1) = 80$

Also $f(1) + f'(1) + f''(1) = 65 \Rightarrow f'(1) = -15$. **Ans.**

10. Let E_1 and E_2 be two events such that the conditional probabilities $P(E_1 | E_2) = \frac{1}{2}$, $P(E_2 | E_1) = \frac{3}{4}$ and $P(E_1 \cap E_2) = \frac{1}{8}$. Then:

(A) $P(E_1 \cap E_2) = P(E_1) \cdot P(E_2)$

(B) $P(E'_1 \cap E'_2) = P(E'_1) \cdot P(E_2)$

(C) $P(E_1 \cap E'_2) = P(E_1) \cdot P(E_2)$

(D) $P(E'_1 \cap E_2) = P(E_1) \cdot P(E_2)$

Official Ans. by NTA (C)

Sol.

(A) $P(E_1) \cdot P(E_2) = \frac{1}{6} \cdot \frac{1}{4} = \frac{1}{24} \neq P(E_1 \cap E_2)$

(B) $P(E'_1 \cap E'_2) = 1 - P(E_1 \cup E_2)$

$= 1 - \left(P(E_1) + P(E_2) - P(E_1 \cap E_2)\right)$

$= 1 - \left(\frac{1}{6} + \frac{1}{4} - \frac{1}{8}\right) = \frac{17}{24}$

$P(E'_1) P(E_2) = \frac{5}{6} \times \frac{1}{4} = \frac{5}{24}$

(C) $P(E_1 \cap E'_2) = P(E_1) - P(E_1 \cap E_2) = \frac{1}{6} - \frac{1}{8} = \frac{1}{24}$

(D) $P(E'_1 \cap E_2) = P(E_2) - P(E_1 \cap E_2) = \frac{1}{4} - \frac{1}{8} = \frac{1}{8}$

11. Let $A = \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix}$. If M and N are two matrices given by $M = \sum_{k=1}^{10} A^{2k}$ and $N = \sum_{k=1}^{10} A^{2k-1}$ then MN^2 is

(A) a non-identity symmetric matrix

(B) a skew-symmetric matrix

(C) neither symmetric nor skew-symmetric matrix

(D) an identify matrix

Official Ans. by NTA (A)

Sol. $A = \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix}$

$A^2 = \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix}\begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} -4 & 0 \\ 0 & -4 \end{bmatrix} = -4I$

$A^3 = -4A$

$A^4 = (-4I)(-4I) = (-4)^2 I$

$A^5 = (-4)^2 A, \quad A^6 = (-4)^3 I$

$M = \sum_{k=1}^{10} A^{2k} = A^2 + A^4 + ... + A^{20}$

$= [-4 + (-4)^2 + (-4)^3 + ... + (-4)^{20}]I$

$= -4\lambda I$

$\Rightarrow$ M is symmetric matrix

$N = \sum_{k=1}^{10} A^{2k-1} = A + A^3 + ... + A^{19}$

$= A[1 + (-4) + (-4)^2 + ... + (-4)^9]$

$= \lambda A \Rightarrow$ skew symmetric

$\Rightarrow N^2$ is symmetric matrix

$\Rightarrow MN^2$ is non identity symmetric matrix

12. Let $g : (0, \infty) \to R$ be a differentiable function such that

$$\int \left(\frac{x(\cos x - \sin x)}{e^x + 1} + \frac{g(x)(e^x + 1 - xe^x)}{(e^x + 1)^2} \right) dx = \frac{xg(x)}{e^x + 1} + c,$$

for all $x > 0$, where c is an arbitrary constant. Then.

(A) g is decreasing in $\left(0, \frac{\pi}{4}\right)$

(B) g' is increasing in $\left(0, \frac{\pi}{4}\right)$

(C) g + g' is increasing in $\left(0, \frac{\pi}{2}\right)$

(D) g – g' is increasing in $\left(0, \frac{\pi}{2}\right)$

Official Ans. by NTA (D)

Sol.

$$\int\left(\frac{x(\cos x-\sin x)}{e^x+1}+\frac{g(x)(e^x+1-xe^x)}{(e^x+1)^2}\right)dx=\frac{xg(x)}{e^x+1}+c$$

On differentiating both sides w.r.t. x, we get

$$\left(\frac{x(\cos x-\sin x)}{e^x+1}+\frac{g(x)(e^x+1-xe^x}{(e^x+1)^2}\right)$$

$$=\frac{(e^x+1)\big(g(x)+xg'(x)\big)-e^x\cdot x\cdot g(x)}{\left(e^x+1\right)^2}$$

$(e^x + 1)x(\cos x - \sin x) + g(x)(e^x + 1 - xe^x)$

$= (e^x+1)\big(g(x)+xg'(x)\big)-e^x\cdot x\cdot g(x)$

$\Rightarrow g'(x) = \cos x - \sin x$

$\Rightarrow g(x) = \sin x + \cos x + C$

$g(x)$ is increasing in $(0, \pi/4)$

$g''(x) = -\sin x - \cos x < 0$

$\Rightarrow$ $g'(x)$ is decreasing function

let $h(x) = g(x) + g'(x) = 2\cos x + C$

$\Rightarrow h'(x) = g'(x) + g''(x) = -2\sin x < 0$

$\Rightarrow$ h is decreasing

let $\phi(x) = g(x) - g'(x) = 2\sin x + C$

$\Rightarrow \phi'(x) = g'(x) - g''(x) = 2\cos x > 0$

$\Rightarrow$ ϕ is increasing

Hence option D is correct.

13. Let $f : R \to R$ and $g : R \to R$ be two functions defined by $f(x) = \log_e(x^2 + 1) - e^{-x} + 1$ and $g(x)= \frac{1-2e^{2x}}{e^x}$. Then, for which of the following range of α, the inequality

$$f\left(g\left(\frac{(\alpha-1)^2}{3}\right)\right)>f\left(g\left(\alpha-\frac{5}{3}\right)\right) \text{ holds?}$$

(A) (2, 3) (B) (–2, – 1)

(C) (1, 2) (D) (–1, 1)

Official Ans. by NTA (A)

Sol. $f(x) = \log_e(x^2 + 1) - e^{-x} + 1$

$\Rightarrow f'(x) = \frac{2x}{x^2+1}+e^{-x} > 0 \quad \forall\ x \in R$

$\Rightarrow$ f is strictly increasing

$g(x) = \frac{1-2e^{2x}}{e^x} = e^{-x} - 2e^x$

$\Rightarrow g'(x) = -\left(2e^x+e^{-x}\right) < 0 \quad \forall\ x \in R$

$\Rightarrow$ g is decreasing

Now $f\left(g\left(\frac{(\alpha-1)^2}{3}\right)\right)>f\left(g\left(\alpha-\frac{5}{3}\right)\right)$

$\Rightarrow g\left(\frac{(\alpha-1)^2}{3}\right)>g\left(\alpha-\frac{5}{3}\right)$

$\Rightarrow \frac{(\alpha-1)^2}{3}<\alpha-\frac{5}{3}$

$\Rightarrow \alpha^2 - 5\alpha + 6 < 0$

$\Rightarrow (\alpha - 2)(\alpha - 3) < 0$

$\Rightarrow \alpha \in (2, 3)$

14. Let $\vec{a}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$ $a_i > 0$, i = 1, 2, 3 be a vector which makes equal angles with the coordinates axes OX, OY and OZ. Also, let the projection of $\vec{a}$ on the vector $3\hat{i}+4\hat{j}$ be 7. Let $\vec{b}$ be a vector obtained by rotating $\vec{a}$ with 90°. If $\vec{a}$, $\vec{b}$ and x-axis are coplanar, then projection of a vector $\vec{b}$ on $3\hat{i}+4\hat{j}$ is equal to

(A) $\sqrt{7}$ (B) $\sqrt{2}$

(C) 2 (D) 7

Official Ans. by NTA (B)

Sol. $\vec{a}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$

$\vec{a}=\lambda\left(\frac{1}{\sqrt{3}}\hat{i}+\frac{1}{\sqrt{3}}\hat{j}+\frac{1}{\sqrt{3}}\hat{k}\right) = \frac{\lambda}{\sqrt{3}}\left(\hat{i}+\hat{j}+\hat{k}\right)$

Now projection of $\vec{a}$ on $\vec{b}=7$

$\Rightarrow \frac{\vec{a}\cdot\vec{b}}{|\vec{b}|} = 7$

$\frac{\lambda}{\sqrt{3}}\frac{\left(\hat{i}+\hat{j}+\hat{k}\right)\cdot\left(3\hat{i}+4\hat{j}\right)}{5} = 7$

$\lambda = 5\sqrt{3}$

$\vec{a} = 5(\hat{i} + \hat{j} + \hat{k})$

now $\vec{b} = 5\alpha(\hat{i} + \hat{j} + \hat{k}) + \beta(\hat{i})$

$\vec{a} \cdot \vec{b} = 0$

$\Rightarrow 25\alpha(3) + 5\beta = 0$

$\Rightarrow 15\alpha + \beta = 0 \quad \Rightarrow \quad \beta = -15\alpha$

$\vec{b} = 5\alpha(-2\hat{i} + \hat{j} + \hat{k})$

$\left|\vec{b}\right| = 5\sqrt{3}$

$\Rightarrow \quad \alpha = \pm \dfrac{1}{\sqrt{2}}$

$\vec{b} = \pm\dfrac{5}{\sqrt{2}}(-2\hat{i} + \hat{j} + \hat{k})$

Projection of $\vec{b}$ on $3\hat{i} + 4\hat{j}$ is

$$\frac{\vec{b}\cdot(3\hat{i} + 4\hat{j})}{5} = \pm\frac{5}{\sqrt{2}}\left(\frac{-6+4}{5}\right) = \pm\sqrt{2}$$

15. Let y = y(x) be the solution of the differential equation $(x + 1)y' - y = e^{3x}(x + 1)^2$, with $y(0) = \dfrac{1}{3}$. Then, the point $x = -\dfrac{4}{3}$ for the curve y = y(x) is:

(A) not a critical point
(B) a point of local minima
(C) a point of local maxima
(D) a point of inflection

Official Ans. by NTA (B)

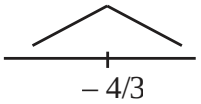
Sol. $(x + 1)dy - y\,dx = e^{3x}(x + 1)^2$

$$\frac{(x+1)dy - ydx}{(x+1)^2} = e^{3x}$$

$$d\left(\frac{y}{x+1}\right) = e^{3x} \quad \Rightarrow \quad \frac{y}{x+1} = \frac{e^{3x}}{3} + C$$

$$\left(0, \frac{1}{3}\right) \quad \Rightarrow \quad C = 0 \quad \Rightarrow \quad y = \frac{(x+1)e^{3x}}{3}$$

$$\frac{dy}{dx} = \frac{1}{3}\left((x+1)3e^{3x} + e^{3x}\right) = \frac{3^{3x}}{3}(3x+4)$$



Clearly, $x = \dfrac{-4}{3}$ is point of local minima

16. If $y = m_1x + c_1$ and $y = m_2x + c_2$, $m_1 \neq m_2$ are two common tangents of circle $x^2 + y^2 = 2$ and parabola $y^2 = x$, then the value of $8|m_1m_2|$ is equal to

(A) $3 + 4\sqrt{2}$ (B) $-5 + 6\sqrt{2}$

(C) $-4 + 3\sqrt{2}$ (D) $7 + 6\sqrt{2}$

Official Ans. by NTA (C)

Sol. C_1: $x^2 + y^2 = 2$

C_2 : $y^2 = x$

Let tangent to parabola be $y = mx + \dfrac{1}{4m}$.

It is also a tangent of circle so distance from centre of circle (0, 0) will be $\sqrt{2}$.

$$\left|\frac{\frac{1}{4m}}{\sqrt{1+m^2}}\right| = \sqrt{2} \quad \Rightarrow \quad 1 = 32m^2 + 32m^4$$

by solving

$m^2 = \dfrac{3\sqrt{2}-4}{8}$, $m^2 = \dfrac{-3\sqrt{2}-4}{8}$ (rejected)

$$m = \pm\sqrt{\frac{3\sqrt{2}-4}{8}}$$

so, $8|m_1m_2| = 3\sqrt{2} - 4$

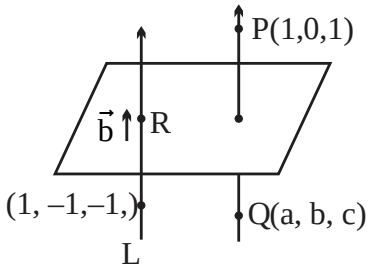
17. Let Q be the mirror image of the point P(1, 0, 1) with respect to the plane S : x + y + z = 5. If a line L passing through (1, –1, –1), parallel to the line PQ meets the plane S at R, then QR^2 is equal to:

(A) 2 (B) 5

(C) 7 (D) 11

Official Ans. by NTA (B)

Sol.



Let parallel vector of L = $\vec{b}$

mirror image of Q on given plane x+y+z=5

$$\frac{a-1}{1}=\frac{b-0}{1}=\frac{c-1}{1}=\frac{-2(2-5)}{3}$$

a = 3, b = 2, c = 3

Q≡(3, 2, 3)

$\because \quad \vec{b} \,||\, \overrightarrow{PQ}$

so, $\vec{b} = (1,1,1)$

Equation of line

$$L : \frac{x-1}{1}=\frac{y+1}{1}=\frac{z+1}{1}$$

Let point R, $(\lambda+1, \lambda-1, \lambda-1)$

lying on plane x + y + z = 5,

so, 3λ – 1 = 5

$\Rightarrow \quad \lambda = 2$

Point R is (3, 1, 1)

$QR^2 = 5$ **Ans.**

18. If the solution curve y = y(x) of the differential equation $y^2dx + (x^2 - xy + y^2)dy = 0$, which passes through the point (1, 1) and intersects the line $y = \sqrt{3}\,x$ at the point $(\alpha, \sqrt{3}\,\alpha)$, then value of $\log_e(\sqrt{3}\,\alpha)$ is equal to

(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{2}$

(C) $\frac{\pi}{12}$ (D) $\frac{\pi}{6}$

Official Ans. by NTA (C)

Sol. $y^2dx - xy\,dy = -(x^2 + y^2)dy$

$y(y\,dx - x\,dy) = -(x^2 + y^2)\,dy$

$-y(x\,dx - y\,dx) = -(x^2 + y^2)dy$

$$\frac{xdy-ydx}{x^2}=\left(1+\frac{y^2}{x^2}\right)\frac{dy}{y}$$

$$\Rightarrow \frac{d(y/x)}{1+\frac{y^2}{x^2}}=\frac{dy}{y}$$

$$\Rightarrow \tan^{-1}\left(\frac{y}{x}\right) = \ln y + C$$

$$\left(\alpha, \sqrt{3}\alpha\right) \Rightarrow \frac{\pi}{3}=\ln\left(\sqrt{3}\alpha\right)+\frac{\pi}{4}$$

$$\therefore \quad \ln\left(\sqrt{3}\alpha\right)=\frac{\pi}{12}$$

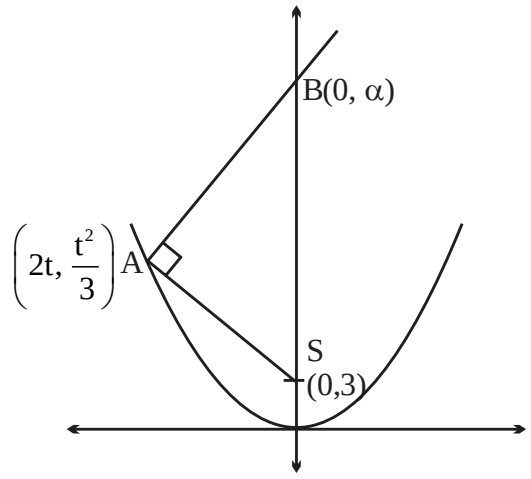
19. Let $x = 2t$, $y=\frac{t^2}{3}$ be a conic. Let S be the focus and B be the point on the axis of the conic such that SA ⊥ BA, where A is any point on the conic. If k is the ordinate of the centroid of ΔSAB, then $\lim\limits_{t\to 1} k$ is equal to

(A) $\frac{17}{18}$ (B) $\frac{19}{18}$

(C) $\frac{11}{18}$ (D) $\frac{13}{18}$

Official Ans. by NTA (D)

Sol.



parabola $x^2 = 12y$

SA ⊥ SB

so, $m_{AS} \cdot m_{AB} = -1$

$$\frac{\left(3-\frac{t^2}{3}\right)}{(0-2t)} \cdot \frac{\left(\alpha-\frac{t^2}{3}\right)}{(0-2t)} = -1$$

by solving

$$3\alpha = \frac{27t^2+t^4}{t^2-9}$$

ordinate of centriod of $\Delta SAB = K = \frac{\alpha+\frac{t^2}{3}+3}{3}$

$$k = \frac{9+3\alpha+t^2}{9}$$

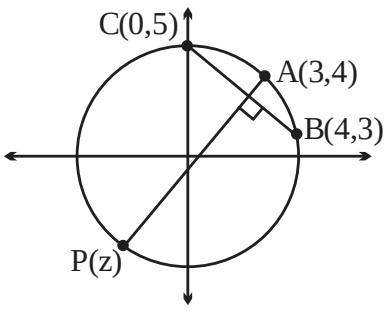
$$\lim_{t\to 1} k = \lim_{t\to 1} \frac{1}{9}\left(9+t^2+\frac{27t^2+t^4}{(t^2-9)}\right) = \frac{13}{18}$$

20. Let a circle C in complex plane pass tltrough the points $z_1 = 3 + 4i$, $z_2 = 4 + 3i$ and $z_3 = 5i$. If $z(\neq z_1)$ is a point on C such that the line through z and z_1 is perpendicular to the line through z_2 and z_3, then arg(z) is equal to :

(A) $\tan^{-1}\left(\frac{2}{\sqrt{5}}\right)-\pi$ (B) $\tan^{-1}\left(\frac{24}{7}\right)-\pi$

(C) $\tan^{-1}(3)-\pi$ (D) $\tan^{-1}\left(\frac{3}{4}\right)-\pi$

Sol.



Slope of BC = $\frac{3-5}{4-0} = -\frac{1}{2}$

Slope of AP = 2

equation of AP : $y - 4 = 2(x - 3)$

$\Rightarrow \quad y = 2(x - 1)$

P lies on circle $x^2 + y^2 = 25$

$\Rightarrow \quad x^2 + (2(x - 1))^2 = 25$

$\Rightarrow \quad x = -\frac{7}{5}$ and $y = -\frac{24}{5}$

$\Rightarrow \quad \arg(z) = \tan^{-1}\left(\frac{24}{7}\right)-\pi$

SECTION-B

1. Let C_r denote the binomial coefficient of x^r in the expansion of $(1 + x)^{10}$. If $\alpha, \beta \in R$. $C_1 + 3\cdot 2C_2 + 5\cdot 3C_3 + \ldots.$ upto 10 terms

$$= \frac{\alpha\times 2^{11}}{2^{\beta}-1}\left(C_0+\frac{C_1}{2}+\frac{C_2}{3}+\ldots..\text{upto 10 terms}\right)$$

then the value of $\alpha + \beta$ is equal to

Official Ans. by NTA (286)

(BONUS)

Sol. $(1 + x)^{10} = C_0 + C_1x + C_2x^2 + \ldots\ldots + C_{10}x^{10}$

Differentiating

$10(1 + x)^9 = C_1 + 2C_2x + 3C_3x^2 + \ldots. + 10C_{10}x^9$

replace $x \to x^2$

$10(1+x^2)^9 = C_1 + 2C_2x^2 + 3C_3x^4 + \ldots.. + 10C_{10}x^{18}$

$10\cdot x(1+x^2)^9 = C_1x + 2C_2x^3 + 3C_3x^5 + \ldots.. + 10C_{10}x^{19}$

Differentiating

$$10\left((1+x^2)^9\cdot 1 + x\cdot 9(1+x^2)^8 2x\right)$$

$= C_1x + 2\,C_2\cdot 3x^3 + 3\cdot 5\cdot C_3x^4 + \ldots.. + 10\cdot 19C_{10}x^{18}$

putting $x = 1$

$10(2^9 + 18\cdot 2^8)$

$= C_1 + 3\cdot 2\cdot C_2 + 5\cdot 3\cdot C_3 + \ldots.. + 19\cdot 10\cdot C_{10}$

$C_1 + 3\cdot 2\cdot C_2 + \ldots\ldots + 19\cdot 10\cdot C_{10}$

$= 10 \cdot 2^9 \cdot 10 = 100 \cdot 2^9$

$$C_0+\frac{C_1}{2}+\frac{C_2}{3}+\ldots..+\underset{\uparrow}{\frac{C_9}{11}}+\underset{\uparrow}{\frac{C_{10}}{11}} = \frac{2^{11}-1}{11}$$

10^{th} term 11^{th} term

$$C_0+\frac{C_1}{2}+\frac{C_2}{3}+\ldots..+\frac{C_9}{11} = \frac{2^{11}-2}{11}$$

Now, $100\cdot 2^9 = \frac{\alpha\cdot 2^{11}}{2^{\beta}-1}\left(\frac{2^{11}-2}{11}\right)$

Eqn. of form $y = k(2^x - 1)$.

It has infinite solutions even if we take $x, y \in N$.

2. The number of 3-digit odd numbers, whose sum of digits is a multiple of 7, is _____.

Official Ans. by NTA (63)

Sol. x y z ← odd number

z = 1,3,5,7,9

x+y+ z = 7, 14, 21 [sum of digit multiple of 7]

$\underset{1\text{to}9}{x} + \underset{0\text{to}9}{y}$ = 6, 4, 2, 13, 11, 9, 7, 5, 20, 18, 16, 14, 12

$x + y = 6 \Rightarrow$ (1,5), (2, 4), (3, 3), (4, 2), (5, 1), (6, 0)

$\rightarrow$ T.N. = 6

$x + y = 4 \Rightarrow (1,3), (2, 2), (3, 1), (4,0)$

$\rightarrow$ T.N = 4

$x + y = 2 \Rightarrow (1,1), (2,0)$

$\rightarrow$ T.N. = 2

$x + y = 13 \Rightarrow (4,9), (5,8), (6,7), (7,6), (8,5), (9,4)$

$\rightarrow$ T.N. = 6

$x + y = 11 \Rightarrow (2,9), (3,8), (4,7), (5,6), (6,5), (6,5), (7,4), (8,3), (9,2)$

$\rightarrow$ T.N. = 8

$x + y = 9 \Rightarrow (1,8),(2,7),(3,8),(4,5),(5,4),....(8,1),(9,0)$

$\rightarrow$ T.N. = 9

$x + y = 7 \Rightarrow (1,8),(2,5),(3,4),....(8, 1), (7,0)$

$\rightarrow$ T.N. = 7

$x + y = 5 \Rightarrow (1,4),(2,3), (3, 2), (4,1),(5,0)$

$\rightarrow$ T.N. = 5

$x + y = 20 \Rightarrow$ Not possible

$x + y = 18 \Rightarrow (9,9)$ $\rightarrow$ T.N. = 1

$x + y = 16 \Rightarrow (7,9),(8,8),(9,7)$

$\rightarrow$ T.N. = 3

$x + y = 14 \Rightarrow (5,9),(6,8),(7,7),(8,6),(9,5)$

$\rightarrow$ T.N. = 5

$x + y = 12 \Rightarrow (3,9),(4,8),(5,7),(6,6)....(9,3)$

$\rightarrow$ T.N. = 7

3. Let θ be the angle between the vectors $\vec{a}$ and $\vec{b}$, where $|\vec{a}| = 4$, $|\vec{b}| = 3$ $\theta \in \left(\frac{\pi}{4}, \frac{\pi}{3}\right)$. Then

$\left|(\vec{a} - \vec{b}) \times (\vec{a} + \vec{b})\right|^2 + 4(\vec{a} \cdot \vec{b})^2$ is equal to ____

Official Ans. by NTA (576)

Sol. $|\vec{a}| = 4, |\vec{b}| = 3 \quad \theta \in \left(\frac{\pi}{4}, \frac{\pi}{3}\right)$

$\left|(\vec{a} - \vec{b}) \times (\vec{a} + \vec{b})\right|^2 + 4(\vec{a} \cdot \vec{b})^2$

$\left|\vec{a} \times \vec{b} - \vec{b} \times \vec{a}\right|^2 + 4a^2b^2\cos^2\theta$

$2\left|\vec{a} \times \vec{b}\right|^2 + 4a^2b^2\cos^2\theta$

$4a^2b^2\sin^2\theta + 4a^2b^2\cos^2\theta$

$4a^2b^2 = 4 \times 16 \times 9 = 576$

4. Let the abscissae of the two points P and Q be the roots of $2x^2 - rx + p = 0$ and the ordinates of P and Q be the roots of $x^2 - sx - q = 0$. If the equation of the circle described on PQ as diameter is $2(x^2 + y^2) - 11x - 14y - 22 = 0$, then $2r + s - 2q + p$ is equal to

Official Ans. by NTA (7)

Sol. $2x^2 - rx + p = 0 \begin{cases} x_1 \\ x_2 \end{cases}$

$y^2 - sy - q = 0 \begin{cases} y_1 \\ y_2 \end{cases}$

Equation of the circle with PQ as diameter is $2(x^2 + y^2) - rx - 2sy + p - 2q = 0$

on comparing with the given equation

r = 11, s = 7

$p - 2q = -22$

$\therefore 2r + s - 2q + p = 22 + 7 - 22 = 7$

5. The number of values of x in the interval $\left(\frac{\pi}{4}, \frac{7\pi}{4}\right)$ for which $14\text{cosec}^2x - 2\sin^2x = 21 - 4\cos^2x$ holds, is _____

Official Ans. by NTA (4)

Sol. $x \in \left(\frac{\pi}{4}, \frac{7\pi}{4}\right)$

$14\text{cosec}^2x - 2\sin^2x = 21 - 4\cos^2x$

$= 21 - 4(1 - \sin^2x)$

$= 17 + 4\sin^2x$

$14\text{cosec}^2x - 6\sin^2x = 17$

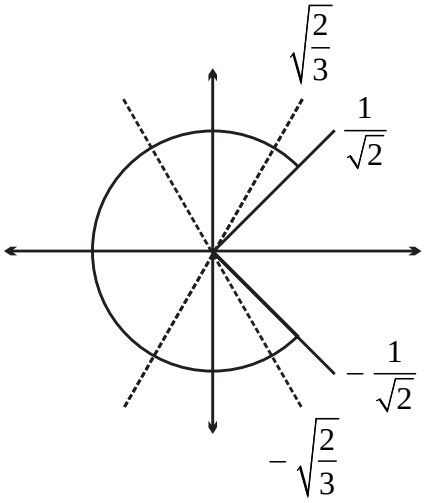
let $\sin^2x = p$

$$\frac{14}{p}-6p=17\Rightarrow 14-6p^2=17p$$

$$6p^2+17p-14=0$$

$$p=-3.5,\ \frac{2}{3}\ \Rightarrow\ \sin^2x=\frac{2}{3}$$

$$\Rightarrow\ \sin x=\pm\sqrt{\frac{2}{3}}$$



$\therefore$ Total 4 solutions

6. For a natural number n, let $a_n = 19^n - 12^n$. Then, the value of $\frac{31\alpha_9-\alpha_{10}}{57\alpha_8}$ is

Official Ans. by NTA (4)

Sol. $a_n=19^n-12^n$

$$\frac{31\alpha_9-\alpha_{10}}{57\alpha_8}=\frac{31\left(19^9-12^9\right)-\left(19^{10}-12^{10}\right)}{57\alpha_8}$$

$$=\frac{19^9\left(31-19\right)-12^9\left(31-12\right)}{57\alpha_8}$$

$$=\frac{19^9\cdot 12-12^{19}\cdot 19}{57\alpha_8}$$

$$=\frac{12\cdot 19\left(19^8-12^8\right)}{57\alpha_8}=4$$

7. Let f : R $\to$ R be a function defined by $f(x)=\left(2\left(1-\frac{x^{25}}{2}\right)\left(2+x^{25}\right)\right)^{\frac{1}{50}}$. If the function $g(x)=f\left(f\left(f(x)\right)\right)+f\left(f(x)\right)$, the the greatest integer less than or equal to g (1) is _____

Official Ans. by NTA (2)

Sol. $f(x)=\left[2\left(1-\frac{x^{25}}{2}\right)\left(2+x^{25}\right)\right]^{\frac{1}{50}}$

$$f(x)=\left[\left(2-x^{25}\right)\left(2+x^{25}\right)\right]^{\frac{1}{50}}$$

$$= (4 - x^{50})^{1/50}$$

$$f\left(f(x)\right)=\left(4-\left(\left(4-x^{50}\right)^{1/50}\right)^{50}\right)^{1/50}=x$$

$$g(x) = f\left(f\left(f(x)\right)\right)+f\left(f(x)\right)$$

$$= f(x) + x$$

$$g(1) = f(1) + 1 = 3^{1/50} + 1$$

$$[g(1)] = [3^{1/50} + 1] = 2$$

8. Let the lines

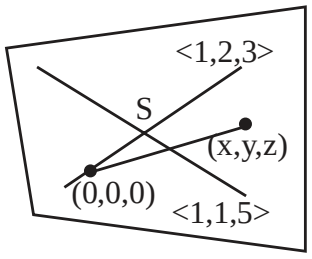
$L_1 : \vec{r}=\lambda(\hat{i}+2\hat{j}+3\hat{k}),\ \lambda\in R$

$L_2 : \vec{r}=(\hat{i}+3\hat{j}+\hat{k})+\mu\left(\hat{i}+\hat{j}+5\hat{k}\right);\ \mu\in R$

intersect at the point S. If a plane ax + by – z + d = 0 passes through S and is parallel to both the lines L_1 and L_2, then the value of a + b + d is equal to _____

Official Ans. by NTA (5)

Sol. Both the lines lie in the same plane



$\therefore$ equation of the plane

$$\begin{vmatrix} x & y & z \\ 1 & 2 & 3 \\ 1 & 1 & 5 \end{vmatrix}=0$$

$\Rightarrow 7x - 2y - z = 0$

$\therefore$ a + b + d = 5

9. Let A be a 3 × 3 matrix having entries from. the set {–1, 0, 1}. The number of all such matrices A having sum of all the entries equal to 5, is _____

Official Ans. by NTA (414)

Sol. **Case-I:** $1 \rightarrow 7$ times

and $-1 \rightarrow 2$ times

number of possible matrix = $\frac{9!}{7!\,2!} = 36$

Case-II: $1 \rightarrow 6$ times,

$-1 \rightarrow 1$ times

and $0 \rightarrow 2$ times

number of possible matrix = $\frac{9!}{6!\,2!} = 252$

Case-III: $1 \rightarrow 5$ times,

and $0 \rightarrow 4$ times

number of possible matrix = $\frac{9!}{5!\,4!} = 126$

Hence total number of all such matrix A = 414

10. The greatest integer less than or equal to the sum of first 100 terms of the sequence $\frac{1}{3}, \frac{5}{9}, \frac{19}{27}, \frac{65}{81}, \ldots\ldots$ is equal to

Official Ans. by NTA (98)

Sol. $\frac{1}{3}+\frac{5}{9}+\frac{19}{27}+\frac{65}{81}+\ldots.$

$\left(1-\frac{2}{3}\right)+\left(1-\frac{4}{9}\right)+\left(1-\frac{8}{27}\right)+\left(1-\frac{16}{81}\right)$....100 terms

$100-\left[\frac{2}{3}+\left(\frac{2}{3}\right)^2+\ldots.\right]$

$100-\frac{\frac{2}{3}\left(1-\left(\frac{2}{3}\right)^{100}\right)}{1-\frac{2}{3}}$

$100-2\left(1-\left(\frac{2}{3}\right)^{100}\right)$

$S=98+2\left(\frac{2}{3}\right)^{100}$

$\Rightarrow [S]=98$

FINAL JEE–MAIN EXAMINATION – JUNE, 2022

(Held On Friday 24th June, 2022) **TIME : 3 : 00 PM to 6 : 00 PM**

PHYSICS

TEST PAPER WITH SOLUTION

SECTION-A

1. Identify the pair of physical quantities that have same dimensions :

(A) velocity gradient and decay constant

(B) wien's constant and Stefan constant

(C) angular frequency and angular momentum

(D) wave number and Avogadro number

Official Ans. by NTA (A)

Sol. Velocity gradient $= \frac{dV}{dx} = \frac{1}{S}$

$\lambda = \frac{1}{S}$

2. The distance between Sun and Earth is R. The duration of year if the distance between Sun and Earth becomes 3R will be :

(A) $\sqrt{3}$ years (B) 3 years

(C) 9 years (D) $3\sqrt{3}$ years

Official Ans. by NTA (D)

Sol. $T' = T\left(\frac{3R}{R}\right)^{3/2} = 3\sqrt{3}\,T$

3. A stone of mass m, tied to a string is being whirled in a vertical circle with a uniform speed. The tension in the string is :

(A) the same throughout the motion

(B) minimum at the highest position of the circular path

(C) minimum at the lowest position of the circular path

(D) minimum when the rope is in the horizontal position

Official Ans. by NTA (B)

Sol. Theory

4. Two identical charged particles each having a mass 10 g and charge 2.0×10^{-7} C area placed on a horizontal table with a separation of L between then such that they stay in limited equilibrium. If the coefficient of friction between each particle and the table is 0.25, find the value of L. [Use g = 10 ms^{-2}]

(A) 12 cm (B) 10 cm

(C) 8 cm (D) 5 cm

Official Ans. by NTA (A)

Sol. $\frac{kq^2}{L^2} = \mu mg \Rightarrow L = \sqrt{\frac{k}{\mu mg}}q$

5. A Carnot engine take 5000 kcal of heat from a reservoir at 727°C and gives heat to a sink at 127°C. The work done by the engine is :

(A) 3×10^6 J (B) Zero

(C) 12.6×10^6 J (D) 8.4×10^6 J

Official Ans. by NTA (C)

Sol. $L = \frac{WD}{Q_H}$

$\Rightarrow WD = Q_H\left(1 - \frac{T_L}{T_H}\right)$

$= 5 \times 10^3\left(1 - \frac{400}{1000}\right)$

$= 3000$ kcal

6. Two massless springs with spring constants 2 k and 2 k, carry 50 g and 100 g masses at their free ends. These two masses oscillate vertically such that their maximum velocities are equal. Then, the ratio of their respective amplitudes will be :

(A) 1 : 2 (B) 3 : 2

(C) 3 : 1 (D) 2 : 3

Official Ans. by NTA (B)

Sol. $V_{max} = \omega A$

$\Rightarrow \frac{A_1}{A_2} = \frac{\omega_2}{\omega_1} = \sqrt{\frac{9}{2} \times \frac{1}{2}} = \frac{3}{2}$

7. What will be the most suitable combination of three resistors A = 2Ω, B = 4Ω, C = 6 Ω so that $\left(\frac{22}{3}\right)\Omega$ is equivalent resistance of combination?

(A) Parallel combination of A and C connected in series with B.

(B) Parallel combination of A and B connected in series with C.

(C) Series combination of A and C connected in parallel with B.

(D) Series combination of B and C connected in parallel with A.

Official Ans. by NTA (B)

Sol. $\Rightarrow \frac{4}{3}+6=\frac{22}{3}$

8. The soft-iron is a suitable material for making an electromagnet. This is because soft-iron has :

(A) low coercively and high retentively

(B) low coercively and low permeability

(C) high permeability and low retentively

(D) high permeability and high retentively

Official Ans. by NTA (C)

Sol. Theory

9. A proton, a deuteron and an α-particle with same kinetic energy enter into a uniform magnetic field at right angle to magnetic field. The ratio of the radii of their respective circular paths is :

(A) $1:\sqrt{2}:\sqrt{2}$ (B) $1:1:\sqrt{2}$

(C) $\sqrt{2}:1:1$ (D) $1:\sqrt{2}:1$

Official Ans. by NTA (D)

Sol. $R=\frac{\sqrt{2km}}{qB}\propto\frac{\sqrt{m}}{q}$

$\frac{\sqrt{m}}{e}:\frac{\sqrt{2m}}{e}:\frac{\sqrt{4m}}{2e}$

$1:\sqrt{2}:1$

10. Given below are two statements :

Statement-I : The reactance of an ac circuit is zero. It is possible that the circuit contains a capacitor and an inductor.

Statement-II : In ac circuit, the average poser delivered by the source never becomes zero.

In the light of the above statements, choose the correct answer from the options given below :

(A) Both Statement I and Statement II are true.

(B) Both Statement I and Statement II are false.

(C) Statement I is true but Statement II in false.

(D) Statement I is false but Statement II is true.

Official Ans. by NTA (C)

Sol. if R = 0, P = 0

11. Potential energy as a function of r is given by $U=\frac{A}{r^{10}}-\frac{B}{r^5}$, where r is the interatomic distance, A and B are positive constants. The equilibrium distance between the two atoms will be :

(A) $\left(\frac{A}{B}\right)^{\frac{1}{5}}$ (B) $\left(\frac{B}{A}\right)^{\frac{1}{5}}$

(C) $\left(\frac{2A}{B}\right)^{\frac{1}{5}}$ (D) $\left(\frac{B}{2A}\right)^{\frac{1}{5}}$

Official Ans. by NTA (C)

Sol. $\frac{-10A}{r^{11}}+\frac{5B}{r^6}=0$

$r^5=\frac{10A}{5B}=\frac{2A}{B}$

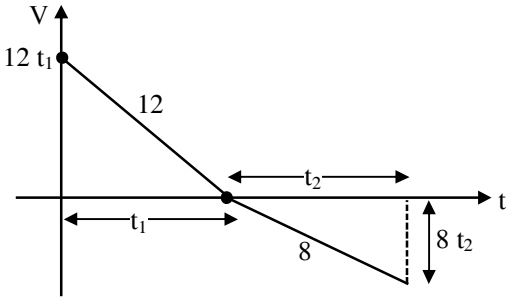
12. An object of mass 5 kg is thrown vertically upwards from the ground. The air resistance produces a constant retarding force of 10 N throughout the motion. The ratio of time of ascent to the time of descent will be equal to : [Use g = 10 ms^{-2}]

(A) 1 : 1 (B) $\sqrt{2}:\sqrt{3}$

(C) $\sqrt{3}:\sqrt{2}$ (D) 2 : 3

Official Ans. by NTA (B)

Sol.



$6t_1^2 = 4t_2^2$

13. A fly wheel is accelerated uniformly from rest and rotates through 5 rad in the first second. The angle rotated by the fly wheel in the next second, will be :

(A) 7.5 rad (B) 15 rad

(C) 20 rad (D) 30 rad

Official Ans. by NTA (B)

Sol. $5 = \frac{1}{2}\alpha(1)^2$

$\theta = \frac{1}{2}\alpha(2)^2$

$\theta - 5 = 15$

14. A 100 g of iron nail is hit by a 1.5 kg hammer striking at a velocity of 60 ms^{-1}. What will be the rise in the temperature of the nail if one fourth of energy of the hammer goes into heating the nail? [Specific heat capacity of iron = 0.42 $Jg^{-1}\,°C^{-1}$]

(A) 675°C (B) 1600°C

(C) 160.7°C (D) 6.75°C

Official Ans. by NTA (C)

Sol. $\frac{1}{2} \times 1.5 \times 60^2 \times \frac{1}{4} = 0.1 \times 420 \times \Delta T$

15. If the charge on a capacitor is increased by 2 C, the energy stored in it increases by 44%. The original charge on the capacitor is (in C) :

(A) 10 (B) 20

(C) 30 (D) 40

Official Ans. by NTA (A)

Sol. $U \propto q^2$

$\Rightarrow q_f = 1.2\ q$

$q_f - q = 2$

$\Rightarrow 1.2\ q - q = 2$

$q = 10$

16. A long cylindrical volume contains a uniformly distributed charge of density ρ. The radius of cylindrical volume is R. A charge particle (q) revolves around the cylinder in a circular path. The kinetic of the particle is :

(A) $\frac{\rho q R^2}{4\varepsilon_0}$ (B) $\frac{\rho q R^2}{2\varepsilon_0}$

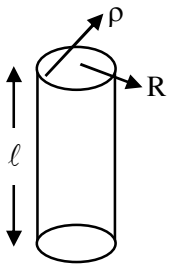
(C) $\frac{q\rho}{4\varepsilon_0 R^2}$ (D) $\frac{4\varepsilon_0 R^2}{q\rho}$

Official Ans. by NTA (A)

Sol. $E = 2\pi r\ell = \frac{\rho\pi r^2 \ell}{\varepsilon_0}$

$qE = \frac{q\rho R^2}{2\varepsilon_0 r} = \frac{mv^2}{r}$

$mv^2 = \frac{q\rho R^2}{2\varepsilon_0}$



17. An electric bulb is rated as 200 W. What will be the peak magnetic field at 4 m distance produced by the radiations coming from this bulb? Consider this bulb as a point source with 3.5% efficiency.

(A) 1.19×10^{-8} T (B) 1.71×10^{-8} T

(C) 0.84×10^{-8} T (D) 3.36×10^{-8} T

Official Ans. by NTA (B)

Sol. $\frac{\eta P}{4\pi r^2} = \frac{cB_0^2}{2\mu_0}$

$B_0 = \sqrt{\frac{\mu_0}{4\pi}\frac{\eta P}{c}}\frac{1}{r}$

$\Rightarrow B_0 = \frac{1}{4}\sqrt{\frac{10^{-7} \times 4 \times 3.5}{3 \times 10^8}} = 1.71 \times 10^{-8}$ T

18. The light of two different frequencies whose photons have energies 3.8 eV and 1.4 eV respectively, illuminate a metallic surface whose work function is 0.6 eV successively. The ratio of maximum speeds of emitted electrons for the two frequencies respectivly will be :

(A) 1 : 1 (B) 2 : 1

(C) 4 : 1 (D) 1 : 4

Official Ans. by NTA (B)

Sol. $\sqrt{\frac{3.8-0.6}{1.4-0.6}}=\sqrt{\frac{3.2}{0.8}}=2$

19. Two light beams of intensities in the ratio of 9 : 4 are allowed to interfere. The .ratio of the intensity of maxima and minima will be :

(A) 2 : 3 (B) 16 : 81

(C) 25 : 169 (D) 25 : 1

Official Ans. by NTA (D)

Sol. $\sqrt{\frac{I_1}{I_2}}=\sqrt{\frac{9}{4}}=\frac{3}{2}$

$$\left(\frac{\sqrt{I_1}+\sqrt{I_2}}{\sqrt{I_1}-\sqrt{I_2}}\right)^2=5^2=25$$

20. In Bohr's atomic model of hydrogen, let K. P and E are the kinetic energy, potential energy and total energy of the electron respectively. Choose the correct option when the electron undergoes transitions to a higher level :

(A) All K. P and E increase.

(B) K decreases. P and E increase.

(C) P decreases. K and E increase.

(D) K increases. P and E decrease.

Official Ans. by NTA (B)

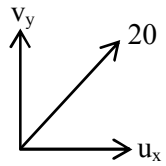
Sol. Based on theory

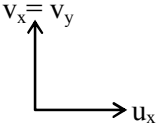
SECTION-B

1. A body is projected from the ground at an angle of 45° with the horizontal. Its velocity after 2s is 20 ms^{-1}. The maximum height reached by the body during its motion is _____m. (use g = 10ms^{-2})

Official Ans. by NTA (20)

Sol.





$v_y = v_x - 20$

$\sqrt{(u_x-20)^2+u_x^2}=20$

$\Rightarrow 2u_x^2-40u_x=0$

$\therefore\ u_x=20$

2. An antenna is placed in a dielectric medium of dielectric constant 6.25. If the maximum size of that antenna is 5.0 mm. it can radiate a signal of minimum frequency of _____GHz.

(Given μ_r = 1 for dielectric medium)

Official Ans. by NTA (6)

Sol. $C'=\frac{C}{\sqrt{\mu_r\varepsilon_r}}=\frac{3\times10^8}{\sqrt{6.25}}=\frac{3\times10^8}{2.5}$

$f\lambda = 1.25\times10^8$ s

$\Rightarrow f\left(5\times10^{-3}\times4\right)=1.25\times10^8$

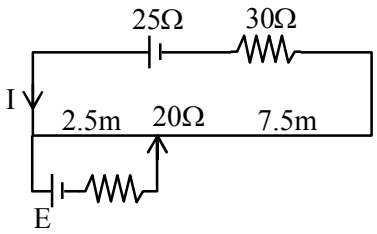
f = 6.25 GHz

So $f\approx6$

3. A potentiometer wire of length 10 m and resistance 20 Ω is connected in series with a 25 V battery and an external resistance 30 Ω . A cell of emf E in secondary circuit is balanced by 250 cm long potentiometer wire. The value of E (in volt) is $\frac{x}{10}$. The value of x is _____.

Official Ans. by NTA (25)

Sol.



$I = \frac{25}{50} = \frac{1}{2}A$

$\therefore \Delta V = 10\ V$

$10\ m \rightarrow 10V$

$2.5m \rightarrow 2.5V$

4. Two travelling waves of equal amplitudes and equal frequencies move in opposite directions along a string. They interfere to produce a stationary wave whose equation is given by

$y = (10 \cos \pi x \sin \frac{2\pi t}{T})$ cm

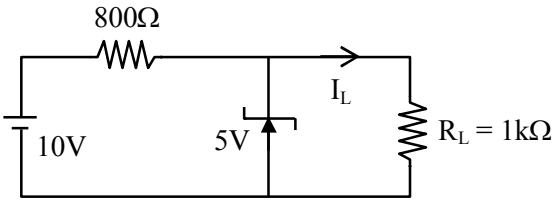
The amplitude of the particle at $x = \frac{4}{3}$ cm will be _____ cm.

Official Ans. by NTA (5)

Sol. $10\cos\left(\frac{4\pi}{3}\right)$

5. In the given circuit- the value of current I_L will be _____ mA.

(When $R_L = 1k\Omega$)



Official Ans. by NTA (5)

Sol. $I_L = \frac{5}{1000} = 5mA$

6. A sample contains 10^{-2} kg each of two substances A and B with half lives 4 s and 8 s respectively. The ratio of then atomic weights is 1 : 2. The ratio of the amounts of A and B after 16 s is $\frac{x}{100}$. the value of x is _____.

Official Ans. by NTA (25)

Sol. $N_t = N_0 (0.5)^{\frac{t}{t_{1/2}}}$

$= \frac{m}{M} \times N_A (0.5)^{\frac{t}{t_{1/2}}}$

$\frac{N_1}{N_2} = \frac{M_2}{M_1}(0.5)^{t\left[\frac{1}{T_A} - \frac{1}{T_B}\right]}$

$= 2(0.5)^{16 \times \frac{1}{8}} = \frac{2}{4} = \frac{1}{2} = \frac{x}{100}$

7. A ray of ligh is incident at an angle of incidence 60° on the glass slab of refractive index $\sqrt{3}$. After refraction, the light ray emerges out from other parallel faces and lateral shift between incident ray and emergent ray is $4\sqrt{3}$ cm. The thickness of the glass slab is _____ cm.

Official Ans. by NTA (12)

Sol. $\ell = t\sin i\left[1-\frac{\cos i}{\sqrt{\mu^2-\sin^2 i}}\right]$

$\Rightarrow 4\sqrt{3} = t\sin 60°\left[1-\frac{\cos 60°}{\sqrt{3-\frac{3}{4}}}\right]$

8. A circular coil of 1000 turns each with area $1m^2$ is rotated about its vertical diameter at the rate of one revolution per second in a uniform horizontal magnetic field of 0.07T. The maximum voltage generation will be _____V.

Official Ans. by NTA (440)

Sol. $\in_{max} = BAN\omega$

$= 0.07 \times 1 \times 10^3 \times 2\pi$

$= 140\pi \approx 440$

9. A monoatomic gas performs a work of $\frac{Q}{4}$ where Q is the heat supplied to it. The molar heat capaticy of the gas will be _____R during this transformation.

Where R is the gas constant.

Official Ans. by NTA (2)

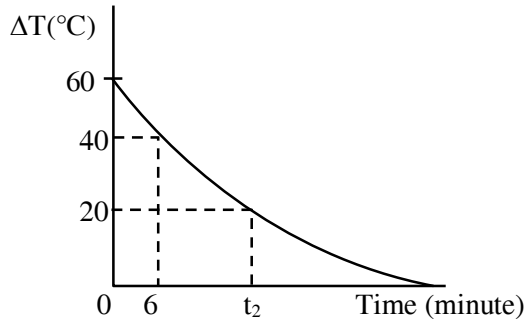
Sol. $\Delta Q = \Delta E + WD \Rightarrow Q = \Delta E + \frac{Q}{4}$

$\Rightarrow n\frac{3R}{2}\Delta T = \Delta E = \frac{3Q}{4}$

$\therefore n\Delta T = \frac{Q}{2R}$

$\therefore C = 2R$

10. In an experment ot verify Newton's law of cooling, a graph is plotted between, the temperature difference (ΔT) of the water and surroundings and time as shown in figure. The initial temperature of water is taken as 80°C. The value of t_2 as mentioned in the graph will be _____.



Official Ans. by NTA (16)

Sol. $T - T_0 (T_i - T_0)\, e^{-\frac{Bt}{ms}}$

$6\lambda = \ln 1.5$

$40 = 60e^{-\lambda(6)} \Rightarrow 6\lambda = \ln 1.5$

$20 = 60e^{-\lambda t_2} \Rightarrow t_2\,\lambda = \ln 3$

$\frac{t_2}{6} = \frac{\ln 3}{\ln 1.5}$

$\therefore t_2 = 16.25$ min

So ≈ 16

FINAL JEE–MAIN EXAMINATION – JUNE, 2022

(Held On Friday 24th June, 2022) **TIME : 3 : 00 PM to 6 : 00 PM**

CHEMISTRY

TEST PAPER WITH SOLUTION

SECTION-A

1. 120 of an organic compound that contains only carbon and hydrogen gives 330g of CO_2 and 270g of water on complete combustion. The percentage of carbon and hydrogen, respectively are.

(A) 25 and 75 (B) 40 and 60

(C) 60 and 40 (D) 75 and 25

Official Ans. by NTA (D)

Sol. Given mass of organic compound = 120

mass of $CO_2(g)$ = 330 g

mass of H_2O (ℓ) = 270 g

mass of carbon = $n_{CO_2} \times 12$

$= \frac{330}{44} \times 12 = 90g$

% of carbon = $\frac{90}{120} \times 100 = 75\%$

mass of hydrogen = $n_{H_2O} \times 2$

$= \frac{270}{18} \times 2 = 30g$

% of hydrogen = $\frac{30}{120} \times 100 = 25\%$

2. The energy of one mole of photons of radiation of wavelength 300 nm is

(Given : h = 6.63×10^{-34} Js, $N_A = 6.02 \times 10^{23} mol^{-1}$, $c = 3 \times 10^8$ ms^{-1})

(A) 235 kJ mol^{-1} (B) 325 kJ mol^{-1}

(C) 399 kJ mol^{-1} (D) 435 kJ mol^{-1}

Official Ans. by NTA (C)

Sol. Energy of one mole of photons = $\frac{hc}{\lambda} \times N_A$

$= \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{300 \times 10^{-9}} \times 6.02 \times 10^{23}$

= 399.13×10^3 Joule/mole

= 399 kJ / mole

3. The correct order of bound orders of C_2^{2-}, N_2^{2-} and O_2^{2-} is, respectively.

(A) $C_2^{2-} < N_2^{2-} < O_2^{2-}$ (B) $O_2^{2-} < N_2^{2-} < C_2^{2-}$

(C) $C_2^{2-} < O_2^{2-} < N_2^{2-}$ (D) $N_2^{2-} < C_2^{2-} < O_2^{2-}$

Official Ans. by NTA (B)

Sol.

Species	Bond order
C_2^{2-}	3
N_2^{2-}	2
O_2^{2-}	1

4. At 25°C and 1 atm pressure, the enthalpies of combustion are as given below:

Substance	H_2	C(graphite)	$C_2H_6(g)$
$\frac{\Delta_C H^\Theta}{kJmol^{-1}}$	–286.0	–394.0	–1560.0

The enthalpy of formation of ethane is

(A) +54.0 kJ mol^{-1} (B) –68.0 kJ mol^{-1}

(C) –86.0 kJ mol^{-1} (D) +97.0 kJ mol^{-1}

Official Ans. by NTA (C)

Sol. $C_2H_6(g) + \frac{7}{2}O_2(g) \rightarrow 2CO_2(g) + 3H_2O(\ell)$

$\Delta_C H(C_2H_6) = 2\Delta_f H\ CO_2(g) + 3\Delta_f H(H_2O,\ell)$

$- \Delta_f H(C_2H_6,g)$

$-1560 = 2(-394) + 3(-286) - \Delta_f H(C_2H_6,g)$

$\Delta_f H(C_2H_6,g) = -86$ kJ/mole

5. For a first order reaction, the time required for completion of 90% reaction is 'x' times the half life of the reaction. The value of 'x' is

(Given: ln 10 = 2.303 and log 2 = 0.3010)

(A) 1.12 (B) 2.43

(C) 3.32 (D) 33.31

Official Ans. by NTA (C)

Sol. Given $t_{0.90} = t_{0.90} = xt_{1/2}$

First order rate constant

$$K = \frac{\ln 2}{t_{1/2}} = \frac{1}{xt_{1/2}} \ln \frac{A_0}{A_0 - A_0 \times \frac{90}{100}}$$

$$\frac{\ln 2}{t_{1/2}} = \frac{\ln 10}{xt_{1/2}}$$

$$x = \frac{\ln 10}{\ln 2} = \frac{2.303}{2.303 \times 0.3010} = 3.32$$

6. Metals generally melt at very high temperature. Amongst the following, the metal with the highest melting point will be

(A) Hg (B) Ag

(C) Ga (D) Cs

Official Ans. by NTA (B)

Sol. Hg, Ga, Cs are liquid near room temperature But Ag(silver) is solid.

7. Which of the following chemical reactions represents Hall-Heroult Process?

(A) $Cr_2O_3 + 2Al \rightarrow Al_2O_3 + 2Cr$

(B) $2Al_2O_3 + 3C \rightarrow 4Al + 3CO_2$

(C) $FeO + CO \rightarrow Fe + CO_2$

(D) $2\left[Au(CN)_2\right]^-_{(aq)} + Zn(s) \rightarrow 2Au(s) + \left[Zn(CN_4)\right]^{2-}$

Official Ans. by NTA (B)

Sol. Hall Heroult process is the major industrial process for extraction of aluminium.

8. In the industrial production of which of the following, molecular hydrogen is obtained as a byproduct?

(A) NaOH (B) NaCl

(C)Na metal (D) Na_2CO_3

Official Ans. by NTA (A)

Sol. Sodium hydroxide is generally prepared commercially by electrolysis of sodium chloride in castner Kellner cell.

at cathode : $Na + e^- \xrightarrow{Hg} Na-$ amalgum

Anode : $Cl^- \longrightarrow \frac{1}{2}Cl_2 + e^-$

The Na–amalgam is treated with water to give sodium hydroxide and hydrogen gas :

$2Na\ (amalgam) + H_2O \rightarrow 2NaOH + H_2 + 2Hg$

9. Which one of the following compounds is used as a chemical in certain type of fire extinguishers?

(A) Baking Soda (B) Soda ash

(C) Washing Soda (D) Caustic Soda

Official Ans. by NTA (A)

Sol. Sodium hydrogencarbonate (Baking soda), $NaHCO_3$ is used in the fire extinguishers.

10. PCl_5 is well known. but NCl_5 is not. Because.

(A) nitrogen is less reactive than phosphorous.

(B) nitrogen doesn't have d-orbitals in its valence shell.

(C) catenation tendency is weaker in nitrogen than phosphorous.

(D) size of phosphorous is larger than nitrogen.

Official Ans. by NTA (B)

Sol. PCl_5 forms five bonds by using the d–orbitals to "expand the octet". But NCl_5 does not exist because there are no d–orbitals in the valence shell (2^{nd} shell). Therefore there is no way to expand the octet.

11. Transition metal complex with highest value of crystal field splitting (Δ_0) will be

(A) $\left[Cr(H_2O)_6\right]^{3+}$ (B) $\left[Mo(H_2O)_6\right]^{3+}$

(C) $\left[Fe(H_2O)_6\right]^{3+}$ (D) $\left[Os(H_2O)_6\right]^{3+}$

Official Ans. by NTA (D)

Sol. CFSE of octahedral complexes with water is greater for 5d series metal centre ion as compared to 3d and 4d series metal centre.

12. Some gases are responsible for heating of atmosphere (green house effect). Identify from the following the gaseous species which does not cause it.

(A) CH_4 (B) O_3

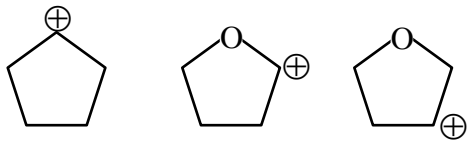
(C) H_2O (D)N_2

Official Ans. by NTA (D)

Sol. CH_4, O_3 and H_2O causes global warming in Tropospheric level.

N_2 does not cause global warming.

13. Arrange the following carbocations in decreasing order of stability.



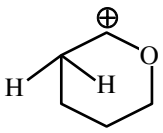
A B C

(A) A > C > B (B) A > B > C

(C) C > B > A (D) C > A > B

Official Ans. by NTA (B)

Sol. Carbocation is stabilised by resonance with lone pairs on oxygen atom and +H effect of 2a hydrogens



B > A > C

14. Given below are two statements.

Statement I : The presence of weaker π- bonds make alkenes less stable than alkanes.

Statement II : The strength of the double bond is greater than that of carbon-carbon single bond.

In the light of the above statements, choose the *correct* answer from the options given below.

(A) Both Statement I and Statement II are correct.

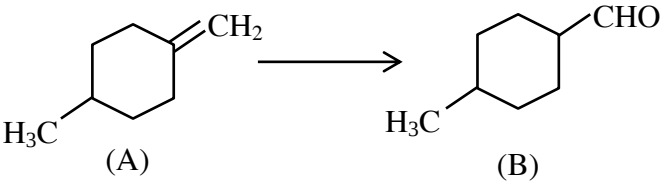
(B) Both Statement I and Statement II are incorrect.

(C) Statement I is correct but Statement II is incorrect.

(D) Statement I is incorrect but Statement II is correct.

Official Ans. by NTA (A)

15. Which of the following reagents/ reactions will convert 'A' to 'B'?



(A) (B)

(A) PCC oxidation

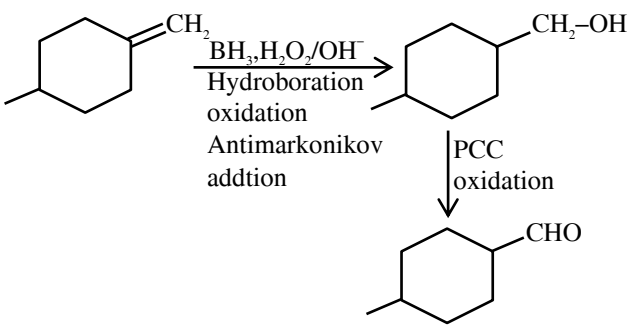
(B) Ozonolysis

(C) $BH_3, H_2O_2 / {}^-OH$ followed by PCC oxidation

(D)HBr, hydrolysis followed by oxidation by $K_2Cr_2O_7$.

Official Ans. by NTA (C)

Sol. BH_3, $H_2O_2/^-OH$ followed by PCC oxidation.



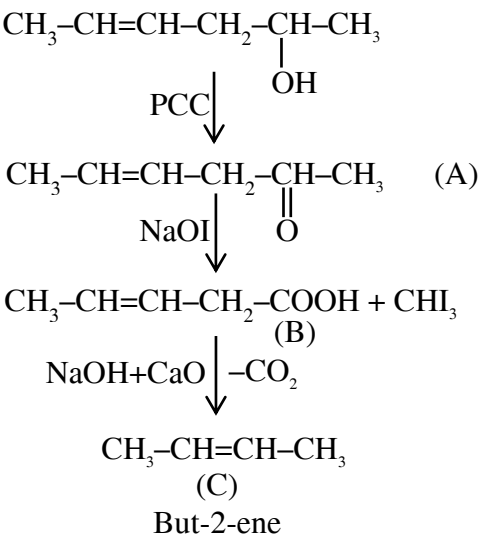
16. Hex-4-ene-2-ol on treatment with PCC gives 'A'. 'A' on reaction with sodium hypoiodite gives 'B', which on further heating with soda lime gives 'C'. The compound 'C' is

(A) 2- pentene (B) proponaldehyde

(C) 2-butene (D) 4-methylpent-2-ene

Official Ans. by NTA (C)

Sol.



17. The conversion of propan-1-ol to n-butylamine involves the sequential addition of reagents. The correct sequential order of reagents is.

(A)(i) $SOCl_2$ (ii) KCN (iii) $H_2/Ni,Na(Hg)/C_2H_5OH$

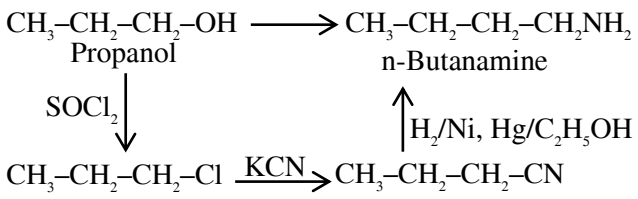
(B) (i) HCl (ii) H_2/Ni, $Na(Hg)/C_2H_5OH$

(C) (i) $SOCl_2$ (ii) KCN (iii) CH_3NH_2

(D) (i) HCl (ii) CH_3NH_2

Official Ans. by NTA (A)

Sol.



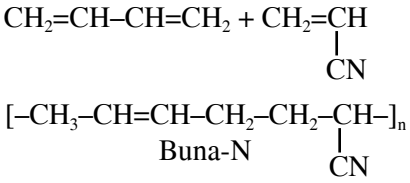
18. Which of the following is **not** an example of a condensation polymer?

(A) Nylon 6,6 (B) Decron

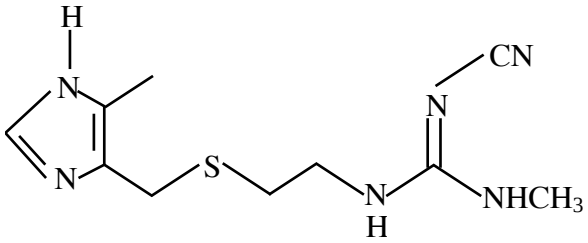
(C) Buna-N (D) Silicone

Official Ans. by NTA (C)

Sol. Buna-N is an addition copolymer of 1,3-butadiene and acrylonitrile.



19. The structure shown below is of which well-known drug molecule?



(A) Ranitidine (B) Seldane

(C) Cimetidine (D) Codeine

Official Ans. by NTA (C)

20. In the flame test of a mixture of salts, a green flame with blue centre was observed. Which one of the following cations may be present?

(A) Cu^{2+} (B) Sr^{2+}

(C) Ba^{2+} (D) Ca^{2+}

Official Ans. by NTA (A)

Sol.

Ion	Colour of the flame
(A) Cu^{+2}	green flame with blue centre
(B) Sr^{2+}	Crimson Red
(C) Ba^{2+}	Apple green

SECTION-B

1. At 300 K, a sample of 3.0 g of gas A occupies the same volume as 0.2 g of hydrogen at 200 K at the same pressure. The molar mass of gas A is____ g mol^{-1} (nearest integer) Assume that the behaviour of gases as ideal. (Given: The molar mass of hydrogen (H_2) gas is 2.0 g mol^{-1})

Official Ans. by NTA (45)

Sol. Given : Ideal gas A and H_2 gas at same pressure and volume.

From ideal gas equation pv = nRT

$n_1T_1 = n_2T_2$

$$\frac{3}{\text{GMM of A}} \times 300 = \frac{0.2}{2} \times 200$$

GMM of A = 45 g/mole

2. A company dissolves 'X' amount of CO_2 at 298 K in 1 litre of water to prepare soda water

X =____ $\times 10^{-3}$g. (nearest integer)

(Given: partial pressure of CO_2 at 298 K= 0.835 bar.

Henry's law constant for CO_2 at 298 K = 1.67 kbar.

Atomic mass of H,C and O is 1, 12 and 6 g mol^{-1}, respectively)

Official Ans. by NTA (1221 OR 1222)

Sol. From Henry law

$P = K_H X_{CO_2}$

$$0.835 = 1.67 \times 10^3 \times 1.67 \times 10^3 \times \frac{w_{CO_2}/44}{\frac{w_{CO_2}}{44} + \frac{1000}{18}}$$

$$w_{CO_2} = 1.2228\text{g} = 1222.8 \times 10^{-3}\text{g}$$

Or

$P = K_H X_{CO_2}$

$$0.835 = 1.67 \times 10^3 \times \frac{n_{CO_2}}{n_{CO_2} + n_{H_2O}}$$

$$0.835 = 1.67 \times 10^3 \times \frac{w_{CO_2}/44}{\frac{1000}{18}}$$

$w_{CO_2} = 1.2222\text{g} = 1222.2 \times 10^{-3}\text{g}$

3. PCl_5 dissociates as

$$PCl_5(g) \rightleftharpoons PCl_3(g) + Cl_2(g)$$

5 moles of PCl_5 are placed in a 200 litre vessel which contains 2 moles of N_2 and is maintained at 600 K. The equilibrium pressure is 2.46 atm. The equilibrium constant K_p for the dissociation of PCl_5 is_____ $\times 10^{-3}$. (nearest integer)

(Given: R = 0.082 L atm K^{-1} mol^{-1} : Assume ideal gas behaviour)

Official Ans. by NTA (1107)

Sol. Given : 2 mole of N_2 gas was present as inert gas.

Equilibrium pressure = 2.46 atm

$$PCl_5(g) \rightleftharpoons PCl_3(g) + C\ell_2(g)$$

t = 0	5	0	0
t = Eq^m	5 – x	x	x

from ideal gas equation

PV = nRT

2.46 × 200 = (5 – x + x + x + 2) × 0.082 × 600

x = 3

$$K_P = \frac{n_{PCl_3} \times n_{Cl_2}}{n_{PCl_5}} \times \left[\frac{P_{total}}{n_{total}}\right]$$

$$\frac{3 \times 3}{2} \times \frac{2.46}{10} = 1.107 = 1107 \times 10^{-3}$$

4. The resistance of conductivity cell containing 0.01 M KCl solution at 298 K is 1750 Ω. If the conductively of 0.01 M KCl solution at 298 K is 0.152×10^{-3} S cm^{-1}, then the cell constant of the conductivity cell is_____ $\times 10^{-3}$ cm^{-1}.

Official Ans. by NT

Sol. $K = \frac{1}{R} \times \text{cell constant}$

$0.152 \times 10^{-3} = \frac{1}{1750}$ cell constant

cell constant = 266×10^{-3}

5. When 200 mL of 0.2 M acetic acid is shaken with 0.6 g of wood charcoal, the final concentration of acetic after adsorption is 0.1 M. The mass of acetic acid adsorbed per garm of carbon is_____ g.

Official Ans. by NTA (2)

Sol. weight of wood charcoal = 0.6 g

Mass of acetic acid adsorbed = $\frac{M_1V_1 - M_2V_2}{1000} \times 60$

$= \frac{0.2 \times 200 - 0.1 \times 200}{1000} \times 60$

= 1.2 g

Mass of acetic acid adsorbed per gram of carbon = $\frac{1.2}{0.6} = 2$

6. (a) Baryte, (b) Galena, (c) Zinc blende and (d) Copper pyrites. How many of these minerals are sulphide based?

Official Ans. by NTA (3)

Sol.

(1) Baryte : $BaSO_4$

(2) Galena : PbS

(3) Zinc blende : ZnS

(4) Copper pyrite : $CuFeS_2$

] sulphide (S^{2-}) ores

7. Manganese (VI) has ability to disproportionate in acidic solution. The difference in oxidation states of two ions it forms in acidic solution is_____

Official Ans. by NTA (3)

Sol. MnO_4^{2-} disproportionates in a neutral or acidic solution to give MnO_4^- and Mn^{+4}

$3MnO_4^{2-} + 3H^+ \longrightarrow 2MnO_4^- + MnO_2 + 2H_2O$

O.S. of Mn in MnO_4^- = +7

O.S. of Mn in MnO_2 = +4

difference = 3

8. 0.2 g of an organic compound was subjected to estimation of nitrogen by Dumas method in which volume of N_2 evolved (at STP) was found to be 22.400 mL. The percentage of nitrogen in the compound is____.[nearest integer]

(Given: Molar mass of N_2 is 28 mol^{-1}. Molar volume of N_2 at STP : 22.4 L)

Official Ans. by NTA (14)

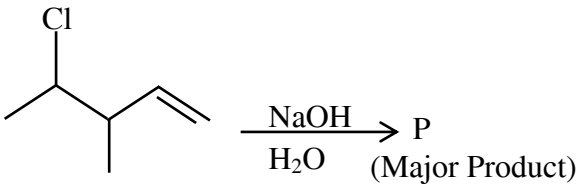
Sol. weight of organic compound = 0.2g

mass of N_2(g) evolved = $\frac{22.4 \times 10^{-3}}{22.4} \times 28$

$= 28 \times 10^{-3}$g

% of N = $\frac{28 \times 10^{-3}}{0.2} \times 100 = 14$

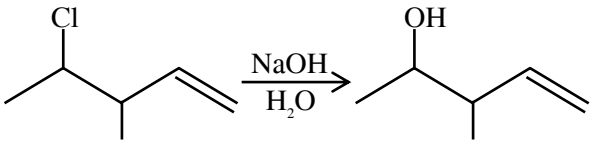
9.



Consider the above reaction. The number of π electrons present in the product 'P' is____.

Official Ans. by NTA (2)

Sol. Number of π electron = 2



10. In alanylglycylleucylalanylvaline, the number of peptide linkages is_____.

Official Ans. by NTA (4)

Sol. There are Five amino acids and four peptide linkages.

FINAL JEE–MAIN EXAMINATION – JUNE, 2022

(Held On Friday 24th June, 2022) **TIME : 3 : 00 PM to 6 : 00 PM**

MATHEMATICS

TEST PAPER WITH SOLUTION

SECTION-A

1. Let $x*y = x^2 + y^3$ and $(x*1)*1 = x*(1*1)$.

Then a value of $2\sin^{-1}\left(\frac{x^4+x^2-2}{x^4+x^2+2}\right)$ is

(A) $\frac{\pi}{4}$ (B) $\frac{\pi}{3}$

(C) $\frac{\pi}{2}$ (D) $\frac{\pi}{6}$

Official Ans. by NTA (B)

Sol. $\because (x * 1) * 1 = x * (1 * 1)$

$(x^2 + 1) * 1 = x * (2)$

$(x^2 + 1)^2 + 1 = x^2 + 8$

$x^4 + x^2 - 6 = 0 \Rightarrow (x^2 + 3)(x^2 - 2) = 0$

$x^2 = 2$

$\Rightarrow 2\sin^{-1}\left(\frac{x^4+x^2-2}{x^4+x^2+2}\right) = 2\sin^{-1}\left(\frac{1}{2}\right)$

$= \frac{\pi}{3}$

2. The sum of all the real roots of the equation $(e^{2x} - 4)(6e^{2x} - 5e^x + 1) = 0$ is

(A) $\log_e 3$ (B) $-\log_e 3$

(C) $\log_e 6$ (D) $-\log_e 6$

Official Ans. by NTA (B)

Sol. $(e^{2x} - 4)(6e^{2x} - 3e^x - 2e^x + 1) = 0$

$(e^{2x} - 4)(3e^x - 1)(2e^x - 1) = 0$

$e^{2x} = 4$ or $e^x = \frac{1}{3}$ or $e^x = \frac{1}{2}$

$\Rightarrow$ sum of real roots $= \frac{1}{2}\ell n4 + \ell n\frac{1}{3} + \ell n\frac{1}{2}$

$= -\ell n3$

3. Let the system of linear equations

$x + y + \alpha z = 2$

$3x + y + z = 4$

$x + 2z = 1$

have a unique solution (x^*, y^*, z^*). If (α, x^*), (y^*, α) and $(x^*, -y^*)$ are collinear points, then the sum of absolute values of all possible values of α is :

(A) 4 (B) 3

(C) 2 (D) 1

Official Ans. by NTA (C)

Sol. $\Delta = \begin{vmatrix} 1 & 1 & \alpha \\ 3 & 1 & 1 \\ 1 & 0 & 2 \end{vmatrix} = -(\alpha + 3)$

$\Delta_1 = \begin{vmatrix} 2 & 1 & \alpha \\ 4 & 1 & 1 \\ 1 & 0 & 2 \end{vmatrix} = -(3 + \alpha)$

$\Delta_2 = \begin{vmatrix} 1 & 2 & \alpha \\ 3 & 4 & 1 \\ 1 & 1 & 2 \end{vmatrix} = -(\alpha + 3)$

$\Delta_3 = \begin{vmatrix} 1 & 1 & 2 \\ 3 & 1 & 4 \\ 1 & 0 & 1 \end{vmatrix} = 0$

$\alpha \neq -3$, $x = 1$, $y = 1$, $z = 0$,

Now points $(\alpha, 1)$, $(1, \alpha)$ & $(1, -1)$ are collinear

$\begin{vmatrix} \alpha & 1 & 1 \\ 1 & \alpha & 1 \\ 1 & -1 & 1 \end{vmatrix} = 0$

$\Rightarrow \alpha(\alpha + 1) - 1(1 - 1) + 1(-1 - \alpha) = 0$

$\alpha^2 + \alpha - 1 - \alpha = 0$

$\alpha = \pm 1$

4. Let $x, y > 0$. If $x^3y^2 = 2^{15}$, then the least value of $3x + 2y$ is

(A) 30 (B) 32

(C) 36 (D) 40

Official Ans. by NTA (D)

Sol. Using AM ≥ GM

$$\frac{x+x+x+y+y}{5} \geq \left(x^3.y^2\right)^{\frac{1}{5}}$$

$$\frac{3x+2y}{5} \geq \left(2^{15}\right)^{\frac{1}{5}}$$

$(3x + 2y)_{min} = 40$

5. Let $f(x)=\begin{cases} \dfrac{\sin\left(x-[x]\right)}{x-[x]} & , \quad x\in(-2,-1) \\ \max\left\{2x, 3[|x|]\right\} & , \quad |x|<1 \\ 1 & , \quad \text{otherwise} \end{cases}$

where [t] denotes greatest integer ≤ t. If m is the number of points where f is not continuous and n is the number of points where f is not differentiable, then the ordered pair (m, n) is :

(A) (3, 3) (B) (2, 4)

(C) (2, 3) (D) (3, 4)

Official Ans. by NTA (C)

Sol. $f(x)=\begin{cases} \dfrac{\sin(x+2)}{x+2} & , \quad x\in(-2,-1) \\ \max\{2x,0\} & , \quad x\in(-1,1) \\ 1 & , \quad \text{otherwise} \end{cases}$

$$f(-2^+) = \lim_{h\to 0} f(-2+h) = \lim_{h\to 0}\frac{\sinh}{h} = 1$$

f is continuous at x = –2

$$f(-1^-) = \lim_{h\to 0}\frac{\sin(-1-h+2)}{(-1-h+2)} = \sin 1$$

$f(-1) = f(-1^+) = 0$

$f(1^+) = 1$ & $f(1^-) = 0 \Rightarrow f$ is not continuous at x = 1

f is continuous but not diff. at x = 0

$$\left.\begin{array}{l} \Rightarrow \text{f is discontinuous at } x = -1 \text{ \& } 1 \\ \text{\& f is not diff. at } x = -1, 0 \text{ \& } 1 \end{array}\right\} \Rightarrow \begin{array}{l} m = 2 \\ n = 3 \end{array}$$

6. The value of the integral $\int_{-\pi/2}^{\pi/2}\frac{dx}{\left(1+e^x\right)\left(\sin^6 x+\cos^6 x\right)}$ is equal to

(A) 2π (B) 0

(C) π (D) $\frac{\pi}{2}$

Official Ans. by NTA (C)

Sol. $I=\int_{-\pi/2}^{0}\frac{dx}{\left(1+e^x\right)\left(\sin^6 x+\cos^6 x\right)}+\int_{0}^{\pi/2}\frac{dx}{\left(1+e^x\right)\left(\sin^6 x+\cos^6 x\right)}$

Put x = –t

$$=\int_{\pi/2}^{0}\frac{-dt}{\left(1+e^{-t}\right)\left(\sin^6 t+\cos^6 t\right)}+\int_{0}^{\pi/2}\frac{dx}{\left(1+e^x\right)\left(\sin^6 x+\cos^6 x\right)}$$

$$=\int_{0}^{\pi/2}\frac{\left(e^x+1\right)dx}{\left(1+e^x\right)\left(\sin^6 x+\cos^6 x\right)}$$

$$=\int_{0}^{\pi/2}\frac{dx}{\left(\sin^2 x+\cos^2 x\right)\left(\sin^4 x-\sin^2 x\cos^2 x+\cos^4 x\right)}$$

$$=\int_{0}^{\pi/2}\frac{\left(1+\tan^2 x\right)\sec^2 x\,dx}{\left(\tan^4 x-\tan^2 x+1\right)}$$

Put tanx = t

$$=\int_{0}^{\infty}\frac{\left(1+t^2\right)dt}{\left(t^4-t^2+1\right)}$$

$$=\int_{0}^{\infty}\frac{\left(1+\frac{1}{t^2}\right)dt}{t^2-1+\frac{1}{t^2}}=\int_{0}^{\infty}\frac{\left(1+\frac{1}{t^2}\right)dt}{\left(t-\frac{1}{t}\right)^2+1}$$

Put $t-\frac{1}{t}=z$

$$\left(1+\frac{1}{t^2}\right)dt = dz$$

$$=\int_{-\infty}^{\infty}\frac{dz}{1+z^2}=\left(\tan^{-1}z\right)_{-\infty}^{\infty}$$

$$=\frac{\pi}{2}-\left(-\frac{\pi}{2}\right)=\pi$$

7. $\lim\limits_{n\to\infty}\left(\frac{n^2}{(n^2+1)(n+1)}+\frac{n^2}{(n^2+4)(n+2)}+\frac{n^2}{(n^2+9)(n+3)}+\ldots+\frac{n^2}{(n^2+n^2)(n+n)}\right)$ is equal to

(A) $\frac{\pi}{8}+\frac{1}{4}\log_e 2$ (B) $\frac{\pi}{4}+\frac{1}{8}\log_e 2$

(C) $\frac{\pi}{4}-\frac{1}{8}\log_e 2$ (D) $\frac{\pi}{8}+\log_e\sqrt{2}$

Official Ans. by NTA (A)

Sol. $\lim\limits_{n\to\infty}\left(\sum\limits_{r=1}^{n}\frac{n^2}{(n^2+r^2)(n+r)}\right)$

$=\lim\limits_{n\to\infty}\left(\sum\limits_{r=1}^{n}\frac{1}{n\left(1+\left(\frac{r}{n}\right)^2\right)\left(1+\left(\frac{r}{n}\right)\right)}\right)$

$=\int_0^1\frac{dx}{(1+x^2)(1+x)}=\frac{1}{2}\int_0^1\frac{1-x}{1+x^2}dx+\frac{1}{2}\int_0^1\frac{1}{1+x}dx$

$=\frac{1}{2}\int\left(\frac{1}{1+x^2}-\frac{x}{1+x^2}\right)dx+\frac{1}{2}\left(\ln(1+x)\right)_0^1$

$=\frac{1}{2}\left[\tan^{-1}x-\frac{1}{2}\ell n\left(1+x^2\right)\right]_0^1+\frac{1}{2}\ell n2$

$=\frac{1}{2}\left[\frac{\pi}{4}-\frac{1}{2}\ell n2\right]+\frac{1}{2}\ell n2$

$=\frac{\pi}{8}+\frac{1}{4}\ell n\,2$

8. A particle is moving in the xy-plane along a curve C passing through the point (3, 3). The tangent to the curve C at the point P meets the x-axis at Q. If the y-axis bisects the segment PQ, then C is a parabola with

(A) length of latus rectum 3

(B) length of latus rectum 6

(C) focus $\left(\frac{4}{3},0\right)$

(D) focus $\left(0,\frac{3}{4}\right)$

Official Ans. by NTA (A)

Sol. Let Point P(x,y)

$Y-y=y'(X-x)$

$Y=0\Rightarrow X=x-\frac{y}{y'}$

$Q\left(x-\frac{y}{y'},0\right)$

Mid Point of PQ lies on y axis

$x-\frac{y}{y'}+x=0$

$y'=\frac{y}{2.x}\Rightarrow 2\frac{dy}{y}=\frac{dx}{x}$

$2\ell ny=\ell nx+\ell nk$

$y^2=kx$

It passes through (3, 3) $\Rightarrow k=3$

curve c $\Rightarrow y^2=3x$

Length of L.R. = 3

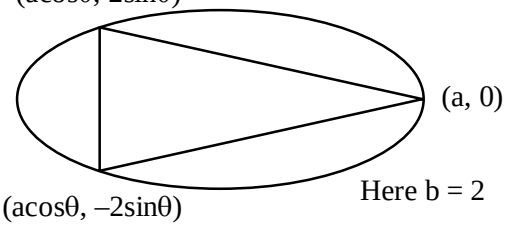
Focus = $\left(\frac{3}{4},0\right)$ **Ans. (A)**

9. Let the maximum area of the triangle that can be inscribed in the ellipse $\frac{x^2}{a^2}+\frac{y^2}{4}=1$, a > 2, having one of its vertices at one end of the major axis of the ellipse and one of its sides parallel to the y-axis, be $6\sqrt{3}$. Then the eccentricity of the ellispe is :

(A) $\frac{\sqrt{3}}{2}$ (B) $\frac{1}{2}$ (C) $\frac{1}{\sqrt{2}}$ (D) $\frac{\sqrt{3}}{4}$

Official Ans. by NTA (A)

Sol.



$A=\frac{1}{2}a\,(1-\cos\theta)\,(4\sin\theta)$

$A = 2a(1-\cos\theta)\sin\theta$

$\frac{dA}{d\theta} = 2a(\sin^2\theta + \cos\theta - \cos^2\theta)$

$\frac{dA}{d\theta} = 0 \Rightarrow 1 + \cos\theta - 2\cos^2\theta = 0$

$\cos\theta = 1$ (Reject)

OR

$\cos\theta = \frac{-1}{2} \Rightarrow \theta = \frac{2\pi}{3}$

$\frac{d^2A}{d\theta^2} = 2a(2\sin^2\theta - \sin\theta)$

$\frac{d^2A}{d\theta^2} < 0$ for $\theta = \frac{2\pi}{3}$

Now, $A_{max} = \frac{3\sqrt{3}}{2}a = 6\sqrt{3}$

$\boxed{a = 4}$

Now, $e = \sqrt{\frac{a^2 - b^2}{a^2}} = \frac{\sqrt{3}}{2}$ **Ans. (A)**

10. Let the area of the triangle with vertices A(1, α), B(α, 0) and C(0, α) be 4 sq. units. If the point (α, –α), (–α, α) and (α^2, β) are collinear, then β is equal to

(A) 64 (B) –8

(C) –64 (D) 512

Official Ans. by NTA (C)

Sol. $\frac{1}{2}\begin{vmatrix} \alpha & 0 & 1 \\ 1 & \alpha & 1 \\ 0 & \alpha & 1 \end{vmatrix} = \pm 4$

$\alpha = \pm 8$

Now given points (8, –8), (–8, 8), (64, β)

OR (–8, 8), (8, –8), (64, β)

are collinear $\Rightarrow$ Slope = –1.

$\boxed{\beta = -64}$ **Ans. (C)**

11. The number of distinct real roots of the equation $x^7 - 7x - 2 = 0$ is

(A) 5 (B) 7 (C) 1 (D) 3

Official Ans. by NTA (D)

Sol. $x^7 - 7x - 2 = 0$

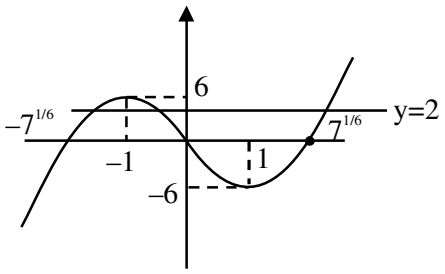
$x^7 - 7x = 2$

$f(x) = x^7 - 7x$ (odd) & $y = 2$

$f(x) = x\,(x^2 - 7^{1/3})\,(x^4 + x^2 \cdot 7^{1/3} + 7^{2/3})$

$f'(x) = 7(x^6 - 1) = 7\,(x^2 - 1)\,(x^4 + x^2 + 1)$

$f'(x) = 0 \Rightarrow x = \pm 1$



$f(x) = 2$ has 3 real distinct solution.

12. A random variable X has the following probability distribution :

X	0	1	2	3	4
P(X)	k	2k	4k	6k	86

The value of $P(1 < X < 4 \mid X \le 2)$ is equal to :

(A) $\frac{4}{7}$ (B) $\frac{2}{3}$

(C) $\frac{3}{7}$ (D) $\frac{4}{5}$

Official Ans. by NTA (A)

Sol. $P\left(\frac{1<x<4}{x\le 2}\right) = \frac{P(1<x<4 \cap x \le 2)}{P(x \le 2)}$

$= \frac{P(1<x\le 2)}{P(x\le 2)} = \frac{P(x=2)}{P(x\le 2)}$

$= \frac{4k}{k + 2k + 4k} = \frac{4}{7}$

13. The number of solutions of the equation $\cos\left(x+\frac{\pi}{3}\right)\cos\left(\frac{\pi}{3}-x\right)=\frac{1}{4}\cos^2 2x$, $x \in [-3\pi, 3\pi]$ is :

(A) 8 (B) 5

(C) 6 (D) 7

Official Ans. by NTA (D)

Sol. $\cos\left(\frac{\pi}{3}+x\right)\cos\left(\frac{\pi}{3}-x\right)=\frac{1}{4}\cos^2 2x$

$x\in[-3\pi,3\pi]$

$4\left(\cos^2\left(\frac{\pi}{3}\right)-\sin^2 x\right)=\cos^2 2x$

$4\left(\frac{1}{4}-\sin^2 x\right)=\cos^2 2x$

$1 - 4\sin^2 x = \cos^2 2x$

$1 - 2(1 - \cos 2x) = \cos^2 2x$

let $\cos 2x = t$

$-1 + 2\cos 2x = \cos^2 2x$

$t^2 - 2t + 1 = 0$

$(t-1)^2 = 0$

$\boxed{t=1}$ $\boxed{\cos 2x = 1}$

$2x = 2n\pi$

$\boxed{x = n\pi}$

$n = -3, -2, -1, 0, 1, 2, 3$

(D) option is correct.

14. If the shortest distance between the lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{\lambda}$ and $\frac{x-2}{1}=\frac{y-4}{4}=\frac{z-5}{5}$ is $\frac{1}{\sqrt{3}}$, then the sum of all possible values of λ is :

(A) 16 (B) 6

(C) 12 (D) 15

Official Ans. by NTA (A)

Sol. SHORTEST distance $\frac{|(a_2-a_1)\cdot(b_1\times b_2)|}{|b_1\times b_2|}$

$a_1 = (1, 2, 3)$

$a_2 = (2, 4, 5)$

$\vec{b}_2 = 2\hat{i}+3\hat{j}+\lambda\hat{k}$

$\vec{b}_2 = \hat{i}+4\hat{j}+5\hat{k}$

$\text{S.D.} = \frac{\left|\left((2-1)\hat{i}+(4-2)\hat{j}+(5-3)\hat{k}\right)\cdot(\vec{b}_1\times\vec{b}_2)\right|}{|b_1\times b_2|}$

$\vec{b}_1\times\vec{b}_2=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\2&3&\lambda\\1&4&5\end{vmatrix}$

$= \hat{i}(15 - 4\lambda) + \hat{j}(\lambda - 10) + \hat{k}(5)$

$= (15 - 4\lambda)\hat{i} + (\lambda - 10)\hat{j} + 5\hat{k}$

$|\vec{b}_1\times\vec{b}_2| = \sqrt{(15-4\lambda)^2+(\lambda-10)^2+25}$

Now

$\text{S.D.} = \frac{\left|\left(\hat{i}+2\hat{j}+2\hat{k}\right)\cdot\left[(15-4\lambda)\hat{i}+(\lambda-10)\hat{j}+5\hat{k}\right]\right|}{\sqrt{(15-4\lambda)^2+(\lambda-10)^2+25}}$

$\frac{|15-4\lambda+2\lambda-20+10|}{\sqrt{(15-4\lambda)^2+(\lambda-10)^2+25}}=\frac{1}{\sqrt{3}}$

square both side

$3(5-2\lambda)^2 = 225 + 16\lambda^2 - 120\lambda + \lambda^2 + 100 - 20\lambda + 25$

$12\lambda^2 + 75 - 60\lambda = 17\lambda^2 - 140\lambda + 350$

$5\lambda^2 - 80\lambda + 275 = 0$

$\lambda^2 - 16\lambda + 55 = 0$

$(\lambda - 5)(\lambda - 11) = 0$

$\Rightarrow \lambda = 5, 11$

(A) is correct option.

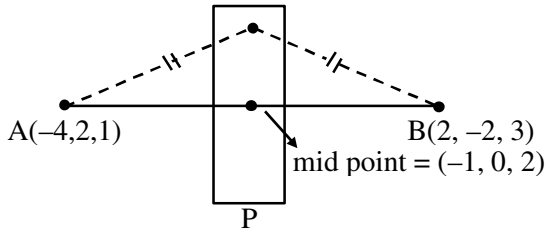
15. Let the points on the plane P be equidistant from the points (–4, 2, 1) and (2, –2, 3). Then the acute angle between the plane P and the plane $2x + y + 3z = 1$ is

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{4}$

(C) $\frac{\pi}{3}$ (D) $\frac{5\pi}{12}$

Official Ans. by NTA (C)

Sol.



Normal vector = $\overrightarrow{AB} = \left(\overrightarrow{OB} - \overrightarrow{OA}\right)$

$= \left(6\hat{i} - 4\hat{j} + 2\hat{k}\right)$

or $2\left(3\hat{i} - 2\hat{j} + \hat{k}\right)$

$P \equiv 3(x + 1) - 2(y) + 1\,(z - 2) = 0$

$P \equiv 3x - 2y + z + 1 = 0$

$P' \equiv 2x + y + 3z - 1 = 0$

angle between P & P' $= \left|\dfrac{\hat{n}_1 \cdot \hat{n}_2}{|n_1||n_2|}\right| = \cos\theta$

$$\theta = \cos^{-1}\left(\frac{6 - 2 + 3}{\sqrt{14} \times \sqrt{14}}\right)$$

$$\theta = \cos^{-1}\left(\frac{7}{14}\right) = = \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$$

Option C is correct.

16. Let $\hat{a}$ and $\hat{b}$ be two unit vectors such that $\left|\left(\hat{a} + \hat{b}\right) + 2\left(\hat{a} \times \hat{b}\right)\right| = 2$. If $\theta \in (0, \pi)$ is the angle between $\hat{a}$ and $\hat{b}$, then among the statements :

(S1) : $2\left|\hat{a} \times \hat{b}\right| = \left|\hat{a} - \hat{b}\right|$

(S2) : The projection of $\hat{a}$ on $\left(\hat{a} + \hat{b}\right)$ is $\dfrac{1}{2}$

(A) Only (S1) is true

(B) Only (S2) is true

(C) Both (S1) and (S2) are true

(D) Both (S1) and (S2) are false

Official Ans. by NTA (C)

Sol. $\left|\left(\hat{a} + \hat{b}\right) + 2\left(\hat{a} \times \hat{b}\right)\right| = 2$, $\theta \in (0, \pi)$

$\left(\left(\hat{a} + \hat{b}\right) + 2\left(\hat{a} \times \hat{b}\right)\right).\left(\left(\hat{a} + \hat{b}\right) + 2\left(\hat{a} \times \hat{b}\right)\right) = 4$

$\left|\hat{a} + \hat{b}\right|^2 + 4\left|\left(\hat{a} \times \hat{b}\right)\right|^2 + 0 = 4$

Let the angle be θ between $\hat{a}$ and $\hat{b}$

$2 + 2\cos\theta + 4\sin^2\theta = 4$

$2 + 2\cos\theta - 4\cos^2\theta = 0$

Let $\cos\theta = t$ then

$2t^2 - t - 1 = 0$

$2t^2 - 2t + t - 1 = 0$

$2t(t - 1) + (t - 1) = 0$

$(2t + 1)(t - 1) = 0$

$t = -\dfrac{1}{2}$ or $t = 1$

$\cos\theta = -\dfrac{1}{2}$ | not possible as $\theta \in (0, \pi)$

$\boxed{\theta = \dfrac{2\pi}{3}}$

Now,

S_1 $2\left|\vec{a} \times \vec{b}\right| = 2\sin\left(\dfrac{2\pi}{3}\right)$

$$\left|\hat{a} - \hat{b}\right| = \sqrt{1 + 1 - 2\cos\left(\frac{2\pi}{3}\right)}$$

$$= \sqrt{2 - 2 \times \left(-\frac{1}{2}\right)}$$

$$= \sqrt{3}$$

S_1 is correct.

S_2 projection of $\hat{a}$ on $\left(\hat{a} + \hat{b}\right)$.

$$\frac{\hat{a}.\left(\hat{a} + \hat{b}\right)}{\left|\hat{a} + \hat{b}\right|} = \frac{1 + \cos\left(\frac{2\pi}{3}\right)}{\sqrt{2 + 2\cos\frac{2\pi}{3}}}$$

$$= \frac{1 - \frac{1}{2}}{\sqrt{1}}$$

$$= \frac{1}{2}$$

C Option is true.

17. If $y = \tan^{-1}(\sec x^3 - \tan x^3)$. $\frac{\pi}{2} < x^3 < \frac{3\pi}{2}$, then

(A) $xy'' + 2y' = 0$

(B) $x^2y'' - 6y + \frac{3\pi}{2} = 0$

(C) $x^2y'' - 6y + 3\pi = 0$

(D) $xy'' - 4y' = 0$

Official Ans. by NTA (B)

Sol. $y = \tan^{-1}(\sec x^3 - \tan x^3)$

$= \tan^{-1}\left(\frac{1-\sin x^3}{\cos x^3}\right)$

$= \tan^{-1}\left(\frac{1-\cos\left(\frac{\pi}{2}-x^3\right)}{\sin\left(\frac{\pi}{2}-x^3\right)}\right)$

$= \tan^{-1}\left(\tan\left(\frac{\pi}{4}-\frac{x^3}{2}\right)\right)$

Since $\frac{\pi}{4}-\frac{x^3}{2} \in \left(-\frac{\pi}{2}, 0\right)$

$y = \left(\frac{\pi}{4}-\frac{x^3}{2}\right)$

$y' = \frac{-3x^2}{2}$, $y'' = -3x$

$4y = \pi - 2x^3$

$4y = \pi - 2x^2\left(\frac{-y''}{3}\right)$

$12y = 3\pi + 2x^2y''$

$x^2y'' - 6y + \frac{3\pi}{2} = 0$

18. Consider the following statements :

A : Rishi is a judge.

B : Rishi is honest.

C : Rishi is not arrogant.

The negation of the statement "if Rishi is a judge and he is not arrogant, then he is honest" is

(A) $B \rightarrow (A \vee C)$

(B) $(\sim B) \wedge (A \wedge C)$

(C) $B \rightarrow ((\sim A) \vee (\sim C))$

(D) $B \rightarrow (A \wedge C)$

Official Ans. by NTA (B)

Sol. $\sim((A \wedge C) \rightarrow B)$

$\sim(\sim(A \wedge C) \vee B)$

Using De-Morgan's law

$(A \wedge C) \wedge (\sim B)$

Option B is correct.

19. The slope of normal at any point (x, y), $x > 0$, $y > 0$ on the curve $y = y(x)$ is given by $\frac{x^2}{xy - x^2y^2 - 1}$.

If the curve passes through the point (1, 1), then e.y(e) is equal to

(A) $\frac{1-\tan(1)}{1+\tan(1)}$ (B) $\tan(1)$

(C) 1 (D) $\frac{1+\tan(1)}{1-\tan(1)}$

Official Ans. by NTA (D)

Sol. Slope of normal $= \frac{-dx}{dy} = \frac{x^2}{xy - x^2y^2 - 1}$

$x^2y^2dx + dx - xydx = x^2dy$

$x^2y^2dx + dx = x^2dy + xydx$

$x^2y^2dx + dx = x(xdy + ydx)$

$x^2y^2dx + dx = xd(xy)$

$\frac{dx}{x} = \frac{d(xy)}{1+x^2y^2}$

$\ln kx = \tan^{-1}(xy)$ … (i)

passes though (1, 1)

$\ln k = \frac{\pi}{4} \Rightarrow k = e^{\frac{\pi}{4}}$

equation (i) be becomes

$\frac{\pi}{4} + \ln x = \tan^{-1}(xy)$

$xy = \tan\left(\frac{\pi}{4} + \ell n\, x\right)$

$xy = \left(\frac{1+\tan(\ell n\, x)}{1-\tan(\ell n\, x)}\right)$ … (ii)

put x = e in (ii)

$\therefore$ $ey\,(e) = \frac{1+\tan 1}{1-\tan 1}$

20. Let λ^* be the largest value of λ for which the function $f_\lambda(x) = 4\lambda x^3 - 36\lambda x^2 + 36x + 48$ is increasing for all $x \in R$. Then $f_\lambda^*(1) + f_\lambda^*(-1)$ is equal to :

(A) 36 (B) 48

(C) 64 (D) 72

Official Ans. by NTA (D)

Sol. $f_\lambda(x) = 4\lambda x^3 - 36\lambda x^2 + 36x + 48$

$f_\lambda'(x) = 12\lambda x^2 - 72\lambda x + 36$

$f_\lambda'(x) = 12(\lambda x^2 - 6\lambda x + 3) \geq 0$

$\therefore \lambda > 0$ & $D \leq 0$

$36\lambda^2 - 4 \times \lambda \times 3 \leq 0$

$9\lambda^2 - 3\lambda \leq 0$

$3\lambda(3\lambda - 1) \leq 0$

$\lambda \in \left[0, \frac{1}{3}\right]$

$\therefore \lambda_{largest} = \frac{1}{3}$

$f(x) = \frac{4}{3}x^3 - 12x^2 + 36x + 48$

$\therefore f(1) + f(1) = 72$

SECTION-B

1. Let $S = \left\{z \in \mathbb{C} : |z-3| \leq 1 \text{ and } z(4+3i) + \bar{z}(4-3i) \leq 24\right\}$.

If $\alpha + i\beta$ is the point in S which is closest to 4i, then $25(\alpha + \beta)$ is equal to _____.

Official Ans. by NTA (80)

Sol. $|z - 3| \leq 1$

represent pt. i/s circle of radius 1 & centred at (3, 0)

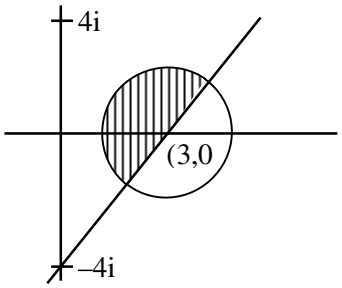
$z(4 + 3i) + \bar{z}(4 - 3i) \leq 24$

$(x + iy)(4 + 3i) + (x - iy)(4 - 3i) \leq 24$

$4x + 3xi + 4iy - 3y + 4x - 3ix - 4iy - 3y \leq 24$

$8x - 6y \leq 24$

$4x - 3y \leq 12$



minimum of (0, 4) from circle = $\sqrt{3^2+4^2} - 1 = 4$

will lie along line joining (0, 4) & (3, 0)

$\therefore$ equation line

$\frac{x}{3} + \frac{y}{4} = 1 \Rightarrow 4x + 3y = 12$ … (i)

equation circle $(x - 3)^2 + y^2 = 1$ … (ii)

$\left(\frac{12-3y}{4} - 3\right)^2 + y^2 = 1$

$\left(\frac{-3y}{4}\right)^2 + y^2 = 1$

$\frac{25y^2}{16} = 1 \Rightarrow y = \pm \frac{4}{5}$

for minimum distance $y = \frac{4}{5}$

$\therefore x = \frac{12}{5}$

$\therefore 25(\alpha + \beta) = 25\left(\frac{4}{5} + \frac{12}{5}\right)$

$= 16 \times 5 = 80$

2. Let $S = \left\{\begin{pmatrix} -1 & a \\ 0 & b \end{pmatrix}; a, b \in \{1, 2, 3, ...100\}\right\}$ and let $T_n = \{A \in S : A^{n(n+1)} = I\}$. Then the number of elements in $\bigcap_{n=1}^{100} T_n$ is _____.

Official Ans. by NTA (100)

Sol. $A = \begin{bmatrix} -1 & a \\ 0 & b \end{bmatrix}$

$A^2 = \begin{bmatrix} -1 & a \\ 0 & b \end{bmatrix}\begin{bmatrix} -1 & a \\ 0 & b \end{bmatrix}$

$= \begin{bmatrix} 1 & -a + ab \\ 0 & b^2 \end{bmatrix}$

$\therefore T_n = \{A \in S; A^{n(n+1)} = I\}$

$\therefore$ b must be equal to 1

$\therefore$ In this case A^2 will become identity matrix and a can take any value from 1 to 100

$\therefore$ Total number of common element will be 100.

3. The number of 7-digit numbers which are multiples of 11 and are formed using all the digits 1, 2, 3, 4, 5, 7 and 9 is _____.

Official Ans. by NTA (576)

Sol. Digits are 1, 2, 3, 4, 5, 7, 9

Multiple of 11 $\rightarrow$ Difference of sum at even & odd place is divisible by 11.

Let number of the form abcdefg

$\therefore (a + c + e + g) - (b + d + f) = 11x$

$a + b + c + d + e + f = 31$

$\therefore$ either $a + c + e + g = 21$ or 10

$\therefore b + d + f = 10$ or 21

Case- 1

$a + c + e + g = 21$

$b + d + f = 10$

$(b, d, f) \in \{(1, 2, 7) (2, 3, 5) (1, 4, 5)\}$

$(a, c, e, g) \in \{(1, 4, 7, 9), (3, 4, 5, 9), (2, 3, 7, 9)\}$

$\therefore$ Total number in case-1 $= (3! \times 3)(4!) = 432$

Case- 2

$a + c + e + g = 10$

$b + d + f = 21$

$(a, b, e, g) \in \{1, 2, 3, 4)\}$

(b, d, f) & $\{(5, 7, 9)\}$

$\therefore$ Total number in case 2 $= 3! \times 4! = 144$

$\therefore$ Total numbers $= 144 + 432 = 576$

4. The sum of all the elements of the set $\{\alpha \in \{1, 2, ..., 100\} : HCF(\alpha, 24) = 1\}$ is _____.

Official Ans. by NTA (1633)

Sol. HCF $(\alpha, 24) = 1$

Now, $24 = 2^3.3$

$\rightarrow \alpha$ is not the multiple of 2 or 3

Sum of values of α

$= S(U) - \{S(\text{multiple of } 2) + S(\text{multiple of } 3) - S(\text{multiple of } 6)\}$

$= (1 + 2 + 3 + ... 100) - (2 + 4 + 6 ..... + 100) - (3 + 6 + ... 99) + (6 + 12 + ... + 96)$

$= \frac{100 \times 101}{2} - 50 \times 51 - \frac{33}{2} \times (3 + 99) + \frac{16}{2}(6 + 96)$

$= 5050 - 2550 - 1683 + 816 = 1633$ **Ans.**

5. The remainder on dividing $1 + 3 + 3^2 + 3^3 + ... + 3^{2021}$ by 50 is _____.

Official Ans. by NTA (4)

Sol. $\frac{1.(3^{2022}-1)}{2}=\frac{9^{1011}-1}{2}$

$=\frac{(10-1)^{1011}-1}{2}$

$=\frac{100\lambda+10110-1-1}{2}$

$=50\lambda+\frac{10108}{2}$

$= 50\lambda + 5054$

$= 50\lambda + 50 \times 101 + 4$

Rem (50) = 4.

6. The area (in sq. units) of the region enclosed between the parabola $y^2 = 2x$ and the line $x + y = 4$ is _____.

Official Ans. by NTA (18)

Sol. $x = 4 - y$

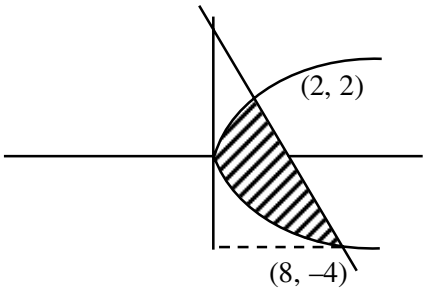
$y^2 = 2(4 - y)$

$y^2 = 8 - 2y$

$y^2 + 2y - 8 = 0$

$y = -4, y = 2$

$x = 8, x = 2$



$\int_{-4}^{2}\left[(4-y)-\frac{y^2}{2}\right]dy$

$=\left[4y-\frac{y^2}{2}-\frac{y^3}{6}\right]_{-4}^{2}$

$=8-2-\frac{8}{6}+16+\frac{16}{2}-\frac{64}{6}$

$=22+8-\frac{72}{6}$

$= 30 - 12 = 18$

7. Let a circle C : $(x - h)^2 + (y - k)^2 = r^2$, $k > 0$, touch the x-axis at (1, 0). If the line $x + y = 0$ intersects the circle C at P and Q such that the length of the chord PQ is 2, then the value of $h + k + r$ is equal to _____.

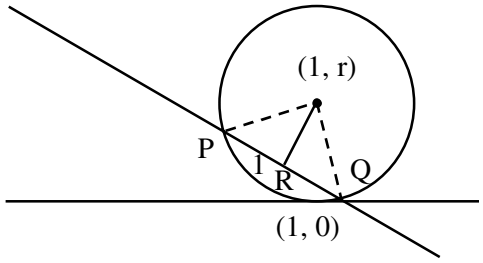
Official Ans. by NTA (7)

Sol. $k = r$

$h = 1$

$OP = r, PR = 1$

$OR = \left|\frac{r+1}{\sqrt{2}}\right|$



$r^2=1+\frac{(r+1)^2}{2}$

$2r^2 = 2 + r^2 + 1 + 2r$

$r^2 - 2r - 3 = 0$

$(r - 3)(r + 1) = 0$

$\boxed{r = 3}, -1$

$h + k + r = 1 + 3 + 3$

$= 7$

8. In an examination, there are 10 true-false type questions. Out of 10, a student can guess the answer of 4 questions correctly with probability $\frac{3}{4}$ and the remaining 6 questions correctly with probability $\frac{1}{4}$. If the probability that the student guesses the answers of exactly 8 questions correctly out of 10 is $\frac{27k}{4^{10}}$, then k is equal to _____.

Official Ans. by NTA (479)

Sol. $A = \{1, 2, 3, 4\} : P(A) = \frac{3}{4} \rightarrow$ Correct

$B = \{5, 6, 7, 8, 9, 10\} ; P(B) = \frac{1}{4}$ Correct

8 Correct Ans.:

(4, 4): ${}^4C_4\left(\frac{3}{4}\right)^4 .{}^6C_4.\left(\frac{1}{4}\right)^4.\left(\frac{3}{4}\right)^2$

(3, 5): ${}^4C_3\left(\frac{3}{4}\right)^3.\left(\frac{1}{4}\right)^1.{}^6C_5\left(\frac{1}{4}\right)^5.\left(\frac{3}{4}\right)$

(2, 6): ${}^4C_2\left(\frac{3}{4}\right)^2\left(\frac{1}{4}\right)^2 . {}^6C_6\left(\frac{1}{4}\right)^6$

Total = $\frac{1}{4^{10}}[3^4\times15\times3^2+4\times3^3\times6\times3+6\times3^2]$

$=\frac{27}{4^{10}}[2.7\times15+72+2]$

$\Rightarrow K = 479$

9. Let the hyperbola H : $\frac{x^2}{a^2}-y^2=1$ and the ellipse E : $3x^2 + 4y^2 = 12$ be such that the length of latus rectum of H is equal to the length of latus rectum of E. If e_H and e_E are the eccentricities of H and E respectively, then the value of $12\left(e_H^2+e_E^2\right)$ is equal to _____.

Official Ans. by NTA (42)

Sol. $\frac{x^2}{a^2}-\frac{y^2}{1}=1$ $\qquad$ $\frac{x^2}{4}+\frac{y^2}{3}=1$

$e_H=\sqrt{1+\frac{1}{a^2}}$ $\qquad$ $e_E=\sqrt{1-\frac{3}{4}}=\frac{1}{2}$

$\ell.R.=\frac{2}{a}$ $\qquad$ $\ell R=\frac{2\times3}{2}=3$

$\frac{2}{a}=3$

$\boxed{a=\frac{2}{3}}$

$e_H=\sqrt{1+\frac{9}{4}}=\frac{\sqrt{13}}{2}$

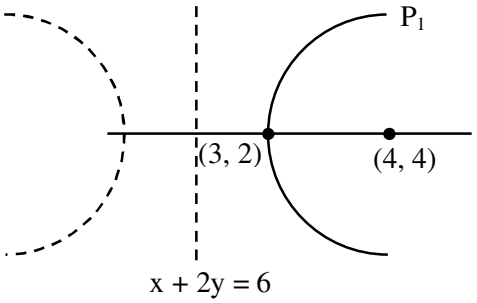
$12\left(e_H^2+e_E^2\right)=12\left(\frac{13}{4}+\frac{1}{4}\right)$

$=\frac{12\times14}{4}=42$

10. Let P_1 be a parabola with vertex (3, 2) and focus (4, 4) and P_2 be its mirror image with respect to the line $x + 2y = 6$. Then the directrix of P_2 is $x + 2y =$ _____.

Official Ans. by NTA (10)

Sol.



P_1: Directorix :

$x + 2y = k$

$x + 2y - k = 0$

$\left|\frac{3+4-K}{\sqrt{5}}\right|=\sqrt{5}$

$|7-k|=5$

$7 - K = 5$ $\qquad$ $7 - K = -5$

$\boxed{k=2}$ $\qquad$ $\boxed{k=12}$

Accepted $\qquad$ Rejected

Passes through focus

$D_1 = x + 2y = 2$

$\ell = x + 2y = 6$ $\Rightarrow$ d

$D_2 = x + 2y = C$ $\Rightarrow$ d $\Rightarrow$ $\boxed{c=10}$

FINAL JEE–MAIN EXAMINATION – JUNE, 2022

(Held On Friday 24th June, 2022) **TIME : 9 : 00 AM to 12 : 00 PM**

PHYSICS

TEST PAPER WITH SOLUTION

SECTION-A

1. The bulk modulus of a liquid is 3×10^{10} Nm^{-2}. The pressure required to reduce the volume of liquid by 2% is :

(A) 3×10^{8} Nm^{-2} (B) 9×10^{8} Nm^{-2}

(C) 6×10^{8} Nm^{-2} (D) 12×10^{8} Nm^{-2}

Official Ans. by NTA (C)

Sol. $B = 3 \times 10^{10}$

$$-\frac{\Delta V}{V} = 0.02$$

$$B = \frac{\Delta P}{-\frac{\Delta V}{v}} \Rightarrow \Delta P = -B\left(\frac{\Delta V}{V}\right)$$

$$= (3\times10^{10})(0.02)$$

$$= 6\times10^{8}\,N/m^{2}$$

2. Given below are two statements : One is labelled as Assertion (A) and the other is labelled as Reason (R).

Assertion (A) : In an uniform magnetic field, speed and energy remains the same for a moving charged particle.

Reason (R) : Moving charged particle experiences magnetic force perpendicular to its direction of motion.

(A) Both (A) and (R) are true and (R) is the correct explanation of (A)

(B) Both (A) and (R) are true but (R) is NOT the correct explanation of (A)

(C) (A) is true but (R) is false

(D) (A) is false but (R) is true.

Official Ans. by NTA (A)

Sol. $\vec{F} = q(\vec{v}\times\vec{B})$

$\boxed{\vec{F} \perp \vec{v}}$

Work done $= \vec{F}\cdot\vec{S}$

Work done = 0

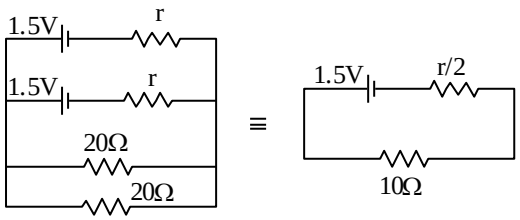
3. Two identical cells each of emf 1.5 V are connected in parallel across a parallel combination of two resistors each of resistance 20Ω. A voltmeter connected in the circuit measures 1.2 V. The internal resistance of each cell is

(A) 2.5Ω (B) 4Ω

(C) 5Ω (D) 10Ω

Official Ans. by NTA (C)

Sol.



$V = E - ir/2$

$$1.2 = 1.5 - i\left(\frac{r}{2}\right)$$

$$i\frac{r}{2} = 0.3$$

$$i = \frac{1.5}{10+\frac{r}{2}} \Rightarrow 10i + \frac{ir}{2} = 1.5$$

$10i = 1.5 - 0.3$

$i = 0.12$ A

$$\Rightarrow \quad r = \frac{0.6}{0.12} = 5\Omega$$

4. Identify the pair of physical quantities which have different dimensions :

(A) Wave number and Rydberg's constant

(B) Stress and Coefficient of elasticity

(C) Coercivity and Magnetisation

(D) Specific heat capacity and Latent heat

Official Ans. by NTA (D)

Sol. $S = \frac{Q}{m\Delta T} = \frac{J}{Kg°C}$

$L = \frac{Q}{m} = \frac{J}{Kg}$

5. A projectile is projected with velocity of 25 m/s at an angle θ with the horizontal. After t seconds its inclination with horizontal becomes zero. If R represents horizontal range of the projectile, the value of θ will be : [use g = 10 m/s^2]

(A) $\frac{1}{2}\sin^{-1}\left(\frac{5t^2}{4R}\right)$ (B) $\frac{1}{2}\sin^{-1}\left(\frac{4R}{5t^2}\right)$

(C) $\tan^{-1}\left(\frac{4t^2}{5R}\right)$ (D) $\cot^{-1}\left(\frac{R}{20t^2}\right)$

Official Ans. by NTA (D)

Sol. $R = \frac{V^2(2\sin\theta\cos\theta)}{g}$

$t = \frac{V\sin\theta}{g} \Rightarrow V = \frac{gt}{\sin\theta}$

$\Rightarrow R = \frac{g^2t^2}{\sin^2\theta}.\frac{2\sin\theta\cos\theta}{g}$

$\tan\theta = \frac{2gt^2}{R} = \frac{20t^2}{R}$

$\cot\theta = \frac{R}{20t^2}$

6. A block of mass 10 kg starts sliding on a surface with an initial velocity of 9.8 ms^{-1}. The coefficient of friction between the surface and bock is 0.5. The distance covered by the block before coming to rest is : [use g = 9.8 ms^{-2}]

(A) 4.9 m (B) 9.8 m

(C) 12.5 m (D) 19.6 m

Official Ans. by NTA (B)

Sol. $a = -\mu g = -0.5\times 9.8 = -4.9 m/s^2$

$d = \frac{v^2}{2a} = \frac{9.8\times 9.8}{2(4.9)}$

$= 9.8$ m

7. A boy ties a stone of mass 100 g to the end of a 2 m long string and whirls it around in a horizontal plane. The string can withstand the maximum tension of 80 N. If the maximum speed with which the stone can revolve is $\frac{K}{\pi}$ rev. / min . The value of K is : (Assume the string is massless and unstretchable)

(A) 400 (B) 300

(C) 600 (D) 800

Official Ans. by NTA (C)

Sol. $T = M\omega^2R$

$T = 80$ N $M = 0.1$ $\omega = ?$ $R = 2$m

$80 = 0.1\ \omega^2(2)$

$\omega^2 = 400$

$\omega = 20$

$2\pi f = 20$

$f = \frac{10}{\pi}\frac{rev}{s}$

$= \frac{600}{\pi}\frac{rev}{min}$

8. A vertical electric field of magnitude 4.9×10^5 N/C just prevents a water droplet of a mass 0.1 g from falling. The value of charge on the droplet will be : (Given g = 9.8 m/s^2)

(A) 1.6×10^{-9} C (B) 2.0×10^{-9} C

(C) 3.2×10^{-9} C (D) 0.5×10^{-9} C

Official Ans. by NTA (B)

Sol. $Mg = qE$

$(0.1\times10^{-3})(9.8) = 4.9\times10^5 q$

$\frac{2\times10^{-4}}{10^5} = q$

$\boxed{q = 2\times10^{-9}\,C}$

9. A particle experiences a variable force $\vec{F} = (4x\hat{i} + 3y^2\hat{j})$ in a horizontal x-y plane. Assume distance in meters and force is newton. If the particle moves from point (1, 2) to point (2, 3) in the x-y plane, the Kinetic Energy changes by

(A) 50.0 J (B) 12.5 J

(C) 25.0 J (D) 0 J

Official Ans. by NTA (C)

Sol. $F = 4x\hat{i} + 3y^2\hat{j}$

$WD = \Delta KE$

$W = \int \vec{F}.(dx\hat{i} + dy\hat{j})$

$= \int_1^2 4x\,dx + \int_2^3 3y^2\,dx$

$= (2x^2)_1^2 + (y^3)_2^3$

$= (8-2) + (27-8)$

$= 6 + 19 = 25J$

10. The approximate height from the surface of earth at which the weight of the body becomes $\frac{1}{3}$ of its weight on the surface of earth is : [Radius of earth R = 6400 km and $\sqrt{3} = 1.732$]

(A) 3840 km (B) 4685 km

(C) 2133 km (D) 4267 km

Official Ans. by NTA (B)

Sol. $Mg' = \frac{M}{3}g$

$g' = \frac{g}{3}$

$g' = g\left(\frac{R}{R+h}\right)^2 = \frac{g}{3}$

$\frac{R}{R+h} = \frac{1}{\sqrt{3}}.$

$h = (\sqrt{3}-1)R$

$= (1.732-1)6400$

$\boxed{h = 4685 km}$

11. A resistance of 40 Ω is connected to a source of alternating current rated 220 V, 50 Hz. Find the time taken by the current to change from its maximum value to rms value :

(A) 2.5 ms (B) 1.25 ms

(C) 2.5 s (D) 0.25 s

Official Ans. by NTA (A)

Sol. Considering sinusoidal AC.

Phase at maximum value $= \frac{\pi}{2}$

Phase at rms value $= \frac{3\pi}{4}$

Thus phase change $= \frac{3\pi}{4} - \frac{\pi}{2} = \frac{\pi}{4}$

Now $\omega = 2\pi f$

$= 2\pi \times 50$

$= 100\,\pi$

time taken $t = \frac{\theta}{\omega} = \frac{\pi/4}{100\pi} = \frac{1}{400}$ s

$t = 2.5 \times 10^{-3} = 2.5$ ms

12. The equations of two waves are given by :

$y_1 = 5\sin 2\pi(x - vt)$cm

$y_2 = 3\sin 2\pi(x - vt + 1.5)$cm

These waves are simultaneously passing through a string. The amplitude of the resulting wave is

(A) 2 cm (B) 4 cm

(C) 5.8 cm (D) 8 cm

Official Ans. by NTA (A)

Sol. $A_1 = 5 \quad A_2 = 3$

$\Delta\theta = 2\pi(1.5) = 3\pi$

$A_{net} = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos(3\pi)}$

$= |A_1 - A_2|$

= 2cm

13. A plane electromagnetic wave travels in a medium of relative permeability 1.61 and relative permittivity 6.44. If magnitude of magnetic intensity is 4.5×10^{-2} Am^{-1} at a point, what will be the approximate magnitude of electric field intensity at that point ?

(Given : permeability of free space $\mu_0 = 4\pi \times 10^{-7}$ NA^{-2}, speed of light in vacuum $c = 3 \times 10^8$ ms^{-1})

(A) 16.96 Vm^{-1} (B) 2.25×10^{-2} Vm^{-1}

(C) 8.48 Vm^{-1} (D) 6.75×10^6 Vm^{-1}

Official Ans. by NTA (C)

Sol. $\mu_r = 1.61 \quad \in_r = 6.44$

$B = 4.5\times10^{-2}$

E = ?

$C = \frac{1}{\sqrt{\mu_0 \in_0}} \quad V = \frac{1}{\sqrt{\mu \in}}$

$\frac{C}{V} = \sqrt{\mu_r \in_r} = \sqrt{1.61\times6.44}$

$\frac{E}{B} = V = \frac{3\times10^8}{\sqrt{1.61\times6.44}} = 9.32\times10^7 \text{ m/s}$

$E = 4.5\times10^{-2}\times9.32\times10^7$

$= 4.2\times10^6$

14. Choose the correct option from the following options given below :

(A) In the ground state of Rutherford's model electrons are in stable equilibrium. While in Thomson's model electrons always experience a net-force.

(B) An atom has a nearly continuous mass distribution in a Rutherford's model but has a highly non-uniform mass distribution in Thomson's model

(C) A classical atom based on Rutherford's model is doomed to collapse.

(D) The positively charged part of the atom possesses most of the mass in Rutherford's model but not in Thomson's model.

Official Ans. by NTA (C)

Sol. According to Rutherford, e^- revolves around nucleus in circular orbit. Thus e^- is always accelerating (centripetal acceleration). An accelerating change emits EM radiation and thus e^- should loose energy and finally should collapse in the nucleus.

15. Nucleus A is having mass number 220 and its binding energy per nucleon is 5.6 MeV. It splits in two fragments 'B' and 'C' of mass numbers 105 and 115. The binding energy of nucleons in 'B' and 'C' is 6.4 MeV per nucleon. The energy Q released per fission will be :

(A) 0.8 MeV (B) 275 MeV

(C) 220 MeV (D) 176 MeV

Official Ans. by NTA (D)

Sol. $Q = (B.E)_P - (B.E)_R$

$= (105+115)(6.4) - (220)(5.6)$

= 176 MeV

16. A baseband signal of 3.5 MHz frequency is modulated with a carrier signal of 3.5 GHz frequency using amplitude modulation method. What should be the minimum size of antenna required to transmit the modulated signal ?

(A) 42.8 m (B) 42.8 mm

(C) 21.4 mm (D) 21.4 m

Official Ans. by NTA (C)

Sol. $f_c = 3.5GHz \quad f_m = 3.5MHz$

Side band frequencies are $f_c - f_m$ & $f_c + f_m$. which are almost f_c

$$\lambda = \frac{c}{f_c}$$

Minimum length of antenna =

$$\frac{c}{f_c 4} = \frac{\lambda}{4} = \frac{3\times10^8}{3.5\times10^9\times4}$$

= 21.4 mm

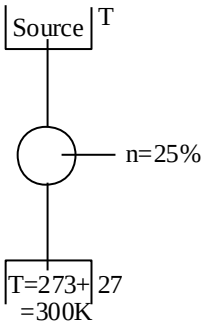
17. A Carnot engine whose heat sinks at 27°C, has an efficiency of 25%. By how many degrees should the temperature of the source be changed to increase the efficiency by 100% of the original efficiency ?

(A) Increases by 18°C (B) Increase by 200°C

(C) Increase by 120°C (D) Increase by 73°

Official Ans. by NTA (B)

Sol.



$$1-\frac{300}{T}=0.25$$

$$\frac{300}{T}=0.75$$

T = 400K

If efficiency increased by 100% then new efficiency $\Rightarrow$ n' = 50%

$$1-\frac{300}{T'}=0.5$$

$$T'=600K$$

Increase in temp = 600 – 400

= 200 K or 200°C

18. A parallel plate capacitor is formed by two plates each of area $30\pi\ cm^2$ separated by 1 mm. A material of dielectric strength $3.6 \times 10^7\ Vm^{-1}$ is filled between the plates. If the maximum charge that can be stored on the capacitor without causing any dielectric breakdown is 7×10^{-6} C, the value of dielectric constant of the material is :

$$\left\{\text{Use}: \frac{1}{4\pi\varepsilon_0}=9\times10^9\,Nm^2C^{-2}\right\}$$

(A) 1.66 (B) 1.75

(C) 2.25 (D) 2.33

Official Ans. by NTA (D)

Sol. $$K=\frac{q}{A\in_0 E}=\frac{7\times10^{-6}}{30\pi\times10^{-4}\times\frac{1}{4\pi\times9\times10^9}\times3.6\times10^7}$$

$$K=\frac{36\times7}{30\times3.6}=2.33$$

19. The magnetic field at the centre of a circular coil of radius r, due to current I flowing through it, is B. The magnetic field at a point along the axis at a distance $\frac{r}{2}$ from the centre is :

(A) B/2 (B) 2B

(C) $\left(\frac{2}{\sqrt{5}}\right)^3$ B (D) $\left(\frac{2}{\sqrt{3}}\right)^3$ B

Official Ans. by NTA (C)

Sol. $$B_C=\frac{\mu_0 I}{2r}, B_a=\frac{\mu_0 I r^2}{2(x^2+r^2)^{3/2}}$$

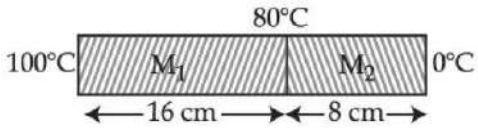
At $x=\frac{r}{2}$

$$B_a=\frac{\mu_0 I r^2}{2\left(\frac{r^2}{4}+r^2\right)^{3/2}}$$

$$=\frac{\mu_0 I r^2}{2\left(\frac{5}{4}r^2\right)^{3/2}}=\frac{\mu_0 I}{2r}\left(\frac{4}{5}\right)^{3/2}$$

$$=\frac{\mu_0 I}{2r}\left(\frac{2}{\sqrt{5}}\right)^3$$

20. Two metallic blocks M_1 and M_2 of same area of cross-section are connected to each other (as shown in figure). If the thermal conductivity of M_2 is K then the thermal conductivity of M_1 will be : [Assume steady state heat conduction]



(A) 10 K (B) 8 K

(C) 12.5 K (D) 2 K

Official Ans. by NTA (B)

Sol. $\Delta T \ \alpha \ R \ \alpha \ \frac{\ell}{k}$,

$$\frac{\Delta T_1}{\Delta T_2} = \frac{\ell_1}{k_1} \times \frac{k_2}{\ell_2} = \frac{16}{k_1} \times \frac{k}{8}$$

$$\frac{20}{80} = \frac{16}{k_1} \times \frac{k}{8} \rightarrow k_1 = 8k$$

SECTION-B

1. 0.056 kg of Nitrogen is enclosed in a vessel at a temperature of 127°C. The amount of heat required to double the speed of its molecules is _____ k cal. (Take R = 2 cal $mole^{-1}K^{-1}$)

Official Ans. by NTA (12)

Sol. 0.056 kg N_2 = 56 gm of N_2 = 2 mole of N_2

$T_1 = 400$ K, $v \alpha \sqrt{T}$ so $T_2 = 4T_1 = 1600K$

$$Q = \frac{f}{2} nR\Delta T$$

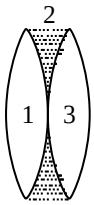
$f = 5$

Q = 12 k cal

2. Two identical thin biconvex lenses of focal length 15 cm and refractive index 1.5 are in contact with each other. The space between the lenses is filled with a liquid of refractive index 1.25. The focal length of the combination is _____ cm.

Official Ans. by NTA (10)

Sol.



$$\frac{1}{f_1} = \frac{1}{15} = \left(\frac{3}{2} - 1\right)\left[\frac{2}{R}\right]$$

$$\frac{1}{R} = \frac{1}{15}$$

$$\frac{1}{f_{eq}} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3}$$

$$= \frac{1}{15} + \left(\frac{5}{4} - 1\right)\left[\frac{-2}{R}\right] + \frac{1}{15}$$

$$= \frac{1}{15} - \frac{1}{30} + \frac{1}{15}$$

$$= \frac{2 - 1 + 2}{30}$$

$$= \frac{3}{30} = \frac{1}{10}$$

= 10

3. A transistor is used in common-emitter mode in an amplifier circuit. When a signal of 10 mV is added to the base-emitter voltage, the base current changes by 10 μA and the collector current changes by 1.5 mA. The load resistance is 5 kΩ. The voltage gain of the transistor will be _____ .

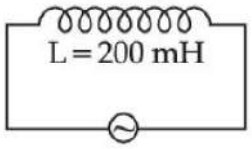
Official Ans. by NTA (750)

Sol. $r_i = \frac{10mV}{10\mu A} = 10^3 \Omega$

$$\beta = \frac{1.5mA}{10\mu A} = 150$$

$$A_V = \left(\frac{R_0}{r_i}\right)\beta = \left(\frac{5000}{1000}\right) \times 150 = 750$$

4. As shown in the figure an inductor of inductance 200 mH is connected to an AC source of emf 220 V and frequency 50 Hz. The instantaneous voltage of the source is 0 V when the peak value of current is $\frac{\sqrt{a}}{\pi}$A. The value of a is _____.



Official Ans. by NTA (242)

Sol. $f = 50Hz$

$X_L = 2\pi fL$

$= 2\pi(50)(200\times10^{-3})$

$= 20\pi\Omega$

$i_0 = \frac{V_0}{X_L} \Rightarrow \frac{V_{rms}\sqrt{2}}{X_L}$

$= \frac{(220)\sqrt{2}}{20\pi} = \frac{11\sqrt{2}}{\pi}$

$\boxed{i_o = \frac{\sqrt{242}}{\pi}}$

5. Sodium light of wavelengths 650 nm and 655 nm is used to study diffraction at a single slit of aperture 0.5 mm. The distance between the slit and the screen is 2.0 m. The separation between the positions of the first maxima of diffraction pattern obtained in the two cases is _____ $\times 10^{-5}$ m.

Official Ans. by NTA (3)

Sol. $a\sin\theta = \frac{3\lambda}{2}$

$\frac{y}{L} = \theta = \frac{3\lambda}{2a}$ L = 2m

$y_1 = \frac{3\lambda_1 L}{2a}$ λ_2 = 655 nm

$y_2 = \frac{3\lambda_2 L}{2a}$ λ_1 = 650 nm

a = 0.5 nm

$\Delta y = y_2 - y_1 = \frac{3(\lambda_2 - \lambda_1)}{2a}L$

$= \frac{3(655-650)}{2\times0.5\times10^{-3}}\times2\times10^{-9}$

$= \frac{3\times5\times2}{1\times10^{-3}}\times10^{-9}$

$= 3\times10^{-5}$

6. When light of frequency twice the threshold frequency is incident on the metal plate, the maximum velocity of emitted election is v_1. When the frequency of incident radiation is increased to five times the threshold value, the maximum velocity of emitted electron becomes v_2. If $v_2 = x\ v_1$, the value of x will be _____.

Official Ans. by NTA (2)

Sol. $hv = hv_{th} + \frac{1}{2}mv^2$

$v = 2v_{th}$

$2hv_{th} = hv_{th} + \frac{1}{2}mv_1^2$ ..... (1)

$v = 5v_{th}$

$5hv_{th} = hv_{th} + \frac{1}{2}mv_2^2$ ....(2)

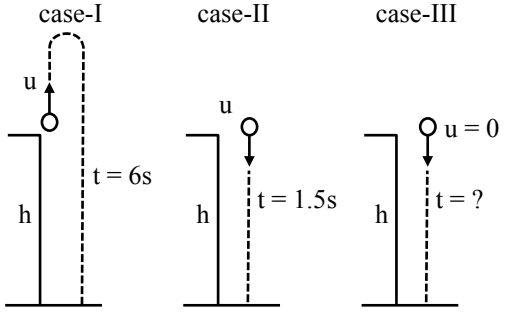
$\frac{\frac{1}{2}mv_1^2}{\frac{1}{2}mv_2^2} = \frac{hv_{th}}{4hv_{th}}$

$\left(\frac{v_1}{v_2}\right)^2 = \frac{1}{4} \Rightarrow \boxed{v_2 = 2v_1}$

7. From the top of a tower, a ball is thrown vertically upward which reaches the ground in 6 s. A second ball thrown vertically downward from the same position with the same speed reaches the ground in 1.5 s. A third ball released, from the rest from the same location, will reach the ground in _____ s.

Official Ans. by NTA (3)

Sol. Let height of tower be h and speed of projection in first two cases be u.



For case-I : 2nd equation $s = ut + \frac{1}{2}at^2$

$h = -u(6) + \frac{1}{2}g(6)^2$

H = – 6u + 18 g …. (i)

For case-II : $h = u(1.5) + \frac{1}{2}g(1.5)^2$

$h = 1.5u + \frac{2.25g}{2}$ … (ii)

Multiplying equation (ii) by 4 we get

4h = 6u + 4.5 g …. (iii)

equation (i) + equation (iii) we get 5h = 22.5g

h = 4.5g … (iv)

For case-III :

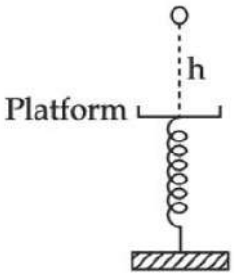
$h = 0 + \frac{1}{2}gt^2$ … (v)

Using equation (4) & equation (5)

$4.5g = \frac{1}{2}gt^2$

$t^2 = 9 \Rightarrow t = 3s$

8. A ball of mass 100 g is dropped from a height h = 10 cm on a platform fixed at the top of vertical spring (as shown in figure). The ball stays on the platform and the platform is depressed by a distance $\frac{h}{2}$. The spring constant is _____ Nm^{-1}. (Use g = 10 ms^{-2})



Official Ans. by NTA (120)

Sol. By energy conservation

PE = KE

$mg\left(H + \frac{H}{2}\right) = \frac{1}{2}kx^2 \quad \left(x = \frac{H}{2}\right)$

$0.100 \times 10 \times \frac{3}{2}(0.10) = \frac{1}{2}k(0.05 \times 0.05)$

$k = \frac{3 \times 0.10}{0.05 \times 0.05}$

$= \frac{3 \times 1000}{25} = 120 \text{ N/m}$

9. In a potentiometer arrangement, a cell gives a balancing point at 75 cm length of wire. This cell is now replaced by another cell of unknown emf. If the ratio of the emf's of two cells respectively is 3 : 2, the difference in the balancing length of the potentiometer wire in above two cases will be _____ cm.

Official Ans. by NTA (25)

Sol. $\frac{\varepsilon_1}{\varepsilon_2} = \frac{\ell_1}{\ell_2}$

$\frac{3}{2} = \frac{75\text{cm}}{\ell_2}$

$\ell_2 = 50\text{cm}$

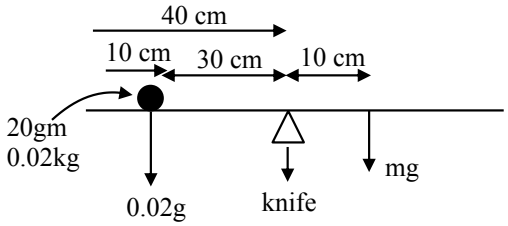
$\ell_1 - \ell_2 = 75 - 50$

= 25 cm

10. A metre scale is balanced on a knife edge at its centre. When two coins, each of mass 10 g are put one on the top of the other at the 10.0 cm mark the scale is found to be balanced at 40.0 cm mark. The mass of the metre scale is found to be x × 10^{-2} kg. The value of x is

Official Ans. by NTA (6)

Sol. Let mass of meter scale be m.



Balancing torque about knife edge

$(0.02g) \times (30 \times 10^{-2}) = mg \times (10 \times 10^{-2})$

$m = 0.06 \text{ kg} = 6 \times 10^{-2} \text{ kg}$

FINAL JEE–MAIN EXAMINATION – JUNE, 2022

(Held On Friday 24th June, 2022) **TIME : 9 : 00 AM to 12 : 00 PM**

CHEMISTRY — TEST PAPER WITH SOLUTION

SECTION-A

1. If a rocket runs on a fuel ($C_{15}H_{30}$) and liquid oxygen, the weight of oxygen required and CO_2 released for every litre of fuel respectively are: (Given: density of the fuel is 0.756 g/mL)

(A) 1188 g and 1296 g (B) 2376 g and 2592 g

(C) 2592g and 2376 g (D) 3429 g and 3142 g

Official Ans. by NTA (C)

Sol. $C_{15}H_{30} + \frac{45}{2}O_2 \rightarrow 15CO_2 + 15H_2O$

Mass of fuel = 0.756×1000 g

No. of moles of fuel = $\frac{0.756 \times 1000}{210}$

Wt. of oxygen = $\frac{0.756 \times 1000}{210} \times \frac{45}{2} \times 32 = 2592$g

Wt of CO_2 = $\frac{0.756 \times 1000}{210} \times 15 \times 44 = 2376$ g

2. Consider the following pairs of electrons

(A) (a) $n = 3, l = 1, m_l = 1, m_s = +\frac{1}{2}$

(b) $n = 3, l = 2, m_l = 1, m_s = +\frac{1}{2}$

(B) (a) $n = 3, l = 2, m_l = -2, m_s = -\frac{1}{2}$

(b) $n = 3, l = 2, m_l = -1, m_s = -\frac{1}{2}$

(C) (a) $n = 4, l = 2, m_l = 2, m_s = +\frac{1}{2}$

(b) $n = 3, l = 2, m_l = 2, m_s = +\frac{1}{2}$

The pairs of electron present in degenerate orbitals is/are:

(A) Only A

(B) Only B

(C) Only C

(D) (B) and (C)

Official Ans. by NTA (B)

Sol. Based on "n + l" rule only (B) has pair of electron in degenerate orbitals

3. Match List – I with List - II

List – I		List – II	
(A)	$[PtCl_4]^{2-}$	**(I)**	sp^3d
(B)	BrF_5	**(II)**	d^2sp^3
(C)	PCl_5	**(III)**	dsp^2
(D)	$[Co(NH_3)_6]^{3+}$	**(IV)**	sp^3d^2

(A) (A)→(II), (B)→(IV), (C)→(I), (D)→(III)

(B) (A)→(III), (B)→(IV), (C)→(I), (D)→(II)

(C) (A)→(III), (B)→(I), (C)→(IV), (D)→(II)

(D) (A)→(II), (B)→(I), (C)→(IV), (D)→(III)

Official Ans. by NTA (B)

Sol. **Answer (B)**

List – I		List – II	
(A)	$[PtCl_4]^{2-}$	(III)	dsp^2
(B)	BrF_5	(IV)	sp^3d^2
(C)	PCl_5	(I)	sp^3d
(D)	$[Co(NH_3)_6]^{3+}$	(II)	d^2sp^3

4. For a reaction at equilibrium

$$A(g) \rightleftharpoons B(g) + \frac{1}{2}C(g)$$

the relation between dissociation constant (K), degree of dissociation (α) and equilibrium pressure (p) is given by :

(A) $K = \dfrac{\alpha^{\frac{1}{2}} p^{\frac{3}{2}}}{\left(1+\frac{3}{2}\alpha\right)^{\frac{1}{2}}(1-\alpha)}$

(B) $K = \dfrac{\alpha^{\frac{3}{2}} p^{\frac{1}{2}}}{(2+\alpha)^{\frac{1}{2}}(1-\alpha)}$

(C) $K = \dfrac{(\alpha p)^{\frac{3}{2}}}{\left(1+\frac{3}{2}\alpha\right)^{\frac{1}{2}}(1-\alpha)}$

(D) $K = \dfrac{(\alpha p)^{\frac{3}{2}}}{(1+\alpha)(1-\alpha)^{\frac{1}{2}}}$

Official Ans. by NTA (B)

Sol.

	A(g)	$\rightleftharpoons$	B(g) + $\frac{1}{2}$ C (g)
Initial : P_i		**0**	**0**
At eq.:	$P_i(1-\alpha)$	$P_i.\alpha$	$P_i\frac{\alpha}{2}$

Now, equilibrium pressure (p) ,

$$P = P_i \times \left(1+\frac{\alpha}{2}\right)$$

$$\therefore\ P_A = \left(\frac{1-\alpha}{1+\frac{\alpha}{2}}\right)P$$

$$P_B = \left(\frac{\alpha}{1+\frac{\alpha}{2}}\right)P$$

$$P_C = \left(\frac{\frac{\alpha}{2}}{1+\frac{\alpha}{2}}\right)P$$

$$\therefore K = \frac{P_c^{\frac{1}{2}} \times P_B}{P_A}$$

$$K = \frac{\alpha^{\frac{3}{2}} p^{\frac{1}{2}}}{(2+\alpha)^{\frac{1}{2}}(1-\alpha)}$$

5. Given below are two statements :

Statement I : Emulsions of oil in water are unstable and sometimes they separate into two layers on standing.

Statement II :For stabilisation of an emulsion, excess of electrolyte is added.

In the light of the above statements, choose the most appropriate answer from the options given below :

(A) Both Statement I and Statement II are correct.

(B) Both Statement I and Statement II are incorrect.

(C) Statement I is correct but Statement II is incorrect.

(D) Statement I is incorrect but Statement II is correct.

Official Ans. by NTA (C)

Sol. Statement I : Fact

Statement II: The principle emulsifying agents for O/W emulsions are proteins, gums natural and synthetic soaps etc…

6. Given below are the oxides:

Na_2O, As_2O_3, N_2O, NO and Cl_2O_7

Number of amphoteric oxides is:

(A) 0 (B) 1

(C) 2 (D) 3

Official Ans. by NTA (B)

Sol. Na_2O = Basic As_2O_3 = Amphoteric

N_2O = Neutral NO = Neutral

Cl_2O_7 = Acidic

7. Match List – I with List – II

	List - I		List - II
(A)	Sphalerite	(I)	$FeCO_3$
(B)	Calamine	(II)	PbS
(C)	Galena	(III)	$ZnCO_3$
(D)	Siderite	(IV)	ZnS

Choose the most appropriate answer from the options given below:

(A) (A) - (IV), (B) – (III), (C) – (II), (D) – (I)

(B) (A) - (IV), (B) – (I), (C) – (II), (D) – (III)

(C) (A) - (II), (B) – (III), (C) – (I), (D) – (IV)

(D) (A) - (III), (B) – (IV), (C) – (II), (D) – (I)

Official Ans. by NTA (A)

Sol.

	List - I		List - II
(A)	Sphalerite	(IV)	ZnS
(B)	Calamine	(III)	$ZnCO_3$
(C)	Galena	(II)	PbS
(D)	Siderite	(I)	$FeCO_3$

8. The highest industrial consumption of molecular hydrogen is to produce compounds of element:

(A) Carbon (B) Nitrogen

(C) Oxygen (D) Chlorine

Official Ans. by NTA (B)

Sol. Nitrogen . Around 55% of hydrogen around would goes to ammonia production

9. Which of the following statements are correct ?

(A) Both LiCl and $MgCl_2$ are soluble in ethanol.

(B) The oxides Li_2O and MgO combine with excess of oxygen to give superoxide.

(C) LiF is less soluble in water than other alkali metal fluorides.

(D) Li_2O is more soluble in water than other alkali metal oxides.

Choose the most appropriate answer from the options given below:

(A) (A) and (C) only (B) (A), (C) and (D) only

(C) (B) and (C) only (D) (A) and (C) only

Official Ans. by NTA (A)

Sol. (A) Both LiCl and $MgCl_2$ are soluble in ethanol

(B) Li and Mg do not form superoxide

(C) LiF has high lattice energy

(D) Li_2O is least soluble in water than other alkali metal oxides

10. Identify the correct statement for B_2H_6 from those given below.

(A) In B_2H_6, all B-H bonds are equivalent.

(B) In B_2H_6 there are four 3-centre-2-electron bonds.

(C) B_2H_6 is a Lewis acid.

(D) B_2H_6 can be synthesized form both BF_3 and $NaBH_4$.

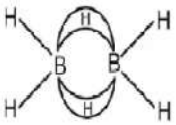
(E) B_2H_6 is a planar molecule.

Choose the most appropriate answer from the options given below :

(A) (A) and (E) only (B) (B), (C) and (E) only

(C) (C) and (D) only (D) (C) and (E) only

Official Ans. by NTA (C)



Sol. (A) (B)

Two 3 centre – 2 – electron bonds

(C) B_2H_6 is e^- deficient species

(E) B_2H_6 is non – Planar molecule

(D) $BF_3 + LiAlH_4 \rightarrow 2B_2H_6 + 3LiF + 3AlF_3$

$NaBH_4 + I_2 \rightarrow B_2H_6 + 2NaI + H_2$

11. The most stable trihalide of nitrogen is:

(A) NF_3 (B) NCl_3

(C) NBr_3 (D) NI_3

Official Ans. by NTA (A)

Sol. Order of stability: -

$NF_3 > NCl_3 > NBr_3 > NI_3$

12. Which one of the following elemental forms is not present in the enamel of the teeth?

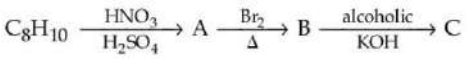
(A) Ca^{2+} (B) P^{3+}

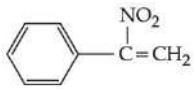
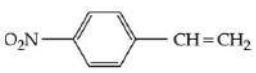
(C) F^- (D) P^{5+}

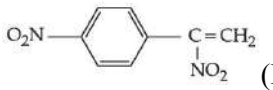
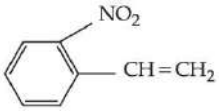
Official Ans. by NTA (B)

Sol. Calcium and phosphate are the major components of teeth enamel

13. In the given reactions sequence, the major product 'C' is :

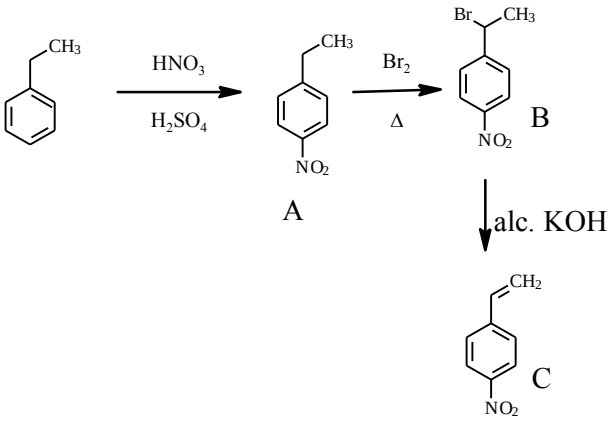


(A)  (B) 

(C)  (D) 

Official Ans. by NTA (B)

Sol. C_8H_{10} DU = 9 –5 = 4



14. Two statements are given below :

Statement I: The melting point of monocarboxylic acid with even number of carbon atoms is higher than that of with odd number of carbon atoms acid immediately below and above it in the series.

Statement II : The solubility of monocarboxylic acids in water decreases with increase in molar mass.

Choose the most appropriate option:

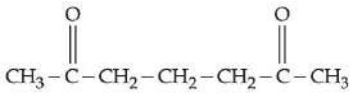
(A) Both Statement I and Statement II are correct.
(B) Both Statement I and Statement II are incorrect.
(C) Statement I is correct but Statement II is incorrect.
(D) Statement I is incorrect but Statement II is correct.

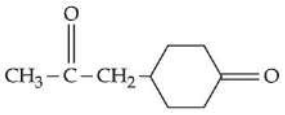
Official Ans. by NTA (A)

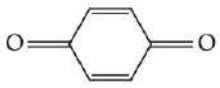
Sol. I . Better packing efficiency of monocarboxylic acids with even number of carbon atoms results in higher M.P

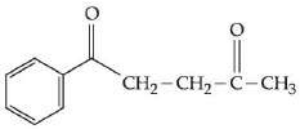
II. As molar mass increases hydrophobic part size increase hence solubility decreases.

15. Which of the following is an example of conjugated diketone?

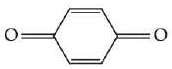
(A) 

(B) 

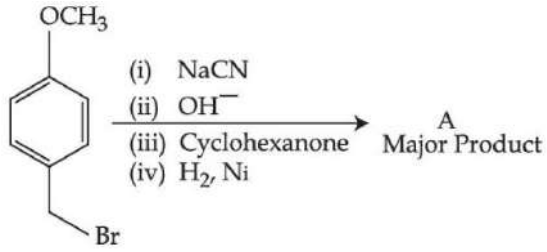
(C) 

(D) 

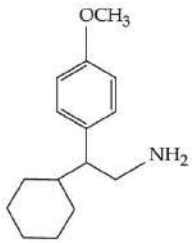
Official Ans. by NTA (C)

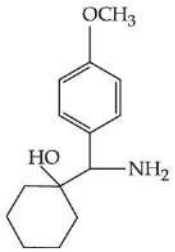
Sol.  **is a conjugated diketone**

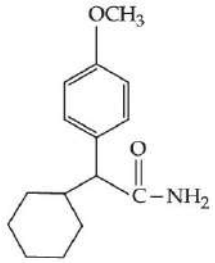
16.

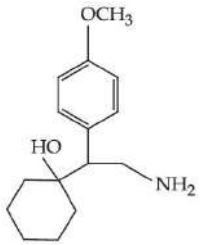


The major product of the above reaction is

(A) 

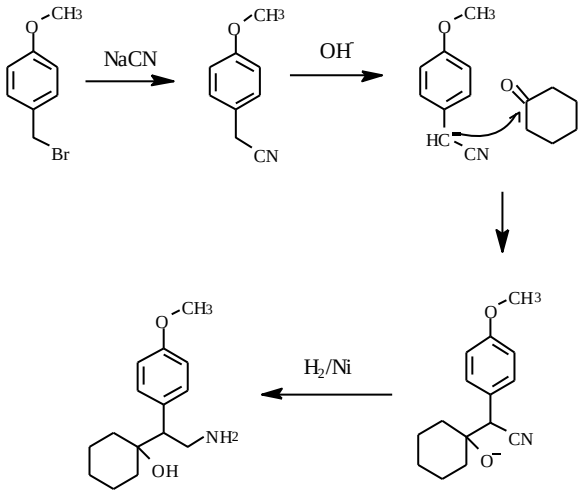
(B) 

(C) 

(D) 

Official Ans. by NTA (D)

Sol.



17. Which of the following is an example of polyester?

(A) Butadiene-styrene copolymer

(B) Melamine polymer

(C) Neoprene

(D) Poly-β-hydroxybutyrate-co-β-hydroxy valerate

Official Ans. by NTA (D)

Sol. **Factual**

18. A polysaccharide 'X' on boiling with dil H_2SO_4 at 393 K under 2-3 atm pressure yields 'Y'.

'Y' on treatment with bromine water gives gluconic acid. 'X' contains β-glycosidic linkages only. Compound 'X' is :

(A) starch (B) cellulose

(C) amylose (D) amylopectin

Official Ans. by NTA (B)

Sol. Cellulose contains β – glycosidic linkages only

19. Which of the following is not a broad spectrum antibiotic?

(A) Vancomycin (B) Ampicillin

(C) Ofloxacin (D) Penicillin G

Official Ans. by NTA (D)

Sol. Penicillin G following is a narrow spectrum antibiotic

20. During the qualitative analysis of salt with cation y^{2+} , addition of a reagent (X) to alkaline solution of the salt gives a bright red precipitate. The reagent (X) and the cation (y^{2+}) present respectively are:

(A) Dimethylglyoxime and Ni^{2+}

(B) Dimethylglyoxime and Co^{2+}

(C) Nessler's reagent and Hg^{2+}

(D) Nessler's reagent and Ni^{2+}

Official Ans. by NTA (A)

Sol. $Ni^{2+} + DMG^{-} \rightarrow [Ni\ (DMG)_2] \downarrow$

(Bright red precipitate)

SECTION-B

1. Atoms of element X form hcp lattice and those of element Y occupy $\frac{2}{3}$ of its tetrahedral voids. The percentage of element X in the lattice is_____

(Nearest integer)

Official Ans. by NTA (43)

Sol. $X \to 6 \qquad Y \to \frac{2}{3} \times 2 \times 6 = 8$

$$\% X = \frac{6}{14} \times 100 = 42.8 \simeq 43\%$$

2. $2O_3(g) \rightleftharpoons 3O_2(g)$

At 300 K, ozone is fifty percent dissociated. The standard free energy change at this temperature and 1 atm pressure is (–) __J mol $^{-1}$ (Nearest integer)

[Given: ln 1.35 = 0.3 and R = 8.3 J K^{-1} mol^{-1}]

Official Ans. by NTA (747)

Sol. $2O_3 \rightleftharpoons 3O_2(g)$

$\frac{2}{5} \qquad \frac{3}{5}$

$$k_p = \frac{P_{O_2}^3}{P_{O_3}^2}$$

$k_p = 1.35$

$\Delta G° = -RT \ln k_p$

$= -8.3 \times 300 \times \ln 1.35$

$= -747$ J/mol

3. The osmotic pressure of blood is 7.47 bar at 300 K. To inject glucose to a patient intravenously, it has to be isotonic with blood. The concentration of glucose solution in gL^{-1} is _____(Molar mass of glucose = 180 g mol^{-1}

R = 0.083 L bar K^{-1} mol^{-1}) (Nearest integer)

Official Ans. by NTA (54)

Sol. $\pi = C.R.T$

$7.47 = C \times 0.083 \times 300$

$C = 0.3$ M

$= 0.3 \times 180$ gL^{-1}

$= 54$ gL^{-1}

4. The cell potential for the following cell

$Pt\,|\,H_2(g)\,|\,H^+(aq)\,||\,Cu^{2+}(0.01M)\,|\,Cu(s)$

is 0.576 V at 298 K. The pH of the solution is ___. (Nearest integer)

Official Ans. by NTA (5)

Sol. $\text{Anode} : H_2 \to 2H^+ + 2e^-$

$\text{Cathode} : Cu^{2+} + 2e^- \to Cu$

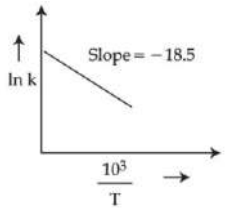
$Cu^{2+} + H_2 \to 2H^+ + Cu$

$$E_{cell} = E^0_{cell} - \frac{0.06}{2} \log \frac{[H^+]^2}{[Cu^{2+}]}$$

$$0.576 = 0.34 - \frac{0.06}{2} \log \left\{ \frac{[H^+]^2}{(0.01)} \right\}$$

$+ 3.93 \; -\log(H^+) + \log 0.1 \Rightarrow pH = 4.93 \simeq 5$

5. The rate constants for decomposition of acetaldehyde have been measured over the temperature range 700 –1000 K. The data has been analysed by plotting In k vs $\frac{10^3}{T}$ graph.The value of activation energy for the reaction is___ kJ mol^{-1}. (Nearest integer) (Given : R = 8.31 J K^{-1} mol^{-1})



Official Ans. by NTA (154)

Sol. $\ln k = \ln A - \frac{Ea}{10^3 RT} \times 10^3 = \ell nA + \frac{10^3}{T}\left[-\frac{Ea}{10^3 RT}\right]$

From the graph

$$\frac{-Ea}{10^3 \times R} = -18.5$$

Ea = 153.735 kJ/mol

~154

6. The difference in oxidation state of chromium in chromate and dichromate salts is _____

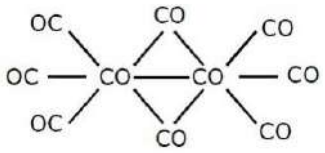
Official Ans. by NTA (0)

Sol. $\overset{+6}{Cr}O_4^{2-}, \overset{+6}{Cr}_2O_7^{2-}$ difference is zero

7. In the cobalt-carbonyl complex: $[Co_2(CO)_8]$, number of Co-Co bonds is "X" and terminal CO ligands is "Y". X + Y =____

Official Ans. by NTA (7)

Sol.



X = 1

Y = 6

8. A 0.166 g sample of an organic compound was digested with cone. H_2SO_4 and then distilled with NaOH. The ammonia gas evolved was passed through 50.0 mL of 0.5 N H_2SO_4. The used acid required 30.0 mL of 0.25 N NaOH for complete neutralization. The mass percentage of nitrogen in the organic compound is____.

Official Ans. by NTA (63)

Sol. m_{eq} of NaOH used = 30 ×0.25

m_{eq} of H_2SO_4 taken = 50 × 0.5

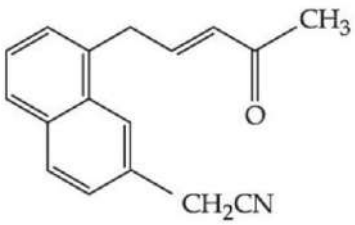
$\therefore$ m_{eq} of H_2SO_4 used

= 50 × 0.25 ×30 ×0.25 = 17.5 m mol of NH_3

$$\therefore \% N = \frac{17.5 \times 10^{-3} \times 14}{0.166} \times 100 = 147.59\%$$

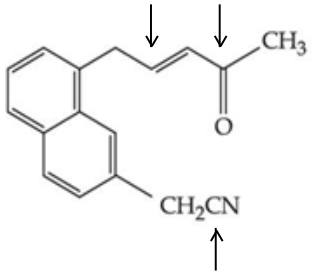
(Not possible)

9. Number of electrophilic centre in the given compound is ____



Official Ans. by NTA (3)

Sol.



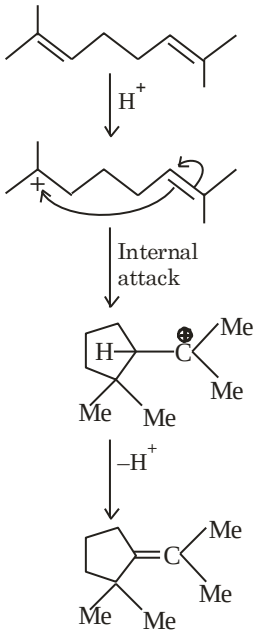
10. The major product 'A' of the following given reaction has _____ sp^2 hybridized carbon atoms.

2,7 – Dimethyl1 – 2, 6 – octadiene

$\xrightarrow{H^+}$ A (Mojor Pr oduct)

Official Ans. by NTA (2)

Sol. Answer (2)



FINAL JEE–MAIN EXAMINATION – JUNE, 2022

(Held On Friday 24th June, 2022) **TIME : 9 : 00 AM to 12 : 00 PM**

MATHEMATICS — TEST PAPER WITH SOLUTION

SECTION-A

1. Let $A=\{z\in C:1\le|z-(1+i)|\le 2\}$ and $B=\{z\in A:|z-(1-i)|=1\}$. Then, B :

(A) is an empty set

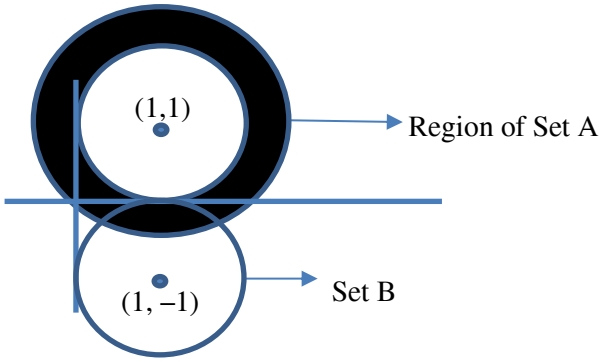
(B) contains exactly two elements

(C) contains exactly three elements

(D) is an infinite set

Official Ans. by NTA (D)

Sol. $A=\{z\in C:1\le|z-(1+i)|\le 2\}$



$B=\{z\in A:|z-(1-i)|=1\}$.

$A\cap B$ has infinite set.

2. The remainder when 3^{2022} is divided by 5 is

(A) 1 (B) 2

(C) 3 (D) 4

Official Ans. by NTA (D)

Sol. $3^{2022}=9^{1011}=(10-1)^{1011}=10\,m-1=10\,m-5+4$

$=5(2m-1)+4$ (m is integer)

Remainder = 4

3. The surface area of a balloon of spherical shape being inflated, increases at a constant rate. If initially, the radius of balloon is 3 units and after 5 seconds,, it becomes 7 units, then its radius after 9 seconds is :

(A) 9 (B) 10

(C) 11 (D) 12

Official Ans. by NTA (A)

Sol. Let r be the radius of spherical balloon

S = Surface area

$S=4\pi r^2$

$\frac{dS}{dt}=8\pi r\times\frac{dr}{dt}=k$ (constant)

$4\pi r^2=kt+C$ (C is constant of integration)

For $t=0$, $r=3\Rightarrow 36\pi=C$

For $t=5$, $r=7\Rightarrow K=32\pi$

$4\pi r^2=32\pi t+36\pi$

$r^2=8t+9$

for t = 9

$r^2=81$

r = 9

4. Bag A contains 2 white, 1 black and 3 red balls and bag B contains 3 black, 2 red and n white balls. One bag is chosen at random and 2 balls drawn from it at random, are found to be 1 red and 1 black. If the probability that both balls come from Bag A is $\frac{6}{11}$, then n is equal to _____ .

(A) 13 (B) 6

(C) 4 (D) 3

Official Ans. by NTA (C)

Sol. E_1 = denotes selection for 1st bag

E_2 = denotes selection for 2nd bag

$P(E_1)=\frac{1}{2}$, $P(E_2)=\frac{1}{2}$

A = selected balls are 1 red & 1 black

$P\left(\frac{A}{E_1}\right)=\frac{{}^3C_1\times{}^1C_1}{{}^6C_2}=\frac{1}{5}$

$$P\left(\frac{A}{E_1}\right)=\frac{{}^3C_1\times{}^2C_1}{(n+5)_{C_2}}=\frac{12}{(n+5)(n+4)}$$

$$P\left(\frac{E_1}{A}\right)=\frac{P(E_1)\times P\left(\frac{A}{E_1}\right)}{P(E_1)\times P\left(\frac{A}{E_1}\right)+P(E_2)\times P\left(\frac{A}{E_2}\right)}$$

$$=\frac{\frac{1}{10}}{\frac{1}{10}+\frac{6}{(n+5)(n+4)}}=\frac{6}{11}$$

$\Rightarrow n=4$

5. Let $x^2 + y^2 + Ax + By + C = 0$ be a circle passing through (0, 6) and touching the parabola $y = x^2$ at (2, 4). Then A + C is equal to_____ .

(A) 16 (B) 88/5

(C) 72 (D) –8

Official Ans. by NTA (A)

Sol. $x^2+y^2+Ax+By+C=0$ is passing through (0,6)

$\Rightarrow 6B + C = -36$

The tangent of the parabola $y = x^2$ at (2, 4) is

$4x - y - 4 = 0$ ----(1)

The tangent of circle $x^2+y^2+Ax+By+C=0$ at (2, 4) is

$(4 + A)x + (8 + B)y + 2A + 4B + 2C = 0$ -----(2)

From Equation (1) and (2)

$$\frac{4+A}{4}=\frac{8+B}{-1}=\frac{2A+4B+2C}{-4}$$

$A + 4B = -36$ ---(3)

$3A + 4B + 2C = -4$ ---(4)

From equation (3) and (4)
A+ C = 16

6. The number of values of α for which the system of equations :
$x + y + z = \alpha$
$\alpha x + 2\alpha y + 3z = -1$
$x + 3\alpha y + 5z = 4$
is inconsistent, is

(A) 0 (B) 1

(C) 2 (D) 3

Official Ans. by NTA (B)

Sol. $x+y+z=\alpha$

$\alpha x+2\alpha y+3z=-1$

$x+3\alpha y+5z=4$

Has inconsistent solution

$$D=\begin{vmatrix}1 & 1 & 1\\ \alpha & 2\alpha & 3\\ 1 & 3\alpha & 5\end{vmatrix}=0$$

$\Rightarrow(\alpha-1)^2=0$

$\alpha=1$

For $\alpha=1$

$$D_1=\begin{vmatrix}1 & 1 & 1\\ -1 & 2 & 3\\ 4 & 3 & 5\end{vmatrix}$$

= (10 – 9) – (–5–12) + (–3–8)

= 1+17 –11 $\neq 0$

For $\alpha=1$ the system of equation has Inconsistent solution

7. If the sum of the squares of the reciprocals of the roots α and β of the equation $3x^2 + \lambda x - 1 = 0$ is 15, then $6(\alpha^3 + \beta^3)^2$ is equal to :

(A) 18 (B) 24

(C) 36 (D) 96

Official Ans. by NTA (B)

Sol. **Here** α,β roots of equation $3x^2+\lambda x-1=0$

$$\alpha+\beta=\frac{-\lambda}{3},\ \alpha\beta=\frac{-1}{3}$$

$$\frac{1}{\alpha^2}+\frac{1}{\beta^2}=\frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha^2\beta^2}=15$$

$\lambda^2=9$

Now $6(\alpha^3+\beta^3)^2=6\left((\alpha+\beta)\left((\alpha+\beta)^2-3\alpha\beta\right)\right)^2$

$$=6\left(\frac{\lambda^2}{9}\right)\left\{\frac{\lambda^2}{9}+1\right\}^2=24$$

8. The set of all values of k for which $(\tan^{-1}x)^3+(\cot^{-1}x)^3=k\pi^3, x\in R$, is the interval :

(A) $\left[\frac{1}{32},\frac{7}{8}\right)$ (B) $\left(\frac{1}{24},\frac{13}{16}\right)$

(C) $\left[\frac{1}{48},\frac{13}{16}\right]$ (D) $\left[\frac{1}{32},\frac{9}{8}\right)$

Official Ans. by NTA (A)

Sol. **Let S =** $(\tan^{-1}x)^3+(\cot^{-1}x)^3$

$=\left(\tan^{-1}x+\cot^{-1}x\right)-3\tan^{-1}x.\cot^{-1}x\left(\tan^{-1}x+\cot^{-1}x\right)$

$=\frac{\pi^3}{8}-\frac{3\pi}{2}\tan^{-1}x\left(\frac{\pi}{2}-\tan^{-1}x\right)$

$=\frac{3\pi}{2}\left(\tan^{-1}x-\frac{\pi}{4}\right)^2+\frac{\pi^3}{32}$

$\Rightarrow \frac{\pi^3}{32}\le S<\frac{7}{8}\pi^3$

$=\frac{\pi^3}{32}\le K\pi^3<\frac{7}{8}\pi^3$

$\frac{1}{32}\le K<\frac{7}{8}$

9. Let $S=\{\sqrt{n}:1\le n\le 50$ and n is odd$\}$

Let $a\in S$ and $A=\begin{bmatrix}1&0&a\\-1&1&0\\-a&0&1\end{bmatrix}$

If $\sum_{a\in S}\det(\text{adj}A)=100\lambda$, then λ is equal to

(A) 218 (B) 221

(C) 663 (D) 1717

Official Ans. by NTA (B)

Sol. $S=\{\sqrt{n}:1\le n\le 50$ and n is odd$\}$

$=\left\{\sqrt{1},\sqrt{3},\sqrt{5}.....\sqrt{49}\right\}$, 25 terms

$|A| = 1 + a^2$

$\sum_{a\in S}\det(\text{adj}A)=\sum_{a\in S}|A|^2=\sum(1+a^2)^2$

$= 22100 = 100\lambda$

$\lambda = 221$

10. $f(x) = 4\log_e(x-1) - 2x^2 + 4x + 5$, $x > 1$, which one of the following is NOT correct ?

(A) f is increasing in (1, 2) and decreasing in (2, ∞)

(B) f(x)= –1 has exactly two solutions

(C) f'(e) –f'' (2) < 0

(D) f(x) = 0 has a root in the interval (e, e +1)

Official Ans. by NTA (C)

Sol. $f(x) = 4\log_e(x-1) - 2x^2 + 4x + 5$, $x > 1$

$f'(x)=\frac{4}{x-1}-4(x-1)$

For $1<x<2\Rightarrow f'(x)>0$

For $x>2\Rightarrow f'(x)<0$ (option 1 is correct)

$f(x)=-1$ has two solution (option 2 is correct)

$f(e)>0$

$f(e+1)<0$

$f(e).f(e+1)<0$ (option 4 is correct)

$f'(e)-f''(2)=\frac{4}{e-1}-4(e-1)+8>0$

(option C is incorrect)

11. the tangent at the point (x_1, y_1) on the curve $y = x^3 + 3x^2 + 5$ passes through the origin, then (x_1, y_1) does NOT lie on the curve :

(A) $x^2+\frac{y^2}{81}=2$ (B) $\frac{y^2}{9}-x^2=8$

(C) $y=4x^2+5$ (D) $\frac{x}{3}-y^2=2$

Official Ans. by NTA (D)

Sol. **The tangent at (x_1, y_1) to the curve**

$y=x^3+3x^2+5$

$y-y_1=\left(3x_1^2+6x_1\right)\left(x-x_1\right)$ passing through origin

$-y_1=\left(3x_1^3+6x_1\right)\left(-x_1\right)$

$y_1=\left(3x_1^3+6x_1^2\right)$ --------(1)

And (x_1, y_1) lies on the curve

$y = x^3 + 3x^2 + 5$

$y_1 = x_1^3 + 3x_1^2 + 5$ ----(2)

From equation (1) and (2)

$2y_1 = 3x_1^2 + \frac{15}{2}$

Hence the equation of curve $y = \frac{3}{2}x^2 + \frac{15}{2}$

This curve does not intersect $\frac{x}{3} - y^2 = 2$

12. The sum of absolute maximum and absolute minimum values of the function $f(x) = |2x^2 + 3x - 2| + \sin x \cos x$ in the interval $[0, 1]$ is :

(A) $3 + \frac{\sin(1)\cos^2(1/2)}{2}$ (B) $3 + \frac{1}{2}(1 + 2\cos(1))\sin(1)$

(C) $5 + \frac{1}{2}(\sin(1) + \sin(2))$ (D) $2 + \sin\left(\frac{1}{2}\right)\cos\left(\frac{1}{2}\right)$

Official Ans. by NTA (B)

Sol. $f(x) = |2x^2 + 3x - 2| + \sin x \cos x$

$f(x) = |(2x - 1)(x + 2)| + \sin x \cos x$

$$f'(x) = \begin{cases} 4x + 3 + \frac{\cos 2x}{4}, & \frac{1}{2} < x < 1 \\ -(4x + 3) + \frac{\cos 2x}{4}, & 0 \le x < \frac{1}{2} \end{cases}$$

For $0 \le x < \frac{1}{2} \Rightarrow f'(x) < 0$

For $\frac{1}{2} < x \le 1 \Rightarrow f'(x) > 0$

f (x) local minima at x = $\frac{1}{2}$ and

local maxima at x = 1

$f\left(\frac{1}{2}\right) + f(1) = 3 + \frac{1}{2}(1 + 2\cos 1)\sin 1$

13. If $\{a_i\}_{i=1}^{n}$ where n is an even integer , is an arithmetic progression with common difference 1, and $\sum_{i=1}^{n} a_i = 192$, $\sum_{i=1}^{n/2} a_{2i} = 120$, then n is equal to:

(A) 48 (B) 96

(C) 92 (D) 104

Official Ans. by NTA (B)

Sol. $\sum_{i=1}^{n} a_i = \frac{n}{2}\{2a_1 + (n+1)\} = 192$

$\Rightarrow 2a_1 + (n-1) = \frac{384}{n}$ ----(1)

$\sum_{i=1}^{n/2} a_{2i} = \frac{n}{4}\left[2a_1 + 2 + \left(\frac{n}{2} - 1\right)2\right] = 120$

$2a_1 + n = \frac{480}{n}$ ----(2)

From equation (2) and (1)

$1 = \frac{480}{n} - \frac{384}{n}$

n = 480 –384 = 96

14. If x = x(y) is the solution of the differential equation $y\frac{dx}{dy} = 2x + y^3(y+1)e^y, x(1) = 0$; then x(e) is equal to :

(A) $e^3(e^e - 1)$ (B) $e^e(e^3 - 1)$

(C) $e^2(e^e + 1)$ (D) $e^e(e^2 - 1)$

Official Ans. by NTA (A)

Sol. $y\frac{dx}{dy} = 2x + y^3(y+1)e^y, x(1) = 0$

$\frac{dx}{dy} - \frac{2}{y}x = y^2(y+1)e^y$

I.f = $e^{\int \frac{-2}{y}dy} = \frac{1}{y^2}$

$x.\frac{1}{y^2} = \int (y+1)e^y dy$

$\frac{x}{y^2} = (y+1)e^y - e^y + c = y.e^y + c$

$x = y^3e^y + cy^2$

For x = 0, y = 1 $\Rightarrow c = -e$

$x = y^3e^y - e.y^2$

$x(e) = e^3(e^e - 1)$

15. Let $\lambda x - 2y = \mu$ be a tangent to the hyperbola $a^2x^2 - y^2 = b^2$. Then $\left(\frac{\lambda}{a}\right)^2 - \left(\frac{\mu}{b}\right)^2$ is equal to:

(A) –2 (B) –4

(C) 2 (D) 4

Official Ans. by NTA (D)

Sol. $\lambda x - 2y = \mu$ is a tangent to the curve

$a^2x^2 - y^2 = b^2$ then

$$a^2x^2 - \left(\frac{\lambda x - \mu}{2}\right)^2 = b^2$$

$$\left(4a^2 - \lambda^2\right)x^2 + 2\lambda\mu x - \mu^2 - 4b^2 = 0$$

Disc. = 0

$$4\lambda^2\mu^2 + 4\left(4a^2 - \lambda^2\right)\left(\mu^2 + 4b^2\right) = 0$$

$$4\lambda^2b^2 - 4a^2\mu^2 = 16a^2b^2$$

$$\frac{\lambda^2}{a^2} - \frac{\mu^2}{b^2} = 4$$

16. Let $\hat{a}, \hat{b}$ be unit vectors. If $\vec{c}$ be a vector such that the angle between $\hat{a}$ and $\vec{c}$ is $\frac{\pi}{12}$, and $\hat{b} = \vec{c} + 2(\vec{c} \times \hat{a})$, then $\left|6\vec{c}\right|^2$ is equal to

(A) $6(3 - \sqrt{3})$ (B) $3 + \sqrt{3}$

(C) $6(3 + \sqrt{3})$ (D) $6(\sqrt{3} + 1)$

Official Ans. by NTA (C)

Sol. $\left|\hat{b}\right|^2 = \left|\vec{c} + 2(\vec{c} \times \hat{a})\right|^2$

$$\left|\hat{b}\right|^2 = |c|^2 + 4\left|\vec{c} \times \hat{a}\right|^2 + 4\vec{c}.(\vec{c} \times \hat{a})$$

$$1 = |c|^2 + 4|c|^2 \sin^2\frac{\pi}{12} + 0$$

$$1 = |c|^2 + 4|c|^2\left(\frac{\sqrt{3} - 1}{2\sqrt{2}}\right)^2$$

$$|c|^2 = \frac{1}{3 - \sqrt{3}} = \frac{3 + \sqrt{3}}{6}$$

So $6^2|c|^2 = 6(3 + \sqrt{3})$

17. If a random variable X follows the Binomial distribution B (33, p) such that 3P(X = 0) = P(X = 1), then the value of $\frac{P(X=15)}{P(X=18)} - \frac{P(X=16)}{P(X=17)}$ is equal to

(A) 1320 (B) 1088

(C) $\frac{120}{1331}$ (D) $\frac{1088}{1089}$

Official Ans. by NTA (A)

Sol. n = 33, let probability of success is p and q = 1 – p

$$3p(x=0) = p(x=1)$$

$$3.{}^{33}C_0(q)^{33} = {}^{33}C_1pq^{32}$$

$$p = \frac{1}{12},\ q = \frac{11}{12},\ \frac{q}{p} = 11$$

$$\frac{p(x=15)}{p(x=18)} - \frac{p(x=16)}{p(x=17)}$$

$$\frac{{}^{33}C_{15}p^{15}q^{18}}{{}^{33}C_{18}p^{18}q^{15}} - \frac{{}^{33}C_{16}p^{16}q^{17}}{{}^{33}C_{17}p^{17}q^{16}} = \left(\frac{q}{p}\right)^3 - \left(\frac{q}{p}\right)$$

$$= (11)^3 - 11$$

= 1320

18. The domain of the function

$$f(x) = \frac{\cos^{-1}\left(\frac{x^2 - 5x + 6}{x^2 - 9}\right)}{\log_e(x^2 - 3x + 2)}$$ is

(A) $(-\infty, 1) \cup (2, \infty)$

(B) $(2, \infty)$

(C) $\left[-\frac{1}{2}, 1\right) \cup (2, \infty)$

(D) $\left[-\frac{1}{2}, 1\right) \cup (2, \infty) - \left\{\frac{3 + \sqrt{5}}{2}, \frac{3 - \sqrt{5}}{2}\right\}$

Official Ans. by NTA (DROP)

Sol. $-1 \le \frac{x^2-5x+6}{x^2-9} \le 1$

$\frac{x^2-5x+6}{x^2-9}-1 \le 0$

$\frac{1}{x+3} \ge 0$

$x \in (-3,\infty)$ **.....(1)**

$\frac{x^2-5x+6}{x^2-9}+1 \ge 0$

$\frac{2x+1}{x+3} \ge 0$

$x \in (-\infty,-3) \cup \left[-\frac{1}{2},\infty\right)$**.....(2)**

after taking intersection

$x \in \left[-\frac{1}{2},\infty\right)$

$x^2-3x+2>0$

$x \in (-\infty,1) \cup (2,\infty)$

$x^2-3x+2 \ne 1$

$x \ne \frac{3 \pm \sqrt{5}}{2}$

after taking intersection of each solution

$\left[-\frac{1}{2},1\right) \cup (2,\infty) - \left\{\frac{3+\sqrt{5}}{2},\frac{3-\sqrt{5}}{2}\right\}$

19. Let

$S = \left\{\theta \in [-\pi,\pi] - \left\{\pm\frac{\pi}{2}\right\} : \sin\theta\tan\theta + \tan\theta = \sin 2\theta\right\}$.

If $T = \sum_{\theta \in S} \cos 2\theta$, then T + n(S) is equal

(A) $7+\sqrt{3}$ (B) 9

(C) $8+\sqrt{3}$ (D) 10

Official Ans. by NTA (B)

Sol. $\sin\theta\tan\theta + \tan\theta = \sin 2\theta$

$\tan\theta(\sin\theta+1) = \frac{2\tan\theta}{1+\tan^2\theta}$

$\tan\theta = 0 \Rightarrow \theta = -\pi, 0, \pi$

$(\sin\theta+1) = 2.\cos^2\theta = 2(1+\sin\theta)(1-\sin\theta)$

$\sin\theta = -1$ which is not possible

$\sin\theta = \frac{1}{2} \quad \theta = \frac{\pi}{6},\frac{5\pi}{6}$

n(s) = 5

$T = \cos 0 + \cos 2\pi + \cos 2\pi + \cos\frac{\pi}{3} + \cos\frac{5\pi}{3}$

T = 4

T + n(s) = 9

20. The number of choices of $\Delta \in \{\wedge, \vee, \Rightarrow, \Leftrightarrow\}$, such that $(p\Delta q) \Rightarrow ((p\Delta \sim q) \vee ((\sim p)\Delta q))$ is a tautology, is

(A) 1 (B) 2

(C) 3 (D) 4

Official Ans. by NTA (B)

Sol. For tautology $((p\Delta \sim q) \vee ((\sim p)\Delta q))$ must be true.

This is possible only when $\Delta = \vee \& \Rightarrow$

SECTION-B

1. The number of one-one function f : {a, b, c, d} → {0, 1, 2, … .,10} such that 2f(a) – f(b) + 3f(c) + f(d) = 0 is _____ .

Official Ans. by NTA (31)

Sol. 2f(a) + 3f(c) = f(d) – f(b)

Using fundamental principle of counting

Number of one-one function is 31

2. In an examination, there are 5 multiple choice questions with 3 choices, out of which exactly one is correct There are 3 marks for each correct answer, -2 marks for each wrong answer and 0 mark if the question is not attempted. Then, the number of ways a student appearing in the examination gets 5 marks is_.

Official Ans. by NTA

Sol. $x_1 + x_2 + x_3 + x_4 + x_5 = 5$

Only one possibilities 3, 3, 3, -2, -2

Number of ways is = $\frac{5!}{3!2!} \times 2 \times 2 = 40$

3. Let $A\left(\frac{3}{\sqrt{a}},\sqrt{a}\right) a > 0$, be a fixed point in the xy-plane. The image of A in y-axis be B and the

image of B in x-axis be C. If D(3 cos θ, a sin θ) is a point in the fourth quadrant such that the maximum area of ΔACD is 12 square units, then a is equal to _____ .

Official Ans. by NTA (8)

Sol. $A=\left(\frac{3}{\sqrt{a}},\sqrt{a}\right)$

$B=\left(\frac{-3}{\sqrt{a}},\sqrt{a}\right)$

$C=\left(-\frac{3}{\sqrt{a}},-\sqrt{a}\right)$

Area of ACD

$$\frac{1}{2}\begin{vmatrix}\frac{3}{\sqrt{a}} & \sqrt{a}\\ -\frac{3}{\sqrt{a}} & -\sqrt{a}\\ 3\cos\theta & a\sin\theta\\ \frac{3}{\sqrt{a}} & \sqrt{a}\end{vmatrix}$$

$\frac{1}{2}6\sqrt{a}(\cos\theta-\sin\theta)$

$3\sqrt{a}(\cos\theta-\sin\theta)$

max values of function is $3\sqrt{a}\sqrt{2}$

$3\sqrt{a}\sqrt{2}=12$

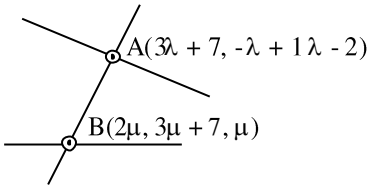
$2a = 16$

$a = 8$

4. Let a line having direction ratios 1, -4, 2 intersect the lines $\frac{x-7}{3}=\frac{y-1}{-1}=\frac{z+2}{1}$ and $\frac{x}{2}=\frac{y-7}{3}=\frac{z}{1}$ at the point A and B. Then $(AB)^2$ is equal to ____ .

Official Ans. by NTA (84)

Sol.



DR's of AB

$(3\lambda-2\mu+7,-\lambda-3\mu-6,\lambda-\mu-2)$

$\frac{3\lambda-2\mu+7}{1}=\frac{-\lambda-3\mu-6}{-4}=\frac{\lambda-\mu-2}{2}$

Taking first (2) $-12\lambda+8\mu-28=-\lambda-3\mu-6$

$\lambda-\mu+2=0$

Taking second & third

$-2\lambda-6\mu-12=-4\lambda+4\mu+8$

$\lambda-5\mu-10=0$

After solving above two equation $\lambda=-5,\mu=-3$

A = (-8, 6, 7)

B = (-6, -2, -3)

$(AB)^2 = 4 + 64 + 16 = 84$

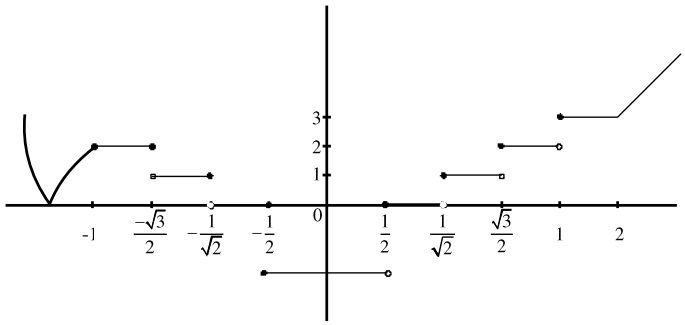
5. The number of points where the function

$$f(x)=\begin{cases}|2x^2-3x-7| & \text{if} \quad x\le -1\\ [4x^2-1] & \text{if} \quad -1<x<1\\ |x+1|+|x-2| & \text{if} \quad x\ge 1\end{cases}$$

[t] denotes the greatest integer $\le t$, is discontinuous is _____ .

Official Ans. by NTA (7)

Sol.



6. Let $f(\theta)=\sin\theta+\int_{-\pi/2}^{\pi/2}(\sin\theta+t\cos\theta)f(t)dt$. Then the value of $\left|\int_0^{\pi/2}f(\theta)d\theta\right|$ is _____ .

Official Ans. by NTA (1)

Sol. $f(\theta)=\sin\theta+\int_{-\pi/2}^{\pi/2}(\sin\theta+t\cos\theta)\,f(t)\,dt$

$f(\theta)=\sin\theta+\sin\theta\int_{-\pi/2}^{\pi/2}f(t)dt+\cos\theta\int_{-\pi/2}^{\pi/2}tf(t)dt$

Let $A=\int_{-\pi/2}^{\pi/2}f(t)dt,\ B=\int_{-\pi/2}^{\pi/2}tf(t)dt$

$f(\theta)=\sin\theta+A\sin\theta+B\cos\theta$

$f(\theta)=(A+1)\sin\theta+B\cos\theta$

$A=\int_{-\pi/2}^{\pi/2}(A+1)\sin t+B\cos t\,dt$

$A=2B \qquad \ldots\ldots(1)$

$B=\int_{-\pi/2}^{\pi/2}t((A+1)\sin t+B\cos t)$

$B=\int_{-\pi/2}^{\pi/2}t(A+1)\sin t$

$B=(A+1)2\int_{0}^{\pi/2}t\sin t\,dt$

$B=(A+1)\,2.1$

$2A+2-B=0$ .....(2)

After solving

$B=-\frac{2}{3},A=-\frac{4}{3}$

$\left|\int_0^{\pi/2}f(\theta)d\theta\right|=\left|\int_0^{\pi/2}-\frac{1}{3}\sin\theta-\frac{2}{3}\cos\theta\right|$

$=1$

7. Let $\underset{0\le x\le 2}{\text{Max}}\left\{\frac{9-x^2}{5-x}\right\}=\alpha$ and $\underset{0\le x\le 2}{\text{Min}}\left\{\frac{9-x^2}{5-x}\right\}=\beta$

If $\int_{\beta-\frac{8}{3}}^{2\alpha-1}\text{Max}\left\{\frac{9-x^2}{5-x},x\right\}dx=\alpha_1+\alpha_2\log_e\left(\frac{8}{15}\right)$ then

$\alpha_1+\alpha_2$ is equal to _____

Official Ans. by NTA (34)

Sol. $y=\frac{9-x^2}{5-x}=5+x+\frac{16}{x-5}$

$\frac{dy}{dx}=1-\frac{16}{(x-5)^2}$

So critical point is x = 1 in [0, 2]

$y(0)=\frac{9}{5},\ y(1)=2,\ y(2)=\frac{5}{3}$

So $\alpha=2$ and $\beta=\frac{5}{3}$

$I=\int_{-1}^{3}\max\left(\frac{9-x^2}{5-x},x\right)$

$I=\int_{-1}^{9/5}\frac{9-x^2}{5-x}dx+\int_{9/5}^{3}x\,dx$

$I=\int_{-1}^{9/5}5+x+\frac{16}{x-5}dx+\int_{9/5}^{3}x\,dx$

After solving

$I=14+\frac{28}{25}+16\ln\left(\frac{8}{15}\right)+\frac{72}{25}$

$\alpha_1=18$ and $\alpha_2=16$

8. If two tangents drawn from a point (α, β) lying on the ellipse $25x^2+4y^2=1$ to the parabola $y^2=4x$ are such that the slope of one tangent is four times the other, then the value of

$(10\alpha+5)^2+(16\beta^2+50)^2$ equals _____

Official Ans. by NTA (2929)

Sol. $\alpha=\frac{1}{5}\cos\theta,\ \beta=\frac{1}{2}\sin\theta$

Equation of tangent to $y^2=4x$

$y=mx+\frac{1}{m}$

It passes through (α,β)

$\frac{1}{2}\sin\theta=m\frac{1}{5}\cos\theta+\frac{1}{m}$

$m^2\left(\frac{\cos\theta}{5}\right)-m\left(\frac{1}{2}\sin\theta\right)+1=0$

It has two roots m_1 and m_2 where $m_1=4m_2$

$m_1+m_2=\frac{\frac{1}{2}\sin\theta}{\frac{\cos\theta}{5}}$

$m_1m_2=\frac{5}{\cos\theta}$

After eliminating m_1 and m_2

$$\cos\theta = \frac{-5 \pm \sqrt{29}}{2}$$

$$\alpha = \frac{-5 \pm \sqrt{29}}{10} \Rightarrow 10\alpha + 5 = \pm\sqrt{29}$$

$$\beta^2 = \frac{1}{4}\sin^2\theta \Rightarrow 16\beta^2 = -50 \pm 10\sqrt{29}$$

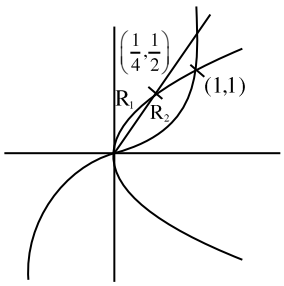
$$(10\alpha + 5)^2 + (16\beta^2 + 50)^2 = 2929$$

9. Let S be the region bounded by the curves $y = x^3$ and $y^2 = x$. The curve $y = 2|x|$ divides S into two regions of areas R_1 and R_2.

If $\max\{R_1, R_2\} = R_2$, then $\frac{R_2}{R_1}$ is equal to ____ .

Official Ans. by NTA (19)

Sol.



$$S = \int_0^1 \sqrt{x} - x^3$$

$$= \left[\frac{2x^{3/2}}{3} - \frac{x^4}{4}\right]_1^0$$

$$= \frac{5}{12}$$

$$R_1 = \int_0^{1/4} (\sqrt{x} - 2x)dx$$

$$= \left[\frac{2x^{3/2}}{3} - x^2\right]_0^{1/4} = \frac{1}{48}$$

$$\therefore\ R_2 = \frac{19}{48}$$

So, $\frac{R_2}{R_1} = 19$

10. If the shortest distance between the line $\vec{r} = \left(-\hat{i} + 3k\right) + \lambda\left(\hat{i} - a\hat{j}\right)$ and $\vec{r} = \left(-\hat{j} + 2k\right) + \mu\left(\hat{i} - \hat{j} + k\right)$ is $\sqrt{\frac{2}{3}}$, then the integral value of a is equal to

Official Ans. by NTA (2)

Sol. $a_1 = (-1,0,3)$

$a_2 = (0,-1,2)$

$b_1 = (1,-a,0)$ dr's of line (1)

$b_2 = (1,-1,1)$ dr's of line (2)

$\bar{a}_2 - \bar{a}_1 = (1,-1,-1)$

$$\bar{b}_1 \times \bar{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & k \\ 1 & -a & 0 \\ 1 & -1 & 1 \end{vmatrix}$$

$$\bar{b}_1 \times \bar{b}_2 = \hat{i}(-a) - \hat{j} + k(a-1)$$

$$\left|\bar{b}_1 \times \bar{b}_2\right| = \sqrt{a^2 + 1 + (a-1)^2}$$

$$a_2 - a_1.\bar{b}_1 \times \bar{b}_2 = 2 - 2a$$

$$\frac{2(1-a)}{\sqrt{a^2 + 1 + (a-1)^2}} = \sqrt{\frac{2}{3}}$$

Squaring an both the side

After solving a = 2, $\frac{1}{2}$