JEE (Main)-2025 (Online) Session-2

Question Paper with Solutions

(Mathematics, Physics, And Chemistry)

3 April 2025 Shift – 2

Time: 3 hrs. **M.M : 300**

IMPORTANT INSTRUCTIONS:

(1) The test is of **3 hours** duration.

(2) This test paper consists of 75 questions. Each subject (PCM) has 25 questions. The maximum marks are 300.

(3) This question paper contains Three Parts. Part-A is Physics, Part-B is Chemistry and Part-C is Mathematics. Each part has only two sections: Section-A and Section-B.

(4) Section - A : Attempt all questions.

(5) Section - B : Attempt all questions.

(6) Section - A (01 - 20) contains 20 multiple choice questions which have only one correct answer. Each question carries +4 marks for correct answer and -1 mark for wrong answer.

(7) Section - B (21 – 25) contains 5 Numerical value based questions. The answer to each question should be rounded off to the nearest integer. Each question carries +4 marks for correct answer and -1 mark for wrong answer.

SECTION-A

1. Let f : R → R be a function defined by $f(x) = \left\| x+2 \right| - 2\left| x \right| \right\|$. If m is the number of points of local minima and n is the number of points of local maxima of f, then m + n is

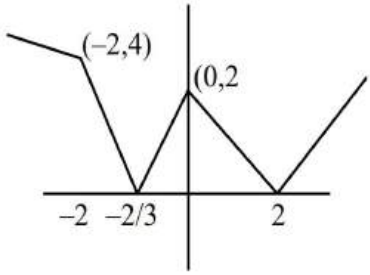
(1) 5 (2) 3

(3) 2 (4) 4

Ans. (2)

Sol. $f(x) = \left\| x+2 \right| - 2\left| x \right| \right\|$

Critical points, $0, -2, 2, -\frac{2}{3}$



No. of maxima = 1

No. of minima = 2

option (2)

2. Each of the angles β and γ that a given line makes with the positive y– and z–axes, respectively, is half of the angle that this line makes with the positive x-axes. Then the sum of all possible values of the angle β is

(1) $\frac{3\pi}{4}$ (2) π

(3) $\frac{\pi}{2}$ (4) $\frac{3\pi}{2}$

Ans. (1)

Sol. $\beta = \frac{\alpha}{2}, \gamma = \frac{\alpha}{2}$

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\cos^2\alpha + 2\cos^2\frac{\alpha}{2} = 1$

$\cos^2\alpha + \cos\alpha = 0$

$\cos\alpha(\cos\alpha + 1) = 0$

$\cos\alpha = 0, -1$

$\alpha = \frac{\pi}{2}, \pi$

Now $\beta = \frac{\alpha}{2} \Rightarrow \frac{\pi}{4}, \frac{\pi}{2}$

so sum is $\frac{3\pi}{4}$

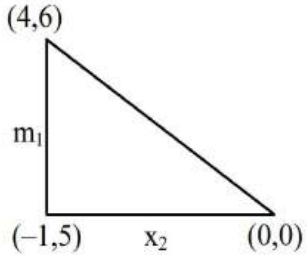
3. If the four distinct points (4, 6), (–1,5), (0,0) and (k, 3k) lie on a circle of radius r, then $10k + r^2$ is equal to

(1) 32 (2) 33

(3) 34 (4) 35

Ans. (4)

Sol.



$m_1m_2 = -1$ so right angle equation circle is

$(x-4)(x-0) + (y-6)(y-0) = 0$

$x^2 + y^2 - 4x - 6y = 0$

(k,3k) lies on it so

$k^2 + 9k^2 - 4k - 18k = 0$

$10k^2 - 22k = 0$

$k = 0, \frac{11}{5}$

k = 0 is not possible so $k = \frac{11}{5}$

also $r = \sqrt{4+9} = \sqrt{13}$

so $10k + r^2 = 10.\frac{11}{5} + \left(\sqrt{13}\right)^2 = 35$

4. Let the Mean and Variance of five observations $x_1 = 1$, $x_2 = 3$, $x_3 = a$, $x_4 = 7$ and $x_5 = b$, $a > b$, be 5 and 10 respectively. Then the Variance of the observations $n + x_n$, $n = 1, 2, \ldots\ldots 5$ is

(1) 17 (2) 16.4

(3) 17.4 (4) 16

Ans. (4)

Sol. $\bar{x} = \frac{\sum x_i}{n} = \frac{1+3+a+7+b}{5} = 5$

$a + b = 14$

$\sigma^2 = \frac{\Sigma x_i^2}{n} - (\bar{x})^2$

$\Rightarrow \frac{1^2+3^2+a^2+7^2+b^2}{5} - 25 = 10$

$a^2 + b^2 = 116$

$a > b \quad a = 10 \quad b = 4$

$n + x_n : 2,5,13,11,9$

$\sigma^2 = \frac{2^2+5^2+13^2+11^2+9^2}{5} - \left(\frac{2+5+13+11+9}{5}\right)^2$

$= 80 - 64 = 16$

option 4

5. Consider the lines $x(3\lambda + 1) + y(7\lambda+2) = 17\lambda + 5$, λ being a parameter, all passing through a point P. One of these lines (say L) is farthest from the origin. If the distance of L from the point (3, 6) is d, then the value of d^2 is

(1) 20 (2) 30

(3) 10 (4) 15

Ans. (1)

Sol. $x(3\lambda + 1) + y(7\lambda + 2) = 17\lambda + 5$

$(x + 2y - 5) + \lambda(3x + 7y - 17) = 0$

intersection of family of lines

P(1,2)

Let Q(3,6)

$d = PQ = \sqrt{2^2 + 4^2} = \sqrt{20}$

$d^2 = 20$

option (1)

6. Let A = {–2,–1,0, 1, 2, 3}. let R be a relation on A defined by xRy if and only if y = max{x, 1}. Let l be the number of elements in R. Let m and n be the minimum number of elements required to be added in R to make it reflexive and symmetric relations, respectively. Then $l + m + n$ is equal to

(1) 12 (2) 11

(3) 13 (4) 14

Ans. (1)

Sol. A = {–2,–1,0,1,2,3}

R = {(–2,1),(–1,1), (0,1), (1,1), (2,2), (3,3)}

$\ell = 6$

$m = 3$

$n = 3$

$\ell + m + n = 12$

7. let the equation $x(x + 2)(12-k) = 2$ have equal roots. Then the distance of the point $\left(k, \frac{k}{2}\right)$ from the line $3x + 4y + 5 = 0$ is

(1) 15 (2) $5\sqrt{3}$

(3) $15\sqrt{5}$ (4) 12

Ans. (1)

Sol. $(x^2 + 2x)(12 - k) = 2$

$\lambda x^2 + 2\lambda x - 2 = 0 \quad k \neq 12$ Let $12 - k = \lambda$

$D = 0$

$4\lambda^2 + 8\lambda = 0$

$\lambda = 0$ or $\lambda = -2$

$\Rightarrow 12 - k = -2$

$k = 14$

So $P\left(k, \frac{k}{2}\right) = (14,7)$

$d = \left|\frac{3\times14+4\times7+5}{5}\right| = 15$

option (1)

8. Line L_1 of slope 2 and line L_2 of slope $\frac{1}{2}$ intersect at the origin O. In the first quadrant, $P_1, P_2, \ldots P_{12}$ are 12 points on line L_1 and $Q_1, Q_2, \ldots Q_9$ are 9 points on line L_2. Then the total number of triangles, that can be formed having vertices at three of the 22 points O, $P_1, P_2, \ldots P_{12}$, $Q_1, Q_2, \ldots Q_9$, is:

(1) 1080 (2) 1134

(3) 1026 (4) 1188

Ans. (2)

Sol. Total number of $\Delta$are

$= {}^9C_1\,{}^{12}C_2 + {}^9C_2\,{}^{12}C_1 + {}^1C_1\,{}^9C_1\,{}^{12}C_1$

$= 594 + 432 + 108$

$= 1134$

9. The integral $\int_0^{\pi} \frac{8x\,dx}{4\cos^2 x + \sin^2 x}$ is equal to

(1) $2\pi^2$ (2) $4\pi^2$

(3) π^2 (4) $\frac{3\pi^2}{2}$

Ans. (1)

Sol. $I = \int_0^{\pi} \frac{8x\,dx}{4\cos^2 x + \sin^2 x}$

$I = \int_0^{\pi} \frac{8(\pi - x)\,dx}{4\cos^2 x + \sin^2 x}$

$2I = 8\pi \int_0^{\pi} \frac{dx}{4\cos^2 x + \sin^2 x}$

$2I = 8\pi \times 2 \int_0^{\pi/2} \frac{\sec^2 x}{4 + \tan^2 x}\,dx$

$I = 8\pi \int_0^{\infty} \frac{dt}{4 + t^2} = 8\pi \times \frac{1}{2}\left(\tan^{-1}\frac{t}{2}\right]_0^{\infty}$

$= 4\pi \times \frac{\pi}{2} = 2\pi^2$

option (1)

10. Let f be a function such that $f(x) + 3f\left(\frac{24}{x}\right) = 4x,\ x \neq 0$. Then f(3) + f(8) is equal to

(1) 11 (2) 10

(3) 12 (4) 13

Ans. (1)

Sol. $f(x) + 3\,f\left(\frac{24}{x}\right) = 4x$

Put x = 3 $\quad f(3) + 3f(8) = 12$

Put x = 8 $\quad f(8) + 3f(3) = 32$

Add both $\quad 4(f(3) + f(8)) = 44$

$f(3) + f(8) = 11$

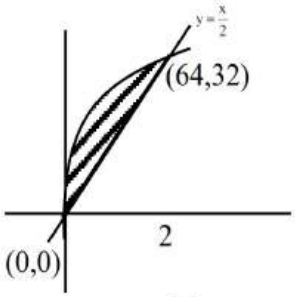
11. The area of the region $\left\{(x,y) : |x - y| \le y \le 4\sqrt{x}\right\}$ is

(1) 512 (2) $\frac{1024}{3}$

(3) $\frac{512}{3}$ (4) $\frac{2048}{3}$

Ans. (2)

Sol.



$|x - y| \le y \le 4\sqrt{x}$

Now $y = |x - y|$

$y^2 = (x - y)^2$

$\Rightarrow y = \frac{x}{2}$ and $x = 0$

Now area $= \int_0^{64} \left(4\sqrt{x} - \frac{x}{2}\right)dx$

$= \left[\frac{4x^{3/2}}{3/2} - \frac{x^2}{4}\right]_0^{64} = \frac{8}{3}.8^3 - \frac{64^2}{4} = 64^2\left(\frac{1}{12}\right)$

$= \frac{1024}{3}$

12. If the domain of the function $f(x) = \log_7(1 - \log_4(x^2 - 9x + 18))$ is $(\alpha, \beta) \cup (\gamma, \delta)$, then $\alpha + \beta + \gamma + \delta$ is equal to

(1) 18 (2) 16

(3) 15 (4) 17

Ans. (1)

Sol. Domain $1 - \log_4(x^2 - 9x + 18) > 0$

Also $x^2 - 9x + 18 > 0$

$(x - 3)(x - 6) > 0$

$x \in (-\infty, 3) \cup (6, \infty)$ ....(1)

also $x^2 - 9x + 18 < 4$

$x^2 - 9x + 14 < 0$

$x \in (2,7)$ ....(2)

$(1) \cap (2)$ $(2,3) \cup (6,7) = (\alpha,\beta) \cup (\gamma,\delta)$

$\Rightarrow \alpha + \beta + \gamma + \delta = 18$

13. If the probability that the random variable X takes the value x is given by $P(X = x) = k(x + 1)3^{-x}$, $x = 0,1,2,3.....$, where k is a constant, then $P(X \geq 3)$ is equal to

(1) $\frac{7}{27}$ (2) $\frac{4}{9}$

(3) $\frac{8}{27}$ (4) $\frac{1}{9}$

Ans. (4)

Sol. $\sum_{x=0}^{\infty} k(x+1)3^{-x} = 1$

$\Rightarrow \frac{1}{k} = 1 + \frac{2}{3} + \frac{3}{3^2} + \frac{4}{3^3} + ...$ (i)

$\frac{1}{3k} = \frac{1}{3} + \frac{2}{3^2} + \frac{3}{3^3} + ...$ ...(ii)

(i)- (ii) $\Rightarrow \frac{1}{k} - \frac{1}{3k} = 1 + \frac{1}{3} + \frac{1}{3^2} + ...$

$\Rightarrow k = \frac{4}{9}$

$P(x \geq 3) = 1 - P(x = 0) - P(x = 1) - P(x = 2)$

$= 1 - k\left(1 + \frac{2}{3} + \frac{3}{9}\right) = \frac{1}{9}$

14. Let $y = y(x)$ be the solution of the differential equation $\frac{dy}{dx} + 3(\tan^2 x)\, y + 3y = \sec^2 x$, $y(0) = \frac{1}{3} + e^3$. Then $y\left(\frac{\pi}{4}\right)$ is equal to

(1) $\frac{2}{3}$ (2) $\frac{4}{3}$

(3) $\frac{4}{3} + e^3$ (4) $\frac{2}{3} + e^3$

Ans. (2)

Sol. $\frac{dy}{dx} + 3\left(\sec^2 x\right)y = \sec^2 x, y(0) = \frac{1}{3} + e^3$

If $= e^{3\int \sec^2 x dx} = e^{3\tan x}$

$\therefore$ Solution is

$e^{3\tan x} y = \int e^{3\tan x} \sec^2 x dx$

$e^{3\tan x} y = \frac{e^{3\tan x}}{3} + c$

$\because\ y(0) = \frac{1}{3} + e^3 \Rightarrow c = e^3$

$\therefore\ y\left(\frac{\pi}{4}\right) = \frac{\frac{e^3}{3} + e^3}{e^3} = \frac{4}{3}$

15. If $z_1, z_2, z_3 \in C$ are the vertices of an equilateral triangle, whose centroid is z_0, then $\sum_{k=1}^{3}(z_k - z_0)^2$ is equal to

(1) 0 (2)

(3) i (4) –i

Ans. (1)

Sol. $z_1 + z_2 + z_3 = 3z_0$

$(z_1 + z_2 + z_3)^2 = 9{z_0}^2$

$\Rightarrow z_1^2 + z_2^2 + z_3^2 + 2\left(z_1^2 + z_2^2 + z_3^2\right) = 9z_0^2$

$\Rightarrow z_1^2 + z_2^2 + z_3^2 = 3z_6^2$

$\sum_{k=1}^{3}\left(z_k - z_0\right)^2 = \left(z_1 - z_0\right)^2 + \left(z_2 - z_0\right)^2 + \left(z_3 - z_0\right)^2$

$= z_1^2 + z_2^2 + z_3^2 + 3z_0^2 - 2\left(z_1 + z_2 + z_3\right)z_0$

$= 6z_0^2 - 6z_0^2$

$= 0$

16. The number of solutions of equation $4-\sqrt{3}\ \sin x - 2\sqrt{3}\cos^2 x = -\frac{4}{1+\sqrt{3}}, x \in \left[-2\pi, \frac{5\pi}{2}\right]$ is

(1) 4 (2) 3

(3) 6 (4) 5

Ans. (4)

Sol. $\left(4-\sqrt{3}\right)\sin x - 2\sqrt{3}\cos^2 x = \frac{-4}{1+\sqrt{3}}, x \in \left[-2\pi, \frac{5\pi}{2}\right]$

$\Rightarrow \left(4-\sqrt{3}\right)\sin x - 2\sqrt{3}\left(1-\sin^2 x\right) = 2\left(1-\sqrt{3}\right)$

$\Rightarrow 2\sqrt{3}\sin^2 x + 4\sin x - \sqrt{3}\sin x - 2 = 0$

$\Rightarrow \left(2\sin x - 1\right)\left(\sqrt{3}\sin x + 2\right) = 0$

$\Rightarrow \sin x = \frac{1}{2}$

$\therefore$ Number of solution = 5

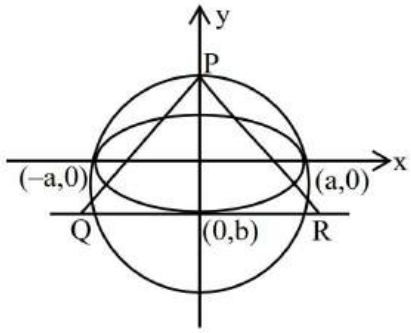
17. Let C be the circle of minimum area enclosing the ellipse E : $\frac{x^2}{a^2}+\frac{y^2}{b^2} = 1$ with eccentricity $\frac{1}{2}$ and foci(±2, 0). Let PQR be a variable triangle, whose vertex P is on the circle C and the side QR of length 29 is parallel to the major axis of E and contains the point of intersection of E with the negative y-axis. Then the maximum area of the triangle PQR is :

(1) $6\left(3+\sqrt{2}\right)$ (2) $8\left(3+\sqrt{2}\right)$

(3) $6\ 2+\sqrt{3}$ (4) $8\ 2+\sqrt{3}$

Ans. (4)

Sol.



Area of ΔPQR

$= \frac{1}{2}(2a)(a\sin\theta + b)$

$\therefore$ maximum area = a(a + b)

$= 4(4 + 2\sqrt{3}) = 8\left(2+\sqrt{3}\right)$

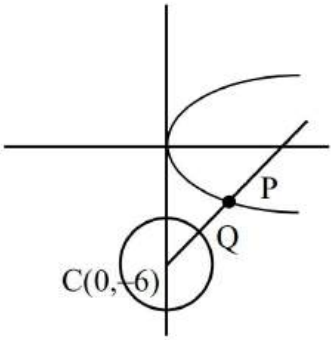
18. The shortest distance between the curves $y^2 = 8x$ and $x^2 + y^2 + 12y + 35 = 0$ is :

(1) $2\sqrt{3} - 1$ (2) $\sqrt{2}$

(3) $3\sqrt{2} - 1$ (4) $2\sqrt{2} - 1$

Ans. (4)

Sol.



Equation of normal to parabola

$y^2 = 8x$ is $y = mx - 4m - 2m^3$

passes through (0,–6) we get

$-6 = -4m - 2m^3$

$\Rightarrow m^3 + 2m - 3 = 0$

$\Rightarrow (m-1)(m^2 + m + 3) = 0 \Rightarrow m = -1$

$P = (am^2, -2am) = (2, -4)$

$\therefore$ Shortest distance = PC – r

$= \left(2\sqrt{2} - 1\right)$

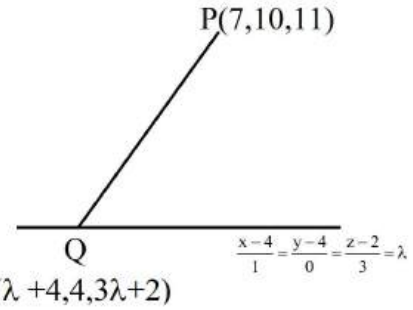
19. The distance of the point (7, 10, 11) from the line $\frac{x-4}{1} = \frac{y-4}{0} = \frac{z-2}{3}$ along the line $\frac{x-9}{2} = \frac{y-13}{3} = \frac{z-17}{6}$ is

(1) 18 (2) 14

(3) 12 (4) 16

Ans. (2)

Sol.



$\because$ line PQ is parallel to line $\frac{x-9}{2}=\frac{y-3}{3}=\frac{z-17}{6}$

$\therefore \frac{\lambda-3}{2}=\frac{-6}{3}=\frac{3\lambda-9}{6}\Rightarrow\lambda=-1$

$Q=(3,4,-1)$

$\therefore PQ=\sqrt{16+36+144}=14$

20. The sum $1+\frac{1+3}{2!}+\frac{1+3+5}{3!}+\frac{1+3+5+7}{4!}+...$ upto ∞ terms, is equal to

(1) $6e$ (2) $4e$

(3) $3e$ (4) $2e$

Ans. (4)

Sol. $S=1+\frac{1+3}{2!}+\frac{1+3+5}{3!}+...$

$=\sum_{r=1}^{\infty}\frac{r^2}{r!}$

$=\sum_{r=1}^{\infty}\frac{(r-1+1)}{(r-1)!}=\sum_{r=2}^{\infty}\frac{1}{(r-2)!}+\sum_{r=1}^{\infty}\frac{1}{(r-1)!}$

$=2e$

SECTION-B

21. Let I be the identity matrix of order 3 × 3 and for the matrix $A=\begin{bmatrix}\lambda & 2 & 3\\ 4 & 5 & 6\\ 7 & -1 & 2\end{bmatrix}$, $|A|=-1$. Let B be the inverse of the matrix $adj(A\ adj(A^2))$. Then $|(\lambda B+1)|$ is equal to _____

Ans. (38)

Sol. $|A|=\begin{vmatrix}\lambda & 2 & 3\\ 4 & 5 & 6\\ 7 & -1 & 2\end{vmatrix}=-1$

$\lambda(16)-2(-34)+3(-39)=-1$

$16\lambda=48\Rightarrow\lambda=3$

$B^{-1}=adj(A.adj\ (A^2))$

Let $C=A.\ adj\ (A^2)$

$AC=A^2\ adj(A^2)=|A|^2.I=I\Rightarrow C=A^{-1}$

Now $B^{-1}=adj(A^{-1})=B=adj(A)$

Now $\lambda B+I\Rightarrow 3B+I$

Let $P=3B+I$

$P=3adj\ (A)+I$

$AP=3Aadj(A)+A$

$AP=3|A|.I+A$

$AP=A-3I$

$|AP|=|A-3I|$

$|A|.\ |P|=\begin{vmatrix}0 & 2 & 3\\ 4 & 2 & 6\\ 7 & -1 & -1\end{vmatrix}=38$

$|P|=-38$

22. Let $(1+x+x^2)^{10}=a_0+a_1x+a_2x^2+....+a_{20}x^{20}$. If $(a_1+a_3+a_5+....+a_{19})-11a_2=121k$, then k is equal to_____.

Ans. (239)

Sol. $(1+x+x^2)^{10}=a_0+a_1x+a_2x^2+....+a_{20}x^{20}$

$\therefore 3^{10}=a_0+a_1+a_2+....+a_{20}$ ....(i)

$1=a_0-a_1+a_2.....+a_{20}$ ...(ii)

(i) - (ii) $\Rightarrow a_1+a_3+....+a_{19}=\frac{3^{10}-1}{2}=29524$

Also $\{1+x(1+x)\}^{10}=1$

$+{}^{10}C_1x\ (1+x)+{}^{10}C_2x^2(1+x)^2+.....$

$\therefore a_2={}^{10}C_1+{}^{10}C_2=55$

$\therefore \frac{(a_1+a_3+...+a_{19})-11a_2}{121}=239$

23. If $\lim\limits_{x\to 0}\left(\frac{\tan x}{x}\right)^{\frac{1}{x^2}} = p$, then $96 \log_e p$ is equal to ___

Ans. (32)

Sol. $P = \lim\limits_{x\to 0}\left(\frac{\tan x}{x}\right)^{\frac{1}{x^2}}$

$\Rightarrow P = e^{\lim\limits_{x\to 0}\left(\frac{\tan x - x}{x^3}\right)}$

$= e^{\lim\limits_{x\to 0}\frac{\left(x+\frac{x^3}{3}+\frac{2x^5}{15}+.....-x\right)}{x^3}}$

$= e^{1/3}$

$\therefore 96\log_e^p = 96 \times \frac{1}{3} = 32$

24. Let $\vec{a} = \hat{i}+2\hat{j}+\hat{k}, \vec{b} = 3\hat{i}-3\hat{j}+3\hat{k}$, $\vec{c} = 2\hat{i}-\hat{j}+2\hat{k}$ and $\vec{d}$ be a vector such that $\vec{b}\times\vec{d} = \vec{c}\times\vec{d}$ and $\vec{a}\cdot\vec{d} = 4$. Then $|(\vec{a}\times\vec{d})|^2$ is equal to _____.

Ans. (128)

Sol. $\vec{b}\times\vec{d} = \vec{c}\times\vec{d}$ -and $\vec{a}.\vec{d} = 4$

$\Rightarrow \vec{d} = \lambda\left(\vec{b}-\vec{c}\right) = \lambda\left(\hat{i}-2\hat{j}+\hat{k}\right)$

$\because \vec{a}.\vec{d} = 4 \Rightarrow \lambda = -2$

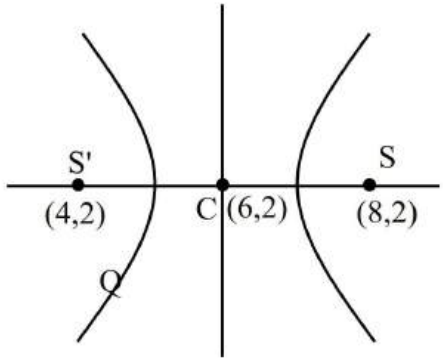
Also. $\left|\vec{a}\times\vec{d}\right|^2 + \left|\vec{a}.\vec{d}\right|^2 = \left|\vec{a}\right|^2\left|\vec{d}\right|^2$

$\Rightarrow \left|\vec{a}\times\vec{d}\right|^2 = 6\times 4\times 6 - 16 = 128$

25. If the equation of the hyperbola with foci (4, 2) and (8, 2) is $3x^2 - y^2 - \alpha x + \beta y + \gamma = 0$, then $\alpha + \beta + \gamma$ is equal to _____.

Ans. (141)

Sol.



Equation of hyperbola is

$\frac{(x-6)^2}{a^2} - \frac{(y-2)^2}{4-a^2} = 1$

$\Rightarrow (4-a^2)(x-6)^2 - a^2(y-2)^2 = a^2(4-a^2)$

comparing with $3x^2 - y^2 - \alpha x + \beta y + \gamma = 0$, we get

$a^2 = 1$ and $\alpha = 36$, $\beta = 4$ and $\gamma = 101$

$\therefore \alpha + \beta + \gamma = 141$

SECTION-A

26. A magnetic dipole experiences a torque of $80\sqrt{3}$ N m when placed in uniform magnetic field in such a way that dipole moment makes angle of 60° with magnetic field. The potential energy of the dipole is :

(1) 80 J (2) $-40\sqrt{3}$ J

(3) – 60 J (4) – 80 J

Ans. (4)

Sol. $\tau = M \times B = MB\sin 60 = \frac{\sqrt{3}}{2}MB = 80\sqrt{3}$

$MB = 160$

$U = -M.B = -MB\cos 60$

$U = -160 \times 1/2 = -80$ J

27. In the resonance experiment, two air columns (closed at one end) of 100 cm and 120 cm long, give 15 beats per second when each one is sounding in the respective fundamental modes. The velocity of sound in the air column is :

(1) 335 m/s (2) 370 m/s

(3) 340 m/s (4) 360 m/s

Ans. (4)

Sol. Fundamental frequency in close/organ pipe

$$(f) = \frac{v}{4\ell}$$

$$f_1 = \frac{v}{4\ell_1} \ \& \ f_2 = \frac{v}{4\ell_2}$$

$$\text{Beat} = (f_1 - f_2) = \frac{v}{4}\left(\frac{1}{\ell_1} - \frac{1}{\ell_2}\right)$$

$$15 = \frac{v}{4}\left(\frac{1}{1} - \frac{1}{1.2}\right)$$

$$v = \left(\frac{15 \times 4 \times 1.2}{0.2}\right) = 60 \times 6 = 360 \text{ m/s}$$

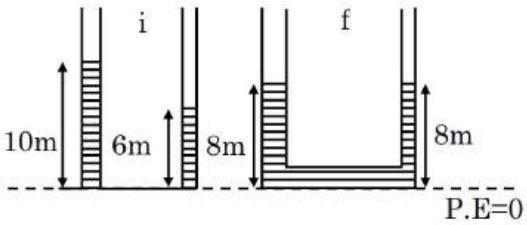
28. Two cylindrical vessels of equal cross sectional area of $2m^2$ contain water upto height 10m and 6m, respectively. If the vessels are connected at their bottom then the work done by the force of gravity is : (Density of water is 10^3 kg/m^3 and g = 10 m/s^2)

(1) 1×10^5 J (2) 4×10^4 J

(3) 6×10^4 J (4) 8×10^4 J

Ans. (4)

Sol.



$U_i = (\rho A \times 10)g \times 5 + (\rho A 6)g \times 3$

$U_i = \rho Ag(50 + 18)$

$U_i = 68\rho Ag$

$U_f = (\rho A \times 16)g \times 4$

$= (\rho Ag) \times 64$

$\omega = \Delta U = 4 \times \rho Ag$

$= 4 \times 1000 \times 2 \times 10 = 8 \times 10^4$ J

29. Width of one of the two slits in a Young's double slit interference experiment is half of the other slit. The ratio of the maximum to the minimum intensity in the interference pattern is :

(1) $(2\sqrt{2}+1):(2\sqrt{2}-1)$ (2) $(3+2\sqrt{2}):(3-2\sqrt{2})$

(3) 9 : 1 (4) 3 : 1

Ans. (2)

Sol. $I \propto$ width $\quad I_{max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$

$\therefore I_1 = I_0, I_2 = 2I_0 \quad I_{min} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2$

$$\frac{I_{max}}{I_{min}} = \frac{\left(\sqrt{2}+1\right)^2}{\left(\sqrt{2}-1\right)^2} \Rightarrow \frac{3+2\sqrt{2}}{3-2\sqrt{2}}$$

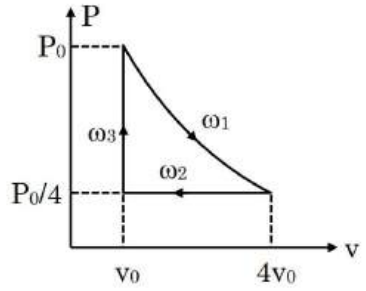
30. An ideal gas exists in a state with pressure P_0, volume V_0.It is isothermally expanded to 4 times of its initial volume (V_0), then isobarically compressed to its original volume. Finally the system is heated isochorically to bring it to its initial state. The amount of heat exchanged in this process is :

(1) $P_0V_0(2\ln 2 - 0.75)$ (2) $P_0V_0(\ln 2 - 0.75)$

(3) $P_0V_0(\ln 2 - 0.25)$ (4) $P_0V_0(2\ln 2 - 0.25)$

Ans. (1)

Sol.



$\omega_1 = P_0v_0\ell n4$

$\omega_2 = \frac{P_0}{4}(-3v_0) = -\frac{3P_0v_0}{4}$

$\omega_3 = 0$

$Q_T = \Delta U_{cyclic} + \omega$

$Q_T = \omega \qquad (\Delta U_{cyclic} = 0)$

$Q_T = P_0v_0\left(\ell n4 - \frac{3}{4}\right)$

$= P_0v_0(2\ell n2 - 0.75)$

31. Two monochromatic light beams have intensities in the ratio 1:9. An interference pattern is obtained by these beams. The ratio of the intensities of maximum to minimum is

(1) 8 : 1 (2) 9 : 1

(3) 3 : 1 (4) 4 : 1

Ans. (4)

Sol. $\frac{I_{max}}{I_{min}} = \frac{\left(\sqrt{I_1} + \sqrt{I_2}\right)^2}{\left(\sqrt{I_1} - \sqrt{I_2}\right)^2} \Rightarrow \frac{(4)^2}{(2)^2} \Rightarrow \frac{16}{4} = 4$

32. Given below are two statements : one is labelled as **Assertion A** and the other is labelled as **Reason R**.

Assertion A : The Bohr model is applicable to hydrogen and hydrogen-like atoms only.

Reason R : The formulation of Bohr model does not include repulsive force between electrons.

In the light of the above statements, choose the ***correct*** answer from the options given below :

(1) Both **A** and **R** are true but **R** is **NOT** the correct explanation of **A**.

(2) **A** is false but **R** is true.

(3) Both **A** and **R** are true and **R** is the correct explanation of **A**.

(4) **A** is true but **R** is false.

Ans. (3)

Sol. Conceptual

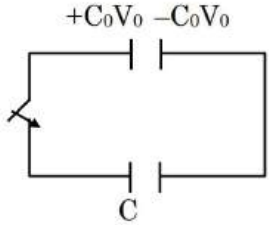
33. Using a battery, a 100 pF capacitor is charged to 60 V and then the battery is removed. After that, a second uncharged capacitor is connected to the first capacitor in parallel. If the final voltage across the second capacitor is 20 V, its capacitance is : (in pF)

(1) 600 (2) 200

(3) 400 (4) 100

Ans. (2)

Sol.



New potential $= \frac{C_0V_0}{C_0 + C} = \frac{V_0}{3}$

$3C_0V_0 = C_0V_0 + CV_0$

$2C_0V_0 = CV_0$

$C \Rightarrow 2C_0$

34. A monochromatic light of frequency 5×10^{14} Hz travelling through air, is incident on a medium of refractive index '2'. Wavelength of the refracted light will be :

(1) 300 nm (2) 600 nm

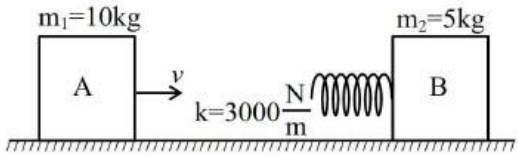
(3) 400 nm (4) 500 nm

Ans. (1)

Sol. $f\lambda = v \quad \lambda_{medium} = \dfrac{\lambda_{vacuum}}{\mu}$

$$\lambda_{medium} \Rightarrow \frac{3\times10^8}{2\times5\times10^{14}} \Rightarrow 0.3 \times 10^{-6} \Rightarrow 300 \text{ nm}$$

35.



Consider two blocks A and B of masses $m_1 = 10$ kg and $m_2 = 5$ kg that are placed on a frictionless table. The block A moves with a constant speed $v = 3$ m/s towards the block B kept at rest. A spring with spring constant k = 3000 N/m is attached with the block B as shown in the figure. After the collision, suppose that the blocks A and B, along with the spring in constant compression state, move together, then the compression in the spring is, (Neglect the mass of the spring)

(1) 0.2 m (2) 0.4 m

(3) 0.1 m (4) 0.3 m

Ans. (3)

Sol. $m_1v_1 + m_2v_2 = (m_1 + m_2)v_{cm}$

$$v_{cm} \Rightarrow \frac{10\times3}{10+5} \Rightarrow \frac{30}{15} = 2\text{m/s}$$

$$\frac{1}{2}kx^2 = \frac{1}{2}(10)(3)^2 - \left[\frac{1}{2}(15)(2)^2\right]$$

$$\Rightarrow 90 - 60 = 30 = 3000\,x^2$$

$$x^2 \Rightarrow \frac{30}{3000} = \frac{1}{100}$$

$$x \Rightarrow \frac{1}{10}\text{m}.$$

36. A particle is projected with velocity u so that its horizontal range is three times the maximum height attained by it. The horizontal range of the projectile is given as $\dfrac{nu^2}{25g}$, where value of n is :

(Given 'g' is the acceleration due to gravity).

(1) 6 (2) 18

(3) 12 (4) 24

Ans. (4)

Sol. Range = $3H_{max}$

$$\frac{u^2\sin2\theta}{g} = \frac{3u^2\sin^2\theta}{2g}$$

$$2\sin\theta\cos\theta = \frac{3}{2}\sin^2\theta$$

$$\tan\theta = \frac{4}{3} \Rightarrow \theta = 53°$$

$$R = \frac{u^2\left(2\times\frac{3}{5}\times\frac{4}{5}\right)}{g} \Rightarrow \frac{24u^2}{25g}$$

37. A solid steel ball of diameter 3.6 mm acquired terminal velocity 2.45×10^{-2} m/s while falling under gravity through an oil of density 925 kg m^{-3}. Take density of steel as 7825 kg m^{-3} and g as 9.8 m/s^2. The viscosity of the oil in SI unit is

(1) 2.18 (2) 2.38

(3) 1.68 (4) 1.99

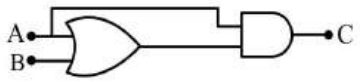
Ans. (4)

Sol. $v_T \Rightarrow \dfrac{2}{9}\dfrac{(\rho_0 - \rho_\ell)r^2g}{\eta}$

$$\eta = \frac{2}{9}\left(\frac{7825-925}{2.45\times10^{-2}}\right)\times(1.8)^2\times10^{-6}\times9.8$$

$$\eta \approx 1.99$$

38. The truth table corresponding to the circuit given below is



(1)

A	B	C
0	0	0
1	0	0
0	1	0
1	1	1

(2)

A	B	C
0	0	0
0	1	0
1	0	1
1	1	1

(3)

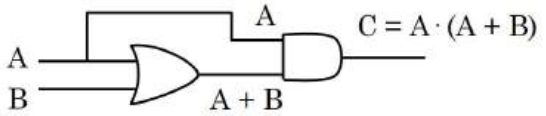
A	B	C
0	0	1
1	0	0
0	1	0
1	1	0

(4)

A	B	C
0	0	1
0	1	0
1	0	0
1	1	0

Ans. (2)

Sol.



A	B	A+B	C
0	0	0	0
1	0	1	1
0	1	1	0
1	1	1	1

39. A particle moves along the x-axis and has its displacement x varying with time t according to the equation

$$x = c_0(t^2 - 2) + c(t-2)^2$$

where c_0 and c are constants of appropriate dimensions. Then, which of the following statements is correct?

(1) the acceleration of the particle is $2c_0$

(2) the acceleration of the particle is 2c

(3) the initial velocity of the particle is 4c

(4) the acceleration of the particle is $2(c + c_0)$

Ans. (4)

Sol. $v = \frac{dx}{dt} = 2tC_0 + 2C(t-2)$

$a = \frac{dv}{dt} = 2C_0 + 2C$

40. An electric bulb rated as 100 W-220 V is connected to an ac source of rms voltage 220 V. The peak value of current through the bulb is :

(1) 0.64 A (2) 0.45 A

(3) 2.2 A (4) 0.32 A

Ans. (1)

Sol. $P = v_{rms} i_{rms}$

$i_{rms} = \frac{100}{220}$

$i_0 = \sqrt{2} i_{rms} = 0.64A$

41. Match the **LIST-I** with **LIST-II**

LIST-I		LIST-II	
A.	Boltzmann constant	I.	ML^2T^{-1}
B.	Coefficient of viscosity	II.	$MLT^{-3}K^{-1}$
C.	Planck's constant	III.	$ML^2T^{-2}K^{-1}$
D.	Thermal conductivity	IV.	$ML^{-1}T^{-1}$

Choose the ***correct*** answer from the options given below :

(1) A-III, B-IV, C-I, D-II

(2) A-II, B-III, C-IV, D-I

(3) A-III, B-II, C-I, D-IV

(4) A-III, B-IV, C-II, D-I

Ans. (1)

Sol. (A) $[k] = \frac{PV}{NT} = \frac{ML^2T^{-2}}{K} = ML^2T^{-2}K^{-1}$

(B) $[\eta] = \frac{F}{6\pi rv} = \frac{MLT^{-2}}{L^2T^{-1}} = ML^{-1}T^{-1}$

(C) $[h] = \frac{E}{f} = \frac{ML^2T^{-2}}{T^{-1}} = ML^2T^{-1}$

(D) $\frac{dQ}{dt} = k\frac{AdT}{dx}$

$k = \frac{(ML^2T^{-3})L}{L^2.K} = MLT^{-3}K^{-1}$

42. Pressure of an ideal gas, contained in a closed vessel, is increased by 0.4% when heated by 1°C. Its initial temperature must be :

(1) 25°C (2) 2500 K
(3) 250 K (4) 250°C

Ans. (3)

Sol. Isochoric process

$P \propto T$

$$\frac{\Delta P}{P} = \frac{\Delta T}{T}$$

$$\frac{0.4}{100} = \frac{1}{T}$$

$T = 250$ K

43. A motor operating on 100 V draws a current of 1 A. If the efficiency of the motor is 91.6%, then the loss of power in units of cal/s is

(1) 4 (2) 8.4
(3) 2 (4) 6.2

Ans. (3)

Sol. $P_{input} = Vi = 100$ W

$$\eta = \frac{P_{out}}{P_{input}} = 0.916$$

$P_{out} = 91.6$ W

Loss = 100 – 91.6 = 8.4 J/s = 2 cal/s

44. A block of mass 1 kg, moving along x with speed v_i = 10 m/s enters a rough region ranging from x = 0.1 m to x = 1.9 m. The retarding force acting on the block in this range is $F_r = -kx$ N, with k = 10 N/m. Then the final speed of the block as it crosses rough region is

(1) 10 m/s (2) 4 m/s
(3) 6 m/s (4) 8 m/s

Ans. (4)

Sol. $a = \frac{F}{m} = -10x$

$$v\frac{dv}{dx} = -10x$$

$$\int_{10}^{v} v\,dv = -10\int_{0.1}^{1.9} x\,dx$$

$$\frac{v^2 - 100}{2} = -10\left(\frac{1.9^2 - 0.1}{2}\right)^2$$

v = 8 m/s

45. Given below are two statements : one is labelled as **Assertion A** and the other is labelled as **Reason R**.

Assertion A : If oxygen ion (O^{-2}) and Hydrogen ion (H^+) enter normal to the magnetic field with equal momentum, then the path of O^{-2} ion has a smaller curvature than that of H^+.

Reason R : A proton with same linear momentum as an electron will form a path of smaller radius of curvature on entering a uniform magnetic field perpendicularly.

In the light of the above statement, choose the ***correct*** answer from the options given below

(1) **A** is true but **R** is false
(2) Both **A** and **R** are true but **R** is **NOT** the correct explanation of **A**
(3) **A** is false but **R** is true
(4) Both **A** and **R** are true and **R** is the correct explanation of **A**

Ans. (1)

Sol. $r = \frac{mv}{qB} = \frac{p}{qB}$

$$r \propto \frac{1}{q}$$

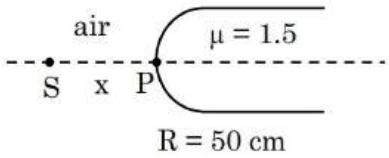
Assertion is true reason is false

SECTION-B

46. Light from a point source in air falls on a spherical glass surface (refractive index, μ = 1.5 and radius of curvature = 50 cm). The image is formed at a distance of 200 cm from the glass surface inside the glass. The magnitude of distance of the light source from the glass surface is _____m.

Ans. (4)

Sol.



$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

$$\frac{1.5}{200} - \frac{1}{-x} = \frac{1.5 - 1}{50}$$

$$\frac{1}{x} = \frac{1}{100} - \frac{3}{400}$$

x = 400 cm

x = 4m

47. The excess pressure inside a soap bubble A in air is half the excess pressure inside another soap bubble B in air. If the volume of the bubble A is *n* times the volume of the bubble B, then, the value of *n* is _____.

Ans. (8)

Sol. $\Delta P = \frac{4T}{R}$

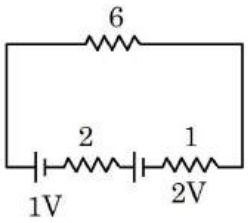
$$\frac{R_A}{R_B} = \frac{\Delta P_B}{\Delta P_A} = 2$$

$$\frac{V_A}{V_B} = \left(\frac{R_A}{R_B}\right)^3 = 8$$

48. Two cells of emf 1V and 2V and internal resistance 2 Ω and 1 Ω, respectively, are connected in series with an external resistance of 6 Ω. The total current in the circuit is I_1. Now the same two cells in parallel configuration are connected to same external resistance. In this case, the total current drawn is I_2. The value of $\left(\frac{I_1}{I_2}\right)$ is $\frac{x}{3}$. The value of x is _____.

Ans. (4)

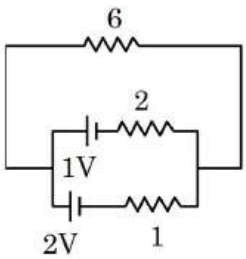
Sol.



$\varepsilon_{eq} = 3$

$R_{eq} = 9$

$i_1 = \frac{3}{9} = \frac{1}{3}$



$$\varepsilon_{eq} = \frac{\frac{\varepsilon_1}{r_1} + \frac{\varepsilon_2}{r_2}}{\frac{1}{r_1} + \frac{1}{r_2}}$$

$$\varepsilon_{eq} = \frac{\frac{1}{2} + \frac{2}{1}}{\frac{1}{2} + \frac{1}{1}} = \frac{5}{3}$$

$$r_{equ} = \frac{2 \times 1}{3} + 6 = \frac{20}{3}$$

$$i_2 = \frac{1}{4} \Rightarrow \frac{i_1}{i_2} = \frac{4}{3}$$

49. An electron in the hydrogen atom initially in the fourth excited state makes a transition to n^{th} energy state by emitting a photon of energy 2.86 eV. The integer value of n will be _____.

Ans. (2)

Sol. $E = 13.6\left(\frac{1}{n_1^2} - \frac{1}{n_1^2}\right)$

$$2.86 = 13.6\left(\frac{1}{n^2} - \frac{1}{5^2}\right)$$

$$\frac{1}{n^2} = 0.21 + \frac{1}{2.5}$$

$n^2 = 4$

$n = 2$

Ans. (2)

50. A physical quantity C is related to four other quantities p, q, r and s as follows

$$C = \frac{pq^2}{r^3\sqrt{s}}$$

The percentage errors in the measurement of p, q, r and s are 1%, 2% 3% and 2% respectively.

The percentage error in the measurement of C will be _____%.

Ans. (15)

Sol. $C = P^1 q^2 r^{-3} s^{1/2}$

$$\left(\frac{dC}{C}\right)_{max} = \frac{dP}{P} + \frac{2dq}{q} + \frac{3dr}{r} + \frac{1}{2}\frac{ds}{s}$$

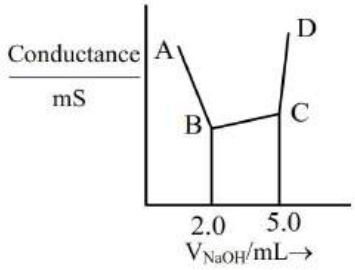
$$= (1 + 2 \times 2 + 3 \times 3 + \frac{1}{2} \times 2)\%$$

$= 15\ \%$

Ans. 15

SECTION-A

51. 40 mL of a mixture of CH_3COOH and HCl (aqueous solution) is titrated against 0.1 M NaOH solution conductometrically. Which of the following statement is **correct**?



(1) The concentration of CH_3COOH in the original mixture is 0.005 M

(2) The concentration of HCl in the original mixture is 0.005 M

(3) CH_3COOH is neutralised first followed by neutralisation of HCl

(4) Point 'C' indicates the complete neutralisation HCl

Ans. (2)

Sol. From the given graph 2 ml NaOH solution is used for neutralisation of HCl and 3 ml NaOH solution is used for neutralisation of CH_3COOH.

∴ Mole of HCl = Mole of NaOH used

$M \times 40 = 0.1 \times 2$

$M = 0.005$

∴ Mole of CH_3COOH = Mole of NaOaH used

$M \times 40 = 0.1 \times 3$

$M = 0.0075$

HCl is strong acid and will be neutralised first.

52. 10 mL of 2 M NaOH solution is added to 20 mL of 1 M HCl solution kept in a beaker. Now, 10 mL of this mixture is poured into a volumetric flask of 100 mL containing 2 moles of HCl and made the volume upto the mark with distilled water. The solution in this flask is :

(1) 0.2 M NaCl solution

(2) 20 M HCl solution

(3) 10 M HCl solution

(4) Neutral solution

Ans. (2)

Sol. When 10 ml, 2M NaOH solution is added to 20 ml of 1M HCl solution :

	NaOH	+	HCl → NaCl + H_2O
Initial :	MV = 2 × 0.1 = 0.2 mole		MV = 1 × 0.2 = 0.2 mole
Final	0		0

∴ Resulting solution becomes neutral.

Now when 10 mol of above solution is poured into a flask containing 2 mole HCl and made solution 100 ml will distilled water.

Molarity of HCl = $\frac{2}{100} \times 1000 = 20$

53. Fat soluble vitamins are :

A. Vitamin B_1

B. Vitamin C

C. Vitamin E

D. Vitamin B_{12}

E. Vitamin K

Choose the ***correct*** answer from the options given below :

(1) C & D Only

(2) A & B Only

(3) B & C Only

(4) C & E Only

Ans. (4)

Sol. Vit D, E, K. A are fat soluble vitamins.

54. Match the **LIST-I** with **LIST-II**.

	LIST-I (Family)		LIST-II (Symbol of Element)
A.	Pnicogen (group 15)	I.	Ts
B.	Chalcogen	II.	Og
C.	Halogen	III.	Lv
D.	Noble gas	IV.	Mc

Choose the ***correct*** answer from the options given below :

(1) A-IV, B-I, C-II, D-III

(2) A-IV, B-III, C-I, D-II

(3) A-III, B-I, C-IV, D-II

(4) A-II, B-III, C-IV, D-I

Ans. (2)

Sol. (A) Pnictogen ⇒ Mc (Moscovium),

Atomic No. = 115

(B) Chalcogen ⇒ Lv (Livermorium),

Atomic No. = 116

(C) Halogen ⇒ Ts (Tennessine),

Atomic No. = 117

(D) Noble gas ⇒ Og (Oganesson),

Atomic No. = 118

55. For electron in '2s' and '2p' orbitals, the orbital angular momentum values, respectively are :

(1) $\sqrt{2}\frac{h}{2\pi}$ and 0 (2) $\frac{h}{2\pi}$ and $\sqrt{2}\frac{h}{2\pi}$

(3) 0 and $\sqrt{6}\frac{h}{2\pi}$ (4) 0 and $\sqrt{2}\frac{h}{2\pi}$

Ans. (4)

Sol. Orbital angular momentum = $\sqrt{\ell(\ell+1)}\frac{h}{2\pi}$

∴ For 2s orbital : $\ell = 0$

Orbital angular momentum = 0

∴ For 2p orbital : $\ell = 1$

Orbital angular momentum = $\sqrt{1(1+2)}\frac{h}{2\pi}$

$= \sqrt{2}\frac{h}{2\pi}$

56. Compounds that should not be used as primary standards in titrimetric analysis are :

A. $Na_2Cr_2O_7$

B. Oxalic acid

C. NaOH

D. $FeSO_4 . 6H_2O$

E. Sodium tetraborate

Choose the ***most appropriate*** answer from the options given below:

(1) B and D Only (2) D and E Only

(3) C, D and E Only (4) A, C and D Only

Ans. (4)

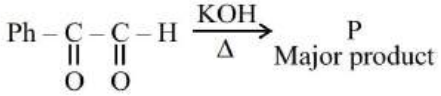
Sol. The primary standard is a highly pure stable compound with a known exact composition that can be accurately weighed and dissolved to creat a solution of known concentration.

NaOH is hygroscopic and can't be used.

$FeSO_4.6H_2O$ is unstable and can be easily oxidised.

$Na_2Cr_2O_7$ is hygroscopic and can't be used.

57. The major product (P) in the following reaction is :

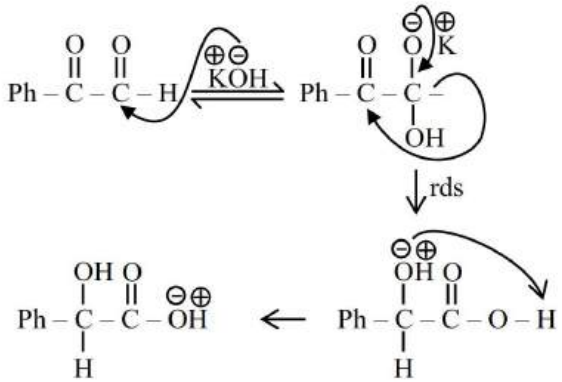


(1) $Ph-\underset{}{\overset{OH}{CH}}-CH_2OH$ (2) $Ph-\underset{OH}{CH}-COO^-K^+$

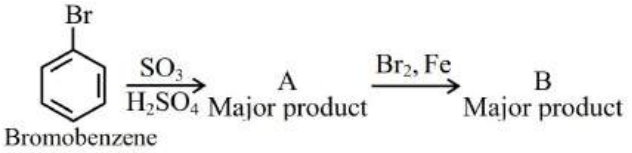
(3) $Ph-\underset{O}{\underset{\|}{C}}-COO^-K^+$ (4) $Ph-\underset{O}{\underset{\|}{C}}-CH_2OH$

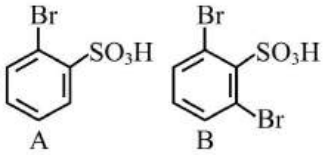
Ans. (2)

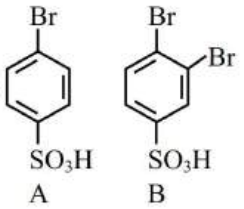
Sol.



58. In the following series of reactions identify the major products A & B respectively.



(1)


(2)


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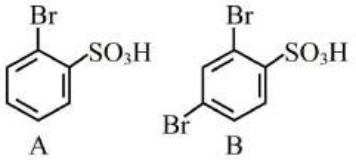
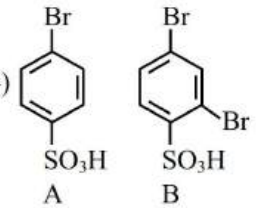
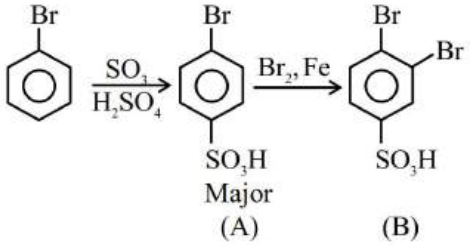


(4)


Ans. (2)

Sol.


Both reactions are electrophilic substitution reaction, I^{st} is sulphonation and IInd is halogenation :

59. The standard cell potential ($E^{\ominus}_{cell}$) of a fuel cell based on the oxidation of methanol in air that has been used to power television relay station is measured as 1.21 V. The standard half cell reduction potential for O_2 ($E^{o}_{O_2/H_2O}$) is 1.229 V.

Choose the correct statement:

(1) The standard half cell reduction potential for the reduction of CO_2 ($E^{o}_{CO_2/CH_3OH}$) is 19 mV

(2) Oxygen is formed at the anode.

(3) Reactants are fed at one go to each electrode.

(4) Reduction of methanol takes place at the cathode.

Ans. (1)

Sol. $\because E^{o}_{cell} = E^{o}_{cathode} - E^{o}_{Anode}$

$1.21 = 1.229 - E^{o}_{Anode}$

$\because$ Fuel cell involves oxidation of methanol which will occur at anode and reduction of O_2 will occur at cathode.

60. Identify the diamagnetic octahedral complex ions from below ;

A. $[Mn(CN)_6]^{3-}$ B. $[Co(NH_3)_6]^{3+}$

C. $[Fe(CN)_6]^{4-}$ D. $[Co(H_2O)_3F_3]$

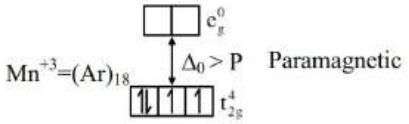
Choose the ***correct*** answer from the options given below :

(1) B and D Only (2) A and D Only

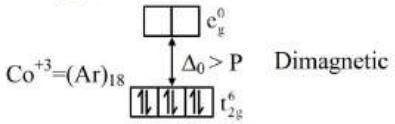
(3) A and C Only (4) B and C Only

Ans. (4)

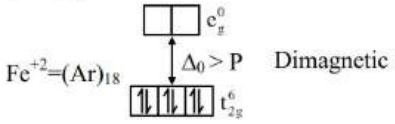
Sol. (A) $[Mn(CN)_6]^{3-}$



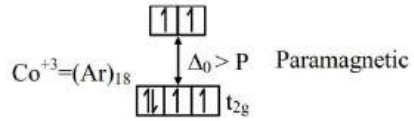
(B) $[Co(NH_3)_6]^{3+}$



(C) $[Fe(CN)_6]^{4-}$



(D) $[Co(H_2O)_3F_3]$



61. In Dumas' method for estimation of nitrogen 0.4 g of an organic compound gave 60 mL of nitrogen collected at 300 K temperature and 715 mm Hg pressure. The percentage composition of nitrogen in the compound is

(Given : Aqueous tension at 300 K = 15 mm Hg)

(1) 15.71% (2) 20.95%

(3) 17.46% (4) 7.85%

Ans. (1)

Sol. Pressure of N_2 gas evolved = 715 – 15

= 700 mm Hg

$= \frac{700}{760}$ atm.

$\therefore$ Mole of N_2 evolved $= \frac{PV}{RT}$

$= \frac{700 \times 60 \times 10^{-3}}{760 \times 0.0821 \times 300}$

= 0.0022 mole

$\therefore$ wt. of N_2 evolved = 0.0022 × 28 = 0.063 gm

$\therefore$ wt. % of nitrogen in compound

$= \frac{\text{wt. of nitrogen}}{\text{wt. of compound}} \times 100$

$= \frac{0.063}{0.4} \times 100$

= 15.71%

62. Mass of magnesium required to produce 220 mL of hydrogen gas at STP on reaction with excess of dil. HCl is

Given : Molar mass of Mg is 24 g mol^{-1}.

(1) 235.7 g (2) 0.24 mg

(3) 236 mg (4) 2.444 g

Ans. (3)

Sol. $Mg + 2HCl \rightarrow MgCl_2 + H_2$

Volume H_2 evolved = 220 ml

Mole of $H_2 = \frac{220 \times 10^{-3}}{22.4}$ = mole of Mg used

$\therefore$ Mass of Mg used $= \frac{220 \times 10^{-3}}{22.4} \times 24$

$= 235.7 \times 10^{-3}$ gm

= 235.7 mg

63. Given below are two statements :

Statement I : Wet cotton clothes made of cellulose based carbohydrate takes comparatively longer time to get dried than wet nylon polymer based clothes.

Statement II : Intermolecular hydrogen bonding with water molecule is more in nylon-based clothes than in the case of cotton clothes.

In the light of above statements, choose the ***Correct*** answer from the options given below

(1) Statement I is false but Statement II is true

(2) Statement I is true but Statement II is false

(3) Both Statement I and Statement II are true

(4) Both Statement I and Statement II are false

Ans. (2)

Sol. Cellulose derivcative has more number of hydroxy groups, so more H-bonding is present with water in cellulose derivatives cotton cloths.

64. Given below are two statements :

Statement I : CrO_3 is a stronger oxidizing agent than MoO_3

Statement II : Cr(VI) is more stable than Mo(VI)

In the light of the above statements, choose the ***correct*** answer from the options given below

(1) Statement I is false but Statement II is true

(2) Statement I is true but Statement II is false

(3) Both Statement I and Statement II are true

(4) Both Statement I and Statement II are false

Ans. (2)

Sol. Statement-I is true but statement II is false.

Cr(VI) is less stable than Mo(VI)

Hence, CrO_3 easily reduce into Cr^{+3} as compared to MoO_3 and show stronger oxidizing nature.

65. Given below are two statements :

Statement I : Hyperconjugation is not a permanent effect.

Statement II : In general, greater the number of alkyl groups attached to a positively charged C-atom, greater is the hyperconjugation interaction and stabilization of the cation.

In the light of the above statements, choose the ***correct*** answer from the options given below

(1) Statement I is true but Statement II is false

(2) Both Statement I and Statement II are false

(3) Statement I is false but Statement II is true

(4) Both Statement I and Statement II are true

Ans. (3)

Sol. Hyper conjugation is permanent effect because external reagent is not required, so Statement-I is false and Statement-II is true. because moore alkyl group, more α-H, so more hyperconjugation which results more stability of carbocation.

66. Given below are two statements :

Statement I : When a system containing ice in equilibrium with water (liquid) is heated, heat is absorbed by the system and there is no change in the temperature of the system until whole ice gets melted.

Statement II : At melting point of ice, there is absorption of heat in order to overcome intermolecular forces of attraction within the molecules of water in ice and kinetic energy of molecules is not increased at melting point.

In the light of the above statements, choose the **correct** answer from the options given below:

(1) Statement I is true but Statement II is false

(2) Both Statement I and Statement II are false

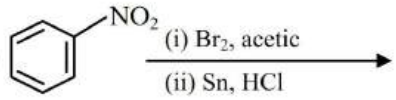
(3) Both Statement I and Statement II are true

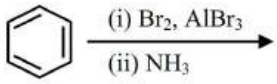
(4) Statement I is false but Statement II is true

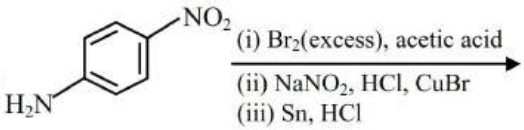
Ans. (3)

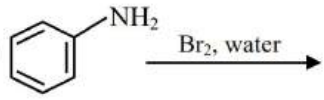
Sol. At melting point when ice melts, supplied heat is utilised to overcome intermolecular attraction within the molecules so temperature remain constant.

67. The sequence from the following that would result in giving predominantly 3, 4, 5 –Tribromoaniline is :

(1) 

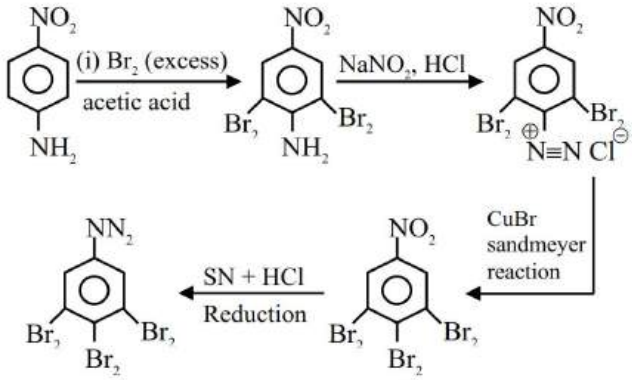
(2) 

(3) 

(4) 

Ans. (3)

Sol.



3,4,5-tri bromo aniline

68. The correct orders among the following are

Atomic radius : B < Al < Ga < In < Tl

Electronegativity : Al < Ga < In < Tl < B

Density : Tl < In < Ga < Al < B

1st Ionisation Energy : In < Al < Ga < Tl < B

Choose the correct answer from the options given below :

(1) B and D Only (2) A and C Only

(3) C and D Only (4) A and B Only

Ans. (1)

Sol.

	B	Al	Ga	In	Tl
Atomic radius (pm)	88	143	135	167	170
Electronegativity	2	1.5	1.6	1.7	1.8
Density (g/cm^3)	2.35	2.7	5.9	7.31	11.85
Ionisation Energy (kJ/mol)	801	577	579	558	589

Radius Order $T\ell > In > A\ell > Ga > B$

EN Order $B > T\ell > In > Ga > Al$

Density Order $T\ell > In > Ga > A\ell > B$

IE_1 Order $B > T\ell > Ga > A\ell > In$

69. What is the correct IUPAC name of

?

(1) 3-Bromo-2-hydroxy-5-nitrobenzoic acid

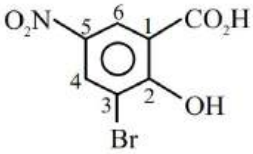
(2) 3-Bromo-4-hydroxy-1-nitrobenzoic acid

(3) 2-Hydroxy-3-bromo-5-nitrobenzoic acid

(4) 5-Nitro-3-bromo-2-hydroxybenzoic acid

Ans. (1)

Sol.



IUPAC 3-Bromo-2-hydroxy-5-nitro-Benzoic acid

70. Consider the following statements related to temperature dependence of rate constants.

Identify the correct statements,

A. The Arrhenius equation holds true only for an elementary homogenous reaction.

B. The unit of A is same as that of k in Arrhenius equation.

C. At a given temperature, a low activation energy means a fast reaction.

D. A and Ea as used in Arrhenius equation depend on temperature.

E. When Ea >> RT. A and Ea become interdependent.

Choose the **correct** answer from the options given below :

(1) A, C and D Only (2) B, D and E Only

(3) B and C Only (4) A and B Only

Ans. (3)

Sol. Arrhenious equation hold true for elementary as well as complex reactions.

Unit of A is same as unit of k. Rate of reaction is high if activation energy is low,

A and Ea are temperature independent.

SECTION-B

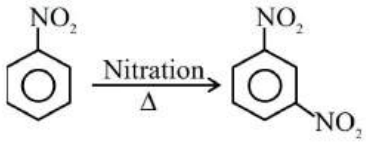
71. X g of nitrobenzene on nitration gave 4.2 g of m-dinitrobenzene.

X = _____ g. (nearest integer)

[Given : molar mass (in g mol^{-1}) C : 12, H : 1, O : 16, N : 14]

Ans. (3)

Sol.



4.2 gm

$C_6H_5NO_2$ $MF = C_6H_4N_2O_4$

MW = 123 MW = 168

$$\therefore \frac{4.2}{168} = 0.025\,mol$$

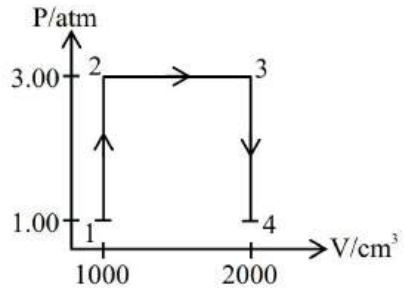
$\because$ required gm of nitro benzene

$= 123 \times 0.025$

$= 3.075$

$\therefore$ Nearest integer is 3

72.



A perfect gas (0.1 mol) having $\overline{C}_v$ = 1.50 R (independent of temperature) undergoes the above transformation from point 1 to point 4. If each step is reversible, the total work done (w) while going from point 1 to point 4 is (–) ___ J (nearest integer) [Given : R = 0.082 L atm K^{-1} mol^{-1}]

Ans. (304)

Sol. $W_{1\to2} = 0$

$W_{2\to3} = -P\Delta V$

$= -3\ [2-1]$

$= -3$ atm – ℓ

$W_{3\to4} = 0$

Total work done

$= -3$ atm – ℓ

$= -3 \times 101.3$ Joule

$= -304$ Joule

73. A sample of n-octane (1.14 g) was completely burnt in excess of oxygen in a bomb calorimeter, whose heat capacity is 5 kJ K^{-1}. As a result of combustion reaction, the temperature of the calorimeter is increased by 5 K. The magnitude of the heat of combustion of octane at constant volume is _____ kJ mol^{-1} (nearest integer).

Ans. (2500)

Sol. Mole of octane = $\frac{1.14}{114}$ = 0.01 mole

Heat evolved = C × ΔT

= 5 × 5 kJ

=25 kJ

∴ Magnitude of Heat of combustion = $\frac{25}{0.01}$ =2500 kJ/mole

74. Among, Sc, Mn, Co and Cu, identify the element with highest enthalpy of atomisation. The spin only magnetic moment value of that element in its +2 oxidation state is _____ BM (in nearest integer).

Ans. (4)

Sol.

	Sc	Mn	Co	Cu
Enthalpy of Atomisation (kJ/mole)	326	281	425	339

Highest Co

Co^{+2}=(Ar)$3d^7$

↑↓	↑↓	↑	↑	↑

n = 3

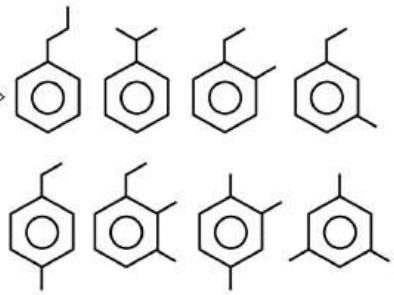
$\mu = \sqrt{15} = 3.87$

Nearest integer = 4

75. The total number of structural isomers possible for the substituted benzene derivatives with the molecular formula C_9H_{12} is _____.

Ans. (8)

Sol. MF = C_9H_{12}

⇒ 

JEE (Main)-2025 (Online) Session-2

Question Paper with Solutions

(Mathematics, Physics, And Chemistry)

3 April 2025 Shift – 1

Time: 3 hrs. **M.M : 300**

IMPORTANT INSTRUCTIONS:

(1) The test is of **3 hours** duration.

(2) This test paper consists of 75 questions. Each subject (PCM) has 25 questions. The maximum marks are 300.

(3) This question paper contains Three Parts. Part-A is Physics, Part-B is Chemistry and Part-C is Mathematics. Each part has only two sections: Section-A and Section-B.

(4) Section - A : Attempt all questions.

(5) Section - B : Attempt all questions.

(6) Section - A (01 - 20) contains 20 multiple choice questions which have only one correct answer. Each question carries +4 marks for correct answer and -1 mark for wrong answer.

(7) Section - B (21 – 25) contains 5 Numerical value based questions. The answer to each question should be rounded off to the nearest integer. Each question carries +4 marks for correct answer and -1 mark for wrong answer.

SECTION-A

1. Let A be a matrix of order 3 x 3 and $|A| = 5$. If $\left|2\text{adj}\left(3A\,\text{adj}(2A)\right)\right| = 2^{\alpha}.3^{\beta}.5^{\gamma}$ $\alpha,\ \beta,\ \gamma \in N$ then $\alpha + \beta + \gamma$ is equal to

(1) 25 (2) 26

(3) 27 (4) 28

Ans. (3)

Sol. |2 adj (3A adj(2A))|

$2^3.|3A\ \text{adj}(2A)|^2$

$2^3.(3^3)^2.|A|^2.|\text{adj}(2A)|^2$

$2^3.3^6.|A|^2.(|2A|^2)^2$

$2^3.3^6.|A|^2[(2^3)^2.|A|^2]^2$

$2^3.3^6.|A|^2.2^{12}.|A|^4$

$2^{15}.3^6.|A|^6$

$2^{15}.3^6.5^6 = 2^{\alpha}.3^{\beta}.5^{\gamma}$

$\alpha = 15,\quad \beta = 6,\quad \gamma = 6$

$\alpha + \beta + \gamma = 27$

Option (3)

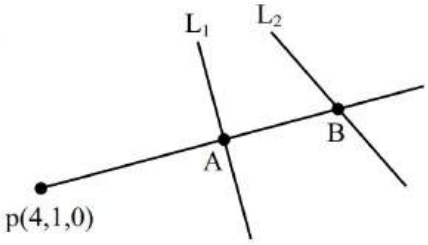
2. Let a line passing through the point (4,1,0) intersect the line $L_1; \frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ at the point A $(\alpha,\ \beta,\ \gamma)$ and the line $L_2 : x - 6 = y = -z + 4$ at the point B (a, b, c). Then $\begin{vmatrix} 1 & 0 & 1 \\ \alpha & \beta & \gamma \\ a & b & c \end{vmatrix}$ is equal to

(1) 8 (2) 16

(3) 12 (4) 6

Ans. (1)

Sol.



$L_1 = \frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} = p$

$L_2 = \frac{x-6}{1} = \frac{y}{1} = \frac{z-4}{-1} = q$

A(2p + 1, 3p + 2, 4p + 3)

B(q + 6, q, 4 – q)

D.R. of PA = 2p – 3, 3P + 1, 4p + 3

D.R. of PB = q + 2, q – 1, 4 – q

$\frac{2p-3}{q+2} = \frac{3p+1}{q-1} = \frac{4p+3}{4-q}$

2pq – 2p – 3q + 3 = 3pq + 6p + q + 2

pq + rp + 4q – 1 = 0 ....(1)

12p – 3pq + 4 – q = 4pq + 3q – 4p – 3

7pq – 16p + 4q – 7 = 0 ....(2)

8p – 2pq – 12 + 3q = 4pq + 8p + 3q + 6

6pq = –18 $\therefore$ $\boxed{pq = -3}$

8p + 4q = 4 $\Rightarrow$ 2p + q = 1

–21 – 16p + 4q – 7 $\Rightarrow$ 4p – q = –7

16p – 4q = –28 $\therefore$ p = –1, q = 3

A(–1, –1, –1) B(9, 3, 1)

$\begin{vmatrix} 1 & 0 & 1 \\ -1 & -1 & -1 \\ 9 & 3 & 1 \end{vmatrix} = \begin{vmatrix} 0 & -1 & 0 \\ -1 & -1 & -1 \\ 9 & 3 & 1 \end{vmatrix} = 1(-1+9) = 8$

Option (1)

3. Let α and β be the roots of $x^2+\sqrt{3x}-16=0$, and γ and δ be the roots of $x^2 + 3x - 1 = 0$. If $P_n = \alpha^n + \beta^n$ and $Q_n = \gamma^n + \delta^n$, then $\frac{P_{25}+\sqrt{3P_{24}}}{2P_{23}} + \frac{Q_{25}-Q_{23}}{Q_{24}}$ is equal to

(1) 3 (2) 4

(3) 5 (4) 7

Ans. (3)

Sol. $x^2+\sqrt{3}x-16=0 \begin{cases} \alpha \\ \beta \end{cases}$ $P_n = \alpha^n + \beta^n$

$P_n + \sqrt{3}P_{n-1} - 16P_{n-2} = 0$

$P_{25} + \sqrt{3}P_{24} - 16P_{23} = 0$

$\therefore \frac{P_{25}+\sqrt{3}P_{24}}{2P_{23}} = 8$

Similarly

$x^2 + 3x - 1 = 0 \begin{cases} \gamma \\ \delta \end{cases}$ $Q_n = \gamma^n + \delta^n$

$Q_{25} - Q_{23} = \gamma^{25} + \delta^{25} - \gamma^{23} - \delta^{23}$

$= \gamma^{23}(\gamma^2 - 1) + \delta^{23}(\delta^2 - 1)$

$= \gamma^{23}(-3\gamma) + \delta^{23}(-3\gamma)$

$= -3[\gamma^{24} + \delta^{24}]$

$= -3\, Q_{24}$

$\therefore \frac{Q_{25}-Q_{23}}{Q_{24}} = -3$

$\frac{P_{25}+\sqrt{3}P_{24}}{2P_{23}} + \frac{Q_{25}-Q_{23}}{Q_{24}} = 8 - 3 = 5$

Option (3)

4. The sum of all rational terms in the expansion of $\left(2+\sqrt{3}\right)^8$ is

(1) 16923 (2) 3763

(3) 33845 (4) 18817

Ans. (4)

Sol. $S = \left(2+\sqrt{3}\right)^8$

For sum of rational terms

$= {}^8C_0(2)^8 + {}^8C_2(2)^6.\left(\sqrt{3}\right)^2 + {}^8C_4(2)^4\left(\sqrt{3}\right)^4$

$+ {}^8C_6(2)^2\left(\sqrt{3}\right)^6 + {}^8C_8\left(\sqrt{3}\right)^8$

$= 2^8 + 28 \times 2^6.3 + 70.2^4.9 + 28.2^2.27 + 81$

$= 256 + 5376 + 10080 + 3024 + 81$

$= 18817$

Option (4)

5. Let $A = \{-3,-2,-1,0,1,2,3,\}$. Let R be a relation on A defined by xRy if and only if $0 \le x^2 + 2y \le 4$. Let l be the number of elements in R and m be the minimum number of elements required to be added in R to make it a reflexive relation. then $l + m$ is equal to

(1) 19 (2) 20

(3) 17 (4) 18

Ans. (4)

Sol. $A = \{-3, -2, -1, 0, 1, 2, 3\}$

$-2y \le x^2 \le 4 - 2y$

$y = -3$ $\quad 6 \le x^2 \le 10$ $\quad \Rightarrow x \in \{-3, 3\}$

$y = -2$ $\quad 4 \le x^2 \le 8$ $\quad \Rightarrow x \in \{-2, 2\}$

$y = -1$ $\quad 2 \le x^2 \le 6$ $\quad \Rightarrow x \in \{-2, 2\}$

$y = 0$ $\quad 0 \le x^2 \le 4$ $\quad \Rightarrow x \in \{-2,-1,0,1,2\}$

$y = 1$ $\quad -2 \le x^2 \le 2$ $\quad \Rightarrow x \in \{-1, 0, 1\}$

$y = 2$ $\quad -4 \le x^2 \le 0$ $\quad \Rightarrow x \in \{0\}$

$y = 3$ $\quad -6 \le x^2 \le -2$ $\quad \Rightarrow$ No x-Exist

R = {(–3,–3) (–3,3), (–2,–2) (–2,2) (–1,–2) (–1,2) (0,–2) (0,–1) (0, 0) (0,1) (0,2) (1, –1) (1,0) (1,1) (2,0)}

$\therefore \ell = 15$

To make it reflexive we will add

{(–1, –1), (2,2), (3,3)} $\quad \therefore m = 3$

$\therefore \ell + m = 15 + 3 = 18$

Option (4)

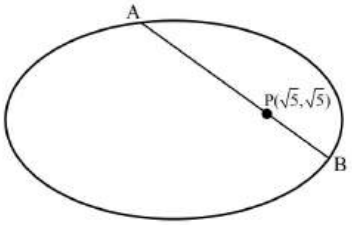
6. A line passing through the point $P\left(\sqrt{5},\sqrt{5}\right)$ intersects the ellipse $\frac{x^2}{36}+\frac{y^2}{25}=1$ at A and B such that $(PA).(PB)$ is maximum. Then $5(PA^2+PB^2)$ is equal to :

(1) 218 (2) 377

(3) 290 (4) 338

Ans. (4)

Sol. Given ellipse is $\frac{x^2}{36}+\frac{y^2}{25}=1$



Any point on line AB can be assumed as

$Q(\sqrt{5}+r\cos\theta,\sqrt{5}+r\sin\theta)$

Putting this in equation of ellipse, we get

$25(\sqrt{5}+r\cos\theta)^2+36(\sqrt{5}+r\sin\theta)^2=900$

Simplifying, we get

$r^2(25\cos^2\theta+36\sin^2\theta)+2\sqrt{5}r(25\cos\theta+36\sin\theta)-595=0$

$|r| = PA, PB$

Thus, $PA\cdot PB=\frac{595}{25\cos^2\theta+36\sin^2\theta}=\frac{595}{25+11\sin^2\theta}$

$=$ maximum, if $\sin^2\theta = 0$

This means line AB must be parallel to x-axis

$\Rightarrow y_A = y_B = \sqrt{5}$

Putting $y=\sqrt{5}$ in equation of ellipse, we get

$\frac{x^2}{36}+\frac{1}{5}=1\Rightarrow x^2=36.\frac{4}{5}$

Hence,

$PA^2+PB^2=\left(\sqrt{5}-\frac{12}{\sqrt{5}}\right)^2+\left(\sqrt{5}+\frac{12}{\sqrt{5}}\right)^2$

$=2\left(5+\frac{144}{5}\right)=\frac{338}{5}$

$5(PA^2+PB^2)=338$

7. The sum 1 + 3 + 11 + 25 + 45 + 71 + .. upto 20 terms, is equal to

(1) 7240 (2) 7130

(3) 6982 (4) 8124

Ans. (1)

Sol. Given sum is

$S_n = 1+3+11+25+45+71+...+T_n$

First order differences are in A.P.

Thus, we can assume that

$T_n = an^2+bn+c$

Solving $\begin{Bmatrix} T_1=1=a+b+c \\ T_2=3=4a+2b+c \\ T_3=11=9a+3b+c \end{Bmatrix}$,

we get a = 3, b = –7, c = 5

Hence, general term of given series is

$T_n = 3n^2-7n+5$

Hence, required sum equals

$\sum_{n=1}^{n=20}(3n^2-7n+5)=3\left(\frac{20\cdot21\cdot41}{6}\right)-7\left(\frac{20\cdot21}{2}\right)+5(20)=7240$

8. If the domain of the function $f(x)=\log_e\left(\frac{2x-3}{5+4x}\right)+\sin^{-1}\left(\frac{4+3x}{2-x}\right)$ is $[\alpha, \beta)$, then $\alpha^2+4\beta$ is equal to

(1) 5 (2) 4

(3) 3 (4) 7

Ans. (2)

Sol. Given function is

$f(x)=\log_e\left(\frac{2x-3}{5+4x}\right)+\sin^{-1}\left(\frac{4+3x}{2-x}\right)$

For domain, the conditions are

$\frac{2x-3}{5+4x}>0$ and $\left|\frac{4+3x}{2-x}\right|\le 1$

Now, $\frac{2x-3}{5+4x}>0 \Rightarrow x\in\left(-\infty,-\frac{5}{4}\right)\cup\left[\frac{3}{2},\infty\right)$

and $-1\le\frac{4+3x}{2-x}\le 1$

$\Rightarrow\left(-1\le\frac{4+3x}{2-x}\right)\cap\left(\frac{4+3x}{2-x}\le 1\right)$

$\Rightarrow\left(\frac{6+2x}{2-x}\ge 0\right)\cap\left(\frac{2+4x}{2-x}\le 0\right)$

$$\Rightarrow \frac{6+2x}{2-x}\cdot\frac{2+4x}{2-x} \le 0$$

$$\Rightarrow x \in \left[-3, -\frac{1}{2}\right]$$

Hence, we get the domain of f as $x \in \left[-3, -\frac{5}{4}\right)$

This means that $\alpha = -3, \beta = -\frac{5}{4}$

Thus, $\alpha^2 + 4\beta = 9 - 5 = 4$

9. If $\sum_{r=1}^{9}\left(\frac{r+3}{2^r}\right).{}^9C_r = \alpha\left(\frac{3}{2}\right)^9 - \beta$, $\alpha, \beta \in N$, then $(\alpha+\beta)^2$ is equal to

(1) 27 (2) 9

(3) 81 (4) 18

Ans. (3)

Sol. Given that

$$\sum_{r=1}^{9}\left(\frac{r+3}{2^r}\right)\cdot{}^9C_r = \alpha\left(\frac{3}{2}\right)^9 - \beta, \alpha, \beta \in N$$

Now,

$$\sum_{r=1}^{9}\left(\frac{r+3}{2^r}\right)\cdot{}^9C_r = \sum_{r=1}^{9}\left(\frac{r}{2^r}\right)\cdot{}^9C_r + \sum_{r=1}^{9}\left(\frac{3}{2^r}\right)\cdot{}^9C_r$$

$$= \sum_{r=1}^{9}\left(\frac{9}{2^r}\right)\cdot{}^8C_{r-1} + 3\sum_{r=1}^{9}{}^9C_r\left(\frac{1}{2}\right)^r \quad \left[\text{Using } \frac{{}^9C_r}{{}^8C_{r-1}} = \frac{9}{r}\right]$$

$$= \frac{9}{2}\sum_{r=1}^{9}{}^8C_{r-1}\left(\frac{1}{2}\right)^{r-1} + 3\left(\sum_{r=0}^{9}\left({}^9C_r\left(\frac{1}{2}\right)^r\right) - 1\right)$$

$$= \frac{9}{2}\left(1+\frac{1}{2}\right)^8 + 3\left(\left(1+\frac{1}{2}\right)^9 - 1\right)$$

$$= \frac{9}{2}\cdot\left(\frac{3}{2}\right)^8 + 3\left(\frac{3}{2}\right)^9 - 3 = 6.\left(\frac{3}{2}\right)^9 - 3$$

Hence, $\alpha = 6, \beta = 3$

Thus $(\alpha+\beta)^2 = 81$

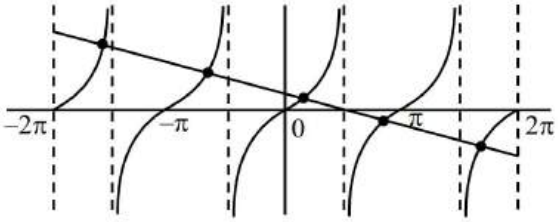
10. The number of solutions of the equation $2x + 3\tan x = \pi$, $x \in [-2\pi, 2\pi] - \left\{\pm\frac{\pi}{2}, \pm\frac{3\pi}{2}\right\}$ is

(1) 6 (2) 5

(3) 4 (4) 3

Ans. (2)

Sol. $\tan x = \frac{\pi}{3} - \frac{2x}{3}$



5 solutions

11. If $y(x) = \begin{vmatrix} \sin x & \cos x & \sin x + \cos x + 1 \\ 27 & 28 & 27 \\ 1 & 1 & 1 \end{vmatrix}, x \in \mathbb{R}$,

then $\frac{d^2y}{dx^2} + y$ is equal to

(1) –1 (2) 28

(3) 27 (4) 1

Ans. (1)

Sol. $C_3 \to C_3 - C_1$

$$y(x) = \begin{vmatrix} \sin x & \cos x & 1+\cos x \\ 27 & 28 & 0 \\ 1 & 1 & 0 \end{vmatrix}$$

$y(x) = -(1 + \cos x)$

$\frac{dy}{dx} = \sin x$

$\frac{d^2y}{dx^2} = \cos x$

$$\boxed{\frac{d^2y}{dx^2} + y = -1}$$

12. Let g be a differentiable function such that $\int_0^x g(t)dt = x - \int_0^x tg(t)dt$, $x \ge 0$ and let $y = y(x)$ satisfy the differential equation $\frac{dy}{dx} - y\tan x = 2(x+1)\sec x\, g(x)$, $x \in \left[0, \frac{\pi}{2}\right)$. If $y(0) = 0$, then $y\left(\frac{\pi}{3}\right)$ is equal to

(1) $\frac{2\pi}{3\sqrt{3}}$ (2) $\frac{4\pi}{3}$

(3) $\frac{2\pi}{3}$ (4) $\frac{4\pi}{3\sqrt{3}}$

Ans. (2)

Sol. Diff. w.r.t. x

$g(x) = 1 - xg(x)$

$g(x) = \frac{1}{1+x}$

so $\frac{dy}{dx} - y\tan x = 2\sec x$

$IF = e^{-\int \tan dx} = e^{\log \cos x} = \cos x$

solution of D.E.

$y\cos x = \int 2dx + c$

$y\cos x = 2x + c$

$y(0) = 0$

$\boxed{c = 0}$

$y = \frac{2x}{\cos x}$

$y = 2x \sec x$

$\boxed{y\left(\frac{\pi}{3}\right) = 2.\frac{\pi}{3}.2 = \frac{4\pi}{3}}$

13. A line passes through the origin and makes equal angles with the positive coordinate axes. It intersects the lines

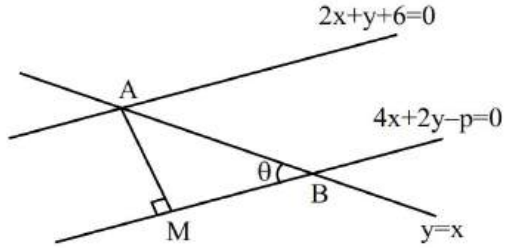
$L_1 : 2x + y + 6 = 0$ and $L_2 : 4x + 2y - p = 0, p > 0,$ at the points A and B, respectively. If $AB = \frac{9}{\sqrt{2}}$ and the foot of the perpendicular from the point A on the line L_2 is M, then $\frac{AM}{BM}$ is equal to

(1) 5 (2) 4

(3) 2 (4) 3

Ans. (4)

Sol.



Line is $y = x$

$m_1 = 1, m_2 = -2$

so $\tan\theta = \left|\frac{1+2}{1-2}\right|$

$\boxed{\tan\theta = \frac{AM}{BM} = 3}$

14. Let $z \in C$ be such that $\frac{z^2 + 3i}{z - 2 + i} = 2 + 3i$. Then the sum of all possible values of z^2 is

(1) $19 - 2i$ (2) $-19 - 2i$

(3) $19 + 2i$ (4) $-19 + 2i$

Ans. (2)

Sol. $z^2 + 3i = z(2 + 3i) - 7 - 4i$

$z^2 - z(2 + 3i) + 7 + 7i = 0 \begin{cases} z_1 \\ z_2 \end{cases}$

$z_1^2 + z_2^2 = (z_1 + z_2)^2 - 2z_1z_2$

$= 4 - 9 + 12i - 14 - 14i$

$= -19 - 2i$

15. Let $f(x) = \int x^3\sqrt{3 - x^2}\, dx$. If $5f\left(\sqrt{2}\right) = -4$, then $f(1)$ is equal to

(1) $-\frac{2\sqrt{2}}{5}$ (2) $-\frac{8\sqrt{2}}{5}$

(3) $-\frac{4\sqrt{2}}{5}$ (4) $-\frac{6\sqrt{2}}{5}$

Ans. (4)

Sol. Let $3 - x^2 = t^2$

$+x\, dx = -t\, dt$

$f(x) = \int (3 - t^2).t(-t\, dt) + c$

$= \int (t^4 - 3t^2) dt + c$

$= \frac{t^5}{5} - t^3 + c$

$f(x) = \frac{(3 - x^2)^{5/2}}{5} - (3 - x^2)^{3/2} + c$

$f\left(\sqrt{2}\right) = \frac{1}{5} - 1 + c = -\frac{4}{5}$

$\boxed{c = 0}$

$f(1) = \frac{2^{5/2}}{5} - 2^{3/2}$

$= 2^{1/2}\left(\frac{4}{5} - 2\right)$

$\boxed{f(1) - \frac{-6\sqrt{2}}{5}}$

16. Let $a_1, a_2, a_3, \ldots$ be a G. P. of increasing positive numbers. If $a_3a_5 = 729$ and $a_2 + a_4 = \frac{111}{4}$, then $24(a_1 + a_2 + a_3)$ is equal to

(1) 131 (2) 130

(3) 129 (4) 128

Ans. (3)

Sol. Let the Ist term of G.P. be a & common ratio be r

$a_3a_5 = ar^2.ar^4 = 729$

$= a^2r^6 = 729$

$= ar^3 = 27$ ...(i)

$a_2 + a_4 = ar + ar^3 = \frac{111}{4}$

$= ar = \frac{3}{4}$ ...(ii)

(i) ÷ (ii)

$\frac{ar^3}{ar} = \frac{27}{3/4}$

$r^2 = 36$

$r = 6$

from (ii)

$a(6) = \frac{3}{4} \Rightarrow a = \frac{1}{8}$

Now, $24(a_1 + a_2 + a_3)$

$= 24(a + ar + ar^2)$

$= 24a(1 + r + r^2)$

$= 24 \times \frac{1}{8}(1 + 6 + 36)$

$= 3(43)$

$= 129$

17. Let the domain of the function

$f(x) = \log_2 \log_4 \log_6 (3 + 4x - x^2)$ be (a, b). If

$\int_0^{b-a} \left[x^2\right] dx = p - \sqrt{q} - \sqrt{r}, p, q, r \in \mathbb{N}$, gcd(p, q, r) = 1,

where [·] is the greatest integer function,

then p + q + r is equal to

(1) 10 (2) 8

(3) 11 (4) 9

Ans. (1)

Sol. $\log_4\log_6(3 + 4x - x^2) > 0$

$\log_6(3 + 4x - x^2) > 1$

$3 + 4x - x^2 > 6$

$x^2 - 4x + 3 < 0$

$(x - 1)(x - 3) < 0$

$x \in (1, 3)$

so a = 1 & b = 3

$\Rightarrow \int_0^2 [x^2]dx = ?$

$I = \int_0^1 [x^2]dx + \int_1^{\sqrt{2}} [x^2]dx + \int_{\sqrt{2}}^{\sqrt{3}} [x^2]dx + \int_{\sqrt{3}}^{\sqrt{4}} [x^2]dx$

$= 0 + |x|_1^{\sqrt{2}} + 2|x|_{\sqrt{2}}^{\sqrt{3}} + 3|x|_{\sqrt{3}}^{\sqrt{4}}$

$= (\sqrt{2} - 1) + 2(\sqrt{3} - \sqrt{2}) + 3(2 - \sqrt{3})$

$= 5 - \sqrt{2} - \sqrt{3} \Rightarrow p + q + r = 10$

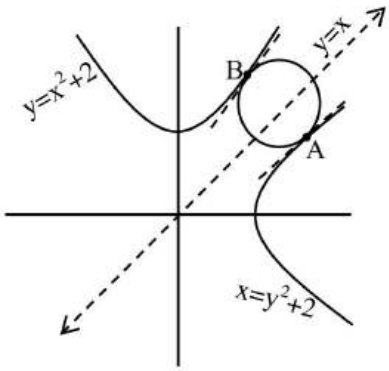
18. The radius of the smallest circle which touches the parabolas $y = x^2 + 2$ and $x = y^2 + 2$ is

(1) $\frac{7\sqrt{2}}{2}$ (2) $\frac{7\sqrt{2}}{16}$

(3) $\frac{7\sqrt{2}}{4}$ (4) $\frac{7\sqrt{2}}{8}$

Ans. (4)

Sol. The given parabolas are symmetric about the line y = x.



Tangents at A & B must be parallel to y= x line, so slope of the tangents = 1

$\left(\frac{dy}{dx}\right)_{\min A} = 1 = \left(\frac{dy}{dx}\right)_{\min B}$

For point B, $y = x^2 + 2$

$$\frac{dy}{dx} = 2x = 1$$

$$x = \frac{1}{2} \Rightarrow y = \frac{9}{4}$$

$$\therefore \text{ Point B} = \left(\frac{1}{2}, \frac{9}{4}\right) \Rightarrow \text{Point A} = \left(\frac{9}{4}, \frac{1}{2}\right)$$

$$AB = \sqrt{\left(\frac{1}{2} - \frac{9}{4}\right)^2 + \left(\frac{9}{4} - \frac{1}{2}\right)^2}$$

$$= \sqrt{\frac{98}{16}} = \frac{7\sqrt{2}}{4}$$

$$\text{Radius} = = \frac{7\sqrt{2}}{8}$$

19. Let $f(x) = \begin{cases} (1+ax)^{1/x} & , \ x < 0 \\ 1+b & , \ x = 0 \\ \dfrac{(x+4)^{1/2} - 2}{(x+c)^{1/3} - 2} & , \ x > 0 \end{cases}$

be continuous at x = 0. Then e^a bc is equal to

(1) 64 (2) 72

(3) 48 (4) 36

Ans. (3)

Sol. $f(0^-) = e^{\lim_{x \to 0} \frac{ax}{x}} = e^a$

$f(0) = 1 + b$

$$f(0^+) = \frac{\dfrac{1}{2\sqrt{x+4}}}{\dfrac{1}{3}(x+c)^{-\frac{2}{3}}} = \frac{\dfrac{1}{2(2)}}{\dfrac{1}{3}.c^{-\frac{2}{3}}}$$

$$= \frac{3}{4}c^{2/3}$$

Also at x = 0 ;

$c^{1/3} = 2 \Rightarrow c = 8$

So $f(0^+) = \frac{3}{4}(8)^{2/3} = 3$

Now, $e^a = b + 1 = 3$

$e^a.b.c = 3.2.8 = 48$

20. Line L_1 passes through the point (1, 2, 3) and is parallel to z-axis. Line L_2 passes through the point (λ, 5, 6) and is parallel to y-axis. Let for $\lambda = \lambda_1, \lambda_2, \lambda_2 < \lambda_1$, the shortest distance between the two lines be 3. Then the square of the distance of the point (λ_1, λ_2, 7) from the line L_1 is

(1) 40 (2) 32

(3) 25 (4) 37

Ans. (3)

Sol. $L_1 \equiv \frac{x-1}{0} = \frac{y-2}{0} = \frac{z-3}{1}$

$L_2 \equiv \frac{x-\lambda}{0} = \frac{y-5}{1} = \frac{z-6}{0}$

$$SD = \frac{\begin{vmatrix} \lambda - 1 & 3 & 3 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{vmatrix}}{\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{vmatrix}}$$

$= |\lambda - 1| = 3$

$\lambda = 4, -2$

$\lambda_1 = 4$

$\lambda_2 = -2$

Let foot of perpendicular from

P(4, –2, 7) is Q(1, 2, t + 3)

So (3, –4, 4 – t) . (0, 0, 1) = 0

$\boxed{t = 4}$

So Q(1, 2, 7)

$PQ^2 = 9 + 16$

$\boxed{PQ^2 = 25}$

SECTION-B

21. All five letter words are made using all the letters A, B, C, D, E and arranged as in an English dictionary with serial numbers. Let the word at serial number n be denoted by W_n. Let the probability $P(W_n)$ of choosing the word W_n satisfy $P(W_n) = 2P(W_{n-1}), n > 1$.

If $P(CDBEA) = \frac{2^{\alpha}}{2^{\beta}-1}, \alpha, \beta \in \mathbb{N}$, then $\alpha + \beta$ is equal to : _____

Ans. (183)

Sol. Let $P(W_1) = x$

$$\sum_{i=1}^{120} P(W_i) = 1$$

$x + 2x + 2^2x + 2^3x + ... + 2^{119}x = 1$

$$\frac{x(2^{120}-1)}{(2-1)} = 1 \Rightarrow x = \frac{1}{2^{120}-1} \quad ....(1)$$

Rank of CDBEA

A _ _ _ _ = $\lfloor 4$ = 24

B _ _ _ _ = $\lfloor 4$ = 24

C A _ _ _ _ = $\lfloor 3$ = 6

C B _ _ _ _ = $\lfloor 3$ = 6

C D A _ _ _ = $\lfloor 2$ = 2

C D B A E = 1

C D B E A = 1

64

So, $P(W_{64}) = 2P(W_{63}) = ... = 2^{63} P(W_1)$

$$= \frac{2^{63}}{2^{120}-1}$$

$\alpha + \beta = 63 + 120 = 183$

22. Let the product of the focal distances of the point $P(4, 2\sqrt{3})$ on the hyperbola $H: \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ be 32. Let the length of the conjugate axis of H be p and the length of its latus rectum be q. Then $p^2 + q^2$ is equal to ……

Ans. (120)

Sol. $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \quad ....(1)$

$P(4, 2\sqrt{3})$

$PS_1 . PS_2 = 32$

$|PS_1 - PS_2| = 2a$

$P(4, 2\sqrt{3})$ lies on H

$\therefore \frac{16}{a^2} - \frac{12}{b^2} = 1$

$16b^2 - 12a^2 = a^2b^2 \quad ....(2)$

$|PS_1 - PS_2|^2 = 4a^2$

$PS_1^2 + PS_2^2 - 2PS_1 . PS_2 = 4a^2$

$(ae - 4)^2 + 12 + (ae + 4)^2 + 12 - 64 = 4a^2$

$2a^2e^2 - 8 = 4a^2$

$a^2 + b^2 - 4 = 2a^2$

$b^2 - a^2 = 4$

(2) & (3) $\Rightarrow 16(a^2 + 4) - 12a^2 = a^2(a^2 + 4)$

$\Rightarrow 16a^2 + 64 - 12a^2 = a^4 + 4a^2$

$\Rightarrow a^4 = 64$

$\Rightarrow a^2 = 8$

$\therefore b^2 = 12$

$$p^2 + q^2 = 4b^2 + \frac{4b^4}{a^2}$$

$= 120$

23. Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{b} = 3\hat{i} + 2\hat{j} - \hat{k}, \vec{c} = \lambda\hat{j} + \mu\hat{k}$ and $\hat{d}$ be a unit vector such that $\vec{a} \times \hat{d} = \vec{b} \times \hat{d}$ and $\vec{c}.\hat{d} = 1$, If $\vec{c}$ is perpendicular to $\vec{a}$, then $|3\lambda\hat{d} + \mu\vec{c}|^2$ is equal to _____.

Ans. (5)

Sol. $\vec{a}\times\vec{d}-\vec{b}\times\vec{d}=0$

$(\vec{a}-\vec{b})\times\vec{d}=0$

$\vec{d}=t(\vec{a}-\vec{b})$

$\vec{d}=t(-2\hat{i}-\hat{j}+2\hat{k})$

$|\vec{d}|=1$

$\boxed{|t|=\frac{1}{3}}$

$\vec{c}\,.\,\vec{a}=0$

$\lambda+\mu=0$

$\mu=-\lambda$

$\vec{c}=\lambda(\hat{j}-\hat{k})$, $\boxed{|\vec{c}|^2=2\lambda^2}$

$\vec{c}\,.\,\hat{d}=1$

$t(-2,-1,2)\,.\,\lambda(0,1,-1)=1$

$\lambda t=\frac{-1}{3}\quad\Rightarrow\boxed{\lambda^2=1}$

$|3\lambda\hat{d}+\mu\vec{c}|^2=9\lambda^2|\hat{d}|^2+\mu^2|\vec{c}|^2+6\lambda\mu(\hat{d}\,.\,\vec{c})$

$=3\lambda^2+2\lambda^4$

$=5$

24. If the number of seven-digit numbers, such that the sum of their digits is even, is $m\cdot n\cdot 10^n$; $m, n \in \{1, 2, 3, \ldots, 9\}$, then $m + n$ is equal to _____

Ans. **(14)**

Sol. Total 7 digit nos. = 9000000

7 digit nos. having sum of digits

Even = 4500000

$= 9.5.10^5$

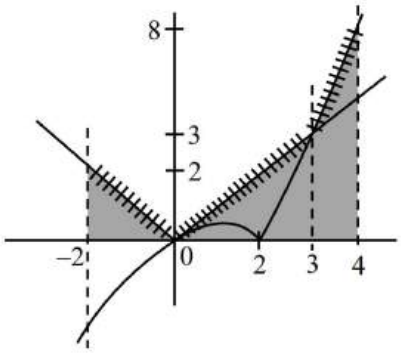
m = 9, n = 5

m + n = 14

25. The area of the region bounded by the curve $y = \max\{|x|, x|x-2|\}$, then x-axis and the lines $x = -2$ and $x = 4$ is equal to _____.

Ans. **(12)**

Sol.



Required Area $=\frac{1}{2}\times 2\times 2+\frac{1}{2}\times 3\times 3+\frac{1}{2}\times 1\times 11$

$= 12$

SECTION-A

26. During the melting of a slab of ice at 273 K at atmospheric pressure :

(1) Internal energy of ice-water system remains unchanged.

(2) Positive work is done by the ice-water system on the atmosphere.

(3) Internal energy of the ice-water system decreases.

(4) Positive work is done on the ice-water system by the atmosphere.

Ans. (4)

Sol. Volume decreases during melting of ice so positive work is done on ice water system by atmosphere

Heat absorbed by ice water so ΔQ is positive, work done by ice water system is negative

Hence by first law of thermodynamics

$\Delta U = \Delta Q + \Delta W$ = Positive

So internal energy increases

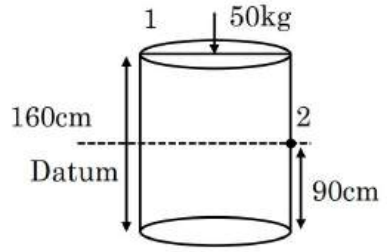
27. Consider a completely full cylindrical water tank of height 1.6 m and cross-sectional area 0.5 m^2. It has a small hole in its side at a height 90 cm from the bottom. Assume, the cross-sectional area of the hole to be negligibly small as compared to that of the water tank. If a load 50 kg is applied at the top surface of the water in the tank then the velocity of the water coming out at the instant when the hole is opened is : ($g = 10\ m/s^2$)

(1) 3 m/s (2) 5 m/s

(3) 2 m/s (4) 4 m/s

Ans. (4)

Sol.



Apply Bernouli equation between points 1 & 2

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho gh = P_2 + \frac{1}{2}\rho v_2^2 + 0$$

$$P_0 + \frac{mg}{A} + \rho g\frac{70}{100} = P_0 + \frac{1}{2}\rho v_2^2$$

$$\frac{5000}{0.5} + 10^3 \times 10\frac{70}{100} = \frac{1}{2} \times 10^3 v_2^2$$

$$10^3 + 10^3 \times 7 = \frac{10^3}{2} v_2^2$$

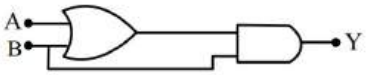
$$v_2^2 = 16$$

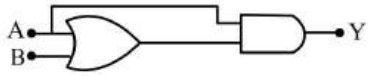
$$v_2 = 4 m/s$$

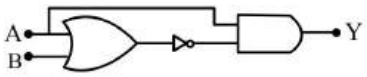
As the tank area is large v_1 is negligible compared to v_2

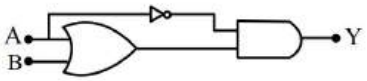
28. Choose the correct logic circuit for the given truth table having inputs A and B.

Inputs		Output
A	B	Y
0	0	0
0	1	0
1	0	1
1	1	1

(1) 

(2) 

(3) 

(4) 

Ans. (2)

Sol. Only option (2) matches with the truth table

29. The radiation pressure exerted by a 450 W light source on a perfectly reflecting surface placed at 2m away from it, is :

(1) 1.5×10^{-8} Pascals

(2) 0

(3) 6×10^{-8} Pascals

(4) 3×10^{-8} Pascals

Ans. (3)

Sol. $P_{rad} = \frac{2I}{C}$

Where I = intensity at surface

C = Speed of light

$$I = \frac{\text{Power}}{\text{Area}} = \frac{450}{4\pi r^2}$$

$$= \frac{450}{4\pi \times 4} = \frac{450}{16\pi}$$

$$P_{rad} = \frac{2 \times 450}{16\pi \times 3 \times 10^8} = \frac{150}{8\pi \times 10^8}$$

$= 5.97 \times 10^{-8} \approx 6 \times 10^{-8}$ Pascals

30. A wire of length 25 m and cross-sectional area 5 mm^2 having resistivity of $2 \times 10^{-6}\ \Omega$ m is bent into a complete circle. The resistance between diametrically opposite points will be

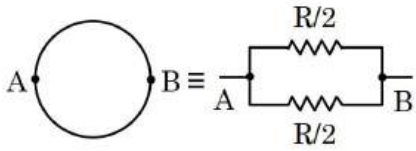
(1) 12.5 Ω (2) 50 Ω

(3) 100 Ω (4) 25 Ω

Allen Ans. (Bonus)

NTA Ans. (4)

Sol.



L = 25 m, A = 5 $mm^2 = 5 \times 10^{-6}\ m^2$

$\rho = 2 \times 10^{-6}\ \Omega m$

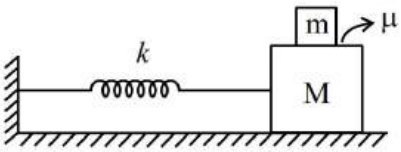
$$R_{wire} = \frac{\rho L}{A} = \frac{2 \times 10^{-6} \times 25}{5 \times 10^{-6}} = 10$$

$$R_{eq} = \frac{R}{4} = \frac{10}{4} = 2.5\Omega$$

Answer does not match with NTA option.

31. Two blocks of masses m and M, (M > m), are placed on a frictionless table as shown in figure. A massless spring with spring constant k is attached with the lower block. If the system is slightly displaced and released then

(μ = coefficient of friction between the two blocks)



(A) The time period of small oscillation of the two blocks is $T = 2\pi\sqrt{\frac{(m+M)}{k}}$

(B) The acceleration of the blocks is $a = \frac{kx}{M+m}$

(x = displacement of the blocks from the mean position)

(C) The magnitude of the frictional force on the upper block is $\frac{m\mu|x|}{M+m}$

(D) The maximum amplitude of the upper block, if it does not slip, is $\frac{\mu(M+m)g}{k}$

(E) Maximum frictional force can be $\mu(M+m)g$.

Choose the ***correct*** answer from the options given below :

(1) A, B, D Only

(2) B, C, D Only

(3) C, D, E Only

(4) A, B, C Only

Ans. (1)

Sol. (A) As both blocks moving together so

Time period = $2\pi\sqrt{\frac{m}{K}}$; where m = M + m

$$T = 2\pi\sqrt{\frac{M+m}{K}}$$

(B) Let block is displaced by x in (+ve) direction so force on block will be in(–ve) direction

$F = -Kx$

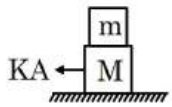
$(M + m)a = -Kx$

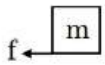
$$a = -\frac{Kx}{(M+m)}$$

(C) As upper block is moving due to friction thus

$$f = ma = \frac{mKx}{(M+m)}$$

(D) This option is like two block problem in friction for maximum amplitude, force on block is also maximum, for which both blocks are moving together.





$KA = (M + m)a$

$$a = \frac{KA}{(M+m)}$$

$$f = ma = \frac{mKA}{(M+m)}$$

$f_{max} = f_L = \mu mg$

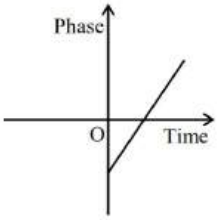
$f = \mu mg$

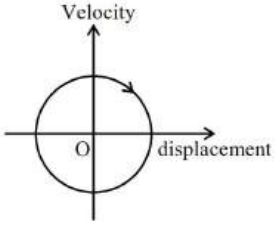
$$\frac{mKA}{(M+m)} = \mu mg$$

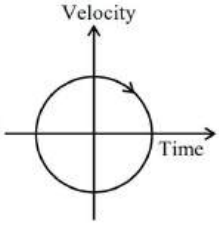
$$A = \frac{\mu(M+m)g}{K}$$

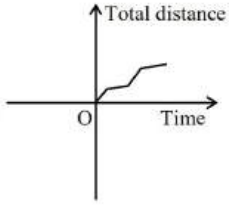
(E) Maximum friction can be μmg as force is acting between blocks & normal force here is mg.

32. Which of the following curves possibly represent one-dimensional motion of a particle ?

(A)


(B)


(C)


(D)


Choose the **correct** answer from the options given below :

(1) A, B and D only (2) A, B and C only

(3) A and B only (4) A, C and D only

Ans. (1)

Sol. For option (A)

$\phi = kt + C$ it can be 1D motion

eg $\rightarrow x = A \sin \phi$ (SHM)

For option (B)

$v^2 + x^2$ = constant yes 1D

For option (C)

time can't be negative Not possible

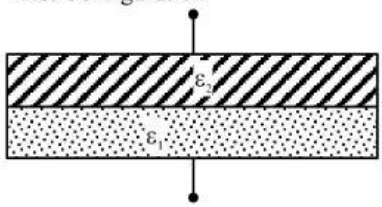
For option (D)

Possible

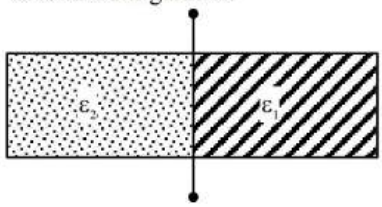
A, B & D only

33. A parallel plate capacitor is filled equally (half) with two dielectrics of dielectric constant ε_1 and ε_2, as shown in figures. The distance between the plates is d and area of each plate is A. If capacitance in first configuration and second configuration are C_1 and C_2 respectively, then $\frac{C_1}{C_2}$ is :

First Configuration



Second Configuration

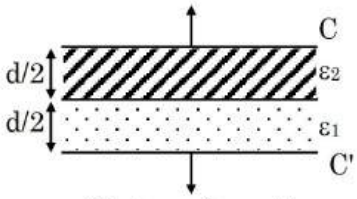


(1) $\frac{\varepsilon_1 \varepsilon_2^2}{(\varepsilon_1 + \varepsilon_2)^2}$ (2) $\frac{4\varepsilon_1 \varepsilon_2}{(\varepsilon_1 + \varepsilon_2)^2}$

(3) $\frac{\varepsilon_1 \varepsilon_2}{\varepsilon_1 + \varepsilon_2}$ (4) $\frac{\varepsilon_0 (\varepsilon_1 + \varepsilon_2)}{2}$

Ans. (2)

Sol.



First configuration

Area of plate is A.

then

$$C = \frac{\varepsilon_2 \varepsilon_0 A}{d/2} = \frac{2\varepsilon_2 \varepsilon_0 A}{d}$$

$$C' = \frac{\varepsilon_1 \varepsilon_0 A}{d/2} = \frac{2\varepsilon_1 \varepsilon_0 A}{d}$$

Let $C_0 = \frac{\varepsilon_0 A}{d}$

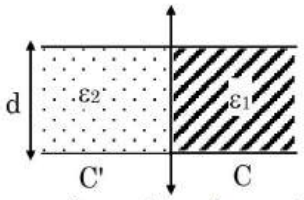
$C = 2\varepsilon_2 C_0$

$C' = 2\varepsilon_1 C_0$

C & C' are in series

$$C_1 = \frac{CC'}{C+C'} = \frac{4\varepsilon_2\varepsilon_1 C_0^2}{2C_0(\varepsilon_2+\varepsilon_1)}$$

$$= \frac{2\varepsilon_2\varepsilon_1 C_0}{(\varepsilon_2+\varepsilon_1)}$$



Second configuration

Here $C = \frac{\varepsilon_1\varepsilon_0 A}{2d} = \frac{\varepsilon_1 C_0}{2}$

$C' = \frac{\varepsilon_2 C_0}{2}$

C & C' are inparallel

$$C_2 = C' + C = (\varepsilon_1+\varepsilon_2)\frac{C_0}{2}$$

Thus $\frac{C_1}{C_2} = \frac{2\varepsilon_2\varepsilon_1 C_0}{(\varepsilon_2+\varepsilon_1)} \times \frac{2}{(\varepsilon_1+\varepsilon_2)C_0}$

$$= \frac{4\varepsilon_2\varepsilon_1}{(\varepsilon_2+\varepsilon_1)^2}$$

34. Match the **LIST-I** with **LIST-II**

	LIST-I		LIST-II
A.	Gravitational constant	I.	$[LT^{-2}]$
B.	Gravitational potential energy	II.	$[L^2T^{-2}]$
C.	Gravitational potential	III.	$[ML^2T^{-2}]$
D.	Acceleration due to gravity	IV.	$[M^{-1}L^3T^{-2}]$

Choose the ***correct*** answer from the options given below :

(1) A-IV, B-III, C-II, D-I (2) A-III, B-II, C-I, D-IV

(3) A-II, B-IV, C-III, D-I (4) A-I, B-III, C-IV, D-II

Ans. (1)

Sol. **(A)** $G = \frac{Fr^2}{m^2}$

$$[G] = \frac{[MLT^{-2}][L^2]}{[M^2]} = [M^{-1}L^3T^{-2}](IV)$$

(B) P.E. = mgh = $[MLT^{-2}L]$

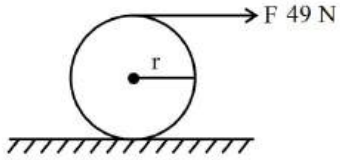
$= [ML^2T^{-2}]$ (III)

(C) Gravitational Potential = $\frac{GM}{r}$

$$= \frac{[M^{-1}L^3T^{-2}][M]}{[L]} = [M^0L^2T^{-2}] = [L^2T^{-2}](II)$$

(D) Acceleration due to gravity = [g] = $[LT^{-2}]$ (I)

35. A force of 49 N acts tangentially at the highest point of a sphere (solid) of mass 20 kg, kept on a rough horizontal plane. If the sphere rolls without slipping, then the acceleration of the center of the sphere is

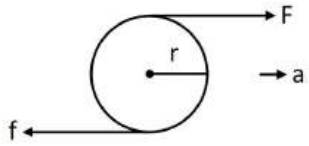


(1) 3.5 m/s^2 (2) 0.35 m/s^2

(3) 2.5 m/s^2 (4) 0.25 m/s^2

Ans. (1)

Sol.



Torque about bottom point

$F \times 2r = I\alpha$

$49 \times 2r = \frac{7}{5}mr^2\alpha$

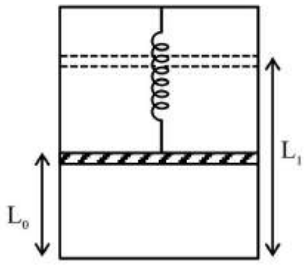
$14 = 4r\alpha$

As sphere rolls without slipping

$a = r\alpha$

$a = \frac{14}{4} = \frac{7}{2} = 3.5 m/s^2$

36. A piston of mass M is hung from a massless spring whose restoring force law goes as $F = -kx^3$, where k is the spring constant of appropriate dimension. The piston separates the vertical chamber into two parts, where the bottom part is filled with 'n' moles of an ideal gas. An external work is done on the gas isothermally (at a constant temperature T) with the help of a heating filament (with negligible volume) mounted in lower part of the chamber, so that the piston goes up from a height L_0 to L_1, the total energy delivered by the filament is (Assume spring to be in its natural length before heating)



(1) $3nRT\ln\left(\frac{L_1}{L_0}\right) + 2Mg(L_1 - L_0) + \frac{k}{3}(L_1^3 - L_0^3)$

(2) $nRT\ln\left(\frac{L_1^2}{L_0^2}\right) + \frac{Mg}{2}(L_1 - L_0) + \frac{k}{4}(L_1^4 - L_0^4)$

(3) $nRT\ln\left(\frac{L_1}{L_0}\right) + Mg(L_1 - L_0) + \frac{k}{4}(L_1^4 - L_0^4)$

(4) $nRT\ln\left(\frac{L_1}{L_0}\right) + Mg(L_1 - L_0) + \frac{3k}{4}(L_1^4 - L_0^4)$

Ans. (3)

Sol. Using WET

Total energy supplied = gravitational potential energy + spring potential energy + work done by gas

$Mg\ (L_1 - L_0) + \int_{L_0}^{L_1} kx^3dx + nRT\ell n\left[\frac{L_1A}{L_0A}\right] + W_{ext} = 0$

$\frac{K}{4}\left[x^4\right]_{L_0}^{L_1} + Mg\ (L_1 - L_0) + \int_{L_0}^{L_1} kx^3dx + nRT\ell n\left[\frac{L_1}{L_0}\right] + W_{ext} = 0$

$\frac{k}{4}\left(L_1^4 - L_0^4\right) + Mg\ (L_1 - L_0) + nRT\ell n\left[\frac{L_1}{L_0}\right] + W_{ext} = 0$

$W_{ext} = \frac{k}{4}\left(L_1^4 - L_0^4\right) + Mg\ (L_1 - L_0) + nRT\ell n\left[\frac{L_1}{L_0}\right]$

37. A gas is kept in a container having walls which are thermally non-conducting. Initially the gas has a volume of 800 cm^3 and temperature 27°C. The change in temperature when the gas is adiabatically compressed to 200 cm^3 is :

(Take $\gamma = 1.5$: γ is the ratio of specific heats at constant pressure and at constant volume)

(1) 327 K

(2) 600 K

(3) 522 K

(4) 300 K

Ans. (4)

Sol. $V_1 = 800\ cm^3$ $V_2 = 200\ cm^3$

$T_1 = 300$ K

for adiabatic

$TV^{\gamma-1} = \text{const.}$

$(300)\ (800)^{1.5-1} = T_2(200)^{1.5-1}$

$T_2 = 300\left[\frac{800}{200}\right]^{0.5} = 300 \times \left(2^2\right)^{1/2}$

$T_2 = 600$ K

$\Delta T = 600 - 300 = 300$ K

38. Match the **LIST-I** with **LIST-II**

	LIST-I		LIST-II
A.	${}_{0}^{1}n + {}_{92}^{235}U \rightarrow {}_{54}^{140}Xe + {}_{38}^{94}Sr + 2{}_{0}^{1}n$	I.	Chemical reaction
B.	$2H_2 + O_2 \rightarrow 2H_2O$	II.	Fusion with +ve Q value
C.	${}_{1}^{2}H + {}_{1}^{2}H \rightarrow {}_{2}^{3}He + {}_{0}^{1}n$	III.	Fission
D.	${}_{1}^{1}H + {}_{1}^{3}H \rightarrow {}_{1}^{2}H + {}_{1}^{2}H$	IV.	Fusion with -ve Q value

Choose the **correct** answer from the options given below :

(1) A-II, B-I, C-III, D-IV

(2) A-III, B-I, C-II, D-IV

(3) A-II, B-I, C-IV, D-III

(4) A-III, B-I, C-IV, D-II

Ans. (2)

Sol. Conceptual

39. The electrostatic potential on the surface of uniformly charged spherical shell of radius R = 10 cm is 120 V. The potential at the centre of shell, at a distance r = 5 cm from centre, and at a distance r = 15 cm from the centre of the shell respectively, are :

(1) 120V, 120V, 80V (2) 40V, 40V, 80V

(3) 0V, 0V, 80V (4) 0V, 120V, 40V

Ans. (1)

Sol. Potential inside shell is equal to potential on surface

$$V_{in} = V_{surface} = \frac{kQ}{R} = 120V$$

at r = 15 cm

$$V = \frac{kQ}{r} = \frac{120 \times 10}{15} = 80V$$

40. The work function of a metal is 3 eV. The color of the visible light that is required to cause emission of photoelectrons is

(1) Green (2) Blue

(3) Red (4) Yellow

Ans. (2)

Sol. $(KE)_{max} = \frac{hc}{\lambda} - \phi$

$$\frac{hc}{\lambda} > \phi[\text{for emission}]$$

$$\lambda < \frac{hc}{\phi} \Rightarrow \lambda < \frac{1242}{3}\text{nm}$$

So blue light option (B)

41. A particle is released from height S above the surface of the earth. At certain height its kinetic energy is three times its potential energy. The height from the surface of the earth and the speed of the particle at that instant are respectively.

(1) $\frac{S}{2}, \sqrt{\frac{3gS}{2}}$ (2) $\frac{S}{2}, \frac{3gS}{2}$

(3) $\frac{S}{4}, \frac{3gS}{2}$ (4) $\frac{S}{4}, \sqrt{\frac{3gS}{2}}$

Ans. (4)

Sol. $V^2 = 0 + 2g(S - x)$

$V^2 = 2g(S - x)$

At B, Potential energy = mgx

$$mgx = 3 \times \frac{1}{2}mv^2$$

$$gx = \frac{3}{2} \times 2g(S - x)$$

$4x = S$

$$x = \frac{S}{4}$$

$$\Rightarrow V = \sqrt{2g \times \frac{3S}{4}} = \sqrt{\frac{3gS}{2}}$$

42. A person measures mass of 3 different particles as 435.42 g, 226.3 g and 0.125 g. According to the rules for arithmetic operations with significant figures, the additions of the masses of 3 particles will be.

(1) 661.845 g

(2) 662 g

(3) 661.8 g

(4) 661.84 g

Ans. (3)

Sol. $m_1 + m_2 + m_3 = 435.42 + 226.3 + 0.125$

According to least significant digits m = 661.8 g

43. The radii of curvature for a thin convex lens are 10 cm and 15 cm respectively. The focal length of the lens is 12 cm. The refractive index of the lens material is

(1) 1.2

(2) 1.4

(3) 1.5

(4) 1.8

Ans. (3)

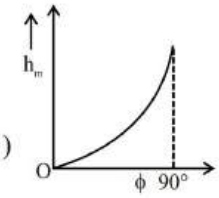
Sol. $$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

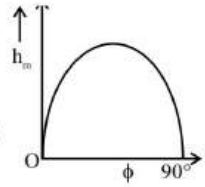
$$\frac{1}{12} = (\mu - 1)\left(\frac{1}{10} - \frac{1}{-15}\right)$$

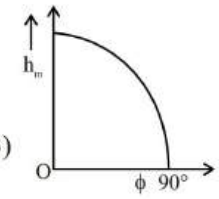
$$\frac{1}{12} = (\mu - 1)\left(\frac{3+2}{30}\right)$$

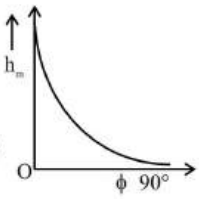
$$\mu = \frac{3}{2}$$

44. The angle of projection of a particle is measured from the vertical axis as ϕ and the maximum height reached by the particle is h_m. Here h_m as function of ϕ can be presented as

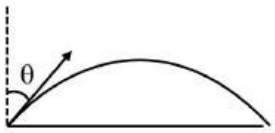
(1) 

(2) 

(3) 

(4) 

Ans. (3)

Sol. 

$$H_{max} = \frac{u^2 \cos^2 \phi}{2g}$$

45. Consider following statements for refraction of light through prism, when angle of deviation is minimum.

(A) The refracted ray inside prism becomes parallel to the base.

(B) Larger angle prisms provide smaller angle of minimum deviation.

(C) Angle of incidence and angle of emergence becomes equal.

(D) There are always two sets of angle of incidence for which deviation will be same except at minimum deviation setting.

(E) Angle of refraction becomes double of prism angle.

Choose the correct answer from the options given below.

(1) A, C and D Only (2) B, C and D Only

(3) A, B and E Only (4) B, D and E Only

Ans. (1)

Sol. $\delta = I + e - A$

For $\delta_{min} \Rightarrow I = e$

and refracted ray is parallel to base

A, C, D are correct

SECTION-B

46. Three identical spheres of mass m, are placed at the vertices of an equilateral triangle of length a. When released, they interact only through gravitational force and collide after a time T = 4 seconds. If the sides of the triangle are increased to length 2a and also the masses of the spheres are made 2m, then they will collide after _____ seconds.

Ans. (8)

Sol. $T \propto m^x G^y a^z$

$T \propto M^x \left[M^{-1}L^3T^{-2}\right]^y [L]^z$

$T \propto M^{x-y} L^{3y+z} T^{-2y}$

$x - y = 0 \Rightarrow x = y$

$-2y = 1 \Rightarrow y = -\frac{1}{2}, x = -\frac{1}{2}$

$\Rightarrow 3y + z = 0$

$z = -3y = \frac{3}{2}$

Hence

$T \propto m^{-1/2} G^{-1/2} a^{3/2}$

$T \propto \left(\frac{a^3}{m}\right)^{1/2}$

$T = 4 \times \left(\frac{2^3}{2}\right)^{1/2} = 8s$

47. A 4.0 cm long straight wire carrying a current of 8A is placed perpendicular to an uniform magnetic field of strength 0.15 T. The magnetic force on the wire is _____ mN.

Ans. (48)

Sol. $F = I\ell B$

$= 8 \times \frac{4}{100} \times 0.15$

$= 48 \times 10^{-3}$ N = 48 mN

48. Two coherent monochromatic light beams of intensities 4I and 9I are superimposed. The difference between the maximum and minimum intensities in the resulting interference pattern is xI. The value of x is _____ .

Ans. (24)

Sol. $I_{max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$

$= \left(\sqrt{4I} + \sqrt{9I}\right)^2 = 25I$

$I_{min} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2$

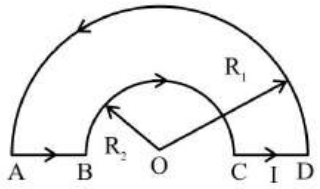
$= \left(\sqrt{4I} - \sqrt{9I}\right)^2 = I$

$I_{max} - I_{min} = 24\ I$

$x = 24$

49. A loop ABCDA, carrying current I = 12 A, is placed in a plane, consists of two semi-circular segments of radius $R_1 = 6\pi$ m and $R_2 = 4\pi$ m. The magnitude of the resultant magnetic field at center O is $k \times 10^{-7}$ T. The value of k is _____ .

(Given $\mu_0 = 4\pi \times 10^{-7}$ Tm A^{-1})



Ans. (1)

Sol. Magnetic field due to AB & CD = 0

$B_0 = \left|B_{R_1} - B_{R_2}\right|$

$= \frac{\mu_0 I}{4R_2} - \frac{\mu_0 I}{4R_1}$

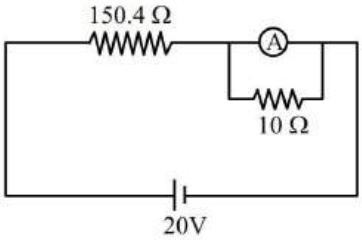
$= \frac{4\pi \times 10^{-7} \times 12}{4}\left(\frac{1}{4\pi} - \frac{1}{6\pi}\right)$

$= 12\pi \times 10^{-7}\left(\frac{1}{12\pi}\right)$

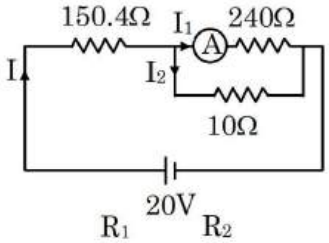
$= 1 \times 10^{-7}$

K = 1

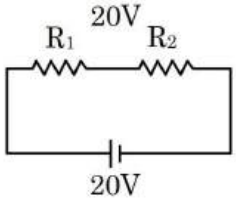
50. In the figure shown below, a resistance of 150.4 Ω is connected in series to an ammeter A of resistance 240 Ω. A shunt resistance of 10 Ω is connected in parallel with the ammeter. The reading of the ammeter is _____ mA.



Ans. (125)



Sol.



$$R_{eq} = R_1 + R_2$$

$$R_{eq} = 150.4 + \frac{240 \times 10}{250}$$

$$= 150.4 + 9.6 = 160\ \Omega$$

$$I_1 = \frac{IR_2}{240}$$

$$I_1 = \frac{I \times 9.6}{240}$$

$$= \frac{20}{160} \times \frac{9.6}{2400} = \frac{1}{200} = 5 \times 10^{-3}\ A = 5mA$$

SECTION-A

51. Which of the following postulate of Bohr's model of hydrogen atom in not in agreement with quantum mechanical model of an atom ?

(1) An atom in a stationary state does not emit electromagnetic radiation as long as it stays in the same state

(2) An atom can take only certain distinct energies E_1, E_2, E_3, etc. These allowed states of constant energy are called the stationary states of atom

(3) When an electron makes a transition from a higher energy stationary state to a lower energy stationary state, then it emits a photon of light

(4) The electron in a H atom's stationary state moves in a circle around the nucleus

Ans. (4)

Sol. The electron in a H-atom's stationary state moves in a spherical path.

52. Given below are two statements

Statement I : The N–N single bond is weaker and longer than that of P–P single bond

Statement II : Compounds of group 15 elements in +3 oxidation states readily undergo disproportionation reactions.

In the light of above statements, choose the correct answer from the options given below

(1) Statement I is true but statement II is false

(2) Both statement I and statement II are false

(3) Statement I is false but statement II is true

(4) Both statement I and statement II are true

Ans. (2)

Sol. $\ddot{N}-\ddot{N}$ single bond weaker than $\ddot{P}-\ddot{P}$ due to more ℓp-ℓp repulsion.

Bond length $\Rightarrow d_{p-p} > d_{N-N}$ (size$\uparrow$, B.L.$\uparrow$)

In group 15 elements only N & P show disproportionation in +3 oxidation state, **As**, **Sb** & **Bi** have almost inert for disproportionation in +3 oxidation state.

So both statements are false.

53. Given below are two statements

Statement I : A catalyst cannot alter the equilibrium constant (K_C) of the reaction, temperature remaining constant

Statement II : A homogenous catalyst can change the equilibrium composition of a system temperature remaining constant

In the light of the above statements, choose the **correct** answer from the options given below

(1) Statement I is false but Statement II is true

(2) Both Statement I and Statement II are true

(3) Both Statement I and Statement II is false

(4) Statement I is true but Statement II is false

Ans. (2)

Sol. A catalyst can change equilibrium composition if it is added at constant pressure, but it can not change equilibrium constant.

54. The metal ions that have the calculated spin only magnetic moment value of 4.9 B.M. are

A. Cr^{2+}

B. Fe^{2+}

C. Fe^{3+}

D. Co^{2+}

E. Mn^{3+}

Choose the **correct** answer from the options given below

(1) A, C and E only (2) A, D and E only

(3) B and E only (4) A, B and E only

Ans. (4)

Sol. Given magnetic moment = 4.9 B.M.

We know M.M = $\sqrt{n(n+2)}$B.M.

Where, n $\rightarrow$ No. of unpaired e^-

$4.9 = \sqrt{n(n+2)}$

We get n = 4

(A) $_{24}Cr^{2+} \Rightarrow [Ar]3d^4$ (4 unpaired e^-)

(B) $_{26}Fe^{2+} \Rightarrow [Ar]3d^6$ (4 unpaired e^-)

(C) $_{26}Fe^{3+} \Rightarrow [Ar]3d^5$ (5 unpaired e^-)

(D) $_{27}Co^{2+} \Rightarrow [Ar]3d^7$ (3 unpaired e^-)

(E) $_{25}Mn^{3+} \Rightarrow [Ar]3d^4$ (4 unpaired e^-)

55. In a reaction $A + B \rightarrow C$, initial concentrations of A and B are related as $[A]_0 = 8[B]_0$. The half lives of A and B are 10 min and 40 min. respectively. If they start to disappear at the same time, both following first order kinetics, after how much time will the concentration of both the reactants be same?

(1) 60 min (2) 80 min

(3) 20 min (4) 40 min

Ans. (4)

Sol. Given : $[A]_0 = 8[B]_0$

$[t_{1/2}]_A = 10$ min.

$[t_{1/2}]_B = 40$ min.

I^{st} order kinetics

t = ?

$[A]_t = [B]_t$

$-k_A \times t \quad -k_B \times t$

$\Rightarrow \quad [A]_0 e = [B]_0 e$

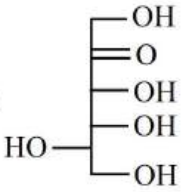
$$\Rightarrow \quad \frac{[A]_0}{[B]_0} = e^{(k_A - k_B)t}$$

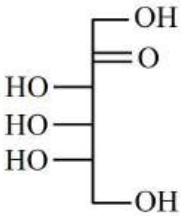
$$\Rightarrow \quad 8 = e^{(k_A - k_B) \times t}$$

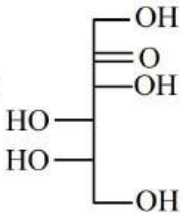
$$\Rightarrow \quad \ell n8 = (k_A - k_B) \times t$$

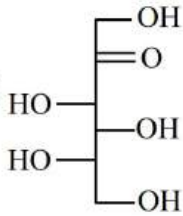
$$\Rightarrow \quad \ell n8 = \ell n2\left(\frac{1}{(ta_2)_A} - \frac{1}{(ta_2)_B}\right) \times t$$

56. Which of the following is the correct structure of L-fructose ?

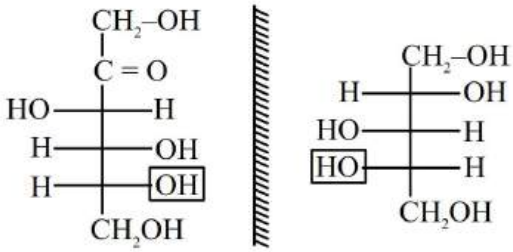
(1) 

(2) 

(3) 

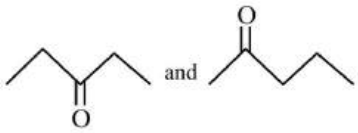
(4) 

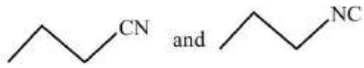
Ans. (3)

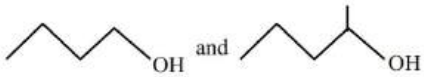
Sol. 

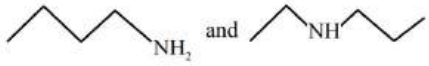
D - Fructose L - Fructose

57. Identify the correct statements from the following

A.  are metamers

B.  are functional isomers

C.  are position isomers

D.  are homologous

Choose the ***correct*** answer from the options given below

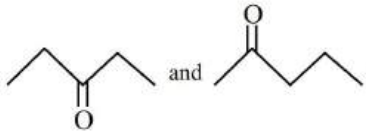
(1) C & D only

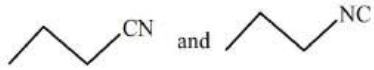
(2) B & C only

(3) A & B only

(4) A, B & C only

Ans. (3)

Sol. A.  = Metamer

B.  = F. G. isomer

In option C are momologoes to each – other and option D are only organic molecule not isomers.

58. Among 10^{-9} g (each) of the following elements, which one will have the highest number of atoms?

Element : Pb, Po, Pr and Pt

(1) Po

(2) Pr

(3) Pb

(4) Pt

Ans. (2)

Sol. No. of atoms = $\frac{\text{Mass in g}}{\text{Molar Mas(g / mol)}} \times N_A$

Therefore for the same Mass element having the least Molar mass will have the higher no. of atoms.

- $M_{Po} = 209$
- $\boxed{M_{Pr} = 141}$
- $M_{Pb} = 207$
- $M_{Pt} = 195$

59. Which of the following statements are <u>correct</u>?

A. The process of the addition an electron to a neutral gaseous atom is always exothermic

B. The process of removing an electron from an isolated gaseous atom is always endothermic

C. The 1st ionization energy of the boron is less than that of the beryllium

D. The electronegativity of C is 2.5 in CH_4 and CCl_4

E. Li is the most electropositive among elements of group I

Choose the ***correct*** answer from the options gives below

(1) B and C only

(2) A, C and D only

(3) B and D only

(4) B, C and E only

Ans. (1)

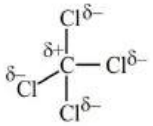
Sol. (A) The process of adding an e^- to a neutral gaseous atom is not always exothermic it may be exothermic or endothermic.

(C) Be B

$1s^2 2s^2$ $1s^2 2s^2 2p^1$

In Be 2s subshell in fully filled

So, need high energy to remove e^- as compared to B.

(D) In $CCl_4 \rightarrow$ 

due to partially positive charge $z_{eff}\uparrow$, EN$\uparrow$

So, EN of C $\Rightarrow CCl_4 > CH_4$

(E) Cs is most electropositive.

60. Which of the following properties will change when system containing solution 1 will become solution 2 ?

10 mol of solute x + 10 L of water	$\longrightarrow$	1 L of solution 1 + 1 mol of solute x + 1 L of water
(Solution 1)		(Solution 2)

(1) Molar heat capacity

(2) Density

(3) Concentration

(4) Gibbs free energy

Ans. (4)

Sol. Both solutions are having same composition, which is 1 mole of 'x' in 1'ℓ' water, so all the intensive properties will remain same, but as total amount is greater in solution '1' compared to solution '2'. So extensive properties will be different hence Gibbs free energy will be different.

61. Number of molecules from below which <u>cannot</u> give ioddoform reaction is :

Ethanol, Isopropyl alcohol, Bromoacetone, 2-Butanol, 2-Butanone, Butanal, 2-Pentanone, 3-Pentanone, Pentanal and 3-Pentanol

(1) 5

(2) 4

(3) 3

(4) 2

Ans. (2)

Sol. Following will not give iodoform reaction/test.

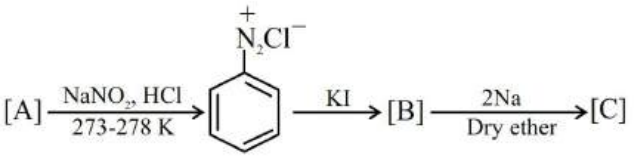
(1) Butanal

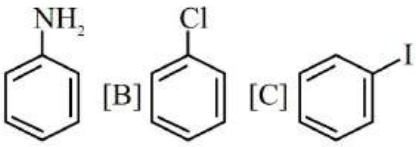
(2) 2-Pentanone

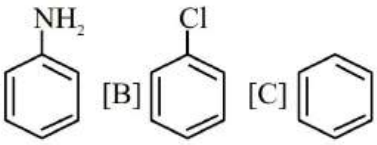
(3) Pentanal

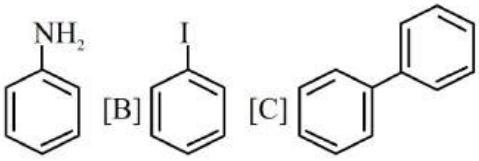
(4) 3- Pentanol

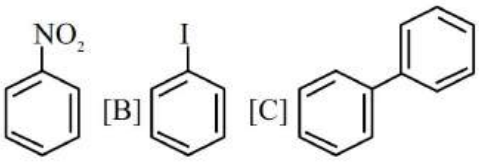
62. Identify [A], [B], and [C], respectively in the following reaction sequence :



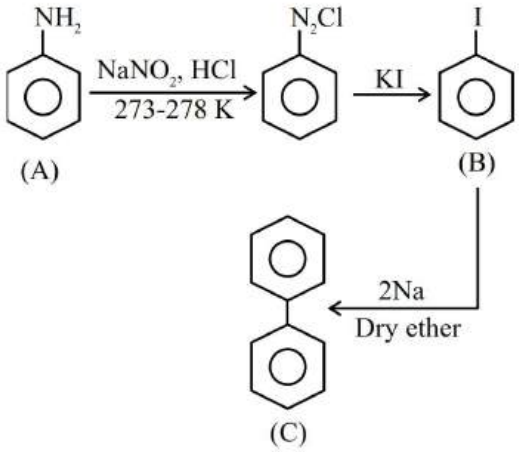
(1) [A] 

(2) [A] 

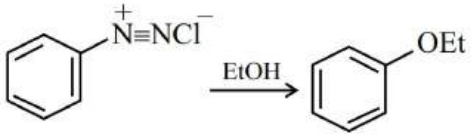
(3) [A] 

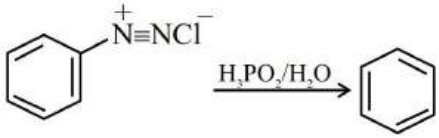
(4) [A] 

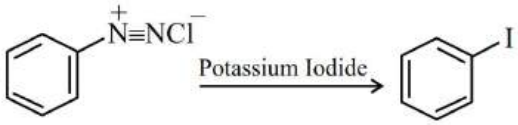
Ans. (3)

Sol. 

63. In the following reactions, which one is NOT correct?

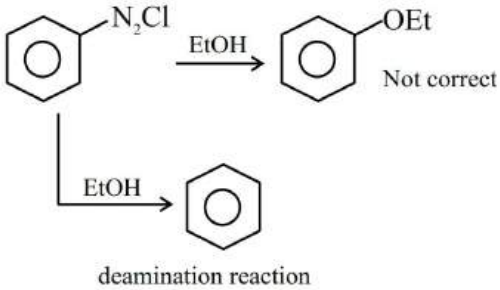
(1) 

(2) 

(3) 

(4)

Ans. (1)

Sol. 

64. The correct order of the complexes $[Co(NH_3)_5(H_2O)]^{3+}$ (A), $[Co(NH_3)_6]^{3+}$ (B), $[Co(CN)_6]^{3-}$ (C) and $[CoCl(NH_3)_5]^{2+}$ (D) in terms wavelength of light absorbed is :

(1) D > A > B > C

(2) C > B > D > A

(3) D > C > B > A

(4) C > B > A > D

Ans. (1)

Sol. We know $E = h\nu = \frac{hC}{\lambda}$

$E \propto \frac{1}{\lambda}$

Here all Co in +3 oxidation state.

So, as the ligand field strength ↑, CFSE ↑

Order of field strength of ligand :

$CN^- > NH_3 > H_2O > Cl^-$

CFSE order : C > B > A > D

Wavelength order : D > A > B > C

65. In the following system,

$PCl_5(g) \rightleftharpoons PCl_3(g) + Cl_2(g)$ at equilibrium, upon addition of xenon gas at constant T & p, the concentration of

(1) PCl_5 will increase

(2) Cl_2 will decrease

(3) PCl_5, PCl_3 & Cl_2 remain constant

(4) PCl_3 will increase

Ans. (2)

Sol. On addition of inert gas at constant P & T, reaction moves in the direction of greater no. of moles so it will shift in forward direction, so $[PCl_5]$ decrease and $[PCl_3]$ & $[Cl_2]$ will increase.

66. 2 moles each of ethylene glycol and glucose are dissolved in 500 g of water. The boiling point of the resulting solution is :

(Given : Ebullioscopic constant of water = 0.52 K kg mol^{-1})

(1) 379.2 K

(2) 377.3 K

(3) 375.3 K

(4) 277.3 K

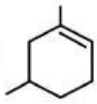
Ans. (2)

Sol. $\Delta T_b = i_1 m_1 k_b + i_2 m_2 k_b$

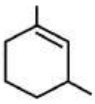
$= 1 \times \frac{2}{0.5} \times 0.52 + \frac{1 \times 2}{0.5} \times 0.52 = 4.16$

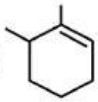
$(T_b)_{solution} = 373.16 + 4.16 = 377.3K.$

67. Which compound would give 3-methyl-6-oxoheptanal upon ozonolysis ?

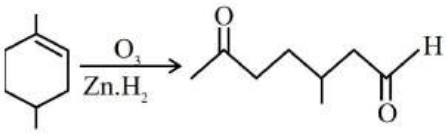
(1) 

(2)

(3) 

(4) 

Ans. (2)

Sol. 

3-Methyl-6-ketoheptanal

68. Match the **LIST-I** with **LIST-II**

LIST-I (Molecules/ion)		LIST-II (Hybridisation of central atom)	
A.	PF_5	I	dsp^2
B.	SF_6	II.	sp^3d
C.	$Ni(CO)_4$	III.	sp^3d^2
D.	$[PtCl_4]^{2-}$	IV.	sp^3

Choose the ***correct*** answer from the options given below :

(1) A-II, B-III, C-IV, D-I

(2) A-IV, B-I, C-II, D-III

(3) A-I, B-II, C-III, D-IV

(4) A-III, B-I, C-IV, D-II

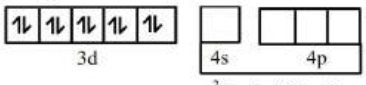
Ans. (1)

Sol. $PF_5 : 5\sigma + 0\ \ell p \rightarrow sp^3d$

$SF_6 : 6\sigma + 0\ \ell p \rightarrow sp^3d^2$

$Ni(CO)_4 : Ni \rightarrow 0$

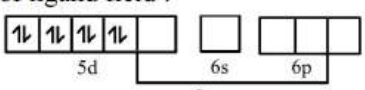
In presence of ligand field :-

Ni(0) : [Ar] 

sp^3 hybridisation

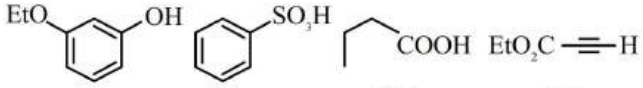
$[PtCl_4]^{2-} : Pt \rightarrow +2$

In presence of ligand field :-

Pt^{2+} : [Kr] 

dsp^2 hybridisation

69. The least acidic compound, among the following is:



(A) (B) (C) (D)

(1) D

(2) A

(3) B

(4) C

Ans. (1)

Sol. $EtO_2C-\equiv-H$

C.B. of terminal alkyne will be sp hybridisation and loculised. In other C.B. will be resonance stablised.

70. Correct order of limiting molar conductivity for cations in water at 298 K is :

(1) $H^+ > Na^+ > K^+ > Ca^{2+} > Mg^{2+}$

(2) $H^+ > Ca^{2+} > Mg^{2+} > K^+ > Na^+$

(3) $Mg^{2+} > H^+ > Ca^{2+} > K^+ > Na^+$

(4) $H^+ > Na^+ > Ca^{2+} > Mg^{2+} > K^+$

Ans. (2)

Sol. Limiting Molar Conductivities of Ions :

- $\overset{\oplus}{H} : 349.8 Scm^2mol^{-1}$
- $Na^+ : 50.11\ Scm^2mol^{-1}$
- $K^+ : 73.52\ Scm^2mol^{-1}$
- $Ca^{+2} : 119\ Scm^2mol^{-1}$
- $Mg^{+2} : 106.12\ Scm^2mol^{-1}$

Therefore correct order of limiting molar conductivity of cations will be –

$\overset{\oplus}{H} > Ca^{+2} > Mg^{+2} > K^+ > Na^+$

SECTION-B

71. During estimation of nitrogen by Dumas' method of compound X (0.42 g) :

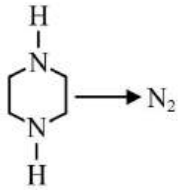
X

_____ mL of N_2 gas will be liberated at STP. (nearest integer)

(Given molar mass in g mol^{-1} : C : 12, H : 1, N : 14)

Ans. (111)

Sol. M.wt. of given compound = 86



Applying POAC on 'N'

$n_X \times 2 = n_{N_2} \times 2$

$\frac{0.42}{86} = n_{N_2}$

$\Rightarrow (Volume)_{N_2}$ at STP $= \frac{0.42}{86} \times 22.4\,L$

$= 0.1108\ L = 110.8\ m\ell$

72. 0.5 g of an organic compound on combustion gave 1.46 g of CO_2 and 0.9 g of H_2O. The percentage of carbon in the compound is ____. (Nearest integer)

[Given : Molar mass (in g mol^{-1}) C : 12, H : 1, O : 16]

Ans. (80)

Sol. Organic $\rightarrow CO_2$

Compound

Applying POAC on 'C'

(mole) of 'C' in compound $= n_{CO_2} \times 1$

So mass of 'C' in compound

$= \frac{1.46}{44} \times 12$

So, % of 'C' in compound $= \frac{1.46}{44} \times \frac{12}{0.5} \times 100$

$= 79.63$

73. The number of optical isomers exhibited by the iron complex (A) obtained from the following reaction is _____.

$FeCl_3 + KOH + H_2C_2O_4 \rightarrow A$

Ans. **(2)**

Sol. $FeCl_3 + KOH + H_2SO_4 \rightarrow \underset{(A)}{K_3[Fe(C_2O_4)_3]}$

$\Rightarrow [Fe(C_2O_4)_3]^{3-}$ is $[M(AA)_3]$ type complex.

So total optical isomers = 2

74. Given :

$\Delta H^{\ominus}_{sub}[C(graphite)] = 710 \text{ kJ mol}^{-1}$

$\Delta_{C-H}H^{\ominus} = 414 \text{ kJ mol}^{-1}$

$\Delta_{H-H}H^{\ominus} = 436 \text{ kJ mol}^{-1}$

$\Delta_{C=C}H^{\ominus} = 611 \text{ kJ mol}^{-1}$

The $\Delta H^{\ominus}_f$ for $CH_2=CH_2$ is _____ kJ mol^{-1} (nearest integer value)

Ans. **(25)**

Sol.

$$\begin{array}{ccc} 2C_{(S)} + 2H_{2(g)} & \longrightarrow & CH_2 = CH_{2(g)} : \left[\Delta H^{\circ}_f\right] C_2H_{4(g)} \\ \downarrow & \nearrow & \\ 2C_{(g)} + 4H_{(g)} & & \end{array}$$

$$\left[\Delta H^{o}_f\right]_{C_2H_{4(g)}} = 2\times\left[\Delta H^{o}_{sub}\right]_{C_{(s)}} + 2\times\Delta H^{o}_{H-H} - 1\times\Delta H^{o}_{C=C} - 4\times\Delta H^{o}_{C-H}$$

$$\Rightarrow \quad \left[\Delta H^{o}_f\right]_{C_2H_{4(g)}} = (2\times 710) + (2\times 436) - 611 - 4\times 414$$

$$\Rightarrow \quad \left[\Delta H^{o}_f\right]_{C_2H_{4(g)}} = 25 \text{ kJ / mol}$$

75. Consider the following reactions

$A + NaCl + H_2SO_4 \rightarrow CrO_2Cl_2 + \text{Side Products}$

Little amount

$CrO_2Cl_{2(Vapour)} + NaOH \rightarrow B + NaCl + H_2O$

$B + H^+ \rightarrow C + H_2O$

The number of terminal 'O' present in the compound 'C' is _____.

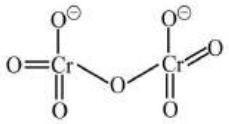
Ans. **(6)**

Sol. $Cr_2O_7^{2-} + NaCl + H_2SO_4 \rightarrow CrO_2Cl_2$

$CrO_2Cl_2 \text{ (Vapour)} + NaOH \rightarrow Na_2CrO_4 + NaCl + H_2O$

$Na_2CrO_4 + H^{\oplus} \rightarrow \underset{(C)}{Na_2Cr_2O_7} + H_2O$

$Na_2Cr_2O_7 \rightarrow 2Na^{\oplus} + Cr_2O_7^{2-}$



No of terminal "O" = 6

JEE (Main)-2025 (Online) Session-2

Question Paper with Solutions

(Mathematics, Physics, And Chemistry)

2 April 2025 Shift – 2

Time: 3 hrs. **M.M : 300**

IMPORTANT INSTRUCTIONS:

(1) The test is of **3 hours** duration.

(2) This test paper consists of 75 questions. Each subject (PCM) has 25 questions. The maximum marks are 300.

(3) This question paper contains Three Parts. Part-A is Physics, Part-B is Chemistry and Part-C is Mathematics. Each part has only two sections: Section-A and Section-B.

(4) Section - A : Attempt all questions.

(5) Section - B : Attempt all questions.

(6) Section - A (01 - 20) contains 20 multiple choice questions which have only one correct answer. Each question carries +4 marks for correct answer and -1 mark for wrong answer.

(7) Section - B (21 – 25) contains 5 Numerical value based questions. The answer to each question should be rounded off to the nearest integer. Each question carries +4 marks for correct answer and -1 mark for wrong answer.

SECTION-A

1. If the image of the point P(1, 0, 3) in the line joining the points A(4, 7, 1) and B(3, 5, 3) is Q(α, β, γ), then α + β + γ is equal to

(1) $\frac{47}{3}$ (2) $\frac{46}{3}$

(3) 18 (4) 13

Ans. (2)

Sol. P(1, 0, 3)

A(4, 7, 1), B(3, 5, 3)

Line AB $\Rightarrow \frac{x-3}{1}=\frac{y-5}{2}=\frac{z-3}{-2}=\lambda$

Let foot of perpendicular of P on AB be

$R \equiv (\lambda + 3, 2\lambda + 5, -2\lambda + 3)$

$\Rightarrow (\lambda + 3 - 1)(1) + (2\lambda + 5 - 0)(2) + (-2\lambda + 3 - 3)(-2) = 0$

$\Rightarrow \lambda + 2 + 4\lambda + 10 + 4\lambda = 0$

$\Rightarrow \lambda = -\frac{4}{3}$

$\Rightarrow R \equiv \left(\frac{5}{3}, \frac{7}{3}, \frac{17}{3}\right)$

$Q \equiv \left(\frac{10}{3}-1, \frac{14}{3}-0, \frac{34}{3}-3\right) \equiv \left(\frac{7}{3}, \frac{14}{3}, \frac{25}{3}\right)$

$\Rightarrow \alpha+\beta+\gamma = \frac{7+14+25}{3} = \frac{46}{3}$

2. Let $f : [1, \infty) \to [2, \infty)$ be a differentiable function, If $10\int_1^x f(t)dt = 5xf(x) - x^5 - 9$ for all $x \geq 1$, then the value of $f(3)$ is :

(1) 18 (2) 32

(3) 22 (4) 26

Ans. (2)

Sol. $10\frac{d}{dx}\int_1^x f(t)dt = \frac{d}{dx}\left(5xf(x) - x^5 - 9\right)$

$\Rightarrow 10f(x) = 5f(x) + 5x\, f'(x) - 5x^4$

$\Rightarrow f(x) + x^4 = x\, f'(x)$

$\Rightarrow y + x^4 = x\frac{dy}{dx}$

$\Rightarrow \frac{dy}{dx} + y\left(-\frac{1}{x}\right) = x^3$

$\Rightarrow ye^{\int -\frac{1}{x}dx} = \int x^3 e^{\int -\frac{1}{x}dx} + c$

$\Rightarrow ye^{-\ell n|x|} = \int x^3 e^{-\ell n|x|} + c$

$\Rightarrow \frac{y}{|x|} = \int \frac{x^3}{|x|} + c$

$\Rightarrow \frac{y}{x} = \int x^2 + c$

$\Rightarrow \frac{y}{x} = \frac{x^3}{3} + c$

Put x = 1 in given equation

$\Rightarrow 0 = 5f(1) - 1 - 9 \Rightarrow f(1) = 2$

$\Rightarrow \frac{2}{1} = \frac{1}{3} + c \Rightarrow c = \frac{5}{3}$

$\Rightarrow \frac{f(3)}{3} = \frac{27}{3} + \frac{5}{3}$

$\Rightarrow f(3) = 32$

3. The number of terms of an A.P. is even; the sum of all the odd terms is 24, the sum of all the even terms is 30 and the last term exceeds the first by $\frac{21}{2}$. Then the number of terms which are integers in the A.P. is :

(1) 4 (2) 10

(3) 6 (4) 8

Ans. (1)

Sol. $a_2 + a_4 + ... + a_n = 30$ ....(1)

$a_1 + a_3 + ... + a_{n-1} = 24$ ....(2)

(1) – (2)

$(a_2 - a_1) + (a_4 - a_3) ... (a_n - a_{n-1}) = 6$

$\Rightarrow \frac{n}{2} d = 6 \Rightarrow nd = 12$

$a_n - a_1 = (n-1)d = \frac{21}{2}$

$\Rightarrow nd - d = \frac{21}{2} \Rightarrow 12 - \frac{21}{2} = d$

$\Rightarrow d = \frac{3}{2}$, $n = 8$

Sum of odd terms $= \frac{4}{2}\left[2a + (4-1)3\right] = 24$

$\Rightarrow a = \frac{3}{2}$

A.P. $\Rightarrow \frac{3}{2}, 3, \frac{9}{2}, 6, \frac{15}{2}, 9, \frac{21}{2}, 12$

no. of integer terms = 4

4. Let A = {1, 2, 3,...., 10} and R be a relation on A such that R = {(a, b) : a = 2b + 1}. Let (a_1, a_2), (a_2, a_3), (a_3, a_4), ..., (a_k, a_{k+1}) be a sequence of k elements of R such that the second entry of an ordered pair is equal to the first entry of the next ordered pair. Then the largest integer k, for which such a sequence exists, is equal to :

(1) 6 (2) 7

(3) 5 (4) 8

Ans. (3)

Sol. a = 2b + 1

2b = a – 1

R = {(3, 1), (5, 2), ..., (99, 49)}

Let (2m + 1, m), (2λ – 1, λ) are such ordered pairs.

According to the condition

$m = 2\lambda - 1 \Rightarrow m$ = odd number

$\Rightarrow 1^{st}$ element of ordered pair (a, b)

$a = 2(2\lambda - 1) + 1 = 4\lambda - 1$

Hence $a \in \{3, 7, ..., 99\}$

$\Rightarrow \lambda \in \{1, 2, ..., 25\}$

$\Rightarrow$ set of sequence

$\left\{(4\lambda-1, 2\lambda-1), (2\lambda-1, \lambda-1), \left(\lambda-1, \frac{\lambda-2}{2}\right), ...\right\}$

2nd element of each ordered pair $= \frac{\lambda - 2^{r-2}}{2^{r-2}}$

For maximum number of ordered pairs in such sequence

$\frac{\lambda - 2^{r-2}}{2^{r-2}} = 1$ or 2 ; $1 \le \lambda \le 25$

$\lambda = 2^{r-1}$ or $\lambda = 3.2^{r-2}$

Case-I : $\lambda = 2r - 1$

$\lambda = 2, 2^2, 2^3, 2^4$

r = 2, 3, 4, 5

Hence maximum value of r is 5 when λ = 16

Case-II : $\lambda = 3.2^{r-2}$

λ = 3, 6, 12, 24

r = 2, 3, 4, 5

Hence maximum value of r is 5 when λ = 24

5. If the length of the minor axis of an ellipse is equal to one fourth of the distance between the foci, then the eccentricity of the ellipse is :

(1) $\frac{4}{\sqrt{17}}$ (2) $\frac{\sqrt{3}}{16}$

(3) $\frac{3}{\sqrt{19}}$ (4) $\frac{\sqrt{5}}{7}$

Ans. (1)

Sol. $2b = \frac{1}{4}(2ae)$

$\frac{b}{a} = \frac{e}{4}$

$e = \sqrt{1 - \frac{b^2}{a^2}}$

$e = \sqrt{1 - \frac{e^2}{16}}$

$e^2\left(1 + \frac{1}{16}\right) = 1$

$e = \frac{4}{\sqrt{17}}$

6. The line L_1 is parallel to the vector $\vec{a} = -3\hat{i} + 2\hat{j} + 4\hat{k}$ and passes through the point (7, 6, 2) and the line L_2 is parallel to the vector $\vec{b} = 2\hat{i} + \hat{j} + 3\hat{k}$ and passes through the point (5, 3, 4). The shortest distance between the lines L_1 and L_2 is :

(1) $\frac{23}{\sqrt{38}}$ (2) $\frac{21}{\sqrt{57}}$

(3) $\frac{23}{\sqrt{57}}$ (4) $\frac{21}{\sqrt{38}}$

Ans. (1)

Sol. $L_1 : \left(7\hat{i} + 6\hat{j} + 2k\right) + \lambda\left(-3\hat{i} + 2\hat{j} + 4k\right)$

$L_2 : \left(5\hat{i} + 3\hat{j} + 4k\right) + \lambda\left(2\hat{i} + \hat{j} + 3k\right)$

Distance between skew lines

$$= \frac{\left(2\hat{i} + 3\hat{j} - 2\hat{k}\right).\left(2\hat{i} + 17\hat{j} - 7\hat{k}\right)}{\sqrt{342}}$$

$$= \frac{69}{\sqrt{342}} = \frac{69}{3\sqrt{38}} = \frac{23}{\sqrt{38}}$$

7. Let (a, b) be the point of intersection of the curve $x^2 = 2y$ and the straight line $y - 2x - 6 = 0$ in the second quadrant. Then the integral $I = \int_a^b \frac{9x^2}{1+5^x} dx$ is equal to :

(1) 24 (2) 27

(3) 18 (4) 21

Ans. (1)

Sol. $x^2 = 2y$ & $y = 2x + 6$

$x^2 = 4x + 12$

$$x^2 - 4x - 12 = 0 \Rightarrow \begin{matrix} x = 6 \\ y = 18 \end{matrix} \Bigg| \begin{matrix} \text{if } x = -2 \\ y = 2 \end{matrix}$$

$\therefore$ (6, 18) & (–2, 2)

Here (6, 18) Rejected because (a, b) lies in 2nd quadrant

$\therefore a = -2$ & $b = 2$

$$\therefore I = \int_{-2}^{2} \frac{9x^2}{1+5^x} dx = \int_{-2}^{2} \frac{9.5^x.x^2}{1+5^x} dx$$

$$\therefore 2I = \int_{-2}^{2} 9x^2 dx = 18\int_0^2 x^2 dx = 18\left(\frac{x^3}{3}\right)_0^2$$

$2I = 48$

$\therefore \boxed{I = 24}$

8. If the system of equation

$2x + \lambda y + 3z = 5$

$3x + 2y - z = 7$

$4x + 5y + \mu z = 9$

has infinitely many solutions, then $(\lambda^2 + \mu^2)$ is equal to :

(1) 22 (2) 18

(3) 26 (4) 30

Ans. (3)

Sol. $$\Delta = 0 \Rightarrow \begin{vmatrix} 2 & \lambda & 3 \\ 3 & 2 & -1 \\ 4 & 5 & \mu \end{vmatrix} = 0$$

$\Rightarrow 2(2\mu + 5) + \lambda(-4 - 3\mu) + 3(7) = 0$

$\Rightarrow 4\mu - 3\lambda\mu - 4\lambda + 31 = 0$ .....(1)

$$\Delta_3 = 0 \Rightarrow \begin{vmatrix} 2 & \lambda & 5 \\ 3 & 2 & 7 \\ 4 & 5 & 9 \end{vmatrix} = 0$$

$\Rightarrow 2(-17) + \lambda(1) + 5(7) = 0$

$\Rightarrow \lambda = -1$

from equation (1)

$4\mu + 3\mu + 4 + 31 = 0 \Rightarrow \boxed{\mu = -5}$

$\therefore \boxed{\lambda^2 + \mu^2 = 26}$

9. If $\theta \in \left[-\frac{7\pi}{6}, \frac{4\pi}{3}\right]$, then the number of solutions of $\sqrt{3}\operatorname{cosec}^2\theta - 2\left(\sqrt{3} - 1\right)\operatorname{cosec}\theta - 4 = 0$, is equal to

(1) 6 (2) 8

(3) 10 (4) 7

Ans. (1)

Sol. $$\operatorname{cosec}\theta = \frac{2\left(\sqrt{3} - 1\right) \pm \sqrt{4\left(3 + 1 - 2\sqrt{3}\right) + 16\sqrt{3}}}{2\sqrt{3}}$$

$$= \frac{2\left(\sqrt{3-1}\right) \pm \sqrt{16 + 8\sqrt{3}}}{2\sqrt{3}}$$

$$= \frac{2\left(\sqrt{3} - 1\right) \pm \left(2 + 2\sqrt{3}\right)}{2\sqrt{3}}$$

$\text{cosec}\theta = 2$ or $\frac{-2}{\sqrt{3}}$

$\therefore \sin\theta = \frac{1}{2}$ or $\frac{-\sqrt{3}}{2}$

$\therefore \sin\theta = \frac{1}{2}$ has 3 solutions & also $\sin\theta = \frac{-\sqrt{3}}{2}$

has 3 solutions in $\left[\frac{-7\pi}{6}, \frac{4\pi}{3}\right]$

10. Given three indentical bags each containing 10 balls, whose colours are as follows :

	Red	Blue	Green
Bag I	3	2	5
Bag II	4	3	3
Bag III	5	1	4

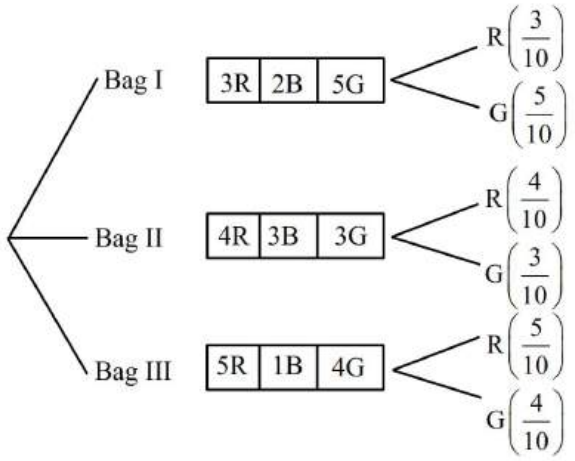
A person chooses a bag at random and takes out a ball. If the ball is Red, the probability that it is from bag I is p and if the balls is Green, the probability that it is from bag III is q, then the value of $\left(\frac{1}{p} + \frac{1}{q}\right)$ is :

(1) 6 (2) 9

(3) 7 (4) 8

Ans. (3)

Sol.



$$p = P\left(\frac{B_I}{R}\right) = \frac{\frac{1}{3}\left(\frac{3}{10}\right)}{\frac{1}{3}\left(\frac{3}{10} + \frac{4}{10} + \frac{5}{10}\right)} = \frac{1}{4}$$

$$q = \left(\frac{B_{III}}{G}\right) = \frac{\frac{1}{3}\left(\frac{4}{10}\right)}{\frac{1}{3}\left(\frac{5}{10} + \frac{3}{10} + \frac{4}{10}\right)} = \frac{1}{3}$$

$\therefore \frac{1}{p} + \frac{1}{q} = 7$

11. If the mean and the variance of 6, 4, a, 8, b, 12, 10, 13 are 9 and 9.25 respectively, then a + b + ab is equal to :

(1) 105 (2) 103

(3) 100 (4) 106

Ans. (2)

Sol. $\because$ mean = 9

$\therefore 53 + a + b = 72$

$\Rightarrow a + b = 19$

$\because \sigma^2 = \frac{37}{4}$ and $(\bar{X})^2 + \sigma^2 = \frac{\sum x_i^2}{N}$

$\Rightarrow 81 + \frac{37}{4} = \frac{529 + a^2 + b^2}{8}$

$\Rightarrow 648 + 74 = 529 + a^2 + b^2$

$\Rightarrow a^2 + b^2 = 193$

$\because a + b = 19 \Rightarrow a^2 + b^2 + 2ab = 361$

$\Rightarrow 2ab = 168$

$\Rightarrow ab = 84$

$\therefore a + b + ab = 103$

12. If the domain of the function

$f(x) = \frac{1}{\sqrt{10 + 3x - x^2}} + \frac{1}{\sqrt{x + |x|}}$ is (a, b), then $(1 + a)^2 + b^2$ is equal to :

(1) 26 (2) 29

(3) 25 (4) 30

Ans. (1)

Sol. $x + |x| > 0 \Rightarrow x \in (0, \infty)$ ....(1)

& $10 + 3x - x^2 > 0$

$\Rightarrow x^2 - 3x - 10 < 0$

$\Rightarrow x \in (-2, 5)$ ....(2)

from (1) & (2) $x \in (0, 5)$

$\therefore a = 0$ & $b = 5$

$\therefore (1 + a^2) + b^2 = 1 + 25 = 26$

13. $4\int_0^1\left(\frac{1}{\sqrt{3+x^2}+\sqrt{1+x^2}}\right)dx - 3\log_e\left(\sqrt{3}\right)$ is equal to :

(1) $2+\sqrt{2}+\log_e\left(1+\sqrt{2}\right)$

(2) $2-\sqrt{2}-\log_e\left(1+\sqrt{2}\right)$

(3) $2+\sqrt{2}-\log_e\left(1+\sqrt{2}\right)$

(4) $2-\sqrt{2}+\log_e\left(1+\sqrt{2}\right)$

Ans. (2)

Sol. $4\int_0^1\frac{1}{\sqrt{3+x^2}+\sqrt{1+x^2}}dx - 3\ln\sqrt{3}$

$= 4\int_0^1\frac{\sqrt{3+x^2}-\sqrt{1+x^2}}{(3+x^2)-(1-x^2)}dx - \frac{3}{2}\ln 3$

$= 2\left[\left\{\frac{x}{2}\sqrt{3+x^2}+\frac{3}{2}\ln\left(x+\sqrt{3+x^2}\right)\right\}_0^1 - \left\{\frac{x}{2}\sqrt{1+x^2}+\frac{1}{2}\ln(x+\sqrt{1+x^2})\right\}_0^1\right] - \frac{3}{2}\ln 3$

$= 2\left[\left\{\frac{1}{2}\sqrt{4}+\frac{3}{2}\ln(1+\sqrt{4})\right\}-\left\{0+\frac{3}{2}\ln\sqrt{3}\right\} - \left\{\frac{1}{2}\sqrt{2}+\frac{1}{2}\ln(1+\sqrt{2})\right\}+\left\{0+\frac{1}{2}(0)\right\}\right]-\frac{3}{2}\ln 3$

$=2\left[1+\frac{3}{2}\ln 3-\frac{3}{4}\ln 3-\frac{1}{\sqrt{2}}-\frac{1}{2}\ln(1+\sqrt{2})\right]-\frac{3}{2}\ln 3$

$=2+3\ln 3-\frac{3}{2}\ln 3-\sqrt{2}-\ln(1+\sqrt{2})-\frac{3}{2}\ln 3$

$=2-\sqrt{2}-\ln(1+\sqrt{2})$

14. If $\lim_{x\to 0}\frac{\cos(2x)+a\cos(4x)-b}{x^4}$ is finite, then (a+b) is equal to :

(1) $\frac{1}{2}$ (2) 0

(3) $\frac{3}{4}$ (4) –1

Ans. (1)

Sol. $\lim_{x\to 0}\frac{\cos 2x+a\cos 4x-b}{x^4}$ = finite

$L = \frac{\left\{1-\frac{(2x)^2}{\lfloor 2}+\frac{(2x)^4}{\lfloor 4}....\right\}+a\left\{1-\frac{(4x)^2}{\lfloor 2}+\frac{(4x)^4}{\lfloor 4}....\right\}-b}{x^4}$

$L = \frac{(1+a-b)-x^2(2+8a)+x^4\left(\frac{2}{3}+\frac{32}{3}a\right)+x^6(\,)....}{x^4}$

$\therefore 1+a-b=0$ and $2+8a=0 \Rightarrow a=-\frac{1}{4}$

$b=a+1$

$=-\frac{1}{4}+1=\frac{3}{4}$

$\therefore a+b=-\frac{1}{4}+\frac{3}{4}=\frac{1}{2}$

15. If $\sum_{r=0}^{10}\left(\frac{10^{r+1}-1}{10^r}\right).{}^{11}C_{r+1}=\frac{\alpha^{11}-11^{11}}{10^{10}}$, then α is equal to :

(1) 15 (2) 11

(3) 24 (4) 20

Ans. (4)

Sol. $\sum_{r=0}^{10}\left(\frac{10^{r-1}-1}{10^r}\right){}^{11}C_{r+1}$

$=\sum_{r=0}^{10}\left(10-\frac{1}{10^r}\right){}^{11}C_{r+1}$

$=10\sum_{r=0}^{10}{}^{11}C_{r+1}-10\sum\left({}^{11}C_{r+1}\left(\frac{1}{10}\right)^{r+1}\right)$

$=10\left[{}^{11}C_1+{}^{11}C_2+.....+{}^{11}C_{11}\right]$

$-10\left[{}^{11}C_1\left(\frac{1}{10}\right)^1+{}^{11}C_2\left(\frac{1}{10}\right)^2+.....+{}^{11}C_{11}\left(\frac{1}{10}\right)^{11}\right]$

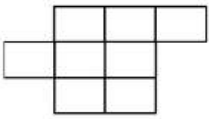
$=10\left[2^{11}-1\right]-10\left[\left(1+\frac{1}{10}\right)^{11}-1\right]$

$=10(2)^{11}-10-\frac{11^{11}}{10^{10}}+10$

$=\frac{(20)^{11}-11^{11}}{10^{10}}$

$\therefore \alpha=20$

16. The number of ways, in which the letters A, B, C, D, E can be placed in the 8 boxes of the figure below so that no row remains empty and at most one letter can be placed in a box, is :

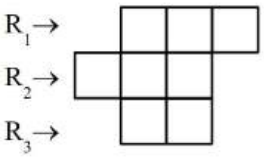


(1) 5880 (2) 960

(3) 840 (4) 5760

Ans. (4)

Sol.



= Total – [(All in R_1 and R_3) + (All in R_2 and R_3) + (All in R_1 and R_2)]

$= {}^8C_5 \cdot \lfloor 5 - \left\{ \lfloor 5 + \lfloor 5 + {}^6C_5 \cdot \lfloor 5 \right\}$

$= \lfloor 5(56-1-1-6) = 120(48)$

$= 5760$

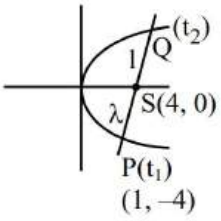
17. Let the point P of the focal chord PQ of the parabola $y^2 = 16x$ be (1, –4). If the focus of the parabola divides the chord PQ in the ratio m : n, gcd(m, n) = 1, then $m^2 + n^2$ is equal to :

(1) 17 (2) 10

(3) 37 (4) 26

Ans. (1)

Sol. $y^2 = 16x$; a = 4 focus S ≡ (4, 0)



$2at_1 = -4$

$\Rightarrow 2(4)t_1 = -4$

$\Rightarrow t_1 = -\frac{1}{2}$

$\because t_1t_2 = -1$

$\Rightarrow t_2 = 2$

$\therefore Q\left(at_2^2, 2at_2\right) = (16, 16)$

Let, S divides PQ internally in λ : 1 ratio

$\therefore \frac{16\lambda - 4}{\lambda + 1} = 0$

$\lambda = \frac{1}{4} = \frac{m}{n}$

$\therefore m^2 + n^2 = 1 + 16 = 17$

18. Let $\vec{a} = 2\hat{i} - 3\hat{j} + k$, $\vec{b} = 3\hat{i} + 2\hat{j} + 5k$ and a vector $\vec{c}$ be such that $\left(\vec{a} - \vec{c}\right) \times \vec{b} = -18\hat{i} - 3\hat{j} + 12k$ and $\vec{a}.\vec{c} = 3$. If $\vec{b} \times \vec{c} = \vec{d}$, then $\left|\vec{a}.\vec{d}\right|$ is equal to :

(1) 18 (2) 12

(3) 9 (4) 15

Ans. (4)

Sol. $\vec{a} = 2\hat{i} - 3\hat{j} + \hat{k}$, $\vec{b} = 3\hat{i} + 2\hat{j} + 5\hat{k}$

$$\vec{a} \times \vec{b} = \begin{vmatrix} i & j & k \\ 2 & -3 & 1 \\ 3 & 2 & 5 \end{vmatrix}$$

$= -17\hat{i} - 7\hat{j} + 13\hat{k}$

$(\vec{a} - \vec{c}) \times \vec{b} = -18\hat{i} - 3j + 12\hat{k}$

$\Rightarrow (\vec{a} \times \vec{b}) - (\vec{c} \times \vec{b}) = -18\hat{i} - 3\hat{j} + 12\hat{k}$

$\Rightarrow \vec{b} \times \vec{c} = (-18\hat{i} - 3\hat{j} + 12\hat{k}) - (\vec{a} \times \vec{b})$

$= (-18\hat{i} - 3\hat{j} + 12\hat{k}) - (-17i - 7\hat{j} + 13\hat{k})$

$\vec{b} \times \vec{c} = -\hat{i} + 4\hat{j} - \hat{k}$

$\therefore \vec{a} \cdot \vec{d} = \vec{a} \cdot (\vec{b} \times \vec{c}) = (2i - 3j + k) \cdot (-\hat{i} + 4\hat{j} - \hat{k})$

$= -2 - 12 - 1 = -15$

$\therefore \left|\vec{a} \cdot \vec{d}\right| = 15$

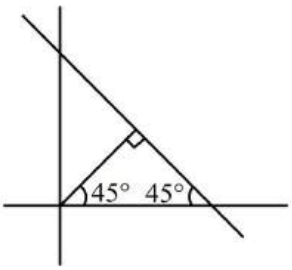
19. Let the area of the triangle formed by a straight Line L : x + by + c = 0 with co-ordinate axes be 48 square units. If the perpendicular drawn from the origin to the line L makes an angle of 45° with the positive x-axis, then the value of b^2+c^2 is:

(1) 90 (2) 93

(3) 97 (4) 83

Ans. (3)

Sol. $\frac{x}{-c}+\frac{y}{-c/b}=1$



$\therefore$ area of triangle $=\frac{1}{2}\left|\frac{c^2}{b}\right|=48$

$\left|\frac{c^2}{b}\right|=96$

$\because -c=-\frac{c}{b}$

$\Rightarrow b=1 \quad \therefore c^2=96$

$\therefore b^2+c^2=97$

20. Let A be a 3×3 real matrix such that $A^2(A-2I)-4(A-I)=O$, where I and O are the identity and null matrices, respectively. If $A^5=\alpha A^2+\beta A+\gamma I$, where α, β and γ are real constants, then $\alpha+\beta+\gamma$ is equal to:

(1) 12 (2) 20

(3) 76 (4) 4

Ans. (1)

Sol. $A^3-2A^2-4A+4I=0$

$A^3=2A^2+4A-4I$

$A^4=2A^3+4A^2-4A$

$=2(2A^2+4A-4I)+4A^2-4A$

$A^4=8A^2+4A-8I$

$A^5=8A^3+4A^2-8A$

$=8(2A^2+4A-4I)+4A^2-8A$

$A^5=20A^2+24A-32I$

$\therefore \alpha=20, \beta=24, \gamma=-32$

$\therefore \alpha+\beta+\gamma=12$

SECTION-B

21. Let $y=y(x)$ be the solution of the differential equation $\frac{dy}{dx}+2y\sec^2 x=2\sec^2 x+3\tan x.\sec^2 x$ such that $y(0)=\frac{5}{4}$. Then $12\left(y\left(\frac{\pi}{4}\right)-e^{-2}\right)$ is equal to _____.

Ans. (21)

Sol. $\text{I.F.}=e^{\int 2\sec^2 x dx}$

$=e^{2\tan x}$

Solution of diff. eq.

$y.e^{2\tan x}=\int e^{2\tan x}(2\sec^2 x+3\tan x.\sec^2 x)dx$

$y.e^{2\tan x}=\int e^{2\tan x}.(2\sec^2 x)dx+\int e^{2\tan x}.(3\tan x.\sec^2 x)dx$

$y.e^{2\tan x}=e^{2\tan x}.2\tan x-\int e^{2\tan x}.2\sec^2 x\times 2\tan x dx+\int e^{2\tan x}.3\tan x.\sec^2 x dx$

$y.e^{2\tan x}=2\tan x.e^{2\tan x}-\int e^{2\tan x}.\tan x\sec^2 x dx$

$y.e^{2\tan x}=2\tan x.e^{2\tan x}-\frac{\tan x.e^{2\tan x}}{2}+\frac{e^{2\tan x}}{4}+C$

$y=2\tan x-\frac{\tan x}{2}+\frac{1}{4}+Ce^{-2\tan x}$

$x=0, y=\frac{5}{4}$

$c=1$

$y\left(\frac{\pi}{4}\right)=\frac{7}{4}+e^{-2}$

Then $12\left(y\left(\frac{\pi}{4}\right)-e^{-2}\right)=12\left(\frac{7}{4}\right)=21$

22. If the sum of the first 10 terms of the series $\frac{4.1}{1+4.1^4}+\frac{4.2}{1+4.2^4}+\frac{4.3}{1+4.3^4}+\ldots$ is $\frac{m}{n}$, where gcd(m, n) = 1, then m + n is equal to _____.

Ans. (441)

Sol. $T_r=\frac{4.r}{1+4.r^4}$

$T_r=\frac{4.r}{(2r^2+2r+1)(2r^2-2r+1)}$

$T_r=\frac{(2r^2+2r+1)-(2r^2-2r+1)}{(2r^2+2r+1)(2r^2-2r+1)}$

$$T_r = \frac{1}{2r^2 - 2r + 1} - \frac{1}{2r^2 + 2r + 1}$$

$$T_1 = \frac{1}{1} - \frac{1}{5}$$

$$T_2 = \frac{1}{5} - \frac{1}{13}$$

$\vdots$

$$T_{10} = \frac{1}{181} - \frac{1}{221}$$

$$S_{10} = 1 - \frac{1}{221} = \frac{220}{221} = \frac{m}{n}$$

$m + n = 441$

23. If $y = \cos\left(\frac{\pi}{3} + \cos^{-1}\frac{x}{2}\right)$, then $(x - y)^2 + 3y^2$ is equal to _____.

Ans. **(3)**

Sol. $y = \cos\left(\cos^{-1}\frac{1}{2} + \cos^{-1}\frac{x}{2}\right)$

$$y = \frac{1}{2} \times \frac{x}{2} - \sqrt{1 - \frac{1}{4}}\sqrt{1 - \frac{x^2}{4}}$$

$4y = x - \sqrt{3}\sqrt{4 - x^2}$

$3(4 - x^2) = x^2 + 16y^2 - 8xy$

$12 - 3x^2 = x^2 + 16y^2 - 8xy$

$4x^2 + 16y^2 - 8xy = 12$

$x^2 + 4y^2 - 2xy = 3$

$x^2 + y^2 - 2xy - 3y^2 = 3$

$(x - y)^2 + 3y^2 = 3$

24. Let A(4, –2), B(1, 1) and C(9, –3) be the vertices of a triangle ABC. Then the maximum area of the parallelogram AFDE, formed with vertices D, E and F on the sides BC, CA and AB of the triangle ABC respectively, is _____.

Ans. **(3)**

Sol. Area of $\Delta ABC = \frac{1}{2}\begin{vmatrix} 4 & -2 & 1 \\ 1 & 1 & 1 \\ 9 & -3 & 1 \end{vmatrix}$

= 6 square units

Maximum area of AFDE = $\frac{1}{2} \times 6$ = 3 sq. units

25. If the set of all $a \in R - \{1\}$, for which the roots of the equation $(1 - a)x^2 + 2(a - 3)x + 9 = 0$ are positive is $(-\infty, -\alpha] \cup [\beta, \gamma)$, then $2\alpha + \beta + \gamma$ is equal to _____.

Ans. **(7)**

Sol. Both the roots are positive

$D \geq 0$

$4(a - 3)^2 - 4 \times 9(1 - a) \geq 0$

$a^2 - 6a + 9 - 9 + 9a \geq 0$

$a^2 + 3a \geq 0$

$a(a + 3) \geq 0$

$a \in (-\infty, -3] \cup [0, \infty)$ .....(i)

$-\frac{b}{2a} > 0$

$\frac{2(a-3)}{2(a-1)} > 0$

$a \in (-\infty, 1) \cup (3, \infty)$ .....(ii)

$f(0) = 9 > 0$

Equation (i) $\cap$ (ii)

$a \in (-\infty, -3] \cup [0, 1)$

$2\alpha + \beta + \gamma - 6 + 0 + 1 = 7$

SECTION-A

26. Given below are two statements : one is labelled as **Assertion (A)** and the other is labelled as **Reason (R).**

Assertion (A) : Net dipole moment of a polar linear isotropic dielectric substance is not zero even in the absence of an external electric field.

Reason (R) : In absence of an external electric field, the different permanent dipoles of a polar dielectric substance are oriented in random directions.

In the light of the above statements, choose the **most appropriate answer** from the options given below :

(1) **(A)** is correct but **(R)** is not correct

(2) Both **(A)** and **(R)** are correct but **(R)** is **not** the correct explanation of **(A)**

(3) Both **(A)** and **(R)** are correct and **(R)** is the correct explanation of **(A)**

(4) **(A)** is not correct but **(R)** is correct

Ans. (4)

Sol. A : Since polar dielectrics are randomly oriental $\vec{P}_{net} = \vec{0}$.

R : If $\vec{E}$ is absent, polar dielectric remain polar & are randomly oriented.

27. In a moving coil galvanometer, two moving coils M_1 and M_2 have the following particulars :

$R_1 = 5\ \Omega$, $N_1 = 15$, $A_1 = 3.6 \times 10^{-3}\ m^2$, $B_1 = 0.25\ T$

$R_2 = 7\ \Omega$, $N_2 = 21$, $A_2 = 1.8 \times 10^{-3}\ m^2$, $B_2 = 0.50\ T$

Assuming that torsional constant of the springs are same for both coils, what will be the ratio of voltage sensitivity of M_1 and M_2 ?

(1) 1 : 1 (2) 1 : 4

(3) 1 : 3 (4) 1 : 2

Ans. (1)

Sol. Voltage sensitivity $= \dfrac{\theta}{V} = \dfrac{NAB}{cR}$

$$\text{Ratio} = = \left(\frac{N_1A_1B_1}{N_2A_2B_2}\right)\frac{R_2}{R_1} = \frac{15\times3.6\times0.25}{21\times1.8\times0.5}\times\frac{7}{5} = \frac{1}{1}$$

28. The moment of inertia of a circular ring of mass M and diameter r about a tangential axis lying in the plane of the ring is :

(1) $\frac{1}{2}Mr^2$ (2) $\frac{3}{8}Mr^2$

(3) $\frac{3}{2}Mr^2$ (4) $2\ Mr^2$

Ans. (2)

Sol. Diameter is given as R.

$\therefore$ Radius = R/2

$$I_{tangent} = \frac{3}{2}m\left(\frac{R}{2}\right)^2 = \frac{3}{8}mR^2$$

29. Two water drops each of radius 'r' coalesce to from a bigger drop. If 'T' is the surface tension, the surface energy released in this process is :

(1) $4\pi r^2T\left[2-2^{\frac{2}{3}}\right]$ (2) $4\pi r^2T\left[2-2^{\frac{1}{3}}\right]$

(3) $4\pi r^2T\left[1+\sqrt{2}\right]$ (4) $4\pi r^2T\left[\sqrt{2}-1\right]$

Ans. (1)

Sol. $2\times\frac{4}{3}\pi R^3 = \frac{4}{3}\pi r^3 \Rightarrow r = 2^{1/3}R$

$U_i = 2\times4\pi R^2T$

$U_f = 4\pi r^2T = 4\pi R^2T2^{2/3}$

$\therefore$ Heat lost $= u_i - u_f = 4\pi R^2T[2-2^{2/3}]$

30. An electron with mass 'm' with an initial velocity (t = 0) $\vec{v} = v_0\hat{i}$ ($v_0 > 0$) enters a magnetic field $\vec{B} = B_0\hat{j}$. If the initial de-Broglie wavelength at t = 0 is λ_0 then its value after time 't' would be :

(1) $\dfrac{\lambda_0}{\sqrt{1-\frac{e^2B_0^2t^2}{m^2}}}$ (2) $\dfrac{\lambda_0}{\sqrt{1+\frac{e^2B_0^2t^2}{m^2}}}$

(3) $\lambda_0\sqrt{1+\frac{e^2B_0^2t^2}{m^2}}$ (4) λ_0

Ans. (4)

Sol. Magnetic field does not work

$\therefore$ Speed will not charge, so De-Broglie wavelength remains same.

31. A sinusoidal wave of wavelength 7.5 cm travels a distance of 1.2 cm along the x-direction in 0.3 sec. The crest P is at x = 0 at t = 0 sec and maximum displacement of the wave is 2 cm. Which equation correctly represents this wave ?

(1) $y = 2\cos(0.83x - 3.35t)$ cm

(2) $y = 2\sin(0.83x - 3.5t)$ cm

(3) $y = 2\cos(3.35x - 0.83t)$ cm

(4) $y = 2\cos(0.13x - 0.5t)$ cm

Ans. (1)

Sol. $v = \frac{\text{distance}}{\text{time}}$

$v = \frac{12}{0.3} = 4 \text{ cm/s}$

$k = \frac{2\pi}{\lambda} = \frac{2\pi}{7.5} = \frac{4\pi}{15} = 0.83$

$v = \frac{\omega}{k} \Rightarrow \omega = vk = 4 \times \frac{4\pi}{15} = 3.35$

So $y = A\cos(kx - \omega t)$

32. Given a charge q, current I and permeability of vacuum μ_0. Which of the following quantity has the dimension of momentum ?

(1) qI/μ_0 (2) $q\mu_0 I$

(3) $q^2\mu_0 I$ (4) $q\mu_0/I$

Ans. (2)

Sol. $Q = AT$

$I = A$

$\mu_0 = MLT^{-2}A^{-2}$

$P = Q^x \mu_0^y I^z = [AT]^x [MLT^{-2}A^{-2}]^y [A]^z$

$MLT^{-1} = M^yL^yT^{x-2y}A^{-2y+z+x}$

Now; $y = 1$

$x - 2y = -1$

$-2y + z = 0$

$\therefore x = y = z = 1$

33. A solenoid having area A and length 'l' is filled with a material having relative permeability 2. The magnetic energy stored in the solenoid is :

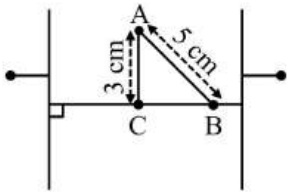
(1) $\frac{B^2Al}{\mu_0}$ (2) $\frac{B^2Al}{2\mu_0}$

(3) B^2Al (4) $\frac{B^2Al}{4\mu_0}$

Ans. (4)

Sol. $\frac{U}{V} = \frac{B^2}{2\mu_r\mu_0} \Rightarrow U = \frac{B^2}{4\mu_0}V = \frac{B^2}{4\mu_0}A\ell$

34. Two large plane parallel conducting plates are kept 10 cm apart as shown in figure. The potential difference between them is V. The potential difference between the points A and B (shown in the figure) is :

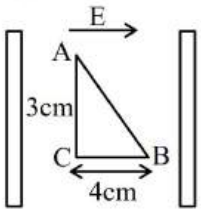


(1) $\frac{1}{4}V$ (2) $\frac{2}{5}V$

(3) $\frac{3}{4}V$ (4) 1 V

Ans. (2)

Sol.



Using $\Delta V = E(\Delta d)$

$V = E(10)$

$V_{AB} = E.4 = \frac{V}{10} \times 4 = \frac{2V}{5}$

35. Identify the characteristics of an adiabatic process in a monoatomic gas.

(A) Internal energy is constant.

(B) Work done in the process is equal to the charge in internal energy.

(C) The product of temperature and volume is a constant.

(D) The product of pressure and volume is a constant.

(E) The work done to change the temperature from T_1 to T_2 is proportional to $(T_2 - T_1)$

Choose the **correct** answer from the options given below :

(1) (A), (C), (D) only (2) (A), (C), (E) only

(3) (B), (E) only (4) (B), (D) only

Ans. (3)

Sol. $Q = \Delta U + W = 0 \Rightarrow -\Delta U = W$

$WD = -nC_v\Delta T \Rightarrow |WD| = nC_v\Delta T \propto T_2 - T_1$

$\therefore$ B & E [Only possibility]

36. Assuming the validity of Bohr's atomic model for hydrogen like ions the radius of Li^{++} ion in its ground state is given by $\frac{1}{X}a_0$, where X = _____.

(Where a_0 is the first Bohr's radius.)

(1) 2 (2) 1

(3) 3 (4) 9

Ans. (3)

Sol. $r = r_0\frac{n^2}{z}$ & z = 3 for Li^{+2} and n = 1

$\therefore r = r_0\frac{1^2}{3} = \frac{r_0}{3}$ $\therefore x = 3$

37. Energy released when two deuterons ($_1H^2$) fuse to form a helium nucleus ($_2He^4$) is :

(Given : Binding energy per nucleon of $_1H^2$ = 1.1 MeV and binding energy per nucleon of $_2He^4$ = 7.0 MeV)

(1) 8.1 MeV (2) 5.9 MeV

(3) 23.6 MeV (4) 26.8 MeV

Ans. (3)

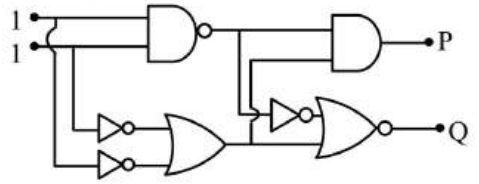
Sol. $\underset{1.1MeV}{_1H^2} + {_1H^2} \rightarrow \underset{7.0MeV}{_1He^4}$

$E_B = BE_{reactant} - BE_{product}$

$= 1.1 \times 2 + 1.1 \times 2 - 7 \times 4 = -23.6$ MeV

$= Q = 23.6$ MeV

38. In the digital circuit shown in the figure, for the given inputs the P and Q values are :

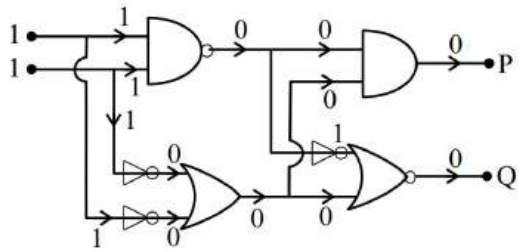


(1) P = 1, Q = 1 (2) P = 0, Q = 0

(3) P = 0, Q = 1 (4) P = 1, Q = 0

Ans. (2)

Sol.



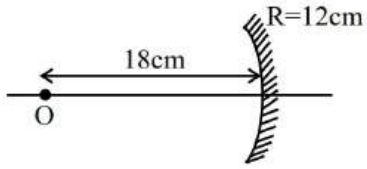
39. Two identical objects are placed in front of convex mirror and concave mirror having same radii of curvature of 12 cm, at same distance of 18 cm from the respective mirrors. The ratio of sizes of the images formed by convex mirror and by concave mirror is :

(1) 1/2 (2) 2

(3) 3 (4) 1/3

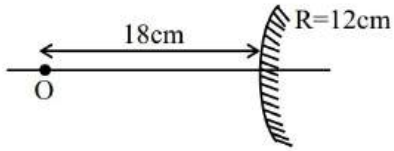
Ans. (1)

Sol.



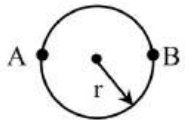
Using $m = \frac{f}{u-f}$

$m_1 = \frac{6}{18-6} = \frac{1}{2}$



$m_2 = \frac{6}{18+6} = \frac{1}{4}$ $\therefore \frac{m_2}{m_1} = \frac{1}{2}$

40. A sportsman runs around a circular track of radius r such that he traverses the path ABAB. The distance travelled and displacement, respectively, are



(1) 2r, 3πr (2) 3πr, πr

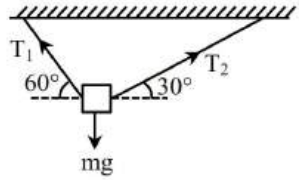
(3) πr, 3r (4) 3πr, 2r

Ans. (4)

Sol. Displacement = 2r

Distance = 2πr + πr = 3πr

41.



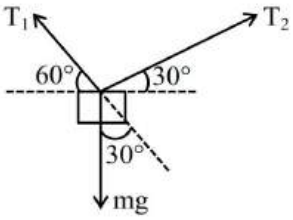
A body of mass 1kg is suspended with the help of two strings making angles as shown in figure. Magnitude of tensions T_1 and T_2, respectively, are (in N) :

(1) 5, $5\sqrt{3}$ (2) $5\sqrt{3}$, 5

(3) $5\sqrt{3}$, $5\sqrt{3}$ (4) 5, 5

Ans. (2)

Sol.



$T_1 = mg\cos 30°$

$T_2 = mg\sin 30°$

42. A bi–convex lens has radius of curvature of both the surfaces same as 1/6 cm. If this lens is required to be replaced by another convex lens having different radii of curvatures on both sides ($R_1 \neq R_2$), without any change in lens power then possible combination of R_1 and R_2 is :

(1) $\frac{1}{3}$cm and $\frac{1}{3}$cm (2) $\frac{1}{5}$cm and $\frac{1}{7}$cm

(3) $\frac{1}{3}$cm and $\frac{1}{7}$cm (4) $\frac{1}{6}$cm and $\frac{1}{9}$cm

Ans. (2)

Sol. This will happen when

$$\frac{1}{f_1} = \frac{1}{f_2}$$

$$(\mu - 1)\left(\frac{1}{R_1} - \frac{1}{-R_2}\right) = (\mu - 1)\left(\frac{2}{R}\right)$$

$$\frac{1}{R_1} + \frac{1}{R_2} = \frac{2}{R}$$

43. If μ_0 and ε_0 are the permeability and permittivity of free space, respectively, then the dimension of $\left(\frac{1}{\mu_0\varepsilon_0}\right)$ is :

(1) L/T^2 (2) L^2/T^2

(3) T^2/L (4) T^2/L^2

Ans. (2)

Sol. $C = \frac{1}{\sqrt{\mu_0\varepsilon_0}} \Rightarrow \frac{1}{\mu_0\varepsilon_0} = C^2 = L^2T^{-2}$

44. Match **List-I with List-II**.

List-I	**List-II**
(A) Heat capacity of body	(I) $J\,kg^{-1}$
(B) Specific heat capacity of body	(II) JK^{-1}
(C) Latent heat	(III) $J\,kg^{-1}\,K^{-1}$
(D)Thermal conductivity	(IV) $Jm^{-1}K^{-1}s^{-1}$

Choose the **correct** answer from the options given below :

(1) (A)–(III), (B)–(I), (C)–(II), (D)–(IV)

(2) (A)–(IV), (B)–(III), (C)–(II), (D)–(I)

(3) (A)–(III), (B)–(IV), (C)–(I), (D)–(II)

(4) (A)–(II), (B)–(III), (C)–(I), (D)–(IV)

Ans. (4)

Sol. $C' = \frac{\Delta Q}{\Delta T} = JK^{-1}$

$$S = \frac{\Delta Q}{m\Delta T} = Jkg^{-1}K^{-1}$$

$$L = \frac{\Delta Q}{m} = Jkg^{-1}$$

$$\Delta Q = \frac{KA\Delta T}{L} \Rightarrow K = \frac{\Delta Q(L)}{A\Delta T} = Jm^{-1}K^{-1}s^{-1}$$

45. Consider a circular loop that is uniformly charged and has a radius $a\sqrt{2}$. Find the position along the positive z–axis of the cartesian coordinate system where the electric field is maximum if the ring was assumed to be placed in xy–plane at the origin :

(1) $\frac{a}{\sqrt{2}}$ (2) $\frac{a}{2}$

(3) a (4) 0

Ans. **(3)**

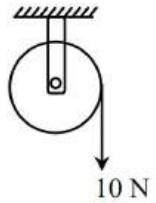
Sol. $E = \frac{KQr}{(x^2 + R^2)^{3/2}}$

$\frac{dE}{dx} = 0$

$\therefore x = \frac{R}{\sqrt{2}} = \frac{\sqrt{2}a}{\sqrt{2}} = a$

SECTION-B

46.

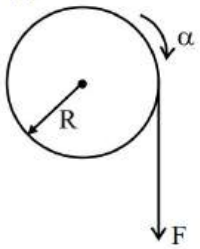


A wheel of radius 0.2 m rotates freely about its center when a string that is wrapped over its rim is pulled by force of 10 N as shown in figure. The established torque produces an angular acceleration of 2 rad/s^2. Moment of inertia of the wheel is _____ kg m^2.

(Acceleration due to gravity = 10 m/s^2)

Ans. **(1)**

Sol.



$FR = I\alpha$

$\Rightarrow I = \frac{FR}{\alpha} = \frac{10 \times 0.2}{2} = 1$ kg–m^2

47. The internal energy of air in 4 m × 4 m × 3 m sized room at 1 atmospheric pressure will be ___ × 10^6 J. (Consider air as diatomic molecule)

Ans. **(12)**

Sol.

To find the internal energy of gas in the room.

$U = nC_vT = n\frac{5RT}{2}$

$= \frac{5}{2}PV = \frac{5}{2} \times 10^5 \times 48 = 12 \times 10^6$ J

48. A ray of light suffers minimum deviation when incident on a prism having angle of the prism equal to 60°. The refractive index of the prism material is $\sqrt{2}$. The angle of incidence (in degrees) is _____.

Ans. **(45)**

Sol. $\mu = \frac{\sin\left(\frac{A + \delta_m}{2}\right)}{\sin\left(\frac{A}{2}\right)}$, since A = 60° $\therefore \delta m = 30°$

$\delta_m = 2i - A$ [as i = e]

$\Rightarrow i = 45°$

49. The length of a light string is 1.4 m when the tension on it is 5 N. If the tension increases to 7 N, the length of the string is 1.56 m. The original length of the string is _____ m.

Ans. **(1)**

Sol. $T = K(\ell - \ell_0)$

$\Rightarrow 5 = K(1.4 - \ell_0)$

$\Rightarrow 7 = K(1.56 - \ell_0)$

$\Rightarrow \frac{5}{1.4 - \ell_0} = \frac{7}{1.56 - \ell_0}$

$\therefore \ell_0 = 1$m

50. A satellite of mass 1000 kg is launched to revolve around the earth in an orbit at a height of 270 km from the earth's surface. Kinetic energy of the satellite in this orbit is _____ × 10^{10} J.

(Mass of earth = 6 × 10^{24} kg, Radius of earth = 6.4 × 10^6 m, Gravitational constant = 6.67 × 10^{-11} Nm^2 kg^{-2})

Ans. **(3)**

Sol. $KE = \frac{1}{2}mv^2 = \frac{1}{2}m\frac{GM_e}{r} = \frac{GM_em}{2r} = \frac{GM_em}{2(R_E + h)}$

$= \frac{6.67 \times 10^{-11} \times 6 \times 10^{24} \times 6.4 \times 10^6}{2(6.4 \times 10^6 + 2.7 \times 10^5)} = 3 \times 10^{10}$ J

SECTION-A

51. When a concentrated solution of sulphanilic acid and 1-naphthylamine is treated with nitrous acid (273 K) and acidified with acetic acid, the mass (g) of 0.1 mole of product formed is :

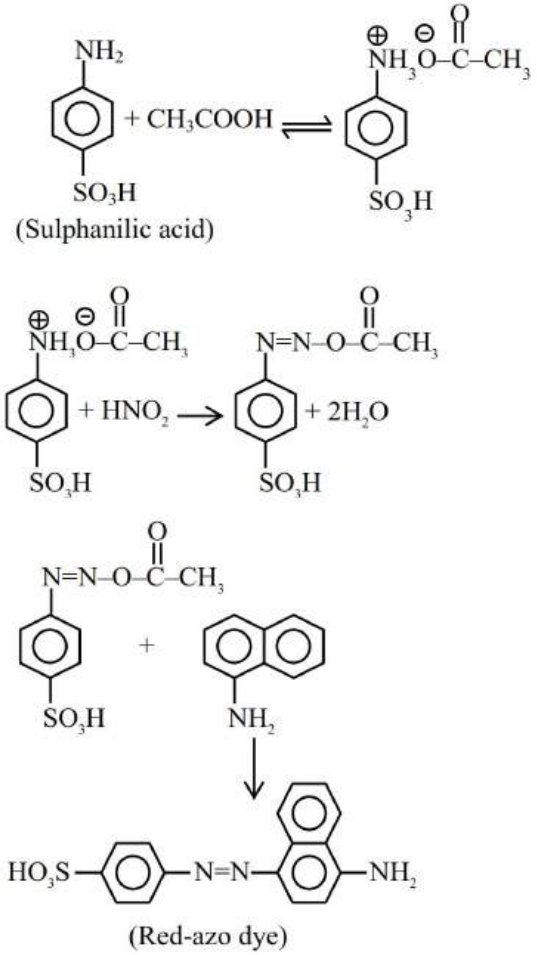
(Given molar mass in g mol^{-1} H : 1, C : 12, N : 14, O : 16, S : 32)

(1) 343 (2) 330

(3) 33 (4) 66

Ans. (3)

Sol.



0.1 mole of red-azo dye (Molar Mass = 327 gm/mol) will have 32.7 gm mass. Nearly 33 gm.

52. The d-orbital electronic configuration of the complex among $[Co(en)_3]^{3+}$, $[CoF_6]^{3-}$, $[Mn(H_2O)_6]^{2+}$ and $[Zn(H_2O)_6]^{2+}$ that has the highest CFSE is :

(1) $t_{2g}^6e_g^0$ (2) $t_{2g}^6e_g^4$

(3) $t_{2g}^3e_g^2$ (4) $t_{2g}^4e_g^2$

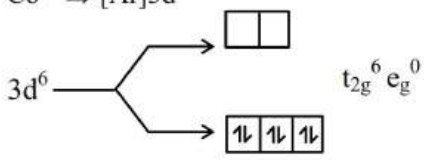
Ans. (1)

Sol. In $[Co(en)_3]^{+3}$ S.F.L. is present and hence highest value of CFSE

In rest all complexes WFL is present hence CFSE will be low.

$[Co(en)_3]^{+3} \Rightarrow Co^{+3}$ and en (SFL)

$Co^{+3} \Rightarrow [Ar]3d^6$



53. Given below are two statements :

Statement (I) : Neopentane forms only one monosubstituted derivative.

Statement (II) : Melting point of neopentane is higher than n-pentane

In the light of the above statements, choose the **most appropriate answer** from the options given below :

(1) **Statement I** is correct but **Statement II** is incorrect

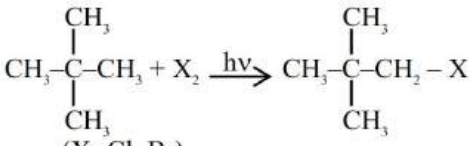
(2) Both **Statement I** and **Statement II** are correct

(3) Both **Statement I** and **Statement II** are incorrect

(4) **Statement I** is incorrect but **Statement II** is correct

Ans. (2)

Sol. Both Statement-I and Statement-II are correct.



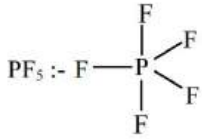
54. Which among the following molecules is (a) involved in sp^3d hybridization, (b) has different bond lengths and (c) has lone pair of electrons on the central atom ?

(1) PF_5 (2) XeF_4

(3) SF_4 (4) XeF_2

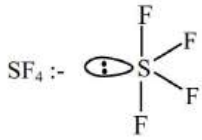
Ans. **(3)**

Sol.

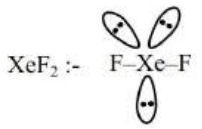


(a) Hybridisation = sp^3d
(b) All bonds are not identical
(c) No lone pair on central atom

XeF_4 :- sp^3d^2 Hybridisation



(a) Hybridisation = sp^3d
(b) All bonds are not identical
(c) 1 lone pair on central atom



(a) Hybridisation = sp^3d
(b) All bonds are identical
(c) 3 lone pair on central atom

55. Formation of $Na_4[Fe(CN)_5NOS]$, a purple coloured complex formed by addition of sodium nitroprusside in sodium carbonate extract of salt indicates the presence of :

(1) Sodium ion (2) Sulphate ion

(3) Sulphide ion (4) Sulphite ion

Ans. **(3)**

Sol. $\underset{\text{Sodium sulphide}}{Na_2S} + \underset{\text{Sodium nitro Prusside}}{Na_2[Fe(CN)_5NO]} \rightarrow \underset{\text{Violet Colour}}{Na_4[Fe(CN)_5NOS]}$

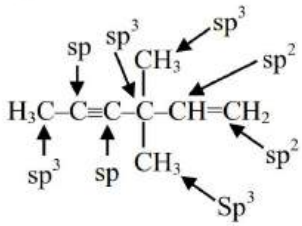
56. In 3, 3-dimethylhex-1-ene-4-yne, there are ____ sp^3, _____ sp^2 and _____ sp hybridised carbon atoms respectively :

(1) 4, 2, 2 (2) 3, 3, 2

(3) 2, 4, 2 (4) 2, 2, 4

Ans. **(1)**

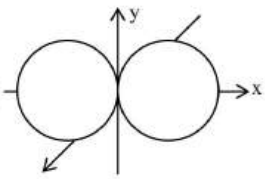
Sol.



4-sp^3, 2-sp^2, 2-sp

57. Which of the following statements are **true** ?

(A) The subsidiary quantum number l describes the shape of the orbital occupied by the electron.

(B) 

is the boundary surface diagram of the $2p_x$ orbital.

(C) The + and – signs in the wave function of the $2p_x$ orbital refer to charge.

(D) The wave function of $2p_x$ orbital is zero everywhere in the xy plane.

(1) (B) and (D) only

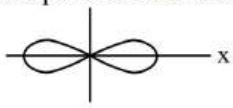
(2) (A), (B) and (C) only

(3) (C) and (D) only

(4) (A) and (B) only

Ans. **(4)**

Sol. (A) Azimuthal quantum number (ℓ) indicates the shape of orbital occupied by the electron

(B) 

p_x orbital

(C) The + and – sign in the wave function of $2p_x$ orbital refer to the sign (Phase) of the wave function , not the charge

(D) The wave function of $2p_x$ orbital will be zero in yz plane (Nodal plane).

58. The type of hybridization and the magnetic property of $[MnCl_6]^{3-}$ are :

(1) d^2sp^3, paramagnetic with four unpaired electrons

(2) sp^3d^2, paramagnetic with four unpaired electrons

(3) d^2sp^3, paramagnetic with two unpaired electrons

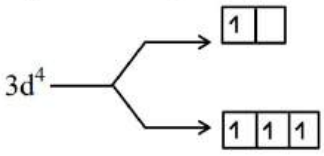
(4) sp^3d^2, paramagnetic with two unpaired electrons

Ans. (2)

Sol. $[MnCl_6]^{3-}$ contains Mn^{+3}

Mn^{+3} :- $[Ar]3d^4$

Ligand $\Rightarrow Cl^-$ (WFL)



Hybridisation = sp^3d^2

4 unpaired electrons

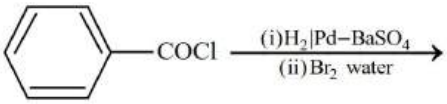
59. Consider the following reactions. From these reactions which reaction will give carboxylic acid as a major product ?

(A) $R-C\equiv N \xrightarrow[\text{mild condition}]{(i)H^+/H_2O}$

(B) $R-MgX \xrightarrow[(ii)H_3O^+]{(i)CO_2}$

(C) $R-C\equiv N \xrightarrow[(ii)H_3O^+]{(i)SnCl_2/HCl}$

(D) $R{\cdot}CH_2{\cdot}OH \xrightarrow{PCC}$

(E) 

Choose the **correct** answer from the options given below :

(1) A and D only

(2) A, B and E only

(3) B, C and E only

(4) B and E only

Ans. (4)

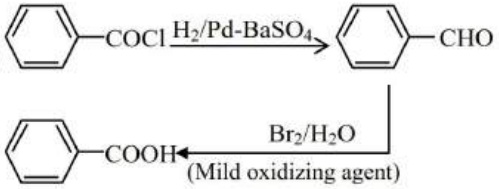
Sol. (A) $R-C\equiv N \xrightarrow[\text{mild condition}]{(i)H^+/H_2O} R-CONH_2$

Under mild condition amide is formed because this reaction is typically slow if further more heat will supplied then it gets convert in to –COOH.

(B) $R-MgX \xrightarrow[(ii)H_3O^+]{(i)CO_2} R-COOH$

(C) $R-C\equiv N \xrightarrow[(ii)H_3O^+]{(i)SnCl_2/HCl} R-CHO$

(D) $R{\cdot}CH_2{\cdot}OH \xrightarrow{PCC} R-CHO$

(E) 

60. Electronic configuration of four elements A, B, C and D are given below :

(A) $1s^22s^22p^3$ (B) $1s^22s^22p^4$

(C) $1s^22s^22p^5$ (D) $1s^22s^22p^2$

Which of the following is the correct order of increasing electronegativity (Pauling's scale) ?

(1) A < D < B < C

(2) A < C < B < D

(3) A < B < C < D

(4) D < A < B < C

Ans. (4)

Sol. N :- $1s^2\ 2s^2\ 2p^3$ (Electronegativity = 3)

O :- $1s^2\ 2s^2\ 2p^4$ (Electronegativity = 3.5)

F :- $1s^2\ 2s^2\ 2p^5$ (Electronegativity = 4)

C :- $1s^2\ 2s^2\ 2p^2$ (Electronegativity = 2.55)

Correct order = C > B > A > D

61. Match **List-I** with **List-II**

List-I (Purification technique)		**List-II (Mixture of organic compounds)**	
(A)	Distillation (simple)	(I)	Diesel + Petrol
(B)	Fractional distillation	(II)	Aniline + Water
(C)	Distillation under reduced pressure	(III)	Chloroform + Aniline
(D)	Steam distillation	(IV)	Glycerol + Spent-lye

Choose the **correct** answer from the options given below :

(1) (A)-(II), (B)-(III), (C)-(IV), (D)-(I)

(2) (A)-(II), (B)-(IV), (C)-(I), (D)-(III)

(3) (A)-(III), (B)-(IV), (C)-(II), (D)-(I)

(4) (A)-(III), (B)-(I), (C)-(IV), (D)-(II)

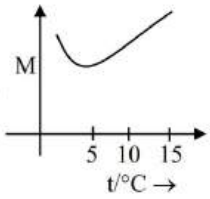
Ans. (4)

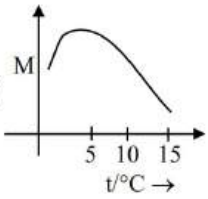
Sol.

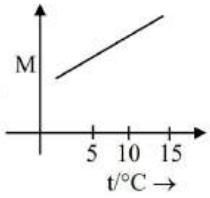
List-I (Purification technique)		**List-II (Mixture of organic compounds)**	
(A)	Distillation (simple)	(III)	Chloroform + Aniline
(B)	Fractional distillation	(I)	Diesel + Petrol
(C)	Distillation under reduced pressure	(IV)	Glycerol + Spent-lye
(D)	Steam distillation	(II)	Aniline + Water

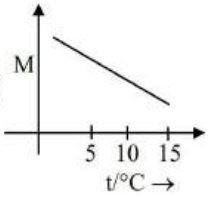
62. 'x' g of NaCl is added to water in a beaker with a lid. The temperature of the system is raised from 1°C to 25°C. Which out of the following plots, is best suited for the change in the molarity (M) of the solution with respect to temperature ?

[Consider the solubility of NaCl remains unchanged over the temperature range]

(1) 

(2) 

(3) 

(4) 

Ans. (2)

Sol. $\underset{1°C}{\text{Water}} \xrightarrow[\text{decreases}]{\text{Volume}} \underset{4°C}{\text{Water}} \xrightarrow[\text{thermal expansion}]{\text{Volume increases}} \underset{25°C}{\text{Water}}$

$$\text{Molarity} = \frac{n_{solute}}{(\text{Volume of solution})_{\ell}}$$

Hence

$1°C \xrightarrow[\text{increases}]{\text{molarity}} 4°C \xrightarrow[\text{decreases}]{\text{molarity}} 25°C$

63. Arrange the following in order of magnitude of work done by the system / on the system at constant temperature :

(a) $|w_{reversible}|$ for expansion in infinite stage.

(b) $|w_{irreversible}|$ for expansion in single stage.

(c) $|w_{reversible}|$ for compression in infinite stage.

(d) $|w_{irreversible}|$ for compression in single stage.

Choose the **correct** answer from the options given below :

(1) a > b > c > d

(2) d > c = a > b

(3) c = a > d > b

(4) a > c > b > d

Ans. (2)

Sol. For isothermal process

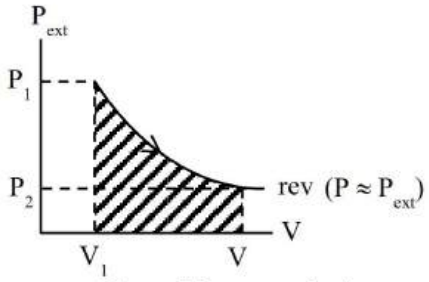
$|W_{reversible}|_{expansion} = |W_{reversible}|_{compression}$

$= -nRT \ln \frac{V_f}{V_i}$

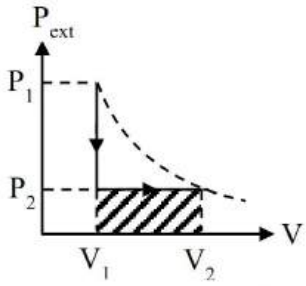
$|W_{irreversible}|_{expansion} < |W_{irreversible}|_{compression}$

d > c = a > b

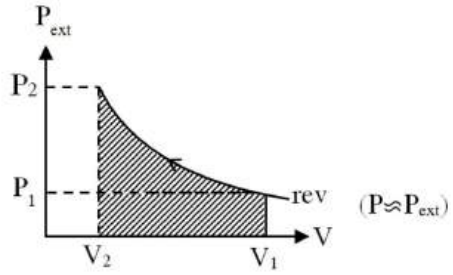
$|W_{irreversible}|_{expansion} = -P_{ext}(V_f - V_i)$



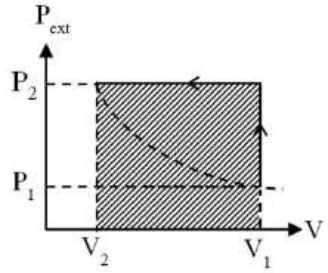
(Reversible expansion)



(Single step irreversible)



(Reversible)



(Single step irreversible)

Graphical representation

We can compare work by area of PV graph.

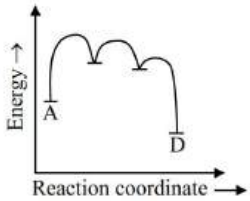
64. Reactant A converts to product D through the given mechanism (with the net evolution of heat) :

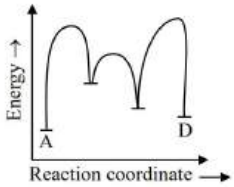
$A \rightarrow B$ slow ; $\Delta H = +ve$

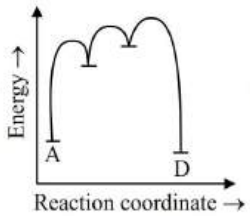
$B \rightarrow C$ fast ; $\Delta H = -ve$

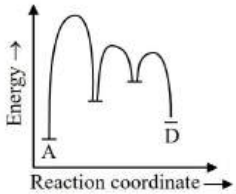
$C \rightarrow D$ fast ; $\Delta H = -ve$

Which of the following represents the above reaction mechanism ?

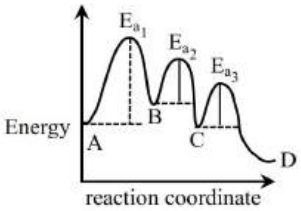
(1)


(2)


(3)


(4)


Ans. (1)

Sol.


$A \rightarrow B$ Slow

$\Delta H = +ve$ $E_{a_1} \rightarrow$ High

$B \rightarrow C$ Fast

$\Delta H = -ve$ $E_{a_2} \rightarrow$ Low

$C \rightarrow D$

$\Delta H = -ve$ $E_{a_3} \rightarrow$ Low

65. The nature of oxide (TeO_2) and hydride (TeH_2) formed by Te, respectively are :

(1) Oxidising and acidic

(2) Reducing and basic

(3) Reducing and acidic

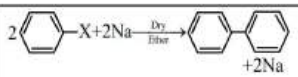
(4) Oxidising and basic

Ans. (1)

Sol. TeO_2 is oxidizing in nature because it can be reduced from +4 oxidation state to lower oxidation state.

TeH_2 due to less bond dissociation energy easily breaks and hence acidic in nature.

66. Match **List-I** with **List-II**

	List-I (Reaction)		**List-II (Name of reaction)**
(A)		(I)	Lucas reaction
(B)	$ArN_2^+X^- \xrightarrow[HCl]{Cu} ArCl + N_2 \uparrow + CuX$	(II)	Finkelstein reaction
(C)	$C_2H_5Br + NaI \xrightarrow[Acetone]{Dry} C_2H_5I + NaBr$	(III)	Fittig reaction
(D)	$CH_3C(OH)(CH_3)CH_3 \xrightarrow[ZnCl_2]{HCl} CH_3C(Cl)(CH_3)CH_3$	(IV)	Gatterman reaction

Choose the correct answer from the options given below :

(1) (A)-(III), (B)-(II), (C)-(IV), (D)-(I)

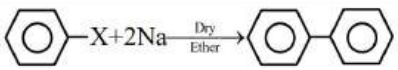
(2) (A)-(III), (B)-(IV), (C)-(II), (D)-(I)

(3) (A)-(IV), (B)-(III), (C)-(I), (D)-(II)

(4) (A)-(IV), (B)-(I), (C)-(II), (D)-(III)

Ans. (2)

Sol.

	List-I (Reaction)		**List-II (Name of reaction)**
(A)		(III)	Fittig reaction
(B)	$ArN_2^+X^- \xrightarrow[HCl]{Cu} ArCl + N_2 \uparrow + CuX$	(IV)	Gatterman reaction
(C)	$C_2H_5Br + NaI \xrightarrow[Acetone]{Dry} C_2H_5I + NaBr$	(II)	Finkelstein reaction
(D)	$CH_3C(OH)(CH_3)CH_3 \xrightarrow[ZnCl_2]{HCl} CH_3C(Cl)(CH_3)CH_3$	(I)	Lucas reaction

67. Consider the following chemical equilibrium of the gas phase reaction at a constant temperature : $A(g) \rightleftharpoons B(g) + C(g)$

If p being the total pressure, K_p is the pressure equilibrium constant and α is the degree of dissociation, then which of the following is true at equilibrium ?

(1) If p value is extremely high compared to K_p, $\alpha \approx 1$

(2) When p increases α decreases

(3) If k_p value is extremely high compared to p, α becomes much less than unity

(4) When p increases α increases

Ans. (2)

Sol.

$$A(g) \rightleftharpoons B(g) + C(g)$$

t = 0	a	0	0
t = t	$a(1-\alpha)$	$a\alpha$	$a\alpha$

a moles of A(g) taken initially and at time

Now moles fraction of A(g), B(g) and C(g) are

$$X_A = \frac{a - a\alpha}{a + a\alpha} = \frac{1-\alpha}{1+\alpha}$$

$$X_B = \frac{a\alpha}{a + a\alpha} = \frac{\alpha}{1+\alpha}$$

$$X_C = \frac{a\alpha}{a + a\alpha} = \frac{\alpha}{1+\alpha}$$

Now if P is total pressure then partial pressure of A(g), B(g) and C(g) are

$$P_A = \left(\frac{1-\alpha}{1+\alpha}\right)P$$

$$P_B = \left(\frac{\alpha}{1+\alpha}\right)P$$

$$P_C = \left(\frac{\alpha}{1+\alpha}\right)P$$

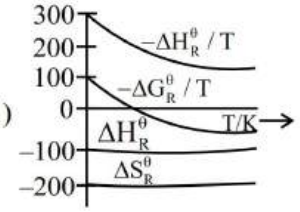
$$K_P = \frac{\left(\frac{\alpha}{1+\alpha}\right)P \left(\frac{\alpha}{1+\alpha}\right)P}{\left(\frac{1-\alpha}{1+\alpha}\right)P}$$

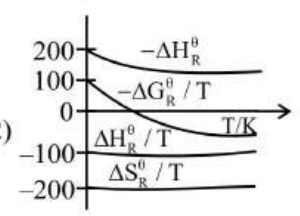
$$K_P = \frac{\alpha^2 P}{1-\alpha^2}$$

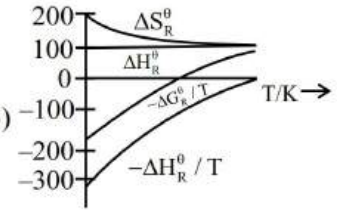
As K_P is only function of temperature.

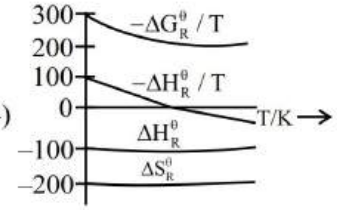
So as $P \uparrow \quad \alpha \downarrow$

68. Which of the following graphs correctly represents the variation of thermodynamic properties of Haber's process ?

(1)


(2)


(3)


(4)


Ans. (1)

Sol. $N_2(g) + 3H_2(g) \rightarrow 2NH_3$

$\Delta H° = -ve$

$\Delta S° = -ve$

(As gaseous moles decreases).

(1) As temperature increases $\frac{-\Delta H_R^o}{T}$, decreases

(2) $\Delta G° = -RT \ln K_{eq}$

$R \ln K_{eq} = -\frac{\Delta G^o}{T}$

(on increasing temperature in exothermic reaction K_{eq} decreases)

$\Delta H°$ and $\Delta S°$ are almost constant with temperature.

69. A tetrapeptide "*x*" on complete hydrolysis produced glycine (Gly), alanine (Ala), valine (Val), leucine (Leu) in equimolar proportion each. The number of tetrapeptides (sequences) possible involving each of these amino acids is

(1) 16 (2) 32

(3) 8 (4) 24

Ans. (24)

Sol. The number of tetrapeptides (sequences) possible involving each of these amino acids (glycine, alanine, valine, leucine) ; It has three (3) peptides linkage the number of permutations in which they can be arranged

$= 4 \times 3 \times 2 \times 1$

$= 24$

70. In Dumas' method for estimation of nitrogen, 0.5 gram of an organic compound gave 60 mL of nitrogen collected at 300 K temperature and 715 mm Hg pressure. The percentage composition of nitrogen in the compound (Aqueous tension at 300 K = 15 mm Hg) is

(1) 1.257 (2) 20.87

(3) 18.67 (4) 12.57

Ans. (4)

Sol. Pressure of N_2 gas = (715 – 15)

= 700 mmHg

$n_{N_2} = \frac{PV}{RT}$

$n_{N_2} = \frac{700 \times 60 \times 10^{-3}}{760 \times 0.0821 \times 300}$

$= 2.24 \times 10^{-3}$ mol

Mass of $N_2 = 2.24 \times 10^{-3} \times 28g$

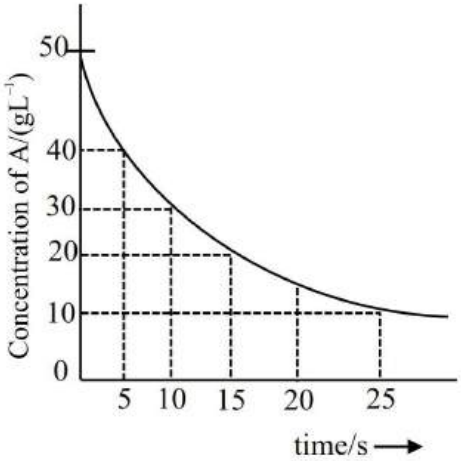
$= 0.06272$ g

$\%N_2 = \frac{0.06272}{0.5} \times 100 \simeq 12.57$

SECTION-B

71. For the reaction A $\rightarrow$ B the following graph was obtained. The time required (in seconds) for the concentration of A to reduce to 2.5 g L^{-1} (if the initial concentration of A was 50 g L^{-1}) is _____. (Nearest integer)

Given : log 2 = 0.3010



Ans. (43)

Sol. As it is difficult to predict order using data provided in graph.

For specific time interval 0 - 5 sec, 5 – 10 sec and 10 – 15 sec. order comes to be zero, but graph is not a straight line.

Assuming 1st order kinetics

$$K = \frac{1}{t}\ln\frac{A_0}{A_t}$$

$$K = \frac{1}{10}\ln\frac{40}{20}$$

Time required to reduce to 2.5 g/L

$$K = \frac{1}{t}\ln\frac{50}{2.5}$$

$$\frac{1}{10}\ln 2 = \frac{1}{t}\ln 20$$

$$t = \frac{1.3010\times 10}{0.3010} = 43.3 \text{ sec.}$$

72. 0.2 % (w/v) solution of NaOH is measured to have resistivity 870.0 mΩ m. The molar conductivity of the solution will be _____$\times 10^2$ mS dm^2 mol^{-1}. (Nearest integer)

Ans. (23)

Sol. Given : Concentration of NaOH = 0.2% (w/v)

$\therefore$ 0.2 g of NaOH in 100 ml of solution.

Molarity of NaOH solution

$$= \frac{\text{moles of solute}}{V_{ml}} \times 1000$$

$$= \frac{0.2/40}{100}\times 1000 = = \frac{0.2}{40\times 100}\times 1000 = \frac{2}{40} \text{ M}$$

Given resistivity of solution = 870 m ohm m

$= 870 \times 10^{-3}$ ohm m

$= 870 \times 10^{-3} \times 10$ ohm dm

$= 870 \times 10^{-2}$ ohm dm

= 8.7 ohm dm

Now conductivity

$$K = \frac{1}{\rho} = \frac{1}{8.7} \text{ ohm}^{-1} \text{ dm}^{-1}$$

Now molar conductivity of solution is

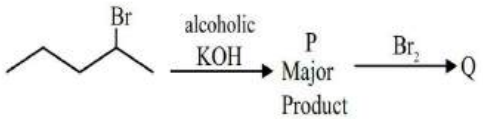
$$\lambda_m = \frac{K}{M} = \frac{\frac{1}{8.7}}{\frac{2}{40}} = \frac{40}{2\times 8.7} = 2.29 \text{ S dm}^2 \text{ mol}^{-1}$$

2.29×10^3 m S dm^2 mol^{-1}

$= 22.9 \times 10^2$ m S dm^2 mol^{-1}

$= 23 \times 10^2$ m S dm^2 mol^{-1}

73.



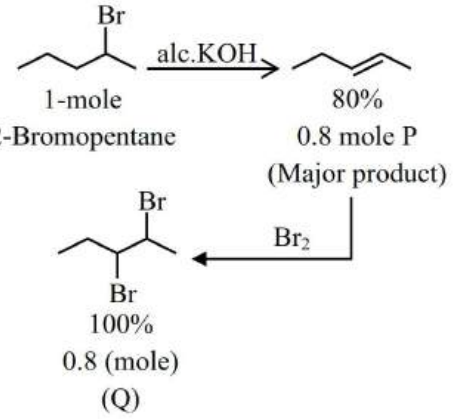
Consider the above sequence of reactions. 151 g of 2-bromopentane is made to react. Yield of major product P is 80% whereas Q is 100%.

Mass of product Q obtained is _____g.

(Given molar mass in g mol^{-1} H: 1, C : 12, O : 16, Br : 80)

Ans. (184)

Sol.



Molecular mass of Q = 230g mol^{-1}

Mass of Q = 0.8 × 230

= 184 g

74. When 1 g each of compounds AB and AB_2 are dissolved in 15 g of water separately, they increased the boiling point of water by 2.7 K and 1.5 K respectively. The atomic mass of A (in amu) is_____$\times 10^{-1}$(Nearest integer)

(Given : Molal boiling point elevation constant is 0.5 K kg mol^{-1})

Ans. (25)

Sol. For AB

$\Delta T_b = 2.7$ K

$2.7 = 1 \times 0.5 \times m$

$m = \frac{27}{5}$

Let molar mass of AB = x.

So $\frac{1/x}{15} \times 1000 = \frac{27}{5}$

x = 12.34

For AB_2

$\Delta T_b = 1.5$ K

$1.5 = 1 \times 0.5 \times m$

m = 3

Let molar mass of AB_2 = y

So $\frac{1/y}{15} \times 1000 = 3$

$y = \frac{1000}{45}$

y = 22.22

Now let a and b be atomic masses of A and B respectively, then

A + b = 12.34 .... (i)

A + 2b = 22.22 .... (ii)

B = 22.22 – 12.34 = 9.88

Now a = 12.34 – 9.88 = 2.46

$= 24.6 \times 10^{-1} = 25 \times 10^{-1}$

75. The spin-only magnetic moment value of M^{n+} ion formed among Ni, Zn Mn and Cu that has the least enthalpy of atomisation is_____. (in nearest integer)

Here n is equal to the number of diamagnetic complexes among $K_2[NiCl_4]$, $[Zn(H_2O)_6]Cl_2$, $K_3[Mn(CN)_6]$ and $[Cu(PPh_3)_3I]$

Ans. (0)

Sol. $K_2[NiCl_4] \Rightarrow sp^3$, Paramagnetic

$[Zn(H_2O)_6]Cl_2 \Rightarrow sp^3d^2$, Diamagnetic

$K_3[Mn(CN)_6] \Rightarrow d^2sp^3$, Paramagnetic

$[Cu(PPh_3)_3I] \Rightarrow sp^3$, Diamagnetic

Hence the value of n is 2

Least value of enthalpy of atomisation among Ni, Zn, Mn and Cu is of Zn

Zn^{+2} :- $[Ar]3d^{10}$

$\mu = 0$

JEE (Main)-2025 (Online) Session-2

Question Paper with Solutions

(Mathematics, Physics, And Chemistry)

2 April 2025 Shift – 1

Time: 3 hrs. **M.M : 300**

IMPORTANT INSTRUCTIONS:

(1) The test is of **3 hours** duration.

(2) This test paper consists of 75 questions. Each subject (PCM) has 25 questions. The maximum marks are 300.

(3) This question paper contains Three Parts. Part-A is Physics, Part-B is Chemistry and Part-C is Mathematics. Each part has only two sections: Section-A and Section-B.

(4) Section - A : Attempt all questions.

(5) Section - B : Attempt all questions.

(6) Section - A (01 - 20) contains 20 multiple choice questions which have only one correct answer. Each question carries +4 marks for correct answer and -1 mark for wrong answer.

(7) Section - B (21 – 25) contains 5 Numerical value based questions. The answer to each question should be rounded off to the nearest integer. Each question carries +4 marks for correct answer and -1 mark for wrong answer.

SECTION-A

1. The largest $n \in N$ such that 3^n divides 50! is:

(1) 21 (2) 22

(3) 20 (4) 23

Ans. (2)

Sol. $2^{\alpha} \cdot 3^{\beta} \cdot 5^{\gamma}$

$$B = \left[\frac{50}{3}\right] + \left[\frac{50}{3^2}\right] + \left[\frac{50}{3^3}\right] + \left[\frac{50}{3^4}\right]$$

$= 16 + 5 + 1$

$= 2$

Maximum value of n is 22

2. Let one focus of the hyperbola H: $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ be at $\left(\sqrt{10}, 0\right)$ and the corresponding directrix be $x = \frac{9}{\sqrt{10}}$. If e and l respectively are the eccentricity and the length of the latus rectum of H, then $9(e^2 + l)$ is equal to:

(1) 14 (2) 15

(3) 16 (4) 12

Ans. (3)

Sol. $ae = \sqrt{10}$ and $\frac{a}{e} = \frac{9}{10}$

$\Rightarrow a^2 = 9$ and $e = \frac{\sqrt{10}}{3}$

Now $(ae)^2 = a^2 + b^2$

$10 = 9 + b^2 \Rightarrow b^2 = 1$

$\ell = \frac{2b^2}{a} = \frac{2(1)}{3}$

$\Rightarrow 9(e^2 + \ell)$

$= 9\left(\frac{10}{9} + \frac{2}{3}\right)$

$= 10 + 6$

$= 16$

3. The number of sequences of ten terms, whose terms are either 0 or 1 or 2, that contain exactly five 1s and exactly three 2s, is equal to

(1) 360 (2) 45

(3) 2520 (4) 1820

Ans. (3)

Sol. 11111 2 2 2 0 0

No. of sequences $= \frac{10!}{5!3!2!} = 2520$

Note : Sequence can start with 0.

4. Let $f : \mathbf{R} \to \mathbf{R}$ be a twice differentiable function such that $(\sin x \cos y)(f(2x+2y) - f(2x-2y)) = (\cos x \sin y)(f(2x+2y) + f(2x-2y))$, for all $x, y \in \mathbf{R}$.

If $f'(0) = \frac{1}{2}$, then the value of $24\ f''\left(\frac{5\pi}{3}\right)$ is:

(1) 2 (2) –3

(3) 3 (4) –2

Ans. (2)

Sol. $(\sin x \cos y)\,(f(2x + 2y) - f(2x - 2y)) = (\cos x \sin y)\,(f(2x + 2y) + f(2x - 2y))$

$f(2x + 2y)(\sin(x-y)) = f(2x - 2y)\sin(x + y)$

$$\frac{f(2x+2y)}{\sin(x+y)} = \frac{f(2x-2y)}{\sin(x-y)}$$

Put $2x + 2y = m,\ 2x - 2y = n$

$$\frac{f(m)}{\sin\left(\frac{m}{2}\right)} = \frac{f(n)}{\sin\left(\frac{n}{2}\right)} = K$$

$$\Rightarrow f(m) = K \sin\left(\frac{m}{2}\right)$$

$$\therefore f(x) = K \sin\left(\frac{x}{2}\right)$$

$$f'(x) = \frac{K}{2}\cos\left(\frac{x}{2}\right)$$

Put $x = 0$; $\frac{1}{2} = \frac{K}{2} \Rightarrow K = 1$

$$f'(x) = \frac{1}{2}\cos\frac{x}{2}$$

$$f''(x) = -\frac{1}{4}\sin\frac{x}{2}$$

$$4f''\left(\frac{5\pi}{3}\right) = \left(-\frac{1}{4}\sin\left(\frac{5\pi}{6}\right)\right)24$$

$$= \frac{-24}{8} = -3$$

5. Let $A = \begin{bmatrix} \alpha & -1 \\ 6 & \beta \end{bmatrix}$, $\alpha > 0$, such that det(A) = 0 and $\alpha + \beta = 1$. If I denotes 2×2 identity matrix, then the matrix $(1 + A)^8$ is:

(1) $\begin{bmatrix} 4 & -1 \\ 6 & -1 \end{bmatrix}$ (2) $\begin{bmatrix} 257 & -64 \\ 514 & -127 \end{bmatrix}$

(3) $\begin{bmatrix} 1025 & -511 \\ 2024 & -1024 \end{bmatrix}$ (4) $\begin{bmatrix} 766 & -255 \\ 1530 & -509 \end{bmatrix}$

Ans. (4)

Sol. $|A| = 0$

$\alpha\beta + 6 = 0$

$\alpha\beta = -6$

$\alpha + \beta = 1$

$\Rightarrow \alpha = 3, \beta = -2$

$$A = \begin{bmatrix} 3 & -1 \\ 6 & -2 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 3 & -1 \\ 6 & -2 \end{bmatrix}\begin{bmatrix} 3 & -1 \\ 6 & -2 \end{bmatrix} = \begin{bmatrix} 3 & -1 \\ 6 & -2 \end{bmatrix}$$

$\therefore A^2 = A$

$A = A^2 = A^3 = A^4 = A^5$

$(I + A)^8$

$= I + {}^8C_1A^7 + {}^8C_2A^6 + ...+ {}^8C_8A^8$

$= I + A\,({}^8C_1 + {}^8C_2 + ...+ {}^8C_8)$

$= I + A(2^8 - 1)$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 765 & -255 \\ 1530 & -510 \end{bmatrix}$$

$$= \begin{bmatrix} 766 & -255 \\ 1530 & -509 \end{bmatrix}$$

6. The term independent of x in the expansion of $\left(\frac{(x+1)}{\left(x^{2/3}+1-x^{1/3}\right)} - \frac{(x+1)}{\left(x-x^{1/2}\right)}\right)^{10}$, $x > 1$ is:

(1) 210 (2) 150

(3) 240 (4) 120

Ans. (1)

Sol. $$\left(\frac{(x+1)}{\left(x^{\frac{2}{3}}+1-x^{\frac{1}{3}}\right)} - \frac{(x-1)}{\left(x-x^{\frac{1}{2}}\right)}\right)^{10}$$

$$= \left(\left(x^{\frac{1}{3}}+1\right) - \left(\frac{\sqrt{x}+1}{\sqrt{x}}\right)\right)^{10}$$

$$= \left(x^{\frac{1}{3}} - \frac{1}{\sqrt{x}}\right)^{10}$$

$$T_{r+1} = {}^{10}C_r (x)^{\frac{10-r}{3}} (-1)^r (x)^{-\frac{r}{2}}$$

$$\frac{10-r}{3} - \frac{r}{2} = 0$$

$(20 - 2r) - 3r = 0$

$r = 4$

$\Rightarrow {}^{10}C_4(-1)^4 = 210$

7. If $\theta \in [-2\pi, 2\pi]$, then the number of solutions of $2\sqrt{2}\cos^2\theta + \left(2-\sqrt{6}\right)\cos\theta - \sqrt{3} = 0$, is equal to:

(1) 12 (2) 6

(3) 8 (4) 10

Ans. (3)

Sol. $2\sqrt{2}\cos^2\theta + 2\cos\theta - \sqrt{6}\cos\theta - \sqrt{3} = 0$

$(2\cos\theta - \sqrt{3})\,(\sqrt{2}\cos\theta + 1) = 0$

$$\cos\theta = \frac{\sqrt{3}}{2}, \frac{-1}{\sqrt{2}}$$

Number of solution = 8

8. Let a_1, a_2, a_3.... be in an A.P. such that $\sum_{k=1}^{12} a_{2k-1} = -\frac{72}{5}a_1, a_1 \neq 0$. If $\sum_{k=1}^{n} a_k = 0$, then n is:

(1) 11 (2) 10

(3) 18 (4) 17

Ans. (1)

Sol. Let $a_1 = a$, common difference = d

$a_1 + a_3 + a_5 + \ldots\ldots + a_{23} = -\frac{72}{5}a$

$\frac{12}{2}[2a + 11 \times 2d] = -\frac{72}{5}a$

$12a + 132d = -\frac{72}{5}a$

$132a + 132 \times 5d = 0$

$a = -5d$

$\frac{n}{2}(2a + (n-1)d) = 0 \Rightarrow -10d + nd - d = 0$

$n = 11$

9. If the function $f(x) = 2x^3 - 9ax^2 + 12a^2x + 1$, where $a > 0$, attains its local maximum and local minimum values at p and q, respectively, such that $p^2 = q$, then $f(3)$ is equal to:

(1) 55 (2) 10

(3) 23 (4) 37

Ans. (4)

Sol. $f'(x) = 6x^2 - 18ax + 12a^2$

$f'(x) = 6(x^2 - 3ax + 2a^2)$

roots are a, 2a

$p^2 = q \Rightarrow a^2 = 2a$

$a = 2$

$f(x) = 2x^3 - 18x^2 + 48x + 1$

$f(3) = 37$

10. Let z be a complex number such that $|z| = 1$. If $\frac{2+k^2z}{k+\bar{z}} = kz$, $k \in \mathbf{R}$, then the maximum distance of $k + ik^2$ from the circle $|z - (1 + 2i)| = 1$ is:

(1) $\sqrt{5}+1$ (2) 2

(3) 3 (4) $\sqrt{3}+1$

Ans. (1)

Sol. $\frac{2+k^2z}{k+\bar{z}} = kz$

$|z|^2 k = 2$

$k = 2$

point p(2, 4); center (1, 2)

distance from circle

$(x-1)^2 + (y-2)^2 = 1$ is max.

if $(OP + r) = \sqrt{1+4} + 1 = \sqrt{5} + 1$

11. If $\vec{a}$ is nonzero vector such that its projections on the vectors $2\hat{i} - \hat{j} + 2\hat{k}$, $\hat{i} + 2\hat{j} - 2\hat{k}$ and $\hat{k}$ are equal, then a unit vector along $\vec{a}$ is:

(1) $\frac{1}{\sqrt{155}}(-7\hat{i} + 9\hat{j} + 5\hat{k})$ (2) $\frac{1}{\sqrt{155}}(-7\hat{i} + 9\hat{j} - 5\hat{k})$

(3) $\frac{1}{\sqrt{155}}(7\hat{i} + 9\hat{j} + 5\hat{k})$ (4) $\frac{1}{\sqrt{155}}(7\hat{i} + 9\hat{j} - 5\hat{k})$

Ans. (3)

Sol. Let $\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$a_1^2 + a_2^2 + a_3^2 = 1$

Let $\bar{b} = 2\bar{i} - \hat{j} + 2\hat{k}$, $\bar{c} = \bar{i} - 2\hat{j} - 2\hat{k}$

$\bar{d} = \hat{k}$

$\frac{\bar{a}.\bar{b}}{|b|} = \frac{\bar{a}.\bar{c}}{|c|} = \frac{\bar{a}.\bar{d}}{|d|}$

$\frac{2a_1 - a_2 + 2a_3}{3} = \frac{a_1 + 2a_2 - 2a_3}{3} = a_3$

By solving

$a_1 = \frac{7}{\sqrt{155}}$, $a_2 = \frac{9}{\sqrt{155}}$, $a_3 = \frac{5}{\sqrt{155}}$

12. Let A be the set of all functions $f: \mathbf{Z} \to \mathbf{Z}$ and R be a relation on A such that R = $\{(f, g) : f(0) = g(1)$ and $f(1) = g(0)\}$. Then R is:

(1) Symmetric and transitive but not reflective

(2) Symmetric but neither reflective nor transitive

(3) Reflexive but neither symmetric nor transitive

(4) Transitive but neither reflexive nor symmetric

Ans. (2)

Sol. $R = \{(f, g) : f(0) = g(1) \text{ and } f(1) = g(0)\}$

Reflexive: $(f, f) \in R$

$= f(0) = f(1)$ and $f(1) = f(0) \rightarrow$ must hold

$\Rightarrow$ but this is not true for all function

so not reflexive

Symmetric: If $(f, g) \in R \Rightarrow (g, f) \in R$

Now, $g(0) = f(1)$ and $g(1) = f(0) \rightarrow$ true

$\therefore$ symmetric

Transitive : If$(f, g) \in R$ and $(g, h) \in R$

$\Rightarrow (f, h) \in R$

Now $(f, g) \in R \Rightarrow f(0) = g(1)$ and $f(1) = g(0)$

$(g, h) \in R \Rightarrow g(0) = h(1)$ and $g(1) = h(0)$

For $(f, h) \in R$ we need $f(0) = h(1)$ and $f(1) = h(0)$

Now $f(0) = g(1) = h(0)$ and $f(1) = g(0) = h(1)$

Hence not transitive

13. For $\alpha, \beta, \gamma, \in \mathbf{R}$, if $\lim\limits_{x\to 0}\dfrac{x^2\sin\alpha x+(\gamma-1)e^{x^2}}{\sin 2x-\beta x}=3$,

then $\beta + \gamma - \alpha$ is equal to:

(1) 7 (2) 4

(3) 6 (4) –1

Ans. (1)

Sol. $$\lim_{x\to 10}\frac{x^2(\alpha x)+(\gamma-1)\left(1+\frac{x^2}{1}\right)}{2x-\frac{8x^3}{6}-\beta x}=3$$

$$\lim_{x\to 0}\frac{(\gamma-1)+(\gamma-1)x^2+\alpha x^3}{(2-\beta)x-\frac{4}{3}x^3}=3$$

$$\gamma-1,\ \beta=2,\ \frac{-3\alpha}{4}=+3 \Rightarrow \alpha=-4$$

$\beta + \gamma - \alpha = 7$

14. If the system of linear equations

$3x + y + \beta z = 3$

$2x + \alpha y - z = -3$

$x + 2y + z = 4$

has infinitely many solutions, then the value of $22\beta - 9\alpha$ is :

(1) 49 (2) 31

(3) 43 (4) 37

Ans. (2)

Sol. $$\Delta=\begin{vmatrix}3 & 1 & \beta\\ 2 & \alpha & -1\\ 1 & 2 & 1\end{vmatrix}=0$$

$3\alpha + 4\beta - \alpha\beta + 3 = 0$

$$\Delta_3=\begin{vmatrix}3 & 1 & 3\\ 2 & \alpha & -3\\ 1 & 2 & 4\end{vmatrix}=0$$

$9\alpha + 19 = 0$

$$\alpha=\frac{-19}{9},\ \beta=\frac{6}{11}$$

$\Rightarrow 22\beta - 9\alpha = 31$

15. Let $P_n = \alpha^n + \beta^n$, $n \in \mathbf{N}$. If $P_{10} = 123$, $P_9 = 76$, $P_8 = 47$ and $P_1 = 1$, then the quadratic equation having roots $\frac{1}{\alpha}$ and $\frac{1}{\beta}$ is :

(1) $x^2 - x + 1 = 0$ (2) $x^2 + x - 1 = 0$

(3) $x^2 - x - 1 = 0$ (4) $x^2 + x + 1 = 0$

Ans. (2)

Sol. $\alpha^{10} + \beta^{10} = 123$

$\alpha + \beta = 1$

$\alpha^9 + \beta^9 = 76$

$\alpha^8 + \beta^8 = 47$

$P_{10} = P_9 + P_8$

$x^2 = x + 1 \Rightarrow x^2 - x - 1 = 0$

$\alpha + \beta = 1,\ \alpha\beta = -1$

$$\frac{1}{\alpha}+\frac{1}{\beta}=\frac{\alpha+\beta}{\alpha\beta}=\frac{1}{-1}=-1,\ \frac{1}{\alpha\beta}=-1$$

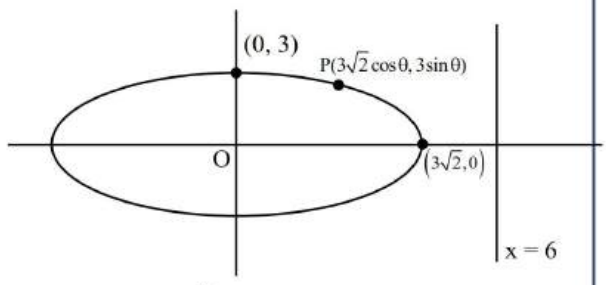
16. If S and S' are the foci of the ellipse $\frac{x^2}{18}+\frac{y^2}{9}=1$ and P be a point on the ellipse, then min(SP.S'P) + max(SP.S'P) is equal to :

(1) $3(1+\sqrt{2})$ (2) $3(6+\sqrt{2})$

(3) 9 (4) 27

Ans. (4)

Sol.



$PS + PS' = 2 \times 3\sqrt{2}$

$b^2 = a^2(1 - e^2) \Rightarrow 9 = 18(1 - e^2)$

$\Rightarrow e = \frac{1}{\sqrt{2}}$

Directrix $x = \frac{a}{e} = \frac{3\sqrt{2}}{\frac{1}{\sqrt{2}}} = 6$

$PS \cdot PS' = \left|\frac{1}{\sqrt{2}}(3\sqrt{2}\cos\theta - 6)\frac{1}{\sqrt{2}}(3\sqrt{2}\cos\theta + 6)\right|$

$= \frac{1}{2}\left|18\cos^2\theta - 36\right|$

$(PS \cdot PS')_{max} = 18$; $(PS \cdot PS')_{min} = 9$

sum = 27

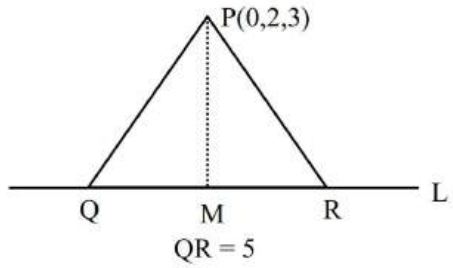
17. Let the vertices Q and R of the triangle PQR lie on the line $\frac{x+3}{5} = \frac{y-1}{2} = \frac{z+4}{3}$, QR = 5 and the coordinates of the point P be (0, 2, 3). If the area of the triangle PQR is $\frac{m}{n}$ then :

(1) $m - 5\sqrt{21}n = 0$ (2) $2m - 5\sqrt{21}n = 0$

(3) $5m - 2\sqrt{21}n = 0$ (4) $5m - 21\sqrt{2}n = 0$

Ans. (2)

Sol.



$M(5\lambda - 3, 2\lambda + 1, 3\lambda - 4)$

Drs of PM $\Rightarrow 5\lambda - 3, 2\lambda - 1, 3\lambda - 7$

Drs of line L $\Rightarrow 5, 2, 3$

$PM \perp L$

$\Rightarrow (5\lambda - 3)5 + (2\lambda - 1)2 + (3\lambda - 7)3 = 0$

$\Rightarrow \lambda = 1$

$\therefore M(2, 3, -1)$

$PM = \sqrt{4+1+16} = \sqrt{21}$

$\text{Area} = \frac{1}{2} \times 5 \times \sqrt{21} = \frac{m}{n}$

$2m - 5\sqrt{21}n = 0$

18. Let ABCD be a tetrahedron such that the edges AB, AC and AD are mutually perpendicular. Let the areas of the triangles ABC, ACD and ADB be 5, 6 and 7 square units respectively. Then the area (in square units) of the ΔBCD is equal to :

(1) $\sqrt{340}$ (2) 12

(3) $\sqrt{110}$ (4) $7\sqrt{3}$

Ans. (3)

Sol. $Ar(\Delta BCD)$

$= \sqrt{(Ar(\Delta ABC))^2 + (Ar(ACD))^2 + (Ar(\Delta ADB))^2}$

$= \sqrt{5^2 + 6^2 + 7^2}$

$= \sqrt{110}$

19. Let $a \in \mathbf{R}$ and A be a matrix of order 3×3 such that $\det(A) = -4$ and $A + I = \begin{bmatrix} 1 & a & 1 \\ 2 & 1 & 0 \\ a & 1 & 2 \end{bmatrix}$, where I is the identity matrix of order 3×3.

If det $((a + 1)\text{adj}((a-1)A))$ is $2^m 3^n$, m, n $\in \{0,1,2,\ldots..20\}$, then m + n is equal to :

(1) 14 (2) 17

(3) 15 (4) 16

Ans. (4)

Sol. $A = \begin{bmatrix} 1 & a & 1 \\ 2 & 1 & 0 \\ a & 1 & 2 \end{bmatrix} - I = \begin{bmatrix} 0 & a & 1 \\ 2 & 0 & 0 \\ a & 1 & 1 \end{bmatrix}$

$|A| = -4 \Rightarrow 2 - 2a = -4 \Rightarrow a = 3$

$|(a+1)\text{ adj }(a-1)A| = |4\text{ adj }3A|$

$= 4^3|\text{adj }3A|$

$= 4^3 \times |3A|^{3-1} = 64|3A|^2$

$= 64 \times (3^3)^2\,|A|^2$

$= 2^6 \times 3^6 \times 16$

$2^m \times 3^n = 2^{10} \times 3^6$

$\therefore\ m = 10, n = 6$

$\Rightarrow\ m + n = 16$

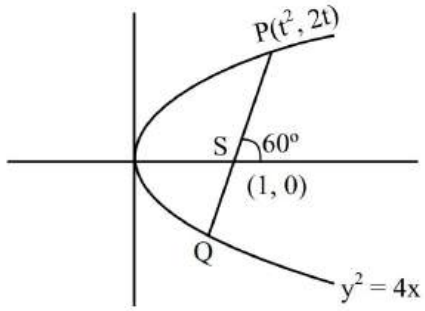
20. Let the focal chord PQ of the parabola $y^2 = 4x$ make an angle of 60° with the positive x-axis, where P lies in the first quadrant. If the circle, whose one diameter is PS, S being the focus of the parabola, touches the y-axis at the point $(0, \alpha)$, then $5\alpha^2$ is equal to :

(1) 15 (2) 25

(3) 30 (4) 20

Ans. (1)

Sol.



$\tan 60° = \dfrac{2t - 0}{t^2 - 1} = \sqrt{3} \Rightarrow t = \sqrt{3}$

$\therefore\ P(3, 2\sqrt{3})$

Circle :

$(x-1)(x-3) + (y-0)(y-2\sqrt{3}) = 0$

at $x = 0$

$\Rightarrow 3 + y^2 - 2\sqrt{3}\,y = 0$

$\Rightarrow y = \sqrt{3} = \alpha$

$5\alpha^2 = 15$

SECTION-B

21. Let [·] denote the greatest integer function. If $\int_0^{e^3}\left[\dfrac{1}{e^{x-1}}\right]dx = \alpha - \log_e 2$, then α^3 is equal to _____.

Ans. (8)

Sol. $f(x) = \dfrac{1}{e^{x-1}} = e^{1-x}$

$f(x) = 2$	$f(x) = 1$
$\dfrac{1}{e^{x-1}} = 2$	$x = 1$
$x = 1 - \ell n2$	

$f(0) = e^1 = 2.71$

$f(e^3) = e^{1-e^3} \in (0,1)$

$I = \int_0^{1-\ell n2} 2dx + \int_{1-\ell n2}^{1} 1dx + \int_1^{e^3} 0\,dx$

$= 2(1 - \ell n2 - 0) + 1(1 - 1 + \ell n2) + 0$

$\alpha - \ell n2 = 2 - \ell n2$

$\alpha = 2$

$\alpha^3 = 8$

22. Let f : **R→R** be a thrice differentiable odd function satisfying

$f'(x) \geq 0,\ f'(x) = f(x), f(0) = 0, f'(0) = 3$. Then $9f(\log_e 3)$ is equal to _____.

Ans. (36)

Sol. $f''(x) = f(x)$

$\Rightarrow f'(x) \cdot f''(x) = f'(x) \cdot f(x)$

$\Rightarrow \dfrac{(f'(x))^2}{2} = \dfrac{(f(x))^2}{2} + C$

$\Rightarrow (f'(x))^2 = (f(x))^2 + C'$

$f(0) = 0,\ f'(0) = 3 \qquad \Rightarrow C' = 9$

$\therefore\ (f'(x))^2 = (f(x))^2 + 9$

$f'(x) = \sqrt{(f(x))^2 + 9} \qquad \because\ f'(x) \geq 0$

$\int \dfrac{dy}{\sqrt{y^2+9}} = \int dx \Rightarrow \ell n\left|y + \sqrt{y^2+9}\right| = x + C$

$\Rightarrow f(0) = 0 \Rightarrow C = \ell n3$

$\Rightarrow\ y + \sqrt{y^2+9} = 3e^x$

at $x = \ell n3$; $y = 4$

$\therefore\ 9f(\ell n3) = 36$

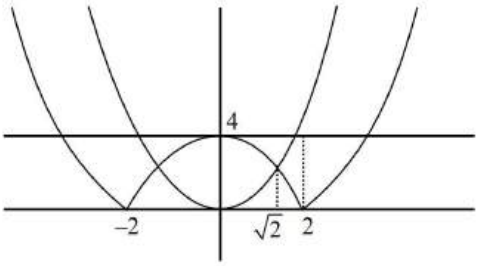
23. If the area of the region

$$\left\{(x,y):\left|4-x^2\right|\le y\le x^2, y\le 4, x\ge 0\right\}$$

is $\left(\frac{80\sqrt{2}}{\alpha}-\beta\right)$, $\alpha, \beta \in \mathbf{N}$, then $\alpha+\beta$ is equal to _____.

Ans. (22)

Sol.



$$A=\int_0^4\sqrt{4+y}\,dy-\int_0^2\sqrt{4-y}\,dy-\int_2^4\sqrt{y}\,dy$$

$$=\left(\frac{(4+y)^{\frac{3}{2}}}{\frac{3}{2}}\right)_0^4+\left(\frac{(4-y)^{\frac{3}{2}}}{\frac{3}{2}}\right)_0^2-\left(\frac{y^{\frac{3}{2}}}{\frac{3}{2}}\right)_0^4$$

$$\frac{80\sqrt{2}}{3}-16=\frac{40\sqrt{2}}{3}-16$$

$\alpha=6, \beta=16$

$\alpha+\beta=22$

24. Three distinct numbers are selected randomly from the set $\{1,2,3,\ldots\ldots,40\}$. If the probability, that the selected numbers are in an increasing G.P. is $\frac{m}{n}$, $\gcd(m,n)=1$, then $m+n$ is equal to ____.

Ans. (4949)

Sol. $1\le a<ar<ar^2\le 40$

(If $r\in N$)

<u>If r = 2</u>

$1\le a<2a<4a\le 40$

$a\in\{1, \ldots\ldots, 10\}$ (10 GP)

<u>If r = 3</u>

$1\le a<3a<9a\le 40$

$a\in\{1, 2, 3, 4\}$ (4 GP)

<u>If r = 4</u>

$1\le a<4a<16a\le 40$

$a\in\{1, 2\}$ (2 GP)

<u>If r = 5</u>

$1\le a<5a<25a\le 40$

$a\in\{1\}$ (1 GP)

<u>If r = 6</u>

$1\le a<6a<36a\le 40$

$a\in\{1\}$ (1 GP)

$\left(P=\frac{18}{9880}=\frac{9}{4940}\right)$ as per NTA for $r\in N$

$m+n=4949$

If <u>$r\notin N$</u> (also possible)

$r=\frac{3}{2}$

$ar^2=\frac{9a}{4}$; $a=4k$

$$\left.\begin{matrix}(4,6,9)\\(8,12,18)\\(12,18,27)\\(16,24,36)\end{matrix}\right\}4GP$$

$r=\frac{5}{2}$ $\quad ar^2=\frac{25a}{4}$; $a=4k$

(4, 10, 25) .....(1) GP

$r=\frac{4}{3}$ $\quad ar^2=\frac{16a}{9}\rightarrow a=9k$

(9, 12, 16), (18, 24, 32) .....(2) GP

$r=\frac{5}{3}$ $\quad ar^2=\frac{25a}{9}$; $a=9k$

(9, 15, 25) .....(1) GP

$r=\frac{5}{4}$ $\quad ar^2=\frac{25a}{16}$; $a=16k$

(16, 20, 25) .....(1) GP

$r=\frac{6}{5}$ $\quad ar^2=\frac{36a}{25}$; $a=25k$

(25, 30, 36) .....(1) GP

Total = 18 + 10 = 28

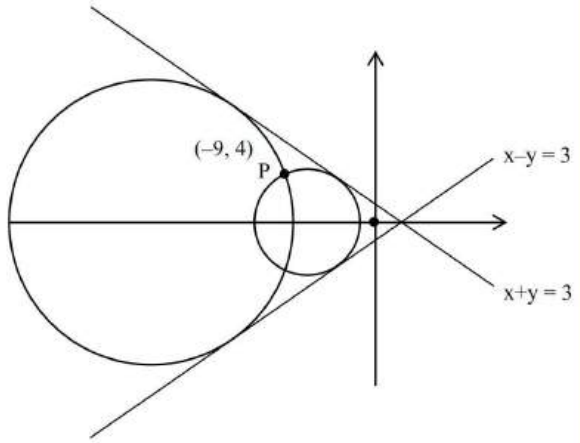
$$P=\frac{28}{{}^{40}C_3}=\frac{28}{9880}=\frac{7}{2470}$$

$m+n=2477$

25. The absolute difference between the squares of the radii of the two circles passing through the point (–9, 4) and touching the lines $x + y = 3$ and $x - y = 3$, is equal to _____.

Ans. (768)

Sol.



Centre (a, 0)

$$r = \left|\frac{a-0-3}{\sqrt{2}}\right|$$

circle $(x-a)^2 + y^2 = \left(\frac{a-3}{\sqrt{2}}\right)^2$

passes through (–9, 4)

$2(a^2 + 18a + 81 + 16) = (a^2 - 6a + 9)$

$a^2 + 42a + 185 = 0$

$(a + 37)(a + 5) = 0$

$\Rightarrow a = -37, -5$

$$r_1 = \left|\frac{-37-3}{\sqrt{2}}\right| = 20\sqrt{2}$$

$$r_2 = \left|\frac{-5-3}{\sqrt{2}}\right| = 4\sqrt{2}$$

$$\left|r_1^2 - r_2^2\right| = |800 - 32| = 768$$

SECTION-A

26. A light wave is propagating with plane wave fronts of the type x + y + z = constant. The angle made by the direction of wave propagation with the x-axis is :

(1) $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$ (2) $\cos^{-1}\left(\frac{2}{3}\right)$

(3) $\cos^{-1}\left(\frac{1}{3}\right)$ (4) $\cos^{-1}\left(\sqrt{\frac{2}{3}}\right)$

Ans. (1)

Sol. The direction of propagation of light is perpendicular to the wave front and is symmetric about x, y and z axis.

$\therefore$ Angle made by the light with x, y & z axis is same.

$\therefore$ $\cos\alpha = \cos\beta = \cos\gamma$ (α, β & γ are angle made by light with x, y & z axis respectively)

Also $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$ [Sum of direction cosines]

$\therefore\ \alpha = \cos^{-1}\frac{1}{\sqrt{3}}$

27. The equation for real gas is given by $\left(P + \frac{a}{V^2}\right)(V - b) = RT$, where P,V,T and R are the pressure, volume, temperature and gas constant, respectively. The dimension of ab^{-2} is equivalent to that of :

(1) Planck's constant (2) Compressibility

(3) Strain (4) Energy density

Ans. (4)

Sol. $\left[P + \frac{a}{V^2}\right](V - b) = RT$

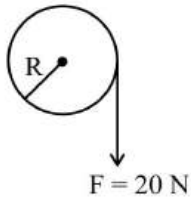
$\therefore [a] = [P][V^2] = ML^{-1}T^{-2}L^6 = ML^5T^{-2}$

$[b] = [V] = L^3$

$[ab^{-2}] = ML^5T^{-2}L^{-6} = ML^{-1}T^{-2}$

Dimension of energy density.

28. A cord of negligible mass is wound around the rim of a wheel supported by spokes with negligible mass. The mass of wheel is 10 kg and radius is 10 cm and it can freely rotate without any friction. Initially the wheel is at rest. If a steady pull of 20 N is applied on the cord, the angular velocity of the wheel, after the cord is unwound by 1 m, would be :



(1) 20 rad/s (2) 30 rad/s

(3) 10 rad/s (4) 0 rad/s

Ans. (1)

Sol. $W_F = 20 \times 1 = 20$ J

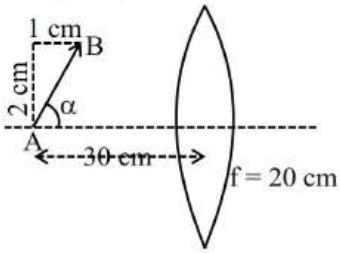
$\therefore\ \Delta KE = 20\text{ J} = \frac{1}{2}I\omega^2$

$I = MR^2 = 10 \times 0.1^2 = 0.1$ kg m^2

$\therefore\ 20 = \frac{1}{2} \times 0.1 \times \omega^2$

$\Rightarrow \omega = 20$ rad/sec

29. A slanted object AB is placed on one side of convex lens as shown in the diagram. The image is formed on the opposite side. Angle made by the image with principal axis is :

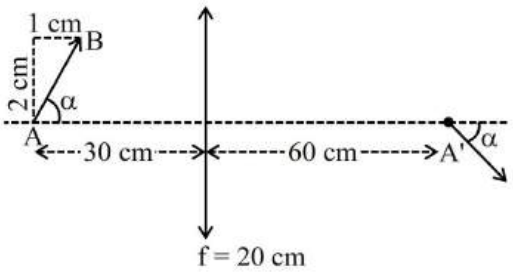


(1) $-\frac{\alpha}{2}$ (2) $-45°$

(3) $+45°$ (4) $-\alpha$

Ans. (2)

Sol.



Location of image of A :-

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}\Rightarrow\frac{1}{v}-\frac{1}{-30}=\frac{1}{20}\Rightarrow\frac{1}{v}=\frac{1}{60}\Rightarrow v=60\text{cm}$$

$\therefore$ m = 2

Since size of object is small wrt the location

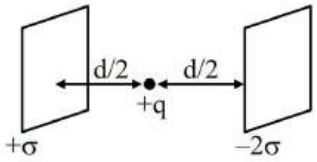
hence

$dv = m^2du \Rightarrow dv = 4 \times 1 = 4$ cm

$h_i = mh_0 \Rightarrow h_i(dy) = 2 \times 2 = 4$ cm

$\therefore$ Angle made with principle axis = –45°

30. Consider two infinitely large plane parallel conducting plates as shown below. The plates are uniformly charged with a surface charge density $+\sigma$ and -2σ. The force experienced by a point charge +q placed at the mid point between two plates will be :

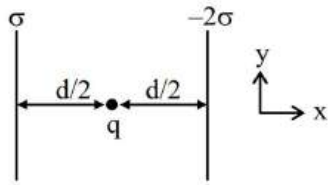


(1) $\frac{\sigma q}{4\epsilon_0}$ (2) $\frac{3\sigma q}{2\epsilon_0}$

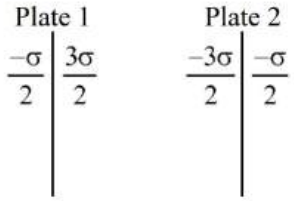
(3) $\frac{3\sigma q}{4\epsilon_0}$ (4) $\frac{\sigma q}{2\epsilon_0}$

Ans. (2)

Sol.



Final charge distribution will be



$$\therefore F_{net} = \frac{3\sigma}{2\epsilon_0}q$$

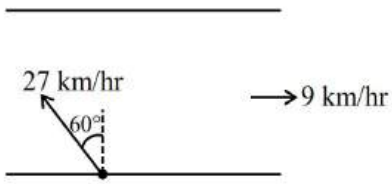
31. A river is flowing from west to east direction with speed of 9 km h^{-1}. If a boat capable of moving at a maximum speed of 27 km h^{-1} in still water, crosses the river in half a minute, while moving with maximum speed at an angle of 150° to direction of river flow, then the width of the river is :

(1) 300 m (2) 112.5 m

(3) 75 m (4) $112.5 \times \sqrt{3}$ m

Ans. (2)

Sol.



$$\therefore V_\perp = \text{river flow} = 27 \times \cos 60° = \frac{27}{2}\text{km/hr.}$$

Time taken = 30 sec.

$$\therefore S = Vt = \frac{27}{2}\times\frac{5}{18}\times 30\text{m} = 112.5\text{m}$$

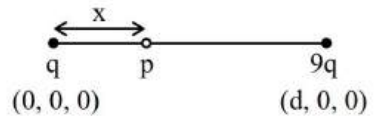
32. A point charge +q is placed at the origin. A second point charge +9q is placed at (d, 0, 0) in Cartesian coordinate system. The point in between them where the electric field vanishes is :

(1) (4d/3, 0, 0) (2) (d/4, 0, 0)

(3) (3d/4, 0, 0) (4) (d/3, 0, 0)

Ans. (2)

Sol.



Let $E_p = 0$

$$\therefore \frac{kq}{x^2} = \frac{k9q}{(d-x)^2}$$

$$\Rightarrow \frac{d-x}{x} = 3 \Rightarrow x = \frac{d}{4}$$

$\therefore$ co-ordinate of P is $\left(\frac{d}{4}, 0, 0\right)$

33. The battery of a mobile phone is rated as 4.2 V, 5800 mAh. How much energy is stored in it when fully charged ?

(1) 43.8 kJ (2) 48.7 kJ

(3) 87.7 kJ (4) 24.4 kJ

Ans. (3)

Sol. Given V = 4.2 volt

$\therefore$ Energy supplied by battery

$= vq = 4.2 \times 5800 \times 3600 \times 10^{-3}$ J = 87.696 kJ

$\therefore$ Energy stored in the battery when fully charged

$= 87.696$ kJ ≈ 87.7 kJ

34. A particle is subjected two simple harmonic motions as :

$x_1 = \sqrt{7}\sin 5t$ cm

and $x_2 = 2\sqrt{7}\sin\left(5t + \frac{\pi}{3}\right)$ cm

where x is displacement and *t* is time in seconds. The maximum acceleration of the particle is $x \times 10^{-2}$ ms^{-2}. The value of x is :

(1) 175 (2) $25\sqrt{7}$

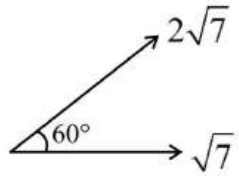
(3) $5\sqrt{7}$ (4) 125

Ans. (1)

Sol. $x_1 = \sqrt{7}\sin 5t$

$\mathbf{x_2} = 2\sqrt{7}\sin\left(5t + \frac{\pi}{3}\right)$

From phasor,



$\therefore$ Amplitude of resultant SHM = 7

$$\phi = \tan^{-1}\frac{2\sqrt{7}\times\sqrt{3}/2}{\sqrt{7}+2\sqrt{7}\times\frac{1}{2}} = \tan^{-1}\frac{\sqrt{21}}{2\sqrt{7}} = \tan^{-1}\frac{\sqrt{3}}{2}$$

$\therefore X_R = 7\sin(5t + \phi)$

$a_R = -7 \times 25 \sin(5t + \phi)$

$\therefore a_{max} = 175$ cm/sec $= 175 \times 10^{-2}$ m/sec

35. The relationship between the magnetic susceptibility (χ) and the magnetic permeability (μ) is given by :

(μ_0 is the permeability of free space and μ_r is relative permeability)

(1) $\chi = \frac{\mu}{\mu_0} - 1$ (2) $\chi = \frac{\mu_r}{\mu_0} + 1$

(3) $\chi = \mu_r + 1$ (4) $\chi = 1 - \frac{\mu}{\mu_0}$

Ans. (1)

Sol. We have

$\mu_r = (1 + \chi) \Rightarrow \chi = (\mu_r - 1)$

$\mu = \mu_0\mu_r \Rightarrow \mu_r = \frac{\mu}{\mu_0}$

$\therefore \chi = \left(\frac{\mu}{\mu_0} - 1\right)$

36. A zener diode with 5V zener voltage is used to regulate an unregulated dc voltage input of 25 V. For a 400 Ω resistor connected in series, the zener current is found to be 4 times load current. The load current (I_L) and load resistance (R_L) are :

(1) $I_L = 20$ mA; $R_L = 250\ \Omega$

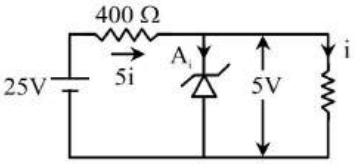
(2) $I_L = 10$ A; $R_L = 0.5\ \Omega$

(3) $I_L = 0.02$ mA; $R_L = 250\ \Omega$

(4) $I_L = 10$ mA; $R_L = 500\ \Omega$

Ans. (4)

Sol.



From the circuit diagram,

$$5i = \frac{20}{400} = \frac{1}{20}\text{A}$$

$$\therefore\ i = \frac{1}{100}\text{A} = 10\text{ mA} = \text{Load current}$$

Also, $V_L = 5$ V

$$\therefore\ R_L = \frac{5}{10\times10^{-3}}\Omega = 500\ \Omega$$

37. In an adiabatic process, which of the following statements is true ?

(1) The molar heat capacity is infinite

(2) Work done by the gas equals the increase in internal energy

(3) The molar heat capacity is zero

(4) The internal energy of the gas decreases as the temperature increases

Ans. (3)

Sol. For adiabatic process, dQ = 0

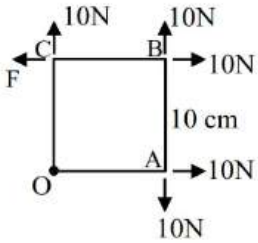
∴ Molar heat capacity = 0

$\because\ dQ = 0 \Rightarrow dU = -dW$

Also $dU = \frac{f}{2}nRdT$

∴ Only option (3) is correct.

38. A square Lamina OABC of length 10 cm is pivoted at 'O'. Forces act at Lamina as shown in figure. If Lamina remains stationary, then the magnitude of F is :



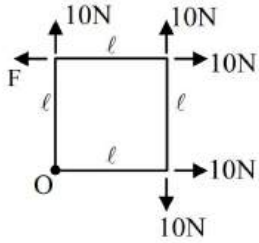
(1) 20 N (2) 0 (zero)

(3) 10 N (4) $10\sqrt{2}$ N

Ans. (3)

Sol. Since the lamina is equilibrium.

$\therefore\ F_{net} = 0\ \&\ \tau_{net} = 0$



$T_O = 10\ell - F\ell \Rightarrow F = 10$ N

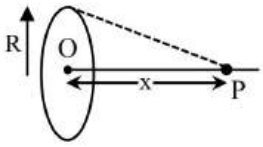
39. Let B_1 be the magnitude of magnetic field at center of a circular coil of radius R carrying current I. Let B_2 be the magnitude of magnetic field at an axial distance 'x' from the center. For x : R = 3 : 4, $\frac{B_2}{B_1}$ is :

(1) 4 : 5 (2) 16 : 25

(3) 64 : 125 (4) 25 : 16

Ans. (3)

Sol.



$B_1 = \frac{\mu_0 i}{2R}$ $B_2 = B_1\sin^3\theta$

$$\therefore\ \frac{B_2}{B_1} = \sin^3\theta = \left(\frac{4}{5}\right)^3 = \frac{64}{125}$$

40. Considering Bohr's atomic model for hydrogen atom :

(A) the energy of H atom in ground state is same as energy of He^+ ion in its first excited state.

(B) the energy of H atom in ground state is same as that for Li^{++} ion in its second excited state.

(C) the energy of H atom in its ground state is same as that of He^+ ion for its ground state.

(D) the energy of He^+ ion in its first excited state is same as that for Li^{++} ion in its ground state

Choose the **correct** answer from the options given below :

(1) (B), (D) only (2) (A), (B) only

(3) (A), (D) only (4) (A), (C) only

Ans. (2)

Sol. $E \propto \frac{Z}{n^2}$

$Z_H = 1 \qquad Z_{He^+} = 2 \qquad Z_{Li^{+2}} = 3$

1st excited state $\Rightarrow n = 2$

2nd excited state $\Rightarrow n = 3$

From the given statements only A & B are correct.

41. Moment of inertia of a rod of mass 'M' and length 'L' about an axis passing through its center and normal to its length is 'α'. Now the rod is cut into two equal parts and these parts are joined symmetrically to form a cross shape. Moment of inertia of cross about an axis passing through its center and normal to plane containing cross is :

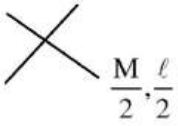
(1) α (2) $\alpha/4$

(3) $\alpha/8$ (4) $\alpha/2$

Ans. (2)

Sol.

$$\alpha = \frac{M\ell^2}{12} \quad \ldots (i)$$

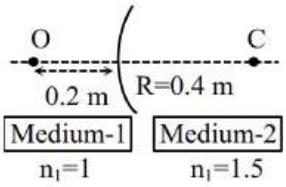


$$\alpha' = 2\left[\frac{\frac{M}{2}\left(\frac{\ell}{2}\right)^2}{12}\right]$$

$$\alpha' = \frac{M\ell^2}{48} = \frac{\alpha}{4}$$

Correct option is (2)

42.



A spherical surface separates two media of refractive indices 1 and 1.5 as shown in figure. Distance of the image of an object 'O', is :

(C is the center of curvature of the spherical surface and R is the radius of curvature)

(1) 0.24 m right to the spherical surface

(2) 0.4 m left to the spherical surface

(3) 0.24 m left to the spherical surface

(4) 0.4 m right to the spherical surface

Ans. (2)

Sol. $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

$$\frac{1.5}{v} - \frac{1}{(-0.2)} = \frac{1.5-1}{0.4}$$

$$\frac{1.5}{v} = \frac{0.5}{0.4} - \frac{1}{0.2}$$

$$\frac{1.5}{v} = -\frac{1.5}{0.4}$$

$v = -0.4$ m

43. Match **List–I** with **List–II**.

List–I	**List–II**
(A) Coefficient of viscosity	(I) $[ML^0T^{-3}]$
(B) Intensity of wave	(II) $[ML^{-2}T^{-2}]$
(C) Pressure gradient	(III) $[M^{-1}LT^2]$
(D) Compressibility	(IV) $[ML^{-1}T^{-1}]$

Choose the **correct** answer from the options given below :

(1) (A)–(I), (B)–(IV), (C)–(III), (D)–(II)

(2) (A)–(IV), (B)–(I), (C)–(II), (D)–(III)

(3) (A)–(IV), (B)–(II), (C)–(I), (D)–(III)

(4) (A)–(II), (B)–(III), (C)–(IV), (D)–(I)

Ans. (2)

Sol. (A) Coefficient of viscosity

$[\eta] = [M^1L^{-1}T^{-1}]$

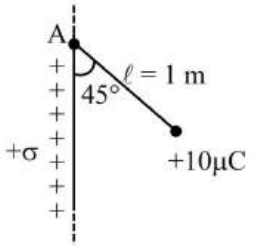
(B) Intensity $[I] = [M^1L^0T^{-3}]$

(C) Pressure gradient $= [ML^{-2}T^{-2}]$

(D) Compressibility $[K] = [M^{-1}L^1T^2]$

44. A small bob of mass 100 mg and charge +10 μC is connected to an insulating string of length 1 m. It is brought near to an infinitely long non–conducting sheet of charge density 'σ' as shown in figure. If string subtends an angle of 45° with the sheet at equilibrium the charge density of sheet will be :

(Given, $\varepsilon_0 = 8.85 \times 10^{-12}\ \frac{F}{m}$ and acceleration due to gravity, $g = 10\ m/s^2$)

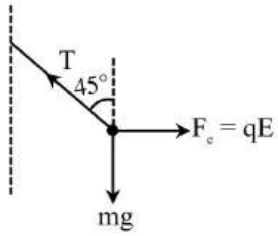


(1) $0.885\ nC/m^2$ (2) $17.7\ nC/m^2$

(3) $885\ nC/m^2$ (4) $1.77\ nC/m^2$

Ans. (4)

Sol.



$qE = mg$

$$q\left[\frac{\sigma}{2\varepsilon_0}\right] = mg$$

$$\sigma = \frac{2\varepsilon_0 mg}{q}$$

$$\sigma = \frac{2 \times 8.85 \times 10^{-12} \times 100 \times 10^{-6} \times 10}{10 \times 10^{-6}}$$

$\sigma = 17.7 \times 10^{-10}\ C/m^2$

$\sigma = 1.77\ nC/m^2$

45. A monochromatic light is incident on a metallic plate having work function ϕ. An electron, emitted normally to the plate from a point A with maximum kinetic energy, enters a constant magnetic field, perpendicular to the initial velocity of electron. The electron passes through a curve and hits back the plate at a point B. The distance between A and B is :

(Given : The magnitude of charge of an electron is e and mass is m, h is Planck's constant and c is velocity of light. Take the magnetic field exists throughout the path of electron)

(1) $\sqrt{2m\left(\frac{hc}{\lambda} - \phi\right)} / eB$ (2) $\sqrt{m\left(\frac{hc}{\lambda} - \phi\right)} / eB$

(3) $\sqrt{8m\left(\frac{hc}{\lambda} - \phi\right)} / eB$ (4) $2\sqrt{m\left(\frac{hc}{\lambda} - \phi\right)} / eB$

Ans. (3)

Sol. $KE_{max} = \frac{hc}{\lambda} - \phi$

$p = \sqrt{2mK_{max}}$

$p = \sqrt{2m\left(\frac{hc}{\lambda} - \phi\right)}$

$d_{A-B} = 2R$

$= 2\left[\frac{p}{qB}\right]$

$d_{AB} = \frac{2\sqrt{2m\left(\frac{hc}{\lambda} - \phi\right)}}{eB} = \frac{\sqrt{8m\left(\frac{hc}{\lambda} - \phi\right)}}{eB}$

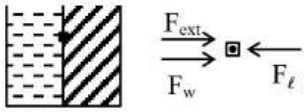
SECTION-B

46. A vessel with square cross–section and height of 6 m is vertically partitioned. A small window of 100 cm^2 with hinged door is fitted at a depth of 3 m in the partition wall. One part of the vessel is filled completely with water and the other side is filled with the liquid having density 1.5×10^3 kg/m^3. What force one needs to apply on the hinged door so that it does not get opened ?

(Acceleration due to gravity = 10 m/s^2)

Ans. (150)

Sol.



in equilibrium

$F_{ext} + F_w = F_\ell$

$\Rightarrow F_{ext} = F_\ell - F_w$

$= (P_0 + \rho_\ell gh)A - (P_0 + \rho_w gh)A$

$= (\rho_\ell - \rho_w)ghA$

$= (1500 - 1000) \times 10 \times 3 \times (100 \times 10^{-4})$

$= 150$ m

47. A steel wire of length 2 m and Young's modulus 2.0×10^{11} Nm^{-2} is stretched by a force. If Poisson ratio and transverse strain for the wire are 0.2 and 10^{-3} respectively, then the elastic potential energy density of the wire is ____ $\times 10^5$ (in SI units)

Ans. (25)

Sol. $\ell = 2m$; $Y = 2 \times 10^{11}\ \frac{N}{m^2}$

$\mu = -\frac{\left(\frac{\Delta r}{r}\right)}{\left(\frac{\Delta \ell}{\ell}\right)} \Rightarrow \frac{\Delta \ell}{\ell} = \frac{1}{\mu} \times \left(\frac{\Delta r}{r}\right)$

$= \frac{1}{0.2} \times (10^{-3})$

$\Rightarrow \frac{\Delta \ell}{\ell} = 5 \times 10^{-3}$

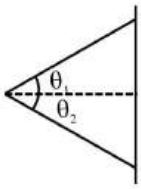
$u = \frac{1}{2} y \varepsilon_\ell^2 = \frac{1}{2} \times 2 \times 10^{11} \times [5 \times 10^{-3}]^2$

$= 25$

48. If the measured angular separation between the second minimum to the left of the central maximum and the third minimum to the right of the central maximum is 30° in a single slit diffraction pattern recorded using 628 nm light, then the width of the slit is ____ μm.

Ans. (6)

Sol.



$\theta_1 = \sin^{-1}\left(\frac{2\lambda}{a}\right)$

$\theta_2 = \sin^{-1}\left(\frac{3\lambda}{a}\right)$

$\because\ \theta_1 + \theta_2 = 30°$

$$\Rightarrow \sin^{-1}\left(\frac{2\lambda}{a}\right)+\sin^{-1}\left(\frac{3\lambda}{a}\right)=\frac{\pi}{6}$$

$$\Rightarrow \frac{2\lambda}{a}\sqrt{1-\left(\frac{3\lambda}{a}\right)^2}+\frac{3\lambda}{a}\sqrt{1+\left(\frac{2\lambda}{a}\right)^2}=\sin\frac{\pi}{6}$$

Here $\lambda = 628$ nm

After solving

$A = 6.07\ \mu m$

Approximate Method :

$\theta = \theta_1 + \theta_2$

$$\Rightarrow \frac{\pi}{6}=\frac{2\lambda}{a}+\frac{3\lambda}{a}$$

$$\Rightarrow \frac{\pi}{6}=\frac{5}{a}(628\text{nm})$$

$\Rightarrow a = 6\ \mu m$

49. γ_A is the specific heat ratio of monoatomic gas A having 3 translational degrees of freedom. γ_B is the specific heat ratio of polyatomic gas B having 3 translational, 3 rotational degrees of freedom and 1 vibrational mode. If $\frac{\gamma_A}{\gamma_B}=\left(1+\frac{1}{n}\right)$, then the value of n is _____.

Ans. (3)

Sol. $$\frac{\gamma_A}{\gamma_B}=\frac{f_A+2}{f_A}\times\frac{f_B}{f_B+2}$$

$$=\frac{3+2}{3}\times\frac{(6+2)}{(6+2)+2}$$

$$=\frac{5}{3}\times\frac{8}{10}=\frac{40}{30}$$

$$\because \frac{40}{30}=1+\frac{1}{n}$$

$$\Rightarrow \frac{40}{30}-1=\frac{1}{n}$$

$\Rightarrow n = 3$

50. A person travelling on a straight line moves with a uniform velocity v_1 for a distance x and with a uniform velocity v_2 for the next $\frac{3}{2}x$ distance. The average velocity in this motion is $\frac{50}{7}$ m/s. If v_1 is 5 m/s then v_2 = _____ m/s.

Ans. (10)

Sol. $$v_{avg}=\frac{x_1+x_2}{t_1+t_2}$$

$$\Rightarrow \frac{50}{7}=\frac{x+\frac{3x}{2}}{\frac{x}{5}+\frac{3x}{2v_2}}$$

$$\Rightarrow \frac{50}{7}=\frac{5/2}{\frac{1}{5}+\frac{3}{2v_2}}$$

$$\Rightarrow \frac{1}{5}+\frac{3}{2v_2}=\frac{7}{20}$$

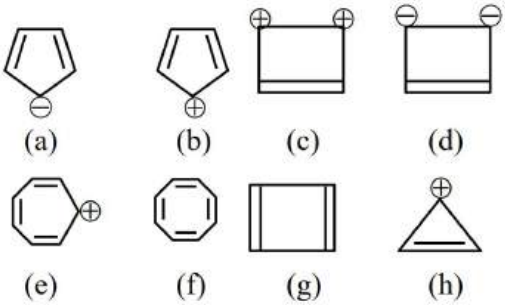
$$\Rightarrow \frac{3}{2v_2}=\frac{7}{20}-\frac{1}{5}=\frac{7-4}{20}$$

$$\Rightarrow \frac{3}{2v_2}=\frac{3}{20}$$

$\Rightarrow v_2 = 10$ m/s

SECTION-A

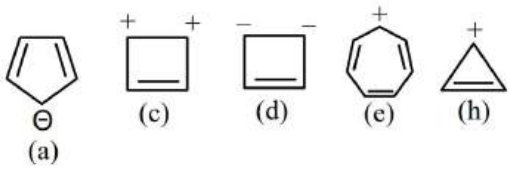
51. Designate whether each of the following compounds is aromatic or not aromatic.



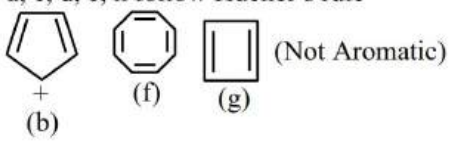
(1) e, g aromatic and a, b, c, d, f, h not aromatic
(2) b, e, f, g aromatic and a, c, d, h not aromatic
(3) a, b, c, d aromatic and e, f, g, h not aromatic
(4) a, c, d, e, h aromatic and b, f, g not aromatic

Ans. (4)

Sol. Aromatic compounds



a, c, d, e, h follow Huckel's rule



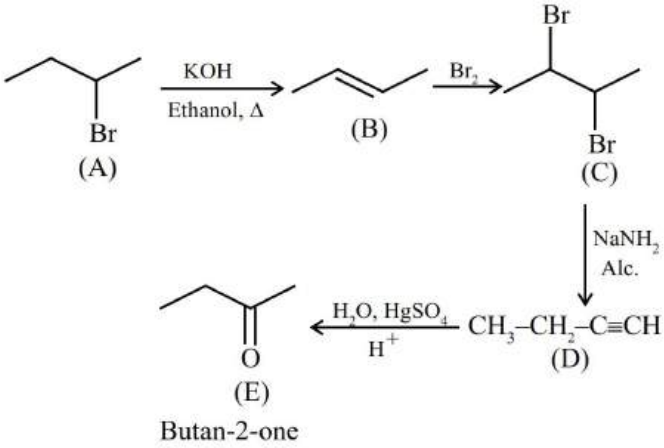
b, f, g, are not aromatic, these compounds do not follow Huckel's rule

52. An optically active alkyl halide C_4H_9Br [A] reacts with hot KOH dissolved in ethanol and forms alkene [B] as major product which reacts with bromine to give dibromide [C]. The compound [C] is converted into a gas [D] upon reacting with alcoholic $NaNH_2$. During hydration 18 gram of water is added to 1 mole of gas [D] on warming with mercuric sulphate and dilute acid at 333 K to form compound [E]. The IUPAC name of compound [E] is :

(1) But-2-yne (2) Butan-2-ol
(3) Butan-2-one (4) Butan-1-al

Ans. (3)

Sol.



53. The property/properties that show irregularity in first four elements of group-17 is/are :

(A) Covalent radius

(B) Electron affinity

(C) Ionic radius

(D) First ionization energy

Choose the **correct** answer from the options given below:

(1) B and D only (2) A and C only
(3) B only (4) A, B, C and D

Ans. (3)

Sol. The order of first four elements of group-17 are as follows.

F < Cl < Br < I (Covalent radius)

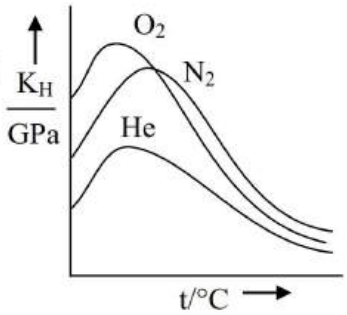
Cl > F > Br > I (Electron affinity)

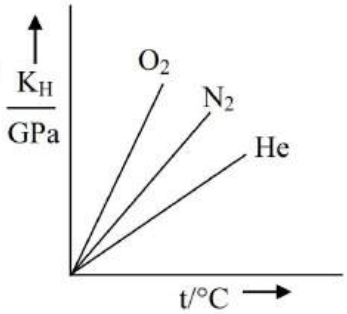
$F^- < Cl^- < Br^- < I^-$ (Ionic radius)

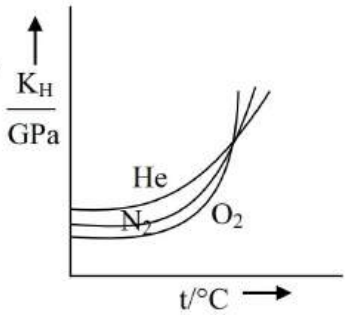
F > Cl > Br > I (I^{st} ionization energy)

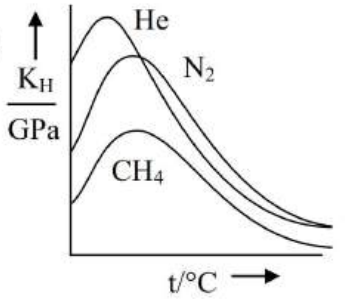
Electron affinity order is irregular.

54. Which of the following graph correctly represents the plots of K_H at 1 bar gases in water versus temperature ?

(1)


(2)


(3)


(4)


Ans. (4)

Sol. As temperature increases solubility first decrease then increase hence K_H first increase than decrease also at moderate temperature K_H value He > N_2 > CH_4.

55. According to Bohr's model of hydrogen atom, which of the following statement is **incorrect**?

(1) Radius of 3rd orbit is nine times larger than that of 1st orbit.

(2) Radius of 8th orbit is four times larger than that of 4th orbit.

(3) Radius of 6th orbit is three time larger than that of 4th orbit.

(4) Radius of 4th orbit is four times larger than that of 2nd orbit.

Ans. (3)

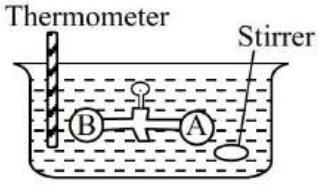
Sol. $r \propto n^2$

(1) $\frac{r_3}{r_1} = \frac{9}{1}$

(2) $\frac{r_8}{r_4} = \frac{64}{16} = 4$

(3) $\frac{r_6}{r_4} = \left(\frac{6}{4}\right)^2 = \frac{9}{4}$

(4) $\frac{r_4}{r_2} = \left(\frac{4}{2}\right)^2 = 4$

56.


Two vessels A and B are connected via stopcock. The vessel A is filled with a gas at a certain pressure. The entire assembly is immersed in water and is allowed to come to thermal equilibrium with water. After opening the stopcock the gas from vessel A expands into vessel B and no change in temperature is observed in the thermometer. Which of the following statement is **true**?

(1) $dw \neq 0$

(2) $dq \neq 0$

(3) $dU \neq 0$

(4) The pressure in the vessel B before opening the stopcock is zero.

Ans. (4)

Sol. It is free expansion of gas $\Rightarrow P_{ext} = 0$

Where $w = 0$, $q = 0$ and $\Delta U = 0$

57. A solution is made by mixing one mole of volatile liquid A with 3 moles of volatile liquid B. The vapour pressure of pure A is 200 mm Hg and that of the solution is 500 mm Hg. The vapour pressure of pure B and the least volatile component of the solution, respectively, are :

(1) 1400 mm Hg, A (2) 1400 mm Hg, B

(3) 600 mm Hg, B (4) 600 mm Hg, A

Ans. (4)

Sol. $P_S = P_A^o.X_A + P_B^o.X_B$

$500 = 200 \times \frac{1}{4} + P_B^o.\frac{3}{4}$

$P_B^o = 600$ mm Hg

As $P_A^o < P_B^o \Rightarrow$ A is least volatile.

58. $CaCO_3(s) + 2HCl(aq) \rightarrow CaCl_2(aq) + CO_2(g)\ H_2O(l)$

Consider the above reaction, what mass of $CaCl_2$ will be formed if 250 mL of 0.76 M HCl reacts with 1000 g of $CaCO_3$?

(Given : Molar mass of Ca, C, O, H and Cl are 40, 12, 16, 1 and 35.5 g mol^{-1}, respectively)

(1) 3.908 g

(2) 2.636 g

(3) 10.545 g

(4) 5.272 g

Ans. (3)

Sol. $CaCO_3 + 2HCl \rightarrow CaCl_2 + CO_2 + H_2O$

Moles of $CaCO_3 = \frac{1000}{100} = 10$

Moles of HCl $= 0.76 \times \frac{250}{1000} = 0.19$ (L.R.)

Moles of $CaCl_2$ formed $= \frac{0.19}{2}$

Mass of $CaCl_2 = \frac{0.19}{2} \times 111 = 10.545$ gm

59. If equal volumes of AB_2 and XY (both are salts) aqueous solutions are mixed, which of the following combination will give a precipitate of AY_2 at 300 K?

(Given K_{sp} (at 300 K) for $AY_2 = 5.2 \times 10^{-7}$)

(1) 3.6×10^{-3} M AB_2, 5.0×10^{-4} M XY

(2) 2.0×10^{-4} M AB_2, 0.8×10^{-3} M XY

(3) 2.0×10^{-2} M AB_2, 2.0×10^{-2} M XY

(4) 1.5×10^{-4} M AB_2, 1.5×10^{-3} M XY

Ans. (3)

Sol. When equal volumes are mixed molarity reduce to half.

For precipitation $Q_{SP} = [A^{+2}]\,[Y^-]^2 > K_{SP}$

(1) $Q_{SP} = (1.8 \times 10^{-3})\left(\frac{5}{2} \times 10^{-4}\right)^2 < K_{SP}$

(2) $Q_{SP} = (10^{-4})\,(0.4 \times 10^{-3})^2 < K_{SP}$

(3) $Q_{SP} = (10^{-2})\,(10^{-2})^2 > K_{SP}$

(4) $Q_{SP} = \left(\frac{1.5}{2} \times 10^{-4}\right)\left(\frac{1.5}{2} \times 10^{-3}\right)^2 < K_{SP}$

60. Among SO_2, NF_3, NH_3, XeF_2, ClF_3 and SF_4, the hybridization of the molecule with non-zero dipole moment and highest number of lone-pairs of electrons on the central atom is

(1) sp^3 (2) dsp^2

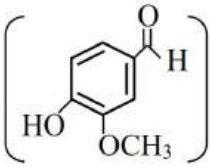
(3) sp^3d^2 (4) sp^3d

Ans. (4)

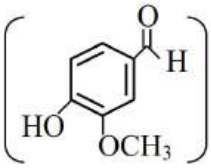
Sol.

Molecule	Hybridisation	Dipole Moment	Lone pair on the central atom
SO_2	sp^2	Non-zero	**1**
NF_3	sp^3	Non-zero	**1**
NH_3	sp^3	Non-zero	**1**
XeF_2	sp^3d	zero	**3**
$C\ell F_3$	sp^3d	Non-zero	**2**
SF_4	sp^3d	Non-zero	**1**

61. Given below are two statements :

Statement (I) : Vanillin 

will react with NaOH and also with Tollen's reagent.

Statement (II) : Vanillin 

will undergo self aldol condensation very easily.

In the light of the above statements, choose the **most appropriate answer** from the options given below :

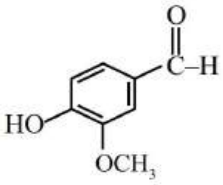
(1) **Statement I** is incorrect but **Statement II** is correct

(2) **Statement I** is correct but **Statement II** is incorrect

(3) Both **Statement I** and **Statement II** are incorrect

(4) Both **Statement I** and **Statement II** are correct

Ans. (2)

Sol. 

Phenolic group soluble in NaOH

Benzaldehyde derivative react with Tollen's reagent.

Vanillin does not give self-aldol reaction due to lack of acidic H for condensation.

62. Identify the correct statement among the following:

(1) All naturally occurring amino acids except glycine contain one chiral centre.

(2) All naturally occurring amino acids are optically active.

(3) Glutamic acid is the only amino acid that contains a –COOH group at the side chain.

(4) Amino acid, cysteine easily undergo dimerization due to the presence of free SH group.

Ans. (4)

Sol. * Isoleucine has 2 chiral centre

* Glycine is optically inactive

* Aspartic acid also contain COOH group at the side chain.

* Cysteine easily dimmerise due to free SH group

63. The correct order of basic nature on aqueous solution for the bases NH_3, $H_2N–NH_2$, $CH_3CH_2NH_2$, $(CH_3CH_2)_2NH$ and $(CH_3CH_2)_3N$ is :

(1) $NH_3 < H_2N–NH_2 < (CH_3CH_2)_3N < CH_3CH_2NH_2 < (CH_3CH_2)_2NH$

(2) $NH_3 < H_2N–NH_2 < CH_3CH_2NH_2 < (CH_3CH_2)_2NH < (CH_3CH_2)_3N$

(3) $H_2N–NH_2 < NH_3 < (CH_3CH_2)_3N < CH_3CH_2NH_2 < (CH_3CH_2)_2NH$

(4) $NH_2–NH_2 < NH_3 < CH_3CH_2NH_2 < (CH_3CH_2)_3N < (CH_3CH_2)_2NH$

Ans. (4)

Sol. Basic strength of amine depends on hydrogen bonding and electronic inductive effect.

$$NH(Et)_2 > N(Et)_3 > NH_2Et > \ddot{N}H_3 > NH_2 - NH_2$$

64. Given below are two statements :

Statement (I) : The metallic radius of Al is less than that of Ga.

Statement (II) : The ionic radius of Al^{3+} is less than that of Ga^{3+}.

In the light of the above statements, choose the **most appropriate answer** from the options given below :

(1) Both **Statement I** and **Statement II** are incorrect

(2) **Statement I** is incorrect but **Statement II** is correct

(3) **Statement I** is correct but **Statement II** is incorrect

(4) Both **Statement I** and **Statement II** are correct

Ans. (2)

Sol. $\Rightarrow$ The metallic radius order of Al & Ga is

$$B < \underbrace{Ga < Al}_{\downarrow} < In < T\ell$$

(due to poor shielding of d-subshell electrons)

$\Rightarrow$ The ionic radius order of Al^{+3} & Ga^{+3} is

$B^{+3} < Al^{+3} < Ga^{+3} < In^{+3} < T\ell^{+3}$

65. Given below are two statements :

Statement (I) : In octahedral complexes, when $\Delta_o < P$ high spin complexes are formed. When $\Delta_o > P$ low spin complexes are formed.

Statement (II) : In tetrahedral complexes because of $\Delta_t < P$, low spin complexes are rarely formed.

In the light of the above statements, choose the **most appropriate answer** from the options given below :

(1) **Statement I** is correct but **Statement II** is incorrect.

(2) Both **Statement I** and **Statement II** are incorrect

(3) **Statement I** is incorrect but **Statement II** is correct

(4) Both **Statement I** and **Statement II** are correct

Ans. (4)

Sol. In octahedral complex (CN = 6)

If $\Delta_0 <$ P.E. , then high spin complexes are formed

If $\Delta_0 >$ P.E. , then low spin complexes are formed

But in tetrahedral complex (CN = 4)

$\Delta_t <$ P.E. , then mainly high spin complexes are formed and rarely low spin complexes are formed.

66. Choose the correct tests with respective observations.

(A) $CuSO_4$ (acidified with acetic acid) + $K_4[Fe(CN)_6] \rightarrow$ Chocolate brown precipitate.

(B) $FeCl_3 + K_4[Fe(CN)_6] \rightarrow$ Prussian blue precipitate.

(C) $ZnCl_2 + K_4[Fe(CN)_6]$, neutralised with NH_4OH $\rightarrow$ White or bluish white precipitate.

(D) $MgCl_2 + K_4[Fe(CN)_6] \rightarrow$ Blue precipitate.

(E) $BaCl_2 + K_4[Fe(CN)_6]$, neutralised with NaOH $\rightarrow$ White precipitate.

Choose the **correct** answer from the options given below :

(1) A, D and E only (2) B, D and E only

(3) A, B and C only (4) C, D and E only

Ans. (3)

Sol.

$$2CuSO_4 + K_4[Fe(CN)_6] \xrightarrow{CH_3COOH} \underset{\text{(Chocolate brown ppt.)}}{Cu_2[Fe(CN)_6]} + 2K_2SO_4$$

$$4FeCl_3 + 3K_4[Fe(CN)_6] \longrightarrow \underset{\text{(Prussian Blue ppt.)}}{Fe_4[Fe(CN)_6]_3} + 12KCl$$

$$3ZnCl_4 + 2K_4[Fe(CN)_6] \xrightarrow{NH_4OH} \underset{\text{(White or bluish white ppt.)}}{K_2Zn_3[Fe(CN)_6]_2} + 6KCl$$

67. On complete combustion 1.0 g of an organic compound (X) gave 1.46 g of CO_2 and 0.567 g of H_2O. The empirical formula mass of compound (X) is _____ g.

(Given molar mass in g mol^{-1} C : 12, H : 1, O : 16)

(1) 30 (2) 45 (3) 60 (4) 15

Ans. (1)

Sol. Moles of 'C' = $n_{CO_2} = \frac{1.46}{44} = 0.033$

Moles of 'C' = $W_C = 0.033 \times 12$

Moles of 'H' = $2 \times n_{H_2O} = 2 \times \frac{0.567}{18} = 0.063$

Mass of 'H' = 0.0063

Mass of Oxygen (O) = $1 - (W_C + W_H)$

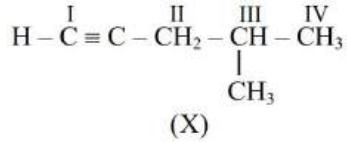
$= 1 - (0.033 \times 12 + 0.063 \times 1) = 0.541$ gm

Moles of 'O' = $\frac{0.541}{16} = 0.033$

Empirical formula = CH_2O

Empirical formula mass = 30.

68. Consider the following compound (X)



(X)

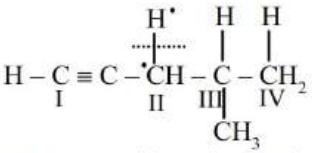
The most stable and least stable carbon radicals, respectively, produced by homolytic cleavage of corresponding C – H bond are :

(1) II, IV (2) III, II

(3) I, IV (4) II, I

Ans. (4)

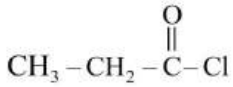
Sol.



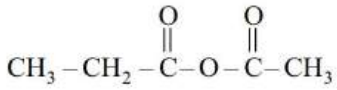
II most stable carbon radical due to resonance stablise

I least stable carbon radical due to no stabilising factor.

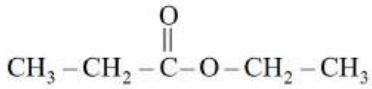
69. Consider the following molecules :



(p)



(q)



(r)

(s)

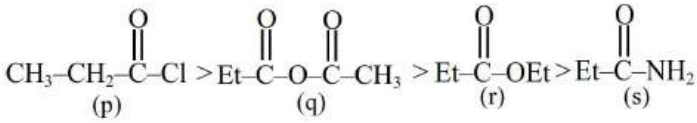
The correct order of rate of hydrolysis is :

(1) r > q > p > s (2) q > p > r > s

(3) p > r > q > s (4) p > q > r > s

Ans. (4)

Sol. Rate of hydrolysis ∝ Leaving group ability



70. A molecule with the formula AX_4Y has all it's elements from p-block. Element A is rarest, monoatomic, non-radioactive from its group and has the lowest ionization enthalpy value among A, X and Y. Elements X and Y have first and second highest electronegativity values respectively among all the known elements. The shape of the molecule is :

(1) Square pyramidal

(2) Octahedral

(3) Pentagonal planar

(4) Trigonal bipyramidal

Ans. (1)

Sol. Given A is rarest, monoatomic, non-radioactive p-block element and form AX_4Y type of molecule.

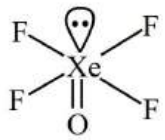
∴ It is concluded that it is Xe

It is given the electronegativity of A is less than X & Y

It is given the electronegativity of X & Y is highest and second highest respectively among all element.

∴ X & Y are F & O

∴ Compound is consider as $XeOF_4$ with square pyramidal shape.



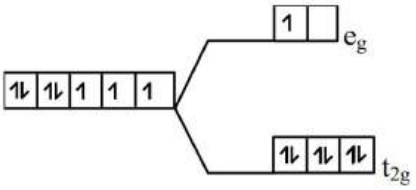
SECTION-B

71. A transition metal (M) among Mn, Cr, Co and Fe has the highest standard electrode potential (M^{3+}/M^{2+}). It forms a metal complex of the type $[M(CN)_6]^{4-}$. The number of electrons present in the e_g orbital of the complex is _____.

Ans. (1)

Sol. Co has highest standard electrode potential (M^{+3}/M^{+2}) among Mn, Cr, Co, Fe

$\therefore$ Complex is $[Co(CN)_6]^{4-}$ and its splitting is as follows.



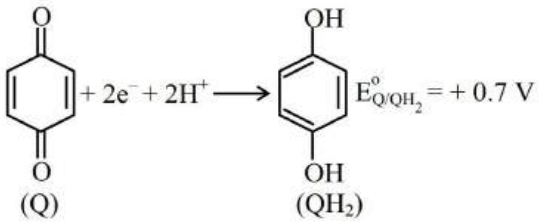
$\therefore$ electron in e_g orbital is one.

72. Consider the following electrochemical cell at standard condition.

$Au(s)|QH_2,Q|NH_4X(0.01M)||Ag^+(1M)|Ag(s)$

$E_{cell} = +0.4V$

The couple QH_2/Q represents quinhydrone electrode, the half cell reaction is given below



$$\left[\text{Given : } E^o_{Ag^+/Ag} = +0.8V \text{ and } \frac{2.303RT}{F} = 0.06V\right]$$

The pK_b value of the ammonium halide salt (NH_4X) used here is _____. (nearest integer)

Ans. (6)

Sol. $QH_2 + 2Ag^+ \rightarrow 2Ag + Q + 2H^+$

$$E = E° - \frac{0.06}{2} \log [H^+]^2$$

$$E = E° - 0.06 \times \log [H^+]$$

$$pH = -\log (H^+) = \frac{E - E^o}{0.06} = \frac{0.4 - 0.1}{0.06}$$

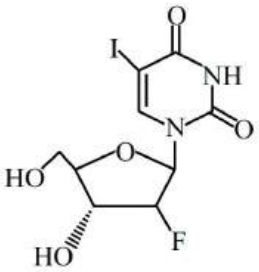
$$= \frac{0.3}{0.06} = 5$$

$$pH + NH_4X = 7 - \frac{1}{2} pK_b - \frac{1}{2} \log C$$

$$5 = 7 - \frac{1}{2} \times pK_b - \frac{1}{2} \log (10^{-2})$$

$pK_b = 6.$

73. 0.1 mol of the following given antiviral compound (P) will weigh _____ $\times 10^{-1}$ g

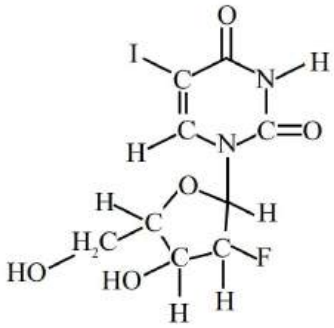


(P)

(Given : molar mass in g mol^{-1} H: 1, C : 12, N : 14, O : 16, F : 19, I : 127)

Ans. (372)

Sol.



Molar mass = 372 gm

$\therefore$ 0.1 mole has = 372×10^{-1} gm

74. Consider the following equilibrium,

$$CO(g) + 2H_2(g) \rightleftharpoons CH_3OH(g)$$

0.1 mol of CO along with a catalyst is present in a 2 dm^3 flask maintained at 500 K. Hydrogen is introduced into the flask until the pressure is 5 bar and 0.04 mol of CH_3OH is formed. The K_p^0 is _____ $\times 10^{-3}$ (nearest integer).

Given : R = 0.08 dm^3 bar K^{-1} mol^{-1}

Assume only methanol is formed as the product and the system follows ideal gas behaviour.

Ans. (74)

Sol.

	$CO(g)$	$+ 2H_2(g)$	$\rightleftharpoons CH_3OH(g)$
t = 0	0.1 mol	a mol	–
t_{eq}	0.1 – x = 0.06	a – 2x = a – 0.08 = 0.23 – 0.08 = 0.15 mole	x = 0.04

V = 2L

T = 500 K

P_{total} = 5 bar

$n_{Total} = 0.25 = \frac{1}{4}$ mol.

$$P_{total} = n_{total} \times \frac{RT}{V}$$

$$\Rightarrow 5 = (0.06 + a - 0.08 + 0.04) \times \frac{0.08 \times 500}{2}$$

$$\Rightarrow 10 = (0.02 + a) \times 0.08 \times 500$$

$$\Rightarrow a = 0.25 - 0.02 = 0.23 \text{ mol.}$$

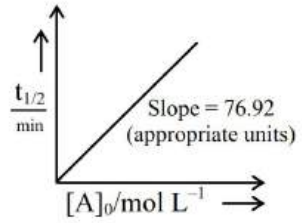
$$K_P = \frac{X_{CH_3OH}}{X_{CO} \times X_{H_2}^2} \times \frac{1}{(P_T)^2} = \frac{0.04}{0.06 \times (0.15)^2} \times \left[\frac{1/4}{5}\right]^2$$

$$= \frac{4}{6 \times (0.15)^2 \times 16} \times \frac{1}{25}$$

$$= \frac{100 \times 100}{24 \times 225 \times 25} = \frac{100 \times 100}{135000}$$

$$= 0.074 = 74 \times 10^{-3}$$

75. For the reaction A → products.



The concentration of A at 10 minutes is _____ $\times 10^{-3}$ mol L^{-1} (nearest integer).

The reaction was started with 2.5 mol L^{-1} of A.

Ans. (2435)

Sol. $t_{1/2} \propto [A]_0 \Rightarrow$ Order = zero

$$t_{1/2} = \frac{A_0}{2K} \Rightarrow \text{Slope} = \frac{1}{2K} = 76.92$$

$$K = \frac{1}{2 \times 76.92}$$

$$[A]_{10} = -Kt + A_0 = -\frac{1}{2 \times 76.92} \times 10 + 2.5 = 2.435$$

$$= 2435 \times 10^{-3} \text{ mol/L}$$

JEE–MAIN EXAMINATION – JANUARY 2025

(HELD ON THURSDAY 23rd JANUARY 2025)

TIME : 3:00 PM TO 6:00 PM

MATHEMATICS

SECTION-A

1. If in the expansion of $(1 + x)^p (1 - x)^q$, the coefficients of x and x^2 are 1 and –2, respectively, then $p^2 + q^2$ is equal to :

(1) 8 (2) 18

(3) 13 (4) 20

Ans. (3)

Sol. $(1+x)^p(1-x)^q = \left({}^pC_0 + {}^pC_1x + {}^pC_2x^2 + ...\right)\left({}^qC_0 - {}^qC_1x + {}^qC_2x^2 + ...\right)$

coff of x $\equiv {}^pC_0\,{}^qC_1 - {}^pC_1\,{}^qC_0 = 1$

$p - q = 1$

coff of $x^2 \equiv {}^pC_0\,{}^qC_2 - {}^pC_1\,{}^qC_1 + {}^pC_2\,{}^qC_0 = -2$

$\frac{q(q-1)}{2} - pq + \frac{p(p-1)}{2} = -2$

$q^2 - q - 2pq + p^2 - p = -4$

$(p - q)^2 - (p + q) = -4$

$p + q = 5$

$p = 3$

$q = 2$

so $p^2 + q^2 = 13$

2. Let $A = \{(x, y) \in \mathbf{R} \times \mathbf{R} : |x + y| \geq 3\}$ and

$B = \{(x, y) \in \mathbf{R} \times \mathbf{R} : |x| + |y| \leq 3\}$.

If $C = \{(x, y) \in \mathbf{A} \cap \mathbf{B} : x = 0 \text{ or } y = 0\}$, then

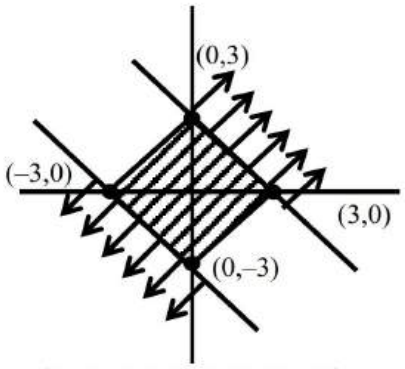
$\sum_{(x,y)\in C} |x+y|$ is :

(1) 15 (2) 18

(3) 24 (4) 12

Ans. (4)

Sol.



$C = \{(3,0), (-3,0), (0,3), (0,-3)\}$

$\Sigma|x + y| = 12$

3. The system of equations

$x + y + z = 6,$

$x + 2y + 5z = 9,$

$x + 5y + \lambda z = \mu,$

has no solution if

(1) $\lambda = 17, \mu \neq 18$

(2) $\lambda \neq 17, \mu \neq 18$

(3) $\lambda = 15, \mu \neq 17$

(4) $\lambda = 17, \mu = 18$

Ans. (1)

Sol. $D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 5 \\ 1 & 5 & \lambda \end{vmatrix} = 0$

$\lambda = 17$

$D_z = \begin{vmatrix} 1 & 1 & 6 \\ 1 & 2 & 9 \\ 1 & 5 & \mu \end{vmatrix} \neq 0$

$\mu \neq 18$

4. Let $\int x^3 \sin x dx = g(x) + C$, where C is the constant of integration.

If $8\left(g\left(\frac{\pi}{2}\right) + g'\left(\frac{\pi}{2}\right)\right) = \alpha\pi^3 + \beta\pi^2 + \gamma, \alpha, \beta, \gamma \in Z,$

Then $\alpha + \beta - \gamma$ equals :

(1) 55 (2) 47

(3) 48 (4) 62

Ans. (1)

Sol. $\int x^3 \sin x dx = -x^3 \cos x + \int 3x^2 \cos x dx$

$= -x^3 \cos x + 3x^2 \sin x - \int 6x \sin x dx$

$= -x^3 \cos x + 3x^2 \sin x + 6x \cos x - 6 \sin x + c$

So $g(x) = -x^3 \cos x + 3x^2 \sin x + 6x \cos x - 6 \sin x$

$g\left(\frac{\pi}{2}\right) = \frac{3\pi^2}{4} - 6$

$g'(x) = x^3 \sin x$

$g'\left(\frac{\pi}{2}\right) = \frac{\pi^3}{8}$

$8\left(g\left(\frac{\pi}{2}\right) + g'\left(\frac{\pi}{2}\right)\right) = \pi^3 + 6\pi^2 - 48$

So $\alpha + \beta - \gamma = 55$

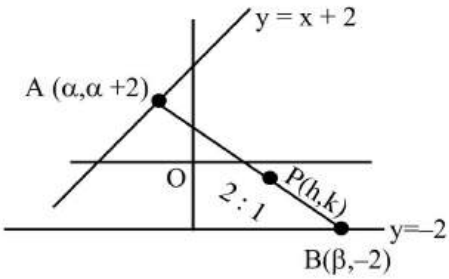
5. A rod of length eight units moves such that its ends A and B always lie on the lines $x - y + 2 = 0$ and $y + 2 = 0$, respectively. If the locus of the point P, that divides the rod AB internally in the ratio 2 : 1 is $9(x^2 + \alpha y^2 + \beta xy + \gamma x + 28y) - 76 = 0$, then $\alpha - \beta - \gamma$ is equal to :

(1) 24 (2) 23

(3) 21 (4) 22

Ans. (2)

Sol.



$h = \frac{3\beta + \alpha}{3}$

$k = \frac{-4 + \alpha + 2}{3}$

$\alpha = 3k + 2$

$2\beta = 3h - a = 3h - 3k - 2$

so AB = 8

$(\alpha - \beta)^2 + (\alpha + 4)^2 = 64$

$\left(3k + 2 - \left(\frac{3h - 3k - 2}{2}\right)\right)^2 + (3k + 2 + 4)^2 = 64$

$\frac{(9k - 3h + 6)^2}{4} + (3k + 6)^2 = 64$

$9\left[(3k - h + 2)^2 + 4(k + 2)^2\right] = 64 \times 4$

$9(x^2 + 13y^2 - 6xy - 4x + 28y) = 76$

$\alpha - \beta - \gamma = 13 + 6 + 4 = 23$

6. The distance of the line $\frac{x-2}{2} = \frac{y-6}{3} = \frac{z-3}{4}$ from the point (1, 4, 0) along the line $\frac{x}{1} = \frac{y-2}{2} = \frac{z+3}{3}$ is :

(1) $\sqrt{17}$ (2) $\sqrt{14}$

(3) $\sqrt{15}$ (4) $\sqrt{13}$

Ans. (2)

Sol. Let the parallel line is

$\frac{x-1}{1} = \frac{y-4}{2} = \frac{z-0}{3}$

so their point of intersection is

$(\lambda + 1, 2\lambda + 4\ 3\lambda) = (2t + 2, 3t + 6, 4t + 3)$

$\lambda = 2t + 1$

$2\lambda + 4 = 3t + 6 \Rightarrow t = 0$

so POI is (2,6,3)

so distance $= \sqrt{(2-1)^2 + (6-4)^2 + (3-0)^2} = \sqrt{14}$

7. Let the point A divide the line segment joining the points P(–1, –1, 2) and Q(5, 5,10) internally in the ratio r : 1 (r > 0). If O is the origin and $\left(\overrightarrow{OQ}.\overrightarrow{OA}\right) - \frac{1}{5}\left|\overrightarrow{OP} \times \overrightarrow{OA}\right|^2 = 10$, then the value of r is :

(1) 14 (2) 3

(3) $\sqrt{7}$ (4) 7

Ans. (4)

Sol. $A \equiv \left(\frac{5r-1}{r+1}, \frac{5r-1}{r+1}, \frac{10r+2}{r+1}\right)$

$\left(\overrightarrow{OQ}.\overrightarrow{OA}\right) - \frac{\left|\overrightarrow{OP} \times \overrightarrow{OA}\right|^2}{5} = 10$ ....(1)

$\overrightarrow{OQ}.\overrightarrow{OA} = \frac{10}{r+1}(15r + 1)$

$\left|\overrightarrow{OP} \times \overrightarrow{OA}\right|^2 = \frac{r^2}{(r+1)^2}(800)$

so by equation (1)

$\frac{10}{r+1}(15r + 1) - \frac{1}{5}\frac{r^2(800)}{(r+1)^2} = 10$

$2r^2 - 14r = 0$

$r = 7, r \neq 0$

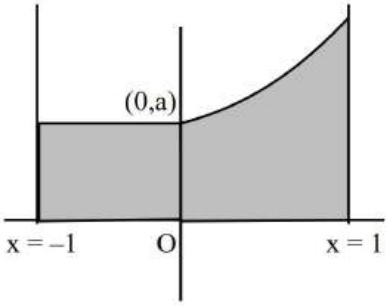
8. If the area of the region $\{(x, y) : -1 \le x \le 1, 0 \le y \le a + e^{|x|} - e^{-x}, a > 0\}$ is $\frac{e^2 + 8e + 1}{e}$, then the value of a is :

(1) 7 (2) 6

(3) 8 (4) 5

Ans. (4)

Sol.



required area is $a+\int_0^1\left(a+e^x-e^{-x}\right)dx$

$a+\left[a+e^x+e^{-x}\right]_0^1$

$2a+e-1+e^{-1}-1=e+8+\frac{1}{e}$

$2a=10 \Rightarrow a=5$

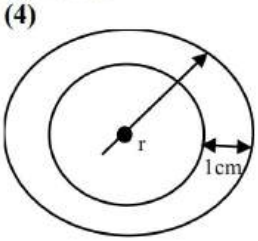
9. A spherical chocolate ball has a layer of ice-cream of uniform thickness around it. When the thickness of the ice-cream layer is 1 cm, the ice-cream melts at the rate of 81 cm^3/min and the thickness of the ice-cream layer decreases at the rate of $\frac{1}{4\pi}$ cm/min. The surface area (in cm^2) of the chocolate ball (without the ice-cream layer) is :

(1) 225 π (2) 128 π

(3) 196 π (4) 256 π

Ans. (4)

Sol



$v=\frac{4}{3}\pi r^3$

$\frac{dv}{dt}=4\pi r^2\frac{dr}{dt}$

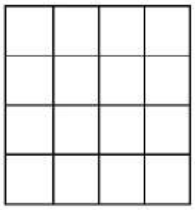
$81=4\pi r^2\times\frac{1}{4\pi}$

$r^2=81$

$r=9$

surface area of chocolate $=4\pi(r-1)^2=256\pi$

10. A board has 16 squares as shown in the figure :



Out of these 16 squares, two squares are chosen at random. The probability that they have no side in common is :

(1) $\frac{4}{5}$ (2) $\frac{7}{10}$

(3) $\frac{3}{5}$ (4) $\frac{23}{30}$

Ans. (1)

Sol. Total ways for selecting any two squares $={}^{16}C_2$

$=120$

Total ways for selecting common side squares

$=\underbrace{3\times4}_{\text{Horizontal side}}+\underbrace{3\times4}_{\text{vertical side}}$

$=24$

so required probability

$=1-\frac{24}{120}$

$=\frac{4}{5}$

11. Let x = x(y) be the solution of the differential equation $y=\left(x-y\frac{dx}{dy}\right)\sin\left(\frac{x}{y}\right)$, $y>0$ and $x(1)=\frac{\pi}{2}$.

Then cos(x(2)) is equal to :

(1) $1-2(\log_e 2)^2$ (2) $2(\log_e 2)^2-1$

(3) $2(\log_e 2)-1$ (4) $1-2(\log_e 2)$

Ans. (2)

Sol. $ydy=(xdy-ydx)\sin\left(\frac{x}{y}\right)$

$\frac{dy}{y}=\left(\frac{xdy-ydx}{y^2}\right)\sin\left(\frac{x}{y}\right)$

$\frac{dy}{y}=\sin\left(\frac{x}{y}\right)d\left(-\frac{x}{y}\right)$

$\Rightarrow \quad \ell ny=\cos\frac{x}{y}+C$

$x(1) = \frac{\pi}{2} \Rightarrow 0 = \cos\frac{\pi}{2} + C \Rightarrow C=0$

$\ell ny = \cos\frac{x}{y}$

but $y = 2 \Rightarrow \cos\frac{x}{2} = \ell n2$

$\cos x = 2\cos^2\frac{x}{2} - 1$

$= 2(\ell n2)^2 - 1$

12. Let the range of the function

$f(x) = 6 + 16 \cos x \,.\, \cos\left(\frac{\pi}{3} - x\right).\ \cos\left(\frac{\pi}{3} + x\right).$

$\sin 3x \,.\, \cos 6x$, $x \in R$ be $[\alpha, \beta]$. Then the distance of the point (α, β) from the line $3x + 4y + 12 = 0$ is :

(1) 11 (2) 8

(3) 10 (4) 9

Ans. (1)

Sol. $f(x) = 6 + 16\left(\frac{1}{4}\cos 3x\right)\sin 3x.\cos 6x$

$= 6 + 4 \cos 3x \sin 3x \cos 6x$

$= 6 + \sin 12x$

Range of f(x) is [5, 7]

$(\alpha, \beta) \equiv (5, 7)$

distance $= \left|\frac{15+28+12}{5}\right| = 11$

13. Let the shortest distance from (a, 0), a > 0, to the parabola $y^2 = 4x$ be 4. Then the equation of the circle passing through the point (a, 0) and the focus of the parabola, and having its centre on the axis of the parabola is:

(1) $x^2 + y^2 - 6x + 5 = 0$

(2) $x^2 + y^2 - 4x + 3 = 0$

(3) $x^2 + y^2 - 10x + 9 = 0$

(4) $x^2 + y^2 - 8x + 7 = 0$

Ans. (1)

Sol. Normal at P

$y + tx = 2t + t^3$

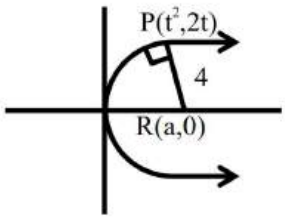
$\uparrow$

(a, 0)

$at = 2t + t^3$

$a = 2 + t^2$

$R\ (2 + t^2, 0)$



$PR = 4 \Rightarrow 4 + 4t^2 = 16$

$4t^2 = 12 \Rightarrow t^2 = 3$

$a = 5$, $R(5, 0)$

Focus (1, 0)

(1, 0) & (5, 0) will be the end points of diameter

$\Rightarrow$ Eq^n of circle is

$(x - 1)(x - 5) + y^2 = 0$

$x^2 + y^2 - 6x + 5 = 0$

14. Let $X = R \times R$. Define a relation R on X as:

$(a_1, b_1)\ R\ (a_2, b_2) \Leftrightarrow b_1 = b_2$.

Statement-I: R is an equivalence relation.

Statement-II: For some $(a, b) \in X$, the set $S = \{(x, y) \in X : (x, y)\ R\ (a, b)\}$ represents a line parallel to $y = x$.

In the light of the above statements, choose the **correct** answer from the options given below:

(1) Both **Statement-I** and **Statement-II** are false.

(2) **Statement-I** is true but **Statement-II** is false.

(3) Both **Statement-I** and **Statement-II** are true.

(4) **Statement-I** is false but **Statement-II** is true.

Ans. (2)

Sol. Statement – I :

Reflexive : $(a_1,b_1)\ R(a_1, b_1) \Rightarrow b_1 = b_1$ True

Symmetric : $\left.\begin{array}{l}(a_1,b_1)\ R\ (a_2, b_2) \Rightarrow b_1 = b_2 \\ (a_2,b_2)\ R\ (a_1, b_1) \Rightarrow b_2 = b_1\end{array}\right\}$ True

Transitive : $\left.\begin{array}{l}(a_1, b_1)\ R\ (a_2, b_2) \Rightarrow b_1 = b_2 \\ \&\ (a_2, b_2)\ R\ (a_3, b_3)\ b_2 = b_3 \\ \Rightarrow (a_1, b_1)\ R\ (a_3, b_3) \Rightarrow \text{True}\end{array}\right\} b_1 = b_3$

Hence Relation R is an equivence relation Statement-I is true.

For statement – II $\Rightarrow$ y = b so False

15. The length of the chord of the ellipse $\frac{x^2}{4}+\frac{y^2}{2}=1$, whose mid-point is $\left(1,\frac{1}{2}\right)$, is:

(1) $\frac{2}{3}\sqrt{15}$ (2) $\frac{5}{3}\sqrt{15}$

(3) $\frac{1}{3}\sqrt{15}$ (4) $\sqrt{15}$

Ans. (1)

Sol. $T = S_1$

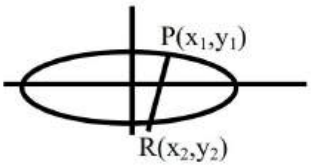
$$\frac{x.1}{4}+\frac{y.\frac{1}{2}}{2}=\frac{1}{4}+\frac{1}{8}$$

$$x+y=\frac{3}{2}$$

solve with ellipse

$$PR=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$=\sqrt{2}|x_2-x_1|$$



$$y_2=\frac{3}{2}-x_2$$

$$y_1=\frac{3}{2}-x_1$$

$$y_2-y_1=x_2-x_1$$

$$x^2+2y^2=4$$

$$x^2+2\left(\frac{3}{2}-x\right)^2=4$$

$$6x^2-12x+1=0$$

$$x_1+x_2=2$$

$$x_1x_2=1/6$$

$$|x_2-x_1|=\sqrt{(x_2+x_1)^2-4x_1x_2}$$

$$=\sqrt{4-4/6}$$

$$PR=\sqrt{2}.2.\frac{\sqrt{5}}{\sqrt{2}\sqrt{3}}=\frac{2}{3}\sqrt{15}$$

16. Let $A=[a_{ij}]$ be a 3×3 matrix such that $A\begin{bmatrix}0\\1\\0\end{bmatrix}=\begin{bmatrix}0\\0\\1\end{bmatrix}$, $A\begin{bmatrix}4\\1\\3\end{bmatrix}=\begin{bmatrix}0\\1\\0\end{bmatrix}$ and $A\begin{bmatrix}2\\1\\2\end{bmatrix}=\begin{bmatrix}1\\0\\0\end{bmatrix}$, then a_{23} equals:

(1) –1 (2) 0

(3) 2 (4) 1

Ans. (1)

Sol. Let $A=\begin{bmatrix}a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\\ a_{31} & a_{32} & a_{33}\end{bmatrix}$

$$A\begin{bmatrix}0\\1\\0\end{bmatrix}=\begin{bmatrix}0\\0\\1\end{bmatrix}\Rightarrow\begin{bmatrix}a_{12}\\a_{22}\\a_{32}\end{bmatrix}=\begin{bmatrix}0\\1\\0\end{bmatrix}\Rightarrow \begin{matrix}a_{22}=0;a_{12}=0\\ a_{32}=1\end{matrix}$$

$$A\begin{bmatrix}4\\1\\3\end{bmatrix}=\begin{bmatrix}0\\1\\0\end{bmatrix}\Rightarrow \begin{matrix}4a_{11}+a_{12}+3a_{13}=0\\ 4a_{21}+a_{22}+3a_{23}=1\\ 4a_{31}+a_{32}+3a_{33}=0\end{matrix}\Rightarrow 4a_{21}+3a_{23}=1$$

$$A\begin{bmatrix}2\\1\\2\end{bmatrix}=\begin{bmatrix}1\\0\\0\end{bmatrix}\Rightarrow \begin{matrix}2a_{11}+a_{12}+2a_{13}=1\\ 2a_{21}+a_{22}+2a_{23}=0\\ 2a_{31}+a_{32}+2a_{33}=0\end{matrix}\Rightarrow a_{21}+a_{23}=0$$

$$-4a_{23}+3a_{23}=1\Rightarrow a_{23}=-1$$

17. The number of complex numbers z, satisfying $|z|=1$ and $\left|\frac{z}{\bar{z}}+\frac{\bar{z}}{z}\right|=1$, is :

(1) 6 (2) 4

(3) 10 (4) 8

Ans. (4)

Sol. $z=e^{i\theta}$

$$\frac{z}{\bar{z}}=e^{i2\theta}$$

$$\left|\frac{z}{\bar{z}}+\frac{\bar{z}}{z}\right|=1\Rightarrow\left|e^{i2\theta}+e^{-i2\theta}\right|=1\Rightarrow|\cos 2\theta|=\frac{1}{2}$$

8 solution in $[0, 2\pi)$

18. If the square of the shortest distance between the lines $\frac{x-2}{1}=\frac{y-1}{2}=\frac{z+3}{-3}$ and $\frac{x+1}{2}=\frac{y+3}{4}=\frac{z+5}{-5}$ is $\frac{m}{n}$, where m, n are coprime numbers, then m + n is equal to:

(1) 6 (2) 9

(3) 21 (4) 14

Ans. (2)

Sol. $\vec{a}=(2,1,-3)$

$\vec{b}=(-1,-3,-5)$

$$\vec{p}\times\vec{q}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 2 & -3\\ 2 & 4 & -5\end{vmatrix}$$

$=2\hat{i}-\hat{j}$

$\vec{b}-\vec{a}=-3\hat{i}-4\hat{j}-2\hat{k}$

$$S_d=\frac{\left|(\vec{b}-\vec{a}).(\vec{p}\times\vec{q})\right|}{|\vec{p}\times\vec{q}|}$$

$=\frac{2}{\sqrt{5}}$

$(S_d)^2=\frac{4}{5}$

$m=4, n=5 \Rightarrow m+n=9$

19. If $I=\int_0^{\frac{\pi}{2}}\frac{\sin^{\frac{3}{2}}x}{\sin^{\frac{3}{2}}x+\cos^{\frac{3}{2}}x}dx$,

then $\int_0^{2l}\frac{x\sin x\cos x}{\sin^4 x+\cos^4 x}dx$ equals:

(1) $\frac{\pi^2}{16}$ (2) $\frac{\pi^2}{4}$

(3) $\frac{\pi^2}{8}$ (4) $\frac{\pi^2}{12}$

Ans. (1)

Sol. For I

Apply king (P– 5) and add

$$2I=\int_0^{\pi/2}dx=\frac{\pi}{2}\Rightarrow I=\frac{\pi}{4}$$

$$I_2=\int_0^{\pi/2}\frac{x\sin x\cos x}{\sin^4 x+\cos^4 x}dx$$

Apply king and add

$$I_2=\frac{\pi}{4}\int_0^{\pi/2}\frac{\tan x\sec^2 x dx}{\tan^4 x+1}$$

put $\tan^2 x = t$

$$\frac{\pi}{8}\int_0^{\infty}\frac{dt}{t^2+1}$$

$$=\frac{\pi}{8}.\frac{\pi}{2}=\frac{\pi^2}{16}$$

20. $\lim_{x\to\infty}\frac{(2x^2-3x+5)(3x-1)^{\frac{x}{2}}}{(3x^2+5x+4)\sqrt{(3x+2)^x}}$ is equal to:

(1) $\frac{2}{\sqrt{3e}}$ (2) $\frac{2e}{\sqrt{3}}$

(3) $\frac{2e}{3}$ (4) $\frac{2}{3\sqrt{e}}$

Ans. (4)

Sol. $$\lim_{x\to\infty}\frac{\left(2-\frac{3}{x}+\frac{5}{x^2}\right)\left(1-\frac{1}{3x}\right)^{x/2}}{\left(3+\frac{5}{x}+\frac{4}{x^2}\right)\left(1+\frac{2}{3x}\right)^{x/2}}$$

$$=\lim_{x\to\infty}\frac{2}{3}.\frac{e^{\frac{x}{2}\left(1-\frac{1}{3x}-1\right)}}{e^{\frac{x}{2}\left(1+\frac{2}{3x}-1\right)}}$$

$$=\frac{2}{3}.\frac{e^{-\frac{1}{6}}}{e^{1/3}}=\frac{2}{3}e^{-\frac{1}{2}}$$

SECTION-B

21. The number of ways, 5 boys and 4 girls can sit in a row so that either all the boys sit together or no two boys sit together, is _____.

Ans. (17280)

Sol. A : number of ways that all boys sit together = 5! × 5!

B : number of ways if no 2 boys sit together = 4! × 5!

$A\cap B=\phi$

Required no. of ways = 5! × 5! + 4! × 5! = 17280

22. Let α, β be the roots of the equation $x^2 - ax - b = 0$ with Im(α) < Im(β). Let $P_n = \alpha^n - \beta^n$. If $P_3=-5\sqrt{7}i$, $P_4=-3\sqrt{7}i$, $P_5=11\sqrt{7}i$ and $P_6=45\sqrt{7}i$, then $|\alpha^4+\beta^4|$ is equal to _____.

Ans. (31)

Sol. $\alpha+\beta=a \quad \alpha\beta=-b$

$P_6=aP_5+bP_4$

$45\sqrt{7}i=a\times 11\sqrt{7}i+b(-3\sqrt{7})i$

$45 = 11a - 3b$ ...(1)

and

$P_5 = aP_4 + bP_3$

$11\sqrt{7}i = a\left(-3\sqrt{7}i\right) + b\left(-5\sqrt{7}i\right)$

$11 = -3a - 5b$ ...(2)

$a = 3, b = -4$

$\left|\alpha^4 + \beta^4\right| = \sqrt{\left(\alpha^4 - \beta^4\right)^2 + 4\alpha^4\beta^4}$

$= \sqrt{-63 + 4.4^4}$

$= \sqrt{-63 + 1024} = \sqrt{961} = 31$

23. The focus of the parabola $y^2 = 4x + 16$ is the centre of the circle C of radius 5. If the values of λ, for which C passes through the point of intersection of the lines $3x - y = 0$ and $x + \lambda y = 4$, are λ_1 and λ_2, $\lambda_1 < \lambda_2$, then $12\lambda_1 + 29\lambda_2$ is equal to _____.

Ans. (15)

Sol. $y^2 = 4(x + 4)$

Equation of circle

$(x + 3)^2 + y^2 = 25$

Passes through the point of intersection of two lines $3x - y = 0$ and $x + \lambda y = 4$ which is

$\left(\frac{4}{3\lambda + 1}, \frac{12}{3\lambda + 1}\right)$, after solving with circle,

we get

$\lambda = -\frac{7}{6}, 1$

$12\lambda_1 + 29\lambda_2$

$-14 + 29 = 15$

24. The variance of the numbers 8, 21, 34, 47, …, 320, is _____.

Ans. (8788)

Sol. Var(8, 21, 34, 47, ..., 320)

Var(0, 13, 26, 39, ..., 312)

13^2.Var(0, 1, 2, ..., 24)

13^2.Var(1, 2, 3, ..., 25)

So, $\sigma^2 = 13^2 \times \left(\frac{25^2 - 1}{12}\right) = 8788$

Alternate solution

$8 + (n - 1)13 = 320$

$13n = 325$

$n = 25$

no. of terms = 25

$\text{mean} = \frac{\Sigma x_i}{n} = \frac{8 + 21 + ... + 320}{25} = \frac{\frac{25}{2}(8 + 320)}{25}$

$\text{variance } \sigma^2 = \frac{\Sigma x_i^2}{n} - (\text{mean})^2$

$= \frac{8^2 + 21^2 + ... + 320^2}{13} - (164)^2$

$= 8788$

25. The roots of the quadratic equation $3x^2 - px + q = 0$ are 10th and 11th terms of an arithmetic progression with common difference $\frac{3}{2}$. If the sum of the first 11 terms of this arithmetic progression is 88, then $q - 2q$ is equal to _____.

Ans. (474)

Sol. $S_{11} = \frac{11}{2}(2a + 10d) = 88$

$a + 5d = 8$

$a = 8 - 5 \times \frac{3}{2} = \frac{1}{2}$

Roots are

$T_{10} = a + 9d = \frac{1}{2} + 9 \times \frac{3}{2} = 14$

$T_{11} = a + 10d = \frac{1}{2} + 10 \times \frac{3}{2} = \frac{31}{2}$

$\frac{p}{3} = T_{10} + T_{11} = 14 + \frac{31}{2} = \frac{59}{2}$

$p = \frac{177}{2}$

$\frac{q}{3} = T_{10} \times T_{11} = 7 \times 31 = 217$

$q = 651$

$q - 2p$

$= 651 - 177$

$= 474$

PHYSICS

SECTION-A

26. A ball having kinetic energy KE, is projected at an angle of 60° from the horizontal. What will be the kinetic energy of ball at the highest point of its flight?

(1) $\frac{(KE)}{8}$ (2) $\frac{(KE)}{4}$

(3) $\frac{(KE)}{16}$ (4) $\frac{(KE)}{2}$

Ans. (2)

Sol. Initial K.E,

$K.E. = \frac{1}{2} mu^2$

Speed at heighest point

$V = u \cos 60° = \frac{u}{2}$

$\therefore \; KE_2 = \frac{1}{2} m\left(\frac{u}{2}\right)^2$

$= \frac{1}{4} \times \frac{1}{2} mu^2$

$= \frac{KE}{4}$

27. Two charges 7 μc and – 4 μc are placed at (– 7 cm, 0, 0) and (7 cm, 0, 0) respectively. Given, $\epsilon_0 = 8.85 \times 10^{-12} C^2 N^{-1} m^{-2}$, the electrostatic potential energy of the charge configuration is :

(1) – 1.5 J (2) – 2.0 J

(3) – 1.2 J (4) – 1.8 J

Ans. (4)

Sol. P.E. of two charges

$u = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r}$

$r = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

= 14 cm

$\therefore u = \frac{9\times10^9 \times 7\times10^{-6} \times (-4)\times10^{-6}}{14\times10^{-2}}$

= – 1.8 J

28. The refractive index of the material of a glass prism is $\sqrt{3}$. The angle of minimum deviation is equal to the angle of the prism. What is the angle of the prism?

(1) 50° (2) 60°

(3) 58° (4) 48°

Ans. (2)

Sol. $\mu = \frac{\sin\left(\frac{A+\delta_{min}}{2}\right)}{\sin\frac{A}{2}}$

Given $\delta_{min} = A$

$\sqrt{3} = \frac{\sin A}{\sin\frac{A}{2}} = \frac{2\sin\frac{A}{2}\cos\frac{A}{2}}{\sin\frac{A}{2}}$

$\cos\frac{A}{2} = \frac{\sqrt{3}}{2}$

A = 60°

29. The equation of a transverse wave travelling along a string is $y(x, t) = 4.0 \sin [20 \times 10^{-3} x + 600t]$ mm, where x is in the mm and t is in second. The velocity of the wave is :

(1) + 30 m/s (2) – 60 m/s

(3) – 30 m/s (4) + 60 m/s

Ans. (3)

Sol. $y = 4 \sin (20 \times 10^{-3} x + 600 t)$

Here $\omega = 600 s^{-1}$

$k = 20 \times 10^{-3} mm^{-1}$

$\therefore \; v = \frac{w}{k} = \frac{600}{20\times10^{-3}}$

$= 30 \times 10^3$ mm/s

= 30 m/s

& direction is towards -ve x axis

$\therefore$ v = – 30 m/s

30. The energy of a system is given as $E(t) = \alpha^3 e^{-\beta t}$, where t is the time and $\beta = 0.3\ s^{-1}$. The errors in the measurement of α and t are 1.2% and 1.6%, respectively. At t = 5 s, maximum percentage error in the energy is :

(1) 4% (2) 11.6%

(3) 6% (4) 8.4%

Ans. (3)

Sol. $E = \alpha^3 e^{-\beta t}$

$\ln E = 3 \ln \alpha - \beta t$

$$\left(\frac{dE}{E}\right)_{max} = \frac{3d\alpha}{\alpha} + \beta\frac{dt}{t} \times t$$

$= 3 \times 1.2\% + (0.3 \times 1.6 \times 5)\%$

$= 6\%$

31. In photoelectric effect an em-wave is incident on a metal surface and electrons are ejected from the surface. If the work function of the metal is 2.14 eV and stopping potential is 2V, what is the wavelength of the em-wave?

(Given hc = 1242 eVnm where h is the Planck's constant and c is the speed of light in vaccum.)

(1) 400 nm (2) 600 nm

(3) 200 nm (4) 300 nm

Ans. (4)

Sol. $eV_s = E - \phi$

$2\ eV = E - 2.14\ eV$

$E = 4.14\ eV$

$$E = \frac{hc}{\lambda}$$

$$\lambda = \frac{1242}{4.14} = 300\ nm$$

32. A circular disk of radius R meter and mass M kg is rotating around the axis perpendicular to the disk. An external torque is applied to the disk such that $\theta(t) = 5t^2 - 8t$, where $\theta(t)$ is the angular position of the rotating disc as a function of time t.

How much power is delivered by the applied torque, when t = 2s?

(1) $60\ MR^2$ (2) $72\ MR^2$

(3) $108\ MR^2$ (4) $8\ MR^2$

Ans. (1)

Sol. $\theta = 5t^2 - 8t$

$$\omega = \frac{d\theta}{dt} = 10\,t - 8$$

$$\alpha = \frac{d\omega}{dt} = 10$$

$\therefore\ p = \tau\omega$

$= (I\alpha)\,\omega$

$$= \left(\frac{mR^2}{2}\right)\alpha\omega$$

$$= \left(\frac{mR^2}{2}\right)(10)\,(10t - 8)$$

Put t = 2

$p = 60\ mR^2$

33. Water flows in a horizontal pipe whose one end is closed with a valve. The reading of the pressure gauge attached to the pipe is P_1. The reading of the pressure gauge falls to P_2 when the valve is opened. The speed of water flowing in the pipe is proportional to

(1) $\sqrt{P_1 - P_2}$ (2) $(P_1 - P_2)^2$

(3) $(P_1 - P_2)^4$ (4) $P_1 - P_2$

Ans. (1)

Sol. By Bernoulli equation

$$P_1 + \frac{1}{2} \times \rho \times 0^2 = P_2 + \frac{1}{2}\rho V^2$$

$$v = \sqrt{2\rho(P_1 - P_2)}$$

34. Match **List-I** with **List-II**.

List-I		List-II
(A) Permeability of free space	(I)	$[M\,L^2T^{-2}]$
(B) Magnetic field	(II)	$[M\,T^{-2}A^{-1}]$
(C) Magnetic moment	(III)	$[M\,L\,T^{-2}\,A^{-2}]$
(D) Torsional constant	(IV)	$[L^2\,A]$

Choose the **correct** answer from the options given below :

(1) (A)-(I), (B)-(IV), (C)-(II), (D)-(III)

(2) (A)-(II), (B)-(I), (C)-(III), (D)-(IV)

(3) (A)-(IV), (B)-(III), (C)-(I), (D)-(II)

(4) (A)-(III), (B)-(II), (C)-(IV), (D)-(I)

Ans. (4)

Sol. $B = \frac{\mu_0 I}{2\pi r}$

$\Rightarrow [\mu_0] = \left[\frac{B \times r}{I}\right] = \left[\frac{MT^{-2}A^{-1} \times L}{A}\right] = [MLT^{-2}A^{-2}]$

magnetic field $F = qvB$

$B = \left[\frac{MLT^{-2}}{AT\ L/T}\right] = [MT^{-2}A^{-1}]$

$[M] = [NTA] = [M] = [ML^2]$

$\tau = c\theta \Rightarrow c = \left[\frac{\tau}{\theta}\right] = [ML^2T^{-2}]$

35. If a satellite orbiting the Earth is 9 times closer to the Earth than the Moon, what is the time period of rotation of the satellite? Given rotational time period of Moon = 27 days and gravitational attraction between the satellite and the moon is neglected.

(1) 1 day (2) 81 days

(3) 27 days (4) 3 days

Ans. (1)

Sol. $T^2 \propto R^3$

$\left(\frac{T_m}{T_s}\right)^2 = \left(\frac{R}{R/9}\right)^3$

$\frac{T_m}{T_s} = (3)^3$

$\Rightarrow T_s = \left(\frac{27}{27}\right) = 1$ day

36. Two point charges – 4 μc and 4 μc, constituting an electric dipole, are placed at (– 9, 0, 0) cm and (9, 0, 0) cm in a uniform electric field of strength $10^4\ NC^{-1}$. The work done on the dipole in rotating it from the equilibrium through 180° is :

(1) 14.4 mJ (2) 18.4 mJ

(3) 12.4 mJ (4) 16.4 mJ

Ans. (1)

Sol. $U = -PE\cos\theta$

$w_{ext} = \Delta U = U_f - U_i = -PE\cos 180° + PE\cos 0°$

$w_{ext} = 2PE$

$= 2 \times (4 \times 10^{-6})(18) \times 10^4$

$= 144 \times 10^{-2}$

$= 14.4$ mJ

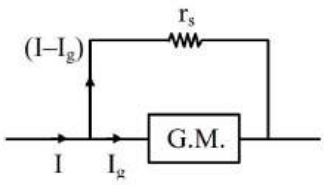
37. A galvanometer having a coil of resistance 30 Ω need 20 mA of current for full-scale deflection. If a maximum current of 3 A is to be measured using this galvanometer, the resistance of the shunt to be added to the galvanometer should be $\frac{30}{X}\Omega$, where X is

(1) 447 (2) 298

(3) 149 (4) 596

Ans. (3)

Sol.



$I_g R_g = (I - I_g)\, r_s$

$20 \times 10^{-3} \times 30 = (3 - 0.02) \times r_s$

$r_s = \left(\frac{0.6}{2.98}\right) = \frac{30}{x}$

$x = \left(\frac{2.98 \times 30}{0.6}\right) = 149$

38. The width of one of the two slits in Young's double slit experiment is d while that of the other slit is xd. If the ratio of the maximum to the minimum intensity in the interference pattern on the screen is 9 : 4 then what is the value of x?

(Assume that the field strength varies according to the slit width.)

(1) 2 (2) 3

(3) 5 (4) 4

Ans. (3)

Sol. $I \propto (\text{width})^2$

$\left(\frac{\sqrt{I_1} + \sqrt{I_2}}{\sqrt{I_1} - \sqrt{I_2}}\right)^2 = \frac{9}{4}$

$\frac{\sqrt{I_1} + \sqrt{I_2}}{\sqrt{I_1} - \sqrt{I_2}} = \frac{3}{2}$

$\frac{(x+1)d}{(x-1)d} = \frac{3}{2}$

$\Rightarrow 3x - 3 = 2x + 2$

$x = 5$

39. Given below are two statements. One is labelled as **Assertion (A)** and the other is labelled as **Reason (R)**.

Assertion (A) : The binding energy per nucleon is found to be practically independent of the atomic number A, for nuclei with mass numbers between 30 and 170.

Reason (R) : Nuclear force is long range.

In the light of the above statements, choose the **correct** answer from the options given below :

(1) **(A)** is false but **(R)** is true

(2) **(A)** is true but **(R)** is false

(3) Both **(A)** and **(R)** are true and **(R)** is the correct explanation of **(A)**

(4) Both **(A)** and **(R)** are true but **(R)** is **NOT** the correct explanation of **(A)**

Ans. (2)

Sol. Conceptual

40. Water of mass m gram is slowly heated to increase the temperature from T_1 to T_2. The change in entropy of the water, given specific heat of water is 1 $Jkg^{-1}K^{-1}$, is :

(1) zero (2) $m(T_2-T_1)$

(3) $m\,ln\left(\frac{T_1}{T_2}\right)$ (4) $m\,ln\left(\frac{T_2}{T_1}\right)$

Ans. (4)

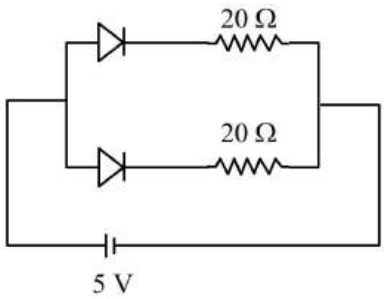
Sol. $dQ = msdT$

$$dS = \frac{dQ}{T} = \frac{msdT}{T}$$

$$\Delta S = \int \frac{msdT}{T} = ms\,ln\frac{T_f}{T_i}$$

$$\Delta S = m\,ln\frac{T_2}{T_1}$$

41. What is the current through the battery in the circuit shown below?



(1) 1.0 A (2) 1.5 A

(3) 0.5 A (4) 0.25 A

Ans. (3)

Sol. Both are forward biased

hence $R_{eq} = 10\ \Omega$

$$i = \frac{V}{R} = \frac{5}{10} = \frac{1}{2}A$$

42. A plane electromagnetic wave of frequency 20 MHz travels in free space along the +x direction. At a particular point in space and time, the electric field vector of the wave is $E_y = 9.3\ Vm^{-1}$. Then, the magnetic field vector of the wave at that point is-

(1) $B_z = 9.3 \times 10^{-8}$ T (2) $B_z = 1.55 \times 10^{-8}$ T

(3) $B_z = 6.2 \times 10^{-8}$ T (4) $B_z = 3.1 \times 10^{-8}$ T

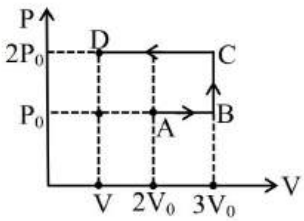
Ans. (4)

Sol. $E = BC$

$9.3 = B \times 3 \times 10^8$

$$B = \frac{9.3}{3\times10^8} = 3.1 \times 10^{-8}\ T$$

43.



Using the given P-V diagram, the work done by an ideal gas along the path ABCD is-

(1) $4\,P_0V_0$ (2) $3\,P_0V_0$

(3) $-4\,P_0V_0$ (4) $-3\,P_0V_0$

Ans. (4)

Sol. $w_{ABCD} = w_{AB} + w_{BC} + w_{CD}$

$= P_0V_0 + 0 + (-2P_0 \times 2V_0)$

$= P_0V_0 - 4P_0V_0$

$= -3P_0V_0$

44. A concave mirror of focal length f in air is dipped in a liquid of refractive index μ. Its focal length in the liquid will be :

(1) $\frac{f}{\mu}$ (2) $\frac{f}{(\mu-1)}$

(3) μf (4) f

Ans. **(4)**

Sol. Focal length of mirror will not change because focal length of mirror doesn't depend on medium.

45. A massless spring gets elongated by amount x_1 under a tension of 5N. Its elongation is x_2 under the tension of 7N. For the elongation of $(5x_1 - 2x_2)$, the tension in the spring will be,

(1) 15 N (2) 20 N

(3) 11 N (4) 39 N

Ans. **(3)**

Sol. $kx_1 = 5N$

$kx_2 = 7N$

$k(5x_1 - 2x_2) = 5kx_1 - 2kx_2$

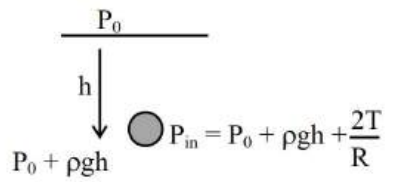
$= 5 \times 5 - 2 \times 7 = 11$ N

SECTION-B

46. An air bubble of radius 1.0 mm is observed at a depth of 20 cm below the free surface of a liquid having surface tension 0.095 J/m^2 and density 10^3 kg/m^3. The difference between pressure inside the bubble and atmospheric pressure_____ N/m^2. (Take g = 10 m/s^2)

Ans. **(2190)**

Sol.



$\Delta P = P_{in} - P_0$

$= \rho gh + \frac{2T}{R} = \frac{1000 \times 10 \times 20}{100} + \frac{2 \times 0.095}{10^{-3}}$

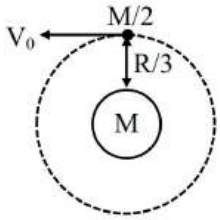
$= 2000 + 190$

$= 2190$

47. A satellite of mass $\frac{M}{2}$ is revolving around earth in a circular orbit at a height of $\frac{R}{3}$ from earth surface. The angular momentum of the satellite is $M\sqrt{\frac{GMR}{x}}$. The value of x is _____, where M and R are the mass and radius of earth, respectively. (G is the gravitational constant)

Ans. **(3)**

Sol. **(i)** If earth is assumed to be stationary



orbital velocity $v_0 = \sqrt{\frac{GM}{4R/3}} = \sqrt{\frac{3GM}{4R}}$

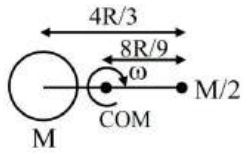
Angular momentum of satellite $= \frac{M}{2} v_0 \frac{4R}{3}$

$= \frac{M}{2} \cdot \sqrt{\frac{3GM}{4R}} \cdot \frac{4R}{3}$

$= M\sqrt{\frac{GMR}{3}}$

x = 3

(ii) Since mass of satellite is comparable to the mass of earth.



$$\frac{G.M.\frac{M}{2}}{\left(\frac{4R}{3}\right)^2} = \frac{M}{2}\omega^2 \cdot \frac{8R}{9}$$

$$\omega = \sqrt{\frac{81GM}{128R^3}}$$

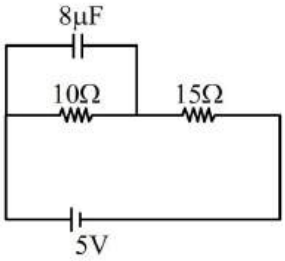
Angular momentum of satellite about common centre of mass,

$$L = \frac{M}{2} \cdot \left(\frac{8R}{9}\right)^2 \cdot \omega$$

$$L = M\sqrt{GMR\left(\frac{8}{81}\right)}$$

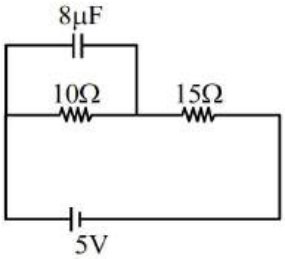
$$x = \frac{81}{8} \simeq 10$$

48. At steady state the charge on the capacitor, as shown in the circuit below, is _____ μC.



Ans. (16)

Sol.



$$i = \left(\frac{5}{25}\right)$$

$$Q = CV$$

$$Q = (8 \times 10^{-6})\left(\frac{5}{25} \times 10\right)$$

$$Q = \left(\frac{8 \times 5 \times 10^{-2}}{25}\right) = 16\ \mu C$$

49. A time varying potential difference is applied between the plates of a parallel plate capacitor of capacitance 2.5 μF. The dielectric constant of the medium between the capacitor plates is 1. It produces an instantaneous displacement current of 0.25 mA in the intervening space between the capacitor plates, the magnitude of the rate of change of the potential difference will be _____ Vs^{-1}.

Ans. (100)

Sol. $$\frac{CdV}{dt} = I_d$$

$$\frac{dV}{dt} = \frac{I_d}{C}$$

$$= \frac{0.25 \times 10^{-3}}{2.5 \times 10^{-6}}$$

$$= 100$$

50. In a series LCR circuit, a resistor of 300 Ω, a capacitor of 25 nF and an inductor of 100 mH are used. For maximum current in the circuit, the angular frequency of the ac source is _____ × 10^4 radians s^{-1}.

Ans. (2)

Sol. $$\omega = \frac{1}{\sqrt{LC}}$$

$$\omega = \frac{1}{\sqrt{25 \times 10^{-9} \times 100 \times 10^{-3}}}$$

$$\omega = \frac{10^{+6}}{5 \times 10} = 2$$

CHEMISTRY

SECTION-A

51. The effect of temperature on spontaneity of reactions are represented as:

	ΔH	ΔS	Temperature	Spontaneity
(A)	+	–	any T	Non spontaneous
(B)	+	+	low T	spontaneous
(C)	–	–	low T	Non spontaneous
(D)	–	+	any T	spontaneous

(1) (B) and (D) only
(2) (A) and (D) only
(3) (B) and (C) only
(D) (A) and (C) only

Ans. **(3)**

Sol. $\because \Delta G = \Delta H - T\Delta S$

For spontaneity of reaction : $\Delta G = -ve$

52. Standard electrode potentials for a few half cells are mentioned below:

$E^o_{Cu^{2+}/Cu} = 0.34V, E^o_{Zn^{2+}/Zn} = -0.76V$

$E^o_{Ag^+/Ag} = 0.80V, E^o_{Mg^{2+}/Mg} = -2.37V$

Which one of the following cells gives the most negative value of ΔG^o ?

(1) $Zn|Zn^{2+}(1M)||Ag^+(1M)|Ag$
(2) $Zn|Zn^{2+}(1M)||Mg^{2+}(1M)|Mg$
(3) $Ag|Ag^+(1M)||Mg^{2+}(1M)|Mg$
(4) $Cu|Cu^{2+}(1M)||Ag^+(1M)|Ag$

Ans. **(1)**

Sol. $\because \Delta G^\circ = -nFE^\circ$

Option (1) $E^\circ = 0.8 + 0.76$
$= 1.56$ V

$\therefore \Delta G^\circ = -2 \times F \times 1.56$
$= -3.12$ V

Option (2) $E^\circ = -2.37 + 0.76$
$= -1.61$ V

$\therefore \Delta G^\circ = -2 \times F \times (-1.61)$
$= +3.22$ V

Option (3) $E^\circ = -2.37 - 0.8$
$= -3.17$ V

$\therefore \Delta G^\circ = -2 \times F \times (-3.17)$
$= +6.34$

Option (4) $E^\circ = 0.8 - 0.34$
$= 0.46$ V

$\Delta G^\circ = -2 \times F \times 0.46$
$= -0.92$ V

53. The α - Helix and β - Pleated sheet structures of protein are associated with its:

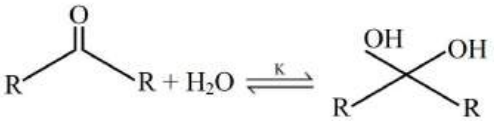
(1) quaternary structure
(2) primary structure
(3) secondary structure
(4) tertiary structure

Ans. **(3)**

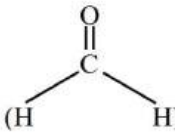
Sol. α-helix and β-pleated sheet belongs to secondary structure of protein, which have hydrogen bonds.

54. Given below are two statements:

Consider the following reaction

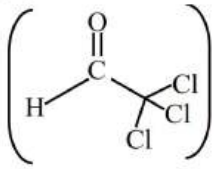


Statement (I) : In the case of formaldehyde



, K is about 2280, due to small substituents, hydration is faster.

Statement (II) : In the case of trichloro acetaldehyde



, K is about 2000 due to – I effect of – Cl.

In the light of the above statements, choose the **correct** answer from the options given below:

(1) **Statement I** true but **Statement II** is false
(2) Both **Statement I** and **Statement II** are true
(3) **Statement I** is false but **Statement II** is true
(4) Both **Statement I** and **Statement II** are false

Ans. **(2)**

Sol. $k_{eq} = 2280$ is for HCHO

$k_{eq} = 2000$ is for chloral

Both data is given in clayden and warren book.

$k_{eq} > 1$ because HCHO and chloral are more electrophilic.

55. Consider the reaction

$$X_2Y(g) \rightleftharpoons X_2(g) + \frac{1}{2}Y_2(g)$$

The equation representing correct relationship between the degree of dissociation (x) of $X_2Y(g)$ with its equilibrium constant Kp is_____.
Assume x to be very very small.

(1) $x = \sqrt[3]{\frac{2Kp}{p}}$ (2) $x = \sqrt[3]{\frac{2Kp^2}{p}}$

(3) $x = \sqrt[3]{\frac{Kp}{2p}}$ (4) $x = \sqrt[3]{\frac{Kp}{p}}$

Ans. (2)

Sol. 1 mole

$$X_2Y_{(g)} \rightleftharpoons X_{2(g)} + \frac{1}{2}Y_{2(g)}$$

1-x mole x mole $\frac{x}{2}$ mole

$$\therefore P_{X_2Y} = \frac{1-x}{1+\frac{x}{2}} \times P$$

$$P_{X_2} = \frac{x}{1+\frac{x}{2}} \times P$$

$$P_{Y_2} = \frac{x/2}{1+\frac{x}{2}} \times P$$

$$\therefore K_p = \left(\frac{x}{1+\frac{x}{2}}P\right)\left(\frac{x}{2\left(1+\frac{x}{2}\right)}P\right)^{\frac{1}{2}} \Bigg/ \left(\frac{1-x}{1+\frac{x}{2}}\right) \times P$$

$$\therefore K_p = \left(\frac{x}{1-x}\right)\left(\frac{x}{2\left(1+\frac{x}{2}\right)}\right)^{\frac{1}{2}} \times p^{\frac{1}{2}}$$

$\because$ x to be very very small

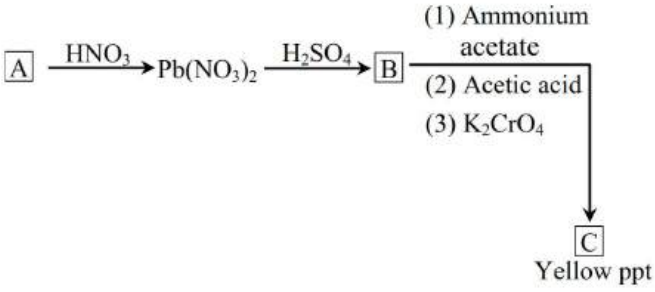
$$\therefore K_p = \frac{x^{3/2}}{(2)^{\frac{1}{2}}} \times P^{\frac{1}{2}}$$

$$\therefore x^{\frac{3}{2}} = \frac{K_p \times 2^{\frac{1}{2}}}{P^{\frac{1}{2}}}$$

$$\therefore x^3 = \frac{K_p^2 \times 2}{P}$$

$$x = \left(\frac{K_p^2 \times 2}{P}\right)^{\frac{1}{3}}$$

56. Identify A, B and C in the given below reaction sequence



(1) $PbCl_2$, $PbSO_4$, $PbCrO_4$

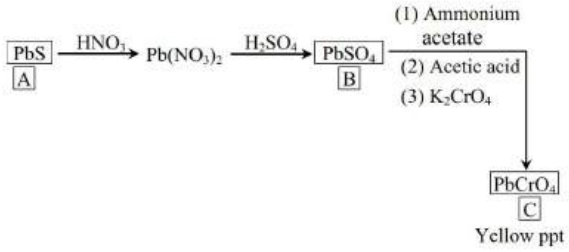
(2) PbS, $PbSO_4$, $PbCrO_4$

(3) PbS, $PbSO_4$, $Pb(CH_3COO)_2$

(4) $PbCl_2$, $Pb(SO_4)_2$, $PbCrO_4$

Ans. (2)

Sol.



57. Given below are two statements:

Statement (I): The boiling points of alcohols and phenols increase with increase in the number of C-atoms.

Statement (II): The boiling points of alcohols and phenols are higher in comparison to other class of compounds such as ethers, haloalkanes.

In the light of the above statements, choose the **correct** answer from the options given below:

(1) Both **Statement I** and **Statement II** are false

(2) **Statement I** is false but **Statement II** is true

(3) **Statement I** is true but **Statement II** is false

(4) Both **Statement I** and **Statement II** are true

Ans. (4)

Sol. B.P. $\propto$ M.W.

B.P. $\propto$ Inter molecular hydrogen bonding

Alcohol & Phenol have intermolecular H-bonding

58. When a non-volatile solute is added to the solvent, the vapour pressure of the solvent decreases by 10 mm of Hg. The mole fraction of the solute in the solution is 0.2. What would be the mole fraction of the solvent if decrease in vapour pressure is 20 mm of Hg ?

(1) 0.6 (2) 0.4

(3) 0.2 (4) 0.8

Ans. (1)

Sol. $\because P^\circ - P \propto X_{solute}$

and $\because 10 \propto 0.2$

$\therefore 20 \propto 0.4$

$\therefore X_{solvent} = 1 - X_{solute}$

$= 1 - 0.4$

$= 0.6$

59. Given below are two statements:

Statement (I) : For a given shell, the total number of allowed orbitals is given by n^2.

Statement (II) : For any subshell, the spatial orientation of the orbitals is given by $-l$ to $+l$ values including zero.

In the light of the above statements, choose the **correct** answer from the options given below:

(1) **Statement I** is true but **Statement II** is false

(2) **Statement I** is false but **Statement II** is true

(3) Both **Statement I** and **Statement II** are true

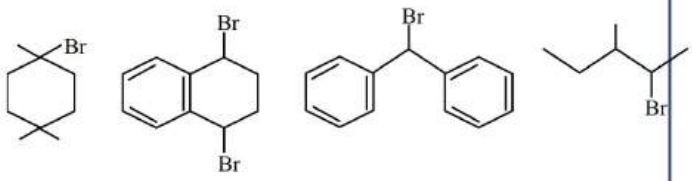
(4) Both **Statement I** and **Statement II** are false

Ans. (3)

Sol. For a shell total number of orbitals = n^2

Magnetic quantum number have values ($-\ell$ to $+\ell$) including 0.

60. The ascending order of relative rate of solvolysis of following compounds is



(A) (B) (C) (D)

(1) (D) < (A) < (B) < (C)

(2) (C) < (B) < (A) < (D)

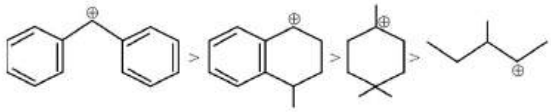
(3) (D) < (B) < (A) < (C)

(4) (C) < (D) < (B) < (A)

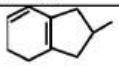
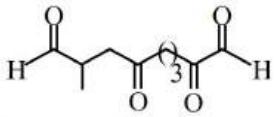
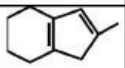
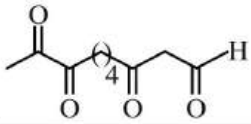
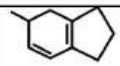
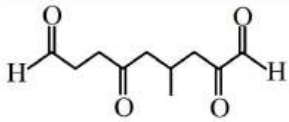
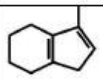
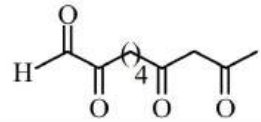
Ans. (1)

Sol. Solvolysis or $S_N1 \propto$ stability of carboccation

Stability order



61. Match **List - I** with **List - II**.

List - I (Isomers of $C_{10}H_{14}$)		List - II (Ozonolysis product)	
(A)		(I)	
(B)		(II)	
(C)		(III)	
(D)		(IV)	

Choose the **correct** answer from the options given below :

(1) (A)-(II), (B)-(III), (C)-(I), (D)-(IV)

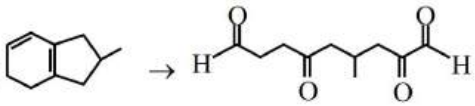
(2) (A)-(III), (B)-(IV), (C)-(I), (D)-(II)

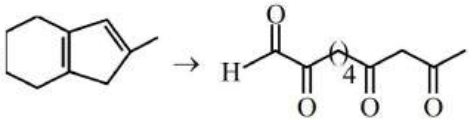
(3) (A)-(III), (B)-(II), (C)-(I), (D)-(IV)

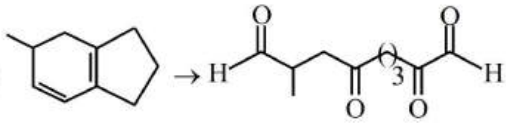
(4) (A)-(I), (B)-(IV), (C)-(III), (D)-(II)

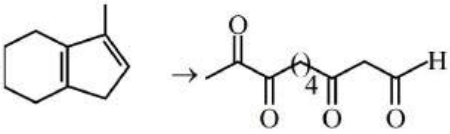
Ans. (2)

Sol. Ozonolysis product

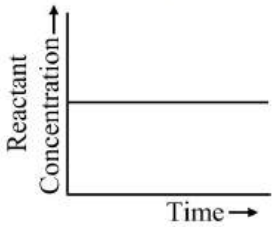
(A) 

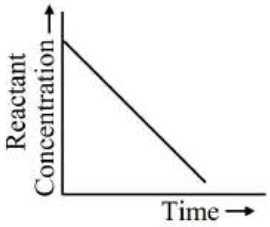
(B) 

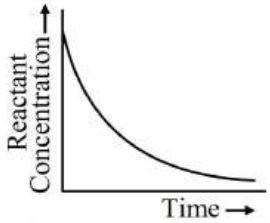
(C) 

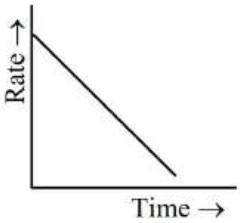
(D) 

62. Which of the following graphs most appropriately represents a zero order reaction ?

(1)


(2)


(3)


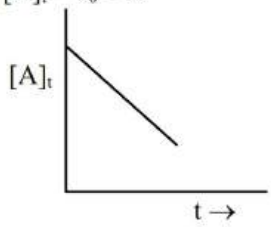
(4)


Ans. (2)

Sol. For zero order reaction : $A \rightarrow P$

Rate = k

$[A]_t = a_0 - kt$



63. Match **List - I** with **List - II**.

	List - I		List - II
(A)	Bronze	(I)	Cu, Ni
(B)	Brass	(II)	Fe, Cr, Ni, C
(C)	UK silver coin	(III)	Cu, Zn
(D)	Stainless Steel	(IV)	Cu, Sn

Choose the **correct** answer from the options given below :

(1) (A)-(IV), (B)-(II), (C)-(III), (D)-(I)

(2) (A)-(IV), (B)-(III), (C)-(I), (D)-(II)

(3) (A)-(III), (B)-(I), (C)-(IV), (D)-(II)

(4) (A)-(III), (B)-(IV), (C)-(II), (D)-(I)

Ans. (2)

Sol. Bronze → Cu, Sn

Brass → Cu, Zn

UK silver coin → Cu, Ni

Stainless steel → Fe, Cr, Ni, C

64. Identify the coordination complexes in which the central metal ion has d^4 configuration.

(A) $[FeO_4]^{2-}$

(B) $[Mn(CN)_6]^{3-}$

(C) $[Fe(CN)_6]^{3-}$

(D) $Cr_2(O-\overset{\overset{\displaystyle O}{\|}}{C}-Me)_4(H_2O)_2$

(E) $[NiF_6]^{2-}$

Choose the **correct** answer from the options given below :

(1) (C) and (E) only (2) (B), (C) and (D) only

(3) (B) and (D) only (4) (A), (B) and (E) only

Ans. (3)

Sol. $Fe^{+6} = [Ar]3d^2$

$Mn^{+3} = [Ar]3d^4$

$Fe^{+3} = [Ar]3d^5$

$Cr^{+2} = [Ar]3d^4$

$Ni^{+4} = [Ar]3d^6$

65. Given below are the atomic numbers of some group 14 elements. The atomic number of the element with lowest melting point is :

(1) 14 (2) 6

(3) 82 (4) 50

Ans. (4)

Sol. Order of M.P. of group 14 : C > Si > Ge > Pb > Sn

element	M.P. (°C)
Z = 6 = C	3730
Z = 14 = Si	1410
Z = 32 = Ge	937
$\boxed{Z=50}$ = Sn	232
Z = 82 = Pb	327

66. pH of water is 7 at 25°C. If water is heated to 80°C, it's pH will :

(1) Decrease

(2) Remains the same

(3) H^+ concentration increases, OH^- concentration decreases

(4) Increase

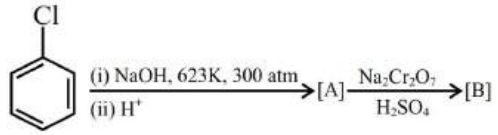
Ans. (1)

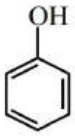
Sol. With increase in temperature, K_w of water increases

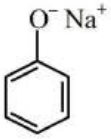
So, degree of dissociation of water increase

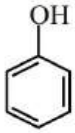
∴ pH as well as pOH of water decrease.

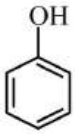
67. Identify the products [A] and [B], respectively in the following reaction :



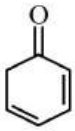
(1) [A] 

, [B] 

(2) [A]
, [B] 

(3) [A] 

, [B]

(4) [A]
, [B] 

Ans. **(3)**

Sol. A is phenol and B is para benzoquinone.

68. Consider a binary solution of two volatile liquid components 1 and 2 x_1 and y_1 are the mole fractions of component 1 in liquid and vapour phase, respectively. The slope and intercept of the linear plot of $\frac{1}{x_1}$ vs $\frac{1}{y_1}$ are given respectively as :

(1) $\frac{P_1^0}{P_2^0}, \frac{P_2^0 - P_1^0}{P_2^0}$ (2) $\frac{P_2^0}{P_1^0}, \frac{P_1^0 - P_2^0}{P_2^0}$

(3) $\frac{P_1^0}{P_2^0}, \frac{P_1^0 - P_2^0}{P_2^0}$ (4) $\frac{P_2^0}{P_1^0}, \frac{P_2^0 - P_1^0}{P_2^0}$

Ans. **(1)**

Sol. $\because$ For liquid solution of two liquids '1' and '2'

$P_1 = P_T y_1 = P_1^o x_1$

$\therefore \frac{P_T}{x_1} = \frac{P_1^o}{y_1}$

$\therefore \frac{P_2^o + x_1(P_1^o - P_2^o)}{x_1} = \frac{P_1^o}{y_1}$

$\therefore \frac{P_2^o}{x_1} + (P_1^o - P_2^o) = \frac{P_1^o}{y_1}$

$\therefore \frac{1}{x_1} = \left(\frac{P_1^o}{P_2^o}\right)\left(\frac{1}{y_1}\right) + \left(\frac{P_2^o - P_1^o}{P_2^o}\right)$

$\therefore$ Slope $= \left(\frac{P_1^o}{P_2^o}\right)$

$\therefore$ Intercept $= \left(\frac{P_2^o - P_1^o}{P_2^o}\right)$

69. Given below are two statements about X-ray spectra of elements :

Statement (I) : A plot of $\sqrt{\nu}$ (ν = frequency of X-rays emitted) vs atomic mass is a straight line.

Statement (II) : A plot of ν(ν = frequency of X-rays emitted) vs atomic number is a straight line.

In the light of the above statements choose the **correct** answer from the options given below :

(1) **Statement I** is true but **Statement II** is false

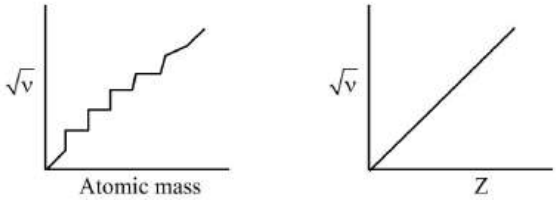
(2) Both **Statement I** and **Statement II** are true

(3) Both **Statement I** and **Statement II** are false

(4) **Statement I** is false but **Statement II** is true

Ans. **(3)**

Sol.



70. Consider the following reactions

$K_2Cr_2O_7 \xrightarrow[-H_2O]{KOH} [A] \xrightarrow[-H_2O]{H_2SO_4} [B] + K_2SO_4$

The products [A] and [B], respectively are :

(1) $K_2Cr(OH)_6$ and Cr_2O_3

(2) K_2CrO_4 and Cr_2O_3

(3) K_2CrO_4 and $K_2Cr_2O_7$

(4) K_2CrO_4 and CrO

Ans. **(3)**

Sol. $K_2Cr_2O_7 \xrightarrow[-H_2O]{KOH} \underset{[A]}{K_2CrO_4} \xrightarrow[-H_2O]{H_2SO_4} \underset{[B]}{K_2Cr_2O_7} + K_2SO_4$

SECTION-B

71. 0.01 mole of an organic compound (X) containing 10% hydrogen, on complete combustion produced 0.9 g H_2O. Molar mass of (X) is _____ g mol^{-1}.

Ans. (100)

Sol. Organic compound $\xrightarrow{\text{combustion}}$ $\underset{0.9\text{gm}}{H_2O}$

$\therefore$ mole of $H_2O = \frac{0.9}{18} = 0.05$ mole

$\therefore$ mole of H in $H_2O = 0.05 \times 2 = 0.1$ mole

= mole of H in 0.01 mole Organic compound

$\therefore$ wt of H atom in 0.01 mole compound $= 0.1 \times 1$ $= 0.1$ gm

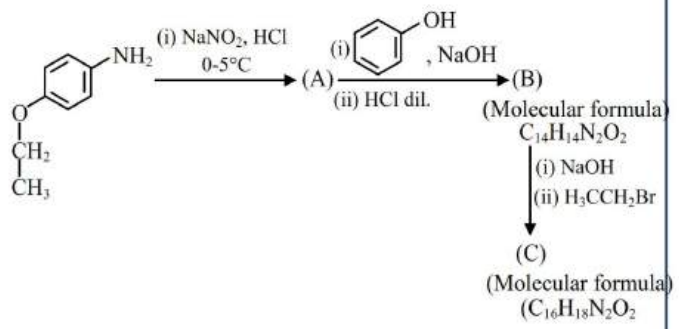
$\therefore$ wt of H atom in one mole compound $= \frac{0.1}{0.01} = 10\text{gm}$

$\because$ wt. % of H $= \frac{\text{wt. of H in one mole compound}}{\text{Molar mass of compound}} \times 1$

$10 = \frac{10}{M} \times 100$

$\therefore$ $\boxed{M = 100}$

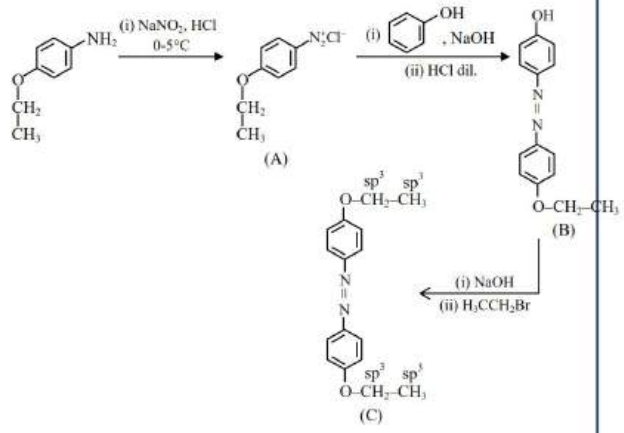
72. Consider the following sequence of reactions.



Total number of sp^3 hybridised carbon atoms in the major product C formed is _____.

Ans. (4)

Sol.



73. When 81.0 g of aluminium is allowed to react with 128.0 g of oxygen gas, the mass of aluminium oxide produced in grams is ____. (Nearest integer)

Given :

Molar mass of Al is 27.0 g mol^{-1}

Molar mass of O is 16.0 g mol^{-1}

Ans. (153)

Sol. $4Al + 3O_2 \longrightarrow 2Al_2O_3$

$\frac{81}{27} = 3\,\text{mole}$ $\quad \frac{128}{32} = 4\,\text{mole}$

Limiting reagent

$\therefore$ mole of Al_2O_3 formed $= \frac{1}{2} \times 3$ mole

$\therefore$ wt. of Al_2O_3 formed $= \frac{3}{2} \times 102$

$= 153$ gm

74. The bond dissociation enthalpy of X_2 ΔH°_{bond} calculated from the given data is _____ kJ mol^{-1}. (Nearest integer)

$M^+X^-(s) \rightarrow M^+(g) + X^-(g)$ $\Delta H^\circ_{lattice} = 800$ kJ mol^{-1}

$M(s) \rightarrow M(g)$ $\Delta H^\circ_{sub} = 100$ kJ mol^{-1}

$M(g) \rightarrow M^+(g)^- + e^-(g)$ $\Delta H^\circ_i = 500$ kJ mol^{-1}

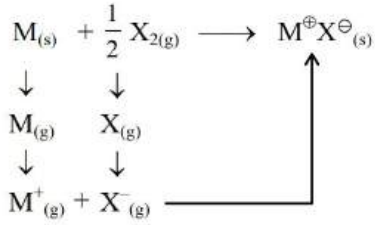
$X(g) + e^-(g) \rightarrow X^-(g)$ $\Delta H^\circ_{eg} = -300$ kJ mol^{-1}

$M(s) + \frac{1}{2}X_2(g) \rightarrow M^+X^-(s)$ $\Delta H^\circ_f = -400$ kJ mol^{-1}

[Given : M^+X^- is a pure ionic compound and X forms a diatomic molecule X_2 is gaseous state]

Ans. (200)

Sol.



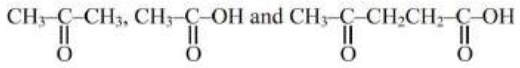
$\therefore$ $\Delta H_f(MX) = \Delta H_{sub}(M) + I.E.(M) + \frac{1}{2}[B.E.(X-X)]$

$+ EG(X) + L.E.(MX)$

$-400 = (100) + (500) + \frac{1}{2}$ (B.E.) $+ (-300) + (-800)$

$\therefore$ B.E. $= 200$ kJ $mole^{-1}$

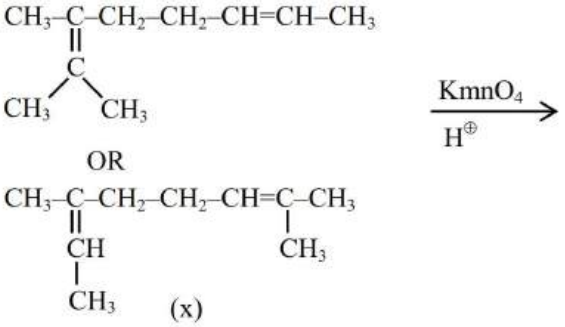
75. A compound 'X' absorbs 2 moles of hydrogen and 'X' upon oxidation with $KMnO_4 \mid H^+$ gives



The total number of σ bonds present in the compound 'X' is _____.

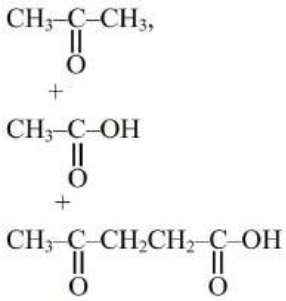
Ans. (27)

Sol.



OR

(x)



JEE–MAIN EXAMINATION – JANUARY 2025

(HELD ON THURSDAY 23rd JANUARY 2025)

TIME : 9:00 AM TO 12:00 NOON

MATHEMATICS

SECTION-A

1. The value of $\int_{e^2}^{e^4} \frac{1}{x}\left(\frac{e^{\left((\log_e x)^2+1\right)^{-1}}}{e^{\left((\log_e x)^2+1\right)^{-1}}+e^{\left((6-\log_e x)^2+1\right)^{-1}}}\right)dx$ is

(1) $\log_e 2$ (2) 2

(3) 1 (4) e^2

Ans. (3)

Sol. Let $\ln x = t \Rightarrow \frac{dx}{x} = dt$

$$I = \int_2^4 \frac{e^{\frac{1}{1+t^2}}}{e^{\frac{1}{1+t^2}}+e^{\frac{1}{1+(6-t)^2}}}dt$$

$$I = \int_2^4 \frac{e^{\frac{1}{1+(6-t)^2}}}{e^{\frac{1}{1+(6-t)^2}}+e^{\frac{1}{1+t^2}}}dt$$

$$2I = \int_2^4 dt = (t)_2^4 = 4-2 = 2$$

$I = 1$

2. Let $I(x) = \int \frac{dx}{(x-11)^{\frac{11}{13}}(x+15)^{\frac{15}{13}}}$.

If $I(37) - I(24) = \frac{1}{4}\left(\frac{1}{b^{\frac{1}{13}}} - \frac{1}{c^{\frac{1}{13}}}\right)$, $b, c \in \mathbb{N}$, then $3(b+c)$ is equal to

(1) 40 (2) 39

(3) 22 (4) 26

Ans. (2)

Sol. $I(x) = \int \frac{dx}{(x-11)^{\frac{11}{13}}(x+15)^{\frac{15}{13}}}$

Put $\frac{x-11}{x+15} = t \Rightarrow \frac{26}{(x+5)^2}dx = dt$

$$I(x) = \frac{1}{26}\int \frac{dt}{t^{11/13}} = \frac{1}{26}.\frac{t^{2/13}}{2/13}$$

$$I(x) = \frac{1}{4}\left(\frac{x-11}{x+15}\right)^{2/13} + C$$

$$I(37) - I(24) = \frac{1}{4}\left(\frac{26}{52}\right)^{2/13} - \frac{1}{4}\left(\frac{13}{39}\right)^{2/13}$$

$$= \frac{1}{4}\left(\frac{1}{2^{2/13}} - \frac{1}{3^{2/13}}\right)$$

$$= \frac{1}{4}\left(\frac{1}{4^{1/13}} - \frac{1}{9^{1/13}}\right)$$

$\therefore b = 4, c = 9$

$3(b + c) = 39$

3. If the function

$$f(x) = \begin{cases} \frac{2}{x}\{\sin(k_1+1)x + \sin(k_2-1)x\} & , \quad x < 0 \\ 4 & , \quad x = 0 \\ \frac{2}{x}\log_e\left(\frac{2+k_1x}{2+k_2x}\right) & , \quad x > 0 \end{cases}$$

is continuous at $x = 0$, then $k_1^2 + k_2^2$ is equal to

(1) 8 (2) 20

(3) 5 (4) 10

Ans. (4)

Sol. $\lim_{x\to 0^-} \frac{2}{x}\{\sin(k_1+1)x + \sin(k_2-1)x\} = 4$

$\Rightarrow 2(k_1 + 1) + 2(k_2 - 1) = 4$

$\Rightarrow k_1 + k_2 = 2$

$$\Rightarrow \lim_{x\to 0^+} \frac{2}{x}\ln\left(\frac{2+k_1x}{2+k_2x}\right) = 4$$

$$\Rightarrow \lim_{x\to 0^+} \frac{1}{x}\ln\left(1+\frac{(k_1-k_2)x}{2+k_2x}\right) = 2$$

$$\Rightarrow \frac{k_1-k_2}{2} = 2$$

$\Rightarrow k_1 - k_2 = 4$

$\therefore k_1 = 3, k_2 = -1$

$k_1^2 + k_2^2 = 9 + 1 = 10$

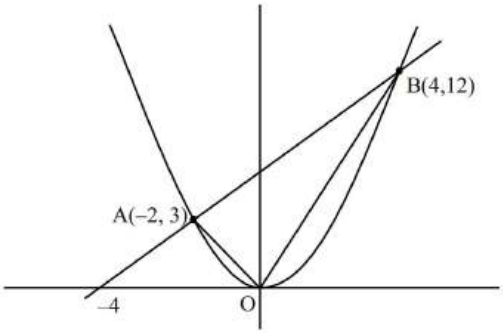
4. If the line $3x - 2y + 12 = 0$ intersects the parabola $4y = 3x^2$ at the points A and B, then at the vertex of the parabola, the line segment AB subtends an angle equal to

(1) $\tan^{-1}\left(\frac{11}{9}\right)$ (2) $\frac{\pi}{2} - \tan^{-1}\left(\frac{3}{2}\right)$

(3) $\tan^{-1}\left(\frac{4}{5}\right)$ (4) $\tan^{-1}\left(\frac{9}{7}\right)$

Ans. (4)

Sol.



$3x - 2y + 12 = 0$

$4y = 3x^2$

$\therefore 2(3x + 12) = 3x^2$

$\Rightarrow x^2 - 2x - 8 = 0$

$\Rightarrow x = -2, 4$

$m_{OA} = -3/2, m_{OB} = 3$

$$\tan\theta = \left(\frac{\frac{-3}{2} - 3}{1 - \frac{9}{2}}\right) = \frac{9}{7}$$

$\theta = \tan^{-1}\left(\frac{9}{7}\right)$ (angle will be acute)

5. Let a curve $y = f(x)$ pass through the points (0, 5) and $(\log_e 2, k)$. If the curve satisfies the differential equation $2(3 + y)e^{2x}dx - (7 + e^{2x})dy = 0$, then k is equal to

(1) 16 (2) 8

(3) 32 (4) 4

Ans. (2)

Sol. $\frac{dy}{dx} = \frac{2(3+y).e^{2x}}{7+e^{2x}}$

$$\frac{dy}{dx} - \frac{2y.e^{2x}}{7+e^{2x}} = \frac{6.e^{2x}}{7+e^{2x}}$$

$$\text{I.F.} = e^{-\int \frac{2e^{2x}}{7+e^{2x}}dx} = \frac{1}{7+e^{2x}}$$

$$\therefore y.\frac{1}{7+e^{2x}} = \int \frac{6e^{2x}}{\left(7+3^{2x}\right)^2}dx$$

$$\frac{y}{7+e^{2x}} = \frac{-3}{7+e^{2x}} + C$$

$(0, 5) \Rightarrow \frac{5}{8} = \frac{-3}{8} + C \Rightarrow C = 1$

$\therefore y = -3 + 7 + e^{2x}$

$y = e^{2x} + 4$

$\therefore k = 8$

6. Let $f(x) = \log_e x$ and $g(x) = \frac{x^4 - 2x^3 + 3x^2 - 2x + 2}{2x^2 - 2x + 1}$. Then the domain of fog is

(1) $\mathbb{R}$ (2) $(0, \infty)$

(3) $[0, \infty)$ (4) $[1, \infty)$

Ans. (1)

Sol. $f(x) = \ln x$

$$g(x) = \frac{x^4 - 2x^3 + 3x^2 - 2x + 2}{2x^2 - 2x + 1}$$

$D_g \in R$

$D_f \in (0, \infty)$

For $D_{fog} \Rightarrow g(x) > 0$

$$\frac{x^4 - 2x^3 + 3x^2 - 2x + 2}{2x^2 - 2x + 1} > 0$$

$\Rightarrow x^4 - 2x^3 + 3x^2 - 2x + 2 > 0$

Clearly $x < 0$ satisfies which are included in option (1) only.

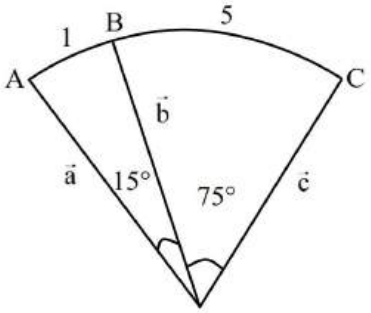
7. Let the arc AC of a circle subtend a right angle at the centre O. If the point B on the arc AC, divides the arc AC such that $\frac{\text{length of arc AB}}{\text{length of arc BC}} = \frac{1}{5}$, and $\overrightarrow{OC} = \alpha\overrightarrow{OA} + \beta\overrightarrow{OB}$, then $\alpha = \sqrt{2}\left(\sqrt{3} - 1\right)\beta$ is equal to

(1) $2 - \sqrt{3}$ (2) $2\sqrt{3}$

(3) $5\sqrt{3}$ (4) $2 + \sqrt{3}$

Ans. (1)

Sol.



$\vec{c} = \alpha\vec{a} + \beta\vec{b}$ ....(1)

$\vec{a}.\vec{c} = \alpha\vec{a}.\vec{a} + \beta\vec{b}.\vec{a}$

$0 = \alpha + \beta \cos 15°$ ....(2)

$(1) \Rightarrow \vec{b}.\vec{c} = \alpha\vec{a}.\vec{b} + \beta\vec{b}.\vec{b}$

$\Rightarrow \cos 75° = \alpha \cos 15° + \beta$ ....(3)

(2) & (3) $\Rightarrow \cos 75° = -\beta \cos^2 15° + \beta$

$$\beta = \frac{\cos 75^\circ}{\sin^2 15^\circ} = \frac{1}{\sin 15^\circ} = \frac{2\sqrt{2}}{\sqrt{3}-1}$$

$$(2) \Rightarrow \alpha = \frac{-\cos 15^\circ}{\sin 15^\circ} = \frac{-\left(\sqrt{3}+1\right)}{\left(\sqrt{3}-1\right)}$$

$$\therefore \vec{c} = \frac{-\left(\sqrt{3}+1\right)}{\left(\sqrt{3}-1\right)}\vec{a} + \left(\frac{2\sqrt{2}}{\sqrt{3}-1}\right)\vec{b}$$

Now

$$\alpha + \sqrt{2}\left(\sqrt{3}-1\right)\beta = \frac{-\left(\sqrt{3}+1\right)}{\left(\sqrt{3}-1\right)} + \frac{\sqrt{2}\left(\sqrt{3}-1\right).2\sqrt{2}}{\sqrt{3}-1}$$

$$= \frac{-\left(\sqrt{3}+1\right)^2}{2} + 4$$

$$= \frac{-3-1-2\sqrt{3}+8}{2}$$

$$= 2 - \sqrt{3}$$

8. If the first term of an A.P. is 3 and the sum of its first four terms is equal to one-fifth of the sum of the next four terms, then the sum of the first 20 terms is equal to

(1) –1200 (2) –1080

(3) –1020 (4) –120

Ans. (2)

Sol. a = 3

$$S_4 = \frac{1}{5}(S_8 - S_4)$$

$\Rightarrow 5S_4 = S_8 - S_4$

$\Rightarrow 6S_4 = S_8$

$$\Rightarrow 6.\frac{4}{2}\left[2\times 3 + (4-1)\times d\right]$$

$$= \frac{8}{2}\left[2\times 3 + (8-1)d\right]$$

$\Rightarrow 12(6 + 3d) = 4(6 + 7d)$

$\Rightarrow 18 + 9d = 6 + 7d$

$\Rightarrow d = -6$

$$S_{20} = \frac{20}{2}\left[2\times 3 + (20-1)(-6)\right]$$

$= 10\,[6 - 114]$

$= -1080$

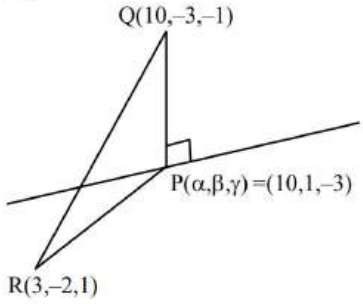
9. Let P be the foot of the perpendicular from the point Q(10, –3, –1) on the line $\frac{x-3}{7} = \frac{y-2}{-1} = \frac{z+1}{-2}$. Then the area of the right angled triangle PQR, where R is the point (3, –2, 1), is

(1) $9\sqrt{15}$ (2) $\sqrt{30}$

(3) $8\sqrt{15}$ (4) $3\sqrt{30}$

Ans. (4)

Sol.



$$\frac{x-3}{7} = \frac{y-2}{-1} = \frac{z+1}{-2} = \lambda$$

$\Rightarrow 7\lambda + 3, -\lambda + 2, -2\lambda - 1$

dr's of QP $\Rightarrow$

$7\lambda - 7, -\lambda + 5, -2\lambda$

Now

$(7\lambda - 7).7 - (-\lambda + 5) + (2\lambda).2 = 0$

$54\lambda - 54 = 0 \Rightarrow \lambda = 1$

$\therefore P = (10, 1, -3)$

$\overrightarrow{PQ} = -4\hat{j} + 2\hat{k}$

$\overrightarrow{PR} = -7\hat{i} - 3\hat{j} + 4\hat{k}$

$$\text{Area} = \left|\frac{1}{2}\begin{vmatrix} i & j & k \\ 0 & -4 & 2 \\ -7 & -3 & 4 \end{vmatrix}\right| = 3\sqrt{30}$$

10. Let $\left|\frac{\overline{z}-i}{2\overline{z}+i}\right|=\frac{1}{3}$, $z \in \mathbb{C}$, be the equation of a circle with center at C. If the area of the triangle, whose vertices are at the points (0, 0), C and (α, 0) is 11 square units, then α^2 equals

(1) 100 (2) 50

(3) $\frac{121}{25}$ (4) $\frac{81}{25}$

Ans. (1)

Sol. $\left|\frac{\overline{z}-i}{2\overline{z}+i}\right|=\frac{1}{3}$

$\left|\frac{\overline{z}-i}{\overline{z}+\frac{i}{2}}\right|=\frac{2}{3}$

$3|x-iy-i| = 2\,|x-iy+\frac{i}{2}|$

$9(x^2+(y+1)^2) = 4\,(x^2+(y-1/3)^2)$

$9x^2+9y^2+18\,y+9 = 4x^2+4y^2-4y+1$

$5x^2+5y^2+22y+8=0$

$x^2+y^2+\frac{22}{5}y+\frac{8}{5}=0$

centre $\Rightarrow (0, -\frac{11}{5})$

$\left|\frac{1}{2}\begin{vmatrix}0 & 0 & 1\\ 0 & -11/5 & 1\\ \alpha & 0 & 1\end{vmatrix}\right|=11$

$\Rightarrow \left(-\frac{11}{5}\alpha\right)^2=(11\times 2)^2$

$\Rightarrow \alpha^2 = 100$

11. Let R = {(1, 2), (2, 3), (3,3)} be a relation defined on the set {1, 2, 3, 4}. Then the minimum number of elements, needed to be added in R so the R becomes an equivalence relation, is :

(1) 10 (2) 8

(3) 9 (4) 7

Ans. (4)

Sol. A = {1, 2, 3, 4}

For relation to be reflexive

R = {(1,2), (2, 3), (3,3)}

Minimum elements added will be

(1,1), (2,2), (4,4) (2,1) (3,2) (3,2) (3,1) (1,3)

$\therefore$ Minimum number of elements = 7

Option : (4)

12. The number of words, which can be formed using all the letters of the word "DAUGHTER", so that all the vowels never come together, is

(1) 34000 (2) 37000

(3) 36000 (4) 35000

Ans. (3)

Sol. DAUGHTER

Total words = 8!

Total words in which vowels are together = 6! × 3!

words in which all vowels are not together

= 8! – 6! × 3!

= 6! [56 – 6]

= 720 × 50

= 36000

Ans.(3)

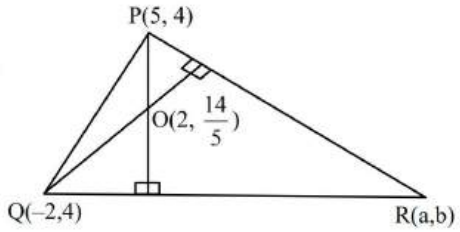
13. Let the area of a ΔPQR with vertices P(5, 4), Q(–2, 4) and R(a, b) be 35 square units. If its orthocenter and centroid are $O\left(2,\frac{14}{5}\right)$ and C(c, d) respectively, then c + 2d is equal to

(1) $\frac{7}{3}$ (2) 3

(3) 2 (4) $\frac{8}{3}$

Ans. (2)

Sol.



Equation of lines QR = 5x + 2y + 2 = 0

Equation of lines PR = 10x –3y – 38 = 0

$\therefore$ Point R (2, –6)

Centroid = $\left(\frac{5-2+2}{3},\frac{4+4-6}{3}\right)$

$=\left(\frac{5}{3},\frac{2}{3}\right)$

$c+2d = \frac{5}{3}+\frac{4}{3}=3$

14. If $\frac{\pi}{2} \le x \le \frac{3\pi}{4}$, then $\cos^{-1}\left(\frac{12}{13}\cos x + \frac{5}{13}\sin x\right)$ is equal to

(1) $x - \tan^{-1}\frac{4}{3}$ (2) $x - \tan^{-1}\frac{5}{12}$

(3) $x + \tan^{-1}\frac{4}{5}$ (4) $x + \tan^{-1}\frac{5}{12}$

Ans. (2)

Sol. $\frac{\pi}{2} \le x \le \frac{3\pi}{4}$

$\cos^{-1}\left(\frac{12}{13}\cos x + \frac{5}{12}\sin x\right)$

$\cos^{-1}(\cos x \cos\alpha + \sin x \sin\alpha)$

$\cos^{-1}(\cos(x-\alpha))$

$\Rightarrow x - \alpha$ because $x - \alpha \in (-\frac{\pi}{2}, \frac{\pi}{2})$

$\Rightarrow x - \tan^{-1}\frac{5}{12}$.

15. The value of $(\sin 70°)(\cot 10° \cot 70° - 1)$ is

(1) 1 (2) 0

(3) 3/2 (4) 2/3

Ans. (1)

Sol. $\sin 70° (\cot 10° \cot 70° - 1)$

$\Rightarrow \frac{\cos(80°)}{\sin 10} = 1$

16. Marks obtains by all the students of class 12 are presented in a frequency distribution with classes of equal width. Let the median of this grouped data be 14 with median class interval 12-18 and median class frequency 12. If the number of students whose marks are less than 12 is 18, then the total number of students is

(1) 48 (2) 44

(3) 40 (4) 52

Ans. (2)

Sol. $\text{medain} = \ell + \left(\frac{\frac{N}{2} - F}{f}\right) \times h$

$= 12 + \left(\frac{\frac{N}{2} - 18}{12}\right) \times 6 = 14$

$\Rightarrow \left(\frac{\frac{N}{2} - 18}{12}\right) \times 6 = 2$

$\frac{N}{2} - 18 = 4 \Rightarrow N = 44$

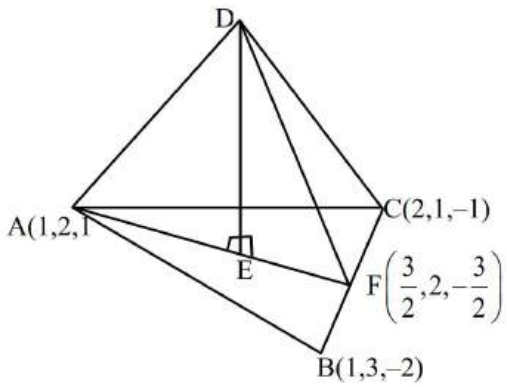
17. Let the position vectors of the vertices A, B and C of a tetrahedron ABCD be $\hat{i} + 2\hat{j} + \hat{k}$, $\hat{i} + 3\hat{j} - 2\hat{k}$ and $2\hat{i} + \hat{j} - \hat{k}$ respectively. The altitude from the vertex D to the opposite face ABC meets the median line segment through A of the triangle ABC at the point E. If the length of AD is $\frac{\sqrt{110}}{3}$ and the volume of the tetrahedron is $\frac{\sqrt{805}}{6\sqrt{2}}$, then the position vector of E is

(1) $\frac{1}{2}(\hat{i} + 4\hat{j} + 7\hat{k})$ (2) $\frac{1}{12}(7\hat{i} + 4\hat{j} + 3\hat{k})$

(3) $\frac{1}{6}(12\hat{i} + 12\hat{j} + \hat{k})$ (4) $\frac{1}{6}(7\hat{i} + 12\hat{j} + \hat{k})$

Ans. (4)

Sol.



Area of $\Delta ABC = \frac{1}{2}\left|\overrightarrow{AB} \times \overrightarrow{AC}\right|$

$= \frac{1}{2}\left|5\hat{i} + 3\hat{j} + \hat{k}\right| = \frac{1}{2}\sqrt{35}$

volume of tetrahedron

$= \frac{1}{3} \times \text{Base area} \times h = \frac{\sqrt{805}}{6\sqrt{2}}$

$\frac{1}{3} \times \frac{1}{2}\sqrt{35} \times h = \frac{\sqrt{805}}{6\sqrt{2}}$

$h = \sqrt{\frac{23}{2}}$

$AE^2 = AD^2 - DE^2 = \frac{13}{18} \therefore AE = \sqrt{\frac{13}{18}}$

$$\overrightarrow{AE} = |AE|.\left(\frac{\hat{i}-5\hat{k}}{\sqrt{26}}\right)$$

$$= \sqrt{\frac{13}{18}}.\left(\frac{\hat{i}-5\hat{k}}{\sqrt{26}}\right)$$

$$= \sqrt{\frac{13}{18}}.\left(\frac{\hat{i}-5\hat{k}}{\sqrt{26}}\right) = \frac{\hat{i}-5\hat{k}}{6}$$

$$\text{P.V. of E} = \frac{\hat{i}-5\hat{k}}{6} + \hat{i} + 2\hat{j} + \hat{k} = \frac{1}{6}\left(7\hat{i}+12\hat{j}+\hat{k}\right)$$

18. If A, B and $(\text{adj}(A^{-1}) + \text{adj}(B^{-1}))$ are non-singular matrices of same order, then the inverse of $A(\text{adj}(A^{-1}) + \text{adj}(B^{-1}))^{-1}B$, is equal to

(1) $AB^{-1} + A^{-1}B$ (2) $\text{adj}(B^{-1}) + \text{adj}(A^{-1})$

(3) $\frac{1}{|AB|}(\text{adj}(B) + \text{adj}(A))$ (4) $\frac{AB^{-1}}{|A|} + \frac{BA^{-1}}{|B|}$

Ans. (3)

Sol. $\left[A\left(\text{adj}\left(A^{-1}\right)+\text{adj}\left(B^{-1}\right)\right)^{-1}.B\right]^{-1}$

$$B^{-1}.\left(\text{adj}\left(A^{-1}\right)+\text{adj}\left(B^{-1}\right)\right).A^{-1}$$

$$B^{-1}\text{adj}\left(A^{-1}\right)A^{-1} + B^{-1}\left(\text{adj}\left(B^{-1}\right)\right).A^{-1}$$

$$B^{-1}\left|A^{-1}\right|I + \left|B^{-1}\right|IA^{-1}$$

$$\frac{B^{-1}}{|A|} + \frac{A^{-1}}{|B|}$$

$$\Rightarrow \frac{\text{adj}B}{|B||A|} + \frac{\text{adj}A}{|A||B|}$$

$$= \frac{1}{|A||B|}\left(\text{adj}B + \text{adj}A\right)$$

19. If the system of equations

$(\lambda - 1)x + (\lambda - 4)y + \lambda z = 5$

$\lambda x + (\lambda - 1)y + (\lambda - 4)z = 7$

$(\lambda + 1)x + (\lambda + 2)y - (\lambda + 2)z = 9$

has infinitely many solutions, then $\lambda^2 + \lambda$ is equal to

(1) 10 (2) 12

(3) 6 (4) 20

Ans. (2)

Sol. $(\lambda-1)x+(\lambda-4)y+\lambda z=5$

$\lambda x+(\lambda-1)y+(\lambda-4)z=7$

$(\lambda+1)x+(\lambda+2)y-(\lambda+2)z=9$

For infinitely many solutions

$$D = \begin{vmatrix} \lambda-1 & \lambda-4 & \lambda \\ \lambda & \lambda-1 & \lambda-4 \\ \lambda+1 & \lambda+2 & -(\lambda+2) \end{vmatrix} = 0$$

$(\lambda - 3)(2\lambda + 1) = 0$

$$D_x = \begin{vmatrix} 5 & \lambda-4 & \lambda \\ 7 & \lambda-1 & \lambda-4 \\ 9 & \lambda+2 & -(\lambda+2) \end{vmatrix} = 0$$

$2(3 - \lambda)(23 - 2\lambda) = 0$

$\lambda = 3$

$\therefore \lambda^2 + \lambda = 9 + 3 = 12$

20. One die has two faces marked 1, two faces marked 2, one face marked 3 and one face marked 4. Another die has one face marked 1, two faces marked 2, two faces marked 3 and one face marked 4. The probability of getting the sum of numbers to be 4 or 5, when both the dice are thrown together, is

(1) $\frac{1}{2}$ (2) $\frac{3}{5}$

(3) $\frac{2}{3}$ (4) $\frac{4}{9}$

Ans. (1)

Sol. a = number or dice 1

b = number on dice 2

(a,b) = (1,3), (3,1), (2,2), (2,3), (3,2), (1,4), (4,1)

Required probability

$$= \frac{2}{6}\times\frac{2}{6}+\frac{1}{6}\times\frac{1}{6}+\frac{2}{6}\times\frac{2}{6}+\frac{2}{6}\times\frac{2}{6}+\frac{1}{6}\times\frac{2}{6}+\frac{2}{6}\times\frac{1}{6}+\frac{1}{6}\times\frac{2}{6}$$

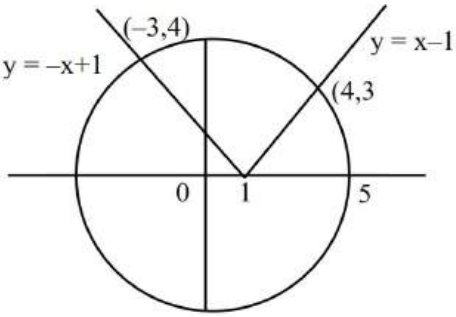
$$= \frac{18}{36} = \frac{1}{2}$$

SECTION-B

21. If the area of the larger portion bounded between the curves $x^2 + y^2 = 25$ and $y = |x - 1|$ is $\frac{1}{4}(b\pi + c)$, $b, c \in \mathbb{N}$, then $b + c$ is equal to _____

Ans. (77)

Sol.



$x^2 + y^2 = 5$

$x^2 + (x - 1)^2 = 25 \Rightarrow x = 4$

$x^2 + (-x + 1)^2 = 5 \Rightarrow x = -3$

$$A = 25\pi - \int_{-3}^{4} \sqrt{25 - x^2}\,dx + \frac{1}{2} \times 4 \times 4 + \frac{1}{2} \times 3 \times 3$$

$$A = 25\pi + \frac{25}{2} - \left[\frac{x}{2}\sqrt{25 - x^2} + \frac{25}{2}\sin^{-1}\frac{x}{5}\right]_{-3}^{4}$$

$$A = 25\pi + \frac{25}{2} - \left[6 + \frac{25}{2}\sin^{-1}\frac{4}{5} + 6 + \frac{25}{2}\sin^{-1}\frac{3}{5}\right]$$

$$A = 25\pi + \frac{1}{2} - \frac{25}{2}.\frac{\pi}{2}$$

$$A = \frac{75\pi}{4} + \frac{1}{2}$$

$$A = \frac{1}{4}(75\pi + 2)$$

$b = 75, c = 2$

$b + c = 75 + 2 = 77$

22. The sum of all rational terms in the expansion of $(1 + 2^{1/3} + 3^{1/2})^6$ is equal to _____

Ans. (612)

Sol. $\left(1 + 2^{\frac{1}{3}} + 3^{\frac{1}{2}}\right)^6$

$$= \frac{\lfloor 6}{\lfloor r_1 \lfloor r_2 \lfloor r_3}(1)^{r_1}(2)^{\frac{r_2}{3}}(3)^{\frac{r_3}{2}}$$

r_1	r_2	r_3
6	0	0
4	0	2
2	0	4
0	0	6
3	3	0
1	3	2
0	6	0

$$\text{sum} = \frac{\lfloor 6}{\lfloor 6 \lfloor 0 \lfloor 0} + \frac{\lfloor 6}{\lfloor 4 \lfloor 0 \lfloor 2}(3) + \frac{\lfloor 6}{\lfloor 2 \lfloor 0 \lfloor 4}(3)^2 + \frac{\lfloor 6}{\lfloor 0 \lfloor 0 \lfloor 6}(3)^3$$

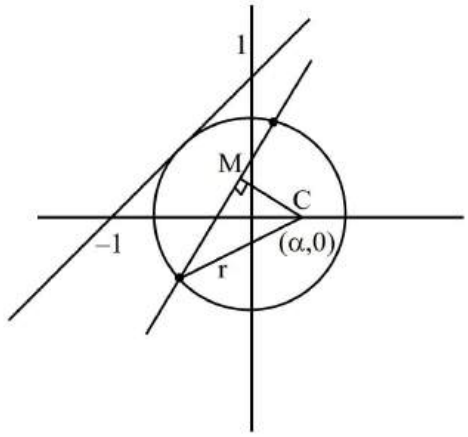
$$+ \frac{\lfloor 6}{\lfloor 3 \lfloor 3 \lfloor 0}(2) + \frac{\lfloor 6}{\lfloor 1 \lfloor 3 \lfloor 2}(2)^1(3)^1 + \frac{\lfloor 6}{\lfloor 0 \lfloor 6 \lfloor 0}(2)^2$$

$= 1 + 45 + 135 + 27 + 40 + 360 + 4 = 612$

23. Let the circle C touch the line $x - y + 1 = 0$, have the centre on the positive x-axis, and cut off a chord of length $\frac{4}{\sqrt{13}}$ along the line $-3x + 2y = 1$. Let H be the hyperbola $\frac{x^2}{\alpha^2} - \frac{y^2}{\beta^2} = 1$, whose one of the foci is the centre of C and the length of the transverse axis is the diameter of C. Then $2\alpha^2 + 3\beta^2$ is equal to _____

Ans. (19)

Sol.



$x - y + 1 = 0$

$p = r$

$$\left|\frac{\alpha - 0 + 1}{\sqrt{2}}\right| = r \Rightarrow (\alpha + 1)^2 = 2r^2 \;....\; (1)$$

now $\left(\frac{-3\alpha+0-1}{\sqrt{9+4}}\right)^2+\left(\frac{2}{\sqrt{13}}\right)^2=r^2$

$\Rightarrow (3\alpha+1)^2+4=13\,r^2$ .....(2)

(1) & (2) $\Rightarrow (3\alpha+1)^2+4=13\,\frac{(\alpha+1)^2}{2}$

$\Rightarrow 18\alpha^2+12\alpha+2+8=13\alpha^2+26\alpha+13$

$\Rightarrow 5\alpha^2-14\alpha-3=0$

$\Rightarrow 5\alpha^2-15\alpha+\alpha-3=0$

$\Rightarrow 5\alpha^2-15\alpha+\alpha-3=0$

$\Rightarrow \alpha=\frac{-1}{5},3$

$\therefore\ r=2\sqrt{2}$

How $\alpha e=3$ and $2\alpha=4\sqrt{2}$

$\alpha^2e^2=9\ \Rightarrow \alpha=2\sqrt{2}\ \Rightarrow \alpha^2=8$

$\alpha^2\left(1+\frac{\beta^2}{\alpha^2}\right)=9$

$\alpha^2+\beta^2=9$

$\therefore \beta^2=1$

$\therefore 2\alpha^2+3\beta^2=2(8)+3(1)=19$

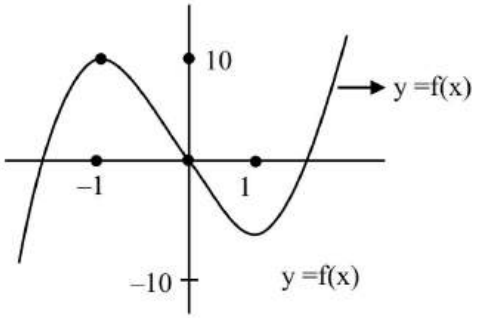
24. If the set of all values of a, for which the equation $5x^3-15x-a=0$ has three distinct real roots, is the interval (α,β), then $\beta-2\alpha$ is equal to _____

Ans. (30)

Sol. $5x^3-15x-a=0$

$f(x)=5x^3-15x$

$f'(x)=15x^2-15=15(x-1)(x+1)$



$a\in(-10,10)$

$\alpha=-10,\ \beta=10$

$\beta-2\alpha=10+20=30$

25. If the equation $a(b-c)x^2+b(c-a)x+c(a-b)=0$ has equal roots, where $a+c=15$ and $b=\frac{36}{5}$, then a^2+c^2 is equal to _____

Ans. (117)

Sol. $a(b-c)\,x^2+b\,(c-a)\,x+c(a-b)=0$

$x=1$ is root $\therefore$ other root is 1

$\alpha+\beta=-\frac{b(c-a)}{a(b-c)}=2$

$\Rightarrow -bc+ab=2ab-2ac$

$\Rightarrow 2ac=ab+bc$

$\Rightarrow 2ac=b(a+c)$

$\Rightarrow 2ac=15b$ ... (1)

$\Rightarrow 2ac=15\left(\frac{36}{5}\right)=108$

$\Rightarrow ac=54$

$a+c=15$

$a^2+c^2+2ac=225$

$a^2+c^2=225-108=117$

PHYSICS

SECTION-A

26. Regarding self-inductance :

A : The self-inductance of the coil depends on its geometry.

B : Self-inductance does not depend on the permeability of the medium.

C : Self-induced e.m.f. opposes any change in the current in a circuit.

D : Self-inductance is electromagnetic analogue of mass in mechanics.

E : Work needs to be done against self-induced e.m.f. in establishing the current.

Choose the correct answer from the options given below:

(1) A, B, C, D only (2) A, C, D, E only

(3) A, B, C, E only (4) B, C, D, E only

Ans. (2)

Sol. Self inductance of coil

$$L = \frac{\mu_0 N^2 A}{2\pi R}$$

27. A light hollow cube of side length 10 cm and mass 10g, is floating in water. It is pushed down and released to execute simple harmonic oscillations. The time period of oscillations is $y\pi \times 10^{-2}$ s, where the value of y is

(Acceleration due to gravity, $g = 10\ m/s^2$, density of water $= 10^3\ kg/m^3$)

(1) 2 (2) 6

(3) 4 (4) 1

Ans. (1)

Sol. $a^2 x\ \rho g = ma_{net}$

$$\frac{L^2 \rho g}{m} x = a_{net}$$

$$T = 2\pi \sqrt{\frac{m}{L^2 \rho g}}$$

where $m = 10g$, $L = 10$ cm, $\rho = 1000\ kg/m^3$

28. Given below are two statements:

Statement-I : The hot water flows faster than cold water.

Statement-II : Soap water has higher surface tension as compared to fresh water.

In the light above statements, choose the ***correct*** answer from the options given below

(1) Statement-I is false but Statement II is true

(2) Statement-I is true but Statement II is false

(3) Both Statement-I and Statement-II are true

(4) Both Statement-I and Statement-II are false

Ans. (2)

Sol. Hot water is less viscous then cold water.

Surfactant reduces surface tension.

29. A sub-atomic particle of mass 10^{-30} kg is moving with a velocity 2.21×10^6 m/s. Under the matter wave consideration, the particle will behave closely like _____. ($h = 6.63 \times 10^{-34}$ J.s)

(1) Infra-red radiation (2) X-rays

(3) Gamma rays (4) Visible radiation

Ans. (2)

Sol. $\lambda = \frac{h}{p} = \frac{6.63 \times 10^{-34}}{10^{-30} \times 2.21 \times 10^6}$

$= 3 \times 10^{-10}$m

Hence particle will behave as x-ray.

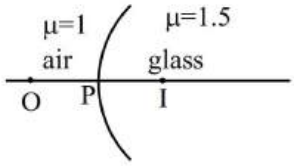
30. A spherical surface of radius of curvature R, separates air from glass (refractive index = 1.5). The centre of curvature is in the glass medium. A point object 'O' placed in air on the optic axis of the surface, so that its real image is formed at 'I' inside glass. The line OI intersects the spherical surface at P and PO = PI. The distance PO equals to-

(1) 5R (2) 3R

(3) 2R (4) 1.5R

Ans. (1)

Sol.



PO = u = –x

PI = v = x

PO = PI

$$\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R}$$

$$\frac{1.5}{x}+\frac{1}{x}=\frac{1}{2R}$$

$$\frac{5}{2x}=\frac{1}{2R}$$

X = 5R

31. A radioactive nucleus n_2 has 3 times the decay constant as compared to the decay constant of another radioactive nucleus n_1. If initial number of both nuclei are the same, what is the ratio of number of nuclei of n_2 to the number of nuclei of n_1, after one half-life of n_1?

(1) 1/4 (2) 1/8

(3) 4 (4) 8

Ans. (1)

Sol. $N_2 = N_0e^{-3\lambda t}$

$N_1 = N_0e^{-\lambda t}$

$$\frac{N_2}{N_1}=e^{-2\lambda t}$$

$$t_{\text{half life of}N_1}\, t=\frac{\ln 2}{\lambda}\ {}_n1$$

$$\frac{N_0}{2}=N_0e^{-\lambda t}$$

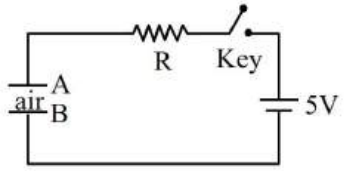
$\lambda t = \ln 2$

$$t=\frac{\ln 2}{\lambda}$$

$$=e^{-2\lambda\frac{\ln 2}{\lambda}}$$

$$\frac{N_2}{N_1}=\frac{1}{4}$$

32. Identify the valid statements relevant to the given circuit at the instant when the key is closed.



A. There will be no current through resistor R.

B. There will be maximum current in the connecting wires.

C. Potential difference between the capacitor plates A and B is minimum.

D. Charge on the capacitor plates is minimum.

Choose the correct answer from the options given below :

(1) C, D only (2) B, C, D only

(3) A, C only (4) A, B, D only

Ans. (2)

Sol. Initially capacitor behave as a short circuit.

So current will be maximum.

Charge on capacitor will be zero.

Potential difference across capacitor will be zero.

33. The position of a particle moving on x-axis is given by $x(t) = A \sin t + B \cos^2 t + Ct^2 + D$, where t is time. The dimension of $\frac{ABC}{D}$ is-

(1) L (2) $L^3 T^{-2}$

(3) $L^2 T^{-2}$ (4) L^2

Ans. (3)

Sol. Dimension [x(t)] = [L]

[A] = [L]

[B] = [L]

[C] = [LT^{-2}]

[D] = [L]

$$\left[\frac{ABC}{D}\right]=\left[\frac{L\times L\times LT^{-2}}{L}\right]=[L^2T^{-2}]$$

34. Match the **List-I** with **List-II**

	List-I		List-II
A.	Pressure varies inversely with volume of an ideal gas.	I.	Adiabatic process
B.	Heat absorbed goes partly to increase internal energy and partly to do work.	II.	Isochoric process
C.	Heat is neither absorbed nor released by a system	III.	Isothermal process
D.	No work is done on or by a gas	IV.	Isobaric process

Choose the ***correct*** answer from the options given below :

(1) A–I, B–IV, C–II, D–III

(2) A–III, B–I, C–IV, D–II

(3) A–I, B–III, C–II, D–IV

(4) A–III, B–IV, C–I, D–II

Ans. (4)

Sol. $A \rightarrow P \propto \frac{1}{V}$

$\Rightarrow PV = \text{constant}$

$\Rightarrow nRT = \text{const.} \Rightarrow T = \text{const.}$

Hence Isothermal III

$B \rightarrow IV$

$W \neq 0, \Delta U \neq 0, \Delta Q \neq 0$ [only isobaric]

$C \rightarrow I\ \Delta Q = 0$ Adiabatic

$D \rightarrow II\ w = 0$ Isochoric

III IV I II

35. Consider a moving coil galvanometer (MCG) :

A : The torsional constant in moving coil galvanometer has dimensions $[ML^2T^{-2}]$

B : Increasing the current sensitivity may not necessarily increase the voltage sensitivity.

C : If we increase number of turns (N) to its double (2N), then the voltage sensitivity doubles.

D : MCG can be converted into an ammeter by introducing a shunt resistance of large value in parallel with galvanometer.

E : Current sensitivity of MCG depends inversely on number of turns of coil.

Choose the correct answer from the options given below :

(1) A, B only (2) A, D, only

(3) B, D, E only (4) A, B, E only

Ans. (1)

Sol. (A) $\tau = C\theta \Rightarrow [ML^2T^{-2}] = [C][1]$

(B) $\textbf{C.S} = \frac{\theta}{I} = \frac{BNA}{C}$;

$V.S. = \frac{BNA}{RC}$ [R also depends on 'N']

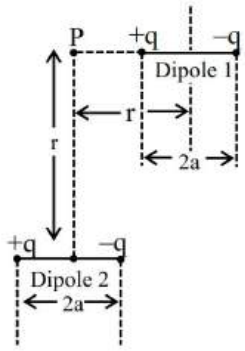
(C) $V.S. \propto \frac{NAB}{CR}$ $R \rightarrow NR$

(D) False [Theory]

(E) E [False] $C.S \propto N$

$\Rightarrow \because C.S. = \frac{NAB}{C}$

36. A point particle of charge Q is located at P along the axis of an electric dipole 1 at a distance r as shown in the figure. The point P is also on the equatorial plane of a second electric dipole 2 at a distance r. The dipoles are made of opposite charge q separated by a distance 2a. For the charge particle at P not to experience any net force, which of the following correctly describes the situation?

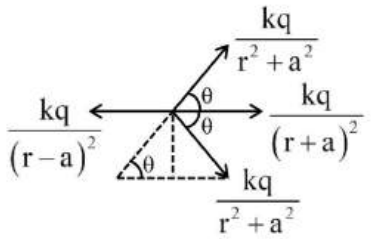


(1) $\frac{a}{r} - 20$ (2) $\frac{a}{r} \sim 10$

(3) $\frac{a}{r} \sim 0.5$ (4) $\frac{a}{r} \sim 3$

Ans. (4)

Sol.



$$\frac{kq}{(r-a)^2}=\frac{kq}{(r+a)^2}+\frac{2kq}{(r^2+a^2)}\cos\theta$$

$$\frac{1}{(r-a)^2}=\frac{1}{(r+a)^2}+\frac{2a}{(r^2+a^2)^{\frac{3}{2}}}$$

$$\frac{1}{(r-a)^2}-\frac{1}{(r+a)^2}=\frac{2a}{(r^2+a^2)^{\frac{3}{2}}}$$

$$\frac{4ra}{(r^2-a^2)^2}=\frac{2a}{(r^2+a^2)^{\frac{3}{2}}}$$

$$\Rightarrow\frac{2r}{(r^2-a^2)^2}=\frac{1}{(r^2+a^2)^{\frac{3}{2}}}$$

$$\frac{4r^2}{(r^2-a^2)^4}=\frac{1}{(r^2+a^2)^3}$$

$$\Rightarrow 4r^2(r^2+a^2)^3=(r^2-a^2)^4$$

$$4r^8\left(1+\frac{a^2}{r^2}\right)^3=r^8\left(1-\frac{a^2}{r^2}\right)^4$$

$$4\left(1+\frac{a^2}{r^2}\right)^3=\left(1-\frac{a^2}{r^2}\right)^4$$

Exact value cannot be solved in exam for this equation to be true

$$\left|\frac{a}{r}\right|>1\Rightarrow a>r$$

But point charge Q lies between charges of dipole 1 hence electric field cannot be zero.

There for it should be bonus.

But by solving from mathematical software we are getting $a/r \approx 3$.

37. A gun fires a lead bullet of temperature 300K into a wooden block. The bullet having melting temperature of 600 K penetrates into the block and melts down. If the total heat required for the process is 625 J, then the mass of the bullet is ___ grams.

(Latent heat of fusion of lead = 2.5×10^4 JKg^{-1} and specific heat capacity of lead = 125 JKg^{-1} K^{-1})

(1) 20 (2) 15

(3) 10 (4) 5

Ans. (3)

Sol. $625 = ms\Delta T + mL$

$625 = m[125 \times 300 + 2.5 \times 10^4]$

$625 = m[37500 + 25000]$

$625 = m[62500]$

$$m=\frac{1}{100}kg$$

M = 10 grams

38. What is the lateral shift of a ray refracted through a parallel-sided glass slab of thickness 'h' in terms of the angle of incidence 'i' and angle of refraction 'r', if the glass slab is placed in air medium ?

(1) $\frac{h\tan(i-r)}{\tan r}$ (2) $\frac{h\cos(i-r)}{\sin r}$

(3) h (4) $\frac{h\sin(i-r)}{\cos r}$

Ans. (4)

Sol. Formula base

$$\frac{h\sin(i-r)}{\cos r}$$

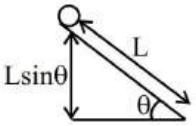
39. A solid sphere of mass 'm' and radius 'r' is allowed to roll without slipping from the highest point of an inclined plane of length 'L' and makes an angle 30° with the horizontal. The speed of the particle at the bottom of the plane is v_1. If the angle of inclination is increased to 45° while keeping L constant. Then the new speed of the sphere at the bottom of the plane is v_2. The ratio of $v_1^2 : v_2^2$ is

(1) $1 : \sqrt{2}$ (2) 1 : 3

(3) 1 : 2 (4) $1 : \sqrt{3}$

Ans. (1)

Sol.



using WET

$W_g = k_f - k_i$

$Mg\,L\sin\theta = k_f - k_i$

K.E. in pure rolling $\frac{1}{2}mV_{cm}^2 + \frac{1}{2}I_{cm}\omega^2$

$= \frac{1}{2}mV^2 + \frac{1}{2}\times\frac{2}{5}mR^2\frac{V^2}{R^2}$

$\frac{7}{10}mV^2$

$mgL\sin\theta = \frac{7}{10}mV_f^2 - 0$

$V_f^2 \propto \sin\theta$

$\left(\frac{V_1}{V_2}\right)^2 = \frac{\sin\theta_1}{\sin\theta_2} = \frac{\sin 30^\circ}{\sin 45^\circ} = \frac{1}{\sqrt{2}}$

40. Refer to the circuit diagram given in the figure, which of the following observation are correct?

A. Total resistance of circuit is 6 Ω.

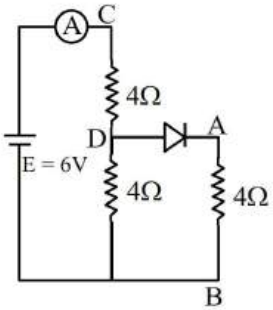
B. Current in Ammeter is 1A

C. Potential across AB is 4 Volts.

D. Potential across CD is 4 Volts.

E. Total resistance of the circuit is 8Ω.

Choose the correct answer from the options given below:

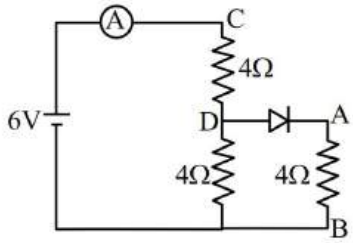


(1) A, B and D only (2) A, C and D only

(3) B, C and E only (4) A, B and C only

Ans. (1)

Sol.



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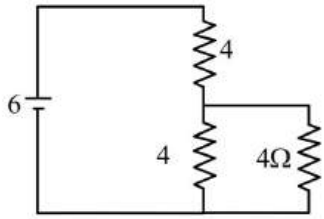
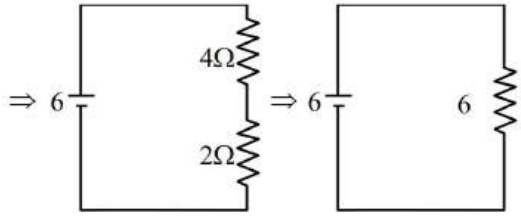
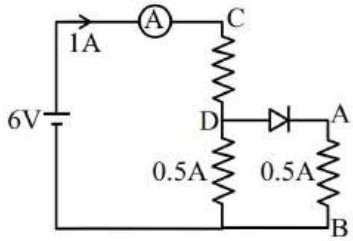




Current through ammeter = 1 A

$R_{net} = 6\Omega$



$V_{AB} = 0.5 \times 4 = 2$ volt

$V_{CD} = 1 \times 4 = 4$ volt

A, B & D are correct

41. The electric flux is $\phi = \alpha\sigma + \beta\lambda$

where λ and σ are linear and surface charge density, respectively, $\left(\frac{\alpha}{\beta}\right)$ represents

(1) charge

(2) electric field

(3) displacement

(4) area

Ans. (3)

Sol. $\phi = \alpha\sigma + \beta\lambda$

$[\phi] = [\alpha\sigma] = [\beta\lambda]$

$$[\alpha] = \frac{[\phi]}{[\sigma]} \qquad \left[\frac{\alpha}{\beta}\right] = \frac{[\lambda]}{[\sigma]}$$

$$[\beta] = \frac{[\phi]}{[\lambda]} = \frac{[Q/L]}{[Q/\text{Area}]} = \left[\frac{\text{Area}}{\text{Length}}\right]$$

$$\left[\frac{\alpha}{\beta}\right] = L$$

42. Given a thin convex lens (refractive index μ_2), kept in a liquid (refractive index μ_1, $\mu_1 < \mu_2$) having radii of curvature $|R_1|$ and $|R_2|$. Its second surface is silver polished. Where should an object be placed on the optic axis so that a real and inverted image is formed at the same place ?

(1) $\dfrac{\mu_1|R_1|.|R_2|}{\mu_2(|R_1|+|R_2|)-\mu_1|R_1|}$

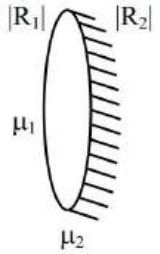
(2) $\dfrac{\mu_1|R_1|.|R_2|}{\mu_2(|R_1|+|R_2|)-\mu_1|R_2|}$

(3) $\dfrac{\mu_1|R_1|.|R_2|}{\mu_2(2|R_1|+|R_2|)-\mu_1\sqrt{|R_1|.|R_2|}}$

(4) $\dfrac{(\mu_2+\mu_1)|R_1|}{(\mu_2-\mu_1)}$

Ans. (2)

Sol.



$$\frac{1}{f_{eq}} = \frac{2}{f_L} - \frac{1}{f_m}$$

$$f_m = -\frac{|R_2|}{2}$$

$$\frac{1}{f_L} = \left(\frac{\mu_2}{\mu_1} - 1\right)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$\frac{1}{f_{eq}} = 2\left(\frac{\mu_2-\mu_1}{\mu_1}\right)\left(\frac{R_1+R_2}{R_1R_2}\right) + \frac{2}{R_2}$$

$$= \frac{2}{R_2}\left[\frac{(\mu_2-\mu_1)(R_1+R_2)+\mu_1R_1}{\mu_1R_1}\right]$$

$$= \frac{2}{R_2}\left[\frac{\mu_2R_1+\mu_2R_2-\mu_1R_1-\mu_1R_2+\mu_1R_1}{\mu_1R_1}\right]$$

$$\frac{1}{f_{eq}} = \frac{2[\mu_2R_1+\mu_2R_2-\mu_1R_2]}{\mu_1R_1R_2}$$

For same size of image

$u = 2f$

$$u = \frac{\mu_1R_1R_2}{\mu_2R_1+\mu_2R_2-\mu_1R_2}$$

43. The electric field of an electromagnetic wave in free space is

$\vec{E} = 57\cos[7.5 \times 10^6\, t - 5 \times 10^{-3}(3x + 4y)]\left(4\hat{i} - 3\hat{j}\right)$ N/C.

The associated magnetic field in Tesla is-

(1) $\vec{B} = \dfrac{57}{3\times10^8}\cos\,[7.5\times10^6t - 5\times10^{-3}(3x+4y)]\left(5\hat{k}\right)$

(2) $\vec{B} = \dfrac{57}{3\times10^8}\cos\,[7.5\times10^6t - 5\times10^{-3}(3x+4y)]\left(\hat{k}\right)$

(3) $\vec{B} = -\dfrac{57}{3\times10^8}\cos\,[7.5\times10^6t - 5\times10^{-3}(3x+4y)]\left(5\hat{k}\right)$

(4) $\vec{B} = -\dfrac{57}{3\times10^8}\cos\,[7.5\times10^6t - 5\times10^{-3}(3x+4y)]\left(\hat{k}\right)$

Ans. (3)

Sol. $\vec{K} = 3\hat{i} + 4\hat{j}$

$$\hat{K} = \frac{3\hat{i}+4\hat{j}}{5}$$

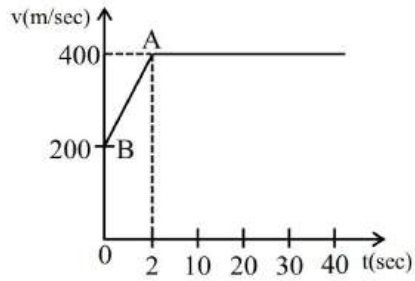
$$\hat{E} = \frac{4\hat{i}-3\hat{j}}{5}$$

$$\hat{B} = \hat{K}\times\hat{E}$$

$$\hat{B} = -\hat{Z}$$

$$B_0 = \frac{E_0}{C} = \frac{57}{3\times10^8}$$

44. The motion of an airplane is represented by velocity-time graph as shown below. The distance covered by airplane in the first 30.5 second is ____ km.



(1) 9 (2) 6

(3) 3 (4) 12

Ans. (4)

Sol. Total Area under curve.

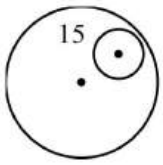
45. Consider a circular disc of radius 20 cm with centre located at the origin. A circular hole of a radius 5 cm is cut from this disc in such a way that the edge of the hole touches the edge of the disc. The distance of centre of mass of residual or remaining disc from the origin will be-

(1) 2.0 cm (2) 0.5 cm

(3) 1.5 cm (4) 1.0 cm

Ans. (4)

Sol.



mass of disc = m

mass of cut part = $\frac{m}{16}$

$$X_{com} = \frac{m \times 0 - \frac{m}{16} \times 15}{m - \frac{m}{16}}$$

= 1 cm.

46. A positive ion A and a negative ion B has charges 6.67×10^{-19} C and 9.6×10^{-10} C, and masses 19.2×10^{-27} kg and 9×10^{-27} kg respectively. At an instant, the ions are separated by a certain distance r. At that instant the ratio of the magnitudes of electrostatic force to gravitational force is $P \times 10^{-13}$, where the value of P is ____.

(Take $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9$ Nm^2C^{-1} and universal gravitational constant as 6.67×10^{-11} $Nm^2 kg^{-2}$)

Ans. (BONUS)

Sol. $$\frac{9\times10^9\times6.67\times10^{-19}\times9.6\times10^{-10}}{6.67\times10^{-11}\times19.2\times10^{-27}\times9\times10^{-27}}$$

$$\frac{1}{2}\times10^{45}$$

Charge is not integral multiple of electron.

47. Two particles are located at equal distance from origin. The position vectors of those are represented by $\vec{A} = 2\hat{i} + 3n\hat{j} + 2\hat{k}$ and $\vec{B} = 2\hat{i} - 2\hat{j} + 4p\hat{k}$, respectively. If both the vectors are at right angle to each other, the value of n^{-1} is____.

Ans. (3)

Sol. $\vec{A}.\vec{B} = 0$

$4 - 6n + 8p = 0$

$|\vec{A}| = |\vec{B}|$

$4 + 9n^2 + 4 = 4 + 4 + 16p^2$

$9n^2 = 16p^2$

$P = +\frac{3}{4}n$

$4 - 6n \pm 6n = 0$

$12n = 4$

$n = \frac{1}{3}$

CHEMISTRY

SECTION-A

51. The element that does not belong to the same period of the remaining elements (modern periodic table) is:

(1) Palladium

(2) Iridium

(3) Osmium

(4) Platinum

Sol. **(1)**

Palladium $\Rightarrow 5^{th}$ period

Iridium, Osmium, Platinum $\Rightarrow 6^{th}$ Period

52. Heat treatment of muscular pain involves radiation of wavelength of about 900 nm. Which spectral line of H atom is suitable for this ?

Given: Rydberg constant

$R_H = 10^5\ cm^{-1}$, $h = 6.6 \times 10^{-34}$ J s, $c = 3 \times 10^8$ m/s)

(1) Paschen series, $\infty \rightarrow 3$

(2) Lyman series, $\infty \rightarrow 1$

(3) Balmer series, $\infty \rightarrow 2$

(4) Paschen series, $5 \rightarrow 3$

Sol. **(1)**

$\lambda = 900$ nm H-atom (Z = 1)

$= 9 \times 10^{-5}$ cm

$R_H = 10^5\ cm^{-1}$

Ryderg eq. = $\frac{1}{\lambda} = R_H Z^2 \times \left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

$\Rightarrow \frac{1}{\lambda \times R_H} = \frac{1}{n_1^2} - \frac{1}{n_2^2}$

$\Rightarrow \frac{1}{9 \times 10^{-5} cm \times 10^5 cm^{-1}} = \left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

$\Rightarrow \frac{1}{n_1^2} - \frac{1}{n_2^2} = \frac{1}{9}$

It is possible when $n_1 = 3$, $n_2 = \infty$

Possible series : $\infty \rightarrow 3$

53. The **incorrect** statements among the following is

(1) PH_3 shows lower proton affinity than NH_3.

(2) PF_3 exists but NF_5 does not.

(3) NO_2 can dimerise easily.

(4) SO_2 can act as an oxidizing agent, but not as a reducing agent.

Sol. **(4)**

SO_2 can oxidise as well as reduce.

Hence it can act as both oxidising and reducing agent.

54. $CrCl_3.xNH_3$ can exist as a complex. 0.1 molal aqueous solution of this complex shows a depression in freezing point of 0.558°C. Assuming 100% ionisation of this complex and coordination number of Cr is 6, the complex will be

(Given $K_f = 1.86$ K kg mol^{-1})

(1) $[Cr(NH_3)_6]\ Cl_3$

(2) $[Cr(NH_3)_4Cl_2]\ Cl$

(3) $[Cr(NH_3)_5Cl]\ Cl_2$

(4) $[Cr(NH_3)_3Cl_3]$

Sol. **(3)**

Given : $\Delta T_f = 0.558°C$

$k_f = 1.86 \frac{K \times kg}{mol}$

0.1 m aq. sol.

$\Rightarrow \Delta T_f = i \times k_f \times m$

$\Rightarrow 0.558 = i \times 1.86 \times 0.1$

$\Rightarrow i = 3$

55. $FeO_4^{2-} \xrightarrow{+2.0V} Fe^{3+} \xrightarrow{0.8V} Fe^{2+} \xrightarrow{-0.5V} Fe^0$

In the above diagram, the standard electrode potentials are given in volts (over the arrow).

The value of $E^{\ominus}_{FeO_4^{2-}/Fe^{2+}}$ is

(1) 1.7 V (2) 1.2 V

(3) 2.1 V (4) 1.4 V

Sol. (1)

$$FeO_4^{-2} \xrightarrow[n_1=3]{E_1^o=2V} Fe^{+3} \xrightarrow[n_2=1]{E_2^o=0.8V} Fe^{+2} \xrightarrow{E_3^o=-.05V} Fe$$

$$E_4^o = ?$$

$$n_4 = 4$$

$$\Delta G_4^o = \Delta G_1^o + \Delta G_2^o$$

$$\Rightarrow -n_4FE_4^o = -n_1FE_1^0 - n_2FE_2^o$$

$$\Rightarrow +4E_4^o = 3\times2 + (1\times0.8)$$

$$\Rightarrow E_4^o = \frac{6.8}{4}V$$

$$\Rightarrow \boxed{E_4^o = 1.7V}$$

56. Match the **LIST-I** with **LIST-II**

LIST-I Name reaction		LIST-II Product obtainable	
A.	Swarts reaction	I.	Ethyl benzene
B.	Sandmeyer's reaction	II.	Ethyl iodide
C.	Wurtz Fittig reaction	III.	Cyanobenzene
D.	Finkelstein reaction	IV.	Ethyl fluoride

Choose the ***correct*** answer from the options given below:

(1) A-II, B-III, C-I, D-IV

(2) A-IV, B-I, C-III, D-II

(3) A-IV, B-III, C-I, D-II

(4) A-II, B-I, C-III, D-IV

Sol. (3)

LIST-I Name reaction		LIST-II Product obtainable	
A.	Swarts reaction	I.	$Et\text{-}I \xrightarrow[DMF]{KF} Et\text{-}F$
B.	Sandmeyer's reaction	II.	$PhN_2^{\oplus}Cl^- \xrightarrow{CuCN/KCN} PhCN + N_2$
C.	Wurtz Fittig reaction	III.	$Ph\text{–}Cl + EtCl \xrightarrow[ether]{Na}$ Ph–Et + Ph–Ph + Et–Et
D.	Finkelstein reaction	IV.	$Et\text{–}Cl \xrightarrow[acetone]{NaI} Et\text{-}I + NaCl$

57. Given below are two statements:

Statement I: Fructose does not contain an aldehydic group but still reduces Tollen's reagent

Statement II : In the presence of base, fructose undergoes rearrangement to give glucose.

In the light of the above statements, choose the ***correct*** answer from the options given below

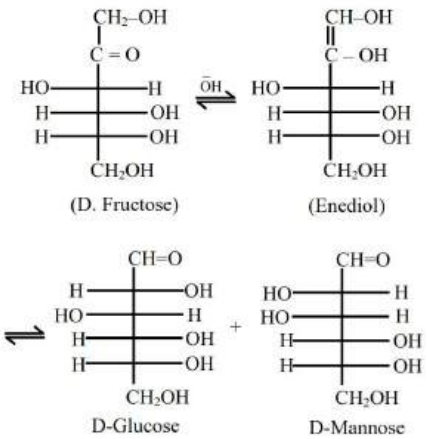
(1) Statement I is false but Statement II is true

(2) Both Statement I and Statement II are true

(3) Both Statement I and Statement II are false

(4) Statement I is true but Statement II is false

Sol. (2)



58. 2.8×10^{-3} mol of CO_2 is left after removing 10^{21} molecules from its 'x' mg sample. The mass of CO_2 taken initially is

Given : $N_A = 6.02 \times 10^{23}\ mol^{-1}$

(1) 196.2 mg (2) 98.3 mg

(3) 150.4 mg (4) 48.2 mg

Sol. (1)

$$(moles)_{initial} = \frac{x\times10^{-3}}{44}$$

$$(moles)_{removal} = \frac{10^{21}}{6.02\times10^{23}}$$

$$(moles)_{left} = (moles)_{initial} - (moles)_{removed}$$

$$2.8\times10^{-3} = \frac{x\times10^{-3}}{44} - \frac{10^{21}}{6.02\times10^{23}}$$

$$\Rightarrow x = 196.2\ mg$$

59. Ice at –5°C is heated to become vapor with temperature of 110°C at atmospheric pressure. The entropy change associated with this process can be obtained from :

(1) $\int_{268K}^{383K} C_p\,dT + \frac{\Delta H_{melting}}{273} + \frac{\Delta H_{boiling}}{373}$

(2) $\int_{268K}^{273K} \frac{C_{p,m}}{T}dT + \frac{\Delta H_m, fusion}{T_f} + \frac{\Delta H_{m,vaporisation}}{T_b}$

$+ \int_{273K}^{373K} \frac{C_{p,m}dT}{T} + \int_{373K}^{383K} \frac{C_{p,m}dT}{T}$

(3) $\int_{268K}^{383K} C_p\,dT + \frac{q_{rev}}{T}$

(4) $\int_{268K}^{273K} C_{p,m}dT + \frac{\Delta H_m, fusion}{T_f} + \frac{\Delta H_{m,vaporisation}}{T_b}$

$+ \int_{273K}^{373K} C_{p,m}dT + \int_{373K}^{383K} C_{p,m}dT$

Sol. (2)

$$\underbrace{\text{Ice}}_{268\text{ K}} \rightarrow \underbrace{\text{Ice}}_{273\text{ K}} \rightleftharpoons \underbrace{\text{Water}}_{273\text{ K}} \rightarrow \underbrace{\text{Water}}_{273\text{ K}} \rightleftharpoons \underbrace{\text{Water vapour}}_{373\text{ K}} \rightarrow \underbrace{\text{Water vapour}}_{383\text{ K}}$$

(1) (2) (3) (4) (5)

$\Delta S_{overall} = \Delta S_1 + \Delta S_2 + \Delta S_3 + \Delta S_4 + \Delta S_5$

$\Delta S_2 = \frac{\Delta H_{m\ fusion}}{273}$ $\quad T_f = 273$ 'K'

$\Delta S_3 = \int_{273}^{373} \frac{C_{p,m}dT}{T}$

$\Delta S_4 = \frac{\Delta H_{m\ vaporisation}}{373}$ $\quad T_b = 373$ 'K'

$\Delta S_5 = \int_{373}^{383} \frac{C_{p,m}dT}{T}$

Answer = (2)

60. The d-electronic configuration of an octahedral Co(II) complex having magnetic moment of 3.95 BM is :

(1) $t_{2g}^6 e_g^1$ (2) $t_{2g}^3 e_g^0$

(3) $t_{2g}^5 e_g^2$ (4) $e^4 t_2^3$

Sol. (3)

$Co^{+2} = (Ar)_{18}\ 3d^7\ 4s^o$

$\underline{\uparrow}\ \ \underline{\uparrow}\ \ e_g^2$

$\underline{\uparrow\downarrow}\ \ \underline{\uparrow\downarrow}\ \ \underline{\uparrow}\ t_{2g}^5$

61. The complex that shows Facial – Meridional isomerism is

(1) $[Co(NH_3)_3Cl_3]$ (2) $[Co(NH_3)_4Cl_2]^+$

(3) $[Co(en)_3]^{3+}$ (4) $[Co(en)_2Cl_2]^+$

Sol. (1)

Ma_3b_3 type complexes show Facial - Meridional isomerism

(i) $[Co(NH_3)_3Cl_3] \Rightarrow Ma_3b_3$

(ii) $[Co(NH_3)_4Cl_2]^+ \Rightarrow Ma_4b_2$

(iii) $[Co(en)_3]^{3+} \Rightarrow M(AA)_3$

(iv) $[Co(en)_2Cl_2]^+ \Rightarrow M(AA)_2b_2$

a, b, = NH_3, Cl^-

AA = en

62. The major product of the following reaction is :

$$CH_3CH_2CH{=}O \xrightarrow[\text{reflux}]{\text{excess HCHO, alkali}} ?$$

(1) $CH_3{-}CH_2{-}CH_2{-}OH$

(2) $CH_3{-}CH(CH_2{-}OH){-}CH{=}O$

(3) $CH_3{-}C(CH_2{-}OH)_2{-}CH_2{-}OH$

(4) $CH_3{-}C(=CH_2){-}CH{=}O$

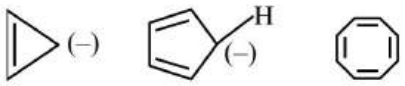
Sol. (3)

This is an example of Tollen's reaction i.e. multiple cross aldol followed by cross Cannizaro reaction

$$CH_3CH_2CH{=}O \xrightarrow[\text{Alkali}]{\text{2HCHO}} CH_3{-}C(CH_2{-}OH)_3{-}CHO$$

$$\xrightarrow[\text{Alkali}]{\text{HCHO}} CH_3{-}C(CH_2{-}OH)_3{-}CH_2{-}OH + HCOO^{\ominus}$$

63. The correct stability order of the following species/molecules is :



p q r

(1) q > r > p (2) r > q > p

(3) q > p > r (4) p > q > r

Sol. **(1)**

q is aromatic r is nonaromatic p is antiaromatic

64. Propane molecule on chlorination under photochemical condition gives two di-chloro products, "x" and "y". Amongst "x" and "y", "x" is an optically active molecule. How many tri-chloro products (consider only structural isomers) will be obtained from "x" when it is further treated with chlorine under the photochemical condition?

(1) 4 (2) 2

(3) 5 (4) 3

Sol. **(4)**

"X" is $CH_3-\overset{*}{C}H(Cl)-CH_2(Cl)$

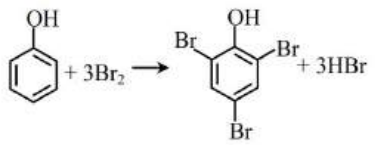
$$X \xrightarrow[h\nu]{Cl_2} CH_3-\overset{*}{C}H(Cl)-CH(Cl)-Cl + CH_3-C(Cl)_2-CH_2-Cl + Cl-CH_2-CH(Cl)-CH_2-Cl$$

65. What amount of bromine will be required to convert 2 g of phenol into 2, 4, 6-tribromophenol ? (Given molar mass in g mol^{-1} of C, H, O, Br are 12, 1, 16, 80 respectively)

(1) 10.22 g (2) 6.0 g

(3) 4.0 g (4) 20.44 g

Sol. **(1)**



Moles of phenol = $\frac{2}{94} = 0.021$

$\therefore$ Moles of bromine = 0.021 × 3 = 0.064

$\therefore$ Mass of bromine = 0.064 × 160 = 10.22 g

66. The correct set of ions (aqueous solution) with same colour from the following is :

(1) V^{2+}, Cr^{3+}, Mn^{3+} (2) Zn^{2+}, V^{3+}, Fe^{3+}

(3) Ti^{4+}, V^{4+}, Mn^{2+} (4) Sc^{3+}, Ti^{3+}, Cr^{2+}

Sol. **(1)**

(1) V^{2+}(Violet), Cr^{3+}(Violet), Mn^{3+}(Violet)

(2) Zn^{2+}(Colourless), V^{3+}(Green), Fe^{3+}(Yellow)

(3) Ti^{4+}(Colourless), V^{4+}(Blue), Mn^{2+} (Pink)

(4) Sc^{3+}(Colourless), Ti^{3+}(Purple), Cr^{2+} (Blue)

67. Given below are two statements :

Statement I : In Lassaigne's test, the covalent organic molecules are transformed into ionic compounds.

Statement II : The sodium fusion extract of an organic compound having N and S gives prussian blue colour with $FeSO_4$ and $Na_4[Fe(CN)_6]$

In the light of the above statements, choose the **correct** answer from the options given below

(1) Both **Statement I** and **Statement II** are true

(2) Both **Statement I** and **Statement II** are false

(3) **Statement I** is false but **Statement II** is true

(4) **Statement I** is true but **Statement II** is false

Sol. **(4)**

The sodium fusion extract of organic compound having N & S gives blood red colour with $FeSO_4$ and $Na_4[Fe(CN)_6]$

68. Which of the following happens when NH_4OH is added gradually to the solution containing 1M A^{2+} and 1M B^{3+} ions ?

Given : $K_{sp}[A(OH)_2] = 9 \times 10^{-10}$ and

$K_{sp}[B(OH)_3] = 27 \times 10^{-18}$ at 298 K.

(1) $B(OH)_3$ will precipitate before $A(OH)_2$

(2) $A(OH)_2$ and $B(OH)_3$ will precipitate together

(3) $A(OH)_2$ will precipitate before $B(OH)_3$

(4) Both $A(OH)_2$ and $B(OH)_3$ do not show precipitation with NH_4OH

Sol. **(1)**

Condition for precipitation $Q_{ip} > K_{sp}$

For $[A(OH)_2]$

$[A^{2+}][OH^-]^2 > 9 \times 10^{-10}$

$[A^{+2}] = 1$ M

$\Rightarrow [OH^-] > 3 \times 10^{-5}$ M

For $[B(OH)_3]$

$[B^{3+}][OH^-]^3 > 27 \times 10^{-18}$

$[B^{3+}] = 1$M

$\Rightarrow [OH^-] > 3 \times 10^{-6}$ M

So, $B(OH)_3$ will precipitate before $A(OH)_2$

69. Match the **LIST-I** with **LIST-II**

LIST-I (Classification of molecules based on octet rule)		**LIST-II (Example)**	
A.	Molecules obeying octet rule	I.	NO, NO_2
B.	Molecules with incomplete octet	II.	BCl_3, $AlCl_3$
C.	Molecules with incomplete octet with odd electron	III.	H_2SO_4, PCl_5
D.	Molecules with expanded octet	IV.	CCl_4, CO_2

Choose the **correct** answer from the options given below :

(1) A-IV, B-II, C-I, D-III

(2) A-III, B-II, C-I, D-IV

(3) A-IV, B-I, C-III, D-II

(4) A-II, B-IV, C-III, D-I

Sol. **(1)**

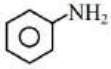
(A) A $\rightarrow$ IV

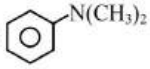
(B) B $\rightarrow$ II

(C) C $\rightarrow$ I

(D) D $\rightarrow$ III

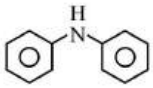
70. Which among the following react with Hinsberg's reagent?

(A) 

(2) 

(C) $CH_3–NH_2$

(4) $N(CH_3)_3$

(E) 

Choose the correct answer from the options given below :

(1) B and D only

(2) C and D only

(3) A, B and E only

(4) A, C and E only

Sol. **(4)**

B and D are 3° amine which does not have replaceable H on N, So does not react.

SECTION-B

71. If 1 mM solution of ethylamine produces pH = 9, then the ionization constant (K_b) of ethylamine is 10^{-x}. The value of x is _____ (nearest integer). [The degree of ionization of ethylamine can be neglected with respect to unity.]

Sol. **(7)**

Sol. $C_2H_5NH_2(aq) + H_2O \rightleftharpoons C_2H_2NH_3^+ + \overset{\ominus}{O}H$

$C = 10^{-3}$ M	-	-
$C(1 - \alpha)$	$C\alpha$	$C\alpha$
$\Rightarrow C = 10^{-3}$	$=10^{-5}$	$= 10^{-5}$

$\boxed{1-\alpha \simeq 1}$

Given, $P^H = 9 \Rightarrow P^{OH} = 5 \Rightarrow \left[\overset{\ominus}{O}H\right] = 10^{-5}$ M

Now, $K_b = \dfrac{[C_2H_5NH_3^+][\overset{\ominus}{O}H]}{[C_2H_5NH_2]}$

$\Rightarrow K_b = \dfrac{10^{-5} \times 10^{-5}}{10^{-3}} = 10^{-7}$

72. During "S" estimation, 160 mg of an organic compound gives 466 mg of barium sulphate. The percentage of Sulphur in the given compound is _____ %.

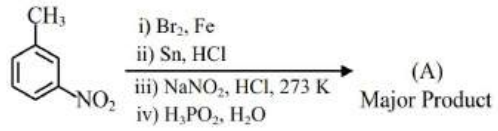
(Given molar mass in g mol^{-1} of Ba : 137, S : 32, O : 16)

Sol. **(40)**

Millimoles of $BaSO_4 = \dfrac{466}{233} = 2$m mol

$\%S = \dfrac{\frac{466}{233} \times 32}{160} \times 100 = 40\%$

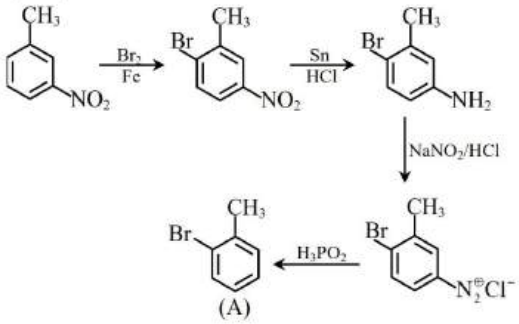
73. Consider the following sequence of reactions to produce major product (A)



Molar mass of product (A) is _____ g mol^{-1}.

(Given molar mass in g mol^{-1} of C : 12, H : 1, O : 16, Br : 80, N : 14, P : 31)

Sol. (171)



Molar mass of product (C_7H_7Br) (A) is 171 g mol^{-1}

74. For the thermal decomposition of $N_2O_5(g)$ at constant volume, the following table can be formed, for the reaction mentioned below :

$2N_2O_5(g) \rightarrow 2N_2O_4(g) + O_2(g)$

S.No.	Time/s	Total pressure / (atm)
1.	0	0.6
2.	100	'x'

x = _____ $\times 10^{-3}$ atm [nearest integer]

Given : Rate constant for the reaction is $4.606 \times 10^{-2} s^{-1}$.

Sol. (900)

NTA. (897)

$K_{N_2O_5} = 2 \times 4.606 \times 10^{-2} S^{-1}$

$2N_2O_5(g) \longrightarrow 2N_2O_4(g) + O_2(g)$

P_i	0.6	0	0
P_f	$0.6 - P$	P	$\frac{P}{2}$

$$2 \times 4.606 \times 10^{-2} = \frac{2.303}{100} \log \frac{0.6}{0.6 - P}$$

$$4 \log_{10} \frac{0.6}{0.6 - P}$$

$$10^4 = \frac{0.6}{0.6 - P}$$

$\Rightarrow 0.6 \times 10^4 - 10^4 P = 0.6$

$\Rightarrow 10^4 P = 0.6(10^4 - 1)$

$P = (6000 - 0.6) \times 10^{-4}$

$= 5999. \times 10^{-4}$

$= 0.59994$

$$P_{Total} = 0.6 + \frac{P}{2}$$

$= 0.6 + 0.29997$

$= 0.89997$

$= 899.97 \times 10^{-3}$

Ans. 900

Given by NTA

Given : $2N_2O_5(g) \rightarrow 2N_2O_4(g) + O_2(g)$

t = 0	0.6	0	0
t = 100s	0.6 - x	x	x/2

$$P_{Total} = 0.6 + \frac{x}{2}$$

As given in equation

$K_r = 4.606 \times 10^{-2} sec^{-1}$

(Here language conflict in question)

$(K_r = \frac{KA}{2}$ not considered)

$$K_r t = \ln \frac{0.6}{0.6 - x}$$

$$4.606 \times 10^{-2} \times 100 = 2.303 \log \frac{0.6}{0.6 - x}$$

$$P_{Total} = 0.6 + \frac{0.594}{2} = 0.897 \text{ atm}$$

$= 897 \times 10^{-3}$ atm

75. The standard enthalpy and standard entropy of decomposition of N_2O_4 to NO_2 are 55.0 kJ mol^{-1} and 175.0 J/K/mol respectively. The standard free energy change for this reaction at 25°C in J mol^{-1} is _____ (Nearest integer)

Sol. (2850)

$\Delta H^o_{rxn} = 55$ kJ/mol, T = 298 K

$\Delta S^o_{rxn} = 175$ J/mol

$\Delta G^o_{rxn} = \Delta H^o_{rxn} - T\Delta S^o_{rxn}$

$\Rightarrow \Delta G^o_{rxn} = 55000$ J/mol $- 298 \times 175$ J/mol

$\Rightarrow \Delta G^o_{rxn} = 55000 - 52150$

$\Rightarrow \Delta G^o_{rxn} = 2850$ J/mol

JEE–MAIN EXAMINATION – JANUARY 2025

(HELD ON WEDNESDAY 22nd JANUARY 2025)

TIME : 3 : 00 PM TO 6 : 00 PM

MATHEMATICS

SECTION-A

1. Let α, β, γ and δ be the coefficients of x^7, x^5, x^3 and x respectively in the expansion of $\left(x+\sqrt{x^3-1}\right)^5+\left(x-\sqrt{x^3-1}\right)^5$, $x > 1$. If u and v satisfy the equations

$\alpha u + \beta v = 18$,

$\gamma u + \delta v = 20$,

then u + v equals :

(1) 5 (2) 4

(3) 3 (4) 8

Ans. (1)

Sol. $\left(x+\sqrt{x^3-1}\right)^5+\left(x-\sqrt{x^3-1}\right)^5$

$= 2\{{}^5C_0.x^5 + {}^5C_2.x^3(x^3-1) + {}^5C_4.x(x^3-1)^2\}$

$= 2\{5x^7 + 10x^6 + x^5 - 10x^4 - 10x^3 + 5x\}$

$\Rightarrow \alpha = 10, \beta = 2, \gamma = -20, \delta = 10$

Now, $10u + 2v = 18$

$-20u + 10v = 20$

$\Rightarrow u = 1, v = 4$

$u + v = 5$

2. In a group of 3 girls and 4 boys, there are two boys B_1 and B_2. The number of ways, in which these girls and boys can stand in a queue such that all the girls stand together, all the boys stand together, but B_1 and B_2 are not adjacent to each other, is :

(1) 144 (2) 72

(3) 96 (4) 120

Ans. (1)

Sol. Total – when B_1 and B_2 are together

$= 2!(3!\,4!) - 2!\,(3!(3!\,2!)) = 144$

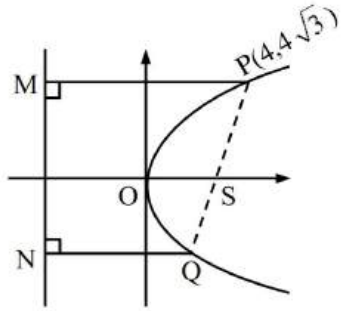
3. Let $P\left(4, 4\sqrt{3}\right)$ be a point on the parabola $y^2 = 4ax$ and PQ be a focal chord of the parabola. If M and N are the foot of perpendiculars drawn from P and Q respectively on the directrix of the parabola, then the area of the quadrilateral PQMN is equal to:

(1) $\frac{263\sqrt{3}}{8}$ (2) $17\sqrt{3}$

(3) $\frac{343\sqrt{3}}{8}$ (4) $\frac{34\sqrt{3}}{3}$

Ans. (3)

Sol.



$(4, 4\sqrt{3})$ lies on $y^2 = 4ax$

$\Rightarrow 48 = 4a.4$

$4a = 12$

$\Rightarrow y^2 = 12x$ is equation of parabola

Now, parameter of P is $t_1 = \frac{2}{\sqrt{3}} \Rightarrow$ Parameters of Q is $t_2 = -\frac{\sqrt{3}}{2} \Rightarrow Q\left(\frac{9}{4}, -3\sqrt{3}\right)$

Area of trapezium PQNM

$= \frac{1}{2}MN.(PM + QN)$

$= \frac{1}{2}MN.(PS + QS)$

$= \frac{1}{2}MN.\,PQ$

$= \frac{1}{2}7\sqrt{3}.\frac{49}{4} = (343)\frac{\sqrt{3}}{8} = 3$

4. For a 3×3 matrix M, let trace (M) denote the sum of all the diagonal elements of M. Let A be a 3×3 matrix such that $|A| = \frac{1}{2}$ and trace (A) = 3. If B = adj(adj(2A)), then the value of |B| + trace (B) equals:

(1) 56 (2) 132

(3) 174 (4) 280

Ans. (4)

Sol. $|A| = \frac{1}{2}$, trace(A) = 3, B = adj(adj(2A)) = $|2A|^{n-2}(2A)$

$n = 3$, $B = |2A|(2A) = 2^3.|A|(2A) = 8A$

$|B| = |8A| = 8^3.|A| = 2^8 = 256$

trace(B) = 8 trace(A) = 24

|B| + trace(B) = 280

5. Suppose that the number of terms in an A.P. is 2k, $k \in N$. If the sum of all odd terms of the A.P. is 40, the sum of all even terms is 55 and the last term of the A.P. exceeds the first term by 27, then k is equal to

(1) 5 (2) 8

(3) 6 (4) 4

Ans. (1)

Sol. $a_1, a_2, a_3, \ldots\ldots, a_{2k}$ $\rightarrow$ A.P.

$\sum_{r=1}^{k} a_{2r-1} = 40$, $\sum_{r=1}^{k} a_{2r} = 55$, $a_{2k} - a_1 = 27$

$\frac{k}{2}[2a_1 + (k-1)2d] = 40$, $\frac{k}{2}[2a_2 + (k-1)2d] = 55$,

$d = \frac{27}{2k-1}$

$a_1 = \frac{40}{k} - (k-1)d = \frac{55}{k} - kd$

$d = \frac{15}{k} \Rightarrow \frac{27}{2k-1} = \frac{15}{k} \Rightarrow 9k = 10k - 5$

$\therefore k = 5$

6. Let a line pass through two distinct points P(–2, –1, 3) and Q, and be parallel to the vector $3\hat{i} + 2\hat{j} + 2k$. If the distance of the point Q from the point R(1, 3, 3) is 5, then the square of the area of ΔPQR is equal to:

(1) 136 (2) 140

(3) 144 (4) 148

Ans. (1)

Sol. $\overrightarrow{PQ}$ parallel to $3\hat{i} + 2\hat{j} + 2\hat{k}$, R(1, 3, 3)

$\Rightarrow Q(3\lambda - 2, 2\lambda - 1, 2\lambda + 3), \lambda \in R - \{0\}$

$\left|\overrightarrow{QR}\right| = 5 = \sqrt{(3\lambda-3)^2 + (2\lambda-4)^2 + (2\lambda)^2}$

$\therefore 17\lambda^2 - 34\lambda + 25 = 25 \Rightarrow \lambda = 2 (\because \lambda \neq 0)$

$\therefore$ Q(4, 3, 7), P(–2, –1, 3), R(1, 3, 3)

Area of $\Delta PQR = [PQR] = \frac{1}{2}\left|\overrightarrow{PQ} \times \overrightarrow{PR}\right|$

$$[PQR] = \frac{1}{2}\left\|\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 6 & 4 & 4 \\ 3 & 4 & 0 \end{vmatrix}\right\| = \left\|\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 2 & 2 \\ 3 & 4 & 0 \end{vmatrix}\right\|$$

$[PQR] = |-8\hat{i} + 6\hat{j} + 6\hat{k}| = \sqrt{136}$

$\therefore [PQR]^2 = 136$

7. If $\lim_{x\to\infty}\left(\left(\frac{e}{1-e}\right)\left(\frac{1}{e} - \frac{x}{1+x}\right)\right)^x = \alpha$, then the value of $\frac{\log_e \alpha}{1 + \log_e \alpha}$ equals :

(1) e (2) e^{-2}

(3) e^2 (4) e^{-1}

Ans. (1)

Sol. $\alpha = \lim_{x\to\infty}\left(\left(\frac{e}{1-e}\right)\left(\frac{1}{e} - \frac{x}{1+x}\right)\right)^x$ (1^∞ form)

$\therefore \alpha = e^L$

Where $L = \lim_{x\to\infty} x\left(\left(\frac{e}{1-e}\right)\left(\frac{1}{e} - \frac{x}{1+x}\right) - 1\right)$

$\Rightarrow L = \lim_{x\to\infty}\left(\frac{e}{1-e}\right)x\left(\frac{1}{e} - \frac{x}{1+x} - \left(\frac{1-e}{e}\right)\right)$

$\Rightarrow L = \frac{e}{1-e}\lim_{x\to\infty} x\left(1 - \frac{x}{1+x}\right)$

$\Rightarrow L = \frac{e}{1-e}\lim_{x\to\infty}\frac{x}{x+1}$

$\Rightarrow L = \frac{e}{1-e}.1$

$\Rightarrow L = \frac{e}{1-e}$

$\therefore \alpha = e^{\frac{e}{1-e}} \Rightarrow \log\alpha = \frac{e}{1-e}$

$\therefore$ Required value $= \dfrac{\frac{e}{1-e}}{1 + \frac{e}{1-e}} = e$

8. Let $f(x) = \int_0^{x^2} \frac{t^2-8t+15}{e^t}dt$, $x \in \mathbf{R}$. Then the numbers of local maximum and local minimum points of f, respectively, are :

(1) 2 and 3 (2) 3 and 2

(3) 1 and 3 (4) 2 and 2

Ans. (1)

Sol. $f'(x) = \left(\frac{x^4-8x^2+15}{e^{x^2}}\right)(2x)$

$= \frac{(x^2-3)(x^2-5)(2x)}{e^{x^2}}$

$= \frac{(x-\sqrt{3})(x+\sqrt{3})(x-\sqrt{5})(x+\sqrt{5})2x}{e^{x^2}}$

$- \quad + \quad - \quad + \quad - \quad +$

$-\sqrt{5} \quad -\sqrt{3} \quad 0 \quad \sqrt{3} \quad \sqrt{5}$

Maxima at $x \in \{-\sqrt{3}, \sqrt{3}\}$

Minima at $x \in \{-\sqrt{5}, 0, \sqrt{5}\}$

2 points of maxima and 3 points of minima.

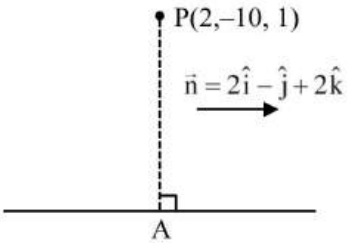
9. The perpendicular distance, of the line $\frac{x-1}{2} = \frac{y+2}{-1} = \frac{z+3}{2}$ from the point P(2, –10,1), is:

(1) 6 (2) $5\sqrt{2}$

(3) $3\sqrt{5}$ (4) $4\sqrt{3}$

Ans. (3)

Sol.



$\frac{x-1}{2} = \frac{y+2}{-1} = \frac{z+3}{2} = \lambda$ (let)

$(2\lambda + 1, -\lambda - 2, 2\lambda - 3)$

$\because \overrightarrow{PA}.\vec{n} = 0$

$\Rightarrow (2\lambda - 1)2 + (-\lambda + 8)(-1) + (2\lambda - 4)2 = 0$

$\Rightarrow 4\lambda - 2 + \lambda - 8 + 4\lambda - 8 = 0$

$\Rightarrow 9\lambda - 18 = 0 \Rightarrow \lambda = 2$

$\therefore A(5, -4, 1)$

$\therefore AP = \sqrt{3^2+6^2+0^2} = \sqrt{45} = 3\sqrt{5}$

10. If x = f(y) is the solution of the differential equation

$(1 + y^2) + \left(x - 2e^{\tan^{-1}y}\right)\frac{dy}{dx} = 0, y \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

with f(0) = 1, then $f\left(\frac{1}{\sqrt{3}}\right)$ is equal to :

(1) $e^{\pi/4}$ (2) $e^{\pi/12}$

(3) $e^{\pi/3}$ (4) $e^{\pi/6}$

Ans. (4)

Sol. $\frac{dx}{dy} + \frac{x}{1+y^2} = \frac{2e^{\tan^{-1}y}}{1+y^2}$

I.F. = $e^{\tan^{-1}y}$

$xe^{\tan^{-1}y} = \int \frac{2(e^{\tan^{-1}y})^2 dy}{1+y^2}$

Put $\tan^{-1}y = t$, $\frac{dy}{1+y^2} = dt$

$xe^{\tan^{-1}y} = \int 2e^{2t}dt$

$xe^{\tan^{-1}y} = e^{2\tan^{-1}y} + c$

$x = e^{\tan^{-1}y} + ce^{-\tan^{-1}y}$

$\because y = 0, x = 1$

$1 = 1 + c \Rightarrow c = 0$

$y = \frac{1}{\sqrt{3}}$, $x = e^{\pi/6}$

11. If $\int e^x \left(\frac{x\sin^{-1}x}{\sqrt{1-x^2}} + \frac{\sin^{-1}x}{(1-x^2)^{3/2}} + \frac{x}{1-x^2}\right)dx = g(x) + C$,

where C is the constant of integration, then $g\left(\frac{1}{2}\right)$ equals :

(1) $\frac{\pi}{6}\sqrt{\frac{e}{2}}$ (2) $\frac{\pi}{4}\sqrt{\frac{e}{2}}$

(3) $\frac{\pi}{6}\sqrt{\frac{e}{3}}$ (4) $\frac{\pi}{4}\sqrt{\frac{e}{3}}$

Ans. (3)

Sol. $\because \frac{d}{dx}\left(\frac{x\sin^{-1}x}{\sqrt{1-x^2}}\right) = \frac{\sin^{-1}x}{(1-x^2)^{3/2}} + \frac{x}{1-x^2}$

$\Rightarrow \int e^x \left(\frac{x\sin^{-1}x}{\sqrt{1-x^2}} + \frac{\sin^{-1}x}{(1-x^2)^{3/2}} + \frac{x}{1-x^2}\right)dx$

$= e^x.\frac{x\sin^{-1}x}{\sqrt{1-x^2}} + c = g(x) + C$

Note : assuming $g(x) = \frac{xe^x\sin^{-1}x}{\sqrt{1-x^2}}$

$g(1/2) = \frac{e^{1/2}}{2}.\frac{\frac{\pi}{6}\times 2}{\sqrt{3}} = \frac{\pi}{6}\sqrt{\frac{e}{3}}$

Comment : In this question we will not get a unique function g(x), but in order to match the answer we will have to assume $g(x) = \frac{xe^x \sin^{-1} x}{\sqrt{1-x^2}}$.

12. Let α_θ and β_θ be the distinct roots of $2x^2 + (\cos\theta)x - 1 = 0$, $\theta \in (0, 2\pi)$. If m and M are the minimum and the maximum values of $\alpha_\theta^4 + \beta_\theta^4$, then 16(M + m) equals :

(1) 24 (2) 25

(3) 27 (4) 17

Ans. (2)

Sol. $(\alpha^2 + \beta^2)^2 - 2\alpha^2\beta^2$

$[(\alpha + \beta)^2 - 2\alpha\beta]^2 - 2(\alpha\beta)^2$

$\left[\frac{\cos^2\theta}{4} + 1\right]^2 - 2.\frac{1}{4}$

$\left(\frac{\cos^2\theta}{4} + 1\right)^2 - \frac{1}{2}$

$M = \frac{25}{16} - \frac{1}{2} = \frac{17}{16}$

$m = \frac{1}{2}$, $16(M + m) = 25$

13. Let A = {1, 2, 3, 4} and B = {1, 4, 9, 16}. Then the number of many-one functions $f : A \to B$ such that $1 \in f(A)$ is equal to :

(1) 127 (2) 151

(3) 163 (4) 139

Ans. (2)

Sol. Total = 4^4

One-one = 4!

Many-one = 256 – 24 = 232

Many-one which $1 \notin f(A)$

= 3.3.3.3 = 81

232 – 81 = 151

14. If the system of linear equations :

$x + y + 2z = 6$,

$2x + 3y + az = a + 1$,

$-x - 3y + bz = 2b$,

where $a, b \in \mathbf{R}$, has infinitely many solutions, then 7a + 3b is equal to :

(1) 9 (2) 12

(3) 16 (4) 22

Ans. (3)

Sol. $\Delta = \begin{vmatrix} 1 & 1 & 2 \\ 2 & 3 & a \\ -1 & -3 & b \end{vmatrix} = 0$

$\Rightarrow 2a + b - 6 = 0$ .....(1)

$\Delta_1 = \begin{vmatrix} 1 & 1 & 6 \\ 2 & 3 & a+1 \\ -1 & -3 & 2b \end{vmatrix} = 0$

$\Rightarrow a + b - 8 = 0$ .....(2)

Solving (1) + (2)

$a = -2, b = 10$

$\Rightarrow 7a + 3b = 16$

15. Let $\vec{a}$ and $\vec{b}$ be two unit vectors such that the angle between them is $\frac{\pi}{3}$. If $\lambda\vec{a} + 2\vec{b}$ and $3\vec{a} - \lambda\vec{b}$ are perpendicular to each other, then the number of values of λ in $[-1, 3]$ is :

(1) 3 (2) 2

(3) 1 (4) 0

Ans. (4)

Sol. $\hat{a}.\hat{b} = \frac{1}{2}$

Now $\left(\lambda\hat{a} + 2\hat{b}\right).\left(3\hat{a} - \lambda\hat{b}\right) = 0$

$3\lambda\hat{a}.\hat{a} - \lambda^2\hat{a}.\hat{b} + 6\hat{a}.\hat{b} - 2\lambda\hat{b}.\hat{b} = 0$

$3\lambda - \frac{\lambda^2}{2} + 3 - 2\lambda = 0$

$\lambda^2 - 2\lambda - 6 = 0$

$\lambda = 1 \pm \sqrt{7}$

$\Rightarrow$ number of values = 0

16. Let E : $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, a > b and H : $\frac{x^2}{A^2}-\frac{y^2}{B^2}=1$. Let the distance between the foci of E and the foci of H be $2\sqrt{3}$. If a – A = 2, and the ratio of the eccentricities of E and H is $\frac{1}{3}$, then the sum of the lengths of their latus rectums is equal to :

(1) 10 (2) 7

(3) 8 (4) 9

Ans. (3)

Sol. $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ foci are (ae, 0) and (–ae, 0)

$\frac{x^2}{A^2}+\frac{y^2}{B^2}=1$ foci are (Ae′, 0) and (–Ae′, 0)

$\Rightarrow 2ae = 2\sqrt{3} \Rightarrow ae = \sqrt{3}$

and $2Ae' = 2\sqrt{3} \Rightarrow Ae' = \sqrt{3}$

$\Rightarrow ae = Ae' \Rightarrow \frac{e}{e'}=\frac{A}{a}$

$\Rightarrow \frac{1}{3}=\frac{A}{a} \quad \Rightarrow a = 3A$

Now $a - A = 2 \Rightarrow a - \frac{a}{3} - 2 \Rightarrow a = 3$ and $A = 1$

$Ae = \sqrt{3} \Rightarrow e = \frac{1}{\sqrt{3}}$ and $e' = \sqrt{3}$

$b^2 = a^2(1 - e^2)$

$b^2 = 6$

and $B^2 = A^2((e')^2 - 1) = (2) \Rightarrow B^2 = 2$

sum of LR $= \frac{2b^2}{a}+\frac{2B^2}{A} = 8$

17. If A and B are two events such that $P(A\cap B) = 0.1$, and $P(A|B)$ and $P(B|A)$ are the roots of the equation $12x^2 - 7x + 1 = 0$, then the value of $\frac{P(\bar{A}\cup\bar{B})}{P(\bar{A}\cap\bar{B})}$ is:

(1) $\frac{5}{3}$ (2) $\frac{4}{3}$

(3) $\frac{9}{4}$ (4) $\frac{7}{4}$

Ans. (3)

Sol. $12x^2 - 7x + 1 = 0$

$x = \frac{1}{3}, \frac{1}{4}$

Let $P\left(\frac{A}{B}\right)=\frac{1}{3}$ & $P\left(\frac{B}{A}\right)=\frac{1}{4}$

$\frac{P(A\cap B)}{P(B)}=\frac{1}{3}$ & $\frac{P(A\cap B)}{P(A)}=\frac{1}{4}$

$\Rightarrow P(B) = 0.3$

& $P(A) = 0.4$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.3 + 0.4 - 0.1 = 0.6$

Now $\frac{P(\bar{A}\cup\bar{B})}{P(\bar{A}\cap\bar{B})} = \frac{P(\overline{A\cap B})}{P(\overline{A\cup B})}$

$= \frac{1-P(A\cap B)}{1-P(A\cup B)} = \frac{1-0.1}{1-0.6} = \frac{9}{4}$

18. The sum of all values of $\theta \in [0, 2\pi]$ satisfying $2\sin^2\theta = \cos2\theta$ and $2\cos^2\theta = 3\sin\theta$ is

(1) $\frac{\pi}{2}$ (2) 4π

(3) $\frac{5\pi}{6}$ (4) π

Ans. (4)

Sol. $2\sin^2\theta = \cos2\theta$

$2\sin^2\theta = 1 - 2\sin^2\theta$

$4\sin^2\theta = 1$

$\sin^2\theta = \frac{1}{4}$

$\sin\theta = \pm\frac{1}{2}$

$2\cos^2\theta = 3\sin\theta$

$2 - 2\sin^2\theta + 3\sin\theta - 2 = 0$

$(2\sin\theta - 1)(2\sin\theta - 2) = 0$

$\sin\theta = \frac{1}{2}$

so common equation which satisfy both equations is $\sin\theta = \frac{1}{2}$

$\theta = \frac{\pi}{6}, \frac{5\pi}{6}$ $(\theta \in [0, 2\pi])$

Sum = π

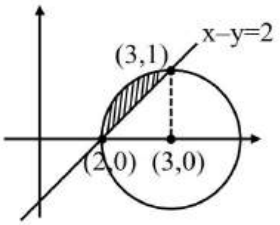
19. Let the curve $z(1 + i) + \bar{z}(1-i) = 4$, $z \in C$, divide the region $|z - 3| \le 1$ into two parts of areas α and β. Then $|\alpha - \beta|$ equals :

(1) $1+\frac{\pi}{2}$ (2) $1+\frac{\pi}{3}$

(3) $1+\frac{\pi}{4}$ (4) $1+\frac{\pi}{6}$

Ans. (1)

Sol.



Let $z = x + iy$

$(x + iy)(1 + i) + (x - iy)(1 - i) = 4$

$x + ix + iy - y + x - ix - iy - y = 4$

$2x - 2y = 4$

$x - y = 2$

$|z - 3| \le 1$

$(x - 3)^2 + y^2 \le 1$

Area of shaded region $= \frac{\pi.1^2}{4} - \frac{1}{2}.1.1 = \frac{\pi}{4} - \frac{1}{2}$

Area of unshaded region inside the circle

$= \frac{3}{4}\pi.1^2 + \frac{1}{2}.1.1 = \frac{3\pi}{4} + \frac{1}{2}$

$\therefore$ difference of area $= \left(\frac{3\pi}{4} + \frac{1}{2}\right) - \left(\frac{\pi}{4} - \frac{1}{2}\right)$

$= \frac{\pi}{2} + 1$

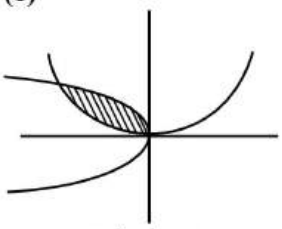
20. The area of the region enclosed by the curves $y = x^2 - 4x + 4$ and $y^2 = 16 - 8x$ is :

(1) $\frac{8}{3}$ (2) $\frac{4}{3}$

(3) 5 (4) 8

Ans. (1)

Sol.



$y = (x - 2)^2$, $y^2 = 8(x - 2)$

$y = x^2$, $y^2 = -8x$

$= \frac{16ab}{3} = \frac{16 \times \frac{1}{4} \times 2}{3} = \frac{8}{3}$

SECTION-B

21. Let $y = f(x)$ be the solution of the differential equation $\frac{dy}{dx} + \frac{xy}{x^2 - 1} = \frac{x^6 + 4x}{\sqrt{1 - x^2}}$, $-1 < x < 1$ such that $f(0) = 0$. If $6\int_{-1/2}^{1/2} f(x)dx = 2\pi - \alpha$ then α^2 is equal to _____.

Ans. (27)

Sol. I.F. $e^{-\frac{1}{2}\int \frac{2x}{1-x^2}dx} = e^{-\frac{1}{2}\ell n(1-x^2)} = \sqrt{1 - x^2}$

$y \times \sqrt{1 - x^2} = \int (x^6 + 4x)dx = \frac{x^7}{7} + 2x^2 + c$

Given $y(0) = 0 \Rightarrow c = 0$

$y = \frac{\frac{x^7}{7} + 2x^2}{\sqrt{1 - x^2}}$

Now, $6\int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{\frac{x^7}{7} + 2x^2}{\sqrt{1 - x^2}}dx = 6\int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{2x^2}{\sqrt{1 - x^2}}dx$

$= 24\int_0^{\frac{1}{2}} \frac{x^2}{\sqrt{1 - x^2}}dx$

Put $x = \sin\theta$

$dx = \cos\theta\, d\theta$

$= 24\int_0^{\frac{\pi}{6}} \frac{\sin^2\theta}{\cos\theta}\cos\theta d\theta$

$= 24\int_0^{\frac{\pi}{6}} \left(\frac{1 - \cos 2\theta}{2}\right)d\theta = 12\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{\frac{\pi}{6}}$

$= 12\left(\frac{\pi}{6} - \frac{\sqrt{3}}{4}\right)$

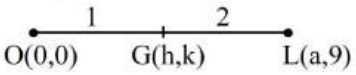
$= 2\pi - 3\sqrt{3}$

$\alpha^2 = (3\sqrt{3})^2 = 27$

22. Let A(6, 8), B(10 cosα, –10 sinα) and C (–10 sinα, 10 cosα), be the vertices of a triangle. If L(a, 9) and G(h, k) be its orthocenter and centroid respectively, then $(5a - 3h + 6k + 100 \sin 2\alpha)$ is equal to_____

Ans. (145)

Sol. All the three points A, B, C lie on the circle $x^2 + y^2 = 100$ so circumcentre is (0, 0)



$\frac{a+0}{3} = h \Rightarrow a = 3h$

and $\frac{9+0}{3} = k \Rightarrow k = 3$

also centroid $\frac{6+10\cos\alpha - 10\sin\alpha}{3} = h$

$\Rightarrow 10(\cos\alpha - \sin\alpha) = 3h - 6$ ....(i)

and $\frac{8+10\cos\alpha - 10\sin\alpha}{3} = k$

$\Rightarrow 10(\cos\alpha - \sin\alpha) = 3k - 8 = 9 - 8 = 1$....(ii)

on squaring $100(1 - \sin 2\alpha) = 1$

$\Rightarrow 100\sin 2\alpha = 99$

from equ. (i) and (ii) we get $h = \frac{7}{3}$

Now $5a - 3h + 6k + 100 \sin 2\alpha$

$= 15h - 3h + 6k + 100 \sin 2\alpha$

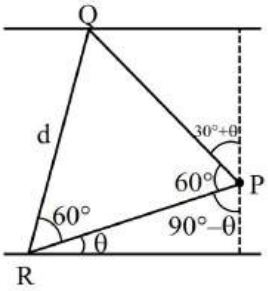
$= 12 \times \frac{7}{3} + 18 + 99$

$= 145$

23. Let the distance between two parallel lines be 5 units and a point P lie between the lines at a unit distance from one of them. An equilateral triangle PQR is formed such that Q lies on one of the parallel lines, while R lies on the other. Then $(QR)^2$ is equal to _____.

Ans. **(28)**

Sol.



$PR = \operatorname{cosec}\theta$, $PQ = 4\sec(30 + \theta)$

For equilateral

$d = PR = PQ$

$\Rightarrow \cos(\theta + 30°) = 4\sin\theta$

$\Rightarrow \frac{\sqrt{3}}{2}\cos\theta - \frac{1}{2}\sin\theta = 4\sin\theta$

$\Rightarrow \tan\theta = \frac{1}{3\sqrt{3}}$

$QR^2 = d^2 = \operatorname{cosec}^2\theta = 28$

24. If $\sum_{r=1}^{30} \frac{r^2\left({}^{30}C_r\right)^2}{{}^{30}C_{r-1}} = \alpha \times 2^{29}$, then α is equal to _____.

Ans. **(465)**

Sol. $\sum_{r=1}^{30} \frac{r^2({}^{30}C_r)^2}{{}^{30}C_{r-1}}$

$= \sum_{r=1}^{30} r^2\left(\frac{31-r}{r}\right).\frac{30!}{r!(30-r)!}$

$\left(\because \frac{{}^{30}C_r}{{}^{30}C_{r-1}} = \frac{30-r+1}{r} = \frac{31-r}{r}\right)$

$= \sum_{r=1}^{30} \frac{(31-r)30!}{(r-1)!(30-r)!}$

$= 30\sum_{r=1}^{30} \frac{(31-r)29!}{(r-1)!(30-r)!}$

$= 30\sum_{r=1}^{30} (30-r+1)\,{}^{29}C_{30-r}$

$= 30\left(\sum_{r=1}^{30}(31-r)\,{}^{29}C_{30-r} + \sum_{r=1}^{30} {}^{29}C_{30-r}\right)$

$= 30(29 \times 2^{28} + 2^{29}) = 30(29 + 2)2^{28}$

$= 15 \times 31 \times 2^{29}$

$= 465(2^{29})$

$\alpha = 465$

25. Let A = {1, 2, 3}. The number of relations on A, containing (1, 2) and (2, 3), which are reflexive and transitive but not symmetric, is _____.

Ans. **(3)**

Sol. Transitivity

$(1, 2) \in R, (2, 3) \in R \Rightarrow (1, 3) \in R$

For reflexive $(1, 1), (2, 2)\ (3, 3) \in R$

Now (2, 1), (3, 2), (3, 1)

(3, 1) cannot be taken

(1) (2, 1) taken and (3, 2) not taken

(2) (3, 2) taken and (2, 1) not taken

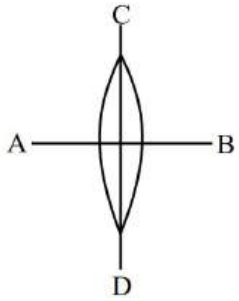
(3) Both not taken

therefore 3 relations are possible.

PHYSICS

SECTION-A

26. A symmetric thin biconvex lens is cut into four equal parts by two planes AB and CD as shown in figure. If the power of original lens is 4D then the power of a part of the divided lens is

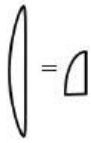


(1) 8D (2) 4D

(3) D (4) 2D

Ans. (4)

Sol. $\frac{1}{f_1} = (\mu - 1)\frac{2}{R} = P = 4D$

 $\frac{1}{f_2} = (\mu - 1)\frac{1}{R} = \frac{P}{2} = 2D$

27. A small rigid spherical ball of mass M is dropped in a long vertical tube containing glycerine. The velocity of the ball becomes constant after some time. If the density of glycerine is half of the density of the ball, then the viscous force acting on the ball will be

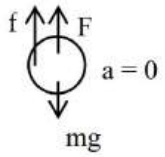
(consider g as acceleration due to gravity)

(1) $\frac{3}{2}Mg$ (2) $\frac{Mg}{2}$

(3) Mg (4) 2 Mg

Ans. (2)

Sol.



$mg - F_B - f = 0$

$\Rightarrow mg - \frac{mg}{2} - f = 0$

$\therefore f = \frac{mg}{2}$

28. The maximum percentage error in the measurment of density of a wire is

[Given, mass of wire = (0.60 ± 0.003)g

radius of wire = (0.50 ± 0.01)cm

length of wire (10.00 ± 0.05)cm]

(1) 4 (2) 5

(3) 8 (4) 7

Ans. (2)

Sol. $d = \frac{m}{vol.} = \frac{m}{\pi R^2 \ell} \Rightarrow \frac{d\rho}{\rho} = \frac{dm}{m} + \frac{2dR}{R} + \frac{d\ell}{\ell}$

$\Rightarrow \frac{d\rho}{\rho} = \left(\frac{0.003}{0.6} + \frac{2 \times 0.01}{0.5} + \frac{0.05}{10}\right)100 = 5\%$

29. A series LCR circuit is connected to an alternating source of emf E. The current amplitude at resonant frequency is I_0. If the value of resistance R becomes twice of its initial value then amplitude of current at resonance will be

(1) I_0 (2) $\frac{I_0}{2}$

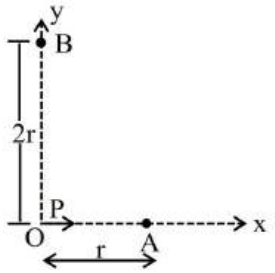
(3) $\frac{I_0}{\sqrt{2}}$ (4) $2I_0$

Ans. (2)

Sol. Initially, $I_0 = \frac{\varepsilon_m}{R}$

Finally, $I_0^1 = \frac{\varepsilon_m}{2R} = \frac{I_0}{2}$

30. For a short dipole placed at origin O, the dipole moment P is along x-axis, as shown in the figure. If the electric potential and electric field at A are V_0 and E_0, respectively, then the correct combination of the electric potential and electric field, respectively, at point B on the y-axis is given by



(1) $\frac{V_0}{2}$ and $\frac{E_0}{16}$ (2) zero and $\frac{E_0}{8}$

(3) zero and $\frac{E_0}{16}$ (4) V_0 and $\frac{E_0}{4}$

Ans. (3)

Sol. $E_A = \frac{2kP}{r^3} = E_0$ & $V_A = \frac{kP}{r^2} = v_0$

$E_B = \frac{kP}{(2r)^3} = \frac{E_0}{16}$ & $V_B = \frac{k\vec{P}.\hat{r}}{r^2} = 0$

31. Which one of the following is the correct dimensional formula for the capacitance in F ? M, L, T and C stand for unit of mass, length, time and charge,

(1) $[F] = [C^2M^{-2}L^2T^2]$

(2) $[F] = [CM^{-2}L^{-2}T^{-2}]$

(3) $[F] = [CM^{-1}L^{-2}T^2]$

(4) $[F] = [C^2M^{-1}L^{-2}T^2]$

Ans. (4)

Sol. $C = \frac{q}{V} = \frac{q.q}{V.q} = \frac{q^2}{WD} = \frac{C^2}{ML^2T^{-2}} = C^2M^{-1}L^{-2}T^2$

32. An electron projected perpendicular to a uniform magnetic field B moves in a circle. If Bohr's quantization is applicable, then the radius of the electronic orbit in the first excited state is :

(1) $\sqrt{\frac{2h}{\pi eB}}$ (2) $\sqrt{\frac{4h}{\pi eB}}$

(3) $\sqrt{\frac{h}{2\pi eB}}$ (4) $\sqrt{\frac{h}{\pi eB}}$

Ans. (4)

Sol. $r = \frac{mv}{eB}$ & $mvr = \frac{nh}{2\pi} \Rightarrow (eBr)r = \frac{nh}{2\pi}$

$\Rightarrow r = \sqrt{\frac{nh}{2\pi eB}}$

first excited state : $n = 2 \therefore r = \sqrt{\frac{h}{\pi eB}}$

33. For a diatomic gas, if $\gamma_1 = \left(\frac{Cp}{Cv}\right)$ for rigid molecules and $\gamma_2 = \left(\frac{Cp}{Cv}\right)$ for another diatomic molecules, but also having vibrational modes. Then, which one of the following options is correct ?

(Cp and Cv are specific heats of the gas at constant pressure and volume)

(1) $\gamma_2 > \gamma_1$ (2) $\gamma_2 = \gamma_1$

(3) $2\gamma_2 = \gamma_1$ (4) $\gamma_2 < \gamma_1$

Ans. (4)

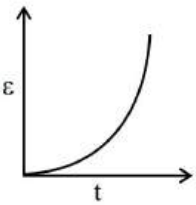
Sol. $\gamma = \frac{2}{f} + 1$

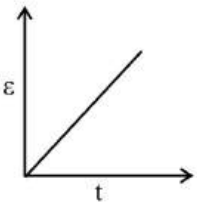
without vibration : $f = 5 : \gamma_1 = 1.4$

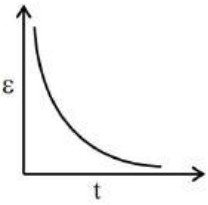
without vibration : $f = 7 : \gamma_2 = 1.14$

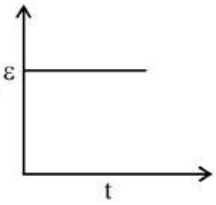
$\therefore \gamma_2 < \gamma_1$

34. A rectangular metallic loop is moving out of a uniform magnetic field region to a field free region with a constant speed. When the loop is partially inside the magnate field, the plot of magnitude of induced emf (ε) with time (t) is given by

(1)


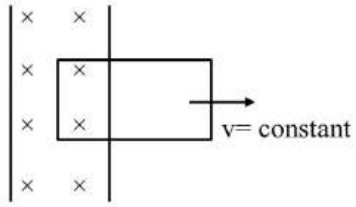
(2)


(3)


(4)


Ans. (4)

Sol.



B = constant

Motional emf : $\varepsilon = B\ell v$ = constant

35. A light source of wavelength λ illuminates a metal surface and electrons are ejected with maximum kinetic energy of 2 eV. If the same surface is illuminated by a light source of wavelength $\frac{\lambda}{2}$, then the maximum kinetic energy of ejected electrons will be (The work function of metal is 1 eV)

(1) 2 eV (2) 6 eV

(3) 5 eV (4) 3 eV

Ans. (3)

Sol. $\frac{hc}{\lambda} = \phi + eV \Rightarrow \frac{hc}{\lambda} = 1 + 2 = 3eV$ .....(1)

$\frac{hc}{\lambda/2} = 6 = 1 + k_{max} \therefore k_{max} = 5eV$

36. Given below are two statements. One is labelled as **Assertion (A)** and the other is labelled as **Reason (R).**

Assertion (A) : A simple pendulum is taken to a planet of mass and radius, 4 times and 2 times, respectively, than the Earth. The time period of the pendulum remains same on earth and the planet.

Reason (R) : The mass of the pendulum remains unchanged at Earth and the other planet. In the light of the above statements, choose the **correct** answer from the options given below :

(1) Both **(A)** and **(R)** are true but **(R)** is **NOT** the correct explanation of **(A)**

(2) **(A)** is true but **(R)** is false

(3) **(A)** is false but **(R)** is true

(4) Both **(A)** and **(R)** are true and **(R)** is the correct explanation of **(A)**

Ans. (1)

Sol. $g = \frac{GM}{R^2}$

$g' = \frac{G(4M)}{(2R)^2} = g$

A is correct, R is correct ; but since $T = 2\pi\sqrt{\frac{\ell}{g}}$ doesn't depend on mass ; R doesn't explain A.

37. The torque due to the force $(2\hat{i} + \hat{j} + 2\hat{k})$ about the origin, acting on a particle whose position vector is $(\hat{i} + \hat{j} + \hat{k})$, would be

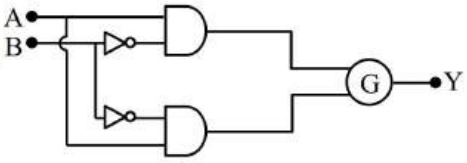
(1) $\hat{i} - \hat{j} + \hat{k}$ (2) $\hat{i} + \hat{k}$

(3) $\hat{i} - \hat{k}$ (4) $\hat{j} - \hat{k}$

Ans. (3)

Sol. $\vec{\tau} = \vec{r} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 2 & 1 & 2 \end{vmatrix} = \hat{i} - 0\hat{j} - \hat{k}$

38.



A	B	Y
0	0	1
0	1	0
1	0	0
1	1	1

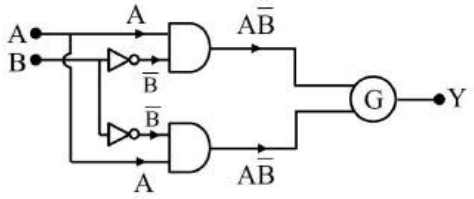
To obtain the given truth table, following logic gate should be placed at G :

(1) NOR Gate (2) AND Gate

(3) NAND Gate (4) OR Gate

NTA Ans. (1)

Sol.



For NOR gate : $\overline{A\bar{B}} = \bar{A} + B$

	A	B	Y
	0	0	1
∴ Truth table	0	1	1
	1	0	0
	1	1	1

∴ Bonus

39. A force $\vec{F} = 2\hat{i} + b\hat{j} + \hat{k}$ is applied on a particle and it undergoes a displacement $\hat{i} - 2\hat{j} - \hat{k}$. What will be the value of b, if work done on the particle is zero.

(1) 0 (2) $\frac{1}{2}$

(3) $\frac{1}{3}$ (4) 2

Ans. (2)

Sol. $WD = \vec{F}.\vec{S} = 2 - 2b - 1 = 0$

$\therefore b = \frac{1}{2}$

40. Given below are two statements. On is labelled as **Assertion (A)** and the other is labelled as **Reason (R).**

Assertion (A) : In Young's double slit experiment, the fringes produced by red light are closer as compared to those produced by blue light.

Reason (R) : The fringe width is directly proportional to the wavelength of light.

In the light of above statements, choose the **correct** answer from the options given below :

(1) Both **(A)** and **(R)** are true and **(R)** is the correct explanation of **(A)**

(2) **(A)** is false but **(R)** is true.

(3) Both **(A)** and **(R)** are true but **(R)** is **NOT** the correct explanation of **(A)**.

(4) **(A)** is true but **(R)** is false.

Ans. (2)

Sol. $\beta = \frac{\lambda D}{d}$ & $\lambda_R > \lambda_b$

$\therefore \beta_R > \beta_b$

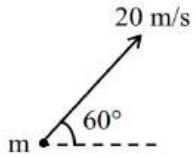
41. A ball of mass 100 g is projected with velocity 20 m/s at 60° with horizontal. The decrease in kinetic energy of the ball during the motion from point of projection to highest point is :

(1) 20 J (2) 15 J

(3) zero (4) 5 J

Ans. (2)

Sol.



$k_i = \frac{1}{2}mv^2$

$k_f = \frac{1}{2}m(v\cos 60°)^2 = \frac{1}{8}mv^2$

$\Delta k = k_i - k_f = \frac{3}{8}mv^2 = \frac{3}{8} \times 0.1 \times 400 = 15J$

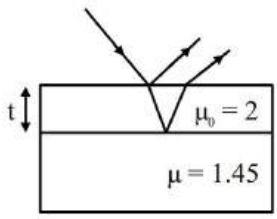
42. A transparent film of refractive index, 2.0 is coated on a glass slab of refractive index, 1.45. What is the minimum thickness of transparent film to be coated for the maximum transmission of Green light of wavelength 550 nm. [Assume that the light is incident nearly perpendicular to the glass surface.]

(1) 94.8 nm (2) 68.7 nm

(3) 137.5 nm (4) 275 nm

Ans. (3)

Sol.

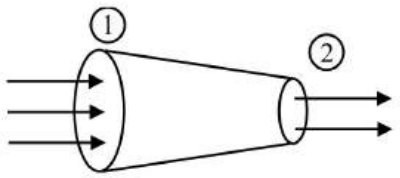


For transmitted green light to be maxima, reflected green should be minima.

$\Delta P = 2\mu_0 t = n\lambda$

$\Rightarrow t = \frac{n\lambda}{2\mu_0} \therefore t_{min} = \frac{\lambda}{2\mu_0} = \frac{550}{2 \times 2} = 137.5$

43.



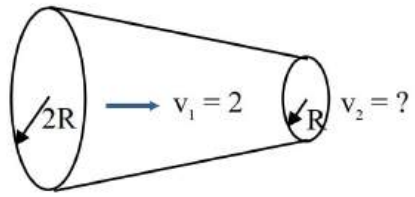
The tube of length L is shown in the figure. The radius of cross section at the point (1) is 2 cm and at the point (2) is 1 cm, respectively. If the velocity of water entering at point (1) is 2 m/s, then velocity of water leaving the point (2) will be :

(1) 2 m/s (2) 4 m/s

(3) 6 m/s (4) 8 m/s

Ans. (4)

Sol.



$$A_1v_1 = A_2V_2 \Rightarrow 2\pi(2R)^2 = V_2\pi R^2$$

$\therefore V_2 = 8$ m/s

44. Given are statements for certain thermodynamic variables,

(A) Internal energy, volume (V) and mass (M) are extensive variables.

(B) Pressure (P), temperature (T) and density (ρ) are intensive variables.

(C) Volume (V), temperature (T) and density (ρ) are intensive variables.

(D) Mass (M), temperature (T) and internal energy are extensive variables.

Choose the **correct** answer from the points given below :

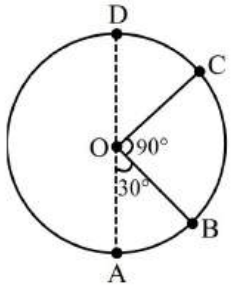
(1) (C) and (D) only (2) (D) and (A) only

(3) (A) and (B) only (4) (B) and (C) only

Ans. (3)

Sol. Extensive variables depends on size or mass of system ex : internal energy, volume, mass

45. A body of mass 100 g is moving in circular path of radius 2 m on vertical plane as shown in figure. The velocity of the body at point A is 10 m/s. The ratio of its kinetic energies at point B and C is :



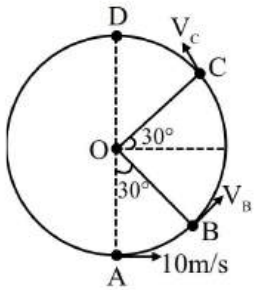
(Take acceleration due to gravity as 10 m/s^2)

(1) $\frac{2+\sqrt{3}}{3}$ (2) $\frac{2+\sqrt{2}}{3}$

(3) $\frac{3+\sqrt{3}}{2}$ (4) $\frac{3-\sqrt{2}}{2}$

Ans. (3)

Sol.



$$\frac{1}{2}m \times 100 + 0 = \frac{1}{2}mV_B^2 + mg\left(R - \frac{R\sqrt{3}}{2}\right)$$

$$100 = V_B^2 + 2gR\left(1 - \frac{\sqrt{3}}{2}\right]$$

$$V_B^2 = 100 - 20(2-\sqrt{3})$$

$$V_B^2 = 60 + 20\sqrt{3})$$

$$K.E_B = \frac{1}{2}mV_B^2 = \frac{m}{2}(60+20\sqrt{3})$$

$$\frac{1}{2}m(100) = \frac{1}{2}mV_C^2 + mg\left(\frac{3R}{2}\right)$$

$$100 = V_C^2 = 60$$

$$V_C^2 = 40$$

$$K.E_C = \frac{1}{2}mV_C^2 = \frac{1}{2}m(40)$$

$$K.E_B = \frac{60+20\sqrt{3}}{40} = \frac{3}{2} + \frac{\sqrt{3}}{2} = \frac{3+\sqrt{3}}{2}$$

SECTION-B

46. A proton is moving undeflected in a region of crossed electric and magnetic fields at a constant speed of 2×10^5 ms^{-1}. When the electric field is switched off, the proton moves along a circular path of radius 2 cm. The magnitude of electric field is $x \times 10^4$ N/C. the value of x is _____.
Take the mass of the proton = 1.6×10^{-27} kg.

Ans. (2)

Sol. For uniform speed $V = \frac{E}{B}$

$$R = \frac{mV}{eB}$$

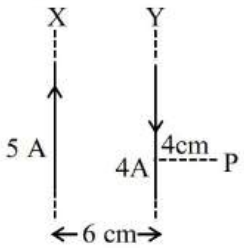
$$= \frac{mV^2}{eE}$$

$$\Rightarrow E = \frac{mV^2}{eR}$$

$$= \frac{1.6\times10^{-27}\times4\times10^{10}}{1.6\times10^{-19}\times2\times10^{-2}}$$

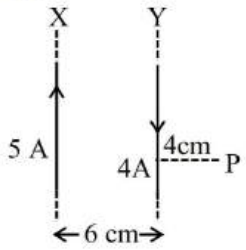
$= 2 \times 10^4$ N/C.

47. Two long parallel wires X and Y, separated by a distance of 6 cm, carry currents of 5A and 4A, respectively, in opposite directions as shown in the figure. Magnitude of the resultant magnetic field at point P at a distance of 4 cm from wire Y is $x \times 10^{-5}$T. The value of x is _____. Take permeability of free space as $\mu_0 = 4\pi \times 10^{-7}$ SI units.



Ans. (1)

Sol.



$$B = \frac{\mu_0(5)}{2\pi\times.01} - \frac{\mu_0 4}{2\pi\times0.04}$$

$$= -\frac{100\mu_0}{4\pi}$$

$= -100 \times 10^{-7}$

$= -1 \times 10^{-5}$ T

48. A parallel plate capacitor of area A = 16 cm^2 and separation between the plates 10 cm, is charged by a DC current. Consider a hypothetical plane surface of area A_0 = 3.2 cm^2 inside the capacitor and parallel to the plates. At an instant, the current through the circuit is 6A. At the same instant the displacement current through A_0 is _____ mA.

Ans. (1200)

Sol. $J_d = \frac{I}{A} = \frac{6}{16}$

$\therefore$ I through small area = $J_d \times A' = \frac{6}{16} \times 3.2 =$ 1.2A =1200 mA

49. A tube of length 1m is filled completely with an ideal liquid of mass 2M, and closed at both ends. The tube is rotated uniformly in horizontal plane about one of its ends. If the force exerted by the liquid at the other end is F then angular velocity of the tube is $\sqrt{\frac{F}{\alpha M}}$ in SI unit. The value of α is _____.

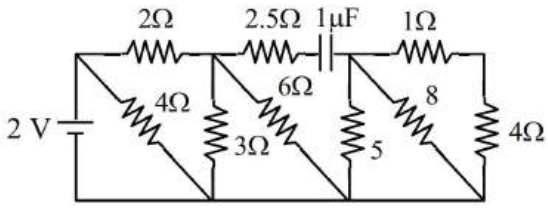
Ans. (1)

Sol.

$$F = 2M\omega^2 \frac{\ell}{2} = Mw^2\ell$$

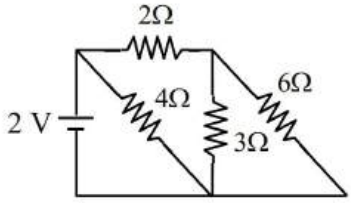
$$\omega = \sqrt{\frac{F}{M\ell}}$$

50. The net current flowing in the given circuit is _____ A.



Ans. (1)

Sol.



$R_{eq} = 2\Omega$

$I = \frac{2}{2} = 1A$

CHEMISTRY

SECTION-A

51. Arrange the following compounds in increasing order of their dipole moment :

HBr, H_2S, NF_3 and $CHCl_3$

(1) $NF_3 < HBr < H_2S < CHCl_3$

(2) $HBr < H_2S < NF_3 < CHCl_3$

(3) $H_2S < HBr < NF_3 < CHCl_3$

(4) $CHCl_3 < NF_3 < HBr < H_2S$

Ans. (1)

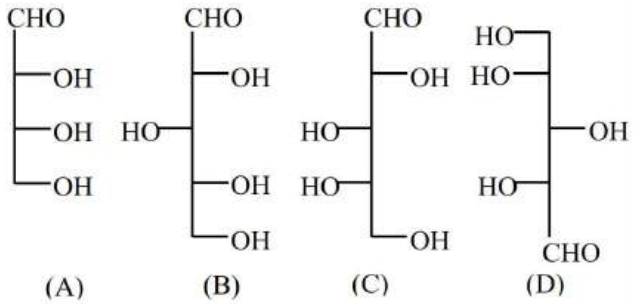
Sol. Increasing order of Dipole moment

$$NF_3 < HBr < H_2S < CHCl_3$$

$$\mu = 0.24D \quad 0.79D \quad 0.95D \quad 1.04D$$

It is NCERT Data Based

52. Identify the number of structure/s from the following which can be correlated to D-glyceraldehyde.



(1) three

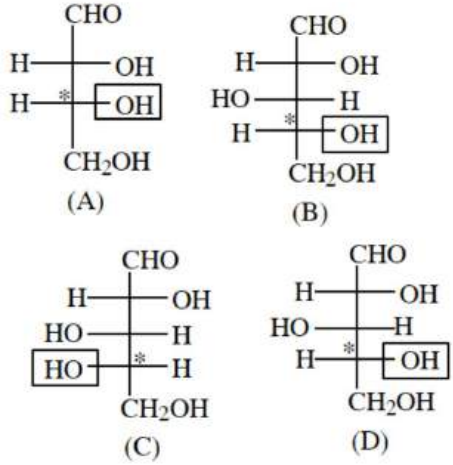
(2) two

(3) four

(4) one

Ans. (1)

Sol.



In A, B, D − OH group in right hand side then D-configuration is assigned

53. The maximum covalency of a non-metallic group 15 element 'E' with weakest E–E bond is :

(1) 5

(2) 3

(3) 6

(4) 4

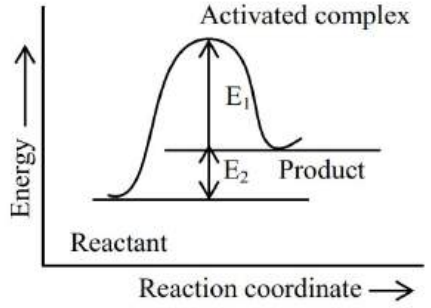
Ans. (4)

Sol. N – N < P – P : single (σ) bond strength

Due to L.P.-L.P. replusion

and maximum possible covalency of nitrogen is 4.

54. Consider the given figure and choose the **correct** option :



(1) Activation energy of backward reaction is E_1 and product is more stable than reactant.

(2) Activation energy of forward reaction is $E_1 + E_2$ and product is more stable than reactant.

(3) Activation energy of forward reaction is $E_1 + E_2$ and product is less stable than reactant.

(4) Activation energy of both forward and backward reaction is $E_1 + E_2$ and reactant is more stable than product.

Ans. (3)

Sol. Activation energy of forward reaction = $E_1 + E_2$

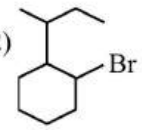
Energy of product > Energy of reactant

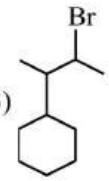
Stability

Reactant > Product

55. When sec-butylcyclohexane reacts with bromine in the presence of sunlight, the major product is :

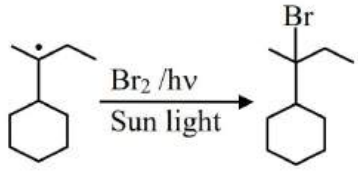
(1)

(2) 

(3) 

(4)

Ans. (4)

Sol. 

Formation of more stable free radical intermediate

56. The species which does not undergo disproportionation reaction is :

(1) ClO_2^- (2) ClO_4^-

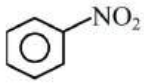
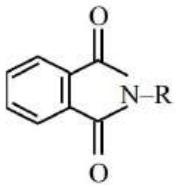
(3) ClO^- (4) ClO_3^-

Ans. (2)

Sol. $ClO_4^- \rightarrow x + \{(-2) \times 4\} = -1 \Rightarrow x = +7$

Chlorine is in its maximum oxidation state, so disproportionation not possible in ClO_4^-.

57. Match the Compounds **(List-I)** with the appropriate Catalyst/Reagents **(List-II)** for their reduction into corresponding amines.

	List-I (Compounds)		**List-II (Catalyst/Reagents)**
(A)	$R-\overset{\overset{O}{\|}}{C}-NH_2$	(I)	NaOH (aqueous)
(B)		(II)	H_2/Ni
(C)	R–C≡N	(III)	$LiAlH_4$, H_2O
(D)		(IV)	Sn, HCl

Choose the **correct** answer from the options given below :

(1) (A)-(III), (B)-(II), (C)-(IV), (D)-(I)

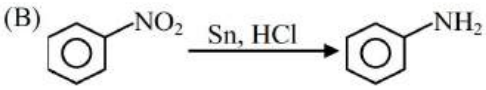
(2) (A)-(II), (B)-(IV), (C)-(III), (D)-(I)

(3) (A)-(II), (B)-(I), (C)-(III), (D)-(IV)

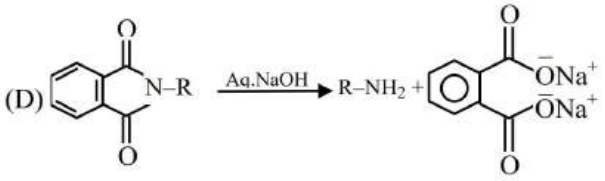
(4) (A)-(III), (B)-(IV), (C)-(II), (D)-(I)

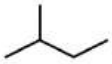
Ans. (4)

Sol. (A) $R-\overset{O}{\overset{\|}{C}}-NH_2 \xrightarrow[H_2O]{LiAlH_4} R-CH_2-NH_2$

(B) 

(C) $R-C\equiv N \xrightarrow{H_2/Ni} R-CH_2NH_2$

(D) 

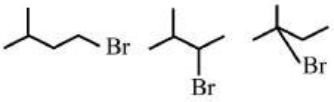
58. RBr $\xrightarrow[\text{(ii) } H_2O]{\text{(i) Mg, dry ether}}$  2-Methylbutane

The maximum number of RBr producing 2-methylbutane by above sequence of reactions is _____. (Consider the structural isomers only)

(1) 4 (2) 5

(3) 3 (4) 1

Ans. (1)



Sol.

Total number of possible structures of RBr = 4

59. Match **List-I** with **List-II**.

	List-I (Partial Derivatives)		**List-II (Thermodynamic Quantity)**
(A)	$\left(\frac{\partial G}{\partial T}\right)_P$	(I)	C_P
(B)	$\left(\frac{\partial H}{\partial T}\right)_P$	(II)	$-S$
(C)	$\left(\frac{\partial G}{\partial P}\right)_T$	(III)	C_V
(D)	$\left(\frac{\partial U}{\partial T}\right)_V$	(IV)	V

Choose the **correct** answer from the options given below :

(1) (A)-(II), (B)-(I), (C)-(III), (D)-(IV)

(2) (A)-(II), (B)-(I), (C)-(IV), (D)-(III)

(3) (A)-(I), (B)-(II), (C)-(IV), (D)-(III)

(4) (A)-(II), (B)-(III), (C)-(I), (D)-(IV)

Ans. (2)

Sol. (A) $dG = VdP - SdT$

Constant pressure

$dG = -SdT$

$$\left(\frac{\partial G}{\partial T}\right)_P = -S$$

(B) $dH = (dq)_P = nC_PdT$

$$\left(\frac{\partial H}{\partial T}\right)_P = C_P$$

(C) $dG = VdP - SdT$

At constant temperature

$dG = VdP$

$$\left(\frac{\partial G}{\partial P}\right)_T = V$$

(D) $dU = nC_VdT = (q)_V$

$$\left(\frac{\partial U}{\partial T}\right)_V = C_V$$

60. The correct order of the following complexes in terms of their crystal field stabilization energies is :

(1) $[Co(NH_3)_4]^{2+} < [Co(NH_3)_6]^{2+} < [Co(en)_3]^{3+} < [Co(NH_3)_6]^{3+}$

(2) $[Co(NH_3)_4]^{2+} < [Co(NH_3)_6]^{2+} < [Co(NH_3)_6]^{3+} < [Co(en)_3]^{3+}$

(3) $[Co(NH_3)_6]^{2+} < [Co(NH_3)_6]^{3+} < [Co(NH_3)_4]^{2+} < [Co(en)_3]^{3+}$

(4) $[Co(en)_3]^{3+} < [Co(NH_3)_6]^{3+} < [Co(NH_3)_6]^{2+} < [Co(NH_3)_4]^{2+}$

Ans. (2)

Sol. Order of CFSE

$$[Co(NH_3)_4]^{+2} < [Co(NH_3)_6]^{+2} < [Co(NH_3)_6]^{+3} < [Co(en)_3]^{+3}$$

Charge↑ CFSE↑ On increasing strength of ligand CFSE↑

SFL : $NH_3 < en$

61. Density of 3 M NaCl solution is 1.25 g/mL. The molality of the solution is :

(1) 1.79 m (2) 2 m

(3) 3 m (4) 2.79 m

Ans. (4)

Sol. 3M NaCl, $d_{sol} = 1.25$ gm/mol

$$\text{Molality} = \frac{M \times 1000}{1000d - M \times M_w}$$

$$= \frac{3000}{1250 - 175.5} = 2.79$$

62. The molar solubility(s) of zirconium phosphate with molecular formula $(Zr^{4+})_3 \left(PO_4^{3-}\right)_4$ is given by relation :

(1) $\left(\frac{K_{sp}}{6912}\right)^{\frac{1}{7}}$ (2) $\left(\frac{K_{sp}}{5348}\right)^{\frac{1}{6}}$

(3) $\left(\frac{K_{sp}}{8435}\right)^{\frac{1}{7}}$ (4) $\left(\frac{K_{sp}}{9612}\right)^{\frac{1}{3}}$

Ans. (1)

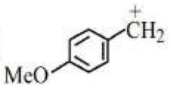
Sol. $Zr_3(PO_4)_4(s) \rightleftharpoons 3Zr^{+4}(aq) + 4PO_4^{-3}(aq)$

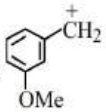
– 3s 4s

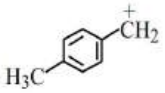
$K_{sp} = (3s)^3 (4s)^4 = 6912\ s^7$

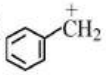
$s = \left(\frac{K_{sp}}{6912}\right)^{1/7}$

63. The most stable carbocation from the following is :

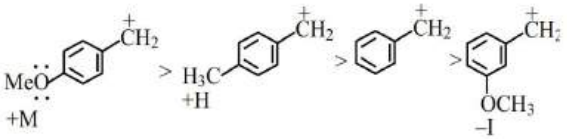
(1) 

(2) 

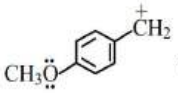
(3) 

(4) 

Ans. (1)

Sol. 

Due to +M effect of –OMe at para position



Will be most stable among these

64. Given below are two statements :

Statement (I) : An element in the extreme left of the periodic table forms acidic oxides.

Statement (II) : Acid is formed during the reaction between water and oxide of a reactive element present in the extreme right of the periodic table.

In the light of the above statements, choose the **correct** answer from the options given below :

(1) **Statement-I** is false but **Statement-II** is true.

(2) Both **Statement-I** and **Statement-II** are false.

(3) **Statement-I** is true but **Statement-II** is false.

(4) Both **Statement-I** and **Statement-II** are true.

Ans. (1)

Sol. Statement-I : False but Statement-II is true.

On moving left to right in periodic table non-metallic character increases and we know that non-metal oxides are acidc in nature.

Non metallic character ↑ Acidic strength of oxide↑

65. Given below are two statements :

Statement (I) : A spectral line will be observed for a $2p_x \rightarrow 2p_y$ transition.

Statement (II) : $2p_x$ and $2p_y$ are degenerate orbitals.

In the light of the above statements, choose the **correct** answer from the options given below :

(1) Both **Statement-I** and **Statement-II** are true.

(2) Both **Statement-I** and **Statement-II** are false.

(3) **Statement-I** is true but **Statement-II** is false.

(4) **Statement-I** is false but **Statement-II** is true.

Ans. (4)

Sol. No spectral line will be observed for a $2p_x \rightarrow 2p_y$ transition because $2p_x$ and $2p_y$ orbitals are degenerate orbitals.

66. Given below are two statements :

Statement (I) : Nitrogen, sulphur, halogen and phosphorus present in an organic compound are detected by Lassaigne's Test.

Statement (II) : The elements present in the compound are converted from covalent form into ionic form by fusing the compound with Magnesium in Lassaigne's test.

In the light of the above statements, choose the **correct** answer from the options given below :

(1) Both Statement I and Statement II are true

(2) Both Statement I and Statement II are false

(3) Statement I is true but Statement II is false

(4) Statement I is false but Statement II is true

Ans. (3)

Sol. The elements present in the compound are converted from covalent form into ionic form by fusing the compound with sodium in Lassigne's test

67. Identify the homoleptic complex(es) that is/are low spin.

(A) $[Fe(CN)_5NO]^{2-}$ (B) $[CoF_6]^{3-}$

(C) $[Fe(CN)_6]^{4-}$ (D) $[Co(NH_3)_6]^{3+}$

(E) $[Cr(H_2O)_6]^{2+}$

Choose the **correct** answer from the options given below :

(1) (B) and (E) only (2) (A) and (C) only

(3) (C) and (D) only (4) (C) only

Ans. (3)

Sol. (A) $[Fe(CN)_5NO]^{-2} \rightarrow$ Heteroleptic, Fe^{+2}, $3d^6$, $t_{2g}^6 e_g^0$, d^2sp^3, Low spin (3d series + SFL)

(B) $[CoF_6]^{-3} \rightarrow$ Homoleptic, sp^3d^2, High spin, Co^{+3}, $3d^6$ (3d series + WFL)

(C) $[Fe(CN)_6]^{-4} \rightarrow$ Homoleptic

Fe^{+2}, $3d^6$, d^2sp^3, $t_{2g}^6\ eg^0$ Low spin

(3d series + SFL)

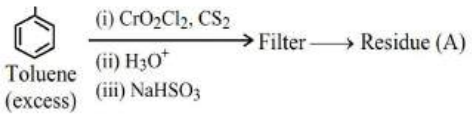
(D) $[Co(NH_3)_6]^{+3} \rightarrow$ Homoleptic, Co^{+3} $3d^6$, d^2sp^3, $t_{2g}^6\ eg^0$, Low spin (3d series + SFL)

(E) $[Cr(H_2O)_6]^{+2} \rightarrow$ Homoleptic

Cr^{+2} $3d^4$, d^2sp^3, High spin $t_{2g}^3\ e_g^1$

(3d series + WFL)

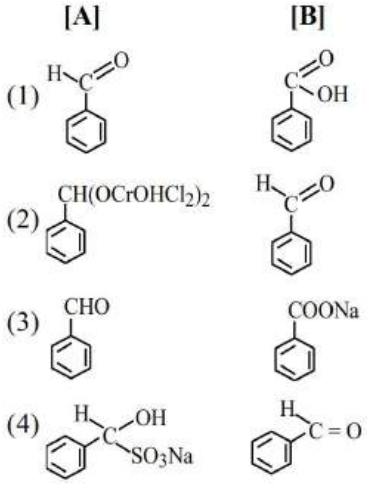
68.



Residue (A) + HCl (dil.) $\rightarrow$ Compound (B)

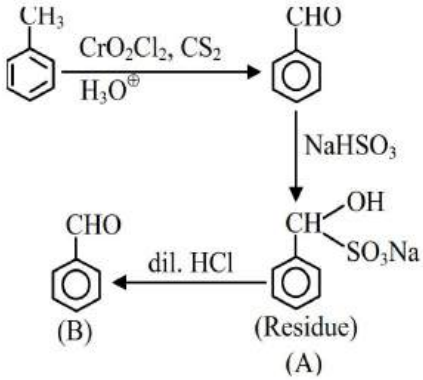
Structure of residue (A) and compound (B)

Formed respectively is :



Ans. (4)

Sol.



69. Given below are two statements :

Statement (I) : Corrosion is an electrochemical phenomenon in which pure metal acts as an anode and impure metal as a cathode.

Statement (II) : The rate of corrosion is more in alkaline medium than in acidic medium.

In the light of the above statements, choose the **correct** answer from the options given below :

(1) Both Statement I and Statement II are false

(2) Statement I is false but Statement II is true

(3) Both Statement I and Statement II are true

(4) Statement I is true but Statement II is false

Ans. (4)

Sol. **Statement I :**

Corrosion is an example of electrochemical phenomenon

In which pure metal act as anode and impure metal (rusted metal) act as cathode.

Statement II :

Corrosion is more favourable in acid medium than alkaline so rate of corrosion is high is acid medium then alkaline.

70. The alkane from below having two secondary hydrogens is :

(1) 4-Ethyl-3,4-dimethyloctane

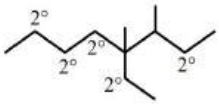
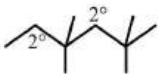
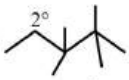
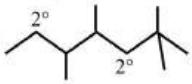
(2) 2,2,4,4-Tetramethylhexane

(3) 2,2,3,3-Tetramethylpentane

(4) 2,2,4,5-Tetramethylheptane

Ans. **(3)**

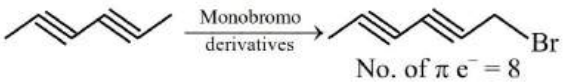
Sol.

Alkane	2°H
1. 	10
2. 	4
3. 	2
4. 	4

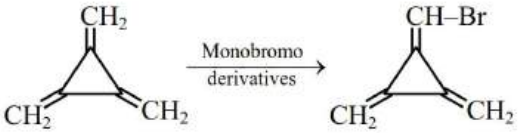
SECTION-B

71. The compound with molecular formula C_6H_6, which gives only one monobromo derivative and takes up four moles of hydrogen per mole for complete hydrogenation has _____ π electrons.

Ans. **(8)**

Sol. 

No. of π e^- = 8



N. of π e^- = 6

72. Niobium (Nb) and ruthenium (Ru) have "x" and "y" number of electrons in their respective 4d orbitals. The value of x + y is _____

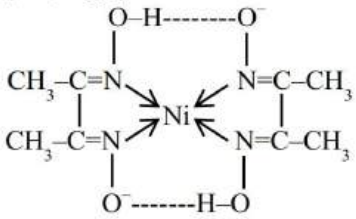
Ans. **(11)**

Sol. Z = 41 → Nb (Niobium) : $[Kr]_{36}\ 4d^4\ 5s^1$
Number of electron in 4d = 4 = x
Z = 44 → Ru (Ruthenium) $[Kr]_{36}\ 4d^7\ 5s^1$
Number of electron in 4d = 7 = y
x + y = 11

73. The complex of Ni^{2+} ion and dimethyl glyoxime contains _____ number of Hydrogen (H) atoms.

Ans. **(14)**

Sol. $[Ni(dmg)_2]$



Number of H-atom = 14

74. Consider the following cases of standard enthalpy of reaction $\left(\Delta H_r^o \text{ in kJ mol}^{-1}\right)$

$$C_2H_6(g)+\frac{7}{2}O_2(g)\rightarrow 2CO_2(g)+3H_2O(\ell)\,\Delta H_1^o=-1550$$

$$C(\text{graphite})+O_2(g)\rightarrow CO_2(g)\,\Delta H_2^o=-393.5$$

$$H_2(g)+\frac{1}{2}O_2(g)\rightarrow H_2O(\ell)\,\Delta H_3^o=-286$$

The magnitude of $\Delta H^o_{f\,C_2H_6(g)}$ is _____ kJ mol^{-1} (Nearest integer).

Ans. **(95)**

Sol. $2C_{(graphite)} + 3H_2(g) \rightarrow C_2H_6(g)$ $\Delta H_f = ?$

$C_2H_6(g) + \frac{7}{2}\ O_2(g) \rightarrow 2CO_2(g) + 3H_2O(l)$

$\Delta H_1 = -1550$

$C_{(graphite)} + O_2(g) \rightarrow CO_2(g)$ $\Delta H_2 = -393.5$

$H_2(g) + \frac{1}{2}O_2(g) \rightarrow H_2O(/)$ $\Delta H_3 = -286$

$\Delta H_f = 2\Delta H_2 + 3\Delta H_3 - \Delta H_1$
$= 95$ kJ/mole.

75. 20 mL of 2 M NaOH solution is added to 400 mL of 0.5 M NaOH solution. The final concentration of the solution is _____ × 10^{-2} M. (Nearest integer).

Ans. **(57)**

Sol. $M_F = \frac{M_1V_1 + M_2V_2}{V_1 + V_2}$

$= \frac{2\times 20 + 0.5\times 400}{420} = 0.571$ M

$= 57.1 \times 10^{-2}$ M

$= 57$

JEE–MAIN EXAMINATION – JANUARY 2025

(HELD ON WEDNESDAY 22nd JANUARY 2025)

TIME : 9:00 AM TO 12:00 NOON

MATHEMATICS

SECTION-A

1. The number of non-empty equivalence relations on the set {1,2,3} is :

(1) 6 (2) 7

(3) 5 (4) 4

Ans. (3)

Sol. Let R be the required relation

A = {(1, 1) (2, 2), (3, 3)}

(i) | R | = 3, when R = A

(ii) | R | = 5, e.g. R = A ∪ {(1, 2), (2, 1)}

Number of R can be [3]

(iii) R = {1, 2, 3} × {1, 2, 3}

Ans. (5)

2. Let f : **R→R** be a twice differentiable function such that $f(x + y) = f(x)\, f(y)$ for all x, y ∈ **R**. If $f'(0) = 4a$ and f staisfies $f''(x) - 3a\, f'(x) - f(x) = 0$, $a > 0$, then the area of the region

$R = \{(x,y) \mid 0 \le y \le f(ax),\ 0 \le x \le 2\}$ is :

(1) $e^2 - 1$ (2) $e^4 + 1$

(3) $e^4 - 1$ (4) $e^2 + 1$

Ans. (1)

Sol. $f(x + y) = f(x).f(y)$

$\Rightarrow f(x) = e^{\lambda x}$ $f'(0) = 4a$

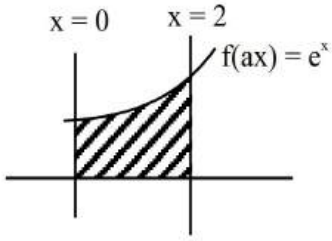
$\Rightarrow f'(x) = \lambda e^{\lambda x} \Rightarrow \lambda = 4a$

So, $f(x) = e^{4ax}$

$f''(x) - 3af'(x) - f(x) = 0$

$\Rightarrow \lambda^2 - 3a\lambda - 1 = 0$

$\Rightarrow 16a^2 - 12a^2 - 1 = 0 \Rightarrow 4a^2 = 1 \Rightarrow \boxed{a = \frac{1}{2}}$



$F(x) = e^{2x}$

$\text{Area} = \int_0^2 e^x dx = \boxed{e^2 - 1}$

3. Let the triangle PQR be the image of the triangle with vertices (1,3), (3,1) and (2, 4) in the line x + 2y = 2. If the centroid of ΔPQR is the point (α, β), then 15(α – β) is equal to :

(1) 24 (2) 19

(3) 21 (4) 22

Ans. (4)

Sol. Let 'G' be the centroid of Δ formed by (1, 3) (3, 1) & (2, 4)

$G \cong \left(2, \frac{8}{3}\right)$

Image of G w.r.t. x + 2y – 2 = 0

$$\frac{\alpha - 2}{1} = \frac{\beta - \frac{8}{3}}{2} = -2\frac{\left(2 + \frac{16}{3} - 2\right)}{1+4}$$

$$= \frac{-2}{5}\left(\frac{16}{3}\right)$$

$$\Rightarrow \alpha = \frac{-32}{15} + 2 = \frac{-2}{15},\ \beta = \frac{-32 \times 2}{15} + \frac{8}{3} = \frac{-24}{15}$$

$15(\alpha - \beta) = -2 + 24 = 22$

4. Let z_1, z_2 and z_3 be three complex numbers on the circle $|z| = 1$ with $\arg(z_1) = \frac{-\pi}{4}$, $\arg(z_2) = 0$ and $\arg(z_3) = \frac{\pi}{4}$.

If $\left|z_1\bar{z}_2 + z_2\bar{z}_3 + z_3\bar{z}_1\right|^2 = \alpha + \beta\sqrt{2}$, α, β ∈ Z, then the value of $\alpha^2 + \beta^2$ is :

(1) 24 (2) 41

(3) 31 (4) 29

Ans. (4)

Sol. $Z_1 = e^{-i\pi/4}$, $Z_2 = 1$, $Z_3 = e^{i\pi/4}$

$$\left|z_1\bar{z}_2 + z_2\bar{z}_3 + z_3\bar{z}_1\right|^2 = \left|e^{-i\frac{\pi}{4}} \times 1 + 1 \times e^{-i\frac{\pi}{4}} + e^{i\frac{\pi}{4}} \times e^{i\frac{\pi}{4}}\right|^2$$

$$\left|e^{-i\frac{\pi}{4}} + e^{-i\frac{\pi}{4}} + e^{i\frac{\pi}{4}}\right|^2$$

$$= \left|2e^{-i\frac{\pi}{4}} + i\right|^2 = = \left|\sqrt{2} - \sqrt{2}i + i\right|^2$$

$$= \left(\sqrt{2}\right)^2 + \left(1 - \sqrt{2}\right)^2 = 2 + 1 + 2 - 2\sqrt{2} = 5 - 2\sqrt{2}$$

$\alpha = 5, \beta = -2$

$\Rightarrow \alpha^2 + \beta^2 = 29$

5. Using the principal values of the inverse trigonometric functions the sum of the maximum and the minimum values of $16((\sec^{-1}x)^2 + (\text{cosec}^{-1}x)^2)$ is :

(1) $24\pi^2$ (2) $18\pi^2$

(3) $31\pi^2$ (4) $22\pi^2$

Ans. (4)

Sol. $16(\sec^{-1}x)^2 + (\text{cosec}^{-1}x)^2$

$\text{Sec}^{-1}x = a \in [0, \pi] - \left\{\frac{\pi}{2}\right\}$

$\text{cosec}^{-1}x = \frac{\pi}{2} - a$

$= 16\left[a^2 + \left(\frac{\pi}{2} - a\right)^2\right] = 16\left[2a^2 - \pi a + \frac{\pi^2}{4}\right]$

$\max]_{a=\pi} = 16[2\pi^2 - \pi^2 + \pi\frac{2}{4}] = 20\pi^2$

$\min]_{a=\frac{\pi}{4}} = 16\left[\frac{2\times\pi^2}{16} - \frac{\pi^2}{4} + \frac{\pi^2}{4}\right] = 2\pi^2$

Sum $= 22\pi^2$

6. A coin is tossed three times. Let X denote the number of times a tail follows a head. If μ and σ^2 denote the mean and variance of X, then the value of $64(\mu + \sigma^2)$ is :

(1) 51 (2) 48

(3) 32 (4) 64

Ans. (2)

Sol. HHH $\to 0$

HHT $\to 0$

HTH $\to 1$

HTT $\to 0$

THH $\to 1$

THT $\to 1$

TTH $\to 1$

TTT $\to 0$

Probability distribution

x_i	0	1
$P(x_i)$	$1/2$	$1/2$

$\mu = \sum x_i p_i = \frac{1}{2}$

$\sigma^2 = \sum x_i^2 p_i - \mu^2$

$= \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$

$64(\mu + \sigma^2) = 64\left(\frac{1}{2} + \frac{1}{4}\right) = 48$

7. Let $a_1, a_2, a_3, \ldots$ be a G.P. of increasing positive terms. If $a_1a_5 = 28$ and $a_2 + a_4 = 29$, the a_6 is equal to

(1) 628 (2) 526

(3) 784 (4) 812

Ans. (3)

Sol. $a_1.a_5 = 28 \Rightarrow a.ar^4 = 28 \Rightarrow a^2r^4 = 28$ ...(1)

$a_2 + a_4 = 29 \Rightarrow ar + ar^3 = 29$

$\Rightarrow ar(1 + r^2) = 29$

$\Rightarrow a^2r^2(1 + r^2)^2 = (29)^2$ ...(2)

By Eq. (1) & (2)

$\frac{r^2}{(1+r^2)^2} = \frac{28}{29\times 29}$

$\Rightarrow \frac{r}{1+r^2} = \frac{\sqrt{28}}{29} \Rightarrow r = \sqrt{28}$

$\because a^2r^4 = 28 \Rightarrow a^2 \times (28)^2 = 28$

$\Rightarrow a = \frac{1}{\sqrt{28}}$

$\therefore a_6 = ar^5 = \frac{1}{\sqrt{28}} \times (28)^2\sqrt{28} = 784$

8. Let $L_1 : \frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and

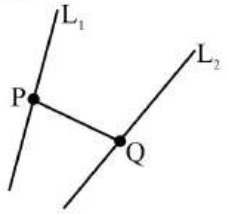
$L_2 : \frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$ be two lines. Then which of the following points lies on the line of the shortest distance between L_1 and L_2 ?

(1) $\left(-\frac{5}{3}, -7, 1\right)$ (2) $\left(2, 3, \frac{1}{3}\right)$

(3) $\left(\frac{8}{3}, -1, \frac{1}{3}\right)$ (4) $\left(\frac{14}{3}, -3, \frac{22}{3}\right)$

Ans. (4)

Sol.



$P(2\lambda + 1, 3\lambda + 2, 4\lambda + 3)$ on L_1

$Q(3\mu + 2, 4\mu + 4, 5\mu + 5)$ on L_2

Dr's of PQ $= 3\mu - 2\lambda + 1, 4\mu - 3\lambda + 2, 5\mu - 4\lambda + 2$

$PQ \perp L_1$

$\Rightarrow (3\mu - 2\lambda + 1)2 + (4\mu - 3\lambda + 2)3 + (5\mu - 4\lambda + 2)4 = 0$

$38\mu - 29\lambda + 16 = 0 \qquad \ldots(1)$

$PQ \perp L_2$

$\Rightarrow (3\mu - 2\lambda + 1)3 + (4\mu - 3\lambda + 2)4 + (5\mu - 4\lambda + 2)5 = 0$

$50\mu - 38\lambda + 21 = 0 \qquad \ldots(2)$

By (1) & (2)

$\lambda = \frac{1}{3};\ \mu = \frac{-1}{6}$

$\therefore\ P\left(\frac{5}{3}, 3, \frac{13}{3}\right)$ & $Q\left(\frac{3}{2}, \frac{10}{3}, \frac{25}{6}\right)$

Line PQ

$$\frac{x - \frac{5}{3}}{\frac{1}{6}} \qquad \frac{y-3}{\frac{-1}{3}} \qquad \frac{z - \frac{13}{3}}{\frac{1}{6}}$$

$$\frac{x - \frac{5}{3}}{1} = \frac{y-3}{-2} = \frac{z - \frac{13}{3}}{1}$$

Point $\left(\frac{14}{3}, -3, \frac{22}{3}\right)$

lies on the line PQ

9. The product of all solutions of the equation $e^{5(\log_e x)^2 + 3} = x^8$, $x > 0$, is :

(1) $e^{8/5}$ (2) $e^{6/5}$

(3) e^2 (4) e

Ans. (1)

Sol. $e^{5(\ell nx)^2 + 3} = x^8$

$\Rightarrow \ell n\, e^{5(\ell nx)^2 + 3} = \ell n\, x^8$

$\Rightarrow 5(\ell nx)^2 + 3 = 8\ell nx$

$(\ell nx = t)$

$\Rightarrow 5t^2 - 8t + 3 = 0$

$t_1 + t_2 = \frac{8}{5}$

$\ell n x_1 x_2 = \frac{8}{5}$

$x_1 x_2 = e^{8/5}$

10. If $\sum_{r=1}^{n} T_r = \frac{(2n-1)(2n+1)(2n+3)(2n+5)}{64}$, then $\lim_{n\to\infty} \sum_{r=1}^{n} \left(\frac{1}{T_r}\right)$ is equal to :

(1) 1 (2) 0

(3) $\frac{2}{3}$ (4) $\frac{1}{3}$

Ans. (3)

Sol. $T_n = S_n - S_{n-1}$

$\Rightarrow T_n = \frac{1}{8}(2n-1)(2n+1)(2n+3)$

$\Rightarrow \frac{1}{T_n} = \frac{8}{(2n-1)(2n+1)(2n+3)}$

$\lim_{n\to\infty} \sum_{r=1}^{n} \frac{1}{T_r} = \lim_{n\to\infty} 8 \sum_{r=1}^{n} \frac{1}{(2n-1)(2n+1)(2n+3)}$

$= \lim_{n\to\infty} \frac{8}{4} \sum \left(\frac{1}{(2n-1)(2n+1)} - \frac{1}{(2n+1)(2n+3)} \right)$

$= \lim_{n\to\infty} 2\left[\left(\frac{1}{1.3} - \frac{1}{3.5}\right) + \left(\frac{1}{3.5} - \frac{1}{5.7}\right) + \ldots\right]$

$= \frac{2}{3}$

11. From all the English alphabets, five letters are chosen and are arranged in alphabetical order. The total number of ways, in which the middle letter is 'M', is :

(1) 14950 (2) 6084

(3) 4356 (4) 5148

Ans. (4)

Sol. $\underbrace{AB}_{12}\ M\ \underbrace{N.....Z}_{13}$

$= \underbrace{{}^{12}C_2}_{\text{Selection of two letters before M}} \times \underbrace{{}^{13}C_2}_{\text{Selection of two letters after M}} = 5148$

12. Let x = x(y) be the solution of the differential equation $y^2 dx + \left(x - \frac{1}{y}\right) dy = 0$. If x(1) = 1, then $x\left(\frac{1}{2}\right)$ is :

(1) $\frac{1}{2} + e$ (2) $\frac{3}{2} + e$

(3) $3 - e$ (4) $3 + e$

Ans. (3)

Sol. $\frac{dx}{dy}+\left(\frac{1}{y^2}\right)x=\frac{1}{y^3}$

$\text{I.F.} = e^{\int \frac{1}{y^2}dy} = e^{-\frac{1}{y}}$

$\Rightarrow x.e^{-\frac{1}{y}} = \int\left(e^{-\frac{1}{t}}\right).\frac{1}{y^3}dy$

Put $-\frac{1}{y}=t$

$+\frac{1}{y^2}dy = dt$

$x.e^{-\frac{1}{y}} = -\int t.e^t dt$

$x.e^{-\frac{1}{y}} = -te^t + e^t + C$

$x.e^{-\frac{1}{y}} = \frac{+1}{y}e^{-\frac{1}{y}} + e^{-\frac{1}{y}} + C$

$x = 1, y = 1$

$\frac{1}{e} = \frac{1}{e} + \frac{1}{e} + C$

$\Rightarrow C = -\frac{1}{e}$

Put $y = \frac{1}{2}$

$\frac{x}{e^2} = \frac{2}{e^2} + \frac{1}{e^2} - \frac{1}{e}$

$x = 3 - e$

13. Let the parabola $y = x^2 + px - 3$, meet the coordinate axes at the points P, Q and R. If the circle C with centre at $(-1, -1)$ passes through the points P, Q and R, then the area of ΔPQR is :

(1) 4 (2) 6

(3) 7 (4) 5

Ans. (2)

Sol. $y = x^2 + px - 3$

Let $P(\alpha, 0)$, $Q(\beta, 0)$, $R(0, -3)$

Circle with centre $(-1, -1)$ is $(x + 1)^2 + (y + 1)^2 = r^2$

Passes through $(0, -3)$

$1^2 + (-2)^2 = r^2$]

$r^2 = 5$

$(x + 1)^2 + (y + 1)^2 = 5$

Put $y = 0$

$(x + 1)^2 = 5 - 1$

$(x + 1)^2 = 4$

$x + 1 = \pm 2$

$x = 1$ or $x = -3$

$\therefore$ P(1, 0) and Q(–3,0)

$\text{Area of } \Delta PQR = \frac{1}{2}\begin{vmatrix} 1 & 0 & 1 \\ -3 & 0 & 1 \\ 0 & -3 & 1 \end{vmatrix} = 6$

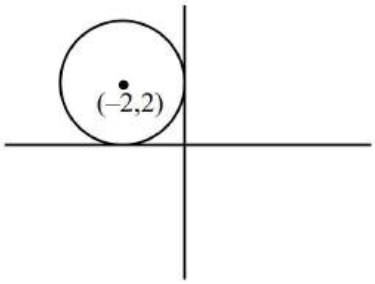
14. A circle C of radius 2 lies in the second quadrant and touches both the coordinate axes. Let r be the radius of a circle that has centre at the point (2, 5) and intersects the circle C at exactly two points. If the set of all possible values of r is the interval (α, β), then $3\beta - 2\alpha$ is equal to :

(1) 15 (2) 14

(3) 12 (4) 10

Ans. (1)

Sol.



$S_1 : (x + 2)^2 + (y - 2)^2 = 2^2$

$S_2 : (x - 2)^2 + (y - 5)^2 = r^2$

Both circle intersect at two points

$\therefore |r_1 - r_2| < c_1c_2 < r_1 + r_2$

$|r - 2| < 5 < 2 + r$

$\Rightarrow 3 < r < 7$

$r \in (3, 7)$

$\alpha = 3, \beta = 7$

$3\beta - 2\alpha = 15$

15. Let for $f(x) = 7\tan^8 x + 7\tan^6 x - 3\tan^4 x - 3\tan^2 x$, $I_1 = \int_0^{\pi/4} f(x)dx$ and $I_2 = \int_0^{\pi/4} x f(x)dx$. Then $7I_1 + 12I_2$ is equal to :

(1) 2π (2) π

(3) 1 (4) 2

Ans. (3)

Sol. $f(x) = (7\tan x - 3\tan x)(\sec x)$

$$I_1 = \int_0^{\pi/4} (7\tan^6 x - 3\tan^2 x)(\sec^2 x)\,dx$$

Put tanx = t

$$I_1 = \int_0^1 (7t^6 - 3t^2)\,dt = \left[t^7 - t^3\right]_0^1 = 0$$

$$I_2 = \int_0^{\pi/4} \underset{I}{x}\underbrace{(7\tan^6 x - 3\tan^2 x)(\sec^2 x)}_{II}\,dx$$

$$= \left[x\left(\tan^7 x - \tan^3 x\right)\right]_0^{\pi/4} - \int_0^{\pi/4} (\tan^7 x - \tan^3 x)\,dx$$

$$= 0 - \int_0^{\pi/4} \tan^3 x\left(\tan^2 x - 1\right)\left(1 + \tan^2 x\right)dx$$

Put tanx = t

$$= -\int_0^1 \left(t^5 - t^3\right)dt = -\left[\frac{t^6}{6} - \frac{t^4}{4}\right] = \frac{1}{12}$$

$7I_1 + 12I_2 = 1$

16. Let $f(x)$ be a real differentiable function such that $f(0) = 1$ and $f(x + y) = f(x)f'(y) + f'(x)\,f(y)$ for all $x, y \in \mathbf{R}$. Then $\sum_{n=1}^{100} \log_e f(n)$ is equal to :

(1) 2384 (2) 2525

(3) 5220 (4) 2406

Ans. (2)

Sol. $f(x + y) = f(x)\,f'(y) + f'(x)\,f(x)$

Put = x = y = 0

$f(0) = f(0)f'(0) + f'(0)f(0)$

$$f'(0) = \frac{1}{2}$$

Put y = 0

$f(x) = f(x)\,f'(0) + f'(x)f(0)$

$$f(x) = \frac{1}{2}f(x) + f'(x)$$

$$f'(x) = \frac{f(x)}{2}$$

$$\frac{dy}{dx} = \frac{y}{2} \Rightarrow \int \frac{dy}{y} = \int \frac{dx}{2}$$

$$\Rightarrow \ell ny = \frac{x}{2} + c$$

$\because f(0) = 1 \Rightarrow C = 0$

$$\ell ny = \frac{\pi}{2} \Rightarrow f(x) = e^{x/2}$$

$$\ell n\, f(n) = \frac{n}{2}$$

$$\sum_{n=1}^{100} \ell f(n) = \frac{1}{2}\sum_{n=1}^{100} n = \frac{5050}{2}$$

= 2525

17. Let A = {1,2,3,.....,10} and

$$B = \left\{\frac{m}{n} : m, n \in A,\ m < n \text{ and } \gcd(m, n) = 1\right\}.$$

Then n(B) is equal to :

(1) 31 (2) 36

(3) 37 (4) 29

Ans. (1)

Sol. A = {1, 2, ….10}

$B\ \{\frac{m}{n} = m, n \in A, m < n, \gcd(m, n) = 1\}$

n(B)

$n = 2 \quad \left\{\frac{1}{2}\right\}$

$n = 3 \quad \left\{\frac{1}{3}, \frac{2}{3}\right\}$

$n = 4 \quad \left\{\frac{1}{4}, \frac{3}{4}\right\}$

$n = 5 \quad \left\{\frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}\right\}$

$n = 6 \quad \left\{\frac{1}{6}, \frac{5}{6}\right\}$

$n = 7 \quad \left\{\frac{1}{7}, \frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}, \frac{6}{7}\right\}$

$n = 8 \quad \left\{\frac{1}{8}, \frac{3}{8}, \frac{5}{8}, \frac{7}{8}\right\}$

$n = 9 \quad \left\{\frac{1}{9}, \frac{2}{9}, \frac{4}{9}, \frac{5}{9}, \frac{7}{9}, \frac{8}{9}\right\}$

$n = 10 \quad \left\{\frac{1}{10}, \frac{3}{10}, \frac{7}{10}, \frac{9}{10}\right\}$

n(B) = 31

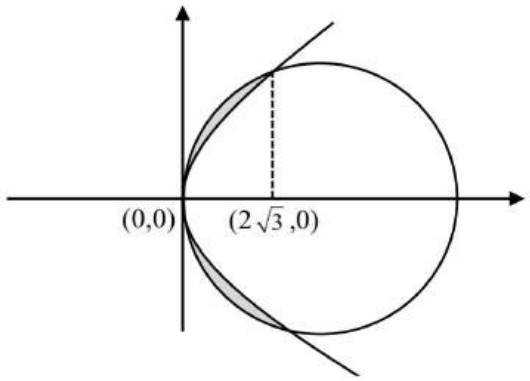
18. The area of the region, inside the circle $\left(x-2\sqrt{3}\right)^2+y^2=12$ and outside the parabola $y^2=2\sqrt{3}x$ is

(1) $6\pi-8$ (2) $3\pi-8$

(3) $6\pi-16$ (4) $3\pi+8$

Ans. (3)

Sol.



$y^2=2\sqrt{3}x$

$\left(x-2\sqrt{3}\right)^2+y^2=\left(2\sqrt{3}\right)^2$

$A=\frac{\pi r^2}{2}-2\int_0^{2\sqrt{3}}\sqrt{2\sqrt{3}x}\,dx$

$\frac{\pi(12)}{2}-2\sqrt{2\sqrt{3}}\frac{\left(x^{3/2}\right)_0^{2/3}}{3/2}$

$=6\pi-16$

19. Two balls are selected at random one by one without replacement from a bag containing 4 white and 6 black balls. If the probability that the first selected ball is black, given that the second selected ball is also black, is $\frac{m}{n}$, where gcd(m, n) = 1, then m + n is equal to :

(1) 14 (2) 4

(3) 11 (4) 13

Ans. (1)

Sol. $P=\frac{\frac{6}{10}\times\frac{5}{9}}{\frac{4}{10}\times\frac{6}{9}+\frac{6}{10}\times\frac{5}{9}}=\frac{5}{9}$

$m=5, n=9$

$m+n=14$

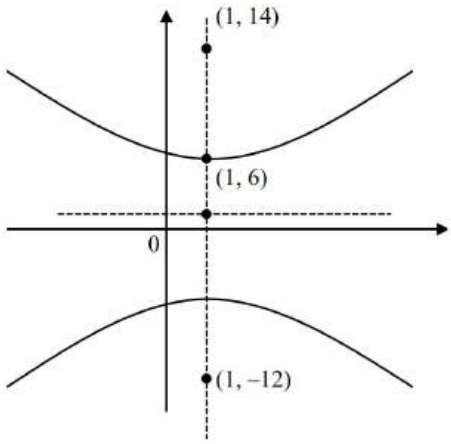
20. Let the foci of a hyperbola be (1, 14) and (1, –12). If it passes through the point (1, 6), then the length of its latus-rectum is :

(1) $\frac{25}{6}$ (2) $\frac{24}{5}$

(3) $\frac{288}{5}$ (4) $\frac{144}{5}$

Ans. (3)

Sol.



$be = 13, b = 5$

$a^2 = b^2(e^2-1)$

$= b^2e^2 - b^2$

$= 169 - 25 = 144$

$\ell(LR)=\frac{2a^2}{b}=\frac{2\times144}{5}=\frac{288}{5}$

SECTION-B

21. Let the function,

$$f(x)=\begin{cases}-3ax^2-2, & x<1\\ a^2+bx, & x\ge1\end{cases}$$

Be differentiable for all $x \in \mathbf{R}$, where $a > 1$, $b \in \mathbf{R}$. If the area of the region enclosed by $y = f(x)$ and the line $y = -20$ is $\alpha+\beta\sqrt{3}$, $\alpha, \beta, \in Z$, then the value of $\alpha+\beta$ is ____.

Ans. (34)

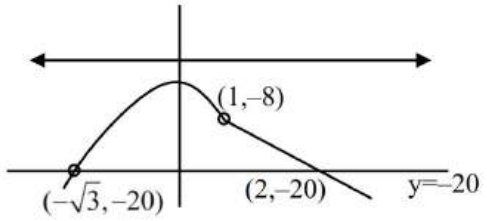
Sol. f(x) is continuous and differentiable

at x = 1; LHL = RHL, LHD = RHD

$-3a-2=a^2+b, -6a=b$

$a=2, 1;\ b=-12$

$$f(x)=\begin{cases}-6x^2-2 & ;\ x<1\\ 4-12x & ;\ x\ge1\end{cases}$$



$$\text{Area} = \int_{-\sqrt{3}}^{1}\left(-6x^2-2+20\right)dx + \int_{1}^{2}(4-12x+20)\,dx$$

$$16+12\sqrt{3}+6 = 22+12\sqrt{3}$$

22. If $\sum_{r=0}^{5}\frac{{}^{11}C_{2r+1}}{2r+2} = \frac{m}{n}$, gcd(m, n) =1, then m – n is equal to _____.

Ans. (2035)

Sol. $\int_0^1 (1+x)^{11}dx = \left[C_0x + \frac{C_1x^2}{2} + \frac{C_2x^3}{3} + \ldots\right]_0^1$

$$\frac{2^{12}-1}{12} = C_0 + \frac{C_1}{2} + \frac{C_2}{3} + \frac{C_3}{4} + \ldots$$

$$\int_{-1}^{0}(1+x)^{11}dx = \left[C_0x + \frac{C_1x^2}{2} + \frac{C_2x^3}{3} + \ldots\right]_{-1}^{0}$$

$$\frac{1}{12} = C_0 - \frac{C_1}{2} + \frac{C_2}{3} - \frac{C_3}{4} + \ldots$$

$$\frac{2^{12}-2}{12} = 2\left(\frac{C_1}{2} + \frac{C_3}{4} + \frac{C_5}{6} + \ldots\right)$$

$$\frac{C_1}{2} + \frac{C_3}{4} - \frac{C_5}{6} + \ldots = \frac{2^{11}-1}{12} = \frac{2047}{12}$$

23. Let A be a square matrix of order 3 such that det(A) = –2 and det(3adj(–6adj(3A))) $=2^{m+n}.3^{mn}$, m > n. Then 4m + 2n is equal to _____.

Ans. (34)

Sol. $|A| = -2$

det(3adj(–6adj(3A)))

$= 3^3\det(\text{adj}(-\text{adj}(3A)))$

$= 3^3(-6)^6(\det(3A))^4$

$= 3^{21} \times 2^{10}$

m + n = 10

mn = 21

m = 7 ; n = 3

24. Let $L_1 : \frac{x-1}{3} = \frac{y-1}{-1} = \frac{z+1}{0}$ and

$L_2 : \frac{x-2}{2} = \frac{y}{0} = \frac{z+4}{\alpha}$, $\alpha \in R$, be two lines, which intersect at the point B. If P is the foot of perpendicular from the point A(1, 1, –1) on L_2, then the value of 26 $\alpha(PB)^2$ is ____.

Ans. (216)

Sol. Point B

$(3\lambda + 1, -\lambda + 1, -1) \equiv (2\mu + 2, 0, \alpha\mu - 4)$

$3\lambda + 1 = 2\mu + 2$

$-\lambda + 1 = 0$

$-1 = \alpha\mu - 4$

$\lambda = 1, \mu = 1, \alpha = 3$

B(4, 0, –1)

Let Point 'P' is $(2\delta + 2, 0, 3\delta - 4)$

Dr's of AP $< 2\delta + 1, -1, 3\delta - 3 >$

$AP \perp L_2 \Rightarrow \delta = \frac{7}{13}$

$P\left(\frac{40}{13}, 0, \frac{-31}{13}\right)$

$2\sigma\delta(PB)^2 = 26 \times 3 \times \left(\frac{144}{169} + \frac{324}{169}\right)$

= 216

25. Let $\vec{c}$ be the projection vector of $\vec{b} = \lambda\hat{i} + 4\hat{k}$, $\lambda > 0$, on the vector $\vec{a} = \hat{i} + 2\hat{j} + 2\hat{k}$. If $|\vec{a} + \vec{c}| = 7$, then the area of the parallelogram formed by the vectors $\vec{b}$ and $\vec{c}$ is _____.

Ans. (16)

Sol. $\vec{c} = \left(\frac{\vec{b}\cdot\vec{a}}{|\vec{a}|}\right)\frac{\vec{a}}{\|\vec{a}|}$

$= \left(\frac{\lambda+8}{9}\right)\left(\hat{i} + 2\hat{j} + 2\hat{k}\right)$

$|\vec{a} + \vec{c}| - = 7 \Rightarrow \lambda = 4$

Area of parallelogram

$$= \left|\vec{b} \times \vec{c}\right| = \left|\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{4}{3} & \frac{8}{3} & \frac{8}{3} \\ 4 & 0 & 4 \end{vmatrix}\right|$$

= 16

PHYSICS

SECTION-A

26. Given below are two statements :

Statement I : In a vernier callipers, one vernier scale division is always smaller than one main scale division.

Statement II : The vernier constant is given by one main scale division multiplied by the number of vernier scale division.

In the light of the above statements, choose the **correct** answer from the options given below.

(1) Both **Statement I** and **Statement II** are false.

(2) **Statement I** is true but **Statement II** is false.

(3) Both **Statement I** and **Statement II** are true.

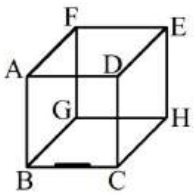
(4) **Statement I** is false but **Statement II** is true.

Ans. (2)

Sol. In general one vernier scale division is smaller than one main scale division but in some modified cases it may be not correct. Also least count is given by one main scale division / number of vernier scale division for normal vernier calliper.

Note: In JA-2016_P-2, Q-6 was present with modified V.C..

27. A line charge of length $\frac{'a'}{2}$ is kept at the center of an edge BC of a cube ABCDEFGH having edge length 'a' as shown in the figure. If the density of line is λC per unit length, then the total electric flux through all the faces of the cube will be ____. (Take, $\in_0$ as the free space permittivity)



(1) $\frac{\lambda a}{8 \in_0}$ (2) $\frac{\lambda a}{16 \in_0}$

(3) $\frac{\lambda a}{2 \in_0}$ (4) $\frac{\lambda a}{4 \in_0}$

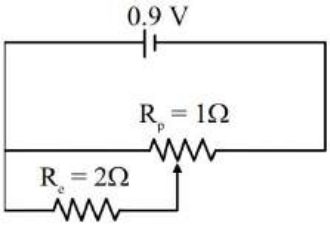
Ans. (1)

Sol. Total charge inside the cube

$$= \frac{\lambda \frac{a}{2}}{4} = \frac{\lambda a}{8}$$

$$\therefore \phi = \frac{q_{in}}{\varepsilon_0} = \frac{\lambda a}{8\varepsilon_0}$$

28.



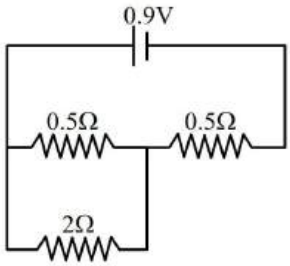
Sliding contact of a potentiometer is in the middle of the potentiometer wire having resistance R_p = 1Ω as shown in the figure. An external resistance of R_e = 2Ω is connected via the sliding contact.

(1) 0.3 A (2) 1.35 A

(3) 1.0 A (4) 0.9 A

Ans. (3)

Sol. The circuit can be considered as



$$\therefore R_{eq} = 0.5 + \frac{0.5 \times 2}{2 + 0.5} = \left(\frac{5}{10} + \frac{10}{25}\right)\Omega$$

$$= \frac{45}{50} = \frac{9}{10} = 0.9$$

$$\therefore i = \frac{0.9}{0.9} = 1A$$

29. Given below are two statements : one is labelled as **Assertion (A)** and the other is labelled as **Reason (R)**.

Assertion (A) : If Young's double slit experiment is performed in an optically denser medium than air, then the consecutive fringes come closer.

Reason (R) : The speed of light reduces in an optically denser medium than air while its frequency does not change.

In the light of the above statements, choose the **most appropriate answer** from the options given below :

(1) Both **(A)** and **(R)** are true and **(R)** is the correct explanation of **(A)**

(2) **(A)** is false but **(R)** is true.

(3) Both **(A)** and **(R)** are true but **(R)** is **not** the correct explanation of **(A)**

(4) **(A)** is true but **(R)** is false.

Ans. (1)

Sol. $\beta(\text{fringe width}) = \frac{\lambda D}{d}$

In denser medium, $\lambda \downarrow \Rightarrow \beta \downarrow$

$\Rightarrow$ fringe come closer

Also, $\mu = \frac{c}{V} \Rightarrow V = \frac{c}{\mu}$

Frequency remains same,

$\Rightarrow \mu = \frac{\lambda_{vac.} f}{\lambda_{med} f} \Rightarrow \lambda_{med} = \frac{\lambda_{vac.}}{\mu}$

30. Two spherical bodies of same materials having radii 0.2 m and 0.8 m are placed in same atmosphere. The temperature of the smaller body is 800 K and temperature of bigger body is 400 K. If the energy radiate from the smaller body is E, the energy radiated from the bigger body is (assume, effect of the surrounding to be negligible)

(1) 256 E (2) E

(3) 64 E (4) 16 E

Ans. (2)

Sol. $\frac{d\theta}{dt} = \sigma e A T^4 \Rightarrow P \propto AT^4$

$\frac{P_{smaller}}{P_{larger}} = \frac{(0.2)^2 \times 800^4}{(0.8)^2 \times 400^4}$

$\frac{1}{16} \times 16 = 1$

$\therefore\ P_{larger} = P_{smaller}$

31. An amount of ice of mass 10^{-3} kg and temperature –10°C is transformed to vapour of temperature 110° by applying heat. The total amount of work required for this conversion is,

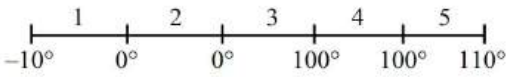
(Take, specific heat of ice = 2100 $Jkg^{-1}K^{-1}$, specific heat of water = 4180 $Jkg^{-1}K^{-1}$, specific heat of steam = 1920 $Jkg^{-1}K^{-1}$, Latent heat of ice = 3.35×10^5 Jkg^{-1} and Latent heat of steam = 2.25×10^6 Jkg^{-1})

(1) 3022 J (2) 3043 J

(3) 3003 J (4) 3024 J

Ans. (2)

Sol.



$\Delta Q_1 = m \times S_I \times \Delta T = 10^{-3} \times 2100 \times 10 = 21$ J

$\Delta Q_2 = m \times L_f = 10^{-3} \times 3.35 \times 10^5 = 335$ J

$\Delta Q_3 = m \times S_W \times \Delta T = 10^{-3} \times 4180 \times 100 = 418$ J

$\Delta Q_4 = m \times L_V = 10^{-3} \times 2.25 \times 10^6 = 2250$ J

$\Delta Q_5 = m \times S_V \times \Delta T = 10^{-3} \times 1920 \times 10 = 19.2$ J

$\Delta Q_{net} = 3043.2$ J

32. An electron in the ground state of the hydrogen atom has the orbital radius of 5.3×10^{-11} m while that for the electron in third excited state is 8.48×10^{-10} m. The ratio of the de Broglie wavelengths of electron in the ground state to that in excited state is

(1) 4 (2) 9

(3) 3 (4) 16

Ans. (1)

Sol. $\lambda = \frac{h}{mv}$

$$mvr = \frac{nh}{2\pi}$$

$$mv = \frac{nh}{2\pi r}$$

$$\lambda = \frac{2\pi rh}{nh}$$

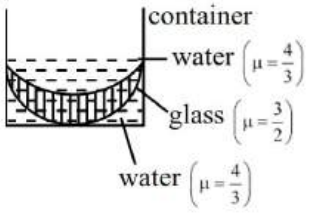
$$\lambda \propto \frac{r}{n}$$

$$\frac{\lambda_1}{\lambda_4} = \frac{r_1 n_4}{n_1 r_4} = \frac{5.3\times10^{-11}\times4}{1\times84.8\times10^{-11}}$$

$$\frac{\lambda_1}{\lambda_4} = \frac{1}{4}$$

Note : Most appropriate answer will be option (1).

33. In the diagram given below, there are three lenses formed. Considering negligible thickness of each of them as compared to $[R_1]$ and $[R_2]$, i.e., the radii of curvature for upper and lower surfaces of the glass lens, the power of the combination is



(1) $-\frac{1}{6}\left(\frac{1}{|R_1|}+\frac{1}{|R_2|}\right)$

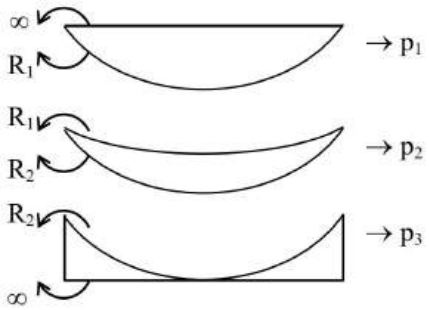
(2) $-\frac{1}{6}\left(\frac{1}{|R_1|}-\frac{1}{|R_2|}\right)$

(3) $\frac{1}{6}\left(\frac{1}{|R_1|}+\frac{1}{|R_2|}\right)$

(4) $\frac{1}{6}\left(\frac{1}{|R_1|}-\frac{1}{|R_2|}\right)$

Ans. (2)

Sol.



$$\Rightarrow p_{eq} = p_1 + p_2 + p_3$$

$$\Rightarrow p_1 = \left(\frac{4}{3}-1\right)\left(\frac{1}{\infty}-\frac{1}{-|R_1|}\right)$$

$$\Rightarrow p_1 = \left(\frac{1}{3|R_1|}\right)$$

$$\Rightarrow p_2 = \left(\frac{1}{2}\right)\left(\frac{1}{-|R_1|}-\frac{1}{-|R_2|}\right)$$

$$\Rightarrow p_2 = \frac{1}{2}\left(\frac{1}{|R_2|}-\frac{1}{|R_1|}\right)$$

$$\Rightarrow p_3 = \left(\frac{1}{3}\right)\left(\frac{1}{-|R_2|}-\frac{1}{\infty}\right) = -\frac{1}{3|R_2|}$$

$$\Rightarrow p_{eq} = \frac{1}{3}\left(\frac{1}{|R_1|}-\frac{1}{|R_2|}\right)-\frac{1}{2}\left(\frac{1}{|R_1|}-\frac{1}{|R_2|}\right)$$

$$= -\frac{1}{6}\left(\frac{1}{|R_1|}-\frac{1}{|R_2|}\right)$$

34. An electron is made to enters symmetrically between two parallel and equally but oppositely charged metal plates, each of 10 cm length. The electron emerges out of the field region with a horizontal component of velocity 10^6 m/s. If the magnitude of the electric between the plates is 9.1 V/cm, then the vertical component of velocity of electron is

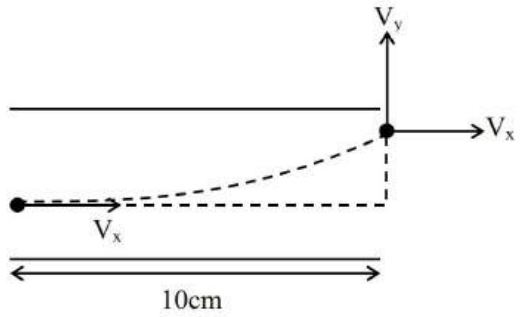
(mass of electron = 9.1×10^{-31} kg and charge of electron = 1.6×10^{-19} C)

(1) 1×10^6 m/s (2) 0

(3) 16×10^6 m/s (4) 16×10^4 m/s

Ans. (3)

Sol.



$$\Rightarrow t = \frac{\ell}{V_x} = \frac{10\times10^{-2}}{10^6} = 10^{-7}$$

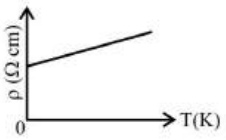
$$\Rightarrow V_y = u_y + a_y t$$

$$\Rightarrow V_y = 0 + \frac{eE}{m}\times10^{-7}$$

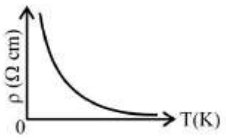
$$\Rightarrow V_y = \frac{1.6\times10^{-19}}{9.1\times10^{-31}}\times9.1\times10^{-2}\times10^{-7}$$

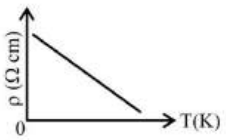
$$\Rightarrow V_y = 16\times10^6$$

35. Which of the following resistivity (ρ) v/s temperature (T) curves is most suitable to be used in wire bound standard resistors?

(1) 

(2)

(3) 

(4) 

Ans. (1)

Sol. Resistivity is independent of temperature for wire bound resistors

36. A closed organ and an open organ tube filled by two different gases having same bulk modulus but different densities ρ_1 and ρ_2 respectively. The frequency of 9th harmonic of closed tube is identical with 4th harmonic of open tube. If the length of the closed tube is 10 cm and the density ratio of the gases is $\rho_1 : \rho_2 = 1 : 16$, then the length of the open tube is :

(1) $\frac{20}{7}$ cm (2) $\frac{15}{7}$ cm

(3) $\frac{20}{9}$ cm (4) $\frac{15}{9}$ cm

Ans. (3)

Sol. 9th harmonic of closed pipe $= \frac{9V_1}{4\ell_1}$

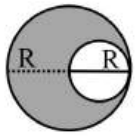
4th harmonic of open pipe $= \frac{2V_2}{\ell_2}$

$$\therefore \frac{9V_1}{4\ell_1} = \frac{2V_2}{\ell_2}$$

$$\therefore \frac{9}{4\ell_1}\sqrt{\frac{B}{\rho_1}} = \frac{2}{\ell_2}\sqrt{\frac{B}{\rho_2}} \Rightarrow \frac{\ell_2}{\ell_1} = \frac{8}{9}\sqrt{\frac{\rho_1}{\rho_2}}$$

$$\ell_2 = \ell_1\times\frac{8}{9}\times\frac{1}{4} = \frac{20}{9}\text{cm}$$

37. A uniform circular disc of radius 'R' and mass 'M' is rotating about an axis perpendicular to its plane and passing through its centre. A small circular part of radius R/2 is removed from the original disc as shown in the figure. Find the moment of inertia of the remaining part of the original disc about the axis as given above.



(1) $\frac{7}{32}MR^2$ (2) $\frac{9}{32}MR^2$

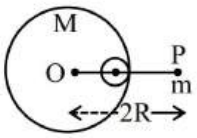
(3) $\frac{17}{32}MR^2$ (4) $\frac{13}{32}MR^2$

Ans. (4)

Sol. $I = \frac{MR^2}{2} - \left[\frac{\frac{M}{4}\left(\frac{R}{2}\right)^2}{2} + \frac{M}{4}\left(\frac{R}{2}\right)^2\right]$

$I = \frac{13}{32}MR^2$

38. A small point of mass m is placed at a distance 2R from the centre 'O' of a big uniform solid sphere of mass M and radius R. The gravitational force on 'm' due to M is F_1. A spherical part of radius R/3 is removed from the big sphere as shown in the figure and the gravitational force on m due to remaining part of M is found to be F_2. The value of ratio $F_1 : F_2$ is



(1) 16 : 9 (2) 11 : 10

(3) 12 : 11 (4) 12 : 9

Ans. (3)

Sol. $F_1 = \frac{GMm}{(2R)^2}$ ...(1)

$F_2 = \frac{GMm}{(2R)^2} - \left(\frac{G\left(\frac{M}{27}\right)m}{\left(\frac{4R}{3}\right)^2}\right)$

$F_2 = \frac{11}{48}\frac{GMm}{R^2}$ ...(2)

$F_1 : F_2 = 12 : 11$

39. The work functions of cesium (Cs) and lithium (Li) metals are 1.9 eV and 2.5 eV, respectively. If we incident a light of wavelength 550 nm on these two metal surface, then photo-electric effect is possible for the case of

(1) Li only (2) Cs only

(3) Neither Cs nor Li (4) Both Cs and Li

Ans. (2)

Sol. $E = \frac{1240}{\lambda} = \frac{1240}{550} \simeq 2.25$

40. If B is magnetic field and $_0$ is permeability of free space, then the dimensions of (B/μ_0) is

(1) $MT^{-2}A^{-1}$ (2) $L^{-1}A$

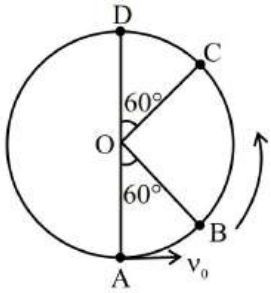
(3) $LT^{-2}A^{-1}$ (4) $ML^2T^{-2}A^{-1}$

Ans. (2)

Sol. $B = \mu_0 ni$

$\left[\frac{B}{\mu_0}\right] = [ni] = L^{-1}A^1$

41. A bob of mass m is suspended at a point O by a light string of length l and left to perform vertical motion (circular) as shown in figure. Initially, by applying horizontal velocity v_0 at the point 'A'. the string becomes slack when, the bob reaches at the point 'D'. The ratio of the kinetic energy of the bob at the points B and C is _____.



(1) 2 (2) 1

(3) 4 (4) 3

Ans. (1)

Sol. $\frac{1}{2}mv_A^2 = \frac{1}{2}mv_B^2 + mgh$

$\Rightarrow \frac{1}{2}m(5g\ell) = \frac{1}{2}mv_B^2 + mg\frac{\ell}{2}$

$\Rightarrow \frac{5mg\ell}{2} - \frac{mg\ell}{2} = KE_B$

$\Rightarrow KE_B = 2mg\ell$

$\frac{1}{2}mv_C^2 = \frac{1}{2}mv_D^2 + mg\frac{\ell}{2}$

$\Rightarrow KE_C = \frac{1}{2}mg\ell + mg\frac{\ell}{2} = mg\ell$

$\Rightarrow \frac{KE_B}{KE_C} = 2$

42. Given below are two statements :

Statement-I : The equivalent emf of two nonideal batteries connected in parallel is smaller than either of the two emfs.

Statement-II : The equivalent internal resistance of two nonideal batteries connected in parallel is smaller than the internal resistance of either of the two batteries.

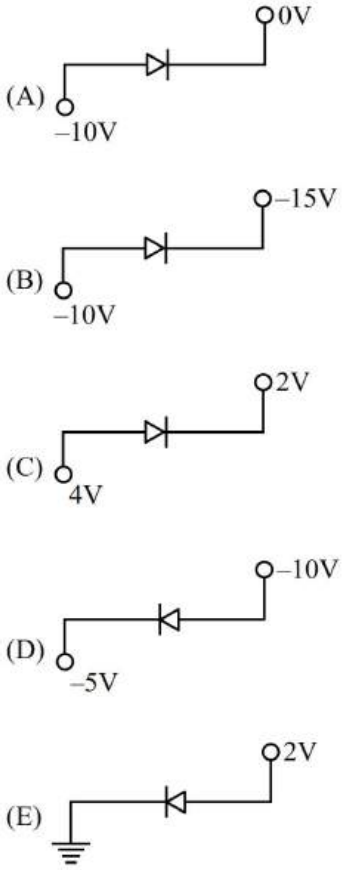
In the light of the above statements, choose the **correct** answer from the options given below.

(1) **Statement-I** is true but **Statement-II** is false
(2) Both **Statement-I** and **Statement-II** are false
(3) Both **Statement-I** and **Statement-II** are true
(4) **Statement-I** is false but **Statement-II** is true

Ans. (4)

Sol. $$= \frac{\frac{\varepsilon_1}{r_1} + \frac{\varepsilon_2}{r_2}}{\frac{1}{r_1} + \frac{1}{r_2}} = \varepsilon$$

43. Which of the following circuits represents a forward biased diode ?



Choose the **correct** answer from the options given below :

(1) (B), (D) and (E) only
(2) (A) and (D) only
(3) (B), (C) and (E) only
(4) (C) and (E) only

Ans. (3)

44. A parallel-plate capacitor of capacitance 40μF is connected to a 100 V power supply. Now the intermediate space between the plates is filled with a dielectric material of dielectric constant K = 2. Due to the introduction of dielectric material, the extra charge and the change in the electrostatic energy in the capacitor, respectively, are -

(1) 2 mC and 0.2 J (2) 8 mC and 2.0 J
(3) 4 mC and 0.2 J (4) 2 mC and 0.4 J

Ans. (3)

Sol. $\Delta q = (KC - C)V$

$= 40 \times 10^{-6} \times 100$

$= 4000 \times 10^{-3} = 4$ mC

$$\Delta U = \frac{1}{2}C'V^2 - \frac{1}{2}CV^2 = \frac{1}{2}(K-1)CV^2$$

$$= \frac{1}{2}CV^2(2-1)$$

$$= \frac{1}{2}CV^2 = \frac{1}{2} \times 40 \times 10^{-6} \times 10000$$

$= 0.2$ J

45. Given is a thin convex lens of glass (refractive index μ) and each side having radius of curvature R. One side is polished for complete reflection. At what distance from the lens, an object be placed on the optic axis so that the image gets formed on the object itself.

(1) R/μ (2) R/(2μ–3)
(3) μR (4) R/(2μ–1)

Ans. (4)

Sol. $P_{eq} = 2P_\ell + P_m$

$$-\frac{1}{f_Q} = \frac{2}{f_\ell} - \frac{1}{f_m}$$

$$= \frac{4(\mu-1)}{R} - \frac{2}{-R} = \frac{1}{R}(4\mu - 4 + 2)$$

$$-\frac{1}{f_{eq}} = \frac{1}{R}(4\mu - 2)$$

$$\Rightarrow \frac{1}{f_{eq}} = \frac{-1}{R}(4\mu - 2)$$

$$f_{eq} = \frac{R}{2}$$

$$R = 2f_{eq} = -2\left(\frac{R}{4\mu - 2}\right) = \frac{-R}{(2\mu - 1)}$$

SECTION-B

46. Two soap bubbles of radius 2 cm and 4 cm, respectively, are in contact with each other. The radius of curvature of the common surface, in cm, is _____ .

Ans. (4)

Sol. $r = \frac{r_1 \cdot r_2}{r_1 - r_2} = \frac{2 \cdot 4}{4-2} = 4\text{cm}$

47. The driver sitting inside a parked car is watching vehicles approaching from behind with the help of his side view mirror, which is a convex mirror with radius of curvature R = 2 m. Another car approaches him from behind with a uniform speed of 90 km/hr. When the car is at a distance of 24 m from him, the magnitude of the acceleration of the image of the side view mirror is 'a'. The value of 100a is _____ m/s^2.

Ans. (8)

Sol. $v = \frac{uf}{u-f} = \frac{-24 \cdot 1}{-24-1} = \frac{24}{25}$

$m = \frac{-v}{u} = -\frac{24}{25(-24)} = \frac{1}{25}$

$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$v_I = -m^2 \cdot v_0 = \frac{-1}{(25)^2} \cdot 25 = \frac{-1}{25}$

Diff.

$\frac{-1}{v^2}\left(\frac{dv}{dt}\right) + \frac{1}{u^2}\left(\frac{du}{dt}\right) = 0 \quad \left[\frac{dv}{dt} = v_I; \frac{du}{dt} = v_0\right]$

$\frac{+2}{v^3} \cdot (v_I)^2 - \frac{1}{v^2} \cdot a_I - \frac{2}{u^3} \cdot (v_0)^2 + \frac{1}{u^2} \cdot a_0 = 0$

$\frac{a_I}{v^2} = \frac{2}{v^3} \cdot v_I^2 - \frac{2}{u^3} \cdot v_0^2$

$a_I = \frac{2}{v} \cdot v_I^2 - \frac{2v^2}{u^3} \cdot v_0^2$

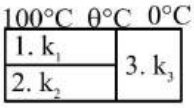
$= \frac{2 \cdot 25}{24} \cdot \frac{1}{25} \cdot \frac{1}{25} - \frac{2}{(24)^3} \cdot \frac{24}{25} \cdot \frac{24}{25} \cdot 25 \cdot 25$

$a_I = \frac{2}{24.25} - \frac{2}{24}$

$a_I = \frac{2}{24} \cdot \frac{-24}{25} = \frac{-2}{25}$

$100a_I = \frac{2}{25} \times 100 = 8$

48. Three conductions of same length having thermal conductivity k_1, k_2 and k_3 are connected as shown in figure.

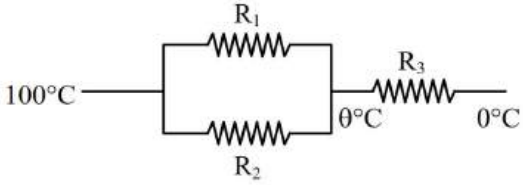


Area of cross sections of 1st and 2nd conductor are same and for 3rd conductor it is double of the 1st conductor. The temperatures are given in the figure. In steady state condition, the value of θ is _____ °C.

(Given : $k_1 = 60\ Js^{-1}m^{-1}K^{-1}$, $k_2 = 120\ Js^{-1}m^{-1}K^{-1}$, $k_3 = 135\ Js^{-1}m^{-1}K^{-1}$)

Ans. (40)

Sol.



$R_1 = \frac{2L}{K_1 A}$

$R_2 = \frac{2L}{K_2 A}$

$R_3 = \frac{L}{K_3 A}$

$\frac{\theta - 100}{\frac{R_1 R_2}{R_1 + R_2}} + \frac{\theta - 0}{R_3} = 0$

$\theta = 40$

49. The position vectors of two 1 kg particles, (A) and (B), are given by

$\vec{r}_A = \left(\alpha_1 t^2 \hat{i} + \alpha_2 t \hat{j} + \alpha_3 t \hat{k}\right)$ m

and $\vec{r}_B = \left(\beta_1 t \hat{i} + \beta_2 t^2 \hat{j} + \beta_3 t \hat{k}\right)$ m, respectively ;

($\alpha_1 = 1\ m/s^2$, $\alpha_2 = 3n\ m/s$, $\alpha_3 = 2\ m/s$, $\beta_1 = 2\ m/s$, $\beta_2 = -1\ m/s^2$, $\beta_3 = 4p\ m/s$), where t is time, n and p are constants, At t = 1s, $|\vec{V}_A| = |\vec{V}_B|$ and velocities $\vec{V}_A$ and $\vec{V}_B$ of the particles are orthogonal to each other. At t = 1 s, the magnitude of angular momentum of particle (A) with respect to the position of particle (B) is $\sqrt{L}$ kgm^2s^{-1}. The value of L is _____.

Ans. (90)

Sol. $V_A = (2t\hat{i} + 3n\hat{j} + 2\hat{k})$

$\vec{V}_B = (2\hat{i} - 2t\hat{j} + 4p\hat{k})$

$\vec{V}_A \cdot \vec{V}_B = 0$

$4 - 6n + 8p = 0$

$2 - 3n + 4p = 0$

$3n = 2 + 4p$

$|\vec{V}_A| = |\vec{V}_B|$

$4 + 9n^2 + 4 = 4 + 4 + 16p^2$

$p = \frac{-1}{4} \quad \Rightarrow n = \frac{1}{3}$

$\vec{L} = m_A\left(\vec{r}_{A/B} \times \vec{V}_A\right)$

$\vec{r}_{A/B} = (\alpha_1 - \beta_1)\hat{i} + (\alpha_2 - \beta_2)\hat{j} + (\alpha_3 - \beta_3)$

$= (1-2)\hat{i} + (1+1)\hat{j} + 3\hat{k}$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 2 & 3 \\ 2 & 1 & 2 \end{vmatrix} = \hat{i} + 8\hat{j} - 5\hat{k}$$

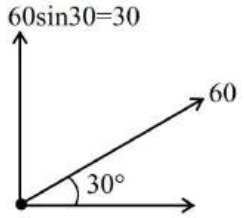
$= \sqrt{1 + 64 + 25} = \sqrt{90}$

50. A particle is projected at an angle of 30° from horizontal at a speed of 60 m/s. The height traversed by the particle in the first second is h_0 and height traversed in the last second, before it reaches the maximum height, is h_1. The ratio $h_0 : h_1$ is _____.

[Take, $g = 10\ m/s^2$]

Ans. (5)

Sol.



$S_1 = 30 \times 1 - \frac{1}{2} \times 10 \times 1 = 25$

$S_3 = 30 + \left(\frac{-10}{2}\right) \times (2 \times 3 - 1) = 5$

$\frac{S_1}{S_3} = \frac{25}{5} = 5$

CHEMISTRY

SECTION-A

51. A solution of aluminium chloride is electrolysed for 30 minutes using a current of 2A. The amount of the aluminium deposited at the cathode is ___ .
[Given : molar mass of aluminium and chlorine are 27 g mol^{-1} and 35.5 g mol^{-1} respectively, Faraday constant = 96500 C mol^{-1}].

(1) 1.660 g (2) 1.007 g
(3) 0.336 g (4) 0.441 g

Ans. (3)

Sol. gm equivalent of Al deposited = $\frac{It}{96500}$

$\frac{w}{27} \times 3 = \frac{2 \times 30 \times 60}{96500}$

w = 0.336 g,

52. Which of the following statement is not true for radioactive decay ?

(1) Amount of radioactive substance remained after three half lives is $\frac{1}{8}$th of original amount.

(2) Decay constant does not depend upon temperature.

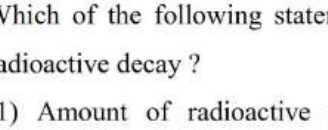
(3) Decay constant increases with increase in temperature.

(4) Half life is ln 2 times of $\frac{1}{\text{rate constant}}$.

Ans. (3)

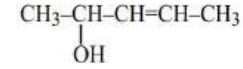
Sol. Decay constant is independent of temperature.

53. How many different stereoisomers are possible for the given molecule ?



(1) 3 (2) 1
(3) 2 (4) 4

Ans. (4)

Sol. 

It has 4 stereoisomers $\begin{bmatrix} \text{R cis} & \text{R trans} \\ \text{S cis} & \text{S trans} \end{bmatrix}$

54. Which of the following electronegativity order is **incorrect**?

(1) Al < Mg < B < N (2) Al < Si < C < N
(3) Mg < Be < B < N (4) S < Cl < O < F

Ans. (1)

Sol.

	Li	Be	B	C	N	O	F
(E.N.)= On pauling scale	1	1.5	2	2.5	3	3.5	4.0

	Na	Mg	Al	Si	P	S	Cl
(E.N.)=	0.9	1.2	1.5	1.8	2.1	2.5	3.0

Correct order Mg < Al < B < N

55. Lanthanoid ions with $4f^7$ configuration are :

(A) Eu^{2+} (B) Gd^{3+} (C) Eu^{3+} (D) Tb^{3+}
(E) Sm^{2+}

Choose the correct answer from the options given below :

(1) (A) and (B) only (2) (A) and (D) only
(3) (B) and (E) only (4) (B) and (C) only

Ans. (1)

Sol. $_{63}Eu^{2+}$ – [Xe] $4f^7 6s^0$

$_{64}Gd^{3+}$ – [Xe] $4f^7\ 5d^0 6s^0$

$_{63}Eu^{3+}$ – [Xe] $4f^6\ 6s^0$

$_{65}Tb^{3+}$ – [Xe] $4f^8\ 6s^0$

$_{62}Sm^{2+}$ – [Xe] $4f^6\ 6s^0$

Eu^{2+} & Gd^{3+}

56. Match **List-I** with **List-II**

	List-I		List-II
(A)	$Al^{3+} < Mg^{2+} < Na^{+} < F^{-}$	(I)	Ionisation Enthalpy
(B)	$B < C < O < N$	(II)	Metallic character
(C)	$B < Al < Mg < K$	(III)	Electronegativity
(D)	$Si < P < S < Cl$	(IV)	Ionic radii

Choose the **correct** answer from the options given below :

(1) A-IV, B-I, C-III, D-II (2) A-II, B-III, C-IV, D-I

(3) A-IV, B-I, C-II, D-III (4) A-III, B-IV, C-II, D-I

Ans. (3)

Sol. Ionic radii – $Al^{3+} < Mg^{2+} < Na^{+} < F^{-}$

Ionisation energy – $B < C < O < N$

Metallic character – $B < Al < Mg < K$

Electron negativity – $Si < P < S < Cl$

57. Which of the following acids is a vitamin ?

(1) Adipic acid (2) Aspartic acid

(3) Ascorbic acid (4) Saccharic acid

Ans. (3)

Sol. Vitamin-C is Ascorbic acid.

58. A liquid when kept inside a thermally insulated closed vessel at 25°C was mechanically stirred from outside. What will be the correct option for the following thermodynamic parameters ?

(1) $\Delta U > 0, q = 0, w > 0$ (2) $\Delta U = 0, q = 0, w = 0$

(3) $\Delta U < 0, q = 0, w > 0$ (4) $\Delta U = 0, q < 0, w > 0$

Ans. (1)

Sol. Thermally insulated $\Rightarrow q = 0$

from Ist law

$\Delta U = q + w$

$\Delta U = w$

$w > 0, \Delta U > 0$

59. Radius of the first excited state of Helium ion is given as :

$a_0 \rightarrow$ radius of first stationary state of hydrogen atom.

(1) $r = \frac{a_0}{2}$ (2) $r = \frac{a_0}{4}$ (3) $r = 4a_0$ (4) $r = 2a_0$

Ans. (4)

Sol. $r = a_0 \frac{n^2}{Z} = a_0 . \frac{(2)^2}{2} = 2a_0.$

60. Given below are two statements :

Statement I : $CH_3 - O - CH_2 - Cl$ will undergo S_N1 reaction though it is a primary halide.

Statement II : $CH_3-\underset{\underset{CH_3}{|}}{\overset{\overset{CH_3}{|}}{C}}-CH_2-Cl$ will not undergo S_N2 reaction very easily though it is a primary halide.

In the light of the above statements, choose the **most appropriate answer** from the options given below :

(1) **Statement I** is incorrect but **Statement II** is correct.

(2) Both **Statement I** and **Statement II** are incorrect

(3) **Statement I** is correct but **Statement II** is incorrect

(4) Both **Statement I** and **Statement II** are correct.

Ans. (4)

Sol. $CH_3–O–CH_2–Cl$ will undergo S_N1 mechanism because $CH_3-O-\overset{+}{C}H_2$ is highly stable.

$H_3C-\underset{\underset{CH_3}{|}}{\overset{\overset{CH_3}{|}}{C}}-CH_2-Cl$ (Neopentyl chloride) will undergo S_N2 mechanism at a slow rate because it's sterically crowded

61. Given below are two statements :

Statement I : One mole of propyne reacts with excess of sodium to liberate half a mole of H_2 gas.

Statement II : Four g of propyne reacts with $NaNH_2$ to liberate NH_3 gas which occupies 224 mL at STP.

In the light of the above statements, choose the **most appropriate answer** from the options given below:

(1) **Statement I** is correct but **Statement II** is incorrect.
(2) Both **Statement I** and **Statement II** are incorrect
(3) **Statement I** is incorrect but **Statement II** is correct
(4) Both **Statement I** and **Statement II** are correct.

Ans. (1)

Sol.

$$\underset{1\,mole}{CH_3-C\equiv CH} + \underset{(excess)}{Na} \rightarrow CH_3-C\equiv \overset{-}{C}\overset{+}{Na} + \underset{\frac{1}{2}\,mole\,H_2}{\frac{1}{2}H_2}\uparrow$$

$$CH_3-C\equiv CH + NaNH_2 \rightarrow CH_3C\equiv \overset{-}{C}\overset{+}{Na} + NH_3$$

4 gm

$$\frac{4}{40} = 0.1\,mole \qquad \frac{0.1\,mole}{2240\,mole}$$

Statement I is correct but **Statement II** is incorrect

62. A vessel at 1000 K contains CO_2 with a pressure of 0.5 atm. Some of CO_2 is converted into CO on addition of graphite. If total pressure at equilibrium is 0.8 atm, then K_P is :

(1) 0.18 atm (2) 1.8 atm (3) 0.3 atm (4) 3 atm.

Ans. (2)

Sol. $CO_2(g) + C(s) \rightleftharpoons 2CO(g)$

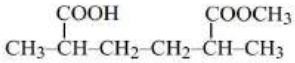
0.5 —
0.5–x 2x

$P_{total} = 0.5 + x = 0.8$

$x = 0.3$

$$K_P = \frac{(0.6)^2}{0.2} = 1.8$$

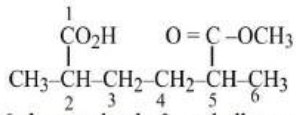
63. The IUPAC name of the following compound is :



(1) 2-Carboxy-5-methoxycarbonylhexane.
(2) Methyl-6-carboxy-2,5-dimethylhexanoate.
(3) Methyl-5-carboxy-2-methylhexanoate.
(4) 6-Methoxycarbonyl-2,5-dimethylhexanoic acid.

Ans. (4)

Sol.



5-Methoxycarbonly-2-methylhexanoic acid

64. Which of the following electrolyte can be sued to obtain $H_2S_2O_8$ by the process of electrolysis?

(1) Dilute solution of sodium sulphate
(2) Dilute solution of sulphuric acid
(3) Concentrated solution of sulphuric acid
(4) Acidified dilute solution of sodium sulphate.

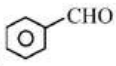
Ans. (3)

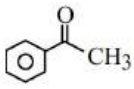
Sol. Theory based.

At anode :

$2HSO_4^- \rightarrow H_2S_2O_8 + 2e^-$

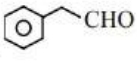
65. The compounds which give positive Fehling's test are :

(A) 

(B) 

(C) $HOCH_2–CO–(CHOH)_3–CH_2–OH$

(D) $CH_3–\overset{O}{\overset{\|}{C}}–H$

(E) 

Choose the **CORRECT** answer from the options given below :

(1) (A),(C) and (D) Only (2) (A),(D) and (E) Only
(3) (C), (D) and (E) Only (4) (A), (B) and (C) Only

Ans. (3)

Sol. $\underset{(C)}{CH_3CH=O}$, $\underset{(D)}{PhCH_2CH=O}$,

$HOCH_2-\underset{\underset{O}{\|}}{C}-(CHOH)_3-CH_2OH$

(E)

All gives positive Fehling test

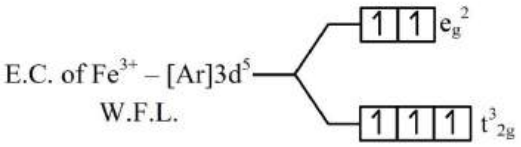
66. In which of the following complexes the CFSE, Δ_0 will be equal to zero?

(1) $[Fe(NH_3)_6]Br_2$ (2) $[Fe(en)_3]Cl_3$

(3) $K_4[Fe(CN)_6]$ (4) $K_3[Fe(SCN)_6]$

Ans. (4)

Sol. For complex $K_3[Fe(SCN)_6]$



Calculation of CFSE

$= (-0.4 \times 3 + 0.6 \times 2)\ \Delta_0$

$= 0\ \Delta_0$

67. Arrange the following solutions in order of their increasing boiling points.

(i) 10^{-4} M NaCl (ii) 10^{-4} M Urea

(iii) 10^{-3} M NaCl (iv) 10^{-2} M NaCl

(1) (ii) < (i) < (iii) < (iv) (2) (ii) < (i) ≅ (iii) < (iv)

(3) (i) < (ii) < (iii) < (iv) (4) (iv) < (iii) < (i) < (ii)

Ans. (1)

Sol. $\Delta T_b = i\ K_b\ .\ m\ .\ \propto$ i.C.

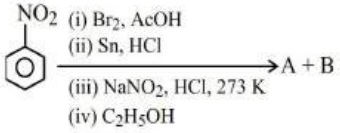
where C = concentration

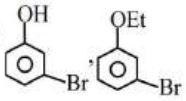
Options	i.C.
(i)	2×10^{-4}
(ii)	1×10^{-4}
(iii)	2×10^{-3}
(iv)	2×10^{-2}

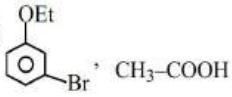
B.P. order :

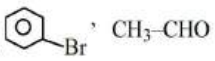
(ii) < (i) < (iii) < (iv)

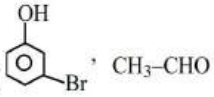
68. The products formed in the following reaction sequence are :



(1) 

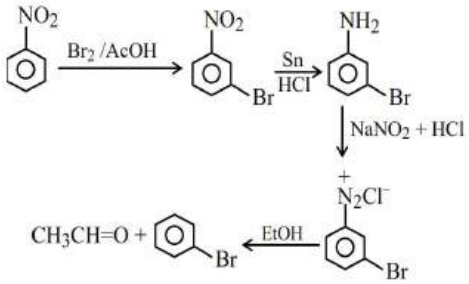
(2) 

(3) 

(4) 

Ans. (3)

Sol.



69. From the magnetic behaviour of $[NiCl_4]^{2-}$ (paramagnetic) and $[Ni(CO)_4]$ (diamagnetic), choose the correct geometry and oxidation state.

(1) $[NiCl_4]^{2-}$: Ni^{II}, square planar

$[Ni(CO)_4]$: Ni(0), square planar

(2) $[NiCl_4]^{2-}$: Ni^{II}, tetrahedral

$[Ni(CO)_4]$: Ni(0), tetrahedral

(3) $[NiCl_4]^{2-}$: Ni^{II}, tetrahedral

$[Ni(CO)_4]$: Ni^{II}, square planar

(4) $[NiCl_4]^{2-}$: Ni(0), tetrahedral

$[Ni(CO)_4]$: Ni(0), square planar

Ans. (2)

Sol. $[NiCl_4]^{2-}$

Ni^{+2} – [Ar] $3d^8\ 4s^0 \rightarrow sp^3$, Tetrahedral

Number of unpaired electron = 2 paramagentic

$[Ni(CO)_4]$,

Ni(0) → [Ar] $3d^{10}\ 4s^0$ (After rearrangement)

No unpaired electron

sp^3, Tetrahedral, Diamagnetic

70. The **incorrect** statements regarding geometrical isomerism are :

(A) Propene shows geometrical isomerism.

(B) Trans isomer has identical atoms/groups on the opposite sides of the double bond.

(C) Cis-but-2-ene has higher dipole moment than trans-but-2-ene.

(D) 2-methylbut-2-ene shows two geometrical isomers.

(E) Trans-isomer has lower melting point that cis isomer.

Choose the **CORRECT** answer from the options given below :

(1) (A), (D) and (E) only (2) (C), (D) and (E) only

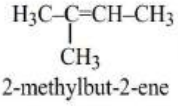
(3) (B) and (C) only (4) (A) and (E) only

Ans. (1)

Sol. (A) $CH_3–CH=CH_2$. GI is not possible

(B) Trans isomer has identical atoms/groups on the opposite side of double bond.

(C) (dipole moment only)

(D)  (does not show GI)

2-methylbut-2-ene

(E) (Melting point)

SECTION-B

71. Some CO_2 gas was kept in a sealed container at a pressure of 1 atm and at 273 K. This entire amount of CO_2 gas was later passed through an aqueous solution of $Ca(OH)_2$. The excess unreacted $Ca(OH)_2$ was later neutralized with 0.1 M of 40 mL HCl. If the volume of the sealed container of CO_2 was x, then x is _____ cm^3 (nearest integer).

[Given : The entire amount of $CO_2(g)$ reacted with exactly half the initial amount of $Ca(OH)_2$ present in the aqueous solution.]

Ans. (45)

Sol. Let moles of CO_2 = n

moles of $Ca(OH)_2$ total initially = 2n

excess $Ca(OH)_2$ = n

gm equivalent of $Ca(OH)_2$ = gm equivalent of HCl

$$n \times 2 = 0.1 \times \frac{40}{1000} \times 1$$

$$n = 2 \times 10^{-3}$$

Volume of $CO_2 = 2 \times 10^{-3} \times 22400 = 44.8\ cm^3$

72. In Carius method for estimation of halogens, 180 mg of an organic compound produced 143.5 mg of AgCl. The percentage composition of chlorine in the compound is _____ %.

[Given : molar mass in g mol^{-1} of Ag : 108, Cl = 35.5]

Ans. (20)

Sol. $$n_{Cl} = n_{AgCl} = \frac{143.5 \times 10^{-3}}{143.5} = 10^{-3}$$

$$\%\ Cl = \frac{10^{-3} \times 35.5}{180 \times 10^{-3}} \times 100 = 19.72$$

73. The number of molecules/ions that show linear geometry among the following is _____ .

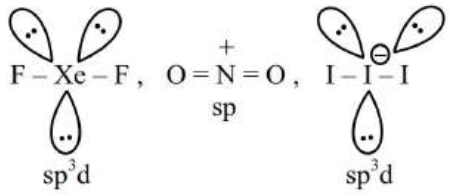
SO_2, $BeCl_2$, CO_2, N_3^-, NO_2, F_2O, XeF_2, NO_2^+, I_3^-, O_3

Ans. (6)

Sol. Linear species are

$Cl – Be – Cl$, $O = C = O$, $N^- = N^+ = N^-$

(sp) (sp) (sp)



74. $A \rightarrow B$

The molecule A changes into its isomeric form B by following a first order kinetics at a temperature of 1000 K. If the energy barrier with respect to reactant energy for such isomeric transformation is 191.48 kJ mol^{-1} and the frequency factor is 10^{20}, the time required for 50%, molecules of A to become B is _____ picoseconds (nearest integer).

[R = 8.314 J K^{-1} mol^{-1}]

Ans. (69)

Sol. $$t_{1/2} = \frac{0.693}{K}$$

$$K = Ae^{-Ea/RT}$$

$$= 10^{20} \times e^{-\frac{191.48 \times 10^3}{8.314 \times 1000}}$$

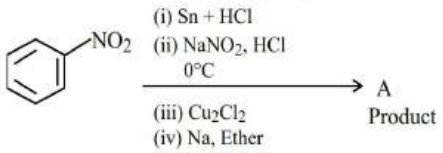
$$= 10^{20} \times e^{-23.031} = 10^{20} \times -e^{\ln 10 \times 10}$$

$$= \frac{10^{20}}{10^{10}} = 10^{10}\ sec.$$

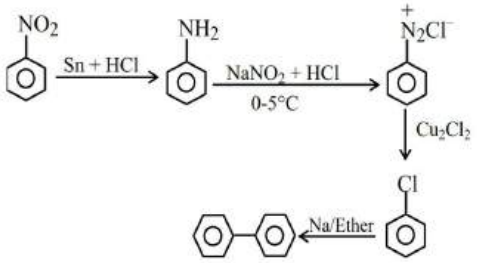
$$t_{1/2} = \frac{0.693}{10^{10}} = 6.93 \times 10^{-11}$$

$$= 69.3 \times 10^{-12}\ sec.$$

75. Consider the following sequence of reactions :



Molar mass of the product formed (A) is _____ g mol^{-1} .

Ans. (154)

Sol.