Solutions to JEE(Main) -2021

Test Date: 20th July 2021 (Second Shift)

PHYSICS, CHEMISTRY & MATHEMATICS

Paper - 1

Time Allotted: 3 Hours **Maximum Marks: 300**

- Please read the instructions carefully. You are allotted 5 minutes specifically for this purpose.

Important Instructions:

1. The test is of 3 hours duration.
2. This test paper consists of 90 questions. Each subject (PCM) has 30 questions. The maximum marks are 300.
3. This question paper contains **Three Parts. Part-A** is Physics, **Part-B** is Chemistry and **Part-C** is Mathematics. Each part has only two sections: **Section-A and Section-B**.
4. **Section – A** : Attempt all questions.
5. **Section – B :** Do any 5 questions out of 10 Questions.
6. **Section-A (01 – 20)** *contains 20 multiple choice questions which have* ***only one correct answer****. Each question carries* ***+4 marks*** *for correct answer and* ***–1 mark*** *for wrong answer.*
7. ***Section-B (01 – 10)*** *contains 10 Numerical based questions with answer as numerical value. Each question carries* ***+4 marks*** *for correct answer. There is no negative marking.*

PART – A (PHYSICS)

SECTION - A

(One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.

Q1. For a series LCR circuit with $R = 100\,\Omega$, $L = 0.5\,mH$ and $C = 0.1\,pF$ connected across $220V - 50Hz$ AC supply, the phase angle between current and supplied voltage and the nature of the circuit is :

(A) $\approx 90^\circ$, predominantly inductive circuit
(B) 0°, resonance circuit
(C) $\approx 90^\circ$, predominantly capacitive circuit
(D) 0°, resistive circuit

Q2. If the kinetic energy of a moving body becomes four times its initial kinetic energy, then the percentage change in its momentum will be :

(A) 100%
(B) 200%
(C) 400%
(D) 300%

Q3. If time (t), velocity (υ), and angular momentum (ℓ) are taken as the fundamental units. Then the dimension of mass (m) in terms of $t, \upsilon,$ and ℓ is :

(A) $\left[t^{-2}\upsilon^{-1}\ell^{1}\right]$
(B) $\left[t^{-1}\upsilon^{1}\ell^{-2}\right]$
(C) $\left[t^{-1}\upsilon^{-2}\ell^{1}\right]$
(D) $\left[t^{1}\upsilon^{2}\ell^{-1}\right]$

Q4. An electron having de-Broglie wavelength λ is indent on a target in a X-ray tube. Cut-off wavelength of emitted X-ray is:

(A) 0
(B) $\frac{2mc\,\lambda^2}{h}$
(C) $\frac{hc}{mc}$
(D) $\frac{2m^2c^2\lambda^2}{h^2}$

Q5. Two small drops of mercury each of radius R coalesce to from a single large drop. The ratio of total surface energy before and after the change is :

(A) $2^{\frac{1}{3}} : 1$
(B) $1 : 2^{\frac{1}{3}}$
(C) $2 : 1$
(D) $1 : 2$

Q6. Two vectors $\vec{P}$ and $\vec{Q}$ have equal magnitudes. If the magnitude of $\vec{P}+\vec{Q}$ is n times the magnitude of $\vec{P}-\vec{Q}$, then angle between $\vec{P}$ and $\vec{Q}$ is :

(A) $\sin^{-1}\left(\frac{n^2-1}{n^2+1}\right)$
(B) $\cos^{-1}\left(\frac{n^2-1}{n^2+1}\right)$
(C) $\sin^{-1}\left(\frac{n-1}{n+1}\right)$
(D) $\cos^{-1}\left(\frac{n-1}{n+1}\right)$

Q7. A boy reaches the airport and finds that the escalator is not working. He walks up the stationary escalator in time t_1. If he remains stationary on a moving escalator then the escalator takes him up in time t_2. The time taken by him to walk up on the moving escalator will be :

(A) $\frac{t_1 t_2}{t_2 + t_1}$ (B) $t_2 - t_1$

(C) $\frac{t_1 + t_2}{2}$ (D) $\frac{t_1 t_2}{t_2 - t_1}$

Q8. The correct relation between the degrees of freedom f and the ratio of specific heat γ is :

(A) $f = \frac{2}{\gamma + 1}$ (B) $f = \frac{2}{\gamma - 1}$

(C) $f = \frac{1}{\gamma + 1}$ (D) $f = \frac{\gamma + 1}{2}$

Q9. For a certain radioactive process the graph between $\ell n(R)$ and t(sec) is obtained as shown in the figure. Then the value of half life for the unknown radioactive material is approximately :

(A) 2.62 sec

(B) 9.15 sec

(C) 4.62 sec

(D) 6.93 sec

Q10. A body rolls down an inclined plane without slipping. The kinetic energy of rotation is 50% of its translational kinetic energy. The body is :

(A) Hollow cylinder (B) Ring

(C) Solid sphere (D) Solid cylinder

Q11. In an electromagnetic wave the electric field vector and magnetic field vector are given as $\vec{E} = E_0\hat{i}$ and $\vec{B} = B_0\hat{k}$ respectively. The direction of propagation of electromagnetic wave is along.

(A) $(-\hat{k})$ (B) $(\hat{k})$

(C) $(-\hat{j})$ (D) $\hat{j}$

Q12. The length of a metal wire is ℓ_1, when the tension in it is T_1 and is ℓ_2 when the tension is T_2. The natural length of the wire is :

(A) $\sqrt{\ell_1 \ell_2}$ (B) $\frac{\ell_1 + \ell_2}{2}$

(C) $\frac{\ell_1 T_2 - \ell_2 T_1}{T_2 - T_1}$ (D) $\frac{\ell_1 T_2 + \ell_2 T_1}{T_2 + T_1}$

Q13. A body at rest is moved along a horizontal straight line by a machine delivering a constant power. The distance moved by the body in time 't' is proportional to :

(A) $t^{\frac{3}{4}}$ (B) $t^{\frac{3}{2}}$

(C) $t^{\frac{1}{2}}$ (D) $t^{\frac{1}{4}}$

Q14. Which of the following graphs represent the behaviour of an ideal gas? Symbols have their usual meaning.

(A)

(B)

(C)

(D)

Q15. With what speed should a galaxy move outward with respect to earth so that the sodium –D line at wavelength 5890 Å is observed at 5896 Å ?

(A) 322 km / sec (B) 296 km / sec

(C) 306 km / sec (D) 336 km / sec

Q16. A particle is making simple harmonic motion along the X-axis. If at a distances x_1 and x_2 from the mean position the velocities of the particle are υ_1 and υ_2 respectively. The time period of its oscillation is given as :

(A) $T = 2\pi\sqrt{\frac{x_2^2 - x_1^2}{\upsilon_1^2 - \upsilon_2^2}}$ (B) $T = 2\pi\sqrt{\frac{x_2^2 + x_1^2}{\upsilon_1^2 - \upsilon_2^2}}$

(C) $T = 2\pi\sqrt{\frac{x_2^2 - x_1^2}{\upsilon_1^2 + \upsilon_2^2}}$ (D) $T = 2\pi\sqrt{\frac{x_2^2 + x_1^2}{\upsilon_1^2 + \upsilon_2^2}}$

Q17. A satellite is launched into a circular orbit of radius R around earth, while a second satellite is launched into a circular orbit of radius 1.02R. The percentage difference in the time periods of the two satellites is :

(A) 0.7 (B) 3.0

(C) 2.0 (D) 1.5

Q18. Consider a binary star system of star A and star B with masses m_A and m_B revolving in a circular orbit of radii r_A and r_B, respectively. If T_A and T_B are the time period of star A and star B, respectively, then :

(A) $T_A = T_B$ (B) $T_A > T_B$ (if $m_A > m_B$)

(C) $\frac{T_A}{T_B} = \left(\frac{r_A}{r_B}\right)^{\frac{3}{2}}$ (D) $T_A > T_B$ (if $r_A > r_B$)

Q19. At an angle of 30° to the magnetic meridian, the apparent dip is 45°. Find the true dip:

(A) $\tan^{-1}\frac{\sqrt{3}}{2}$

(B) $\tan^{-1}\sqrt{3}$

(C) $\tan^{-1}\frac{1}{\sqrt{3}}$

(D) $\tan^{-1}\frac{2}{\sqrt{3}}$

Q20. The magnetic susceptibility of a material of a rod is 499. Permeability in vacuum is $4\pi\times10^{-7}$ H/m. Absolute permeability of the material of the rod is:

(A) $3\pi\times10^{-4}$ H/m

(B) $\pi\times10^{-4}$ H/m

(C) $2\pi\times10^{-4}$ H/m

(D) $4\pi\times10^{-4}$ H/m

SECTION - B

(Numerical Answer Type)

This section contains **10** questions. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the **second decimal place**).

Q1. A radioactive substance decays to $\left(\frac{1}{16}\right)^{th}$ of this initial activity in 80 days. The half life of the radioactive substance expressed in days is _____.

Q2. A certain metallic surface is illuminated by monochromatic radiation of wavelength λ. The stopping potential for photoelectric current for this radiation is $3V_0$. If the same surface is illuminated with a radiation of wavelength 2λ, the stopping potential is V_0. The threshold wavelength of this surface for photoelectric effect is _____ λ.

Q3. Two bodies, a ring and a solid cylinder of same material are rolling down without slipping an inclined plane. The radii of the bodies are same. The ratio of velocity centre of mass at the bottom of the inclined plane of the ring to that of the cylinder is $\frac{\sqrt{x}}{2}$. Then, the value of x is _____.

Q4. A series LCR circuit of $R = 5\Omega$, $L = 20\,mH$ and $C = 0.5\,\mu F$ is connected across an AC supply of 250 V, having variable frequency. The power dissipated at resonance condition is _____ $\times 10^2$ W.

Q5. A body rotating with an angular speed of $600\,rpm$ is uniformly accelerated to $1800\,rpm$ in 10 sec. The number of rotations made in the process is _____.

Q6. For the forward biased diode characteristics shown in the figure, the dynamic resistance at $I_D = 3\,mA$ will be _____ Ω.

Q7. One mole of an ideal gas at $27^{\circ}C$ is taken from A to B as shown in the given PV indicator diagram. The work done by the system will be _____ $\times 10^{-1}$ J.
[Given : $R = 8.3\,J/\text{mole K}$, $\ell n 2 = 0.6931$]
(Round off to the nearest integer)

Q8. In the given figure switches S_1 and S_2 are in open condition. The resistance across ab when the switches S_1 and S_2 are closed is _____ Ω.

Q9. A body of mass 'm' is launched up on a rough inclined plane making an angle of 30° with the horizontal. The coefficient of friction between the body and plane is $\frac{\sqrt{x}}{5}$ if the time of ascent is half of the time of descent. The value of x is _____.

Q10. A zener diode having zener voltage 8 V and power dissipation rating of $0.5\,W$ is connected across a potential divider arranged with maximum potential drop across zener diode is as shown in the diagram. The value of protective resistance R_p is _____ Ω.

PART – B (CHEMISTRY)

SECTION - A

(One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.

Q1.

Major product P of above reaction, is :

(A)

(B)

(C)

(D)

Q2. Outermost electronic configuration of a group 13 element, E, is $4s^2, 4p^1$. The electronic configuration of an element of p-block period-five placed diagonally to element, E is:

(A) $[Kr]4d^{10}5s^2 5p^2$
(B) $[Kr]3d^{10}4s^2 4p^2$
(C) $[Xe]5d^{10}6s^2 6p^2$
(D) $[Ar]3d^{10}4s^2 4p^2$

Q3.

In the above reactions, product A and product B respectively are:

(A)

,

(B)

,

(C)

,

(D)

,

Q4. Bakelite is a cross- linked polymer of formaldehyde and :

(A) PHBV
(B) Buna – S
(C) Novolac
(D) Dacron

Q5. Consider two chemical reactions (A) and (B) that take place during metallurgical process:

(a) $ZnCO_{3(s)} \xrightarrow{\Delta} ZnO_{(s)} + CO_{2(g)}$

(b) $2ZnS_{(s)} + 3O_{2(g)} \xrightarrow{\Delta} 2ZnO_{(s)} + 2SO_{2(g)}$

The correct option of names given to them respectively is :

(A) (a) is calcination and (b) is roasting
(B) Both (a) and (b) are producing same product so both are calcination
(C) Both (a) and (b) are producing same product so both are roasting
(D) (a) is roasting and (b) is calcination

Q6.

(a)

(b)

(c)

(d)

The correct order of their reactivity towards hydrolysis at room temperature is:

(A) (d) > (b) > (a) > (c)
(B) (d) > (a) > (b) > (c)
(C) (a) > (c) > (b) > (d)
(D) (a) > (b) > (c) > (d)

Q7. Spin only magnetic moment of an octahedral complex of Fe^{2+} in the presence of a strong field ligand in BM is :

(A) 2.82 (B) 3.46
(C) 0 (D) 4.89

Q8. Cu^{2+} salt reacts with potassium iodide to give :

(A) Cu_2I_3 (B) Cu_2I_2
(C) $Cu(I_3)_2$ (D) CuI

Q9. Which one of the following statements is not true about enzymes ?

(A) The action of enzymes is temperature and pH specific.
(B) Enzymes are non-specific for a reaction and substrate.
(C) Enzymes work as catalysts by lowering the activation energy of a biochemical reaction.
(D) Almost all enzymes are proteins.

Q10.

Consider the above reaction, compound B is :

(A)

(B)

(C)

(D)

Q11. Which one of the following gases is reported to retard photosynthesis ?

(A) CO_2 (B) CFCs
(C) CO (D) NO_2

Q12. Benzene on nitration gives nitrobenzene in presence of HNO_3 and H_2SO_4 mixture, where:

(A) both H_2SO_4 and HNO_3 act as a bases

(B) both H_2SO_4 and HNO_3 act as an acids

(C) HNO_3 acts as a base and H_2SO_4 acts as an acid

(D) HNO_3 acts as an acid and H_2SO_4 acts as a base

Q13. Which one of the following species doesn't have a magnetic moment of 1.73 BM, (spin only value)?

(A) CuI

(B) $\left[Cu(NH_3)_4\right]Cl_2$

(C) O_2^-

(D) O_2^+

Q14. Which one of the following pairs of isomers is an example of metamerism?

(A)

(B) $CH_3\ CH_2\ CH_2\ CH_2\ CH_3$ and

(C)

(D)

Q15. The hybridisations of the atomic orbitals of nitrogen in NO_2^-, NO_2^+ and NH_4^+ respectively are :

(A) sp^3, sp and sp^2

(B) sp, sp^2 and sp^3

(C) sp^2, sp and sp^3

(D) sp^3, sp^2 and sp

Q16. Metallic sodium does not react normally with :

(A) gaseous ammonia

(B) tert – butyl alcohol

(C) But – 2 – yne

(D) Ethyne

Q17. In Carius method, halogen containing organic compound is heated with fuming nitric acid in the presence of :

(A) $AgNO_3$

(B) HNO_3

(C) $BaSO_4$

(D) $CuSO_4$

Q18. The single largest industrial application of dihydrogen is :

(A) Rocket fuel in space research
(B) In the synthesis of nitric acid
(C) In the synthesis of ammonia
(D) Manufacture of metal hydrides

Q19. The major product (P) in the following reaction is :

(A)

(B)

(C)

(D)

Q20. A solution is 0.1 M in Cl^- and 0.001 M in CrO_4^{2-}. Solid $AgNO_3$ is gradually added to it. Assuming that the addition does not change in volume and $K_{sp}(AgCl) = 1.7 \times 10^{-10} M^2$ and $K_{sp}(Ag_2CrO_4) = 1.9 \times 10^{-12} M^3$.

Select correct statement from the following :

(A) Ag_2CrO_4 precipitates first because the amount of Ag^+ needed is low.
(B) Ag_2CrO_4 precipitates first as its K_{sp} is low.
(C) AgCl will precipitate first as the amount of Ag^+ needed to precipitate is low.
(D) AgCl precipitates first because its K_{sp} is high.

SECTION - B

(Numerical Answer Type)

This section contains **10** questions. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the **second decimal place**).

Q1. 4g equimolar mixture of $NaOH$ and Na_2CO_3 contains x g of $NaOH$ and y g of Na_2CO_3. The value of x is _____ g.(Nearest Integer)

Q2. When 0.15 g of an organic compound was analyzed using carius method for estimation of bromine, 0.2397 g of AgBr was obtained. The percentage of bromine in the organic compound is _____. .(Nearest Integer) (Atomic mass: Ag = 108, Br = 80)

Q3. For a given chemical reaction $A \rightarrow B$ at 300 K the free energy change is – 49.4 kJ mol^{-1} and the enthalpy of reaction is 51.4 kJ mol^{-1}. The entropy change of the reaction is _____ J K^{-1} mol^{-1}.

Q4. The wavelength of electrons accelerated from rest through a potential difference of 40 kV is $x \times 10^{-12}$m. The value of x is _____. .(Nearest Integer)

Given : Mass of electron $= 9.1 \times 10^{-31}$ kg

Charge on an electron $= 1.6 \times 10^{-19}$ C

Planck's constant $= 6.63 \times 10^{-34}$ Js

Q5. 100 ml of 0.0018% (w/v) solution of Cl^- ion was the minimum concentration of Cl^- required to precipitate a negative sol in one h. The coagulating value of Cl^- ion is _____. (Nearest integer)

Q6. An aqueous solution of $NiCl_2$ was heated with excess sodium cyanide in presence of strong oxidizing agent to form $\left[Ni(CN)_6\right]^{2-}$. The total change in number of unpaired electrons on metal centre is _____.

Q7. Diamond has a three dimensional structure of C atoms formed by covalent bonds. The structure of diamond has face centred cubic lattice where 50% of the tetrahedral voids are also occupied by carbon atoms. The number of carbon atoms present per unit cell of diamond is _____.

Q8. The vapour pressures of A and B at 25°C are 90mm Hg and 15 mm Hg respectively. If A and B are mixed such that the mole fraction of A in the mixture is 0.6, then the mole fraction of B in the vapour phase is $x \times 10^{-1}$. The value x is _____. .(Nearest Integer)

Q9. $PCl_5(g) \rightarrow PCl_3(g) + Cl_2(g)$

In the above first order reaction, the concentration of PCl_5 reduces from initial concentration 50 mol L^{-1} to 10 mol L^{-1} in 120 minutes at 300K. The rate constant for the reaction at 300K is $x \times 10^{-2} \text{ min}^{-1}$. the value of x is _____.

$[\text{Given } \log 5 = 0.6989]$

Q10. Potassium chlorate is prepared by electrolysis of KCl in basic solution as shown by following equation.

$$6\,OH^- + Cl^- \rightarrow ClO_3^- + 3H_2O + 6e^-$$

A current of xA has to be passed for 10 h to produce $10.0\,g$ of potassium chlorate. The value of x is _____. (Nearest Integer)

(Molar mass of $KClO_3 = 122.6\,g\,mol^{-1}, F = 96500\,C$)

PART – C (MATHEMATICS)

SECTION - A

(One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.

Q1. Consider the line L given by the equation $\frac{x-3}{2}=\frac{y-1}{1}=\frac{z-2}{1}$. Let Q be the mirror image of the point $(2,3,-1)$ with respect to L. Let a plane P be such that it passes through Q, and the line L is perpendicular to P. Then which of the following points is one the plane P ?

(A) $(1,2,2)$ (B) $(-1,1,2)$

(C) $(1,1,1)$ (D) $(1,1,2)$

Q2. If the real part of the complex number $(1-\cos\theta+2i\sin\theta)^{-1}$ is $\frac{1}{5}$ for $\theta\in(0,\pi)$, then the value of the integral $\int_0^{\theta}\sin x\,dx$ is equal to :

(A) -1 (B) 0

(C) 2 (D) 1

Q3. The value of $k\in R$, for which the following system of linear equations

$3x-y+4z=3,$

$x+2y-3z=-2,$

$6x+5y+kz=-3,$

has infinitely many solutions, is :

(A) 3 (B) -3

(C) -5 (D) 5

Q4. For the natural numbers m,n, if $(1-y)^m(1+y)^n=1+a_1y+a_2y^2+....+a_{m+n}\,y^{m+n}$ and $a_1=a_2=10$, then the value of $(m+n)$ is equal to :

(A) 64 (B) 88

(C) 80 (D) 100

Q5. The value of $\tan\left(2\tan^{-1}\left(\frac{3}{5}\right)+\sin^{-1}\left(\frac{5}{13}\right)\right)$ is equal to :

(A) $\frac{151}{63}$ (B) $\frac{-291}{76}$

(C) $\frac{220}{21}$ (D) $\frac{-181}{69}$

Q6. If $f:R\to R$ is given by $f(x)=x+1$, then the value of

$$\lim_{n\to\infty}\frac{1}{n}\left[f(0)+f\left(\frac{5}{n}\right)+f\left(\frac{10}{n}\right)+....+f\left(\frac{5(n-1)}{n}\right)\right],$$ is :

(A) $\frac{3}{2}$ (B) $\frac{1}{2}$

(C) $\frac{7}{2}$ (D) $\frac{5}{2}$

Q7. Let P be a variable point on the parabola $y=4x^2+1$. Then, the locus of the mid-point of the point P and the foot of the perpendicular drawn from the point P to the line y = x is :

(A) $2(x-3y)^2+(3x-y)+2=0$ (B) $(3x-y)^2+(x-3y)+2=0$

(C) $2(3x-y)^2+(x-3y)+2=0$ (D) $(3x-y)^2+2(x-3y)+2=0$

Q8. The sum of all the local minimum values of the twice differentiable function $f:R\to R$ defined by $f(x)=x^3-3x^2-\frac{3f''(2)}{2}x+f''(1)$ is :

(A) -22 (B) 5

(C) 0 (D) -27

Q9. Let r_1 and r_2 be the radii of the largest and smallest circles, respectively, which pass through the point $(-4,1)$ and having their centres on the circumference of the circle $x^2+y^2+2x+4y-4=0$. If $\frac{r_1}{r_2}=a+b\sqrt{2}$, then $a+b$ is equal to :

(A) 5 (B) 11

(C) 7 (D) 3

Q10. If the mean and variance of six observations 7, 10, 11, 15, a, b are 10 and $\frac{20}{3}$, respectively, then the value of $|a-b|$ is equal to

(A) 9 (B) 7

(C) 11 (D) 1

Q11. Let $y=y(x)$ satisfies the equation $\frac{dy}{dx}-|A|=0$, for all $x>0$, where $A=\begin{bmatrix} y & \sin x & 1 \\ 0 & -1 & 1 \\ 2 & 0 & \frac{1}{x}\end{bmatrix}$. If $y(\pi)=\pi+2$, then the value of $y\left(\frac{\pi}{2}\right)$ is :

(A) $\frac{\pi}{2}+\frac{4}{\pi}$ (B) $\frac{3\pi}{2}-\frac{1}{\pi}$

(C) $\frac{\pi}{2}-\frac{4}{\pi}$ (D) $\frac{\pi}{2}-\frac{1}{\pi}$

Q12. Let $f : R - \left\{\frac{\alpha}{6}\right\} \to R$ be defined by $f(x) = \frac{5x+3}{6x-\alpha}$. Then the value of α for which $(fof)(x) = x$, for all $x \in R - \left\{\frac{\alpha}{6}\right\}$, is :

(A) 6
(B) 8
(C) No such α exists
(D) 5

Q13. The lines $x = ay - 1 = z - 2$ and $x = 3y - 2 = bz - 2, (ab \neq 0)$ are coplanar, if :

(A) $a = 2, b = 2$
(B) $b = 1, a \in R - \{0\}$
(C) $a = 2, b = 3$
(D) $a = 1, b \in R - \{0\}$

Q14. In a triangle ABC, if $|\overrightarrow{BC}| = 3, |\overrightarrow{CA}| = 5$ and $|\overrightarrow{BA}| = 7$, then the projection of the vector $\overrightarrow{BA}$ on $\overrightarrow{BC}$ is equal to :

(A) $\frac{19}{2}$
(B) $\frac{13}{2}$
(C) $\frac{15}{2}$
(D) $\frac{11}{2}$

Q15. Let $g(t) = \int_{-\pi/2}^{\pi/2} \cos\left(\frac{\pi}{4}t + f(x)\right)dx$, where $f(x) = \log_e\left(x + \sqrt{x^2+1}\right), x \in R$. Then which one of the following is correct?

(A) $g(1) = g(0)$
(B) $g(1) = \sqrt{2}g(0)$
(C) $g(1) + g(0) = 0$
(D) $\sqrt{2}g(1) = g(0)$

Q16. Consider the following three statements :
(A) if 3 + 3 = 7 then 4 + 3 = 8.
(B) If 5 + 3 = 8 then earth is flat.
(C) If both (A) and (B) are true then 5 + 6 = 17.
Then, which of the following statements is correct?

(A) (A) and (C) are true while (B) is false
(B) (A) and (B) are false while (C) is true
(C) (A) is true while (B) and (C) are false
(D) (A) is false, but (B) and (C) are true

Q17. If sum of the first 21 terms of the series $\log_{9^{1/2}} x + \log_{9^{1/3}} x + \log_{9^{1/4}} x + ...$, where x > 0 is 504, then x is equal to :

(A) 7
(B) 9
(C) 243
(D) 81

Q18. If [x] denotes the greatest integer less than or equal to x, then the value of the integral $\int_{-\pi/2}^{\pi/2} \left[[x] - \sin x\right] dx$ is equal to :

(A) $-\pi$
(B) 0
(C) 1
(D) π

Q19. Let in a right angled triangle, the smallest angle be θ. If a triangle formed by taking the reciprocal of its sides is also a right angled triangle, then $\sin\theta$ is equal to :

(A) $\frac{\sqrt{2}-1}{2}$

(B) $\frac{\sqrt{5}+1}{4}$

(C) $\frac{\sqrt{5}-1}{2}$

(D) $\frac{\sqrt{5}-1}{4}$

Q20. Let A, B and C be three events such that the probability that exactly one of A and B occurs is (1 – k), the probability that exactly one of B and C occurs is (1 – 2k), the probability that exactly one of C and A occurs is (1 – k) and the probability of all A, B and C occur simultaneously is k^2, where $0 < k < 1$. Then the probability that at least one of A, B and C occur is :

(A) greater than $\frac{1}{2}$

(B) greater than $\frac{1}{4}$ but less than $\frac{1}{2}$

(C) exactly equal to $\frac{1}{2}$

(D) greater than $\frac{1}{8}$ but less than $\frac{1}{4}$

SECTION - B

(Numerical Answer Type)

This section contains **10** questions. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the **second decimal place**).

Q1. Let $\{a_n\}_{n=1}^{\infty}$ be a sequence such that $a_1 = 1, a_2 = 1$ and $a_{n+2} = 2a_{n+1} + a_n$ for all $n \geq 1$. Then the value of $47\sum_{n=1}^{\infty}\frac{a_n}{2^{3n}}$ is equal to.....

Q2. If $\lim_{x\to 0}\frac{axe^x - \beta\log_e(1+x) + \gamma x^2 e^{-x}}{x\sin^2 x} = 10, \alpha, \beta, \gamma \in R$, then the value of $\alpha + \beta + \gamma$ is.....

Q3. Consider a triangle having vertices $A(-2,3), B(1,9)$ and $C(3,8)$. If a line L passing through the circum-center of triangle ABC, bisects line BC, and intersects y-axis at point $\left(0, \frac{\alpha}{2}\right)$, then the value of real number α is.....

Q4. If the point on the curve $y^2 = 6x$, nearest to the point $\left(3, \frac{3}{2}\right)$ is (α, β), then $2(\alpha + \beta)$ is equal to.....

Q5. Let $A = \{a_{ij}\}$ be a 3 x 3 matrix, where

$$a_{ij} = \begin{cases} (-1)^{j-i} & \text{if } i < j, \\ 2 & \text{if } i = j, \\ (-1)^{i+j} & \text{if } i > j, \end{cases}$$

then $\det\left(3\,\text{Adj}\left(2A^{-1}\right)\right)$ is equal to.....

Q6. For $p > 0$, a vector $\vec{v}_2 = 2\hat{i} + (p+1)\hat{j}$ is obtained by rotating the vector $\vec{v}_1 = \sqrt{3}\,p\hat{i} + \hat{j}$ by an angle θ about origin in counter clockwise direction. If $\tan\theta = \frac{\left(\alpha\sqrt{3} - 2\right)}{\left(4\sqrt{3} + 3\right)}$, then the value of α is equal to.....

Q7. Let a curve $y = y(x)$ be given by the solution of the differential equation $\cos\left(\frac{1}{2}\cos^{-1}\left(e^{-x}\right)\right)dx = \sqrt{e^{2x} - 1}\,dy$

If it intersects y-axis at $y = -1$, and the intersection point of the curve with x-axis is $(\alpha, 0)$, then e^{α} is equal to.....

Q8. The number of solutions of the equation $\log_{(x+1)}\left(2x^2+7x+5\right)+\log_{(2x+5)}\left(x+1\right)^2-4=0,\ x>0,$ is.....

Q9. For $k \in N$, let $\dfrac{1}{\alpha(\alpha+1)(\alpha+2).....(\alpha+20)}=\sum_{k=0}^{20}\dfrac{A_k}{a+k}$, where $\alpha>0$. Then the value of $100\left(\dfrac{A_{14}+A_{15}}{A_{13}}\right)^2$ is equal to.....

Q10. Let a function $g:[0,4]\to R$ be defined as $g(x)=\begin{cases}\max\limits_{0\le t\le x}\left\{t^3-6t^2+9t-3\right\}, & 0\le x\le 3\\ 4-x & ,3<x\le 4\end{cases}$, then the number of points in the interval $(0,4)$ where $g(x)$ is NOT differentiable, is.....

KEYS to JEE (Main)-2021

PART – A (PHYSICS)

SECTION - A

1.	**C**	2.	**A**	3.	**C**	4.	**B**
5.	**A**	6.	**B**	7.	**A**	8.	**B**
9.	**C**	10.	**D**	11.	**C**	12.	**C**
13.	**B**	14.	**C**	15.	**C**	16.	**A**
17.	**B**	18.	**A**	19.	**A**	20.	**C**

SECTION - B

1.	**20**	2.	**4**	3.	**3**	4.	**125**
5.	**200**	6.	**25**	7.	**17258**	8.	**10**
9.	**3**	10.	**192**				

PART – B (CHEMISTRY)

SECTION - A

1.	**D**	2.	**A**	3.	**B**	4.	**C**
5.	**A**	6.	**D**	7.	**C**	8.	**B**
9.	**B**	10.	**D**	11.	**D**	12.	**C**
13.	**A**	14.	**C**	15.	**C**	16.	**C**
17.	**A**	18.	**C**	19.	**D**	20.	**C**

SECTION - B

1.	**1**	2.	**68**	3.	**336**	4.	**6**
5.	**1**	6.	**2**	7.	**8**	8.	**1**
9.	**1**	10.	**1**				

PART – C (MATHEMATICS)

SECTION - A

1.	**A**	2.	**D**	3.	**C**	4.	**C**
5.	**C**	6.	**C**	7.	**C**	8.	**D**
9.	**A**	10.	**D**	11.	**A**	12.	**D**
13.	**B**	14.	**D**	15.	**D**	16.	**A**
17.	**D**	18.	**A**	19.	**C**	20.	**A**

SECTION – B

1.	**7**	2.	**3**	3.	**9**	4.	**9**
5.	**108**	6.	**6**	7.	**2**	8.	**8**
9.	**9**	10.	**1**				

Solutions to JEE (Main)-2021

PART – A (PHYSICS)

SECTION - A

Sol1. $\omega = 100\pi$, $R = 100\Omega$, $X_L = L\omega = 0.5\times10^{-3}\times100\pi = 5\pi\times10^{-2}\Omega$, and

$X_C = \frac{1}{C\omega} = \frac{1}{0.1\times10^{-12}\times100\pi} = \frac{10^{11}}{\pi}\Omega$

Since X_C is approximately infinite , so the phase angle between current and supplied voltage and the nature of the circuit is $\approx 90^\circ$, predominantly capacitive circuit

Sol2. $p = \sqrt{2mK} \Rightarrow \frac{p_2}{p_1} = \sqrt{\frac{K_2}{K_1}} \Rightarrow \frac{p_2}{p} = \sqrt{\frac{4K_1}{K_1}} = 2 \Rightarrow p_2 = 2p$

The percentage change in its momentum $= \frac{\Delta p}{p}\times100 = \frac{p}{p}\times100 = 100\%$

Sol3. Let Mass $\alpha\left[t^a \upsilon^b \ell^c\right] \Rightarrow \left[M^1L^0T^0\right] \equiv T^a \times \left[LT^{-1}\right]^b \times \left[ML^2T^{-1}\right]^c$

$\Rightarrow a - b - c = 0 \quad c = 1$, and $b + 2c = 0 \Rightarrow b = -2c = -2$

$\Rightarrow a = b + c = 1 - 2 = -1$

Sol4. According to de-Broglie 's hypothesis

$\lambda = \frac{h}{mv} \Rightarrow p = mv = \frac{h}{\lambda} \Rightarrow K = \frac{p^2}{2m} = \frac{h^2}{2m\lambda^2}$

Cut-off wavelength of emitted X-ray $= \frac{hc}{K} = hc\times\frac{2m\lambda^2}{h^2} = \frac{2mc\lambda^2}{h}$ **[IIT Adv]**

Sol5. Total surface energy before coalesce $= U_i = 2\times4\pi R^2\times\sigma = 8\pi R^2\sigma$ .....(i)

Let new radius becomes r ,so according to conservation energy we can write

$2\times\frac{4\pi R^3}{3} = \frac{4\pi r^3}{3} \Rightarrow r = 2^{\frac{1}{3}}R$

Total surface energy after coalesce $= U_f = 4\pi r^2\times\sigma = 2^{\frac{2}{3}}\times4\pi R^2\sigma$ .....(ii)

$\Rightarrow \frac{U_i}{U_f} = \frac{8\pi R^2\sigma}{2^{\frac{2}{3}}\times4\pi R^2\sigma} = 2^{\frac{1}{3}} : 1$

Sol6. $\left|\vec{P}\right| = \left|\vec{Q}\right| = x$

let the angle between $\vec{P}$ and $\vec{Q}$ is θ, so according to question , we can write

$$\left|\vec{P}+\vec{Q}\right| = n\times\left|\vec{P}-\vec{Q}\right| \Rightarrow \sqrt{P^2+Q^2+2PQ\cos\theta} = n\times\sqrt{P^2+Q^2-2PQ\cos\theta}$$

$$\Rightarrow \sqrt{x^2+x^2+2x^2\cos\theta} = n\times\sqrt{x^2+x^2-2x^2\cos\theta}$$

$$\Rightarrow 2+2\cos\theta = 2n^2-2n^2\cos\theta \Rightarrow \left(1+n^2\right)\cos\theta = n^2-1 \Rightarrow \cos\theta = \frac{n^2-1}{n^2+1}$$

$$\Rightarrow \theta = \cos^{-1}\left(\frac{n^2-1}{n^2+1}\right)$$

Sol7. let the distance travelled is x, so

$\left|\vec{v}_{Eg}\right| = \frac{x}{t_2} \Rightarrow$ speed of escalator with respect to ground

$\left|\vec{v}_{BE}\right| = \frac{x}{t_1} \Rightarrow$ speed of boy with respect to escalator

$\left|\vec{v}_{Bg}\right| = \frac{x}{t_1}+\frac{x}{t_2} \Rightarrow$ speed of boy with respect to ground

The time taken by him to walk up on the moving escalator $= t = \frac{x}{\left|\vec{v}_{Bg}\right|}$

$$\Rightarrow t = \frac{x}{\frac{x}{t_1}+\frac{x}{t_2}} = \frac{t_1t_2}{t_1+t_2}$$

Sol8. $\gamma = 1+\frac{2}{f} \Rightarrow \frac{2}{f} = \gamma-1 \Rightarrow f = \frac{2}{\gamma-1}$

Sol9. $R = -\frac{dN}{dt} = \lambda N = \lambda N_0 e^{-\lambda t} \Rightarrow \ell n(R) = \ell n(\lambda N_0) - \lambda t$, so

$\lambda = \tan\theta = \frac{6}{40} = \frac{3}{20}s^{-1} \Rightarrow$ Slope of graph

$$\Rightarrow t_{\frac{1}{2}} = \frac{0.693}{\lambda} = \frac{0.693\times 20}{3} = 4.62s$$

Sol10. $K_T = \frac{mv^2}{2}$ and $K_R = \frac{I\omega^2}{2} = \frac{Iv^2}{2R^2}$

According to question, we can write

$K_R = \frac{1}{2}K_T \Rightarrow \frac{Iv^2}{2R^2} = \frac{1}{2}\times\frac{mv^2}{2} \Rightarrow I = \frac{mR^2}{2} \Rightarrow$ Solid cylinder

Sol11. **As we know that direction of propagation $(\hat{p})$ of wave is given as**

$\hat{p} = \hat{E}\times\hat{B} = \hat{i}\times\hat{k} = \left(-\hat{j}\right)$

Sol12. Assuming Tension in metal wire suspended from roof varies linearly and 0 and T_0 (developed due its own weight) are tensions are ends of wire of length ℓ. So

$T_1\ \alpha\ (\ell_1-\ell)$, and $T_2\ \alpha\ (\ell_2-\ell)$

$$\frac{T_1}{T_2}=\frac{\ell_1-\ell}{\ell_2-\ell} \Rightarrow \ell=\frac{T_2\,\ell_1-T_1\,\ell_2}{T_2-T_1}$$

Sol13. **According to question, we can write**

$$\text{Power}=Fv=c \Rightarrow mv\frac{dv}{dt}=c \Rightarrow \int_0^v vdv=\frac{c}{m}\int_0^t dt \Rightarrow \frac{v^2}{2}=\frac{c}{m}t \Rightarrow v=\frac{dx}{dt}=\sqrt{\frac{2c}{m}}\,t^{\frac{1}{2}}$$

$$\Rightarrow \int_0^x dx=\sqrt{\frac{2c}{m}}\int_0^d t^{\frac{1}{2}}\,dt \Rightarrow x=\sqrt{\frac{2c}{m}}\left(\frac{t^{\frac{3}{2}}}{\frac{3}{2}}\right)=\frac{2}{3}\sqrt{\frac{2c}{m}}\left(t^{\frac{3}{2}}\right) \Rightarrow x\ \alpha\left(t^{\frac{3}{2}}\right)$$

Sol14. According to ideal gas law we know that

$PV=nRT$

Sol15. **According to Doppler's effect in light , we can write**

$$\frac{v}{c}=\frac{\Delta\lambda}{\lambda} \Rightarrow \frac{v}{c}=\frac{5896-5890}{5890}=\frac{6}{5890}=\frac{3}{2945} \Rightarrow v=\frac{3}{2945}\times 3\times 10^5 \approx 305.6\ \text{km/s}$$

Sol16. As we know that $\upsilon^2=\omega^2\left(A^2-x^2\right)$ for SHM , so

$\upsilon_1^2=\omega^2\left(A^2-x_1^2\right)$.....(i) , and $\upsilon_2^2=\omega^2\left(A^2-x_2^2\right)$.....(ii)

Subtracting equation (ii) from equation (i) , we have

$$\upsilon_1^2-\upsilon_2^2=\omega^2\left(x_2^2-x_1^2\right) \Rightarrow \omega=\frac{2\pi}{T}=\sqrt{\frac{\upsilon_1^2-\upsilon_2^2}{x_2^2-x_1^2}} \Rightarrow T=2\pi\sqrt{\frac{x_2^2-x_1^2}{\upsilon_1^2-\upsilon_2^2}}$$

Sol17. Time period of Satellite $=T=\frac{2\pi R}{v}=\left(\frac{4\pi^2R^3}{GM}\right)^{\frac{1}{2}}$

$$\Rightarrow T\ \alpha\ R^{\frac{3}{2}} \Rightarrow \frac{T_2}{T_1}=\frac{(1.02R)^{\frac{3}{2}}}{R^{\frac{3}{2}}}=(1.02)^{\frac{3}{2}}=(1+0.02)^{\frac{3}{2}}$$

$$\Rightarrow \frac{T_2}{T_1}=\left(1+\frac{3}{2}\times 0.02\right) \Rightarrow T_2=(1+0.03)T_1 \Rightarrow \Delta T=(0.03)T_1$$

The percentage difference in the time periods of the two satellites= $\frac{\Delta T}{T_1}\times 100$

$=(0.03)\times 100=3\%$

Sol18. Since binary mass system performs circular motion about is common centre of mass, so

$$m_A\omega_A^2 r_A = \frac{Gm_Bm_A}{(r_A+r_B)^2} = \frac{Gm_Bm_A}{r^2}$$

$$\Rightarrow m_A\omega_A^2 \times \frac{m_B}{(m_A+m_B)}r = \frac{Gm_Bm_A}{r^2}$$

$$\Rightarrow \omega_A^2 = \frac{G(m_A+m_B)}{r^3}$$

$$\Rightarrow \omega_A = \sqrt{\frac{G(m_A+m_B)}{r^3}}$$

Similarly we can show that

$$\omega_B = \sqrt{\frac{G(m_A+m_B)}{r^3}}$$

Hence their angular velocity will be same , time period will be same, i.e. $T_A = T_B$

Sol19. Let the true dip is δ and apparent dip is δ' , so

$$\tan\delta' = \frac{B_V}{B_H\cos\theta} = \frac{\tan\delta}{\cos\theta} \Rightarrow \tan\delta = \tan\delta'\cos\theta = \tan 45^0 \cos 30^0 = 1\times\frac{\sqrt{3}}{2}$$

$$\Rightarrow \delta = \tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

Sol20. Given $\chi = 499$ and $\mu_0 = 4\pi\times10^{-7}\,H/m$

As we know that

$\mu_r = 1+\chi = 1+499 = 500 \Rightarrow$ Relative permeability

Absolute permeability= $\mu = \mu_r\mu_0 = 500\times4\pi\times10^{-7} = 2\pi\times10^{-4}\,H/m$

SECTION - B

Sol1. $A = -\frac{dN}{dt} = \lambda N = \lambda N_0 e^{-\lambda t} \Rightarrow \lambda\frac{N_0}{16} = \lambda N_0 e^{-80\lambda} \Rightarrow \frac{1}{16} = e^{-80\lambda} \Rightarrow \frac{1}{16} = e^{-80\lambda}$

$$\Rightarrow 80\lambda = \ell n(16) = 4\ell n(2) \Rightarrow t_{\frac{1}{2}} = \frac{\ell n(2)}{\lambda} = \frac{80}{4} = 20\,\text{days}$$

Sol2. Let the work function of metal is ϕ, so

$$K_1 = 3V_0e = \frac{hc}{\lambda} - \phi \text{ .....(i)}$$

$$K_2 = V_0e = \frac{hc}{2\lambda} - \phi \text{ .....(ii)}$$

Doing (i) – 3 X (ii) ,we have

$$0 = \frac{hc}{\lambda} - \frac{3hc}{2\lambda} + 2\phi \Rightarrow \phi = \frac{hc}{4\lambda}$$

The threshold wavelength= $= \frac{hc}{\phi} = 4\lambda$

Sol3. Consider translational motion of a body

$$mg\sin\theta - f_S = ma \text{ .....(i)}$$

Consider rotational motion of body about its centre of mass

$$f_S R = I\alpha \Rightarrow \alpha = \frac{f_S R}{I} \text{ .....(ii)}$$

FBD of Body

Using the condition of rolling without slipping at contact, we have

$$\alpha R = a \Rightarrow \frac{f_S R^2}{I} = \frac{mg\sin\theta - f_S}{m} \Rightarrow f_S = \frac{Img\sin\theta}{I + mR^2}$$

Using the condition of static friction, we have

$$f_S \le \mu N \Rightarrow \frac{Img\sin\theta}{I + mR^2} \le \mu mg\cos\theta \Rightarrow \mu \ge \frac{I\tan\theta}{I + mR^2} \text{ .....(iii)}$$

$$\alpha = \frac{mgR\sin\theta}{I + mR^2} \Rightarrow a = \alpha R = \frac{mgR^2\sin\theta}{I + mR^2} = \frac{gR^2\sin\theta}{k^2 + R^2} \text{ .....(iv)}$$

$$t = \sqrt{\frac{2\ell}{a}} \Rightarrow \text{Time required to reach the ground}$$

$$v = at = \sqrt{2a\ell} = \sqrt{\frac{2g\ell R^2\sin\theta}{k^2 + R^2}} \Rightarrow \text{Speed at bottom}$$

$$\Rightarrow \frac{v_{ring}}{v_{cylinder}} = \sqrt{\frac{2g\ell R^2\sin\theta}{k_{ring}^2 + R^2}} \times \sqrt{\frac{k_{cylinder}^2 + R^2}{2g\ell R^2\sin\theta}} = \sqrt{\frac{\frac{R^2}{2} + R^2}{R^2 + R^2}} = \frac{\sqrt{3}}{2}$$

Sol4. Power dissipated in cycle $= P = \frac{V_0 I_0}{2}\cos\phi = \frac{V_0^2}{2z} \times \frac{R}{z} = \frac{V_0^2 R}{2z^2}$

For resonance, z = R, so

$$\Rightarrow P = \frac{V_0^2}{2R} = \frac{250\sqrt{2} \times 250\sqrt{2}}{2 \times 5} = 12500 \text{ Watt} = 125 \times 10^2 \text{ Watt}$$

Sol5. $\omega^2 = \omega_0^2 + 2\alpha\theta \Rightarrow \left(\frac{2\pi}{60} \times 1800\right)^2 = \left(\frac{2\pi}{60} \times 600\right)^2 + 2\left(\frac{2\pi}{60} \times \frac{1200}{10}\right)\theta$

$$\Rightarrow (60\pi)^2 - (20\pi)^2 = 2(4\pi)\theta \Rightarrow \theta = \frac{80\pi \times 40\pi}{2 \times 4\pi} = 400\pi \text{ rad}$$

Number of rotations $= \frac{\theta}{2\pi} = \frac{400\pi}{2\pi} = 200$

Sol6. $R = \frac{\Delta V}{\Delta I} = \frac{0.75 - 0.70}{(5-3) \times 10^{-3}} = \frac{0.05 \times 1000}{2} = 25\Omega$

Sol7. Since process is isothermal, so

$$W = nRT\ell n\left(\frac{V_f}{V_i}\right) = = 1 \times 8.3 \times 300 \times \ell n\left(\frac{4}{2}\right) = 1 \times 8.3 \times 300 \times \ell n(2)$$

$$\Rightarrow W = 1 \times 8.3 \times 300 \times 0.6931 \approx 1725.82 \text{ J} \approx 17258 \times 10^{-1} \text{ J}$$

Sol8. Here

$$R_{eq} = (12 //^{\ell} 6) + (4 //^{\ell} 4) + (6 //^{\ell} 12)$$

$$= \left(\frac{12 \times 6}{18}\right) + \left(\frac{4 \times 4}{8}\right) + \left(\frac{6 \times 12}{18}\right)$$

$$= 4 + 2 + 4 = 10\,\Omega$$

Sol9. For ascent

$$t_a = \sqrt{\frac{2\ell}{a_{ascent}}} = \sqrt{\frac{2\ell}{g(\sin\theta + \mu\cos\theta)}}$$

For descent

$$t_d = \sqrt{\frac{2\ell}{a_{descent}}} = \sqrt{\frac{2\ell}{g(\sin\theta - \mu\cos\theta)}}$$

According question , we can write

$$t_a = \frac{1}{2}t_d \Rightarrow \frac{1}{\sin\theta + \mu\cos\theta} = \left(\frac{1}{4}\right)\left(\frac{1}{\sin\theta - \mu\cos\theta}\right)$$

$$\Rightarrow \sin\theta + \mu\cos\theta = 4\sin\theta - 4\mu\cos\theta \Rightarrow 5\mu\cos\theta = 3\sin\theta$$

$$\Rightarrow \mu = \frac{3}{5}\tan\theta = \frac{3}{5}\tan 30^0 = \frac{\sqrt{3}}{5}$$

Sol10.

$$P_z = V_z I_z \Rightarrow 0.5 = 8I_z \Rightarrow I_z = I_P = \frac{1}{16}\text{ Amp}$$

When zener is connected across a potential divider arranged with maximum potential drop across zener diode, then

$V_P = V - V_z = 20 - 8 = 12$ volt $\Rightarrow$ Potential difference across protective resistance R_p

$$\Rightarrow R_P = \frac{V_P}{I_P} = \frac{12}{\frac{1}{6}} = 192 \text{ Amp.}$$

PART – B (CHEMISTRY)

SECTION - A

Sol1. Mechanism path is free radical.

Sol2. Given electronic configuration is for Ga and in 5th period diagonally situated element is Sn with respect to Ga. Hence electronic configuration of Sn is [Kr] $4d^{10}\ 5s^2\ 5p^2$.

Sol3.

Sol4. Bakelite is cross linked polymer of formaldehyde and novolac. Novolac is linear polymer of phenol- formaldehyde.

Sol5. Calcinations is the decomposition in absence of air, $ZnCO_3 \xrightarrow[\Delta]{} ZnO + CO_2 \uparrow$

while roasting is the oxidation process, $2ZnS + 3O_2 \longrightarrow 2ZnO + 2SO_2 \uparrow$

Sol6. High leaving tendency corresponds to high reactivity towards hydrolysis. Hence order is.

Sol7. In strong field of octahedral complex of Fe^{2+}, the electronic configuration is , $Fe_{26}^{2+} \rightarrow [Ar]3d^6 4s^0$

Number of unpaired electrons are zero in the presence of strong field ligand.

Hence, spin only magnetic moment $= \sqrt{n(n+2)} = 0BM\ (n = 0)$

Sol8. KI acts as a reducing agent for Cu^{2+} $[2Cu^{++} + 4KI \longrightarrow Cu_2I_2 + I_2 + 4K^+]$

Sol9. Enzymes are the form of proteins and highly specific regarding temperature and pH change. It also lowers the activation energy for biochemical reaction.

Sol10.

After diazotization reaction, A is =

Product formed after EAS reaction (Coupling reaction)

Sol11. NO_2 gas reported to retard photosynthesis

Sol12. Due to the larger acid dissociation constant (Ka) sulphuric acid acts as an acid and HNO_3 acts as a base.

Sol13. In CuI, Cu^+ has $3d^{10}\ 4s^0$, $[n = 0 \text{ and } \mu = 0BM]$

In $[Cu(NH_3)_4]Cl_2, Cu^{++}$ has $3d^9\ 4s^0\ [n = 1 \text{ and } \mu = 1.73BM]$

In O_2^- & O_2^+ only one electron is unpaired in anti bonding MO,

Hence $[n = 1 \text{ and } \mu = 1.73\ BM]$

Sol14. In metamers, distribution of alkyl groups are changed with respect to polyvalent functional groups.

Sol15.

Sol16. Due to the absence of acidic hydrogen in But – 2- yne, metallic sodium does not react with this .

Sol17. In halide anion analysis $AgNO_3$ added in carius method to detect halide anion.

Sol18. Regarding industrial consumption of H_2 manufacturing of NH_3 is the largest applications.

Sol19.

Sol20. $\left[Cl^-\right] = 10^{-1}$ M

$\left[CrO_4^{--}\right] = 10^{-3}$ M

for AgCl ppt[1], $\left[Ag^+\right]_{req} = \frac{1.7\times10^{-10}}{10^{-1}}$ M

$\left[Ag^+\right]_{req} = 1.7\times10^{-9}$ M

For Ag_2CrO_4 ppt[2], $\left[Ag^+\right]_{req} = \sqrt{\frac{1.9\times10^{-12}}{10^{-3}}}$ M

$\left[Ag^+\right]_{req} = 4.3\times10^{-5}$ M

Being lower concentration of $[Ag^+]$ in case of AgCl, it will precipitate first.

SECTION - B

Sol1. Total wt $= 4\,gm = W_{NaOH} + W_{Na_2CO_3}$

Let us suppose moles are ' m ' for each.

4 = 40m + 106 m

$\therefore\ m = \left(\frac{4}{146}\right)$ mole of NaOH and Na_2CO_3 each.

$\therefore$ Mass of NaOH (x gm) $= \frac{4}{146}\times 40 = 1.095 \approx 1.0$

Sol2. 0.15 gm (organic compounds) $\xrightarrow{Ag\,NO_3}$ AgBr (s)

$\downarrow$

0.2397 gm

Weight of Br in AgBr $= \left(\frac{80}{188}\times 0.2397\right) = 0.102\,gm$

% Br in compound $= \frac{0.102\times100}{0.15} = 68\%$

Sol3. From $\Delta H = \Delta G + T\Delta S\ \therefore\ \Delta S = \frac{\Delta H - \Delta G}{T}$

$\therefore\ \Delta S = \left[\frac{51.4-(-49.4)}{300}\right]\times 1000\ JK^{-1}mol^{-1} = 336\,JK^{-1}mol^{-1}$

Sol4. Here, $\lambda = \frac{h}{\sqrt{2\times m\times eV}} = \frac{6.63\times10^{-34}}{\sqrt{2\times1.6\times10^{-19}\times40000\times9.1\times10^{-31}}}$ m

$\therefore\ \lambda = 0.614\times10^{-11}$m

$\lambda = 6.14\times10^{-12}$m [Nearest integer = 6]

Sol5. Wt of Cl^- in 100 ml = 1.8×10^{-3} gm

Mol . of Cl^- in 100 ml $=\dfrac{1.8\times10^{-3}}{35.5}=0.0507\times10^{-3}$ mole

$\therefore\ \left[Cl^-\right]=\dfrac{0.0507\times10^{-3}\times1000}{100}=\ 5.07\times10^{-4}M$

i.e. 0.507 milli mole in one lit required in one hr.
Coagulation value = (milli mole / lit) required in one hr $=0.507$
= 1 (Nearest integer)

Sol6. $NiCl_2+NaCN\xrightarrow[\text{O.A.}]{\text{strong}}\left[Ni(CN)_6\right]^{2-}$

Complex has Ni^{4+} and strong ligand, hence following are the metal ion electronic configuration.

$Ni^{2+}\longrightarrow\ 3d^8\ 4s^0$

Ni^{4+} (strong ligand)

$3d^6\quad 4s^0$

Change of unpaired electron = 2

Sol7. In fcc structure of diamond four C present in fcc lattice and other four C present in tetrahedral voids where 50% of tetrahedral voids are occupied. Hence number of carbon atoms present per unit cell of diamond is 8.

Sol8. Total pressure of mixture $=P_{Total}=X_A\times P_A^o+X_B\times P_B^o\ =\ 90\times0.6+15\times0.4=60mm$

Mole fraction of B in vapour phase , $\left(X'_B\right)=\dfrac{X_B\times P_B^o}{P_{Total}}$

$\therefore X'_B=\dfrac{0.4\times15}{60}=0.1=1\times10^{-1}$

Sol9. $PCl_5(g)\longrightarrow PCl_3(g)+Cl_2(g)$ (first order reaction)

$K=\dfrac{2.303}{120}\log\left(\dfrac{50}{10}\right)$

$\therefore K=\dfrac{2.303\times\log5}{120\,min}=\dfrac{2.303\times0.6989}{120}=0.0134\,min^{-1}$

Rate constant at 300K = $1.34\times10^{-2}\,min^{-1}$ [Nearest integer = 1.0]

Sol10. $6OH^-+Cl^-\longrightarrow ClO_3^-+3H_2O+6e^-$

Mole of ClO_3^- or $KClO_3$ = $\dfrac{10}{122.6}=0.08456$ mol

1 mole of ClO_3^- or $KClO_3$ produced by 6 faraday (F).

$\therefore$ 0.08456 mole of ClO_3^- or $KClO_3$ produced by $(6\times0.08156)F$

$\dfrac{I\times t}{F}=6\times0.8156$

$I=\dfrac{6\times0.08156\times96500}{10\times60\times60}A=1.311A\approx1A$

PART – C (MATHEMATICS)

SECTION - A

Sol1. Equation of plane $\perp^r$ to line $\frac{x-3}{2}=\frac{x-1}{1}=\frac{x-2}{1}$ and passes through the point (2, 3,-1) is $2(x-2)+1(y-3)+1(z+1)=0$

$\Rightarrow\ 2x+y+z-6=0$ .....(i)

Hence point (1,2,2) satisfies equation (i)

Sol2. $z=\frac{1}{1-\cos\theta+2i\sin\theta}=\frac{1-\cos\theta-2i\sin\theta}{(1-\cos\theta)^2-(2i\sin\theta)^2}=\frac{2\sin^2\frac{\theta}{2}-2i\sin\theta}{(1-\cos\theta)^2+4\sin^2\theta}$

$\therefore \text{Re}(z)=\frac{2\sin^2\frac{\theta}{2}}{4\sin^2\frac{\theta}{2}\left(\sin^2\frac{\theta}{2}+4\cos^2\frac{\theta}{2}\right)}=\frac{1}{2\left(1+3\cos^2\frac{\theta}{2}\right)}=\frac{1}{5}\ \Rightarrow \cos^2\frac{\theta}{2}=\frac{1}{2}$

$\Rightarrow \theta=\frac{\pi}{2}$ $\quad\therefore\quad \int_0^{\frac{\pi}{2}}\sin x\,dx=1$

Sol3. $\begin{vmatrix}3 & -1 & 4\\ 1 & 2 & -3\\ 6 & 5 & k\end{vmatrix}=0\ \Rightarrow k=-5$

Sol4. $(1-y)^m(1+y)^n$

The coefficient of y $(a_1)=1.{}^nC_1+{}^mC_1(-1)=n-m=10$ .....(i)

coefficient of $y^2(a_2)=1.\ {}^nC_2-{}^mC_1.\ {}^nC_1.+1.{}^mC_2=10$

$\frac{n(n-1)}{2}-mn+\frac{m(m-1)}{2}=10\Rightarrow m^2+n^2-2mn-(n+m)=20$

$\Rightarrow (n-m)^2-(n+m)=20\Rightarrow n+m=80$

Sol5. $\tan^{-1}\left(\frac{2\times\frac{3}{5}}{1-\frac{9}{25}}\right)+\sin^{-1}\frac{5}{13}\Rightarrow \tan^{-1}\frac{15}{8}+\tan^{-1}\frac{5}{12}=\tan^{-1}\frac{\frac{15}{8}+\frac{5}{12}}{1-\frac{15}{8}\frac{5}{12}}=\tan^{-1}\frac{220}{21}$

$\therefore \tan\left(\tan^{-1}\frac{220}{21}\right)=\frac{220}{21}$

Sol6. $I=\sum_{r=0}^{n-1}f\left(\frac{5r}{n}\right)\frac{1}{n}\Rightarrow I=\int_0^1 f(5x)\,dx=\int_0^1(5x+1)\,dx=\frac{7}{2}$

Sol7. Let mid point of PQ is R(h,k)

$$\therefore\ h = \frac{\alpha + \frac{4\alpha^2+\alpha+1}{2}}{2} \quad \text{and}$$

$$k = \frac{4\alpha^2 + 1 + \frac{4\alpha^2+\alpha+1}{2}}{2}$$

Eliminate α from above these two, we get

$2(3x - y)^2 + (x - 3y) + 2 = 0$

Sol8. $f(x) = x^3 - 3x^2 - \frac{3f''(2)}{2}x + f''(1)$ .....(i)

$f'(x) = 3x^2 - 6x - \frac{3f''(2)}{2}$ .....(ii)

$f''(x) = 6x - 6$ .....(iii)

$\therefore f''(2) = 12 - 6 = 6$

Use (ii) $f''(x) = 6x - 6$

$f''(2) = 12 - 6 = 6$

$f''(-1) = -6 - 6 < 0$

Hence f(x) is local maxima at x = –1

and f(x) is local minima at x = 3

∴ from (i) local minimum value at x = 3 is f(3) = –27

Sol9. Center of the smallest circle is A

Center of the largest circle is B

$r_1 = |CP + CB| = 3\sqrt{2} + 3$ and

$r_2 = |CP - CB| = 3\sqrt{2} - 3$

$\frac{r_1}{r_2} = \frac{3\sqrt{2}+3}{3\sqrt{2}-3} = 3 + 2\sqrt{2}$

Sol10. $10 = \frac{7+10+11+15+a+b}{6} \Rightarrow a + b = 17$ .....(i)

$\Rightarrow \frac{20}{3} = \frac{7^2+10^2+11^2+15^2+a^2+b^2}{6} - 10^2$

$a^2 + b^2 = 145$ .....(ii)

solving (i) and (ii) we get a = 8, b = 9 or a = 9 , b = 8

$|a - b| = 1$

Sol11. $A = -\frac{y}{x} + 2\sin x + 2$

$\therefore \frac{dy}{dx} + \frac{y}{x} = 2\sin x + 2$ .....(i)

$\Rightarrow I.F = e^{\int \ln x dx} = x$ from (i) $d(xy) = 2\int x\sin x dx + 2\int x dx$

$xy = 2[-x\cos x + \sin x] + x^2 + c$ .....(ii)

according to question, we get c = 0 $\therefore y\left(\frac{\pi}{2}\right)=\frac{4}{\pi}+\frac{\pi}{2}$

Sol12. $f(x)=\frac{5x+3}{6x-\alpha}=y$ .....(i)

$\Rightarrow x=\frac{\alpha y+3}{6y-5}\Rightarrow f^{-1}(x)=\frac{\alpha x+3}{6x-5}$ .....(ii)

According to question , $f(x)=f^{-1}(x)$ $\therefore$ from (i) and (ii) we get $\propto = 5$

Sol13. $\frac{x+1}{a}=\frac{y}{1}=\frac{z-1}{a}$ .....(i)

$\frac{x+2}{a}=\frac{y}{1}=\frac{z}{\frac{3}{b}}$ .....(ii)

Let A(-1,0,1) and B(-2,0,0) $\therefore$ direction ratios of AB =-1,0,-1

$\because$ lines are coplanar

$\therefore \begin{vmatrix} a & 1 & a \\ 3 & 1 & \frac{3}{b} \\ -1 & 0 & -1 \end{vmatrix}=0 \Rightarrow$ b = 1 and a $\in$ R-{0}

Sol14. The projection of $\overrightarrow{BA}$ on $\overrightarrow{BC}=\cos\angle ABC=7\left|\frac{7^2+3^2-5^2}{2\times7\times3}\right|=\frac{11}{2}$

Sol15. $\because f(x)=\ln\left(x+\sqrt{1+x^2}\right)\therefore f(-x)=-f(x)$.

Hence f(x) is an odd function .

Now $g(t)=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\cos\left(\frac{\pi}{4}t+f(x)\right)dx$

Put $t=0, g(0)=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\cos(f(x))dx=2\int_{0}^{\frac{\pi}{2}}\cos(f(x))dx$ .....(i)

where cos(f(x)) is an even function .

Now again put t = 1,

$g(1)=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\cos\left(\frac{\pi}{4}+f(x)\right)dx=\frac{1}{\sqrt{2}}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}(\cos(f(x))-\sin(f(x)))dx$

$= \frac{1}{\sqrt{2}}\left[\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \cos(f(x))dx - \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \sin(f(x))dx\right] = \frac{1}{\sqrt{2}} \times 2\int_{0}^{\frac{\pi}{2}} \cos(f(x))dx$, where $\cos(f(x))$ is an even function and $\sin(f(x))$ is an odd function.

$\therefore g(1) = \frac{1}{\sqrt{2}} \times g(0)$, from (i)

$\therefore g(0) = \sqrt{2}g(1)$

Sol16. Truth table

Hence according to option (4) is most appropriate option

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Sol17. $\log_{9^{\frac{1}{2}}} x + \log_{9^{\frac{1}{3}}} x + \log_{9^{\frac{1}{4}}} x + ... + \log_{9^{\frac{1}{22}}} x = 504$

$\Rightarrow 2\log_9 x + 3\log_9 x + 4\log_9 x + ... + 22\log_9 x = 504$

$\Rightarrow \log_9 x + 2\log_9 x + 3\log_9 x + 4\log_9 x + ... + 22\log_9 x - \log_9 x = 504$

$\Rightarrow (1+2+3+.....+22)\log_9 x - \log_9 x = 504 \Rightarrow x = 81$

Sol18. $I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} ([x] + [-\sin x])dx$ .....(i)

$I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} ([-x] + [\sin x])dx$ .....(ii)

by using property $\left(\int_a^b f(x)dx = \int_a^b f(a+b-x)dx\right)$

Adding (i) and (ii) we get $2I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (-2)dx = -2\pi \Rightarrow I = -\pi$

Sol19. Let smallest angle is $C = \theta$

Therefore angle $A = 90^0 - \theta$

And angle $B = 90^0$

i.e. $b > a > c$

Here , $a = 2R\cos\theta$, $b = 2R$, $c = 2R\sin\theta$

$b^2 = a^2 + c^2$

according to question ,

$\frac{1}{c^2} = \frac{1}{a^2} + \frac{1}{b^2}$

$. \Rightarrow \frac{a^2b^2}{c^2} = a^2 + b^2$

$$\Rightarrow \frac{\cos^2\theta}{\sin^2\theta} = \cos^2\theta + 1$$
$$\Rightarrow 1 - \sin^2\theta = \sin^2\theta\left(2 - \sin^2\theta\right)$$
$$\Rightarrow \sin^4\theta - 3\sin^2\theta + 1 = 0$$
$$\Rightarrow \sin^2\theta = \frac{3-\sqrt{5}}{2}$$
$$\Rightarrow \sin\theta = -\frac{\sqrt{5}-1}{2}$$

Sol20. $P(\bar{A} \cap B) + P(A \cap \bar{B}) = 1 - k$,

$P(\bar{A} \cap C) + P(A \cap \bar{C}) = 1 - 2k$,

$P(\bar{B} \cap C) + P(B \cap \bar{C}) = 1 - k$ and

$P(A \cap B \cap C) = k^2$

$P(A) + P(B) - 2P(A \cap B) = 1 - k$ .....(i)

$P(B) + P(C) - 2P(B \cap C) = 1 - k$ .....(ii)

$P(A) + P(C) - 2P(A \cap C) = 1 - 2k$ .....(iii)

Adding (i),(ii) and (iii) we get $P(A \cup B \cup C) = \frac{-4k+3}{2} + k^2$

$$\Rightarrow P(A \cup B \cup C) = \frac{2k^2 - 4k + 2 + 1}{2} = \frac{2(k-1)^2 + 1}{2} \Rightarrow P(A \cup B \cup C) > \frac{1}{2}$$

SECTION - B

Sol1. $a_{n+1} = \frac{a_{n+2} - a_n}{2}$ let $p = \sum_{n=1}^{\infty} \frac{a_n}{8^n}$

$$\Rightarrow 64\frac{a_{n+2}}{8^{n+2}} = \frac{16a_{n+1}}{8^{n+1}} + \frac{a_n}{8^n}$$
$$\Rightarrow 64\sum_{n=1}^{\infty}\frac{a_{n+2}}{8^{n+2}} = \sum_{n=1}^{\infty}\frac{16a_{n+1}}{8^{n+1}} + \sum_{n=1}^{\infty}\frac{a_n}{8^n} \Rightarrow 64\left(p - \frac{a_1}{8} - \frac{a_2}{8^2}\right) = 16\left(p - \frac{a_1}{8}\right) + p$$
$$\Rightarrow 64\left(p - \frac{1}{8} - \frac{1}{8^2}\right) = 16\left(p - \frac{1}{8}\right) + p \Rightarrow 64p - 8 - 1 = 16p - 2 + p \qquad \Rightarrow 47p = 7$$

Sol2. $$\lim_{x\to 0} \frac{\alpha x\left(1 + x + \frac{x^2}{2} + ...\right) - \beta\left(x - \frac{x^2}{2} + \frac{x^3}{3} + ...\right) + \gamma x^2(1-x)}{x^3} = 10$$

For limit to exist $\alpha - \beta = 0$ .....(i) , $\alpha + \frac{\beta}{2} + \gamma = 0$ .....(ii)

and $\frac{\alpha}{2} - \frac{\beta}{2} - \gamma = 10$ .....(iii)

Solving (i),(ii) and (iii) we get $\alpha = 6, \beta = 6$ and $\lambda = -9 \Rightarrow \alpha + \beta + \lambda = 3$

Sol3. $\left(\sqrt{50}\right)^2 = \left(\sqrt{45}\right)^2 + \left(\sqrt{5}\right)^2$

$\therefore \angle B = 90^\circ$ circum centre = $O\left(\frac{1}{2}, \frac{11}{2}\right)$

Mid point of $BC = D\left(2, \frac{17}{2}\right)$

Equation of OD is $y = 2x + \frac{9}{2}$. This line passes through

$\left(0, \frac{\alpha}{2}\right) \therefore \alpha = 9$

Sol4. Let equation of normal PQ is

$y = -tx + 3t + \frac{3}{2}t^3$ and it is passing through

$P\left(3, \frac{3}{2}\right) \therefore t = 1$

$\therefore Q\left(\frac{3}{2}, 3\right) \Rightarrow a = \frac{3}{2}$ and $b = 3$

$2(a+b) = 9$

Sol5. $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} \Rightarrow |A| = 4 \Rightarrow \left|3\text{adj}\left(2A^{-1}\right)\right| = \left|3.2^2\text{adj}\left(A^{-1}\right)\right|$

$\Rightarrow 12^3\left|\text{adj}\left(A^{-1}\right)\right| = 12^3\left|A^{-1}\right|^2 = \frac{12^3}{|A|^2} = \frac{12 \times 12 \times 12}{16} = 108$

Sol6. $\because \left|\vec{v}_1\right| = \left|\vec{v}_2\right| \Rightarrow p^2 - p - 2 = 0 \Rightarrow p = -1, 2$ we take only $p = 2$ $(p > 0)$

$\therefore \cos\theta = \frac{\vec{v_2}.\vec{v_2}}{\left|\vec{v_1}\right|\left|\vec{v_2}\right|} = \frac{4\sqrt{3}+3}{13} \Rightarrow \tan\theta = \frac{6\sqrt{3}-2}{4\sqrt{3}+3} = \frac{\alpha\sqrt{3}-2}{4\sqrt{3}+3} \therefore \alpha = 6$

Sol7. $\int dy = \int \frac{\cos\left(\frac{1}{2}\cos^{-1}(e^{-x})\right)}{e^x\sqrt{1-\left(e^{-x}\right)^2}}dx$ put $\cos 2\theta = e^{-x} \Rightarrow -2\sin 2\theta d\theta = -e^{-x}dx$

$\therefore \int dy = \int \frac{2\cos\theta\sin 2\theta d\theta}{\sqrt{1-\cos^2 2\theta}} \Rightarrow y = 2\sin\theta + c \Rightarrow y = -1, \theta = 0, c = 0$

$\therefore y = 2\sqrt{\frac{1-e^{-x}}{2}} - 1$ at $(\alpha, 0), e^\alpha = 2$

Sol8. $\frac{\log((2x+5)(x+1))}{\log(x+1)} + \frac{\log(x+1)}{\log(2x+5)} - 4 = 0 \Rightarrow \frac{\log(2x+5)}{\log(x+1)} + \frac{\log(x+1)}{\log(2x+5)} - 3 = 0$

$\Rightarrow x \in (-1, 0) \cup (0, \infty)$

Put $t = \dfrac{\log(2x+5)}{\log(x+1)} \Rightarrow t + \dfrac{2}{t} - 3 = 0 \Rightarrow t^2 - 3t + 2 = 0 \Rightarrow t = 1,2$

$\dfrac{\log(2x+5)}{\log(x+1)} = 1 \Rightarrow x = -4$ (not possible) and $\dfrac{\log(2x+5)}{\log(x+1)} = 2 \Rightarrow x = 2$

Hence only one solution is possible.

Sol9. $\dfrac{1}{\alpha(\alpha+1)(\alpha+2).....(\alpha+20)} = \dfrac{A_0}{\alpha} + \dfrac{A_1}{\alpha+1} + \dfrac{A_2}{\alpha+2} + ... + \dfrac{A_{20}}{\alpha+20}$

Solving by partial fraction , we get

$A_{13} = \dfrac{-1}{14!7!}, A_{14} = \dfrac{-1}{14!6!}$ and $A_{15} = \dfrac{-1}{14!5!}$

$\therefore A_{14} + A_{15} = \dfrac{1}{14!6!} - \dfrac{1}{15!5!} = \dfrac{1}{14!5!} \times \dfrac{1}{10}$

$\therefore \dfrac{A_{14} + A_{15}}{A_{13}} = \dfrac{3}{10} \Rightarrow \left(\dfrac{A_{14} + A_{15}}{A_{13}}\right)^2 = \left(\dfrac{3}{10}\right)^2 \Rightarrow 100\left(\dfrac{A_{14} + A_{15}}{A_{13}}\right)^2 = 9$

Sol10. $f(t) = t^3 - 6t^2 + 9t + 3$

$\Rightarrow f'(t) = 3t^2 - 12t + 9 = 3(t-1)(t-3)$

$\therefore f'(t) = 0 \Rightarrow t = 1,3$

$\Rightarrow f(1) = 1, f(3) = -3$

$g(x) = \begin{cases} f(x) & , \quad 0 \le x < 1 \\ 1 & , \quad 1 \le x \le 3 \\ 4 - x & , \quad 3 < x \le 4 \end{cases}$ g(x) is continuous

$g'(x) = \begin{cases} 3(x-1)(x-3) & , \quad 0 \le x < 1 \\ 0 & , \quad 1 \le x \le 3 \\ -1 & , \quad 3 < x \le 4 \end{cases}$.

Hence g(x) is not differentiable at only x = 3.

Solutions to JEE(Main) -2021

Test Date: 20th July 2021 (First Shift)

PHYSICS, CHEMISTRY & MATHEMATICS

Paper - 1

Time Allotted: 3 Hours **Maximum Marks: 300**

- Please read the instructions carefully. You are allotted 5 minutes specifically for this purpose.

Important Instructions:

1. The test is of 3 hours duration.
2. This test paper consists of 90 questions. Each subject (PCM) has 30 questions. The maximum marks are 300.
3. This question paper contains **Three Parts. Part-A** is Physics, **Part-B** is Chemistry and **Part-C** is Mathematics. Each part has only two sections: **Section-A and Section-B**.
4. **Section – A** : Attempt all questions.
5. **Section – B :** Do any 5 questions out of 10 Questions.
6. **Section-A (01 – 20)** *contains 20 multiple choice questions which have* ***only one correct answer****. Each question carries* ***+4 marks*** *for correct answer and* ***–1 mark*** *for wrong answer.*
7. ***Section-B (01 – 10)*** *contains 10 Numerical based questions with answer as numerical value. Each question carries* ***+4 marks*** *for correct answer. There is no negative marking.*

PART – A (PHYSICS)

SECTION - A

(One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.

Q1. The amount of heat needed to raise the temperature of 4 moles of a rigid diatomic gas from $0^\circ C$ to $50^\circ C$ when no work is done is _____. (R is the universal gas constant)

(A) 175 R
(B) 750 R
(C) 250 R
(D) 500 R

Q2. A deuteron and an alpha particle having equal kinetic energy enter perpendicularly into a magnetic field. Let r_d and r_d be their respective radii of circular path. The value of $\frac{r_d}{r_\alpha}$ is equal to :

(A) $\sqrt{2}$
(B) 2
(C) $\frac{1}{\sqrt{2}}$
(D) 1

Q3. If $\vec{A}$ and $\vec{B}$ are two vectors satisfying the relation $\vec{A}\cdot\vec{B} = |\vec{A}\times\vec{B}|$. Then the value of $|\vec{A}-\vec{B}|$ will be :

(A) $\sqrt{A^2+B^2-\sqrt{2}AB}$
(B) $\sqrt{A^2+B^2}$
(C) $\sqrt{A^2+B^2+2AB}$
(D) $\sqrt{A^2+B^2+\sqrt{2}AB}$

Q4. Consider a mixture of gas molecule of types A, B and C having masses $m_A < m_B < m_C$. The ratio of their root mean square speeds at normal temperature and pressure is :

(A) $\frac{1}{\upsilon_A} > \frac{1}{\upsilon_B} > \frac{1}{\upsilon_C}$
(B) $\upsilon_A = \upsilon_B = \upsilon_C = 0$
(C) $\frac{1}{\upsilon_A} < \frac{1}{\upsilon_B} < \frac{1}{\upsilon_C}$
(D) $\upsilon_A = \upsilon_B \neq \upsilon_C$

Q5. A nucleus of mass M emits $\gamma-$ray photon of frequency 'v'. The loss of internal energy by the nucleus is : [Take 'c' as the speed of electromagnetic wave]

(A) $hv\left[1+\frac{hv}{2Mc^2}\right]$
(B) 0
(C) $hv\left[1-\frac{hv}{2Mc^2}\right]$
(D) hv

Q6. A radioactive material decays by simultaneous emissions of two particles with half lives of 1400 years and 700 years respectively. What will be the time after which one third of the material remains? (Take ln 3 = 1.1)

(A) 340 years
(B) 1110 years
(C) 700 years
(D) 740 years

Q7. The value of current in the $6\,\Omega$ resistance is :

(A) 6 A
(B) 10 A
(C) 4 A
(D) 8 A

Q8. The radiation corresponding to $3 \rightarrow 2$ transition of a hydrogen atom falls on a gold surface to generate photoelectrons. These electrons are passed through a magnetic field of 5×10^{-4} T. Assume that the radius of the largest circular circular path followed by these electrons is 7mm, the work function of the metal is : (Mass of electron = 9.1×10^{-31} kg)

(A) 0.82 eV
(B) 1.88 eV
(C) 1.36 eV
(D) 0.16 eV

Q9. For the circuit shown below, calculate the value of I_2 :

(A) 0.15 A
(B) 0.1 A
(C) 25 mA
(D) 0.05 A

Q10. The arm PQ of a rectangular conductor is moving from $x = 0$ to x = 2b outwards and then inwards from x = 2b to x = 0 as shown in the figure. A uniform magnetic field perpendicular to the plane is acting from x = 0 to x = b. Identify the graph the variation of different quantities with distance.

(A) A – Flux, B – EMF, C – Power dissipated
(B) A – EMF, B – Power dissipated, C – Flux
(C) A – Power dissipated, B – Flux, C – EMF
(D) A – Flux, B – Power dissipated, C – EMF

Q11. Region I and II are separated by a spherical surface of radius 25 cm. An object is kept in region I at a distance of 40 cm from the surface. The distance of the image from the surface is :

(A) 9.52 cm
(B) 18.23 cm
(C) 37.58 cm
(D) 55.44 cm

Q12. The normal reaction 'N' for a vehicle of 800 kg mass, negotiating a turn on a 30° banked road at maximum possible speed without skidding is _____ $\times 10^3$ kg m / s^2 .

[Given $\cos 30^\circ = 0.87, \mu_5 = 0.2$]

(A) 6.96
(B) 10.2
(C) 12.4
(D) 7.2

Q13. The entropy of any system is given by $S = \alpha^2 \beta \, \ell n \left[\frac{\mu k R}{J \beta^2} + 3 \right]$ Where α and β are the constants. μ, J, k and R no. of moles, mechanical equivalent of heat, Boltzmann constant and gas constant respectively . [Take $S = \frac{dQ}{T}$]

Choose the **incorrect** option from the following :

(A) S, β, k and μ R have the same dimensions.
(B) α and k have the same dimensions.
(C) S and α have different dimensions.
(D) α and J have the same dimensions.

Q14. A current of 5 A is passing through a non-linear magnesium wire of cross – section $0.04\,m^2$. At every point the direction of current density is at an angle of 60° with unit vector of area of cross -section. The magnitude of electric field at every point of the conductor is : (Resistivity of magnesium $\rho = 44 \times 10^{-8} \Omega m$)

(A) 11×10^{-7} V / m
(B) 11×10^{-2} V / m
(C) 11×10^{-5} V / m
(D) 11×10^{-3} V / m

Q15. The value of tension in a long thin metal wire has been changed from T_1 to T_2. The lengths of the metal wire at two different values of tension T_1 and T_2 are ℓ_1 and ℓ_2 respectively. The actual length of the metal wire is :

(A) $\frac{T_1 \ell_2 - T_2 \ell_1}{T_1 - T_2}$
(B) $\sqrt{T_1 T_2 \ell_1 \ell_2}$
(C) $\frac{\ell_1 + \ell_2}{2}$
(D) $\frac{T_1 \ell_1 - T_2 \ell_2}{T_1 - T_2}$

Q16. AC voltage $V(t) = 20\sin\omega t$ of frequency $50\,Hz$ is applied to a parallel plate capacitor. The separation between the plates is 2mm and the area is $1\,m^2$. The amplitude of the oscillating displacement current for the applied Ac voltage is _____.
[Take $\varepsilon_0 = 8.85\times10^{-12}\,F/m$]

(A) 83.37 μA
(B) 55.58 μA
(C) 21.14 μA
(D) 27.79 μA

Q17. A butterfly is flying with a velocity $4\sqrt{2}\,m/s$ in North – East direction. Wind is slowly blowing at $1\,m/s$ from North to South. The resultant displacement of the butterfly in 3 seconds is :

(A) 15m
(B) 20m
(C) 3 m
(D) $12\sqrt{2}\,m$

Q18. A certain charge Q is divided into parts q and $(Q-q)$. How should the charges Q and q be divided so that q and (Q – q) placed at a certain distance apart experience maximum electrostatic repulsion ?

(A) $Q = \frac{q}{2}$
(B) $Q = 4q$
(C) $Q = 2q$
(D) $Q = 3q$

Q19. A steel block of 10 kg rests on a horizontal floor as shown. When three iron cylinders are placed on it as shown, the block and cylinders go down with an acceleration 0.2 m/s^2. The normal reaction R' by the floor if mass of the iron cylinders are equal and of $20\,kg$ each, is _____N.
[Take $g = 10\,m/s^2$ and $\mu_s = 0.2$]

(A) 714
(B) 684
(C) 716
(D) 686

Q20. A person whose mass is 100 kg travels from Earth to Mars in a spaceship. Neglect all other object in sky and take acceleration due to gravity on the surface of the Earth and mars as $10\,m/s^2$ and $4\,m/s^2$ respectively. Identify from the below figures, the curve that fits best for the weight of the passenger as a function of time.

(A) (c)
(B) (a)
(C) (b)
(D) (d)

SECTION - B

(Numerical Answer Type)

This section contains **10** questions. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the **second decimal place**).

Q1. The amplitude of wave disturbance propagating in the positive x-direction is given by $y = \frac{1}{1+(x)^2}$ at time $t = 0$ and $y = \frac{1}{1+(x-2)^2}$ at $t = 1\,s$, where x and y are in metres. The shape of wave does not change during the propagation. The velocity of the wave will be _____ m /s.

Q2. In the reported figure, heat energy absorbed by a system in going through a cyclic process is _____ πJ.

Q3. In a spring gun having spring constant $100\,N/m$ a small ball 'B' of mass 100 g is put in its barrel (as shown in figure) by compressing the spring through 0.05 m. There should be a box placed at a distance 'd' on the ground so that the ball falls in it. If the ball leaves the gun horizontally at a height of 2 m above the ground . The value of d is _____m.

Q4. The frequency of a car horn encountered a change from 400 Hz to 500 Hz, when the car approaches a vertical wall. If the speed of sound is 330 m /s. Then the speed of car is _____km /h.

Q5. A body having specific charge $8\,\mu C/g$ is resting on a frictionless plane at a distance 10cm from the wall (as shown in the figure). It starts moving towards the wall when a uniform electric field of 100 V/m is applied horizontally towards the wall. If the collision of the body with the wall is perfectly elastic, then the time period of the motion will be _____s.

Q6. A rod of mass M and length L is lying on a horizontal frictionless surface. A particle of mass 'm' travelling along the surface hits at one end of the rod with a velocity 'u' in a direction perpendicular to the rod. The collision is completely elastic. After collision, particle comes to rest. The ratio of masses $\left(\frac{m}{M}\right)$ is $\frac{1}{x}$. The value of 'x' will be _____.

Q7. An object viewed from a near point distance of 25 cm, using a microscopic lens with magnification '6', gives an unresolved image, A resolved image is observed at infinite distance with a total magnification double the earlier using an eyepiece along with the given lens and a tube of length 0.6 m, if the focal length of the eyepiece is equal to _____cm.

Q8. In an LCR series circuit, an inductor 30 mH and a resistor 1Ω are connected to an Ac source of angular frequency 300 rad /s. The value of capacitance for which, the current leads the voltage by 45° is $\frac{1}{x}\times 10^{-3}$ F. Then the value of x is _____.

Q9. A circular disc reaches from top to bottom of an inclined plane of length 'L' when it slips down the plane, it takes time $'t_1'$. When it rolls down the plane, it takes time t_2. The value of $\frac{t_1}{t_2}$ is $\sqrt{\frac{3}{x}}$ The value of x will be _____.

Q10. A carrier wave $V_C(t) = 160\sin\left(2\pi\times 10^6 t\right)$ volts is made to very between $V_{max} = 200\,V$ and $V_{min} = 120\,V$ by a message signal $V_m(t) = A_m \sin\left(2\pi\times 10^3 t\right)$ volts. The peak voltage A_m of the modulating single is _____V.

PART – B (CHEMISTRY)

SECTION - A

(One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.

Q1. Chemical nature of the nitrogen oxide compound obtained from a reaction of concentrated nitric acid and P_4O_{10} (in 4:1 ratio) is:
(A) basic
(B) neutral
(C) amphoteric
(D) acidic

Q2. Compound A is converted to B on reaction with $CHCl_3$ and KOH. The compound B is toxic and can be decomposed by C. A, B and C respectively are:
(A) Primary amine, nitrile compound, conc. HCl
(B) Primary amine, isonitrile compound , conc. HCl
(C) Secondary amine, isonitrile compound, conc NaOH
(D) Secondary amine, nitrile compound, conc. NaOH

Q3. Given below are two statements : One is labelled as **Assertion A** and the other is labelled as **Reason R.**
Assertion A: The dihedral angles in H_2O_2 in gaseous phase is 90.2^o and in solid phase is 111.5^o.
Reason R: The change in dihedral angle in solid and gaseous phase is due to the difference in the intermolecular forces.
Choose the most appropriate answer from the options given below for **A** and **R**.
(A) **A** is correct but **R** is not correct.
(B) Both **A** and **R** are correct but **R** is not the correct explanation of **A**.
(C) Both **A** and **R** are correct and **R** is the correct explanation of **A**
(D) **A** is not correct but **R** is correct.

Q4.

(A) (B) (C) (D)

Among the given species the Resonance stabilized carbocations are:
(A) (A), (B) and (C) only
(B) (C) and (D) only
(C) (A), (B) and (D) only
(D) (A) and (B) only

Q5. A s-block element (M) reacts with oxygen to form an oxide of the formula MO_2. The oxide is pale yellow in colour and paramagnetic. The element (M) is:
(A) Na
(B) K
(C) Mg
(D) Ca

Q6. Green chemistry in day- to day life is in the use of:
(A) Chlorine for bleaching of paper
(B) Large amount of water alone for washing clothes
(C) Liquified CO_2 for dry cleaning of clothes
(D) Tetrachloroethene for laundry

Q7. Identify the incorrect statement from the following:
(A) β-Glycosidic linkage makes cellulose polymer
(B) Amylose is a branched chain polymer of glucose
(C) Starch is a polymer of α- D glucose
(D) Glycogen is called as animal starch

Q8. The metal that can be purified economically by fractional distillation method is:
(A) Ni (B) Cu
(C) Fe (D) Zn

Q9. The set in which compounds have different nature is:
(A) $B(OH)_3$ and $Al(OH)_3$ (B) $B(OH)_3$ and H_3PO_3
(C) NaOH and $Ca(OH)_2$ (D) $Be(OH)_2$ and $Al(OH)_3$

Q10. The conditions given below are in the context of observing Tyndall effect in colloidal solutions:
(a) The diameter of the colloidal particles is comparable to the wavelength of light used.
(b) The diameter of the colloidal particles in much smaller than the wavelength of light used.
(c) The diameter of the colloidal particles is much larger than the wavelength of light used.
(d) The refractive indices of the dispersed phase and the dispersion medium are comparable.
(e) The dispersed phase has a very different refractive index from the dispersion medium.
Choose the most appropriate conditions from the options given below.
(A) (a) and (d) only (B) (a) and (e) only
(C) (c) and (d) only (D) (b) and (e) only

Q11. According to the valence bond theory the hybridization of central metal atom is dsp^2 for which one of the following compounds?
(A) $K_2[Ni(CN)_4]$ (B) $NiCl_2.6H_2O$
(C) $Na_2[NiCl_4]$ (D) $[Ni(CO)_4]$

Q12. In the given reaction 3-Bromo-2,2-dimethyl butane $\xrightarrow{C_2H_5OH}$ 'A' (Major Product)

Product A is:
(A) 2-Ethoxy-2,3-dimethyl butane (B) 2-Hydroxy-3,3-dimethyl butane.
(C) 2-Ethoxy-3,3-dimethyl butane (D) 1-Ethoxy-3,3-dimethyl butane.

Q13. Orlon fibres are made up of:
(A) Polyamide (B) Polyacrylonitrile
(C) Cellulose (D) Polyesters

Q14. The species given below that does NOT show disproportionation reaction is:

(A) BrO^- (B) BrO_3^-

(C) BrO_2^- (D) BrO_4^-

Q15.

For above chemical reactions, identify the correct statement from the following

(A) Both compound 'A' and compound 'B' are dicarboxylic acids.

(B) Compound 'A' is dicarboxylic acid and compound 'B' is diol.

(C) Compound 'A' is diol and compound 'B' is dicarboxylic acid.

(D) Both compound 'A' and compound 'B' are diols.

Q16. An inorganic compound 'X' on treatment with concentrated H_2SO_4 produces brown fumes and gives dark brown ring with $FeSO_4$ in presence of concentrated H_2SO_4. Also compound 'X' gives precipitate 'Y', when its solution in dilute HCl is treated with H_2S gas. The precipitate 'Y' on treatment with concentrated HNO_3 followed by excess of NH_4OH further gives deep blue coloured solution, Compound 'X' is:

(A) $Cu(NO_3)_2$ (B) $Pb(NO_3)_2$

(C) $Co(NO_3)_2$ (D) $Pb(NO_2)_2$

Q17.

Which among the above compound/s does / do not form silver mirror when treated with Tollen's reagent?

(A) Only (IV) (B) (III) and (IV) only

(C) Only (II) (D) (I), (III) and (IV) only

Q18. Given below are two statements. One is labelled as **Assertion A** and the other is labelled as **Reason R**.

Assertion A: Sharp glass edge becomes smooth on heating it upto its melting point.

Reason R: The viscosity of glass decreases on melting.

Choose the most appropriate answer from the options given below.

(A) **A** is false but **R** is true.

(B) Both **A** and **R** are true but **R** is NOT the correct explanation of **A**.

(C) **A** is true but **R** is false

(D) Both **A** and **R** are true and **R** is the correct explanation of **A**.

Q19. The correct structure of Rhumann's Purple, the compound formed in the reaction of ninhydrin with proteins is:

(A)

(B)

(C)

(D)

Q20. The correct order of intensity of colors of the compounds is:

(A) $[NiCl_4]^{2-} > [Ni(H_2O)_6]^{2+} > [Ni(CN)_4]^{2-}$

(B) $[NiCl_4]^{2-} > [Ni(CN)_4]^{2-} > [Ni(H_2O)_6]^{2+}$

(C) $[Ni(H_2O)_6]^{2+} > [NiCl_4]^{2-} > [Ni(CN)_4]^{2-}$

(D) $[Ni(CN)_4]^{2-} > [NiCl_4]^{2-} > [Ni(H_2O)_6]^{2+}$

SECTION - B

(Numerical Answer Type)

This section contains **10** questions. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the **second decimal place**).

Q1. The number of lone pairs of electrons on the central I atom in I_3^- is_____.

Q2. An average person needs abut 10000 KJ energy per day.The amount of glucose (molar mass = 180.0 g mol^{-1}) needed to meet this energy requirement is_____ g, (Nearest integer)
(Use : $\Delta_c H$ (glucose) = – 2700kJ mol^{-1})

Q3. To synthesise 1.0 mole of 2-methylpropan-2-ol from Ethylethanoate _____ equivalents of CH_3MgBr reagent will be required. (Integer value)

Q4. The spin only magnetic moment value for the complex $[Co(CN)_6]^{4-}$ is_____ BM.
[At. No. of Co= 27]

Q5. At 20°C, the vapour pressure of benzene is 70 torr and that of methyl benzene is 20 torr. The mole fraction of benzene in the vapour phase at 20°C above an equimolar mixture of benzene and methyl benzene is _____ $\times 10^{-2}$. (Nearest integer)

Q6. $2SO_2(g) + O_2(g) \rightleftharpoons 2SO_3(g)$
In an equilibrium mixture, the partial pressures are
$P_{SO_3} = 43kPa$; $P_{O_2} = 530$ Pa and
$P_{SO_2} = 45kPa$. The equilibrium constant K_p = _____ $\times 10^{-2}$ (Nearest integer)

Q7. 250 mL of 0.5 M NaOH was added to 500 mL of 1 M HCl. The number of unreacted HCl molecules in the solution after complete reaction is_____ $\times 10^{21}$ (Nearest integer)
$\left(N_A = 6.022 \times 10^{23}\right)$

Q8. The number of nitrogen atoms in a semicarbazone molecule of acetone is_____.

Q9. The inactivation rate of a viral preparation in proportional to the amount of virus. In the first minute after preparation, 10% of the virus is inactivated. The rate constant for viral inactivation is _____$\times 10^{-3} min^{-1}$. (Nearest integer)
[Use: $\ell n10$ = 2.303; $\log_{10} 3$=0.477; property of logarithm: $\log x^y = y \log x$]

Q10. The Azimuthal quantum number for the valence electrons of Ga^+ ion is_____.
(Atomic number of Ga= 31)

PART – C (MATHEMATICS)

SECTION - A

(One Options Correct Type)

This section contains **20 multiple choice questions**. Each question has **four choices** (A), (B), (C) and (D), out of which **ONLY ONE** option is correct.

Q1. Let 'a' be a real number such that the function $f(x) = ax^2 + 6x - 15, x \in R$ is increasing in $\left(-\infty, \frac{3}{4}\right)$ and decreasing in $\left(\frac{3}{4}, \infty\right)$. Then the function $g(x) = ax^2 - 6x + 15, x \in R$ has a :

(A) local minimum at $x = \frac{3}{4}$
(B) local maximum at $x = \frac{3}{4}$
(C) local minimum at $x = -\frac{3}{4}$
(D) local maximum at $x = -\frac{3}{4}$

Q2. Let $A = [a_{ij}]$ be a 3 x 3 matrix, where

$$a_{ij} = \begin{cases} 1 & , \text{ if } i = j \\ -x & , \text{ if } |i-j| = 1 \\ 2x+1 & , \text{ otherwise} \end{cases}$$

Let a function $f : R \to R$ be defined as $f(x) = \det(A)$. Then the sum of maximum and minimum values of f on R is equal to :

(A) $-\frac{88}{27}$
(B) $\frac{20}{27}$
(C) $\frac{88}{27}$
(D) $-\frac{20}{27}$

Q3. If z and ω are two complex numbers such that $|z\omega| = 1$ and $\arg(z) - \arg(w) = \frac{3\pi}{2}$, then $\arg\left(\frac{1 - 2\bar{z}\omega}{1 + 3\bar{z}\omega}\right)$ is :

(Here arg(z) denotes the principal argument of complex number z)

(A) $\frac{\pi}{4}$
(B) $-\frac{3\pi}{4}$
(C) $-\frac{\pi}{4}$
(D) $\frac{3\pi}{4}$

Q4. If in a triangle ABC, AB = 5 units, $\angle B = \cos^{-1}\left(\frac{3}{5}\right)$ and radius of circumcircle of ΔABC is 5 units, then the area (in sq. units) of ΔABC is :

(A) $6 + 8\sqrt{3}$
(B) $8 + 2\sqrt{2}$
(C) $10 + 6\sqrt{2}$
(D) $4 + 2\sqrt{3}$

Q5. Words with or without meaning are to be formed using all the letters of the word EXAMINATION. The probability that the letter M appears at the fourth position in any such word is :

(A) $\frac{1}{9}$ (B) $\frac{1}{11}$

(C) $\frac{2}{11}$ (D) $\frac{1}{66}$

Q6. The coefficient of x^{256} in the expansion of $(1-x)^{101}(x^2+x+1)^{100}$ is :

(A) $-{}^{100}C_{15}$ (B) ${}^{100}C_{16}$

(C) $-{}^{100}C_{16}$ (D) ${}^{100}C_{15}$

Q7. The probability of selecting integers $a \in [-5, 30]$ such that $x^2+2(a+4)x-5a+64>0$, for all $x \in R$ is :

(A) $\frac{2}{9}$ (B) $\frac{1}{6}$

(C) $\frac{7}{36}$ (D) $\frac{1}{4}$

Q8. The Boolean expression $(p \wedge \sim q) \Rightarrow (q \vee \sim p)$ is equivalent to :

(A) $q \Rightarrow p$ (B) $\sim q \Rightarrow p$

(C) $p \Rightarrow q$ (D) $p \Rightarrow \sim q$

Q9. Let a be a positive real numbers such that

$\int_0^a e^{x-[x]}dx = 10e-9$

where [x] is the greatest integer less than or equal to x. Then a is equal to :

(A) $10+\log_e 2$ (B) $10-\log_e(1+e)$

(C) $10+\log_e(1+e)$ (D) $10+\log_e 3$

Q10. Let $A=\begin{bmatrix}2 & 3\\ a & 0\end{bmatrix}, a \in R$ be written as P + Q where P is a symmetric matrix and Q is skew symmetric matrix. If $\det(Q)=9$, then the modulus of the sum of all possible values of determinant of P is equal to :

(A) 18 (B) 36

(C) 24 (D) 45

Q11. If α and β are the distinct roots of the equation $x^2+(3)^{1/4}x+3^{1/2}=0$, then the value of $\alpha^{96}(\alpha^{12}-1)+\beta^{96}(\beta^{12}-1)$ is equal to :

(A) 56×3^{25} (B) 28×3^{25}

(C) 52×3^{24} (D) 56×3^{24}

Q12. The number of real roots of the equation

$\tan^{-1}\sqrt{x(x+1)}+\sin^{-1}\sqrt{x^2+x+1}=\frac{\pi}{4}$ is :

(A) 4
(B) 1
(C) 0
(D) 2

Q13. The value of the integral $\int_{-1}^{1}\log_e\left(\sqrt{1-x}+\sqrt{1+x}\right)dx$ is equal to :

(A) $2\log_e 2+\frac{\pi}{4}-1$
(B) $\log_e 2+\frac{\pi}{2}-1$
(C) $\frac{1}{2}\log_e 2+\frac{\pi}{4}-\frac{3}{2}$
(D) $2\log_e 2+\frac{\pi}{2}-\frac{1}{2}$

Q14. The mean of 6 distinct observations is 6.5 and their variance is 10.25. If 4 out of 6 observations are 2, 4, 5 and 7, then the remaining two observations are :

(A) 10, 11
(B) 3, 18
(C) 1, 20
(D) 8, 13

Q15. Let the tangent to the parabola $S: y^2=2x$ at the point P(2, 2) meet the x-axis at Q and normal at it meet the parabola S at the point R. Then the area (in sq. units) of the triangle PQR is equal to :

(A) $\frac{35}{2}$
(B) $\frac{15}{2}$
(C) 25
(D) $\frac{25}{2}$

Q16. Let a function $f: R\to R$ be defined as

$$f(x)=\begin{cases}\sin x-e^x & \text{if } x\le 0\\ a+[-x] & \text{if } 0<x<1\\ 2x-b & \text{if } x\ge 1\end{cases}$$

where [x] is the greatest integer less than or equal to x. If f is continuous on R, then $(a+b)$ is equal to :

(A) 2
(B) 5
(C) 3
(D) 4

Q17. Let $\vec{a}=2\hat{i}+\hat{j}-2\hat{k}$ and $\vec{b}=\hat{i}+\hat{j}$. If $\vec{c}$ is a vector such that $\vec{a}.\vec{c}=|\vec{c}|, |\vec{c}-\vec{a}|=2\sqrt{2}$ and the angle between $(\vec{a}\times\vec{b})$ and $\vec{c}$ is $\frac{\pi}{6}$, then the value of $\left|(\vec{a}\times\vec{b})\times\vec{c}\right|$ is :

(A) 3
(B) $\frac{2}{3}$
(C) $\frac{3}{2}$
(D) 4

Q18. Let $y = y(x)$ be the solution of the differential equation

$$e^x\sqrt{1-y^2}\,dx + \left(\frac{y}{x}\right)dy = 0,\ y(1) = -1$$

Then the value of $(y(3))^2$ is equal to :

(A) $1+4e^6$
(B) $1-4e^3$
(C) $1+4e^3$
(D) $1-4e^6$

Q19. Let [x] denote the greatest integer $\le x$, where $x \in R$. If the domain of the real valued function $f(x) = \sqrt{\frac{|[x]|-2}{|[x]|-3}}$ is $(-\infty, a) \cup [b, c) \cup [4, \infty)$, $a < b < c$, then the value of $a+b+c$ is:

(A) -3
(B) 8
(C) -2
(D) 1

Q20. Let $y = y(x)$ be the solution of the differential equation

$$x\tan\left(\frac{y}{x}\right)dy = \left(y\tan\left(\frac{y}{x}\right) - x\right)dx,\ -1 \le x \le 1,\ y\left(\frac{1}{2}\right) = \frac{\pi}{6}$$

Then the area of the region bounded by the curves $x = 0$, $x = \frac{1}{\sqrt{2}}$ and $y = y(x)$ in the upper half plane is :

(A) $\frac{1}{6}(\pi-1)$
(B) $\frac{1}{8}(\pi-1)$
(C) $\frac{1}{4}(\pi-2)$
(D) $\frac{1}{12}(\pi-3)$

SECTION - B

(Numerical Answer Type)

This section contains **10** questions. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the **second decimal place**).

Q1. Let $A = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}$ and $B = 7A^{20} - 20A^{7} + 2I$, where I is an identity matrix of order 3 x 3.

If $B = [b_{ij}]$, then b_{13} is equal to.....

Q2. Let $y = mx + c, m > 0$ be the focal chord of $y^2 = -64x$, which is tangent to $(x+10)^2 + y^2 = 4$. Then, the value of $4\sqrt{2}(m+c)$ is equal to .....

Q3. Let a, b, c, d be in arithmetic progression with common difference λ.

If $\begin{vmatrix} x+a-c & x+b & x+a \\ x-1 & x+c & x+b \\ x-b+d & x+d & x+c \end{vmatrix} = 2$, then the value of λ^2 is equal to.....

Q4. Let $\vec{a}, \vec{b}, \vec{c}$ be three mutually perpendicular vectors of the same magnitude and equally inclined at an angle θ, with the vector $\vec{a} + \vec{b} + \vec{c}$. Then $36\cos^2 2\theta$ is equal to.....

Q5. Let T be the tangent to the ellipse $E: x^2 + 4y^2 = 5$ at the point $P(1,1)$. If the area of the region bounded by the tangent T, ellipse E, lines x = 1 and $x = \sqrt{5}$ is $\alpha\sqrt{5} + \beta + \gamma\cos^{-1}\left(\frac{1}{\sqrt{5}}\right)$, then $|\alpha + \beta + \gamma|$ is equal to.....

Q6. There are 15 players in a cricket team, out of which 6 are bowlers, 7 are batsmen and 2 are wicketkeepers. The number of ways, a team of 11 players be selected from them so as to include at least 4 bowlers, 5 batsmen and 1 wicketkeeper, is .....

Q7. The number of rational terms in the binomial expansion of $\left(4^{\frac{1}{4}} + 5^{\frac{1}{6}}\right)^{120}$ is.....

Q8. Let P be a plane passing through the points $(1,0,1)$ $(1,-2,1)$ and $(0,1,-2)$. Let a vector $\vec{a} = \alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$ be such that $\vec{a}$ is parallel to the plane P, perpendicular to $(\hat{i} + 2\hat{j} + 3\hat{k})$ and $\vec{a}.(\hat{i} + \hat{j} + 2\hat{k}) = 2$, then $(\alpha - \beta + \gamma)^2$ equals.....

Q9. If the value of $\lim_{x \to 0}\left(2 - \cos x\sqrt{\cos 2x}\right)^{\left(\frac{x+2}{x^2}\right)}$ is equal to e^a, then a is equal to.....

Q10. If the shortest distance between the lines $\vec{r}_1 = \alpha\hat{i} + 2\hat{j} + 2\hat{k} + \lambda(\hat{i} - 2\hat{j} + 2\hat{k}), \lambda \in R$, $\alpha > 0$ and $\vec{r}_2 = -4\hat{i} - \hat{k} + \mu(3\hat{i} - 2\hat{j} - 2\hat{k}), \mu \in R$ is 9, then α is equal to.....

KEYS to JEE (Main)-2021

PART – A (PHYSICS)

SECTION - A

1.	**D**	2.	**A**	3.	**A**	4.	**C**
5.	**A**	6.	**D**	7.	**B**	8.	**A**
9.	**C**	10.	**A**	11.	**C**	12.	**B**
13.	**B**	14.	**C**	15.	**A**	16.	**D**
17.	**A**	18.	**C**	19.	**D**	20.	**A**

SECTION - B

1.	**2**	2.	**100**	3.	**1**	4.	**132**
5.	**1**	6.	**4**	7.	**25**	8.	**3**
9.	**2**	10.	**40**				

PART – B (CHEMISTRY)

SECTION - A

1.	**D**	2.	**B**	3.	**D**	4.	**D**
5.	**B**	6.	**C**	7.	**B**	8.	**D**
9.	**A**	10.	**B**	11.	**A**	12.	**A**
13.	**B**	14.	**D**	15.	**B**	16.	**A**
17.	**C**	18.	**B**	19.	**B**	20.	**A**

SECTION - B

1.	**3**	2.	**667**	3.	**2**	4.	**2**
5.	**78**	6.	**172**	7.	**226**	8.	**3**
9.	**106**	10.	**0**				

PART – C (MATHEMATICS)

SECTION - A

1.	**D**	2.	**A**	3.	**B**	4.	**A**
5.	**B**	6.	**D**	7.	**A**	8.	**C**
9.	**A**	10.	**B**	11.	**C**	12.	**C**
13.	**B**	14.	**A**	15.	**D**	16.	**C**
17.	**C**	18.	**D**	19.	**C**	20.	**B**

SECTION – B

1.	**910**	2.	**34**	3.	**1**	4.	**4**
5.	**1.25 (As per NTA 1)**			6.	**777**	7.	**21**
8.	**81**	9.	**3**	10.	**6**		

Solutions to JEE (Main)-2021

PART – A (PHYSICS)

SECTION - A

Sol1. Since process is isochoric, so

$$Q = nC_V\Delta T = n\left(\frac{f}{2}R\right)\Delta T = 4\left(\frac{5}{2}\times R\right)(50) = 500R$$

Sol2. As we know that radius of circular path in magnetic field is given as

$r = \frac{mv}{qB} = \frac{\sqrt{2mK}}{qB}$, so

Charged particle	Charge	Mass
Deuteron	e	2m
Alpha particle	2e	4m

$$\frac{r_d}{r_\alpha} = \sqrt{\frac{m_d}{m_\alpha}}\times\frac{q_\alpha}{q_d} = \frac{1}{\sqrt{2}}\times 2 = \sqrt{2}$$

Sol3. $\vec{A}\cdot\vec{B} = \left|\vec{A}\times\vec{B}\right| \Rightarrow AB\cos\theta = AB\sin\theta \Rightarrow \theta = 45^0$

$$\left|\vec{A}-\vec{B}\right| = \sqrt{A^2+B^2-2AB\cos\theta} = \sqrt{A^2+B^2-2AB\times\frac{1}{\sqrt{2}}} = \sqrt{A^2+B^2-\sqrt{2}AB}$$

Sol4. As we know that root mean square speed is given as

$v = \sqrt{\frac{3RT}{M}}$, so

$$v_A > v_B > v_C \Rightarrow \frac{1}{v_A} < \frac{1}{v_B} < \frac{1}{v_C} \quad \text{as} \quad m_A < m_B < m_C.$$

Sol5. **Using conservation linear momentum, we can write**

$$Mv = \frac{h}{\lambda} = \frac{h\nu}{c} \Rightarrow v = \frac{h\nu}{Mc}$$

Loss of internal energy= Gain in kinetic energy+ energy of gamma ray

$$= \frac{1}{2}Mv^2 + h\nu = \frac{1}{2}M\left(\frac{h\nu}{Mc}\right)^2 + h\nu = h\nu\left[1+\frac{h\nu}{2Mc^2}\right]$$

Difficulty Level: Difficult and Very good Question

Sol6. As we know that equivalent half life of radioactive material decays by simultaneous emissions, is given as

$$\frac{1}{T}=\frac{1}{T_1}+\frac{1}{T_2}=\frac{1}{700}+\frac{1}{1400}=\frac{3}{1400}\Rightarrow T=\frac{1400}{3}\text{ years}$$

$$N=N_0e^{-\lambda t}\Rightarrow\frac{N_0}{3}=N_0e^{-\lambda t}\Rightarrow\frac{1}{3}=e^{-\lambda t}\Rightarrow\lambda t=\ell n(3)$$

$$\Rightarrow t=\frac{\ell n(3)}{\lambda}=\frac{\ell n(3)}{\ell n(2)}T=\frac{1.1}{0.693}\times\frac{1400}{3}\approx 740.74\text{ years}$$

Sol7. **Method-I: Using Kirchhoff's Law for loop $\overline{BDABB}$, we can write**

$$-140+20I+6I_1=0\Rightarrow 10I+3I_1=70\ .....(i)$$

Using Kirchhoff's Law for loop $\overline{ACBBA}$, we can write

$$5(I-I_1)+90-6I_1=0$$

$$\Rightarrow -5I+11I_1=90\ .....(ii)$$

Adding equation (1) with twice the equation (2), we have

$$25I_1=250\Rightarrow I_1=\frac{250}{25}=10\,A$$

Method-II: Using concept of equivalent cell, we can write

$$V_{AB}=\frac{\frac{140}{20}+\frac{0}{6}+\frac{90}{5}}{\frac{1}{20}+\frac{1}{6}+\frac{1}{5}}=\frac{2100+0+5400}{15+50+60}$$

$$=\frac{7500}{125}=60\text{ Volt}$$

$$\Rightarrow I_1=\frac{60}{6}=10\,A$$

Sol8. $h\nu=3.4-1.51=1.89\text{ eV}$

As we know that radius of circular path in magnetic field is given as

$$r=\frac{\sqrt{2mK}}{qB}\Rightarrow K=\frac{r^2q^2B^2}{2m}$$

$$\Rightarrow K=\frac{(7\times10^{-3})^2\times(1.6\times10^{-19})\times(5\times10^{-4})^2}{2\times9.1\times10^{-31}}\text{eV}=107.7\times10^{-2}\text{eV}=1.08\,\text{eV}$$

$$\Rightarrow\phi=h\nu-K=1.89-1.08=0.81\text{eV}$$

Difficulty Level: Difficult and Very good Question

Sol9. $I_1 = \frac{50}{2000} = 25\,mA$, and $I = \frac{V_i - V_Z}{1000} = \frac{100-50}{1000} = 50\,mA$

$\Rightarrow I_z = I - I_1 = 25\,mA$

Sol10. When arm PQ of a rectangular conductor is moving from $x = 0$ to x = b, the flux(ϕ =ax) linked with loop increases and while moving from $x = b$ to x = 2b and from $x = 2b$ to x = b, flux (ϕ =ab) remains constant and then flux (ϕ =ax) deceases to zero as it moves from $x = b$ to x = 0 .

$e_{in} = -\frac{d\phi}{dt} = -a\frac{dx}{dt} = -av \Rightarrow$ Induced emf

$P = \frac{(e_{in})^2}{R} \Rightarrow$ Power dissipated

Sol11. **Since direction of incident ray (DIR) is from left to write , so considering refraction at point M , we can write**

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

$$\Rightarrow \frac{1.4}{v} - \frac{1.25}{-40} = \frac{1.4 - 1.25}{-25}$$

$$\Rightarrow -\frac{1.4}{v} = \frac{0.15}{25} + \frac{1.25}{40} = \frac{1.2 + 6.25}{200} = \frac{7.45}{200}$$

$$\Rightarrow v = -\frac{200 \times 1.4}{7.45} = -37.58\,cm$$

Sol12. In the frame of vehicle, vehicle is in equilibrium under the influence of pseudo force F_P

$$F_P = \frac{mv^2}{R}$$

$$N = mg\cos 30^0 + F_P \sin 30^0$$

$$\Rightarrow N = mg\cos 30^0 + \frac{mv^2}{R}\sin 30^0 .(i)$$

, and

$$f_S = \frac{mv^2}{R}\cos 30^0 - mg\sin 30^0$$

FBD in the Frame of vehicle

$$\Rightarrow \mu N = \frac{mv^2}{R}\cos 30^0 - mg\sin 30^0 \ldots.(ii)$$

By doing $(1)\times\cos 30^0 - (2)\times\sin 30^0$, we have

$$N\left(\cos 30^0 - \mu\sin 30^0\right) = mg \Rightarrow N = \frac{mg}{\cos 30^0 - \mu\sin 30^0}$$

$$\Rightarrow N = \frac{800\times 10}{0.87 - 0.2\times 0.5} = \frac{800\times 10}{0.77} = 10.2\times 10^3 \text{ kgm/s}^2$$

Difficulty Level: Difficult and Very good Question

Sol13.

Physical Quantity	Formula	Dimension
Entropy (S)	$S = \frac{dQ}{T}$	$\left[ML^2T^{-2}K^{-1}\right]$
Number of mole (μ)		**mol**
Mechanical equivalent of heat (J)	$J = \frac{W}{Q}$	**Dimensionless**
Boltzmann constant (k)	$k = \frac{\text{Energy}}{\text{Temperature}}$	$\left[ML^2T^{-2}K^{-1}\right]$
Gas constant (R)	$R = kN_a$	$\left[ML^2T^{-2}K^{-1}\right]$

$N_a \Rightarrow$ **Avogadro's Number**

$J\beta^2 \equiv [\mu kR] \Rightarrow \beta^2 \equiv [kR] \Rightarrow \beta \equiv [k] \equiv \left[ML^2T^{-2}K^{-1}\right]$, and

$S \equiv \alpha^2\beta \Rightarrow \left[ML^2T^{-2}K^{-1}\right] \equiv \alpha^2\left[ML^2T^{-2}K^{-1}\right] \Rightarrow \alpha \equiv \left[M^0L^2T^0K^0\right] \Rightarrow$ **Dimensionless**

Sol14. $I = JA\cos\theta \Rightarrow 5 = J\times 0.04\times\cos 60^0 \Rightarrow J = \frac{5}{0.02} = 250\,A/m^2$

$$J = \frac{E}{\rho} \Rightarrow E = \rho J = 44\times 10^{-8}\times 250 = 11\times 10^{-5} V/m$$

Sol15. Assuming Tension in metal wire suspended from roof varies linearly and 0 and T_0 (developed due its own weight) are tensions are ends of of wire of length ℓ. So

$T_1\ \alpha\ (\ell_1 - \ell)$, and $T_2\ \alpha\ (\ell_2 - \ell)$

$$\frac{T_1}{T_2} = \frac{\ell_1 - \ell}{\ell_2 - \ell} \Rightarrow \ell = \frac{T_1\ell_2 - T_2\ell_1}{T_1 - T_2}$$

Sol16. $C = \frac{\varepsilon_0 A}{d} \Rightarrow X_C = \frac{1}{C\omega} = \frac{d}{\varepsilon_0 A\omega} \Rightarrow I_0 = \frac{V_0}{X_C} = \frac{V_0\varepsilon_0 A\omega}{d} = \frac{2\pi f V_0\varepsilon_0 A}{d}$

$$\Rightarrow I_0 = \frac{2\times 3.14\times 50\times 20\times 8.85\times 10^{-12}\times 1}{2\times 10^{-3}} = 27.79\times 10^{-6} A = 27.79\,\mu A$$

Sol17. $\vec{v}_{BW} = 4\sqrt{2}\left(\cos 45^0\,\hat{i} + \sin 45^0\,\hat{j}\right) = 4\hat{i} + 4\hat{j}$, and $\vec{v}_{Wg} = 0\hat{i} - \hat{j}$

$$\vec{v}_{Bg} = \vec{v}_{BW} + \vec{v}_{Wg} = \left(4\hat{i} + 4\hat{j}\right) + \left(0\hat{i} - \hat{j}\right) = 4\hat{i} + 3\hat{j}$$

$\left|\vec{v}_{Bg}\right| = 5\ m/s \Rightarrow$ Speed of Butterfly

Magnitude of displacement of Butterfly= $\left|\vec{v}_{Bg}\right|\times t = 5\times 3 = 15$ m

Sol18. Here r is fixed

$$F = \frac{q(Q-q)}{4\pi\varepsilon_0 r^2} \Rightarrow \frac{dF}{dq} = \frac{1}{4\pi\varepsilon_0 r^2}(Q-2q) \Rightarrow \frac{dF}{dq} = -\frac{1}{2\pi\varepsilon_0 r^2}$$

For maxima or minima of force , its first derivative should be zero.

$$\frac{dF}{dq} = \frac{1}{4\pi\varepsilon_0 r^2}(Q-2q) = 0 \Rightarrow q = \frac{Q}{2}$$

Since Second Derivative is always negative so maxima will occur at this value of q.

Sol19. $70g - N = 70a = 70 \times 0.2 = 14$

$\Rightarrow N = 700 - 14 = 686N$

Sol20. $W_1 = 1000N \Rightarrow$ Weight of person at surface of earth $W_2 = 400N \Rightarrow$ Weight of person at surface of Mars. There will be a neutral point –N where weight will be zero because at this point net gravitational field due to earth and mars is zero.

SECTION - B

Sol1. The equation of wave at any time t will be $y = \frac{1}{1+(x-vt)^2}$, so $v \times 1 = 2 \Rightarrow v = 2m/s$

Sol2. Heat absorbed in cyclic process = Work done = 100 π Joule

Sol3. Let ball starts its motion with horizontal velocity v_0 , so with the help of conservation of mechanical energy , we can write

$$\frac{1}{2}mv_0^2 = \frac{1}{2}mx^2 \Rightarrow v_0 = x\sqrt{\frac{k}{m}} = 0.05 \times \sqrt{\frac{100}{0.1}} = 0.5 \times \sqrt{10}\, m/s$$

t = Time required to fall the ball $= \sqrt{\frac{2h}{g}} = 2 \times \sqrt{\frac{1}{10}}$ s

$$\Rightarrow d = v_0 \times t = 0.5 \times \sqrt{10} \times 2 \times \sqrt{\frac{1}{10}} = 1\,m$$

Sol4. Initially wall will act as observer, so **with the help of Doppler's effect, we can write**

$$f' = \frac{c}{c - v_0} f_0$$

Now wall will act as source of sound of frequency f', so **With the help of Doppler's effect, we can write**

$$f'' = \frac{c+v_0}{c} f' = \frac{c+v_0}{c-v_0} f_0$$

$$\Rightarrow 500 = \left(\frac{c+v_0}{c-v_0}\right) 400 \Rightarrow 5c - 5v_0 = 4c + 4v_0 \Rightarrow v_0 = \frac{c}{9} = \frac{330}{9} m/s = \frac{330}{9} \times \frac{18}{5} km/h = 132\ km/h$$

Sol5. $F = qE \Rightarrow$ Electric force acting on the body

$$a = \frac{F}{m} = \frac{q}{m} E = \frac{8 \times 10^{-6}}{10^{-3}} \times 100 = 0.80\ m/s^2 \Rightarrow \text{Acceleration of body}$$

$$T = 2\sqrt{\frac{2\ell}{a}} = 2\sqrt{\frac{2 \times 0.1}{0.8}} = 1\ s \Rightarrow \text{Time period of Motion}$$

Sol6. **Using conservation of linear momentum, we can write**

$$mu = Mv_1 \Rightarrow v_1 = \frac{mu}{M} \quad .....(1)$$

Using conservation of angular momentum about centre of mass of Rod, we can write

$$\frac{mu\ell}{2} = \frac{M\ell^2\omega}{12} + \frac{mv_2\ell}{2} \Rightarrow 6mu = M\ell\omega \Rightarrow \ell\omega = \frac{6mu}{M} \quad .....(2)$$

Using definition of e, we can write

$$v_1 + \frac{\ell\omega}{2} = u \Rightarrow \frac{mu}{M} + \frac{3mu}{M} = u \Rightarrow \frac{m}{M} = \frac{1}{4}$$

Sol7. **For microscopic lens**

$$m = 1 + \frac{D}{f} \Rightarrow 6 = 1 + \frac{25}{f} \Rightarrow f = 5\,cm$$

Now in second case, this lens acts as objective of compound microscope, so according to question

$$m = m_O \times m_e = \frac{LD}{f_o f_e} \Rightarrow f_e = \frac{LD}{mf_o} = \frac{60 \times 25}{12 \times 5} = 25\ cm$$

Sol8. Here $\phi = 45^0$ and current leads voltage , so $X_C > X_L$

$$\tan\phi = \frac{X}{R} = \tan 45^0 = 1 \Rightarrow X = X_C - X_L = R$$

$$\Rightarrow X_C = R + L\omega \Rightarrow X_C = 1 + 30 \times 10^{-3} \times 300 = 10$$

$$\Rightarrow \frac{1}{C\omega} = 10 \Rightarrow C = \frac{1}{10 \times \omega} = \frac{1}{10 \times 300} = \frac{1}{3} \times 10^{-3}\ F$$

Sol9. **Case-I:** when disk slides down

$$t_1 = \sqrt{\frac{2L}{g\sin\theta}} \text{ .....(i)}$$

Case-II: when disk rolls down

$$t_2 = \sqrt{\frac{2L}{\frac{g\sin\theta}{1+\beta^2}}} = \sqrt{\frac{2L}{\frac{g\sin\theta}{1+\left(\frac{k}{R}\right)^2}}} = \sqrt{\frac{2L}{\frac{g\sin\theta}{1+\left(\frac{R/\sqrt{2}}{R}\right)^2}}} = \sqrt{\frac{2L}{\frac{g\sin\theta}{1+\frac{1}{2}}}} = \sqrt{\frac{4L}{3g\sin\theta}} \text{ .... (ii)}$$

$$\Rightarrow \frac{t_1}{t_2} = \sqrt{\frac{2L}{g\sin\theta}} \times \sqrt{\frac{3g\sin\theta}{4L}} = \sqrt{\frac{3}{2}}$$

Sol10. $\mu = \frac{A_m}{A_c} = \frac{V_1 - V_2}{V_1 + V_2} = \frac{200-120}{200+120} = \frac{1}{4} \Rightarrow$ Modulation index

Here $V_C(t) = 160\sin\left(2\pi \times 10^6 t\right) V$ and $V_m(t) = A_m \sin\left(2\pi \times 10^3 t\right) V$, so

$$A_m = \frac{A_c}{4} = \frac{160}{4} = 40\,V$$

PART – B (CHEMISTRY)

SECTION - A

Sol1. Anhydride of HNO_3 produced by the treatment of P_4O_{10} (anhydring reagent) in ratio of 4:1 is N_2O_5 which is acidic in nature. $4HNO_3 + P_4O_{10} \longrightarrow \underbrace{2N_2O_5}_{\text{Acidic nature}} + (HPO_3)_4$

Sol2. 1° – Amine produced isocyanide (iso nitrile) compound with $CHCl_3$ and KOH

$$\underset{(A)}{R-NH_2} + CHCl_3 + 3KOH \longrightarrow \underset{(B)}{RNC} + 3KCl + 3H_2O$$

Isonitrile (toxic nature)

$$\xrightarrow{H_3O^+ / HCl\ (C)} \underset{(A)}{R-NH_2} + HCOOH$$

Sol3. Following are the dihedral angles of H_2O_2 in gas phase and solid (Reverse with respect to question).

Change of angle is the changing of intermolecular interaction in different phase

Sol4. Only (A) and (B) structures have resonance effect. In (A) benzyllic resonance and in (B) allylic resonance of carbocation.

Sol5. An alkali metal oxide of formula MO_2 is super oxide which is formed by K and Rb. KO_2 has pale yellow colour and paramagnetic nature, while alkaline earth metal oxides formula MO_2 are peroxide and diamagnetic nature.

Sol6. Nowdays liquid CO_2 used as washing clothes, because Cl_2 and $C_2H_2Cl_4$ are toxic

Sol7. Amylose is polysaccharide made of α-D- glucose unit of linear chain polymer not branched

Sol8. Due to volatile nature , Zn – metal can be economically purified by fractional distillation refining process.

Sol9. $B(OH)_3$ is orthoboric acid having acidic nature, while $Al(OH)_3$ has amphoteric nature that is acidic and as well as basic

Sol10. Tyndall effect is the optical characteristics of light in which comparable particle size of colloids and bombarded light wavelength can produce this type of effect. For scattering effect of light in Tyndall effect refractive index of dispersed phase and dispersion medium should be greatly different.

Sol11. $K_2\left[Ni(CN)_4\right]$

$Ni^{2+} \rightarrow 3d^8 4s^0$

Pairing developed due to strong ligand effect.

$NiCl_2.6H_2O$

$Ni^{2+} \rightarrow 3d^8 4s^0$

$Na_2[NiCl_4]$

$Ni^{2+} \rightarrow 3d^8 4s^0$

$[Ni(CO)_4]$

$Ni \rightarrow 3d^8 4s^2$

Pairing developed due to strong ligand (CO).

Sol12.

(2- Ethoxy-2,3- dimethyl butane)

Sol13. Orlon fibres are the polymer of acrylonitrile

Polyacrylonitrile (PAN)

Sol14. In BrO_4^-,Br has +7 O.S, therefore it only acts as a oxidizing reagent , does not undergo disproportionation reaction

Sol15.

Sol16. $X \xrightarrow[H_2SO_4]{Conc}$ Brown fumes + Brown ring test with $FeSO_4$ / H_2SO_4

$X \xrightarrow[HCl]{H_2S} Y(ppt) \xrightarrow[HNO_3]{Conc}$ dissolved $\xrightarrow{NH_4OH}$ Deep blue colour solution

In cation analysis of Cu^{++} ions, precipitate formed is CuS on treating with H_2S and HCl which dissolved in HNO_3 and produced blue colour complex solution $\left[Cu(NH_3)_4\right]^{++}$ in NH_4OH. Brown fumes and brown ring performance given by nitrate sample. Hence salt is $Cu(NO_3)_2$.

Sol17. All structure can produce –CHO functional group which gives +ve test of Tollen's reagent except structure (II), because if forms ketonic group after tautomerisation

Sol18. Both statement are correct for the glass body heating , but reason is not correct explanation during heating process of glass, constituents unit rupture of glass body and gives the edge smoothness.

Sol19.

Sol20. For Ni^{2+}, crystal field CFSE magnitude shows the magnitude of absorbed light.

Following are the energy absorbed order $\left[NiCl_4\right]^{2-} < \left[Ni(H_2O)\right]^{2+} < \left[Ni(CN)_4\right]^{2-}$

Hence order of colour of compounds are.

$\left[NiCl_4\right]^{2-} > \left[Ni(H_2O)_6\right]^{2+} > \left[Ni(CN)_4\right]^{2-}$

SECTION - B

Sol1. I_3^- anion has three lone pair

Sol2. 2700 kJ energy released from 180gm (1 mole) of glucose

$\therefore$ 1kJ energy released from $\frac{180}{2700}$ gm of glucose

$\therefore$ 10000 kJ energy released from $\left(\frac{180\times10000}{2700}\right)$gm of glucose

Amount of glucose = $\frac{18000}{27} = 666.66 \approx 667$gm

Sol3.

Sol4. In $\left[Co(CN)_6\right]^{4-}$, Co has +2 oxidation state

Hence, electronic configuration of Co^{2+} is $[Ar]3d^7 4s^0$. In complex $\left[Co(CN)_6\right]^{4-}$, given ligand CN^- is strong hence, after pairing in d- subshell, total number of unpaired electron = 1

$\therefore$ spin magnetic moment $= \sqrt{1(1+2)} = \sqrt{3}$ = 1.73 BM

= 1.73BM $\approx$ 2.0 BM

Sol5. P_B^o = vapour pressure of benzene at $20^oC = 70$torr

P_M^o = vapour pressure of toluene at $20^oC = 20$torr

Mixture is equimolar, $X_B = 0.5$ and $X_M = 0.5$

Total vapour pressure $(P_T) = 70\times0.5 + 20\times0.5 = 45$torr

Mole fraction of benzene in vapour phase $\left(X_B'\right) = \frac{P_B^o X_B}{P_T} = \left(\frac{70\times0.5}{45}\right)$torr

$= 0.777 = 77.7\times10^{-2}$torr

Ans = 78 (nearest integer)

Sol6. $K_p = \dfrac{(P_{SO_3})^2}{(P_{SO_2})^2 \times (P_{O_2})} = \dfrac{(43)^2}{(45)^2 \times (0.53)} = 1.72(kPa)^{-1}$

$k_P = 172 \times 10^{-2}\,(kPa)^{-1}$

Sol7.

	NaOH +	HCl	$\longrightarrow$ NaCl +	H_2O	
(Milimole) t= 0		250×0.5	500×1.0	0	0
$t = \infty$	–	375		125	125 millimole

unreacted HCl mole = $\dfrac{375}{1000}$

number of HCl molecule un reacted $= \dfrac{375}{1000} \times 6.023 \times 10^{23}$

$= 225.75 \times 10^{21} \approx 226 \times 10^{21}$

Sol8. Reaction of acetone and semicarbazide

Sol9. $Kt = 2.303 \log \dfrac{[A_0]}{[A_t]}$

Here, $[A_0] = 100$, $[A_t] = 90$ and $t = 1$ min

$\therefore K \times 1 = 2.303 \log_{10} \dfrac{100}{90}$

$\therefore K = 2.303 \times [\log 10 - \log 9] = 2.303[1 - 2\log 3]$

$\therefore K = 2.303[1 - 2 \times 0.477] = 0.105938 = 105.938 \times 10^{-3}\,min^{-1}$

Rounded = 106

Sol10. Electronic configuration of Ga^+ ion = [Ar] $3d^{10}$ $4s^2$ $4p^o$

Last electron goes into S- orbital, hence

Azimuthal quantum number (ℓ) for last electron = 0

PART – C (MATHEMATICS)

SECTION - A

Sol1. $f(x) = ax^2 + 6x - 15$

$f'(x) = 2ax + 6 = 0 \Rightarrow x = \dfrac{-3}{a} = \dfrac{3}{4} \Rightarrow a = -4$

$g(x) = -4x^2 - 6x + 15$

$g'(x) = 0 \Rightarrow -8x - 6 = 0 \Rightarrow x = -\dfrac{3}{4}$

$g''(x) = -8 < 0 \Rightarrow x = \dfrac{-3}{4}$ is a point of local maxima for g(x).

Sol2. $\det(A) = \begin{vmatrix} 1 & -x & 2x+1 \\ -x & 1 & -x \\ 2x+1 & -x & 1 \end{vmatrix}$

$= 1(1-x^2) - x(-2x^2 - x + x) + (2x+1)(x^2 - 2x - 1)$

$= 1 - x^2 + 2x^3 + 2x^3 + x^2 - 4x^2 - 2x - 2x - 1$

$f(x) = 4x^3 - 4x^2 - 4x$

$f'(x) = 4(3x^2 - 2x - 1) = 0 \Rightarrow x = 1, \frac{-1}{3}$

$f(1) + f\left(-\frac{1}{3}\right) = -4 + \left(\frac{-4}{27} - \frac{4}{9} + \frac{4}{3}\right)$

$= -4 + \frac{-4 - 12 + 36}{27} = -4 + \frac{20}{27} = \frac{-88}{27}$

Sol3. Let $w = r.e^{i\theta}$

$\arg(w) = \theta$

$\therefore z = \frac{1}{r}(\sin\theta - i\cos\theta)$

$\bar{z}w = \frac{i}{r}e^{-i\theta}.re^{i\theta} = i$

$\arg\left(\frac{1 - 2\bar{z}w}{1 + 3\bar{z}w}\right) = \arg\left(\frac{1-2i}{1+3i}\right)$

$\frac{1-2i}{1+3i} = \frac{(1-2i)(1-3i)}{1+9} = \frac{-5-5i}{10} = -\frac{1}{2}(1+i)$

$\arg\left(-\frac{1}{2}(1+i)\right) = \frac{-3\pi}{4}$

Sol4. $\cos B = \frac{3}{5}, R = 5$

$\frac{5}{\sin C} = 10 \Rightarrow C = 30^\circ$

$\Delta = \frac{1}{2}(4)(3 + 4\sqrt{3}) = 6 + 8\sqrt{3}$

Sol5. EXAMINATION, required probability

$= \frac{10!/2!\,2!\,2!}{11!/2!\,2!\,2!} = \frac{1}{11}$

Sol6. Coefficient of x^{256} in $\left\{(1-x^3)^{100} - x(1-x^3)^{100}\right\}$

= -Coefficient of x^{255} in $(1-x^3)^{100}$

$= -(-1)^{85}.\,{}^{100}C_{85} = {}^{100}C_{15}$

Sol7. D<0

$\Rightarrow 4\left[(a+4)^2-(64-5a)\right]<0$

$\Rightarrow a^2+13a-48<0$

$\Rightarrow (a+16)(a-3)<0$

$\Rightarrow -16<a<3$

but $-5\le a\le 30$

$\Rightarrow -5\le a<3$

$\Rightarrow a=-5,-4,.....,1,2$ (integers)

Probability $=\frac{8}{36}=\frac{2}{9}$

Sol8. $(P\wedge \sim q)\to(q\vee \sim p)$

$\equiv \sim(P\wedge \sim q)\vee(P\vee \sim p)$

$\equiv \sim p\vee q$

$\equiv p\to q$

Sol9. $\int_0^a e^{\{x\}}dx=10(e-9)$

$=\int_0^{[a]}e^{\{x\}}dx+\int_{[a]}^a e^x dx$

$=[a]\int_0^1 e^x dx+\int_0^{\{a\}}e^{[x]}dx$

$=[a](e-1)+e^{\{a\}}-1$

$=[a]e+e^{\{a\}}-([a]+1)$

For $[a]=10$ & $e^{\{a\}}-11=-9$

i.e. $e^{\{a\}}=2$

$a=10+\ln 2$

Sol10. $P^T=P, Q^T=-Q$

$A=\frac{(A+A^T)+(A-A^T)}{2}$

$P=\frac{A+A^T}{2}$

$=\frac{1}{2}\left(\begin{bmatrix}2 & 3\\ a & 0\end{bmatrix}+\begin{bmatrix}2 & a\\ 3 & 0\end{bmatrix}\right)$

$=\frac{1}{2}\begin{bmatrix}4 & 3+a\\ a+3 & 0\end{bmatrix}=\begin{bmatrix}2 & \frac{a+3}{2}\\ \frac{a+3}{2} & 0\end{bmatrix}$

$Q=\frac{A-A^T}{2}=\frac{1}{2}\begin{bmatrix}0 & 3-a\\ a-3 & 0\end{bmatrix}=\begin{bmatrix}0 & \frac{3-a}{2}\\ \frac{a-3}{2} & 0\end{bmatrix}$

$\det(Q) = 9 \Rightarrow \frac{(a-3)^2}{4} = 9 \Rightarrow a-3 = \pm 6 \Rightarrow a = 9, -3$

For a = 9

$\det(P) = \begin{vmatrix} 2 & 6 \\ 6 & 0 \end{vmatrix} = -36$

For a = -3

$\det(P) = \begin{vmatrix} 2 & 0 \\ 0 & 0 \end{vmatrix} = 0$

Sol11. $x^2 + 3^{\frac{1}{4}}x + 3^{\frac{1}{2}} = 0$

Put $3^{\frac{1}{4}} = a$

Equation : $x^2 + ax + a^2 = 0$

$\Rightarrow \left(\frac{x}{a}\right)^2 + \frac{x}{a} + 1 \Rightarrow x = aw, aw^2$

$\alpha = aw, \beta = aw^2$

$\alpha^{96} = a^{96}\ w^{96} = 3^{24}$

$\alpha^{12} - 1 = a^{12}\ w^{12} - 1 = 3^3 - 1 = 26$

$\alpha^{96}\left(\alpha^{12} - 1\right) + \beta^{96}\left(\beta^{12} - 1\right) = 52 \times 3^{24}$

Sol12. Domain of definition

$= \{-1, 0\}$

For $x = 0$ LHS = $\frac{\pi}{2}$

$x = -1$ LHS = $\frac{\pi}{2}$

No solution.

Sol13. $I = 2\int_0^1 \ln\left(\sqrt{1-x} + \sqrt{1+x}\right)dx$

Put $x = \cos\theta$

then $I = 2\int_{\frac{\pi}{2}}^{0} \ln\left(\sqrt{2}\left(\sin\frac{\theta}{2} + \cos\frac{\theta}{2}\right)\right).(-\sin\theta)d\theta$

$= 2\ln\sqrt{2} + 2\left[\left[\ln\left(\sin\frac{\theta}{2} + \cos\frac{\theta}{2}\right).(-\cos\theta)\right]_0^{\frac{\pi}{2}} + \int_0^{\frac{\pi}{2}} \frac{\cos\frac{\theta}{2} - \sin\frac{\theta}{2}}{\sin\frac{\theta}{2} + \cos\frac{\theta}{2}}.\frac{1}{2}\cos\theta\, d\theta\right]$

$I = 2\ln\sqrt{2} + \int_0^{\frac{\pi}{2}} (1 - \sin\theta)d\theta$

$= \ln 2 + \left[\theta + \cos\theta\right]_0^{\frac{\pi}{2}}$

$= \ln 2 + \frac{\pi}{2} - (1)$

$= \ln 2 + \frac{\pi}{2} - 1$

Sol14. $\frac{2+4+5+7+x+y}{6}=6.5$ .....(i)

$\frac{2^2+4^2+5^2+7^2+x^2+y^2}{6}-(6.5)^2=10.25$ .....(ii)

$(i)\Rightarrow x+y=21$

$(ii)\Rightarrow x^2+y^2=221$

$(i)\,\&\,(ii)\Rightarrow (x,y)=(10,11)$

Sol15. Tangent at $P(2,2)$ to $y^2=2x$ is $y(2)=(x+2)$

$\Rightarrow y=\frac{x}{2}+1$

Normal at P is $y=-2x+6$

$R\left(\frac{9}{2},-3\right)$

$Q(-2,0)$

$\Delta PQR=\frac{1}{2}\left|2(3)-2(-3-2)+\frac{9}{2}(2-0)\right|=\frac{1}{2}|6+10+9|=\frac{25}{2}$

Sol16. $f(0^-)=f(0^+)=f(0)$

$\Rightarrow -1=a-1=-1\Rightarrow a=0$

$f(1^-)=f(1^+)=f(1)$

$\Rightarrow a-1=2-b=2-b\Rightarrow a+b=3$

Sol17. $\vec{a}.\vec{c}=|\vec{c}|,\ |\vec{c}|^2+|\vec{a}|^2-2\vec{c}.\vec{a}=8,\ |\vec{a}|=3,\ |\vec{a}\times\vec{b}|=3$

$\Rightarrow |\vec{c}|^2+9-2|\vec{c}|=8\Rightarrow |\vec{c}|=1$

$\left|(\vec{a}\times\vec{b})\times\vec{c}\right|=|\vec{a}\times\vec{b}||\vec{c}|\sin\frac{\pi}{6}=\frac{3}{2}$

Sol18. $xe^x dx+\frac{ydy}{\sqrt{1-y^2}}=0$

$\Rightarrow (x-1)e^x-\sqrt{1-y^2}=c$

Point $(1,-1)\Rightarrow c=0$

$x=3\Rightarrow 2e^3=\sqrt{1-y^2}$

$y^2=1-4e^6$

Sol19. $\frac{|[x]|-2}{|[x]|-3}\geq 0$

$\Rightarrow |[x]|\leq 2$ or $|[x]|>3$

$\Rightarrow -2\leq [x]\leq 2,\qquad [x]<-3$ or $[x]>3$

$\Rightarrow -2\leq x<3,\qquad x<-3$ or $x\geq 4$

$x\in(-\infty,-3)\cup[-2,3)\cup[4,\infty),\ a+b+c=-3-2+3=-2$

Sol20. $y = vx$

$\Rightarrow$ D.E. $\tan v.\left(v + x\dfrac{dv}{dx}\right) = (v\tan V - 1)$

$\Rightarrow \tan vx\dfrac{dv}{dx} + 1 = 0$

$\Rightarrow \tan v dv + \dfrac{1}{x}dx = 0 \Rightarrow x = c\cos\dfrac{y}{x}$

Point $\left(\dfrac{1}{2}, \dfrac{\pi}{6}\right)$

$\Rightarrow C = 1$

$x = \cos\dfrac{y}{x}$

$\Rightarrow y = x\cos^{-1}x$

$A = \int_0^{\frac{1}{\sqrt{2}}} x\cos^{-1}x dx = \int_{\frac{\pi}{2}}^{\frac{\pi}{4}} \cos\theta.\theta.(-\sin\theta)d\theta$

$= \int_{\frac{\pi}{2}}^{\frac{\pi}{4}} -\dfrac{\theta}{2}\sin 2\theta d\theta = \dfrac{\pi - 1}{8}$

SECTION - B

Sol1. $A = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$

$B = 7A^{20} - 20A^7 + 2I$

$A^2 = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$

$A^3 = \begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -3 & 3 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{bmatrix}$

$A^4 = \begin{bmatrix} 1 & -3 & 3 \\ 0 & 1 & -3 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -4 & 6 \\ 0 & 1 & -4 \\ 0 & 0 & 1 \end{bmatrix}$

a_{13} of $A, A^2, A^3, ...$

are 0, 1, 3, 6,…….

For 0, 1, 3, 6, ……..

$S_n = 0 + 1 + 3 + 6 + ... + t_n$

$S_n = \quad 0 + 1 + 3 + ... + t_{n-1} + t_n$

$t_n = 1 + 2 + 3 + ... + (t_n - t_{n-1})$

$= \dfrac{(n-1).n}{2}$

$b_{13} = 7(190) - 20(21) + 0$

$= 1330 - 420 = 910$

Sol2. $0 = -16m + c$ .....(i)

$$\frac{|m(-10)+C|}{\sqrt{m^2+1}} = 2 \text{ .....(ii)}$$

(i) & (ii) $\Rightarrow 36m^2 = 4(m^2+1)$

$$m = \frac{1}{2\sqrt{2}}, C = \frac{8}{\sqrt{2}}$$

$$4\sqrt{2}(m+c) = 34$$

Sol3. $R_1 \to R_1 - R_2,\ R_2 \to R_2 - R_3$

$$\Delta = \begin{vmatrix} a-c+1 & b-c & a-b \\ b-d-1 & c-d & b-c \\ x-b+d & x+d & x+c \end{vmatrix}$$

$C_1 \to C_1 - C_2$ & $C_2 \to C_2 - C_3$

$$\Delta = \begin{vmatrix} a+1-b & 2b-c-a & a-b \\ b-1-c & 2c-b-d & b-c \\ -b & d-c & x+c \end{vmatrix} = \begin{vmatrix} -\lambda+1 & 0 & -\lambda \\ -\lambda-1 & 0 & -\lambda \\ -b & \lambda & x+c \end{vmatrix}, R_1 \to R_1 - R_2$$

$$= \begin{vmatrix} 2 & 0 & 0 \\ -\lambda-1 & 0 & -\lambda \\ -b & \lambda & x+c \end{vmatrix} = 2\lambda^2 = 2 \Rightarrow \lambda^2 = 1$$

Sol4. $|\vec{a}| = |\vec{b}| = |\vec{c}| = \ell$

$$\vec{a}.\vec{b} = \vec{b}.\vec{c} = \vec{c}.\vec{a} = 0$$

$$|\vec{a}+\vec{b}+\vec{c}|^2 = 3\ell^2$$

$$\vec{a}.(\vec{a}+\vec{b}+\vec{c}) = |\vec{a}||\vec{a}+\vec{b}+\vec{c}|\cos\theta$$

$$\Rightarrow |\vec{a}|^2 + 0 + 0 = |\vec{a}||\vec{a}+\vec{b}+\vec{c}|\cos\theta$$

$$\Rightarrow \ell^2 = \sqrt{3}\ell^2\cos\theta \Rightarrow \cos\theta = \frac{1}{\sqrt{3}}$$

$$36\cos^2 2\theta = 36\left(\frac{2}{3}-1\right)^2 = 4$$

Sol5. Area = $\frac{1}{2}(\sqrt{5}-1)\frac{9-\sqrt{5}}{4} - \int_1^{\sqrt{5}} \frac{1}{2}\sqrt{5-x^2}\,dx$

$$= \frac{1}{8}(-14+10\sqrt{5}) - \frac{1}{2}\int_{\cos^{-1}\frac{1}{\sqrt{5}}}^{0} (-\sin^2\theta)\,d\theta$$

$$A = \frac{-14}{8} + \frac{5}{4}\sqrt{5} - \left(\frac{5}{4}\cos^{-1}\frac{1}{\sqrt{5}} - \frac{1}{2}\right)$$

$$= \frac{5}{4}\sqrt{5} - \frac{5}{4} - \frac{5}{4}\cos^{-1}\frac{1}{\sqrt{5}}$$

$$\alpha = \frac{5}{4}, \beta = \frac{-5}{4}, \gamma = \frac{-5}{4}$$

$$|\alpha + \beta + \gamma| = \frac{5}{4}$$

Sol6.

Bowlers	Batsmen	Wicket Keepers
(6)	(7)	(2)
4	5	1
5	6	2
6	7	

Required number of ways

$$= {}^6C_4 \times {}^7C_6 \times {}^2C_1 + {}^6C_5 \times {}^7C_5 \times {}^2C_1 + {}^6C_4 \times {}^7C_5 \times {}^2C_2$$

$$= 15 \times 7 \times 2 + 6 \times 21 \times 2 + 15 \times 21 \times 1$$

$$= 210 + 252 + 315 = 777$$

Sol7. $T_{r+1} = {}^{120}C_r \, 4^{\frac{120-r}{4}} . 5^{r/6}$

For T_{r+1} to be rational

r should be multiple of 6

Number of such terms = 21

Sol8. For the plane P,

$$\vec{n} = \frac{1}{2}\left(-2\hat{j}\right) \times \left(-\hat{i} + \hat{j} - 3\hat{k}\right) = \hat{j} \times \hat{i} + 3\hat{j} \times \hat{k} = -\hat{k} + 3\hat{i}$$

$$\vec{a} . \vec{n} = 0 \Rightarrow 3\alpha - \gamma = 0 \text{ .....(i)}$$

$$\vec{a} . (1,2,3) = 0 \Rightarrow \alpha + 2\beta + 3\gamma = 0 \text{ .....(ii)}$$

$$\vec{a} . (1,1,2) = 2 \Rightarrow \alpha + \beta + 2\gamma = 2 \text{ .....(iii)}$$

(i), (ii) & (iii) $\Rightarrow \alpha = 1, \beta = -5, \gamma = 3$

$$(\alpha - \beta + \gamma)^2 = 81$$

Sol9. $\lim\limits_{x \to 0} \left(2 - \cos x . \sqrt{\cos 2x}\right)^{\frac{x+2}{x^2}}$ $\quad (1^\infty)$

$$= \lim_{x \to 0} e^{\left(1 - \cos x . \sqrt{\cos 2x}\right) . \frac{x+2}{x^2}}$$

$$= \lim_{x \to 0} e^{(x+2) \frac{\left(1 - \cos x . \sqrt{\cos 2x}\right)}{x^2}}$$

$$= \lim_{x \to 0} e^{2 . \left(\frac{\sin x \sqrt{\cos 2x} - \cos x . \frac{1(-2\sin 2x)}{2\sqrt{\cos 2x}}}{2x} \right)}$$

$$= \lim_{x \to 0} e^{2 . \left(\frac{1}{2} + 1\right)} = e^3 = e^a$$

$$\Rightarrow a = 3$$

Sol10. dist. $= \dfrac{\vec{A}_1A_2 \cdot \left(\vec{b}_1 \times \vec{b}_2\right)}{\left|\vec{b}_1 \times \vec{b}_2\right|}$

$= \dfrac{(-4-\alpha, -2, -3).(2,2,1)}{3}$

$= \dfrac{-8-2\alpha-4-3}{3} = -5 - \dfrac{2\alpha}{3}$

$\left|-5 - \dfrac{2\alpha}{3}\right| = 9 \Rightarrow -5 - \dfrac{2\alpha}{3} = \pm 9$

$\Rightarrow \dfrac{2\alpha}{3} = 4, -14$, but $\alpha > 0$

$\alpha = 6$

FINAL JEE–MAIN EXAMINATION – AUGUST, 2021

(Held On Thursday 26th August, 2021) TIME : 3 : 00 PM to 6 : 00 PM

PHYSICS — TEST PAPER WITH SOLUTION

SECTION-A

1. The temperature of equal masses of three different liquids x,y and z are 10ºC, 20ºC and 30ºC respectively. The temperature of mixture when x is mixed with y is 16ºC and that when y is mixed with z is 26ºC. The temperature of mixture when x and z are mixed will be :

(1) 28.32º C (2) 25.62º C

(3) 23.84ºC (4) 20.28ºC

Official Ans. by NTA (3)

Sol.

X	Y	Z
$m_1 = m$	$m_2 = m$	$m_3 = m$
$T_1 = 10°C$	$T_2 = 20°C$	$T_3 = 30°C$
s_1	s_2	s_3

when x & y are mixed, $T_{f_1} = 16°C$

$m_1s_1T + m_2s_2T_2 = (m_1s_1 + m_2s_2)Tf_1$

$s_1 \times 10 + s_2 \times 20 = (s_1 + s_2) \times 16$

$s_1 = \frac{2}{3}s_2$ ….(i)

when y & z are mixex, $T_{f_2} = 26°C$

$m_2s_2T + m_3s_3T_3 = (m_3s_3 + m_3s_3)Tf_2$

$s_2 \times 20 + s_3 \times 30 = (s_2 + s_3) \times 26$

$s_3 = \frac{3}{2}s_2$ .....(ii)

when x & z are mixex

$m_1s_1T_1 + m_3s_3T_3 = (m_1s_1 + m_3s_3)Tf$

$$\frac{2}{3}s_2 \times 10 + \frac{2}{3}s_2 \times 20 = \left(\frac{2}{3}s_2 + \frac{3}{2}s_2\right)T_f$$

$T_f = 23.84°C$

Ans (3)

2. The de-Broglie wavelength of a particle having kinetic energy E is λ. How much extra energy must be given to this particle so that the de-Broglie wavelength reduces to 75% of the initial value ?

(1) $\frac{1}{9}E$ (2) $\frac{7}{9}E$

(3) E (4) $\frac{16}{9}E$

Official Ans. by NTA (2)

Sol. $\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2mE}}$, $mv = \sqrt{2mE}$

$\lambda \propto \frac{1}{\sqrt{E}}$

$\frac{\lambda_2}{\lambda_1} = \sqrt{\frac{E_1}{E_2}} = \frac{3}{4}$, $\lambda_2 = 0.75\,\lambda_1$

$\frac{E_1}{E_2} = \left(\frac{3}{4}\right)^2$

$E_2 = \frac{16}{9}E_1 = \frac{16}{9}E$ $(E_1 = E)$

Extra energy given = $\frac{16}{9}E - E = \frac{7}{9}E$

Ans. 2

3. A particle of mass m is suspended from a ceiling through a string of length L. The particle moves in a horizontal circle of radius r such that $r = \frac{L}{\sqrt{2}}$.The speed of particle will be :

(1) $\sqrt{rg}$ (2) $\sqrt{2rg}$ (3) $2\sqrt{rg}$ (4) $\sqrt{\frac{rg}{2}}$

Official Ans. by NTA (1)

Sol. Conical pendulum

$r = \frac{\ell}{\sqrt{2}}$

$\sin\theta = \frac{r}{\ell} = \frac{1}{\sqrt{2}}$

$\theta = 45°$

$T\sin\theta = \frac{mv^2}{r}$

$T\cos\theta = mg$

$\tan\theta = \frac{v^2}{rg} \Rightarrow v = \sqrt{rg}$

Ans. 1

4. A cylindrical container of volume 4.0×10^{-3} m^3 contains one mole of hydrogen and two moles of carbon dioxide. Assume the temperature of the mixture is 400 K. The pressure of the mixture of gases is :

[Take gas constant as 8.3 J mol^{-1} K^{-1}]

(1) 249×10^1 Pa (2) 24.9×10^3 Pa

(3) 24.9×10^5 Pa (4) 24.9 Pa

Official Ans. by NTA (3)

Sol. $V = 4 \times 10^{-3}\ m^3$

n = 3 moles

T = 400K

$$PV = nRT \Rightarrow P = \frac{nRT}{V}$$

$$P = \frac{3 \times 8.3 \times 400}{4 \times 10^{-3}}$$

$= 24.9 \times 10^5$ Pa

Ans 3

5. The angle between vector $(\vec{A})$ and $(\vec{A} - \vec{B})$ is :

(1) $\tan^{-1}\left(\dfrac{-\frac{B}{2}}{A - B\frac{\sqrt{3}}{2}}\right)$

(2) $\tan^{-1}\left(\dfrac{A}{0.7B}\right)$

(3) $\tan^{-1}\left(\dfrac{\sqrt{3}B}{2A - B}\right)$

(4) $\tan^{-1}\left(\dfrac{B\cos\theta}{A - B\sin\theta}\right)$

Official Ans. by NTA (3)

Sol.

Angle between $\vec{A}$ and $\vec{B}$, θ = 60°

Angle betwenn $\vec{A}$ and $\vec{A} - \vec{B}$

$$\tan\alpha = \frac{B\sin\theta}{A - B\cos\theta}$$

$$= \frac{B\sqrt{\frac{3}{2}}}{A - B \times \frac{1}{2}2}$$

$$\tan\alpha = \frac{\sqrt{3}B}{2A - B}$$

Ans 3

6. A light beam is described by $E = 800 \sin\omega\left(t - \frac{x}{c}\right)$

.An electron is allowed to move normal to the propagation of light beam with a speed of 3×10^7 ms^{-1}. What is the maximum magnetic force exerted on the electron ?

(1) 1.28×10^{-18} N

(2) 1.28×10^{-21} N

(3) 12.8×10^{-17} N

(4) 12.8×10^{-18} N

Official Ans. by NTA (4)

Sol. $\dfrac{E_0}{C} = B_0$

$F_{max} = eB_0V$

$$= 1.6 \times 10^{-19} \times \frac{800}{3 \times 10^8} \times 3 \times 10^7$$

$= 12.8 \times 10^{-18}$N

Ans. 4

7. The two thin coaxial rings, each of radius 'a' and having charges +Q and –Q respectively are separated by a distance of 's'. The potential difference between the centres of the two rings is :

(1) $\frac{Q}{2\pi\varepsilon_0}\left[\frac{1}{a}+\frac{1}{\sqrt{s^2+a^2}}\right]$

(2) $\frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{a}+\frac{1}{\sqrt{s^2+a^2}}\right]$

(3) $\frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{a}-\frac{1}{\sqrt{s^2+a^2}}\right]$

(4) $\frac{Q}{2\pi\varepsilon_0}\left[\frac{1}{a}-\frac{1}{\sqrt{s^2+a^2}}\right]$

Official Ans. by NTA (4)

Sol.

$$V_A=\frac{KQ}{a}-\frac{KQ}{\sqrt{a^2+s^2}}$$

$$V_B=\frac{-KQ}{a}+\frac{KQ}{\sqrt{a^2+s^2}}$$

$$V_A-V_B=\frac{2KQ}{a}-\frac{2KQ}{\sqrt{a^2+s^2}}$$

$$=\frac{Q}{2\pi\varepsilon_0}\left(\frac{1}{a}-\frac{1}{s^2+a^2}\right)$$

Ans 4

8. If you are provided a set of resistances 2Ω, 4Ω, 6Ω and 8Ω. Connect these resistances so as to obtain an equivalent resistance of $\frac{46}{3}\Omega$.

(1) 4Ω and 6Ω are in parallel with 2Ω and 8 Ω in series

(2) 6Ω and 8Ω are in parallel with 2Ω and 4Ω in series

(3) 2Ω and 6Ω are in parallel with 4Ω and 8 Ω in series

(4) 2Ω and 4Ω are in parallel with 6Ω and 8Ω in series

Official Ans. by NTA (4)

Sol.

Ans 4

9. The solid cylinder of length 80 cm and mass M has a radius of 20 cm. Calculate the density of the material used if the moment of inertia of the cylinder about an axis CD parallel to AB as shown in figure is 2.7 kg m^2.

(1) 14.9 kg / m^3 (2) 7.5 × 10^1 kg / m^3

(3) 7.5 × 10^2 kg/m^3 (4) 1.49 × 10^2 kg / m^3

Official Ans. by NTA (4)

Sol. Parallel axis theorem

$I = I_{CM} + Md^2$

$$I=\frac{Mr^2}{2}+M\left(\frac{L}{2}\right)^2$$

$$2.7=M\frac{(0.2)^2}{2}+M\left(\frac{0.8}{2}\right)^2$$

$$2.7=M\left[\frac{2}{100}+\frac{16}{100}\right]$$

M = 15kg

$$\Rightarrow \rho=\frac{M}{\pi r^2 L}=\frac{15}{\pi(0.2)^2\times 0.8}$$

= 0.1492 × 10^3

Ans. 4

10. A parallel - plate capacitor with plate area A has separation d between the plates. Two dielectric slabs of dielectric constant K_1 and K_2 of same area A/2 and thickness d/2 are inserted in the space between the plates. The capacitance of the capacitor will be given by :

(1) $\frac{\varepsilon_0 A}{d}\left(\frac{1}{2}+\frac{K_1K_2}{K_1+K_2}\right)$

(2) $\frac{\varepsilon_0 A}{d}\left(\frac{1}{2}+\frac{K_1K_2}{2(K_1+K_2)}\right)$

(3) $\frac{\varepsilon_0 A}{d}\left(\frac{1}{2}+\frac{K_1+K_2}{K_1K_2}\right)$

(4) $\frac{\varepsilon_0 A}{d}\left(\frac{1}{2}+\frac{2(K_1+K_2)}{K_1K_2}\right)$

Official Ans. by NTA (1)

Sol. $C_{eq}=\frac{\frac{A}{2}\varepsilon_0}{d}+\frac{A\varepsilon_0}{d}\frac{K_1K_2}{K_1+K_2}$

$=\frac{A\varepsilon_0}{d}\left(\frac{1}{2}+\frac{K_1K_2}{K_1+K_2}\right)$

Ans.1

11. A bomb is dropped by fighter plane flying horizontally. To an observer sitting in the plane, the trajectory of the bomb is a :

(1) hyperbola

(2) parabola in the direction of motion of plane

(3) straight line vertically down the plane

(4) parabola in a direction opposite to the motion of plane

Official Ans. by NTA (3)

Sol.

$v_B = u_0\hat{i} - gt\hat{j}$

$\vec{v}_{B/P} = \vec{v}_B - \vec{v}_P$

$\vec{v}_{B/P} = -8t\hat{j}$

straight line vertically down

Ans.3

12. At time t = 0, a material is composed of two radioactive atoms A and B, where $N_A(0) = 2N_B(0)$. The decay constant of both kind of radioactive atoms is λ. However, A disintegrates to B and B disintegrates to C. Which of the following figures represents the evolution of $N_B(t)$ / $N_B(0)$ with respect to time t ?

$\begin{bmatrix} N_A(0) = \text{No. of A atoms at } t=0 \\ N_B(0) = \text{No. of B atoms at } t=0 \end{bmatrix}$

(1)

(2)

(3)

(4)

Official Ans. by NTA (3)

Sol.

$A \to B,\ B \to C$

$$\frac{dN_B}{dt} = \lambda N_A - \lambda N_B$$

$$\frac{dN_B}{dt} = 2\lambda N_{B_0} e^{-\lambda t} - \lambda N_B$$

$$e^{-\lambda t}\left(\frac{dN_B}{dt} + \lambda N_B\right) = 2\lambda N_{B_0} e^{-\lambda t} \times e^{\lambda t}$$

$$\frac{d}{dt}\left(N_B e^{\lambda t}\right) = 2\lambda N_{B_0}$$, on integrating

$$N_B e^{\lambda t} = 2\lambda t N_{B_0} + N_{B_0}$$

$$N_B = N_{B_0}[1 + 2\lambda t]e^{-\lambda t}$$

$$\frac{dN_B}{dt} = 0 \text{ at } -\lambda[1 + 2\lambda t)e^{-\lambda t} + 2\lambda e^{-\lambda t} = 0$$

$$N_{B_{max}} \text{ at } t = \frac{1}{2\lambda}$$

13. A transmitting antenna at top of a tower has a height of 50 m and the height of receiving antenna is 80 m. What is range of communication for Line of Sight (LoS) mode ?

[use radius of earth = 6400 km]

(1) 45.5 km (2) 80.2 km

(3) 144.1 km (4) 57.28 km

Official Ans. by NTA (4)

Sol.

$$d_t = \sqrt{2Rh_1} + \sqrt{2Rh_2}$$

$$= \sqrt{2R}\left(\sqrt{h_1} + \sqrt{h_2}\right)$$

$$= (2 \times 6400 \times 10^3)^{1/2}\left(\sqrt{50} + \sqrt{80}\right)$$

= 3578 (7.07 + 8.94)

= 57.28 Km

14. A refrigerator consumes an average 35 W power to operate between temperature –10°C to 25°C. If there is no loss of energy then how much average heat per second does it transfer ?

(1) 263 J/s (2) 298 J/s (3) 350 J/s (4) 35 J/s

Official Ans. by NTA (1)

Sol. $$\frac{T_L}{T_H - T_L} = \text{C.O.P.} = \frac{\frac{dH}{dt}}{\frac{dW}{dt}}$$

$$\frac{263}{35} \times 35 = \frac{dH}{dt}$$

$$\frac{dH}{dt} = 263 \text{ watts}$$

Ans.1

15. An electric bulb of 500 watt at 100 volt is used in a circuit having a 200 V supply. Calculate the resistance R to be connected in series with the bulb so that the power delivered by the bulb is 500 W.

(1) 20 Ω (2) 30 Ω (3) 5 Ω (4) 10 Ω

Official Ans. by NTA (1)

500 watt at 100 v

Sol.

P = Vi

500 = Vi

i = 5 Amp

V = i × R

R = 20

Ans. 1

16. Four NOR gates are connected as shown in figure.

The truth table for the given figure is :

(1)

A	B	Y
0	0	1
0	1	0
1	0	1
1	1	0

(2)

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

(3)

A	B	Y
0	0	0
0	1	1
1	0	0
1	1	1

(4)

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	1

Official Ans. by NTA (4)

Sol.

$$y=\overline{(\overline{A+\overline{A+B}})+(\overline{B+\overline{A+B}})}$$

$$y=(A+\overline{A+B}).(B+\overline{A+B})$$

A	B	y
0	0	1
0	1	0
1	0	0
1	1	1

Ans.4

17. Match List–I with List–II.

	List-I		List-II
(a)	Magnetic Induction	(i)	$ML^2T^{-2}A^{-1}$
(b)	Magnetic Flux	(ii)	$M^0L^{-1}A$
(c)	Magnetic Permeability	(iii)	$MT^{-2}A^{-1}$
(d)	Magnetization	(iv)	$MLT^{-2}A^{-2}$

Choose the most appropriate answer from the options given below :

(1) (a)-(ii), (b)-(iv), (c)-(i), (d)-(iii)

(2) (a)-(ii), (b)-(i), (c)-(iv), (d)-(iii)

(3) (a)-(iii), (b)-(ii), (c)-(iv), (d)-(i)

(4) (a)-(iii), (b)-(i), (c)-(iv), (d)-(ii)

Official Ans. by NTA (4)

Sol. (a) Magnetic Induction = $MT^{-2}A^{-1}$

(b) Magnetic Flux = $ML^2T^{-2}A^{-1}$

(c) Magnetic Permeability = $MLT^{-2}A^{-2}$

(d) Magnetization = $M^0L^{-1}A$

Ans. 4

18. In the given circuit the AC source has ω = 100 rad s^{-1}. Considering the inductor and capacitor to be ideal, what will be the current I flowing through the circuit?

(1) 5.9 A

(2) 4.24 A

(3) 0.94 A

(4) 6 A

Official Ans. by NTA (2)

Official Ans. by ALLEN (Bonus)

Sol. $Z_C = \sqrt{\left(\frac{1}{\omega C}\right)^2 + R^2}$

$= \sqrt{\left(\frac{1}{100 \times 100 \times 10^{-6}}\right)^2 + 100^2}$

$Z_C = \sqrt{(100)^2 + (100)^2}$

$= 100\sqrt{2}$

$Z_L = \sqrt{(\omega L)^2 + R^2}$

$\sqrt{(100 \times 0.5)^2 + 50^2}$

$= 50\sqrt{2}$

$i_C = \frac{200}{z_C} = \frac{200}{100\sqrt{2}} = \sqrt{2}$

$i_L = \frac{200}{z_L} = \frac{200}{50\sqrt{2}} = 2\sqrt{2}$

$\cos\phi_1 = \frac{100}{10\sqrt{2}} = \frac{1}{\sqrt{2}} \Rightarrow \phi_1 = 45°$

$\cos\phi_2 = \frac{50}{50\sqrt{2}} = \frac{1}{\sqrt{2}} \Rightarrow \phi_2 = 45°$

$I = \sqrt{I_C^2 + I_L^2}$

$= \sqrt{2+8}$

$= \sqrt{10}$

I = 3.16 A

Ans. 3.16

19. If the length of the pendulum in pendulum clock increases by 0.1%, then the error in time per day is:

(1) 86.4 s

(2) 4.32 s

(3) 43.2 s

(4) 8.64 s

Official Ans. by NTA (3)

Sol. $T = 2\pi\sqrt{\frac{\ell}{g}}$

$\frac{\Delta T}{T} = \frac{1}{2}\frac{\Delta \ell}{\ell}$

$\Delta T = \frac{1}{2} \times \frac{0.1}{100} \times 24 \times 3600$

$\Delta T = 43.2$

Ans. 3

20. Two blocks of masses 3 kg and 5 kg are connected by a metal wire going over a smooth pulley. The breaking stress of the metal is $\frac{24}{\pi} \times 10^2$ Nm^{-2}. What is the minimum radius of the wire? (Take g = 10 ms^{-2})

(1) 125 cm (2) 1250 cm

(3) 12.5 cm (4) 1.25 cm

Official Ans. by NTA (3)

Sol.

$T = \frac{2m_1m_2g}{m_1 + m_2} = \frac{2 \times 3 \times 5 \times 10}{8}$

$= \frac{75}{2}$

Stress = $\frac{T}{A}$

$\frac{24}{\pi} \times 10^2 = \frac{75}{2 \times \pi R^2}$

$R^2 = \frac{75}{2 \times 24 \times 100} = \frac{3}{8 \times 24}$

$\Rightarrow$ R = 0.125 m

R = 12.5 cm

SECTION-B

1. Two waves are simultaneously passing through a string and their equations are :

$y_1 = A_1 \sin k(x–vt)$, $y_2 = A_2 \sin k(x–vt + x_0)$. Given amplitudes A_1 = 12 mm and A_2 = 5 mm, x_0 = 3.5 cm and wave number k = 6.28 cm^{-1}. The amplitude of resulting wave will be ..... mm.

Official Ans. by NTA (7)

Sol. $y_1 = A_1 \sin k(x - vt)$

$y_1 = 12 \sin 6.28 (x - vt)$

$y_2 = 5 \sin 6.28 (x - vt + 3.5)$

$$\Delta\phi = \frac{2\pi}{\lambda}(\Delta x)$$

$= K(\Delta x)$

$$= 6.28 \times 3.5 = \frac{7}{2} \times 2\pi = 7\pi$$

$$A_{net} = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\phi}$$

$$A_{net} = \sqrt{(12)^2 + (5)^2 + 2(12)(5)\cos(7\pi)}$$

$$= \sqrt{144 + 25 - 120}$$

Ans. 7

2. A source of light is placed in front of a screen. Intensity of light on the screen is I. Two Polaroids P_1 and P_2 are so placed in between the source of light and screen that the intensity of light on screen is I/2. P_2 should be rotated by an angle of ..... (degrees) so that the intensity of light on the screen becomes $\frac{3I}{8}$.

Official Ans. by NTA (30)

Sol. $$I = \frac{I_0}{2}\cos^2\phi$$

$$\frac{I}{2}\cos^2\phi = \frac{3I}{8}$$

$$\cos^2\phi = \frac{3}{4}$$

$$\cos^2\phi = \frac{\sqrt{3}}{2}$$

$\Rightarrow \phi = 30$

Ans. 30

3. If the maximum value of accelerating potential provided by a ratio frequency oscillator is 12 kV. The number of revolution made by a proton in a cyclotron to achieve one sixth of the speed of light is .....

[$m_p = 1.67 \times 10^{-27}$ kg, $e = 1.6 \times 10^{-19}$ C,

Speed of light = 3×10^8 m/s]

Official Ans. by NTA (543)

Sol. V = 12 kV

Number of revolution = n

$$n[2 \times q_P \times V] = \frac{1}{2} m_P \times v_P^2$$

$n[2\times 1.6 \times 10^{-19} \times 12 \times 10^3$

$$= \frac{1}{2} \times 1.67 \times 10^{-27} \times \left[\frac{3 \times 10^8}{6}\right]^2$$

$n(38.4\times10^{-16}) = 0.2087 \times 10^{-11}$

n = 543.4

Ans. 543

4. The acceleration due to gravity is found upto an accuracy of 4% on a planet. The energy supplied to a simple pendulum to known mass 'm' to undertake oscillations of time period T is being estimated. If time period is measured to an accuracy of 3%, the accuracy to which E is known as .....%

Official Ans. by NTA (14)

Sol. $$T = 2\pi\sqrt{\frac{\ell}{g}} \Rightarrow \ell = \frac{T^2 g}{4\pi^2}$$

$$E = mg\ell\frac{\theta^2}{2} = mg^2\frac{T^2\theta^2}{8\pi^2}$$

$$\frac{dE}{E} = 2\left(\frac{dg}{g} + \frac{dT}{T}\right)$$

= (4 + 3) = 14%

5. A circular coil of radius 8.0 cm and 20 turns is rotated about its vertical diameter with an angular speed of 50 rad s^{-1} in a uniform horizontal magnetic field of 3.0×10^{-2} T. The maximum emf induced the coil will be ..... $\times 10^{-2}$ volt (rounded off to the nearest integer)

Official Ans. by NTA (60)

Sol. Maximum emf $\varepsilon = N\,\omega AB$

$N = 20,\ \omega = 50,\ B = 3 \times 10^{-2}T$

$\varepsilon = 20 \times 50 \times \pi \times (0.08)^2 \times 3 \times 10^{-2} = 60.28 \times 10^{-2}$

Rounded off to nearest integer = 60

Ans.60

6. Two simple harmonic motions are represented by the equations

$x_1 = 5 \sin\left(2\pi t + \frac{\pi}{4}\right)$ and $x_2 = 5\sqrt{2}\,(\sin 2\pi t + \cos 2\pi t)$.

The amplitude of second motion is ..... times the amplitude in first motion.

Official Ans. by NTA (2)

Sol. $x_2 = 5\sqrt{2}\left(\frac{1}{\sqrt{2}}\sin 2\pi t + \frac{1}{\sqrt{2}}\cos 2\pi t\right)\sqrt{2}$

$= 10\sin\left(2\pi t + \frac{\pi}{4}\right)$

$\therefore \frac{A_2}{A_1} = \frac{10}{5} = 2$

Ans. 2

7. A coil in the shape of an equilateral triangle of side 10 cm lies in a vertical plane between the pole pieces of permanent magnet producing a horizontal magnetic field 20 mT. The torque acting on the coil when a current of 0.2 A is passed through it and its plane becomes parallel to the magnetic field will be $\sqrt{x} \times 10^{-5}$ Nm. The value of x is.....

Official Ans. by NTA (3)

Sol.

$\vec{\tau} = \vec{M} \times \vec{B} = MB\sin 90°$

$= MB = \frac{i\sqrt{3}\ell^2}{4}B$

$= \sqrt{3} \times 10^{-5}\,N - m$

Ans. 3

8. For the given circuit, the power across zener diode is ..... mW.

Official Ans. by NTA (120)

Sol.

$i = \frac{10V}{5k\Omega} = 2mA$

$I = \frac{14V}{1k\Omega} = 14mA$

$\therefore\ I_z = 12mA$

$\therefore\ P = I_zV_z = 120\ \ mW$

Ans. 120

9. An object is placed at a distance of 12 cm from a convex lens. A convex mirror of focal length 15 cm is placed on other side of lens at 8 cm as shown in the figure. Image of object coincides with the object.

When the convex mirror is removed, a real and inverted image is formed at a position. The distance of the image from the object will be .....(cm)

Official Ans. by NTA (50)

Sol.

For the object to coincide with image, the light must fall perpendicularly to mirror. Which means that the light will have to converge at C of mirror. Without the mirror also, the light would coverage at C.

So the distance is : 12 + 8 + 30 = 50 cm

10. The coefficient of static friction between two blocks is 0.5 and the table is smooth. The maximum horizontal force that can be applied to move the blocks together is .....N.

(take g = 10 ms^{-2})

Official Ans. by NTA (15)

Sol.

F = 3a (For system) ….(i)

$fs_{max} = 1a$ (for 1kg block) ….(ii)

$\mu \times 1 \times g = a$

$\Rightarrow 5 = a$

F = 15N

FINAL JEE–MAIN EXAMINATION – AUGUST, 2021

(Held On Thursday 26th August, 2021) **TIME : 3 : 00 PM to 6 : 00 PM**

CHEMISTRY — TEST PAPER WITH SOLUTION

SECTION-A

1. Which one of the following phenols does not give colour when condensed with phthalic anhydride in presence of conc. H_2SO_4 ?

(1)

(2)

(3)

(4)

Official Ans. by NTA (2)

Sol. Only p-methyl, phenol does not give any colour with phthalic anhydroxide with cons. H_2SO_4.

2. Given below are two statements : one is labelled as **Assertion (A)** and the other is labelled as **Reason (R)**.

Assertion (A) : Photochemical smog causes cracking of rubber.

Reason (R) : Presence of ozone, nitric oxide, acrolein, formaldehyde and peroxyacetyl nitrate in photochemical smog makes it oxidizing.

Choose the **most appropriate** answer from the options given below :

(1) Both **(A)** and **(R)** are true but **(R)** is not the true explanation of **(A)**

(2) **(A)** is false but **(R)** is true.

(3) **(A)** is true but **(R)** is false

(4) Both **(A)** and **(R)** are true and **(R)** is the true explanation of **(A)**

Official Ans. by NTA (4)

Sol. Photochemical smog causes cracking of rubber, the common component of photochemical smog are ozone, nitric oxide, acrolein, formaldehyde and peroxyacetyle nitrate (PAN).

3. The interaction energy of London forces between two particles is proportional to r^x, where r is the distance between the particles. The value of x is :

(1) 3 (2) –3

(3) –6 (4) 6

Official Ans. by NTA (3)

Sol. For london dispersion forces.

$$E \propto \frac{1}{r^6}$$

Hence x = –6

4. The number of non-ionisable hydrogen atoms present in the final product obtained from the hydrolysis of PCl_5 is :

(1) 0 (2) 2 (3) 1 (4) 3

Official Ans. by NTA (1)

Sol. $PCl_5 + H_2O \rightarrow POCl_3 + 2HCl$

$$\downarrow + 3H_2O$$

$$H_3PO_4 + 3HCl$$

In + HCl

all hydrogens are ionisable

∴ Ans is zero.

5. The bond order and magnetic behaviour of O_2^- ion are, respectively :

(1) 1.5 and paramagnetic

(2) 1.5 and diamagnetic

(3) 2 and diamagnetic

(4) 1 and paramagnetic

Official Ans. by NTA (1)

Sol. $O_2^- = (\sigma_{1s})^2(\sigma_{1s}^*)^2(\sigma_{2s})^2(\sigma_{2s}^*)^2(\sigma_{2p_z})^2$

$\left(\pi_{2p_x}^2 = \pi_{2p_y}^2\right)\left(\pi_{2p_x}^{*2} = \pi_{2p_y}^{*1}\right)$

Bond order $= \dfrac{10-7}{2} = 1.5$

and paramagnetic

6. Given below are two statements : one is labelled as **Assertion (A)** and other is labelled as **Reason (R)**.

Assertion (A) : Sucrose is a disaccharide and a non-reducing sugar.

Reason (R) : Sucrose involves glycosidic linkage between C_1 of β-glucose and C_2 of α-fructose.

Choose the **most appropriate** answer from the options given below :

(1) Both **(A)** and **(R)** are true but **(R)** is not the true explanation of **(A)**

(2) **(A)** is false but **(R)** is true.

(3) **(A)** is true but **(R)** is false

(4) Both **(A)** and **(R)** are true and **(R)** is the true explanation of **(A)**

Official Ans. by NTA (3)

Sol. Surcrose is example of disaccharide & non reducing sugar

Assertion : correct

Sucrose involves glycosidic linkage between C_1 of α-D-glucose C_2 of β-D-fructose

Reason : Incorrect

7. Match **List-I** with **List-II** :

List-I (Chemical Reaction)	**List-II (Reagent used)**
(a) $CH_3COOCH_2CH_3 \rightarrow CH_3CH_2OH$	(i) CH_3MgBr / H_3O^+ (1.equivalent)
(b) $CH_3COOCH_3 \rightarrow CH_3CHO$	(ii) H_2SO_4 / H_2O
(c) $CH_3C \equiv N \rightarrow CH_3CHO$	(iii) DIBAL-H/H_2O
(d) $CH_3C \equiv N \rightarrow CH_3-C(=O)-CH_3$	(iv) $SnCl_2$, HCl/H_2O

Choose the most appropriate match :

(1) a-ii, b-iv, c-iii, d-i

(2) a-iv, b-ii, c-iii, d-i

(3) a-ii, b-iii, c-iv, d-i

(4) a-iii, b-ii, c-i, d-iv

Official Ans. by NTA (3)

Sol. $CH_3-C(=O)-O-CH_2CH_3 \xrightarrow{H_3O^+} CH_3CO_2H + CH_3CH_2OH$

$CH_3-C(=O)-O-CH_3 \xrightarrow[-78^oC]{DIBALH/H_2O} CH_3CHO$

$CH_3-CN \xrightarrow[/H_2O]{SnCl_2+HCl} CH_3CH=O$

$CH_3-C\equiv N \xrightarrow[/H_3O^+]{CH_3MgBr\ (1eq)}$

8. Given below are two statements : one is labelled as **Assertion (A)** and the other is labelled as **Reason (R).**

Assertion (A) : Barium carbonate is insoluble in water and is highly stable.

Reason (R) : The thermal stability of the carbonates increases with increasing cationic size.

(1) Both **(A)** and **(R)** are true but **(R)** is the true explanation of **(A)**

(2) **(A)** is true but **(R)** is false

(3) Both **(A)** and **(R)** are true and **(R)** is not the true explanation of **(A)**

(4) **(A)** is false but **(R)** is true.

Official Ans. by NTA (1)

Sol. In IIA group on moving down the group size of cation increases and show thermal stability of carbonate increases.

9.

$\xrightarrow{(CH_3CO)_2O}$ P (Major product)

The major product in the above reaction is :

(1)

(2)

(3)

(4)

Official Ans. by NTA (4)

Sol.

10. Indicate the complex/complex ion which did not show any geometrical isomerism :

(1) $[CoCl_2(en)_2]$ (2) $[Co(CN)_5(NC)]^{3-}$

(3) $[Co(NH_3)_3(NO_2)_3]$ (4) $[Co(NH_3)_4Cl_2]^+$

Official Ans. by NTA (2)

Sol. (1) $[CoCl_2(en)_2]$ show Cis-trans isomerism

(2) $[Co(CN)_5(NC)]^{-3}$ can't Show G.I.

(3) $[Co(NH_3)_3(NO_2)_3]$ Show fac & mer isomerism

(4) $[Co(NH_3)_4Cl_2]^{\oplus}$ show cis & trans isomerism

11. The sol given below with negatively charged colloidal particles is :

(1) $FeCl_3$ added to hot water

(2) KI added to $AgNO_3$ solution

(3) $AgNO_3$ added to KI solution

(4) $Al_2O_3.xH_2O$ in water

Official Ans. by NTA (3)

Sol.

12. Given below are two statements :

Statement I : Sphalerite is a sulphide ore of zinc and copper glance is a sulphide ore of copper.

Statement II : It is possible to separate two sulphide ores by adjusting proportion of oil to water or by using 'depressants' in a froth flotation method.

Choose the **most appropriate** answer from the options given below :

(1) **Statement I** is true but **Statement II** is false.

(2) Both **Statement I** and **Statement II** are true.

(3) **Statement I** is false but **Statement II** is true.

(4) Both **Statement I** and **Statement II** are false.

Official Ans. by NTA (2)

Sol. Sphalerite-ZnS, copper glance - Cu_2S two sulphide ores can be separated by adjusting proportions of oil to water or by using ' Depressants '

13. Given below are two statements : one is labelled as **Assertion (A)** and the other is labelled as **Reason (R)**.

Assertion (A) : Heavy water is used for the study of reaction mechanism.

Reason (R) : The rate of reaction for the cleavage of O - H bond is slower than that of O-D bond.

Choose the **most appropriate** answer from the options given below :

(1) Both **(A)** and **(R)** are true but **(R)** is not the true explanation of **(A)**.

(2) Both **(A)** and **(R)** are true and **(R)** is the true explanation of **(A)**.

(3) **(A)** is false but **(R)** is true.

(4) **(A)** is true but **(R)** is false.

Official Ans. by NTA (4)

Sol. D_2O in used for the study of reaction mechanism. Rate of reaction for the cleavage of O–H bond > O-D bond.

14. Arrange the following Cobalt complexes in the order of increasing Crystal Field Stabilization Energy (CFSE) value.

Complexes : $\underset{A}{[CoF_6]^{3-}}, \underset{B}{[Co(H_2O)_6]^{2+}}, \underset{C}{[Co(NH_3)_6]^{3+}}$ and $[Co(en)_3]^{3+}$

Choose the **correct** option :

(1) A < B < C < D (2) B < A < C < D

(3) B < C < D < A (4) C < D < B < A

Official Ans. by NTA (2)

Sol. (i) CFSE ∝ charge or oxidation no. of central metal ion.

(ii) CFSE ∝ strength of ligand

en > NH_3 > H_2O > F^-

∴ order of CFSE

$$\overset{III}{[Co(en)_3]^{+3}} > \overset{III}{Co(NH_3)_6]^{+3}} > \overset{III}{[CoF_6]^{-3}} > \overset{II}{[Co(H_2O)_6]^{+2}}$$

15.

Chlordiazepoxide

The class of drug to which chlordiazepoxide with above structure belongs is :

(1) Antacid (2) Analgesic

(3) Tranquilizer (4) Antibiotic

Official Ans. by NTA (3)

Sol. The drug named chlordiate poxide is example of tranquilizer.

16. Chalcogen group elements are :

(1) Se, Tb and Pu. (2) Se, Te and Po.

(3) S, Te and Pm. (4) O, Ti and Po.

Official Ans. by NTA (2)

Sol. Group 16/oxygen family is known as Chalcogens the members are O, S, Se, Te, Po

17. Which one of the following compounds is not aromatic ?

(1) (2)

(3) (4)

Official Ans. by NTA (3)

Sol. : Non aromatic

18. The number of stereoisomers possible for 1,2-dimethyl cyclopropane is :

(1) One (2) Four

(3) Two (4) Three

Official Ans. by NTA (4)

Sol.

19.

Consider the given reaction, Identify **'X'** and **'Y'** :

(1) X – NaOH Y –

(2) X – HNO_3 Y –

(3) X – NaOH Y –

(4) X – HNO_3 Y –

Official Ans. by NTA (3)

Sol.

20.

Consider the given reaction, the product **A** is :

(1)

(2)

(3)

(4)

Official Ans. by NTA (3)

Sol.

SECTION-B

1. In the sulphur estimation, 0.471 g of an organic compound gave 1.44 g of barium sulphate. The percentage of sulphur in the compound is _____%. (Nearest integer)

(Atomic Mass of Ba = 137 u)

Official Ans. by NTA (42)

Sol. Molecular mass of $BaSO_4$ = 233 g

$\because$ 233 $BaSO_4$ contain $\rightarrow$ 32 g sulphur

$\therefore$ 1.44 g $BaSO_4$ contain $\rightarrow \frac{32}{233} \times 1.44$ g sulphur

given : 0.471 g of organic compound

$$\% \text{ of S} = \frac{32 \times 1.44}{233 \times 0.471} \times 100 = 41.98\% \approx 42\%$$

OR

$$\underset{W_{OC}=0.471g}{\boxed{O.C.}} \longrightarrow \underset{1.44g}{BaSO_4}$$

$$\Rightarrow n_s = n_{BaSO_4} = \frac{1.44}{233}$$

$$\Rightarrow w_s = \frac{1.44}{233} \times 32g$$

$$\text{therefore } \%S = \frac{W_s}{W_{O.C.}} \times 100 = \frac{1.44 \times 32}{233 \times 0.471} \times 100$$

$$= \frac{46.08}{109.743} \times 100 = 41.98 \simeq 42$$

2. The equilibrium constant K_c at 298 K for the reaction $A + B \rightleftharpoons C + D$ is 100. Starting with an equimolar solution with concentrations of A, B, C and D all equal to 1M, the equilibrium concentration of D is_____ $\times 10^{-2}$ M. (Nearest integer)

Official Ans. by NTA (182)

Sol. $A + B \rightleftharpoons C + D : K_{eq} = 100$

1M 1M 1M 1M

First check direction of reversible reaction.

Since $Q_C = \frac{[C][D]}{[A][B]} = 1 < K_{eq.} \Rightarrow$ reaction will move in forward direction to attain equilibrium state.

$\Rightarrow A + B \rightleftharpoons C + D : K_{eq} = 100$

to 1 1 1 1

$t_{eq.}$ 1–x 1–x 1+x 1+x

$$\text{Now : } K_{eq} = 100 = \frac{(1+x)(1+x)}{(1-x)(1-x)}$$

$$\Rightarrow \boxed{100 = \left(\frac{1+x}{1-x}\right)^2}$$

(i) $10 = \left(\frac{1+x}{1-x}\right)$

$\Rightarrow 10 - 10x = 1+x$

$\Rightarrow 11x = 9$

$\Rightarrow \boxed{x = \frac{9}{11}}$

(ii) $-10 = \frac{1+x}{1-x}$

$\Rightarrow -10 + 10x = 1 + x$

$\Rightarrow -9x = -11$

$\Rightarrow \boxed{x = \frac{11}{9}}$

$\rightarrow$ 'x' cannot be more than one, therefore not valid.

therefore equation concretion of (D) = 1 + x

$$= 1 + \frac{9}{11} = \frac{20}{11}$$

$$= 1.8181 = 181.81 \times 10^{-2}$$

$$\simeq 182 \times 10^{-2}$$

3. For water Δ_{vap} H = 41 kJ mol^{-1} at 373 K and 1 bar pressure. Assuming that water vapour is an ideal gas that occupies a much larger volume than liquid water, the internal energy change during evaporation of water is_____kJ mol^{-1}

[Use : R = 8.3 J mol^{-1} K^{-1}]

Official Ans. by NTA (38)

Sol. Given equation is

$$H_2O(\ell) \longrightarrow H_2O(g) : \Delta H = 41 \frac{kJ}{mol}$$

$\Rightarrow$ From the relation : $\Delta H = \Delta U + \Delta n_g RT$

$$\Rightarrow 41 \frac{kJ}{mol} = \Delta U + (1) \times \frac{8.3}{1000} \times 373$$

$\Rightarrow$ DU = 41 – 3.0959

= 38 kJ/mol

4. A metal surface is exposed to 500 nm radiation. The threshold frequency of the metal for photoelectric current is 4.3 × 10^{14} Hz. The velocity of ejected electron is _____× 10^5 ms^{-1} (Nearest integer)

[Use : h = 6.63 × 10^{-34} Js, m_e = 9.0 × 10^{-31} kg]

Official Ans. by NTA (5)

Sol.

υ : speed of electron having max. K.E.

$\Rightarrow$ from Einstein equation : E = ϕ + K.E.$_{max}$

$$\Rightarrow \frac{hc}{\lambda} = h\upsilon_0 + \frac{1}{2}mv^2$$

$$\Rightarrow \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{500 \times 10^{-9}} = 6.63 \times 10^{-34} \times 4.3 \times 10^{14} + \frac{1}{2}mv^2$$

$$\Rightarrow \frac{6.63 \times 30 \times 10^{-20}}{5} = 6.63 \times 4.3 \times 10^{-20} + \frac{1}{2}mv^2$$

$$\Rightarrow 11.271 \times 10^{-20} \text{ J} = \frac{1}{2} \times 9 \times 10^{-31} \times \upsilon^2$$

$$\Rightarrow \boxed{\upsilon = 5 \times 10^5 \text{ m/sec.}}$$

5. For the galvanic cell,

$Zn(s) + Cu^{2+}$ (0.02 M) $\rightarrow Zn^{2+}$ (0.04 M) + Cu(s),

E_{cell} =_____ × 10^{-2} V. (Nearest integer)

[Use : $E^0_{Cu/Cu^{2+}}$ = –0.34 V, $E^0_{Zn/Zn^{2+}}$ =+0.76 V, $\frac{2.303\ RT}{F}$ = 0.059 V]

Official Ans. by NTA (109)

Sol. Galvanic cell:

$$Zn_{(s)} + \underset{0.02M}{Cu^{+2}_{(aq.)}} \rightarrow \underset{0.04M}{Zn^{+2}} + Cu(s)$$

Nernst equation = $F_{cell} = E^o_{cell} - \frac{0.059}{2}\log\frac{[2n^{+2}]}{[Cu^{+2}]}$

$$\Rightarrow E_{cell}\left[E^o_{cell} - E^o_{Zn^{+2}/Zn}\right] - \frac{0.059}{2}\log\frac{0.04}{0.02}$$

$$\Rightarrow E_{cell}[0.34 - (-0.76)] - \frac{0.059}{2}\log^2$$

$$\Rightarrow E_{cell} 1 - 1 - \frac{0.059}{2} \times 0.3010$$

$$= 1.0911 = 109.11 \times 10^{-2}$$

$$= 109$$

6. 100 mL of Na_3PO_4 solution contains 3.45 g of sodium. The molarity of the solution is _____×10^{-2} mol L^{-1} . (Nearest integer)

[Atomic Masses - Na : 23.0 u, O : 16.0 u, P : 31.0 u]

Official Ans. by NTA (50)

Sol.

therefore molarity of Na_3PO_4 Solution =

$$\frac{n_{Na_3PO_4}}{\text{volume of solution in L}}$$

$$= \frac{\frac{1}{3} \times \frac{3.45}{23} \text{mol}}{0.1\,\text{L}}$$

$= 0.5 = 50 \times 10^{-2}$

7. The overall stability constant of the complex ion $[Cu(NH_3)_4]^{2+}$ is 2.1×10^{13}. The overall dissociations constant is $y \times 10^{-14}$. Then y is ____.(Nearest integer)

Official Ans. by NTA (5)

Sol. Given $k_f = 2.1 \times 10^{13}$

$$K_d = \frac{1}{k_f} = 4.7 \times 10^{-14}$$

$\therefore y = 4.7 \approx 5$

8. 83 g of ethylene glycol dissolved in 625 g of water. The freezing point of the solution is _____K. (Nearest integer)

[Use : Molal Freezing point depression constant of water = 1.86 K kg mol^{-1}]

Freezing Point of water = 273 K

Atomic masses : C : 12.0 u, O : 16.0 u, H : 1.0 u]

Official Ans. by NTA (269)

Sol. $k_f = 1.86$ k. kg/mol

$T_f^o = 273$ k

solvent : H_2O(625 g)

Solute : 83 g $\begin{pmatrix} CH_2 - CH_2 \\ | \quad\quad | \\ OH \quad\quad OH \end{pmatrix} \Rightarrow$ Non dissociative solute

$\Rightarrow \Delta T_f = k_f \times m$

$$\Rightarrow \left(T_f^o - T_f^1\right) = 1.86 \times \frac{83/62}{624/1000}$$

$$\Rightarrow 273 - T_f^1 = \frac{1.86 \times 83 \times 1000}{62 \times 625} = \frac{154380}{38750}$$

$\Rightarrow 273 - T_f^1 = 4$

$\Rightarrow \boxed{T_f^1 = 259\text{ K}}$

9. The reaction rate for the reaction

$[PtCl_4]^{2-} + H_2O \rightleftharpoons [Pt(H_2O)Cl_3]^- + Cl^-$

was measured as a function of concentrations of different species. It was observed that

$$\frac{-d\left[[PtCl_4]^{2-}\right]}{dt} = 4.8 \times 10^{-5}\left[[PtCl_4]^{2-}\right] - 2.4 \times 10^{-3}\left[[Pt(H_2O)Cl_3]^-\right]\left[Cl^-\right].$$

where square brackets are used to denote molar concentrations. The equilibrium constant K_c=_____. (Nearest integer)

Official Ans. by NTA (50)

Sol. $[PtCl_4]^{-2} + H_2O \rightleftharpoons [Pt(H_2O)Cl_3]^- + Cl^-$

$$\frac{-d[Pt\,Cl_4]^{-2}}{dt} = 4.8 \times 10^{-5}\,[PtCl_4^{-2}] - 2.4 \times 10^{3}\,[Pt(H_2O)Cl_3][\overset{o}{u}]$$

$$\Rightarrow K_{eq} = \frac{k_f}{k_b} = \frac{4.8 \times 10^{-5}}{2.4 \times 10^{-3}} = 0.02$$

10. A chloro compound "A".

(i) forms aldehydes on ozonolysis followed by the hydrolysis.

(ii) when vaporized completely 1.53 g of A, gives 448 mL of vapour at STP.

The number of carbon atoms in a molecule of compound **A** is _____.

Official Ans. by NTA (3)

Sol. 448 ml of A $\Rightarrow$ 1.53 gm A

22400 ml of A $\Rightarrow \frac{1.53}{445} \times 22400$ gm A = 7650

$H_3CHC{-}CH{-}Cl \xrightarrow[Zn/H_2O]{O_3} CH_3{-}CH{=}O$

It has 3 carbon atoms — Aldehyde

& mm is 36 + 5 + 35.5 = 76.5

FINAL JEE–MAIN EXAMINATION – AUGUST, 2021

(Held On Thursday 26th August, 2021) **TIME : 3 : 00 PM to 6 : 00 PM**

MATHEMATICS — TEST PAPER WITH SOLUTION

SECTION-A

1. Let [t] denote the greatest integer less than or equal to t. Let f(x) = x – [x], g(x) = 1 – x + [x], and h(x) = min{f(x), g(x)}, x ∈ [–2, 2]. Then h is :

(1) continuous in [–2, 2] but not differentiable at more than four points in (–2, 2)

(2) not continuous at exactly three points in [–2, 2]

(3) continuous in [–2, 2] but not differentiable at exactly three points in (–2, 2)

(4) not continuous at exactly four points in [–2, 2]

Official Ans. by NTA (1)

Sol. min{x – [x], 1 – x + [x]}

h(x) = min{x – [x], 1 – [x – [x]]}

⇒ always continuous in [–2, 2]

but non differentiable at 7 Points

2. Let $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix}$. Then $A^{2025} - A^{2020}$ is equal to :

(1) $A^6 - A$ (2) A^5

(3) $A^5 - A$ (4) A^6

Official Ans. by NTA (1)

Sol. $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} \Rightarrow A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$

$A^3 = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} \Rightarrow A^4 = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$

$A^n = \begin{bmatrix} 1 & 0 & 0 \\ n-1 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$

$A^{2025} - A^{2020} = \begin{bmatrix} 0 & 0 & 0 \\ 5 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$A^6 - A = \begin{bmatrix} 0 & 0 & 0 \\ 5 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

3. The local maximum value of the function

$f(x) = \left(\frac{2}{x}\right)^{x^2}, x > 0$, is

(1) $(2\sqrt{e})^{\frac{1}{e}}$ (2) $\left(\frac{4}{\sqrt{e}}\right)^{\frac{e}{4}}$

(3) $(e)^{\frac{2}{e}}$ (4) 1

Official Ans. by NTA (3)

Sol. $f(x) = \left(\frac{2}{x}\right)^{x^2}$; $x > 0$

$\ell n\, f(x) = x^2 (\ell n 2 - \ell n x)$

$f'(x) = f(x)\{-x + (\ell n\, 2 - \ell n\, x)2x\}$

$f'(x) = \underbrace{f(x)}_{+}.\underbrace{x}_{+}\underbrace{(2\ell n 2 - 2\ell n x - 1)}_{g(x)}$

$g(x) = 2\ell\, n^2 - 2\ell\, n\, x - 1$

$= \ell n \frac{4}{x^2} - 1 = 0 \Rightarrow x = \frac{2}{\sqrt{e}}$

$LM = \frac{2}{\sqrt{e}}$

Local maximum value $= \left(\frac{2}{2/\sqrt{e}}\right)^{\frac{4}{e}} \Rightarrow e^{\frac{2}{e}}$

4. If the value of the integral $\int_0^5 \frac{x+[x]}{e^{x-[x]}}dx = \alpha e^{-1} + \beta$, where $\alpha, \beta \in \mathbf{R}$, $5\alpha + 6\beta = 0$, and $[x]$ denotes the greatest integer less than or equal to x; then the value of $(\alpha + \beta)^2$ is equal to :

(1) 100 (2) 25

(3) 16 (4) 36

Official Ans. by NTA (2)

Sol. $I = \int_0^5 \frac{x+[x]}{e^{x-[x]}}dx$

$$\int_0^1 \frac{x}{e^x}dx + \int_1^2 \frac{x+1}{e^{x-1}}dx + \int_2^3 \frac{x+2}{e^{x-2}}dx + \ldots\ldots \int_4^5 \frac{x+4}{e^{x-4}}dx$$

$$\Downarrow \qquad \Downarrow \qquad \Downarrow$$

$$x = t+1 \qquad x = z+2 \qquad x = y+4$$

$$\int_0^1 \frac{t+2}{e^t}dt + \int_0^1 \frac{z+4}{e^z}dz + \ldots\ldots + \int_0^1 \frac{y+8}{e^y}dx$$

$$\Rightarrow \int_0^5 \frac{5x+20}{e^x}dt = 5\int_0^1 \frac{x+4}{e^x}dx$$

$$\Rightarrow 5\int_0^1 (x+4)e^{-x}dx$$

$$\Rightarrow 5e^{-x}(-x-5)\Big|_0^1 \Rightarrow -\frac{30}{e} + 25$$

$\alpha = -30$

$\beta = 25 \Rightarrow 5\alpha + 6\beta = 0$

$(\alpha + \beta)^2 = 5^2 = 25$

5. The point $P\left(-2\sqrt{6}, \sqrt{3}\right)$ lies on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ having eccentricity $\frac{\sqrt{5}}{2}$. If the tangent and normal at P to the hyperbola intersect its conjugate axis at the point Q and R respectively, then QR is equal to :

(1) $4\sqrt{3}$ (2) 6

(3) $6\sqrt{3}$ (4) $3\sqrt{6}$

Official Ans. by NTA (3)

Sol. $P(-2\sqrt{6}, \sqrt{3})$ lies on hyperbola

$$\Rightarrow \frac{24}{a^2} - \frac{3}{b^2} = 1 \quad \ldots\ldots.(i)$$

$$e = \frac{\sqrt{5}}{2} \Rightarrow b^2 = a^2\left(\frac{5}{4} - 1\right) \Rightarrow 4b^2 = a^2$$

Put in (i) $\Rightarrow \frac{6}{b^2} - \frac{3}{b^2} = 1 \Rightarrow b = \sqrt{3}$

$$\Rightarrow a = \sqrt{12}$$

$$\frac{x^2}{12} - \frac{y^2}{3} = 1$$

Tangent at P :

$$\frac{-x}{\sqrt{6}} - \frac{y}{\sqrt{3}} = 1 \Rightarrow Q\ (0, \sqrt{3})$$

Slope of $T = -\frac{1}{\sqrt{2}}$

Normal at P :

$$y - \sqrt{3} = \sqrt{2}(x + 2\sqrt{6})$$

$$\Rightarrow R = (0, 5\sqrt{3})$$

$QR = 6\sqrt{3}$

6. Let y(x) be the solution of the differential equation $2x^2dy + (e^y - 2x)dx = 0$, $x > 0$. If $y(e) = 1$, then $y(1)$ is equal to :

(1) 0 (2) 2

(3) $\log_e 2$ (4) $\log_e(2e)$

Official Ans. by NTA (3)

Sol. $2x^2dy + (e^y - 2x)dx = 0$

$$\frac{dy}{dx} + \frac{e^y - 2x}{2x^2} = 0 \Rightarrow \frac{dy}{dx} + \frac{e^y}{2x^2} - \frac{1}{x} = 0$$

$$e^{-y}\frac{dy}{dx} - \frac{e^{-y}}{x} = -\frac{1}{2x^2} \Rightarrow \text{Put } e^{-y} = z$$

$$\frac{-dz}{dx} - \frac{z}{x} = -\frac{1}{2x^2} \Rightarrow xdz + zdx = \frac{dx}{2x}$$

$$d(xz) = \frac{dx}{2x} \Rightarrow xz = \frac{1}{2}\log_e x + c$$

$xe^{-y} = \frac{1}{2}\log_e x + c$, passes through (e,1)

$\Rightarrow C = \frac{1}{2}$

$xe^{-y} = \frac{\log_e ex}{2}$

$e^{-y} = \frac{1}{2} \Rightarrow y = \log_e 2$

7. Consider the two statements :

(S1) : $(p \to q) \vee (\sim q \to p)$ is a tautology.

(S2) : $(p \wedge \sim q) \wedge (\sim p \vee q)$ is a fallacy.

Then :

(1) only (S1) is true.

(2) both (S1) and (S2) are false.

(3) both (S1) and (S2) are true.

(4) only (S2) is true.

Official Ans. by NTA (3)

Sol. S_1 : $(\sim p \vee q) \vee (q \vee p) = (q \vee \sim p) \vee (q \vee p)$

$S_1 = q \vee (\sim p \vee p) = qvt = t =$ tautology

S_2 : $(p \wedge \sim q) \wedge (\sim p \vee q) = (p \wedge \sim q) \wedge \sim (p \wedge \sim q) = C$

= fallacy

8. The domain of the function $\text{cosec}^{-1}\left(\frac{1+x}{x}\right)$ is :

(1) $\left(-1, -\frac{1}{2}\right] \cup (0, \infty)$ (2) $\left[-\frac{1}{2}, 0\right) \cup [1, \infty)$

(3) $\left(-\frac{1}{2}, \infty\right) - \{0\}$ (4) $\left[-\frac{1}{2}, \infty\right) - \{0\}$

Official Ans. by NTA (4)

Sol. $\frac{1+x}{x} \in (-\infty, -1] \cup [1, \infty)$

$\frac{1}{x} \in (-\infty, -2] \cup [0, \infty)$

$x \in \left[-\frac{1}{2}, 0\right) \cup (0, \infty)$

$x \in \left[-\frac{1}{2}, \infty\right) - \{0\}$

9. A fair die is tossed until six is obtained on it. Let X be the number of required tosses, then the conditional probability $P(X \geq 5 \mid X > 2)$ is :

(1) $\frac{125}{216}$ (2) $\frac{11}{36}$ (3) $\frac{5}{6}$ (4) $\frac{25}{36}$

Official Ans. by NTA (4)

Sol. $P(x \geq 5 \mid x > 2) = \frac{P(x \geq 5)}{P(x > 2)}$

$$\frac{\left(\frac{5}{6}\right)^4 \cdot \frac{1}{6} + \left(\frac{5}{6}\right)^5 \cdot \frac{1}{6} + \ldots\ldots + \infty}{\left(\frac{5}{6}\right)^2 \cdot \frac{1}{6} + \left(\frac{5}{6}\right)^3 \cdot \frac{1}{6} + \ldots\ldots + \infty}$$

$$\frac{\frac{\left(\frac{5}{6}\right)^4 \cdot \frac{1}{6}}{1 - \frac{5}{6}}}{\frac{\left(\frac{5}{6}\right)^2 \cdot \frac{1}{6}}{1 - \frac{5}{6}}} = \left(\frac{5}{6}\right)^2 = \frac{25}{36}$$

10. If $\sum_{r=1}^{50} \tan^{-1} \frac{1}{2r^2} = p$, then the value of tan p is :

(1) $\frac{101}{102}$ (2) $\frac{50}{51}$ (3) 100 (4) $\frac{51}{50}$

Official Ans. by NTA (2)

Sol. $\sum_{r=1}^{50} \tan^{-1}\left(\frac{2}{4r^2}\right) = \sum_{r=1}^{50} \tan^{-1}\left(\frac{(2r+1)-(2r-1)}{1+(2r+1)(2r-1)}\right)$

$\sum_{r=1}^{50} \tan^{-1}(2r+1) - \tan^{-1}(2r-1)$

$\tan^{-1}(101) - \tan^{-1} 1 \Rightarrow \tan^{-1} \frac{50}{51}$

11. Two fair dice are thrown. The numbers on them are taken as λ and μ, and a system of linear equations

$x + y + z = 5$
$x + 2y + 3z = \mu$
$x + 3y + \lambda z = 1$

is constructed. If p is the probability that the system has a unique solution and q is the probability that the system has no solution, then :

(1) $p = \frac{1}{6}$ and $q = \frac{1}{36}$ (2) $p = \frac{5}{6}$ and $q = \frac{5}{36}$

(3) $p = \frac{5}{6}$ and $q = \frac{1}{36}$ (4) $p = \frac{1}{6}$ and $q = \frac{5}{36}$

Official Ans. by NTA (2)

Sol. $D \neq 0 \Rightarrow \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & \lambda \end{vmatrix} \neq 0 \Rightarrow \lambda \neq 5$

For no solution $D = 0 \Rightarrow \lambda = 5$

$$D_1 = \begin{vmatrix} 1 & 1 & 5 \\ 1 & 2 & \mu \\ 1 & 3 & 1 \end{vmatrix} \neq 0 \Rightarrow \mu \neq 3$$

$$p = \frac{5}{6}$$

$$q = \frac{1}{6} \times \frac{5}{6} = \frac{5}{36}$$

Option (2)

12. The locus of the mid points of the chords of the hyperbola $x^2 - y^2 = 4$, which touch the parabola $y^2 = 8x$, is :

(1) $y^3(x-2) = x^2$ (2) $x^3(x-2) = y^2$

(3) $y^2(x-2) = x^3$ (4) $x^2(x-2) = y^3$

Official Ans. by NTA (3)

Sol. $T = S_1$

$xh - yk = h^2 - k^2$

$$y = \frac{xh}{k} - \frac{(h^2 - k^2)}{k}$$

this touches $y^2 = 8x$ then $c = \frac{a}{m}$

$$\left(\frac{k^2 - h^2}{k}\right) = \frac{2k}{h}$$

$2y^2 = x(y^2 - x^2)$

$y^2(x-2) = x^3$

13. The value of

$$2\sin\left(\frac{\pi}{8}\right)\sin\left(\frac{2\pi}{8}\right)\sin\left(\frac{3\pi}{8}\right)\sin\left(\frac{5\pi}{8}\right)\sin\left(\frac{6\pi}{8}\right)\sin\left(\frac{7\pi}{8}\right)$$

is :

(1) $\frac{1}{4\sqrt{2}}$ (2) $\frac{1}{4}$

(3) $\frac{1}{8}$ (4) $\frac{1}{8\sqrt{2}}$

Official Ans. by NTA (3)

Sol. $2\sin\left(\frac{\pi}{8}\right)\sin\left(\frac{2\pi}{8}\right)\sin\left(\frac{3\pi}{8}\right)\sin\left(\frac{5\pi}{8}\right)\sin\left(\frac{6\pi}{8}\right)\sin\left(\frac{7\pi}{8}\right)$

$$2\sin^2\frac{\pi}{8}\sin^2\frac{2\pi}{8}\sin^2\frac{3\pi}{8}$$

$$\sin^2\frac{\pi}{8}\sin^2\frac{3\pi}{8}$$

$$\sin^2\frac{\pi}{8}\cos^2\frac{\pi}{8}$$

$$\frac{1}{4}\sin^2\left(\frac{\pi}{4}\right) = \frac{1}{8}$$

14. If $(\sqrt{3}+i)^{100} = 2^{99}(p+iq)$, then p and q are roots of the equation :

(1) $x^2 - (\sqrt{3}-1)x - \sqrt{3} = 0$

(2) $x^2 + (\sqrt{3}+1)x + \sqrt{3} = 0$

(3) $x^2 + (\sqrt{3}-1)x - \sqrt{3} = 0$

(4) $x^2 - (\sqrt{3}+1)x + \sqrt{3} = 0$

Official Ans. by NTA (1)

Sol. $\left(2e^{i\pi/6}\right)^{100} = 2^{99}(p+iq)$

$$2^{100}\left(\cos\frac{50\pi}{3} + i\sin\frac{50\pi}{3}\right) = 2^{99}(p+iq)$$

$$p + iq = 2\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$$

$p = -1, q = \sqrt{3}$

$x^2 - (\sqrt{3} - 1)x - \sqrt{3} = 0.$

15. A hall has a square floor of dimension 10m × 10m (see the figure) and vertical walls. If the angle GPH between the diagonals AG and BH is $\cos^{-1}\frac{1}{5}$, then the height of the hall (in meters) is :

(1) 5 (2) $2\sqrt{10}$

(3) $5\sqrt{3}$ (4) $5\sqrt{2}$

Official Ans. by NTA (4)

Sol. $A(\hat{j})$. $B(10\hat{i})$

H $(h\hat{j} + 10\hat{k})$

G $(10\hat{i} + h\hat{j} + 10\hat{k})$

$\overrightarrow{AG} = 10\hat{i} + h\hat{j} + 10\hat{k}$

$\overrightarrow{BH} = -10\hat{i} + h\hat{j} + 10\hat{k}$

$\cos\theta = \dfrac{\overrightarrow{AG}\,\overrightarrow{BH}}{|\overrightarrow{AG}||\overrightarrow{BH}|}$

$\dfrac{1}{5} = \dfrac{h^2}{h^2 + 200}$

$4h^2 = 200 \Rightarrow h = 5\sqrt{2}$

16. Let P be the plane passing through the point (1,2,3) and the line of intersection of the planes $\vec{r}\cdot(\hat{i}+\hat{j}+4\hat{k}) = 16$ and $\vec{r}\cdot(-\hat{i}+\hat{j}+\hat{k}) = 6$. Then which of the following points does **NOT** lie on P ?

(1) (3, 3, 2) (2) (6, –6, 2)

(3) (4, 2, 2) (4) (–8, 8, 6)

Official Ans. by NTA (3)

Sol. $(x+y+4z-16)+\lambda(-x+y+z-6) = 0$

Passes through (1,2,3)

$-1+\lambda(-2) \Rightarrow \lambda = -\dfrac{1}{2}$

$2(x+y+4z-16)-(-x+y+z-6) = 0$

$3x + y + 7z - 26 = 0$

17. A 10 inches long pencil AB with mid point C and a small eraser P are placed on the horizontal top of a table such that $PC = \sqrt{5}$ inches and $\angle PCB = \tan^{-1}(2)$. The acute angle through which the pencil must be rotated about C so that the perpendicular distance between eraser and pencil becomes exactly 1 inch is :

(1) $\tan^{-1}\left(\dfrac{3}{4}\right)$ (2) $\tan^{-1}(1)$

(3) $\tan^{-1}\left(\dfrac{4}{3}\right)$ (4) $\tan^{-1}\left(\dfrac{1}{2}\right)$

Official Ans. by NTA (1)

Sol.

From figure.

$\sin\beta = \dfrac{1}{\sqrt{5}}$

$\therefore \tan\beta = \dfrac{1}{2}$

$\tan(\alpha+\beta) = 2$

$\dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha\cdot\tan\beta} = 2$

$\dfrac{\tan\alpha + \dfrac{1}{2}}{1 - \tan\alpha\left(\dfrac{1}{2}\right)} = 2$

$\tan\alpha = \dfrac{3}{4}$

$\boxed{\alpha = \tan^{1}\left(\dfrac{3}{4}\right)}$

18. The value of $\displaystyle\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(\frac{1+\sin^2 x}{1+\pi^{\sin x}}\right)dx$ is

(1) $\dfrac{\pi}{2}$ (2) $\dfrac{5\pi}{4}$

(3) $\dfrac{3\pi}{4}$ (4) $\dfrac{3\pi}{2}$

Official Ans. by NTA (3)

Sol. $I = \displaystyle\int_0^{\pi/2} \frac{(1+\sin^2 x)}{(1+\pi^{\sin x})} + \frac{\pi^{\sin x}(1+\sin^2 x)}{(1+\pi^{\sin x})}dx$

$I = \displaystyle\int_0^{\pi/2}(1+\sin^2 x)dx$

$I = \dfrac{\pi}{2} + \dfrac{\pi}{2}\cdot\dfrac{1}{2} = \dfrac{3\pi}{4}$

19. A circle C touches the line x = 2y at the point (2,1) and intersects the circle $C_1 : x^2 + y^2 + 2y - 5 = 0$ at two points P and Q such that PQ is a diameter of C_1. Then the diameter of C is :

(1) $7\sqrt{5}$ (2) 15

(3) $\sqrt{285}$ (4) $4\sqrt{15}$

Official Ans. by NTA (1)

Sol. $(x-2)^2+(y-1)^2+\lambda(x-2y)=0$

$C: x^2+y^2+x(\lambda-4)+y(-2-2\lambda)+5=0$

$C_1: x^2+y^2+2y-5=0$

$S_1-S_2=0$ (Equation of PQ)

$(\lambda-4)x-(2\lambda+4)y+10=0$ Passes through $(0,-1)$

$\Rightarrow \lambda=-7$

$C: x^2+y^2-11x+12y+5=0$

$=\dfrac{\sqrt{245}}{4}$

Diometer $=7\sqrt{5}$

20. $\lim\limits_{x\to 2}\left(\sum\limits_{n=1}^{9}\dfrac{x}{n(n+1)x^2+2(2n+1)x+4}\right)$ is equal to :

(1) $\dfrac{9}{44}$ (2) $\dfrac{5}{24}$

(3) $\dfrac{1}{5}$ (4) $\dfrac{7}{36}$

Official Ans. by NTA (1)

Sol. $S=\lim\limits_{x\to 2}\sum\limits_{n=1}^{9}\dfrac{x}{n(n+1)x^2+2(2n+1)x+4}$

$S=\sum\limits_{n=1}^{9}\dfrac{2}{4(n^2+3n+2)}=\dfrac{1}{2}\sum\limits_{n=1}^{9}\left(\dfrac{1}{n+1}-\dfrac{1}{n+2}\right)$

$S=\dfrac{1}{2}\left(\dfrac{1}{2}-\dfrac{1}{11}\right)=\dfrac{9}{44}$

SECTION-B

1. The sum of all 3-digit numbers less than or equal to 500, that are formed without using the digit "1" and they all are multiple of 11, is _____.

Official Ans. by NTA (7744)

Sol1. 209, 220, 231, ..., 495

Sum $=\dfrac{27}{2}(209+495)=9504$

Number containing 1 at unit place $\begin{array}{ccc}\underline{2} & \underline{3} & \underline{1}\\ \underline{3} & \underline{4} & \underline{1}\\ \underline{4} & \underline{5} & \underline{1}\end{array}$

Number containing 1 at 10th place $\begin{array}{ccc}\underline{3} & \underline{1} & \underline{9}\\ \underline{4} & \underline{1} & \underline{8}\end{array}$

Required = 9501 – (231 + 341 + 451 + 319 + 418)

7744

2. Let a and b respectively be the points of local maximum and local minimum of the function $f(x)=2x^3-3x^2-12x$. If A is the total area of the region bounded by $y=f(x)$, the x-axis and the lines $x=a$ and $x=b$, then 4A is equal to _____.

Official Ans. by NTA (114)

Sol. $f'(x)=6x^2-6x-12=6(x-2)(x+1)$

Point = (2,–20) & (–1,7)

$A=\int_{-1}^{0}(2x^3-3x^2-12x)dx+\int_{0}^{2}(12x+3x^2-2x^3)dx$

$A=\left(\dfrac{x^4}{2}-x^3-6x^2\right)_{-1}^{0}+\left(6x^2+x^3-\dfrac{x^4}{2}\right)_{0}^{2}$

4A = 114

3. If the projection of the vector $\hat{i}+2\hat{j}+\hat{k}$ on the sum of the two vectors $2\hat{i}+4\hat{j}-5\hat{k}$ and $-\lambda\hat{i}+2\hat{j}+3\hat{k}$ is 1, then λ is equal to _____.

Official Ans. by NTA (5)

Sol. $\vec{a}=\hat{i}+2\hat{j}+\hat{k}$

$\vec{b}=(2-\lambda)\hat{i}+6\hat{j}-2\hat{k}$

$\dfrac{\vec{a}\cdot\vec{b}}{|\vec{b}|}=1,\ \vec{a}\cdot\vec{b}=12-\lambda$

$(\vec{a}\cdot\vec{b})=|\vec{b}|^2$

$\lambda^2-24\lambda+144=\lambda^2-4\lambda+4+40$

$20\lambda=100\Rightarrow\lambda=5.$

4. Let $a_1, a_2,....,a_{10}$ be an AP with common difference –3 and $b_1, b_2,...., b_{10}$ be a GP with common ratio 2. Let $c_k=a_k+b_k$, k = 1,2,..., 10. If $c_2=12$ and $c_3=13$, then $\sum\limits_{k=1}^{10}c_k$ is equal to _____.

Official Ans. by NTA (2021)

Sol. $c_2=a_2+b_2=a_1-3+2b_1=12$

$a_1 + 2b_1 = 15$ ____(1)

$c_3 = a_3 + b_3 = a_1 - 6 + 4b_1 = 13$

$a_1 + 4b_1 = 19$ ____(2)

from (1) & (2) $b_1 = 2$, $a_1 = 11$

$$\sum_{k=1}^{10} c_k = \sum_{k=1}^{10}(a_k + b_k) = \sum_{k=1}^{10} a_k + \sum_{k=1}^{10} b_k$$

$$= \frac{10}{2}(2\times 11 + 9\times(-3)) + \frac{2(2^{10}-1)}{2-1}$$

$= 5(22 - 27) + 2(1023)$
$= 2046 - 25 = 2021$

5. Let Q be the foot of the perpendicular from the point P(7,–2,13) on the plane containing the lines $\frac{x+1}{6} = \frac{y-1}{7} = \frac{z-3}{8}$ and $\frac{x-1}{3} = \frac{y-2}{5} = \frac{z-3}{7}$. Then $(PQ)^2$, is equal to _____.

Official Ans. by NTA (96)

Sol. Containing the line $\begin{vmatrix} x+1 & y-1 & z-3 \\ 6 & 7 & 8 \\ 3 & 5 & 7 \end{vmatrix} = 0$

$9(x + 1) - 18(y - 1) + 9(z - 3) = 0$

$x - 2y + z = 0$

$PQ = \left|\frac{7+4+13}{\sqrt{6}}\right| = 4\sqrt{6}$

$PQ^2 = 96$

6. Let $\binom{n}{k}$ denotes ${}^nC_k$ and $\begin{bmatrix} n \\ k \end{bmatrix} = \begin{cases} \binom{n}{k}, & \text{if } 0 \le k \le n \\ 0, & \text{otherwise} \end{cases}$

If $A_k = \sum_{i=0}^{9}\binom{9}{i}\begin{bmatrix} 12 \\ 12-k+i \end{bmatrix} + \sum_{i=0}^{8}\binom{8}{i}\begin{bmatrix} 13 \\ 13-k+i \end{bmatrix}$

and $A_4 - A_3 = 190$ p, then p is equal to :

Official Ans. by NTA (49)

Sol. $A_k = \sum_{i=0}^{9} {}^9C_i \, {}^{12}C_{k-i} + \sum_{i=0}^{8} {}^8C_i \, {}^{13}C_{k-i}$

$A_k = {}^{21}C_k + {}^{21}C_k = 2\cdot {}^{21}C_k$

$A_4 - A_3 = 2({}^{21}C_4 - {}^{21}C_3) = 2(5985 - 1330)$

$190\,p = 2(5985 - 1330) \Rightarrow p = 49.$

7. Let $\lambda \neq 0$ be in **R**. If α and β are the roots of the equation $x^2 - x + 2\lambda = 0$, and α and γ are the roots of equation $3x^2 - 10x + 27\lambda = 0$, then $\frac{\beta\gamma}{\lambda}$ is equal to _____.

Official Ans. by NTA (18)

Sol. $3\alpha^2 - 10\alpha + 27\lambda = 0$ _____(1)

$\alpha^2 - \alpha + 2\lambda = 0$ _____(2)

(1) – 3(2) gives

$-7\alpha + 21\lambda = 0 \Rightarrow \alpha = 3\lambda$

Put $\alpha = 3\lambda$ in equation (1) we get

$9\lambda^2 - 3\lambda + 2\lambda - 0$

$9\lambda^2 = \lambda \Rightarrow \lambda = \frac{1}{9}$ as $\lambda \neq 0$

Now $\alpha = 3\lambda \Rightarrow \lambda = \frac{1}{3}$

$\alpha + \beta = 1 \Rightarrow \beta = 2/3$

$\alpha + \gamma = \frac{10}{3} \Rightarrow \gamma = 3$

$$\frac{\beta\gamma}{\lambda} = \frac{\frac{2}{3}\times 3}{\frac{1}{9}} = 18$$

8. Let the mean and variance of four numbers 3, 7, x and y(x > y) be 5 and 10 respectively. Then the mean of four numbers 3 + 2x, 7 + 2y, x + y and x – y is _____.

Official Ans. by NTA (12)

Sol. $5 = \frac{3+7+x+y}{4} \Rightarrow x + y = 10$

$\text{Var}(x) = 10 = \frac{3^2+7^2+x^2+y^2}{4} - 25$

$140 = 49 + 9 + x^2 + y^2$

$x^2 + y^2 = 82$

$x + y = 10$

$\Rightarrow (x,y) = (9,1)$

Four numbers are 21,9,10,8

$\text{Mean} = \frac{48}{4} = 12$

9. Let A be a 3 × 3 real matrix.

If det(2Adj(2 Adj(Adj(2A)))) = 2^{41}, then the value of det(A^2) equal _____.

Official Ans. by NTA (4)

Sol. adj (2A) = 2^2 adjA

$\Rightarrow$ adj(adj (2A)) = adj(4 adjA) = 16 adj (adj A)

= 16 |A| A

$\Rightarrow$ adj (32 |A| A) = $(32\ |A|)^2$ adj A

$12(32|A|)^2\ |\text{adj } A| = 2^3\ (32|A|)^6\ |\text{adj } A|$

$2^3.2^{30}\ |A|^6\ .\ |A|^2 = 2^{41}$

$|A|^8 = 2^8 \Rightarrow |A| = \pm 2$

$|A|^2 = |A|^2 = 4$

10. The least positive integer n such that

$\frac{(2i)^n}{(1-i)^{n-2}}, i = \sqrt{-1}$ is a positive integer, is _____.

Official Ans. by NTA (6)

Sol. $\frac{(2i)^n}{(1-i)^{n-2}} = \frac{(2i)^n}{(-2i)^{\frac{n-2}{2}}}$

$= \frac{(2i)^{\frac{n+2}{2}}}{(-1)^{\frac{n-2}{2}}} = \frac{2^{\frac{n+2}{2}}; i^{\frac{n+2}{2}}}{(-1)^{\frac{n-2}{2}}}$

This is positive integer for n = 6

FINAL JEE–MAIN EXAMINATION – AUGUST, 2021

(Held On Thursday 26th August, 2021) **TIME : 9 : 00 AM to 12 : 00 NOON**

PHYSICS — TEST PAPER WITH SOLUTION

SECTION-A

1. The fractional change in the magnetic field intensity at a distance 'r' from centre on the axis of current carrying coil of radius 'a' to the magnetic field intensity at the centre of the same coil is : (Take r < a)

(1) $\frac{3}{2}\frac{a^2}{r^2}$ (2) $\frac{2}{3}\frac{a^2}{r^2}$

(3) $\frac{2}{3}\frac{r^2}{a^2}$ (4) $\frac{3}{2}\frac{r^2}{a^2}$

Official Ans. by NTA (4)

Sol. $B_{axis} = \frac{\mu_0 i R^2}{2\left(R^2+x^2\right)^{3/2}}$

$B_{centre} = \frac{\mu_0 i}{2R}$

$\therefore B_{centre} = \frac{\mu_0 i}{2a}$

$\therefore B_{axis} = \frac{\mu_0 i a^2}{2\left(a^2+r^2\right)^{3/2}}$

$\therefore$ fractional change in magnetic field =

$$\frac{\frac{\mu_0 i}{2a} - \frac{\mu_0 i a^2}{2\left(a^2+r^2\right)^{3/2}}}{\frac{\mu_0 i}{2a}} = 1 - \frac{1}{\left[1+\left(\frac{r^2}{a^2}\right)\right]^{3/2}}$$

$$\approx 1 - \left[1 - \frac{3}{2}\frac{r^2}{a^2}\right] = \frac{3}{2}\frac{r^2}{a^2}$$

Note : $\left(1+\frac{r^2}{a^2}\right)^{-3/2} \approx \left(1 - \frac{3}{2}\frac{r^2}{a^2}\right)$

[True only if r << a]

Hence option (4) is the most suitable option

2. The magnitude of vectors $\overrightarrow{OA}$, $\overrightarrow{OB}$ and $\overrightarrow{OC}$ in the given figure are equal. The direction of $\overrightarrow{OA}+\overrightarrow{OB}-\overrightarrow{OC}$ with x-axis will be :-

(1) $\tan^{-1}\frac{(1-\sqrt{3}-\sqrt{2})}{(1+\sqrt{3}+\sqrt{2})}$

(2) $\tan^{-1}\frac{\left(\sqrt{3}-1+\sqrt{2}\right)}{\left(1+\sqrt{3}-\sqrt{2}\right)}$

(3) $\tan^{-1}\frac{\left(\sqrt{3}-1+\sqrt{2}\right)}{\left(1-\sqrt{3}+\sqrt{2}\right)}$

(4) $\tan^{-1}\frac{\left(1+\sqrt{3}-\sqrt{2}\right)}{\left(1-\sqrt{3}-\sqrt{2}\right)}$

Official Ans. by NTA (1)

Sol.

Let magnitude be equal to λ.

$\overrightarrow{OA} = \lambda\left[\cos 30°\hat{i} + \sin 30\hat{j}\right] = \lambda\left[\frac{\sqrt{3}}{2}\hat{i} + \frac{1}{2}\hat{j}\right]$

$\overrightarrow{OB} = \lambda\left[\cos 60°\hat{i} - \sin 60\hat{j}\right] = \lambda\left[\frac{1}{2}\hat{i} - \frac{\sqrt{3}}{2}\hat{j}\right]$

$\overrightarrow{OC} = \lambda\left[\cos 45°\left(-\hat{i}\right) + \sin 45\hat{j}\right] = \lambda\left[-\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j}\right]$

$\therefore \overrightarrow{OA}+\overrightarrow{OB}-\overrightarrow{OC}$

$= \lambda\left[\left(\frac{\sqrt{3}+1}{2} + \frac{1}{\sqrt{2}}\right)\hat{i} + \left(\frac{1}{2} - \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}}\right)\hat{j}\right]$

$\therefore$ Angle with x-axis

$$\tan^{-1}\left[\frac{\frac{1}{2}-\frac{\sqrt{3}}{2}-\frac{1}{\sqrt{2}}}{\frac{\sqrt{3}}{2}+\frac{1}{2}+\frac{1}{\sqrt{2}}}\right]=\tan^{-1}\left[\frac{\sqrt{2}-\sqrt{6}-2}{\sqrt{6}+\sqrt{2}+2}\right]$$

$$=\tan^{-1}\left[\frac{1-\sqrt{3}-\sqrt{2}}{\sqrt{3}+1+\sqrt{2}}\right]$$

Hence option (1)

3. Car B overtakes another car A at a relative speed of 40 ms^{-1}. How fast will the image of car B appear to move in the mirror of focal length 10 cm fitted in car A, when the car B is 1.9 m away from the car A?

(1) 4 ms^{-1} (2) 0.2 ms^{-1}

(3) 40 ms^{-1} (4) 0.1 ms^{-1}

Official Ans. by NTA (4)

Sol.

Mirror used is convex mirror (rear-view mirror)

$\therefore V_{I/m}=-m^2V_{O/m}$

Given,

$V_{O/m}=40m/s$

$$m=\frac{f}{f-u}=\frac{10}{10+190}=\frac{10}{200}$$

$$\therefore V_{I/m}=-\frac{1}{400}\times 40=-0.1m/s$$

$\therefore$ Car will appear to move with speed 0.1 m/s.

Hence option (4)

4. Inside a uniform spherical shell :

(a) the gravitational field is zero

(b) the gravitational potential is zero

(c) the gravitational field is same everywhere

(d) the gravitation potential is same everywhere

(e) all of the above

Choose the most appropriate answer from the options given below :

(1) (a), (c) and (d) only

(2) (e) only

(3) (a), (b) and (c) only

(4) (b), (c) and (d) only

Official Ans. by NTA (1)

Sol. Inside a spherical shell, gravitational field is zero and hence potential remains same everywhere

Hence option (1)

5. Two narrow bores of diameter 5.0 mm and 8.0 mm are joined together to form a U-shaped tube open at both ends. If this U-tube contains water, what is the difference in the level of two limbs of the tube. [Take surface tension of water T = 7.3 × 10^{-2} Nm^{-1}, angle of contact = 0, g = 10 ms^{-2} and density of water = 1.0 × 10^3 kg m^{-3}]

(1) 3.62 mm (2) 2.19 mm

(3) 5.34 mm (4) 4.97 mm

Official Ans. by NTA (2)

Sol.

We have $P_A = P_B$. [Points A & B at same horizontal level]

$$\therefore P_{atm}-\frac{2T}{r_1}+\rho g(x+\Delta h)=P_{atm}-\frac{2T}{r_2}+\rho gx$$

$$\therefore \rho g\Delta h=2T\left[\frac{1}{r_1}-\frac{1}{r_2}\right]$$

$$=2\times 7.3\times 10^{-2}\left[\frac{1}{2.5\times 10^{-3}}-\frac{1}{4\times 10^{-3}}\right]$$

$$\therefore \Delta h=\frac{2\times 7.3\times 10^{-2}\times 10^3}{10^3\times 10}\left[\frac{1}{2.5}-\frac{1}{4}\right]$$

= 2.19 × 10^{-3}m = 2.19 mm

Hence option (2)

6. An electric appliance supplies 6000 J/min heat to the system. If the system delivers a power of 90W. How long it would take to increase the internal energy by 2.5 × 10^3 J ?

(1) 2.5 × 10^2 s (2) 4.1 × 10^1s

(3) 2.4 × 10^3 s (4) 2.5 × 10^1s

Official Ans. by NTA (1)

Sol. $\Delta Q = \Delta U + \Delta W$

$$\frac{\Delta Q}{\Delta t} = \frac{\Delta U}{\Delta t} + \frac{\Delta W}{\Delta t}$$

$$\frac{6000}{60}\frac{J}{sec} = \frac{2.5\times10^3}{\Delta t} + 90$$

$\Delta t = 250$ sec

Option (1)

7. An inductor coil stores 64 J of magnetic field energy and dissipates energy at the rate of 640 W when a current of 8A is passed through it. If this coil is joined across an ideal battery, find the time constant of the circuit in seconds :

(1) 0.4 (2) 0.8 (3) 0.125 (4) 0.2

Official Ans. by NTA (4)

Sol. $U = \frac{1}{2}Li^2 = 64 \Rightarrow L = 2$

$i^2R = 640$

$$R = \frac{640}{(8)^2} = 10$$

$$\tau = \frac{L}{R} = \frac{1}{5} = 0.2$$

Option (4)

8. A series LCR circuit driven by 300 V at a frequency of 50 Hz contains a resistance R = 3 kΩ, an inductor of inductive reactance X_L = 250 πΩ and an unknown capacitor. The value of capacitance to maximize the average power should be : (Take $\pi^2 = 10$)

(1) 4 μF (2) 25 μF (3) 400 μF (4) 40 μF

Official Ans. by NTA (1)

Sol. For maximum average power

$$X_L = X_C$$

$$250\pi = \frac{1}{2\pi(50)C}$$

$$C = 4\times10^{-6}$$

Option (1)

9. Identify the logic operation carried out by the given circuit :-

(1) OR (2) AND (3) NOR (4) NAND

Official Ans. by NTA (3)

Sol.

A	B	X	Y	Z
1	1	0	0	0
1	0	0	1	0
0	1	1	0	0
0	0	1	1	1

Option (3)

10. A particular hydrogen like ion emits radiation of frequency 2.92×10^{15} Hz when it makes transition from n = 3 to n = 1. The frequency in Hz of radiation emitted in transition from n = 2 to n = 1 will be :

(1) 0.44×10^{15} (2) 6.57×10^{15}

(3) 4.38×10^{15} (4) 2.46×10^{15}

Official Ans. by NTA (4)

Sol. $nf_1 = k\left(\frac{1}{1} - \frac{1}{3^2}\right)$

$$nf_2 = k\left(1 - \frac{1}{2^2}\right)$$

$$\frac{f_1}{f_2} = \frac{8/9}{3/4} \Rightarrow f_2 = 2.46\times10^{15}$$

Option (4)

11. In a photoelectric experiment ultraviolet light of wavelength 280 nm is used with lithium cathode having work function ϕ = 2.5 eV. If the wavelength of incident light is switched to 400 nm, find out the change in the stopping potential. ($h = 6.63 \times 10^{-34}$ Js, $c = 3 \times 10^8$ ms^{-1})

(1) 1.3 V (2) 1.1 V (3) 1.9 V (4) 0.6 V

Official Ans. by NTA (1)

Sol. $KE_{max} = eV_S = \frac{hc}{\lambda} - \phi$

$$\Rightarrow eV_S = \frac{1240}{280} - 2.5 = 1.93eV$$

$$\rightarrow V_{S_1} = 1.93V \quad \ldots (i)$$

$$\rightarrow eV_{S_2} = \frac{1240}{400} - 2.5 = 0.6eV$$

$$\Rightarrow V_{S_2} = 0.6V \quad \ldots (ii)$$

$$\Delta V = V_{S_1} - V_{S_2} = 1.93 - 0.6 = 1.33V$$

Option (1)

12. In the given figure, the emf of the cell is 2.2 V and if internal resistance is 0.6Ω. Calculate the power dissipated in the whole circuit :

(1) 1.32 W (2) 0.65 W

(3) 2.2 W (4) 4.4 W

Official Ans. by NTA (3)

Sol.

⇒

$$\frac{1}{R_{eq}} = \frac{1}{4} + \frac{1}{8} + \frac{1}{12} + \frac{1}{6} = \frac{6+3+2+4}{24} = \frac{15}{24}$$

$$R_{eq} = \frac{24}{15} = 1.6 \Rightarrow R_T = 1.6 + 0.6 = 2.2\Omega$$

$$P = \frac{V^2}{R_T} = \frac{(2.2)^2}{2.2} = 2.2W$$

Option (3)

13. A solid metal sphere of radius R having charge q is enclosed inside the concentric spherical shell of inner radius a and outer radius b as shown in figure. The approximate variation electric field $\vec{E}$ as a function of distance r from centre O is given by

(1)

(2)

(3)

(4)

Official Ans. by NTA (1)

Official Ans. by ALLEN (1 or 2)

Sol. Considering outer spherical shell is non-conducting

Electric field inside a metal sphere is zero.

$r < R \Rightarrow E = 0$

$$r > R \Rightarrow E = \frac{kQ}{r^2}$$

Option (2)

Considering outer spherical shell is conducting

$r < R$, $E = 0$

$R \le r < a$ $\quad E = \frac{kQ}{r^2}$

$a \le r < b$, $\quad E = 0$

$r \ge b$ $\quad E = \frac{kQ}{r^2}$

Option (1)

14. The rms speeds of the molecules of Hydrogen, Oxygen and Carbondioxide at the same temperature are V_H, V_O and V_C respectively then :

(1) $V_H > V_O > V_C$ (2) $V_C > V_O > V_H$

(3) $V_H = V_O > V_C$ (4) $V_H = V_O = V_C$

Official Ans. by NTA (1)

Sol. $V_{RMS} = \sqrt{\frac{3RT}{M_W}}$

At the same temperature $V_{RMS} \propto \frac{1}{\sqrt{M_w}}$

$\Rightarrow V_H > V_O > V_C$

Option (1)

15. In a Screw Gauge, fifth division of the circular scale coincides with the reference line when the ratchet is closed. There are 50 divisions on the circular scale, and the main scale moves by 0.5 mm on a complete rotation. For a particular observation the reading on the main scale is 5 mm and the 20th division of the circular scale coincides with reference line. Calculate the true reading.

(1) 5.00 mm (2) 5.25 mm

(3) 5.15 mm (4) 5.20 mm

Official Ans. by NTA (3)

Sol. Least count (L.C) $= \frac{0.5}{50}$

True reading $= 5 + \frac{0.5}{50} \times 20 - \frac{0.5}{50} \times 5$

$= 5 + \frac{0.5}{50}(15) = 5.15\text{mm}$

Option (3)

16. What equal length of an iron wire and a copper-nickel alloy wire, each of 2 mm diameter connected parallel to give an equivalent resistance of 3Ω?

(Given resistivities of iron and copper-nickel alloy wire are 12 μΩ cm and 51 μΩ cm respectively)

(1) 82 m (2) 97 m

(3) 110 m (4) 90 m

Official Ans. by NTA (2)

Sol. $\frac{R_1R_2}{R_1+R_2} = 3$

$$\frac{\frac{(12\times10^{-6}\times10^{-2})\ell\times4}{\pi(2)^2\times10^{-6}} \times \frac{(51\times10^{-6}\times10^{-2})\ell\times4}{\pi(2)^2\times10^{-6}}}{\frac{63\times10^{-6}\times10^{-2}\times\ell\times4}{\pi(2)^2\times10^{-6}}}$$

$\Rightarrow \ell = 97\text{m}$

Option (2)

17. The initial mass of a rocket is 1000 kg. Calculate at what rate the fuel should be burnt so that the rocket is given an acceleration of 20 ms^{-2}. The gases come out at a relative speed of 500 ms^{-1} with respect to the rocket :[Use g = 10 m/s^2]

(1) 6.0×10^2 kg s^{-1} (2) 500 kg s^{-1}

(3) 10 kg s^{-1} (4) 60 kg s^{-1}

Official Ans. by NTA (4)

Sol.

$$F_{thrust} = \left(\frac{dm}{dt} \cdot V_{rel}\right)$$

$$\left(\frac{dm}{dt} V_{rel} - mg\right) = ma$$

$$\Rightarrow \left(\frac{dm}{dt}\right) \times 500 - 10^3 \times 10 = 10^3 \times 20$$

$$\frac{dm}{dt} = (60 kg/s)$$

Option (4)

18. If E, L, M and G denote the quantities as energy, angular momentum, mass and constant of gravitation respectively, then the dimensions of P in the formula $P = EL^2M^{-5}G^{-2}$ are :-

(1) $[M^0 L^1 T^0]$ (2) $[M^{-1} L^{-1} T^2]$

(3) $[M^1 L^1 T^{-2}]$ (4) $[M^0 L^0 T^0]$

Official Ans. by NTA (4)

Sol. $E = ML^2T^{-2}$

$L = ML^2T^{-1}$

$m = M$

$G = M^{-1}L^{+3}T^{-2}$

$$P = \frac{EL^2}{M^5G^2}$$

$$[P] = \frac{(ML^2T^{-2})(M^2L^4T^{-2})}{M^5(M^{-2}L^6T^{-4})} = M^0L^0T^0$$

Option (4)

19. The material filled between the plates of a parallel plate capacitor has resistivity 200 Ωm. The value of capacitance of the capacitor is 2 pF. If a potential difference of 40 V is applied across the plates of the capacitor, then the value of leakage current flowing out of the capacitor is : (given the value of relative permitivity of material is 50)

(1) 9.0 μA (2) 9.0 mA

(3) 0.9 mA (4) 0.9 μA

Official Ans. by NTA (3)

Sol. $\rho = 200\ \Omega m$

$C = 2 \times 10^{-12}$ F

$V = 40$ V

$K = 56$

$$i = \frac{q}{\rho k \varepsilon_0} = \frac{q_0}{\rho k \varepsilon_0} e^{-\frac{t}{\rho k \varepsilon_0}}$$

$$i_{max} = \frac{2 \times 10^{-12} \times 40}{200 \times 50 \times 8.85 \times 10^{-12}}$$

$$= \frac{80}{10^4 \times 8.85} = 903 \mu A = 0.9 mA$$

Option (3)

20. **Statement-I :** By doping silicon semiconductor with pentavalent material, the electrons density increases.

Statement-II : The n-type semiconductor has net negative charge.

In the light of the above statements, choose the most appropriate answer from the options given below :

(1) Statement-I is true but Statement-II is false.

(2) Statement-I is false but Statement-II is true.

(3) Both Statement-I and Statement-II are true.

(4) Both Statement-I and Statement-II are false.

Official Ans. by NTA (1)

Sol. Pentavalent activities have excess free e^-

So e^- density increases but overall semiconductor is neutral.

Option (1)

SECTION-B

1. A uniform chain of length 3 meter and mass 3 kg overhangs a smooth table with 2 meter laying on the table. If k is the kinetic energy of the chain in joule as it completely slips off the table, then the value of k is ..... . (Take g = 10 m/s^2)

Official Ans. by NTA (40)

Sol.

From energy conservation

$K_i + U_i = k_f + U_f$

$$0+\left(-1\times10\times\frac{1}{2}\right)=k_f+\left(-3\times10\times\frac{3}{2}\right)$$

$-5 = k_f - 45$

$k_f = 40$ J

Ans. 40.00

2. The electric field in a plane electromagnetic wave is given by

$$\vec{E}=200\cos\left[\left(\frac{0.5\times10^3}{m}\right)x-\left(1.5\times10^{11}\frac{rad}{s}\times t\right)\right]\frac{V}{m}\hat{j}$$

If this wave falls normally on a perfectly reflecting surface having an area of 100 cm^2. If the radiation pressure exerted by the E.M. wave on the surface during a 10 minute exposure is $\frac{x}{10^9}\frac{N}{m^2}$. Find the value of x.

Official Ans. by NTA (354)

Sol. $E_0 = 200$

$$I=\frac{1}{2}\varepsilon_0E_0^2\cdot C$$

Radiation pressure

$$P=\frac{2I}{C}$$

$$=\left(\frac{2}{C}\right)\left(\frac{1}{2}\varepsilon_0E_0^2C\right)$$

$$=\varepsilon_0E_0^2$$

$$=8.85\times10^{-12}\times200^2$$

$$=8.85\times10^{-8}\times4$$

$$=\frac{354}{10^9}$$

Ans. 354.0

3. A source and a detector move away from each other in absence of wind with a speed of 20 m/s with respect to the ground. If the detector detects a frequency of 1800 Hz of the sound coming from the source, then the original frequency of source considering speed of sound in air 340 m/s will be ..... Hz.

Official Ans. by NTA (2025)

Sol.

$$f'=f\left(\frac{C-V_0}{C+V_s}\right)$$

$$1800=f\left(\frac{340-20}{340+20}\right)$$

f = 2025 Hz

Ans. 2025

4. Two spherical balls having equal masses with radius of 5 cm each are thrown upwards along the same vertical direction at an interval of 3s with the same initial velocity of 35 m/s, then these balls collide at a height of ..... m. (Take g = 10 m/s^2)

Official Ans. by NTA (50)

Sol.

When both balls will collied

$y_1 = y_2$

$$35t-\frac{1}{2}\times10\times t^2=35(t-3)-\frac{1}{2}\times10\times(t-3)^2$$

$$35t-\frac{1}{2}\times10\times t^2=35t-105-\frac{1}{2}\times10\times t^2$$

$$-\frac{1}{2}\times10\times3^2+\frac{1}{2}\times10\times6t$$

$0 = 150 - 30\,t$

t = 5 sec

$\therefore$ Height at which both balls will collied

$$h=35t-\frac{1}{2}\times10\times t^2$$

$$=35\times5-\frac{1}{2}\times10\times5^2$$

h = 50 m

Ans. 50.00

5. A soap bubble of radius 3 cm is formed inside the another soap bubble of radius 6 cm. The radius of an equivalent soap bubble which has the same excess pressure as inside the smaller bubble with respect to the atmospheric pressure is ..... cm.

Official Ans. by NTA (2)

Sol.

Excess pressure inside the smaller soap bubble

$$\Delta P = \frac{4S}{r_1} + \frac{4S}{r_2} \quad \text{.... (i)}$$

The excess pressure inside the equivalent soap bubble

$$\Delta P = \frac{4S}{R_{eq}} \quad \text{... (ii)}$$

From (i) & (ii)

$$\frac{4S}{R_{eq}} = \frac{4S}{r_1} + \frac{4S}{r_2}$$

$$\frac{1}{R_{eq}} = \frac{1}{r_1} + \frac{1}{r_2}$$

$$= \frac{1}{6} + \frac{1}{3}$$

$R_{eq} = 2$ cm

Ans. 2.00

6. An amplitude modulated wave is represented by

$C_m(t) = 10(1 + 0.2\cos 12560t)\sin(111 \times 10^4 t)$ volts.

The modulating frequency in kHz will be ..... .

Official Ans. by NTA (2)

Sol. $W_m = 12560 = 2\pi f_m$

$$f_m = \frac{12560}{2\pi}$$

= 2000 Hz

Ans. 2.00

7. Two short magnetic dipoles m_1 and m_2 each having magnetic moment of 1 Am^2 are placed at point O and P respectively. The distance between OP is 1 meter. The torque experienced by the magnetic dipole m_2 due to the presence of m_1 is ..... $\times 10^{-7}$ Nm.

Official Ans. by NTA (1)

Sol.

$$\vec{\tau} = \vec{M}_2 \times \vec{B}_1$$

$$\tau = M_2 B_1 \sin 90°$$

$$= 1 \times \frac{\mu_0}{4\pi}\frac{M_1}{(1)^3}1$$

$= 10^{-7}$ N.m

Ans. 1.00

8. Two travelling waves produces a standing wave represented by equation,

$y = 1.0$ mm $\cos(1.57\ cm^{-1})$ x $\sin(78.5\ s^{-1})t$.

The node closest to the origin in the region $x > 0$ will be at x = ... cm.

Official Ans. by NTA (1)

Sol. For node

$$\cos\left(1.57cm^{-1}\right)x = 0$$

$$\left(1.57cm^{-1}\right)x = \frac{\pi}{2}$$

$$x = \frac{\pi}{2(1.57)}cm = 1cm$$

Ans. 1.00

9. White light is passed through a double slit and interference is observed on a screen 1.5 m away. The separation between the slits is 0.3 mm. The first violet and red fringes are formed 2.0 mm and 3.5 mm away from the central white fringes. The difference in wavelengths of red and voilet light is ..... nm.

Official Ans. by NTA (300)

Sol. Position of bright fringe $y = n\dfrac{D\lambda}{d}$

y_1 of red $= \dfrac{D\lambda_r}{d} = 3.5\text{mm}$

$\lambda_r = 3.5\times10^{-3}\dfrac{d}{D}$

Similarly $\lambda_v = 2\times10^{-3}\dfrac{d}{D}$

$\lambda_r - \lambda_v = \left(1.5\times10^{-3}\right)\left(\dfrac{0.3\times10^{-3}}{1.5}\right)$

$= 3 \times 10^{-7} = 300$ nm

Ans. 300.0

10. Consider a badminton racket with length scales as shown in the figure.

If the mass of the linear and circular portions of the badminton racket are same (M) and the mass of the threads are negligible, the moment of inertia of the racket about an axis perpendicular to the handle and in the plane of the ring at, $\dfrac{r}{2}$ distance from the end A of the handle will be ..... Mr^2.

Official Ans. by NTA (52)

Sol.

$$I = \left[I_1 + M\left(\frac{5}{2}r\right)^2\right] + \left[I_2 + M\left(\frac{13r}{2}\right)^2\right]$$

$$= \left[\frac{M\left(36r^2\right)}{12} + \frac{M\left(25r^2\right)}{4}\right] + \left[\frac{Mr^2}{2} + \frac{169Mr^2}{4}\right]$$

$= 52\ Mr^2$

Ans. 52.00

FINAL JEE–MAIN EXAMINATION – AUGUST, 2021

(Held On Thursday 26th August, 2021) **TIME : 9 : 00 AM to 12 : 00 NOON**

CHEMISTRY — TEST PAPER WITH SOLUTION

SECTION-A

1. Which one of the following complexes is violet in colour?

(1) $[Fe(CN)_6]^{4-}$ (2) $[Fe(SCN)_6]^{4-}$

(3) $Fe_4[Fe(CN_6)]_3 \cdot H_2O$ (4) $[Fe(CN)_5NOS]^{4-}$

Official Ans. by NTA (4)

Sol. (1) $[Fe(CN)_6]^{4-} \rightarrow$ Pale yellow solution

(2) $[Fe(SCN)_6]^{4-} \rightarrow$ Blood red colour

(3) $Fe_4[Fe(CN_6)]_3 \cdot H_2O \rightarrow$ Prussian blue

(4) $[Fe(CN)_5NOS]^{4-} \rightarrow$ Violet colour

2. Which one of the following is correct for the adsorption of a gas at a given temperature on a solid surface?

(1) $\Delta H > 0, \Delta S > 0$ (2) $\Delta H > 0, \Delta S < 0$

(3) $\Delta H < 0, \Delta S < 0$ (4) $\Delta H < 0, \Delta S > 0$

Official Ans. by NTA (3)

Sol. (i) Adsorption of gas at metal surface is an exothermic process so $\Delta H < 0$

(ii) As the adsorption of gas on metal surface reduces the free movement of gas molecules thus restricting its randomness hences $\Delta S < 0$

3. Which one of the following when dissolved in water gives coloured solution in nitrogen atmosphere?

(1) $CuCl_2$ (2) AgCl

(3) $ZnCl_2$ (4) Cu_2Cl_2

Official Ans. by NTA (1)

Sol. (1) $CuCl_2 + nH_2O \rightarrow Cu^{+2}_{(aq.)}$

blue colour

(2) $AgCl + nH_2O \rightarrow$ Insoluble

(3) $ZnCl_2 + nH_2O \rightarrow Zn^{+2}_{(aq.)}$

Colourless

(4) $Cu_2Cl_2 + nH_2O \rightarrow$ Insoluble

4. The major products formed in the following reaction sequence **A** and **B** are :

(1) **A** =

, **B**=$CHBr_3$

(2) **A**=

(3) **A**=

(4) **A**=

Official Ans. by NTA (1)

Sol.

(1)

, B=$CHBr_3$

5. The major product formed in the following reaction is :

(1)

(2)

(3)

(4)

Official Ans. by NTA (3)

Sol.

6. The major product formed in the following reaction is :

(1)

(2)

(3)

(4)

Official Ans. by NTA (1)

Sol.

7. The polymer formed on heating Novolac with formaldehyde is :

(1) Bakelite (2) Polyester

(3) Melamine (4) Nylon 6,6

Official Ans. by NTA (1)

Sol. Novolac + formaldehyde → Bakelite

8. Given below are two statements :

Statement I : The limiting molar conductivity of KCl (strong electrolyte) is higher compared to that of CH_3COOH (weak electrolyte).

Statement II : Molar conductivity decreases with decrease in concentration of electrolyte.

In the light of the above statements, choose the **most appropriate** answer from the options given below :

(1) **Statement I** is true but **Statement II** is false.

(2) **Statement I** is false but **Statement II** is true.

(3) Both **Statement I** and **Statement II** are true.

(4) Both **Statement I** and **Statement II** are false.

Official Ans. by NTA (4)

Sol.

Ion	H^+	K^+	Cl^-	CH_3COO^-
Λ^{∞}_{m} Scm²/mole	349.8	73.5	76.3	40.9

So $\Lambda^{\infty}_{m\ CH_3COOH} = \Lambda^{\infty}_{m\ (H^+)} + \Lambda^{\infty}_{m\ CH_3COO^-}$

$= 349.8 + 40.9$

$= 390.7\ Scm^2/mole$

$\Lambda^{\infty}_{m\ KCl} = \Lambda^{\infty}_{m\ (K^+)} + \Lambda^{\infty}_{m\ (Cl^-)}$

$= 73.5 + 76.3$

$= 149.3\ Scm^2/mole$

So statement-I is wrong or False.

As the concentration decreases, the dilution increases which increases the degree of dissociation, thus increasing the no. of ions, which increases the molar conductance.

So statement-II is false.

9. The correct options for the products **A** and **B** of the following reactions are :

(1) **A=**

, **B=**

(2) **A=**

, **B=**

(3) **A=**

, **B=**

(4) **A=**

, **B=**

Official Ans. by NTA (2)

Sol.

10. The conversion of hydroxyapatite occurs due to presence of F^- ions in water. The correct formula of hydroxyapatite is:

(1) $[3Ca_3(PO_4)_2 \cdot Ca(OH)_2]$

(2) $[3Ca(OH)_2 \cdot CaF_2]$

(3) $[Ca_3(PO_4)_2 \cdot CaF_2]$

(4) $[3Ca_3(PO_4)_2 \cdot CaF_2]$

Official Ans. by NTA (1)

Sol. The $F^{\ominus}$ ions make the enamel on teeth much harder by converting hydroxyapatite,$[3(Ca_3(PO_4)_2].Ca(OH)_2]$, the enamel on the surface of the teeth into much harder fluroappatite. $[3Ca_3(PO_4)_2.CaF_2]$

11. Given below are two statements.

Statement I: In the titration between strong acid and weak base methyl orange is suitable as an indicator.

Statement II: For titration of acetic acid with NaOH phenolphthalein is not a suitable indicator.

In the light of the above statements, choose the **most appropriate** answer from the options given below:

(1) **Statement I** is false but **Statement II** is true

(2) **Statement I** is true but **Statement II** is false

(3) Both **Statement I** and **Statement II** are true

(4) Both **Statement I** and **Statement II** are false

Official Ans. by NTA (2)

Sol. Titration curve for strong acid and weak base initially a buffer of weak base and conjugate acid is :

Formed, thus pH falls slowly and after equivalence point, so the pH falls sharply so methyl arrange, having pH range of 3.2 to 4.4 will weak as indicator. So statement-I is correct.

Titration curve for weak acid and strong base (NaOH)

Initially weak acid will form a buffer so pH increases slowly but after equivalence point. it rises sharply covering range of phenolphthalein so it will be suitable indicator so statement-II is false.

12. Among the following compounds I-IV, which one forms a yellow precipitate on reacting sequentially with (i) NaOH (ii) dil. HNO_3 (iii) $AgNO_3$?

I II III IV

(1) II (2) IV

(3) I (4) III

Official Ans. by NTA (2)

Sol.

Other compounds halide can't be removed because corresponding C^+ is highly unstable.

13. Which one of the following methods is most suitable for preparing deionized water?

(1) Synthetic resin method

(2) Clark's method

(3) Calgon's method

(4) Permutit method

Official Ans. by NTA (1)

Sol. Pure demineralised (de-ionized) water free from all soluble mineral salts is obtained by passing water successively through a cation exchange (in the H^+ form) and an anion exchange (in the OH^- form) resins.

14. Given below are two statements.

Statement I: The choice of reducing agents for metals extraction can be made by using Ellingham diagram, a plot of ΔG vs temperature.

Statement II: The value of ΔS increases from left to right in Ellingham diagram.

In the light of the above statements, choose the **most appropriate** answer from the options given below:

(1) Both **Statement I** and **Statement II** are true

(2) **Statement I** is false but **Statement II** is true

(3) Both **Statement I** and **Statement II** are false

(4) **Statement I** is true but **Statement II** is false

Official Ans. by NTA (4)

Sol. Given statement-I is true as in a number of processes, one element is used to reduce the oxide of another metal. Any element will reduce the oxide of other metal which lie above it in the Ellingham diagram because the free energy change will become more negative.

Given statement-II is false as the value of ΔS is decreases from left to right in Ellingham diagram.

15. What are the products formed in sequence when excess of CO_2 is passed in slaked lime?

(1) $Ca(HCO_3)_2$, $CaCO_3$

(2) $CaCO_3$, $Ca(HCO_3)_2$

(3) CaO, $Ca(HCO_3)_2$

(4) CaO, $CaCO_3$

Official Ans. by NTA (2)

Sol. $Ca(OH)_2 + CO_2 \longrightarrow CaCO_3\downarrow + H_2O$

$CaCO_3\downarrow + CO_2 + H_2O \rightarrow Ca(HCO_3)_2$

16. Given below are two statements.

Statement I: According to Bohr's model of an atom, qualitatively the magnitude of velocity of electron increases with decrease in positive charges on the nucleus as there is no strong hold on the electron by the nucleus.

Statement II: According to Bohr's model of an atom, qualitatively the magnitude of velocity of electron increases with decrease in principal quantum number.

In the light of the above statements, choose the **most appropriate** answer from the options given below:

(1) Both **Statement I** and **Statement II** are false

(2) Both **Statement I** and **Statement II** are true

(3) **Statement I** is false but **Statement II** is true

(4) **Statement I** is true but **Statement II** is false

Official Ans. by NTA (3)

Sol. Velocity of electron in Bohr's atom is given by

$$V \propto \frac{Z}{n}$$

Z = atomic number of atom, corresponds to +ve charge so as Z increase velocity increases so statement-I is wrong.

and as 'n' decreases velocity increases so statement-II is correct.

17. The correct sequential addition of reagents in the preparation of 3-nitrobenzoic acid from benzene is:

(1) $Br_2/AlBr_3$, HNO_3/H_2SO_4, Mg/ether, CO_2, H_3O^+

(2) $Br_2/AlBr_3$, NaCN, H_3O^+, HNO_3/H_2SO_4

(3) $Br_2/AlBr_3$, HNO_3/H_2SO_4, NaCN, H_3O^+

(4) HNO_3/H_2SO_4, $Br_2/AlBr_3$, Mg/ether, CO_2, H_3O^+

Official Ans. by NTA (4)

Sol.

3–nitrobenzoic acid

18. Given below are two statements.

Statement I: Frenkel defects are vacancy as well as interstitial defects.

Statement II: Frenkel defect leads to colour in ionic solids due to presence of F-centres.

Choose the **most appropriate** answer for the statements from the options given below:

(1) **Statement I** is false but **Statement II** is true

(2) Both **Statement I** and **Statement II** are true

(3) **Statement I** is true but **Statement II** is false

(4) Both **Statement I** and **Statement II** are false

Official Ans. by NTA (3)

Sol. Theory based.

19. The **incorrect** statement is:

(1) Cl_2 is more reactive than ClF.

(2) F_2 is more reactive than ClF.

(3) On hydrolysis ClF froms HOCl and HF.

(4) F_2 is a stronger oxidizing agent than Cl_2 in aqueous solution

Official Ans. by NTA (1)

Sol. (i) Reactivity order :

$F_2 > ClF$ (inter halogen) $> Cl_2$

(ii) $ClF + H_2O \rightarrow HOCl + HF$

(iii) Oxidizing power in aqueous solution

$F_2 > Cl_2 > Br_2 > I_2$

20. Excess of isobutane on reaction with Br_2 in presence of light at 125°C gives which one of the following, as the major product?

(1) $CH_3-C(Br)(CH_3)-CH_2-Br$

(2) $CH_3-CH(CH_2Br)-CH_2Br$

(3) $CH_3-CH(CH_3)-CH_2Br$

(4) $CH_3-C(CH_3)_2-Br$

Official Ans. by NTA (4)

Sol.

SECTION-B

1. AB_3 is an interhalogen T-shaped molecule. The number of lone pairs of electrons on A is _____. (Integer answer)

Official Ans. by NTA (2)

Sol. T-shaped molecule means 3 sigma bond and 2 lone pairs of electron on central atom.

2. These are physical properties of an element

(A) Sublimation enthalpy

(B) Ionisation enthalpy

(C) Hydration enthalpy

(D) Electron gain enthalpy

The total number of above properties that affect the reduction potential is _____ (Integer answer)

Official Ans. by NTA (3)

Sol. Sublimation enthalpy, Ionisation enthalpy and hydration enthalpy affect the reduction potential.

3. Of the following four aqueous solutions, total number of those solutions whose freezing point is lower than that of 0.10 M C_2H_5OH is _____ (Integer answer)

(i) 0.10 M $Ba_3(PO_4)_2$

(ii) 0.10 M Na_2SO_4

(iii) 0.10 M KCl

(iv) 0.10 M Li_3PO_4

Official Ans. by NTA (4)

Sol. As 0.1 M C_2H_5OH is non-dissociative and rest all salt given are electrolyte so in each case effective molarity > 0.1 so each will have lower freezing point.

4. The OH^- concentration in a mixture of 5.0 mL of 0.0504 M NH_4Cl and 2 mL of 0.0210 M NH_3 solution is x × 10^{-6} M. The value of x is _____. (Nearest integer)

[Given $K_w = 1 \times 10^{-14}$ and $K_b = 1.8 \times 10^{-5}$]

Official Ans. by NTA (3)

Sol. $\left[NH_4^+\right] = 0.0504$ & $\left[NH_3\right] = 0.0210$

$$\text{So } K_b = \frac{[NH_4^+][HO^-]}{[NH_3]}$$

$$[HO^-] = \frac{K_b \times [NH_3]}{[NH_4^+]} = 1.8 \times 10^{-5} \times \frac{2}{5} \times \frac{210}{504}$$

$$= 3 \times 10^{-6}$$

5. The number of 4*f* electrons in the ground state electronic configuration of Gd^{2+} is _____.

[Atomic number of Gd = 64]

Official Ans. by NTA (7)

Sol. The electronic configuration of

$_{64}Gd$: [Xe] $4f^7\ 5d^1\ 6s^2$

So the electronic configuration of

$_{64}Gd^{2+}$: [Xe] $4f^7\ 5d^1\ 6s^0$

i.e. the number of 4f electrons in the ground state electronic configuration of Gd^{2+} is 7.

6. The ratio of number of water molecules in Mohr's salt and potash alum is _____ × 10^{-1}. (Integer answer)

Official Ans. by NTA (5)

Sol. **(5)**

Mohr's salt : $(NH_4)_2\ Fe(SO_4)_2.6H_2O$

The number of water molecules in Mohr's salt = 6

Potash alum : $KAl(SO_4)_2.12H_2O$

The number of water molecules in potash alum = 12

So ratio of number of water molecules in Mohr's salt and potash alum = $\frac{6}{12}$

$$= \frac{1}{2}$$

$$= 0.5$$

$$= 5 \times 10^{-1}$$

7. The following data was obtained for chemical reaction given below at 975 K.

$$2NO_{(g)} + 2H_{2(g)} \rightarrow N_{2(g)} + 2H_2O_{(g)}$$

	[NO] mol L^{-1}	$[H_2]$ mol L^{-1}	Rate mol $L^{-1}s^{-1}$
(A)	8×10^{-5}	8×10^{-5}	7×10^{-9}
(B)	24×10^{-5}	8×10^{-5}	2.1×10^{-8}
(C)	24×10^{-5}	32×10^{-5}	8.4×10^{-8}

The order of the reaction with respect to NO is _____. [Integer answer]

Official Ans. by NTA (1)

Sol. $7 \times 10^{-9} = K \times (8 \times 10^{-5})^x\ (8 \times 10^{-5})^y$ .....(1)

$2.1 \times 10^{-8} = K \times (24 \times 10^{-5})^x\ (8 \times 10^{-5})^y$ .....(2)

$$\frac{1}{3} = \left(\frac{1}{3}\right)^x \Rightarrow x = 1$$

8. The Born-Haber cycle for KCl is evaluated with the following data :

$\Delta_f H^\ominus$ for KCl $= -436.7\ kJ\ mol^{-1}$;

$\Delta_{sub} H^\ominus$ for K = 89.2 kJ mol^{-1};

$\Delta_{ionization} H^\ominus$ for K = 419.0 kJ mol^{-1}; $\Delta_{electron\ gain} H^\ominus$ for $Cl_{(g)}$ $= -348.6$ kJ mol^{-1}; $\Delta_{bond} H^\ominus$ for $Cl_2 = 243.0$ kJ mol^{-1}

The magnitude of lattice enthalpy of KCl in kJ mol^{-1} is _____ (Nearest integer)

Official Ans. by NTA (718)

Sol. $\Delta_f H^\ominus_{KCl} = \Delta_{sub} H^\ominus_{(K)} + \Delta_{ionization} H^\ominus_{(K)} + \frac{1}{2}\Delta_{bond} H^\ominus_{(Cl_2)}$

$$+ \Delta_{\text{electron gain}}H^{\ominus}_{(Cl)} + \Delta_{\text{lattice}}H^{\ominus}_{(KCl)}$$

$$\Rightarrow -436.7 = 89.2 + 419.0 + \frac{1}{2}(243.0) + \{-348.6\}$$

$$+ \Delta_{\text{lattice}}H^{\ominus}_{(KCl)}$$

$$\Rightarrow \Delta_{\text{lattice}}H^{\ominus}_{(KCl)} = -717.8 \text{ kJ mol}^{-1}$$

The magnitude of lattice enthalpy of KCl in kJ mol^{-1} is 718 (Nearest integer).

9. The total number of negative charge in the tetrapeptide, Gly-Glu-Asp-Tyr, at pH 12.5 will be _____. (Integer answer)

Official Ans. by NTA (4)

Sol.

Total negative charge produced = 4.

10. An aqueous KCl solution of density 1.20 g mL^{-1} has a molality of 3.30 mol kg^{-1}. The molarity of the solution in mol L^{-1} is _____ (Nearest integer)

[Molar mass of KCl = 74.5]

Official Ans. by NTA (3)

Sol. 1000 kg solvent has 3.3 moles of KCl

1000 kg solvent $\longrightarrow$ 3.3 × 74.5 gm KCl

$\longrightarrow$ 245.85

Weight of solution = 1245.85 gm

Volume of solution = $\frac{1245.85}{1.2}$ ml

So molarity = $\frac{3.3 \times 1.2}{1245.85} \times 1000 = 3.17$

FINAL JEE–MAIN EXAMINATION – AUGUST, 2021

(Held On Thursday 26th August, 2021) **TIME : 9 : 00 AM to 12 : 00 NOON**

MATHEMATICS — TEST PAPER WITH SOLUTION

SECTION-A

1. The sum of solutions of the equation $\frac{\cos x}{1+\sin x} = |\tan 2x|$, $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) - \left\{\frac{\pi}{4}, -\frac{\pi}{4}\right\}$ is :

(1) $-\frac{11\pi}{30}$ (2) $\frac{\pi}{10}$

(3) $-\frac{7\pi}{30}$ (4) $-\frac{\pi}{15}$

Official Ans. by NTA (1)

Sol. $\frac{\cos x}{1+\sin x} = |\tan 2x|$

$\Rightarrow \frac{\cos^2 x/2 - \sin^2 x/2}{(\cos x/2 + \sin x/2)} = |\tan 2x|$

$\Rightarrow \tan^2\left(\frac{\pi}{4} - \frac{x}{2}\right) = \tan^2 2x$

$\Rightarrow 2x = n\pi \pm \left(\frac{\pi}{4} - \frac{x}{2}\right)$

$\Rightarrow x = \frac{-3\pi}{10}, \frac{-\pi}{6}, \frac{\pi}{10}$

or sum $= \frac{-11\pi}{6}$.

2. The mean and standard deviation of 20 observations were calculated as 10 and 2.5 respectively. It was found that by mistake one data value was taken as 25 instead of 35. If α and $\sqrt{\beta}$ are the mean and standard deviation respectively for correct data, then (α, β) is :

(1) (11, 26) (2) (10.5, 25)

(3) (11, 25) (4) (10.5, 26)

Official Ans. by NTA (4)

Sol. Given :

Mean $(\bar{x}) = \frac{\Sigma x_i}{20} = 10$

or $\Sigma x_i = 200$ (incorrect)

or $200 - 25 + 35 = 210 = \Sigma x_i$ (Correct)

Now correct $\bar{x} = \frac{210}{20} = 10.5$

again given S.D = 2.5 (σ)

$\sigma^2 = \frac{\Sigma x_i^2}{20} - (10)^2 = (2.5)^2$

or $\Sigma x_i^2 = 2125$ (incorrect)

or $\Sigma x_i^2 = 2125 - 25^2 + 35^2$

$= 2725$ (Correct)

$\therefore$ correct $\sigma^2 = \frac{2725}{20} - (10.5)^2$

$\underline{\sigma}^2 = 26$

or $\sigma = 26$

$\therefore \underline{\alpha} = 10.5, \beta = 26$

3. On the ellipse $\frac{x^2}{8} + \frac{y^2}{4} = 1$ let P be a point in the second quadrant such that the tangent at P to the ellipse is perpendicular to the line $x + 2y = 0$. Let S and S' be the foci of the ellipse and e be its eccentricity. If A is the area of the triangle SPS' then, the value of $(5 - e^2)$. A is :

(1) 6 (2) 12

(3) 14 (4) 24

Official Ans. by NTA (1)

Sol.

Equation of tangent : $y = 2x + 6$

at P

$\therefore$ P(–8/3, 2/3)

$e = \frac{1}{\sqrt{2}}$

S & S' = (–2, 0) & (2, 0)

Area of Δ SPS' = $\frac{1}{2} \times 4 \times \frac{2}{3}$

$A = \frac{4}{3}$

$\therefore (5 - e^2)A = (5 - \frac{1}{2})\frac{4}{3} = 6$

4. Let y = y(x) be a solution curve of the differential equation $(y + 1)\tan^2 x\, dx + \tan x\, dy + y\, dx = 0$, $x \in \left(0, \frac{\pi}{2}\right)$. If $\lim_{x \to 0+} xy(x) = 1$, then the value of $y\left(\frac{\pi}{4}\right)$ is :

(1) $-\frac{\pi}{4}$ (2) $\frac{\pi}{4} - 1$

(3) $\frac{\pi}{4} + 1$ (4) $\frac{\pi}{4}$

Official Ans. by NTA (4)

Sol. $(y + 1)\tan^2 x\, dx + \tan x\, dy + y\, dx = 0$

or $\frac{dy}{dx} + \frac{\sec^2 x}{\tan x}.y = -\tan x$

$IF = e^{\int \frac{\sec^2 x}{\tan x} dx} = e^{\ln \tan x} = \tan x$

$\therefore y \tan x = -\int \tan^2 x dx$

or $y \tan x = -\tan x + x + C$

or $y = -1 + \frac{x}{\tan x} + \frac{C}{\tan x}$

or $\lim_{x \to 0} xy = -x + \frac{x^2}{\tan x} + \frac{Cx}{\tan x} = 1$

or C = 1

$y(x) = \cot x + x \cot x - 1$

$y\left(\frac{\pi}{4}\right) = \frac{\pi}{4}$

5. Let A and B be independent events such that P(A) = p, P(B) = 2p. The largest value of p, for which P (exactly one of A, B occurs) = $\frac{5}{9}$, is :

(1) $\frac{1}{3}$ (2) $\frac{2}{9}$

(3) $\frac{4}{9}$ (4) $\frac{5}{12}$

Official Ans. by NTA (4)

Sol. P(Exactly one of A or B)

$= P(A \cap \bar{B}) + P(\bar{A} \cap B) = \frac{5}{9}$

$= P(A)P(\bar{B}) + P(\bar{A})P(B) = \frac{5}{9}$

$\Rightarrow P(A)(1-P(B)) + (1-P(A))P(B) = \frac{5}{9}$

$\Rightarrow p(1 - 2p) + (1 - p)\, 2p = \frac{5}{9}$

$\Rightarrow 36p^2 - 27p + 5 = 0$

$\Rightarrow p = \frac{1}{3}$ or $\frac{5}{12}$

$p_{max} = \frac{5}{12}$

6. Let $\theta \in \left(0, \frac{\pi}{2}\right)$. If the system of linear equations

$(1 + \cos^2\theta)x + \sin^2\theta\, y + 4\sin 3\theta\, z = 0$

$\cos^2\theta\, x + (1 + \sin^2\theta)\, y + 4\sin 3\theta\, z = 0$

$\cos^2\theta\, x + \sin^2\theta\, y + (1 + 4\sin 3\theta)\, z = 0$

has a non–trivial solution, then the value of θ is :

(1) $\frac{4\pi}{9}$ (2) $\frac{7\pi}{18}$ (3) $\frac{\pi}{18}$ (4) $\frac{5\pi}{18}$

Official Ans. by NTA (2)

Sol. **Case–I**

$$\begin{vmatrix} 1+\cos^2\theta & \sin^2\theta & 4\sin 3\theta \\ \cos^2\theta & 1+\sin^2\theta & 4\sin 3\theta \\ \cos^2\theta & \sin^2\theta & 1+4\sin 3\theta \end{vmatrix} = 0$$

$C_1 \to C_1 + C_2$

$$\begin{vmatrix} 2 & \sin^2\theta & 4\sin 3\theta \\ 2 & 1+\sin^2\theta & 4\sin 3\theta \\ 1 & \sin^2\theta & 1+4\sin 3\theta \end{vmatrix} = 0$$

$R_1 \to R_1 - R_2,\ R_2 \to R_2 - R_3$

$$\begin{vmatrix} 0 & -1 & 0 \\ 1 & 1 & -1 \\ 1 & \sin^2\theta & 1+4\sin^3\theta \end{vmatrix} = 0$$

or $4 \sin 3\theta = -2$

$\sin 3\theta = -\frac{1}{2}$

$\theta = \frac{7\pi}{18}$

7. Let $f(x) = \cos\left(2\tan^{-1}\sin\left(\cot^{-1}\sqrt{\frac{1-x}{x}}\right)\right)$,

$0 < x < 1$. Then :

(1) $(1-x)^2 f'(x) - 2(f(x))^2 = 0$

(2) $(1+x)^2 f'(x) + 2(f(x))^2 = 0$

(3) $(1-x)^2 f'(x) + 2(f(x))^2 = 0$

(4) $(1+x)^2 f'(x) - 2(f(x))^2 = 0$

Official Ans. by NTA (3)

Sol. $f(x) = \cos\left(2\tan^{-1}\sin\left(\cot^{-1}\sqrt{\frac{1-x}{x}}\right)\right)$

$$\cot^{-1}\sqrt{\frac{1-x}{x}} = \sin^{-1}\sqrt{x}$$

or $f(x) = \cos(2\tan^{-1}\sqrt{x})$

$$= \cos\tan^{-1}\left(\frac{2\sqrt{x}}{1-x}\right)$$

$$f(x) = \frac{1-x}{1+x}$$

Now $f'(x) = \frac{-2}{(1+x)^2}$

or $f'(x)(1-x)^2 = -2\left(\frac{1-x}{1+x}\right)^2$

or $(1-x)^2 f'(x) + 2(f(x))^2 = 0$.

8. The sum of the series

$\frac{1}{x+1} + \frac{2}{x^2+1} + \frac{2^2}{x^4+1} + \ldots + \frac{2^{100}}{x^{2^{100}}+1}$ when $x = 2$ is :

(1) $1 + \frac{2^{101}}{4^{101}-1}$ (2) $1 + \frac{2^{100}}{4^{101}-1}$

(3) $1 - \frac{2^{100}}{4^{100}-1}$ (4) $1 - \frac{2^{101}}{4^{101}-1}$

Official Ans. by NTA (4)

Allen Ans. (BONUS)

Sol. $S = \frac{1}{x+1} + \frac{2}{x^2+1} + \frac{2^2}{x^4+1} + \ldots \frac{2^{100}}{x^{2^{100}}+1}$

$$S + \frac{1}{1-x} = \frac{1}{1-x} + \frac{1}{x+1} + \ldots = \frac{2}{1-x^2} + \frac{2}{1+x^2} + \ldots$$

$$S + \frac{1}{1-x} = \frac{2^{101}}{1-x^{2^{101}}}$$

Put $x = 2$

$$S = 1 - \frac{2^{101}}{2^{2^{101}}-1}$$

Not in option (BONUS)

9. If ${}^{20}C_r$ is the co-efficient of x^r in the expansion of $(1+x)^{20}$, then the value of $\sum_{r=0}^{20} r^2\, {}^{20}C_r$ is equal to :

(1) 420×2^{19} (2) 380×2^{19}

(3) 380×2^{18} (4) 420×2^{18}

Official Ans. by NTA (4)

Sol. $\sum_{r=0}^{20} r^2 . {}^{20}C_r$

$$\sum\left(4(r-1)+r\right).{}^{20}C_r$$

$$\sum r(r-1).\frac{20\times 19}{r(r-1)}.{}^{18}C_r + r.\frac{20}{r}.\sum {}^{19}C_{r-1}$$

$\Rightarrow 20 \times 19.2^{18} + 20.2^{19}$

$\Rightarrow 420 \times 2^{18}$

10. Out of all the patients in a hospital 89% are found to be suffering from heart ailment and 98% are suffering from lungs infection. If K% of them are suffering from both ailments, then K can not belong to the set :

(1) {80, 83, 86, 89} (2) {84, 86, 88, 90}

(3) {79, 81, 83, 85} (4) {84, 87, 90, 93}

Official Ans. by NTA (3)

Sol. $n(A \cup B) \geq n(A) + n(B) - n(A \cap B)$

$100 \geq 89 + 98 - n(A \cup B)$

$n(A \cup B) \geq 87$

$87 \leq n(A \cup B) \leq 89$

Option (3)

11. The equation $\arg\left(\frac{z-1}{z+1}\right)=\frac{\pi}{4}$ represents a circle with:

(1) centre at (0, –1) and radius $\sqrt{2}$

(2) centre at (0, 1) and radius $\sqrt{2}$

(3) centre at (0,0) and radius $\sqrt{2}$

(4) centre at (0,1) and radius 2

Official Ans. by NTA (2)

Sol.

In ΔOAC

$\sin\left(\frac{\pi}{4}\right)=\frac{1}{AC}$

$\Rightarrow AC=\sqrt{2}$

Also, $\tan\frac{\pi}{4}=\frac{OA}{OC}=\frac{1}{OC}$

$\Rightarrow OC = 1$

$\therefore$ centre (0, 1) ; Radius = $\sqrt{2}$

12. Let $\vec{a}=\hat{i}+\hat{j}+\hat{k}$ and $\vec{b}=\hat{j}-\hat{k}$. If $\vec{c}$ is a vector such that $\vec{a}\times\vec{c}=\vec{b}$ and $\vec{a}.\vec{c}=3$, then $\vec{a}.(\vec{b}\times\vec{c})$ is equal to :

(1) –2 (2) –6 (3) 6 (4) 2

Official Ans. by NTA (1)

Sol. $|\vec{a}|=\sqrt{3};\ \vec{a}.\vec{c}=3;\ \vec{a}\times\vec{b}=-2\hat{i}+\hat{j}+\hat{k},\ \vec{a}\times\vec{c}=\vec{b}$

Cross with $\vec{a}$.

$\vec{a}\times(\vec{a}\times\vec{c})=\vec{a}\times\vec{b}$

$\Rightarrow (\vec{a}.\vec{c})\vec{a}-a^2\vec{c}=\vec{a}\times\vec{b}$

$\Rightarrow 3\vec{a}-3\vec{c}=-2\hat{i}+\hat{j}+\hat{k}$

$\Rightarrow 3\hat{i}+3\hat{j}+3\hat{k}-3\vec{c}=-2\hat{i}+\hat{j}+\hat{k}$

$\Rightarrow \vec{c}=\frac{5\hat{i}}{3}+\frac{2\hat{j}}{3}+\frac{2\hat{k}}{3}$

$\therefore\ \vec{a}.(\vec{b}\times\vec{c})=(\vec{a}\times\vec{b}).\vec{c}=\frac{-10}{3}+\frac{2}{3}+\frac{2}{3}=-2$

13. If a line along a chord of the circle $4x^2 + 4y^2 + 120x + 675 = 0$, passes through the point (–30, 0) and is tangent to the parabola $y^2 = 30x$, then the length of this chord is :

(1) 5 (2) 7 (3) $5\sqrt{3}$ (4) $3\sqrt{5}$

Official Ans. by NTA (4)

Sol. Equation of tangent to $y^2 = 30x$

$y = mx + \frac{30}{4m}$

Pass thru (–30, 0) : $a = -30m + \frac{30}{4m} \Rightarrow m^2 = 1/4$

$\Rightarrow m=\frac{1}{2}$ or $m=-\frac{1}{2}$

At $m=\frac{1}{2}$: $y=\frac{x}{2}+15 \Rightarrow x-2y+30=0$

$P=\frac{15}{\sqrt{5}}$

$\ell_{AB}=2\sqrt{R^2-P^2}=2\sqrt{\frac{225}{4}-\frac{225}{5}}$

$\Rightarrow \ell_{AB}=30.\sqrt{\frac{1}{20}}=\frac{15}{\sqrt{5}}=3\sqrt{5}$

14. The value of $\int_{-1/\sqrt{2}}^{1/\sqrt{2}}\left(\left(\frac{x+1}{x-1}\right)^2+\left(\frac{x-1}{x+1}\right)^2-2\right)^{1/2}dx$ is:

(1) $\log_e 4$ (2) $\log_e 16$

(3) $2\log_e 16$ (4) $4\log_e(3+2\sqrt{2})$

Official Ans. by NTA (2)

Sol. $I=\int_{-1/\sqrt{2}}^{1/\sqrt{2}}\left(\left(\frac{x+1}{x-1}-\frac{x-1}{x+1}\right)^2\right)^{1/2}dx$

$I=\int_{-1/\sqrt{2}}^{1/\sqrt{2}}\left|\frac{4x}{x^2-1}\right|dx \Rightarrow I=2.4\int_0^{1/\sqrt{2}}\left|\frac{x}{x^2-1}\right|dx$

$\Rightarrow I=-4\int_0^{1/\sqrt{2}}\frac{2x}{x^2-1}dx \Rightarrow I=-4\ln\left|x^2-1\right|_0^{1/\sqrt{2}}$

$\Rightarrow I = 4\ln 2 \Rightarrow I = \ln 16$

15. A plane P contains the line

$x + 2y + 3z + 1 = 0 = x - y - z - 6$,

and is perpendicular to the plane $-2x + y + z + 8 = 0$.

Then which of the following points lies on P ?

(1) (–1, 1, 2) (2) (0, 1, 1)

(3) (1, 0, 1) (4) (2, –1, 1)

Official Ans. by NTA (2)

Sol. Equation of plane P can be assumed as

$P : x + 2y + 3z + 1 + \lambda(x - y - z - 6) = 0$

$\Rightarrow P : (1 + \lambda)x + (2 - \lambda)y + (3 - \lambda)z + 1 - 6\lambda = 0$

$\Rightarrow \vec{n}_1 = (1+\lambda)\hat{i} + (2-\lambda)\hat{j} + (3-\lambda)\hat{k}$

$\therefore \vec{n}_1.\vec{n}_2 = 0$

$\Rightarrow 2(1 + \lambda) - (2 - \lambda) - (3 - \lambda) = 0$

$\Rightarrow 2 + 2\lambda - 2 + \lambda - 3 + \lambda = 0 \Rightarrow \lambda = \frac{3}{4}$

$\Rightarrow P : \frac{7x}{4} + \frac{5}{4}y + \frac{9z}{4} - \frac{14}{4} = 0$

$\Rightarrow 7x + 5y + 9z = 14$

(0, 1, 1) lies on P

16. If $A = \begin{pmatrix} \frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} \\ \frac{-2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \end{pmatrix}$, $B = \begin{pmatrix} 1 & 0 \\ i & 1 \end{pmatrix}$, $i = \sqrt{-1}$, and

$Q = A^TBA$, then the inverse of the matrix $A\,Q^{2021}\,A^T$ is equal to :

(1) $\begin{pmatrix} \frac{1}{\sqrt{5}} & -2021 \\ 2021 & \frac{1}{\sqrt{5}} \end{pmatrix}$ (2) $\begin{pmatrix} 1 & 0 \\ -2021i & 1 \end{pmatrix}$

(3) $\begin{pmatrix} 1 & 0 \\ 2021i & 1 \end{pmatrix}$ (4) $\begin{pmatrix} 1 & -2021i \\ 0 & 1 \end{pmatrix}$

Official Ans. by NTA (2)

Sol. $AA^T = \begin{pmatrix} \frac{1}{5} & \frac{2}{\sqrt{5}} \\ \frac{-2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{5}} & \frac{-2}{\sqrt{5}} \\ \frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \end{pmatrix}$

$AA^T = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$

$Q^2 = A^TBA\,A^TBA = A^TBIBA$

$\Rightarrow Q^2 = A^T B^2 A$

$Q^3 = A^TB^2AA^TBA \Rightarrow Q^3 = A^TB^3A$

Similarly : $Q^{2021} = A^TB^{2021}A$ ..... (1)

Now $B^2 = \begin{pmatrix} 1 & 0 \\ i & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ i & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 2i & 1 \end{pmatrix}$

$B^3 = \begin{pmatrix} 1 & 0 \\ 2i & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ i & 1 \end{pmatrix} \Rightarrow B^3 = \begin{pmatrix} 1 & 0 \\ 3i & 1 \end{pmatrix}$

Similarly $B^{2021} = \begin{pmatrix} 1 & 0 \\ 2021i & 1 \end{pmatrix}$

$\therefore AQ^{2021}A^T = AA^T B^{2021} AA^T = IB^{2021}I$

$\Rightarrow AQ^{2021}A^T = B^{2021} = \begin{pmatrix} 1 & 0 \\ 2021i & 1 \end{pmatrix}$

$\therefore (AQ^{2021}A^T)^{-1} = \begin{pmatrix} 1 & 0 \\ 2021i & 1 \end{pmatrix}^{-1} = \begin{pmatrix} 1 & 0 \\ -2021i & 1 \end{pmatrix}$

17. If the sum of an infinite GP a, ar, ar^2, ar^3,... is 15 and the sum of the squares of its each term is 150, then the sum of ar^2, ar^4, ar^6, ... is :

(1) $\frac{5}{2}$ (2) $\frac{1}{2}$

(3) $\frac{25}{2}$ (4) $\frac{9}{2}$

Official Ans. by NTA (2)

Sol. Sum of infinite terms :

$\frac{a}{1-r} = 15$ .....(i)

Series formed by square of terms:

$a^2, a^2r^2, a^2r^4, a^2r^6$ .....

Sum = $\frac{a^2}{1-r^2} = 150$

$\Rightarrow \frac{a}{1-r}.\frac{a}{1+r} = 150 \Rightarrow 15.\frac{a}{1+r} = 150$

$\Rightarrow \frac{a}{1+r} = 10$ ..... (ii)

by (i) and (ii) a = 12 ; $r = \frac{1}{5}$

Now series : ar^2, ar^4, ar^6

$$\text{Sum} = \frac{ar^2}{1-r^2} = \frac{12.\left(1/25\right)}{1-1/25} = \frac{1}{2}$$

18. The value of $\lim_{n\to\infty}\frac{1}{n}\sum_{r=0}^{2n-1}\frac{n^2}{n^2+4r^2}$ is:

(1) $\frac{1}{2}\tan^{-1}(2)$ (2) $\frac{1}{2}\tan^{-1}(4)$

(3) $\tan^{-1}(4)$ (4) $\frac{1}{4}\tan^{-1}(4)$

Official Ans. by NTA (2)

Sol. $L = \lim_{n\to\infty}\frac{1}{n}.\sum_{r=0}^{2n-1}\frac{1}{1+4\left(\frac{r}{n}\right)^2}$

$$\Rightarrow L = \int_0^2 \frac{1}{1+4x^2}dx$$

$$\Rightarrow L = \frac{1}{2}\tan^{-1}(2x)\Big|_0^2 \Rightarrow L = \frac{1}{2}\tan^{-1}4$$

19. Let ABC be a triangle with A(–3, 1) and $\angle ACB = \theta$, $0 < \theta < \frac{\pi}{2}$. If the equation of the median through B is 2x + y – 3 = 0 and the equation of angle bisector of C is 7x – 4y – 1 = 0, then tanθ is equal to:

(1) $\frac{1}{2}$ (2) $\frac{3}{4}$

(3) $\frac{4}{3}$ (4) 2

Official Ans. by NTA (3)

Sol.

$\therefore$ $M\left(\frac{a-3}{2}, \frac{b+1}{2}\right)$ lies on 2x + y – 3 = 0

$\Rightarrow$ 2a + b = 11 ..... (i)

$\because$ C lies on 7x – 4y = 1

$\Rightarrow$ 7a – 4b = 1 .....(ii)

$\therefore$ by (i) and (ii) : a = 3, b = 5

$\Rightarrow$ C(3, 5)

$\therefore$ $m_{AC} = 2/3$

Also, $m_{CD} = 7/4$

$$\Rightarrow \tan\frac{\theta}{2} = \left|\frac{\frac{2}{3}-\frac{4}{4}}{1+\frac{14}{12}}\right| \Rightarrow \tan\frac{\theta}{2} = \frac{1}{2}$$

$$\Rightarrow \tan\theta = \frac{2.\frac{1}{2}}{1-\frac{1}{4}} = \frac{4}{3}$$

20. If the truth value of the Boolean expression $\left((p\vee q)\wedge(q\to r)\wedge(\sim r)\right)\to(p\wedge q)$ is false, then the truth values of the statements p, q, r respectively can be:

(1) T F T (2) F F T

(3) T F F (4) F T F

Official Ans. by NTA (3)

Sol.

p	q	r	$\underbrace{p\vee q}_{a}$	$\underbrace{q\to r}_{b}$	$a\wedge b$	$\sim r$	$\underbrace{a\wedge b\wedge(\sim r)}_{c}$	$\underbrace{p\wedge q}_{d}$	$c\to d$
T	F	T	T	T	T	F	F	F	T
F	F	T	F	T	F	F	F	F	T
T	F	F	T	T	T	T	T	F	F
F	T	F	T	F	F	T	F	F	T

SECTION-B

1. Let $z = \frac{1-i\sqrt{3}}{2}$, $i = \sqrt{-1}$. Then the value of

$$21+\left(z+\frac{1}{z}\right)^3+\left(z^2+\frac{1}{z^2}\right)^3+\left(z^3+\frac{1}{z^3}\right)^3+\ldots+\left(z^{21}+\frac{1}{z^{21}}\right)^3$$

is _____.

Official Ans. by NTA (13)

Sol. $Z = \frac{1-\sqrt{3}i}{2} = e^{-i\frac{\pi}{3}}$

$$z^r + \frac{1}{z^r} = 2\cos\left(-\frac{\pi}{3}\right)r = 2\cos\frac{r\pi}{3}$$

$$\Rightarrow\ 21 + \sum_{r=1}^{21}\left(z^r + \frac{1}{z^r}\right)^3 = 8\left(\cos^3\frac{r\pi}{3}\right) = 2\left(\cos r\pi + 3\cos\frac{r\pi}{3}\right)$$

$$\Rightarrow\ 21 + \left(z + \frac{1}{2}\right)^3 + \left(z^2 + \frac{1}{z^2}\right)^3 + \ldots\ldots\left(z^{21} + \frac{1}{z^{21}}\right)^3$$

$$= 21 + \sum_{r=1}^{21}\left(z^r + \frac{1}{z^r}\right)^3$$

$$= 21 + \sum_{r=1}^{21}\left(2\cos r\pi + 6\cos\frac{r\pi}{3}\right)$$

$= 21 - 2 - 6$

$= 13$

2. The sum of all integral values of k ($k \neq 0$) for which the equation $\frac{2}{x-1} - \frac{1}{x-2} = \frac{2}{k}$ in x has no real roots, is _____.

Official Ans. by NTA (66)

Sol. $\frac{2}{x-1} - \frac{1}{x-2} = \frac{2}{k}$

$x \in R - \{1, 2\}$

$\Rightarrow\ k(2x - 4 - x + 1) = 2(x^2 - 3x + 2)$

$\Rightarrow\ k(x - 3) = 2(x^2 - 3x + 2)$

for $x \neq 3$, $\quad k = 2\left(x - 3 + \frac{2}{x-3} + 3\right)$

$$x - 3 + \frac{2}{x-3} \geq 2\sqrt{2},\ \forall x > 3$$

$$\&\ x - 3 + \frac{2}{x-3} \leq -2\sqrt{2},\ \forall x < -3$$

$$\Rightarrow 2\left(x - 3 + \frac{2}{x-3} + 3\right) \in \left(-\infty, 6 - 4\sqrt{2}\right] \cup \left[6 + 4\sqrt{2}, \infty\right)$$

for no real roots

$k \in (6 - 4\sqrt{2}, 6 + 4\sqrt{2}) - \{0\}$

Integral $\quad k \in \{1, 2 \ldots.. 11\}$

Sum of k = 66

3. Let the line L be the projection of the line

$$\frac{x-1}{2} = \frac{y-3}{1} = \frac{z-4}{2}$$

in the plane $x - 2y - z = 3$. If d is the distance of the point (0, 0, 6) from L, then d^2 is equal to _____.

Official Ans. by NTA (26)

Sol. $L_1 : \frac{x-1}{2} = \frac{y-3}{1} = \frac{z-4}{2}$

for foot of $\perp r$ of (1, 3, 4) on $x - 2y - z - 3 = 0$

$(1 + t) - 2(3 - 2t) - (4 - t) - 3 = 0$

$\Rightarrow\ t = 2$

So foot of $\perp r \triangleq (3, -1, 2)$

& point of intersection of L_1 with plane

is (–11, –3, –8)

dr's of L is <14, 2, 10>

$\cong$ <7, 1, 5>

$$d = AB\sin\theta = \left|\frac{\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & -4 \\ 7 & 1 & 5 \end{vmatrix}}{\sqrt{7^2 + 1^2 + 5^2}}\right|$$

$$\Rightarrow\ d^2 = \frac{1^2 + (43)^2 + (10)^2}{49 + 1 + 25} = 26$$

4. If ${}^1P_1 + 2\cdot{}^2P_2 + 3\cdot{}^3P_3 + \ldots + 15\cdot{}^{15}P_{15} = {}^qP_r - s,\ 0 \leq s \leq 1$, then ${}^{q+s}C_{r-s}$ is equal to _____.

Official Ans. by NTA (136)

Sol. ${}^{1}P_1 + 2\cdot{}^{2}P_2 + 3\cdot{}^{3}P_3 + \ldots + 15\cdot{}^{15}P_{15}$

$= 1! + 2.2! + 3.3! + \ldots.15 \times 15!$

$= \sum_{r=1}^{15}(r+1-1)r!$

$= \sum_{r=1}^{15}(r+1)!-(r)!$

$= 16! - 1$

$= {}^{16}P_{16} - 1$

$\Rightarrow q = r = 16, s = 1$

${}^{q+s}C_{r-s} = {}^{17}C_{15} = 136$

5. A wire of length 36 m is cut into two pieces, one of the pieces is bent to form a square and the other is bent to form a circle. If the sum of the areas of the two figures is minimum, and the circumference of the circle is k (meter), then $\left(\frac{4}{\pi}+1\right)k$ is equal to _____.

Official Ans. by NTA (36)

Sol. Let x + y = 36

x is perimeter of square and y is perimeter of circle

side of square = x/4

radius of circle = $\frac{y}{2\pi}$

Sum Areas = $\left(\frac{x}{4}\right)^2 + \pi\left(\frac{y}{2\pi}\right)^2$

$= \frac{x^2}{16} + \frac{(36-x)^2}{4\pi}$

For min Area :

$x = \frac{144}{\pi+4}$

$\Rightarrow$ Radius = $y = 36 - \frac{144}{\pi+4}$

$\Rightarrow k = \frac{36\pi}{\pi+4}$

$\left(\frac{4}{\pi}+1\right)k = 36$

6. The area of the region

$S = \{(x, y) : 3x^2 \le 4y \le 6x + 24\}$ is _____.

Official Ans. by NTA (27)

Sol.

For A & B

$3x^2 = 6x + 24 \Rightarrow x^2 - 2x - 8 = 0$

$\Rightarrow x = -2, 4$

Area = $\int_{-2}^{4}\left(\frac{3}{2}x+6-\frac{3}{4}x^2\right)dx$

$= \left[\frac{3x^2}{4}+6x-\frac{x^3}{4}\right]_{-2}^{4} = 27$

7. The locus of a point, which moves such that the sum of squares of its distances from the points (0, 0), (1, 0), (0, 1) (1, 1) is 18 units, is a circle of diameter d. Then d^2 is equal to _____.

Official Ans. by NTA (16)

Sol. Let P(x, y)

$x^2 + y^2 + x^2 + (y-1)^2 + (x-1)^2 + y^2 + (x-1)^2 + (y-1)^2;$

$\Rightarrow 4(x^2 + y^2) - 4y - 4x = 14$

$\Rightarrow x^2 + y^2 - x - y - \frac{7}{2} = 0$

$d = 2\sqrt{\frac{1}{4}+\frac{1}{4}+\frac{7}{2}}$

$\Rightarrow d^2 = 16$

8. If y = y(x) is an implicit function of x such that $\log_e(x + y) = 4xy$, then $\frac{d^2y}{dx^2}$ at x = 0 is equal to _____.

Official Ans. by NTA (40)

Sol. $\ln(x + y) = 4xy$ (At $x = 0$, $y = 1$)

$x + y = e^{4xy}$

$$\Rightarrow 1+\frac{dy}{dx}=e^{4xy}\left(4x\frac{dy}{dx}+4y\right)$$

At $x = 0$ $\boxed{\frac{dy}{dx}=3}$

$$\frac{d^2y}{dx^2}=e^{4xy}\left(4x\frac{dy}{dx}+4y\right)^2+e^{4xy}\left(4x\frac{d^2y}{dx^2}+4y\right)$$

At $x = 0$, $\frac{d^2y}{dx^2} = e^0(4)^2 + e^0(24)$

$$\Rightarrow \frac{d^2y}{dx^2} = 40$$

9. The number of three-digit even numbers, formed by the digits 0, 1, 3, 4, 6, 7 if the repetition of digits is not allowed, is _____.

Official Ans. by NTA (52)

Sol. (i) When '0' is at unit place

↑	↑	0
5	4	

Number of numbers = 20

(ii) When 4 or 6 are at unit place

OX		4,6
4 × 4		↑ 2

Number of numbers = 32

So number of numbers = 52

10. Let $a, b \in \mathbf{R}$, $b \neq 0$, Define a function

$$f(x)=\begin{cases} a\sin\frac{\pi}{2}(x-1), & \text{for } x\le 0 \\ \frac{\tan 2x-\sin 2x}{bx^3}, & \text{for } x>0. \end{cases}$$

If f is continuous at $x = 0$, then $10 - ab$ is equal to _____.

Official Ans. by NTA (14)

Sol. $$f(x)=\begin{cases} a\sin\frac{\pi}{2}(x-1), & x\le 0 \\ \frac{\tan 2x-\sin 2x}{bx^3}, & x>0 \end{cases}$$

For continuity at '0'

$$\lim_{x\to 0^+} f(x)=f(0)$$

$$\Rightarrow \lim_{x\to 0^+}\frac{\tan 2x-\sin 2x}{bx^3}=-a$$

$$\Rightarrow \lim_{x\to 0^+}\frac{\frac{8x^3}{3}+\frac{8x^3}{3!}}{bx^3}=-a$$

$$\Rightarrow 8\left(\frac{1}{3}+\frac{1}{3!}\right)=-ab$$

$\Rightarrow 4 = -ab$

$\Rightarrow 10 - ab = 14$

FINAL JEE–MAIN EXAMINATION – MARCH, 2021

(Held On Tuesday 16th March, 2021) TIME : 3 : 00 PM to 6 : 00 PM

PHYSICS — TEST PAPER WITH ANSWER & SOLUTION

SECTION-A

1. The following logic gate is equivalent to :

(1) NOR Gate (2) OR Gate

(3) AND Gate (4) NAND Gate

Official Ans. by NTA (1)

Sol. Truth table for the given logic gate :

A	B	Y
0	0	1
0	1	1
1	0	1
1	1	0

The truth table is similar to that of a NOR gate.

2. A large block of wood of mass M = 5.99 kg is hanging from two long massless cords. A bullet of mass m = 10g is fired into the block and gets embedded in it. The (block + bullet) then swing upwards, their centre of mass rising a vertical distance h = 9.8 cm before the (block + bullet) pendulum comes momentarily to rest at the end of its arc. The speed of the bullet just before collision is : (Take g = 9.8 ms^{-2})

(1) 841.4 m/s (2) 811.4 m/s

(3) 831.4 m/s (4) 821.4 m/s

Official Ans. by NTA (3)

Sol. From energy conservation,

[after bullet gets embedded till the system comes momentarily at rest]

$$(M + m)g\,h = \frac{1}{2}(M+m)v_1^2$$

[v_1 is velocity after collision]

$$\therefore\ v_1 = \sqrt{2gh}$$

Applying momentum conservation, (just before and just after collision)

$mv = (M + m)v_1$

$$v = \left(\frac{M+m}{m}\right)v_1 = \frac{6}{10\times10^{-3}}\times\sqrt{2\times9.8\times9.8\times10^{-2}}$$

≈ 831.55 m / s

3. A charge Q is moving $\vec{dl}$ distance in the magnetic field $\vec{B}$. Find the value of work done by $\vec{B}$.

(1) 1 (2) Infinite

(3) Zero (4) –1

Official Ans. by NTA (3)

Sol. Since force on a point charge by magnetic field is always perpendicular to $\vec{v}\left[\vec{F} = q\vec{V}\times\vec{B}\right]$

$\therefore$ Work by magnetic force on the point charge is zero.

4. What will be the nature of flow of water from a circular tap, when its flow rate increased from 0.18 L/min to 0.48 L/min ? The radius of the tap and viscosity of water are 0.5 cm and 10^{-3} Pa s, respectively.

(Density of water : 10^3 kg/m^3)

(1) Unsteady to steady flow

(2) Remains steady flow

(3) Remains turbulent flow

(4) Steady flow to unsteady flow

Official Ans. by NTA (4)

Sol. The nature of flow is determined by Reynolds Number.

$$R_e = \frac{\rho v D}{\eta}$$

$\left[\begin{array}{ll} \rho \rightarrow \text{density of fluid} \quad ; & \eta \rightarrow \text{coefficient of viscosity} \\ v \rightarrow \text{velocity of flow} & \\ D \rightarrow \text{Diameter of pipe} & \end{array}\right]$

From NCERT

If $R_e < 1000 \rightarrow$ flow is steady

$1000 < R_e < 2000 \rightarrow$ flow becomes unsteady

$R_e > 2000 \rightarrow$ flow is turbulent

$$R_{e\,initial} = 10^3 \times \frac{0.18 \times 10^{-3}}{\pi \times (0.5 \times 10^{-2})^2 \times 60} \times \frac{1 \times 10^{-2}}{10^{-3}}$$

$= 382.16$

$$R_{e\,final} = 10^3 \times \frac{0.48 \times 10^{-3}}{\pi \times (0.5 \times 10^{-2})^2 \times 60} \times \frac{1 \times 10^{-2}}{10^{-3}}$$

$= 1019.09$

5. A mosquito is moving with a velocity $\vec{v} = 0.5t^2\,\hat{i} + 3t\,\hat{j} + 9\hat{k}$ m/s and accelerating in uniform conditions. What will be the direction of mosquito after 2s ?

(1) $\tan^{-1}\left(\frac{2}{3}\right)$ from x-axis

(2) $\tan^{-1}\left(\frac{2}{3}\right)$ from y-axis

(3) $\tan^{-1}\left(\frac{5}{2}\right)$ from y-axis

(4) $\tan^{-1}\left(\frac{5}{2}\right)$ from x-axis

Official Ans. by NTA (2)

Official Ans. by ALLEN (Bonus)

Sol. Given :

$$\vec{v} = 0.5t^2\,\hat{i} + 3t\,\hat{j} + 9\hat{k}$$

$$\vec{v}_{at\,t=2} = 2\hat{i} + 6\hat{j} + 9\hat{k}$$

$\therefore$ Angle made by direction of motion of mosquito will be,

$\cos^{-1}\frac{2}{11}$ (from x-axis) = $\tan^{-1}\frac{\sqrt{117}}{2}$

$\cos^{-1}\frac{6}{11}$ (from y-axis) = $\tan^{-1}\frac{\sqrt{85}}{6}$

$\cos^{-1}\frac{9}{11}$ (from z-axis) = $\tan^{-1}\frac{\sqrt{40}}{9}$

None of the option is matching.

Hence this question should be bonus.

6. Find out the surface charge density at the intersection of point x = 3 m plane and x-axis, in the region of uniform line charge of 8 nC/m lying along the z-axis in free space.

(1) 0.424 nC m^{-2} (2) 47.88 C/m

(3) 0.07 nC m^{-2} (4) 4.0 nC m^{-2}

Official Ans. by NTA (1)

Sol. $\frac{2K\lambda}{r} = \frac{\sigma}{\varepsilon_0}$ (x = 3m)

$$\sigma = 0.424 \times 10^{-9}\,\frac{C}{m^2}$$

7. The de-Broglie wavelength associated with an electron and a proton were calculated by accelerating them through same potential of 100 V. What should nearly be the ratio of their wavelengths ? ($m_P = 1.00727$ u, $m_e = 0.00055$u)

(1) 1860 : 1 (2) $(1860)^2$: 1

(3) 41.4 : 1 (4) 43 : 1

Official Ans. by NTA (4)

Sol. $\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2mK}} = \frac{h}{\sqrt{2mqV}}$

$$\frac{\lambda_1}{\lambda_2} = \sqrt{\frac{m_2}{m_1}}$$

$$\frac{\lambda_e}{\lambda_P} = \sqrt{\frac{m_P}{m_e}} = \sqrt{1831.4} = 42.79$$

8. For the given circuit, comment on the type of transformer used :

(1) Auxilliary transformer
(2) Auto transformer
(3) Step-up transformer
(4) Step down transformer

Official Ans. by NTA (3)

Sol. $V_S = \frac{P}{i} = \frac{60}{0.11} = 545.45$

$V_P = 220$

$V_S > V_P$

$\Rightarrow$ Step up transformer

9. The half-life of Au^{198} is 2.7 days. The activity of 1.50 mg of Au^{198} if its atomic weight is 198 g mol^{-1} is, ($N_A = 6 \times 10^{23}$/mol)

(1) 240 Ci (2) 357 Ci
(3) 535 Ci (4) 252 Ci

Official Ans. by NTA (2)

Sol. $A = \lambda N$

$N = nN_A$ $\left(t_{1/2} = \frac{\ln 2}{\lambda}\right)$

$N = \left(\frac{1.5 \times 10^{-3}}{198}\right) N_A$

$A = \left(\frac{\ln 2}{t_{1/2}}\right) N$

1 Curie = 3.7×10^{10} Bq

A = 365 Bq

10. Calculate the value of mean free path (λ) for oxygen molecules at temperature 27°C and pressure 1.01×10^5 Pa. Assume the molecular diameter 0.3 nm and the gas is ideal. ($k = 1.38 \times 10^{-23}$ JK^{-1})

(1) 58 nm (2) 32 nm
(3) 86 nm (4) 102 nm

Official Ans. by NTA (4)

Sol. $\lambda = \frac{RT}{\sqrt{2}\pi d^2 N_A P}$

λ =102 nm

11. The refractive index of a converging lens is 1.4. What will be the focal length of this lens if it is placed in a medium of same refractive index ? (Assume the radii of curvature of the faces of lens are R_1 and R_2 respectively)

(1) 1 (2) Infinite
(3) $\frac{R_1R_2}{R_1 - R_2}$ (4) Zero

Official Ans. by NTA (2)

Sol. $\frac{1}{F} = \left[\frac{\mu_L}{\mu_S} - 1\right]\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$

If $\mu_L = \mu_S \Rightarrow \frac{1}{F} = 0 \Rightarrow F = \infty$

12. In order to determine the Young's Modulus of a wire of radius 0.2 cm (measured using a scale of least count = 0.001 cm) and length 1m (measured using a scale of least count = 1 mm), a weight of mass 1kg (measured using a scale of least count = 1g) was hanged to get the elongation of 0.5 cm (measured using a scale of least count 0.001 cm). What will be the fractional error in the value of Young's Modulus determined by this experiment ?

(1) 0.14%
(2) 0.9%
(3) 9%
(4) 1.4%

Official Ans. by NTA (4)

Sol. $Y = \frac{\text{Stress}}{\text{Strain}} = \frac{FL}{Al} = \frac{mg.L}{\pi R^2.\ell}$

$\frac{\Delta Y}{Y} = \frac{\Delta m}{m} + \frac{\Delta L}{L} + 2.\frac{\Delta R}{R} + \frac{\Delta \ell}{\ell}$

$\frac{\Delta Y}{Y} \times 100 = 100\left[\frac{1}{1000} + \frac{1}{1000} + 2\left(\frac{0.001}{0.2}\right) + \frac{0.001}{0.5}\right]$

$= \frac{1}{10} + \frac{1}{10} + 1 + \frac{1}{5} = \frac{14}{10} = 1.4\%$

13. A bimetallic strip consists of metals A and B. It is mounted rigidly as shown. The metal A has higher coefficient of expansion compared to that of metal B. When the bimetallic strip is placed in a cold both, it will :

(1) Bend towards the right
(2) Not bend but shrink
(3) Neither bend nor shrink
(4) Bend towards the left

Official Ans. by NTA (4)

Sol. $\alpha_A > \alpha_B$

Length of both strips will decrease

$\Delta L_A > \Delta L_B$

14. A resistor develops 500 J of thermal energy in 20s when a current of 1.5 A is passed through it. If the current is increased from 1.5 A to 3A, what will be the energy developed in 20 s.

(1) 1500 J (2) 1000 J
(3) 500 J (4) 2000 J

Official Ans. by NTA (4)

Sol. $500 = (1.5)^2 \times R \times 20$

$E = (3)^2 \times R \times 20$

$E = 2000$ J

15. **Statement I :** A cyclist is moving on an unbanked road with a speed of 7 kmh^{-1} and takes a sharp circular turn along a path of radius of 2m without reducing the speed. The static friction coefficient is 0.2. The cyclist will not slip and pass the curve ($g = 9.8$ m/s^2)

Statement II : If the road is banked at an angle of 45°, cyclist can cross the curve of 2m radius with the speed of 18.5 kmh^{-1} without slipping.

In the light of the above statements, choose the correct answer from the options given below.

(1) Statement I is incorrect and statement II is correct
(2) Statement I is correct and statement II is incorrect
(3) Both statement I and statement II are false
(4) Both statement I and statement II are true

Official Ans. by NTA (4)

Sol. **Statement I :**

$$v_{max} = \sqrt{\mu R g} = \sqrt{(0.2) \times 2 \times 9.8}$$

$v_{max} = 1.97$ m/s

7 km/h = 1.944 m/s

Speed is lower than v_{max}, hence it can take safe turn.

Statement II

$$v_{max} = \sqrt{Rg\left[\frac{\tan\theta + \mu}{1 - \mu\tan\theta}\right]}$$

$$= \sqrt{2 \times 9.8\left[\frac{1+0.2}{1-0.2}\right]} = 5.42 \text{ m/s}$$

18.5 km/h = 5.14 m/s

Speed is lower than v_{max}, hence it can take safe turn.

16. Two identical antennas mounted on identical towers are separated from each other by a distance of 45 km. What should nearly be the minimum height of receiving antenna to receive the signals in line of sight ? (Assume radius of earth is 6400 km)

(1) 19.77 m (2) 39.55 m
(3) 79.1 m (4) 158.2 m

Official Ans. by NTA (2)

Sol. $D = 2\sqrt{2Rh}$

$$h = \frac{D^2}{8R} = \frac{45^2}{8 \times 6400} \text{km} \cong 39.55 \text{m}$$

17. The magnetic field in a region is given by $\vec{B} = B_0\left(\frac{x}{a}\right)\hat{k}$. A square loop of side d is placed with its edges along the x and y axes. The loop is moved with a constant velocity $\vec{v} = v_0\hat{i}$. The emf induced in the loop is :

(1) $\frac{B_0 v_0^2 d}{2a}$ (2) $\frac{B_0 v_0 d}{2a}$

(3) $\frac{B_0 v_0 d^2}{a}$ (4) $\frac{B_0 v_0 d^2}{2a}$

Official Ans. by NTA (3)

Sol. $E_1 = \frac{B_0(x+d)}{a} v_0 d$

$E_2 = \frac{B_0(x)}{a} v_0 d$

$E_{net} = E_1 - E_2$

$E_{net} = \frac{B_0 v_0 d^2}{a}$

18. Amplitude of a mass-spring system, which is executing simple harmonic motion decreases with time. If mass = 500g, Decay constant = 20 g/s then how much time is required for the amplitude of the system to drop to half of its initial value ? (ln 2 = 0.693)

(1) 34.65 s (2) 17.32 s

(3) 0.034 s (4) 15.01 s

Official Ans. by NTA (1)

Sol. $A = A_0 e^{-\gamma t} = A_0 e^{-\frac{bt}{2m}}$

$\frac{A_0}{2} = A_0 e^{-\frac{bt}{2m}}$

$\frac{bt}{2m} = \ln 2$

$t = \frac{2m}{b}\ln 2 = \frac{2 \times 500 \times 0.693}{20}$

t = 34.65 second.

19. Calculate the time interval between 33% decay and 67% decay if half-life of a substance is 20 minutes.

(1) 60 minutes (2) 20 minutes

(3) 40 minutes (4) 13 minutes

Official Ans. by NTA (2)

Sol. $N_1 = N_0 e^{-\lambda t_1}$

$\frac{N_1}{N_0} = e^{-\lambda t_1}$

$0.67 = e^{-\lambda t_1}$

$\ln(0.67) = -\lambda t_1$

$N_2 = N_0 e^{-\lambda t_2}$

$\frac{N_2}{N_0} = e^{-\lambda t_2}$

$0.33 = e^{-\lambda t_2}$

$\ln(0.33) = -\lambda t_2$

$\ln(0.67) - \ln(0.33) = \lambda t_1 - \lambda t_2$

$\lambda(t_1 - t_2) = \ln\left(\frac{0.67}{0.33}\right)$

$\lambda(t_1 - t_2) \cong \ln 2$

$t_1 - t_2 \simeq \frac{\ln 2}{\lambda} = t_{1/2}$

Half life = $t_{1/2}$ = 20 minutes.

20. Red light differs from blue light as they have :

(1) Different frequencies and different wavelengths

(2) Different frequencies and same wavelengths

(3) Same frequencies and same wavelengths

(4) Same frequencies and different wavelengths

Official Ans. by NTA (1)

Sol. Red light and blue light have different wavelength and different frequency.

SECTION-B

1. The energy dissipated by a resistor is 10 mJ in 1s when an electric current of 2 mA flows through it. The resistance is _____ Ω. (Round off to the Nearest Integer)

Official Ans. by NTA (2500)

Sol. **Ans. (2500)**

$Q = i^2 RT$

$R = \frac{Q}{i^2 t} = \frac{10 \times 10^{-3}}{4 \times 10^{-6} \times 1} = 2500\ \Omega$

2. In a parallel plate capacitor set up, the plate area of capacitor is 2 m^2 and the plates are separated by 1m. If the space between the plates are filled with a dielectric material of thickness 0.5 m and area 2m^2 (see fig.) the capacitance of the set-up will be _____ ε_0.

(Dielectric constant of the material = 3.2)

(Round off to the Nearest Integer)

Official Ans. by NTA (3)

Sol. **Ans. (3)**

$$C = \frac{\varepsilon_0 A}{\frac{d}{2K}+\frac{d}{2}} = \frac{2\varepsilon_0 A}{\frac{d}{K}+d}$$

$$= \frac{2\times 2\varepsilon_0}{\frac{1}{3.2}+1} = \frac{4\times 3.2}{4.2}\varepsilon_0$$

$$= 3.04\ \varepsilon_0$$

3. A force $\vec{F} = 4\hat{i}+3\hat{j}+4\hat{k}$ is applied on an intersection point of x = 2 plane and x-axis. The magnitude of torque of this force about a point (2, 3, 4) is _____. (Round off to the Nearest Integer)

Official Ans. by NTA (20)

Sol. **Ans. (20)**

$$\vec{\tau} = \vec{r}\times\vec{F}$$

$$\vec{r} = (2\hat{i}) - (2\hat{i}+3\hat{j}+4\hat{k}) = -3\hat{j}-4\hat{k}$$

$$\& \ \vec{F} = 4\hat{i}+3\hat{j}+4\hat{k}$$

$$\vec{\tau} = \vec{r}\times\vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & -3 & -4 \\ 4 & 3 & 4 \end{vmatrix}$$

$$= \hat{i}(-12+12) - \hat{j}(0+16) + \hat{k}(0+12)$$

$$= -16\hat{i}+12\hat{k}$$

$$\therefore \ |\vec{\tau}| = \sqrt{16^2+12^2} = 20$$

4. If one wants to remove all the mass of the earth to infinity in order to break it up completely. The amount of energy that needs to be supplied will be $\frac{x}{5}\frac{GM^2}{R}$ where x is ____ (Round off to the Nearest Integer)

(M is the mass of earth, R is the radius of earth, G is the gravitational constant)

Official Ans. by NTA (3)

Sol. **Ans. (3)**

Energy given = $U_f - U_i$

$$= 0 - \left(-\frac{3}{5}\frac{GM^2}{R}\right)$$

$$= \frac{3}{5}\frac{GM^2}{R}$$

x = 3

5. A deviation of 2° is produced in the yellow ray when prism of crown and flint glass are achromatically combined. Taking dispersive powers of crown and flint glass are 0.02 and 0.03 respectively and refractive index for yellow light for these glasses are 1.5 and 1.6 respectively. The refracting angles for crown glass prism will be _____° (in degree)

(Round off to the Nearest Integer)

Official Ans. by NTA (12)

Sol. **Ans. (12)**

$\omega_1 = 0.02$; $\mu_1 = 1.5$; $\omega_2 = 0.03$; $\mu_2 = 1.6$

Achromatic combination

$\therefore\ \theta_{net} = 0$

$\theta_1 - \theta_2 = 0$

$\theta_1 = \theta_2$

$\omega_1\delta_1 = \omega_2\delta_2$

& $\delta_{net} = \delta_1 - \delta_2 = 2°$

$$\delta_1 - \frac{\omega_1\delta_1}{\omega_2} = 2°$$

$$\delta_1\left(1 - \frac{\omega_1}{\omega_2}\right) = 2°$$

$$\delta_1\left(1 - \frac{2}{3}\right) = 2°$$

$\delta_1 = 6°$

$\delta_1 = (\mu_1 - 1)\ A_1$

$6° = (1.5 - 1)\ A_1$

$A_1 = 12°$

6. A body of mass 2kg moves under a force of $(2\hat{i} + 3\hat{j} + 5\hat{k})$ N. It starts from rest and was at the origin initially. After 4s, its new coordinates are (8, b, 20). The value of b is _____. (Round off to the Nearest Integer)

Official Ans. by NTA (12)

Sol. **Ans. (12)**

$$\vec{a} = \frac{\vec{F}}{m} = \frac{2\hat{i} + 3\hat{j} + 5\hat{k}}{2}$$

$= \hat{i} + 1.5\hat{j} + 2.5\hat{k}$

$$\vec{\tau} = \vec{u}t + \frac{1}{2}\vec{a}t^2$$

$= 0 + \frac{1}{2}(\hat{i} + 1.5\hat{j} + 2.5\hat{k})\ (16)$

$= 8\hat{i} + 12\hat{j} + 20\hat{k}$

b = 12

7. A swimmer can swim with velocity of 12 km/h in still water. Water flowing in a river has velocity 6 km/h. The direction with respect to the direction of flow of river water he should swim in order to reach the point on the other bank just opposite to his starting point is _____°. (Round off to the Nearest Integer) (find the angle in degree)

Official Ans. by NTA (120)

Sol. **Ans. (12)**

$12\sin\theta = v_r$

$\sin\theta = \frac{1}{2}$

$\theta = 30°$

$\therefore\ \alpha = 120°$

8. A closed organ pipe of length L and an open organ pipe contain gases of densities ρ_1 and ρ_2 respectively. The compressibility of gases are equal in both the pipes. Both the pipes are vibrating in their first overtone with same frequency. The length of the open pipe is $\frac{x}{3}L\sqrt{\frac{\rho_1}{\rho_2}}$ where x is _____. (Round off to the Nearest Integer)

Official Ans. by NTA (4)

Sol. **Ans. (4)**

$f_c = f_0$

$$\frac{3V_C}{4L} = \frac{2V_0}{2L'}$$

$$\frac{3V_C}{4L} = \frac{V_0}{L'}$$

$$L' = \frac{4L}{3}\frac{V_0}{V_C} = \frac{4L}{3}\sqrt{\frac{B.\rho_1}{\rho_2.B}}$$ (B is bulk modulus)

$$= \frac{4L}{3}\sqrt{\frac{\rho_1}{\rho_2}}$$

x = 4

9. A solid disc of radius 'a' and mass 'm' rolls down without slipping on an inclined plane making an angle θ with the horizontal. The acceleration of the disc will be $\frac{2}{b}g\sin\theta$ where b is _____. (Round off to the Nearest Integer)

(g = acceleration due to gravity)

(θ = angle as shown in figure)

Official Ans. by NTA (3)

Sol. Ans. (3)

$$a = \frac{g\sin\theta}{1+\frac{I}{mR^2}} = \frac{g\sin\theta}{1+\frac{1}{2}} = \frac{2}{3}g\sin\theta$$

b = 3

10. For an ideal heat engine, the temperature of the source is 127°C. In order to have 60% efficiency the temperature of the sink should be _____°C. (Round off to the Nearest Integer)

Official Ans. by NTA (113)

Official Ans. by ALLEN (–113)

Sol. Ans. (–113)

$$n = 0.60 = 1 = \frac{T_L}{T_H}$$

$$\frac{T_L}{T_H} = 0.4 \Rightarrow T_L = 0.4 \times 400$$

$$= 160 \text{ K}$$

$$= -113°C$$

FINAL JEE–MAIN EXAMINATION – MARCH, 2021

(Held On Tuesday 16th March, 2021) TIME : 3 : 00 PM to 6 : 00 PM

CHEMISTRY — TEST PAPER WITH ANSWER & SOLUTION

SECTION-A

1. The green house gas/es is (are) :

(A) Carbon dioxide
(B) Oxygen
(C) Water vapour
(D) Methane

Choose the most appropriate answer from the options given below :

(1) (A) and (C) only
(2) (A) only
(3) (A), (C) and (D) only
(4) (A) and (B) only

Official Ans. by NTA (3)

Sol. The green house gases are CO_2, $H_2O_{(vapour)}$ & CH_4.

2.

In the above reaction, the reagent "A" is :

(1) $NaBH_4$, H_3O^+
(2) $LiAlH_4$
(3) Alkaline $KMnO_4$, H^+
(4) HCl, Zn-Hg

Official Ans. by NTA (3)

Sol.

;

3. Which of the following reduction reaction CANNOT be carried out with coke ?

(1) $Al_2O_3 \rightarrow Al$
(2) $ZnO \rightarrow Zn$
(3) $Fe_2O_3 \rightarrow Fe$
(4) $Cu_2O \rightarrow Cu$

Official Ans. by NTA (1)

Sol. Reduction of $Al_2O_3 \rightarrow Al$ is carried out by electrolytic reduction of its fused salts.
ZnO, Fe_2O_3 & Cu_2O can be reduce by carbon.

4. Identify the elements X and Y using the ionisation energy values given below :

	Ionization energy (kJ/mol)	
	1st	2nd
X	495	4563
Y	731	1450

(1) X = Na ; Y = Mg
(2) X = Mg ; Y = F
(3) X = Mg ; Y = Na
(4) X = F ; Y = Mg

Official Ans. by NTA (1)

Sol. Na $\rightarrow$ [Ne] $3s^1$ IE_1 is very low but IE_2 is very high due to stable noble gas configuration of Na^+.
Mg $\rightarrow$ [Ne] $3s^2$ IE_1 & IE_2 $\rightarrow$ Low
IE_3 is very high.

5.

Identify the reagent(s) 'A' and condition(s) for the reaction :

(1) A = HCl; Anhydrous $AlCl_3$
(2) A = HCl, $ZnCl_2$
(3) A = Cl_2; UV light
(4) A = Cl_2; dark, Anhydrous $AlCl_3$

Official Ans. by NTA (3)

Sol.

For substitution at allylic position in the given compound, the reagent used is Cl_2/uv light. The reaction is free radical halogenation.

6. The secondary structure of protein is stabilised by:
(1) Peptide bond
(2) glycosidic bond
(3) Hydrogen bonding
(4) van der Waals forces
Official Ans. by NTA (3)

Sol. The secondary structure of protein includes two type :
(a) α-Helix (b) β-pleated sheet
In α-Helix structure, the poly peptide chain is coil around due to presence of Intramolecular H-Bonding.

7. Fex_2 and Fey_3 are known when x and y are :
(1) x = F, Cl, Br, I and y = F, Cl, Br
(2) x = F, Cl, Br and y = F, Cl, Br, I
(3) x = Cl, Br, I and y = F, Cl, Br, I
(4) x = F, Cl, Br, I and y = F, Cl, Br, I
Official Ans. by NTA (1)

Sol. $\underset{\text{(Unstable)}}{2FeI_3} \longrightarrow \underset{\text{(Stable)}}{2FeI_2} + I_2$
Due to strong reducing nature of I^-
$2Fe^{3+} + 2I^- \longrightarrow 2Fe^{2+} + I_2$
remaining halides of Fe^{2+} & Fe^{3+} are stable.

8. Which of the following polymer is used in the manufacture of wood laminates ?
(1) *cis*-poly isoprene
(2) Melamine formaldehyde resin
(3) Urea formaldehyde resin
(4) Phenol and formaldehyde resin
Official Ans. by NTA (3)

Sol. Urea –HCHO resin is used in manufacture of wood laminates.

9. **Statement I :** Sodium hydride can be used as an oxidising agent.
Statement II : The lone pair of electrons on nitrogen in pyridine makes it basic.
Choose the CORRECT answer from the options given below :
(1) Both statement I and statement II are false
(2) Statement I is true but statement II is false
(3) Statement I is false but statement II is true
(4) Both statement I and statement II are true
Official Ans. by NTA (3)

Sol. (1) NaH (sodium Hydride) is used as a reducing reagent.
(2)

In pyridine, due to free electron on N atom, it is basic in nature.
Hence statement I is false & II is true.

10. The INCORRECT statement regarding the structure of C_{60} is :
(1) The six-membered rings are fused to both six and five-membered rings.
(2) Each carbon atom forms three sigma bonds.
(3) The five-membered rings are fused only to six-membered rings.
(4) It contains 12 six-membered rings and 24 five-membered rings.
Official Ans. by NTA (4)

Sol. Structure of C_{60}

It contain 20 hexagons (20) and 12 pentagons (12) so option 4 is incorrect.

11. The correct statements about H_2O_2 are :
(A) used in the treatment of effluents.
(B) used as both oxidising and reducing agents.
(C) the two hydroxyl groups lie in the same plane.
(D) miscible with water.
Choose the correct answer from the options given below :
(1) (A), (B), (C) and (D)
(2) (A), (B) and (D) only
(3) (B), (C) and (D) only
(4) (A), (C) and (D) only
Official Ans. by NTA (2)

Sol.

Structure of H_2O_2
(Open book type) → Non planar

H_2O_2 is used in the treatment of effluents.

$H_2O_2 \xrightarrow{R.A.} O_2$
$H_2O_2 \xrightarrow{O.A.} H_2O$
act as both O.A & R.A.

H_2O_2 is miscible in water due to hydrogen bonding.

12. Ammonolysis of Alkyl halides followed by the treatment with NaOH solution can be used to prepare primary, secondary and tertiary amines. The purpose of NaOH in the reaction is :

(1) to remove basic impurities

(2) to activate NH_3 used in the reaction

(3) to remove acidic impurities

(4) to increase the reactivity of alkyl halide

Official Ans. by NTA (3)

Sol.

So the purpose of NaOH in the above reactions in to remove acidic impurities.

13. An unsaturated hydrocarbon X on ozonolysis gives A. Compound A when warmed with ammonical silver nitrate forms a bright silver mirror along the sides of the test tube. The unsaturated hydrocarbon X is :

(1) $CH_3–C(CH_3) = C(CH_3)–CH_3$

(2) $CH_3–C(CH_3)=$

(3) $HC≡C–CH_2–CH_3$

(4) $CH_3–C≡C–CH_3$

Official Ans. by NTA (3)

Sol. (X) Unsaturated Hydrocarbon $\xrightarrow{\text{Ozonolysis}}$ (A) $\xrightarrow[\text{(Tollen's regent)}]{\text{Ammonical } AgNO_3}$ Ag↓ silver mirror

As (A) compound given positive tollen's test hence it may consist –CHO (aldehyde group). or it can be HCOOH

So for the given option :

(3) $CH_3–CH_2–C≡CH \xrightarrow{\text{Ozonolysis}} CH_3CH_2COOH + \underset{\substack{\text{(A)}\\ \text{+ve tollen's test}}}{HCOOH}$

and for other compounds (options):

(1)

(Does not show tollen's Test)

(2)

(Both do not show tollen's test)

(3) $CH_3–C≡C–CH_3 \xrightarrow{\text{Ozonolysis}} 2CH_3–COOH$ (Does not show tollen's test)

14. Which of the following is least basic ?

(1) $(CH_3CO)\ddot{N}HC_2H_5$

(2) $(C_2H_5)_3\ddot{N}$

(3) $(CH_3CO)_2\ddot{N}H$

(4) $(C_2H_5)_2\ddot{N}H$

Official Ans. by NTA (3)

Sol. For the given compounds :

(1) $CH_3–C(=O)–\ddot{N}H–C_2H_5$; L.P. on Nitrogen is delocalised.

(2) $CH_3CH_2–\ddot{N}(CH_2CH_3)–CH_2CH_3$; L.P. on Nitrogen is localised.

(3) $CH_3–C(=O)–\ddot{N}H–C(=O)–CH_3$; L.P. on Nitrogen is delocalised due to conjugation with both –C(=O)–

(Hence least basic)

(4) $CH_3–CH_2–\ddot{N}H–CH_2–CH_3$; L.P. on Nitrogen is localised.

15. The characteristics of elements X, Y and Z with atomic numbers, respectively, 33, 53 and 83 are :

(1) X and Y are metalloids and Z is a metal.

(2) X is a metalloid, Y is a non-metal and Z is a metal.

(3) X, Y and Z are metals.

(4) X and Z are non-metals and Y is a metalloid

Official Ans. by NTA (2)

Sol. X = $_{33}$As $\rightarrow$ Metalloid

Y = $_{53}$I $\rightarrow$ Nonmetal

Z = $_{83}$Bi $\rightarrow$ Metal

16. Match List-I with List-II

List-I Test/Reagents/Observation(s)	List-II Species detected
(a) Lassaigne's Test	(i) Carbon
(b) Cu(II) oxide	(ii) Sulphur
(c) Silver nitrate	(iii) N, S, P, and halogen
(d) The sodium fusion extract gives black precipitate with acetic acid and lead acetate	(iv) Halogen Specifically

The correct match is :

(1) (a)-(iii), (b)-(i), (c)-(ii), (d)-(iv)

(2) (a)-(i), (b)-(iv), (c)-(iii), (d)-(ii)

(3) (a)-(iii), (b)-(i), (c)-(iv), (d)-(ii)

(4) (a)-(i), (b)-(ii), (c)-(iv), (d)-(iii)

Official Ans. by NTA (3)

Sol. Match list :-

(a) Lassaigne's Test	(iii) N, S, P and Halogen
(b) Cu(II) Oxide	(i) Carbon
(c) $AgNO_3$	(iv) Halogen specifically.
(d) Sodium fusion extract given black precipitate with acetic acid and lead acetate ($CH_3COOH/(CH_3COO)_2Pb$)	(ii) Sulphur

Option-(a)-(iii) ; (b)-(i) ; (c)-(iv) ; (d)-(ii)

17. The INCORRECT statements below regarding colloidal solutions is :

(1) A colloidal solution shows colligative properties.

(2) An ordinary filter paper can stop the flow of colloidal particles.

(3) The flocculating power of Al^{3+} is more than that of Na^+.

(4) A colloidal solution shows Brownian motion of colloidal particles.

Official Ans. by NTA (2)

Sol. * Colloidel solution exhibits colligative properties

* An ordinary filter can not stop the flow of colloidal particles.

* Flocculating power increases with increase the opposite charge of electrolyte.

* Colloidal particles show brownian motion.

18. Arrange the following metal complex/ compounds in the increasing order of spin only magnetic moment. Presume all the three, high spin system.

(Atomic numbers Ce = 58, Gd = 64 and Eu = 63.)

(a) $(NH_4)_2[Ce(NO_3)_6]$ (b) $Gd(NO_3)_3$ and

(c) $Eu(NO_3)_3$

Answer is :

(1) (b) < (a) < (c)

(2) (c) < (a) < (b)

(3) (a) < (b) < (c)

(4) (a) < (c) < (b)

Official Ans. by NTA (4)

Sol. (a) $_{58}Ce \rightarrow [Xe]4f^2\ 5d^0\ 6s^2$

In complex $Ce^{4+} \rightarrow [Xe]\ 4f^0\ 5d^0\ 6s^0$

there is no unpaired electron so $\mu_m = 0$

(b) $_{64}Gd^{3+} \rightarrow [Xe]4f^7\ 5d^0\ 6s^0$

contain seven unpaired electrons so,

$\mu_m = \sqrt{7(7+2)} = \sqrt{63}$ B.M.

(c) $_{63}Eu^{3+} \rightarrow [_{54}Xe]4f^6\ 5d^0\ 6s^0$

contain six unpaired electron

so, $\mu_m = \sqrt{6(6+2)} = \sqrt{48}$ B.M.

Hence, order of spin only magnetic movement

$\boxed{b > c > a}$

19. The exact volumes of 1 M NaOH solution required to neutralise 50 mL of 1 M H_3PO_3 solution and 100 mL of 2 M H_3PO_2 solution, respectively, are :

(1) 100 mL and 100 mL
(2) 100 mL and 50 mL
(3) 100 mL and 200 mL
(4) 50 mL and 50 mL

Official Ans. by NTA (3)

Sol. $H_3PO_3 + 2NaOH \rightarrow Na_2HPO_3 + 2H_2O$

50 ml 1M

1M V = ?

$$\Rightarrow \frac{n_{NaoH}}{n_{H_3PO_3}} = \frac{2}{1}$$

$$\Rightarrow \frac{1\times V}{50\times 1} = \frac{2}{1} \Rightarrow \boxed{V_{NaOH} = 100\,ml}$$

$H_3PO_2 + 2NaOH \rightarrow NaH_2PO_3 + H_2O$

100 ml 1M

2M V = ?

$$\Rightarrow \frac{n_{NaoH}}{n_{H_3PO_3}} = \frac{1}{1} \quad \Rightarrow \frac{1\times V}{2\times 100} = \frac{1}{1} \Rightarrow \boxed{V_{NaOH} = 200\,ml}$$

20.

The structure of X is :

(1)

(2)

(3)

(4)

Official Ans. by NTA (4)

Sol.

SECTION-B

1. Ga (atomic mass 70 u) crystallizes in a hexagonal close packed structure. The total number of voids in 0.581 g of Ga is _____ $\times 10^{21}$. (Round off to the Nearest Integer).

Official Ans. by NTA (15)

Sol. HCP structure : Per atom, there will be one octahedral void (OV) and two tetrahedral voids (TV).

Therefore total three voids per atom are present in HCP structure.

$\rightarrow$ therefore total no of atoms of Ga will be-

$$= \frac{\text{Mass}}{\text{Molar Mass}} \times N_A = \frac{0.581g}{70g/mol} \times 6.023\times 10^{23}$$

$\rightarrow$ Now, total Number of voids = 3 $\times$ total no. of atoms

$$= 3\times\frac{0.581}{70}\times 6.023\times 10^{23} = 14.99 \times 10^{21}$$

$$\simeq 15\times 10^{21}$$

2. A 5.0 m mol dm^{-3} aqueous solution of KCl has a conductance of 0.55 mS when measured in a cell constant 1.3 cm^{-1}. The molar conductivity of this solution is _____ $mSm^2\ mol^{-1}$. (Round off to the Nearest Integer)

Official Ans. by NTA (143)

Official Ans. by ALLEN (14)

Sol. Given conc^n of KCl = $\frac{m.mol}{L}$

: Conductance (G) = 0.55 mS

: Cell constant $\left(\frac{\ell}{A}\right) = 1.3\ cm^{-1}$

To Calculate : Molar conductivity (λ_m) of sol.

$\rightarrow$ Since $\boxed{\lambda_m = \frac{1}{1000}\times\frac{k}{m}}$ .....(1)

$\rightarrow$ Molarity = $5 \times 10^{-3}\ \frac{mol}{L}$

$$\rightarrow \text{Conductivity} = G\times\left(\frac{\ell}{A}\right) = 0.55\ mS \times \frac{1.3}{\frac{1}{100}}\ m^{-1}$$

$= 55 \times 1.3 \quad mSm^{-1}$

eq^n (1) $\lambda_m = \frac{1}{1000}\times\frac{55\times 1.3}{\left(\frac{5}{1000}\right)}\ \frac{mSm^2}{mol}$

$$\Rightarrow \lambda_m = 14.3\frac{mSm^2}{mol}$$

3. A and B decompose via first order kinetics with half-lives 54.0 min and 18.0 min respectively. Starting from an equimolar non reactive mixture of A and B, the time taken for the concentration of A to become 16 times that of B is _____ min. (Round off to the Nearest Integer).

Official Ans. by NTA (108)

Sol. Given t_{2} = 54 min $\quad T_{1/2}$ = 18 min

A $\quad$ B

t = 0 'x' M $\quad$ t = 0 'x' M

$\Rightarrow$ To calculate : $[A_t] = 16 \times [B_t]$ ....(1) time = ?

$\Rightarrow$ For I order kinetic : $\boxed{[A_t] = \frac{A_0}{(2)^n}}$

n $\rightarrow$ no of Half lives

$\Rightarrow$ Now from the relation (1)

$[A_t] = 16 \times [B_t]$

$\Rightarrow \frac{x}{(2)^{n_1}} = \frac{x}{(2)^{n_2}} \times 16 \quad \Rightarrow \quad (2)^{n_2} = (2)^{n_1} \times (2)^4$

$\Rightarrow n_2 = n_1 + 4 \quad \Rightarrow \quad \frac{t}{(t_{1/2})_2} = \frac{t}{(t_{1/2})_1} + 4$

$\Rightarrow t\left(\frac{1}{18} - \frac{1}{54}\right) = 4 \Rightarrow t = \frac{4 \times 18 \times 54}{36}$

$\Rightarrow \boxed{t = 108 \text{min}}$

4. In Duma's method of estimation of nitrogen, 0.1840 g of an organic compound gave 30 mL of nitrogen collected at 287 K and 758 mm of Hg pressure. The percentage composition of nitrogen in the compound is _____. (Round off to the Nearest Integer).

[Given : Aqueous tension at 287 K = 14 mm of Hg]

Official Ans. by NTA (19)

Sol. In Duma's method of estimation of Nitrogen. 0.1840 gm of organic compound gave 30 mL of nitrogen which is collected at 287 K & 758 mm of Hg.

Given ;

Aqueous tension at 287 K = 14 mm of Hg.

Hence actual pressure = (758 – 14)

= 744 mm of Hg.

Volume of nitrogen at STP = $\frac{273 \times 744 \times 30}{287 \times 760}$

V = 27.935 mL

$\because$ 22400 mL of N_2 at STP weighs = 28 gm.

$\therefore$ 27.94 mL of N_2 at STP weighs =

$\left(\frac{28}{22400} \times 27.94\right)$gm

= 0.0349 gm

Hence % of Nitrogen = $\left(\frac{0.0349}{0.1840} \times 100\right)$

= 18.97 %

Rond off. Answer = 19 %

5. The number of orbitals with n = 5, m_l = + 2 is _____. (Round off to the Nearest Integer).

Official Ans. by NTA (3)

Sol. For, n = 5

ℓ = (0, 1, 2, 3, 4)

If ℓ = 0, m = 0

ℓ = 1, m = {–1, 0, +1}

ℓ = 2, m = {–2, –1, 0, +1, **+2**}

ℓ = 3, m = {–3, –2, –1, 0, +1, **+2**, +3}

ℓ = 4, m = {–4, –3, –2, –1, 0, +1, **+2**, +3, +4}

5d, 5f and 5g subshell contain one-one orbital having m_ℓ = +2

6. At 363 K, the vapour pressure of A is 21 kPa and that of B is 18 kPa. One mole of A and 2 moles of B are mixed. Assuming that this solution is ideal, the vapour pressure of the mixture is _____ kPa. (Round of to the Nearest Integer).

Official Ans. by NTA (19)

Sol. Given $P_A^0 = 21\text{kPa} \quad \Rightarrow \quad P_B^0 = 18\text{kPa}$

$\rightarrow$ An Ideal solution is prepared by mixing 1 mol A and 2 mol B.

$\rightarrow X_A = \frac{1}{3}$ and $X_B = \frac{2}{3}$

$\rightarrow$ Acc to Raoult's low

$P_T = X_A P_A^0 + X_B P_B^0$

$\Rightarrow \quad P_T = \left(\frac{1}{3} \times 21\right) + \left(\frac{2}{3} \times 18\right)$

$\Rightarrow \quad P_T$ = 7 +12 = 19 KPa

7. Sulphurous acid (H_2SO_3) has $Ka_1 = 1.7 \times 10^{-2}$ and $Ka_2 = 6.4 \times 10^{-8}$. The pH of 0.588 M H_2SO_3 is _____. (Round off to the Nearest Integer)

Official Ans. by NTA (1)

Sol. H_2SO_3 [Dibasic acid]

c = 0.588 M

⇒ pH of solution Þ due to First dissociation only since $K_{a,} >> Ka_2$

⇒ First dissociation of H_2SO_3

$$H_2SO_3(aq) \rightleftharpoons H^{\oplus}(aq) + HSO_3^-(aq) : ka_1 = 1.7 \times 10^{-2}$$

t = 0 C

t C–x x x

$$\Rightarrow \quad Ka_1 = \frac{1.7}{100} = \frac{[H^{\oplus}][HSO_3^-]}{[H_2SO_3]}$$

$$\Rightarrow \frac{1.7}{100} = \frac{x^2}{(0.58 - x)}$$

⇒ $1.7 \times 0.588 - 1.7x = 100\ x^2$

⇒ $100x^2 + 1.7x - 1 = 0$

$$\Rightarrow \quad [H^{\oplus}] = x = \frac{-1.7 + \sqrt{(1.7)^2 + 4 \times 100 \times 1}}{2 \times 100} = 0.09186$$

Therefore pH of sol. is : pH = – log $[H^{\oplus}]$

⇒ pH = –log (0.09186) = 1.036 ≃ 1

8. When 35 mL of 0.15 M lead nitrate solution is mixed with 20 mL of 0.12 M chromic sulphate solution, _____ × 10^{-5} moles of lead sulphate precipitate out. (Round off to the Nearest Integer).

Official Ans. by NTA (525)

Sol. $3\ Pb\ (NO_3)_2 + Cr_2\ (SO_4)_3 \rightarrow 3PbSO_4 + 2Cr(NO_3)_3$

35 ml 20 ml

0.15 M 0.12 M

= 5.25 m.mol = 2.4 m.mol 5.25 m.mol

= 5.25×10^{-3} mol

therefore moles of $PbSO_4$ formed = 5.25×10^{-3}

= 525×10^{-5}

9. At 25°C, 50 g of iron reacts with HCl to form $FeCl_2$. The evolved hydrogen gas expands against a constant pressure of 1 bar. The work done by the gas during this expansion is _____ J.

(Round off to the Nearest Integer)

[Given : R = 8.314 J mol^{-1} K^{-1}. Assume, hydrogen is an ideal gas]

[Atomic mass off Fe is 55.85 u]

Official Ans. by NTA (2218)

Sol. T = 298 K, R = 8.314 $\frac{J}{mol K}$

→ Chemical reaction is

$Fe + 2HCl \rightarrow FeCl_2 + H_2(g)$

50g P = 1 bar

$= \frac{50}{55.85}$ mol

$\frac{50}{55.85}$ mol

→ Work done for 1 mol gas

$= -P_{ext} \times \Delta V$

$= \Delta ng\ RT$

$= -1 \times 8.314 \times 298$ J

→ Work done for $\frac{50}{55.85}$ mol of gas

$= -1.8314 \times 298 \times \frac{50}{55.85}$ J

= –2218.059 J

≃ –2218 J

10. $[Ti(H_2O)_6]^{3+}$ absorbs light of wavelength 498 nm during a d – d transition. The octahedral splitting energy for the above complex is _____ × 10^{-19} J. (Round off to the Nearest Integer). h = 6.626×10^{-34} Js; c = 3×10^8 ms^{-1}.

Official Ans. by NTA (4)

Sol. $\lambda_{absorbed}$ = 498 nm (given)

The octahedral spilitting energy

$$\Delta_0 \text{ or } E = \frac{hc}{\lambda} = \frac{6.626 \times 10^{-34} \times 3 \times 10^8}{498 \times 10^{-9}}$$

= 0.0399×10^{-17} J

= 3.99×10^{-19} J

= 4.00×10^{-19} J (round off)

FINAL JEE–MAIN EXAMINATION – MARCH, 2021

(Held On Tuesday 16th March, 2021) TIME : 3 : 00 PM to 6 : 00 PM

MATHEMATICS **TEST PAPER WITH SOLUTION**

SECTION-A

1. The maximum value of

$$f(x)=\begin{vmatrix} \sin^2 x & 1+\cos^2 x & \cos 2x \\ 1+\sin^2 x & \cos^2 x & \cos 2x \\ \sin^2 x & \cos^2 x & \sin 2x \end{vmatrix}, x \in R \text{ is:}$$

(1) $\sqrt{7}$ (2) $\frac{3}{4}$ (3) $\sqrt{5}$ (4) 5

Official Ans by NTA (3)

Sol. $C_1 + C_2 \to C_1$

$$\begin{vmatrix} 2 & 1+\cos^2 x & \cos 2x \\ 2 & \cos^2 x & \cos 2x \\ 1 & \cos^2 x & \sin 2x \end{vmatrix}$$

$R_1 - R_2 \to R_1$

$$\begin{vmatrix} 0 & 1 & 0 \\ 2 & \cos^2 x & \cos 2x \\ 1 & \cos^2 x & \sin 2x \end{vmatrix}$$

Open w.r.t. R_1

$-(2 \sin 2x - \cos 2x)$

$\cos 2x - 2 \sin 2x = f(x)$

$$f(x)\big|_{max} = \sqrt{1+4} = \sqrt{5}$$

2. Let A denote the event that a 6-digit integer formed by 0, 1, 2, 3, 4, 5, 6 without repetitions, be divisible by 3. Then probability of event A is equal to :

(1) $\frac{9}{56}$ (2) $\frac{4}{9}$ (3) $\frac{3}{7}$ (4) $\frac{11}{27}$

Official Ans by NTA (2)

Sol. Total cases :

$\underline{6} \cdot \underline{6} \cdot \underline{5} \cdot \underline{4} \cdot \underline{3} \cdot \underline{2}$

$n(s) = 6 \cdot 6!$

Favourable cases :

Number divisible by 3 ≡

Sum of digits must be divisible by 3

Case-I

1, 2, 3, 4, 5, 6

Number of ways = 6!

Case-II

0, 1, 2, 4, 5, 6

Number of ways = 5·5!

Case-III

0, 1, 2, 3, 4, 5

Number of ways = 5·5!

n(favourable) = 6! + 2·5·5!

$$P = \frac{6! + 2 \cdot 5 \cdot 5!}{6 \cdot 6!} = \frac{4}{9}$$

3. Let $\alpha \in R$ be such that the function

$$f(x)=\begin{cases} \dfrac{\cos^{-1}(1-\{x\}^2)\sin^{-1}(1-\{x\})}{\{x\}-\{x\}^3}, & x \neq 0 \\ \alpha, & x = 0 \end{cases}$$

is continuous at x = 0, where {x} = x – [x], [x] is the greatest integer less than or equal to x. Then :

(1) $\alpha = \frac{\pi}{\sqrt{2}}$ (2) $\alpha = 0$

(3) no such α exists (4) $\alpha = \frac{\pi}{4}$

Official Ans by NTA (3)

Sol.

$$\lim_{x \to 0^+} f(x) = f(0) = \lim_{x \to 0^-} (x)$$

$$\lim_{x \to 0^+} \frac{\cos^{-1}(1-x^2) \cdot \sin^{-1}(1-x)}{x(1-x)(1+x)}$$

$$\lim_{x \to 0^+} \frac{\cos^{-1}(1-x^2)}{x \cdot 1 \cdot 1} \cdot \frac{\pi}{2}$$

Let $1 - x^2 = \cos\theta$

$$\frac{\pi}{2}\lim_{x\to 0^+}\frac{\theta}{\sqrt{1-\cos\theta}}$$

$$\frac{\pi}{2}\lim_{\theta\to 0^+}\frac{\theta}{\sqrt{2}\sin\frac{\theta}{2}} = \frac{\pi}{\sqrt{2}}$$

Now, $\lim_{x\to 0^-}\frac{\cos^{-1}\left(1-(1+x)^2\right)\sin^{-1}(-x)}{(1+x)-(1+x)^3}$

$$\lim_{x\to 0^-}\frac{\frac{\pi}{2}\left(-\sin^{-1}x\right)}{(1+x)(2+x)(-x)}$$

$$\lim_{x\to 0^-}\frac{\frac{\pi}{2}}{1\cdot 2}\cdot\frac{\sin^{-1}x}{x} = \frac{\pi}{4}$$

$\Rightarrow$ RHL $\neq$ LHL

Function can't be continuous

$\Rightarrow$ No value of α exist

4. If (x, y, z) be an arbitrary point lying on a plane P which passes through the point (42, 0, 0), (0, 42, 0) and (0, 0, 42), then the value of expression

$$3+\frac{x-11}{(y-19)^2(z-12)^2}+\frac{y-19}{(x-11)^2(z-12)^2}$$

$$+\frac{z-12}{(x-11)^2(y-19)^2}-\frac{x+y+z}{14(x-11)(y-19)(z-12)}$$

(1) 0 (2) 3 (3) 39 (4) –45

Official Ans by NTA (2)

Sol. Plane passing through (42, 0, 0), (0, 42, 0), (0, 0, 42)

From intercept from, equation of plane is

$x + y + z = 42$

$\Rightarrow$ $(x - 11) + (y - 19) + (z - 12) = 0$

let $a = x - 11$, $b = y - 19$, $c = z - 12$

$a + b + c = 0$

Now, given expression is

$$3+\frac{a}{b^2c^2}+\frac{b}{a^2c^2}+\frac{c}{a^2b^2}-\frac{42}{14abc}$$

$$3+\frac{a^3+b^3+c^3-3abc}{a^2b^2c^2}$$

If $a + b + c = 0$

$\Rightarrow$ $a^3 + b^3 + c^3 = 3\,abc$

$\Rightarrow$ 3

5. Consider the integral

$$I=\int_0^{10}\frac{[x]\,e^{[x]}}{e^{x-1}}dx,$$

where [x] denotes the greatest integer less than or equal to x. Then the value of I is equal to:

(1) 9(e – 1) (2) 45(e + 1)

(3) 45(e – 1) (4) 9(e + 1)

Official Ans by NTA (3)

Sol. $I=\int_0^{10}[x]\cdot e^{[x]-x+1}$

$$I=\int_0^1 0dx+\int_1^2 1\cdot e^{2-x}+\int_2^3 2\cdot e^{3-x}+.....+\int_9^{10}9\cdot e^{10-x}dx$$

$$\Rightarrow\ I=\sum_{n=0}^{9}\int_n^{n+1}n\cdot e^{n+1-x}dx$$

$$= -\sum_{n=0}^{9}n\left(e^{n+1-x}\right)_n^{n+1}$$

$$= -\sum_{n=0}^{9}n\cdot(e^0-e^1)$$

$$= (e-1)\sum_{n=0}^{9}n$$

$$= (e-1)\cdot\frac{9\cdot 10}{2}$$

$= 45(e - 1)$

6. Let C be the locus of the mirror image of a point on the parabola $y^2 = 4x$ with respect to the line $y = x$. Then the equation of tangent to C at P(2,1) is :

(1) $x - y = 1$ (2) $2x + y = 5$
(3) $x + 3y = 5$ (4) $x + 2y = 4$

Official Ans by NTA (1)

Sol. Given $y^2 = 4x$

Mirror image on $y = x \Rightarrow C : x^2 = 4y$

$$2x = 4 \cdot \frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{x}{2}$$

$$\left.\frac{dy}{dx}\right|_{P(2,1)} = \frac{2}{2} = 1$$

Equation of tangent at (2, 1)

$\Rightarrow y - 1 = 1(x - 2)$

$\Rightarrow x - y = 1$

7. If $y = y(x)$ is the solution of the differential equation $\frac{dy}{dx} + (\tan x)\, y = \sin x,\ 0 \le x \le \frac{\pi}{3}$, with $y(0) = 0$, then $y\left(\frac{\pi}{4}\right)$ equal to :

(1) $\frac{1}{4}\log_e 2$ (2) $\left(\frac{1}{2\sqrt{2}}\right)\log_e 2$

(3) $\log_e 2$ (4) $\frac{1}{2}\log_e 2$

Official Ans by NTA (2)

Sol. $\frac{dy}{dx} + (\tan x)\, y = \sin x\ ;\ 0 \le x \le \frac{\pi}{3}$

$$\text{I.F.} = e^{\int \tan x\, dx} = e^{\ell n \sec x} = \sec x$$

$$y \sec x = \int \tan x\, dx$$

$$y \sec x = \int \tan x dx$$

$$y \sec x = \ell n\,|\sec x| + C$$

$x = 0,\ y = 0 \quad \Rightarrow \quad \therefore\ c = 0$

$$y \sec x = \ell n\,|\sec x|$$

$$y = \cos x \cdot \ell n\,|\sec x|$$

$$y\big|_{x=\frac{\pi}{4}} = \left(\frac{1}{\sqrt{2}}\right) \cdot \ell n\sqrt{2}$$

$$y\big|_{x=\frac{\pi}{4}} = \frac{1}{2\sqrt{2}}\log_e 2$$

8. Let A = {2, 3, 4, 5, ... , 30} and '$\simeq$' be an equivalence relation on A × A, defined by (a, b) $\simeq$ (c, d), if and only if ad = bc. Then the number of ordered pairs which satisfy this equivalence relation with ordered pair (4, 3) is equal to :

(1) 5 (2) 6 (3) 8 (4) 7

Official Ans by NTA (4)

Sol. A = {2, 3, 4, 5, ... , 30}

(a, b) $\simeq$ (c, d) $\Rightarrow$ ad = bc

(4, 3) $\simeq$ (c, d) $\Rightarrow$ 4d = 3c

$$\Rightarrow \frac{4}{3} = \frac{c}{d}$$

$$\frac{c}{d} = \frac{4}{3} \quad \& \ c, d \in \{2, 3, ..., 30\}$$

(c, d) = {(4, 3), (8, 6), (12, 9), (16, 12), (20, 15), (24, 18), (28, 21)}

No. of ordered pair = 7

9. Let the lengths of intercepts on x-axis and y-axis made by the circle $x^2 + y^2 + ax + 2ay + c = 0$, $(a < 0)$ be $2\sqrt{2}$ and $2\sqrt{5}$, respectively. Then the shortest distance from origin to a tangent to this circle which is perpendicular to the line $x + 2y = 0$, is euqal to :

(1) $\sqrt{11}$ (2) $\sqrt{7}$ (3) $\sqrt{6}$ (4) $\sqrt{10}$

Official Ans by NTA (3)

Sol. $x^2 + y^2 + ax + 2ay + c = 0$

$$2\sqrt{g^2 - c} = 2\sqrt{\frac{a^2}{4} - c} = 2\sqrt{2}$$

$$\Rightarrow \frac{a^2}{4} - c = 2 \qquad ...(1)$$

$$2\sqrt{f^2-c}=2\sqrt{a^2-c}=2\sqrt{5}$$

$\Rightarrow$ $a^2 - c = 5$ ...(2)

(1) & (2)

$$\frac{3a^2}{4}=3 \Rightarrow a = -2 \quad (a < 0)$$

$\therefore$ $c = -1$

Circle $\Rightarrow$ $x^2 + y^2 - 2x - 4y - 1 = 0$

$\Rightarrow$ $(x - 1)^2 + (y - 2)^2 = 6$

Given $x + 2y = 0 \Rightarrow m = -\frac{1}{2}$

$m_{tangent} = 2$

Equation of tangent

$\Rightarrow$ $(y - 2) = 2(x - 1) \pm \sqrt{6}\sqrt{1+4}$

$\Rightarrow 2x - y \pm \sqrt{30} = 0$

Perpendicular distance from (0, 0) = $\left|\frac{\pm\sqrt{30}}{\sqrt{4+1}}\right| = \sqrt{6}$

10. The least value of $|z|$ where z is complex number which satisfies the inequality

$$\exp\left(\frac{(|z|+3)(|z|-1)}{|z|+1}\log_e 2\right) \geq \log_{\sqrt{2}}\left|5\sqrt{7}+9i\right|,$$

$i=\sqrt{-1}$, is equal to :

(1) 3 (2) $\sqrt{5}$ (3) 2 (4) 8

Official Ans by NTA (1)

Sol. $$\exp\left(\frac{(|z|+3)(|z|-1)}{|z|+1}\ell n\, 2\right) \geq \log_{\sqrt{2}}\left|5\sqrt{7}+9i\right|$$

$$\Rightarrow 2^{\frac{(|z|+3)(|z|-1)}{(|z|+1)}} \geq \log_{\sqrt{2}}(16)$$

$$\Rightarrow 2^{\frac{(|z|+3)(|z|-1)}{(|z|+1)}} \geq 2^3$$

$$\Rightarrow \frac{(|z|+3)(|z|-1)}{(|z|+1)} \geq 3$$

$\Rightarrow$ $(|z| + 3)(|z| - 1) \geq 3(|z| + 1)$

$|z|^2 + 2|z| - 3 \geq 3|z| + 3$

$\Rightarrow$ $|z|^2 + |z| - 6 \geq 0$

$\Rightarrow$ $(|z| - 3)(|z| + 2) \geq 0 \Rightarrow |z| - 3 \geq 0$

$\Rightarrow$ $|z| \geq 3$ $\Rightarrow$ $|z|_{min} = 3$

11. Consider a rectangle ABCD having 5, 7, 6, 9 points in the interior of the line segments AB, CD, BC, DA respectively. Let α be the number of triangles having these points from different sides as vertices and β be the number of quadrilaterals having these points from different sides as vertices. Then $(\beta - \alpha)$ is equal to :

(1) 795 (2) 1173 (3) 1890 (4) 717

Official Ans by NTA (4)

Sol.

α = Number of triangles

$\alpha = 5\cdot6\cdot7 + 5\cdot7\cdot9 + 5\cdot6\cdot9 + 6\cdot7\cdot9$

$= 210 + 315 + 270 + 378$

$= 1173$

β = Number of Quadrilateral

$\beta = 5\cdot6\cdot7\cdot9 = 1890$

$\beta - \alpha = 1890 - 1173 = 717$

12. If the point of intersections of the ellipse $\frac{x^2}{16}+\frac{y^2}{b^2}=1$ and the circle $x^2 + y^2 = 4b$, $b > 4$ lie on the curve $y^2 = 3x^2$, then b is equal to:

(1) 12 (2) 5 (3) 6 (4) 10

Official Ans by NTA (1)

Sol. $y^2 = 3x^2$

and $x^2 + y^2 = 4b$

Solve both we get

so $x^2 = b$

$$\frac{x^2}{16}+\frac{3x^2}{b^2}=1$$

$$\frac{b}{16}+\frac{3}{b}=1$$

$b^2 - 16b + 48 = 0$

$(b - 12)(b - 4) = 0$

$b = 12,\ b > 4$

13. Given that the inverse trigonometric functions take principal values only. Then, the number of real values of x which satisfy

$\sin^{-1}\left(\frac{3x}{5}\right)+\sin^{-1}\left(\frac{4x}{5}\right)=\sin^{-1}x$ is equal to:

(1) 2 (2) 1 (3) 3 (4) 0

Official Ans by NTA (3)

Sol. $\sin^{-1}\frac{3x}{5}+\sin^{-1}\frac{4x}{5}=\sin^{-1}x$

$$\sin^{-1}\left(\frac{3x}{5}\sqrt{1-\frac{16x^2}{25}}+\frac{4x}{5}\sqrt{1-\frac{9x^2}{25}}\right)=\sin^{-1}x$$

$$\frac{3x}{5}\sqrt{1-\frac{16x^2}{25}}+\frac{4x}{5}\sqrt{1-\frac{9x^2}{25}}=x$$

$x = 0,\ 3\sqrt{25-16x^2}+4\sqrt{25-9x^2}=25$

$4\sqrt{25-9x^2}=25-3\sqrt{25-16x^2}$ squaring we get

$16(25-9x^2) = 625+9(25-16x^2)-150\sqrt{25-16x^2}$

$400 = 625 + 225 - 150\sqrt{25-16x^2}$

$\sqrt{25-16x^2} = 3 \Rightarrow 25 - 16x^2 = 9$

$\Rightarrow x^2 = 1$

Put x = 0, 1, –1 in the original equation

We see that all values satisfy the original equation.

Number of solution = 3

14. Let A(–1, 1), B(3, 4) and C(2, 0) be given three points. A line y = mx, m > 0, intersects lines AC and BC at point P and Q respectively. Let A_1 and A_2 be the areas of ΔABC and ΔPQC respectively, such that $A_1 = 3A_2$, then the value of m is equal to :

(1) $\frac{4}{15}$ (2) 1 (3) 2 (4) 3

Official Ans by NTA (2)

Sol.

$P \equiv (x_1, mx_1)$

$Q \equiv (x_2, mx_2)$

$$A_1=\frac{1}{2}\begin{vmatrix}3 & 4 & 1\\ 2 & 0 & 1\\ -1 & 1 & 1\end{vmatrix}=\frac{13}{2}$$

$$A_2=\frac{1}{2}\begin{vmatrix}x_1 & mx_1 & 1\\ x_2 & mx_2 & 1\\ 2 & 0 & 1\end{vmatrix}$$

$$A_2=\frac{1}{2}\left|2(mx_1-mx_2)\right|=m\left|x_1-x_2\right|$$

$$A_1 = 3A_2 \Rightarrow \frac{13}{2} = 3m\left|x_1-x_2\right|$$

$$\Rightarrow \left|x_1-x_2\right|=\frac{16}{6m}$$

AC : x + 3y = 2

BC : y = 4x – 8

$P : x + 3y = 2\ \&\ y = mx \Rightarrow x_1 = \frac{2}{1+3m}$

$Q : y = 4x - 8\ \&\ y = mx \Rightarrow x_2 = \frac{8}{4-m}$

$$\left|x_1-x_2\right|=\left|\frac{2}{1+3m}-\frac{8}{4-m}\right|$$

$$=\left|\frac{-26m}{(1+3m)(4-m)}\right| = \frac{26m}{(3m+1)\,|m-4|}$$

$$= \frac{26m}{(3m+1)(4-m)}$$

$$|x_1 - x_2| = \frac{13}{6m}$$

$$\frac{26m}{(3m+1)(4-m)} = \frac{13}{6m}$$

$\Rightarrow$ $12m^2 = -(3m + 1)(m - 4)$

$\Rightarrow$ $12m^2 = -(3m^2 - 11m - 4)$

$\Rightarrow$ $15m^2 - 11m - 4 = 0$

$\Rightarrow$ $15m^2 - 15m + 4m - 4 = 0$

$\Rightarrow$ $(15m + 4)(m - 1) = 0$

$\Rightarrow$ $m = 1$

15. Let f be a real valued function, defined on $R - \{-1, 1\}$ and given by

$$f(x) = 3\log_e\left|\frac{x-1}{x+1}\right| - \frac{2}{x-1}.$$

Then in which of the following intervals, function f(x) is increasing?

(1) $(-\infty,-1)\cup\left(\left[\frac{1}{2},\infty\right)-\{1\}\right)$

(2) $(-\infty, \infty) - \{-1, 1\}$

(3) $\left(-1,\frac{1}{2}\right]$

(4) $\left(-\infty,\frac{1}{2}\right]-\{-1\}$

Official Ans by NTA (1)

Sol. $f(x) = 3\ell n(x-1) - 3\ell n(x+1) - \frac{2}{x-1}$

$$f'(x) = \frac{3}{x-1} - \frac{3}{x+1} + \frac{2}{(x-1)^2}$$

$$f'(x) = \frac{4(2x-1)}{(x-1)^2(x+1)}$$

$f'(x) \geq 0$

$\Rightarrow$ $x \in (-\infty,-1)\cup\left[\frac{1}{2},1\right)\cup(1,\infty)$

16. Let $f : S \to S$ where $S = (0, \infty)$ be a twice differentiable function such that $f(x + 1) = xf(x)$. If $g : S \to R$ be defined as $g(x) = \log_e f(x)$, then the value of $|g''(5) - g''(1)|$ is equal to :

(1) $\frac{205}{144}$ (2) $\frac{197}{144}$ (3) $\frac{187}{144}$ (4) 1

Official Ans by NTA (1)

Sol. $\ln f(x + 1) = \ln(xf(x))$

$\ln f(x + 1) = \ln x + \ln f(x)$

$\Rightarrow$ $g(x + 1) = \ln x + g(x)$

$\Rightarrow$ $g(x + 1) - g(x) = \ln x$

$\Rightarrow$ $g''(x + 1) - g''(x) = -\frac{1}{x^2}$

Put x = 1, 2, 3, 4

$g''(2) - g''(1) = -\frac{1}{1^2}$ ...(1)

$g''(3) - g''(2) = -\frac{1}{2^2}$ ...(2)

$g''(4) - g''(3) = -\frac{1}{3^2}$ ...(3)

$g''(5) - g''(4) = -\frac{1}{4^2}$ ...(4)

Add all the equation we get

$$g''(5) - g''(1) = -\frac{1}{1^2} - \frac{1}{2^2} - \frac{1}{3^2} - \frac{1}{4^2}$$

$$|g''(5) - g''(1)| = \frac{205}{144}$$

17. Let $P(x) = x^2 + bx + c$ be a quadratic polynomial with real coefficients such that $\int_0^1 P(x)dx = 1$ and P(x) leaves remainder 5 when it is divided by (x – 2). Then the value of 9(b + c) is equal to:

(1) 9 (2) 15 (3) 7 (4) 11

Official Ans by NTA (3)

Sol. $\int_0^1 (x^2 + bx + c)dx = 1$

$\frac{1}{3} + \frac{b}{2} + c = 1 \quad \Rightarrow \quad \frac{b}{2} + c = \frac{2}{3}$

$3b + 6c = 4 \qquad ...(1)$

P(2) = 5

4 + 2b + c = 5

2b + c = 1 ...(2)

From (1) & (2)

b = $\frac{2}{9}$ & c = $\frac{5}{9}$

9(b + c) = 7

18. If the foot of the perpendicular from point (4, 3, 8) on the line $L_1 : \frac{x-a}{l} = \frac{y-2}{3} = \frac{z-b}{4}$, $l \neq 0$ is (3, 5, 7), then the shortest distance between the line L_1 and line $L_2 : \frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$ is equal to :

(1) $\frac{1}{2}$ (2) $\frac{1}{\sqrt{6}}$ (3) $\sqrt{\frac{2}{3}}$ (4) $\frac{1}{\sqrt{3}}$

Official Ans by NTA (2)

Sol. (3,5,7) satisfy the line L_1

$\frac{3-a}{\ell} = \frac{5-2}{3} = \frac{7-b}{4}$

$\frac{3-a}{\ell} = 1 \qquad \& \qquad \frac{7-b}{4} = 1$

a + ℓ = 3 ...(1) & b = 3 ...(2)

$\vec{v}_1 = <4,3,8> - <3,5,7>$

$\vec{v}_1 = <1,-2,1>$

$\vec{v}_2 = <\ell,3,4>$

$\vec{v}_1.\vec{v}_2 = 0 \quad \Rightarrow \quad \ell - 6 + 4 = 0 \quad \Rightarrow \quad \ell = 2$

$a + \ell = 3 \quad \Rightarrow \quad a = 1$

$L_1 : \frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$

$L_2 : \frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$

A = <1, 2, 3>

B = <2, 4, 5>

$\overrightarrow{AB} = <1,2,2>$

$\vec{p} = 2\hat{i} + 3\hat{j} + 4\hat{k}$

$\vec{q} = 3\hat{i} + 4\hat{j} + 5\hat{k}$

$\vec{p} \times \vec{q} = -\hat{i} + 2\hat{j} - \hat{k}$

Shortest distance = $\left| \frac{\overrightarrow{AB} \cdot (\vec{p} \times \vec{q})}{|\vec{p} \times \vec{q}|} \right| = \frac{1}{\sqrt{6}}$

19. Let C_1 be the curve obtained by the solution of differential equation $2xy\frac{dy}{dx} = y^2 - x^2$, x > 0. Let the curve C_2 be the solution of $\frac{2xy}{x^2 - y^2} = \frac{dy}{dx}$. If both the curves pass through (1,1), then the area enclosed by the curves C_1 and C_2 is equal to :

(1) $\pi - 1$ (2) $\frac{\pi}{2} - 1$ (3) $\pi + 1$ (4) $\frac{\pi}{4} + 1$

Official Ans by NTA (2)

Sol. $\frac{dy}{dx} = \frac{y^2 - x^2}{2xy}$, $x \in (0, \infty)$

put y = vx

$$x\frac{dv}{dx} + v = \frac{v^2 - 1}{2v}$$

$$\frac{2v}{v^2 + 1}dv = -\frac{dx}{x}$$

Integrate,

$ln(v^2 + 1) = -\, lnx + C$

$$ln\left(\frac{y^2}{x^2} + 1\right) = -lnx + C$$

put x = 1, y = 1, C = $ln2$

$$ln\left(\frac{y^2}{x^2}+1\right) = -lnx + ln2$$

$\Rightarrow$ $x^2 + y^2 - 2x = 0$ (Curve C_1)

Similarly,

$$\frac{dy}{dx} = \frac{2xy}{x^2 - y^2}$$

Put y = vx

$x^2 + y^2 - 2y = 0$

required area = $2\int_0^1 \left(\sqrt{2x - x^2} - x\right)dx = \frac{\pi}{2} - 1$

20. Let $\vec{a} = \hat{i} + 2\hat{j} - 3\hat{k}$ and $\vec{b} = 2\hat{i} - 3\hat{j} + 5\hat{k}$. If $\vec{r} \times \vec{a} = \vec{b} \times \vec{r}$, $\vec{r}.\left(\alpha\hat{i} + 2\hat{j} + \hat{k}\right) = 3$ and $\vec{r}.\left(2\hat{i} + 5\hat{j} - \alpha\hat{k}\right) = -1$, $\alpha \in R$, then the value of $\alpha + |\vec{r}|^2$ is equal to :

(1) 9 (2) 15 (3) 13 (4) 11

Official Ans by NTA (2)

Sol. $\vec{r} \times \vec{a} = \vec{b} \times \vec{r} \Rightarrow \vec{r} \times (\vec{a} + \vec{b}) = 0$

$\vec{r} = \vec{\lambda}(\vec{a} + \vec{b}) \Rightarrow \vec{r} = \vec{\lambda}(\hat{i} + 2\hat{j} - 3\hat{k} + 2\hat{i} - 3\hat{j} + 5\hat{k})$

$\vec{r} = \vec{\lambda}(3\hat{i} - \hat{j} + 2\hat{k})$ ...(1)

$\vec{r} \cdot (\alpha\hat{i} + 2\hat{j} + \hat{k}) = 3$

Put $\vec{r}$ from (1) $\alpha\lambda = 1$ ...(2)

$\vec{r} \cdot (2\hat{i} + 5\hat{j} - \alpha\hat{k}) = -1$

Put $\vec{r}$ from (1) $2\lambda\alpha - \lambda = 1$ ...(3)

Solve (2) & (3)

$\alpha = 1$, $\lambda = 1$

$\Rightarrow$ $\vec{r} = 3\hat{i} - \hat{j} + 2\hat{k}$

$|\vec{r}|^2 = 14$ & $\alpha = 1$

$\alpha + |\vec{r}|^2 = 15$

SECTION-B

1. If the distance of the point (1, –2, 3) from the plane x + 2y – 3z + 10 = 0 measured parallel to the line, $\frac{x-1}{3} = \frac{2-y}{m} = \frac{z+3}{1}$ is $\sqrt{\frac{7}{2}}$, then the value of |m| is equal to _____.

Official Ans by NTA (2)

Sol.

$$\text{DC of line} \equiv \left(\frac{3}{\sqrt{m^2+10}}, \frac{-m}{\sqrt{m^2+10}}, \frac{1}{\sqrt{m^2+10}}\right)$$

$$Q \equiv \left(1 + \frac{3r}{\sqrt{m^2+10}}, -2 + \frac{-mr}{\sqrt{m^2+10}}, 3 + \frac{r}{\sqrt{m^2+10}}\right)$$

Q lies on x + 2y – 3z + 10 = 0

$$1 + \frac{3r}{\sqrt{m^2+10}} - 4 - \frac{2mr}{\sqrt{m^2+10}} - 9 - \frac{3r}{\sqrt{m^2+10}} + 10 = 0$$

$$\Rightarrow \frac{r}{\sqrt{m^2+10}}(3 - 2m - 3) = 2$$

$$\Rightarrow \frac{r}{\sqrt{m^2+10}}(-2m) = 2$$

$r^2m^2 = m^2 + 10$

$\frac{7}{2}m^2 = m^2 + 10 \Rightarrow \frac{5}{2}m^2 = 10 \Rightarrow m^2 = 4$

$|m| = 2$

2. Consider the statistics of two sets of observations as follows :

	Size	Mean	Variance
Observation I	10	2	2
Observation II	n	3	1

If the variance of the combined set of these two observations is $\frac{17}{9}$, then the value of n is equal to _____.

Official Ans by NTA (5)

Sol. $\sigma^2 = \frac{n_1\sigma_1^2 + n_2\sigma_2^2}{n_1 + n_2} + \frac{n_1 n_2}{(n_1 + n_2)}(\bar{x}_1 - \bar{x}_2)^2$

$n_1 = 10,\ n_2 = n,\ \sigma_1^2 = 2,\ \sigma_2^2 = 1$

$\bar{x}_1 = 2,\ \bar{x}_2 = 3,\ \sigma^2 = \frac{17}{9}$

$\frac{17}{9} = \frac{10 \times 2 + n}{n + 10} + \frac{10n}{(n+10)^2}(3-2)^2$

$\Rightarrow \frac{17}{9} = \frac{(n+20)(n+10) + 10n}{(n+10)^2}$

$\Rightarrow 17n^2 + 1700 + 340\,n = 90n + 9(n^2+30n+200)$

$\Rightarrow 8n^2 - 20n - 100 = 0$

$2n^2 - 5n - 25 = 0$

$\Rightarrow (2n + 5)(n - 5) = 0 \Rightarrow n = \frac{-5}{2}, 5$

$\downarrow$

(Rejected)

Hence n = 5

3. Let $A = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$ and $B = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$ be two 2 × 1 matrices with real entries such that A = XB, where $X = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & -1 \\ 1 & k \end{bmatrix}$, and k ∈ R. If $a_1^2 + a_2^2 = \frac{2}{3}\left(b_1^2 + b_2^2\right)$ and $(k^2+1)b_2^2 \neq -2b_1b_2$, then the value of k is _____.

Official Ans by NTA (1)

Sol. A = XB

$\begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & -1 \\ 1 & k \end{bmatrix}\begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$

$\begin{bmatrix} \sqrt{3}a_1 \\ \sqrt{3}a_2 \end{bmatrix} = \begin{bmatrix} b_1 - b_2 \\ b_1 + kb_2 \end{bmatrix}$

$b_1 - b_2 = \sqrt{3}a_1$ ...(1)

$b_1 + kb_2 = \sqrt{3}a_2$ ...(2)

Given, $a_1^2 + a_2^2 = \frac{2}{3}\left(b_1^2 + b_2^2\right)$

$(1)^2 + (2)^2$

$(b_1 + b_2)^2 + (b_1 + kb_2)^2 = 3(a_1^2 + a_2^2)$

$a_1^2 + a_2^2 = \frac{2}{3}b_1^2 + \frac{(1+k^2)}{3}b_2^2 + \frac{2}{3}b_1b_2(k-1)$

Given, $a_1^2 + a_2^2 = \frac{2}{3}b_1^2 + \frac{2}{3}b_2^2$

On comparing we get

$\frac{k^2+1}{3} = \frac{2}{3} \Rightarrow k^2 + 1 = 2$

$\Rightarrow k = \pm 1$ ....(3)

& $\frac{2}{3}(k-1) = 0 \Rightarrow k = 1$ ...(4)

From both we get k = 1

4. For real numbers α, β, γ and δ, if

$$\int \frac{(x^2-1) + \tan^{-1}\left(\frac{x^2+1}{x}\right)}{(x^4+3x^2+1)\tan^{-1}\left(\frac{x^2+1}{x}\right)}dx$$

$$= \alpha \log_e\left(\tan^{-1}\left(\frac{x^2+1}{x}\right)\right)$$

$$+\beta\tan^{-1}\left(\frac{\gamma(x^2-1)}{x}\right)+\delta\tan^{-1}\left(\frac{x^2+1}{x}\right)+C$$

where C is an arbitrary constant, then the value of $10(\alpha + \beta\gamma + \delta)$ is equal to _____.

Official Ans by NTA (6)

Sol. $$\int\frac{(x^2-1)dx}{(x^4+3x^2+1)\tan^{-1}\left(x+\frac{1}{x}\right)}+\int\frac{dx}{x^4+3x^2+1}$$

$$\int\frac{\left(1-\frac{1}{x^2}\right)dx}{\left(\left(x+\frac{1}{x}\right)^2+1\right)\tan^{-1}\left(x+\frac{1}{x}\right)}+\frac{1}{2}\int\frac{(x^2+1)-(x^2-1)dx}{x^4+3x^2+1}$$

Put $\tan^{-1}\left(x+\frac{1}{x}\right)=t$

$$\int\frac{dt}{t}+\frac{1}{2}\int\frac{\left(1+\frac{1}{x^2}\right)dx}{\left(x-\frac{1}{x}\right)^2+5}-\frac{1}{2}\int\frac{\left(1-\frac{1}{x^2}\right)dx}{\left(x+\frac{1}{x}\right)^2+1}$$

Put $x-\frac{1}{x}=y$, $x+\frac{1}{x}=z$

$$\log_e t+\frac{1}{2}\int\frac{dy}{y^2+5}-\frac{1}{2}\int\frac{dz}{z^2+1}$$

$$= \log_e\tan^{-1}\left(x+\frac{1}{x}\right)+\frac{1}{2\sqrt{5}}\tan^{-1}\left(\frac{x^2-1}{\sqrt{5}x}\right)$$

$$-\frac{1}{2}\tan^{-1}\left(\frac{x^2+1}{x}\right)+C$$

$\alpha = 1$, $\beta = \frac{1}{2\sqrt{5}}$, $\gamma = \frac{1}{\sqrt{5}}$, $\delta = \frac{-1}{2}$

or

$\alpha = 1$, $\beta = \frac{-1}{2\sqrt{5}}$, $\gamma = \frac{-1}{\sqrt{5}}$, $\delta = \frac{-1}{2}$

$$10(\alpha + \beta\gamma + \delta) = 10\left(1+\frac{1}{10}-\frac{1}{2}\right) = 6$$

5. Let $f : R \to R$ and $g : R \to R$ be defined as

$$f(x)=\begin{cases} x+a, & x<0 \\ |x-1|, & x\ge 0\end{cases} \text{ and}$$

$$g(x)=\begin{cases} x+1, & x<0 \\ (x-1)^2+b, & x\ge 0\end{cases}$$

where a, b are non-negative real numbers. If (gof)(x) is continuous for all $x \in R$, then $a + b$ is equal to _____.

Official Ans by NTA (1)

Sol. $$g[f(x)] = \begin{cases} f(x)+1 & f(x)<0 \\ (f(x)-1)^2+b & f(x)\ge 0\end{cases}$$

$$g[f(x)]=\begin{cases} x+a+1 & x+a<0 \,\&\, x<0 \\ |x-1|+1 & |x-1|<0 \,\&\, x\ge 0 \\ (x+a-1)^2+b & x+a\ge 0 \,\&\, x<0 \\ (|x-1|-1)^2+b & |x-1|\ge 0 \,\&\, x\ge 0\end{cases}$$

$$g[f(x)]=\begin{cases} x+a+1 & x\in(-\infty,-a) \,\&\, x\in(-\infty,0) \\ |x-1|+1 & x\in\phi \\ (x+a-1)^2+b & x\in[-a,\infty) \,\&\, x\in(-\infty,0) \\ (|x-1|-1)^2+b & x\in R \,\&\, x\in[0,\infty)\end{cases}$$

$$g[f(x)]=\begin{cases} x+a+1 & x\in(-\infty,-a) \\ (x+a-1)^2+b & x\in[-a,0) \\ (|x-1|-1)^2+b & x\in[0,\infty)\end{cases}$$

g(f(x)) is continuous

at $x = -a$ & at $x = 0$

$1 = b + 1$ & $(a - 1)^2 + b = b$

$b = 0$ & $a = 1$

$\Rightarrow \quad a + b = 1$

6. Let $\frac{1}{16}$, a and b be in G.P. and $\frac{1}{a}, \frac{1}{b}, 6$ be in A.P., where a, b > 0. Then 72(a + b) is equal to _____.

Official Ans by NTA (14)

Sol. $a^2 = \frac{b}{16} \Rightarrow \frac{1}{b} = \frac{1}{16a^2}$

$\frac{2}{b} = \frac{1}{a} + 6$

$\frac{1}{8a^2} = \frac{1}{a} + 6$

$\frac{1}{a^2} - \frac{8}{a} - 48 = 0$

$\frac{1}{a} = 12, -4 \quad \Rightarrow \quad a = \frac{1}{12}, -\frac{1}{4}$

$a = \frac{1}{12}, \ a > 0$

$b = 16a^2 = \frac{1}{9}$

$\Rightarrow \quad 72\ (a + b) = 6 + 8 = 14$

7. In ΔABC, the lengths of sides AC and AB are 12 cm and 5 cm, respectively. If the area of ΔABC is 30 cm^2 and R and r are respectively the radii of circumcircle and incircle of ΔABC, then the value of 2R + r (in cm) is equal to _____.

Official Ans by NTA (15)

Sol. $\Delta = \frac{1}{2}.5.12.\sin A = 30$

$\sin A = 1$

$A = 90° \Rightarrow BC = 13$

$BC = 2R = 13$

$r = \frac{\Delta}{S} = \frac{30}{15} = 2$

$2R + r = 15$

8. Let n be a positive integer. Let

$$A = \sum_{k=0}^{n} (-1)^k {}^nC_k \left[\left(\frac{1}{2}\right)^k + \left(\frac{3}{4}\right)^k + \left(\frac{7}{8}\right)^k + \left(\frac{15}{16}\right)^k + \left(\frac{31}{32}\right)^k\right]$$

If $63A = 1 - \frac{1}{2^{30}}$, then n is equal to _____.

Official Ans by NTA (6)

Sol. $A = \sum_{k=0}^{n} {}^nC_k \left[\left(-\frac{1}{2}\right)^k + \left(\frac{-3}{4}\right)^k + \left(\frac{-7}{8}\right)^k + \left(\frac{-15}{16}\right)^k + \left(\frac{-37}{32}\right)^k\right]$

$A = \left(1 - \frac{1}{2}\right)^n + \left(1 - \frac{3}{4}\right)^n + \left(1 - \frac{7}{8}\right)^n + \left(1 - \frac{15}{16}\right)^n + \left(1 - \frac{31}{32}\right)^n$

$A = \frac{1}{2^n} + \frac{1}{4^n} + \frac{1}{8^n} + \frac{1}{16^n} + \frac{1}{32^n}$

$A = \frac{1}{2^n}\left(\frac{1 - \left(\frac{1}{2^n}\right)^5}{1 - \frac{1}{2^n}}\right) \Rightarrow A = \frac{\left(1 - \frac{1}{2^{5n}}\right)}{(2^n - 1)}$

$(2^n - 1)A = 1 - \frac{1}{2^{5n}}$, Given $63A = 1 - \frac{1}{2^{30}}$

Clearly 5n = 30

n = 6

9. Let $\vec{c}$ be a vector perpendicular to the vectors $\vec{a} = \hat{i} + \hat{j} - \hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$.

If $\vec{c}.\left(\hat{i} + \hat{j} + 3\hat{k}\right) = 8$ then the value of $\vec{c}.\left(\vec{a} \times \vec{b}\right)$ is equal to _____.

Official Ans by NTA (28)

Sol. $\vec{c} = \lambda(\vec{a} \times \vec{b})$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -1 \\ 1 & 2 & 1 \end{vmatrix}$$

$(\vec{a} \times \vec{b}) = 3\hat{i} - 2\hat{j} + \hat{k}$

$\vec{c} \cdot (\hat{i} + \hat{j} + 3\hat{k}) = \lambda(3\hat{i} - 2\hat{j} + \hat{k}) \,.\, (\hat{i} + \hat{j} + 3\hat{k})$

$\Rightarrow \lambda(4) = 8 \Rightarrow \lambda = 2$

$\vec{c} = 2(\vec{a} \times \vec{b})$

$\vec{c}.(\vec{a} \times \vec{b}) = 2\,|\vec{a} \times \vec{b}|^2 = 28$

10. Let

$$S_n(x) = \log_{a^{1/2}} x + \log_{a^{1/3}} x + \log_{a^{1/6}} x$$

$$+ \log_{a^{1/11}} x + \log_{a^{1/18}} x + \log_{a^{1/27}} x + ...$$

up to n-terms, where a > 1. If $S_{24}(x) = 1093$ and $S_{12}(2x) = 265$, then value of a is equal to _____.

Official Ans by NTA (16)

Sol. $S_n(x) = (2+3+6+11+18+27+.....+n\text{-terms})\log_a x$

Let $S_1 = 2 + 3 + 6 + 11 + 18 + 27 + ... + T_n$

$S_1 = 2 + 3 + 6 + ... + T_n$

———————————

$T_n = 2 + 1 + 3 + 5 + ... + n$ terms

$T_n = 2 + (n-1)^2$

$$S_1 = \Sigma T_n = 2n + \frac{(n-1)n(2n-1)}{6}$$

$$\Rightarrow S_n(x) = \left(2n + \frac{n(n-1)(2n-1)}{6}\right)\log_a x$$

$S_{24}(x) = 1093$ (Given)

$$\log_a x \left(48 + \frac{23.24.47}{6}\right) = 1093$$

$\log_a x = \frac{1}{4}$ .... (1)

$S_{12}(2x) = 265$

$S_{12}(2x) = 265$

$$\log_a(2x) \left(24 + \frac{11.12.23}{6}\right) = 265$$

$\log_a 2x = \frac{1}{2}$ .... (2)

(2) – (1)

$\log_a 2x - \log_a x = \frac{1}{4}$

$\log_a 2 = \frac{1}{4} \Rightarrow a = 16$

FINAL JEE–MAIN EXAMINATION – MARCH, 2021

(Held On Tuesday 16th March, 2021) TIME : 9 : 00 AM to 12 : 00 NOON

PHYSICS — TEST PAPER WITH ANSWER & SOLUTION

SECTION-A

1. One main scale division of a vernier callipers is 'a' cm and n^{th} division of the vernier scale coincide with $(n-1)^{th}$ division of the main scale. The least count of the callipers in mm is :

(1) $\frac{10na}{(n-1)}$ (2) $\frac{10a}{(n-1)}$

(3) $\left(\frac{n-1}{10n}\right)a$ (4) $\frac{10a}{n}$

Official Ans. by NTA (4)

Sol. $(n-1)a = n(a')$

$$a' = \frac{(n-1)a}{n}$$

$\therefore$ L.C. = 1 MSD – 1 VSD

$= (a - a')$cm

$$= a - \frac{(n-1)a}{n}$$

$$= \frac{na - na + a}{n} = \frac{a}{n}\text{cm}$$

$$= \left(\frac{10a}{n}\right)\text{mm}$$

2. For changing the capacitance of a given parallel plate capacitor, a dielectric material of dielectric constant K is used, which has the same area as the plates of the capacitor. The thickness of the dielectric slab is $\frac{3}{4}d$, where 'd' is the separation between the plates of parallel plate capacitor. The new capacitance (C') in terms of original capacitance (C_0) is given by the following relation :

(1) $C' = \frac{3+K}{4K}C_0$ (2) $C' = \frac{4+K}{3}C_0$

(3) $C' = \frac{4K}{K+3}C_0$ (4) $C' = \frac{4}{3+K}C_0$

Official Ans. by NTA (3)

Sol.

$$C_0 = \frac{\epsilon_0 A}{d}$$

$C' = C_1$ and C_2 in series.

i.e. $\frac{1}{C'} = \frac{1}{C_1} + \frac{1}{C_2}$

$$\frac{1}{C'} = \frac{(3d/4)}{\epsilon_0 KA} + \frac{d/4}{\epsilon_0 A}$$

$$\frac{1}{C'} = \frac{d}{4\epsilon_0 A}\left(\frac{3+K}{K}\right)$$

$$C' = \frac{4KC_0}{(3+K)}$$

3. A block of mass m slides along a floor while a force of magnitude F is applied to it at an angle θ as shown in figure. The coefficient of kinetic friction is μ_K. Then, the block's acceleration 'a' is given by : (g is acceleration due to gravity)

(1) $-\frac{F}{m}\cos\theta - \mu_K\left(g - \frac{F}{m}\sin\theta\right)$

(2) $\frac{F}{m}\cos\theta - \mu_K\left(g - \frac{F}{m}\sin\theta\right)$

(3) $\frac{F}{m}\cos\theta - \mu_K\left(g + \frac{F}{m}\sin\theta\right)$

(4) $\frac{F}{m}\cos\theta + \mu_K\left(g - \frac{F}{m}\sin\theta\right)$

Official Ans. by NTA (2)

Sol.

$N = mg - f \sin\theta$

$F\cos\theta - \mu_k N = ma$

$F\cos\theta - \mu_k (mg - F\sin\theta) = ma$

$$a = \frac{F}{m}\cos\theta - \mu_K\left(g - \frac{F}{m}\sin\theta\right)$$

4. The pressure acting on a submarine is 3×10^5 Pa at a certain depth. If the depth is doubled, the percentage increase in the pressure acting on the submarine would be : (Assume that atmospheric pressure is 1×10^5 Pa density of water is 10^3 kg m^{-3}, g = 10 ms^{-2})

(1) $\frac{200}{3}\%$ (2) $\frac{200}{5}\%$

(3) $\frac{5}{200}\%$ (4) $\frac{3}{200}\%$

Official Ans. by NTA (1)

Sol. $P_1 = \rho gd + P_0 = 3 \times 10^5$ Pa

$\therefore \rho gd = 2 \times 10^5$ Pa

$P_2 = 2\rho gd + P_0$

$= 4 \times 10^5 + 10^5 = 5 \times 10^5$ Pa

$$\%\text{increase} = \frac{P_2 - P_1}{P_1} \times 100$$

$$= \frac{5\times10^5 - 3\times10^5}{3\times10^5}\times100 = \frac{200}{3}\%$$

5. The angle of deviation through a prism is minimum when

(A) Incident ray and emergent ray are symmetric to the prism

(B) The refracted ray inside the prism becomes parallel to its base

(C) Angle of incidence is equal to that of the angle of emergence

(D) When angle of emergence is double the angle of incidence

Choose the correct answer from the options given below :

(1) Statements (A), (B) and (C) are true

(2) Only statement (D) is true

(3) Only statements (A) and (B) are true

(4) Statements (B) and (C) are true

Official Ans. by NTA (1)

Sol. Deviation is minimum in a prism when :

i = e, $r_1 = r_2$ and ray (2) is parallel to base of prism.

6. A plane electromagnetic wave of frequency 500 MHz is travelling in vacuum along y-direction. At a particular point in space and time, $\vec{B} = 8.0\times10^{-8}\,\hat{z}\,T$. The value of electric field at this point is :

(speed of light = 3×10^8 ms^{-1})

$\hat{x}, \hat{y}, \hat{z}$ are unit vectors along x, y and z direction.

(1) $-24\hat{x}$ V / m (2) $2.6\hat{x}$ V / m

(3) $24\hat{x}$ V / m (4) $-2.6\hat{y}$ V / m

Official Ans. by NTA (1)

Sol. $f = 5 \times 10^8$ Hz

EM wave is travelling towards $+\hat{j}$

$\vec{B} = 8.0 \times 10^{-8}\hat{z}T$

$\vec{E} = \vec{B} \times \vec{C} = (8 \times 10^{-8}\hat{z}) \times (3 \times 10^8\ \hat{y})$

$= -24\hat{x}$ V / m

7. The maximum and minimum distances of a comet from the Sun are 1.6×10^{12} m and 8.0×10^{10} m respectively. If the speed of the comet at the nearest point is 6×10^4 ms^{-1}, the speed at the farthest point is :

(1) 1.5×10^3 m/s (2) 6.0×10^3 m/s
(3) 3.0×10^3 m/s (4) 4.5×10^3 m/s

Official Ans. by NTA (3)

Sol. By angular momentum conservation :

$mv_1r_1 = mv_2r_2$

$$v_1 = \frac{48 \times 10^{14}}{1.6 \times 10^{12}} = 3000 \text{ m/sec}$$

$= 3 \times 10^3$ m/sec.

8. A bar magnet of length 14 cm is placed in the magnetic meridian with its north pole pointing towards the geographic north pole. A neutral point is obtained at a distance of 18 cm from the center of the magnet. If $B_H = 0.4$ G, the magnetic moment of the magnet is (1 G = 10^{-4}T)

(1) 2.880×10^3 J T^{-1} (2) 2.880×10^2 J T^{-1}
(3) 2.880 J T^{-1} (4) 28.80 J T^{-1}

Official Ans. by NTA (3)

Sol.

i.e. $\frac{2\mu_0}{4\pi}\frac{m}{r^2} \times \frac{7}{r} = 0.4 \times 10^{-4}$

$\Rightarrow 2 \times 10^{-7} \times \frac{m \times 7}{(7^2 + 18^2)^{3/2}} \times 10^4$

$= 0.4 \times 10^{-4}$

$$m = \frac{4 \times 10^{-2} \times (373)^{3/2}}{14}$$

$$M = m \times 14 \text{ cm} = m \times \frac{14}{100}$$

$$= \frac{0.04 \times (373)^{3/2}}{14} \times \frac{14}{100}$$

$= 4 \times 10^{-4} \times 7203.82 = 2.88$ J/T

9. The volume V of an enclosure contains a mixture of three gases, 16 g of oxygen, 28 g of nitrogen and 44 g of carbon dioxide at absolute temperature T. Consider R as universal gas constant. The pressure of the mixture of gases is :

(1) $\frac{88RT}{V}$ (2) $\frac{3RT}{V}$

(3) $\frac{5}{2}\frac{RT}{V}$ (4) $\frac{4RT}{V}$

Official Ans. by NTA (3)

Sol. $PV = (n_1 + n_2 + n_3)RT$

$$P \times V = \left[\frac{16}{32} + \frac{28}{28} + \frac{44}{44}\right]RT$$

$$PV = \left[\frac{1}{2} + 1 + 1\right]RT$$

$$P = \frac{5}{2}\frac{RT}{V}$$

10. In thermodynamics, heat and work are :

(1) Path functions

(2) Intensive thermodynamic state variables

(3) Extensive thermodynamic state variables

(4) Point functions

Official Ans. by NTA (1)

Sol. Heat and work are treated as path functions in thermodynamics.

$\Delta Q = \Delta U + \Delta W$

Since work done by gas depends on type of process i.e. path and ΔU depends just on initial and final states, so ΔQ i.e. heat, also has to depend on process is path.

11. Four equal masses, m each are placed at the corners of a square of length (l) as shown in the figure. The moment of inertia of the system about an axis passing through A and parallel to DB would be :

(1) ml^2 (2) $2\,ml^2$

(3) $3\,ml^2$ (4) $\sqrt{3}\,ml^2$

Official Ans. by NTA (3)

Sol. Moment of inertia of point mass

= mass × (Perpendicular distance from axis)^2

Moment of Inertia

$$= m(0)^2 + m(l\sqrt{2})^2 + m\left(\frac{l}{\sqrt{2}}\right)^2 + m\left(\frac{l}{\sqrt{2}}\right)^2$$

$= 3\,ml^2$

12. A conducting wire of length 'l', area of cross-section A and electric resistivity ρ is connected between the terminals of a battery. A potential difference V is developed between its ends, causing an electric current.

If the length of the wire of the same material is doubled and the area of cross-section is halved, the resultant current would be :

(1) $\frac{1}{4}\frac{VA}{\rho l}$ (2) $\frac{3}{4}\frac{VA}{\rho l}$

(3) $\frac{1}{4}\frac{\rho l}{VA}$ (4) $4\frac{VA}{\rho l}$

Official Ans. by NTA (1)

Sol. As per the question

$$\text{Resistance} = \frac{\rho(2l)}{(A/2)} = \frac{4\rho l}{A}$$

$$\Rightarrow \text{Current} = \frac{V}{R} = \frac{VA}{4\rho l}$$

13. Time period of a simple pendulum is T inside a lift when the lift is stationary. If the lift moves upwards with an acceleration g/2, the time period of pendulum will be :

(1) $\sqrt{3}\,T$

(2) $\frac{T}{\sqrt{3}}$

(3) $\sqrt{\frac{3}{2}}\,T$

(4) $\sqrt{\frac{2}{3}}\,T$

Official Ans. by NTA (4)

Sol. When lift is stationary

$$T = 2\pi\sqrt{\frac{L}{g}}$$

When lift is moving upwards

$\Rightarrow$ Pseudo force acts downwards

$$\Rightarrow g_{eff} = g + \frac{g}{2} = \frac{3g}{2}$$

$\Rightarrow$ New time period

$$T' = 2\pi\sqrt{\frac{L}{g_{eff}}} = 2\pi\sqrt{\frac{2L}{3g}}$$

$$\boxed{T' = \sqrt{\frac{2}{3}}\,T}$$

14. The velocity-displacement graph describing the motion of a bicycle is shown in the figure.

The acceleration-displacement graph of the bicycle's motion is best described by :

(1)

(2)

(3)

(4)

Official Ans. by NTA (1)

Official Ans. by ALLEN (1 or Bonus)

Sol. For $0 \le x \le 200$

$v = mx + C$

$$v = \frac{1}{5}x + 10$$

$$a = \frac{vdv}{dx} = \left(\frac{x}{5}+10\right)\left(\frac{1}{5}\right)$$

$$a = \frac{x}{25}+2 \Rightarrow \text{Straight line till } x = 200$$

for $x > 200$

v = constant

$\Rightarrow a = 0$

$\Rightarrow$

Hence most approriate option will be (1), otherwise it would be BONUS.

15. A 25 m long antenna is mounted on an antenna tower. The height of the antenna tower is 75 m. The wavelength (in meter) of the signal transmitted by this antenna would be :

(1) 300 (2) 400

(3) 200 (4) 100

Official Ans. by NTA (4)

Sol. Length of Antena = 25m = $\frac{\lambda}{4}$

$$\Rightarrow \boxed{\lambda = 100\,m}$$

16. For an electromagnetic wave travelling in free space, the relation between average energy densities due to electric (U_e) and magnetic (U_m) fields is :

(1) $U_e = U_m$ (2) $U_e > U_m$

(3) $U_e < U_m$ (4) $U_e \neq U_m$

Official Ans. by NTA (1)

Sol. In EMW, Average energy density due to electric (U_e) and magnetic (U_m) fields is same.

17. An RC circuit as shown in the figure is driven by a AC source generating a square wave. The output wave pattern monitored by CRO would look close to :

(1)

(2)

(3)

(4)

Official Ans. by NTA (3)

Sol.

For $t_1 - t_2$ Charging graph

$t_2 - t_3$ Discharging graph

18. The stopping potential in the context of photoelectric effect depends on the following property of incident electromagnetic radiation :

(1) Phase (2) Intensity

(3) Amplitude (4) Frequency

Official Ans. by NTA (4)

Sol. Stopping potential changes linearly with frequency of incident radiation.

19. A block of 200 g mass moves with a uniform speed in a horizontal circular groove, with vertical side walls of radius 20 cm. If the block takes 40 s to complete one round, the normal force by the side walls of the groove is :

(1) 0.0314 N

(2) 9.859×10^{-2} N

(3) 6.28×10^{-3} N

(4) 9.859×10^{-4} N

Official Ans. by NTA (4)

Sol. $N = m\omega^2 R$

$$N = m\left[\frac{4\pi^2}{T^2}\right]R \qquad .....(1)$$

Given m = 0.2 kg, T = 40 S, R = 0.2 m

Put values in equation (1)

$N = 9.859 \times 10^{-4}$ N

20. A conducting bar of length L is free to slide on two parallel conducting rails as shown in the figure

Two resistors R_1 and R_2 are connected across the ends of the rails. There is a uniform magnetic field $\vec{B}$ pointing into the page. An external agent pulls the bar to the left at a constant speed *v*.

The correct statement about the directions of induced currents I_1 and I_2 flowing through R_1 and R_2 respectively is :

(1) Both I_1 and I_2 are in anticlockwise direction

(2) Both I_1 and I_2 are in clockwise direction

(3) I_1 is in clockwise direction and I_2 is in anticlockwise direction

(4) I_1 is in anticlockwise direction and I_2 is in clockwise direction

Official Ans. by NTA (3)

Sol.

SECTION-B

1. In the figure given, the electric current flowing through the 5 kΩ resistor is 'x' mA.

The value of x to the nearest integer is _____.

Official Ans. by NTA (3)

Sol.

=

$$I = \frac{21}{5+1+1} = 3 \text{ mA}$$

2. A fringe width of 6 mm was produced for two slits separated by 1 mm apart. The screen is placed 10 m away. The wavelength of light used is 'x' nm. The value of 'x' to the nearest integer is _____.

Official Ans. by NTA (600)

Sol. $\beta = \dfrac{\lambda D}{d}$

$$\lambda = \frac{\beta d}{D}$$

$$\lambda = \frac{6\times10^{-3}\times10^{-3}}{10}$$

$\lambda = 6 \times 10^{-7}$ m $= 600 \times 10^{-9}$ m

$\lambda = 600$ nm

3. Consider a 20 kg uniform circular disk of radius 0.2 m. It is pin supported at its center and is at rest initially. The disk is acted upon by a constant force F = 20 N through a massless string wrapped around its periphery as shown in the figure.

Suppose the disk makes n number of revolutions to attain an angular speed of 50 rad s^{-1}. The value of n, to the nearest integer, is _____. [Given : In one complete revolution, the disk rotates by 6.28 rad]

Official Ans. by NTA (20)

Sol. $\alpha = \dfrac{\tau}{I} = \dfrac{F.R.}{mR^2/2} = \dfrac{2F}{mR}$

$$\alpha = \frac{2\times200}{20\times(0.2)} = 10 \text{ rad/s}^2$$

$\omega^2 = \omega_0^2 + 2\alpha\Delta\theta$

$(50)^2 = 0^2 + 2(10)\,\Delta\theta \Rightarrow \Delta\theta = \dfrac{2500}{20}$

$\Delta\theta = 125$ rad

No. of revolution $= \dfrac{125}{2\pi} \approx 20$ revolution

4. The first three spectral lines of H-atom in the Balmer series are given λ_1, λ_2, λ_3 considering the Bohr atomic model, the wave lengths of first and third spectral lines $\left(\frac{\lambda_1}{\lambda_3}\right)$ are related by a factor of approximately 'x' × 10^{-1}. The value of x, to the nearest integer, is _____.

Official Ans. by NTA (15)

Sol. For 1st line

$$\frac{1}{\lambda_1} = Rz^2\left(\frac{1}{2^2} - \frac{1}{3^2}\right)$$

$$\frac{1}{\lambda_1} = Rz^2\,\frac{5}{36} \qquad .....(i)$$

For 3rd line

$$\frac{1}{\lambda_3} = Rz^2\left(\frac{1}{2^2} - \frac{1}{5^2}\right)$$

$$\frac{1}{\lambda_3} = Rz^2\,\frac{21}{100} \qquad .....(ii)$$

(ii) + (i)

$$\frac{\lambda_1}{\lambda_3} = \frac{21}{100} \times \frac{36}{5} = 1.512 = 15.12 \times 10^{-1}$$

$x \approx 15$

5. The value of power dissipated across the zener diode (V_z = 15 V) connected in the circuit as shown in the figure is x × 10^{-1} watt.

The value of x, to the nearest integer, is _____.

Official Ans. by NTA (5)

Sol.

Voltage across R_S = 22 – 15 = 7V

Current through R_S = I = $\frac{7}{35} = \frac{1}{5}$ A

Current through 90Ω = I_2 = $\frac{15}{90} = \frac{1}{6}$ A

Current through zener = $\frac{1}{5} - \frac{1}{6} = \frac{1}{30}$ A

Power through zener diode

P = VI

$$P = 15 \times \frac{1}{30} = 0.5 \text{ watt}$$

P = 5 × 10^{-1} watt

6. A sinusoidal voltage of peak value 250 V is applied to a series LCR circuit, in which R = 8Ω, L = 24 mH and C = 60μF. The value of power dissipated at resonant condition is 'x' kW. The value of x to the nearest integer is _____.

Official Ans. by NTA (4)

Sol. At resonance power (P)

$$P = \frac{(V_{rms})^2}{R}$$

$$P = \frac{(250/\sqrt{2})^2}{8} = 3906.25 \text{ W}$$

≈ 4 kW

7. In the logic circuit shown in the figure, if input A and B are 0 to 1 respectively, the output at Y would be 'x'. The value of x is _____.

Official Ans. by NTA (0)

Sol.

8. The resistance $R = \frac{V}{I}$, where $V = (50 \pm 2)V$ and $I = (20 \pm 0.2)A$. The percentage error in R is 'x' %. The value of 'x' to the nearest integer is _____.

Official Ans. by NTA (5)

Sol. $\frac{\Delta R}{R} \times 100 = \frac{\Delta V}{V} \times 100 + \frac{\Delta I}{I} \times 100$

% error in $R = \frac{2}{50} \times 100 + \frac{0.2}{20} \times 100$

% error in R = 4 + 1

% error in R = 5%

9. Consider a frame that is made up of two thin massless rods AB and AC as shown in the figure. A vertical force $\vec{p}$ of magnitude 100 N is applied at point A of the frame.

Suppose the force is $\vec{p}$ resolved parallel to the arms AB and AC of the frame. The magnitude of the resolved component along the arm AC is xN. The value of x, to the nearest integer, is _____.

[Given : sin(35°) = 0.573, cos(35°) = 0.819
sin(110°) = 0.939, cos(110°) = –0.342]

Official Ans. by NTA (82)

Sol.

Component along AC

= 100 cos 35°N

= 100 × 0.819 N

= 81.9 N

≈ 82 N

10. A ball of mass 10 kg moving with a velocity $10\sqrt{3}\,ms^{-1}$ along X-axis, hits another ball of mass 20 kg which is at rest. After collision, the first ball comes to rest and the second one disintegrates into two equal pieces. One of the pieces starts moving along Y-axis at a speed of 10 m/s. The second piece starts moving at a speed of 20 m/s at an angle θ (degree) with respect to the X-axis.

The configuration of pieces after collision is shown in the figure. The value of θ to the nearest integer is _____.

Official Ans. by NTA (30)

Sol. **Before Collision**

After Collision

From conservation of momentum along x axis;

$\vec{P}_i = \vec{P}_f$

$10 \times 10\sqrt{3} = 200 \cos \theta$

$\cos \theta = \frac{\sqrt{3}}{2}$

$\theta = 30°$

FINAL JEE–MAIN EXAMINATION – MARCH, 2021

(Held On Tuesday 16th March, 2021) TIME : 9 : 00 AM to 12 : 00 NOON

CHEMISTRY **TEST PAPER WITH ANSWER & SOLUTION**

SECTION-A

1. Given below are two statement : one is labelled as Assertion A and the other is labelled as Reason R :

Assertion A : Size of Bk^{3+} ion is less than Np^{3+} ion.

Reason R : The above is a consequence of the lanthanoid contraction.

In the light of the above statements, choose the correct answer from the options given below :

(1) A is false but R is true

(2) Both A and R are true but R is not the correct explanation of A

(3) Both A and R are true and R is the correct explanation of A

(4) A is true but R is false

Official Ans. by NTA (3)

Official Ans. by ALLEN (4)

Sol. Size of $_{97}Bk^{3+}$ ion is less than that of $_{93}Np^{3+}$ due to actinoid contraction.

As we know that in a period from left to right ionic radius decreases and in actinide series it is due to actinoid contraction.

2. Which among the following pairs of Vitamins is stored in our body relatively for longer duration ?

(1) Thiamine and Vitamin A

(2) Vitamin A and Vitamin D

(3) Thiamine and Ascorbic acid

(4) Ascorbic acid and Vitamin D

Official Ans. by NTA (2)

Sol. Vitamin-A & Vitamin-D

3. Given below are two statements :

Statement I : Both $CaCl_2.6H_2O$ and $MgCl_2.8H_2O$ undergo dehydration on heating.

Statement II : BeO is amphoteric whereas the oxides of other elements in the same group are acidic.

In the light of the above statements, choose the correct answer from the options given below :

(1) Statement I is false but statement II is true

(2) Both statement I and statement II are false

(3) Both statement I and statement II are true

(4) Statement I is true but statement II is false

Official Ans. by NTA (2)

Sol. (a) $CaCl_2.6H_2O \xrightarrow{\Delta} \underset{\text{(Anhydrous)}}{CaCl_2} + 6H_2O$

(b) $MgCl_2.8H_2O \xrightarrow{\Delta} MgO + 2HCl + 6H_2O$

The dehydration of hydrated chloride of calcium can be achieved. The corresponding hydrated chloride of magnesium on heating suffer hydrolysis.

(c) BeO → Amphoteric

$\left.\begin{array}{l} MgO \\ CaO \\ SrO \\ BaO \end{array}\right\} \Rightarrow$ All are basic oxide

4.

The product "P" in the above reaction is :

(1)

(2)

(3)

(4)

Official Ans. by NTA (2)

Sol.

DIBAL can not reduce double bond

It can reduce cyclic ester.

5. Match List-I with List-II :

List-I Industrial process	List-II Application
(a) Haber's process	(i) HNO_3 synthesis
(b) Ostwald's process	(ii) Aluminium extraction
(c) Contact process	(iii) NH_3 synthesis
(d) Hall-Heroult process	(iv) H_2SO_4 synthesis

Choose the correct answer from the options given below :

(1) (a)-(ii), (b)-(iii), (c)-(iv), (d)-(i)

(2) (a)-(iii), (b)-(iv), (c)-(i), (d)-(ii)

(3) (a)-(iii), (b)-(i), (c)-(iv), (d)-(ii)

(4) (a)-(iv), (b)-(i), (c)-(ii), (d)-(iii)

Official Ans. by NTA (3)

Sol. (a) Haber's process is used for NH_3 synthesis.

(b) Ostwald's process is used for HNO_3 synthesis.

(c) Contact process is used for H_2SO_4 synthesis.

(d) In Hall-Heroult process, electrolytic reduction of impure alumina can be done. (Aluminium extraction)

6. Among the following, the aromatic compounds are :

(A)

(B)

(C)

(D)

Choose the correct answer from the following options :

(1) (A) and (B) only

(2) (B) and (C) only

(3) (B), (C) and (D) only

(4) (A), (B) and (C) only

Official Ans. by NTA (2)

Sol. (A) Non-Aromatic (B) Aromatic

(C) Aromatic (D) Anti-Aromatic

7.

In the above chemical reaction, intermediate "X" and reagent/condition "A" are :

(1) X-

; A- H_2O/NaOH

(2) X-

; A- H_2O/Δ

(3) X-

; A- H_2O/Δ

(4) X-

; A- H_2O/NaOH

Official Ans. by NTA (3)

Sol.

8. Given below are two statements :

Statement I : The E° value of Ce^{4+} / Ce^{3+} is + 1.74 V.

Statement II : Ce is more stable in Ce^{4+} state than Ce^{3+} state.

In the light of the above statements, choose the most appropriate answer from the options given below :

(1) Both statement I and statement II are correct

(2) Statement I is incorrect but statement II is correct

(3) Both statement I and statement II are incorrect

(4) Statement I is correct but statement II is incorrect

Official Ans. by NTA (4)

Sol. The E° value for Ce^{4+}/Ce^{3+} is +1.74 V because the most stable oxidation state of lanthanide series elements is +3.
It means Ce^{3+} is more stable than Ce^{4+}.

9. The functions of antihistamine are :

(1) Antiallergic and Analgesic

(2) Antacid and antiallergic

(3) Analgesic and antacid

(4) Antiallergic and antidepressant

Official Ans. by NTA (2)

10. Which of the following is Lindlar catalyst ?

(1) Zinc chloride and HCl

(2) Cold dilute solution of $KMnO_4$

(3) Sodium and Liquid NH_3

(4) Partially deactivated palladised charcoal

Official Ans. by NTA (4)

Sol. Partially deactivated palladised charcoal (H_2/pd/$CaCO_3$) is lindlar catalyst.

11.

The product "A" and "B" formed in above reactions are :

(1) A- B-

(2) A- B-

(3) A- B-

(4) A- B-

Official Ans. by NTA (3)

Sol.

12. Given below are two statements :
Statement I : H_2O_2 can act as both oxidising and reducing agent in basic medium.
Statement II : In the hydrogen economy, the energy is transmitted in the form of dihydrogen.
In the light of the above statements, choose the correct answer from the options given below :
(1) Both statement I and statement II are false
(2) Both statement I and statement II are true
(3) Statement I is true but statement II is false
(4) Statement I is false but statement II is true
Official Ans. by NTA (2)

Sol. (a) H_2O_2 can acts as both oxidising and reducing agent in basic medium.
(i) $2Fe^{2+} + H_2O_2 \rightarrow 2Fe^{3+} + 2OH^-$
In this reaction, H_2O_2 acts as oxiding agent.
(ii) $2\overset{+7}{M}nO_4^- + 3H_2O_2 \rightarrow 2\overset{+4}{M}nO_2 + 3O_2 + 2H_2O + 2OH^-$
In this reaction, H_2O_2 acts as reducing agent.
(b) The basic principle of hydrogen economy is the transportation and storage of energy in the form of liquids or gaseous dihydrogen.
Advantage of hydrogen economy is that energy is transmitted in the form of dihydrogen and not as electric power

13. The type of pollution that gets increased during the day time and in the presence of O_3 is :
(1) Reducing smog (2) Oxidising smog
(3) Global warming (4) Acid rain
Official Ans. by NTA (2)

Sol. In presence of ozone(O_3), oxidising smog gets increased during the day time because automobiles and factories produce main components of the photochemcial smog (oxidising smog) results from the action of sunlight on unsaturated hydrocarbon and nitrogen oxide.
Ozone is strong oxidising agent and can react with the unburnt hydrocarbons in the polluted air to produce chemicals.

14. Assertion A : Enol form of acetone $[CH_3COCH_3]$ exists in < 0.1% quantity. However, the enol form of acetyl acetone $[CH_3COCH_2OCCH_3]$ exists in approximately 15% quantity.

Reason R : enol form of acetyl acetone is stabilized by intramolecular hydrogen bonding, which is not possible in enol form of acetone.

Choose the correct statement :

(1) A is false but R is true

(2) Both A and R are true and R is the correct explanation of A

(3) Both A and R are true but R is not the correct explanation of A

(4) A is true but R is false

Official Ans. by NTA (2)

Sol.

enol from of acetone is very less (< 0.1 %)

enol from (more than 50%)

15. Which of the following reaction DOES NOT involve Hoffmann Bromamide degradation ?

(1)

(2)

(3)

(4)

Official Ans. by NTA (3)

Sol.

⇒ This reaction does not involve haffmann bromanide degradation.

⇒ Rest all options involve haffmann bromamide degradation during the reaction of Br_2+NaOH with amide.

16. The process that involves the removal of sulphur from the ores is :

(1) Smelting

(2) Roasting

(3) Leaching

(4) Refining

Official Ans. by NTA (2)

Sol. In roasting process, metal sulphide (MS) ore are converted into metal oxide and sulphur is remove in the form of SO_2 gas.

$$2MS + 3O_2 \xrightarrow{\Delta} 2MO + 2SO_2\uparrow$$

17. Match List-I with List-II :

	List-I	**List-II**
	Name of oxo acid	Oxidation state of 'P'
(a)	Hypophosphorous acid	(i) +5
(b)	Orthophosphoric acid	(ii) +4
(c)	Hypophosphoric acid	(iii) +3
(d)	Orthophosphorous acid	(iv) +2
		(v) +1

Choose the correct answer from the options given below :

(1) (a)-(v), (b)-(i), (c)-(ii), (d)-(iii)

(2) (a)-(iv), (b)-(i), (c)-(ii), (d)-(iii)

(3) (a)-(iv), (b)-(v), (c)-(ii), (d)-(iii)

(4) (a)-(v), (b)-(iv), (c)-(ii), (d)-(iii)

Official Ans. by NTA (1)

Sol. (a) Hypophosphorus acid : $H_3\underline{P}O_2$

$(+1)\ 3 + x + (-2)2 = 0$

$x = +1$

(b) Orthophosphoric acid : $H_3\underline{P}O_4$

$(+1)\ 3 + x + (-2)4 = 0$

$x = +5$

(c) Hypophosphoric acid : $H_4P_2O_6$

$(+1)\ 4 + 2x + (-2)6 = 0$

$x = +4$

(d) Orthophosphorous acid : $H_3\underline{P}O_3$

$(+1)3 + x + (-2)3 = 0$

$x = +3$

18. Given below are two statements : one is labelled as Assertion A and the other is labelled as Reason R :

Assertion A : The H–O–H bond angle in water molecule is 104.5°.

Reason R : The lone pair – lone pair repulsion of electrons is higher than the bond pair - bond pair repulsion.

(1) A is false but R is true

(2) Both A and R are true, but R is not the correct correct explanation of A

(3) A is true but R is false

(4) Both A and R are true, and R is the correct explanation of A

Official Ans. by NTA (4)

Sol. H_2O

$\theta = 104.5°$

the hybridisation of oxygen is water molecule is sp^3.

So electron geometry of water molecule is tetrahedral and the bond angle should be 109°28" but as we know that lone pair-lone pair repulsion of electrons is higher than the bond pair-bond pair repulsion because lone pair is occupied more space areound central atom than that of bond pair.

19. In chromotography technique, the purification of compound is independent of :

(1) Mobility or flow of solvent system

(2) Solubility of the compound

(3) Length of the column or TLC Plate

(4) Physical state of the pure compound

Official Ans. by NTA (4)

Sol. In chromotography technique, the purification of a compound is independent of the physical state of the pure compound.

20. A group 15 element, which is a metal and forms a hydride with strongest reducing power among group 15 hydrides. The element is :

(1) Sb (2) P (3) As (4) Bi

Official Ans. by NTA (4)

Sol. In group 15

N, P] → Non metal

As, Sb] → Metalloid

Bi] → Metal

Hydrides of group 15 elements are

NH_3

PH_3

AsH_3

SbH_3

BiH_3

In NH_3, hydrogen atom gets partial positive charge due to less electronegativity.

But in BiH_3, hydrogen atom gets partial negative charge because hydrogen is more electronegative than bismuth.

i.e. BiH_3 is a strong reducing agent than others because we know that H^- is a strong reducing agent.

SECTION-B

1. For the reaction $A(g) \rightleftharpoons B(g)$ at 495 K, $\Delta_rG^o = -9.478$ kJ mol^{-1}.

If we start the reaction in a closed container at 495 K with 22 millimoles of A, the amount of B is the equilibrium mixture is _____ millimoles. (Round off to the Nearest Integer).

[R = 8.314 J mol^{-1} K^{-1}; ℓn 10 = 2.303]

Official Ans. by NTA (20)

Sol. $\Delta G° = -RT \ell n K_{eq}$

Given $\Delta G° = -9.478$ KJ/mole

$T = 495K \quad R = 8.314 \text{ J mol}^{-1}$

So $-9.478 \times 10^3 = -495 \times 8.314 \times \ell n K_{eq}$

$\ell n K_{eq} = 2.303$

$= \ell n 10$

So $K_{eq} = 10$

Now $A(g) \rightleftharpoons B(g)$

t = 0 22 0

t = t 22–x x

$$K_{eq} = \frac{[B]}{[C]} = \frac{x}{22-x} = 10$$

or x = 20

So millmoles of B = 20

2. Complete combustion of 750 g of an organic compound provides 420 g of CO_2 and 210 g of H_2O. The percentage composition of carbon and hydrogen in organic compound is 15.3 and _____ respectively. (Round off to the Nearest Integer)

Official Ans. by NTA (3)

Sol. 44 gm CO_2 have 12 gm carbon

So, 420 gm $CO_2 \Rightarrow \frac{12}{44} \times 420$

$\Rightarrow \frac{1260}{11}$ gm carbon

$\Rightarrow$ 114.545 gram carbon

So, % of carbon = $\frac{114.545}{750} \times 100$

$\simeq 15.3\%$

18 gm $H_2O \Rightarrow$ 2 gm H_2

210 gm $\Rightarrow \frac{2}{18} \times 210$

= 23.33 gm H_2

So, % $H_2 \Rightarrow \frac{23.33}{750} \times 100 = 3.11\%$

$\approx 3\%$

3. $2\, MnO_4^- + b\, C_2O_4^{2-} + c\, H^+ \rightarrow x\, Mn^{2+} + y\, CO_2 + z\, H_2O$

If the above equation is balanced with integer coefficients, the value of c is _____. (Round off to the Nearest Integer).

Official Ans. by NTA (16)

Sol. Writting the half reaction

oxidation half reaction

$MnO_4^- \rightarrow Mn^{2+}$

balancing oxygen

$MnO_4^- \rightarrow Mn^{2+} + 4H_2O$

balancing Hydrogen

$8H^+ + MnO_4^- \rightarrow Mn^{2+} + 4H_2O$

balancing charge

$5e^- + 8H^+ + MnO_4^- \rightarrow Mn^{2+} + 4H_2O$

Reduction half

$C_2O_4^{2-} \rightarrow CO_2$

Balancing carbon

$C_2O_4^{2-} \rightarrow 2CO_2$

Balancing charge

$C_2O_4^{2-} \rightarrow 2CO_2 + 2e^-$

Net equation

$16H^+ + 2MnO_4^- + 5C_2O_4^{2-} \rightarrow 10CO_2 + 2Mn^{2+} + 8H_2O$

So c = 16

4. AB_2 is 10% dissociated in water to A^{2+} and B^-. The boiling point of a 10.0 molal aqueous solution of AB_2 is _____°C. (Round off to the Nearest Integer).

[Given : Molal elevation constant of water $K_b = 0.5$ K kg mol^{-1} boiling point of pure water = 100°C]

Official Ans. by NTA (106)

Sol. $AB_2 \rightarrow A^{2+} + 2B^-$

t = 0 a 0 0

t = t a –aα aα 2aα

$n_T = a - a\alpha + a\alpha + 2a\alpha$

$= a(1 + 2\alpha)$

so $i = 1 + 2\alpha$

Now $\Delta T_b = i \times m \times K_b$

$\Delta T_b = (1 + 2\alpha) \times m \times K_b$

$\alpha = 0.1 \quad m = 10 \quad K_b = 0.5$

$\Delta T_b = 1.2 \times 10 \times 0.5$

$= 6$

So boiling point = 106

5. The equivalents of ethylene diamine required to replace the neutral ligands from the coordination sphere of the trans-complex of $CoCl_3.4NH_3$ is _____. (Round off to the Nearest Integer).

Official Ans. by NTA (2)

Sol. trans - $CoCl_3.4NH_3$

or

trans-$[Co(NH_3)_4Cl_2]C\ell$

As we know that ethylene diamine is a bidentate ligand and ammonia is a mono dentate ligand.

It means overall two ethylene diamine is required to replace the all neutral ligands (four ammonia) from the coordination sphere of this complex.

6. A 6.50 molal solution of KOH (aq.) has a density of 1.89 g cm^{-3}. The molarity of the solution is _____ mol dm^{-3}. (Round off to the Nearest Integer).

[Atomic masses: K :39.0 u; O :16.0 u; H :1.0 u]

Official Ans. by NTA (9)

Sol. 6.5 molal KOH = 1000gm solvent has 6.5 moles KOH

so wt of solute = 6.5 × 56

= 364 gm

wt of solution = 1000 + 364 = 1364

Volume of solution = $\frac{1364}{1.89} m\ell$

Molarity = $\frac{\text{mole of solute}}{V_{solution} \text{ in Litre}}$

$= \frac{6.5 \times 1.89 \times 1000}{1364}$

= 9.00

7. When light of wavelength 248 nm falls on a metal of threshold energy 3.0 eV, the de-Broglie wavelength of emitted electrons is _____ Å. (Round off to the Nearest Integer).

[Use : $\sqrt{3}$ = 1.73, h = 6.63 × 10^{-34} Js

m_e = 9.1 × 10^{-31} kg ; c = 3.0 × 10^8 ms^{-1} ; 1eV = 1.6 × 10^{-19}J]

Official Ans. by NTA (9)

Sol. Energy incident = $\frac{hc}{\lambda}$

$= \frac{6.63 \times 10^{-34} \times 3.0 \times 10^8}{248 \times 10^{-9} \times 1.6 \times 10^{-19}}$ eV

$= \frac{6.63 \times 3 \times 100}{248 \times 1.6}$

= 0.05 eV × 100 = 5 eV

Now using

E = ϕ + K.E.

5 = 3 + K.E.

K.E. = 2eV = 3.2 × 10^{-19} J

for debroglie wavelength $\lambda = \frac{h}{mv}$

K.E = $\frac{1}{2}mv^2$

so $v = \sqrt{\frac{2KE}{m}}$

hence $\lambda = \frac{h}{\sqrt{2KE \times m}}$

$= \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 3.2 \times 10^{-19} \times 9.1 \times 10^{-31}}}$

$= \frac{6.63}{7.6} \times \frac{10^{-34}}{10^{-25}} = \frac{66.3 \times 10^{-10} m}{7.6}$

= 8.72 × 10^{-10} m

$\approx 9 \times 10^{-10} m$

$= 9Å$

8. Two salts A_2X and MX have the same value of solubility product of 4.0×10^{-12}. The ratio of their molar solubilities i.e. $\frac{S(A_2X)}{S(MX)}$ = _____.

(Round off to the Nearest Integer).

Official Ans. by NTA (50)

Sol. For A_2X

$A_2X \rightarrow 2A^+ + X^{2-}$

$2S_1 \quad S_1$

$K_{sp} = 4S_1^3 = 4 \times 10^{-12}$

$S_1 = 10^{-4}$

for MX

$MX \rightarrow M^+ + X^-$

$S_2 \quad S_2$

$K_{sp} = S_2^2 = 4 \times 10^{-12}$

$S_2 = 2 \times 10^{-6}$

so $\frac{S_{A_2X}}{S_{MX}} = \frac{10^{-4}}{2 \times 10^{-6}} = 50$

9. A certain element crystallises in a bcc lattice of unit cell edge length 27 Å. If the same element under the same conditions crystallises in the fcc lattice, the edge length of the unit cell in Å will be _____. (Round off to the Nearest Integer).

[Assume each lattice point has a single atom]

[Assume $\sqrt{3} = 1.73$, $\sqrt{2} = 1.41$]

Official Ans. by NTA (33)

Sol. For BCC $\sqrt{3}\,a = 4r$

so $r = \frac{\sqrt{3}}{4} \times 27$

for FCC $a = 2\sqrt{2}\,r$

$= 2 \times \sqrt{2} \times \frac{\sqrt{3}}{4} \times 27$

$= \frac{\sqrt{3}}{\sqrt{2}} \times 27$

$= 33$

10. The decomposition of formic acid on gold surface follows first order kinetics. If the rate constant at 300 K is $1.0 \times 10^{-3}\ s^{-1}$ and the activation energy $E_a = 11.488$ kJ mol^{-1}, the rate constant at 200 K is _____ $\times 10^{-5}\ s^{-1}$. (Round of to the Nearest Integer).

(Given : R = 8.314 J mol^{-1} K^{-1})

Official Ans. by NTA (10)

Sol. $K_{300} = 10^{-4}$ $\quad K_{200} = ?$

$E_a = 11.488$ KJ/mole $\quad R = 8.314$ J/mole-K

so $\ell n\left(\frac{K_{300}}{K_{200}}\right) = \frac{E_a}{R}\left(\frac{1}{200} - \frac{1}{300}\right)$

$\ell n\left(\frac{K_{300}}{K_{200}}\right) = \frac{11.488 \times 1000 \times 100}{8.314 \times 200 \times 300}$

$= 2.303$

$= \ell n10$

so $\frac{K_{300}}{K_{200}} = 10$

$K_{200} = \frac{1}{10} \times K_{300} = 10^{-4}$

$= 10 \times 10^{-5}\ sec^{-1}$

FINAL JEE–MAIN EXAMINATION – MARCH, 2021

(Held On Tuesday 16th March, 2021) TIME : 9 : 00 AM to 12 : 00 NOON

MATHEMATICS **TEST PAPER WITH SOLUTION**

SECTION-A

1. The number of elements in the set $\{x \in \mathbb{R} : (|x| - 3)\,|x + 4| = 6\}$ is equal to

(1) 3 (2) 2 (3) 4 (4) 1

Official Ans. by NTA (2)

Sol. $x \neq -4$

$(|x| - 3)(|x + 4|) = 6$

$\Rightarrow \quad |x| - 3 = \dfrac{6}{|x+4|}$

No. of solutions = 2

2. Let a vector $\alpha\hat{i} + \beta\hat{j}$ be obtained by rotating the vector $\sqrt{3}\hat{i} + \hat{j}$ by an angle 45° about the origin in counterclockwise direction in the first quadrant. Then the area of triangle having vertices (α, β), $(0, \beta)$ and $(0, 0)$ is equal to

(1) $\frac{1}{2}$ (2) 1 (3) $\frac{1}{\sqrt{2}}$ (4) $2\sqrt{2}$

Official Ans. by NTA (1)

Sol.

Area of $\Delta(OA'B) = \frac{1}{2} OA'\cos 15° \times OA'\sin 15°$

$= \frac{1}{2}(OA')^2 \frac{\sin 30°}{2}$

$= (3+1) \times \frac{1}{8} = \frac{1}{2}$

3. If for $a > 0$, the feet of perpendiculars from the points $A(a, -2a, 3)$ and $B(0, 4, 5)$ on the plane $lx + my + nz = 0$ are points $C(0, -a, -1)$ and D respectively, then the length of line segment CD is equal to :

(1) $\sqrt{31}$ (2) $\sqrt{41}$

(3) $\sqrt{55}$ (4) $\sqrt{66}$

Official Ans. by NTA (4)

Sol.

C lies on plane $\Rightarrow -ma - n = 0 \Rightarrow \frac{m}{n} = -\frac{1}{a}$ ....(1)

$\overrightarrow{CA} \parallel l\hat{i} + m\hat{j} + n\hat{k}$

$\frac{a-0}{l} = \frac{-a}{m} = \frac{4}{n} \Rightarrow \frac{m}{n} = -\frac{a}{4}$ ....(2)

From (1) & (2)

$-\frac{1}{a} = \frac{-a}{4} \Rightarrow a^2 = 4 \Rightarrow a = 2$ (since $a > 0$)

From (2) $\frac{m}{n} = \frac{-1}{2}$

Let $m = -t \Rightarrow n = 2t$

$\frac{2}{l} = \frac{-2}{-t} \Rightarrow l = t$

So plane : $t(x - y + 2z) = 0$

$BD = \frac{6}{\sqrt{6}} = \sqrt{6}$ $\quad C \cong (0, -2, -1)$

$CD = \sqrt{BC^2 - BD^2}$

$= \sqrt{(0^2 + 6^2 + 6^2) - (\sqrt{6})^2}$

$= \sqrt{66}$

4. Consider three observations a, b and c such that b = a + c. If the standard deviation of a + 2, b + 2, c + 2 is d, then which of the following is true ?

(1) $b^2 = 3(a^2 + c^2) + 9d^2$

(2) $b^2 = a^2 + c^2 + 3d^2$

(3) $b^2 = 3(a^2 + c^2 + d^2)$

(4) $b^2 = 3(a^2 + c^2) - 9d^2$

Official Ans. by NTA (4)

Sol. For a, b, c

mean = $\frac{a+b+c}{3}$ $(=\overline{x})$

b = a + c

$\Rightarrow \quad \overline{x} = \frac{2b}{3}$ .....(1)

S.D. (a + 2, b + 2, c + 2) = S.D. (a, b, c) = d

$\Rightarrow \quad d^2 = \frac{a^2+b^2+c^2}{3} - (\overline{x})^2$

$\Rightarrow \quad d^2 = \frac{a^2+b^2+c^2}{3} - \frac{4b^2}{9}$

$\Rightarrow \quad 9d^2 = 3(a^2 + b^2 + c^2) - 4b^2$

$\Rightarrow \quad b^2 = 3(a^2 + c^2) - 9d^2$

5. If for $x \in \left(0, \frac{\pi}{2}\right)$, $\log_{10}\sin x + \log_{10}\cos x = -1$

and $\log_{10}(\sin x + \cos x) = \frac{1}{2}(\log_{10}n - 1)$, n > 0,

then the value of n is equal to :

(1) 20 (2) 12 (3) 9 (4) 16

Official Ans. by NTA (2)

Sol. $x \in \left(0, \frac{\pi}{2}\right)$

$\log_{10}\sin x + \log_{10}\cos x = -1$

$\Rightarrow \quad \log_{10}\sin x.\cos x = -1$

$\Rightarrow \quad \sin x.\cos x = \frac{1}{10}$ .....(1)

$\log_{10}(\sin x + \cos x) = \frac{1}{2}(\log_{10} n - 1)$

$\Rightarrow \quad \sin x + \cos x = 10^{\left(\log_{10}\sqrt{n} - \frac{1}{2}\right)} = \sqrt{\frac{n}{10}}$

by squaring

$1 + 2\sin x.\cos x = \frac{n}{10}$

$\Rightarrow \quad 1 + \frac{1}{5} = \frac{n}{10} \quad \Rightarrow \quad n = 12$

6. Let $A = \begin{bmatrix} i & -i \\ -i & i \end{bmatrix}$, $i = \sqrt{-1}$ .Then, the system of linear equations $A^8\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ 64 \end{bmatrix}$ has :

(1) A unique solution

(2) Infinitely many solutions

(3) No solution

(4) Exactly two solutions

Official Ans. by NTA (3)

Sol. $A = \begin{bmatrix} i & -i \\ -i & i \end{bmatrix}$

$A^2 = \begin{bmatrix} -2 & 2 \\ 2 & -2 \end{bmatrix} = 2\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}$

$A^4 = 2^2\begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} = 8\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$

$A^8 = 64\begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} = 128\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$

$A^8\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ 64 \end{bmatrix}$

$\Rightarrow \quad 128\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ 64 \end{bmatrix}$

$\Rightarrow \quad 128\begin{bmatrix} x-y \\ -x+y \end{bmatrix} = \begin{bmatrix} 8 \\ 64 \end{bmatrix}$

$\Rightarrow \quad x - y = \frac{1}{16}$ .....(1)

& $\quad -x + y = \frac{1}{2}$ .....(2)

$\Rightarrow$ From (1) & (2) : No solution.

7. If the three normals drawn to the parabola, $y^2 = 2x$ pass through the point (a, 0) $a \neq 0$, then 'a' must be greater than :

(1) $\frac{1}{2}$ (2) $-\frac{1}{2}$ (3) -1 (4) 1

Official Ans. by NTA (4)

Sol. For standard parabola

For more than 3 normals (on axis)

$x > \frac{L}{2}$ (where L is length of L.R.)

For $y^2 = 2x$
L.R. = 2
for (a, 0)

$$a > \frac{\text{L.R.}}{2} \Rightarrow a > 1$$

8. Let the position vectors of two points P and Q be $3\hat{i} - \hat{j} + 2\hat{k}$ and $\hat{i} + 2\hat{j} - 4\hat{k}$, respectively. Let R and S be two points such that the direction ratios of lines PR and QS are (4, –1, 2) and (–2, 1, –2), respectively. Let lines PR and QS intersect at T. If the vector $\overrightarrow{TA}$ is perpendicular to both $\overrightarrow{PR}$ and $\overrightarrow{QS}$ and the length of vector $\overrightarrow{TA}$ is $\sqrt{5}$ units, then the modulus of a position vector of A is :

(1) $\sqrt{482}$ (2) $\sqrt{171}$

(3) $\sqrt{5}$ (4) $\sqrt{227}$

Official Ans. by NTA (2)

Sol. P(3, –1, 2)
Q(1, 2, –4)

$\overrightarrow{PR} \parallel 4\hat{i} - \hat{j} + 2\hat{k}$

$\overrightarrow{QS} \parallel -2\hat{i} + \hat{j} - 2\hat{k}$

dr's of normal to the plane containing P, T & Q will be proportional to :

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -1 & 2 \\ -2 & 1 & -2 \end{vmatrix}$$

$$\therefore \quad \frac{\ell}{0} = \frac{m}{4} = \frac{n}{2}$$

For point, T : $\overrightarrow{PT} = \frac{x-3}{4} = \frac{y+1}{-1} = \frac{z-2}{2} = \lambda$

$$\overrightarrow{QT} = \frac{x-1}{-2} = \frac{y-1}{1} = \frac{z+4}{-2} = \mu$$

T : $(4\lambda + 3, -\lambda - 1, 2\lambda + 2)$
$\cong (2\mu + 1, \mu + 2, -2\mu - 4)$

$4\lambda + 3 = -2\mu + 1 \Rightarrow 2\lambda + \mu = -1$
$\lambda + \mu = -3 \Rightarrow \lambda = 2$
& $\mu = -5$ $\lambda + \mu = -3 \Rightarrow \lambda = 2$
So point T : (11, –3, 6)

$$\overrightarrow{OA} = \left(11\hat{i} - 3\hat{j} + 6\hat{k}\right) \pm \left(\frac{2\hat{j} + \hat{k}}{\sqrt{5}}\right)\sqrt{5}$$

$$\overrightarrow{OA} = \left(11\hat{i} - 3\hat{j} + 6\hat{k}\right) \pm \left(2\hat{j} + \hat{k}\right)$$

$$\overrightarrow{OA} = 11\hat{i} - \hat{j} + 7\hat{k}$$

or

$$9\hat{i} - 5\hat{j} + 5\hat{k}$$

$$|\overrightarrow{OA}| = \sqrt{121 + 1 + 49} = \sqrt{171}$$

or

$$\sqrt{81 + 25 + 25} = \sqrt{131}$$

9. Let the functions $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ be defined as :

$$f(x) = \begin{cases} x + 2, & x < 0 \\ x^2, & x \geq 0 \end{cases} \text{ and } g(x) = \begin{cases} x^3, & x < 1 \\ 3x - 2, & x \geq 1 \end{cases}$$

Then, the number of points in $\mathbb{R}$ where $(fog)(x)$ is NOT differentiable is equal to :

(1) 3 (2) 1

(3) 0 (4) 2

Official Ans. by NTA (2)

Sol. $$f(g(x)) = \begin{cases} g(x) + 2, & g(x) < 0 \\ (g(x))^2, & g(x) \geq 0 \end{cases}$$

$$= \begin{cases} x^3 + 2, & x < 0 \\ x^6, & x \in [0,1) \\ (3x-2)^2, & x \in [1, \infty) \end{cases}$$

$$(fog(x))' = \begin{cases} 3x^2, & x < 0 \\ 6x^5, & x \in (0,1) \\ 2(3x-2) \times 3, & x \in (1, \infty) \end{cases}$$

At 'O'
L.H.L. ≠ R.H.L. (Discontinuous)
At '1'
L.H.D. = 6 = R.H.D.
$\Rightarrow$ $fog(x)$ is differentiable for $x \in \mathbb{R} - \{0\}$

10. Which of the following Boolean expression is a tautology ?

(1) $(p \wedge q) \vee (p \vee q)$

(2) $(p \wedge q) \vee (p \rightarrow q)$

(3) $(p \wedge q) \wedge (p \rightarrow q)$

(4) $(p \wedge q) \rightarrow (p \rightarrow q)$

Official Ans. by NTA (4)

Sol.

p	q	$p \wedge q$	$p \rightarrow q$	$(p \wedge q) \rightarrow (p \rightarrow q)$
T	T	T	T	T
T	F	F	F	T
F	T	F	T	T
F	F	F	T	T

$(p \wedge q) \rightarrow (p \rightarrow q)$ is tautology

11. Let a complex number z, $|z| \neq 1$, satisfy $\log_{\frac{1}{\sqrt{2}}}\left(\frac{|z|+11}{(|z|-1)^2}\right) \leq 2$. Then, the largest value of $|z|$ is equal to _____ .

(1) 8 (2) 7 (3) 6 (4) 5

Official Ans. by NTA (2)

Sol. $\log_{\frac{1}{\sqrt{2}}}\left(\frac{|z|+11}{(|z|-1)^2}\right) \leq 2$

$\frac{|z|+11}{(|z|-1)^2} \geq \frac{1}{2}$

$2|z| + 22 \geq (|z| - 1)^2$

$2|z| + 22 \geq |z|^2 + 1 - 2|z|$

$|z|^2 - 4|z| - 21 \leq 0$

$\Rightarrow \quad |z| \leq 7$

$\therefore$ Largest value of $|z|$ is 7

12. If n is the number of irrational terms in the expansion of $(3^{1/4} + 5^{1/8})^{60}$, then$(n - 1)$ is divisible by :

(1) 26 (2) 30 (3) 8 (4) 7

Official Ans. by NTA (1)

Sol. $(3^{1/4} + 5^{1/8})^{60}$

${}^{60}C_r(3^{1/4})^{60-r}.(5^{1/8})^r$

${}^{60}C_r(3)^{\frac{60-r}{4}}.5^{\frac{r}{8}}$

For rational terms.

$\frac{r}{8} = k$; $\quad 0 \leq r \leq 60$

$0 \leq 8k \leq 60$

$0 \leq k \leq \frac{60}{8}$

$0 \leq k \leq 7.5$

k = 0, 1, 2, 3, 4, 5, 6, 7

$\frac{60-8k}{4}$ is always divisible by 4 for all value of k.

Total rational terms = 8

Total terms = 61

irrational terms = 53

n – 1 = 53 – 1 = 52

52 is divisible by 26.

13. Let P be a plane $lx + my + nz = 0$ containing the line, $\frac{1-x}{1} = \frac{y+4}{2} = \frac{z+2}{3}$. If plane P divides the line segment AB joining points A(–3, –6, 1) and B(2, 4, –3) in ratio k : 1 then the value of k is equal to :

(1) 1.5 (2) 3 (3) 2 (4) 4

Official Ans. by NTA (3)

Sol.

Point C is

$\left(\frac{2k-3}{k+1}, \frac{4k-6}{k+1}, \frac{-3k+1}{k+1}\right)$

$\frac{x-1}{-1} = \frac{y+4}{2} = \frac{z+2}{3}$

Plane $lx + my + nz = 0$

$l(-1) + m(2) + n(3) = 0$

$-l + 2m + 3n = 0$ .....(1)

It also satisfy point (1, –4, –2)

$l - 4m - 2n = 0$ .....(2)

Solving (1) and (2)

$2m + 3n = 4m + 2n$

$n = 2m$

$l - 4m - 4m = 0$

$l = 8m$

$\frac{l}{8} = \frac{m}{1} = \frac{n}{2}$

$l : m : n = 8 : 1 : 2$

Plane is $8x + y + 2z = 0$

It will satisfy point C

$8\left(\frac{2k-3}{k+1}\right) + \left(\frac{4k-6}{k+1}\right) + 2\left(\frac{-3k+1}{k+1}\right) = 0$

$16k - 24 + 4k - 6 - 6k + 2 = 0$

$14k = 28 \quad \therefore \quad k = 2$

14. The range of $a \in \mathbb{R}$ for which the function

$$f(x) = (4a-3)(x+\log_e 5) + 2(a-7)\cot\left(\frac{x}{2}\right)\sin^2\left(\frac{x}{2}\right),$$

$x \neq 2n\pi, n \in \mathbb{N}$, has critical points, is :

(1) $(-3, 1)$ (2) $\left[-\frac{4}{3}, 2\right]$

(3) $[1, \infty)$ (4) $(-\infty, -1]$

Official Ans. by NTA (2)

Sol. $f(x) = (4a - 3)(x + \log_e 5) + (a - 7)\sin x$

$f(x) = (4a - 3)(1) + (a - 7)\cos x = 0$

$$\Rightarrow \quad \cos x = \frac{3-4a}{a-7}$$

$$-1 \le \frac{3-4a}{a-7} < 1$$

$$\frac{3-4a}{a-7} + 1 \ge 0 \qquad \frac{3-4a}{a-7} < 1$$

$$\frac{3-4a+a-7}{a-7} \ge 0 \qquad \frac{3-4a}{a-7} - 1 < 0$$

$$\frac{-3a-4}{a-7} \ge 0 \qquad \frac{3-4a-a+7}{a-7} < 0$$

$$\frac{3a+4}{a-7} \le 0 \qquad \frac{-5a+10}{a-7} < 0$$

$$\frac{5a-10}{a-7} > 0$$

$$\frac{5(a-2)}{a-7} > 0$$

$$\alpha \in \left[-\frac{4}{3}, 2\right)$$

Check end point $\left[-\frac{4}{3}, 2\right)$

15. A pack of cards has one card missing. Two cards are drawn randomly and are found to be spades. The probability that the missing card is not a spade, is :

(1) $\frac{3}{4}$ (2) $\frac{52}{867}$

(3) $\frac{39}{50}$ (4) $\frac{22}{425}$

Official Ans. by NTA (3)

Sol. E_1 : Event denotes spade is missing

$$P(E_1) = \frac{1}{4};\ P(\bar{E}_1) = \frac{3}{4}$$

A : Event drawn two cards are spade

$$P(A) = \frac{\frac{1}{4}\times\left(\frac{{}^{12}C_2}{{}^{51}C_2}\right) + \frac{3}{4}\times\left(\frac{{}^{13}C_2}{{}^{51}C_2}\right) + \frac{3}{4}\times\left(\frac{{}^{13}C_2}{{}^{51}C_2}\right)}{\frac{1}{4}\times\left(\frac{{}^{12}C_2}{{}^{51}C_2}\right) + \frac{3}{4}\times\left(\frac{{}^{13}C_2}{{}^{51}C_2}\right)}$$

$$= \frac{39}{50}$$

16. Let [x] denote greatest integer less than or equal to x. If for $n \in \mathbb{N}$, $(1-x+x^3)^n = \sum_{j=0}^{3n} a_j x^j$,

then $\sum_{j=0}^{\left[\frac{3n}{2}\right]} a_{2j} + 4\sum_{j=0}^{\left[\frac{3n-1}{2}\right]} a_{2j} + 1$ is equal to :

(1) 2 (2) 2^{n-1}

(3) 1 (4) n

Official Ans. by NTA (3)

Sol. $(1-x+x^3)^n = \sum_{j=0}^{3n} a_j x^j$

$(1 - x + x^3)^n = a_0 + a_1x + a_2x^2 \ldots\ldots + a_{3n}x^{3n}$

$\sum_{j=0}^{\left[\frac{3n}{2}\right]} a_{2j}$ = Sum of $a_0 + a_2 + a_4 \ldots\ldots$

$\sum_{j=0}^{\left[\frac{3n-1}{2}\right]} a_{2j} + 1$ = Sum of $a_1 + a_3 + a_5 \ldots\ldots$

put x = 1

$1 = a_0 + a_1 + a_2 + a_3 \ldots\ldots\ldots + a_{3n}$ .....(A)

Put x = –1

$1 = a_0 - a_1 + a_2 - a_3 \ldots\ldots\ldots + (-1)^{3n}a_{3n}$ .....(B)

Solving (A) and (B)

$a_0 + a_2 + a_4 \ldots\ldots = 1$

$a_1 + a_3 + a_5 \ldots\ldots = 0$

$$\sum_{j=0}^{\left[\frac{3n}{2}\right]} a_{2j} + 4\sum_{j=0}^{\left[\frac{3n-1}{2}\right]} a_{2j+1} = 1$$

17. If y = y(x) is the solution of the differential equation, $\frac{dy}{dx}+2y\tan x=\sin x,\ y\left(\frac{\pi}{3}\right)=0$, then the maximum value of the function y(x) over $\mathbb{R}$ is equal to :

(1) 8 (2) $\frac{1}{2}$ (3) $-\frac{15}{4}$ (4) $\frac{1}{8}$

Official Ans. by NTA (4)

Sol. $\frac{dy}{dx}+2y\tan x=\sin x$

I.F. = $e^{\int 2\tan x dx}=e^{2\ell n\sec x}$

I.F. = $\sec^2 x$

$y.(\sec^2 x)=\int \sin x.\sec^2 x dx$

$y.(\sec^2 x)=\int \sec x\tan x dx$

$y.(\sec^2 x)=\sec x+C$

$x=\frac{\pi}{3};\ y=0$

$\Rightarrow$ C = –2

$\Rightarrow$ $y=\frac{\sec x-2}{\sec^2 x}=\cos x-2\cos^2 x$

$y = t - 2t^2 \Rightarrow \frac{dy}{dt}=1-4t=0 \Rightarrow t=\frac{1}{4}$

$\therefore$ max $=\frac{1}{4}-\frac{1}{8}=\frac{2-1}{8}=\frac{1}{8}$

18. The locus of the midpoints of the chord of the circle, $x^2 + y^2 = 25$ which is tangent to the hyperbola, $\frac{x^2}{9}-\frac{y^2}{16}=1$ is :

(1) $(x^2 + y^2)^2 - 16x^2 + 9y^2 = 0$

(2) $(x^2 + y^2)^2 - 9x^2 + 144y^2 = 0$

(3) $(x^2 + y^2)^2 - 9x^2 - 16y^2 = 0$

(4) $(x^2 + y^2)^2 - 9x^2 + 16y^2 = 0$

Official Ans. by NTA (4)

Sol.

Equation of chord

$y - k = -\frac{h}{k}(x-h)$

$ky - k^2 = -hx + h^2$

$hx + ky = h^2 + k^2$

$y=-\frac{hx}{k}+\frac{h^2+k^2}{k}$

tangent to $\frac{x^2}{9}-\frac{y^2}{16}=1$

$c^2 = a^2m^2 - b^2$

$\left(\frac{h^2+k^2}{k}\right)^2=9\left(-\frac{h}{k}\right)^2-16$

$(x^2 + y^2)^2 = 9x^2 - 16y^2$

19. The number of roots of the equation,

$(81)^{\sin^2 x}+(81)^{\cos^2 x}=30$

in the interval [0, π] is equal to :

(1) 3 (2) 4 (3) 8 (4) 2

Official Ans. by NTA (2)

Sol. $(81)^{\sin^2 x}+(81)^{\cos^2 x}=30$

$(81)^{\sin^2 x}+\frac{(81)^1}{(18)^{\sin^2 x}}=30$

$(81)^{\sin^2 x}=t$

$t+\frac{81}{t}=30$

$t^2 - 30t + 81 = 0$

$(t - 3)(t - 27) = 0$

$(81)^{\sin^2 x}=3^1$ or $(81)^{\sin^2 x}=3^3$

$3^{4\sin^2 x}=3^1$ or $3^{4\sin^2 x}=3^3$

$\sin^2 x=\frac{1}{4}$ or $\sin^2 x=\frac{3}{4}$

Total sol. = 4

20. Let $S_k=\sum_{r=1}^{k}\tan^{-1}\left(\frac{6^r}{2^{2r+1}+3^{2r+1}}\right)$. Then $\lim_{k\to\infty}S_k$ is equal to :

(1) $\tan^{-1}\left(\frac{3}{2}\right)$ (2) $\frac{\pi}{2}$

(3) $\cot^{-1}\left(\frac{3}{2}\right)$ (4) $\tan^{-1}(3)$

Official Ans. by NTA (3)

Sol. $S_k = \sum_{r=1}^{k} \tan^{-1}\left(\frac{6^r}{2^{2r+1}+3^{2r+1}}\right)$

Divide by 3^{2r}

$$\sum_{r=1}^{k} \tan^{-1}\left(\frac{\left(\frac{2}{3}\right)^r}{\left(\frac{2}{3}\right)^{2r}.2+3}\right)$$

$$\sum_{r=1}^{k} \tan^{-1}\left(\frac{\left(\frac{2}{3}\right)^r}{3\left(\left(\frac{2}{3}\right)^{2r+1}+1\right)}\right)$$

Let $\left(\frac{2}{3}\right)^r = t$

$$\sum_{r=1}^{k} \tan^{-1}\left(\frac{\frac{t}{3}}{1+\frac{2}{3}t^2}\right)$$

$$\sum_{r=1}^{k} \tan^{-1}\left(\frac{t-\frac{2t}{3}}{1+t.\frac{2t}{3}}\right)$$

$$\sum_{r=1}^{k}\left(\tan^{-1}(t)-\tan^{-1}\left(\frac{2t}{3}\right)\right)$$

$$\sum_{r=1}^{k}\left(\tan^{-1}\left(\frac{2}{3}\right)^r-\tan^{-1}\left(\frac{2}{3}\right)^{r+1}\right)$$

$$S_k = \tan^{-1}\left(\frac{2}{3}\right)-\tan^{-1}\left(\frac{2}{3}\right)^{k+1}$$

$$S_\infty = \lim_{k\to\infty}\left(\tan^{-1}\left(\frac{2}{3}\right)-\tan^{-1}\left(\frac{2}{3}\right)^{k+1}\right)$$

$$= \tan^{-1}\left(\frac{2}{3}\right)-\tan^{-1}(0)$$

$$\therefore \quad S_\infty = \tan^{-1}\left(\frac{2}{3}\right) = \cot^{-1}\left(\frac{3}{2}\right)$$

SECTION-B

1. Consider an arithmetic series and a geometric series having four initial terms from the set {11, 8, 21, 16, 26, 32, 4}. If the last terms of these series are the maximum possible four digit numbers, then the number of common terms in these two series is equal to _____ .

Official Ans. by NTA (3)

Sol. **GP :** 4, 8, 16, 32, 64, 128, 256, 512, 1024, 2048, 4096, 8192

AP : 11, 16, 21, 26, 31, 36

Common terms : 16, 256, 4096 only

2. Let $f : (0, 2) \to \mathbb{R}$ be defined as

$$f(x) = \log_2\left(1+\tan\left(\frac{\pi x}{4}\right)\right).$$

Then, $\lim_{n\to\infty}\frac{2}{n}\left(f\left(\frac{1}{n}\right)+f\left(\frac{2}{n}\right)+....+f(1)\right)$ is equal to _____ .

Official Ans. by NTA (1)

Sol. $E = 2\lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{n}f\left(\frac{r}{n}\right)$

$$E = \frac{2}{\ell n2}\int_0^1 \ell n\left(1+\tan\frac{\pi x}{4}\right)dx \qquad .....(i)$$

replacing $x \to 1 - x$

$$E = \frac{2}{\ell n2}\int_0^1 \ell n\left(1+\tan\frac{\pi}{4}(1-x)\right)dx$$

$$E = \frac{2}{\ell n2}\int_0^1 \ell n\left(1+\tan\left(\frac{\pi}{4}-\frac{\pi}{4}x\right)\right)dx$$

$$E = \frac{2}{\ell n2}\int_0^1 \ell n\left(1+\frac{1+\tan\frac{\pi}{4}x}{1+\tan\frac{\pi}{4}x}\right)dx$$

$$E = \frac{2}{\ell n2}\int_0^1 \ell n\left(\frac{2}{1+\tan\frac{\pi x}{4}}\right)dx$$

$$E = \frac{2}{\ell n2}\int_0^1 \left(\ell n2-\ell n\left(1+\tan\frac{\pi x}{4}\right)\right)dx \qquad .....(ii)$$

equation (i) + (ii)

E = 1

3. Let ABCD be a square of side of unit length. Let a circle C_1 centered at A with unit radius is drawn. Another circle C_2 which touches C_1 and the lines AD and AB are tangent to it, is also drawn. Let a tangent line from the point C to the circle C_2 meet the side AB at E. If the length of EB is $\alpha+\sqrt{3}\beta$, where α, β are integers, then α + β is equal to_____.

Official Ans. by NTA (1)

Sol.

Here AO + OD = 1 or $(\sqrt{2}+1)r = 1$

$\Rightarrow \quad r = \sqrt{2-1}$

equation of circle $(x - r)^2 + (y - r)^2 = r^2$

Equation of CE

$y - 1 = m(x - 1)$

$mx - y + 1 - M = 0$

It is tangent to circle

$$\therefore \quad \left|\frac{mr-r+1-m}{\sqrt{m^2+1}}\right| = r$$

$$\left|\frac{(m-1)r+1-m}{\sqrt{m^2+1}}\right| = r$$

$$\frac{(m-1)^2(r-1)^2}{m^2+1} = r^2$$

Put $r=\sqrt{2}-1$

On solving $m = 2 - \sqrt{3}$, $2 + \sqrt{3}$

Taking greater slope of CE as

$2 + \sqrt{3}$

$y - 1 = (2 + \sqrt{3})(x - 1)$

Put $y = 0$

$-1 = (2 + \sqrt{3})(x - 1)$

$$\frac{-1}{2+\sqrt{3}} \times \left(\frac{2-\sqrt{3}}{2-\sqrt{3}}\right) = x-1$$

$x - 1 = \sqrt{3} - 1$

$EB = 1 - x = 1 - (\sqrt{3} - 1)$

$EB = 2 - \sqrt{3}$

4. If $\lim\limits_{x\to 0}\dfrac{ae^x - b\cos x + ce^{-x}}{x\sin x} = 2$, then a + b + c is equal to _____.

Official Ans. by NTA (4)

Sol. $\lim\limits_{x\to 0}\dfrac{ae^x - b\cos x + ce^{-x}}{x\sin x} = 2$

$$\Rightarrow \lim_{x\to 0}\frac{a\left(1+x+\frac{x^2}{2!}\ldots\right)-b\left(1-\frac{x^2}{2!}+\ldots\right)+c\left(1-x+\frac{x^2}{2!}\right)}{\left(\frac{x\sin x}{x}\right)x} = 2$$

$a - b + c = 0$ .....(1)

$a - c = 0$ .....(2)

& $\dfrac{a+b+c}{2} = 2$

$\Rightarrow \boxed{a+b+c=4}$

5. The total number of 3 × 3 matrices A having enteries from the set (0, 1, 2, 3) such that the sum of all the diagonal entries of AA^T is 9, is equal to _____.

Official Ans. by NTA (766)

Sol. Let $A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$

diagonal elements of

AA^T, $a^2 + b^2 + c^2$, $d^2 + e^2 + f^2$, $g^2 + b^2 + c^2$

Sum $= a^2 + b^2 + c^2 + d^2 + e^2 + f^2 + g^2 + h^2 + i^2 = 9$

a, b, c, d, e, f, g, h, i ∈ {0, 1, 2, 3}

	Case	No. of Matrices
(1)	All – 1s	$\frac{9!}{9!}=1$
(2)	One $\to$ 3 remaining-0	$\frac{9!}{1!\times 8!}=9$
(3)	One-2 five-1s three-0s	$\frac{9!}{1!\times 5!\times 3!}=8\times 63$
(4)	two – 2's one-1 six-0's	$\frac{9!}{2!\times 6!}=63\times 4$

Total no. of ways = 1 + 9 + 8 × 63 + 63 × 4

= $\boxed{766}$

6. Let

$$P=\begin{bmatrix}-30 & 20 & 56\\ 90 & 140 & 112\\ 120 & 60 & 14\end{bmatrix} \text{ and } A=\begin{bmatrix}2 & 7 & \omega^2\\ -1 & -\omega & 1\\ 0 & -\omega & -\omega+1\end{bmatrix}$$

where $\omega=\frac{-1+i\sqrt{3}}{2}$, and I_3 be the identity matrix of order 3. If the determinant of the matrix $(P^{-1}AP - I_3)^2$ is $\alpha\omega^2$, then the value of α is equal to _____ .

Official Ans. by NTA (36)

Sol. Let $M = (P^{-1}AP - I)^2$

$= (P^{-1}AP)^2 - 2P^{-1}AP + I$

$= P^{-1}A^2P - 2P^{-1}AP + I$

$PM = A^2P - 2AP + P$

$= (A^2 - 2A.I + I^2)P$

$\Rightarrow$ Det(PM) = Det((A – I)^2 × P)

$\Rightarrow$ DetP.DetM = Det(A – I)^2 × Det(P)

$\Rightarrow$ Det M = (Det(A – I))^2

$$\text{Now } A-I=\begin{bmatrix}1 & 7 & w^2\\ -1 & -w-1 & 1\\ 0 & -w & -w\end{bmatrix}$$

Det(A – I) = $(w^2 + w + w) + 7(-w) + w^3 = -6w$

Det((A – I))^2 = $36w^2$

$\Rightarrow$ $\alpha = 36$

7. If the normal to the curve $y(x)=\int_0^x (2t^2-15t+10)\,dt$ at a point (a,b) is parallel to the line x + 3y = –5, a > 1, then the value of |a + 6b| is equal to _____ .

Official Ans. by NTA (406)

Sol. $y(x)=\int_0^x (2t^2-15t+10)\,dt$

$$y'(x)\big]_{x=a}=\left[2x^2-15x+10\right]_a=2a^2-15a+10$$

Slope of normal $=-\frac{1}{3}$

$\Rightarrow$ $2a^2-15a+10=3 \Rightarrow a=7$

& $a = \frac{1}{2}$ (rejected)

$b = y(7) = \int_0^7 (2t^2-15t+10)dt$

$$=\left[\frac{2t^3}{3}-\frac{15t^2}{2}+10t\right]_0^7$$

$\Rightarrow$ $6b=4\times 7^3-45\times 49+60\times 7$

|a + 6b| = 406

8. Let the curve y = y(x) be the solution of the differential equation, $\frac{dy}{dx}=2(x+1)$. If the numerical value of area bounded by the curve y = y(x) and x-axis is $\frac{4\sqrt{8}}{3}$, then the value of y(1) is equal to _____ .

Official Ans. by NTA (2)

Sol. $\frac{dy}{dx}=2(x+1)$

$\Rightarrow$ $\int dy=\int 2(x+1)dx$

$\Rightarrow$ $y(x) = x^2 + 2x + C$

Area $=\frac{4\sqrt{8}}{3}$

$-1+\sqrt{1-C}$

$\Rightarrow$ $2\int_{-1}^{-1+\sqrt{1-C}} (-(x+1)^2-C+1)dx=\frac{4\sqrt{8}}{3}$

$\Rightarrow$ $2\left[-\frac{(x+1)^3}{3}-Cx+x\right]_{-1}^{-1+\sqrt{1-C}}=\frac{4\sqrt{8}}{3}$

$\Rightarrow$ $-(\sqrt{1-C})^3+3c-3C\sqrt{1-C}$

$-3+3\sqrt{1-C}-3C+3=2\sqrt{8}$

$\Rightarrow$ C = –1

$\Rightarrow$ $f(x) = x^2 + 2x - 1$, $f(1) = 2$

9. Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous function such that $f(x) + f(x + 1) = 2$, for all $x \in \mathbb{R}$. If $I_1 = \int_0^8 f(x)dx$ and $I_2 = \int_{-1}^3 f(x)dx$, then the value of $I_1 + 2I_2$ is equal to _____ .

Official Ans. by NTA (16)

Sol. $f(x) + f(x + 1) = 2$

$\Rightarrow$ $f(x)$ is periodic with period = 2

$$I_1 = \int_0^8 f(x)dx = 4\int_0^2 f(x)dx$$

$$= 4\int_0^1 \left(f(x) + f(1+x)\right)dx = 8$$

Similarly $I_2 = 2 \times 2 = 4$

$I_1 + 2I_2 = 16$

10. Let z and w be two complex numbers such that $w = z\bar{z} - 2z + 2$, $\left|\frac{z+i}{z-3i}\right| = 1$ and Re(w) has minimum value. Then, the minimum value of $n \in \mathbb{N}$ for which w^n is real, is equal to _____.

Official Ans. by NTA (4)

Sol. $\omega = z\bar{z} - 2z + 2$

$\left|\frac{z+i}{z-3i}\right| = 1$

$\Rightarrow$ $|z+i| = |z-3i|$

$\Rightarrow$ $z = x + i$, $x \in \mathbb{R}$

$\omega = (x + i)(x - i) - 2(x + i) + 2$

$= x^2 + 1 - 2x - 2i + 2$

$Re(\omega) = x^2 - 2x + 3$

For min $(Re(\omega))$, $x = 1$

$\Rightarrow$ $\omega = 2 - 2i = 2(1 - i) = 2\sqrt{2}\, e^{-i\frac{\pi}{4}}$

$\omega^n = (2\sqrt{2})^n e^{-i\frac{n\pi}{4}}$

For real & minimum value of n,

n = 4

24th Feb. 2021 | Shift - 2
PHYSICS

1. Zener breakdown occurs in a p-n junction having p and n both :
(1) lightly doped and have wide depletion layer.
(2) heavily doped andhave narrow depletion layer.
(3) heavily doped and have wide depletion layer.
(4) lightly doped and have narrow depletion layer.

Ans. **(2)**

Sol. The zener breakdown occurs in the heavily doped p-n junction diode. Heavily doped p-n junction diodes have narrow depletion region.

2. According to Bohr atom model, in which of the following transitions will the frequency be maximum?
(1)n=2 to n=1 (2)n=4 to n =3 (3)n=5 to n=4 (4)n=3 to n=2

Ans. **(1)**

Sol.

f is more for transition from n = 2 to n = 1.

3. An X-ray tube is operated at 1.24 million volt. The shortest wavelength of the produced photon will be :
(1) 10^{-2} nm (2)10^{-3} nm (3)10^{-4} nm (4)10^{-1} nm

Ans. **(2)**

Sol. $\lambda_{min} = \frac{hc}{eV}$

$\lambda_{min} = \frac{1240nm - eV}{1.24 \times 10^6}$

$\lambda_{min} = 10^{-3}nm$

4. On the basis of kinetic theory of gases, the gas exerts pressure because its molecules:
(1) suffer change in momentum when impinge on the walls of container.
(2) continuously stick to the walls of container.
(3) continuously lose their energy till it reaches wall.
(4) are attracted by the walls of container.

Ans. **(1)**

Sol. On the basis of kinetic theory of gases, the gas pressure is due to the molecules suffering change in momentum when impinge on the walls of container.

5. A circular hole of radius $\left(\frac{a}{2}\right)$ is cut out of a circular disc of radius 'a' shown in figure. The centroid of the remaining circular portion with respect to point 'O' will be :

(1) $\frac{10}{11}a$ (2) $\frac{2}{3}a$ (3) $\frac{1}{6}a$ (4) $\frac{5}{6}a$

Ans. **(4)**

Sol. Let σ is the surface mass density of disc.

$$X_{com} = \frac{(\sigma \times \pi a^2 \times a) - (\sigma \frac{\pi a^2}{4} \times \frac{3a}{2})}{\sigma\pi a^2 - \frac{\sigma\pi a^2}{4}}$$

$$X_{com} = \frac{a - 3\frac{a}{8}}{1 - \frac{1}{4}}$$

$$X_{com} = \frac{\frac{5a}{8}}{\frac{3}{4}}$$

$$X_{com} = \frac{5a}{6}$$

6. Given below are two statements :

Statement I : PN junction diodes can be used to function as transistor, simply by connecting two diodes, back to back, which acts as the base terminal.

Statement II : In the study of transistor, the amplification factor β indicates ratio of the collector current to the base current.

In the light of the above statements, choose the correct answer from the options given below.

(1) Statement I is false but Statement II is true.
(2) Both Statement I and Statement II are true
(3) Statement I is true but Statement II is false.
(4) Both Statement I and Statement II are false

Ans. **(1)**

Sol. **S-1**

Statement 1 is false because in case of two discrete back to back connected diodes, there are four doped regions instead of three and there is nothing that resembles a thin base region between an emitter and a collector.

S-2

Statement-2 is true, as

$$\beta = \frac{I_C}{I_B}$$

7. When a particle executes SHM, the nature of graphical representation of velocity as a function of displacement is :

(1)elliptical (2)parabolic (3)straight line (4)circular

Ans. **(1)**

Sol. We know that is SHM;

$$V = \omega\sqrt{A^2 - x^2}$$

elliptical

8. Match List – I with List – II.

	List – I	List - II
(a)	Source of microwave frequency	(i) Radioactive decay of nucleus
(b)	Source of infrared frequency	(ii) Magnetron
(c)	Source of Gamma Rays	(iii) Inner shell electrons
(d)	Source of X-rays	(iv) Vibration of atoms and molecules
		(v) LASER
		(vi) RC circuit

Choose the correct answer from the options given below :

(1) (a)-(ii),(b)-(iv),(c)-(i),(d)-(iii)
(2) (a)-(vi),(b)-(iv),(c)-(i),(d)-(v)
(3) (a)-(ii),(b)-(iv),(c)-(vi),(d)-(iii)
(4) (a)-(vi),(b)-(v),(c)-(i),(d)-(iv)

Ans. **(1)**

Sol. (a) Source of microwave frequency – (ii) Magnetron
(b) Source of infra red frequency – (iv) Vibration of atom and molecules
(c) Source of gamma ray – (i) Radio active decay of nucleus
(d) Source of X-ray – (iii) inner shell electron

9.

The logic circuit shown above is equivalent to :

(1)

(2)

(3)

(4)

Ans. **(2)**

Sol.

$C = \overline{A + \overline{B}}$

$C = \overline{A}.B$

10. If the source of light used in a Young's double slit experiment is changed from red to violet:
(1)the fringes will become brighter.
(2)consecutive fringe lineswill come closer.
(3)the central bright fringe will become a dark fringe.
(4)the intensity of minima will increase.

Ans. **(2)**

Sol. $\beta = \frac{\lambda D}{d}$

As $\lambda_v < \lambda_R$

$\Rightarrow \beta_v < \beta_R$

$\therefore$ Consecutive fringe line will come closer.

$\therefore$ (2)

11. A body weighs 49 N on a spring balance at the north pole. What will be its weight recorded on the same weighing machine, if it is shifted to the equator?

[Use $g = \frac{GM}{R^2}$ = 9.8 ms^{-2} and radius of earth, R = 6400 km.]

(1)49 N (2) 49.83 N (3) 49.17 N (4) 48.83 N

Ans. **(4)**

Sol. At north pole, weight

Mg = 49

Now, at equator

$g' = g - \omega^2 R$

$\Rightarrow Mg' = M(g - \omega^2 R)$

$\Rightarrow$ weight will be less than Mg at equator.

12. If one mole of an ideal gas at (P_1, V_1) is allowed to expand reversibly and isothermally (A to B) its pressure is reduced to one-half of the original pressure (see figure). This is followed by a constant volume cooling till its pressure is reduced to one-fourth of the initial value (B→C). Then it is restored to its initial state by a reversible adiabatic compression (C to A). The net workdone by the gas is equal to :

(1)0 (2)$-\frac{RT}{2(\gamma-1)}$ (3)$RT\left[\ln 2-\frac{1}{2(\gamma-1)}\right]$ (4)RT ln2

Ans. **(3)**

Sol. AB → Isothermal process

$W_{AB} \rightarrow nRT \ln 2 = RT \ln 2$

BC → Isochoric process

$W_{BC} = 0$

CA → Adiabatic process

$$W_{CA} = \frac{P_1V_1 - \frac{P_1}{4} X 2V_1}{1-\gamma} = \frac{P_1V_1}{2(1-\gamma)} = \frac{RT}{2(1-\gamma)}$$

$$W_{ABCA} = RT\ell n2 + \frac{RT}{2(1-\gamma)}$$

$$= RT\left[\ell n2 - \frac{1}{2(\gamma-1)}\right]$$

13. The period of oscillation of a simple pendulum is $T = 2\pi\sqrt{\frac{L}{g}}$. Measured value of 'L' is 1.0 m from meter scale having a minimum division of 1 mm and time of one complete oscillation is 1.95 s measured from stopwatch of 0.01 s resolution. The percentage error in the determination of 'g' will be :

(1)1.33 % (2)1.30 % (3)1.13 % (4)1.03 %

Ans. **(3)**

Sol. $T = 2\pi\sqrt{\frac{\ell}{g}}$

$$T^2 = 4\pi^2\left[\frac{\ell}{g}\right]$$

$$g = 4\pi^2\left[\frac{\ell}{T^2}\right]$$

$$\frac{\Delta g}{g} = \frac{\Delta\ell}{\ell} + \frac{2\Delta T}{T}$$

$$= \left[\frac{1mm}{1m} + \frac{2(10\times10^{-3})}{1.95}\right]\times 100$$

= 1.13 %

14. In the given figure, a body of mass M is held between two massless springs, on a smooth inclined plane. The free ends of the springs are attached to firm supports. If each spring has spring constant k, the frequency of oscillation of given body is :

(1) $\frac{1}{2\pi}\sqrt{\frac{2k}{Mg\sin\alpha}}$ (2) $\frac{1}{2\pi}\sqrt{\frac{k}{Mg\sin\alpha}}$

(3) $\frac{1}{2\pi}\sqrt{\frac{2k}{M}}$ (4) $\frac{1}{2\pi}\sqrt{\frac{k}{2M}}$

Ans. **(1)**

Sol. Equivalent K = K + K = 2K

Now, $T = 2\pi\sqrt{\frac{m}{K_{eq}}}$

$\Rightarrow T = 2\pi\sqrt{\frac{m}{2k}}$

$\therefore f = \frac{1}{2\pi}\sqrt{\frac{2k}{m}}$

15. Figure shows a circuit that contains four identical resistors with resistance R = 2.0 Ω. Two identical inductors with inductance L = 2.0 mH and an ideal battery with emf E = 9.V. The current 'i' just after the switch 's' is closed will be :

(1) 9A (2) 3.0 A (3) 2.25 A (4) 3.37 A

Ans. **(3)**

Sol. When switch S is closed-

Given : v = 9v
From V = IR

$I = \frac{v}{R}$

$R_{eq.} = 2+2 = 4\Omega$

$I = \frac{9}{4} = 2.25$ A

16. The de Broglie wavelength of a proton andα-particle are equal. The ratio of their velocities is :
(1) 4:2 (2) 4:1 (3) 1:4 (4) 4:3

Ans. **(2)**

Sol. From De-broglie's wavelength :-

$\lambda = \frac{h}{mv}$

Given $\lambda_P = \lambda_\alpha$

$v \alpha \frac{1}{m}$

$\frac{v_p}{v_\alpha} = \frac{m_\alpha}{m_p} = \frac{4m_p}{m_p} = \frac{4}{1}$

17. Two electrons each are fixed at a distance '2d'. A third charge proton placed at the midpoint is displaced slightly by a distance x (x<<d) perpendicular to the line joining the two fixed charges. Proton will execute simple harmonic motion having angular frequency:
(m = mass of charged particle)

(1) $\left(\frac{q^2}{2\pi\varepsilon_0 md^3}\right)^{\frac{1}{2}}$ (2) $\left(\frac{\pi\varepsilon_0 md^3}{2q^2}\right)^{\frac{1}{2}}$ (3) $\left(\frac{2\pi\varepsilon_0 md^3}{q^2}\right)^{\frac{1}{2}}$ (4) $\left(\frac{2q^2}{\pi\varepsilon_0 md^3}\right)^{\frac{1}{2}}$

Ans. **(1)**

Sol.

Restoring force on proton :-

$$F_r = \frac{2Kq^2y}{\left[d^2 + y^2\right]^{\frac{3}{2}}}$$

Y <<< d

$$F_r = \frac{2kq^2y}{d^3} = \frac{q^2y}{2\pi\varepsilon_o d^3} = ky$$

$$K = \frac{q^2}{2\pi\varepsilon_0 d^3}$$

Angular Frequency :-

$$\omega = \sqrt{\frac{k}{m}}$$

$$\omega = \sqrt{\frac{q^2}{2\pi\varepsilon_0 md^3}}$$

18. A soft ferromagnetic material is placed in an external magnetic field. The magnetic domains :

(1) decrease in size and changes orientation.
(2) may increase or decrease in size and change its orientation.
(3) increase in size but no change in orientation.
(4) have no relation with external magnetic field.

Ans. **(2)**

Sol. Atoms of ferromagnetic material in unmagnetised state form domains inside the ferromagnetic material. These domains have large magnetic moment of atoms. In the absence of magnetic field, these domains have magnetic moment in different directions. But when the magnetic field is applied, domains aligned in the direction of the field grow in size and those aligned in the direction opposite to the field reduce in size and also its orientation changes.

19. Which of the following equations represents a travelling wave?

(1) $y = Ae^{-x^2}(vt + \theta)$
(2) $y = A\sin(15x - 2t)$
(3) $y = Ae^{x}\cos(\omega t - \theta)$
(4) $y = A\sin x\cos\omega t$

Ans. **(2)**

Sol. Y = F(x,t)

For travelling wave y should be linear function of x and t and they must exist as (x±vt)

Y = A sin (15x-2t) $\rightarrow$ linear function in x and t.

20. A particle is projected with velocity v_0 along x-axis. A damping forceis acting on the particle which is proportional to the square of the distance from the origin i.e. ma = $-\alpha x^2$. The distance at which the particle stops :

(1) $\left(\frac{2v_0}{3\alpha}\right)^{\frac{1}{3}}$ (2) $\left(\frac{3v_0^2}{2\alpha}\right)^{\frac{1}{2}}$ (3) $\left(\frac{3v_0^2}{2\alpha}\right)^{\frac{1}{3}}$ (4) $\left(\frac{2v_0^2}{3\alpha}\right)^{\frac{1}{2}}$

Ans. **Bonus**

Sol. $a = \frac{vdv}{dx}$

$\int_{v_i}^{v_f} Vdv = \int_{x_i}^{x_f} adx$

Given :- $v_i = v_0$

$V_f = 0$

$X_i = 0$

$X_f = x$

From Damping Force : $a = -\frac{\alpha x^2}{m}$

$\int_{V_0}^{O} VdV = -\int_{o}^{x} \frac{\alpha x^2}{m} dx$

$-\frac{v_0^2}{2} = \frac{-\alpha}{m}\left[\frac{x^3}{3}\right]$

$x = \left[\frac{3mv_0^2}{2\alpha}\right]^{\frac{1}{3}}$

1. A uniform metallic wire is elongated by 0.04 m when subjected to a linear force F. The elongation, if its length and diameter is doubled andsubjected to the same force will be _____ cm.

Ans. **2**

Sol.

$y = \frac{F/A}{\Delta\ell / \ell}$

$\Rightarrow \frac{F}{A} = y\frac{\Delta\ell}{\ell}$

$\Rightarrow \frac{F}{A} = y \times \frac{0.04}{\ell}$ ...(1)

When length & diameter is doubled.

$\Rightarrow \frac{F}{4A} = y \times \frac{\Delta\ell}{2\ell}$ ...(2)

$(1) \div (2)$

$\frac{F/A}{F/4A} = \frac{y \times \frac{0.04}{\ell}}{y \times \frac{\Delta\ell}{2\ell}}$

$4 = \frac{0.04 \times 2}{\Delta\ell}$

$\Delta\ell = 0.02$

$\Delta\ell = 2 \times 10^{-2}$

$\therefore x = 2$

2. A cylindrical wire of radius 0.5 mm and conductivity 5×10^7 S/m is subjected to an electric field of 10 mV/m. The expected value of current in the wire will be $x^3\pi$ mA. The value of x is ____.

Ans. **5**

Sol. We know that

$J = \sigma E$

$\Rightarrow J = 5\times10^7 \times 10 \times 10^{-3}$

$\Rightarrow J = 50\times10^4\ A/m^2$

Currentflowing ;

$I = J \times \pi R^2$

$I = 50\times10^4 \times\pi(0.5\times10^{-3})^2$

$I = 5\times10^4\times\pi\times0.25\times10^{-6}$

$I = 125\times10^{-3}\pi$

X = 5

3. Two cars are approaching each other at an equal speed of 7.2 km/hr. When they see each other, both blow horns having frequency of 676 Hz. The beat frequency heard by each driver will be _____ Hz. [Velocity of sound in air is 340 m/s.]

Ans. **8**

Sol.

Speed = 7.2 km/h = 2 m/s

Frequency as heard by A

$$f_A' = f_B\left(\frac{v+v_0}{v-v_s}\right)$$

$$f_A' = 676\left(\frac{340+2}{340-2}\right)$$

$$f_A' = 684\text{Hz}$$

$$\therefore f_{Beat} = f_A' - f_B$$

=684-676

= 8 Hz

4. A uniform thin bar of mass 6 kg and length 2.4 meter is bent to make an equilateral hexagon. The moment of inertia about an axis passing through the centre of mass and perpendicular to the plane of hexagon is _____$\times10^{-1}$ kg m^2.

Ans. **8**

Sol.

MOI of AB about P : $I_{AB_P} = \dfrac{\frac{M}{6}\left(\frac{\ell}{6}\right)^2}{12}$

MOI of AB about O,

$$I_{AB_O} = \left[\frac{\frac{M}{6}\left(\frac{\ell}{6}\right)^2}{12} + \frac{M}{6}\left(\frac{\ell}{6}\frac{\sqrt{3}}{2}\right)^2\right]$$

$$I_{Hexagon_0} = 6I_{AB_0} = M\left[\frac{\ell^2}{12\times 36} + \frac{\ell^2}{36}\times\frac{3}{4}\right]$$

$$= \frac{6}{100}\left[\frac{24\times 24}{12\times 36} + \frac{24\times 24}{36}\times\frac{3}{4}\right]$$

= 0.8 kgm^2

= 8×10^{-2} kg/m^2

5. A point charge of +12 μC is at a distance 6 cm vertically above the centre of a square of side 12 cm as shown in figure. The magnitude of the electric flux through the square will be _____ $\times10^3$ Nm^2/C.

Ans. **226**

Sol. Using Gauss law, it is a part of cube of side 12 cm and charge at centre so;

$$\phi = \frac{Q}{6\varepsilon_0} = \frac{12\mu c}{6\varepsilon_0} = 2\times 4\pi\times 9\times 10^9\times 10^{-6}$$

$$= 226\times 10^3 \ Nm^2/C$$

6. Two solids A and B of mass 1 kg and 2 kg respectively are moving withequal linear momentum. The ratio of their kinetic energies $(K.E.)_A$: $(K.E.)_B$ will be $\frac{A}{1}$. So the value of A will be _____.

Ans. **2**

Sol. Given that, $\frac{M_1}{M_2} = \frac{1}{2}$

Also, $p_1 = p_2 = p$

$\Rightarrow M_1V_1 = M_2V_2 = p$

Also, we know that

$$K = \frac{p^2}{2M} \Rightarrow K_1 = \frac{p^2}{2M_1} \& \Rightarrow K_2 = \frac{p^2}{2M_2}$$

$$\Rightarrow \frac{K_1}{K_2} = \frac{p^2}{2M_1} \times \frac{2M_2}{p^2} \Rightarrow \frac{K_1}{K_2} = \frac{M_2}{M_1} = \frac{2}{1}$$

$$\Rightarrow \frac{A}{1} = \frac{2}{1} \Rightarrow \therefore A = 2$$

7. The root mean square speed of molecules of a given mass of a gas at 27°C and 1 atmosphere pressure is 200 ms^{-1}. The root mean square speed of molecules of the gas at 127°C and 2 atmosphere pressure is $\frac{x}{\sqrt{3}}$ ms^{-1}. The value of x will be _____.

Ans. **400 m/s**

Sol. $V_{rms}\sqrt{\frac{3RT_1}{M_0}}$

$$200 = \sqrt{\frac{3R \times 300}{M_0}} \quad ...(1)$$

Also, $$\frac{x}{\sqrt{3}} = \sqrt{\frac{3R \times 400}{M_0}} \quad ...(2)$$

$(1) \div (2)$

$$\frac{200}{x/\sqrt{3}} = \sqrt{\frac{300}{400}} = \sqrt{\frac{3}{4}}$$

$\Rightarrow x = 400$ m/s

8. A series LCR circuit is designed to resonate at an angular frequency $\omega_0 = 10^5 rad/s$. The circuit draws 16W power from 120 V source at resonance. The value of resistance 'R' in the circuit is _____ Ω.

Ans. **900**

Sol. $P = \frac{V^2}{R}$

$$16 = \frac{120^2}{R} \Rightarrow R = \frac{14400}{16}$$

$\Rightarrow R = 900\Omega$

9. An electromagnetic wave of frequency 3 GHz enters a dielectric medium of relative electric permittivity 2.25 from vacuum. The wavelength of this wave in that medium wil be _____ $\times 10^{-2}$ cm.

Ans. 667

Sol. f = 3GHz, ε_r = 2.25

$$v = \lambda f \Rightarrow \lambda = \frac{v}{f}$$

$$C = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

$$v = \frac{1}{\sqrt{\mu_0 \mu_r \varepsilon_0 \varepsilon_r}} \Rightarrow \lambda = \frac{1}{f.\sqrt{\mu_0 \varepsilon_0}.\sqrt{\mu_r \varepsilon_r}.f}$$

$$\Rightarrow \lambda = \frac{C}{f.\sqrt{\mu_r}.\sqrt{\varepsilon_r}} \Rightarrow \lambda = \frac{3 \times 10^8}{3 \times 10^9 \times \sqrt{1} \times \sqrt{2.25}}$$

$$\Rightarrow \lambda = 667 \times 10^{-2} \text{ cm}$$

10. A signal of 0.1 kW is transmitted in a cable. The attenuation of cable is -5 dB per km and cable length is 20 km. the power received at receiver is 10^{-x}W. The value of x is _____.

[Gain in dB = 10 $\log_{10}\left(\frac{P_0}{P_i}\right)$]

Ans. **8**

Sol. Power of signal transmitted : P_i = 0.1 Kw = 100w

Rate of attenuation = -5 dB/Km

Total length of path = 20 km

Total loss suffered = -5×20=-100dB

Gain in dB = 10 $\log_{10}\frac{P_0}{P_i}$

$$-100 = 10\log_{10}\frac{P_0}{P_i}$$

$$\Rightarrow \log_{10}\frac{P_i}{P_0} = 10$$

$$\Rightarrow \log_{10}\frac{P_i}{P_0} = \log_{10} 10^{10}$$

$$\Rightarrow \frac{100}{P_0} = 10^{10}$$

$$\Rightarrow P_0 = \frac{1}{10^8} = 10^{-8}$$

$\therefore$ x = 8

24th Feb. 2021 | Shift - 2
CHEMISTRY

1. The correct order of the following compounds showing increasing tendency towards nucleophilic substitution reaction is :

(i) (ii) (iii) (iv)

(1) (iv) < (i) < (iii) < (ii) (2) (iv) < (i) < (ii) < (iii)
(3) (i) < (ii) < (iii) < (iv) (4) (iv) < (iii) < (ii) < (i)

Ans. **(3)**

Sol.

(i) (ii) (iii) (iv)

Reactivity ∝ – m group present at O/P position.

2. Match List-I with List-II

List- I (Metal)	List-II (Ores)
(a) Aluminium	(i) Siderite
(b) Iron	(ii) Calamine
(c) Copper	(iii) Kaolinite
(d) Zinc	(iv) Malachite

Choose the correct answer from the options given below :

(1) (a)-(iv), (b)-(iii), (c)-(ii), (d)-(i) (2) (a)-(i), (b)-(ii), (c)-(iii), (d)-(iv)
(3) (a)-(iii), (b)-(i), (c)-(iv), (d)-(ii) (4) (a)-(ii), (b)-(iv), (c)-(i), (d)-(iii)

Ans. **(3)**

Sol.

Siderite	$FeCO_3$
Calamine	$ZnCO_3$
Kaolinite	$Si_2Al_2O_5(OH)_4$ or $Al_2O_3.2SiO_2.2H_2O$
Malachite	$CuCO_3.Cu(OH)_2$

3. Match List-I with List-II

List- I (Salt)	List-II (Flame colour wavelength)
(a) LiCl	(i) 455.5 nm
(b) NaCl	(ii) 970.8 nm
(c) RbCl	(iii) 780.0 nm
(d) CsCl	(iv) 589.2 nm

Choose the correct answer from the options given below :

(1) (a)-(ii), (b)-(i), (c)-(iv), (d)-(iii) (2) (a)-(ii), (b)-(iv), (c)-(iii), (d)-(i)

(3) (a)-(iv), (b)-(ii), (c)-(iii), (d)-(i) (4) (a)-(i), (b)-(iv), (c)-(ii), (d)-(iii)

Ans. **(2)**

Sol. Range of visible region : - 390nm – 760nm

VIBGYOR

Violet Red

LiCl Crimson Red

NaCl Golden yellow

RbCl Violet

CsCl Blue

So Licl Which is crimson have wave length closed to red in the spectrum of visible region which is as per given data is.

4. Given below are two statements : one is labelled as Assertion A and the other is labelled as Reason R.

Assertion A : Hydrogen is the most abundant element in the Universe, but it is not the most abundant gas in the troposphere.

Reason R : Hydrogen is the lightest element.

In the light of the above statements, choose the correct answer from the given below

(1) A is false but R is true

(2) Both A and R are true and R is the correct explanation of A

(3) A is true but R is false

(4) Both A and R are true but R is NOT the correct explanation of A

Ans. **(2)**

Sol. Hydrogen is most abundant element in universe because all luminous body of universe i.e. stars & nebulae are made up of hydrogen which acts as nuclear fuel & fusion reaction is responsible for their light.

5. Given below are two statements :

Statement I : The value of the parameter ''Biochemical Oxygen Demand (BOD)'' is important for survival of aquatic life.

Statement II : The optimum value of BOD is 6.5 ppm.

In the light of the above statements, choose the most appropriate answer from the options given below.

(1) Both Statement I and Statement II are false

(2) Statement I is false but Statement II is true

(3) Statement I is true but Statement II is false

(4) Both Statement I and Statement II are true

Ans. **(3)**

Sol. For survival of aquatic life dissolved oxygen is responsible its optimum limit 6.5 ppm and optimum limit of BOD ranges from 10-20 ppm & BOD stands for biochemical oxygen demand.

6. Wich one of the following carbonyl compounds cannot be prepared by addition of wate on an alkyne in the presence of $HgSO_4$ and H_2SO_4 ?

(1) $CH_3-CH_2-\overset{\overset{\displaystyle O}{||}}{C}-H$

(2)

(3) $CH_3-\overset{\overset{\displaystyle O}{||}}{C}-H$

(4) $CH_3-\overset{\overset{\displaystyle O}{||}}{C}-CH_2CH_3$

Ans. **(1)**

Sol. Reaction of Alkyne with $HgSO_4$ & H_2SO_4 follow as

$$CH \equiv CH \xrightarrow[H_2O]{HgSO_4, H_2SO_4} CH_3CHO$$

$$CH_3 - C \equiv CH \xrightarrow[H_2O]{HgSO_4, H_2SO_4} CH_3-\overset{\overset{\displaystyle O}{||}}{C}-CH_3$$

Hence, by this process preparation of CH_3CH_2CHO

Cann't possible.

7. Which one of the following compounds is non-aromatic ?

(1)

(2)

(3)

(4)

Ans. **(2)**

Sol.

sp^3 carbon atom (Not planer)

Hence It is non-aromatic.

8. The incorrect statement among the following is :

(1) $VOSO_4$ is a reducing agent

(2) Red colour of ruby is due to the presence of CO^{3+}

(3) Cr_2O_3 is an amphoteric oxide

(4) RuO_4 is an oxidizing agent

Ans. **(2)**

Sol. Red colour of ruby is due to presence of CrO_3 or Cr^{+6} not CO^{3+}

9. According to Bohr's atomic theory :

(A) Kinetic energy of electron is $\propto \frac{Z^2}{n^2}$

(B) The product of velocity (v) of electron and principal quantum number (n). 'vn' $\propto Z^2$.

(C) Frequency of revolution of electron in an orbit is $\propto \frac{Z^3}{n^3}$.

(D) Coulombic force of attraction on the electron is $\propto \frac{Z^3}{n^4}$.

Choose the most appropriate answer from the options given below:

(1) (C) only
(2) (A) and (D) only
(3) (A) only
(4) (A), (C) and (D) only

Ans. **(2) Correction on NTA**

Sol. (A) KE = –TE = $13.6 \times \frac{Z^2}{n^2}$ eV

$KE \propto \frac{Z^2}{n^2}$

(B) $V = 2.188 \times 10^6 \times \frac{Z}{n}$ m/sec.

So, $Vn \propto Z$

(C) Frequency = $\frac{V}{2\pi r}$

So, $F \propto \frac{Z^2}{n^3}$ $\left[\therefore r \propto \frac{n^2}{z} \text{ and } v \propto \frac{Z}{n}\right]$

(D) Force $\propto \frac{z}{r^2}$

So, $F \propto \frac{Z^3}{n^4}$

So, only statement (A) is correct

10. Match List-I with List-II

List- I	List-II
(a) Valium	(i) Antifertility drug
(b) Morphine	(ii) Pernicious anaemia
(c) Norethindrone	(iii) Analgesic
(d) Vitamin B_{12}	(iv) Tranquilizer

(1) (a)-(iv), (b)-(iii), (c)-(ii), (d)-(i)
(2) (a)-(i), (b)-(iii), (c)-(iv), (d)-(ii)
(3) (a)-(ii), (b)-(iv), (c)-(iii), (d)-(i)
(4) (a)-(iv), (b)-(iii), (c)-(i), (d)-(ii)

Ans. **(4)**

Sol.

(a) Valium	(iv) Tranquilizer
(b) Morphine	(iii) Analgesic
(c) Norethindrone	(i) Antifertility drug
(d) Vitamin B_{12}	(ii) Pernicious anaemia

11. The Correct set from the following in which both pairs are in correct order of melting point is :

(1) LiF > LiCl ; NaCl > MgO (2) LiF > LiCl ; MgO > NaCl

(3) LiCl > LiF ; NaCl > MgO (4) LiCl > LiF ; MgO > NaCl

Ans. **(2)**

Sol. Generally

M.P. $\propto$ Lattice energy = $\frac{KQ_1Q_2}{r^+ + r^-}$

$\propto$ (packing efficiency)

12. The calculated magnetic moments (spin only value) for species $\left[FeCl_4\right]^{2-}$, $\left[Co\left(C_2O_4\right)_3\right]^{3-}$ and MnO_4^{2-} respectively are :

(1) 5.92, 4.90 and 0 BM (2) 5.82, O and 0 BM

(3) 4.90, 0 and 1.73 BM (4) 4.90, 0 and 2.83 BM

Ans. **(3)**

Sol. $\left[FeCl_4\right]^{2-}$ Fe^{2+} $3d^6 \rightarrow$ 4 unpaired electron. as Cl^- in a weak field liquid.

$\mu_{spin} = \sqrt{24}\,8M$

= 4.9 BM

$\left[Co\left(C_2O_4\right)_3\right]^{3-}$ Co^{3+} $3d^6 \rightarrow$ for Co^{3+} with coodination no. 6 $C_2O_4^{2-}$ is strong field ligend & causes pairing & hence no. unpaired electron

μ_{spin} = 0

$\left[MnO_4\right]^{2-}$ Mn^{+6} it has one unpaired electron.

$\mu_{spin} = \sqrt{3}\,BM$

13.

Which of the following reagent is suitable for the preparation of the product in the above reaction.

(1) Red P + Cl_2 (2) $NH_2-NH_2/C_2H_5\overset{\ominus\oplus}{O}Na$

(3) Ni/H_2 (4) $NaBH_4$

Ans. **(2)**

Sol.

It is wolf-kishner reduction of carbonyl compounds.

14. The diazonium salt of which of the following compounds will form a coloured dye on reaction with β-Naphthol in NaOH ?

(1)

(2)

(2)

(4)

Ans. **(3)**

Sol.

Orange bright dye.

15. What is the correct sequence of reagents used for converting nitrobenzene into m-dibromobenzene ?

(1) $\xrightarrow{Sn/HCl}$ / $\xrightarrow{Br_2}$ / $\xrightarrow{NaNO_2}$ / $\xrightarrow{NaBr}$

(2) $\xrightarrow{Sn/HCl}$ / $\xrightarrow{KBr}$ / $\xrightarrow{Br_2}$ / $\xrightarrow{H^+}$

(3) $\xrightarrow{NaNO_2}$ / $\xrightarrow{HCl}$ / $\xrightarrow{KBr}$ / $\xrightarrow{H^+}$

(4) $\xrightarrow{Br_2/Fe}$ / $\xrightarrow{Sn/HCl}$ / $\xrightarrow{NaNO_2/HCl}$ / $\xrightarrow{CuBr/HBr}$

Ans. **(4)**

Sol.

16. The correct shape and I-I-I bond angles respectively in I_3^- ion are :

(1) Trigonal planar; 120º

(2) Distorted trigonal planar; 135º and 90º

(3) Linear; 180º

(4) T-shaped; 180º and 90º

Ans. **(3)**

Sol. I_3^- sp^3d hybridisation (2BP + 3L.P.) Linear geometry

17. What is the correct order of the following elements with respect to their density ?

(1) Cr < Fe < Co < Cu < Zn

(2) Cr < Zn < Co < Cu < Fe

(3) Zn < Cu < Co < Fe < Cr

(4) Zn < Cr < Fe < Co < Cu

Ans. **(4)**

Sol. Fact Based

Density depend on many factor like atomic mass. atomic radius and packing efficiency.

18. **Match List-I and List-II.**

List – I	List-II
(a) $R-\overset{\overset{\displaystyle O}{\|}}{C}-Cl \rightarrow R-CHO$	(i) Br_2/NaOH
(b) $R-CH_2-COOH \rightarrow R-\underset{\underset{\displaystyle Cl}{\|}}{CH}-COOH$	(ii) H_2/Pd-$BaSO_4$
(c) $R-\overset{\overset{\displaystyle O}{\|}}{C}-NH_2 \rightarrow R-NH_2$	(iii) Zn(Hg)/Conc. HCl
(d) $R-\overset{\overset{\displaystyle O}{\|}}{C}-CH_3 \rightarrow R-CH_2-CH_3$	(iv) Cl_2/Red P, H_2O

Choose the correct answer from the options given below :

(1) (a)-(ii), (b)-(i), (c)-(iv), (d)-(iii) (2) (a)-(iii), (b)-(iv), (c)-(i), (d)-(ii)

(3) (a)-(iii), (b)-(i), (c)-(iv), (d)-(ii) (4) (a)-(ii), (b)-(iv), (c)-(i), (d)-(iii)

Ans. **(4)**

Sol. (a) $R-\overset{\overset{\displaystyle O}{\|}}{C}-Cl \xrightarrow{H_2/Pd-BaSO_4} R-CHO$ (Rosenmunt reaction)

(b) $R-CH_2-COOH \xrightarrow{Cl_2/Red\ P,\ H_2O} R-\underset{\underset{\displaystyle Cl}{\|}}{CH}-COOH$ (HVZ reaction)

(c) $R-\overset{\overset{\displaystyle O}{\|}}{C}-NH_2 \xrightarrow{Br_2/NaOH} R-NH_2$ (Hoffmann Bromamide reaction)

(d) $R-\overset{\overset{\displaystyle O}{\|}}{C}-CH_3 \xrightarrow{Zn(Hg)/conc.HCl} R-CH_2-CH_3$ (Clemmensen reaction)

19. In polymer Buna-S ; 'S' stands for :

(1) Styrene (2) Sulphur (3) Strength (4) Sulphonation

Ans. **(1)**

Sol. Buna-S is the co-polymer of buta- 1, 3 diene & styrene.

20. Most suitable salt which can be used for efficient clotting of blood will be :

(1) $Mg(HCO_3)_2$ (2) $FeSO_4$ (3) $NaHCO_3$ (4) $FeCl_3$

Ans. **(4)**

Sol. Blood is a negative sol. According to hardy-Schulz's rule, the cation with high charge has high coagulation power. Hence, $FeCl_3$ can be used for clotting blood.

Section -B

1. The magnitude of the change in oxidising power of the MnO_4^- / Mn^{2+} couple is x × 10^{-4} V, if the H^+ concentration is decreased from 1M to 10^{-4} M at 25ºC. (Assume concentration of MnO_4^- and Mn^{2+} to be same on change in H^+ concentration). The value of x is _____.
(Rounded off to the nearest integer)

$$\left[\text{Given : } \frac{2303RT}{F} = 0.059\right]$$

Ans. **3776**

Sol. $5e^- + MnO_4^- + 8H^+ \longrightarrow Mn^{+2} + 4H_2O$

$$Q = \frac{\left[Mn^{+2}\right]}{\left[H^+\right]^8\left[MnO_4^-\right]} \quad \Rightarrow \quad E_1 = E^\circ - \frac{0.059}{5}\log\left(Q_1\right)$$

$$E_2 = E^\circ - \frac{0.059}{5}\log\left(Q_2\right) \quad \Rightarrow \quad E_2 - E_1 = \frac{0.059}{5}\log\left(\frac{Q_1}{Q_2}\right)$$

$$= \frac{0.059}{5}\log\left\{\frac{\left[H^+\right]_{II}}{\left[H^+\right]_{I}}\right\}^8 \quad \Rightarrow \quad = \frac{0.059}{5}\log\left(\frac{10^{-4}}{1}\right)^8$$

$$\left(E_2 - E_1\right) = \frac{0.059}{5}\times\left(-32\right) \quad \Rightarrow \quad \left|\left(E_2 - E_1\right)\right| = 32\times\frac{0.059}{5} = x\times10^{-4}$$

$$= \frac{32\times590}{5}\times10^{-4} = x\times10^{-4} \quad \Rightarrow \quad = 3776 \times 10^{-4} \qquad x = 3776$$

2. Among the following allotropic forms of sulphur, the number of allotropic forms, which will show paramagnetism is _____.

(1) α-sulphur (2) β-sulphur (3) S_2-form

Ans. **(1)**

Sol. S_2 is like O_2 i;e paramagnetic as per molecular orbital theory.

3. C_6H_6 freezes at 5.5ºC. The temperature at which a solution of 10 g of C_4H_{10} in 200 g of C_6H_6 freeze is _____ºC. (The molal freezing point depression constant of C_6H_6 is) 5.12ºC/m)

Ans. **1**

Sol. $\Delta T_f = i \times K_f \times m$

$= (1) \times 5.12 \times \frac{10/58}{200} \times 1000 \quad \Rightarrow \quad \Delta T_f = \frac{5.12 \times 50}{58} = 4.414$

$T_{f(solution)} = T_{K(solvent)} - \Delta T_f$

$= 5.5 - 4.414$

$= 1.086°C$

$\approx 1.09°C = 1$ (nearest integer)

4. The volume occupied by 4.75 g of acetylene gas at 50ºC and 740 mmHg pressure is _____L. (Rounded off to the nearest integer)
(Given R = 0.0826 L atm K^{-1} mol^{-1})

Ans. **5**

Sol. T = 50ºC = 323.15 K

P = 740 mm of Hg = $\frac{740}{760}$ atm

V = ?

moles (n) = $\frac{4.75}{26}$

$V = \frac{4.75}{26} \times \frac{0.0821 \times 323.15}{740} \times 760$

$V = 4.97 \simeq 5$ Lit

5. The solubility product of PbI_2 is 8.0×10^{-9} . The solubility of lead iodide in 0.1 molar solution of lead nitrate is $x \times 10^{-6}$ mol/L. The value of x is _____(Rounded off to the nearest integer)
[Given $\sqrt{2} = 1.41$]

Ans. **141**

Sol. $K_{SP}(PbI_2) = 8 \times 10^{-9}$

$PbI_2(s) \rightleftharpoons Pb^{+2}(aq) + 2I^-(aq)$

S + 0.1 2S

$K_{SP} = [Pb^{+2}][I^-]^2$

$8 \times 10^{-9} = (S + 0.1)(2S)^2 \Rightarrow 8 \times 10^{-9} \simeq 0.1 \times 4S^2$

$\Rightarrow S^2 = 2 \times 10^{-8}$

$S = 1.414 \times 10^{-4}$ mol/Lit

$= x \times 10^{-6}$ mol/Lit $\quad \therefore \quad x = 141.4 \simeq 141$

6. The total number of amines among the following which can be synthesized by Gabriel synthesis is _____.

(1) $(CH_3)_2CH-CH_2-NH_2$ (2) $CH_3CH_2NH_2$

(3) (4)

Ans. **(3)**

Sol. Only aliphatic amines can be prepared by Gabriel synthesis.

7. 1.86 g of aniline completely reacts to form acetanilide. 10% of the product is lost during purificatiion. Amount of acetanilide obtained after purification (in g) is ____$\times 10^{-2}$.

Ans. **243**

Sol. $Ph-NH_2 \longrightarrow Ph-NH-\overset{\overset{O}{||}}{C}-CH_3$

(C_6H_7N) $(\text{Acetanilide})(C_8H_9NO)$

Molar mass = 93 Molar mass = 135

93 g Aniline produce 135 g acetanilide

1.86 g produce $\frac{135 \times 1.86}{93} = 2.70\,g$

At 10% loss, 90% product will be formed after purification.

$\therefore$ Amount of product obtained = $\frac{2.70 \times 90}{100} = 2.43\,g$ = 243×10^{-2} g

8. The formula of a gaseous hydrocarbon which requires 6 times of its own volume of O_2 for complete oxidation and produces 4 times its own volume of CO_2 is C_xH_y. The value of y is .....

Ans. **8**

Sol. $C_xH_y + 6O_2 \longrightarrow 4CO_2 + \frac{y}{2} H_2O$

Applying POAC on 'O' atoms

$6 \times 2 = 4 \times 2 + y/2 \times 1$

$y/2 = 4 \Rightarrow y = 8$

9. Sucrose hydrolyses in acid solution into glucose and fructose following first order rate law with a half-life of 3.33 h at 25ºC. After 9h, the fraction of sucrose remaining is f. The value of $\log_{10}\left(\frac{1}{f}\right)$ is _____$\times 10^{-2}$ (Rounded off to the nearest integer)
[Assume: ln10 = 2.303, ln2 = 0.693]

Ans. **81**

Sol. Sucose $\xrightarrow{\text{Hydrolysis}}$ Glucose + Fructose

$t_{1/2} = 3.33h = \frac{10}{3}h \quad \Rightarrow \quad C_t = \frac{C_o}{2^{t/t_{1/2}}}$

Fraction of sucrose remaining = f = $\frac{C_t}{C_o} = \frac{1}{2^{t/t_{1/2}}}$

$\frac{1}{f} = 2^{t/t_{1/2}}$

$\log(1/f) = \log\left(2^{t/t_{1/2}}\right) = \frac{t}{t_{1/2}}\log(2)$

$= \frac{9}{10/3} \times 0.3 = \frac{8.1}{10} = 0.81 \quad = x \times 10^{-2} \quad x = 81$

10. Assuming ideal behaviour, the magnitude of log K for the following reaction at 25ºC is $x \times 10^{-1}$. The value of x is _____.(Integer answer)

$$3HC \equiv CH_{(g)} \rightleftharpoons C_6H_{6(\ell)}$$

[Given : $\Delta_f G°(HC \equiv CH) = -2.04 \times 10^5$] mol^{-1}; $\Delta_f G°(C_6H_6) = -1.24 \times 10^5$ J mol^{-1} ;
R = 8.314 J K^{-1} mol^{-1}]

Ans. **855**

Sol. $3HC \equiv CH(g) \rightleftharpoons C_6H_6(\ell)$

$\Delta G_r^\circ = \Delta G_f^\circ\left[C_6H_8(\ell)\right] - 3 \times \Delta G_f^\circ\left[HC \equiv CH\right]$

$= [-1.24 \times 10^5 - 3x(-2.04 \times 10^5)]$

$= 4.88 \times 10^5$ J/mol

$\Delta G_r^\circ = -RT\ln(K_{eq})$

$\log(K_{eq}) = \frac{-\Delta G^\circ}{2.303RT}$

$= \frac{-4.88 \times 10^5}{2.303 \times 8.314 \times 298}$

$= -8.55 \times 10^1 = 855 \times 10^{-1}$

24th Feb. 2021 | Shift - 2
MATHEMATICS

1. Let $a, b \in R$. If the mirror image of the point $P(a, 6, 9)$ with respect to the line $\frac{x-3}{7} = \frac{y-2}{5} = \frac{z-1}{-9}$ is $(20, b, -a-9)$, then $|a+b|$ is equal to :

(1) 86 (2) 88 (3)84 (4)90

Ans. (2)

Sol. P(a, 6, 9), Q (20, b, –a–9)

mid point of $PQ = \left(\frac{a+20}{2}, \frac{b+6}{2}, -\frac{a}{2}\right)$

lie on line

$$\frac{\frac{a+20}{2}-3}{7} = \frac{\frac{b+6}{2}-2}{5} = \frac{-\frac{a}{2}-1}{-9}$$

$$\frac{a+20-6}{14} = \frac{b+6-4}{10} = \frac{-a-2}{-18}$$

$$\frac{a+14}{14} = \frac{a+2}{18}$$

$$18a + 252 = 14a + 28$$

$$4a = -224$$

$$\boxed{a = -56}$$

$$\frac{b+2}{10} = \frac{a+2}{18}$$

$$\frac{b+2}{10} = \frac{-54}{18}$$

$$\frac{b+2}{10} = -3 \Rightarrow b = -32$$

$$|a+b| = |-56-32| = 88$$

2. Let f be a twice differentiable function defined on R such that $f(0) = 1, f'(0) = 2$ and $f'(x) \neq 0$ for all x∈R. If $\begin{vmatrix} f(x) & f'(x) \\ f'(x) & f''(x) \end{vmatrix} = 0$, for all x∈R then the value of $f(1)$ lies in the interval:

(1) (9, 12) (2) (6, 9) (3) (3, 6) (4) (0, 3)

Ans. (2)

Sol. Given f(x) f ”(X) – $(f'(x))^2 = 0$

Let h (x) = $\frac{f(x)}{f'(x)}$

$\Rightarrow$ h’(x) = 0 $\Rightarrow$ h(x) = k

$\Rightarrow \frac{f(x)}{f'(x)} = k$ $\Rightarrow$ f (x) = k f’(x)

$\Rightarrow$ f(0) = k f’(0) $\Rightarrow$ 1 = k(2) $\Rightarrow$ k = $\frac{1}{2}$

Now f(x) = $\frac{1}{2}$ f’(x) $\Rightarrow \int 2dx = \int \frac{f'(x)}{f(x)} dx$

$\Rightarrow$ 2x = $ln|f(x)|$ + C

As f(0) = 1 $\Rightarrow$ C = 0

$\Rightarrow$ 2x = $ln|f(X)|$ $\Rightarrow$ f(x) = ± e^{2x}

As f(0) = 1 $\Rightarrow$ f(x) = e^{2x} $\Rightarrow$ f(1) = e^2

3. A possible value of $\tan\left(\frac{1}{4}\sin^{-1}\frac{\sqrt{63}}{8}\right)$ is:

(1) $\frac{1}{2\sqrt{2}}$ (2) $\frac{1}{\sqrt{7}}$ (3) $\sqrt{7}-1$ (4) $2\sqrt{2}-1$

Ans. (2)

Sol. $\tan\left(\frac{1}{4}\sin^{-1}\frac{\sqrt{63}}{8}\right)$

Let $\sin^{-1}\left(\frac{\sqrt{63}}{8}\right) = \theta$ $\sin\theta = \frac{\sqrt{63}}{8}$

$\cos\theta = \frac{1}{8}$

$$2\cos^2\frac{\theta}{2}-1=\frac{1}{8}$$

$$\cos^2\frac{\theta}{2}=\frac{9}{16}$$

$$\cos\frac{\theta}{2}=\frac{3}{4}$$

$$\frac{1-\tan^2\frac{\theta}{4}}{1+\tan^2\frac{\theta}{4}}=\frac{3}{4}$$

$$\tan\frac{\theta}{4}=\frac{1}{\sqrt{7}}$$

4. The probability that two randomly selected subsets of the set {1,2,3,4,5} have exactly two elements in their intersection, is:

(1) $\frac{65}{2^7}$ (2) $\frac{135}{2^9}$ (3) $\frac{65}{2^8}$ (4) $\frac{35}{2^7}$

Ans. (2)

Sol. Required probability

$$= \frac{{}^5C_2 \times 3^3}{4^5}$$

$$= \frac{10\times 27}{2^{10}} = \frac{135}{2^9}$$

5. The vector equation of the plane passing through the intersection of the planes $\vec{r}\cdot(\hat{i}+\hat{j}+\hat{k})=1$ and $\vec{r}\cdot(\hat{i}-2\hat{j})=-2$, and the point (1,0,2) is :

(1) $\vec{r}.\left(\hat{i}-7\hat{j}+3\hat{k}\right)=\frac{7}{3}$ (2) $\vec{r}.\left(\hat{i}+7\hat{j}+3\hat{k}\right)=7$

(3) $\vec{r}.\left(3\hat{i}+7\hat{j}+3\hat{k}\right)=7$ (4) $\vec{r}.\left(\hat{i}+7\hat{j}+3\hat{k}\right)=\frac{7}{3}$

Ans. (2)

Sol. Plane passing through intersection of plane is

$$\left\{\vec{r}\cdot\left(\hat{i}+\hat{j}+\hat{k}\right)-1\right\}+\lambda\left\{\vec{r}\cdot\left(\hat{i}-2\hat{j}\right)+2\right\}=0$$

Passes through $\hat{i}+2\hat{k}$, we get

$$(3-1)+\lambda(1+2)=0 \Rightarrow \lambda=-\frac{2}{3}$$

Hence, equation of plane is $3\left\{\vec{r}\cdot\left(\hat{i}+\hat{j}+\hat{k}\right)-1\right\}-2\left\{\vec{r}\cdot\left(\hat{i}-2\hat{j}\right)+2\right\}=0$

$$\Rightarrow \quad \vec{r}\cdot\left(\hat{i}+7\hat{j}+3\hat{k}\right)=7$$

6. If P is a point on the parabola $y=x^2+4$ which is closest to the straight line $y=4x-1$, then the co-ordinates of P are :

(1) (–2, 8) (2) (1, 5) (3) (3, 13) (4) (2, 8)

Ans. (4)

Sol. $\frac{dy}{dx}\Big|_p=4$

$\therefore\ 2x_1 = 4$

$\Rightarrow\ x_1 = 2$

$\therefore$ Point will be (2, 8)

7. Let a, b, c be in arithmetic progression. Let the centroid of the triangle with vertices $(a,c),(2,b)$ and (a,b) be $\left(\frac{10}{3},\frac{7}{3}\right)$. If α,β are the roots of the equation $ax^2+bx+1=0$, then the value of $\alpha^2+\beta^2-\alpha\beta$ is:

(1) $\frac{71}{256}$ (2) $-\frac{69}{256}$ (3) $\frac{69}{256}$ (4) $-\frac{71}{256}$

Ans. (4)

Sol. 2b = a + c

$\frac{2a+2}{3}=\frac{10}{3}$ and $\frac{2b+c}{3}=\frac{7}{3}$

a = 4, $\left.\begin{matrix}2b+c=7\\2b-c=4\end{matrix}\right\}$, solving

$b=\frac{11}{4}$

$c=\frac{3}{2}$

$\therefore$ Quadratic Equation is $4x^2+\frac{11}{4}x+1=0$

$\therefore$ The value of $(\alpha+\beta)^2-3\alpha\beta=\frac{121}{256}-\frac{3}{4}=-\frac{71}{256}$

8. The value of the integral, $\int_1^3\left[x^2-2x-2\right]dx$, where [x] denotes the greatest integer less than or equal to x, is:

(1) –4 (2) –5 (3) $-\sqrt{2}-\sqrt{3}-1$ (4) $-\sqrt{2}-\sqrt{3}+1$

Ans. (3)

Sol. $I=\int_1^3-3dx+\int_1^3\left[(x-1)^2\right]dx$

Put x –1 = t ; dx = dt

$I=(-6)+\int_0^2\left[t^2\right]dt$

$I=-6+\int_0^1 0\,dt+\int_1^{\sqrt{2}}1\,dt+\int_{\sqrt{2}}^{\sqrt{3}}2\,dt+\int_{\sqrt{3}}^{2}3\,dt$

$I=-6+\left(\sqrt{2}-1\right)+2\sqrt{3}-2\sqrt{2}+6-3\sqrt{3}$

$I=-1-\sqrt{2}-\sqrt{3}$

9. Let $f:\mathbf{R}\to\mathbf{R}$ be defined as

$$f(x)=\begin{cases}-55x, & \text{if } x<-5\\ 2x^3-3x^2-120x, & \text{if } -5\le x\le 4\\ 2x^3-3x^2-36x-336, & \text{if } x>4\end{cases}$$

Let $A=\{x\in R: f$ is increasing$\}$. Then A is equal to :

(1) $(-5,-4)\cup(4,\infty)$ (2) $(-5,\infty)$

(3) $(-\infty,-5)\cup(4,\infty)$ (4) $(-\infty,-5)\cup(-4,\infty)$

Ans. (1)

Sol. $$f(x)=\begin{cases}-55 & ; \quad x<-5\\ 6\left(x^2-x-20\right) & ; \quad -5<x<4\\ 6\left(x^2-x-6\right) & ; \quad x>4\end{cases}$$

$$f(x)=\begin{cases}-55 & ; \quad x<-5\\ 6(x-5)(x+4) & ; \quad -5<x<4\\ 6(x-3)(x+2) & ; \quad x>4\end{cases}$$

Hence, f(x) is monotonically increasing in interval $(-5, -4)\cup(4, \infty)$

10. If the curve $y=ax^2+bx+c, x\in R,$ passes through the point (1,2) and the tangent line to this curve at origin is $y=x$, then the possible values of a, b, c are :

(1). a =1, b=1, c=0 (2) a = –1, b=1, c =1

(3) a =1, b=0, c =1 (4) $a=\frac{1}{2}, b=\frac{1}{2}, c=1$

Ans. (1)

Sol. 2 = a + b + c .....(i)

$\frac{dy}{dx}$ = 2ax + b $\Rightarrow \left.\frac{dy}{dx}\right|_{(0,0)}$ = 1

$\Rightarrow$ b = 1 $\Rightarrow$ a + c = 1

(0,0) lie on curve

$\therefore$ c=0, a=1

11. The negation of the statement

$\sim p \wedge (p \vee q)$ is :

(1) $\sim p \wedge q$ (2) $p \wedge \sim q$ (3) $\sim p \vee q$ (4) $p \vee \sim q$

Ans. (4)

Sol.

p	q	$\sim$ p	p $\vee$ q	($\sim$ p) $\wedge$ (p $\vee$ q)	$\sim$ q	p $\vee$ $\sim$ q
T	T	F	T	F	F	T
T	F	F	T	F	T	T
F	T	T	T	T	F	F
F	F	T	F	F	T	T

$\therefore \sim p \wedge (p \vee q) \equiv p \vee \sim q$

12. For the system of linear equations:

$x - 2y = 1, x - y + kz = -2, ky + 4z = 6, k \in \mathbf{R}$

consider the following statements:

(A) The system has unique solution if $k \neq 2, k \neq -2$.

(B) The system has unique solution if $k = -2$.

(C) The system has unique solution if $k = 2$.

(D) The system has no-solution if $k = 2$.

(E) The system has infinite number of solutions if $k \neq -2$.

Which of the following statements are correct?

(1) (B) and (E) only (2)(C) and (D) only

(3) (A) and (D) only (4) (A) and (E) only

Ans. (3)

Sol. $x - 2y + 0.z = 1$

$x - y + kz = -2$

$0.x + ky + 4z = 6$

$$\Delta = \begin{vmatrix} 1 & -2 & 0 \\ 1 & -1 & k \\ 0 & k & 4 \end{vmatrix} = 4 - k^2$$

For unique solution $4 - k^2 \neq 0$

$$\boxed{k \neq \pm 2}$$

For k=2

$x - 2y + 0.z = 1$

$x - y + 2z = -2$

$0.x + 2y + 4z = 6$

$$\Delta x = \begin{vmatrix} 1 & -2 & 0 \\ -2 & -1 & 2 \\ 6 & 2 & 4 \end{vmatrix} = (-8) + 2[-20]$$

$\Delta x = -48 \neq 0$

For k=2 $\Delta x \neq 0$

For K=2; The system has no solution

13. For which of the following curves, the line $x + \sqrt{3}y = 2\sqrt{3}$ is the tangent at the point $\left(\frac{3\sqrt{3}}{2}, \frac{1}{2}\right)$?

(1) $x^2 + 9y^2 = 9$

(2) $2x^2 - 18y^2 = 9$

(3) $y^2 = \frac{1}{6\sqrt{3}}x$

(4) $x^2 + y^2 = 7$

Ans. (1)

Sol. Tangent to $x^2 + 9y^2 = 9$ at point $\left(\frac{3\sqrt{3}}{2}, \frac{1}{2}\right)$ is $x\left|\frac{3\sqrt{3}}{2}\right| + 9y\left(\frac{1}{2}\right) = 9$

$3\sqrt{3}x + 9y = 18 \Rightarrow x + \sqrt{3}y = 2\sqrt{3}$

$\Rightarrow$ option (1) is true

14. The angle of elevation of a jet plane from a point A on the ground is 60°. After a flight of 20 seconds at the speed of 432 km/ hour, the angle of elevation changes to 30°. If the jet plane is flying at a constant height, then its height is:

(1) $1200\sqrt{3}$m (2) $1800\sqrt{3}$m (3) $3600\sqrt{3}$m (4) $2400\sqrt{3}$m

Ans. (1)

Sol.

$v = 432 \times \frac{1000}{60 \times 60}$ m/sec = 120 m/sec

Distance AB = v × 20 = 2400 meter

In ΔPAC

$\tan 60° = \frac{h}{PC} \Rightarrow PC = \frac{h}{\sqrt{3}}$

In ΔPBD

$\tan 30° = \frac{h}{PD} \Rightarrow PD = \sqrt{3}h$

PD = PC + CD

$\sqrt{3}h = \frac{h}{\sqrt{3}} + 2400 \Rightarrow \frac{2h}{\sqrt{3}} = 2400$

$h = 1200\sqrt{3}$ meter

15. For the statements p and q, consider the following compound statements:

(a) $(\sim q \wedge (p \to q)) \to \sim p$

(b) $((p \vee q)) \wedge \sim P) \to P$

Then which of the following statements is correct?

(1) (a) is a tautology but not (b)

(2) (a) and (b) both are not tautologies.

(3) (a) and (b) both are tautologies.

(4) (b) is a tautology but not (a).

Ans. (3)

Sol. (a)

p	q	$\sim q$	$p \to q$	$\sim q \wedge (p \to q)$	$\sim p$	$(\sim q) \wedge (p \to q) \to \sim p$
T	T	F	T	F	F	T
T	F	T	F	F	F	T
F	T	F	T	F	T	T
F	F	T	T	T	T	T

(a) is tautologies

(b)

p	q	$p \vee q$	$\sim p$	$(p \vee q) \wedge \sim p$	$((p \vee q) \wedge \sim p) \to q$
T	T	T	F	F	T
T	F	T	F	F	T
F	T	T	T	T	T
F	F	F	T	F	T

(b) is tautologies

$\therefore$ a & b are both tautologies.

16. Let A and B be 3×3 real matrices such that A is symmetric matrix and B is skew-symmetric matrix. Then the system of linear equations $(A^2 B^2 - B^2 A^2) X = O,$ where X is a 3×1 column matrix of unknown variables and O is a 3×1 null matrix, has :

(1) a unique solution

(2) exactly two solutions

(3) infinitely many solutions

(4) no solution

Ans. (3)

Sol. $A^T = A, B^T = -B$

Let $A^2B^2 - B^2A^2 = P$

$P^T = (A^2B^2 - B^2A^2)^T = (A^2B^2)^T - (B^2A^2)^T$

$= (B^2)^T (A^2)^T - (A^2)^T (B^2)^T$

$= B^2A^2 - A^2B^2$

$\Rightarrow$ P is skew-symmetric matrix

$$\begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\therefore$ $ay + bz = 0$ ...(1)

$-ax + cz = 0$ ...(2)

$-bx - cy = 0$ ...(3)

From equation 1,2,3

$\Delta = 0$ & $\Delta_1 = \Delta_2 = \Delta_3 = 0$

$\therefore$ equation have infinite number of solution

17. If $n \geq 2$ is a positive integer, then the sum of the series

${}^{n+1}C_2 + 2\left({}^2C_2 + {}^3C_2 + {}^4C_2 + \ldots + {}^nC_2\right)$ is :

(1) $\dfrac{n(n+1)^2(n+2)}{12}$

(2) $\dfrac{n(n-1)(2n+1)}{6}$

(3) $\dfrac{n(n+1)(2n+1)}{6}$

(4) $\dfrac{n(2n+1)(3n+1)}{6}$

Ans. (3)

Sol. ${}^2C_2 = {}^3C_3$

$S = {}^3C_3 + {}^3C_2 + \ldots\ldots + {}^nC_2 = {}^{n+1}C_3$

$\because {}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

$\therefore {}^{n+1}C_2 + {}^{n+1}C_3 + {}^{n+1}C_3 = {}^{n+2}C_3 + {}^{n+1}C_3$

$= \dfrac{(n+1)!}{3!(n-1)!} + \dfrac{(n+1)!}{3!(n-2)!}$

$= \dfrac{(n+2)(n+1)n}{6} + \dfrac{(n+1)(n)(n-1)}{6} = \dfrac{n(n+1)(2n+1)}{6}$

18. If a curve $y = f(x)$ passes through the point (1,2) and satisfies $x\frac{dy}{dx} + y = bx^4$, then for what value of b, $\int_1^2 f(x)dx = \frac{62}{5}$?

(1) 5 (2) $\frac{62}{5}$ (3) $\frac{31}{5}$ (4) 10

Ans. (4)

Sol. $\frac{dy}{dx} + \frac{y}{x} = bx^3$, I.F. = $e^{\int \frac{dx}{x}} = x$

$\therefore yx = \int bx^4 dx = \frac{bx^5}{5} + C$

Passes through (1,2), we get

$2 = \frac{b}{5} + C$ .....(i)

Also, $\int_1^2 \left(\frac{bx^4}{5} + \frac{c}{x}\right) dx = \frac{62}{5}$

$\Rightarrow \frac{b}{25} \times 32 + C\ln 2 - \frac{b}{25} = \frac{62}{5} \Rightarrow C = 0$ & $b = 10$

19. The area of the region : $R = \{(x, y) : 5x^2 \le y \le 2x^2 + 9\}$ is:

(1) $9\sqrt{3}$ square units (2) $12\sqrt{3}$ square units (3) $11\sqrt{3}$ square units (4) $6\sqrt{3}$ square units

Ans. (2)

Sol.

Required area

$= 2\int_0^{\sqrt{3}} \left(2x^2 + 9 - 5x^2\right) dx$

$= 2\int_0^{\sqrt{3}} \left(9 - 3x^2\right) dx$

$= 2 \mid 9x - x^3 \mid_0^{\sqrt{3}} = 12\sqrt{3}$

20. Let $f(x)$ be a differentiable function defined on [0,2] such that $f'(x) = f'(2-x)$ for all $x \in (0,2)$, $f(0) = 1$ and $f(2) = e^2$. Then the value of $\int_0^2 f(x)dx$ is:

(1) $1 + e^2$ (2) $1 - e^2$ (3) $2(1 - e^2)$ (4) $2(1 + e^2)$

Ans. (1)

Sol. f '(x) = f ' (2–x)

On integrating both side f(x) = –f(2–x) +c

put x = 0

f(0) + f(2) = c $\Rightarrow$ c = 1 + e^2

$\Rightarrow$ f(x) + f(2–x) = 1+e^2 .....(i)

$I = \int_0^2 f(x)\,dx = \int_0^1 \{f(x) + f(2-x)\}\,dx = \left(1 + e^2\right)$

Section B

1. The number of the real roots of the equation $(x+1)^2 + |x-5| = \frac{27}{4}$ is_____.

Ans. 2

Sol. $x \geq 5$

$(x+1)^2 + (x-5) = \frac{27}{4}$

$\Rightarrow x^2 + 3x - 4 = \frac{27}{4}$

$\Rightarrow x^2 + 3x - \frac{43}{4} = 0$

$\Rightarrow 4x^2 + 12x - 43 = 0$

$$x = \frac{-12 \pm \sqrt{144+688}}{8}$$

$$x = \frac{-12 \pm \sqrt{832}}{8} = \frac{-12 \pm 28.8}{8}$$

$$= \frac{-3 \pm 7.2}{2}$$

$$= \frac{-3+7.2}{2}, \frac{-3-7.2}{2}$$ (Therefore no solution)

For $x \le 5$

$$(x+1)^2 - (x-5) = \frac{27}{4}$$

$$x^2 + x + 6 - \frac{27}{4} = 0$$

$$4x^2 + 4x - 3 = 0$$

$$x = \frac{-4 \pm \sqrt{16+48}}{8}$$

$$x = \frac{-4 \pm 8}{8} \Rightarrow x = -\frac{12}{8}, \frac{4}{8}$$

$\therefore$ 2 Real Root's

2. The students $S_1, S_2, \ldots, S_{10}$ are to be divided into 3 groups A, B and C such that each group has at least one student and the group C has at most 3 students. Then the total number of possibilities of forming such groups is_____.

Ans. 31650

Sol.

$C \to 1$ $\quad 9 \begin{cases} A \\ B \end{cases}$

$C \to 2$ $\quad 8 \begin{cases} A \\ B \end{cases}$

$C \to 3$ $\quad 7 \begin{cases} A \\ B \end{cases}$

$= {}^{10}C_1\,[2^9 - 2] + {}^{10}C_2[2^8 - 2] + {}^{10}C_3\,[2^7 - 2]$

$= 2^7[{}^{10}C_1 \times 4 + {}^{10}C_2 \times 2 + {}^{10}C_3] - 20 - 90 - 240$

$= 128\ [40 + 90 + 120\ ] - 350$

$= (128 \times 250) - 350$

$= 10[3165] = 31650$

3. If $a+\alpha=1, b+\beta=2$ and $af(x)+\alpha f\left(\frac{1}{x}\right)=bx+\frac{\beta}{x}, x\neq 0,$ then the value of the expression $\dfrac{f(x)+f\left(\frac{1}{x}\right)}{x+\frac{1}{x}}$ is_____.

Ans. 2

Sol. $af(x) + \alpha f\left(\frac{1}{x}\right) = bx + \frac{\beta}{x}$ .....(i)

$x \to \frac{1}{x}$

$af\left(\frac{1}{x}\right) + \alpha f(x) = \frac{b}{x} + \beta x$ .....(ii)

(i) + (ii)

$(a+\alpha)\left[f(x)+f\left(\frac{1}{x}\right)\right] = \left(x+\frac{1}{x}\right)(b+\beta)$

$\dfrac{f(x)+f\left(\frac{1}{x}\right)}{x+\frac{1}{x}} = \frac{2}{1} = 2$

4. If the variance of 10 natural numbers $1,1,1,\ldots,1,k$ is less than 10, then the maximum possible value of k is_____.

Ans. **11**

Sol. $\sigma^2 = \frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2$

$\sigma^2 = \frac{\left(9+k^2\right)}{10} - \left(\frac{9+k}{10}\right)^2 < 10$

$(90 + k^2)\ 10 - (81 + k^2 + 8k) < 1000$

$90 + 10k^2 - k^2 - 18k - 81 < 1000$

$9k^2 - 18k + 9 < 1000$

$(k-1)^2 < \frac{1000}{9} \Rightarrow k - 1 < \frac{10\sqrt{10}}{3}$

$k < \frac{10\sqrt{10}}{3} + 1$

Maximum integral value of k = 11

5. Let λ be an integer. If the shortest distance between the lines $x - \lambda = 2y - 1 = -2z$ and $x = y + 2\lambda = z - \lambda$ is $\frac{\sqrt{7}}{2\sqrt{2}}$, then the value of $|\lambda|$ is

Ans. **1**

Sol. $\frac{x-\lambda}{1} = \frac{y-\frac{1}{2}}{\frac{1}{2}} = \frac{z}{-\frac{1}{2}}$

$\frac{x-\lambda}{2} = \frac{y-\frac{1}{2}}{1} = \frac{2}{-1}$ .....(1) Point on line = $\left(\lambda, \frac{1}{2}, 0\right)$

$\frac{x}{1} = \frac{y+2\lambda}{1} = \frac{z-\lambda}{1}$ .....(2) Point on line = $\left(0, -2\lambda, \lambda\right)$

Distance between skew lines = $\frac{\left[\vec{a}_2 - \vec{a}_1\ \vec{b}_1\ \vec{b}_2\right]}{\left|\vec{b}_1 \times \vec{b}_2\right|}$

$$\frac{\begin{vmatrix} \lambda & \frac{1}{2}+2\lambda & -\lambda \\ 2 & 1 & -1 \\ 1 & 1 & 1 \end{vmatrix}}{\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -1 \\ 1 & 1 & 1 \end{vmatrix}}$$

$$= \frac{\left|-5\lambda-\frac{3}{2}\right|}{\sqrt{14}} = \frac{\sqrt{7}}{2\sqrt{2}} \text{ (given)}$$

$$= \left|10\lambda+3\right| = 7 \Rightarrow \lambda = -1$$

$$\Rightarrow \left|\lambda\right| = 1$$

6. Let $i=\sqrt{-1}$. If $\frac{(-1+i\sqrt{3})^{21}}{(1-i)^{24}}+\frac{(1+i\sqrt{3})^{21}}{(1+i)^{24}}=k$, and $n=\left[\left|k\right|\right]$ be the greatest integral part of |k|. Then $\sum_{j=0}^{n+5}(j+5)^2-\sum_{j=0}^{n+5}(j+5)$ is equal to_____.

Ans. 310

Sol. $$\frac{\left(2e^{i\frac{2\pi}{3}}\right)^{21}}{\left(\sqrt{2}e^{-i\frac{\pi}{4}}\right)^{24}}+\frac{\left(2e^{i\frac{\pi}{3}}\right)^{21}}{\left(\sqrt{2}e^{i\frac{\pi}{4}}\right)^{24}}$$

$$\Rightarrow \frac{2^{21}.e^{i14\pi}}{2^{12}.e^{-i6\pi}}+\frac{2^{21}\left(e^{i7\pi}\right)}{2^{12}\left(e^{i6\pi}\right)}$$

$$\Rightarrow 2^9e^{i(20\pi)}+2^9e^{i\pi}$$

$$\Rightarrow 2^9+2^9\left(-1\right)=0$$

n= 0

$$\sum_{j=0}^{5}\left(j+5\right)^2-\sum_{j=0}^{5}\left(j+5\right)$$

$\Rightarrow \left[5^2+6^2+7^2+8^2+9^2+10^2\right]-\left[5+6+7+8+9+10\right]$

$\Rightarrow \left[\left(1^2+2^2+....+10^2\right)-\left(1^2+2^2+3^2+4^2\right)\right]-\left[\left(1+2+3+.....+10\right)-\left(1+2+3+4\right)\right]$

$\Rightarrow \left(385-30\right)-\left[55-10\right]$

$\Rightarrow$ 355 – 45 $\Rightarrow$ 310 ans.

7. Let a point P be such that its distance from the point (5,0) is thrice the distance of P from the point $(-5,0)$. If the locus of the point P is a circle of radius r, then $4r^2$ is equal to

Ans. 56.25

Sol. Let P(h,k)

Given

PA = 3PB

$PA^2 = 9PB^2$

$\Rightarrow (h-5)^2 + k^2 = 9[(h+5)^2+k^2]$

$\Rightarrow 8h^2 + 8k^2 + 100h + 200 = 0$

$\therefore$ Locus

$x^2 + y^2 + \left(\frac{25}{2}\right)x + 25 = 0$

$\therefore c \equiv \left(\frac{-25}{4},0\right)$

$\therefore r^2 = \left(\frac{-25}{4}\right)^2 - 25$

$= \frac{625}{16} - 25$

$= \frac{225}{16}$

$\therefore 4r^2 = 4\times\frac{225}{16} = \frac{225}{4} = 56.25$

8. For integers n and r, let $\binom{n}{r} = \begin{cases} {}^nC_r, & \text{if } n \geq r \geq 0 \\ 0, & \text{otherwise} \end{cases}$

The maximum value of k for which the sum

$\sum_{i=0}^{k} \binom{10}{i}\binom{15}{k-i} + \sum_{i=0}^{k+1} \binom{12}{i}\binom{13}{k+1-i}$ exists, is equal to_____.

Ans. 12

Sol. $(1+x)^{10} = {}^{10}C_0 + {}^{10}C_1x + {}^{10}C_2x^2 + \ldots\ldots + {}^{10}C_{10}x^{10}$

$(1+x)^{15} = {}^{15}C_0 + {}^{15}C_1x + \ldots.{}^{15}C_{k-1}\, x^{k-1} + {}^{15}C_kx^k + {}^{15}C_{k+1}x^{k+1} + \ldots\ldots\ {}^{15}C_{15}x^{15}$

$\sum_{i=0}^{k}(10C_i)(15C_{k-i}) = {}^{10}C_0.\ {}^{15}C_k + {}^{10}C_1.{}^{15}C_{k-1} + \ldots.. + {}^{10}C_k.{}^{15}C_0$

Coefficient of x_k in $(1+x)^{25}$

$= {}^{25}C_k$

$\sum_{i=0}^{k+1}\left({}^{12}C_i\right)\left({}^{13}C_{k+1-i}\right) = {}^{12}C_0.{}^{13}C_{k+1} + {}^{12}C_1.\ {}^{13}C_k + \ldots\ldots + {}^{12}C_{k+1}.\ {}^{13}C_0$

Coefficient of x^{k+1} in $(1+x)^{25}$

$= {}^{25}C_{k+1}$

${}^{25}C_k + {}^{25}C_{k+1} = {}^{26}C_{k+1}$

For maximum value

k+1= 13

K= 12

9. The sum of first four terms of a geometric progression (G.P.) is $\frac{65}{12}$ and the sum of their respective reciprocals is $\frac{65}{18}$. If the product of first three terms of the G.P. is 1, and the third term is α, then 2α is_____.

Ans. 3

Sol. a, ar, ar^2, ar^3

$a + ar + ar^2 + ar^3 = \frac{65}{12}$ .....(1)

$\frac{1}{a} + \frac{1}{ar} + \frac{1}{ar^2} + \frac{1}{ar^3} = \frac{65}{18}$

$$\frac{1}{a}\left(\frac{r^3+r^2+r+1}{r^3}\right)=\frac{65}{18} \qquad \text{.....(2)}$$

$$\frac{(i)}{(ii)}, a^2r^3=\frac{18}{12}=\frac{3}{2}$$

$$a^3r^3=1 \Rightarrow a\left(\frac{3}{2}\right)=1 \Rightarrow a=\frac{2}{3}$$

$$\frac{4}{9}r^3=\frac{3}{2} \Rightarrow r^3=\frac{3^3}{2^3} \Rightarrow r=\frac{3}{2}$$

$$\alpha=ar^2=\frac{2}{3}.\left(\frac{3}{2}\right)^2=\frac{3}{2}$$

$2\alpha = 3$

10. If the area of the triangle formed by the positive x-axis, the normal and the tangent to the circle $(x-2)^2+(y-3)^2=25$ at the point (5,7) is A, then 24A is equal to_____.

Ans. 1225

Sol.

Equation of normal at P

$$(y-7)=\left(\frac{7-3}{5-2}\right)(x-5)$$

$3y - 21 = 4x - 20$

$\Rightarrow 4x - 3y + 1 = 0$ .....(i)

$$\Rightarrow \quad M\left(-\frac{1}{4},0\right)$$

Equation of tangent at P

$$(y-7) = -\frac{3}{4}(x-5)$$

$4y - 28 = -3x + 15$

$\Rightarrow 3x + 4y = 43$ .....(ii)

$\Rightarrow \quad N\left(\frac{43}{3}, 0\right)$

Hence ar (ΔPMN) = $\frac{1}{2} \times MN \times 7$

$$\lambda = \frac{1}{2} \times \frac{175}{12} \times 7$$

$\Rightarrow 24\lambda = 1225$

24th Feb. 2021 | Shift - 1
PHYSICS

Section - A

1. Four identical particles of equal masses 1 kg made to move along the circumference of a circle of radius 1 m under the action of their own mutual gravitational attraction. The speed of each particle will be -

(1) $\frac{\sqrt{(1+2\sqrt{2})G}}{2}$ (2) $\sqrt{G(1+2\sqrt{2})}$ (3) $\sqrt{\frac{G}{2}\left(2\sqrt{2}-1\right)}$ (4) $\sqrt{\frac{G}{2}\left(1+2\sqrt{2}\right)}$

Sol. **(1)**

$\Rightarrow$ By resolving force F_2, we get

$\Rightarrow F_1 + F_2 \cos 45° + F_2 \cos 45°$

$\Rightarrow F_1 + 2F_2 \cos 45° = F_c$

F_c = centripital force = $\frac{MV^2}{R}$

$$\Rightarrow \frac{GM^2}{(2R)^2} + \left[\frac{2GM^2}{\left(\sqrt{2}R\right)^2}\cos 45°\right] = \frac{MV^2}{R}$$

$$\Rightarrow \frac{GM^2}{4R^2} + \frac{2GM^2}{2\sqrt{2}R^2} = \frac{MV^2}{R}$$

$$\Rightarrow \frac{GM}{4R} + \frac{GM}{\sqrt{2}.R} = V^2$$

$$\Rightarrow V = \sqrt{\frac{GM}{4R} + \frac{GM}{\sqrt{2}.R}}$$

$$\Rightarrow V = \sqrt{\frac{GM}{R}\left[\frac{1+2\sqrt{2}}{4}\right]}$$

$$\Rightarrow V = \frac{1}{2}\sqrt{\frac{GM}{R}\left(1+2\sqrt{2}\right)}$$

(given : mass = 1 kg, radius = 1 m)

$$\Rightarrow v = \frac{1}{2}\sqrt{G(1+2\sqrt{2})}$$

2. Consider two satellites S_1 and S_2 with periods of revolution 1 hr. and 8 hr. respectively revolving around a planet in circular orbits.The ratio of angular velocity of satellite S_1 to the angular velocity of satellite S_2 is -

(1)8 : 1 (2)1 : 8 (3)2 : 1 (4) 1 : 4

Sol. **(1)**

We know that $\omega = \frac{2\pi}{T}$

given : Ratio of time period

$\frac{T_1}{T_2} = \frac{1}{8}$

$\Rightarrow \omega \propto \frac{1}{T}$

$\Rightarrow \frac{\omega_1}{\omega_2} = \frac{T_2}{T_1}$

$\Rightarrow \frac{\omega_1}{\omega_2} = \frac{8}{1}$

$\Rightarrow \omega_1 : \omega_2 = 8 : 1$

3. n mole of a perfect gas undergoes a cyclic process ABCA (see figure) consisting of the following processes –

A→B : Isothermal expansion at temperature T so that the volume is doubled from V_1 to $V_2 = 2V_1$ and pressure changes from P_1 to P_2.

B → C : Isobaric compression at pressure P_2 to initial volume V_1.

C → A : Isochoric change leading to change of pressure from P_2 to P_1.

Total workdone in the complete cycle ABCA is –

(1)0 (2)$nRT\left(\ln 2 + \frac{1}{2}\right)$ (3)nRTln2 (4) $nRT\left(\ln 2 - \frac{1}{2}\right)$

Sol. **(4)**

$A \rightarrow B$ = isotheraml process

$B \rightarrow C$ = isobaric process

$C \rightarrow A$ = isochoric process

also, $V_2 = 2V_1$

work done by gas in the complete cycle ABCA is –

$\Rightarrow w = w_{AB} + w_{BC} + w_{CA}$ ....(1)

$\Rightarrow w_{CA} = 0$, as isochoric process

$\Rightarrow w_{AB} = 2P_1V_1 \ln\left(\frac{V_2}{V_1}\right) = 2\ nRT \ln(2)$

$\Rightarrow w_{BC} = P_2(V_1 - V_2) = P_2(V_1 - 2V_1) = -P_2V_1 = -nRT$

$\Rightarrow$Now put the value of w_{AB}, w_{BC} and w_{CA} in equation, we get

$\Rightarrow w = 2nRT \ln(2) - nRT + 0$

$\Rightarrow w = nRT[2\ln(2) - 1]$

$\Rightarrow w = nRT[\ln(2) - \frac{1}{2}]$

4. Two equal capacitors are first connected in series and then in parallel. The ratio of the equivalent capacities capacities in the two cases will be -

(1)2 : 1 (2)1 : 4 (3)4 : 1 (4) 1 : 2

Sol. **(2)**

Given that first connection

$\Rightarrow \frac{1}{C_{12}} = \frac{1}{C} + \frac{1}{C} \Rightarrow C_{12} = \frac{C}{2}$

Second connection

$C_{34} = C + C = 2\ C$

Now, the ratio of equivalent capacities in the two cases will be –

$$\Rightarrow \frac{C_{12}}{C_{34}} = \frac{C/2}{2C} \Rightarrow \frac{C_{12}}{C_{34}} = \frac{1}{4}$$

5. A cell E_1 of emf 6V and internal resistance 2Ω is connected with another cell E_2 of emf 4V and internal resistance 8Ω (as shown in the figure). The potential difference across points X and Y is –

(1)3.6V (2)10.0V (3)5.6V (4) 2.0V

Sol. **(3)**

emf of E_1 = 6v

$r_1 = 2\ \Omega$

emf of $E_2 = 4\ \Omega$

$r_2 = 8\Omega$

$|v_x - v_y|$ = potential difference across points x and y

$E_{eff} = 6 - 4 = 2$ V

$R_{eq} = 2 + 8 = 10\ \Omega$

So, current in the circuit will be

$$\Rightarrow I = \frac{E_{eff}}{R_{eq}} \Rightarrow I = \frac{2}{10} = 0.2 \text{ A}$$

Now, potential difference across points X and Y is

$|v_x - v_y| = E + iR$

$\Rightarrow |v_x - v_y| = 4 + 0.2 \times 8 = 5.6$ V

$\Rightarrow |v_x - v_y| = 5.6$ v

6. If Y,K and η are the values of Young's modulus, bulk modulus and modulus of rigidity of any material respectively. Choose the correct relation for these parameters.

(1) $K = \dfrac{Y\eta}{9\eta - 3Y}$ N/m^2 (2) $\eta = \dfrac{3YK}{9K + Y}$ N/m^2

(3) $Y = \dfrac{9K\eta}{3K - \eta}$ N/m^2 (4) $Y = \dfrac{9K\eta}{2\eta + 3K}$ N/m^2

Sol. **(1)**

$\Rightarrow y = 3k\,(1 - 2\sigma)$

$\Rightarrow \sigma = \dfrac{1}{2}\left(1 - \dfrac{y}{3k}\right)$ ....(1)

$\Rightarrow y = 2\eta\,(1 + \sigma)$

$\Rightarrow \sigma = \dfrac{y}{2\eta} - 1$ ....(2)

by comparing equation (1) and (2), we get

$\Rightarrow \dfrac{y}{2\eta} - 1 = \dfrac{1}{2}\left(1 - \dfrac{y}{3k}\right)$

$\Rightarrow \dfrac{y}{\eta} - 2 = 1 - \dfrac{y}{3k}$

$\Rightarrow \dfrac{y}{\eta} = 1 + 2 - \dfrac{y}{3k} \Rightarrow \dfrac{y}{\eta} = 3 - \dfrac{y}{3k}$

$\Rightarrow \dfrac{y}{3k} = 3 - \dfrac{y}{\eta} \Rightarrow \dfrac{y}{3k} = \dfrac{3\eta - y}{\eta}$

$\Rightarrow k = \dfrac{\eta y}{9\eta - 3y}$

7. Two stars of masses m and 2m at a distance d rotate about their common centre of mass in free space. The period of revolution is -

(1) $2\pi\sqrt{\dfrac{d^3}{3Gm}}$ (2) $\dfrac{1}{2\pi}\sqrt{\dfrac{3Gm}{d^3}}$ (3) $\dfrac{1}{2\pi}\sqrt{\dfrac{d^3}{3Gm}}$ (4) $2\pi\sqrt{\dfrac{3Gm}{d^3}}$

Sol. **(1)**

$\Rightarrow \dfrac{G(m)(2m)}{d^2} = m\omega^2 \times \dfrac{2d}{3}$

$\Rightarrow \dfrac{2Gm}{d^2} = \omega^2 \times \dfrac{2d}{3}$

$\Rightarrow \omega^2 = \frac{3Gm}{d^3}$

$\Rightarrow \omega = \sqrt{\frac{3Gm}{d^3}}$

we know that, $\omega = \frac{2\pi}{T}$ so $T = \frac{2\pi}{\omega}$

$\Rightarrow T = \frac{2\pi}{\sqrt{\frac{3Gm}{d^3}}} \Rightarrow T = 2\pi\sqrt{\frac{d^3}{3Gm}}$

8. If the velocity-time graph has the shape AMB, what would be the shape of the corresponding acceleration-time graph ?

(1)

(2)

(3)

(4)

Sol. **(1)**

$a = \frac{dv}{dt}$ = slope of (v – t) curve

If m = +ve, then equation of straight line is

$y = mx + c \Rightarrow v = mt + c$ (for MB)

If m =-ve, then equation of straight line is

$y = -mx + c \Rightarrow v = -mt + c$ (for AM)

If we differentiate equation (1) and (2), we get

a_{MB} = +ve = m

a_{AM} =-ve = -m, so graph of (a-t) will be

9. Given below are two statements :

Statement – I: Two photons having equal linear momenta have equal wavelengths.

Statement-II: If the wavelength of photon is decreased, then the momentum and energy of a photon will also decrease.

In the light of the above statements, choose the correct answer from the options given below.

(1)Statemnet-I is false but Statement-II is true

(2)Both Statement-I and Statement-II are true

(3)Both Statement-I and Statement-II are false

(4) Statement-I is true but Statement-II is false

Sol. **(4)**

By theory

10. A current through a wire depends on time as $i = \alpha_0 t + \beta t^2$

where α_0 = 20 A/s and β = 8 As^{-2}. Find the charge crossed through a section of the wire in 15 s.

(1)2100 C (2)260 C (3)2250 C (4) 11250 C

Sol. **(4)**

given :$i = \alpha_0 t + \beta t^2$

α = 20 A/s and β = 8As^{-2}

t = 15 sec

we know that, $i = \frac{dq}{dt} \Rightarrow \int_0^t i\,dt = \int_0^Q dq$

$$\Rightarrow \int_0^{15} (\alpha_0 t + \beta t^2)dt = \int_0^Q dq$$

$$\Rightarrow Q = \left[\frac{\alpha_0 t^2}{2} + \frac{\beta t^3}{3}\right]_0^{15}$$

$\Rightarrow Q = \frac{20 \times 15 \times 15}{2} + \frac{8 \times 15 \times 15 \times 15}{3} - 0$

$\Rightarrow Q = 11250$ C

11. match List I with List II

List-I	List-II
(a) Isothermal	(i) Pressure constant
(b) Isochoric	(ii) Temperature constant
(c) Adiabatic	(iii) Volume constant
(d) Isobaric	(iv) Heat content is constant

Choose the correct answer from the options given below –

(1)(a) – (ii), (b) – (iv), (c) – (iii), (d) – (i)

(2)(a) – (ii), (b) – (iii), (c) – (iv), (d) – (i)

(3)(a) – (i), (b) – (iii), (c) – (ii), (d) – (iv)

(4) (a) – (iii), (b) – (ii), (c) – (i), (d) – (iv)

Sol. (**2**)

(a)→(ii), (b)→(iii), (c)→(iv), (d)→(i),

By theory

In isothermal process, temperature is constant.

In isochoric process, volume is constant.

In adiabatic process, heat content is constant.

In isobaric process, pressure is constant.

12. In the given figure, the energy levels of hydrogen atom have been shown along with some transitions marked A,B,C,D and E.

The transitions A,B and C respectively represents –

(1)The series limit of Lyman series, third member of balmer series and second member of paschen series

(2)The first member of the Lyman series, third member of Balmer series and second member of paschen series

(3)The ionization potential of hydrogen, second member of Balmer series and third member of Paschen series

(4) The series limit of Lyman series, second memebr of Balmer series and second member of Paschen series.

Sol. **(1)**

A $\rightarrow$ series limit of lyman.

B $\rightarrow$ 3rd member of Balmer series.

C $\rightarrow$ 2nd member of Paschen series.

13. The focal length f is related to the radius of curvature r of the spherical convex mirror by -

(1) f = r (2) $f = -\frac{1}{2}r$ (3) $f = +\frac{1}{2}r$ (4) f = -r

Sol. **(3)**

So, $\frac{R}{2} = f$

$F = +\frac{1}{2} R$

14. Moment of inertia (M.I.) of four bodies, having same mass and radius, are reported as –

I_1 = M.I. of thin circular ring about its diameter,

I_2 = M.I. of circular disc about an axis perpendicular to disc and going through the centre,

I_3 = M.I. of solid cylinder about its axis and

I_4 = M.I. of solid sphere about its diameter.

Then :-

(1)$I_1 = I_2 = I_3 < I_4$ (2)$I_1 + I_2 = I_3 + \frac{5}{2} I_4$

(3)$I_1 + I_3 < I_2 + I_4$ (4) $I_1 = I_2 = I_3 > I_4$

Sol. **(4)**

Given$\Rightarrow$ I_1 = M.I. of thin circular ring about its diameter

I_2 = M.I. circular disc about an axis perpendicular to disc and going through the centre.

I_3 = M.I. of solid cylinder about its axis

I_4 = M.I. of solid sphere about its diameter

we know that,

$I_1 = \frac{MR^2}{2}$, $I_2 = \frac{MR^2}{2}$, $I_3 = \frac{MR^2}{2}$

$I_4 = \frac{2}{5} MR^2$

So, $I_1 = I_2 = I_3 > I_4$

15. The workdone by a gas molecule in an isolated system is given by, $W = \alpha\beta^2 e^{-\frac{x^2}{\alpha kT}}$, where x is the displacement, k is the Boltzmann constant and T is the temperature.α and β are constants. Then the dimensions of β will be -

(1)$[M^0LT^0]$ (2)$[M^2LT^2]$ (3)$[MLT^{-2}]$ (4) $[ML^2T^{-2}]$

Sol. **(3)**

given : work = $\alpha.\beta^2.e^{-\frac{x^2}{\alpha.k.T}}$

k = boltzmann constant

T = temperature

x = displacement

we know that, $\frac{x^2}{\alpha.k.T}$ = dimensionless

$\left[\frac{x^2}{\alpha.k.T}\right] = [M^0L^0T^0]$

$[\alpha] = \left[\frac{L^2}{K.T}\right]$

$\Rightarrow [K] = [M^1L^2T^{-2}K^{-1}]$

$[T] = [K]$

$\Rightarrow [\alpha] = \left[\frac{L^2}{M^1L^2T^{-2}K^{-1} \times K}\right] \Rightarrow [\alpha] = [M^{-1}T^2]$

$\Rightarrow \omega = \alpha.\beta^2$

$\Rightarrow \frac{[M^1L^1T^{-2}][L^{-1}]}{[M^{-1}T^2]} = [\beta^2] = [M^2L^2T^{-4}]$

$[\beta] = [MLT^{-2}]$

16. If an emitter current is changed by 4mA, the collector current changes by 3.5 mA.The value of β will be -

(1)7 (2)0.875 (3)0.5 (4) 3.5

Sol. **(1)**

Given :

ΔI_E = 4 mA

ΔI_C = 3.5 mA

we know that, $\alpha = \frac{\Delta I_C}{\Delta I_E}$

$\Rightarrow \alpha = \frac{3.5}{4} = \frac{7}{8}$

Also, $\beta = \frac{\alpha}{1-\alpha}$, so

$\beta = \frac{\frac{7}{8}}{1-\frac{7}{8}} = \frac{7}{1}$

$\beta = 7$

17. In a Young's double slit experiment, the width ofthe one of the slit is three times the other slit. The amplitude of the light coming from a slit is proportional to the slit-width. Find the ratio of the maximum to the minimum intensity in the interference pattern.

(1)4 : 1 (2)2 : 1 (3)3 : 1 (4) 1 : 4

Sol. **(1)**

given : $\omega_2 = 3\omega_1$

also, $A \propto \omega$

$\frac{\omega_1}{\omega_2} = \frac{1}{3}$ ...(1)

Assume $\omega_1 = x$, $\omega_2 = 3x$

we know that

$I_{max} = (A_1 + A_2)^2$, and

$I_{min} = (A_1 - A_2)^2$

$$\frac{A_1}{A_2} = \frac{\omega_1}{\omega_2} \qquad ....(2)$$

from equation (2) we can say that

$A_1 = A$ and $A_2 = 3A$

Now, $\frac{I_{max}}{I_{min}} = \frac{(A+3A)^2}{(A-3A)^2} = \frac{16A^2}{4A^2} = \frac{4}{1}$

$\Rightarrow \frac{I_{max}}{I_{min}} = \frac{4}{1}$

18. In the given figure, a mass M is attached to a horizontal spring which is fixed on one side to a rigid support. The spring constant of the spring is k. The mass oscillates on a frictionless surface with time period T and amplitude A. When the mass is in equilibrium position, as shown in the figure, another mass m is gently fixed upon it. The new amplitude of oscillation will be -

(1) $A\sqrt{\frac{M}{M+m}}$ (2) $A\sqrt{\frac{M}{M-m}}$ (3) $A\sqrt{\frac{M-m}{M}}$ (4) $A\sqrt{\frac{M+m}{M}}$

Sol. **(1)**

Before placing

After placing

We know that $\omega = \sqrt{\frac{k}{m}}$ and $\omega_i = \sqrt{\frac{k}{M}}$ $\qquad A_i = A$

Also, momentum is conserved just before and just after the block of mass (m) is placed because there is no implusive force. So -

$MA_i\omega_i = (M + m)\, v'$

$v' = \frac{MA_i\omega_i}{(M+m)} \Rightarrow v' = A_f\omega_f$

$\frac{MA\omega_i}{(M+m)} = A_f\sqrt{\frac{K}{(M+m)}}$

$\Rightarrow \frac{MA\sqrt{\frac{K}{M}}}{M+m} \times \sqrt{\frac{M+m}{K}} = A_f$

$\Rightarrow A_f = A\sqrt{\frac{M}{(M+m)}}$

19. A cube of side 'a' has point charges +Q located at each of its vertices except at the origin where the charge is –Q. The electric field at the centre of cube is :

(1) $\frac{2Q}{3\sqrt{3}\pi\varepsilon_0 a^2}\left(\hat{x}+\hat{y}+\hat{z}\right)$

(2) $\frac{Q}{3\sqrt{3}\pi\varepsilon_0 a^2}\left(\hat{x}+\hat{y}+\hat{z}\right)$

(3) $\frac{-2Q}{3\sqrt{3}\pi\varepsilon_0 a^2}\left(\hat{x}+\hat{y}+\hat{z}\right)$

(4) $\frac{-Q}{3\sqrt{3}\pi\varepsilon_0 a^2}\left(\hat{x}+\hat{y}+\hat{z}\right)$

Sol. **(3)**

If only +Q charges are placedat the corners of cube of side a then electric field at the centre of the cube will be zero.

But in the given condition one (-Q) is placed at one corner of cube so here

$E_1 = E_6$, $E_2 = E_5$ and $E_3 = E_8$ (So it will cancel out each other so electric field at centre is due to Q_4 and Q_7.

Here electric field at centre = 2 $(E.f.)_4$

As, $|E_4| = |E_7|$

$$(E.F)_C = \frac{2kQ}{\left(\frac{\sqrt{3}a}{2}\right)^2} = \frac{8KQ}{3a^2} \qquad \left\{\because K = \frac{1}{4\pi\varepsilon_0}\right\}$$

$$(E.F)_C = \frac{2Q}{3a^2\pi\varepsilon_0}$$

In vector form $\Rightarrow \vec{E} = \frac{-2Q}{3a^2\pi\varepsilon_0} \times \left(\frac{\hat{x}+\hat{y}+\hat{z}}{\sqrt{3}}\right)$

20. Each side of a box made of metal sheet in cubic shape is 'a' at room temperature 'T', the coefficient of linear expansion of the metal sheet is 'α'. The metal sheet is heated uniformly, by a small temperature ΔT, so that its new temeprature is $T+\Delta T$. Calculate theincrease in the volume of the metal box-

(1) $\frac{4}{3}\pi a^3\alpha\Delta T$ (2) $4\pi a^3\alpha\Delta T$ (3) $3a^3\alpha\Delta T$ (4) $4a^3\alpha\Delta T$

Sol. **(3)**

volume expansion $\gamma = 3\alpha$

$$\frac{\Delta V}{V} = \gamma\Delta T$$

$\Delta V = V.\gamma\Delta T$

$\Delta V = a^3.\ 3\alpha\Delta T$

SECTION-B

1. A resonance circuit having inductance and resistance 2×10^{-4} H and 6.28 Ω respectively oscillates at 10 MHz frequency. The value of quality factor of this resonator is_____.
[π = 3.14]

Sol. **2000**

Given : R = 6.28 Ω

f = 10 MHz

L = 2×10^{-4} Henry

we know that quality factor Q is given by

$$\Rightarrow Q = \frac{X_L}{R} = \frac{\omega L}{R}$$

also, $\omega = 2\pi f$, so

$$\Rightarrow Q = \frac{2\pi f L}{R}$$

$$\Rightarrow Q = \frac{2\pi \times 10 \times 10^6 \times 2 \times 10^{-4}}{6.28} = 2000$$

Q = 2000

2. A ball with a speed of 9 m/s collides with another identical ball at rest. After thecollision, the direction of each ball makes an angle of 30° with the original direction. The ratio of velocities of the balls after collision is x : y, where x is _____.

Sol. **1**

Momentum is conserved just before and just after the collision in both x-y direction.

In y-direction

$p_i = 0$

$$P_f = m \times \frac{1}{2} v_1 - m \times \frac{1}{2} v_2$$

$p_i = p_f$, so

$$= \frac{mv_1}{2} - \frac{mv_2}{2} = 0$$

$\Rightarrow \frac{mv_1}{2} = \frac{mv_2}{2} \Rightarrow v_1 = v_2$

$\frac{v_1}{v_2} = 1$

3. An audio signal $\upsilon_m = 20\sin 2\pi(1500t)$ amplitude modulates a carrier $\upsilon_c = 80 \sin 2\pi\,(100{,}000t)$. The value of percent modulation is _____.

Sol. **25**

Given : $\upsilon_m = 20 \sin\left[100\pi t + \frac{\pi}{4}\right]$

$\upsilon_c = 80 \sin\left[10^4 \pi t + \frac{\pi}{6}\right]$

we know that, modulation index = $\frac{A_m}{A_c}$

from given equations, $A_m = 20$ and $A_c = 80$

percentage modulation index = $\frac{A_m}{A_c} \times 100$

$\Rightarrow \frac{20}{80} \times 100 = 25\%$

The value of percentage modulation index is

=25

4. The coefficient of static friction between a wooden block of mass 0.5 kg and a vertical rough wall is 0.2. The magnitude of horizontal force that should be applied on the block to keep it adhere to the wall will be _____ N.

[$g = 10\ ms^{-2}$]

Sol. **25**

Given : $\mu_s = 0.2$

m = 0.5 kg

g = 10 m/s^2

we know that

$f_s = \mu N$ and ....(1)

To keep the block adhere to the wall

here N = F ...(2)

f_s = mg ....(3)

from equation (1), (2), and (3), we get

$\Rightarrow mg = \mu F$

$\Rightarrow F = \frac{mg}{\mu} \Rightarrow F = \frac{0.5 \times 10}{0.2}$

F = 25 N

5. An inclined plane is bent in such a way that the vertical cross-section is given by $y = \frac{x^2}{4}$ where y is in vertical and x in horizontal direction. If the upper surface of this curved plane is rough with coefficient of friction μ = 0.5, the maximum height in cm at which a stationary block will not slip downward is _____ cm.

Sol. **25**

given

$y = \frac{x^2}{4}$

$\mu = 0.5$

condition for block will not slip downward

$mg \sin\theta = \mu mg \cos\theta$

$\Rightarrow \tan\theta = \mu$

and we know that

$\Rightarrow \tan\theta = \frac{dy}{dx}$

$\Rightarrow \frac{dy}{dx} = \mu \Rightarrow \frac{x}{2} = 0.5$ $\left[\begin{array}{l} y = \frac{x^2}{4} \\ \frac{dy}{dx} = \frac{x}{2} \end{array}\right]$

$\Rightarrow x = 1,$

put x = 1 in equation $y = x^2/4$

$\Rightarrow y = \frac{(1)^2}{4} \Rightarrow y = \frac{1}{4} \Rightarrow y = 0.25$

y = 25 cm

6. An electromagnetic wave of frequency 5 GHz, is travelling in a medium whose relative electric permittivity and relative magnetic permeability bothare 2. Its velocity in this medium is _____ $\times 10^7$ m/s.

Sol. **15**

Given : f = 5 GHz

$\varepsilon_r = 2$

$\mu_r = 2$

velocity of wave $\Rightarrow$ $v = \frac{c}{n}$ ....(1)

where, $n = \sqrt{\mu_r \varepsilon_r}$ and c = speed of light = 3×10^8 m/s

$n = \sqrt{2 \times 2} = 2$

put the value of n in we get

$\Rightarrow v = \frac{3 \times 10^8}{2} = 15 \times 10^7$ m/s

$\Rightarrow X \times 10^7 = 15 \times 10^7$

X = 15

7. A hydraulic press can lift 100 kg when a mass 'm' is placed on the smaller piston. It can lift _____ kg when the diameter of the larger piston is increased by 4 times and that of the smaller piston is decreased by 4 times keeping the same mass 'm' on the smaller piston.

Sol. **25600**

Atmospheric pressure P_0 will be acting on both the limbs of hydraulic lift.

Applying pascal's law for same liquid level

$$\Rightarrow P_0 + \frac{mg}{A_1} = P_0 + \frac{(100)g}{A_2}$$

$$\Rightarrow \frac{Mg}{A_1} = \frac{(100)g}{A_2} \Rightarrow \frac{m}{100} = \frac{A_1}{A_2} \qquad ...(1)$$

Diameter of piston on side of 100 kg is increased by 4 times so new area = $16A_2$

Diameter of piston on side of (m) kg is decreasing

$$A_1 = \frac{A_1}{16}$$

(In order to increasing weight lifting capacity, diameter of smaller piston must be reduced)

Again, $\frac{mg}{\left(\frac{A_1}{16}\right)} = \frac{M'g}{16A_2} \Rightarrow \frac{256m}{M'} = \frac{A_1}{A_2}$

From equation (1) = $\frac{256m}{M'} = \frac{m}{100} \Rightarrow \therefore$ M' = 25600 kg

8. A common transistor radio set requires 12 V (D.C.) for its operation. The D.C. source is constructed by using a transformer and a rectifier circuit, which are operated at 220 V (A.C.) on standard domestic A.C. supply. The number of turns of secondary coil are 24, then the number of turns of primary are _____.

Sol. **440**

Given

Primary voltage, V_p = 220 V

Secondary voltage, v_s = 12 V

No. of turns in secondary coil is N_s = 24

no. of turns in primary coil, N_p = ?

We know that for a transformer

$$\Rightarrow \frac{N_p}{N_s} = \frac{V_p}{V_s}$$

$$\Rightarrow N_p = \frac{V_p \times N_S}{V_s} = \frac{220 \times 24}{12}$$

$$\Rightarrow N_p = 440$$

9. An unpolarized light beam is incident on the polarizer of a polarization experiment and the intensity of light beam emerging fromthe analyzer is measured as 100 Lumens. Now, if the analyzer is rotated around the horizontal axis (direction of light) by 30° in clockwise direction, the intensity of emerging light will be_____ Lumens.

Sol. **75**

Given :I_0 = 100 lumens, θ = 30'

$I_{net} = I_0 \cos^2\theta$

$$I_{net} = 100 \times \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{100 \times 3}{4}$$

I_{net} = 75 lumens

10. In connection with the circuit drawn below, the value of current flowing through 2kΩresistor is _____ × 10^{-4} A.

Sol. **25**

In zener diode there will be o change in current after 5V

zener diode breakdown

$$\Rightarrow i = \frac{5}{2 \times 10^3}$$

$\Rightarrow i = 2.5 \times 10^{-3}$ A

$\Rightarrow i = 25 \times 10^{-4}$ A

24th Feb. 2021 | Shift - 1
CHEMISTRY

SECTION - A

1. The gas released during anaerobic degradation of vegetation may lead to:
(1) Global warming and cancer (2) Acid rain
(3) Corrosion of metals (4) Ozone hole

Ans. **(1)**

Sol. Biogas is the mixtrue of gases produced by the breakdown of organic matter in the absence of oxygen (anaerobically), primary consisting of methane and carbondioxide. Biogas can be produced from raw material such as agricultural waste, manure, municiple waste, plant material, sewage, green waste or good waste. Due to release of CH_4 gas during anaerobic vegetative degradstion which caueses globle warming and cancer.

2. Out of the following, which type of interaction is responsible for the stabilisation α–helix structure of proteins ?
(1) Ionic bonding (2) Hydrogen bonding
(3) vander Waals forces (4) Covalent bonding

Ans. **(2)**

Sol. The α-helix is stabilized by hydrogen bond between the NH and CO group of the main chain.

3. Which of the following are isostructural pairs ?
(A) SO_4^{2-} and CrO_4^{2-}
(B) $SiCl_4$ and $TiCl_4$
(c) NH_3 and NO_3^-
(D) BCl_3 and $BrCl_3$
1. A and C only
2. A and B only
3. B and C only
4. C and D only

Ans. **(2)**

Sol. (1) SO_4^{-2} and CrO_4^{2-} both have tetrahedral structure.

Tetrahedral

Tetrahedral

(2) $SiCl_4$ and $TiCl_4$ both have Tetrahedral structure also.

4. Identify products A and B.

(1) A :

B :

(2) A :

B :

(3) A : $OHC-CH_2CH_2CH_2-\overset{\overset{\displaystyle O}{\|}}{C}-CH_3$ B : $HOOC-CH_2CH_2CH_2-\overset{\overset{\displaystyle O}{\|}}{C}-CH_3$

(4) A :

B :

Ans. **(2)**

Sol.

5. The product formed in the first step of the reaction of

with excess Mg/Et_2O ($Et = C_2H_5$) is :

(1)

(2)

(3)

(4)

Ans. **(3)**

Sol.

6. The electrode potential of M^{2+}/M of 3d– series elements shows positive value for:

(1) Zn (2) Co (3) Fe (4) Cu

Ans. **(4)**

Sol. (A) Zn –0.76

(B) CO –0.28

(C) Fe –0.44

(D) Cu +0.34

7. In the following reaction the reason why meta–nitro product also formed is:

[A] 51% [B] 47% [C] 2%

(1) Formation of anilinium ion

(2) $-NO_2$ substitution always takes place at meta–position

(3) low temperature

(4) $-NH_2$ group is highly meta–directive

Ans. **(1)**

Sol.

In acidic medium the $-NH_2$ group in aniline converts into anilinium ion which is meta directing.

8. (A) $HOCl + H_2O_2 \rightarrow H_3O^+ + Cl^- + O_2$

(B) $I_2 + H_2O_2 + 2OH^- \rightarrow 2I^- + 2H_2O + O_2$

Choose the correct option.

(1) H_2O_2 act as oxidizing and reducing agent respectively in equations (A) and (B).

(2) H_2O_2 acts as oxidizing agent in equations (A) and (B).

(3) H_2O_2 acts as reducing agent in equations (A) and (B).

(4) H_2O_2 acts as reducing and oxidising agent respectively in equation (A) and (B).

Ans. **(3)**

Sol. When H_2O_2 acts a reducing agent it liberates the O_2.

$H_2O_2 \rightleftharpoons 2H^+ + O_2 + 2e^-$

9. Which of the following ore is concentrated using group 1 cyanide salt ?

(1) Sphalerite (2) Siderite

(3) Malachite (4) Calamine

Ans. **(1)**

Sol. Conc. of sphalerite, first by cyanide salt as a depressant to remove the impurity of galena

$$Zns + Pbs + \underset{\text{excess}}{NaCN} \longrightarrow \underset{\text{solution}}{Na_2[Zn(CN)_4]} + PbS\uparrow$$

10. Which is the final product (major) 'A' in the given reaction?

(1)

(2)

(3)

(4)

Ans. **(3)**

Sol.

11. What is the major product formed by HI on reaction with ?

(1)

(2)

(3)

(4)

Ans. **(3)**

Sol.

12. Which of the following reagent is used for the following reaction ?

$CH_3CH_2CH_3 \xrightarrow{?} CH_3CH_2CHO$

(1) Potassium permanganate

(2) Molybdenum oxide

(3) Copper at high temperature and pressure

(4) Manganese acetate

Ans. **(2)**

Sol. $CH_3–CH_2–CH_3 \xrightarrow{MO_2O_3} CH_3–CH_2–CH=0$

13. In Freundlich adsorption isotherm, slope of AB line is :

(1) $\frac{1}{n}$ with $\left(\frac{1}{n} = 0 \text{ to } 1\right)$

(2) log $\frac{1}{n}$ with (n<1)

(3) log n with (n>1)

(4) n with (n, 0.1 to 0.5)

Ans. **(1)**

Sol. Freundlich adsorption isotherm is :

$\frac{x}{m} = kp^{1/n}$

x = mass of adsorbate

m = mass of adsorbent

P = eq. pressure

$k_1 n = \frac{1}{n}$ log p + log k

y = mx + c

compairing

m = $\frac{1}{n}$ = slope $[\frac{1}{n} = 0 \text{ to } 1]$

n > 1

14. The major components in "Gun Metal" are:

(1) Al, Cu, Mg and Mn

(2) Cu, Sn and Zn

(3) Cu, Zn and Ni

(4) Cu, Ni and Fe

Ans. **(2)**

Sol. "Gun metal" is alloy of copper with tin and zinc.

15. 'A' and 'B' in the following reactions are :

(1) (A) :

(B) :

(2) (A) :

(B) :

(3) (A) :

(B) :

(4) (A) :

(B) :

Ans. **(3)**

Sol.

(A) (Stephen's rexn) (B)

16. Which of the following compound gives pink colour on reaction with phthalic anhydride in conc.H_2SO_4 followed by treatment with NaOH ?

(1)

(2)

(3)

(4)

Ans. **(2)**

Sol.

Phenophthalein

(Pink colour)

17. Consider the elements Mg, Al, S, P and Si, the correct increasing order of their first ionization enthalpy is:

(1) Al < Mg < Si < S < P (2) Al < Mg < S < Si < P

(3) Mg < Al < Si < S < P (4) Mg < Al < Si < P < S

Ans. **(1)**

Sol. Order of IE, in 3rd period is

Na < Mg > Al < Si < P > S < Cl < Ar

Na < Al < Mg < Si < S < P < Cl < Ar

due to stable full filed 3s-orbital and more penetrating power

due to half filed $3p^3$-orbital of phosphorous

18. Given below are two statements :

Statement I : Colourless cupric metaborate is reduced to cuprous metaborate in a luminous flame.

Statement II : Cuprous metaborate is obtained by heating boric anhydride and copper sulphate in a non-luminous flame.

In the light of the above statements, choose the most appropriate answer from the options given below.

(1) Statement I is false but statement II is true.

(2) Statement I is true but Statement II is false.

(3) Both Statement I and Statement II are true.

(4) Both Statement I and Statement II are false.

Ans. **(4)**

Sol. Both are False

(1) Copper sulphate form copper meta boric with beric an hydride

$CuSO_4 \longrightarrow CuO + SO_3$

$CuO + B_2O_3 \longrightarrow Cu(BO_2)_2$

blue in cold oxidising flame (non luminous flame)

(2) Blue coloured metal borate is reduced to copper in a luminous flame.

19. Al_2O_3 was leached with alkali to get X. The solution of X on passing of gas Y, forms Z. X, Y and Z respectively are :

(1) X = $Na[Al(OH)_4]$, Y=CO_2, Z = $Al_2O_3.xH_2O$

(2) X=$Na[Al(OH)_4]$, Y=SO_2, Z = Al_2O_3

(3) X=$Al(OH)_3$, Y=SO_2, Z = $Al_2O_3.xH_2O$

(4) X = $Al(OH)_3$, Y=CO_2, Z= Al_2O_3

Ans. **(1)**

Sol. (1) $Al_2O_3 + NaOH \longrightarrow \underset{\text{"X"}}{Na[Al(OH)_4]}$

(2) $Na[Al(OH)_4] \xrightarrow[\text{"Y"}]{CO_2} \underset{\text{"Z"}}{Al(OH)_3 \text{ or } Al_2O_3 \,.\, xH_2O}$

20. Match List I with List II.

List I	**List II**
(Monomer Unit)	(Polymer)
(a) Caprolactum	(i) Natural rubber
(b) 2-Chloro-1,3-butadiene	(ii) Buna-N
(c) Isoprene	(iii) Nylon 6
(d) Acrylonitrile	(iv) Neoprene

Choose the correct answer from the options given below :

(1) (a) $\rightarrow$ (iii), (b) $\rightarrow$ (iv), (c) $\rightarrow$ (i), (d) $\rightarrow$ (ii)

(2) (a) $\rightarrow$ (i), (b) $\rightarrow$ (ii), (c) $\rightarrow$ (iii), (d) $\rightarrow$ (iv)

(3) (a) $\rightarrow$ (ii), (b) $\rightarrow$ (i), (c) $\rightarrow$ (iv), (d) $\rightarrow$ (iii)

(4) (a) $\rightarrow$ (iv), (b) $\rightarrow$ (iii), (c) $\rightarrow$ (ii), (d) $\rightarrow$ (i)

Ans. **(1)**

Sol. (1) Polymer of caprolactum is nylon-6

(2) Polymer of 2-chloro-1,3-butadiene is neoprene.

(3) Polymer of isoprene is natureal rubber

(4) Polymer of acrylonitrile and 1,3-butadiene is buna-N

SECTION – B

1. The stepwise formation of $[Cu(NH_3]^{2+}$ is given below:

$$Cu^{2+} + NH_3 \xrightleftharpoons{K_1} \left[Cu\left(NH_3\right)_4\right]^{2+}$$

$$\left[Cu\left(NH_3\right)_2\right]^{2+} + NH_3 \xrightleftharpoons{K_2} \left[Cu\left(NH_3\right)_2\right]^{2+}$$

$$\left[Cu\left(NH_3\right)_2\right]^{2+} + NH_3 \xrightleftharpoons{K_3} \left[Cu\left(NH_3\right)_3\right]^{2+}$$

$$\left[Cu\left(NH_3\right)_3\right]^{2+} + NH_3 \xrightleftharpoons{K_4} \left[Cu\left(NH_3\right)_4\right]^{2+}$$

The value of stability constants K_1, K_2, K_3 and K_4 are 10^4, 1.58×10^2 , 5×10^2 and 10^2 respectively. The overall equilibrium constants for dissociation of $[Cu(NH_3)_4]^{2+}$ is $x \times 10^{-12}$. The value of x is _____. (Rounded off to the nearest integer)

Ans. **(1)**

Sol. $\left[Cu\left(NH_3\right)_4\right]^{+2} \xrightleftharpoons{k} cu^{+2} + 4NH_3$ .....(A)

For this :

$$Cu^{+2} + NH_3 \xrightleftharpoons{k1} \left[Cu\left(NH_3\right)\right]^{+2} \text{.....(1)}$$

$$\left[Cu\left(NH_3\right)\right]^{+2} + NH_3 \xrightleftharpoons{k_2} \left[cu\left(NH_3\right)_2\right]^{+2} \text{.....(2)}$$

$$\left[Cu\left(NH_3\right)\right]^{+2} + NH_3 \xrightleftharpoons{k_3} \left[cu\left(NH_3\right)_3\right]^{+2} \text{.....(3)}$$

$$\left[Cu\left(NH_3\right)_3\right]^{+2} + NH_3 \xrightleftharpoons{k_4} \left[Cu\left(NH_3\right)_4\right]^{+2} \text{.....(4)}$$

(1) + (2) + (3) + (4)

$$Cu^{+2} + 4NH_3 \xrightleftharpoons{k_1.k_2.k_3.k_4} \left[C_4\left(NH_3\right)_4\right]^{+4} \text{.....(B)}$$

So for (A)

$$K = \frac{1}{k_1.K_2.K_3.K_4}$$

Putting the value of k_1,k_2,k_3 and k_4.

$$K = \frac{1}{(10)^4.\left(1.58 \times 10^3\right)\left(5 \times 10^2\right)(10)^2} = 1.26 \times 10^{-12}$$

x = 1.

2. At 1990 K and 1 atrm pressure, there are equal number of Cl_2 molecules and Cl atoms in the reaction mixture. The value of K_p for the reaction $Cl_{2(g)} \rightleftharpoons 2Cl_{(g)}$ under the above conditions is x × 10^{-1}. The value of x is _____. (Rounded off to the nearest integer)

Ans. **(5)**

Sol.

$$Cl_2 \rightleftharpoons 2Cl^-$$

Lets mole of eq. x x

P.P. at eq. $\frac{x}{2x} \times 1$ $\frac{x}{2x} \times 1$

$\frac{1}{2}$ $\frac{1}{2}$

$$K_p = \frac{[P_{c1}]^2}{[P_{Cl_2}]} = \frac{\left[\frac{1}{2}\right]^2}{\frac{1}{2}} = \frac{1}{2} = 0.5 = 5 \times 10^{-1}$$

X = 5.

3. 4.5 g of compound A (MW = 90) was used to make 250 mL of its aqueous solution. The molarity of the solution in M is x × 10^{-1}. The value of x is _____. (Rounded off to the nearest integer)

Ans. **(2)**

Sol. Moles of A = $\frac{\text{Weight}}{\text{M.w}}$

$= \frac{4.5}{90} = \frac{1}{20} = 0.05$

Volume (Lit) = $= \frac{250}{1000} = 0.250$ lit lit

Moles of A = $\frac{\text{Weight}}{\text{M.w}}$

$= \frac{4.5}{90} = \frac{1}{20} = 0.05$

Volume (Lit) = $= \frac{250}{1000} = 0.250$ lit lit

Molarity (M) = $\frac{\text{Mole}}{\text{(Lit)volume}} = \frac{0.05}{0.250} = 0.2$

$= 2 \times 10^{-1} \frac{\text{mol}}{\text{Lit}}$

x = 2

Molarity (M) = $\frac{\text{Mole}}{\text{(Lit)volume}} = \frac{0.05}{0.250} = 0.2$

$= 2 \times 10^{-1} \frac{\text{mol}}{\text{Lit}}$ x = 2

4. The coordination number of an atom in a body-centered cubic structure is _____.
[Assume that the lattice is made up of atoms]

Ans. **(4)**

Sol. Fact

5. Number of amphoteric compounds among the following is _____.
(A) BeO (B) BaO (C) $Be(OH)_2$ (D) $Sr(OH)_2$

Ans. **(2)**

Sol. BeO and $Be(OH)_2$ are amphoteric in nature

6. When 9.45g of $ClCH_2COOH$ is added to 500 mL of water, its freezing point drops by 0.5°C. The dissociation constant of $ClCH_2COOH$ is $x \times 10^{-3}$. The value of x is _____. (Rounded off to the nearest integer)
$[K_{f(H_2O)} = 1.86\,K\,kg\,mol^{-1}]$

Ans. **(35)**

Sol.

	$ClCH_2COOH$	$\longrightarrow$	$ClCH_2COO^{\ominus}$	+ $H^{\oplus}$
t = 0	C		O	O
t = t	C−cα		cα	cα

Total no. of moles = c+ cα = c(1+ α)

$$i = \frac{\text{observed}}{\text{calculate}} = \frac{c(1+\alpha)}{c} = (1+\alpha)$$

M.W. = 94.5

$\Delta T_f = i \times k_f \times m$ $\Delta T_f = 0.5°C$

$i = 1 + \alpha$

$$0.5 = (1+\alpha) \times 1.86 \times \frac{9.45}{\frac{94.5}{\frac{500}{1000}}}$$

$$m = \frac{\text{mole}}{\text{k.g(Solvent)}}$$

$k_t = 1.86k$ kg/mol

$$(1+\alpha) = \frac{2.5}{1.86}$$

$$\alpha = \frac{0.64}{1.86} = \frac{32}{93}$$

$$K_a = \frac{C\alpha^2}{1-\alpha} = \frac{0.2 \times 1024}{93 \times 93 \times \frac{61}{93}}$$

$K_a = 0.0351 = 35.1 \times 10^{-3}$

7. A proton and a Li^{3+} nucleus are accelerated by the same potential. If λ_{Li} and λ_p denote the de Broglie wavelengths of Li^{3+} and proton respectively, then the value of $\frac{\lambda_{Li}}{\lambda_p}$ is $x \times 10^{-1}$. The value of x is _____. [Rounded off to the nearest integer]
[Mass of Li^{3+} = 8.3 mass of proton]

Ans. **(2)**

Sol. De Brogir Davelength

$$\lambda = \frac{h}{\sqrt{2m\ k.E.}}$$

$$\frac{\lambda_{Li^{+3}}}{\lambda_p} = \sqrt{\frac{m_p \times (e^- v)_p}{m_{Li^{+3}} \times 3e_p v}}$$

$m_{Li^{+3}}$ = 8.3 mp

$$\frac{\lambda_{Li^{+3}}}{\lambda_p} = \sqrt{\frac{m_p}{3 \times 8.3 m_p}} = \sqrt{\frac{1}{25}}$$

$$= \frac{1}{5} = 0.2 = 2 \times 10^{-1}$$

x = 2.

8. Gaseous cyclobutene isomerizes to butadiene in a first order process which has a 'k' value of $3.3 \times 10^{-4}\ s^{-1}$ at 153°C. The time in minutes it takes for the isomerization to proceed 40% to completion at this temperature is _____. (Rounded off to the nearest integer)

Ans. **(26)**

Sol. For firdst order Rxn :-

$$t = \frac{2.303}{k} \log\left[\frac{100}{100 - x}\right]$$

X = 40, k =3.3×10^{-4}

$$t = \frac{2.303}{3.3 \times 10^{-4}} \log\left[\frac{100}{60}\right]$$

For firdst order Rxn :-

$$t = \frac{2.303}{k} \log\left[\frac{100}{100 - x}\right]$$

X = 40, k =3.3×10^{-4}

$$t = \frac{2.303}{3.3 \times 10^{-4}} \log\left[\frac{100}{60}\right]$$

$$t = \frac{2.303}{3.3 \times 10^{-4}} \times 0.22$$

t = $0.1535.3 \times 10^4$

t = 1535 sec.

t = $0.1535.3 \times 10^4$

t = 1535 sec = 25.6 Min.

9. For the reaction $A_{(g)} \rightarrow B_{(g)}$, the value of the equilibrium constant at 300 K and 1 atm is equal to 100.0. The value of $\Delta_r G$ for the reaction at 300 K and 1 atm in J mol^{-1} is –xR, where x is _____. (Rounded off to the nearest integer)
[R = 8.31 J $mol^{-1}K^{-1}$ and ln 10 = 2.3]

Ans. **(1380)**

Sol. $\Delta G° = -RT \ln Keq.$
$= -R \times 300 \times \ln(10^2)$
$= 300 \times 2 \times 2.3 \times (-R)$
$= -1380R$
x = 1380 ans.

10. The reaction of sulphur in alkaline medium is given below:
$S_{8(s)} + a\ OH^-_{(aq)} \longrightarrow b\ S^{2-}_{(aq)} + c\ S_2O_3^{2-}{}_{(aq)} + d\ H_2O_{(\ell)}$
The values of 'a' is _____. (Integer answer)

Ans. **(12)**

Sol. $S_8 + aOH^- \longrightarrow bs^{-2} + CdS_2O_3^{-2} + dH_2O$
$S_8 + bOH^- \longrightarrow 4S^{-2} + 2S_2O_3^{-2} + dH_2O$
$S_8 + 12OH^- \longrightarrow 4S^{-2} + 2S_2O_3^{-2} + 6H_2O$
a = 12

24th Feb. 2021 | Shift - 1
MATHEMATICS

1. The locus of the mid-point of the line segment joining the focus of the parabola $y^2=4ax$ to a moving point of the parabola, is another parabola whose directrix is:.

(1) $x = a$ (2) $x = 0$ (3) $x = -\frac{a}{2}$ (4) $x = \frac{a}{2}$

Ans. (2)

Sol. $h = \frac{at^2 + a}{2}$, $k = \frac{2at + 0}{2}$

$\Rightarrow t^2 = \frac{2h - a}{a}$ and $t = \frac{k}{a}$

$\Rightarrow \frac{k^2}{a^2} = \frac{2h - a}{a}$

$\Rightarrow$ Locus of (h, k) is $y^2 = a(2x - a)$

$\Rightarrow y^2 = 2a\left(x - \frac{a}{2}\right)$

Its directrix is $x - \frac{a}{2} = -\frac{a}{2} \Rightarrow x = 0$

2. A scientific committee is to formed from 6 Indians and 8 foreigners, which includes at least 2 Indians and double the number of foreigners as Indians. Then the number of ways, the committee can be formed is:

(1) 560 (2) 1050 (3) 1625 (4) 575

Ans. (3)

Sol. (2I, 4F) + (3I, 6F) + (4I, 8F)

$= {}^6C_2\,{}^8C_4 + {}^6C_3\,{}^8C_6 + {}^6C_4\,{}^8C_8$

$= 15 \times 70 + 20 \times 28 + 15 \times 1$

$= 1050 + 560 + 15 = 1625$

3. The equation of the plane passing through the point (1, 2, –3) and perpendicular to the planes $3x + y - 2z = 5$ and $2x - 5y - z = 7$, is:

(1) $3x - 10y - 2z + 11 = 0$ (2) $6x - 5y - 2z - 2 = 0$

(3) $11x + y + 17z + 38 = 0$ (4) $6x - 5y + 2z + 10 = 0$

Ans. (3)

Sol. Normal vector of required plane is $\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & -2 \\ 2 & -5 & -1 \end{vmatrix} = -11\hat{i} - \hat{j} - 17\hat{k}$

$\therefore$ $11(x - 1) + (y - 2) + 17(z + 3) = 0$

$11x + y + 17z + 38 = 0$

4. A man is walking on a straight line. The arithmetic mean of the reciprocals of the intercepts of this line on the coordinate axes is $\frac{1}{4}$. Three stones A, B and C are placed at the points (1, 1), (2, 2) and (4, 4) respectively. Then which of these stones is/are on the path of the man?

(1) B only (2) A only (3) All the three (4) C only

Ans. (1)

Sol. $\frac{x}{a} + \frac{y}{b} = 1$

$\frac{h}{a} + \frac{k}{b} = 1$ .....(1)

$\frac{\frac{1}{a} + \frac{1}{b}}{2} = \frac{1}{4}$

$\therefore \frac{1}{a} + \frac{1}{b} = \frac{1}{2}$ .....(ii)

$\therefore$ Line passes through fixed point B(2, 2)

(from (1) and (2))

5. The statement among the following that is a tautology is:

(1) $A \wedge (A \vee B)$ (2) $B \rightarrow [A \wedge (A \rightarrow B)]$ (3) $A \vee (A \wedge B)$ (4) $[A \wedge (A \rightarrow B)] \rightarrow B$

Ans. (4)

Sol. $A \wedge (\sim A \vee B) \rightarrow B$

$= [(A \wedge \sim A) \vee (A \wedge B)] \rightarrow B$

$= (A \wedge B) \rightarrow B$

$= \sim A \vee \sim B \vee B$

$= t$

6. Let $f : R \rightarrow R$ be defined as $f(x) = 2x-1$ and $g : R - \{1\} \rightarrow R$ be defined as $g(x) = \dfrac{x - \frac{1}{2}}{x-1}$.

Then the composition function $f(g(x))$ is :

(1) both one-one and onto (2) onto but not one-one

(3) neither one-one nor onto (4) one-one but not onto

Ans. (4)

Sol. $f(g(x)) = 2g(x) - 1$

$= 2\dfrac{\left(x - \frac{1}{2}\right)}{x-1} = \dfrac{x}{x-1}$

$f(g(x)) = 1 + \dfrac{1}{x-1}$

one-one, into

7. If f:R $\rightarrow$ R is a function defined by f(x) = [x–1] $\cos\left(\frac{2x-1}{2}\right)\pi$, where [.] denotes the greatest integer function, then f is :

(1) discontinuous only at x = 1

(2) discontinuous at all integral values of x except at x = 1

(3) continuous only at x = 1

(4) continuous for every real x

Ans. (4)

Sol. Doubtful points are x = n, n $\in$ I

L.H.L = $\lim_{x\to n^-}[x-1]\cos\left(\frac{2x-1}{2}\right)\pi$ = (n – 2)$\cos\left(\frac{2n-1}{2}\right)\pi$ = 0

R.H.L = $\lim_{x\to n^+}[x-1]\cos\left(\frac{2x-1}{2}\right)\pi$ = (n – 1)$\cos\left(\frac{2n-1}{2}\right)\pi$ = 0

f(n) = 0

Hence continuous.

8. The function $f(x) = \frac{4x^3-3x^2}{6} - 2\sin x + (2x-1)\cos x$:

(1) increases in $\left[\frac{1}{2},\infty\right)$

(2) decreases $\left(-\infty, \frac{1}{2}\right]$

(3) increases in $\left(-\infty, \frac{1}{2}\right]$

(4) decreases $\left[\frac{1}{2},\infty\right)$

Ans. (1)

Sol. f'(x) = (2x – 1) (x –sinx)

$\Rightarrow$ f′(x) $\geq$ 0 in x $\in \left[\frac{1}{2},\infty\right)$

and f′(x) $\leq$ 0 in x $\in \left(-\infty,\frac{1}{2}\right]$

9. The distance of the point (1, 1, 9) from the point of intersection of the line $\frac{x-3}{1} = \frac{y-4}{2} = \frac{z-5}{2}$ and the plane x + y + z = 17 is:

(1) $\sqrt{38}$

(2) $19\sqrt{2}$

(3) $2\sqrt{19}$

(4) 38

Ans. (1)

Sol. $\frac{x-3}{1}=\frac{y-4}{2}=\frac{z-5}{2}=\lambda$

$\Rightarrow x = \lambda+3,\ y = 2\lambda+4,\ z = 2\lambda+5$

Which lines on given plane hence

$\Rightarrow \lambda + 3 + 2\lambda + 4 + 2\lambda + 5 = 17$

$\Rightarrow \lambda=\frac{5}{5}=1$

Hence, point of intersection is Q (4, 6, 7)

$\therefore$ Required distance = PQ

$= \sqrt{9+25+4}$

$= \sqrt{38}$

10. $\lim\limits_{x\to 0}\frac{\int_0^{x^2}\left(\sin\sqrt{t}\right)dt}{x^3}$ is equal to :

(1) $\frac{2}{3}$ (2) 0 (3) $\frac{1}{15}$ (4) $\frac{3}{2}$

Ans. (1)

Sol. $\lim\limits_{x\to 0}\frac{\int_0^{x^2}\sin\sqrt{t}\,dt}{x^3}=\lim\limits_{x\to 0}\frac{\left(\sin|x|\right)2x}{3x^2}=\lim\limits_{x\to 0}\left(\frac{\sin x}{x}\right)\times\frac{2}{3}=\frac{2}{3}$

11. Two vertical poles are 150 m apart and the height of one is three times that of the other. If from the middle point of the line joining their feet, an observer finds the angles of elevation of their tops to be complementary, then the height of the shorter pole (in meters) is:

(1) 25 (2) $20\sqrt{3}$ (3) 30 (4) $25\sqrt{3}$

Ans. (4)

Sol. $\tan\theta = \frac{h}{75}=\frac{75}{3h}$

$\Rightarrow h^2 = \frac{(75)^2}{3}$

$h = 25\sqrt{3}m$

12. If the tangent to the curve $y = x^3$ at the point $P(t, t^3)$ meets the curve again at Q, then the ordinate of the point which divides PQ internally in the ratio 1 : 2 is :

(1) $-2t^3$ (2) $-t^3$ (3) 0 (4) $2t^3$

Ans. (1)

Sol. Equation of tangent at $P(t, t^3)$

$(y - t^3) = 3t^2(x - t)$ .....(1)

Now solve the above equation with

$y = x^3$ .....(2)

By (1) & (2)

$x^3 - t^3 = 3t^2 (x - t)$

$x^2 + xt + t^2 = 3t^2$

$x^2 + xt - 2t^2 = 0$

$(x - t)(x + 2t) = 0$

$\Rightarrow x = -2t \Rightarrow Q(-2t, -8t^3)$

Ordinate of required point = $\dfrac{2t^3 + \left(-8t^3\right)}{3} = -2t^3$

13. The area (in sq. units) of the part of the circle $x^2+y^2=36$, which is outside the parabola $y^2=9x$, is :

(1) $24\pi + 3\sqrt{3}$

(2) $12\pi + 3\sqrt{3}$

(3) $12\pi - 3\sqrt{3}$

(4) $24\pi - 3\sqrt{3}$

Ans. (4)

Sol. The curves intersect at point $(3, \pm 3\sqrt{3})$

Required area

$$= \pi r^2 - 2\left[\int_0^3 \sqrt{9x}\,dx + \int_3^6 \sqrt{36 - x^2}\,dx\right]$$

$$= 36\pi - 12\sqrt{3} - 2\left(\frac{x}{2}\sqrt{36 - x^2} + 18\sin^{-1}\left(\frac{x}{6}\right)\right)_3^6$$

$$= 36\pi - 12\sqrt{3} - 2\left(9 - \left(\frac{9\sqrt{3}}{2} + 3\pi\right)\right) = 24\pi - 3\sqrt{3}$$

14. If $\int \frac{\cos x - \sin x}{\sqrt{8 - \sin 2x}}\,dx = a\sin^{-1}\left(\frac{\sin x + \cos x}{b}\right) + c$, where c is a constant of integration, then the ordered pair (a, b) is equal to :

(1) (1, –3) (2) (1, 3) (3) (–1, 3) (4) (3, 1)

Ans. (2)

Sol. put $\sin x + \cos x = t \Rightarrow 1 + \sin 2x = t^2$

$\Rightarrow (\cos x - \sin x)\,dx = dt$

$$\therefore I = \int \frac{dt}{\sqrt{8 - (t^2 - 1)}} = \int \frac{dt}{\sqrt{9 - t^2}} = \sin^{-1}\left(\frac{t}{3}\right) + C = \sin^{-1}\left(\frac{\sin x + \cos x}{3}\right) + C$$

$\Rightarrow a = 1$ and $b = 3$

15. The population P = P(t) at time 't' of a certain species follows the differential equation $\frac{dP}{dt}$ = 0.5P – 450. If P(0) = 850, then the time at which population becomes zero is :

(1) $\frac{1}{2}\log_e 18$ (2) $2\log_e 18$ (3) $\log_e 9$ (4) $\log_e 18$

Ans. (2)

Sol. $\frac{dp}{dt} = \frac{p-900}{2}$

$$\int_{850}^{0} \frac{dp}{p-900} = \int_{0}^{t} \frac{dt}{2}$$

$$\ell n\left|P-900\right|\Big|_{850}^{0} = \frac{t}{2}$$

$$\ell n|900| - \ell n|50| = \frac{t}{2}$$

$$\frac{t}{2} = \ell n|18|$$

$$\Rightarrow t = 2\ell n18$$

16. The value of

$-{}^{15}C_1 + 2.{}^{15}C_2 - 3.{}^{15}C_3 + ... -15.{}^{15}C_{15} + {}^{14}C_1 + {}^{14}C_3 + {}^{14}C_5 + ...+ {}^{14}C_{11}$ is:

(1) 2^{14} (2) $2^{13} - 13$ (3) $2^{16} - 1$ (4) $2^{13} - 14$

Ans. (4)

Sol. $S_1 = -{}^{15}C_1 + 2.{}^{15}C_2 - ..... - 15\ {}^{15}C_{15}$

$$= \sum_{r=1}^{15}(-1)^r .r.{}^{15}C_r = 15\sum_{r=1}^{15}(-1)^r\ {}^{14}C_{r-1}$$

$= 15\ (-{}^{14}C_0 + {}^{14}C_1 - .....-{}^{14}C_{14}) = 15\ (0) = 0$

$S_2 = {}^{14}C_1 + {}^{14}C_3 + ...+{}^{14}C_{11}$

$= ({}^{14}C_1 + {}^{14}C_3 + ...+{}^{14}C_{11} + {}^{14}C_{13}) - {}^{14}C_{13}$

$= 2^{13} - 14$

$= S_1 + S_2 = 2^{13} - 14$

17. An ordinary dice is rolled for a certain number of times. If the probability of getting an odd number 2 times is equal to the probability of getting an even number 3 times, then the probability of getting an odd number for odd number of times is :

(1) $\frac{3}{16}$ (2) $\frac{1}{2}$ (3) $\frac{5}{16}$ (4) $\frac{1}{32}$

Ans. (2)

Sol. P(odd no. twice) = P(even no. thrice)

$\Rightarrow {}^nC_2 \left(\frac{1}{2}\right)^n = {}^nC_3 \left(\frac{1}{2}\right)^n \Rightarrow n = 5$

Success is getting an odd number then P(odd successes) = P(1) + P(3) + P(5)

$= {}^5C_1\left(\frac{1}{2}\right)^5 + {}^5C_3\left(\frac{1}{2}\right)^5 + {}^5C_5\left(\frac{1}{2}\right)^5$

$= \frac{16}{2^5} = \frac{1}{2}$

18. Let p and q be two positive number such that p + q = 2 and $p^4 + q^4 = 272$. Then p and q are roots of the equation :

(1) $x^2 - 2x + 2 = 0$ (2) $x^2 - 2x + 8 = 0$

(3) $x^2 - 2x + 136 = 0$ (4) $x^2 - 2x + 16 = 0$

Ans. (4)

Sol. $(p^2 + q^2)^2 - 2p^2q^2 = 272$

$((p + q)^2 - 2pq)^2 - 2p^2q^2 = 272$

$16 + 16pq + 2p^2 q^2 = 272$

$(pq)^2 - 8pq - 128 = 0$

$pq = \frac{8 \pm 24}{2} = 16, -8$

$pq = 16$

Now

$x^2 - (p + q)x + pq = 0$

$x^2 - 2x + 16 = 0$

19. If $e^{\left(\cos^2 x+\cos^4 x+\cos^6 x+\ldots\infty\right)\log_e 2}$ satisfies the equation $t^2 - 9t + 8 = 0$, then the value of $\frac{2\sin x}{\sin x+\sqrt{3}\cos x}\left(0 < x < \frac{\pi}{2}\right)$ is :

(1) $\frac{3}{2}$ (2) $2\sqrt{3}$ (3) $\frac{1}{2}$ (4) $\sqrt{3}$

Ans. (3)

Sol. $e^{\left(\cos^2 x+\cos^4 x+\ldots\ldots\infty\right)\ell n2} = 2^{\cos^2 x+\cos^4 x+\ldots\ldots\infty}$

$= 2^{\cot^2 x}$

$t^2 - 9t + 8 = 0 \Rightarrow t = 1,8$

$\Rightarrow 2^{\cot^2 x} = 1, 8 \Rightarrow \cot^2 x = 0, 3$

$0 < x < \frac{\pi}{2} \Rightarrow \cot x = \sqrt{3}$

$\Rightarrow \frac{2\sin x}{\sin x+\sqrt{3}\cos x} = \frac{2}{1+\sqrt{3}\cot x} = \frac{2}{4} = \frac{1}{2}$

20. The system of linear equations

$3x - 2y - kz = 10$

$2x - 4y - 2z = 6$

$x + 2y - z = 5m$

is inconsistent if :

(1) $k = 3,\ m = \frac{4}{5}$ (2) $k \neq 3, m \in R$

(3) $k \neq 3,\ m \neq \frac{4}{5}$ (4) $k = 3,\ m \neq \frac{4}{5}$

Ans. (4)

Sol. $\Delta = \begin{vmatrix} 3 & -2 & -k \\ 1 & -4 & -2 \\ 1 & 2 & -1 \end{vmatrix} = 0$

$3(4 + 4) + 2(-2 + 2) - k(4 + 4) = 0$

$\Rightarrow k = 3$

$\Delta_x = \begin{vmatrix} 10 & -2 & -3 \\ 6 & -4 & -2 \\ 5m & 2 & -1 \end{vmatrix} \neq 0$

$10(4 + 4) + 2(-6 + 10m) - 3(12 + 20m) \neq 0$

$80 - 12 + 20m - 36 - 60m \neq 0$

$40m \neq 32 \Rightarrow m \neq \frac{4}{5}$

$$\Delta_y = \begin{vmatrix} 3 & 10 & -3 \\ 2 & 6 & -2 \\ 1 & 5m & -1 \end{vmatrix} \neq 0$$

$3(-6 + 10m) - 10(-2 + 2) - 3(10m - 6) \neq 0$

$-18 + 30m - 30m + 18 \neq 0 \Rightarrow 0$

$$\Delta_z = \begin{vmatrix} 3 & -2 & 10 \\ 2 & -4 & 6 \\ 1 & 2 & 5m \end{vmatrix} \neq 0$$

$3(-20m - 12) + 2(10m - 6) + 10(4 + 4) - 40m + 32 \neq 0 \Rightarrow m \neq \frac{4}{5}$

Section – B

1. Let $P = \begin{bmatrix} 3 & -1 & -2 \\ 2 & 0 & \alpha \\ 3 & -5 & 0 \end{bmatrix}$, where $\alpha \in R$. Suppose $Q = [q_{ij}]$ is a matrix satisfying $PQ = kI_3$ for some non-zero $k \in R$. If $q_{23} = -\frac{k}{8}$ and $|Q| = \frac{k^2}{2}$, then $\alpha^2 + k^2$ is equal to _____

Ans. 17

Sol. As $PQ = KI$ $\Rightarrow$ $Q = kP^{-1}I$

now $Q = \frac{k}{|P|}(adjP)\,I$ $\Rightarrow$ $Q = \frac{k}{(20+12\alpha)}\begin{bmatrix} - & - & - \\ - & - & (-3\alpha - 4) \\ - & - & - \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\because q_{23} = \frac{-k}{8}$ $\Rightarrow$ $\frac{k}{(20+12\alpha)}(-3\alpha - 4) = \frac{-k}{8} \Rightarrow 2(3\alpha + 4) = 5 + 3\alpha$

$3\alpha = -3$ $\Rightarrow$ $\alpha = -1$

also $|Q| = \frac{k^3|I|}{|P|}$ $\Rightarrow$ $\frac{k^2}{2} = \frac{k^3}{(20+12\alpha)}$

$(20+12\alpha) = 2k \Rightarrow 8 = 2k \Rightarrow k = 4$

2. Let $B_i (i=1, 2, 3)$ be three independent events in a sample space. The probability that only B_1 occur is α, only B_2 occurs is β and only B_3 occurs is γ. Let p be the probability that none of the events B_i occurs and these 4 probabilities satisfy the equations $(\alpha - 2\beta)p = \alpha\beta$ and $(\beta - 3\gamma)p = 2\beta\gamma$ (All the probabilities are assumed to lie in the interval (0, 1)). Then $\frac{P(B_1)}{P(B_3)}$ is equal to ____

Ans. **6**

Sol. Let x, y, z be probability of B_1, B_2, B_3 respectively

$\Rightarrow x(1 - y)(1 - z) = \alpha$

$\Rightarrow y(1 - x)(1 - z) = \beta$

$\Rightarrow z(1 - x)(1 - y) = \gamma$

$\Rightarrow (1 - x)(1 - y)(1 - z) = p$

$(\alpha - 2\beta)p = \alpha\beta$

$(x(1-y)(1-z) - 2y(1-x)(1-z))(1-x)(1-y)(1-z) = xy(1-x)(1-y)(1-z)$

$x - xy - 2y + 2xy = xy$

$x = 2y$ ...(1)

Similarly $(\beta - 3r)p = 2\beta r$

$\Rightarrow y = 3z$ ...(2)

From (1) & (2)

$x = 6z$

Now

$\frac{x}{z} = 6$

3. The minimum value of α for which the equation $\frac{4}{\sin x} + \frac{1}{1 - \sin x} = \alpha$ has at least one solution in $\left(0, \frac{\pi}{2}\right)$ is _____

Ans. **9**

Sol. $f(x) = \frac{4}{\sin x} + \frac{1}{1 - \sin x}$

Let sinx = t $\because x \in \left(0, \frac{\pi}{2}\right) \Rightarrow 0 < t < 1$

$$f(t) = \frac{4}{t} + \frac{1}{1-t}$$

$$f'(t) = \frac{-4}{t^2} + \frac{1}{(1-t)^2}$$

$$= \frac{t^2 - 4(1-t)^2}{t^2(1-t)^2}$$

$$= \frac{(t - 2(1-t))(t + 2(1-t))}{t^2(1-t)^2}$$

$$= \frac{(3t-2)(2-t)}{t^2(1-t)^2}$$

f_{min} at $t = \frac{2}{3}$

$$\alpha_{min} = f\left(\frac{2}{3}\right) = \frac{4}{\frac{2}{3}} + \frac{1}{1-\frac{2}{3}}$$

$= 6 + 3$

$= 9$

4. If one of the diameters of the circle $x^2 + y^2 - 2x - 6y + 6 = 0$ is a chord of another circle 'C' whose center is at (2,1), then its radius is _____

Ans. **3**

distance between (1, 3) and (2, 1) is $\sqrt{5}$

$\therefore \left(\sqrt{5}\right)^2 + (2)^2 = r^2$

$\Rightarrow r = 3$

5. $\lim_{x\to\infty} \tan\left\{\sum_{r=1}^{n} \tan^{-1}\left(\frac{1}{1+r+r^2}\right)\right\}$ is equal to _____

Ans. **1**

Sol. $\tan\left(\lim_{n\to\infty}\sum_{r=1}^{n}\left[\tan^{-1}(r+1)-\tan^{-1}(r)\right]\right)$

$= \tan\left(\lim_{n\to\infty}\left(\tan^{-1}(n+1)-\frac{\pi}{4}\right)\right)$

$= \tan\left(\frac{\pi}{4}\right) = 1$

6. If $\int_{-a}^{a}(|x|+|x-2|)dx = 22$,(a > 2) and [x] denotes the greatest integer $\le$ x, then $\int_{a}^{-a}(x+[x])dx$ is equal to _____

Ans. **3**

Sol. $\int_{-a}^{0}(-2x+2)dx + \int_{0}^{2}(x+2-x)dx + \int_{2}^{a}(2x-2)dx = 22$

$x^2 - 2x\,|_0^{-a} + 2x\,|_0^2 + x^2 - 2x\,|_2^a = 22$

$a^2 + 2a + 4 + a^2 - 2a - (4-4) = 22$

$2a^2 = 18 \Rightarrow a = 3$

$\int_{3}^{-3}(x+[x])dx = -\left(\int_{-3}^{3}(x+[x])dx\right) = -\left(\int_{-3}^{3}[x]dx\right)$

$= -(-3-2-1+0+1+2) = 3$

7. Let three vectors $\vec{a}, \vec{b}$ and $\vec{c}$ be such that $\vec{c}$ is coplanar with $\vec{a}$ and $\vec{b}$, $\vec{a}.\vec{c} = 7$ and $\vec{b}$ is perpendicular to $\vec{c}$, where $\vec{a} = -\hat{i}+\hat{j}+\hat{k}$ and $\vec{b} = 2\hat{i}+\hat{k}$, then the value of $2\left|\vec{a}+\vec{b}+\vec{c}\right|^2$ is _____

Ans. **75**

Sol. $\vec{c} = \lambda\left(\vec{b}\times\left(\vec{a}\times\vec{b}\right)\right)$

$= \lambda\left(\left(\vec{b}.\vec{b}\right)\vec{b} - \left(\vec{b}.\vec{a}\right)\vec{b}\right)$

$= \lambda\left(5\left(-\hat{i}+\hat{j}+\hat{k}\right) + 2\hat{i}+\hat{k}\right)$

$= \lambda\left(-3\hat{i}+5\hat{j}+6\hat{k}\right)$

$\vec{c}.\vec{a}=7 \Rightarrow 3\lambda + 5\lambda + 6\lambda = 7$

$\lambda=\frac{1}{2}$

$\therefore 2\left|\left(\frac{-3}{2}-1+2\right)\hat{i}+\left(\frac{5}{2}+1\right)\hat{j}+(3+1+1)\hat{k}\right|^2$

$= 2\left(\frac{1}{4}+\frac{49}{4}+25\right)=25+50=75$

8. Let A = {n∈N : n is a 3-digit number}

B = {9k + 2 : k∈N}

and C : {9k + ℓ : k∈N} for some ℓ (0 < ℓ < 9)

If the sum of all the elements of the set A ∩ (B ∪ C) is 274×400, then ℓ is equal to __

Ans. 5

Sol. 3 digit number of the form 9K + 2 are {101, 109, ...992}

$\Rightarrow$ Sum equal to $\frac{100}{2}$ (1093) = s_1 = 54650

274 × 400 = $s_1 + s_2$

274 × 400 = $\frac{100}{2}$ [101 + 992] + s_2

274 × 400 = 50 × 1093 + s_2

s_2 = 109600 – 54650

s_2 = 54950

$s_2 = 54950 = \frac{100}{2}\left[(99+\ell)+(990+\ell)\right]$

1099 = 2ℓ + 1089

ℓ = 5

9. If the least and the largest real values of α, for which the equation $z+\alpha|z-1|+2i=0$ $\left(z \in C \text{ and } i=\sqrt{-1}\right)$ has a solution, are p and q respectively; then $4(p^2 + q^2)$ is equal to ___

Ans. **10**

Sol. $x + iy + \alpha\sqrt{(x-1)^2+y^2} + 2i = 0$

$\therefore y + 2 = 0$ and $x + \alpha\sqrt{(x-1)^2+y^2} = 0$

$y = -2$ & $x^2 = \alpha^2(x^2 - 2x + 1 + 4)$

$\alpha^2 = \dfrac{x^2}{x^2-2x+5} \Rightarrow x^2(\alpha^2-1)-2x\alpha^2+5\alpha^2 = 0$

$x \in R \Rightarrow D \geq 0$

$4\alpha^4 - 4(\alpha^2-1)5\alpha^2 \geq 0$

$\alpha^2\left[4\alpha^2 - 2\alpha^2 + 20\right] \geq 0$

$\alpha^2\left[-16\alpha^2 + 20\right] \geq 0$

$\alpha^2\left[\alpha^2 - \dfrac{5}{4}\right] \leq 0$

$0 \leq \alpha^2 \leq \dfrac{5}{4}$

$\therefore \alpha^2 \in \left[0, \dfrac{5}{4}\right]$

$\therefore \alpha \in \left[-\dfrac{\sqrt{5}}{2}, \dfrac{\sqrt{5}}{2}\right]$

then $4[(q)^2+(p)^2] = 4\left[\dfrac{5}{4}+\dfrac{5}{4}\right] = 10$

10. Let M be any 3×3 matrix with entries from the set {0, 1, 2}. The maximum number of such matrices, for which the sum of diagonal elements of M^TM is seven, is __

Ans. **540**

Sol. $\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}\begin{bmatrix} a & d & g \\ b & e & h \\ c & f & i \end{bmatrix}$

$a^2 + b^2 + c^2 + d^2 + e^2 + f^2 + g^2 + h^2 + i^2 = 7$

Case I : Seven (1's) and two (0's)

${}^{9}C_{2} = 36$

Case II : One (2) and three (1's) and five (0's)

$\frac{9!}{5!3!} = 504$

$\therefore$ Total = 540