Principle of Programming Languages

Haskell

Evaluation of Functions

In mathematics, functions do not have side-effects. This is clearly not true in conventional programming languages, Scheme included. Haskell is a **purely functional programming language**. We have already seen that, in absence of side effects (purely functional computations) from the point of view of the result the order in which functions are applied does not matter (almost). A function application ready to be performed is called a reducible expression (or redex). **Prelude** is the standard library for Haskell.

Evaluation strategies

- **Call-by-Value**: in this strategy, arguments of functions are always evaluated before evaluating the function itself, this corresponds to passing arguments by value (innermost strategy).
- **Call-by-Name**: functions are always applied before their arguments, this corresponds to passing arguments by name (outermost fashion). If the argument is not used, it is never evaluated; if the argument is used several times, it is re-evaluated each time.
- **Call-by-Need**: is a memoized version of call-by-name where, if the function argument is evaluated, that value is stored for subsequent uses (lazy evaluation). This is what Haskell uses.

In Scheme `delay` is used to return a promise to execute a computation (implements call-by-name). Moreover, it caches the result (memoization) of the computation on its first evaluation and returns that value on subsequent calls (implements call-by-need). Then `force` is used to force the evaluation of a promise.

Currying

In Haskell, functions have only one argument. This is not a limitation, because functions with more arguments are automatically **curried** and consequently managed.

Types and Functions

Static Typing: the type of everything must be known at compile time. There is type inference, so usually we do not need to explicitly declare types. "Has type" is written `::` instead of `:` (the latter is cons). Functions are declared through a sequence of equations and they use pattern matching, which means that arguments are matched with the right parts of equations, top to bottom, and if the match succeeds, the function body is called.
Parametric Polymorphism: lower case letters are type variables, so [a] stands for a list of elements of type a, for any a. Every well-typed expression is guaranteed to have a unique principal type, and the principal type can be inferred automatically.
`(.)` is used for composing functions, e.g. `(f.g)(x)` is `f(g(x))`. `$` can be used for avoiding parentheses, e.g. `(10*) (5+3) = (10*) $ 5+3`.

User-Defined Types

Data and type constructors live in separate name-spaces, so it is possible (and common) to use the same name for both. If we apply a data constructor we obtain a value (e.g. Pnt 2.3 5.7), while with a type constructor we obtain a type (e.g. Pnt Bool)

```
--User-defined types
data Bool = False | True -- a "sum" type
     (union in C)
data Point a = Point a a -- a "product" type
     (struct in C)
```

Recursive Types

```
-- Trees
data Tree a = Leaf a | Branch (Tree a) (Tree
     a)
Branch :: Tree a -> Tree a -> Tree a
aTree = Branch (Leaf 'a') (Branch (Leaf 'b')
     (Leaf 'c'))
-- Lists
data List a = Null | Cons a ( List a)
data [a] = [] | a : [a]
```

Haskell has special syntax for lists: `[]` is a data and type constructor, while `:` is an infix data constructor. `(++)` denotes list concatenation.
In product types, the access is positional, for instance we may define accessors using _ as a blank and there is also a C-like syntax to have named fields. This declaration automatically defines two field names `pointx`, `pointy` and their corresponding selector functions:

```
pointx Point x _ = x
pointy Point _ y = y
-- OR
data Point = Point {pointx, pointy :: Float}
```

Type Synonyms

Type Synonyms are defined with the keyword `type`, used usually for readability and shortness.

Map

Haskell has map, and it can be defined as:

```
map f [] = []
map f (x:xs) = f x : map f xs
```

We can partially apply also infix operators, by using parentheses: `(+ 1)` or `(1 +)` or `(+)`

Infinite Computations

Call-by-need is very convenient for dealing with never-ending computations that provide data. Clearly, we cannot evaluate them, but there is take to get finite slices from them.

```
ones = 1 : ones
numsFrom n = n : numsFrom (n + 1)
allNums = [1..] -- Same as ()numsFrom 1)
squares = map (^2) (numsFrom 0)
zip :: [a] -> [b] -> [(a, b)]
zip [1, 2, 3] "ciao" ---> [(1, 'c'), (2,
    'i'), (3, 'a')]
[(x,y) | x <- [1 ,2] , y <- "ab"] ---> [(1,
    'a'), (1, 'b'), (2, 'a'), (2, 'b')]
fibonacci = 1 : 1 : [a + b | (a, b) <- zip
    fibonacci (tail fibonacci)]
```

Error

`bottom` (aka $\perp$) is defined as `bot = bot`. All errors have value `bot`, a value shared by all types. `error :: String -> a` is strange because is polymorphic only in the output. The reason is that it returns `bot` (in practice, an exception is raised).

Pattern matching

The matching process proceeds top-down, left-to-right. Patterns may have boolean guards. _ stands for don't care. Definition of take:

```
take 0 _ = []
take _ [] = []
take n (x:xs) = x : take (n -1) xs
```

`let` is like Scheme's `letrec*`. The layout is like in Python, with meaningful whitespaces, but we can also use a C-like syntax. `where` can be convenient to scope binding over equations.

```
let x = 3
    y = 12
in x+y ---> 15
let {x = 3; y = 12} in x+y ---> 15
```

Strictness

There are various ways to enforce strictness in Haskell (analogously there are classical approaches to introduce laziness in strict languages). For example on data with **bang patterns** (a datum marked with `!` is considered strict). There are extensions for using `!` also in function parameters. Canonical operator to force evaluation is `seq :: a -> t -> t`. `seq x y` returns `y`, only if the evaluation of `x` terminates (i.e. it performs `x` then returns `y`). Strict versions of standard functions are usually primed. There is a convenient strict variant of `$` (function application) called `$!`.

```
data Complex = Complex !Float !Float --
     Strict Float
($!) :: (a -> b) -> a -> b
f $! x = seq x (f x) -- Strict function
     application
```

Modules and Abstract Data Types (ADT)

Haskell has a simple module system, with import, export and namespaces. Modules provide the only way to build abstract data types (ADT). The characteristic feature of an ADT is that the representation type is hidden: all operations on the ADT are done at an abstract level which does not depend on the representation.

Type classes and overloading

We already saw parametric polymorphism in Haskell (e.g. in length). Type classes are the mechanism provided by Haskell for ad hoc polymorphism (aka overloading). The first, natural example is that of numbers: 6 can represent an integer, a rational, a floating point number, etc...

Class Eq

Also numeric operators and equality work with different kinds of numbers. Let's start with equality: it is natural to define equality for many types. Type classes are used for overloading: a class is a "container" of overloaded operations. We can declare a type to be an instance of a type class, meaning that it implements its operations. Eq a is a constraint on type a, it means that a must be an instance of Eq. An implementation of `(==)` is called a **method**. Eq offers also a standard definition of `(/=)`, derived from`(==)`: We can also extend Eq with comparison operations. Ord is also called a subclass of Eq. It is possible to have multiple inheritance: `class (X a, Y a) => Z a`.

```
class Eq a where -- Class Eq
  (==) :: a -> a -> Bool
(==) :: (Eq a) => a -> a -> Bool -- Type of
     (==)
class Eq a where -- Inequality
  (==), (/=) :: a -> a -> Bool
  x /= y = not (x == y)
class (Eq a) => Ord a where -- Class Ord,
     comparison operations
  (<), (<=), (>=), (>) :: a -> a -> Bool
  max, min :: a -> a -> a
```

Class Show

It is used for showing: to have an instance we must implement `show`. Functions do not have a standard representation, but we can give them one. We can also represent binary trees:

```
instance Show a => Show (Tree a) where
  show (Leaf a) = show a
  show (Branch x y) = "<" ++ show x ++ " | "
     ++ show y ++ ">"
```

Usually it is not necessary to explicitly define instances of some classes, e.g. Eq and Show. Haskell can be quite smart and do it automatically, by using **deriving**. For example we may define binary trees using an infix syntax and automatic Eq, Show like this:

```
infixr 5 :^:
data Tr a = Lf a | Tr a :^: Tr a
     deriving (Show, Eq)
```

An example with class Num, Rational Numbers:

```
data Rat = Rat !Integer !Integer deriving Eq

simplify (Rat x y) = let g = gcd x y
     in Rat (x 'div' g) (y 'div' g)

makeRat x y = simplify (Rat x y)

instance Num Rat where
  (Rat x y) + (Rat x' y') = makeRat
     (x*y'+x'*y) (y*y')
  (Rat x y) - (Rat x' y') = makeRat
     (x*y'-x'*y) (y*y')
  (Rat x y) * (Rat x' y') = makeRat (x*x')
     (y*y')
  abs (Rat x y) = makeRat (abs x) (abs y)
  signum (Rat x y) = makeRat (signum x *
     signum y) 1
  fromInteger x = makeRat x 1

instance Ord Rat where
  (Rat x y) <= (Rat x' y') = x*y' <= x'*y

instance Show Rat where
  show (Rat x y) = show x ++ "/" ++ show y
```

Input/Output is dysfunctional

IO computation is based on state change (e.g. of a file), hence if we perform a sequence of operations, they must be performed in order (and this is not easy with call-by-need). For example `getChar` is not referentially transparent: two different calls of `getChar` could return different characters. `main` is the default entry point of the program (like in C). "`do`" is used to define an IO action as an ordered sequence of IO actions. "`<-`" is used to obtain a value from an IO action.

```
main = do {
  putStr "Please, tell me something>";
  thing <- getLine;
  putStrLn $ "You told me \"" ++ thing ++
     "\".";
}
```

Of course, purely functional Haskell code can raise exceptions. But if we want to catch them, we need an IO action: `handle :: Exception e => (e -> IO a) -> IO a -> IO a;`, where the 1st argument is the handler. IO is a type constructor, instance of **Monad**.

Other Classical Data Structures

Traditional data structures are imperative. If really needed, there are libraries with imperative implementations living in the IO monad. Idiomatic approach: use immutable arrays (**Data.Array**), and maps (**Data.Map**, implemented with balanced binary trees). `find` are respectively O(1) and O(log n); `update` O(n) for arrays, O(log n) for maps. Of course, the `update` operations copy the structure, it does not change it.

Class Foldable

Foldable is a class used for folding. The main idea is the one we know from `foldl` and `foldr` for lists: we have a container, a binary operation f , and we want to apply f to all the elements in the container, starting from a value z. A minimal implementation of Foldable requires `foldr`. `foldl` can be expressed in term of `foldr` (id is the identity function). The converse is not true, since `foldr` may work on infinite lists, unlike `foldl`:

```
foldr f z [] = z -- Definition
foldr f z (x:xs) = f x (foldr f z xs)

foldl f z [] = z -- Definition
foldl f z (x:xs) = foldl f (f z x) xs

foldl f a bs = foldr (\b g x -> g (f x b)) id
    bs a
```

Implementation of Foldabel for **Tree**:

```
tfoldr f z Empty = z
tfoldr f z (Leaf x) = f x z
tfoldr f z (Node l r) = tfoldr f (tfoldr f z
    r) l

instance Foldable Tree where
  foldr = tfoldr
```

Maybe is used to represent computations that may fail: we either have Just v, if we are lucky, or Nothing (Optional class in Java). It is basically a simple "conditional container". It is adopted in many recent languages, to avoid NULL and limit exceptions usage.

```haskell
data Maybe a = Nothing | Just a

instance Foldable Maybe where
  foldr _ z Nothing = z
  foldr f z (Just x) = f x z
```

Class Functor

Functor is the class of all the types that offer a map operation. The map operation of functors is called `fmap`. It is quite natural to define map for a container, for example Maybe.

```haskell
fmap :: (a -> b) -> f a -> f b

instance Functor Maybe where
  fmap _ Nothing = Nothing
  fmap f (Just a) = Just (f a)
```

Well-defined functors should obey the following laws:

- fmap id = id (where id is the identity function)
- fmap (f . g) = fmap f . fmap g (homomorphism)

Implementation of Functor for **Tree**:

```haskell
tmap f Empty = Empty
tmap f (Leaf x) = Leaf $ f x
tmap f (Node l r) = Node (tmap f l) (tmap f r)

instance Functor Tree where
  fmap = tmap
```

Class Applicative

In our voyage toward monads, we must consider also an extended version of functors: **Applicative** functors. The definition of Applicative and the class Maybe implementing it is:

```haskell
class (Functor f) => Applicative f where
  pure :: a -> f a
  (<*>) :: f (a -> b) -> f a -> f b

instance Applicative Maybe where
  pure = Just
  Just f <*> m = fmap f m
  Nothing <*> _ = Nothing
```

Note that `f` is a type constructor, and `f` a is a Functor type. Moreover, `f` must be parametric with one parameter. If f is a container, the idea is not too complex: pure takes a value and returns an f containing it; `<*>` is like fmap, but instead of taking a function, takes an f containing a function, to apply it to a suitable container of the same kind.

Of course, **Lists** are instances of Foldable and Functor. What about Applicative? For that, it is first useful to introduce `concat`. So we start from a container of lists, and get a list with the concatenation of them trough `concat`. Its composition with map is called `concatMap`. With `concatMap`, we get the standard implementation of `<*>` for lists. Note that we map the operations in sequence, then we concatenate the resulting lists.

```haskell
concat :: Foldable t => t [a] -> [a]
concat l = foldr (++) [] l

concatMap f l = concat $ map f l

instance Applicative [] where
  pure x = [x]
  fs <*> xs = concatMap (\f -> map f xs) fs
```

Following the list approach, we can make our binary trees an instance of Applicative Functors. First, we need to define what we mean by tree concatenation with `tconc`. Then, concat and concatMap (here `tconcmap` for short) are like those of lists.

```haskell
tconc Empty t = t
tconc t Empty = t
tconc t1 t2 = Node t1 t2

tconcat t = tfoldr tconc Empty t

tconcmap f t = tconcat $ tmap f t

instance Applicative Tree where
  pure x = Leaf x
  fs <*> xs = tconcmap (\f -> tmap f xs) fs
```

Class Monad

A **Monad** is a kind of algebraic data. Type used to represent computations (instead of data in the domain model), we will often call these computations **actions**. Monads allow the programmer to chain actions together to build an ordered sequence, in which each action is decorated with additional processing rules provided by the monad and performed automatically. Monads are **flexible** and **abstract**. Monads can also be used to make imperative programming easier in a pure functional language. In practice, through them it is possible to define an imperative sub-language on top of a purely functional one.
The Monad class definition:

```haskell
class Applicative m => Monad m where
  -- Sequentially compose two actions,
     passing any value produced
  -- by the first as an argument to the
     second.
  (>>=) :: m a -> (a -> m b) -> m b
  -- Sequentially compose two actions,
     discarding any value produced
  -- by the first, like sequencing operators
     (such as the semicolon)
  -- in imperative languages.
  (>>) :: m a -> m b -> m b
  m >> k = m >>= \_ -> k
  -- Inject a value into the monadic type.
  return :: a -> m a
  return = pure
  -- Fail with a message.
  fail :: String -> m a
  fail s = error s
```

Note that only `>>=` is required, all the other methods have standard definitions. `>>=` and `>>` are called **bind**. `m` a is a computation (or action) resulting in a value of type `a`. `return` is by default pure, so it is used to create a single monadic action. Bind operators are used to compose actions: `x >>= y` performs the computation `x`, takes the resulting value and passes it to `y`, then performs `y`; `x >> y` is analogous, but "throws away" the value obtained by `x`.
Maybe is a Monad: the information managed automatically by the monad is the "bit" which encodes the success (i.e. Just) or failure (i.e. Nothing) of the action sequence.

```haskell
instance Monad Maybe where
  (Just x) >>= k = k x
  Nothing >>= _ = Nothing
  fail _ = Nothing
```

for a monad to behave correctly, method definitions must obey the following laws:

- return is the identity element: `(return x) >>= f <=> f x` and `m >>= return <=> m`.
- associativity for binds: `(m >>= f) >>= g <=> m >>= (\x -> (f x >>= g))`.

The **do** syntax is used to avoid the explicit use of `>>=` and `>>`. The essential translation of do is captured by the following two rules: `do e1 ; e2 <=> e1 >> e2` and `do p <- e1 ; e2 <=> e1 >>= \p -> e2`. IO is a build-in monad in Haskell: indeed, we used the do notation for performing IO.
The List Monad: In lists, monadic binding involves joining together a set of calculations for each value in the list. In practice, bind is concatMap. The underlying idea is to represent non-deterministic computations. List comprehensions can be expressed in do notation.

```haskell
instance Monad [] where
  xs >>= f = concatMap f xs
```

```
    fail _ = []
```

Monadic Trees:

```
instance Monad Tree where
    xs >>= f = tconcmap f xs
    fail _ = Empty
```

Monads are abstract, so monadic code is very flexible, because it can work with any instance of Monad. Monads can be used to implement parsers, continuations, and, of course, IO.

The State Monad

We saw that monads are useful to automatically manage state. We now define a general monad to do it. First of all, we define a type to represent our **State** class. The idea is having a type that represent a computation with a state. The "container" has now type constructor State st, because State has two parameters. We can then make State an istance of Functor, Applicative and Monad.

```
data State st a = State (st -> (st, a))

instance Functor (State st) where
    fmap f (State g) = State (\s -> let (s', x)
          = g s
        in (s', f x))

instance Applicative (State st) where
    pure x = State (\t -> (t, x))
    (State f) <*> (State g) =
        State (\state -> let (s, f') = f state
                (s', x) = g s
            in (s', f' x))

instance Monad (State state) where
    State f >>= g = State (\olds ->
        let (news, value) = f olds
            State f' = g value
        in f' news)
```

An important aspect of this monad is that monadic code does not get evaluated to data, but to a function. Note that State is a function and bind is function composition. In particular, we obtain a function of the initial state. To get a value out of it, we need to call it.

```
runStateM :: State state a -> state ->
    (state, a)
runStateM (State f) st = f st

example = runStateM
    (do x <- return 5
      return (x+1))
    333
```

Also after the example, it should be clear that, as it is, the state is not really used in a computation, it is only passed around unchanged. The point is to move the state to the data part and back, if we want to access and modify it in the program. 3 this is easily done with the two utilities `getState = State (\state -> (state, state))` and `putState new = State (\_ -> (new, ()))`.

State Applications: Trees and Logging

Trees: The idea is to visit a tree and to give a number (e.g. a unique identifier) to each leaf. It is of course possible to do it directly, but we need to define functions passing the current value of the current id value around, to be assigned and then incremented for the next leaf. But we can also see this id as a state, and obtain we a more elegant and general definition by using our State monad. First we need a monadic map for trees.

```
mapTreeM :: (a -> State state b) -> Tree a ->
    State state (Tree b)
mapTreeM f (Leaf a) = do
  b <- f a
  return (Leaf b)
mapTreeM f (Branch lhs rhs) = do
  lhs' <- mapTreeM f lhs
  rhs' <- mapTreeM f rhs
  return (Branch lhs' rhs')

numberTree tree = runStateM (mapTreeM number
    tree) 1
  where number v = do cur <- getState
        putState (cur+1)
        return (v,cur)
```

Logging: In this case, instead of changing the tree, we want to implement a logger, that, while visiting the data structure, keeps track of the found data. This is quite easy, if we see the log text as the state of the computation.

```
logTree tree = runStateM (mapTreeM collectLog
    tree) "Log\n"
  where collectLog v = do
        cur <- getState
        putState (cur ++ "Found node: " ++
            [v] ++ "\n")
        return v
```

Exams

2020-07-17

Define a data type that stores an m by n matrix as a list of lists by row. In your implementation you can use the following functions: splitAt, unzip, (!!). After defining an appropriate data constructor, do the following:

- Define a function 'new' that takes as input two integers m and n and a value 'fill', and returns an m by n matrix whose elements are all equal to 'fill'.
- Define function 'replace' such that, given a matrix m, the indices i, j of one of its elements, and a new element, it returns a new matrix equal to m except for the element in position i, j, which is replaced with the new one.
- Define function 'lookup', which returns the element in a given position of a matrix.
- Make the data type an instance of Functor and Foldable.
- Make the data type an instance of Applicative.

Solution:

```
newtype Matrix a = Matrix [[a]] deriving (Eq,
    Show)

new :: Int -> Int -> a -> Matrix a
new m n fill = Matrix [[fill | _ <- [1..n]] | _
    <- [1..m]]

replace :: Int -> Int -> a -> Matrix a -> Matrix
    a
replace i j x (Matrix rows) = let (rowsHead,
    r:rowsTail) = splitAt i rows
(rHead, x':rTail) = splitAt j r
in Matrix $ rowsHead ++ ((rHead ++
    (x:rTail)):rowsTail)

lookup :: Int -> Int -> Matrix a -> a
lookup i j (Matrix rows) = (rows !! i) !! j

instance Functor Matrix where
fmap f (Matrix rows) = Matrix $ map (\r -> map f
    r) rows

instance Foldable Matrix where
foldr f e (Matrix rows) = foldr (\r acc -> foldr
    f acc r) e rows

hConcat :: Matrix a -> Matrix a -> Matrix a
hConcat (Matrix []) m2 = m2
hConcat m1 (Matrix []) = m1
hConcat (Matrix (r1:r1s)) (Matrix (r2:r2s)) =
```

```
let (Matrix tail) = hConcat (Matrix r1s) (Matrix
    r2s)
in Matrix $ (r1 ++ r2) : tail

vConcat :: Matrix a -> Matrix a -> Matrix a
vConcat (Matrix rows1) (Matrix rows2) = Matrix $
    rows1 ++ rows2

concatMapM :: (a -> Matrix b) -> Matrix a ->
    Matrix b
concatMapM f (Matrix rows) =
let empty = Matrix []
in foldl
(\acc r -> vConcat acc $ foldl (\acc x ->
    hConcat acc (f x)) empty r)
empty
rows

instance Applicative Matrix where
pure x = Matrix [[x]]
fs <*> xs = concatMapM (\f -> fmap f xs) fs
```

2020-06-29

We want to implement a queue, i.e. a FIFO container with the two operations enqueue and dequeue with the obvious meaning. A functional way of doing this is based on the idea of using two lists, say L1 and L2, where the first one is used for dequeuing (popping) and the second one is for enqueing (pushing). When dequeing, if the first list is empty, we take the second one and put it in the first, reversing it. This last operation appears to be O(n), but suppose we have n enqueues followed by n dequeues; the first dequeue takes time proportional to n (reverse), but all the other dequeues take constant time. This makes the operation O(1) amortised that is why it is acceptable in many applications.

- Define Queue and make it an instance of Eq.
- Define enqueue and dequeue, stating their types.
- Make Queue an instance of Functor and Foldable.
- Make Queue an instance of Applicative.

Solution:

```
data Queue a = Queue [a] [a] deriving Show

to_list (Queue x y) = x ++ reverse y

instance Eq a => Eq (Queue a) where
  q1 == q2 = (to_list q1) == (to_list q2)

enqueue :: a -> Queue a -> Queue a
enqueue x (Queue pop push) = Queue pop
    (x:push)

dequeue :: Queue a -> (Maybe a, Queue a)
dequeue q@(Queue [] []) = (Nothing, q)
dequeue (Queue (x:xs) v) = (Just x, Queue xs
    v)
dequeue (Queue [] v) = dequeue (Queue
    (reverse v) [])

instance Functor Queue where
  fmap f (Queue x y) = Queue (fmap f x) (fmap
      f y)

instance Foldable Queue where
  foldr f z q = foldr f z $ to_list q

q1 +++ (Queue x y) = Queue ((to_list q1) ++
    x) y

qconcat q = foldr (+++) (Queue [][]) q

instance Applicative Queue where
  pure x = Queue [x] []
  fs <*> xs = qconcat $ fmap (\f -> fmap f
      xs) fs
```

2020-02-07

Consider a data type PriceList that represents a list of items, where each item is associated with a price, of type Float: data PriceList a = PriceList [(a, Float)]

- Make PriceList an instance of Functor and Foldable.
- Make PriceList an instance of Applicative, with the constraint that each application of a function in the left hand side of a `<*>` must increment a right hand side value's price by the price associated with the function.

Solution:

```
pmap :: (a -> b) -> Float -> PriceList a ->
    PriceList b
pmap f v (PriceList prices) = PriceList $
    fmap (\x -> let (a, p) = x
           in (f a, p+v)) prices

instance Functor PriceList where
  fmap f prices = pmap f 0.0 prices

instance Foldable PriceList where
  foldr f i (PriceList prices) = foldr (\x y
      -> let (a, p) = x
           in f a y) i prices

(PriceList x) +.+ (PriceList y) = PriceList $
    x ++ y

plconcat x = foldr (+.+) (PriceList []) x

instance Applicative PriceList where
  pure x = PriceList [(x, 0.0)]
  (PriceList fs) <*> xs = plconcat (fmap (\ff
      -> let (f, v) = ff
           in pmap f v xs) fs)
```

2020-01-15

The following data structure represents a cash register. As it should be clear from the two accessor functions, the first component represents the current item, while the second component is used to store the price (not necessarily of the item: it could be used for the total).

```
data CashRegister a = CashRegister  getReceipt ::  (a,
Float)  deriving (Show, Eq)
getCurrentItem = fst .  getReceipt
getPrice = snd .  getReceipt
```

- Make CashRegister an instance of Functor and Applicative
- Make CashRegister an instance of Monad.

Solution:

```
instance Functor CashRegister where
    fmap f cr = CashRegister (f $
        getCurrentItem cr, getPrice cr)

instance Applicative CashRegister where
    pure x = CashRegister (x, 0.0)
    CashRegister (f, pf) <*> CashRegister (x,
        px) = CashRegister (f x, pf + px)

instance Monad CashRegister where
    CashRegister (oldItem, price) >>= f =
        let newReceipt = f oldItem
        in CashRegister (getCurrentItem
            newReceipt, price + (getPrice
            newReceipt))
```

2019-09-03

Consider the data structure Tril, which is a generic container consisting of three lists

- 1) Give a data definition for Tril.
- 2) Define list2tril, a function which takes a list and 2 values x and y, say x ¡ y, and builds a Tril, where the last component is the ending sublist of length x, and the middle component is the middle sublist of length y-x. Also, list2tril L x y = list2tril L y x.

E.g. list2tril [1,2,3,4,5,6] 1 3 should be a Tril with first component [1,2,3], second component [4,5], and third component [6].

- 3) Make Tril an instance of Functor and Foldable.
- 4) Make Tril an instance of Applicative, knowing that the concatenation of 2 Trils has first component which is the concatenation of the first two components of the first Tril, while the second component is the concatenation of the ending component of the first Tril and the beginning one of the second Tril (the third component should be clear at this point).

```haskell
data Tril a = Tril [a] [a] [a] deriving
    (Show, Eq)

instance Functor Tril where
    fmap f (Tril x y z) = Tril (fmap f
        x)(fmap f y)(fmap f z)

instance Foldable Tril where
    foldr f i (Tril x y z) = foldr f (foldr f
        (foldr f i z) y) x

(Tril x y z) +++ (Tril a b c) = Tril (x ++
    y) (z ++ a) (b ++ c)

trilconcat t = foldr (+++) (Tril [][][]) t
trilcmap f t = trilconcat $ fmap f t

instance Applicative Tril where
    pure x = Tril [x][][]
    x <*> y = trilcmap (\f -> fmap f y) x

list2tril lst n1 n2 = let (_,_,[x,y,z]) =
    foldr helper (n1, n2, [[]]) lst
                    in Tril x y z
    where
        helper el (0, m, next) = (-1, m-1,
            [el]:next)
        helper el (n, 0, next) = (n-1, -1,
            [el]:next)
        helper el (n, m, (x:xs)) = (n-1, m-1,
            (el:x):xs)
```

2019-07-24

Consider a non-deterministic finite state automaton (NFSA) and assume that its states are values of a type State defined in some way. An NFSA is encoded in Haskell through three functions:
i) transition :: Char → State → [State], i.e. the transition function.
ii) end :: State → Bool, i.e. a functions stating if a state is an accepting state (True) or not.
ii) start :: [State], which contains the list of starting states.

- 1) Define a data type suitable to encode the configuration of an NSFA.
- 2) Define the necessary functions (providing also all their types) that, given an automaton A (through transition, end, and start) and a string s, can be used to check if A accepts s or not.

```haskell
data Config = Config String [State] deriving
    (Show, Eq)

steps :: (Char -> State -> [State]) ->
    Config -> Bool
steps trans (Config "" confs) = not . null $
    filter end confs
steps trans (Config (a:as) confs) = steps
    trans $ Config as (concatMap (trans a)
    confs)
```

2019-06-28

- 1) Define a Tritree data structure, i.e. a tree where each node has at most 3 children, and every node contains a value.
- 2) Make Tritree an instance of Foldable and Functor.
- 3) Define a Tritree concatenation t1 +++ t2, where t2 is appended at the bottom-rightmost position of t1.
- 4) Make Tritree an instance of Applicative.

```haskell
data Tritree a = Nil | Tritree a (Tritree
    a)(Tritree a)(Tritree a) deriving (Eq,
    Show)

instance Functor Tritree where
    fmap f Nil = Nil
    fmap f (Tritree x t1 t2 t3) = Tritree (f
        x)(fmap f t1)(fmap f t2)(fmap f t3)

instance Foldable Tritree where
    foldr f i Nil = i
    foldr f i (Tritree x t1 t2 t3) = f x $
        foldr f (foldr f (foldr f i t3) t2) t1

x +++ Nil = x
Nil +++ x = x
(Tritree x t1 t2 Nil) +++ t = (Tritree x t1
    t2 t)
(Tritree x t1 t2 t3) +++ t = (Tritree x t1
    t2 (t3 +++ t))

ttconcat t = foldr (+++) Nil t
ttconcmap f t = ttconcat $ fmap f t

instance Applicative Tritree where
    pure x = (Tritree x Nil Nil Nil)
    x <*> y = ttconcmap (\f -> fmap f y) x
```

2019-02-08

We want to define a data structure, called BFlist (Back/Forward list), to define lists that can either be "forward" (like usual list, from left to right), or "backward", i.e. going from right to left.
We want to textually represent such lists with a plus or a minus before, to state their direction: e.g. +[1,2,3] is a forward list, -[1,2,3] is a backward list.
Concatenation (let us call it $<++>$) for BFlist has this behavior: if both lists have the same direction, the returned list is the usual concatenation. Otherwise, forward and backward elements of the two lists delete each other, without considering their stored values.
For instance: +[a,b,c] $<++>$ -[d,e] is +[c], and -[a,b,c] $<++>$ +[d,e] is -[c].

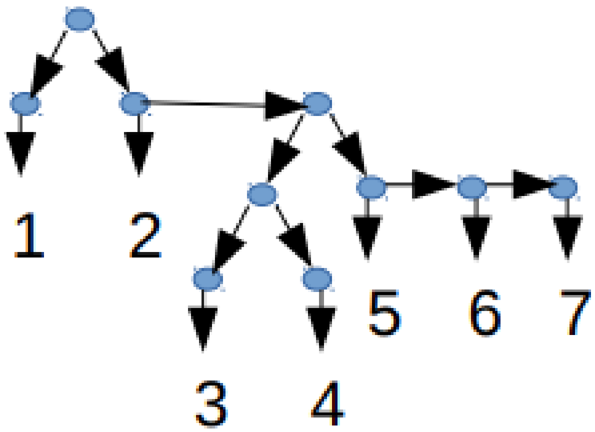
- 1) Define a datatype for BFlist.
- 2) Make BFList an instance of Eq and Show, having the representation presented above.
- 3) Define $<++>$, i.e. concatenation for BFList.
- 4) Make BFList an instance of Functor.
- 5) Make BFList an instance of Foldable.
- 6) Make BFList an instance of Applicative.

```haskell
data Dir = Fwd | Bwd deriving Eq
data BFlist a = BFlist Dir [a] deriving Eq

instance Show Dir where
    show Fwd = "+"
    show Bwd = "-"

instance (Show a) => Show (BFlist a) where
    show (BFlist x y) = show x ++ show y
instance Functor BFlist where
    fmap f (BFlist d x) = BFlist d (fmap f x)

instance Foldable BFlist where
    foldr f i (BFlist d x) = foldr f i x

(BFlist _ []) <++> x = x
x <++> (BFlist _ []) = x
(BFlist d1 x) <++> (BFlist d2 y) | d1 == d2
    = BFlist d1 (x ++ y)
(BFlist d1 (x:xs)) <++> (BFlist d2 (y:ys)) =
    (BFlist d1 xs) <++> (BFlist d2 ys)
```

```
bflconcat (BFlist d v) = foldr (<++>)
     (BFlist d []) (BFlist d v)
bflconcatmap f x = bflconcat $ fmap f x

instance Applicative BFlist where
    pure x = BFlist Fwd [x]
    x <*> y = bflconcatmap (\f -> fmap f y) x
```

2019-01-16

We want to define a data structure, called Listree, to define structures working both as lists and as binary trees, like in the next figure.



- 1) Define a datatype for Listree.
- 2) Write the example of the figure with the defined data structure.
- 3) Make Listree an instance of Functor.
- 4) Make Listree an instance of Foldable.
- 5) Make Listree an instance of Applicative.

```
data Listree a = Nil | Cons a (Listree a) |
     Branch (Listree a)(Listree a) deriving
     (Eq, Show)

exfig = Branch (Cons 1 Nil) (Cons 2 (Branch
     (Branch (Cons 3 Nil) (Cons 4 Nil)) (Cons
     5 (Cons 6 (Cons 7 Nil)))))

instance Functor Listree where
    fmap f Nil = Nil
    fmap f (Cons x y) = Cons (f x) (fmap f y)
    fmap f (Branch x y) = Branch (fmap f x)
        (fmap f y)

instance Foldable Listree where
    foldr f i Nil = i
    foldr f i (Cons x y) = f x (foldr f i y)
    foldr f i (Branch x y) = foldr f (foldr f
        i x) y

x <++> Nil = x
Nil <++> x = x
(Cons x y) <++> z = (Cons x (y <++> z))
(Branch x y) <++> z = (Branch x (y <++> z))

ltconcat t = foldr (<++>) Nil t
ltconcmap f t = ltconcat $ fmap f t

instance Applicative Listree where
    pure x = (Cons x Nil)
    x <*> y = ltconcmap (\f -> fmap f y) x
```

2018-07-06

Consider this datatype: `data Blob a = Blob a (a -> a)`
Note: in this exercise, do not consider the practical meaning of Blob; the only constraint is to use all the available data, and the types must be right!
E.g.

```
instance Show a => Show (Blob a) where
show (Blob x f) = "Blob " ++ (show (f x))
```

- 1) Can Blob automatically derive Eq? Explain how, why, and, if the answer is negative, make it an instance of Eq.
- 2) Make Blob an instance of the following classes: Functor, Foldable, and Applicative.

Solution:

```
instance Eq a => Eq (Blob a) where
    (Blob x f) == (Blob y g) = (f x) == (g y)
instance Functor Blob where
    fmap f (Blob x g) = Blob (f (g x)) id

instance Foldable Blob where
    foldr f z (Blob x g) = f (g x) z
instance Applicative Blob where
    pure x = Blob x id
    (Blob fx fg) <*> (Blob x g) = Blob (((fg
        fx) . g) x) id
```

Exercise session

2019-10-29

```
module ES20191029 where

-- Factorial in Haskell
fact :: Int -> Int
fact 0 = 1
fact n = n * fact (n-1)

facti :: Integer -> Integer
facti 0 = 1
facti n = n * facti (n-1)
-- Int is a fixed-precision integer type with at
   least the range [-2^29 .. 2^29-1].
-- Integer are arbitrary-precision integers.

-- Fibonacci in Haskell
fib :: Integer -> Integer
fib 0 = 0
fib 1 = 1
fib n = fib (n-1) + fib (n-2)

-- We can use guards, too
fibg :: Integer -> Integer
fibg n | n == 0 = 0
       | n == 1 = 1
       | otherwise = fibg (n-1) + fibg (n-2)

-- A few functions with lists
myLength :: [a] -> Int
myLength [] = 0
myLength (x : xs) = 1 + myLength xs

empty :: [a] -> Bool
empty [] = True
empty (_:_) = False

myReverse :: [a] -> [a]
myReverse [] = []
myReverse (x:xs) = myReverse xs ++ [x]

range :: Integer -> Integer -> [Integer]
range a b = if a > b
  then error "Min > Max"
  else if a < b
       then a : (range (a+1) b)
       else [a]

range2 :: Integer -> Integer -> [Integer]
range2 a b | a > b = error "Min > Max"
           | a < b = a : (range (a+1) b)
           | otherwise = [a]

-- List comprehensions
```

```
rightTriang n = [(a, b, c) | a <- [1..n], b <-
    [1..a], c <- [1..b], a^2 == b^2 + c^2]

allRightTriang = [(a, b, c) | a <- [1..], b <-
    [1..a], c <- [1..b], a^2 == b^2 + c^2]

-- We can make an infinite list this way, too.
numsfrom :: Integer -> [Integer]
numsfrom n = n : (numsfrom $ n+1)

-- Alternatively:
numsfrom2 n = [n, n+1 ..]

-- fib 100 is very slow...
fibinf = 0 : 1 : [x + y | (x,y) <- zip fibinf $
    tail fibinf]
-- try take 10 $ zip fibinf $ tail fibinf
-- try fib 100 and take 100 fibinf

fibb n = fibb' n (0,1)
  where fibb' n (f1, f2) | n == 0 = f1
                         | otherwise = (fibb' $!
                             (n-1)) $! (f2, f1+f2)

-- ($!) f $! x = seq x (f x)

-- Higher order functions
myMap :: (a -> b) -> [a] -> [b]
myMap _ [] = []
myMap f (x:xs) = f x : (myMap f xs)

myTakeWhile :: (a -> Bool) -> [a] -> [a]
myTakeWhile _ [] = []
myTakeWhile p (x:xs) = if p x
  then x : myTakeWhile p xs
  else []


data TrafficLight = Red | Green | Yellow
    deriving (Show, Eq)

-- instance Show TrafficLight where
--   show Red = "Red light"
--   show Yellow = "Yellow light"
--   show Green = "Green light"

-- instance Eq TrafficLight where
--   Red == Red = True
--   Yellow == Yellow = True
--   Green == Green = True
--   _ == _ = False

-- Sum type
data Point = Point Float Float deriving (Eq,
    Show)

pointx (Point x _) = x
pointy (Point _ y) = y

distance :: Point -> Point -> Float
distance (Point x1 x2) (Point y1 y2) =
  let d1 = x1-y1
      d2 = x2-y2
  in sqrt $ (d1*d1) + (d2*d2)

type TPoint = (Float, Float)

tdistance :: TPoint -> TPoint -> Float
tdistance (x1, x2) (y1, y2) =
  let d1 = x1-y1
      d2 = x2-y2
  in sqrt $ (d1*d1) + (d2*d2)

data APoint = APoint {apx, apy :: Float}
    deriving (Eq, Show)
```

2019-11-12

```
module ES5 where

-- A few more higher order functions

-- map
-- map (+1) [1..10]

-- filter
-- filter even [1..100]
myFilter :: (a -> Bool) -> [a] -> [a]
myFilter _ [] = []
myFilter p (x:xs) | p x = x : myFilter p xs
                  | otherwise = myFilter p xs

-- zip
-- zip [1..10] ['a'..'j']
myZip :: [a] -> [b] -> [(a, b)]
myZip l [] = []
myZip [] l = []
myZip (x:xs) (y:ys) = (x, y) : myZip xs ys

-- myZip [1,2,3] ['a','b','c']

-- zipWith
-- zipWith (*) [1..10] [1..10]

myZipWith _ _ [] = []
myZipWith _ [] _ = []
myZipWith f (x:xs) (y:ys) = f x y : myZipWith f
    xs ys

myFoldL :: (b -> a -> b) -> b -> [a] -> b
myFoldL _ acc [] = acc
myFoldL f acc (x:xs) = myFoldL f (f acc x) xs
-- myFoldL (+) 0 [1,2,3]

myFoldR :: (a -> b -> b) -> b -> [a] -> b
myFoldR _ acc [] = acc
myFoldR f acc (x:xs) = f x $ myFoldR f acc xs

-- we can use folds to redefine many higher
    order functions
sumf :: Num n => [n] -> n
sumf = foldl (+) 0

elem' e = foldl (\acc x -> x == e || acc) False
-- what's the type of elem'?
-- elem' 2 [1..10]
-- elem' 100 [1..10]

filter' :: (a -> Bool) -> [a] -> [a]
filter' p = foldr
  (\x acc -> if p x then x : acc else acc)
  []
-- filterf odd [1..10]

map' f = foldr (\x acc -> (f x): acc) []

map'' f = foldl (\acc x -> acc ++ [f x]) []
-- map'' (+3) [1..10]

-- Try foldr (:) [] [1..10]
-- this is the identity

lapp :: [a] -> [a] -> [a]
lapp l1 l2 = foldr (:) l2 l1
-- lapp [1..10] [20..100]
-- [1..10] ++ [20..100]
```

```haskell
takeWhile' p = foldr (\x acc -> if p x then
    x:acc else []) []
-- it works with infinite lists too
-- takeWhile' (<10) [1..]

-- Binary Tree
data BTree a = BEmpty | BNode a (BTree a) (BTree
    a)

instance Eq a => Eq (BTree a) where
  BEmpty == BEmpty = True
  BNode x1 l1 r1 == BNode x2 l2 r2 =
    x1 == x2 && l1 == l2 && r1 == r2
  _ == _ = False

instance Show a => Show (BTree a) where
  show BEmpty = "Empty"
  show (BNode x BEmpty BEmpty) = "BNode " ++
      show x
  show (BNode x l r) = "BNode " ++ show x ++ "
      (" ++ show l ++ ") (" ++ show r ++ ")"

bleaf x = BNode x BEmpty BEmpty

isbleaf (BNode _ BEmpty BEmpty) = True
isbleaf _ = False

btmap :: (a -> b) -> BTree a -> BTree b
btmap _ BEmpty = BEmpty
btmap f (BNode x l r) =
  BNode (f x) (btmap f l) (btmap f r)
-- btmap (*2) (BNode 1 BEmpty BEmpty)
-- btmap (*2) (BNode 1 (bleaf 2) (bleaf 3))

instance Functor BTree where
  fmap = btmap

-- Functor laws:
-- fmap id = id
-- fmap (f . g) = fmap f . fmap g
-- Indeed:
-- fmap id (BNode 1 BEmpty BEmpty)
-- fmap id (BNode 1 (bleaf 2) (bleaf 3))
-- and
-- fmap (*2) $ fmap (+1) (BNode 1 (bleaf 2)
    (bleaf 3))
-- fmap ((*2) . (+1)) (BNode 1 (bleaf 2) (bleaf
    3))

btfoldr :: (a -> b -> b) -> b -> BTree a -> b
btfoldr _ acc BEmpty = acc
btfoldr f acc (BNode x l r) =
  f x (btfoldr f (btfoldr f acc r) l)
-- btfoldr (+) 0 (BNode 6 (bleaf 1) (bleaf 2))

instance Foldable BTree where
  foldr = btfoldr

-- Count nodes:
-- foldr (\_ acc -> acc + 1) 0 (BNode 6 (bleaf
    1) (bleaf 2))

-- Depth-First Visit:
-- foldr (:) [] (BNode 6 (bleaf 1) (bleaf 2))

-- DFS:
btelem x = foldr (\y acc -> x == y || acc) False

-- infinite trees
inftree n = BNode n (inftree (n+1)) (inftree
    (n+1))

bttake _ BEmpty = BEmpty
bttake h (BNode x l r)
  | h <= 0 = BEmpty
  | otherwise = BNode x (bttake (h-1) l) (bttake
      (h-1) r)

btZipWith _ _ BEmpty = BEmpty
btZipWith _ BEmpty _ = BEmpty
btZipWith f (BNode x1 l1 r1) (BNode x2 l2 r2) =
  BNode (f x1 x2) (btZipWith f l1 l2) (btZipWith
      f r1 r2)
```

2019-11-19

```haskell
module ES6 where

import qualified Data.Map as M

data BTree a = BEmpty | BNode a (BTree a) (BTree
    a) deriving Eq

instance (Show a) => Show (BTree a) where
  show BEmpty = "Bempty"
  show (BNode x BEmpty BEmpty) = "BNode " ++
      show x
  show (BNode x l r) = "BNode " ++ show x ++ "
      (" ++ show l ++ ") (" ++ show r ++ ")"

bleaf x = BNode x BEmpty BEmpty

isbleaf (BNode _ BEmpty BEmpty) = True
isbleaf _ = False

btmap :: (a -> b) -> BTree a -> BTree b
btmap _ BEmpty = BEmpty
btmap f (BNode x l r) = BNode (f x) (btmap f l)
    (btmap f r)

instance Functor BTree where
  fmap = btmap

-- fmap can be seen as "apply f to all elements
    of a container/context"
-- fmap (+1) $ Just 42
-- fmap (+1) Nothing
-- fmap (+1) [1..10]
-- fmap (+1) (BNode 1 (bleaf 2) (bleaf 3))

-- but also as a way to lift a unary function,
    so that it works between functors
-- :t fmap
-- :t fmap (+1)
incAll :: (Functor f, Num b) => f b -> f b
incAll = fmap (+1)
-- inc $ Just 1
-- inc [1..9]
-- inc (BNode 1 (bleaf 2) (bleaf 3))

-- class (Functor f) => Applicative f where
--   pure :: a -> f a
--   (<*>) :: f (a -> b) -> f a -> f b

-- pure encloses something into an applicative
    in a default way
-- pure 42 :: Maybe Integer
-- pure 42 :: [Integer]

-- <*> takes an applicative containing a
    function, and applies it to the content of
    another applicative
-- pure (*2) <*> Just 42
-- pure (*2) <*> [1..10]
-- now we can apply general functions to
    containers/contexts
```

```haskell
-- pure (*) <*> Just 6 <*> Just 7
-- pure (\x y z -> x*y*z) <*> Just 2 <*> Just 3
    <*> Just 4
-- pure (\x y z -> x*y*z) <*> Just 2 <*> Nothing
    <*> Just 4

-- (fmap (\x y z -> x*y*z) (Just 2)) <*> Just 3
    <*> Just 4
-- (fmap (\x y z -> x*y*z) (Just 2)) <*> Nothing
    <*> Just 4

-- To make this simpler, we have <$>:
-- (<$>) :: (Functor f) => (a -> b) -> f a -> f b
-- f <$> x = fmap f x

-- (*) <$> Just 6 <*> Just 7
-- (\x y z -> x*y*z) <$> Just 2 <*> Just 3 <*>
    Just 4
-- (\x y z -> x*y*z) <$> Just 2 <*> Nothing <*>
    Just 4

-- With lists:
-- (+2) <$> [1..3]
-- (+) <$> [1..3] <*> [2]
-- (+) <$> [1..3] <*> [1..10]
-- [(+1), (+2), (+3)] <*> [1..10]

-- this is due to the peculiar implementation of
    Applicative for lists
concat' [] = []
concat' (x:xs) = x ++ concat' xs

concatMap' f l = concat' $ map f l

-- instance Applicative [] where
--   pure x = [x]
--   fs <*> xs = concatMap' (\f -> map f xs) fs

-- But this is not the only way of implementing
    Applicative for lists
data ZipList a = ZEmpty | ZL a (ZipList a)

instance (Eq a) => Eq (ZipList a) where
  ZEmpty == ZEmpty = True
  ZL x xs == ZL y ys = x == y && xs == ys
  _ == _ = False

showZipList ZEmpty = "[]"
showZipList (ZL x ZEmpty) = "[" ++ show x ++ "]"
showZipList (ZL x xs) = "[" ++ show x ++ "," ++
    (drop 1 $ showZipList xs)

instance (Show a) => Show (ZipList a) where
  show l = "ZipList " ++ showZipList l

instance Functor ZipList where
  fmap _ ZEmpty = ZEmpty
  fmap f (ZL x xs) = ZL (f x) $ fmap f xs

toZipList :: [a] -> ZipList a
toZipList [] = ZEmpty
toZipList (x:xs) = ZL x (toZipList xs)

instance Applicative ZipList where
  pure x = ZL x $ pure x
  ZEmpty <*> _ = ZEmpty
  _ <*> ZEmpty = ZEmpty
  ZL f fs <*> ZL y ys = ZL (f y) (fs <*> ys)

-- (toZipList [(+1), (+2), (*3)]) <*> (toZipList
    [1,2,3])
-- pure (*1) <*> toZipList [1..10] -- meh

-- Let us make binary trees Applicative
btcat BEmpty t2 = t2
btcat t1 BEmpty = t1
btcat t1@(BNode x l r) t2 = BNode x l new_r
  where new_r = if isbleaf t1
                then t2
                else btcat r t2
-- btcat (BNode 1 (bleaf 2) (bleaf 3)) (BNode 4
    (bleaf 5) (bleaf 6))

btfoldr _ acc BEmpty = acc
btfoldr f acc (BNode x l r) =
  f x (btfoldr f (btfoldr f acc r) l)

btconcat t = btfoldr btcat BEmpty t

btcatmap f t = btconcat $ fmap f t
-- btcatmap (\x -> BNode x (bleaf x) (bleaf x))
    (BNode 1 (bleaf 2) (bleaf 3))

-- instance Applicative BTree where
--   pure = bleaf
--   fs <*> xs = btcatmap (\f -> fmap f xs) fs

-- we can also do it with the Zip semantics
instance Applicative BTree where
  pure x = BNode x (pure x) (pure x)
  BEmpty <*> _ = BEmpty
  _ <*> BEmpty = BEmpty
  BNode f lf rf <*> BNode x lx rx = BNode (f x)
      (lf <*> lx) (rf <*> rx)


-- An exercise on Data.Map
-- map(personNames, PhoneNumbers)
-- map(PhoneNumber, MobileCarriers)
-- map(MobileCarriers, BillingAddresses)

type PersonName = String
type PhoneNumber = String
type BillingAddress = String
data MobileCarrier = TIM | Vodafone | Wind |
    Iliad deriving (Eq, Show, Ord)


findCarrierBillingAddress ::
  PersonName
  -> M.Map PersonName PhoneNumber
  -> M.Map PhoneNumber MobileCarrier
  -> M.Map MobileCarrier BillingAddress
  -> Maybe BillingAddress
findCarrierBillingAddress person phoneMap
    carrierMap addressMap =
  case M.lookup person phoneMap of
    Nothing -> Nothing
    Just number ->
      case M.lookup number carrierMap of
        Nothing -> Nothing
        Just carrier -> M.lookup carrier
            addressMap

findCarrierBillingAddress' person phoneMap
    carrierMap addressMap =
  do
    number <- M.lookup person phoneMap
    carrier <- M.lookup number carrierMap
    M.lookup carrier addressMap
```

2019-11-26

```haskell
module ES7 where

-- We saw:
```

```haskell
-- Functors, to lift a function so that it works
    on a container/context
-- fmap (+2) (Just 5)
-- :t fmap (+2)
-- Applicative, to do the same with functions
    with multiple arguments
-- (+) <$> Just 5 <*> Just 2

-- What if we have a function that *returns* a
    value wrapped in a container/context?
apply42 f x = let s = f x
            in if s > 42 then Just s else
                Nothing

-- we want to apply a sequence of functions to
    an initial value,
-- but none of them can return a value lower
    than 42.
sequence42 x = case apply42 (+12) x of
  Nothing -> Nothing
  Just x1 -> case apply42 (\x -> x-6) x1 of
    Nothing -> Nothing
    Just x2 -> apply42 (*2) x2

-- Try sequence42 42
-- sequence42 30

-- We must combine the information in the
    context containing the input value
-- with the context generated by the function in
    the output value.

-- class Monad m where
--   return :: a -> m a
--   (>>=) :: m a -> (a -> m b) -> m b
--   (>>) :: m a -> m b -> m b
--   x >> y = x >>= \_ -> y
--   fail :: String -> m a
--   fail msg = error msg

-- instance Monad Maybe where
--   return x = Just x
--   Nothing >>= f = Nothing
--   Just x >>= f = f x
--   fail _ = Nothing

sequence42' x = return x
  >>= apply42 (+12)
  >>= apply42 (\x -> x-6)
  >>= apply42 (*2)

-- do notation:
sequence42do x = do
  x1 <- apply42 (+12) x
  x2 <- apply42 (\x -> x-6) x1
  x3 <- apply42 (*2) x2
  return x3

-- this gets translated to
sequence42do' x = apply42 (+12) x >>=
  (\x1 -> apply42 (\x -> x-6) x1 >>=
    (\x2 -> apply42 (*2) x2 >>=
      (\x3 -> return x3)))

sequenceDiscard x = do
  apply42 (+1) x
  x1 <- apply42 (*2) x
  return x1

-- this is the same as
sequenceDiscard' x = apply42 (+1) x >> apply42
    (*2) x
  >>= (\x1 -> return x1)

sequenceDiscard'' x = apply42 (+1) x >>=
  (\_ -> apply42 (*2) x >>= (\x1 -> return x1))

-- Let's make a Monad ourselves!
-- The Log Monad
type Log = [String]
newtype Logger a = Logger { unwrap :: (a, Log) }

getContent l = x where (x, _) = unwrap l
getLog l = log where (_, log) = unwrap l

instance (Eq a) => Eq (Logger a) where
  Logger (x, _) == Logger (y, _) = x == y

instance (Show a) => Show (Logger a) where
  show l = show (getContent l)
    ++ "\n\nLog:"
    ++ foldr (\line acc -> "\n\t" ++ line ++ acc)
        ""
        (getLog l)

instance Functor Logger where
  fmap f l = let (x, log) = unwrap l
          in Logger (f x, log)

-- Functor laws:
-- fmap id = id
-- fmap (f . g) = fmap f . fmap g
-- what if we modified the log?

instance Applicative Logger where
  pure x = Logger (x, [])
  Logger (f, lf) <*> Logger (x, lx) =
    Logger (f x, lf ++ lx)

-- (+) <$> Logger (2, ["first operand: 2"]) <*>
    Logger (3, ["second operand: 3"])

instance Monad Logger where
  return = pure
  Logger (x, log) >>= f = let (y, newLog) =
      unwrap $ f x
                    in Logger (y, log ++
                        newLog)

logPlusOne :: (Num a) => a -> Logger a
logPlusOne x = Logger (x+1, ["Add one."])

logMultiplyTwo x = Logger (x*2, ["Multiply by
    two."])

doOps x = do
  x1 <- logPlusOne x
  logMultiplyTwo x1
  x3 <- logPlusOne x1
  return x3

-- Monadic Laws
-- Left identity: return a >>= f 'equivalent to'
    f a
-- f has the type (a -> m b) so it returns a
    monad
-- this means that the minimal context to return
-- is just applying f to a

-- return 42 >>= logPlusOne
-- logPlusOne 42

-- Right identity: m >>= return 'equivalent to' m
-- When we feed monadic values to functions by
    using >>=,
-- those functions take normal values and return
    monadic ones.
```

```
-- return is also one such function, if you
    consider its type.

-- Logger (1,["barnibalbi"]) >>= return

-- Associativity: (m >>= f) >>= g 'equivalent
    to' m >>= (\x -> f x >>= g)

doOps' x = logPlusOne x >>=
  (\x1 -> logMultiplyTwo x1 >>=
    (\x2 -> logPlusOne x2 >>=
      (\x3 -> return x3)))
-- doOps 3 is the same as
-- doOps' 3


-- Let us take our binary trees again
data BTree a = BEmpty | BNode a (BTree a) (BTree
    a) deriving Eq

instance (Show a) => Show (BTree a) where
  show BEmpty = "BEmpty"
  show (BNode x BEmpty BEmpty) = "BNode " ++
      show x
  show (BNode x l r) = "BNode "++ show x ++ " ("
      ++ show l ++ ") (" ++ show r ++ ")"

bleaf x = BNode x BEmpty BEmpty

putLog :: String -> Logger ()
putLog msg = Logger ((), [msg])

bleafM x = do
  putLog $ "Create leaf " ++ show x
  return $ bleaf x

treeReplaceM :: (Show a) => (BTree a) -> (a ->
    Bool) -> a
  -> Logger (BTree a)
treeReplaceM BEmpty _ _ = return BEmpty
treeReplaceM (BNode x l r) p y = do
  newL <- treeReplaceM l p y
  newR <- treeReplaceM r p y
  if p x
  then do
    putLog $ "Replaced " ++ show x ++ " with "
        ++ show y
    return $ BNode y newL newR
  else do
    return $ BNode x newL newR

-- This is the same as
-- treeReplaceM (BNode x l r) p y =
--   treeReplaceM l p y >>=
--   (\newL -> treeReplaceM r p y >>=
--             (\newR -> if p x
--               then do
--                 putLog $ "Replaced " ++ show x
    ++ " with " ++ show y
--                 return $ BNode y newL newR
--               else
--                 return $ BNode x newL newR))
```

2019-12-03

```
module ES8 where

import Control.Monad.Fail

-- Exam 2017 02 27
data LolStream x = LolStream Int [x]

-- The list [x] must always be an infinite list
-- (also called a stream), while the first
    parameter, of type Int,
-- when positive represents the fact that the
    stream is periodic, while it is not periodic
    if
-- negative (0 is left unspecified). E.g.
-- LolStream -1 [1,2..]
-- LolStream 2 [1,2,1,2...]
period12 = [1,2] ++ period12
-- LolStream 2 period12

isperiodic (LolStream n _) = n > 0
destream ls@(LolStream n l) = if isperiodic ls
  then take n l
  else l

instance (Eq a) => Eq (LolStream a) where
  ls1 == ls2 = destream ls1 == destream ls2

instance (Show a) => Show (LolStream a) where
  show = show . destream

-- Define a function lol2lolstream which takes a
    finite list of finite lists
-- [h 1 , h 2 , ... h n ], and returns
-- LolStream (|h 1 | + |h 2 | + ... + |h n |) (h
    1 ++ h 2 ++ ... ++ h n ++ h 1 ++ h 2 ++ ...)
lolrepeat xs = xs ++ lolrepeat xs

lol2lolstream :: [[a]] -> LolStream a
lol2lolstream ls = let lscat = foldr (++) [] ls
                   in LolStream (length lscat)
                        (lolrepeat lscat)

instance Functor LolStream where
  fmap f (LolStream n l) = LolStream n $ map f l

instance Foldable LolStream where
  foldr f e ls = foldr f e $ destream ls

instance Applicative LolStream where
  pure x = lol2lolstream [[x]]
  ls1@(LolStream nf fs) <*> ls2@(LolStream nx
      xs) =
    LolStream (nf * nx) $ lolrepeat (destream
        ls1 <*> destream ls2)

-- lol2lolstream [[(+2),(*3)]] <*> lol2lolstream
    [[1..4]]

instance Monad LolStream where
  ls >>= f = lol2lolstream [destream ls >>= \x
      -> destream (f x)]


something = do
  x <- lol2lolstream [[1..3]]
  y <- lol2lolstream [[2..4]]
  return (x,y)

somethingButWithLists = do
  x <- [1..3]
  y <- [2..4]
  return (x,y)


-- Let us try to implement a stack in Haskell.
type Stack = [Int]

pop :: Stack -> (Stack, Int)
pop [] = error "Popping an empty stack!"
pop (x:xs) = (xs, x)

push :: Int -> Stack -> (Stack, ())
```

```haskell
push x xs = (x:xs, ())

-- Define a function that executes the following
    operations on the stack:
-- pop an element
-- pop another element
-- push 100
-- pop an element
-- push 42
stackManip :: Stack -> (Stack, ())
stackManip stack = let
  (newStack1, a) = pop stack
  (newStack2, b) = pop newStack1
  (newStack3, ()) = push 100 newStack2
  (newStack4, c) = pop newStack3
  in push 42 newStack4
-- try stackManip [1..10]

data State st a = State (st -> (st, a))

instance Functor (State st) where
  fmap f (State g) = State (\s -> let (s', x) =
      g s
                              in  (s', f x))

instance Applicative (State st) where
  pure x = State (\t -> (t, x))
  (State f) <*> (State g) =
    State (\state -> let (s, f') = f state
                         (s', x) = g s
                     in  (s', f' x))

instance Monad (State state) where
  State f >>= g = State (\olds ->
                          let (news, value) = f
                              olds
                              State f' = g value
                          in f' news)

-- We need this to use the do notation with
    pattern matching since GHC 8.6.1.
instance MonadFail (State a) where
  fail s = error s

runStateM :: State state a -> state -> (state, a)
runStateM (State f) st = f st

getState = State (\state -> (state, state))
putState new = State (\_ -> (new, ()))

-- define pop using the State monad
popM :: State Stack Int
popM = do
  (x:xs) <- getState
  putState xs
  return x

-- define push using the State monad
pushM :: Int -> State Stack ()
pushM x = do
  xs <- getState
  putState (x:xs)
  return ()

stackManipM :: State Stack ()
stackManipM = do
  popM
  popM
  pushM 100
  popM
  pushM 42

-- runStateM stackManipM [1..10]


-- Exam 2015 09 22
-- Define the Bilist data-type, which is a
    container of two homogeneous lists.
-- Define an accessor for Blist, called
    bilist_ref, that, given an index i,
-- returns the pair of values at position i in
    both lists.
-- E.g. bilist_ref (Bilist [1,2,3] [4,5,6]) 1
    should return (2,5).
data Bilist a = Bilist [a] [a] deriving (Eq,
    Show)

bilist_ref (Bilist l1 l2) n = (l1 !! n, l2 !! n)

-- Define a function, called oddeven, that is
    used to build a Bilist x y from a simple
    list.
-- oddeven takes all the elements at odd
    positions and puts them in y,
-- while all the other elements are put in x,
    maintaining their order.
-- You may assume that the given list has an
    even length (or 0).
-- Write also all the types of the functions you
    define.
-- E.g. oddeven [1,2,3,4] must be Bilist [1,3]
    [2,4].
oddeven :: [a] -> Bilist a
oddeven l = oddevenh l [] []
  where oddevenh [] ev od = Bilist ev od
        oddevenh (x:xs) ev od = oddevenh xs od
             (ev ++ [x])

inv_oddeven :: Bilist a -> [a]
inv_oddeven (Bilist l r) = concat $ map (\(x,y)
    -> [x,y]) $ zip l r


bilist_maxh :: (Num a, Ord a) => Bilist a -> Int
    -> a -> Int -> Int
bilist_maxh (Bilist (l:ls) (r:rs)) pos max maxpos
  | l+r > max = bilist_maxh (Bilist ls rs)
      (pos+1) (l+r) pos
  | otherwise = bilist_maxh (Bilist ls rs)
      (pos+1) max maxpos
bilist_maxh _ _ _ maxpos = maxpos

bilist_max (Bilist (l:ls) (r:rs)) =
  bilist_maxh (Bilist ls rs) 1 (l+r) 0

-- Try to make Bilist an instance of Functor,
    Foldable, Applicative, Monad.
```