

Date:

Student's Name:

Exam #2

* there are 10 applications in this exam; each problem carries 10% "weight" from the exam grade (5% will be earned by the integrity - shown on this exam paper - of your complete work/process, whereas the other "half" of the "credit" associated with each application will come from the accuracy of the final answer - units, and all - also, to be shown on this exam paper)

1. Our YCP soccer team has 2 games scheduled for one weekend. It has a 0.3 probability of not losing the first game, and a 0.8 probability of not losing the second game, independent of the first. If it does not lose a particular game, the team is equally likely to win or tie, independent of what happens in the other game. The YCP team will receive 2 points for a win, 1 for a tie, and 0 for a loss. Find the PMF of the number of points that the team earns over the weekend.

2. Let $Y = |X|$. Find the PMF (p_Y), for the case where:

$1/7$, if x is an integer in the range $[-3, 3]$

$p_{X(x)} =$

0, otherwise.

3. A family has 3 natural children and has adopted 1 girl. Each natural child has equal probability of being a girl or a boy, independent of the other children. Find the PMF of the number of girls out of the 4 children.
4. A city's temperature is modeled as a random variable with mean and standard deviation both equal to 7 degrees Celsius. A day is described as "normal" if the temperature during that day ranges within one standard deviation from the mean. What would be the temperature range for a normal day if temperature were expressed in degrees Fahrenheit?
5. Consider a quiz game where a person is given two questions and must decide which one to answer first. Question 1 will be answered correctly with probability 0.9, and the person will then receive as prize \$50, while question 2 will be answered correctly with probability 0.3, and the person will then receive as prize \$100. If the first question attempted is answered incorrectly, the quiz ends (the person is not allowed to attempt the second question). If the first question is answered correctly, the person is allowed to attempt the second question. Which question should be answered first to maximize the expected value of the total prize money received (and what is that "expected value"?)?

6. A stock market trader buys 50 shares of stock A and 100 shares of stock B. Let X and Y be the price changes of A and B, respectively, over a certain time period, and assume that the joint PMF of X and Y is uniform over the set of integers satisfying:

$$-3 \leq x \leq 5 \text{ and } -1 \leq (y - x) \leq 1.$$

- (a) Find the marginal PMFs and the means of X and Y .
(b) Find the mean of the trader's profit.

7. Professor May B Right often has her facts wrong, and answers each of her students' questions incorrectly with probability $1/8$, independent of other questions. In each lecture, professor May is asked 0, 1, or 2 questions with equal probability $1/3$. Let X and Y be the number of questions professor May is asked and the number of questions she answers wrong in a given lecture, respectively. Construct the joint PMF $p_{X,Y}(x,y)$.

8. Messages transmitted by a computer in Boston through a data network are destined for New York with probability 0.2, for Chicago with probability 0.5, and for San Francisco with probability 0.3. The transit time X of a message is random. Its mean is 0.07 seconds if it is destined for New York, 0.01 seconds if it is destined for Chicago, and 0.1 seconds if it is destined for San Francisco. Find $E[X]$.

9. Consider four independent rolls of a fair 6-sided die. Let X be the number of 2s and Y be the number of 3s obtained. What is the joint PMF of X and Y ?
10. Alice passes through five traffic lights on her way to work, and each light is equally likely to be green or red, independent of the others.
- (a) What is the PMF, the mean, and the variance of the number of red lights that Alice encounters?
 - (b) Suppose that each red light delays Alice by exactly one minute. What is the variance of Alice's commuting time?