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$$\mathcal{L}_{primal} = \sum_{i=1}^{n} a_i - \frac{1}{2}\left[\sum_{i=1}^{n}\sum_{j=1}^{n} a_i a_j y_i y_j x_i^T x^j\right]$$

$x_2 = \begin{pmatrix} -3 \\ 2 \end{pmatrix}$ $x_1 = \begin{pmatrix} 0 \\ -3 \end{pmatrix}$

$y_2 = -1$

$y_1 = +1$

we need to optimize $\mathcal{L}_{primal}$ where we need to meet te constraint $\sum_{i=1}^{n} a_i y_i = 0$

Firsty, let's start as

$n = 2$

$$\mathcal{L}_{primal} = a_1 + a_2 - \frac{1}{2}\Big[a_1 a_1 y_1 y_1 x_1^T x^1 + a_1 a_2 y_1 y_2 x_1^T x^2 + a_2 a_1 y_2 y_1 x_2^T x^1 + a_2 a_2 y_2 y_2 x_2^T x^2\Big]$$

$/ n=2$

$\sum_{i=1} a_i y_i = 0 \Rightarrow a_1 y_1 + a_2 y_2 = 0 \Rightarrow a_1(-1) + a_2(1) = 0$

$\Rightarrow -a_1 + a_2 = 0 \Rightarrow \boxed{a_1 = a_2}$

since $a_1 = a_2$ let's just say a

$$L_{primal} = 2a - \frac{1}{2}\Bigg[a^2(+1)^2 (0, -3)\begin{pmatrix}0\\-3\end{pmatrix} = -9a^2$$

$$+ \qquad\qquad +$$

$$a^2(1)(-1)\ (0,-3)\begin{pmatrix}-3\\2\end{pmatrix} = +6a^2$$

$$+ \qquad\qquad +$$

$$a^2(-1)(1)\ (-3\ \ 2)\begin{pmatrix}0\\-3\end{pmatrix} = +6a^2$$

$$+$$

$$a^2(-1)^2\ (-3,2)\begin{pmatrix}-3\\2\end{pmatrix} = 13a^2$$

$$L_{primal} = 2a - \frac{1}{2}\left[+9a^2 + 12a^2 + 13a^2\right]$$

$$L_{primal} = 2a - \frac{1}{2}(34a^2) = 2a - 17a^2$$

$$\nabla L = 2 - 34a = 0 \to \boxed{a = \frac{1}{17}}$$

$$w^\circ = \sum_{i=1}^{n=2} a_i y_i x^i = \frac{1}{17}(1)\begin{pmatrix}0\\-3\end{pmatrix} + \frac{1}{17}(-1)\begin{pmatrix}-3\\2\end{pmatrix}$$

$$w^\circ = \begin{pmatrix}0\\-\frac{3}{17}\end{pmatrix} + \begin{pmatrix}-\frac{3}{17}\\-\frac{2}{17}\end{pmatrix} = \begin{pmatrix}+\frac{3}{17}\\-\frac{5}{17}\end{pmatrix} = w^\circ$$

$$w^* = \begin{pmatrix} \frac{3}{17} \\ \frac{-5}{17} \end{pmatrix} \rightarrow \quad ax + by = c$$

$$by = c - ax$$

$$y = \frac{c}{b} - \underbrace{\frac{a}{b}}\, x$$

$$b^* = y - x^T w^* \qquad \frac{\frac{2}{17}}{\frac{-5}{17}} \qquad \frac{\frac{3}{17}}{\frac{-5}{17}} \Rightarrow +\frac{3}{5}$$

$$\underline{b^* = -\frac{2}{5}}$$

$$x_1 = \begin{pmatrix} 0 \\ -3 \end{pmatrix} \Rightarrow \quad c = 1 - (0 \ -3) \begin{pmatrix} \frac{3}{17} \\ \frac{-5}{17} \end{pmatrix}$$

$$y_1 = 1$$

$$c = 1 - \frac{15}{17} = \frac{2}{17}$$

So hyperplane is = $\boxed{y = +\underset{\sim}{\frac{3}{5}}x - \frac{2}{5}}$

Since our two points are bound

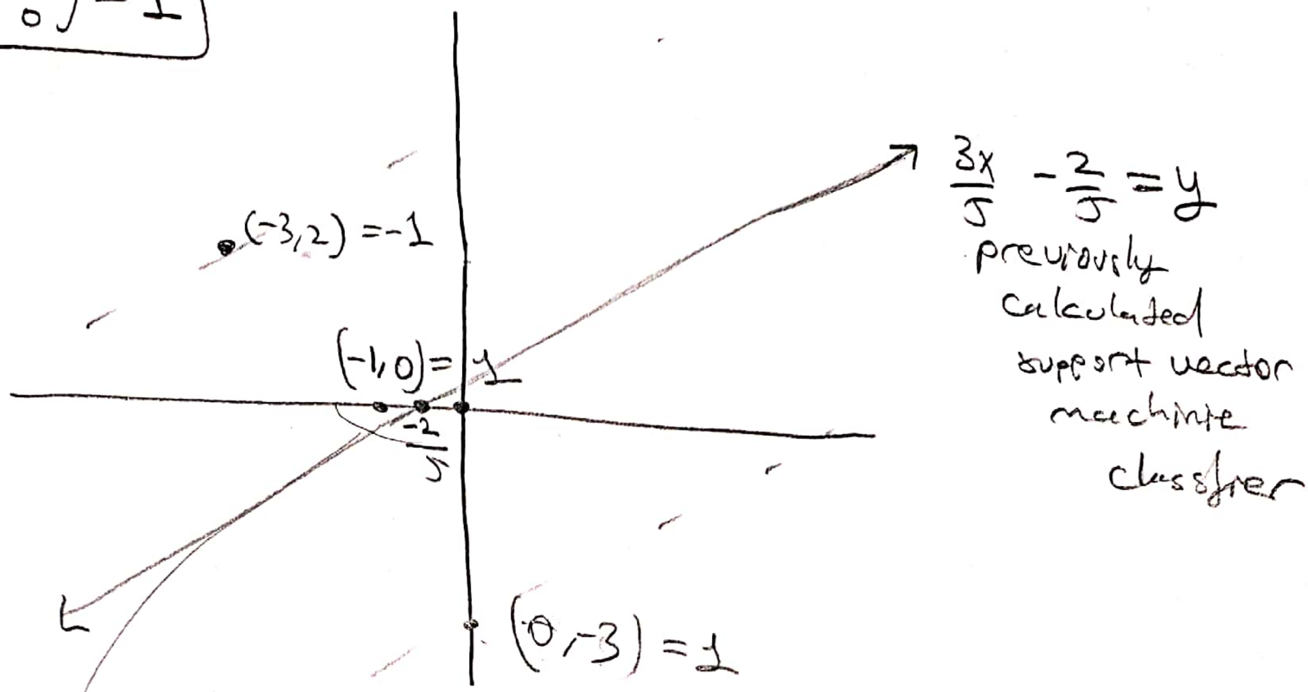
Upper bound : $\boxed{y = \frac{3}{9}x + \frac{19}{5}}$ $(-3,2) \rightarrow$ support vector 1

Lower bound

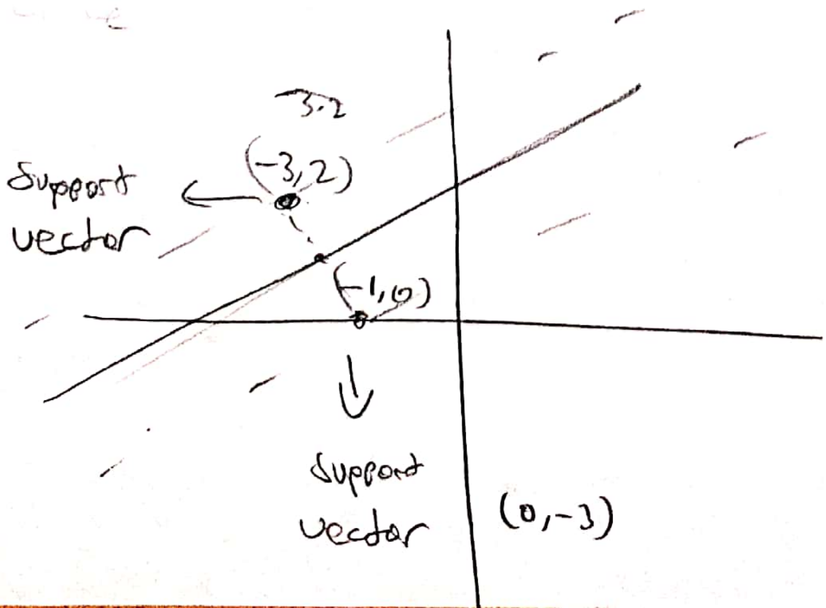
$\boxed{y = \frac{3x}{5} - 3}$ $(0-3) \rightarrow$ support vector 2

② Let's draw

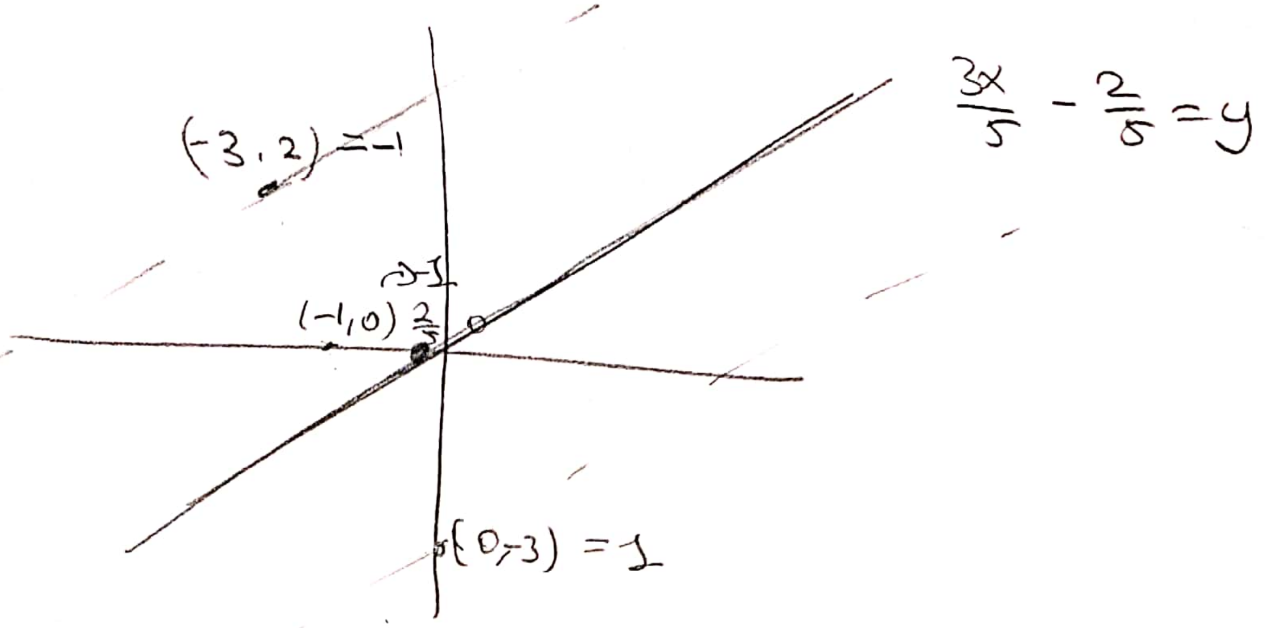
for $\begin{pmatrix}-1\\0\end{pmatrix} = 1$



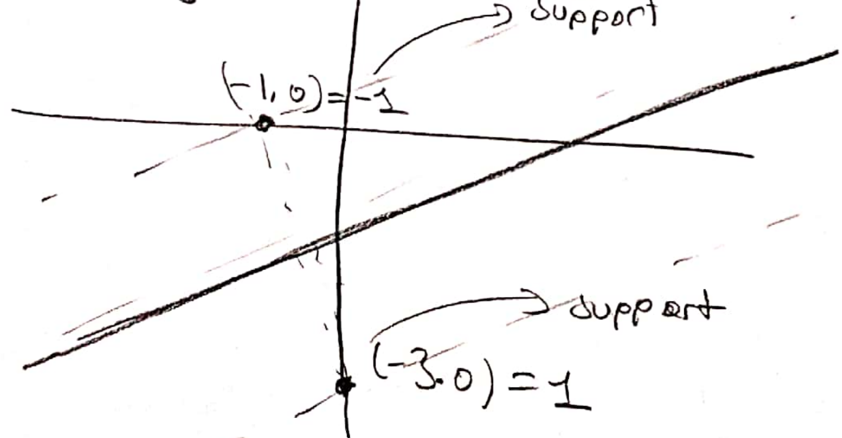
→ As we can see It will change our SVM and It is going to be a support vector for new SVM.



for $\begin{pmatrix} -1 \\ 0 \end{pmatrix} = -1$



In this situtation. It is near to decision line thus it will become support vector of new SVM.



(3)

we have found upper bound as $y = \frac{3x}{5} + \frac{19}{5}$

and Lower bound as $y = \frac{3x}{5} - 3$

So let's test the $(1,-4)$

$$-4 \overset{?}{=} \frac{3}{5} + \frac{19}{5} \Rightarrow -4 \overset{?}{=} \frac{22}{5} \quad \boxed{-4 < \frac{22}{5}}$$

point $(1,-4)$ is lower side of upper bound.

$$-4 \overset{?}{=} \frac{3}{5} - 3 \Rightarrow \boxed{-4 < \frac{-12}{5}}.$$

point $(1,-4)$ is lower side of lower bound

More general way solution so It belongs to $\boxed{y=1}$

$$\tilde{x} = \begin{bmatrix} 1 \\ 1 \\ -4 \end{bmatrix} = \begin{bmatrix} 1 \\ x \end{bmatrix} \quad \tilde{w} = \begin{bmatrix} \frac{2}{17} \\ \frac{3}{17} \\ \frac{-5}{17} \end{bmatrix} = \begin{bmatrix} b \\ w \end{bmatrix}$$

$$\tilde{x}^T \tilde{w} = \begin{bmatrix} 1 & 1 & -4 \end{bmatrix} \begin{bmatrix} \frac{2}{17} \\ \frac{3}{17} \\ \frac{-5}{17} \end{bmatrix} = \frac{2}{17} + \frac{3}{17} + \frac{20}{17} = \frac{25}{17}$$

$$\cong 1.47 > 1$$

Therefore It is $\boxed{\text{class } y=1}$