Calculus I - Quiz 1 Name: Solutions

(All work must be shown clearly to get full credit. Calculators are not allowed in this quiz.)

1.[5 pts] Find the right-hand slope of the function $\sqrt{x^2+6x+11}$ at $x=1$.

2.[5 pts] Let $f(x) = \frac{x^2+2x-8}{x-2}$. Is $f(x)$ continuous everywhere ? If not, can the discontinuity be removed ?

1

$$\lim_{x \to 1^+} \frac{f(x)-f(1)}{x-1} = \lim_{h \to 0^+} \frac{f(1+h)-f(1)}{h}$$

$$= \lim_{h \to 0^+} \frac{\sqrt{(1+h)^2+6(1+h)+11} - \sqrt{1+6+11}}{h}$$

$$= \lim_{h \to 0^+} \frac{\sqrt{h^2+6h+18} - \sqrt{18}}{h}$$

$$= \lim_{h \to 0^+} \frac{h^2+6h}{h\left(\sqrt{h^2+6h+18}+\sqrt{18}\right)}$$ (by multiplying and dividing by the conjugate)

$$= \lim_{h \to 0^+} \frac{h+6}{\sqrt{h^2+6h+18}+\sqrt{18}}$$

$$= \frac{6}{2\sqrt{18}} = \frac{1}{\sqrt{2}}$$

(2) Note that $f(x) = \dfrac{x^2+2x-8}{x-2}$ is valid everywhere except at 2

So $f(x) = \dfrac{(x-2)(x+4)}{(x-2)} = x+4$ at $\mathbb{R} - \{2\}$

Also note that $\lim_{x \to 2} x+4 = 6$

Hence the discontinuity can be removed by re-defining the $f(2) = 6$.