Math 165 Midterm Exam Section:_________ Name:________________

Part A: Pencil & Paper Only

This part of the exam has 10 problems on 3 pages. Each problem is worth 5 points. You may NOT use a calculator on this section. You must show all work. This part of the exam will be collected after 45 minutes.

1. Find dy/dx if
$$y = \frac{3x^3}{5x^2+2}.$$
You do not need to simplify your answer.

Solution:
$$\frac{dy}{dx} = \frac{(9x^2)(5x^2+2) - (3x^3)(10x)}{(5x^2+2)^2}$$

2. Suppose
$$g(x) = \frac{2}{x^2}.$$
Find $g''(2)$. Simplify your answer.

Solution:
$$g'(x) = -\frac{4}{x^3} \qquad g''(x) = \frac{12}{x^4} \qquad g''(2) = \frac{3}{4}$$

3. If
$$y = t^3 \ln t$$
find dy/dt. Simplify your answer.

Solution:
$$\begin{aligned}\frac{dy}{dx}(t^3 \ln t) &= 3t^2 \ln t + t^3 \frac{1}{t} \\ &= 3t^2 \ln t + t^2 \\ &= t^2(3 \ln t + 1)\end{aligned}$$

4. Find

$$\lim_{x \to 1^-} \frac{x^2 + 4x + 3}{x^2 - 1}$$

Solution:

$$\lim_{x \to 1^-} \frac{x^2 + 4x + 3}{x^2 - 1} = \lim_{x \to 1^-} \frac{(x+1)(x+3)}{(x+1)(x-1)} = \lim_{x \to 1^-} \frac{x+3}{x-1} = -\infty$$

5. Let

$$y = \frac{3x^2 - 5x - 2}{x^2 - 4}.$$

Give the equations of all the horizontal asymptotes.

Solution:

$$\lim_{x \to \infty} \frac{3x^2 - 5x - 2}{x^2 - 4} = \lim_{x \to \infty} \frac{3 - 5/x - 2/x^2}{1 - 4/x^2} = \frac{3 - 0 - 0}{1 - 0} = 3$$

Therefore $y = 3$ is the horizontal asymptote.

6. Let

$$y = \frac{3x^2 - 5x - 2}{x^2 - 4}$$

Give the equations of all the vertical asymptotes.

Solution:

$$y = \frac{3x^2 - 5x - 2}{x^2 - 4} = \frac{(x-2)(3x+1)}{(x-2)(x+2)} = \frac{3x+1}{x+2}$$

The function is not defined when the denominator is 0, therefore, $x = -2$ is the vertical asymptote.

7. Find the derivative of $4x^3 \tan^2(x)$. You do not need to simplify your answer.

Solution:

$$\begin{aligned}\frac{dy}{dx}(4x^3 \tan^2(x)) &= 12x^2 \tan^2(x) + 4x^3 \frac{dy}{dx} \tan^2(x) \\ &= 12x^2 \tan^2(x) + 4x^3 (2 \tan(x) \sec^2(x))\end{aligned}$$

8. Find the slope of the line tangent to the graph of $y = \sqrt{x^2 + 4x - 5}$ at $x = 3$. Simplify your answer.

Solution:

$$y' = f'(x) = (1/2)(x^2 + 4x - 5)^{-1/2}(2x + 4),$$

so the slope at $x = 3$ is

$$f'(3) = (1/2)(3^2 + 4(3) - 5)^{-1/2}(2(3) + 4) = \frac{10}{2\sqrt{16}} = \frac{5}{4}.$$

9. Suppose $f(x) = \sin^4(3e^x + 1)$. Find $f'(x)$. You do not need to simplify your answer.

Solution:

$$\begin{aligned}f'(x) &= 4 \sin^3(3e^x + 1) \frac{dy}{dx} \sin(3e^x + 1) \\ &= 4 \sin^3(3e^x + 1) \cos(3e^x + 1) \frac{dy}{dx}(3e^x + 1) \\ &= 4 \sin^3(3e^x + 1) \cos(3e^x + 1) 3e^x\end{aligned}$$

10. Let $f(x) = x^3 + 2x^2 + 1$ for $x \geq 0$. Using that $f(1) = 4$, find $(f^{-1})'(4)$.

Solution:

$$\begin{aligned}(f^{-1})'(y) &= \frac{1}{f'(f^{-1}(y))} \\ &= \frac{1}{3(f^{-1}(y))^2 + 4f^{-1}(y)}\end{aligned}$$

Since $f^{-1}(4) = 1$,

$$(f^{-1})'(4) = \frac{1}{3 + 4}. = \frac{1}{7}$$