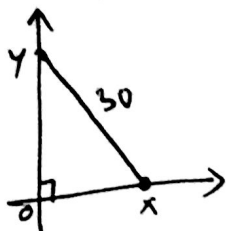


(All work must be shown clearly to get full credit. Calculators are *not* allowed in this quiz.)

1. [10 pts] The top of a ladder 30 feet long leans against a vertical wall and the lower end rests on a level pavement. The ladder begins to slide outwards so that the lower end is being moved away from the wall at a rate equal to twice its distance from the wall. How fast is the top sliding downwards at the instant when the lower end is 10 feet away from the wall. How far is the lower end from the wall when it and the top are moving at the same rate?



Given  $\frac{dx}{dt} = 2x$ .

(a) To find  $\frac{dy}{dt}$  at  $x=10$

We know  $x^2 + y^2 = 30^2$

$$\Rightarrow 2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 0 \quad \text{--- (1)}$$

$$\Rightarrow \frac{dy}{dt} = \frac{-2x \left(\frac{dx}{dt}\right)}{2y} = \frac{-2x \cdot 2x}{2y} = -2 \frac{x^2}{y}$$

Now at  $x=10$ ,  $y = \sqrt{900 - 100} = \sqrt{800}$

$$\therefore \frac{dy}{dt} = 2 \cdot \frac{10^2}{\sqrt{800}} = \frac{20}{\sqrt{8}} = \frac{10}{\sqrt{2}}$$

(b) Find  $x$  when  $\left|\frac{dx}{dt}\right| = \left|\frac{dy}{dt}\right|$

by equation (1) we get  $x = y$ .

$$\Rightarrow x^2 + x^2 = 900 \Rightarrow x = \sqrt{450}$$