Calculus I - Quiz 1 Name: Solutions

(All work must be shown clearly to get full credit. Calculators are not allowed in this quiz.)

1.[5 pts] Evaluate $\lim_{x\to 0} \dfrac{1-\cos x}{x^2}$.

2.[5 pts] Consider the function $f(x) = \dfrac{x(x^2-1)}{|x^2-1|}$. Is $f(x)$ continuous everywhere ? If not, can the discontinuities be removed ?

(1) $\dfrac{1-\cos x}{x^2} = \dfrac{1-\cos x}{x^2}\cdot\dfrac{(1+\cos x)}{(1+\cos x)} = \dfrac{\sin^2 x}{x^2}\cdot\dfrac{1}{(1+\cos x)}$

$\therefore \lim_{x\to 0} \left(\dfrac{\sin x}{x}\right)^2 \cdot \dfrac{1}{1+\cos x} = 1^2 \cdot \dfrac{1}{1+1} = \dfrac{1}{2}$

OR

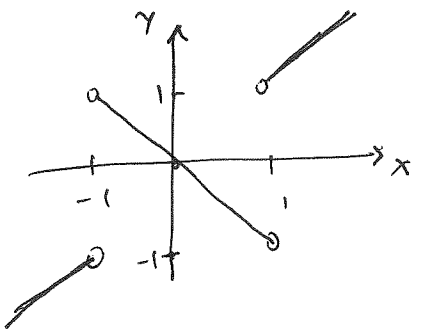
(1) $\dfrac{1-\cos x}{x^2} = \dfrac{2\sin^2\left(\frac{x}{2}\right)}{x^2} = \left[\dfrac{\sin\left(\frac{x}{2}\right)}{\left(\frac{x}{2}\right)}\right]^2 \cdot \dfrac{1}{2} \xrightarrow{x\to 0} 1^2 \cdot \dfrac{1}{2} = 1/2$

(2) ~~$f(x)$~~ = Note $|x^2-1| \begin{cases} x^2-1 & \text{if } x \geq 1 \text{ or } x \leq -1 \\ -(x^2-1) & \text{if } -1 < x < 1 \end{cases}$

$\therefore f(x) = x$ if $x \geq 1$ or $x \leq -1$

$= -x$ if ~~$-1 < x < 1$~~

= undefined at $x = 1$ and -1



The discontinuities cannot be removed because

$\lim_{x\to 1^+} f(x) = 1 \neq -1 = \lim_{x\to 1^-} f(x)$