PRACTICE WITH RELATED RATES

The following problems are adapted from Paul's Online Course Notes: http://tutorial.math.lamar.edu /Classes/CalcI/RelatedRates.aspx. You can just google "related rates problems," and this should be the first page to pop up.

Example Air is being pumped into a spherical balloon at a rate of $5\ \text{cm}^3/\text{min}$. Determine the rate at which the radius of the balloon is increasing when the diameter of the balloon is 20 cm.

Solution Recall that when we did implicit differentiation, we applied $\frac{d}{dx}$ to expressions of y's and x's. With these problems, the idea is to *relate rates* by differentiating multiple variables with respect to time (so the operator we'll employ is $\frac{d}{dt}$). In other words, we want to use what we know about the **rate of change in volume** to find the **rate of change of the radius** at a specific point in time.

How can we relate the volume of a sphere to it's radius?

$V(t) =$

Let's implicitly differentiate:

$\frac{dV}{dt} =$

Now, using the components of our two expressions, let's list our known quantities and unknown quantities from the problem statement .

To finish the problem, simply plug your knowns into the expression of $\frac{dV}{dt}$, and solve for $\frac{dr}{dt}$:

Problem 1 A 15 foot ladder is resting against the wall. The bottom is initially 10 feet away from the wall and is being pushed towards the wall at a rate of $\frac{1}{4}$ ft/sec. How fast is the top of the ladder moving up the wall 12 seconds after we start pushing?

Problem 2 Two people are 50 feet apart. One of them starts walking north at a rate so that the angle shown in the diagram below is changing at a constant rate of 0.01 rad/min. At what rate is distance between the two people changing when $\theta = 0.5$ radians?