$$R_{d^1,d^2,\ldots,d^p} = \Pr\left\{ \bigcup_{\text{all } (d^1,d^2,\ldots,d^p)\text{-MP } X} \{Y \in \Omega \mid Y \geq X\} \right\} = \Pr\{B_1 \cup B_2 \cup \cdots \cup B_r\}.$$

Hence, the multicommodity reliability can be calculated by applying the inclusion-exclusion method [8–11]. Note that $\Pr\{Y \geq X\} = \Pr\{y_1 \geq x_1\} \times \Pr\{y_2 \geq x_2\} \times \cdots \times \Pr\{y_n \geq x_n\}$ by Assumption 4.

4. NUMERICAL EXAMPLES

Two numerical examples are presented to illustrate the proposed procedure.

4.1. Example 1

The network and its arc-data are given in Figure 2 and Table 1, respectively. In this example, $p = 2$, $n = 4$, and $(C_1, C_2, C_3, C_4) = (2, 3, 3, 2)$. All (d^1, d^2)-MPs for $(d^1, d^2) = (2, 2)$ and the multicommodity reliability $R_{2,2}$ can be derived as follows.

STEP 1. There are four MPs: $P_1 = \{a_1, a_3\}$, $P_2 = \{a_1, a_4\}$, $P_3 = \{a_2, a_3\}$, $P_2 = \{a_2, a_4\}$.

STEP 2. Generate all flow vectors $F = (f_1^1, f_2^1, f_3^1, f_4^1, f_1^2, f_2^2, f_3^2, f_4^2)$ of the following integer-programming by applying the implicit enumeration method:

$$\begin{aligned} &\lceil f_1^1 + f_2^1 + f_1^2 + f_2^2 \rceil \leq 2, \\ &\lceil f_3^1 + f_4^1 + 1.5 f_3^2 + 1.5 f_4^2 \rceil \leq 3, \\ &\lceil f_1^1 + f_3^1 + 2 f_1^2 + 2 f_3^2 \rceil \leq 3, \\ &\lceil f_2^1 + f_4^1 + 0.5 f_2^1 + 0.5 f_4^1 \rceil \leq 2, \end{aligned} \tag{2}$$

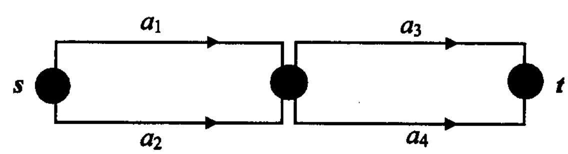

Figure 2. A simple network for Example 1.

Table 1. The arc data of Example 1.

Arc	Capacity	Probability	α_i^1	α_i^2
a_1	0	0.05	1	1
	1	0.10		
	2	0.85		
a_2	0	0.05	1	1.5
	1	0.10		
	2	0.15		
	3	0.70		
a_3	0	0.05	1	2
	1	0.05		
	2	0.10		
	3	0.80		
a_4	0	0.05	1	0.5
	1	0.15		
	2	0.80		