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# Image retrieval based on fuzzy color histogram processing

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**Abstract**

Content-based image retrieval (CBIR) is a collection of techniques for retrieving images on the basis of features, such as color, texture and shape. An efficient tool, which is widely used in CBIR, is that of color image histograms. The classic method of color histogram creation results in very large histograms with large variations between neighboring bins. Thus, small changes in the image might result in great changes in the histogram. Moreover, the fact that each color space consists of three components leads to 3-dimensional histograms. Manipulating and comparing 3D histograms is a complicated and computationally expensive procedure. The need, therefore, for reduction of the three dimensions to one could lead to efficient approaches. This procedure of projecting the 3D histogram onto one single-dimension histogram is called histogram linking. In this paper, a new fuzzy linking method of color histogram creation is proposed based on the $L^*a^*b^*$ color space and provides a histogram which contains only 10 bins. The histogram creation method in hand was assessed based on the performances achieved in retrieving similar images from a widely diverse image collection. The experimental results prove that the proposed method is less sensitive to various changes in the images (such as lighting variations, occlusions and noise) than other methods of histogram creation.

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## 1. Introduction

Nowadays and in the days to come, a large number of applications, including military, industrial and civilian generate, and will continue to generate, even more gigabytes of color images per day. As a result, there is a huge amount of

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information which cannot be accessed or made use of unless it is organized [1]. Organization here means that appropriate indexing is available in order to allow efficient browsing, searching and retrieving as in keyword searches of text databases. The easiest way to search is with the use of query by example, which means that the user has to present an image to the system and the latter searches for others alike by extracting features from the query image and comparing them to the ones stored in the database. The extraction of meaningful features is critical in CBIR (content-based image retrieval) and therefore an open and active field of research [2,3]. Of the features usually employed by researchers are color [4–10], texture [4,5,11] and shape [4,5,12]. Among the methods that utilize color as a retrieval feature, the most popular one is probably that of color histograms [6,7]. The classic histogram is a global statistical feature, which describes the intensity distribution for a given image [13]. Its main advantage is that it is fast to manipulate, store and compare and insensitive to rotation and scale. On the other hand, it is also quite unreliable as it is sensitive to even small changes in the scene of the image. In color image processing, the histogram consists of three components, respect to the three components of the color space.

A histogram-based retrieval system requires the following components in order to work: A suitable color space such as HSV, $L^*a^*b^*$ or $L^*u^*v^*$, a histogram representation such as classic [6,7] or joint [8] histograms, and a similarity metric like the L2 (Euclidean Distance), the Matusita distance [14] or the Histogram Intersection method [9]. A histogram is created by firstly dividing a color space into a number of bins and then by counting the number of pixels of the image that belong to each bin. It is usually thought that in order for an image retrieval system to perform satisfyingly, the number of regions that the color space is divided into is quite large and thus the colors represented by neighboring regions have relatively small differences. As a result, the “perceptually similar colors” problem appears [10], ergo, images which are similar to each other but have small differences in scene or contain noise will produce histograms with dissimilar adjacent bins and vice versa due to the small distance that the regions are separated from each other. In order to present a solution to this problem, the method proposed uses a small number of bins produced by linking the triplet from the $L^*a^*b^*$ color space into a single histogram by means of a fuzzy expert system. The $a^*$ and $b^*$ components are considered to have more weight than $L^*$ as it is mostly the combination of the two which provides the color information of an image. The Mathworks’ Matlab fuzzy logic toolbox was used to assess a Mamdani style inference system and hence the fuzzy color histogram creation system in hand. The main reason why fuzzy logic [3,7,11,15,16] was selected is that since it was first introduced by Zadeh [15] in 1965 it has proven effective in many applications such as automatic control and image understanding due to its flexibility and tolerance to imprecise data. This is exactly the case when dealing with the triplet of $L^*a^*b^*$ as many different combinations will lead to approximately the same color. As one might notice in the experimental results, the proposed method performs much better than the straightforward implementation using the $L^*a^*b^*$ color space. Thus, its ability to incorporate the expertise we acquired by studying the color spaces easily and efficiently, was another reason to choose fuzzy logic. The main advantage of the proposed method is the retrieval accuracy and its main disadvantage is the retrieval time. However, taking into consideration that most existing CBIR systems create their feature databases beforehand, since the proposed method produces histograms with just 10 bins, then the similarity procedure in this case is significantly quicker.

In the length of this paper, the proposed fuzzy linking histogram creation method is described in Section 2, the experimental results and comparisons to previous methods are shown in Section 3 and finally the conclusions are expressed in Section 4.

## 2. The fuzzy linking histogram creation method

One of the reasons why the $L^*a^*b^*$ color space was selected is that it is a perceptually uniform color space which approximates the way that humans perceive color. However, the main reason is that $L^*a^*b^*$ was found to perform better than other color spaces in various retrieval tests performed in the laboratory for this exact purpose [6]. In $L^*a^*b^*$, $L^*$ stands for luminance, $a^*$ represents relative greenness-redness and $b^*$ represents relative blueness-yellowness. All colors and grey levels can be expressed throughout a combination of the three components. However, $L^*$ does not contribute in providing any unique color but for shades of colors, white, black and grey. Thus, the $L^*$ component receives a lower weight with respect to the other two components of the triplet.

After a large number of tests performed on the regions of the $L^*a^*b^*$ color space, we reached to the conclusion that in order for the CBIR system to work effectively the $a^*$ and $b^*$ components should be subdivided into five regions representing green, greenish, the middle of the component, reddish and red for $a^*$, blue, bluish, the middle of the component, yellowish and yellow for $b^*$, whereas $L^*$ should be subdivided into only three regions: dark dim and bright areas. The fuzzification of the input is accomplished by using triangular-shaped built-in membership functions (MF) for the three input components ($L^*$, $a^*$, $b^*$) which represent the regions as shown in Fig. 1. The reason for which the middle MF exists both in $a^*$ and $b^*$, is that in order to represent black, grey and white as seen in $L^*$, then $a^*$ and $b^*$ must be very close to the middle of their regions; this is a well-known fact about the $L^*a^*b^*$ space [13].

The Mamdani type of fuzzy inference is used in which the fuzzy sets from the output MFs of each rule are combined through the aggregation operator which is set to max and the resulting fuzzy set is defuzzified to produce the output of the system. The implication factor which determines the process of shaping the fuzzy set in the output MFs based on the results of the input MFs is set to min and the OR and AND operators are set to max and min, respectively. The output of the system has only 10 equally divided MFs, as shown in Fig. 2. So, the final fuzzy histogram consists of only 10 bins approximately representing black, dark grey, red, brown, yellow, green, blue, cyan, magenta and white. The defuzzification phase is performed using the lom (largest of maximum) method along with the 10 trapezoidal MFs, thus producing 2500 clustered bin values ($50 \times 50$) which lead to the 10 bin final fuzzy histogram.

The fuzzy linking of the three components is made according to 27 fuzzy rules (Appendix A), which leads to the output of the system. The rules were established through empirical conclusion which arose through thorough examination of the properties of a series of colors and images in the $L^*a^*b^*$ color space.

In Figs. 3–5, three query images and the respective resulting fuzzy histograms are presented. These images were chosen as they are characteristic of the database used, since there are several images in the database having similar semantic content (i.e., landscapes, sunsets, cats, etc.) and are taken by using different setups, from several different angles and distances, and under varying lighting conditions, making it difficult for most simple systems to retrieve them.

The bins of the histograms shown in Figs. 3–5 are in respect to: (1) black, (2) dark grey, (3) red, (4) brown, (5) yellow, (6) green, (7) blue, (8) cyan, (9) magenta and (10) white. One can easily notice the dominant colors in each of the images. In the first image, bins 1, 6 and 7 are mostly activated because of the black shadows, the grass and the sky, respectively. In the second image, bin 1 is activated because of the dark portion in the low part of the image, bin 3 because of the red sky and bin 7 due to the large portions of blue scattered in the image. Last, in the third image, bin 3 is activated because of the red chair, brown because of the brown cat and magenta because the marble in the background is not actually white but a tone between blue, white and pink. The histograms in the proposed scheme, though apparently rough, have proved to be an efficient tool for accurate image retrieval, as presented in the succeeding section.

## 3. Experimental results

The proposed method of histogram creation was compared with Liang's et al. [7], Swain's and Ballard's [9], Tico's et al. [10] and the straightforward histogram creation method. Their performance was compared through the color based

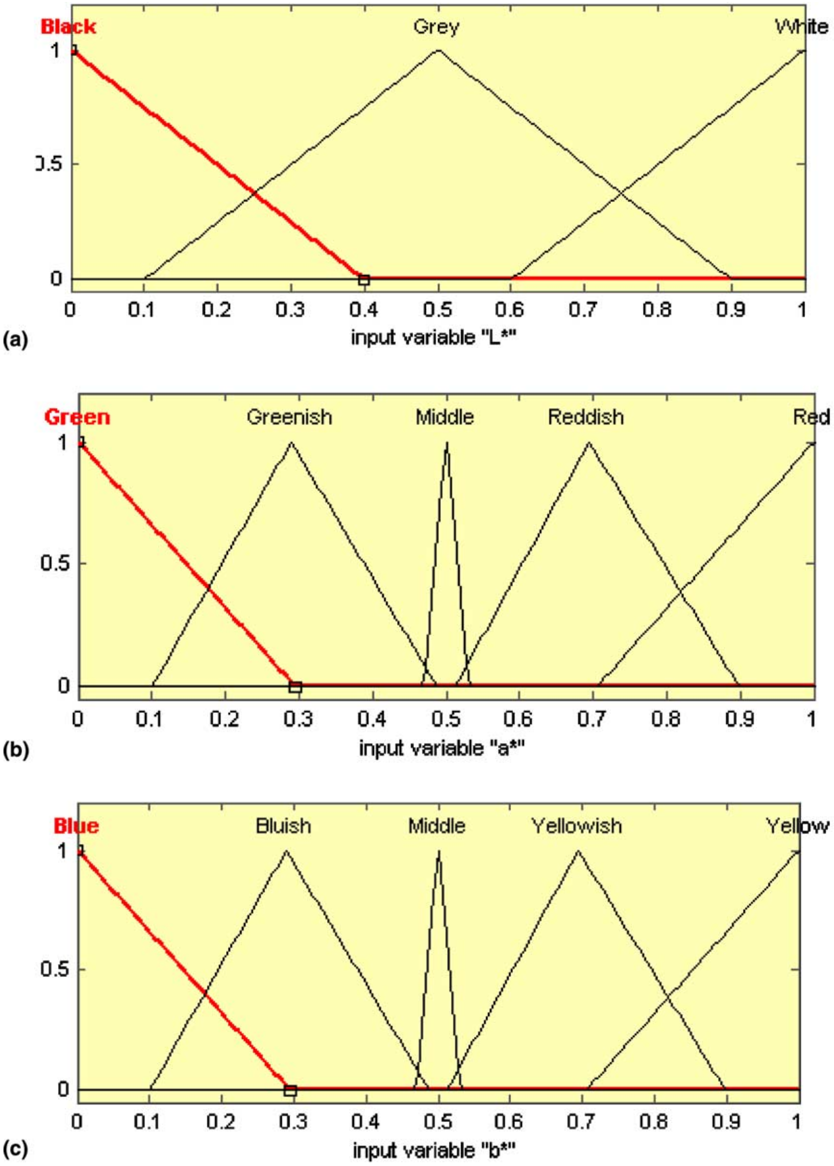

Fig. 1. Membership functions of $L^*$, $a^*$ and $b^*$.

retrieval of images 1, 2 and 3 from a collection of 1000 images, some selected from different sites on the internet, others scanned from personal photographs and a large amount of images taken with several different digital cameras. The images are online, available at the following URL: http://-utopia.duth.gr/~konkonst. The images in the collection are representative for the general requirements of an image retrieval system over the internet. The range of topics present in the image database is quite wide and varies from several different landscapes to sports, concerts and other computer graphics which usually confuse image retrieval systems. The similarity metric used is that of histogram intersection, introduced by Swain and Ballard [9] and which is very robust in respect

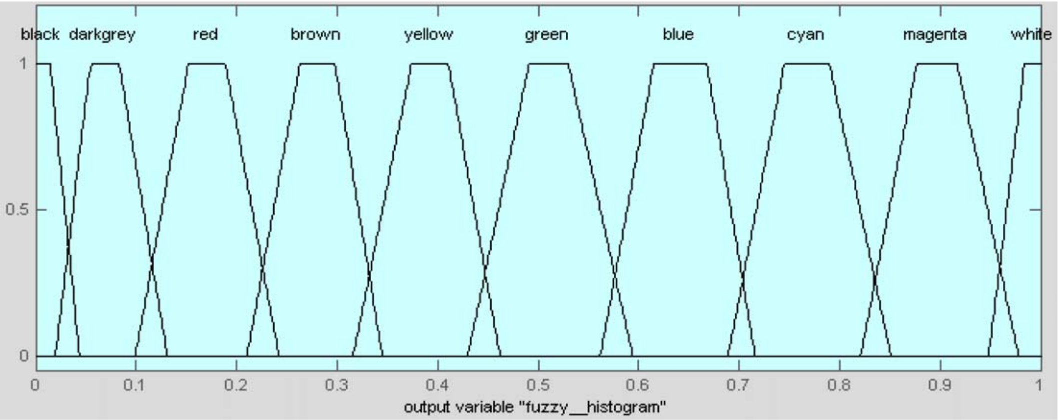

Fig. 2. Membership functions of the output of the fuzzy system.

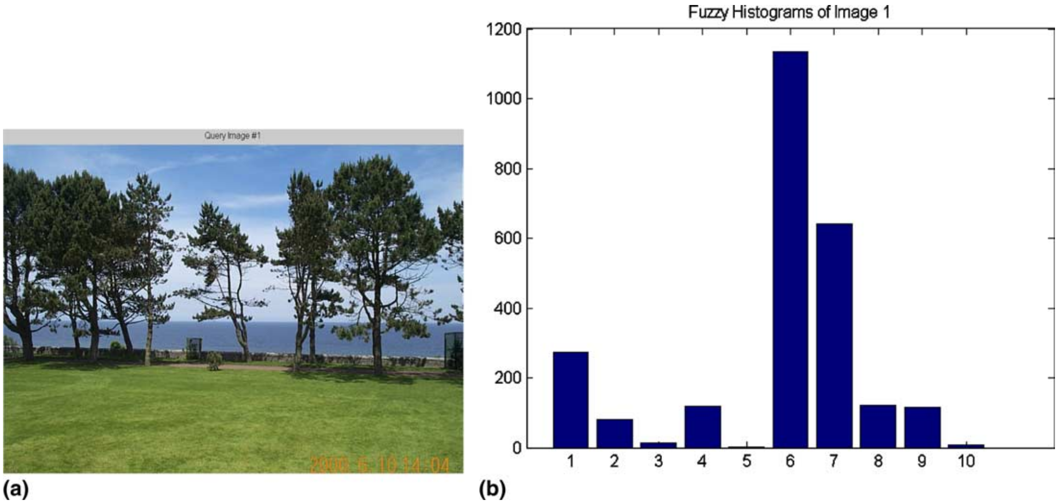

Fig. 3. (a) Query image 1 and (b) its fuzzy linked histogram.

to changes in image resolution, histogram size, occlusion, depth, and viewing point. The similarity ratio which belongs to the interval [0,1] can be expressed through the following equation:

$$H(H_\mathrm{Q}, H_\mathrm{C}) = \frac{\sum_{i=1}^{N} \min\left(H_\mathrm{Q}(i), H_\mathrm{C}(i)\right)}{\min\left(\sum_{i=1}^{N} H_\mathrm{Q}(i), \sum_{i=1}^{N} H_\mathrm{C}(i)\right)}, \qquad (1)$$

where $H_\mathrm{Q}$ and $H_\mathrm{C}$ are the query and challenging histograms, respectively, and $N$ is the number of bins.

Despite the fact that their paper was published in 1991, Histogram Intersection still remains one of the best methods for comparing histograms in image retrieval applications.

The experiments were all run on Mathworks' Matlab on an AMD 2400+processor with 1 GB of RAM. All the images were scaled to a $50 \times 50$ pixel size using the nearest neighbor interpolation method in order to make the algorithms faster and to avoid later normalization of the histograms, which might result in loss of color quantity information. For example, in images 2 and 3 the distribution of red (bin 3) is strong, but there is also a difference of 100 pixels between them. In

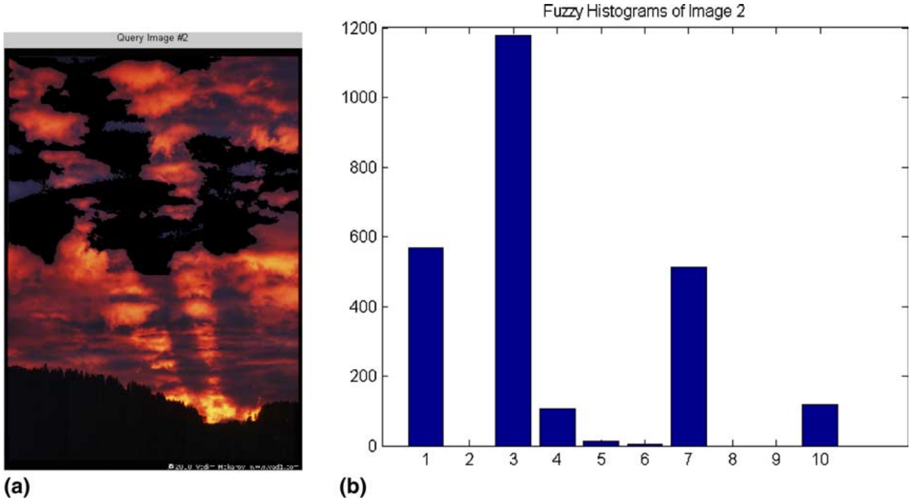

Fig. 4. (a) Query image 2 and (b) its fuzzy linked histogram.

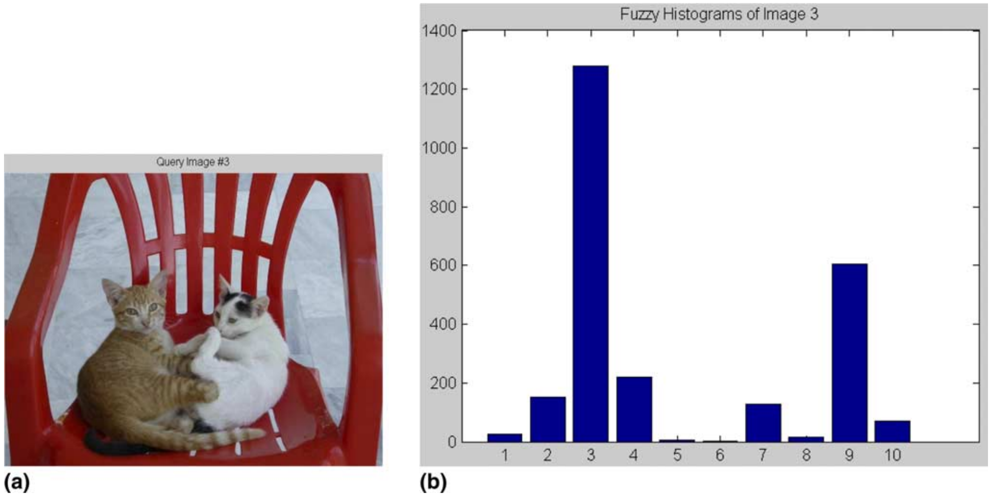

Fig. 5. (a) Query image 3 and (b) its fuzzy linked histogram.

the case normalization was adopted (division by the largest bin) both bins would be set to 1 resulting in loss of quantity information.

Tico et al. [10] work on a different color space, that of HSI and from its three components they select only H and I, from which to obtain the histogram. This histogram consists of 20 bins, 16 for hue and 4 for intensity. In addition, Tico's et al. method was also implemented using the $L^*a^*b^*$ color space instead for the HSI for testing and comparison purposes.

Using Swain and Ballard's approach of histogram creation [9], a histogram was created from the opponent color space (derived from RGB via linear combination of R, G and B) by subdividing the three derived components rg, by and wb into 8, 8 and 4 sections, respectively, resulting in a 256 bin histogram.

Furthermore, Liang's et al. [7] system was implemented. This system uses crisp values produced by a Gaussian MF to characterize a similarity fuzzy set centered at a given color vector in the RGB color space. Simple histograms are created and are compared using the previously mentioned Gaussian MF. A match or mismatch of the colors (bins) is defined by an $\alpha$-cut defuzzification process. In a few words, if the value produced by the MFs proves to be above $\alpha$, then is accepted, if not it is rejected. Next, in order to save calculations, they introduce a fuzzy search procedure, in which a number of the largest histogram peaks is selected by the user to express all the colors that are empirically determined as being characteristic of the image class considered, and only those are compared by use of the Gaussian MF as mentioned beforehand. However, even though the authors suggest RGB as the suitable color space to be used in their method, following a series of tests it was found that it performed much better when implemented by using the HSV color space (Table 1).

The first set of images similar to image 1 consists of 17 landscape images dominated mostly by light blue, light green and black. It is a tricky image as the human mind interprets most leaves of the trees to be green when they are truly dominated by black. The second one consists of four dark red sky images dominated by red, black and blue. Again, a big percentage of the image is blue, but the human sight fails as it is very dark. The third set consists of 20 images mainly dominated by red, very light cyan (almost light magenta) and brown. One should also notice that the first and third sets are quite bright and the second one is very dark.

A query session produces 20 images (the figures of the image sets 1 and 3 are shown in Appendix B) ranked in similarity according to the value produced by the metric. The smaller the number that belongs to the image is, the higher the similarity of that specific image. Based on the diversity which exists in the image database the ranking of the histogram intersection value can be considered as quite an objective criterion to compare the query image to a random image in the database. In the second and the third images, the concentration of red is very high; nonetheless the system does not confuse them due to the differences in the rest of the colors and the brightness in the two images proving, qualitatively though, its robustness.

Furthermore, three more tasks were executed to test the robustness of the systems on various images. For the first task salt and pepper of density 0.15 (an example is depicted in Fig. 6(b)) and random noise was inserted to the query image (Fig. 6(a)), then the brightness of the images was increased and decreased (example of an increase is shown in Fig. 6(c)), and last the images were blurred (Fig. 6(d)) by a filter which approximates, once convolved with the image, the linear motion of a camera. The filter becomes a vector for horizontal and vertical motions. Again the system that performed best was the proposed one. The accuracy percentages were decreased in the region of 5–10% in the tests but nonetheless most of the images were retrieved successfully demonstrating that the algorithm presented is robust to extreme changes in the images in controversy to the other methods which had losses in the region of 15–60%.

The retrieval performance measurement used in order to compare these five methods is the filtering performance percentage, which is the percentage of similar images produced in the 20 first most similar images retrieved by the system. On Table 1, one can see a synopsis of the comparisons of the five methods' performances for the three different image sets described above and notice the significant

Table 1
Filtering Performance achieved by the five methods. As one might notice the proposed method dominates the other four. This is due to the fact that the proposed method's fuzziness makes it even less sensitive to changes of scene, noise or illumination

| Image set | Swain and Ballard | Classic $L^*a^*b^*$ histogram | Tico (HSI) | Tico ($L^*a^*b^*$) | Liang (RGB) | Liang (HSI) | Fuzzy linking ($L^*a^*b^*$) |
|---|---|---|---|---|---|---|---|
| 1 | 80% | 75% | 75% | 55% | 70% | 90% | 95% |
| 2 | 70% | 75% | 70% | 75% | 50% | 67% | 85% |
| 3 | 90% | 90% | 75% | 90% | 65% | 80% | 95% |
| Time (s) | 511.8 | 226.95 | 303.52 | 367.53 | 221.73 | 318.99 | 525.54 |

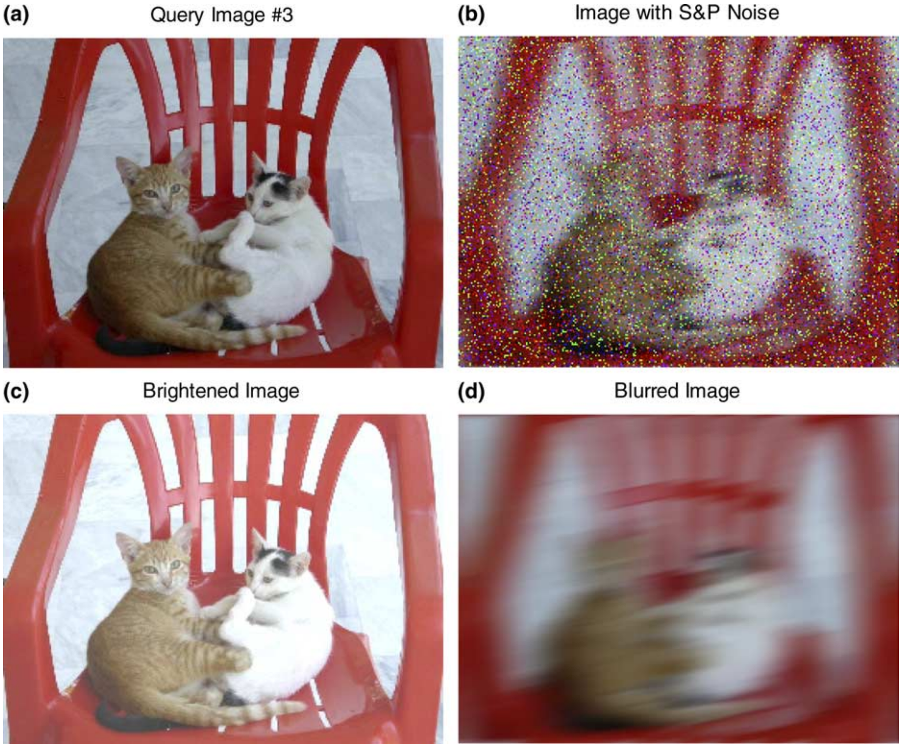

Fig. 6. Query image 3 (a) with (b) salt and pepper noise of density 0.15, brightened (c), and (d) blurred through a filter which approximates the linear motion of a camera by len 31 pixels, with an angle of theta 11° in a counterclockwise direction.

advantage which the proposed method holds above the rest. These image sets were selected to be presented in this paper as they are characteristic of the image database. The majority of methods performed satisfyingly for image set 1, they performed worst for image set 2 and best for image set 3. From the wide range of tests performed on various image sets, those three were selected because they present the rough width of accuracy percentage of the methods in hand. Swain and Ballard's method ranges from 70% to 90%, the classic $L^*a^*b^*$ histogram from 75 to 90, Tico's et al. using the HIS color space from 70 to 75, using $L^*a^*b^*$ from 55 to 90, Liang's et al. using the RGB color space from 50 to 70, using HIS from 67 to 90, and finally the proposed method spans from a low accuracy percentage of 85% to the highest 95% stating the clear advantage in accuracy of the proposed method over the rest. However, a drawback in the performance of the proposed method is the time it takes to execute due to the fuzzy logic method it includes. It requires 525 seconds in order to produce the desired images. Nonetheless, as mentioned in the introduction, when the feature database (color histograms) is produced beforehand, then the proposed method becomes the fastest of the methods as only 10 bins are used in order to check the similarity between the images, in which case the drawback becomes an advantage.

In addition to the accuracy Table, another aspect of retrieval performance is that shown in Fig. 7: precision versus recall [17]. Precision is the proportion of relevant images retrieved R (similar to the query image) in respect to the total retrieved A, whereas recall is the proportion of similar images retrieved in respect to the similar images that exist

$$\begin{aligned}\text{Precision} &= \text{Similar retrieved/Total retrieved}\\ &= |\mathrm{A} \cap \mathrm{R}|/\mathrm{A}. \qquad (2)\end{aligned}$$

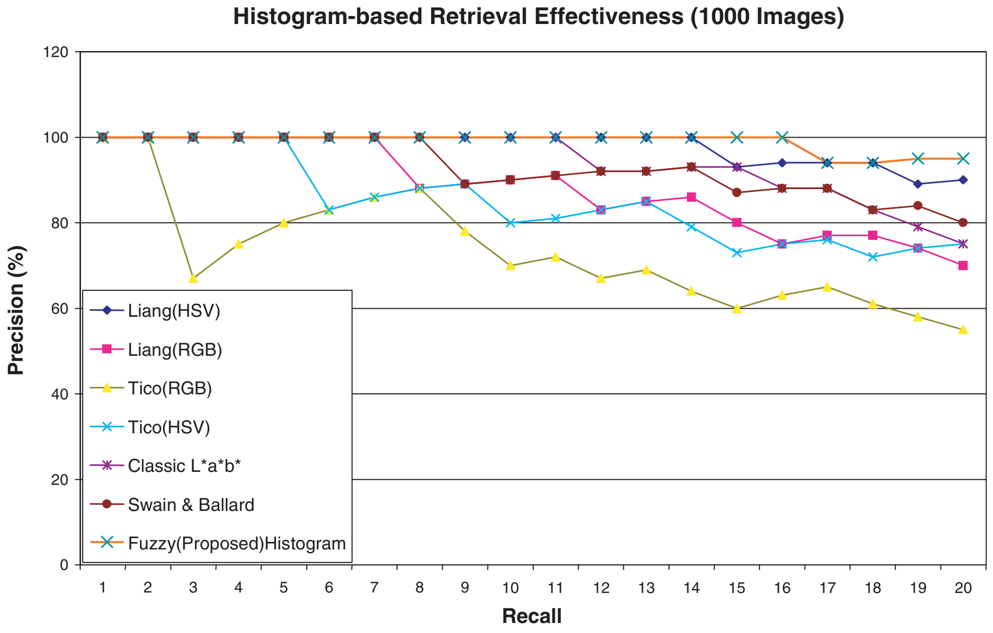

Fig. 7. Comparison of the five methods. Precision against recall for image set 1.

$$\text{Recall} = \text{Similar retrieved}/\text{Similar exist}$$
$$= |\mathrm{A} \cap \mathrm{R}|/\mathrm{R}. \quad (3)$$

Generally, precision and recall are used together in graphs in order to point out the change of the precision in respect to recall (i.e., the change in accuracy percentage each time an image is retrieved). In most typical systems, the precision drops as recall increases, thus an image retrieval system is said to be effective if the precision values are higher at the same recall ones, which is the case in the proposed method. For example, when retrieving the first image set the system only fails once after producing 16 correct images in comparison to the rest of the methods described above, which tend to produce false images earlier.

## 4. Conclusions

A new histogram creation method has been proposed. The histogram is created based on the $L^*a^*b^*$ color space components which are considered to be fuzzy sets. The proposed histogram is acquired through the linking of these fuzzy sets according to 27 fuzzy rules. Very few bins are used to describe the color distribution of the image resulting in much faster comparison between the histograms and greater robustness of the algorithm. The method in hand was compared to other histogram creation methods proving to be much more accurate and robust through several image retrieval tests. However this does not mean that it cannot be enhanced (e.g., insertion of spatial information).

Target applications of the proposed system include internet queries, retrieval of remote sensing images and related computer vision applications.

## Appendix A

Twenty-seven rules used to derive the final histogram

1. If ($L$ is black) and ($a$ is amiddle) and ($b$ is bmiddle) then (fuzzyhist is black) (1).
2. If ($L$ is white) and ($a$ is amiddle) and ($b$ is bmiddle) then (fuzzyhist is white) (1).
3. If ($L$ is Lmiddle) and ($a$ is red) and ($b$ is yellow) then (fuzzyhist is red) (1).
4. If ($a$ is reddish) and ($b$ is yellow) then (fuzzyhist is brown) (1).

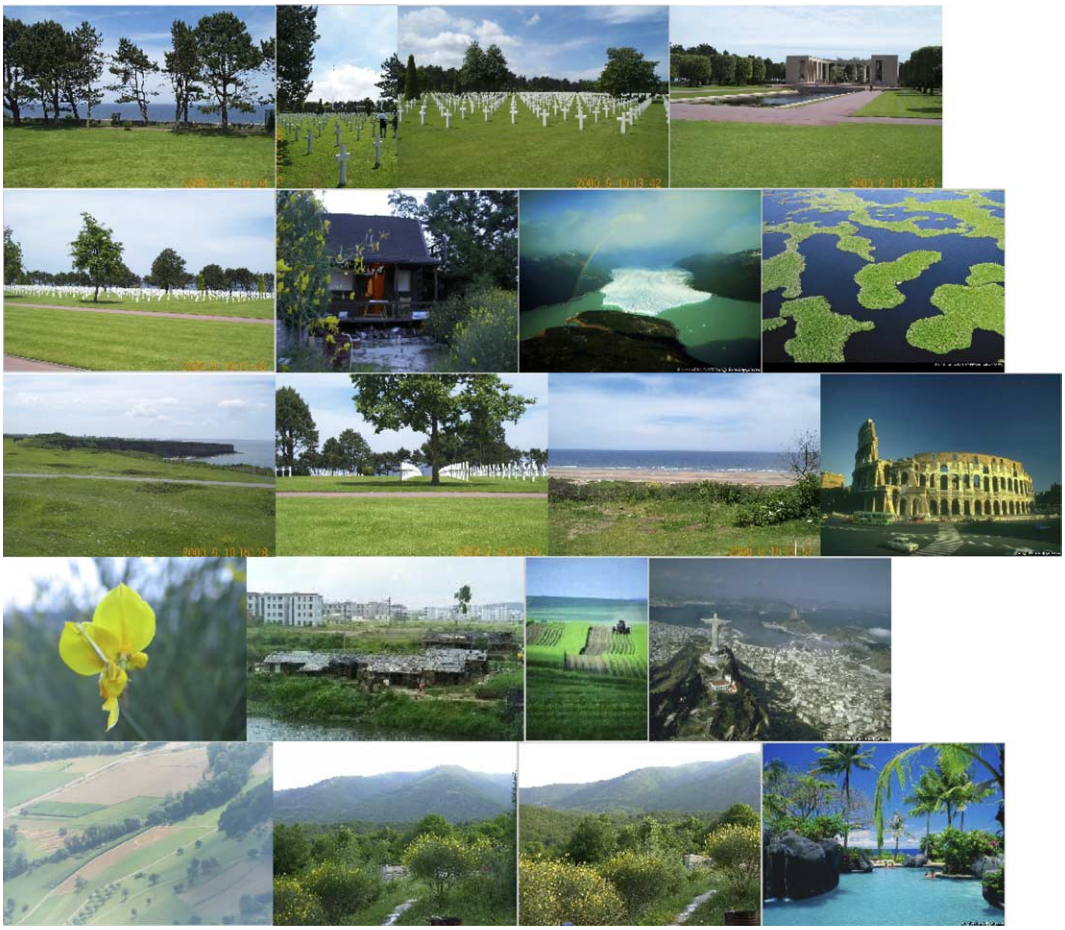

Fig. B.1. The 20 retrieved images from data set 1. Only one is wrongfully retrieved (first image in the fifth row). The first image is also the query image; the images are presented in descending score from left to right and from top to bottom.

5. If (*L* is white) and (*a* is green) and (*b* is yellow) then (fuzzyhist is green) (1).
6. If (*L* is white) and (*a* is green) and (*b* is yellowish) then (fuzzyhist is green) (1).
7. If (*L* is black) and (*b* is blue) then (fuzzyhist is blue) (1).
8. If (*L* is white) and (*a* is green) and (*b* is bluish) then (fuzzyhist is cyan) (1).
9. If (*L* is Lmiddle) and (*a* is amiddle) and (*b* is bmiddle) then (fuzzyhist is darkgrey) (1).
10. If (*a* is greenish) and (*b* is bluish) then (fuzzyhist is blue) (1).
11. If (*a* is red) and (*b* is bluish) then (fuzzyhist is blue) (1).
12. If (*L* is white) and (*b* is yellow)then (fuzzyhist is yellow) (1).
13. If (*L* is black) and (*a* is reddish) and (*b* is bluish) then (fuzzyhist is blue) (1).
14. If (*a* is red) and (*b* is blue) then (fuzzyhist is blue) (1).
15. If (*L* is Lmiddle) and (*a* is reddish) and (*b* is yellowish) then (fuzzyhist is red) (1).
16. If (*L* is white) and (*a* is reddish) and (*b* is yellowish) then (fuzzyhist is yellow) (1).
17. If (*L* is black) and (*a* is reddish) and (*b* is yellowish) then (fuzzyhist is red) (1).
18. If (*a* is reddish) and (*b* is yellow)then (fuzzyhist is yellow) (1).

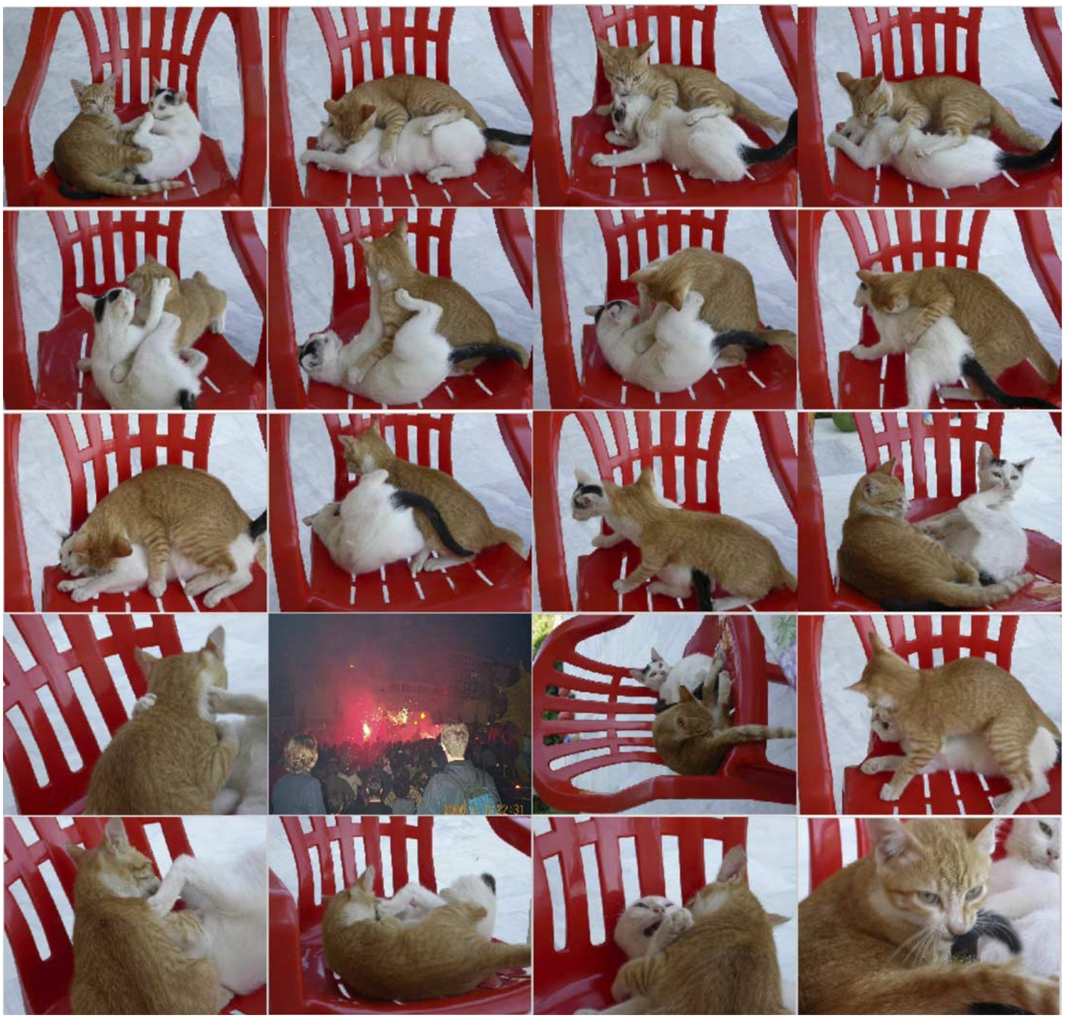

Fig. B.2. The 20 retrieved images from data set 3. Only one is wrongfully retrieved due to the large concentration of red that exists in the image (second image in the fourth row). (The first image is also the query image).

19. If (*L* is black) and (*b* is bluish) then (fuzzyhist is blue) (1).
20. If (*L* is Lmiddle) and (*b* is blue) then (fuzzyhist is blue) (1).
21. If (*L* is Lmiddle) and (*a* is reddish) and (*b* is bluish) then (fuzzyhist is magenta) (1).
22. If (*L* is Lmiddle) and (*a* is amiddle) and (*b* is bluish) then (fuzzyhist is cyan) (1).
23. If (*L* is Lmiddle) and (*a* is amiddle) and (*b* is yellowish) then (fuzzyhist is brown) (1).
24. If (*L* is white) and (*a* is amiddle) and (*b* is yellowish) then (fuzzyhist is yellow) (1).
25. If (*L* is Lmiddle) and (*a* is red) and (*b* is bmiddle) then (fuzzyhist is red) (1).
26. If (*L* is Lmiddle) and (*a* is reddish) and (*b* is bmiddle) then (fuzzyhist is red) (1).
27. If (*L* is white) and (*a* is reddish) and (*b* is bluish) then (fuzzyhist is magenta) (1).

## Appendix B

See Figs. B.1 and B.2.

## References

[1] I. Gagliardi, R. Schettini, New Rev. Hypermed. Multimed. 3 (1997) 201.

[2] A.W.M. Smeulders, M. Worring, S. Santini, A. Gupta, R. Jain, IEEE Trans. Pattern Anal. Mach. Intell. 22 (12) (2000) 1349.

[3] J.P. Eakins, Pattern Recog. 35 (2002) 3.

[4] V. Castelli, L.D. Bergman (Eds.), Image Databases: Search and Retrieval of Digital Imagery, Wiley, 2001.

[5] A. Del Bimbo, in: Visual Information Retrieval, Morgan Kaufman Publishers, San Francisco, CA, 1999.

[6] K. Konstantinidis, I. Andreadis, J. Comput. Methods Sci. Eng. (JCMSE), to appear.

[7] Y. Liang, H. Zhai, P. Chavel, Opt. Commun. 212 (2002) 247.

[8] G. Pass, R. Zabih, Multimed. Syst. 7 (3) (1999) 234.

[9] M.J. Swain, D.H. Ballard, Int. J. Comput. Vis. 7 (1991) 11.

[10] M. Tico, T. Haverinen, P. Kuosmanen, in: Proceedings of the Nordic Sig. Proc. Symposium (NORSIG'2000), Kolmarden, Sweden, 2000, pp. 157–160.

[11] A. Gasteratos, P. Zafeiridis, I. Andreadis, An Intelligent System for Aerial Image Retrieval and Classification, in: G.A. Vouros, T. Panayiotopoulos (Eds.), Lecture Notes in Artificial Intelligence, vol. 3025, Springer, Berlin, Heidelberg, 2004, pp. 63–71.

[12] A.K. Jain, A. Vailaya, Pattern Recog. 29 (1996) 1233.

[13] R.C. Gonzalez, R.E. Woods, Digital Image Processing, Addison-Wesley, Reading, MA, 1992.

[14] K. Fukunaga, Introduction to Statistical Pattern Recognition, second ed., Academic Press, New York, 1990.

[15] L.A. Zadeh, Info. Contr. 8 (1965) 338.

[16] H.J. Zimmerman, in: Fuzzy Sets, Decision Making and Expert Systems, Kluwer Academic Publishers, Boston, MA, 1987.

[17] H. Muller, W. Muller, D.McG. Squire, S. Marchand-Maillet, T. Pun, Pattern Recog. Lett. 22 (5) (2001) 593.