Bone remodelling simulations using the proposed open-source framework

Problem definition:

To benchmark the proposed open-source framework for bone remodelling studies, a numerical scheme proposed by Fernández et al. (2010) for the standard strain-adaptive bone remodelling model (Weinans et al. 1992) was analysed. The model is based on the principle that bone remodelling is induced by a local mechanical signal which activates the regulating bone cells. A classical 2D plate model representing the cross-section of bone tissue was simulated in the open-source framework and the results obtained were compared with those reported in the study by Fernández (2010).

The square plate of size 0.1 x 0.1 m was meshed with crossed-triangle cells using FEniCS and the boundary conditions applied are shown in Fig 1. The plate was assumed to be fixed on its lower horizontal boundary with the left-most node completely clamped, whereas the rest of the nodes on this boundary had only their vertical displacement constrained. The plate was being acted upon by a linearly increasing compressive load on its top boundary with maximum magnitude of 10 MPa (Fernández 2010; Weinans et al. 1992).

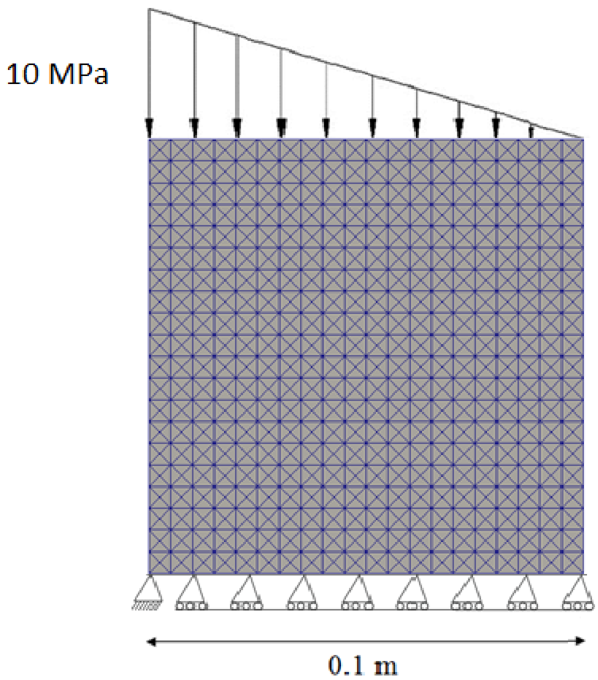

Fig 1: FEM model with boundary conditions relating to bone remodelling (Fernández 2010)

Here, plane stress condition was assumed and the simulations were performed under the assumption of small displacement theory (Fernández et al. 2010; 2012a; Fernández 2010).

Mechanical and variational formulation

The main idea of this bone remodelling model is to use the bone apparent density as the characterization of the internal morphology of bone**.** Here, the bone is considered to be linear-elastic and isotropic and the constitutive law for the stress field can be given as follows (Fernández et al. 2010):

$$\sigma(u) = 2\mu(\rho)\varepsilon(u) + \lambda(\rho)\mathrm{Div}(u)I \tag{1}$$

Where Div represents the divergence operator and I denotes the identity operator. $\mu(\rho)$ and $\lambda(\rho)$ are Lamé coefficients of the material that were assumed to be dependent on the bone apparent density denoted by ρ and given as:

$$\mu(\rho) = \frac{E(\rho)}{2\,(1+\nu)} \text{ and } \lambda(\rho) = \frac{E(\rho)\nu}{1-\nu^2} \tag{2}$$

where ν is the Poisson's ratio and $E(\rho)$ is the Young's modulus. The following function was used for elastic modulus depending on the apparent density:

$$E \;= M\rho^{\gamma} \tag{3}$$

where M and γ are positive constants that characterise bone behaviour. The evolution of apparent density function was obtained from the following first order ordinary differential equation:

$$\dot{\rho} \;= B\left(\frac{U(\sigma(u), \varepsilon(u))}{\rho} - S_r\right) \tag{4}$$

where B and S_r are the experimental constants. Further the strain energy density (SED) as mechanical stimulus $U(\sigma(u), \varepsilon(u))$ can be given as (Weinans et al. 1992):

$$U\big(\sigma(u), \varepsilon(u)\big) = \frac{1}{2}\,\sigma(u){:}\,\varepsilon(u) \tag{5}$$

where : denotes the inner product and it had been assumed that the apparent density is bounded as:

$$\rho_a \;\leq\; \rho \;\leq\; \rho_b \tag{6}$$

where $\rho_a$and ρ_b represent the minimal and maximal density corresponding to the resorbed and cortical bone, respectively. Applying Green's formula, the weak form of strain-adaptive bone remodelling problem can be given as Fernández el at. 2010:

$$\int_{\Omega} 2\mu(\rho)\boldsymbol{\varepsilon}(\boldsymbol{u}) : \boldsymbol{\varepsilon}(\boldsymbol{v}) + \lambda(\rho)\mathrm{Tr}\big(\boldsymbol{\varepsilon}(\boldsymbol{u})\big)\mathrm{Tr}\big(\boldsymbol{\varepsilon}(\boldsymbol{v})\big)\, d\boldsymbol{x} \;=\; \int_{\Omega} \boldsymbol{f}_B(t)\cdot\boldsymbol{v}\, d\boldsymbol{x} + \int_{\Gamma_N} \boldsymbol{f}_N(t)\cdot\boldsymbol{v}\, d\Gamma, \tag{7}$$

where *Tr* denotes the classical trace operator, $\boldsymbol{v}$ is the test function, and $d\boldsymbol{x}$ denotes the differential element for integration over the domain. More details about this model can be found in Fernández el at. 2010.

Flowchart of the implemented algorithm

A schematic illustration of the implemented strain-adaptive bone remodelling algorithm (see Fig. 2) is presented in following figure.

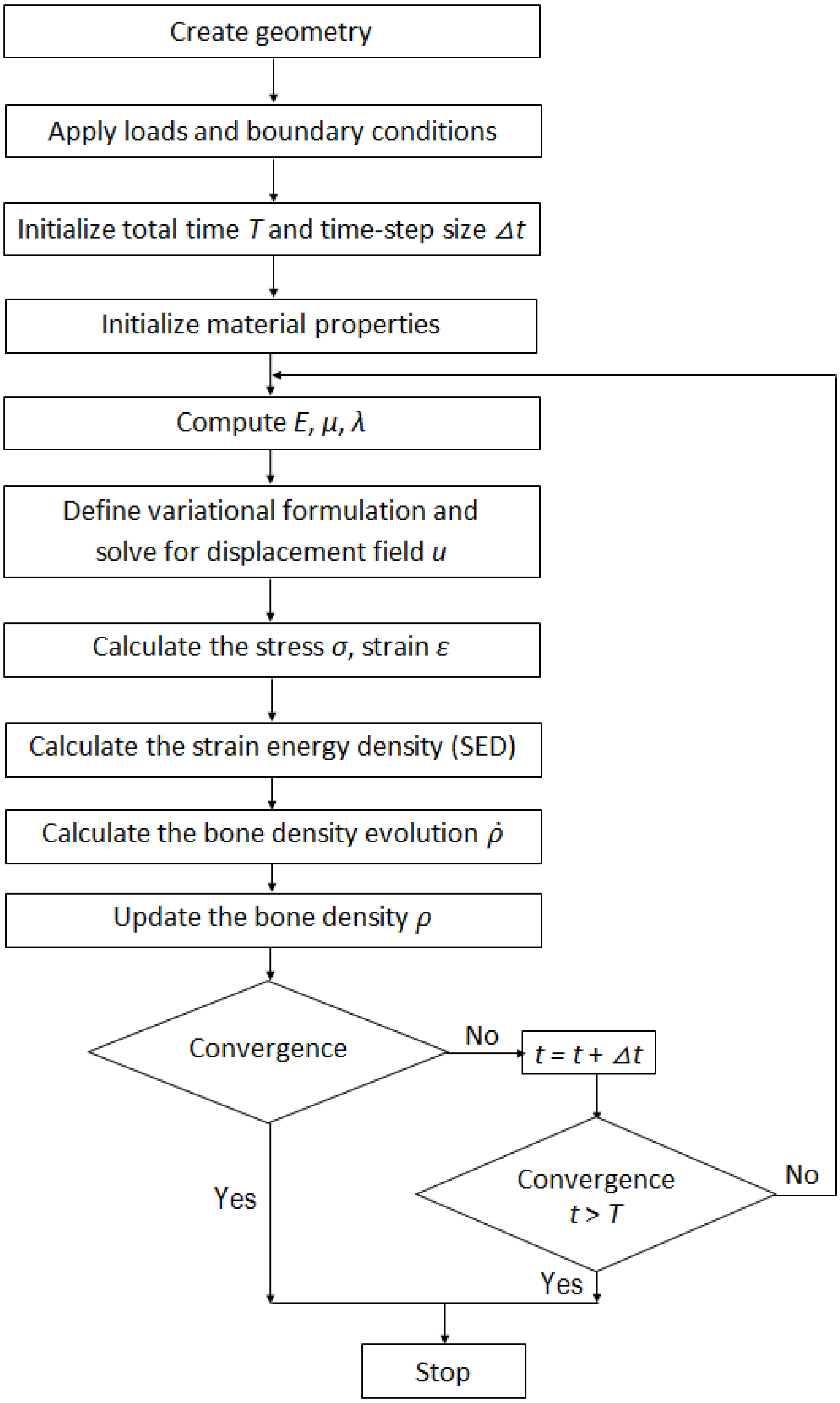

Fig 2: Schematic representation of implemented bone remodelling algorithm

The strain-adaptive bone remodelling algorithm implemented in this study and can be summarized in the following eight steps:

Step 1 Apply the appropriate loading and boundary conditions.
Step 2 Initialize the material properties.
Step 3 Determine elastic modulus, Lame's constants, and constitutive function.
Step 4 Calculate the discrete displacement field by defining and solving the coupled discrete linear variational equations.
Step 5 Evaluate the constitutive equations (stress and strain).
Step 6 Determine the strain energy density (SED) function using FEM.
Step 7 Update the bone density distribution according to the density evolution equation that justifies if the mechanical stimulus could cause bone apposition, bone resorption, or equilibrium.
Step 8 Check for convergence. The convergence criterion is imposed based on the change in bone density during the iterative process. The required configuration is obtained and the process terminates when the convergence criterion is satisfied; otherwise, the iterative process continues from *step 3*.
Finally, the simulation results are visualized using Paraview software.

Proposed open-source framework

Figure 3 represents a flow diagram of the main simulation steps executed using the proposed open-source software framework and more details can be found in Bansod et al. (2019).

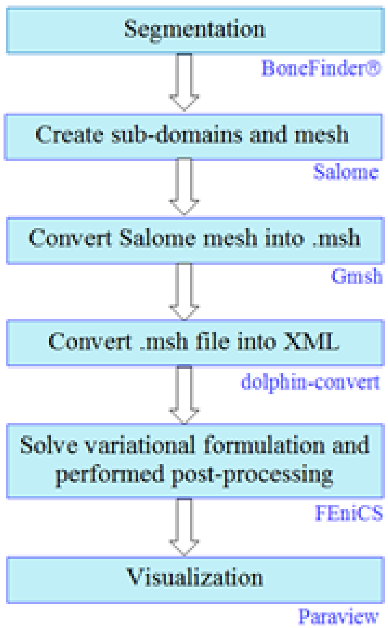

Fig. 3 Open-source software framework used for bone remodelling simulations

The Python scripts and other files created for this study are available for open access on the GitHub repository (https://github.com/YDBansod/Bone_Remodelling).

Simulation results

Starting from a uniform density distribution, the simulation results demonstrate a density distribution (see Fig 4) that is in line with the results obtained by Fernandez (2010).

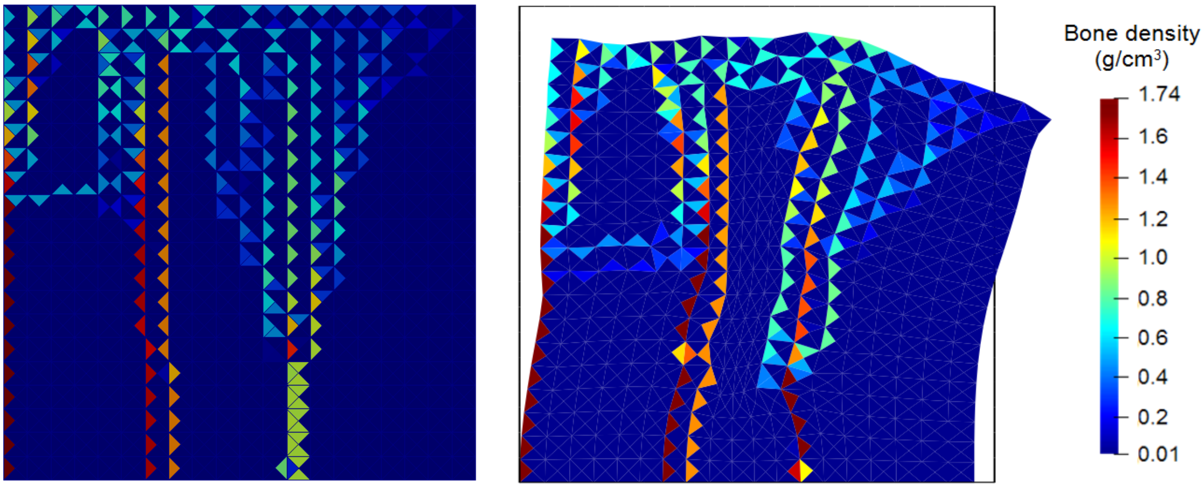

Fig. 4: Density distribution obtained (left) and the corresponding results from Fernandez (2010) (right) (please see Fig 3.4 on pg-136)

This serves as a preliminary validation of the bone remodelling algorithm (see Fig 2) performed using the proposed open-source framework shown in (see Fig 3).

References

Bansod YD, van Rienen U (2019) Numerical analysis of electromechanically driven bone remodeling using the open-source software framework. 41st Annual International Conference of the IEEE Engineering in Medicine and Biology Society (EMBC) 6466-6471. http://dx.doi.org/10.1109/EMBC.2019.8856543

Fernández JR, García-Aznar J, Martínez R (2012a) Numerical analysis of a piezoelectric bone remodelling problem. Eur J Appl Math 23:635-657. http://dx.doi.org/10.1017/S0956792512000150

Fernández JR, Garcia-Aznar J, Martínez R, Viano J (2010) Numerical analysis of a strain-adaptive bone remodelling problem. Comput Method Appl M 199:1549-1557. http://dx.doi.org/10.1016/j.cma.2010.01.005

Fernández RM, (2010) Numerical analysis and simulations in bone remodeling models. Dissertation, Universidade de Santiago de Compostela.

Weinans H, Huiskes R, and Grootenboer H (1992) The behavior of adaptive bone-remodeling simulation models. J Biomech 25:1425-1441. http://dx.doi.org/10.1016/0021-9290(92)90056-7