# Tunnel diode

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## 1 Introduction

IV-curve (Esaki)

$$I\left(V\right)=\frac{em_{e}kT}{2\pi^{2}\hbar^{3}}\int_{0}^{\infty}T\left(E_{\perp}\right)\left[\ln\left(1+\exp\frac{E_{Fn}-E_{\perp}}{kT}\right)-\ln\left(1+\exp\frac{E_{Fp}-eV-E_{\perp}}{kT}\right)\right]dE_{\perp}$$

## 2 Electrons

Distribution of elecrons on the $n$ side:

$$n\left(p_{\perp}\right)=\frac{2\cdot2\pi}{\left(2\pi\hbar\right)^{3}}\int_{0}^{\infty}\frac{p_{||}dp_{\|}}{1+\exp\frac{p_{c}^{2}+p_{\perp}^{2}-p_{F}^{2}+p_{\|}^{2}}{2m_{e}kT}}=\frac{m_{e}kT}{2\pi^{2}\hbar^{3}}\int_{0}^{\infty}\frac{du}{1+\exp\frac{p_{c}^{2}+p_{\perp}^{2}-p_{F}^{2}}{2m_{e}kT}e^{u}}$$

$$n\left(p_{\perp}\right)dp_{\perp}=\frac{m_{e}kT}{2\pi^{2}\hbar^{3}}\ln\left(1+\exp\frac{p_{F}^{2}-p_{c}^{2}-p_{\perp}^{2}}{2m_{e}kT}\right)dp_{\perp}$$

$$dp_{\perp}=\frac{\sqrt{m_{e}}}{\sqrt{2E_{\perp}}}dE_{\perp}$$

$$n\left(E_{\perp}\right)dE_{\perp}=\frac{m_{e}kT}{2\pi^{2}\hbar^{3}}\ln\left(1+\exp\frac{E_{F}-E_{c}-E_{\perp}}{kT}\right)\frac{\sqrt{m_{e}}}{\sqrt{2E_{\perp}}}dE_{\perp} \tag{1}$$

$$v\left(E_{\perp}\right)=\frac{\sqrt{2E_{\perp}}}{\sqrt{m_{e}}}$$

General formula for the current is:

$$I\left(V\right)=e\int_{0}^{\infty}T\left(E_{\perp}\right)n\left(E_{\perp}\right)v\left(E_{\perp}\right)dE_{\perp}$$

The majority electron current:

$$I_{1}\left(V\right)=\frac{em_{e}kT}{2\pi^{2}\hbar^{3}}\int_{0}^{\infty}T_{1}\left(E_{\perp}\right)\ln\left(1+\exp\frac{E_{F}-E_{c}-E_{\perp}}{kT}\right)dE_{\perp}$$

Minority electron current will be different due to $E_c$ on that side being raised by $\Delta\Phi$ — the equilibrium energy barrier or the work function difference between N and P semiconductors.

$$I_2(V)=\frac{em_e kT}{2\pi^2\hbar^3}\int_0^\infty T_2(E_\perp)\ln\left(1+\exp\frac{E_F-E_c-\Delta\Phi-E_\perp}{kT}\right)dE_\perp$$

However, the transmission probabilities $T_1, T_2$ will also be different. There's nothing preventing the minority electrons from reaching the N side, so:

$$T_2(E_\perp)=1$$

Then the minority current doesn't depend on voltage and we can integrate it:

$$I_2(V)=\frac{em_e kT}{2\pi^2\hbar^3}\int_0^\infty \ln\left(1+\exp\frac{E_F-E_c-\Delta\Phi-E_\perp}{kT}\right)dE_\perp$$

Moreover, we have a similar dependence for $T_1$, which gives us:

$$T_1(E_\perp>\Delta\Phi-eV)=1$$

$$\int_{\Delta\Phi-eV}^\infty \ln\left(1+\exp\frac{E_F-E_c-E_\perp}{kT}\right)dE_\perp=\int_0^\infty \ln\left(1+\exp\frac{E_F-E_c-\Delta\Phi+eV-E_\perp}{kT}\right)dE_\perp$$

$$\begin{aligned}I_e(V)=\frac{em_e kT}{2\pi^2\hbar^3}\int_0^{\Delta\Phi-eV} T_1(E_\perp)\ln\left(1+\exp\frac{E_F-E_c-E_\perp}{kT}\right)dE_\perp+\\+\frac{em_e kT}{2\pi^2\hbar^3}\int_0^\infty\left[\ln\left(1+\exp\frac{E_F-E_c+eV-\Delta\Phi-E_\perp}{kT}\right)-\ln\left(1+\exp\frac{E_F-E_c-\Delta\Phi-E_\perp}{kT}\right)\right]dE_\perp.\end{aligned}$$

For simplicity we assume that:

$$\exp\frac{E_F-E_c-\Delta\Phi}{kT}\ll 1$$

Then our expression for current simplifies to:

$$\begin{aligned}I(V)=\frac{em_e kT}{2\pi^2\hbar^3}\int_0^{\Delta\Phi-eV} T_1(E_\perp)\ln\left(1+\exp\frac{E_F-E_c-E_\perp}{kT}\right)dE_\perp+\\+\frac{em_e kT}{2\pi^2\hbar^3}\exp\frac{E_F-E_c-\Delta\Phi}{kT}\left[\exp\left(\frac{eV}{kT}\right)-1\right]\int_0^\infty\exp\left(-\frac{E_\perp}{kT}\right)dE_\perp.\end{aligned}$$

Or:

$$I(V) = I_1(V) + I_2(V)$$

$$I_1(V) = A \int_0^{\Delta\Phi} T_1(kTu) \ln\left(1 + w(0)\, e^{-u}\right) du$$

$$I_2(V) = A \cdot w(-\Delta\Phi) \left[\exp\left(\frac{eV}{kT}\right) - 1\right]$$

$$A = \frac{e m_e k^2 T^2}{2\pi^2 \hbar^3}$$

$$w(eV) = \exp \frac{E_F - E_c + eV}{kT}$$

Which clearly allows us to separate the two current contributions: a regular diode current and the tunneling current.

# 3 Transmission probability

It's easy to see that electrons can tunnel only into the valence band. Then their energy should be below $E_v$ on the P side.

We can see that:

$$E_v = E_c + \Delta\Phi - E_g$$

$$T_1(E_\perp > E_v - E_c - eV) = 0$$

$$T_1(E_\perp > \Delta\Phi - E_g - eV) = 0$$

So we need to rewrite the tunneling current as:

$$I_1(V) = A \int_0^{b} T_1(kTu) \ln\left(1 + w(0)\, e^{-u}\right) du$$

$$b = \frac{\Delta\Phi - E_g - eV}{kT}$$

Then for the rest we can use the usual formula for delta-function potential barrier:

$$T_1(E) = \frac{E}{E + \frac{2m_e a^2}{\hbar^2} (\Delta\Phi - eV)^2}$$

Where $a$ is the width of the PN junction depletion layer.

$$\frac{N_a + N_d}{\sqrt{N_a N_d (N_a + N_d)}} = \sqrt{\frac{N_a + N_d}{N_a N_d}}$$

$$a^2 = \frac{2\varepsilon_0 \varepsilon}{e^2} \frac{N_a + N_d}{N_a N_d} (\Delta\Phi - eV)$$

$$T_1(u) = \frac{u}{u + (\Delta\Phi - eV)^3 / W^3}$$

$$W^3 = \frac{e^2}{4\varepsilon_0\varepsilon} \frac{\hbar^2 kT}{m_e} \frac{N_a N_d}{N_a + N_d}$$

So the final expression will become:

Full current:

$$I(V) = I_1(V) + I_2(V)$$

Tunneling current:

$$I_1(V) = A \int_0^b \frac{u \ln(1 + w_0 e^{-u})}{u + (\Delta\Phi - eV)^3 / W^3} du$$

Diode current:

$$I_2(V) = A \cdot s_0 \left[\exp\left(\frac{eV}{kT}\right) - 1\right]$$

Parameters:

$$A = \frac{e m_e k^2 T^2}{2\pi^2 \hbar^3}$$

$$w_0 = \exp \frac{E_F - E_c}{kT}$$

$$s_0 = \exp \frac{E_F - E_c - \Delta\Phi}{kT}$$

$$W^3 = \frac{e^2}{4\varepsilon_0\varepsilon} \frac{\hbar^2 kT}{m_e} \frac{N_a N_d}{N_a + N_d}$$

Upper integration limit in the tunneling current depends on voltage:

$$b = \frac{\Delta\Phi - E_g - eV}{kT} > 0, \qquad eV < \Delta\Phi - E_g$$

When $b$ becomes negative, the tunneling current disappears! We should set it to 0 and stop calculating the integral.