YData: An Introduction to Data Science

Lecture 27: Sample Averages

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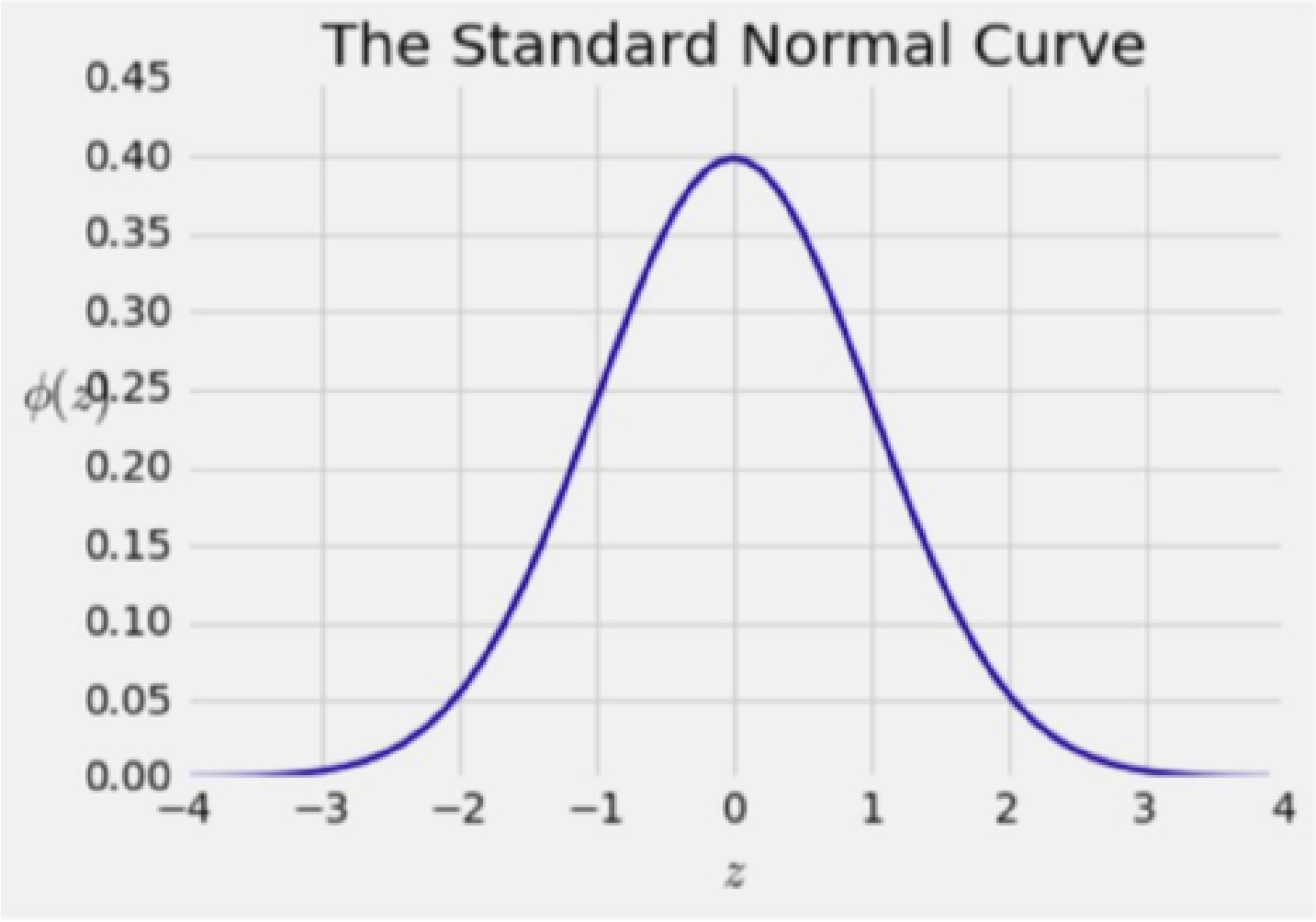
Credit: data8.org

Announcements

Questions for This Week

- How can we quantify natural concepts like "center" and "variability"?
- Why do many of the empirical distributions that we generate come out bell shaped?
- How is sample size related to the accuracy of an estimate?

Bell Curve


The Standard Normal Curve
0.45
0.40
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
$\phi(z)$
−4
−3
−2
−1
0
1
2
3
4
z

Bounds and Normal Approximations

Percent in Range	All Distributions	Normal Distribution
average $\pm$ 1 SD	at least 0%	about 68%
average $\pm$ 2 SDs	at least 75%	about 95%
average $\pm$ 3 SDs	at least 88.888...%	about 99.73%

Sample Averages

- The Central Limit Theorem describes how the normal distribution (a bell-shaped curve) arises in the context of random sampling.
- Many distributions we observed were not bell-shaped, but empirical distributions of sample averages were.
- We care about sample averages because they estimate population averages.

Distribution of the Sample Average

Why is There a Distribution?

- You have only one random sample, and it has only one average.
- But **the sample could have come out differently**.
- And then the sample average might have been different.
- So there are many possible sample averages.

Distribution of the Sample Average

- Imagine all possible random samples of the same size as yours. There are lots of them.
- Each of these samples has an average.
- The **distribution of the sample average** is the distribution of the averages of all the possible samples.

Shape of the Distribution

Central Limit Theorem

If the sample is

- large, and
- drawn at random with replacement,

Then, *regardless of the distribution of the population*,

> **the probability distribution of the sample sum (or of the sample average) is roughly bell-shaped**

(DEMO)

Specifying the Distribution

Suppose the random sample is large.

- We have seen that the distribution of the sample average is roughly bell shaped.

- Important questions remain:
  - Where is the center of that bell curve?
  - How wide is that bell curve?

Center of the Distribution

The Population Average

The distribution of the sample average is roughly a bell curve centered at the population average.

Variability of the Sample Average

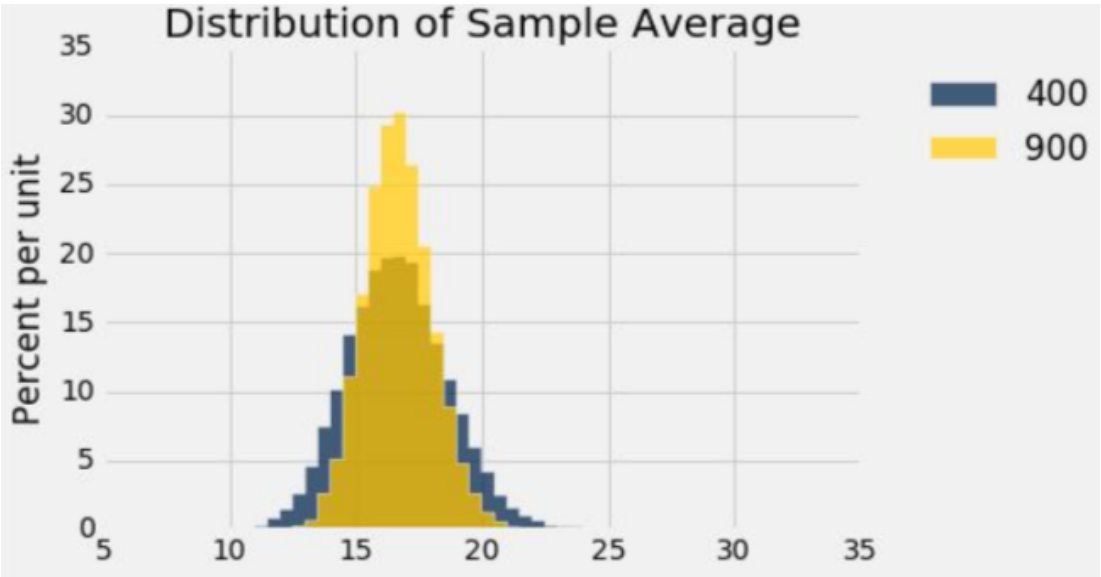
Why Is This Important?

- Along with the center, the spread helps identify exactly which normal curve is the distribution of the sample average.
- The variability of the sample average helps us measure how accurate the sample average is as an estimate of the population average.
- If we want a specified level of accuracy, understanding the variability of the sample mean helps us work out how large our sample has to be.

(DEMO)

Discussion Question

The gold histogram shows the distribution of _____ values, each of which is _____.



(a) 900
(b) 10,000
(c) a randomly sampled flight delay
(d) an average of flight delays

The Two Histograms

- The gold histogram shows the distribution of 10,000 values, each of which is an average of 900 randomly sampled flight delays.
- The blue histogram shows the distribution of 10,000 values, each of which is an average of 400 randomly sampled flight delays.
- Both are roughly bell shaped.
- The larger the sample size, the narrower the bell.

Variability of the Sample Average

- The distribution of all possible sample averages of a given size is called the distribution of the sample average.
- We approximate it by an empirical distribution.
- By the CLT, it's roughly normal:
  - Center = the population average
  - SD = (population SD) $/\sqrt{\text{sample size}}$

(DEMO)

Discussion Question

A city has 500,000 households. The annual incomes of these households have an average of \$65,000 and an SD of \$45,000. The distribution of the incomes [pick one and explain]:

(a) is roughly normal because the number of households is large.

(b) is not close to normal.

(c) may be close to normal, or not; we can't tell from the information given.

Discussion Question

A city has 500,000 households. The annual incomes of these households have an average of \$65,000 and an SD of \$45,000. A random sample of 900 households is taken.

Fill in the blanks and explain: There is about a 68% chance that the average annual income of the sampled households is in the range \$_____ plus or minus \$_____.