Statements

```
more_than_1 = 2 + 3
```

Name — Any expression

- Statements don't have a value; they perform an action
- An assignment statement changes the meaning of the name to the left of the = symbol
- The name is bound to a value (not an equation)

- < and > mean what you expect (less than, greater than)
- <= means "less than or equal"; likewise for >=
- == means "equal"; != means "not equal"
- Comparing strings compares their alphabetical order

Arrays - sequences of the same type that can be manipulated

- Arithmetic and comparisons are applied to each element of an array individually
  - `make_array(1,2,3) > 2 # array([False, False, True])`
- Elementwise operations can be done on arrays of the same size
  - `make_array(1,2) * make_array(5,4) # array([5,8])`

Defining a Function

```
def function_name(arg1, arg2, ...):
    # Body can contain anything inside of it
    return # a value (the output of the function call)
```

`For Statements`

```
total = 0
for i in np.arange(12):
    total = total + i
```

The body is executed **for** every item in a sequence

The body of the statement can have multiple lines

The body should do something: assign, sample, print, etc.

`Conditional Statements`

```
if <if expression>:
    <if body>
elif <elif expression 0>:
    <elif body 0>
elif <elif expression 1>:
    <elif body 1>
...
else:
    <else body>
```

Simulating a Statistic:

- Create an empty array in which to collect the simulated values
- For each repetition of the process
  - Simulate one value of the statistic
  - Append this value to the collection array
- At the end, all simulated values will be in the collection array

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Total Variation Distance between two distributions:

- For each category, compute the difference in proportions between the two distributions
- Take the absolute value of each difference
- Sum, and then divide by 2

- **P-Value**: The chance, **under the null hypothesis**, that the test statistic is equal to the one in the sample, or more extreme in the direction of the alternative:
  - If the null is true and the p-value is low, something unlikely has happened.
  - We can conclude that the data support the alternative hypothesis more than they support the null.

`Operations:` `addition 2+3=5; subtraction 4-2=2; division 9/2=4.5`
`multiplication 2*3=6; division remainder 11%3=2;`
`exponentiation 2**3=8`

`Data Types:` **`string`** `"hello";` **`boolean`** `True, False;`
`int` `1, -5;` **`float`** `- 2.3, -52.52, 7.9`

`Table.where predicates` `(x is a string or number):`

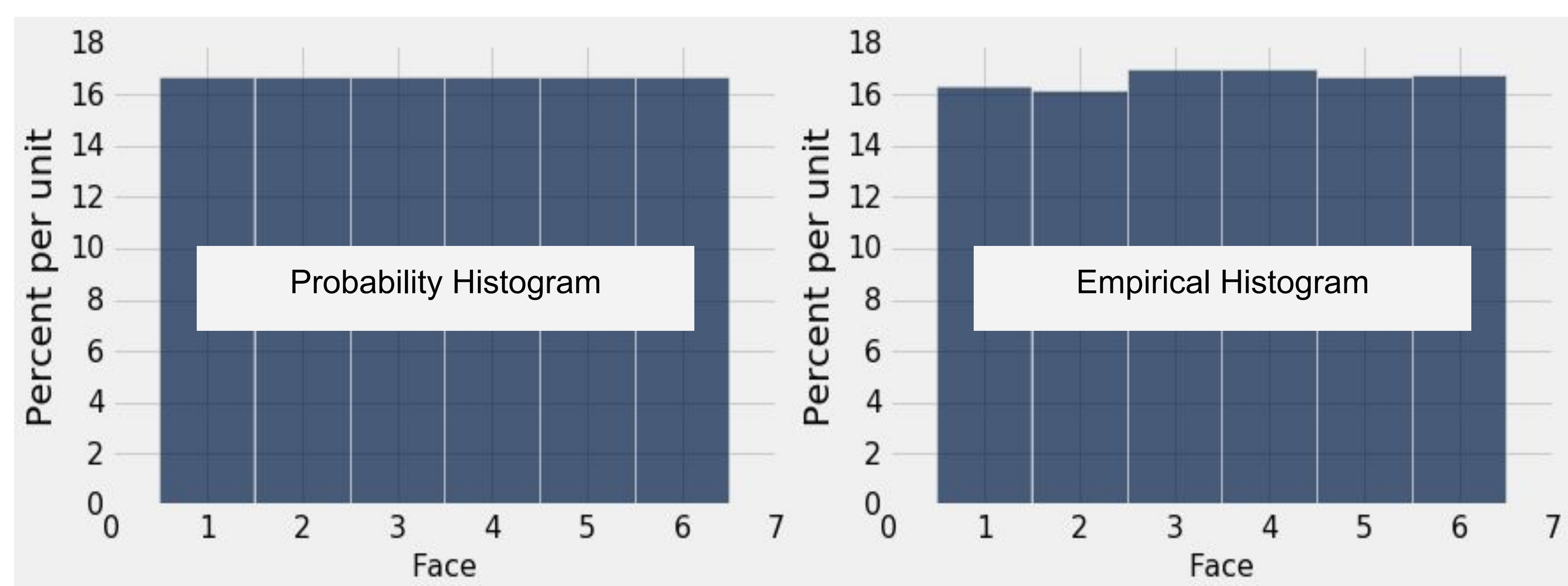
- `are.equal_to(x)`
- `are.not_equal_to(x)`
- `are.above(x) # val > x`
- `are.above_or_equal_to(x) # val >= x`
- `are.below(x) # val < x`
- `are.between(x, y) # x <= val < y`
- `are.containing(val) # val is present in the entry`

A **histogram** has two defining properties:

- The bins are contiguous (though some might be empty) and are drawn to scale
- The **area** of each bar is equal to the percent of entries in the bin

Has total area 100%

- The histogram on the left represents the theoretical probabilities under the distribution of a fair die
- The histogram on the right represents the observed distribution of rolling many dice
- If we roll enough, the right hand histogram will start to look like the one on the left



Calculating Probabilities

Complement Rule: P(event does not happen) = 1 - P(event happens)

Multiplication Rule: P(two events both happen) =
P(one happens) * P(the other happens, given the first happened)

Addition Rule: If an event can ONLY happen in one of two ways:
P(an event happens) =
P(first way it can happen) + P(second way it can happen)

A/B test for comparing two samples

- **Example**: Among babies born at some hospital, is there an association between birth weight and whether the mother smokes?
- **Null hypothesis**: The distribution of birth weights is the same for babies with smoking mothers and non-smoking mothers.
- **Inferential Idea:** If maternal smoking and birth weight were not associated, then we could simulate new samples by replacing each baby's birth weight by a randomly picked value from among all the birth weights.
- **Simulating the test statistic under the null:**
  - Permute (shuffle) the outcome column many times. Each time:
    - Create a shuffled table that pairs each individual with a random outcome.
    - Compute a sampled test statistic that compares the two groups, such as the difference in mean birth weights for the two groups.

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In the examples in the left column, np refers to the NumPy module, as usual. Everything else is a function, a method, an example of an argument to a function or method, or an example of an object we might call the method on. For example, `tbl` refers to a table, `array` refers to an array, and `num` refers to a number. `array.item(0)` is an example call for the method `item`, and in that example, `array` is the name previously given to some array.

Example function call	**Value of a call to the function**
`max(array); min(array)`	Maximum or minimum of a list or array
`sum(array)`	Sum of all elements in a list or array; The sum of boolean values is the number that are True
`len(array)`	Length (num elements) in a list or array
`round(num); np.round(array)`	The nearest integer to a single number or each number in an array
`abs(num); np.abs(array)`	The absolute value of a single number or each number in an array
`np.average(array), np.mean(array)`	The average of the values in an array

`np.arange(start, stop, step)`
`np.arange(start, stop)`
`np.arange(stop)`
	An array of numbers starting with `start`, going up in increments of `step`, and going up to but excluding `stop`. When `start` and/or `step` are left out, default values are used in their place. Default `step` is 1; default `start` is 0.

`array.item(index)`	The item in the array at some index. `array.item(0)` is the first item of `array`.
`np.append(array, item)`	A copy of the array with item appended to the end. If item is another array, all of its elements are appended.
`np.random.choice(array, n)`	An item selected at random from an array. If `n` is specified, an array of n items selected at random with replacement is returned. Otherwise, only one item will be chosen at random.
`np.ones(n)`	An array of length `n` which consists of all ones.
`np.diff(array)`	An array of length `len(array)-1` which contains the difference between consecutive elements.
`np.count_nonzero(array)`	An integer corresponding to the number of non-zero (or `True`) elements in an array.
`sample_proportions(sample_size, model_proportions)`	An array of proportions that add up to 1. The result of sampling `sample_size` elements from a distribution specified by `model_proportions`, and keeping track of the proportion of each element sampled.
`Table()`	An empty table.
`Table.read_table(filename)`	A table with data from a file.
`tbl.num_rows`	The number of rows in a table.
`tbl.num_columns`	The number of columns in a table.
`tbl.labels`	A list of the column labels of a table.
`tbl.with_column(name, values)` `tbl.with_column(n1, v1, n2, v2…)`	A table with an additional or replaced column or columns. `name` is a string for the name of a column, `values` is a list or array.
`tbl.column(column_name_or_index)`	An array containing the values of a column
`tbl.select(col1, col2, …)`	A table with only the selected columns. (Each argument is the label of a column, or a column index.)
`tbl.drop(col1, col2, …)`	A table without the dropped columns. (Each argument is the label of a column, or a column index.)
`tbl.relabeled(old_label, new_label)`	A new table with a label changed.
`tbl.take(row_indices)`	A table with only the rows at the given indices. `row_indices` must be an array of indices.
`tbl.sort(column_name_or_index)`	A table of rows sorted according to the values in a column (specified by name/index). Default order is ascending. For descending order, use argument `descending=True`. For unique values, use `distinct=True.`
`tbl.where(column, predicate)`	A table of the rows for which the column satisfies some predicate. See "`Table.where predicates`".
`tbl.apply(function, column)`	An array of results when a `function` is applied to each item in a `column`.
`tbl.group(column_or_columns)`	A table with the counts of rows grouped by unique values or combinations of values in a column or columns.
`tbl.group(column_or_columns, func)`	A table that groups rows by unique values or combinations of values in a column or columns. The other values are aggregated by `func.` All column names (except the one(s) we group by) will now be `original_name func`. If a column is named 'price', and we group using the `min` function, our new column name will be 'price min'.
`tblA.join(colA, tblB, colB)` `tblA.join(colA, tblB)`	A table with the columns of `tblA` and `tblB`, containing rows for all values of a column that appear in both tables. Default value of `colB` is `colA. colA` is a string specifying a column name, as is `colB.`
`tbl.pivot(col1, col2)` `tbl.pivot(col1, col2, vals, collect)`	A pivot table where each unique value in `col1` has its own column and each unique value in `col2` has its own row. The cells of the grid contain row counts (two arguments) or the values from a third column, aggregated by the collect function.
`tbl.sample(n)` `tbl.sample(n, with_replacement)`	A new table where `n` rows are randomly sampled from the original table. Default is with replacement. For sampling without replacement, use argument `with_replacement=False`. If there is no sample size specified, the default is the number of rows in the table.
`tbl.scatter(x_column, y_column)`	Draws a scatter plot consisting of one point for each row of the table.
`tbl.barh(categories)` `tbl.barh(categories, values)`	Displays a bar chart with bars for each category in a column, with length proportional to the corresponding frequency. If `values` is not specified, overlaid bar charts of all the remaining columns are drawn.
`tbl.bin(column, bins)`	A table of how many values in a column fall into each bin. Bins include lower bounds & exclude upper bounds.
`tbl.hist(column, units, bins, group)`	Displays a histogram of the values in a column. `unit` and `bins` are optional arguments, used to label the axes and group the values into intervals (bins), respectively. Bins include lower bounds & exclude upper bounds. If `group` is specified, the rows are grouped by the values in the column, and histograms for all the groups are overlaid.

Error Probabilities

- Even if the null is true, your random sample might indicate the alternative, just by chance
- The **cutoff value** for P is the chance that your test makes the wrong conclusion when the null hypothesis is true
- Using a small cutoff limits the probability of this kind of error

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Population (fixed) → Sample (random) → Statistic (random)
A 95% **Confidence Interval** is an interval constructed so that it will contain the true population parameter for approximately 95% of samples.
For a particular sample, the generated interval *either contains* the true parameter or *it doesn't*; the process works 95% of the time
Bootstrap: When we wish we could sample again from the population, instead we can sample with replacement from the *original large random sample, the same number of times as there are data-points in the sample.*

Using a confidence interval to test a hypothesis about a numerical parameter:

- Null hypothesis: **Population parameter = *x***
- Alternative hypothesis: **Population parameter ≠ *x***
- Cutoff for P-value: *p*%
- Method:
  - Construct a (100-*p*)% confidence interval for the population parameter
  - If *x* is not in the interval, reject the null
  - If *x* is in the interval, fail to reject the null

The 80th percentile is the value in a set that is at least as large as 80% of the elements in the set

For `s = [1, 7, 3, 9, 5]`, `percentile(80, s)` is 7

The 80th percentile is ordered element 4: `(80/100) * 5`

For a percentile that does not exactly correspond to an element, take the next greater element instead

`percentile(10, s)` is 1 `percentile(20, s)` is 1
`percentile(21, s)` is 3 `percentile(40, s)` is 3

- `minimize` must take in a function whose arguments are numerical, and returns an array of those numerical arguments
- If the function `rmse(a, b)` returns the root mean squared error of estimation using the line "estimate = ax + b",
  - then `minimize(rmse)` returns array [a_0, b_0]
  - a_0 is the slope and b_0 the intercept of the line that minimizes the rmse among lines with arbitrary slope a and arbitrary intercept b (that is, among all lines)

Expression	Description
`percentile(n, arr)`	Returns the n-th percentile of array `arr`
`np.std(array)`	Return the standard deviation of an array of numbers
`minimize(fn)`	Return an array of arguments that minimize a function
`tbl1.append(tbl2)` `tbl1.append(row)`	Append a row or all rows of `tbl2`, mutating `tbl1`. Appended object and `tbl1` must have identical columns.
`table.rows`	All rows of a table; Used in `for row in table.rows:`
`table.row(i)`	Return the row of a table at index *i*
`row.item(j)`	Returns item `j` from some row

The Central Limit Theorem (CLT)

If the sample is large, and drawn at random with replacement,
Then, *regardless of the distribution of the population,*

the probability distribution of the sample average (or sample sum) is roughly bell-shaped

- Fix a large sample size
- Draw all possible random samples of that size
- Compute the mean of each sample
- You'll end up with a lot of means
- The distribution of those is the *probability distribution of the sample mean*
- It's roughly normal, centered at the population mean
- The SD of this distribution is the (population SD) /√sample size

---

Choosing sample size so that the 95% confidence interval is small

- CLT says the distribution of a sample proportion is roughly normal, centered at the true population proportion
- **95% confidence interval:**
  - Sample proportion ± 2 SDs of the sample proportion
- **CI Width** = 4 SDs of the sample proportion

  = 4 x (SD of 0/1 population)/√sample size
- The SD of a 0/1 population is less than or equal to 0.5

The following functions were defined in lecture, but will **not** be available for use during the final exam. If you would like to use one of the functions, you must define it yourself.

- `standard_units`
- `correlation`
- `slope`
- `intercept`
- `fitted_values`
- `residuals`
- `prediction_at`

Mean (or average): Balance point of the histogram

Standard deviation (SD) =

root 5	mean 4	square of 3	deviations from 2	average 1

Measures roughly how far off the values are from average

Most values are within the range "average ± *z* SDs"

- *z* measures "how many SDs above average"
- If *z* is negative, the value is below average
- *z* is a value in **standard units**
- Chebyshev: At most $1/z^2$ are z or more SDs from the mean
- Almost all standard unit values are in the range (-5, 5)
- Convert a value to standard units: (value - average) / SD
- *z* * SD + average is the original value

Percent in Range	All Distributions	Normal Distribution
average ± 1 SD	at least 0%	about 68%
average ± 2 SDs	at least 75%	about 95%
average ± 3 SDs	at least 88.888...%	about 99.73%

Correlation Coefficient (*r*) =

average of	product of	x in standard units	and	y in standard units

Measures how clustered the scatter is around a straight line

- $-1 \le r \le 1$; r = 1 (or -1) if the scatter is a perfect straight line
- *r* is a pure number, with no units

Regression for y and x in standard units: $y_{predicted,su} = r * x_{su}$

The regression line minimizes the root mean square error among all lines used to predict y from x.
The slope and intercept found by linear regression are unique.
Fitted value: height of the regression line at some x: a*x + b
Residual: difference between y and regression line height at x

$$y_{predicted} = slope * x + intercept$$

$$\text{slope of the regression line} = r \cdot \frac{\text{SD of } y}{\text{SD of } x}$$

$$\text{intercept of the regression line} = \text{mean of } y - \text{slope} * \text{mean of } x$$

Properties of fitted values, residuals, and the correlation r:

- mean of fitted values = mean of y
- SD of fitted values = |r| * (SD of y)
- mean of residuals = 0
- SD of residuals = $\sqrt{1 - r^2}$ * (SD of y)

- **Regression Model**: y is a linear function of x + normal "noise"
- The errors are randomly sampled from a normal distribution that has mean 0
- Under this model, residual plot looks like a formless cloud

Prediction Intervals (assuming the regression model)

- Creating an interval of predictions of the true value of *y* based on a specified value of *x*
- Steps for creating an approximate 95% prediction interval:
  - Bootstrap your original sample
  - Calculate the slope and intercept of the regression line based on the new sample
  - Calculate slope * x + intercept, for the given *x*
  - Repeat the above steps many times and keep track of all of your fitted values
  - Create the prediction interval by taking the middle 95% of all the fitted values

Distance between two points

- Two numerical attributes *x* and *y*: $D = \sqrt{(x_0 - x_1)^2 + (y_0 - y_1)^2}$.
- Three numerical attributes *x, y,* and *z:*

$$D = \sqrt{(x_0 - x_1)^2 + (y_0 - y_1)^2 + (z_0 - z_1)^2}$$

***k*-Nearest Neighbors Classifier**

Choose *k* to be odd. To find the *k* nearest neighbors of an example:

- Find the distance between the example and each example in the training set
- Augment the training data table with a column containing all the distances
- Sort the augmented table in increasing order of the distances
- Take the top *k* rows of the sorted table

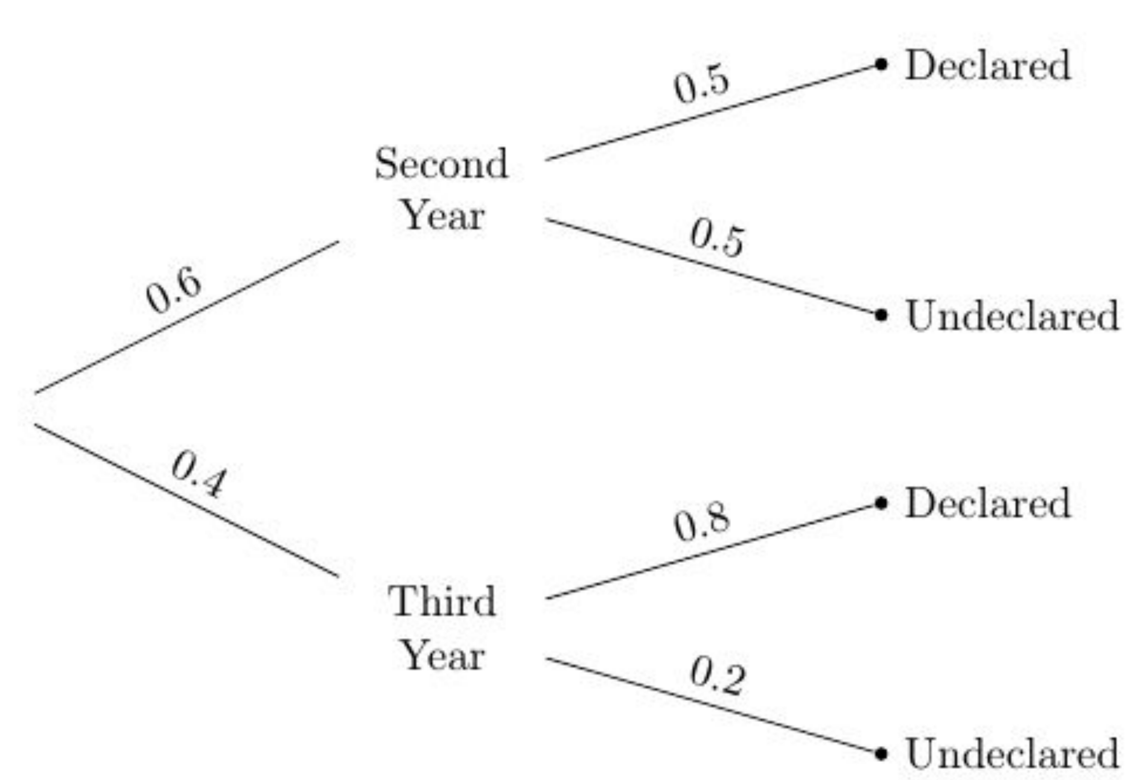
To classify an example into one of two classes:

- Find its *k* nearest neighbors
- Take a majority vote of the *k* nearest neighbors to see which of the two classes appears more often
- Assign the example the class that wins the majority vote

Accuracy of a classifier: The proportion of examples in the test set that are classified correctly

Scenario in which to apply Bayes Rule

- Class consists of second years (60%) and third years (40%)
- 50% of the second years have declared their major
- 80% of the third years have declared their major
- I pick one student at random... **That student has declared a major!**



Posterior probability:

P(Third Year | Declared)

$$= \frac{0.4 \times 0.8}{(0.6 \times 0.5) + (0.4 \times 0.8)}$$