YData: An Introduction to Data Science

Lecture 15: Sampling

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Credit: data8.org

Announcements

- HW05 due Thursday, 2/20
- Project 1 due Friday, 2/21

Probability

Basics

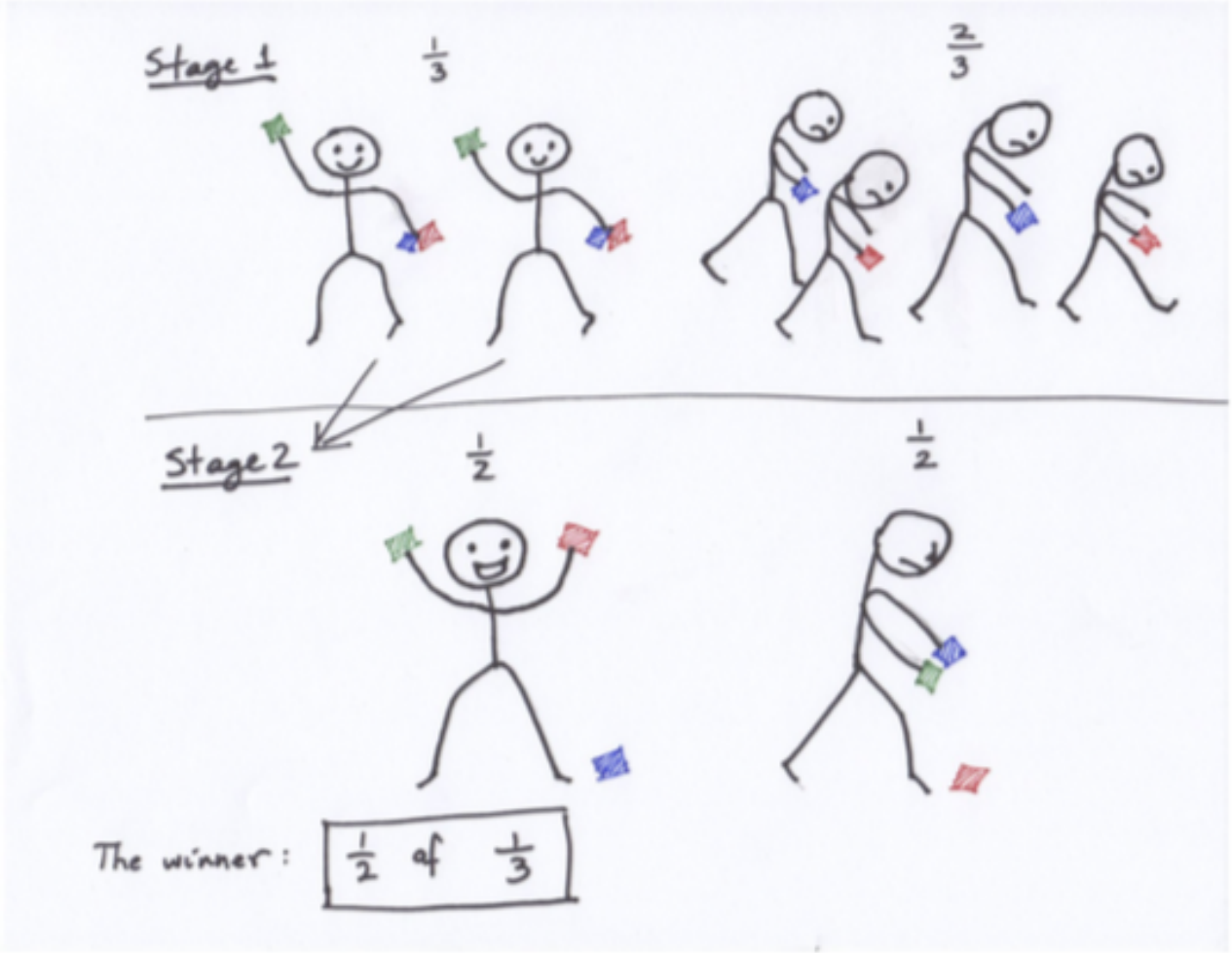
- Lowest value: 0
  - Chance of event that is impossible
- Highest value: 1 (or 100%)
  - Chance of event that is certain
- If an event has chance 70%, then the chance that it doesn't happen is
  - 100% - 70% = 30%
  - 1 - 0.7 = 0.3

Equally Likely Outcomes

Assuming all outcomes are equally likely, chance of an event A is:

$$P(A) = \frac{\text{number of outcomes that make A happen}}{\text{total number of outcomes}}$$

Fraction of a Fraction



Multiplication Rule

Chance that two events A and B both happen

$$= P(\text{A happens}) \times P(\text{B happens given that A has happened})$$

- The answer is less than or equal to each of the two chances being multiplied
- The more conditions you have to satisfy, the less likely you are to satisfy them all

Addition Rule

If event A can happen in *exactly* one of two ways, then

$$P(A) = P(\text{first way}) + P(\text{second way})$$

- The answer is *greater than or equal to* the chance of each individual way

Example: At Least One Head

- In 3 tosses:
  - Any outcome except TTT
  - P(TTT) $= (1/2) \times (1/2) \times (1/2) = 1/8$
  - P(at least one head) = 1 - P(TTT) $= 7/8 = 87.5\%$
- In 10 tosses:
  - 1 - (1/2)**10
  - 99.9%

Discussion Question

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We sample two people at random without replacement.

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(b) P(neither Mo nor Jo is in the sample)
= (98/100) * (97/99)
= 0.9602

Sampling

Sampling

- Deterministic sample:
  - Sampling scheme doesn't involve chance
- Probability sample:
  - Before the sample is drawn, you have to know the selection probability of every group of people in the population
  - Not all individuals have to have equal chance of being selected

Sample of Convenience

- Example: sample consists of whomever walks by
- Just because you think you're sampling "at random", doesn't mean you are.
- If you can't figure out ahead of time
  - what's the population
  - what's the chance of selection, for each group in the population

  then you don't have a random sample

(DEMO)

Distributions

Probability Distribution

- Random quantity with various possible values
- "Probability distribution":
  - All the possible values of the quantity
  - The probability of each of those values
- If you can do the math, you can work out the probability distribution without ever simulating the random quantity

Empirical Distribution

- "Empirical": based on observations
- Observations can be from repetitions of an experiment
- "Empirical Distribution"
  - All observed values
  - The proportion of times each value appears

Large Random Samples

Law of Averages

If a chance experiment is repeated many times, independently and under the same conditions, then the proportion of times that an event occurs gets closer to the theoretical probability of the event

As you increase the number of rolls of a die, the proportion of times you see the face with five spots gets closer to 1/6

Empirical Distribution of a Sample

If the sample size is large,
then the empirical distribution of a uniform random sample
resembles the distribution of the population,
with high probability

(DEMO)

A Statistic

Inference

- **Statistical Inference:**
  Making conclusions based on data in random samples

- **Example:**
  Use the data to guess the value of a fixed, unknown number

  Create an **estimate** [depends on the random sample] of the unknown quantity

Terminology

- **Parameter**
  - A number associated with the population
- **Statistic**
  - A number calculated from the sample

A statistic can be used as an estimate of a parameter

Probability Distribution of a Statistic

- Values of a statistic vary because random samples vary
- "Sampling distribution" or "probability distribution" of the statistic:
  - All possible values of the statistic,
  - and all the corresponding probabilities
- Can be hard to calculate
  - Either have to do the math
  - Or have to generate all possible samples and calculate the statistic based on each sample

Empirical Distribution of a Statistic

- Empirical distribution of the statistic:
  - Based on simulated values of the statistic
  - Consists of all the observed values of the statistic,
  - and the proportion of times each value appeared

- Good approximation to the probability distribution of the statistic
  - if the number of repetitions in the simulation is large

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