S&DS 265 / 565

Introductory Machine Learning

Neural Networks

October 31

Reminders

- Quiz 4 today
- Assn 4 due next week

For today

- Finishing up topic models – Gibbs sampling
- Neural networks

Outline: Neural networks

- Connecting to embeddings and logistic regression
- Alternative view of linear models
- Adding nonlinearities — activation functions
- Biological analogy and inspiration
- Backpropagation — high level
- Examples: Regression

Logistic Regression

Recall:

$$\log\left(\frac{P(y=1\,|\,x)}{P(y=0\,|\,x)}\right) = \beta^T x + \beta_0$$

Equivalently:

$$P(Y=1\,|\,x) \propto e^{\beta^T x + \beta_0}$$

Logistic Regression

In the multi-class case we have

$$P(Y = k \mid x) \propto e^{\beta_k^T x + \beta_{k0}}, \quad k = 1, \ldots, K-1$$

We can write this in ML terminology as

$$\text{Softmax}\left(\left\{\beta_k^T x + \beta_{k0}\right\}\right)$$

Note: Can also use β_k for $k = \underline{0}, \ldots, K-1$. This will be "overparameterized"

Logistic Regression

What if x is an image, represented as pixels? It might be hard to get an accurate classifier.

Want to learn *feature representation* $\phi(x)$.

The model becomes

$$P(Y = k \mid x) \propto e^{\beta_k^T \phi(x) + \beta_{k0}}, \quad k = 0, 1, \ldots, K-1$$

The parameters of ϕ and the parameters β need to be learned/trained.

Word embeddings

Applying this to language modeling, we could have a bigram model

The model becomes

$$P(v \mid w) \propto e^{\beta_v^T \phi(w)}$$

where $\phi(w)$ is a learned feature vector or "embedding vector" for each word.

The parameters β_v are also embeddings. By making them the same as ϕ we get the word2vec model.

Starting with regression

For linear regression, our loss function for an example (x, y) is

$$\begin{aligned}\mathcal{L} &= \frac{1}{2}(y - \beta^T x - \beta_0)^2 \\ &= \frac{1}{2}(y - f)^2\end{aligned}$$

where $f(x) = \beta^T x + \beta_0$.

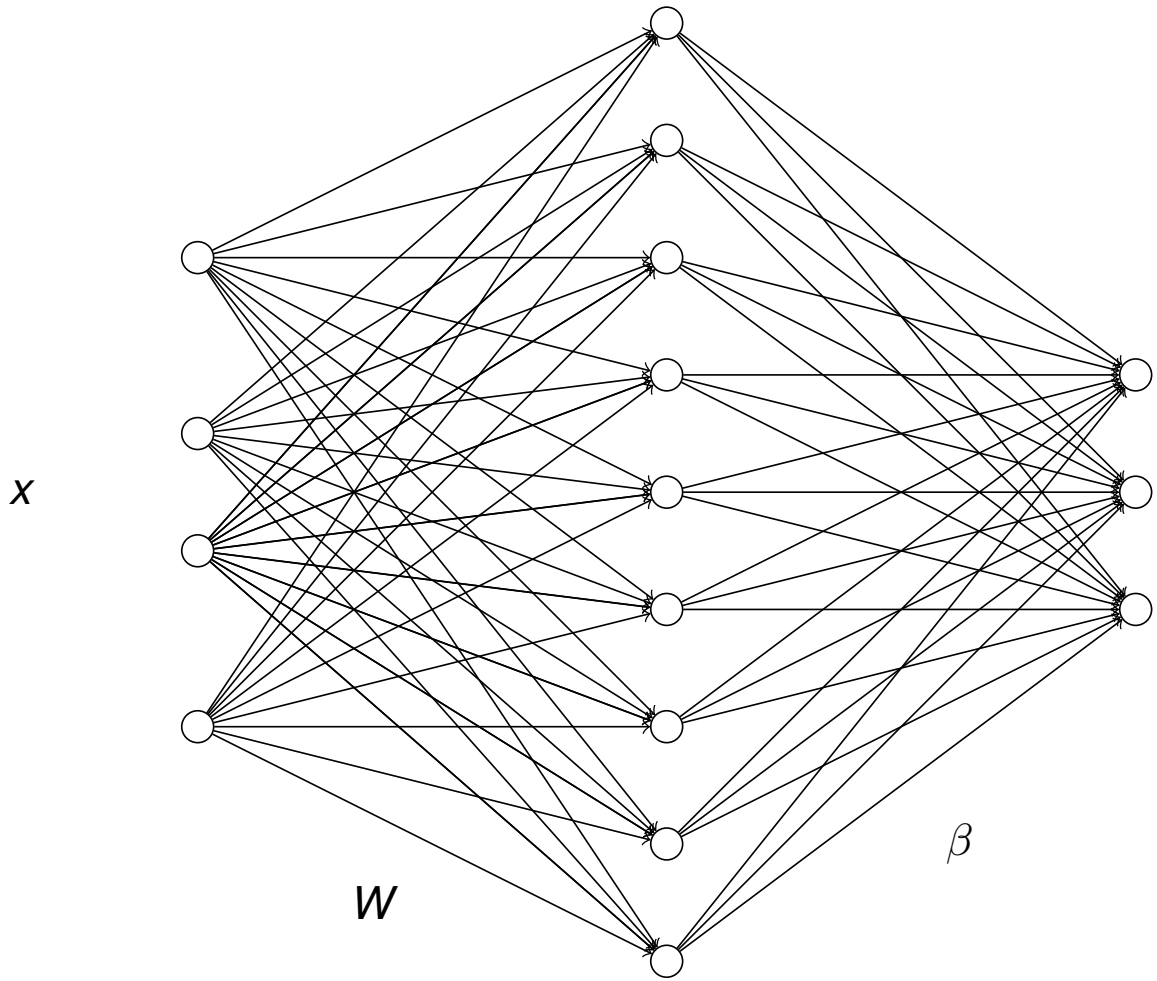
Adding a layer

Loss is

$$\mathcal{L} = \frac{1}{2}(y - f(x))^2$$

where now $f(x) = \beta^T h(x) + \beta_0$ where $h(x) = Wx + b$.

This can be viewed graphically.


x
W
β

Equivalent to linear model

But this is just a linear model

$$f = \widetilde{\beta}^T x + \widetilde{\beta}_0$$

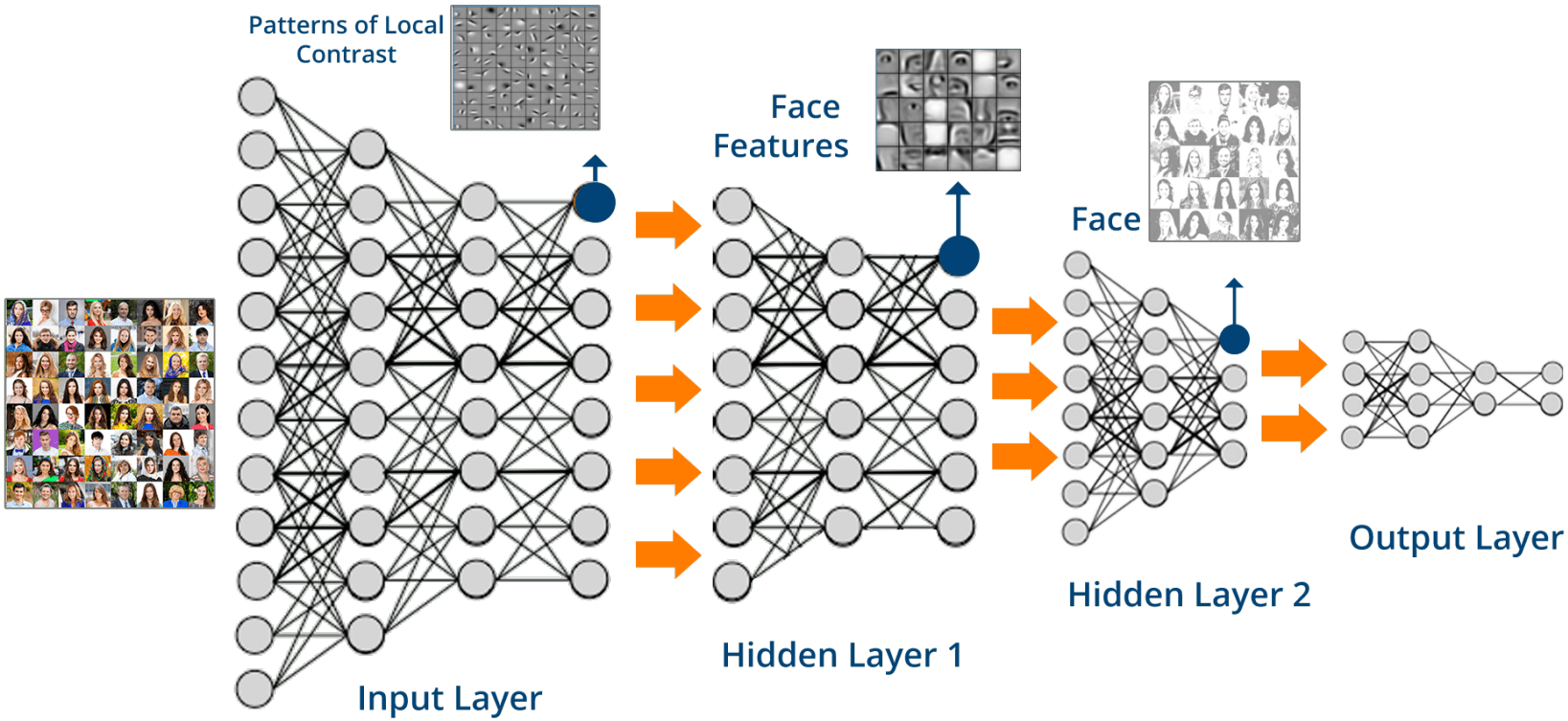
We get a reparameterization of a linear model; nothing new.

Need to add *nonlinearities*

Neural networks

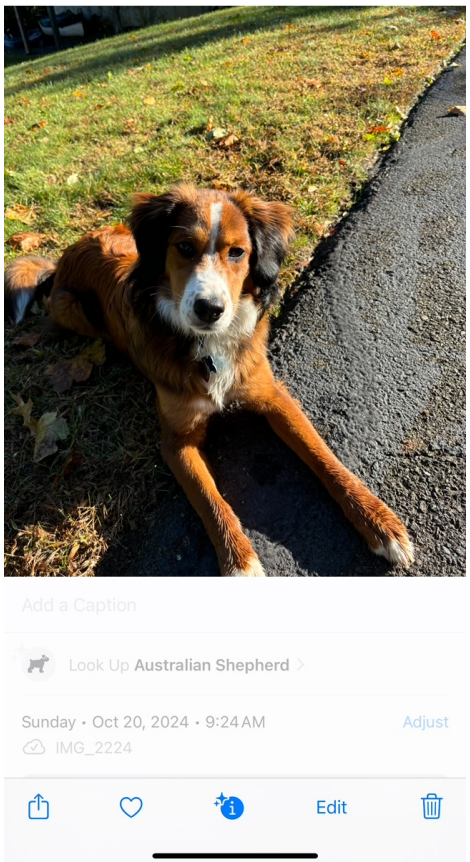
- Linear models are usually too simple to give good prediction rules
- Nonlinear models allow us to fit the data better—but run the risk of *overfitting*
- Neural networks are kind of in-between linear regression and k-nearest neighbors
- *Neural networks are layered linear models with carefully restricted nonlinearities*

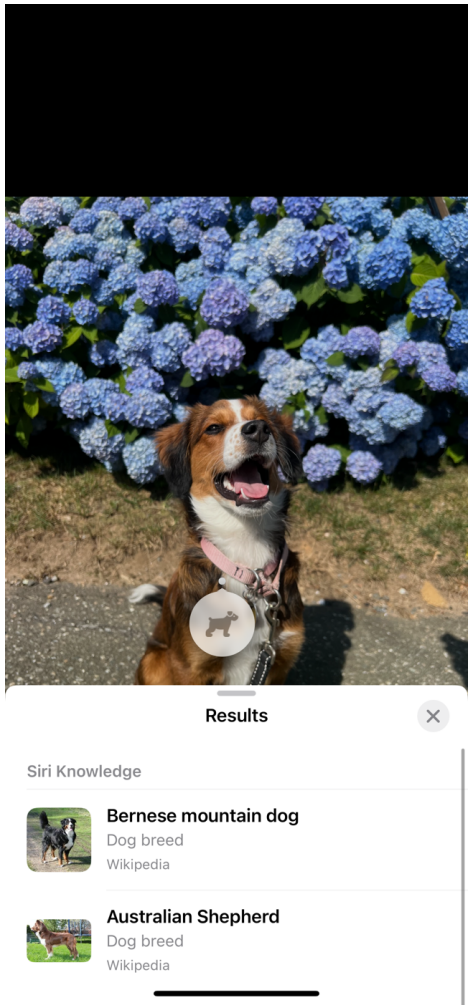
Deep neural networks

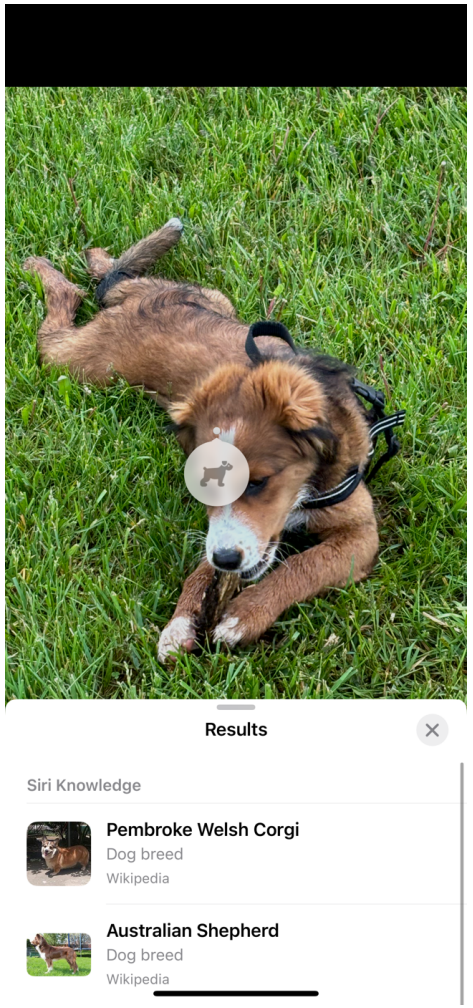


- Heuristics motivated from simplified view of the brain
- A particular form of nonlinear classification/regression

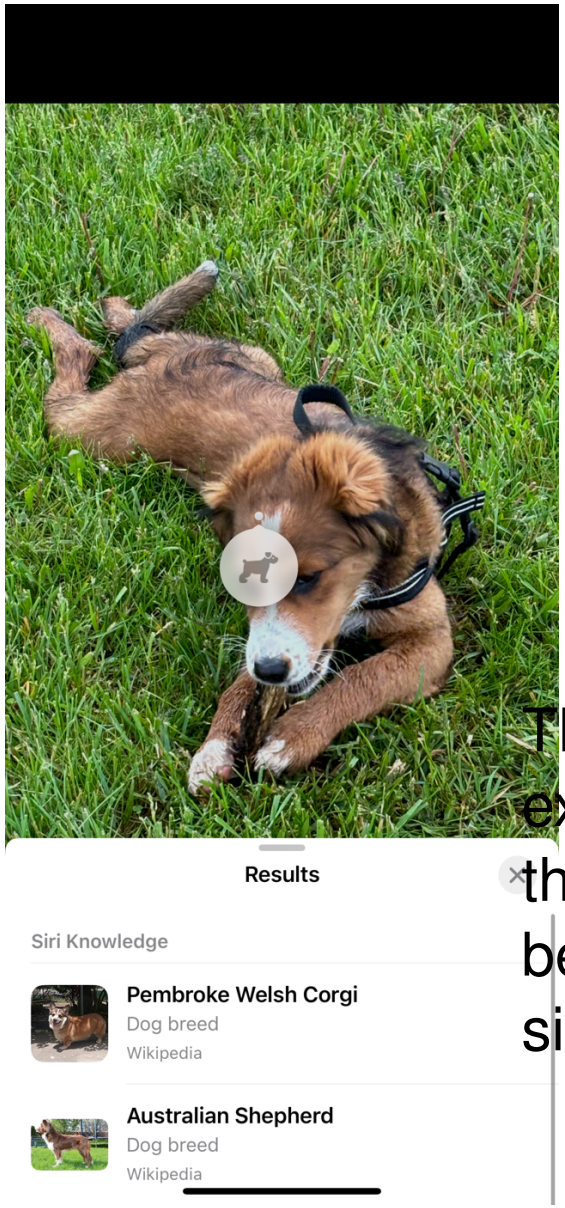
Everyday example







Everyday example



The neural network extracts “features” of the image that can be used to find similar images

Everyday example

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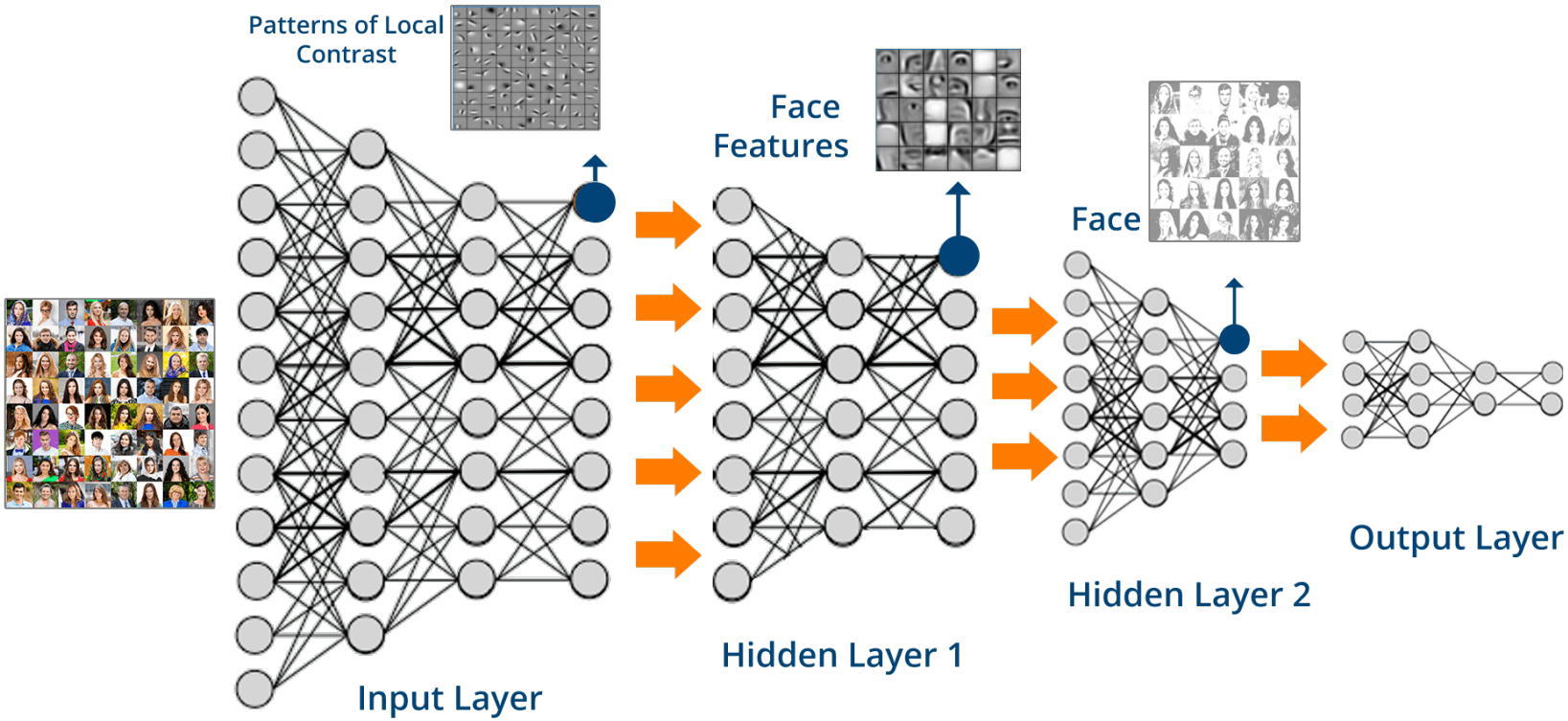
Deep neural networks



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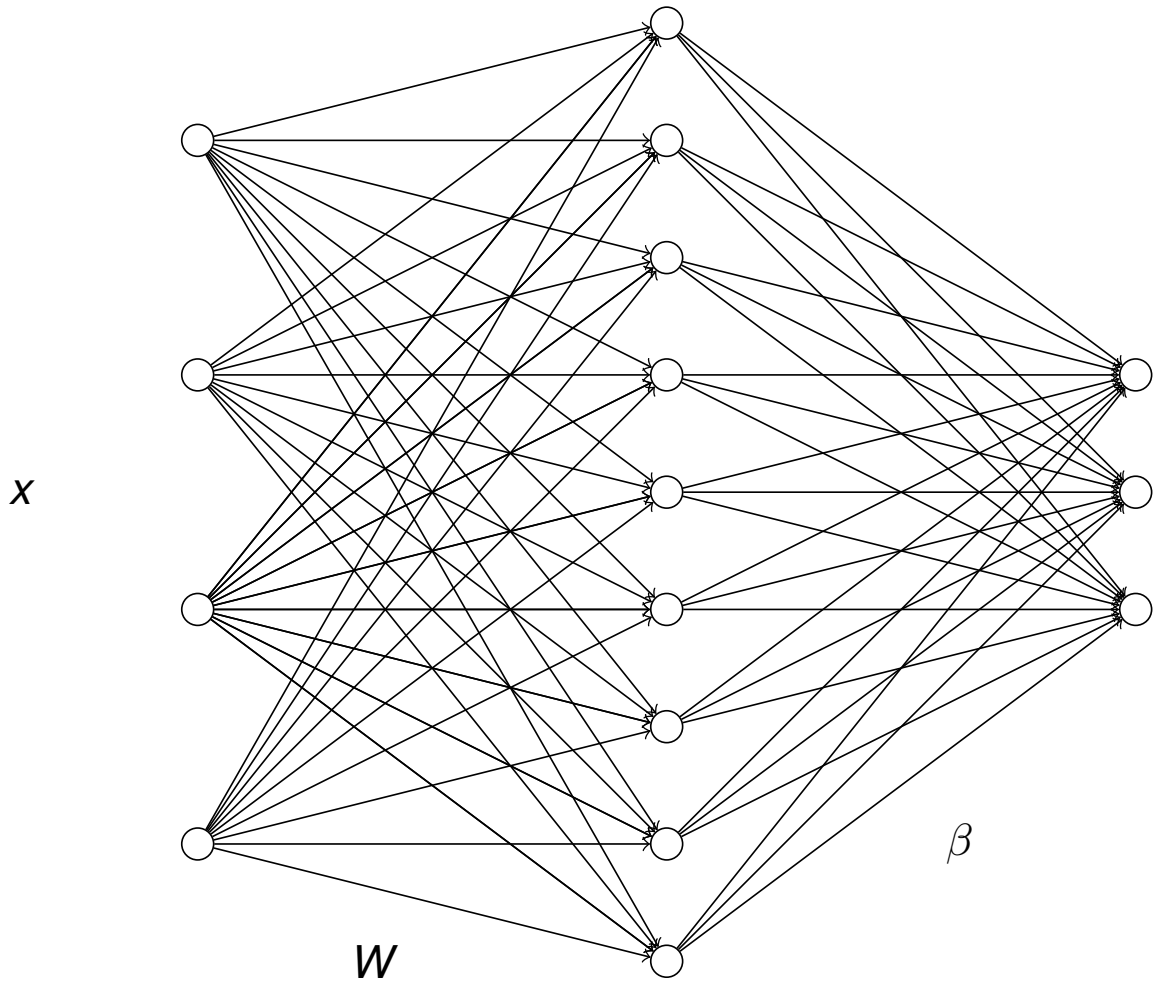

x
W
β

Nonlinearities

Add *fixed nonlinearity*, for example

$$h = \max(Wx + b, 0)$$

applied component-wise to each "neuron"

- Typically the last layer is just linear
- So, a neural net is a linear model, but with complex *predictor variables* or "features" that are automatically learned from data

Nonlinearities

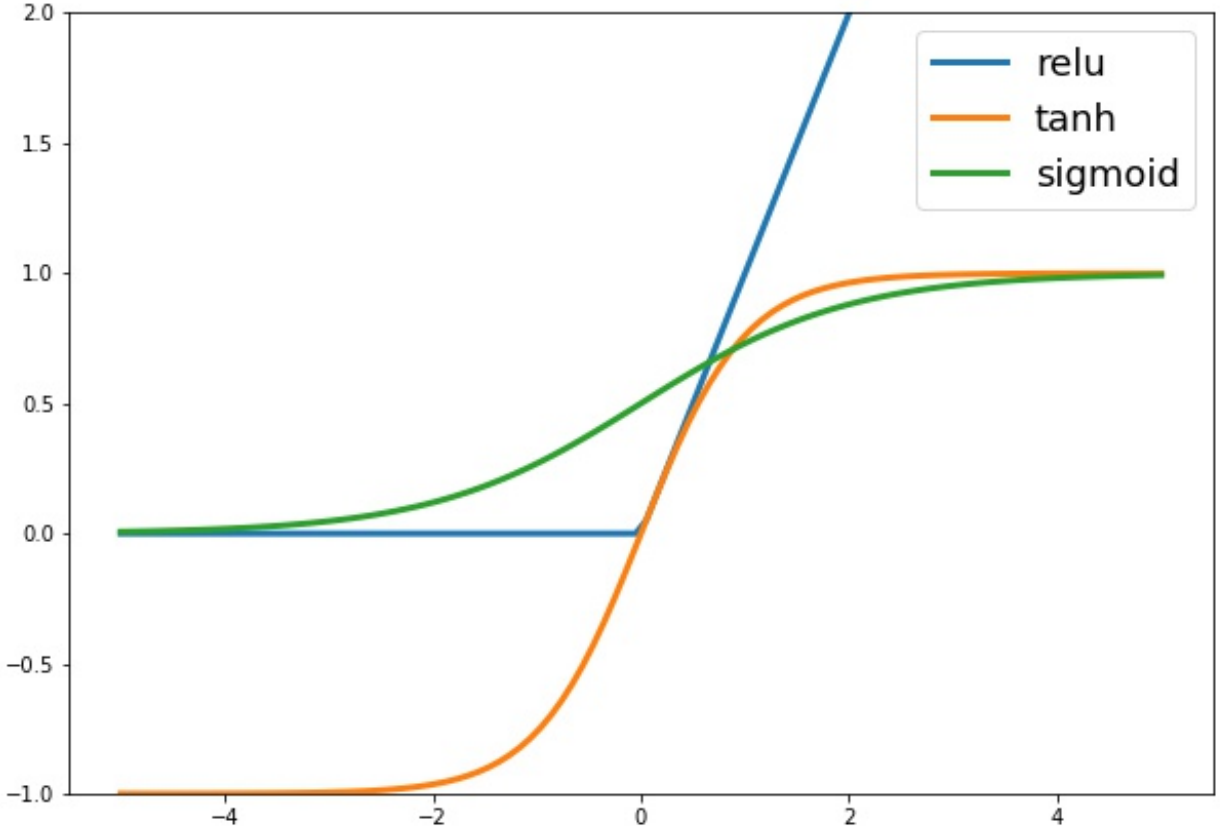
Commonly used nonlinearities:

$$\phi(u) = \tanh(u) = \frac{e^u - e^{-u}}{e^u + e^{-u}}$$

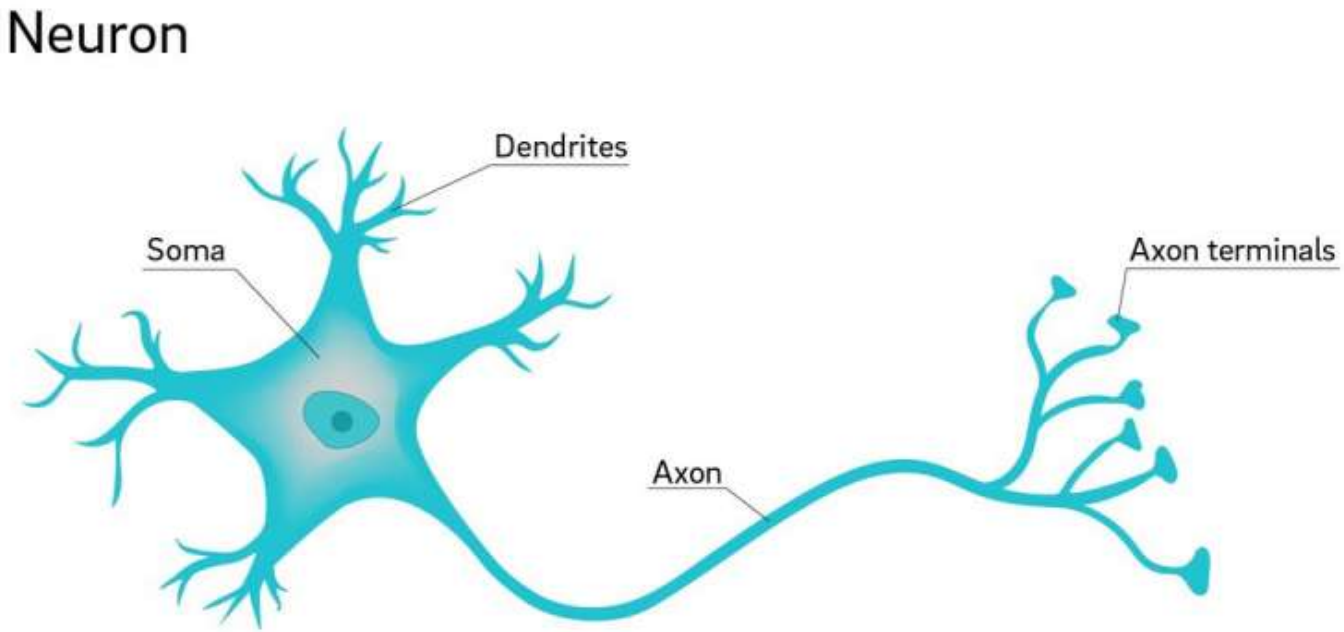
$$\phi(u) = \text{sigmoid}(u) = \frac{1}{1 + e^{-u}}$$

$$\phi(u) = \text{relu}(u) = \max(u, 0)$$

Nonlinearities

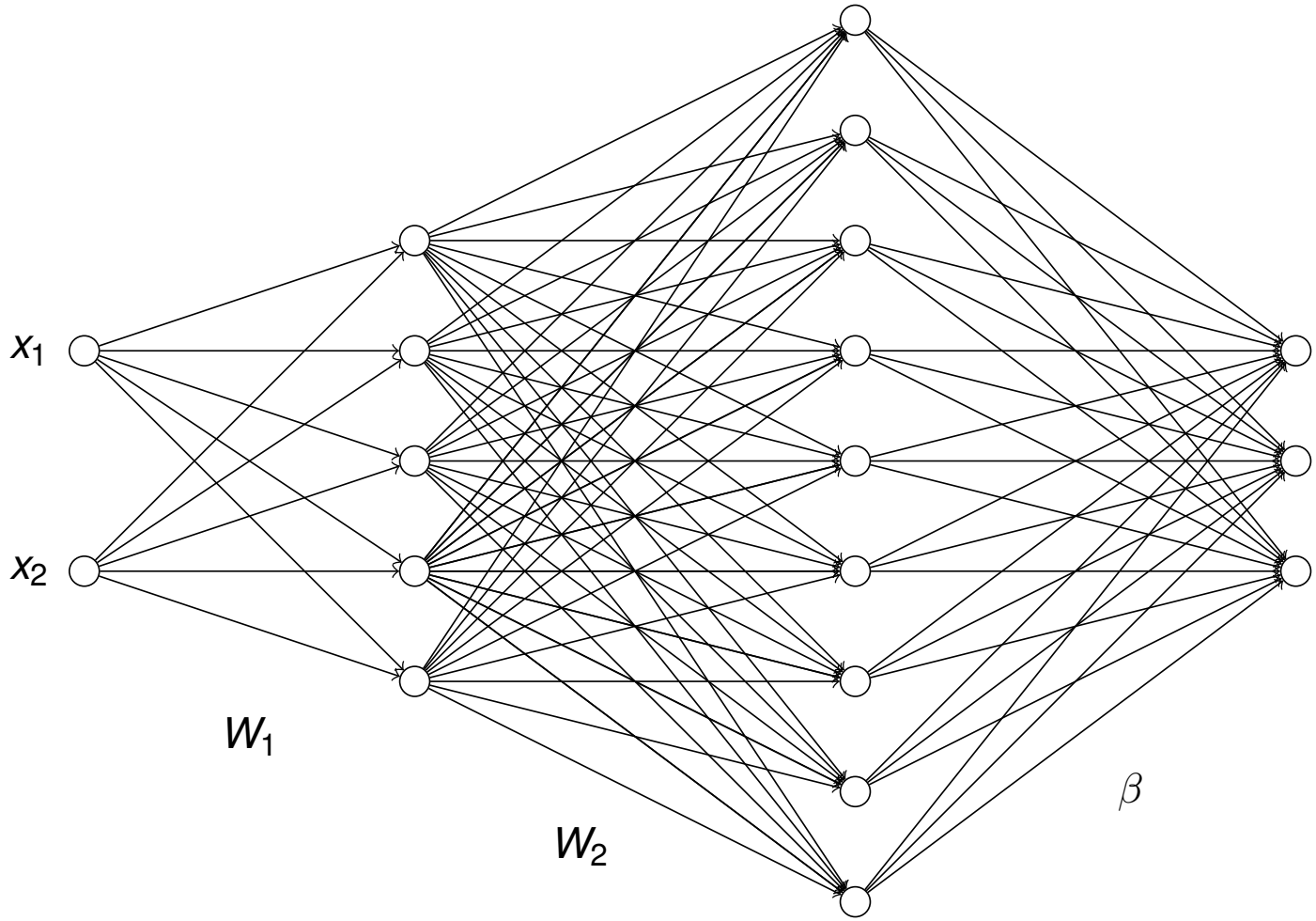


Biological analogy



Biological analogy

- The dendrites play the role of inputs, collecting signals from other neurons and transmitting them to the soma, which is the “central processing unit.”
- If the total input arriving at the soma reaches a threshold, an output is generated — this is the nonlinearity!
- The axon is the output device, which transmits the output signal to the dendrites of other neurons.

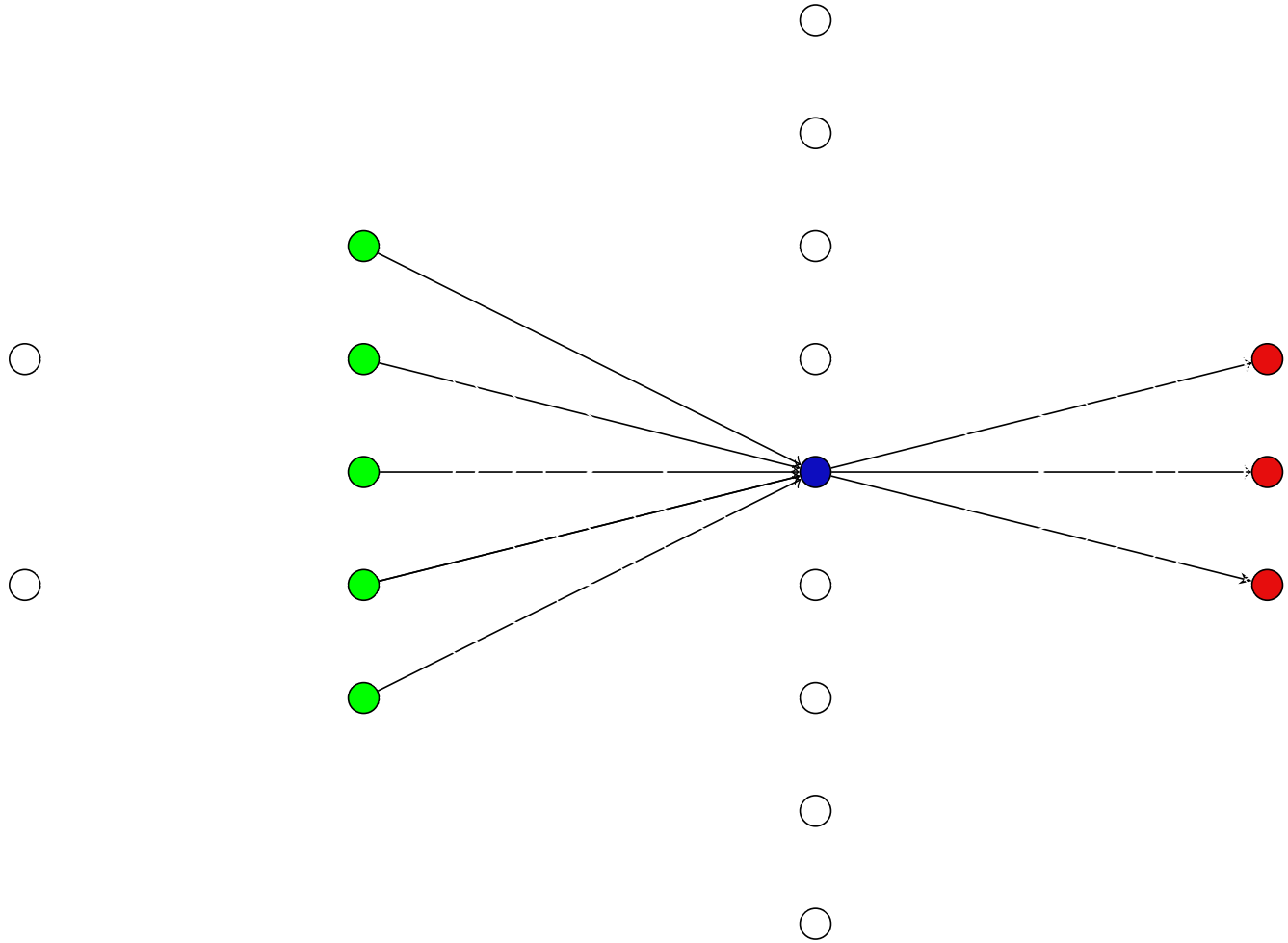


inputs

4 neurons
layer 1

9 neurons
layer 2

3 outputs



upstream neurons
(dentrites)

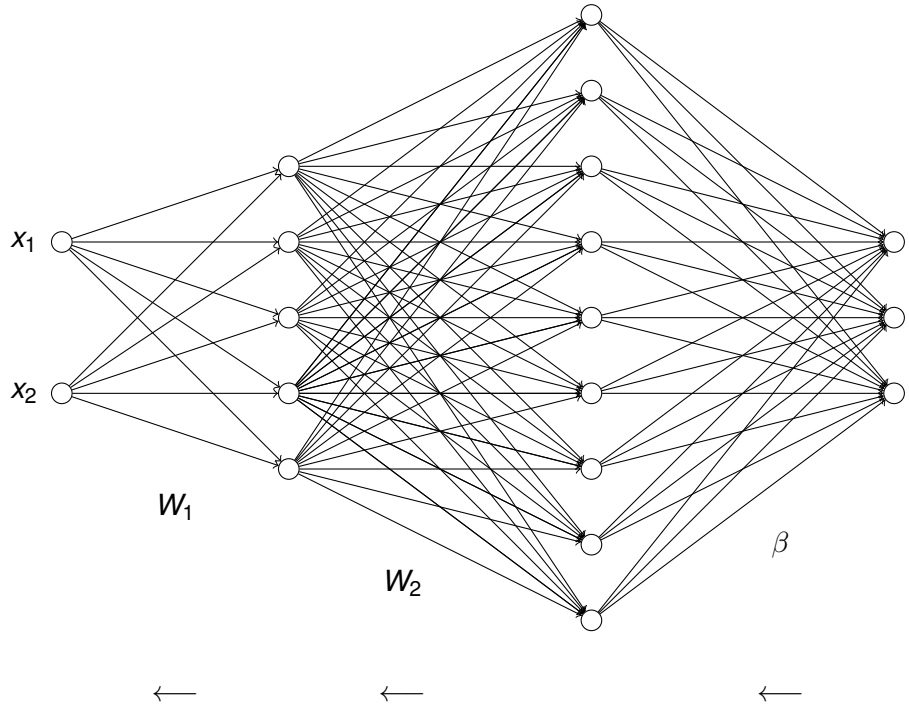
nonlinearity applied
(soma)

downstream neurons
(axon terminals)

Training

- The parameters are trained by “stochastic gradient descent”
  - Stream through the data, using small batches
  - Add a little bit to each network weight — proportional to how sensitive the predictions are to that weight (derivative)
  - Derivatives calculated with automatically — no math!
  - Stop if overfitting is detected!

High level idea: Backpropagation



Start at last layer, send error information back to previous layers

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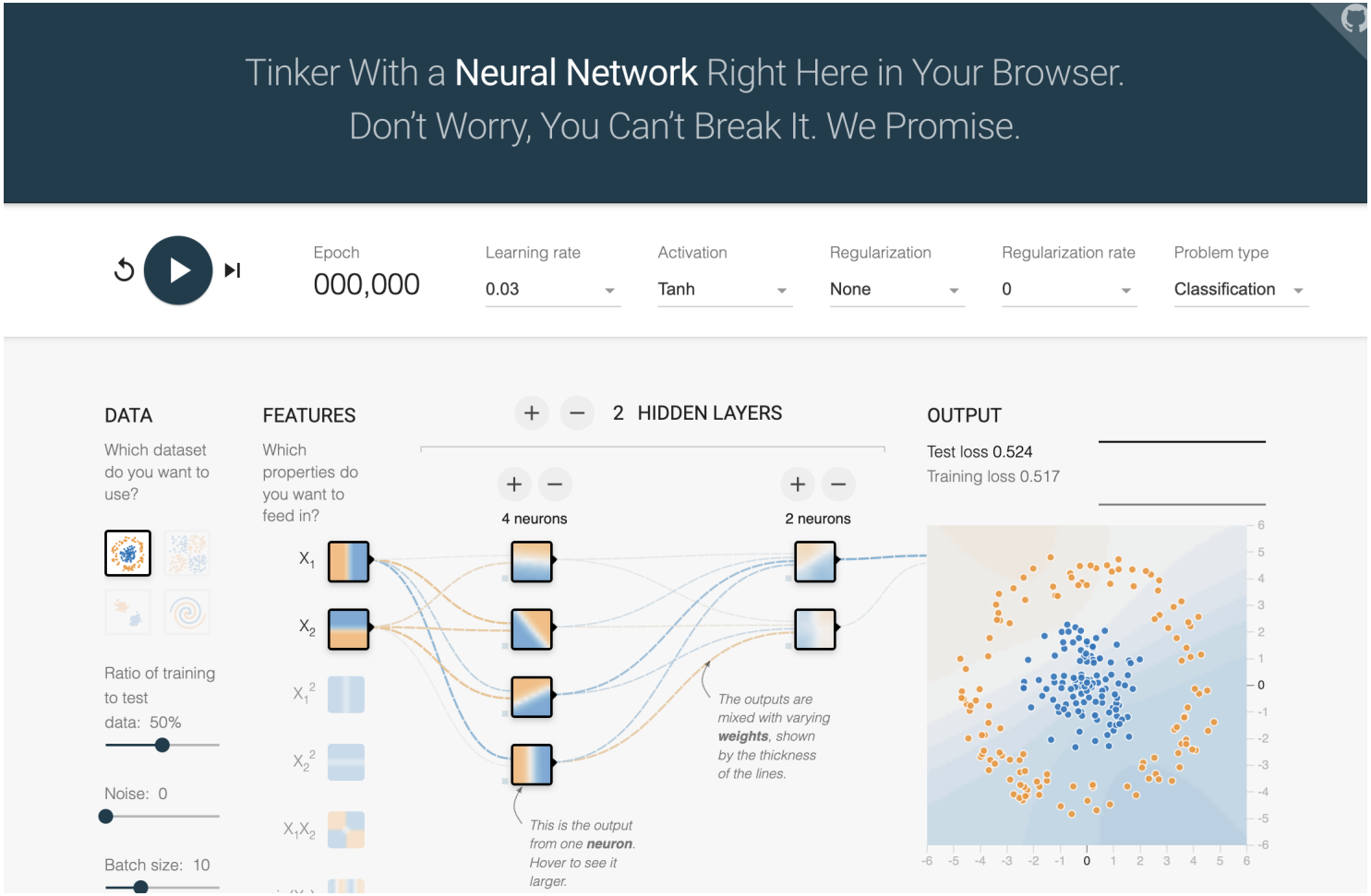
First described by Geoff Hinton, who won the Nobel Prize in Physics a couple weeks ago.

These types of neural networks are called "multilayer perceptrons" (MLPs)

They can be surprisingly robust to overfitting!

Demo



https://playground.tensorflow.org/

Summary

- For complex data we may want to learn features of the inputs
- This representation can be part of a logistic regression model
- Features that are linear transforms followed by a nonlinearity form the building blocks of (artificial) neural networks
- Based on a crude analogy with neurons in biological brains
- Trained using stochastic gradient descent
- Can be thought of as a particular type of nonlinear regression model