S&DS 265 / 565 Introductory Machine Learning

Reinforcement Learning

November 9

Reminders

- Assignment 4 due today
- Assignment 5 posted today (topic models, neural nets and RL)
- Final exam, Dec 15 at 2pm
 https://registrar.yale.edu/general-information/final-exams

For today and next class

- Overview of RL
- Q-learning
- Illustration on taxi problem
- RL concepts
- Deep reinforcement learning
- Neuroscience connection

Main reference: "Reinforcement Learning: An Introduction" (Second Edition), Richard S. Sutton and Andrew G. Barto

Reinforcement learning

- An agent interacts with an environment
- The actions the agent takes change the state of the environment
- The agent receives rewards for each action, and seeks to maximize the total cumulative reward

Reinforcement learning is a framework for sequential decision making to achieve a long-term goal.

Reinforcement learning: Motivating examples

- Learning to walk or ride a bike
- A robot vacuum cleaning up the house
- Playing chess, backgammon, Atari games, etc.
- Drug discovery, personalized health, energy management

Warning! Thinking about RL can change how you think about the world! :-)

In the news





In the news



Reinforcement learning: Formalization

- The environment is in state s at a given time
- The agent takes action a
- The environment transitions to state s' = next(s, a)
- The agent receives reward r = reward(s, a)

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This is said to be a *Markov decision process*. It's "Markov" because the next state only depends on the current state and the action selected. It's a "decision process" because the agent is making choices of actions in a sequential manner.

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Characteristics

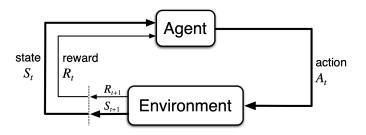
- RL is inherently sequential
- In between supervised and unsupervised learning
- Agent can't act too greedily; needs to be strategic

The aim of RL is to learn to make optimal decisions from experience

Some key RL concepts and principles:

Policy, reward signal, value function, model, Bellman equation

[&]quot;Reinforcement Learning: An Introduction" (Second Edition), Richard S. Sutton and Andrew G. Barto



Rewards and state transitions are probabilistic, in general

[&]quot;Reinforcement Learning: An Introduction" (Second Edition), Richard S. Sutton and Andrew G. Barto

Policy: A mapping from states to actions. An algorithm/rule to make decisions at each time step, designed to maximize the long term reward.

[&]quot;Reinforcement Learning: An Introduction" (Second Edition), Richard S. Sutton and Andrew G. Barto

Reward signal: The sequence of rewards received at each time step. An abstraction of "pleasure" (positive reward) and "pain" (negative reward) in animal behavior.

[&]quot;Reinforcement Learning: An Introduction" (Second Edition), Richard S. Sutton and Andrew G. Barto

Value function: A mapping from states to total reward. The total reward the agent can expect to accumulate in the future, starting from that state.

Rewards are short term. Values are predictions of future rewards.

[&]quot;Reinforcement Learning: An Introduction" (Second Edition), Richard S. Sutton and Andrew G. Barto

Model: Used for planning to mimic the behavior of the environment, to predict rewards and next states.

A *model-free* approach directly estimates a value function, without modeling the environment.

Analogous to distinction between generative and discriminative classification models

[&]quot;Reinforcement Learning: An Introduction" (Second Edition), Richard S. Sutton and Andrew G. Barto

Taxi problem

We'll introduce the important Q-learning algorithm with the toy "Taxi problem"

The code uses OpenAl gym

Taxi problem

A taxicab drives around the environment, picking up and delivering a passenger at four locations

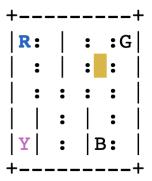


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Taxi problem

A taxicab drives around the environment, picking up and delivering a passenger at four locations

"Ascii art" rendition:



Taxi problem: Description

- Four designated locations: R(ed), G(reen), Y(ellow), and B(lue)
- Taxi starts off at random square and passenger is at random location
- Taxi drives to passenger's location, picks up the passenger, drives to passenger's destination, drops off passenger
- Once the passenger is dropped off, the episode ends.

- 25 taxi positions
- 5 possible locations of passenger: At waiting location or in taxi
- 4 possible destination locations
- Total number of states: $25 \times 5 \times 4 = 500$

Passenger location coded as integers:

- 0: R(ed)
- 1: G(reen)
- 2: Y(ellow)
- 3: B(lue)
- 4: in taxi

Destinations coded as:

0: R(ed)

• 1: G(reen)

• 2: Y(ellow)

• 3: B(lue)

Agent actions coded as:

- 0: move south
- 1: move north
- 2: move east
- 3: move west
- 4: pickup passenger
- 5: drop off passenger

Rewards:

- Default reward per step: -1
- Reward for delivering passenger: +20
- Illegal "pickup" or "drop-off": -10

State space represented as a tuple: state = (taxi row, taxi column, passenger location, destination)

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- Maintains a "quality" variable Q(s, a) for taking action a in state s
- A measure of the cumulative rewards obtained by the algorithm when it takes action a in state s
- Quality should not be assessed purely based on the reward the action has in the current time step
- Need to take into account the future rewards

$$Q(s, a) \leftarrow Q(s, a) + \alpha \left(\text{reward}(s, a) + \gamma \max_{a'} Q(\text{next}(s, a), a') - Q(s, a) \right)$$

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- When action a is taken in state s, reward reward(s, a) is given
- Then, the algorithm moves to a new state next(s, a)

$$Q(s, a) \leftarrow Q(s, a) + \alpha \Big(\text{reward}(s, a) + \gamma \max_{a'} Q(\text{next}(s, a), a') - Q(s, a) \Big)$$

- For example, if the taxi is location (2,2) and takes the "West" action (a=3), then there is a reward of -1, and the taxi moves to the new location (2,1)
- If cab is empty, it remains empty, and if it contains the passenger, the passenger remains.

$$Q(s, a) \leftarrow Q(s, a) + \alpha \Big(\text{reward}(s, a) + \gamma \max_{a'} Q(\text{next}(s, a), a') - Q(s, a) \Big)$$

- Cumulative future reward of this action is $\max_{a'} Q(\text{next}(s, a), a')$
- Future rewards discounted by factor $\gamma < 1$
- Trades off short-term against long-term rewards
- A gradient ascent algorithm, with step size α

Let's go to the notebook!

Bellman equation



The optimal value function is the largest expected discounted long term reward starting from that state.

Bellman equation

Curse of dimensionality [edit]

Main article: Curse of dimensionality

The *curse of dimensionality* is an expression coined by Bellman to describe the problem caused by the exponential increase in volume associated with adding extra dimensions to a (mathematical) space. One implication of the curse of dimensionality is that some methods for numerical solution of the Bellman equation require vastly more computer time when there are more state variables in the value function. For example, 100 evenly spaced sample points suffice to sample a unit interval with no more than 0.01 distance between points; an equivalent sampling of a 10-dimensional unit hypercube with a lattice with a spacing of 0.01 between adjacent points would require 10²⁰ sample points: thus, in some sense, the 10-dimensional hypercube can be said to be a factor of 10¹⁸ "larger" than the unit interval. (Adapted from an example by R. E. Bellman, see below.) ^[15]

Bellman equation: Deterministic case

The optimality condition for the value function v_* is

$$v_*(s) = \max_{a} \Big\{ \operatorname{reward}(s, a) + \gamma v_*(\operatorname{next}(s, a)) \Big\}$$

Bellman equation: Deterministic case

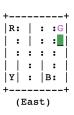
The optimality condition for the Q-function is

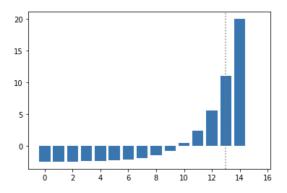
$$\mathbf{Q}_*(\mathbf{s}, \mathbf{a}) = \mathsf{reward}(\mathbf{s}, \mathbf{a}) + \gamma \max_{\mathbf{a}'} \mathbf{Q}_*(\mathsf{next}(\mathbf{s}, \mathbf{a}), \mathbf{a}')$$

and then $v_*(s) = \max_{a'} Q_*(s, a')$

Note how this makes sense in terms of the update rule:

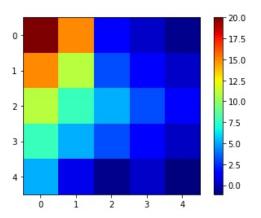
$$Q(s, a) \leftarrow Q(s, a) + \alpha \Big(\text{reward}(s, a) + \gamma \max_{a'} Q(\text{next}(s, a), a') - Q(s, a) \Big)$$





Question

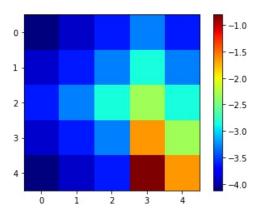
For a fixed passenger location and destination, the value function V(row, col) assigns a value to each of the $25 = 5 \times 5$ grid points.



Is the passenger waiting, or in the taxi?

Question

For a fixed passenger location and destination, the value function v(row, col) assigns a value to each of the $25 = 5 \times 5$ grid points.



Is the passenger waiting, or in the taxi?

Learning (and unlearning) to ride a bike



[&]quot;Learning to Drive a Bicycle Using Reinforcement Learning and Shaping," Jette Randløv and Preben Alstrøm, ICML 1998 Smarter Every Day: Learning to ride a backwards bike: https://www.youtube.com/watch?v=MFzDaBzBlL0

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Summary

- Reinforcement learning is a framework for sequential decision making to achieve a long-term goal
- The agent receives rewards for each action, and seeks to maximize the total cumulative reward
- The value of a state is the total reward the agent can expect to accumulate in the future, starting from that state
- Q-learning is an iterative algorithm that maximizes the "quality" of each state-action pair
- The Bellman equations are optimality conditions that characterize the value function