

S&DS 265 / 565
Introductory Machine Learning

Trees

September 28

Classification and Regression Trees (CART)

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- Response variables can be categorical or quantitative
- Yields a set of **interpretable decision rules**
- Predictive ability is mediocre, *but* can be improved by combining multiple trees (resampling, ensemble methods)


Titanic data

Sign In

Getting Started Prediction Competition

Titanic: Machine Learning from Disaster

Start here! Predict survival on the Titanic and get familiar with ML basics

 Kaggle · 17,691 teams · Ongoing

[Overview](#) [Data](#) [Notebooks](#) [Discussion](#) [Leaderboard](#) [Rules](#) [Join Competition](#)

Overview

Description

Evaluation

Frequently Asked Questions

👋🏠 Ahoy, welcome to Kaggle! You're in the right place.

This is the legendary Titanic ML competition – the best, first challenge for you to dive into ML competitions and familiarize yourself with how the Kaggle platform works.

The competition is simple: use machine learning to create a model that predicts which passengers survived the Titanic shipwreck.

Read on or watch the video below to explore more details. Once you're ready to start competing, click on the ["Join Competition" button](#) to create an account and gain access to the [competition data](#). Then check out [Alexis Cook's Titanic Tutorial](#) that walks you through step by step how to make your first submission!

Titanic data

- **Survived:** Outcome of survival (0 = No; 1 = Yes)
- **Pclass:** Socio-economic class (1 = Upper class; 2 = Middle class; 3 = Lower class)
- **Name:** Name of passenger
- **Sex:** Sex of the passenger
- **Age:** Age of the passenger (Some entries contain NaN)
- **SibSp:** Number of siblings and spouses of the passenger aboard
- **Parch:** Number of parents and children of the passenger aboard
- **Ticket:** Ticket number of the passenger
- **Fare:** Fare paid by the passenger
- **Cabin** Cabin number of the passenger (Some entries contain NaN)
- **Embarked:** Port of embarkation of the passenger (C = Cherbourg; Q = Queenstown; S = Southampton)

Trees



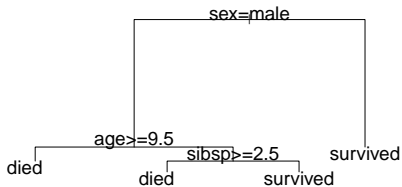
Trees



Trees

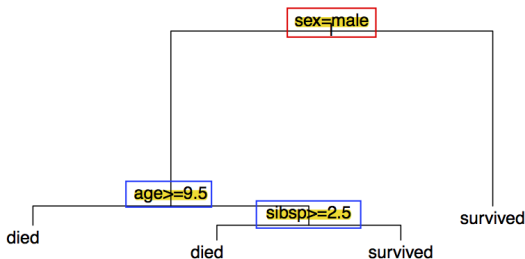


Modeling Titanic survival:



Tree terminology

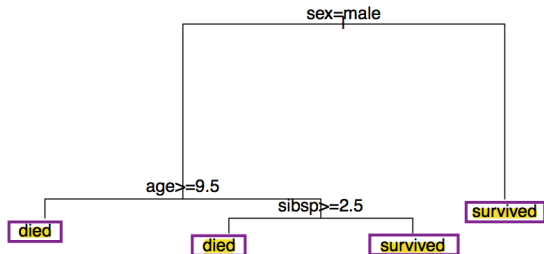
Internal nodes are points where the predictor space is split.



The internal node at the top is the **root** of the tree.

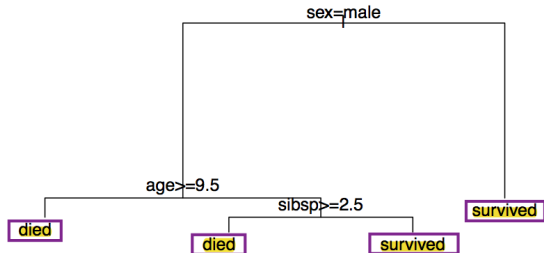
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Terminal nodes (or **leaves**) are the ends of the tree where no further splitting occurs.



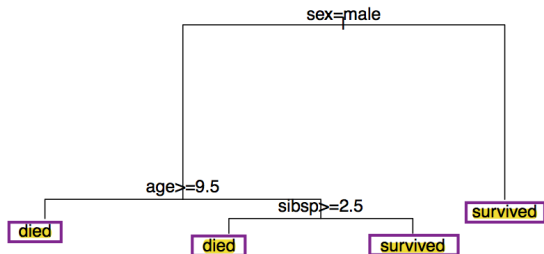
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Denote these J regions as R_1, \dots, R_J .

Tree terminology



- $R_1 = \{i : \text{sex}_i = \text{male} \cap \text{age}_i \geq 9.5\}$
- $R_2 = \{i : \text{sex}_i = \text{male} \cap \text{age}_i < 9.5 \cap \text{sibsp}_i \geq 2.5\}$
- $R_3 = \{i : \text{sex}_i = \text{male} \cap \text{age}_i < 9.5 \cap \text{sibsp}_i < 2.5\}$
- $R_4 = \{i : \text{sex}_i \neq \text{male}\}$

Let's go to the Titanic demo

Bias-variance

- As tree is grown deeper, bias decreases
- But the variance increases
- How to choose the right size of tree?

Stopping criterion

Once we stop, we relabel the terminal nodes to be R_1, \dots, R_J and compute \bar{y}_{R_j} (means within each region) to serve as \hat{y} values.

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Many options – resulting in tuning parameters that are hard to deal with.

Tree pruning

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Tree pruning

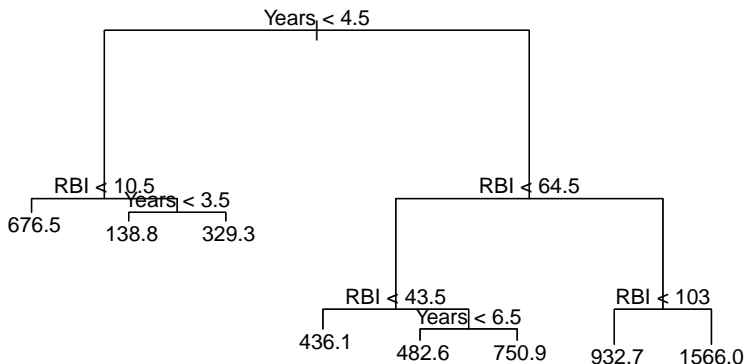
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- cross validation

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- cost-complexity pruning

Cost-complexity pruning

$$C(T) = \sum_{m=1}^{|T|} \sum_{i \in R_m} (y_i - \hat{y}_{R_m})^2 + \alpha |T|$$

α is a tuning parameter that controls for the complexity of the model.

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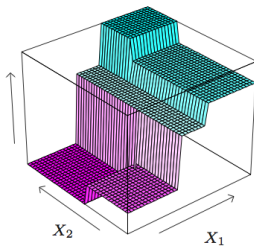
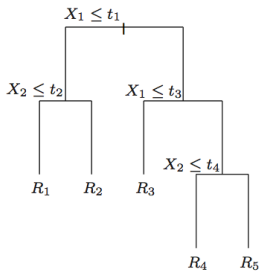
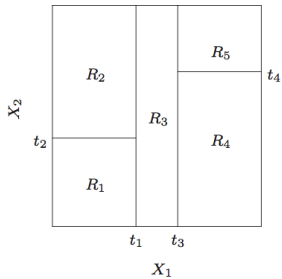
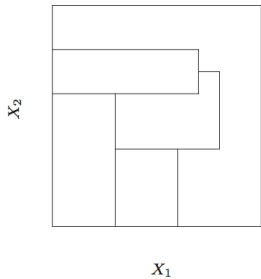
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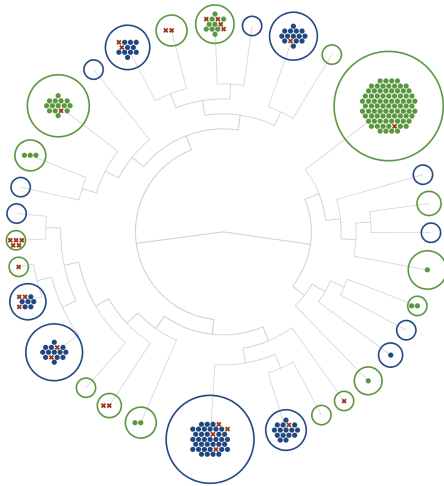
- $\alpha = 0$ implies the full tree
- Larger α implies higher penalty for complexity of model

Tree pruning

- 1 Grow a big tree on a *training set*.
- 2 Obtain a nested set of subtrees $T_L \subset \dots \subset T_2 \subset T_1 \subset T_0$ corresponding to a sequence of α values.
- 3 Use K -fold cross-validation to identify the subtree/ α that does best.



Beautiful demo <http://www.r2d3.us/>



Summary from today

- Trees give interpretable, nonlinear prediction rules
- Deep trees have low bias, high variance
- Shallow trees have high bias, low variance
- Deep trees are pruned back using cross-validation to find best bias/variance tradeoff.