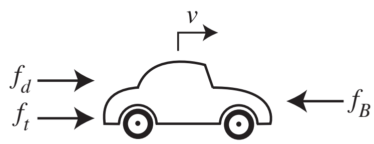
Homework #1

Reading: Chapters 1 and 2

Textbook Problems: None

Special Problem:



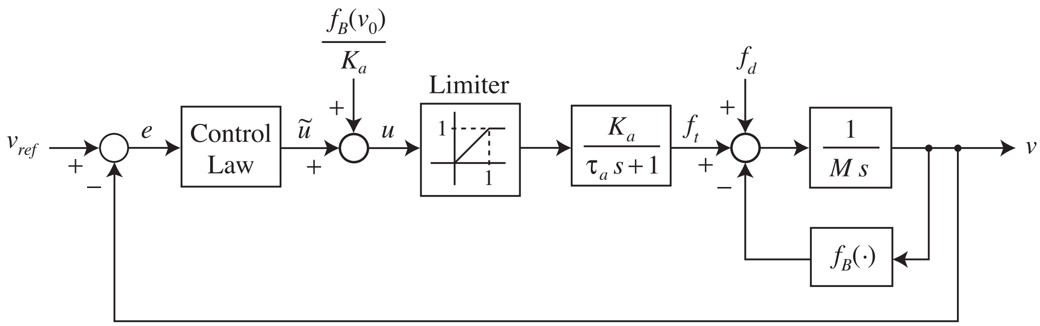


Figure 1. Speed control system block diagram.

1. The dynamics of the speed control system for a car are represented in Figure 1. Here:
    - v_{ref} is the reference or desired forward velocity (m/sec)
    - v_0 is the constant operating point velocity (m/sec)
    - u is the normalized (between 0 and 1) throttle position (dimensionless)
    - K_a is the gain constant for the engine and drive-train thrust response (N)
    - τ_a is the time constant for the engine and drive-train thrust response (sec)
    - f_t is the forward thrust developed by the engine and drive-train (N)
    - M is the mass of the car (kg)
    - v is the actual forward velocity of the car (m/sec)
    - f_d is a disturbance force that pushes the car forward (e.g., when the car goes down a hill) (N)
    - $f_B(\cdot) = B(\cdot)^2 \mathrm{sign}(\cdot)$ is the aerodynamic drag force (N)[1]

The mass of the car is $M = 1670$ kg.

The time constant for the engine and drive-train thrust response is $\tau_a = 0.5$ sec.

The car's maximum horsepower is 115 horsepower = 85,756 W = 85,756 Nm/sec.

The top speed of the car on a level road is 120 miles/hr = 53.63 m/sec.

(a) Show that $B = 0.5559$ N/(m/sec)^2 and $K_a = 1599$ N.

(b) Plot $f_B(v)$ versus v and use a Taylor series to determine a linear approximation to this function for small perturbations of v away from an operating point velocity v_0. Superimpose this linear approximation on your plot.

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1. $$\mathrm{sign}(x) = \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{if } x = 0 \\ -1 & \text{if } x < 0 \end{cases}$$

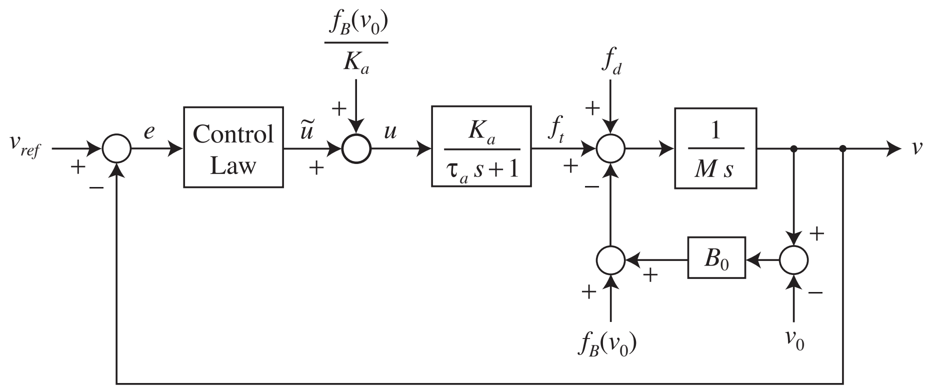

Figure 2. Small-perturbation speed control system dynamics.

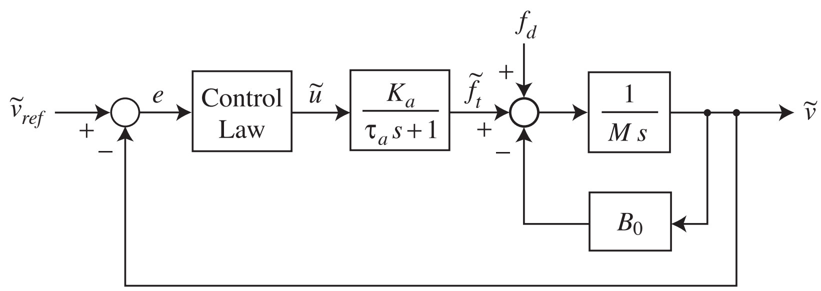

Figure 3. Linear model of small perturbation speed control system dynamics.

(c) Assuming that no limiting of the throttle occurs, show that, for small perturbations of v away from the operating point velocity v_0, the dynamics of the Figure 1 system are approximately represented by the block diagram in Figure 2, with

$$B_0 = 2 B v_0$$

(d) Use the block diagram in Figure 2 to show that:

$$\frac{dv}{dt} = \frac{1}{M}\{f_t + f_d - [f_B(v_0) + B_0(v - v_0)]\} \tag{1}$$

$$\tau_a \frac{df_t}{dt} + f_t = K_a \tilde{u} + f_B(v_0) \tag{2}$$

(e) Use (1) and (2) to show that:

$$\frac{d\tilde{v}}{dt} = \frac{1}{M}(\tilde{f}_t + f_d - B_0 \tilde{v}) \tag{3}$$

$$\tau_a \frac{d\tilde{f}_t}{dt} + \tilde{f}_t = K_a \tilde{u} \tag{4}$$

with:

$$\tilde{v} = v - v_0 \tag{5}$$

$$\tilde{f}_t = f_t - f_B(v_0) \tag{6}$$

(f) Use Figure 2 and (3) and (4) to show that, assuming that no limiting of the throttle occurs, for small perturbations of v away from the operating point velocity v_0, the dynamics of the Figure 1 system are approximately represented by the block diagram in Figure 3 with

$$\tilde{v}_{ref} = v_{ref} - v_0 \tag{7}$$