CSE 450 Assignment 4

13^{rd} November, 2022

Submission Instructions: Deadline is **11:59pm on 11/21/2022**. Late submissions will be penalized, therefore please ensure that you submit (file upload is completed) before the deadline. Additionally, you can download the submitted file to verify if the file was uploaded correctly. **Please TYPE UP YOUR SOLUTIONS and submit a PDF** electronically, via *Canvas*.
Furthermore, please note that the graders will grade 2 out of the 4 questions randomly. Therefore, if the grader decides to check questions 1 and 4,and you haven't answered question 4, you'll lose points for question 4. Hence, please answer all the questions.

1. Suppose that we have a set of activities to schedule among a large number of lecture halls, where any activity can take place in any lecture hall. We wish to schedule all the activities using as few lecture halls as possible. Give an efficient greedy algorithm to determine which activity should use which lecture hall. [**25 points**]

2. One of the basic motivations behind the Minimum Spanning Tree Problem is the goal of designing a spanning network for a set of nodes with minimum total cost. Here we explore another type of objective: designing a spanning network for which the most expensive edge is as cheap as possible. Specifically, let $G = (V, E)$ be a connected graph with n vertices, m edges, and positive edge costs that you may assume are all distinct. Let $T = (V, E')$ be a spanning tree of G; we define the bottleneck edge of T to be the edge of T with the greatest cost. A spanning tree T of G is a minimum-bottleneck spanning tree if there is no spanning tree T' of G with a cheaper bottleneck edge. [**25 points**]

   (a) Is every minimum-bottleneck tree of G a minimum spanning tree of G? Prove or give a counterexample.

   (b) Is every minimum spanning tree of G a minimum-bottleneck tree of G? Prove or give a counterexample.

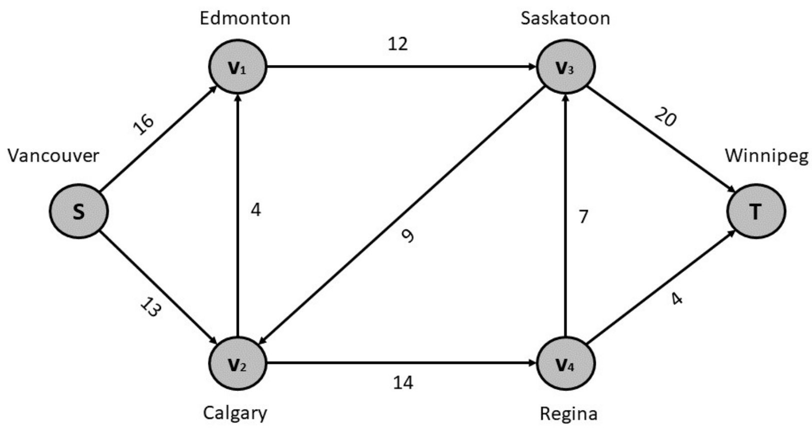

Figure 1: Network for Q3

3. For the network shown in Figure 1, compute the maximum flow from Vancouver to Winnipeg. **Show all your work**. **[25 points]**

4. Will the maximum flow from the source to the destination node in the Ford-Fulkerson Algorithm will be equal to the capacity of the minimum cut, if the capacity of a cut is defined in the following manner? For each definition, if you agree, then please provide arguments as to why. Else, provide a counter example to show that the definition does not hold: **[25 points]**

   (a) $C(S:\overline{S}) = \sum_{e\in(S:\overline{S})} C(e) + \sum_{e\in(\overline{S}:S)} C(e)$

   (b) $C(S:\overline{S}) = \sum_{e\in(S:\overline{S})} C(e)$