

CSE 450 Assignment 2

14th September, 2022

Submission Instructions: Deadline is **11:59pm on 09/20/2022**. Late submissions will be penalized, therefore please ensure that you submit (file upload is completed) before the deadline. Additionally, you can download the submitted file to verify if the file was uploaded correctly. **Please TYPE UP YOUR SOLUTIONS and submit a PDF** electronically, via *Canvas*.

Furthermore, please note that the graders will grade 2 out of the 4 questions randomly. Therefore, if the grader decides to check questions 1 and 4, and you haven't answered question 4, you'll lose points for question 4. Hence, please answer all the questions.

1. The Fibonacci series can be computed as follows,

$$F(n) = F(n-1) + F(n-2) \quad (1)$$

In class, we showed how this can be done in $\mathcal{O}(\log n)$ computation time. Now suppose that the definition is changed in the following way,

$$F'(n) = F'(n-1) + F'(n-2) + F'(n-3) + F'(n-4) \quad (2)$$

Can $F'(n)$ be computed in $\mathcal{O}(\log n)$? If yes, please show how it can be done. If no, show a counterexample where this fails. Please provide your rationale for both.

Assume that $F'(0) = 0, F'(1) = 1, F'(2) = 1, F'(3) = 1$. **[25 Points]**.

2. Find the closed form solution of the following recurrence relation and show the closed form solution in O, Θ , or Ω notation. **[25 points]**

$$T(n) = T(n-1) + n/2, T(1) = 1$$

3. You are to organize a tournament involving n competitors. Each competitor must play exactly once against each of his/her opponents. Moreover, each competitor must play exactly one match every day. If n is a power of 2, design a Divide-and-Conquer based algorithm to construct a time table allowing the tournament to be finished in $n-1$ days. Prove that the algorithm indeed completes the tournaments in $n-1$ days **[25 points]**

4. Design an algorithm to compute the 2nd smallest number in an unordered (unsorted) sequence of numbers $\{a_1, a_2, \dots, a_n\}$ in $n + \lceil \log_2(n) \rceil - 2$ comparisons in the worst case. If you think such an algorithm can be designed, then show how it can be done. If your answer is no, then explain why it cannot be done. **[25 points]**