Coping with NP-Complete Problems

- A combinatorial optimization problem P is either a minimization or a maximization problem and consists of the following three parts
  - A set D_P of instances
  - For each instance I a finite set $S_P(I)$ of candidate solutions for I
  - A function m_P that assigns to each instance I and each candidate solution $s \in S_P(I)$ a positive rational number $m_P(I, s)$ called the solution value for s.

- If a combinatorial optimization problem is NP-Complete,
  it might take an absurdly long time (e.g., 300 centuries) to find the optimal solution for the problem.

- Probably cannot wait 300 centuries to find the solution

- However, the problem does not go away. One still has to find a solution!

Approximation Algorithms

- If the optimal solution is unattainable then it is reasonable to sacrifice optimality and settle for a "good" (close to optimal) feasible solution that can be computed efficiently.

- We would like to sacrifice as little optimality as possible, while gaining as much as possible in efficiency.

- Trading-off optimality in favor of tractability is the paradigm of approximation algorithms

- Dorit. S. Hochbaum

Approximation Algorithms for NP-Hard Problems

Approximation Algorithms

ε-approximation algorithm

Let Π be an optimization (minimization or maximization) problem and A an algorithm which, given an instance I of Π, returns a feasible (but not necessarily optimal) solution denoted by APP(I). Let the optimal solution be denoted by OPT(I). The algorithm A is called an ε-approximation algorithm for Π for some $\varepsilon \geq 0$ if and only if

$| APP(I) - OPT(I) | / OPT(I) \leq \varepsilon$ for all instances I

Yet Another Job Scheduling Problem

- $P_1, \ldots, P_m$: A set of m independent processors with similar (dissimilar) performance characteristics
- $T_1, \ldots, T_n$: A set of n independent tasks with no ordering relationship between them
- If the processors are dissimilar (heterogeneous computing environment), the execution time of a task on different processors are different.
- t_{ij} = Execution time of task T_i on Processor P_j
- If the processors are similar (homogeneous computing environment), the execution time of a task on different processors are same.

Yet Another Job Scheduling Problem

- Total execution time used by Processor P_j is the sum of the execution times of the tasks assigned to this processor
- Makespan of an assignment is maximum of the total execution times of individual processors
- Objective: Find the assignment that minimizes the makespan
- Question: How difficult is it to find the schedule with the minimum makespan?

Algorithm for the Job Scheduling Problem

(Heterogeneous Environment)

Step1: for j:=1 to m do

$F_j := 0$;

Step2: for i:=1 to n do

begin

Step 2.1 Find k where k is the smallest integer j for which $F_j + t_{i,j}$ is minimum ($1 \leq j \leq m$)

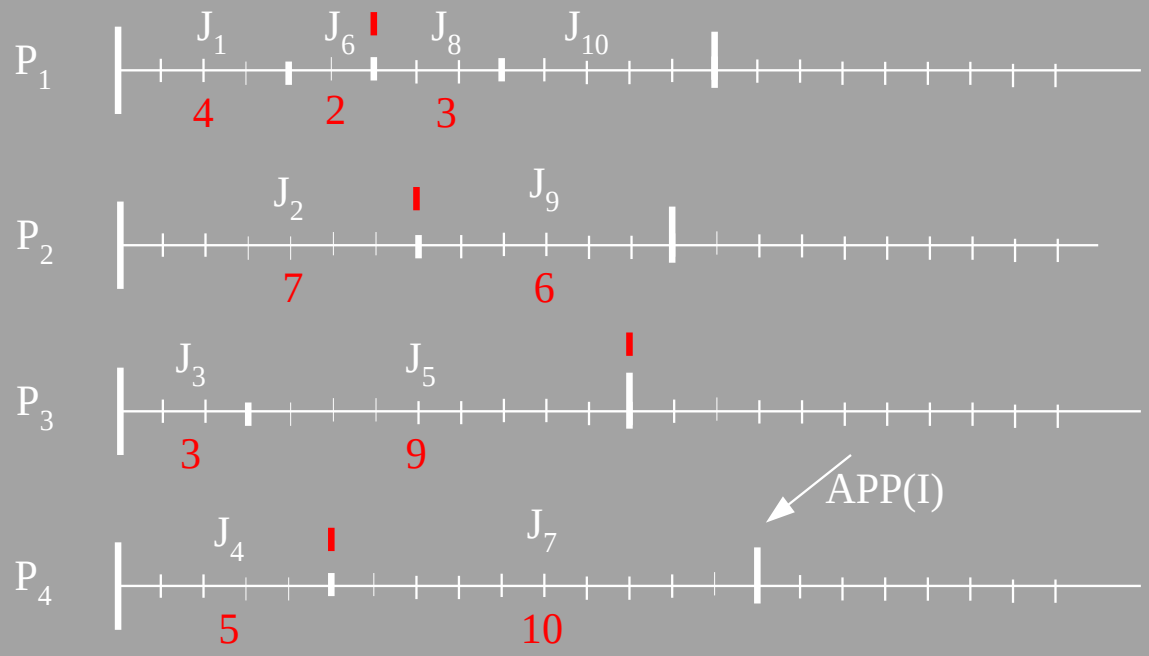
Step 2.2 Assign task T_i to processor P_k

Step 2.3 Update $F_k \Leftarrow F_k + t_{i,j}$

end

Jobs $\Rightarrow \{J_1, J_2, J_3, J_4, J_5, J_6, J_7, J_8, J_9, J_{10}\}$

Execution Time $\Rightarrow \{4, 7, 3, 5, 9, 2, 10, 3, 6, 8/5\}$



Case 1 $t_{10} + F_1 \geq F_4 \Rightarrow APP(I) = F_1 + t_{10}$

Case 2 $t_{10} + F_1 < F_4 \Rightarrow APP(I) = F_4$

Case 1: $App(I) \le F_1 + t_{10}$

$$App(I) \le F_1 + t_{10}$$
$$App(I) \le F_2 + t_{10}$$
$$App(I) \le F_3 + t_{10}$$
$$App(I) \le F_4 + t_{10}$$

$$App(I) \le F_j + t_n$$
$$(1 \le j \le m)$$

We put it in the first one because, F2 + t10, F3 + t10, F4 + t10 is all greater than the approximation. So we just put into first one.

We have m number of processors

$$m\ App(I) \le F_1 + F_2 + \ldots + F_m + mt_n$$

$$= \sum_{i=1}^{n-1} t_i + mt_n$$

Sum all execution time

$$= \sum_{i=1}^{n} t_i + (m-1)t_n$$

or

$$App(I) \le \frac{1}{m}\sum_{i=1}^{n} t_i + \frac{m-1}{m} t_n$$

Divide by m on both sides

That is logically correct if you just take all times and divide equally among processor it should be absolute optimal...

Lower bound for optimal value as you just add up all ti and then divide by m... And optimal value must be greater than this.

$$\le OPT(I) + \frac{m-1}{m} OPT(I)$$

$$App(I) \le (2 - \frac{1}{m}) OPT(I)$$

The m is the number of processor, and if we have large number of processor, then m goes to infinity, so this will result in 2.

as $$OPT(I) \ge \frac{1}{m}\sum_{i=1}^{n} t_i \quad .....(1)$$

and $$OPT(I) \ge \max\{t_i \mid 1 \le i \le m\}$$

Case 2 $App(I) = F_4$

F_4 was the finish time at the end of scheduling jobs J_1,.., J_7. Let the finish time on the processors at this time be $F_1', F_2', ..., F_m'$ (before scheduling job J_7).

$$App(I) \le F_1' + t_7$$
$$App(I) \le F_2' + t_7$$
$$App(I) \le F_3' + t_7$$
$$App(I) \le F_4' + t_7$$

$$App(I) \le F_j' + t_k \quad (1 \le j \le m)$$

$$m\ App(I) \le F_1' + F_2' + \ldots + F_m' + mt_k$$

$$= \sum_{i=1}^{k-1} t_i + mt_k = \sum_{i=1}^{k} t_i + (m-1)t_k$$

$$\le \sum_{i=1}^{n} t_i + (m-1)t_k$$

or $$App(I) \le \frac{1}{m}\sum_{i=1}^{n} t_i + \frac{m-1}{m} t_k$$

$$App(I) \le OPT(I) + \frac{m-1}{m} OPT(I)$$

$$= (2 - \frac{1}{m}) OPT(I)$$