Topics: Normal distribution, Functions of Random Variables

1. The time required for servicing transmissions is normally distributed with μ = 45 minutes and σ = 8 minutes. The service manager plans to have work begin on the transmission of a customer's car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?

   A. 0.3875
   B. 0.2676
   C. 0.5
   D. 0.6987

   Ans) b

2. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean μ = 38 and Standard deviation σ=6. For each statement below, please specify True/False. If false, briefly explain why.
   A. More employees at the processing center are older than 44 than between 38 and 44.
   B. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

   Ans)

A. True.

Given that the distribution is normal with a mean of 38 and a standard deviation of 6, we can infer that the majority of employees would fall within one standard deviation of the mean, which is between 32 and 44 years old. However, since the distribution is symmetric, there would be more employees older than 44 than between 38 and 44.

B. False.

To calculate the number of employees under the age of 30, we need to find the proportion of the normal distribution below 30 years old and then multiply it by the total number of employees (400).

Using the Z-score formula: Z=x-μ/, where x=30, μ=38, and σ=6, we get Z=630−38 =−1.33.

Referring to the standard normal distribution table or using a calculator, we find that the proportion of the distribution below Z=−1.33 is approximately 0.0918.

Thus, the expected number of employees under the age of 30 would be 0918×400=36.72. However, since the number of employees must be a whole number, we would expect about 37 employees, not 36. Therefore, the statement is false.

3. If $X_1 \sim N(\mu, \sigma^2)$ and $X_2 \sim N(\mu, \sigma^2)$ are *iid* normal random variables, then what is the difference between 2 X_1 and $X_1 + X_2$? Discuss both their distributions and parameters.

   Ans)

   the difference between $2X1$ and $X1 \quad +X2$ lies in their variances. The distribution of $2X1$ has a larger variance ($4\sigma 2$) compared to the distribution of $X1 \quad +X2$ ($2\sigma 2$). This is because adding two independent random variables typically results in a smaller variance compared to multiplying a random variable by a constant.

4. Let X ~ N(100, 20^2). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.

   A. 90.5, 105.9
   B. 80.2, 119.8
   C. 22, 78
   D. 48.5, 151.5
   E. 90.1, 109.9

   Ans) d

5. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions $\text{Profit}_1 \sim N(5, 3^2)$ and $\text{Profit}_2 \sim N(7, 4^2)$ respectively. Both the profits are in \$ Million. Answer the following questions about the total profit of the company in Rupees. Assume that \$1 = Rs. 45

   A. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
   B. Specify the 5th percentile of profit (in Rupees) for the company
   C. Which of the two divisions has a larger probability of making a loss in a given year?

A. To specify a Rupee range centered on the mean such that it contains 95% probability for the annual profit of the company, we need to find the mean and standard deviation of the total profit, then use the properties of the normal distribution.

Given: Profit1 ~ N(5, 32) (Mean = 5, Variance = 32) Profit2 ~ N(7, 42) (Mean = 7, Variance = 42)

First, we find the mean and variance of the total profit: Mean of total profit = Mean of Profit1 + Mean of Profit2 = 5 + 7 = 12 Variance of total profit = Variance of Profit1 + Variance of Profit2 = 32 + 42 = 74

Now, we convert the variance from dollars to rupees: 1 dollar = 45 rupees So, variance in rupees = 74 * (45)^2 = 148500

Standard deviation of total profit = sqrt(148500) ≈ 385.75

Now, we can find the range centered on the mean that contains 95% probability using the properties of the normal distribution: For a normal distribution, approximately 95% of the values lie within 1.96 standard deviations from the mean.

So, the Rupee range centered on the mean for 95% probability is: Mean ± (1.96 * standard deviation) = 12 ± (1.96 * 385.75) = 12 ± 755.66

So, the Rupee range is (12 - 755.66, 12 + 755.66), which is approximately (-743.66, 767.66) million rupees.

B. To specify the 5th percentile of profit in rupees for the company, we need to find the value below which 5% of the profits fall.

Using the properties of the normal distribution, the 5th percentile corresponds to approximately 1.645 standard deviations below the mean.

Profit at 5th percentile = Mean - (1.645 * standard deviation) = 12 - (1.645 * 385.75) = 12 - 634.64 ≈ -622.64 million rupees

So, the 5th percentile of profit for the company is approximately -622.64 million rupees.

C. To determine which division has a larger probability of making a loss in a given year, we need to calculate the probability of each division making a loss individually.

For a normal distribution, the probability of making a loss (profit less than 0) can be calculated using the cumulative distribution function (CDF).

For Profit1: Mean = 5 million dollars Standard deviation = sqrt(32) ≈ 5.66 million dollars Using the CDF of a normal distribution, we find the probability of Profit1 being negative.

For Profit2: Mean = 7 million dollars Standard deviation = sqrt(42) ≈ 6.48 million dollars Using the CDF of a normal distribution, we find the probability of Profit2 being negative.

Comparing these probabilities, the division with a larger probability of making a loss in a given year is the one with a higher probability of its profit being negative.