Signal Processing (Örgün Öğretim)
Midterm Make-Up Exam Solutions

Istanbul University - Computer Engineering Department - FALL 2017

December 5$^{\text{th}}$, 2017

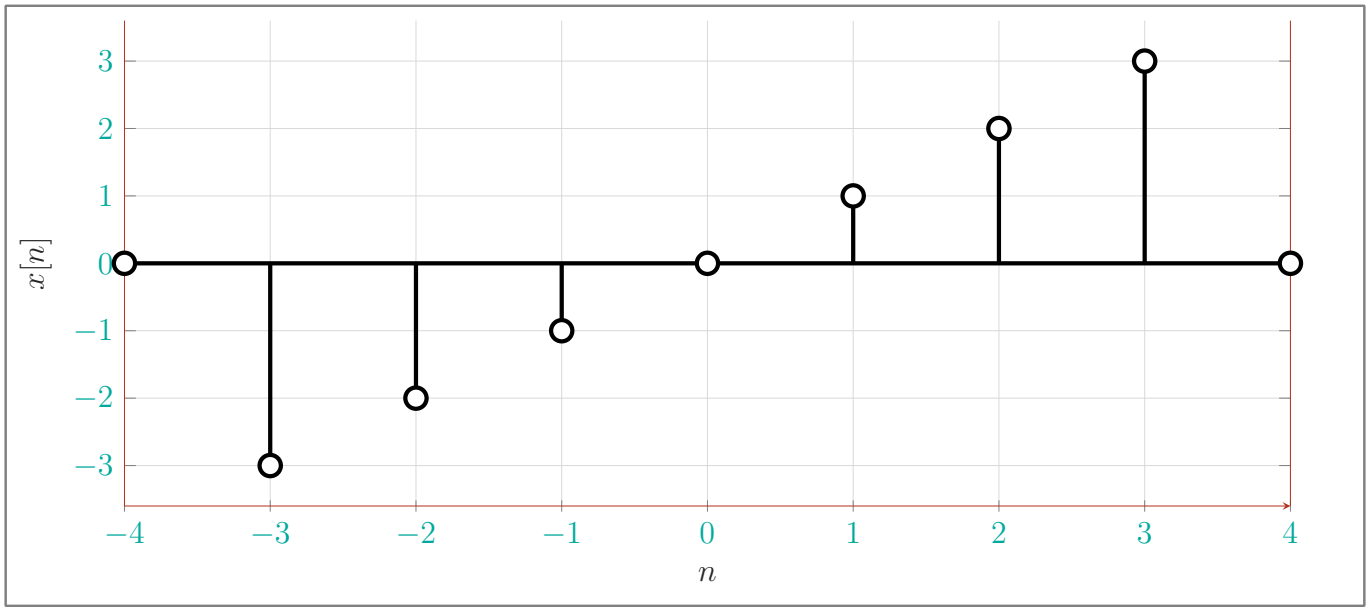
Q1: Consider the following DISCRETE TIME signal. Answer the following questions.

$$x[n] = \sum_{k=-3}^{3} k\ \delta[n-k]$$

Aşağıdaki soruları cevaplayınız.

(a) `(20 pts)` Carefully sketch $x[n]$.

Solution 1a:



(b) `(20 pts)` Is $x[n]$ an even signal, odd signal or neither?

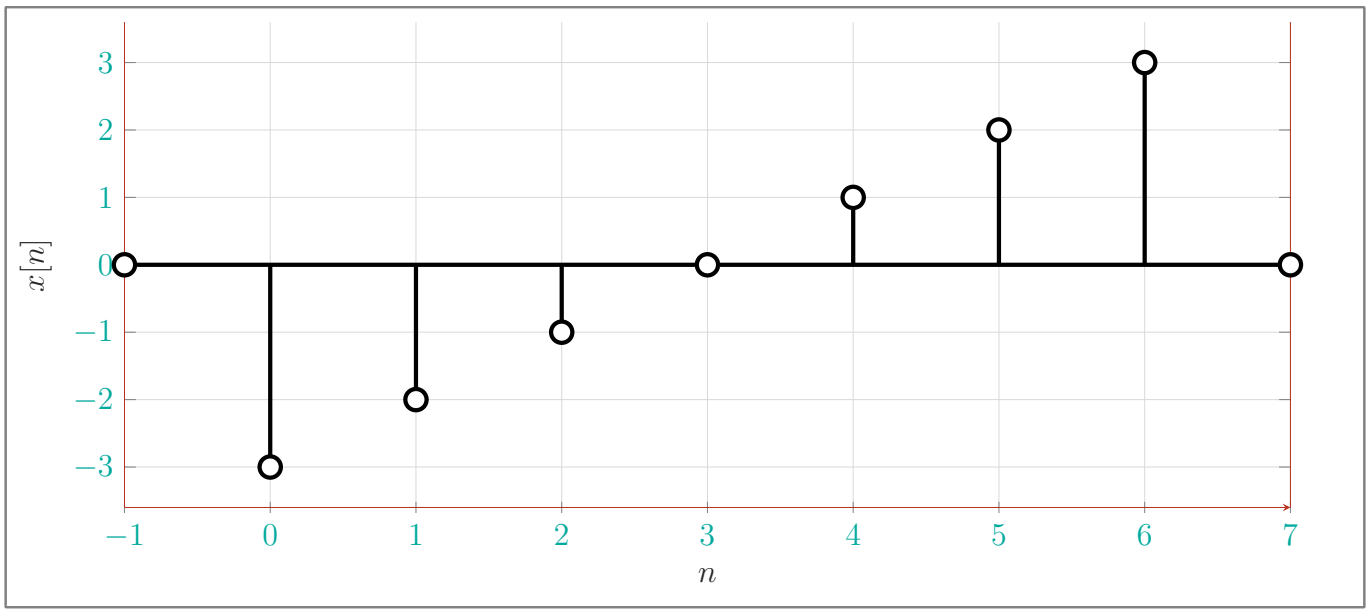
Solution 1b:

It is an odd signal.

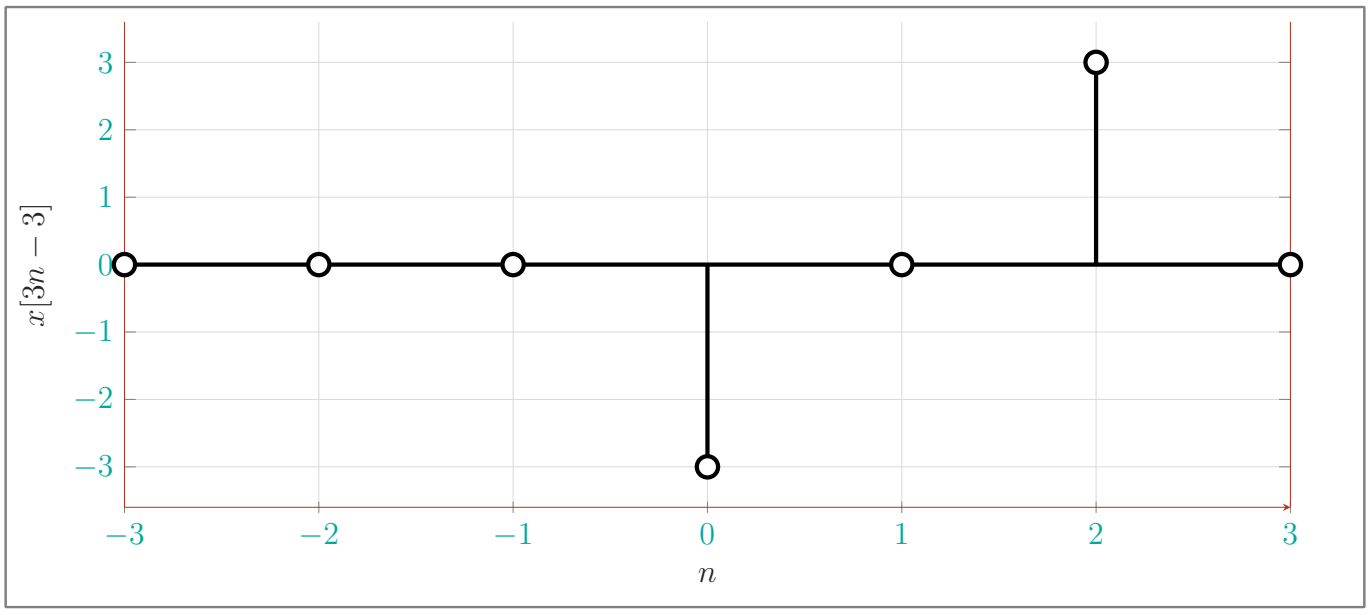
(c) `(20 pts)` Carefully sketch $x[3n-3]+x[2n+2]$.

Solution 1c:

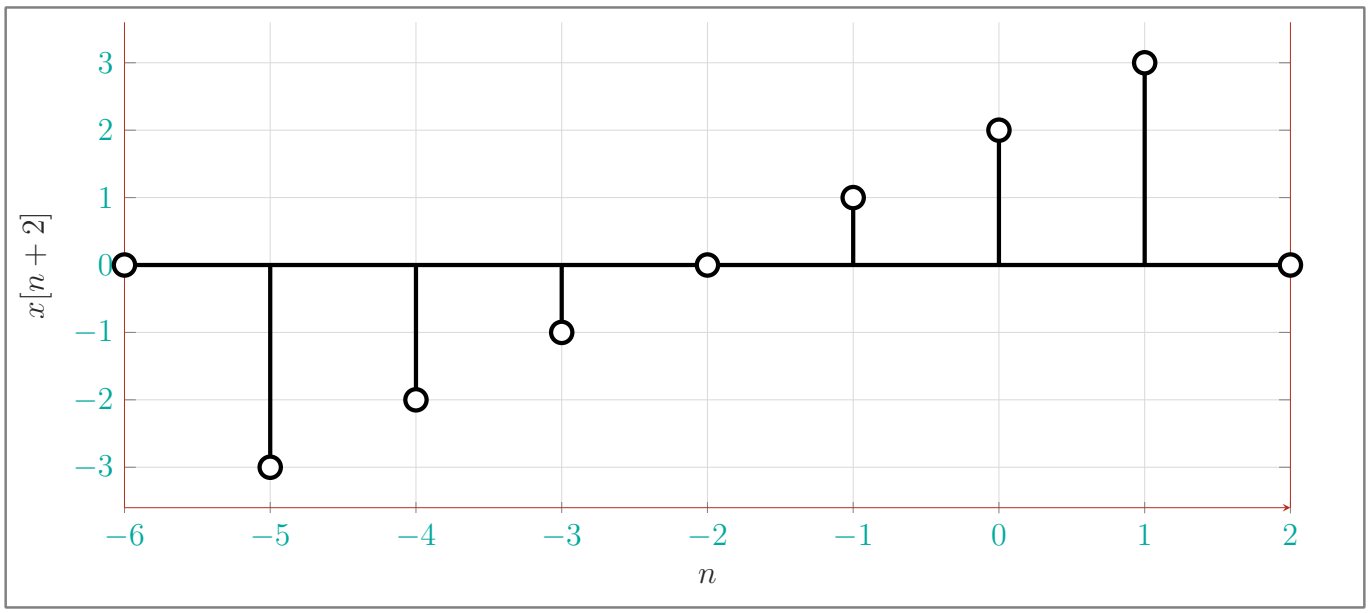
Let's first sketch $x[n-3]$



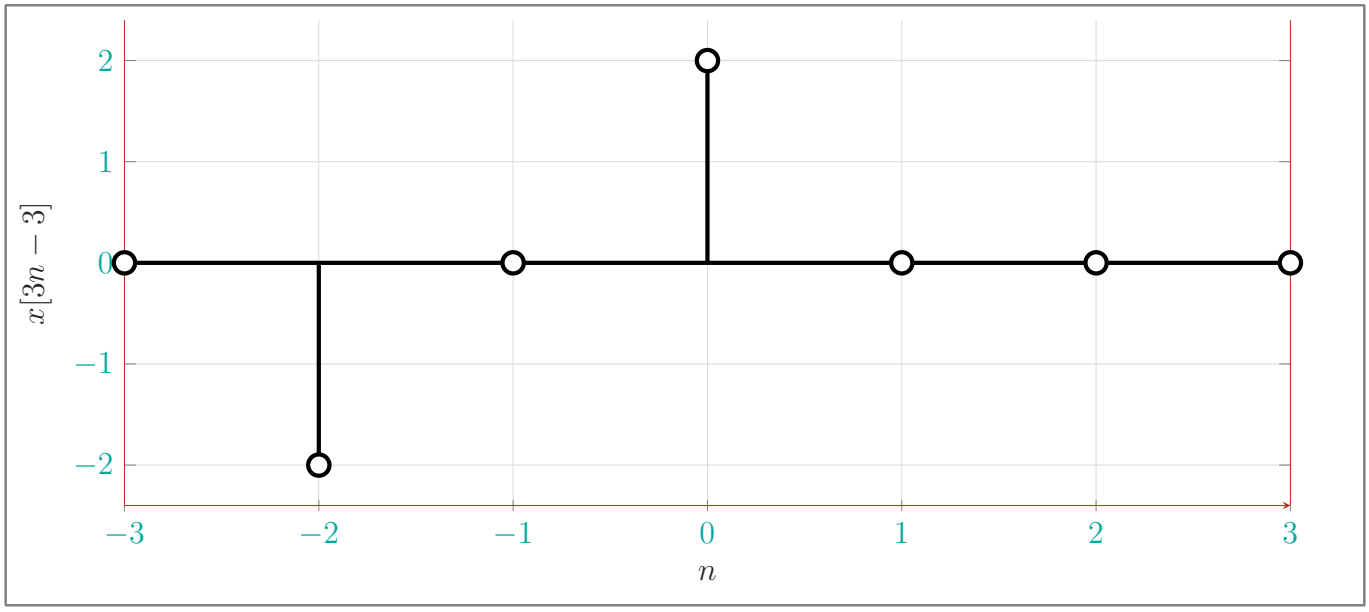
Then, we sketch $x[3n-3]$,



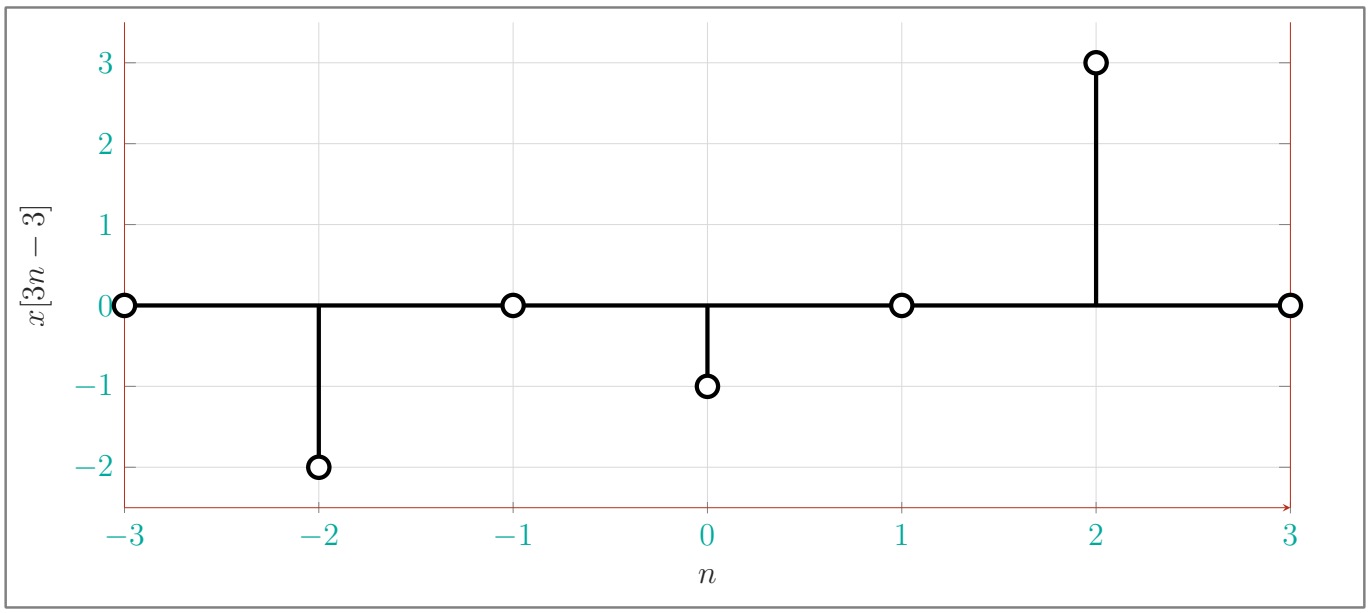
Let's plot $x[n+2]$



Plot $x[2n+2]$



Finally, we put them together,



Q2: Consider the following DISCRETE TIME system. Answer the following questions.

$$y[n] = \mathcal{H}_1\{x[n]\} = \sum_{k=-1}^{1} k\ x[n-k]$$

(a) `(10 pts)` Is $\mathcal{H}_1$ linear? Show your work.

Solution 2a:

We'll check for homogenity first:

$$\begin{aligned} y[n] &= x[n-1] - x[n+1] \\ y_1[n] &= \mathcal{H}_1\{\alpha x[n]\} \\ y_1[n] &= \alpha x[n-1] - \alpha x[n+1] \\ &= \alpha y[n] \end{aligned}$$

Homogenity is satisfied. Checking for superposition, given the signals $x_1[n]$ and $x_2[n]$ and:

$$\begin{aligned} \mathcal{H}\{x_1[n]\} &= y_1[n] \\ \mathcal{H}\{x_2[n]\} &= y_2[n] \end{aligned}$$

$$\begin{aligned}\mathcal{H}_\infty\{x_1[n]+x_2[n]\} &= (x_1[n-1]+x_2[n-1]) - (x_1[n+1]+x_2[n+1]) + \\ &= \underbrace{(x_1[n-1]-x_1[n+1])}_{y_1[n]} + \underbrace{(x_2[n-1]-x_2[n+1])}_{y_2[n]} \\ &= y_1[n]+y_2[n]\end{aligned}$$

Superposition is satisfied. Therefore, $\mathcal{H}_\infty$ is LINEAR. ■

(b) `(10 pts)` Is $\mathcal{H}_1$ time-invariant? Show your work.

Solution 2b:

We'll check for homogenity first:

Let's say $y_1[n] = y[n-n_0]$ and $y_2[n] = \mathcal{H}_\infty\{x[n-n_0]\}$. We'll check if they are equal.

$$\begin{aligned}y_2[n] &= x[n-n_0-1] - x[n-n_0+1] \\ y_1[n] &= x[n-1-n_0] - x[n+1-n_0] \\ y_1[n] &= y_2[n]\end{aligned}$$

Therefore $\mathcal{H}$ is TIME INVARIANT. ■

Q3: Consider the following CONTINUOUS TIME system. Answer the following questions.

$$y(t) = \mathcal{H}_2\{x(t)\} = \int_{-\infty}^{t} t\ x(\tau)\ \mathrm{d}\tau$$

(a) `(10 pts)` Is $\mathcal{H}_2$ stable? Show your work.

(b) `(10 pts)` Is $\mathcal{H}_2$ linear? Show your work.