— DTFS cont'd

$x[n]$: periodic, N

DTFS is represented as

$$\underbrace{(n, N)}_{\text{Discrete Periodic}} \xleftrightarrow{DTFS} \underbrace{(\Omega, k)}_{\text{Discrete Periodic}}$$

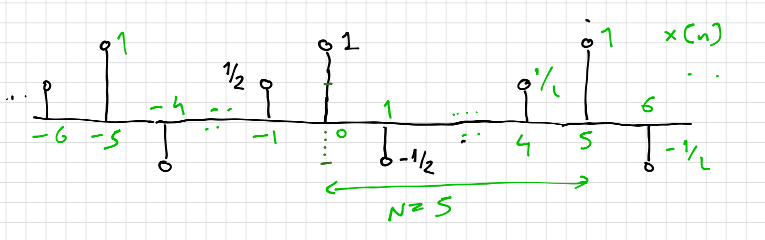
$$x[n] = \sum_{k=0}^{N-1} X[k] \cdot e^{jk\Omega_0 n}$$

$\Omega_0 \to \frac{2\pi}{N}$

$$\rightarrow \underbrace{X[k]}_{\text{DTFS coefficients}} = \frac{1}{N} \sum_{n=0}^{N-1} x[n] \cdot e^{-jk\Omega_0 n}$$

— Each DTFS coefficient is associated with a different frequency.

Example : Determining DTFS coefficients.



$\Omega_0 = \frac{2\pi}{5}$ radians

$$X[k] = \frac{1}{N} \sum_{n=\langle N\rangle} x[n] \cdot e^{-jk\Omega_0 n}$$

$$= \frac{1}{5} \sum_{n=-2}^{2} x[n] \cdot e^{-jk\frac{2\pi}{5}n}$$

$$= \frac{1}{5}\Big(x[-2] \cdot e^{-jk\frac{2\pi}{5}(-2)} + x[-1] \cdot e^{-jk\frac{2\pi}{5}(-1)} + x[0] \cdot e^{-jk\frac{2\pi}{5}\cdot 0} + x[1] \cdot e^{-jk\frac{2\pi}{5}(1)} + x[2] \cdot e^{-jk\frac{2\pi}{5}\cdot 2}\Big)$$

$$= \frac{1}{5}\Big(1 + \frac{1}{2} e^{jk\frac{2\pi}{5}} - \frac{1}{2} e^{-jk\frac{2\pi}{5}}\Big)$$

$$X[k] = \frac{1}{5}\Big(1 + j\sin\big(k\tfrac{2\pi}{5}\big)\Big)$$

$e^{j\theta} = \cos\theta + j\sin\theta$

$\frac{1}{2}(e^{j\theta} - e^{-j\theta}) = j\sin\theta$

$X[k] = |X[k]| \, e^{j\arg\{X[k]\}}$

$$X[-2] = \frac{1}{5}\Big(1 + j\sin\big(-2\tfrac{2\pi}{5}\big)\Big)$$

$$= \frac{1}{5}\Big(1 - j\sin\big(\tfrac{4\pi}{5}\big)\Big)$$

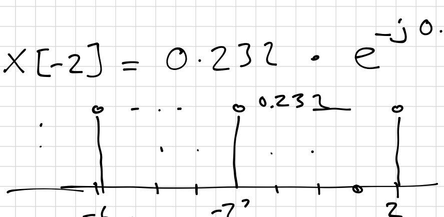
$$|X[-2]| = \frac{1}{5}\left(\sqrt{1^2 + 0.5878}\right)$$

$$= 0.232$$

$$\arg\{X[-2]\} = \arctan\left\{\frac{-\sin(4\pi/5)}{1}\right\}$$

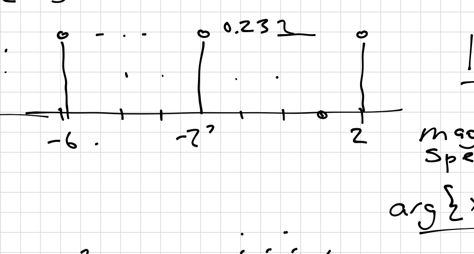
$$= -0.531$$

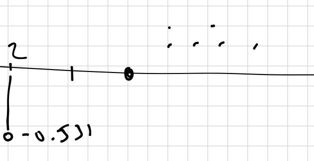
$$X[-2] = 0.232 \cdot e^{-j0.531}$$



$\underbrace{|X[k]|}_{\text{magnitude Spectrum}}$

$\arg\{X[k]\}$ → Phase Spectrum





Example

Computing the DTFS coefficients by inspection.

$$x[n] = \cos\left(\frac{\pi n}{3} + \phi\right)$$

$\Omega_0 = \frac{\pi}{3}$ $\quad N = \frac{2\pi}{\pi/3} = 6$

$$x[n] = \frac{e^{j(\pi/3\, n + \phi)} + e^{-j(\pi/3\, n + \phi)}}{2}$$

$$= \frac{1}{2} e^{-j\phi} \cdot e^{-j\frac{\pi}{3}n} + \frac{1}{2} e^{j\phi} \cdot e^{j\frac{\pi}{3}n}$$

$$x[n] = \sum_{k=-2}^{3} X[k] \, e^{jk\pi n/3}$$

$$= X[-2] \cdot e^{+j2\pi n/3} + X[-1] \cdot e^{-j\cdot 1\pi n/3} + X[0] \cdot e^{-j0\cdot\pi n/3} + X[1] \cdot e^{j1\pi n/3} + \ldots + \ldots$$

$$x[n] \xleftrightarrow{DTFS;\ \pi/3} X[k] = \begin{cases} e^{-j\phi/2}, & k=-1 \\ e^{j\phi/2}, & k=1 \\ 0, & \text{otherwise } k=\{-2,0,2,3\} \end{cases}$$

CIML grubu — Computer Intelligence and Machine Learning

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↳ Seminerler

↳

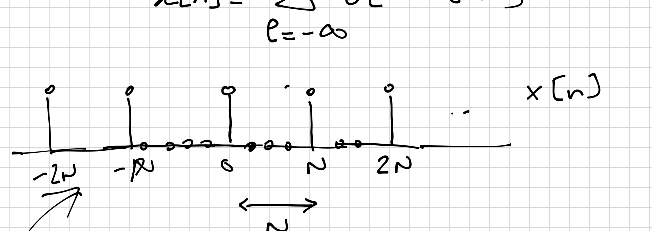
Sınava not kağıdı, fotokopi vs getirmek YASAK

Hesap makinesi serbest.

Example

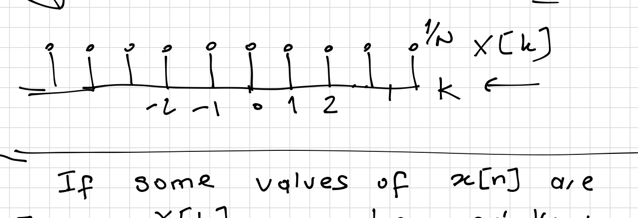
DTFS of an impulse train:

$$x[n] = \sum_{\ell=-\infty}^{\infty} \delta[n-\ell N]$$



$$X[k] = \frac{1}{N}\sum_{n=0}^{N-1} x[n]\, e^{-jk\frac{2\pi}{N}n}$$

$$= \frac{1}{N}\cdot 1 \cdot e^{0} = \frac{1}{N}$$



If some values of $x[n]$ are zero, $X[k]$ may be periodic in k with a period less than N. It may not be possible to derive N from $X[k]$ unless we know N beforehand.

Example 3.5 @ page 247

$X[k]$ periodic, $N=9$, over N samples

$$|X[k]| = \begin{cases} 1, & k=0,3,-3 \\ 2, & k=-2,2 \\ 0, & k=-4,4,1,-1 \end{cases}$$

$$\arg\{X[k]\} = \begin{cases} 0, & n=-4 \\ 2\pi/3, & n=-3 \\ \pi/3, & n=-2 \\ 0, & n=-1 \\ \pi, & n=0 \\ 0, & n=1 \\ -\pi/3, & n=2 \\ -2\pi/3, & n=3 \\ 0, & n=4 \end{cases}$$

$$x[n] = \sum_{k=-4}^{4} X[k]\cdot e^{jk2\pi n/9}$$

$$= 0 + 1\cdot e^{j2\pi/3}\cdot e^{-j6\pi n/9}$$

$$+ 2\, e^{j\pi/3}\cdot e^{-j4\pi n/9}$$

$\vdots$

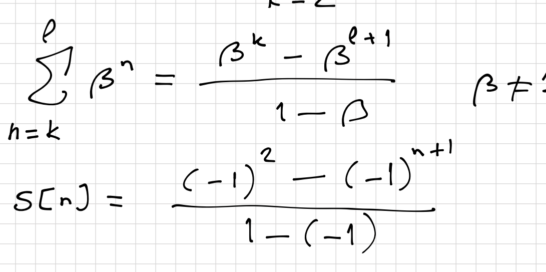
$$x[n] = 2\cos\left(\frac{6\pi n}{9} - \frac{2\pi}{3}\right) + 4\cos\left(\frac{4\pi n}{9} - \pi/3\right) - 1$$

Examples (All Topics)

2020 / Final

1 a) Find the step response of the LTI system represented by the following impulse response

$$h[n] = (-1)^n\{u[n+2] - u[n-3]\}$$



$$s[n] = \sum_{k=-\infty}^{n} h[k]$$

$n<-2$ $\quad s[n]=0$

$-2\leq n<2$ $\quad s[n] = \sum_{k=2}^{n}(-1)^k$

$$\sum_{n=k}^{\ell}\beta^n = \frac{\beta^k - \beta^{\ell+1}}{1-\beta} \quad \beta\neq 1$$

$$s[n] = \frac{(-1)^2 - (-1)^{n+1}}{1-(-1)}$$

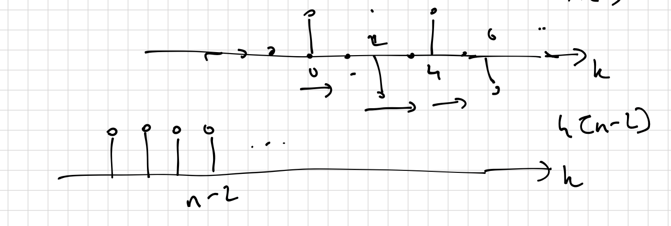
$$= \frac{1}{2}\left[1 + (-1)^n\right]$$

$n\geq 2$ $\quad s[n] = \sum_{k=2}^{2}(-1)^k$

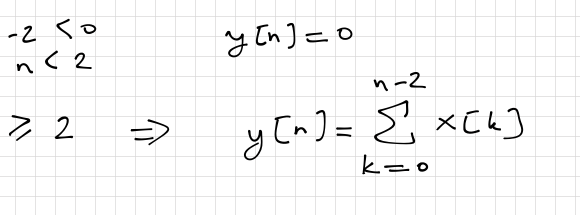
$= 1$

$$s[n] = \begin{cases} 0, & n<2 \\ \frac{1}{2}(1+(-1)^n), & -2\leq n<2 \\ 1, & n\geq 2 \end{cases}$$

$$s[n] = \begin{cases} 0, & n<-2 \\ 0, & -2\leq n<2,\ n \text{ odd} \\ 1, & -2\leq n<2,\ n \text{ even} \\ 1, & n\geq 2 \end{cases}$$

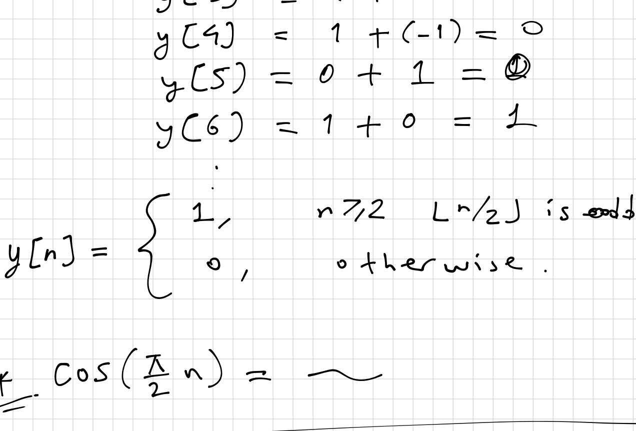


3 $\quad \underbrace{\cos\left(\frac{\pi}{2}n\right)u[n]}_{x[n]} * \underbrace{u[n-2]}_{h[n]}$



$$x[n] = \cos\left(\frac{\pi}{2}n\right) = \begin{cases} 1, & n\geq 0 \text{ and } \lfloor n/2\rfloor \text{ is even} \\ -1, & n\geq 0 \text{ and } \lfloor n/2\rfloor \text{ is odd} \\ 0, & \text{otherwise} \end{cases}$$

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]\, h[n-k]$$



$n-2<0$, $n<2$ $\quad y[n]=0$

$n\geq 2$ $\quad y[n] = \sum_{k=0}^{n-2} x[k]$

$y[2] = 1$

$y[3] = 1+0 = 1$

$y[4] = 1+(-1) = 0$

$y(5) = 0+1 = 1$

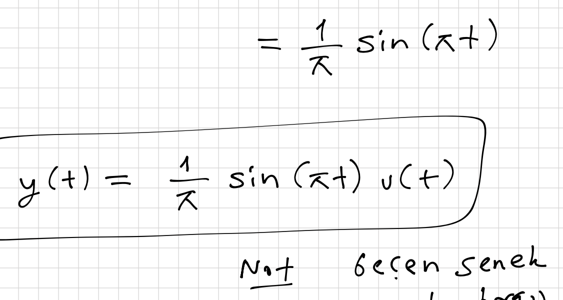
$y(6) = 1+0 = 1$

$$y[n] = \begin{cases} 1, & n\geq 2 \quad \lfloor n/2\rfloor \text{ is odd} \\ 0, & \text{otherwise} \end{cases}$$

$*\ \cos\left(\frac{\pi}{2}n\right) = \sim$

4b $\quad \underbrace{\cos(\pi t)\,u(t)}_{x(t)} * \underbrace{u(t)}_{h(t)}$

$$y(t) = \int_{-\infty}^{+\infty} x(\tau)\, h(t-\tau)\, d\tau$$



$t<0$ $\quad y(t) = 0$

$t\geq 0$ $\quad y(t) = \int_0^t \cos(\pi\tau)\, d\tau$

$$= \frac{1}{\pi}\sin(\pi t)$$

$$\boxed{y(t) = \frac{1}{\pi}\sin(\pi t)\, u(t)}$$

Not: Geçen senek sınavda burası hatalı yazılmış. Düzeltilecek.