④ Deterministic vs Random Signals

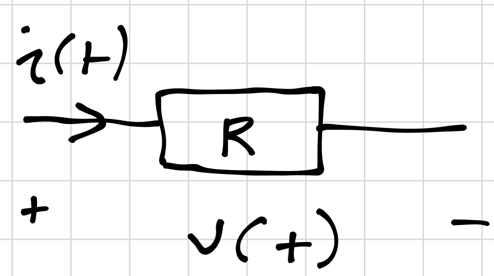
- A deterministic signal is a signal about which there's no uncertainty with respect to its value at any time.

- A random signal is that about which there's uncertainty before it occurs.

  - noise

⑤ Energy Signals vs Power Signals

Ex



The instantenous power dissipated in the resistor is

$$p(t) = \frac{v^2(t)}{R}$$

$$p(t) = R \cdot i^2(t)$$

If $R = 1 \Rightarrow \quad p(t) = v^2(t) = i^2(t)$

Let's define

Instantaneous Power of a signal $x(t)$

$$p(t) = x^2(t)$$

Total Energy

$$E = \lim_{T \to \infty} \int_{-T/2}^{T/2} x^2(t)\, dt$$

$$E = \int_{-\infty}^{+\infty} x^2(t)\, dt$$

Average Power

$$\rightarrow \quad P = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} x^2(t)\, dt$$

If $x(t)$ is periodic with a fundamental period of T

$$P = \frac{1}{T} \int_{-T/2}^{T/2} x^2(t)\, dt$$

If $x(t)$ is periodic

For DT signal

For a DT signal, $x[n]$

Total energy $$E = \sum_{n=-\infty}^{+\infty} x^2[n]$$

Average Power $$P = \lim_{N \to \infty} \frac{1}{2N} \sum_{n=-N}^{N} x^2[n]$$

If $x[n]$ is periodic $\Big\}$ $$P = \frac{1}{N} \sum_{n=0}^{N-1} x^2[n]$$

A signal is referred to as an <u>energy signal</u> if

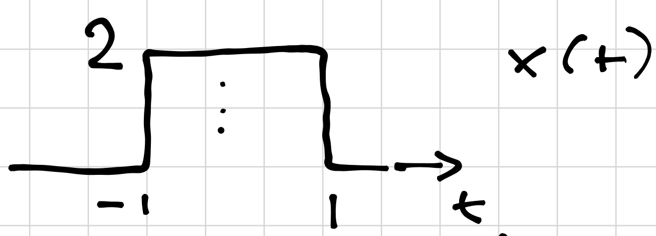
$$0 < E < \infty \Rightarrow \text{Energy Signal}$$

It is a <u>power</u> signal if

$$0 < P < \infty \Rightarrow \text{Power Signal.}$$

• A signal cannot be both an energy and a power signal!

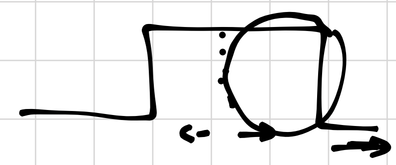
<u>Ex</u>



$$E = \int_{-\infty}^{+\infty} x^2(t)\, dt = \int_{-1}^{1} 2^2 \cdot dt = 4t \Big|_{-1}^{1}$$
$$= 4(1+1) = \underline{\underline{8}}$$

$0 < E < \infty$ $\therefore$ $x(t)$ is an Energy Signal.

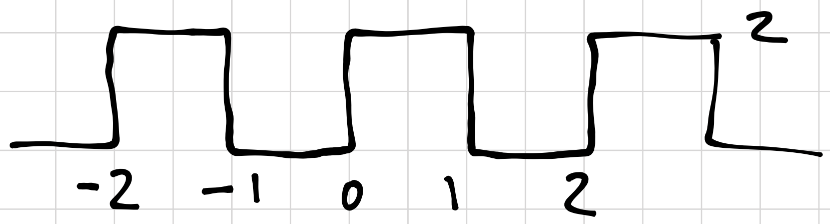
$$P = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} x^2(t)\, dt$$



$$= \lim_{T \to \infty} \frac{1}{T} \int_{-1}^{1} 2^2\, dt = \lim_{T \to \infty} \frac{1}{T} \cdot 8 = \underline{\underline{0}}$$

$$\boxed{\begin{array}{lll} \text{Energy signals} & \rightarrow & P = 0 \\ \text{Power } '' & \rightarrow & E = \infty \end{array}}$$

Ex

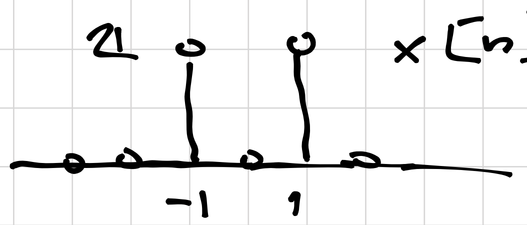


$T = 2$

$$P = \frac{1}{2}\int_0^2 2^2\, dt = 4 \quad \text{Finite}$$

Power signal $\therefore$ $E = \infty$

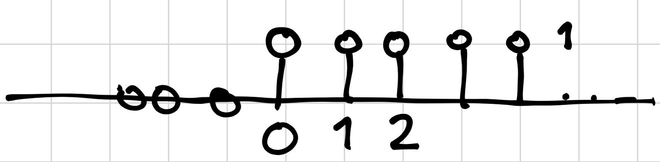
Ex



$$E = 2^2 + 2^2 = 8 \quad \therefore \quad \underline{P = 0}$$

Energy signal.

Ödev



$$x[n] = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}$$

$E = ?$
$P = ?$
Energy or power?

Basic Operations on Signals

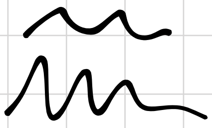
$x[n]$, $x(t)$

independent variable

dependent variable

→ Operation on the dependent variable.

- Amplitude Scaling $\quad c\,x(t) = y(t)$ — c: real



- Addition

$$y(t) = x_1(t) + x_2(t)$$

- multiplication $\quad y(t) = x_1(t) \cdot x_2(t)$

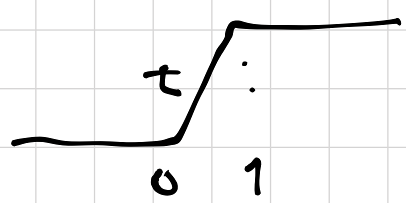
· Differentiation (CT)

$$y(t) = \frac{d}{dt} x(t)$$

· Integration

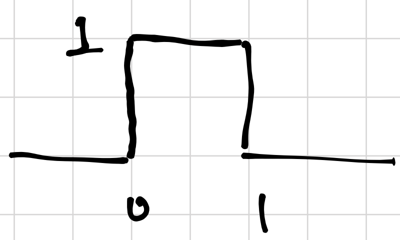
$$y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau$$

Ex



$x(t)$

$$y(t) = \frac{d}{dt} x(t)$$



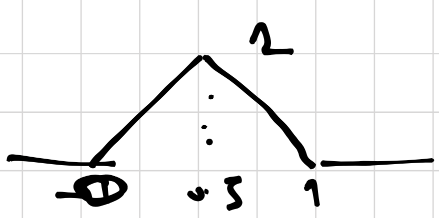
$y(t)$

.. on the independent variable.

Time scaling

$$y(t) = x(a \cdot t) \qquad a \in \mathbb{R}^+$$

$a > 1 \Rightarrow$ örnek

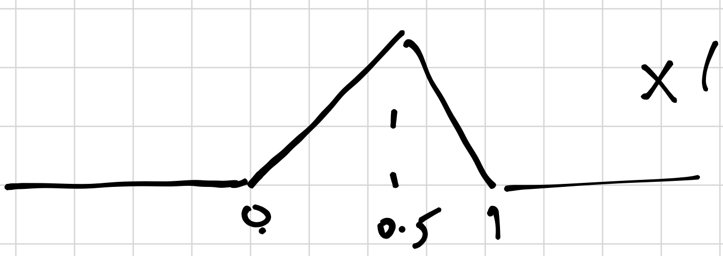


$y(t) = x(\underline{\underline{2t}})$

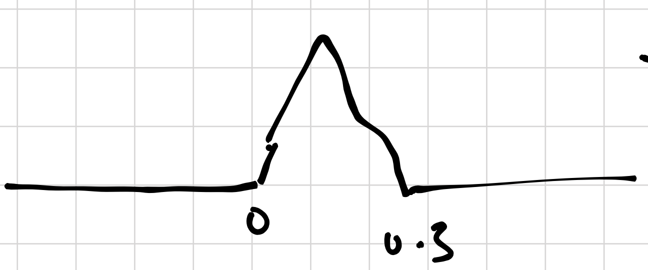
$y(0) = x(2 \cdot 0)$
$= \underline{0}$

$y(1) = x(2 \cdot 1)$
$= x(\underline{\underline{2}}) = 0$

$y(0.5) = x(2 \cdot 0.5)$
$= \underline{x(1)}$



$x(t)$

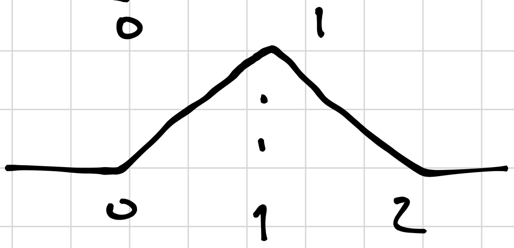


$y(t)$

$a > 1 \Rightarrow$ the signal $y(t)$ is a ~~called~~ compressed version of $x(t)$

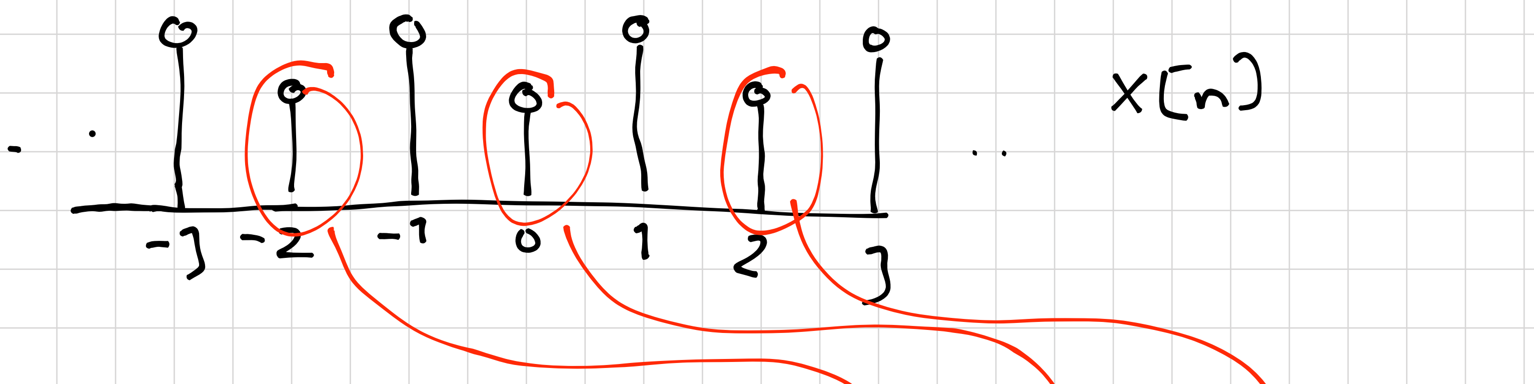
$0 < a < 1$ $\qquad y(0.5t)$

$\Rightarrow$ $y(t)$ is an expanded - stretched. version of $x(t)$

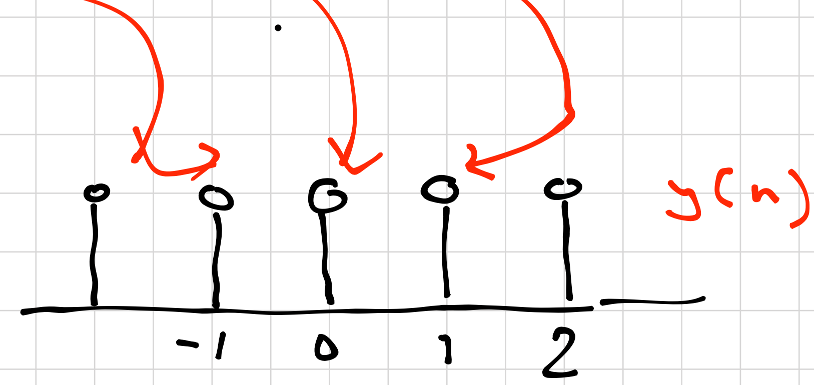


DT

$$y[n] = x[kn] \quad , \; k > 0 \; , \; k \in \mathbb{Z}^+$$



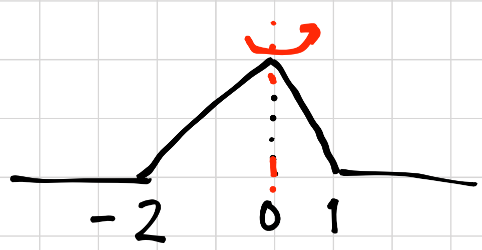
$$y[n] = x[2n]$$



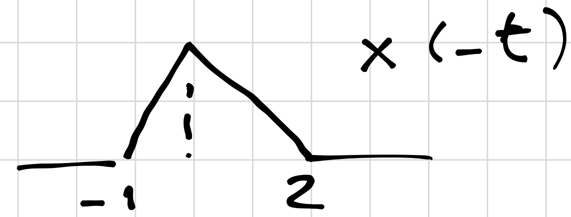
$$y[0] = x[0]$$
$$y[1] = x[2]$$
$$y[-1] = x[-2]$$

Reflection

$$y(t) = x(-t)$$
$$y[n] = x[-n]$$



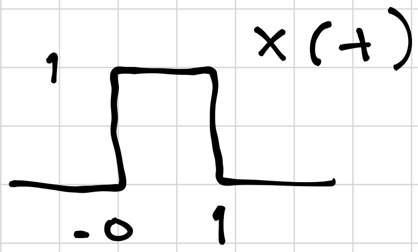
$x(t) \Rightarrow$



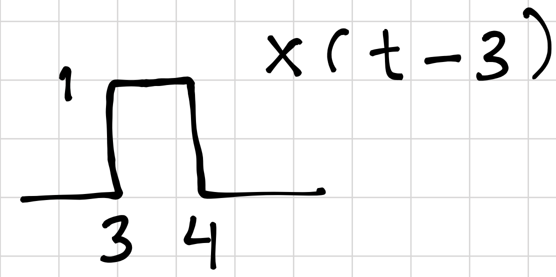
Time Shifting

$$y(t) = x(t - t_0)$$
$$y[n] = x[n - n_0]$$

Ex

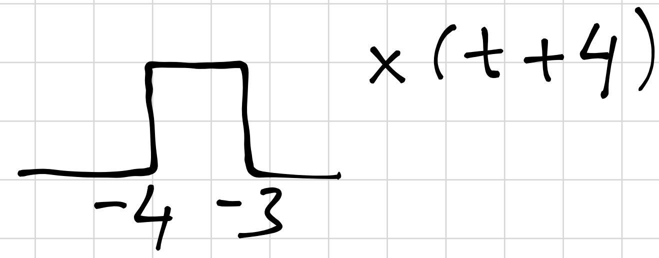


$\Rightarrow$

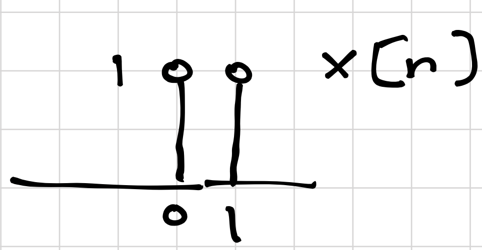


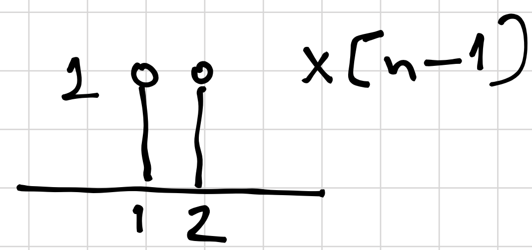
$t_0 > 0 \Rightarrow$ shift right

$t_0 < 0 \Rightarrow$ shift left



Ex





Precedence Rule for Time-Shifting and Time-Scaling

$$y(t) = x(at-b)$$

$$\boxed{\begin{aligned} y(0) &= x(-b) \\ y\left(\tfrac{b}{a}\right) &= x(0) \end{aligned}}$$

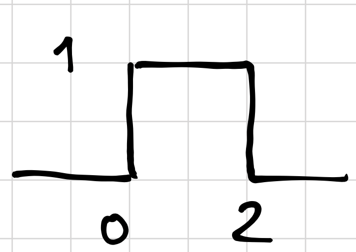
$$v(t) = x(t-b) \qquad \underline{\text{Shift}}$$

$$y(t) = v(\underset{\smile}{at}) = x(at-b) \qquad \underline{\text{Scale}}$$

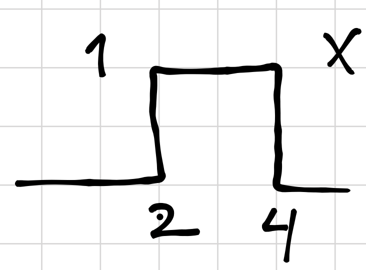
Shift then Scale

ex

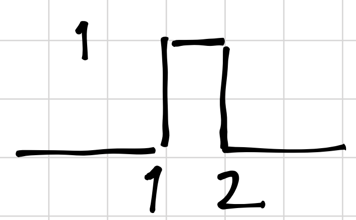
$$y(t) = x(2t-2)$$



$$v(t) = x(t-2)$$



$$y(t) = v(2t)$$



Elementary Signals

1 Exponential Signals

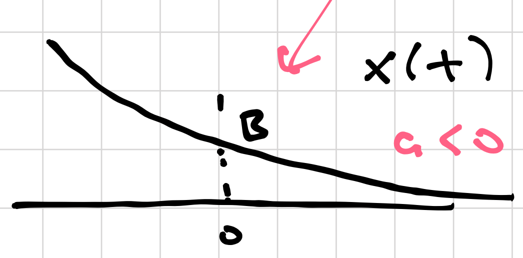
CT

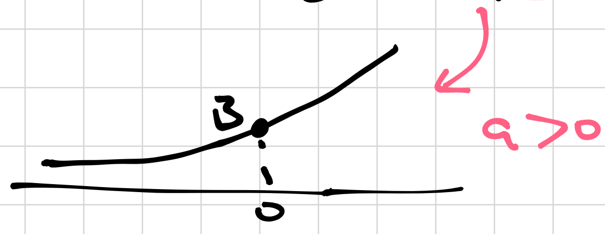
$$x(t) = B \cdot e^{at} \qquad B, a \in \mathbb{R}$$

B: amplitude

$a<0 \Rightarrow x(t)$ is a "decaying exponential"

$a>0 \Rightarrow x(t)$ is a "growing exponential"





DT

$x[n] = B \cdot r^n \qquad (r = e^{a})$

$0 < |r| < 1 \rightarrow$ decaying exp.

$|r| > 1 \rightarrow$ growing "