BIMU3009
Signal Processing
Midterm Exam Solutions

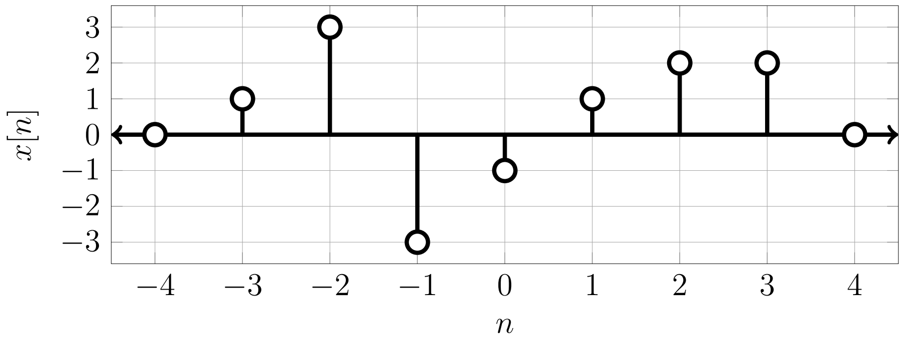
Istanbul University - Cerrahpaşa
Computer Engineering Department - FALL 2021

November 8th, 2021 12:30-13:45

`Son güncelleme: 2022-01-10 14:39`

QUESTIONS and THEIR SOLUTIONS

Q1: Consider the following DISCRETE TIME signal. Answer the following questions.

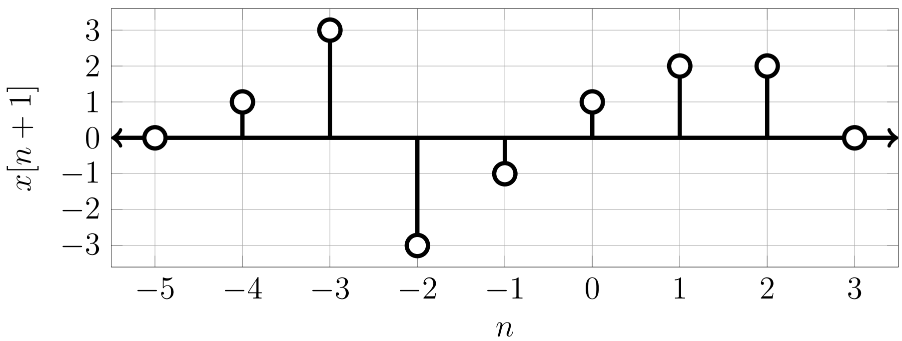


(a) `(10 pts)` Step by step, sketch $2\,x[2n+1]$. Show your work.

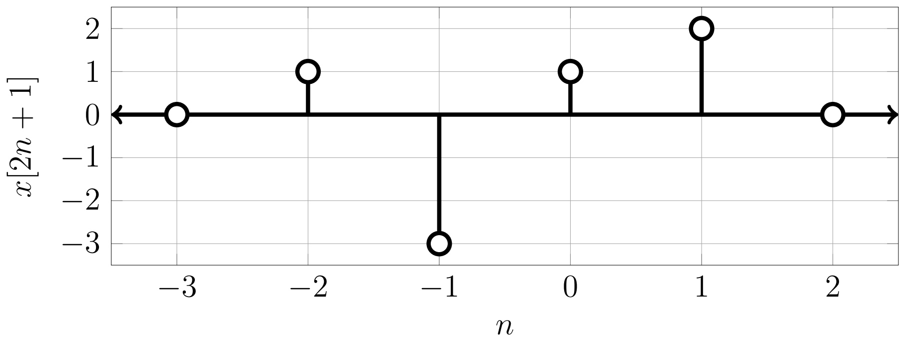
Solution 1a:

The following steps can be followed:

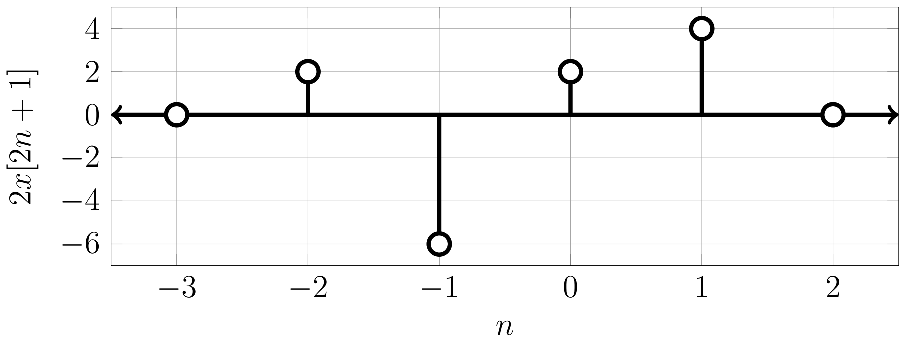
i. We first shift the signal to the right by 1.



ii. Then shrink the signal by 2:



iii. Finally amplify by 2:



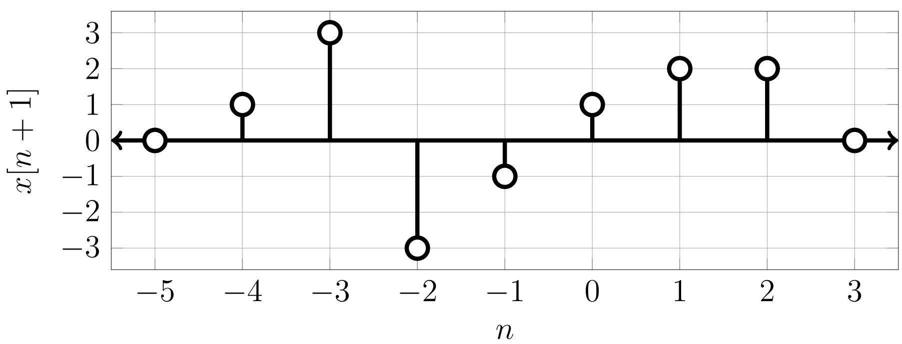
We can also use tables to find the result.

n	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6
$x[n]$	0	0	0	1	3	-3	-1	1	2	3	0	0	0
$x[n+1]$	0	0	1	3	-3	-1	1	2	3	0	0	0	0
$x[2n+1]$	0	0	0	0	1	-3	1	2	0	0	0	0	0
$2x[2n+1]$	0	0	0	0	2	-6	2	4	0	0	0	0	0

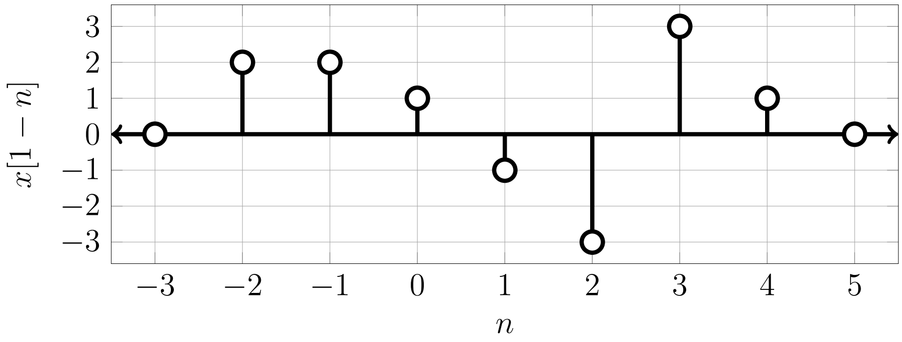
(b) `(10 pts)` Step by step, sketch $3\,x[1-n]$. Show your work.

Solution 1b:

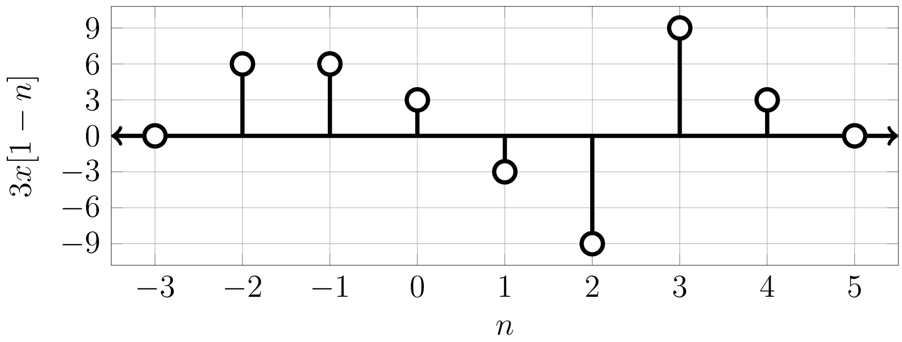
i. First find $x[n+1]$ by shifting the signal to the left by 1.



ii. Then reflect around the y axis to find $x[1-n]$.



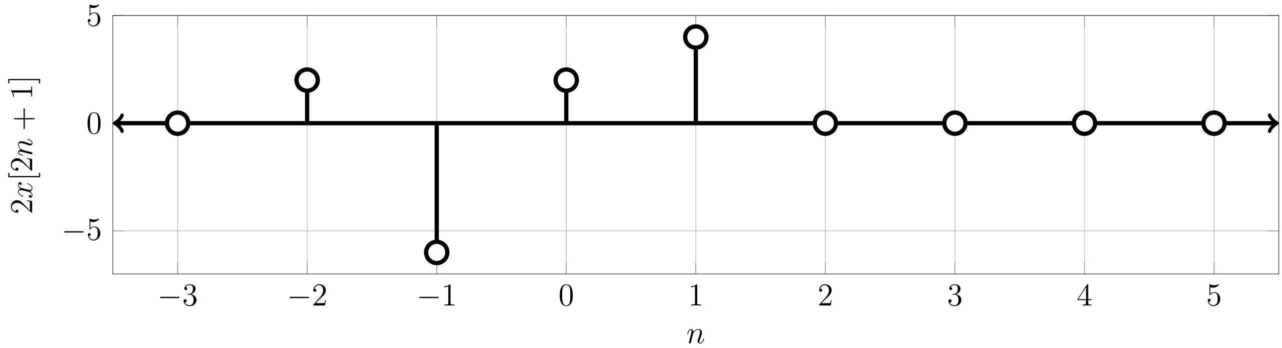
iii. Then, amplify by 3.

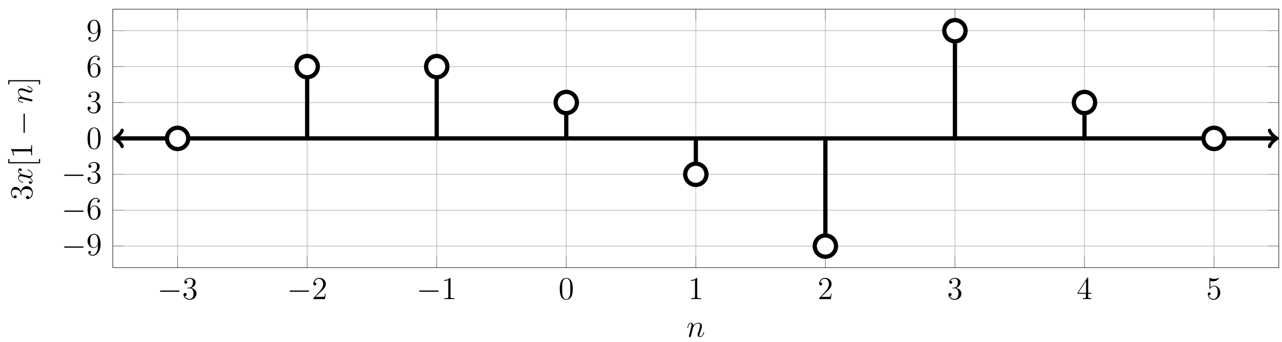


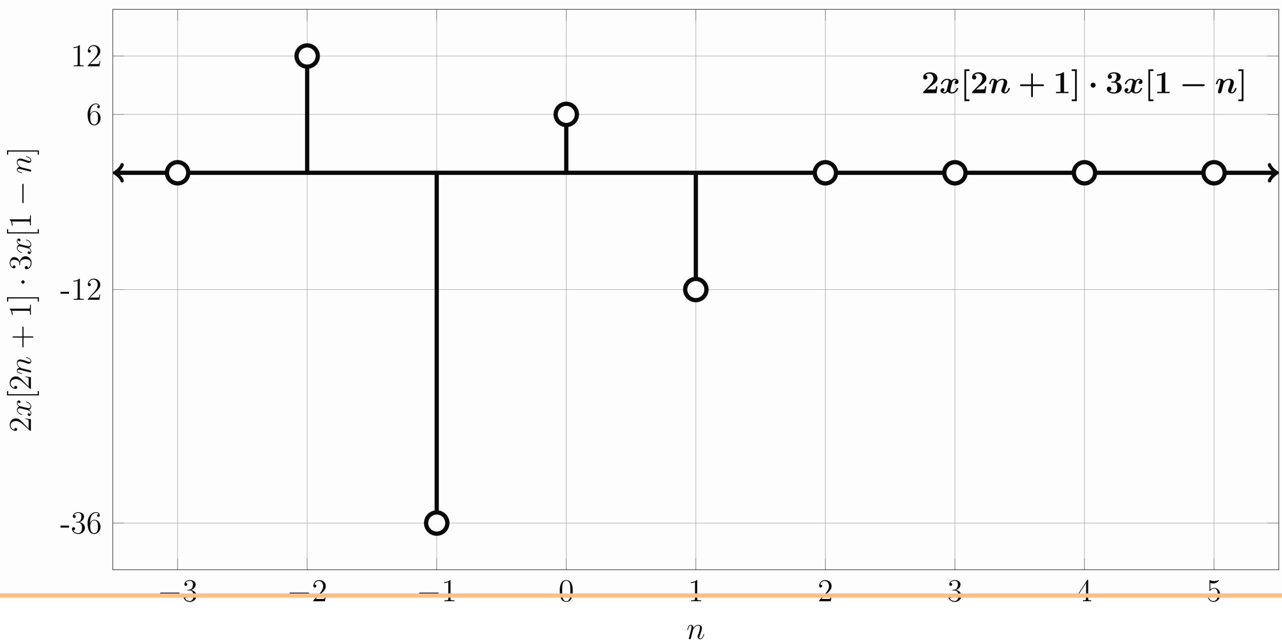
(c) `(10 pts)` Carefully sketch $(2\,x[2n+1]) \times (3\,x[1-n])$. ($\times$ symbol denotes *multiplication* operation). Show your work step by step.

Solution 1c:

In this case we should multiply the product terms for each time n.







(d) `(10 pts)` Calculate the average power and total energy of $x[n]$. Is $x[n]$ an energy signal, power signal or neither? Please provide explanation.

Solution (1d):

and

$$
\begin{aligned}
E &= \sum_{n=-\infty}^{\infty} |x[n]|^2 \\
E &= 1^2 + 3^2 + (-3)^2 + (-1)^2 + 1^2 + 2^2 + 2^2 \\
E &= 29
\end{aligned}
$$

So, $x(t)$ has finite non-zero energy, therefore it is an energy signal. The average power of $x(t)$ is therefore *zero*.

(e) `(10 pts)` Find and sketch the even and odd components of this signal. Show your work step by step.

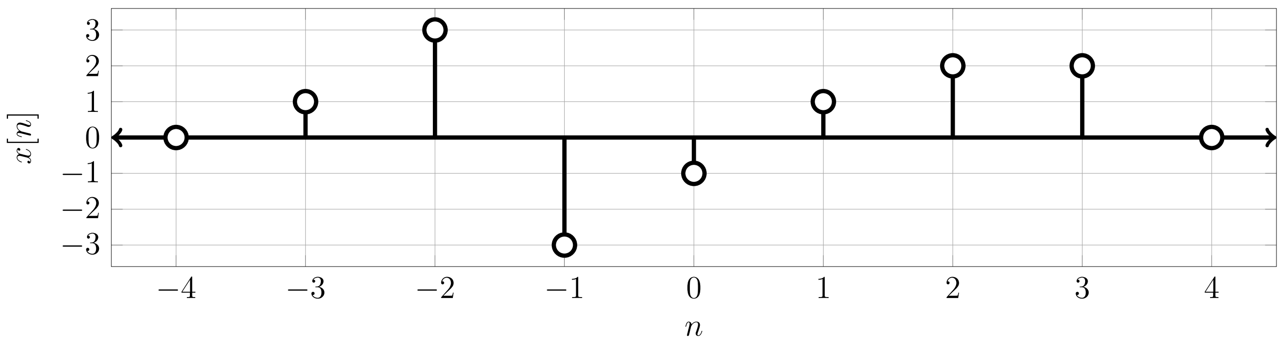
Solution (1e):

The signal $x(t)$ can be decomposed into its even and odd components by the following:

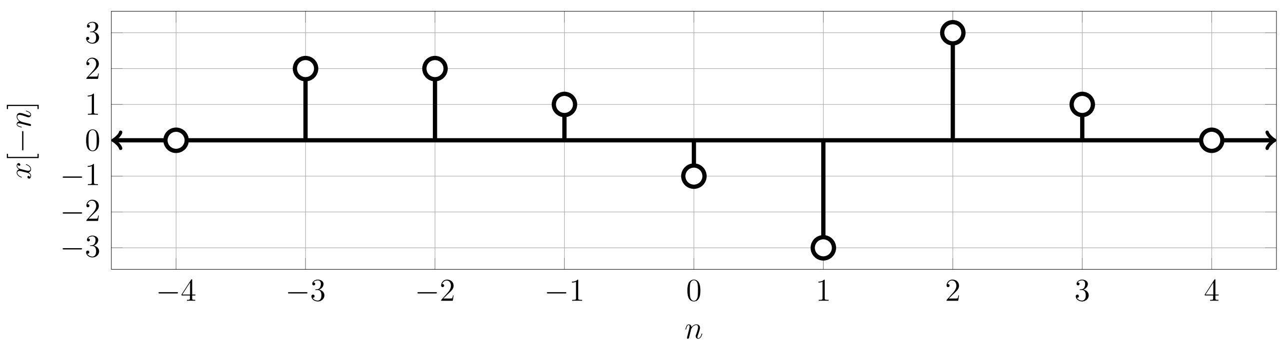
$$
\begin{aligned}
x_e(t) &= \frac{1}{2}\Big[x(t) + x(-t)\Big] \\
x_o(t) &= \frac{1}{2}\Big[x(t) - x(-t)\Big]
\end{aligned}
$$

We can plot $x(t)$ and $x(-t)$:

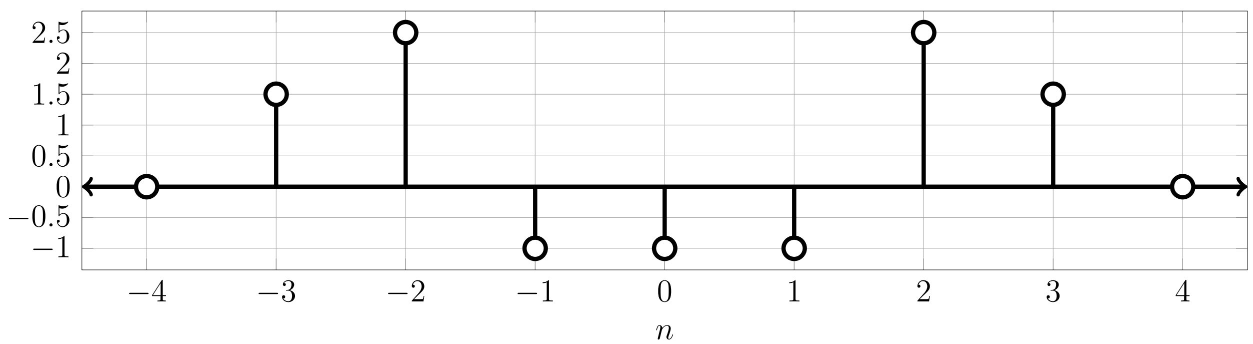
$x[n]$



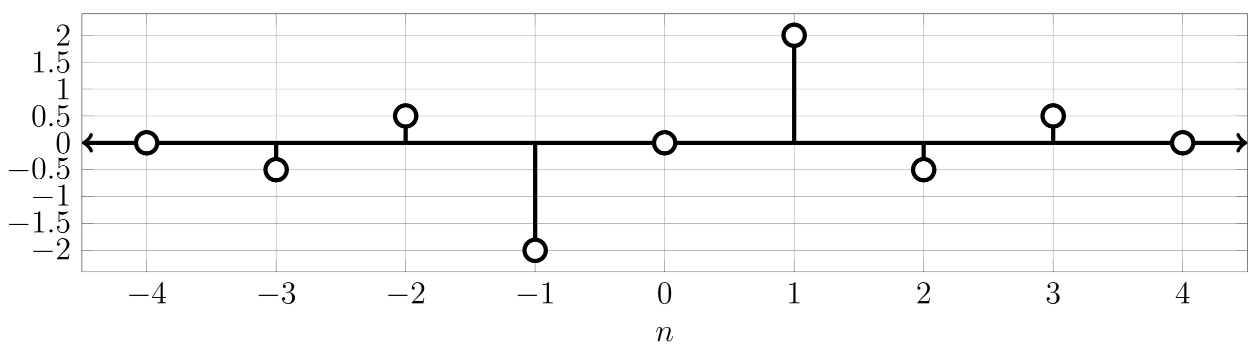
$x[-n]$



$x_e[n] = 0.5(x[n]) + x[-n])$



$x_o[n] = 0.5(x[n]) - x[-n])$



Q2: `(10 pts)` Consider the following CT system. Is $\mathcal{H}$ stable? Show your work.

$$y(t) = \mathcal{H}\{x(t)\} = \mathrm{e}^{-2t}\ x(t)\ u(t)$$

Solution (2):

First, let's assume $x(t)$ is finite, that is, $|x(n) \leq M_x < \infty$. So,

$$\begin{aligned} |y(t)| &= \mathrm{e}^{-2t}\ x(t)\ u(t) \\ &\leq M_x\ \mathrm{e}^{-2t} u(t) \end{aligned}$$

Since, $\mathrm{e}^{-2t} u(t) < 1$ for $\forall t$,

$$|y(t)| \leq M_x < \infty$$

Therefore $\mathcal{H}$ is BIBO-stable. ■

Q3: `(10 pts)` Determine whether the following signal is periodic. If it is, determine its fundamental period. Show your work.

$$x(t) = \cos\left(\frac{2\pi}{3}t + \frac{\pi}{3}\right) + \sin\left(\frac{3\pi}{7}t + \frac{2\pi}{5}\right)$$

Solution (3):

Let

$$\begin{aligned} x_1(t) &= \cos\left(\frac{2\pi}{3}t + \frac{\pi}{3}\right) \\ x_2(t) &= \sin\left(\frac{3\pi}{7}t + \frac{2\pi}{5}\right) \\ x(t) &= x_1(t) + x_2(t) \end{aligned}$$

Let T_1 be the fundamental period of x_1 and T_2 be the fundamental period of x_2. So,

$$\omega_1 = \frac{2\pi}{3}$$

$$T_1 = \frac{2\pi}{\omega_1}$$

$$T_1 = \frac{2\pi}{\frac{2\pi}{3}}$$

$$T_1 = 3$$

$$T_2 = \frac{2\pi}{\frac{3\pi}{7}}$$

$$T_2 = \frac{14}{3}$$

We need to find the smallest integer pair (m, k) that satisfies the following:

$$\begin{aligned} T &= m\ T_1 = k\ T_2 \\ \frac{m}{k} &= \frac{T_2}{T_1} \\ &= \frac{14}{9} \end{aligned}$$

So $(m, k) = (14, 9)$ is the smallest pair we can find. Then the period of $x(t)$ will be:

$$T = m\ T_1$$

$$T = 14 \times 3$$

$$\boxed{T = 42 \text{ seconds}} \quad \blacksquare$$

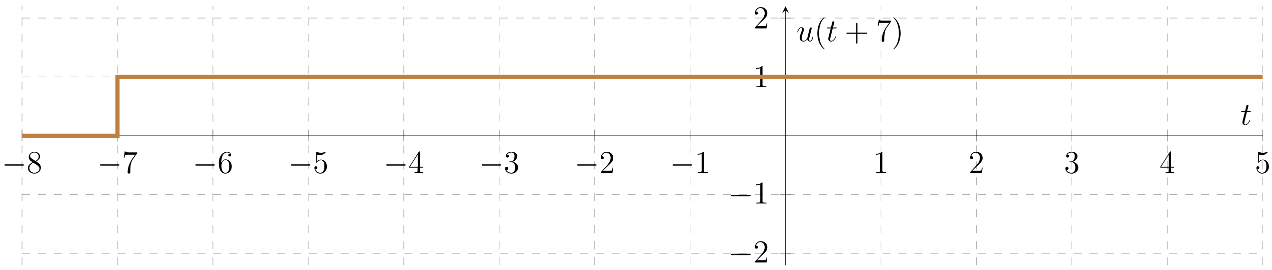
Q4: Consider the following signal. Answer the following questions.

$$x(t) = u(t+7) - 2\ u(t+1) + u(t-3)$$

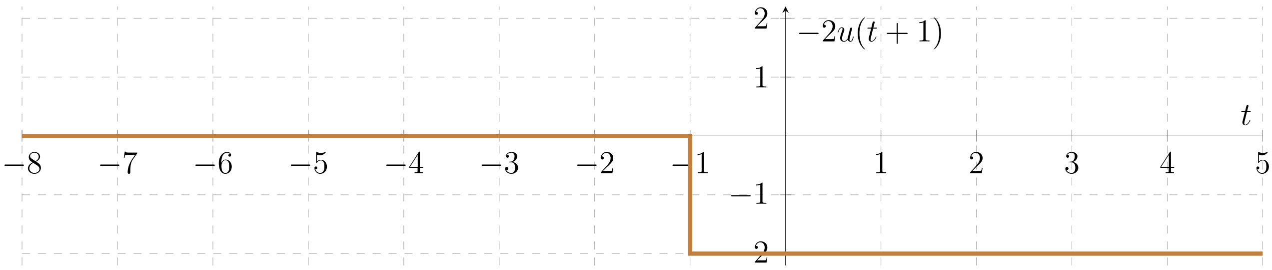
(a) `(10 pts)` Carefully sketch $x(t)$. Show your work.

Solution 4a:

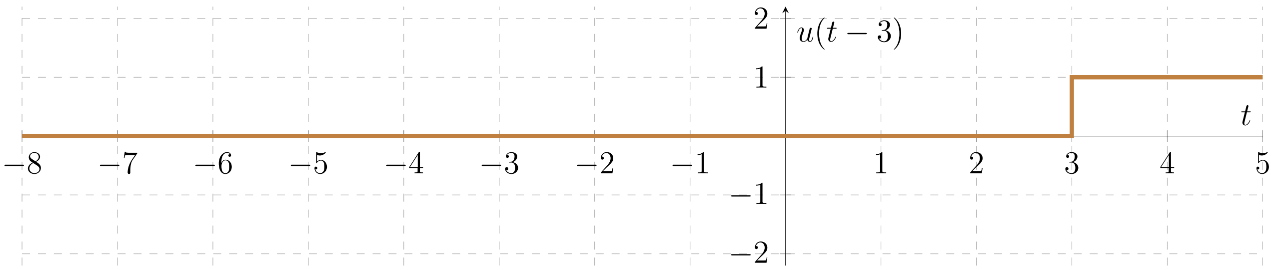
$u(t+7)$



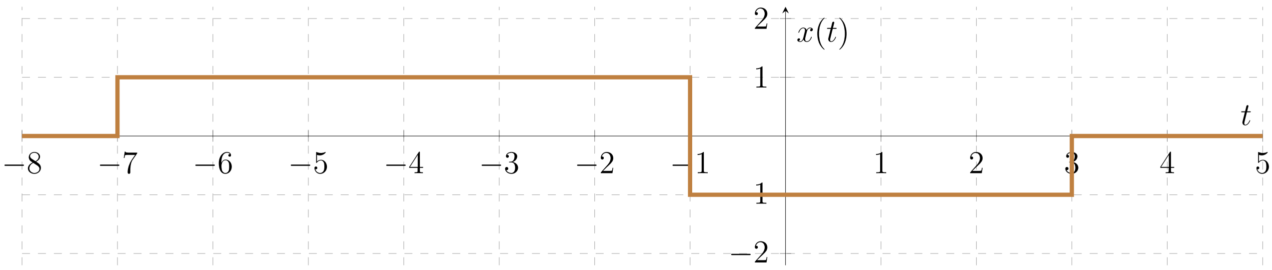
$-2u(t+1)$



$u(t-3)$



$x(t)$

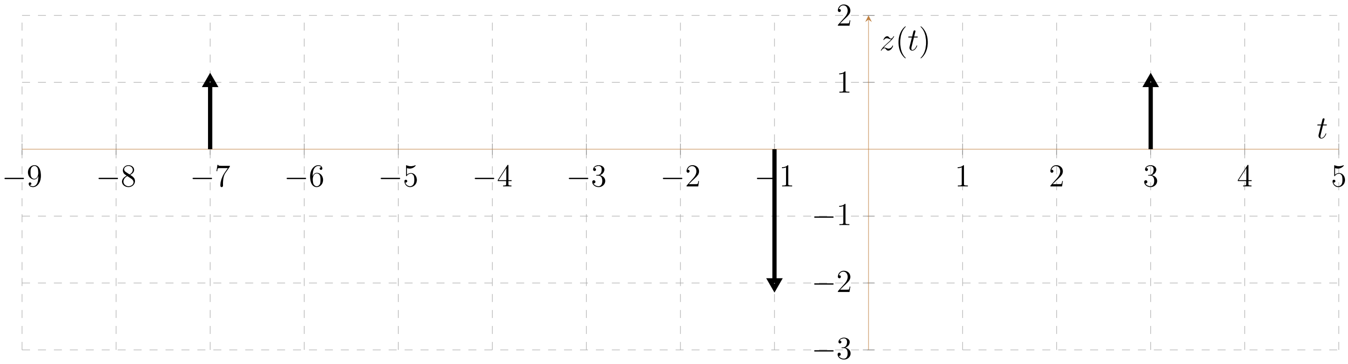


(b) `(10 pts)` Find and sketch the following signal, show your work.

$$z(t) = \frac{\mathrm{d}}{\mathrm{d}t}\, x(t)$$

Solution 4b:

$$
\begin{aligned}
z(t) &= \frac{\mathrm{d}}{\mathrm{d}t}\, x(t) \\
&= \frac{\mathrm{d}}{\mathrm{d}t}[u(t+7) - 2u(t+1) + u(t-3)] \\
&= \delta(t+7) - 2\delta(t+2) + \delta(t-3)
\end{aligned}
$$

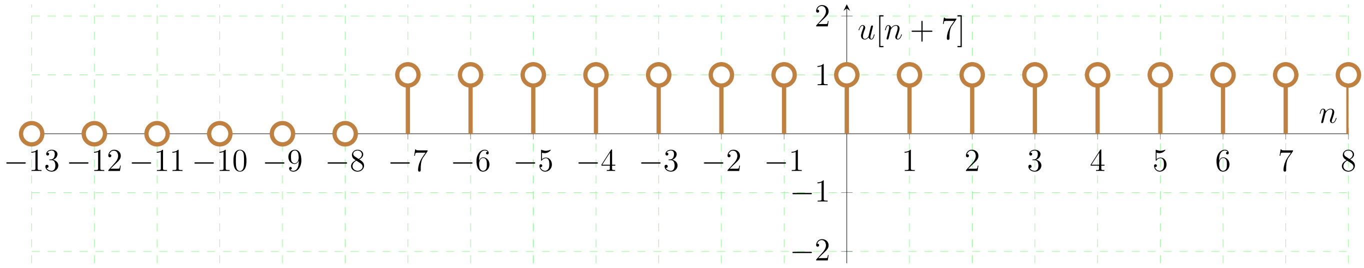


Q5: `(10 pts)` Carefully sketch the following signal. Show your work.

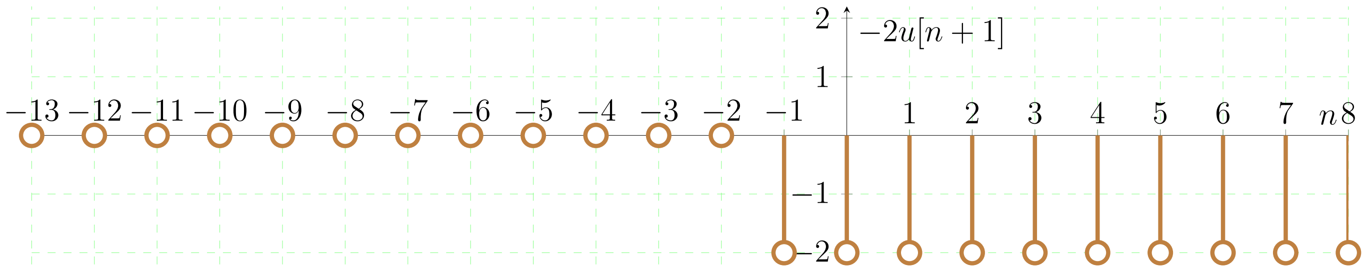
$$x[n] = u[n+7] - 2\ u[n+1] + u[n-3]$$

Solution 5:

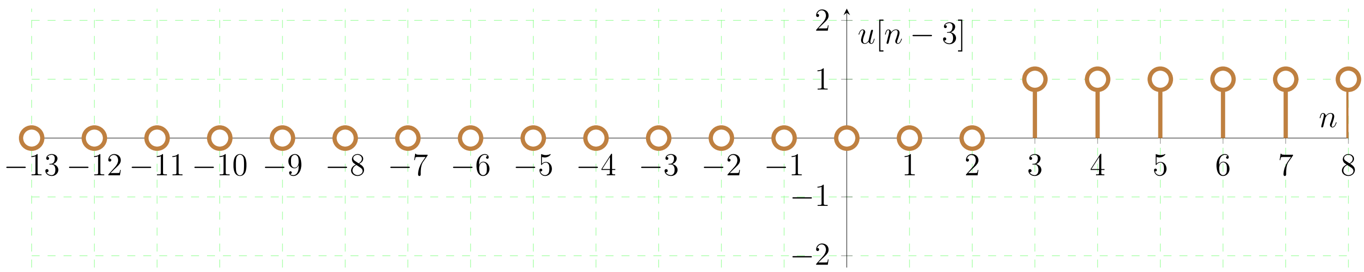
$u[n+7]$



$-2u[n+1]$



$u[n-3]$



$x[n]$