ELEMENTARY SIGNALS

① Exponential Signals

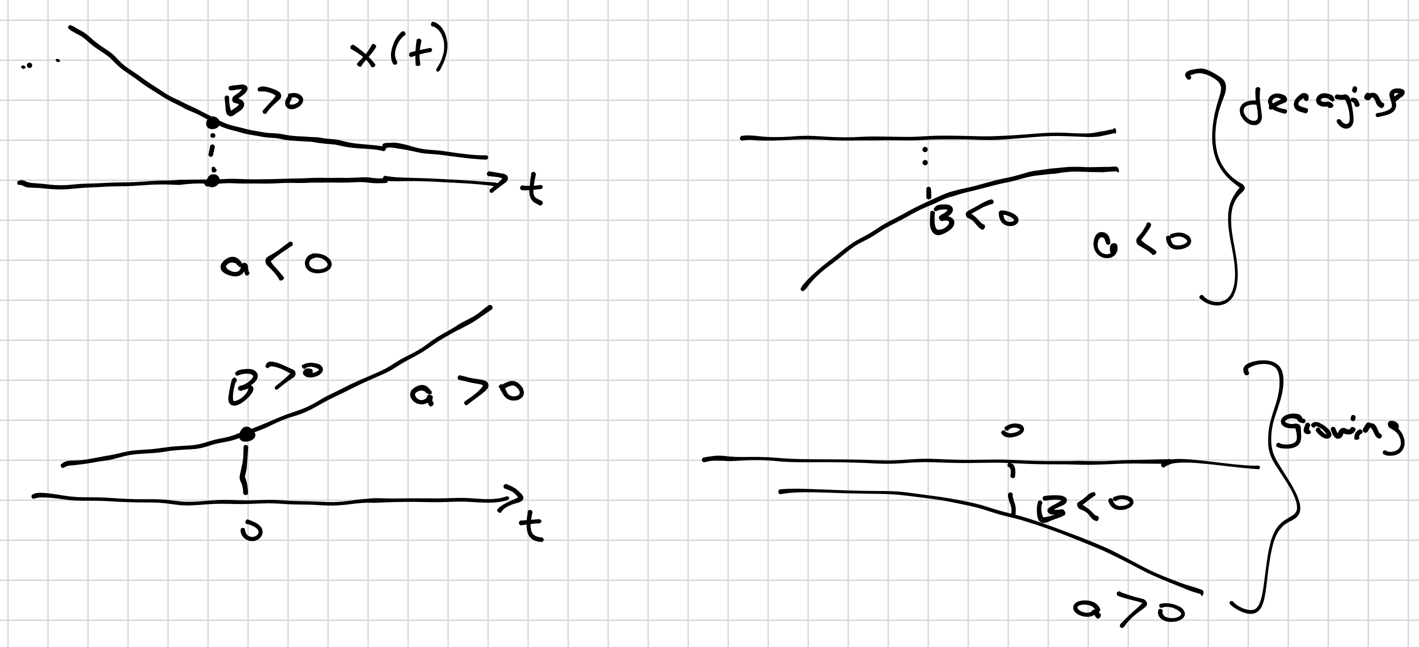
CT

$$x(t) = B \cdot e^{at} \qquad B, a \in \mathbb{R}$$

(B ← amplitude)

If $a < 0 \Rightarrow x(t)$ is a "decaying exponential"

If $a > 0 \Rightarrow x(t)$ is a "growing exponential"

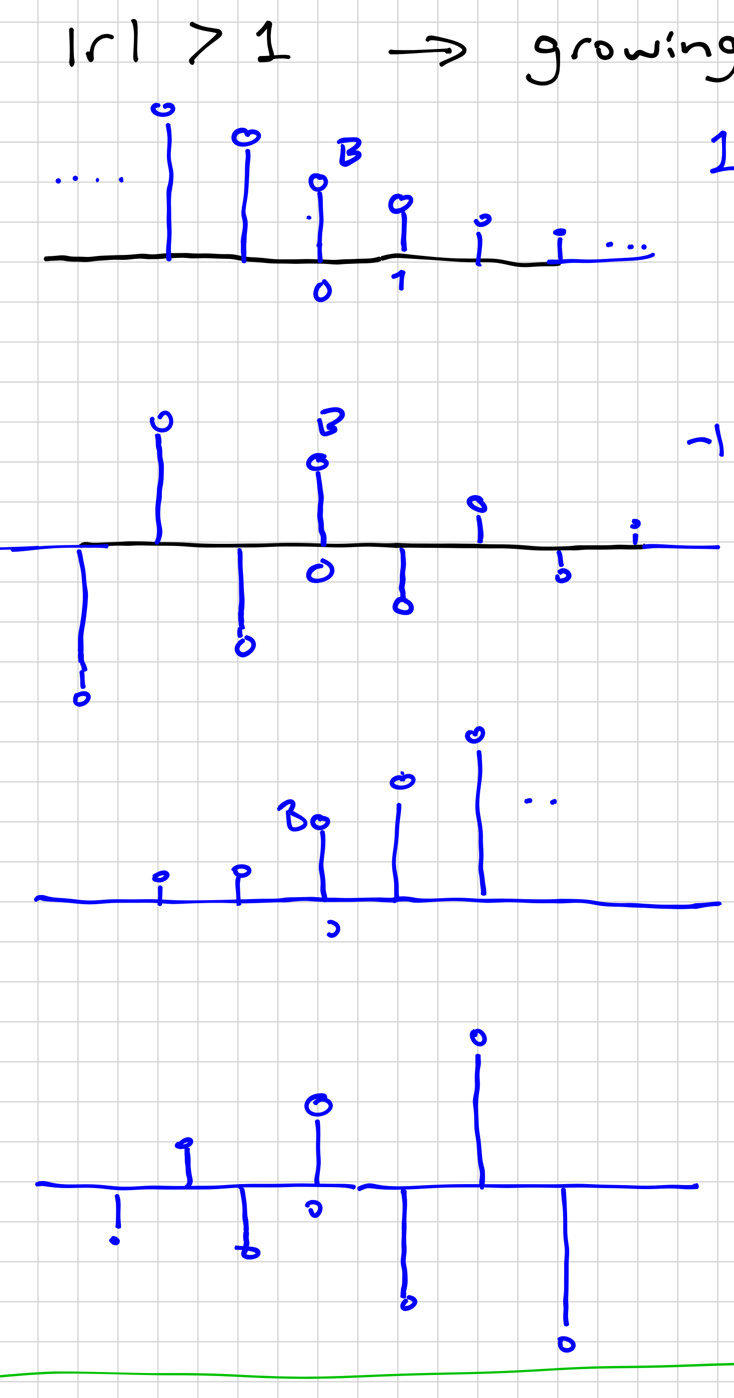


DT

$$x[n] = B \cdot r^n \qquad (r = e^a)$$

$0 < |r| < 1 \longrightarrow$ decaying exponential

$|r| > 1 \longrightarrow$ growing "



$1 > r > 0$

Decaying exponential

$-1 < r < 0$

"

$r > 1$

Growing expon.

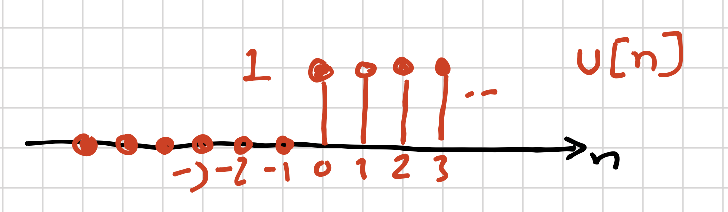
$r < -1$

Growing exponential

② Step Function

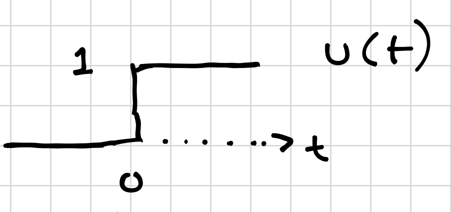
DT A D.T. Unit Step Function

$$u[n] = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}$$



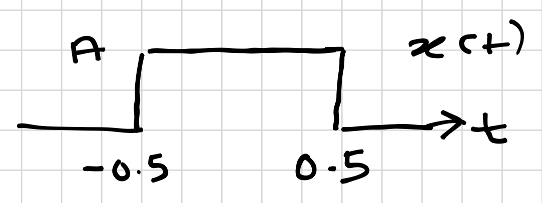
CT

$$u(t) = \begin{cases} 1, & t > 0 \\ 0, & t < 0 \end{cases}$$

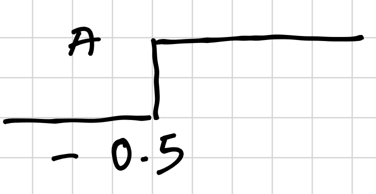


CT unit step function is undefined at $t = 0$

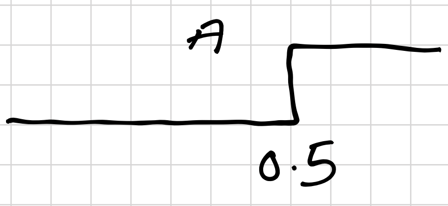
Ex



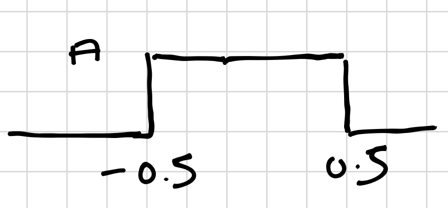
Express $x(t)$ as a weighted sum of two step functions.



$x_1(t) = A \cdot u(t + 0.5)$



$x_2(t) = A\, u(t - 0.5)$



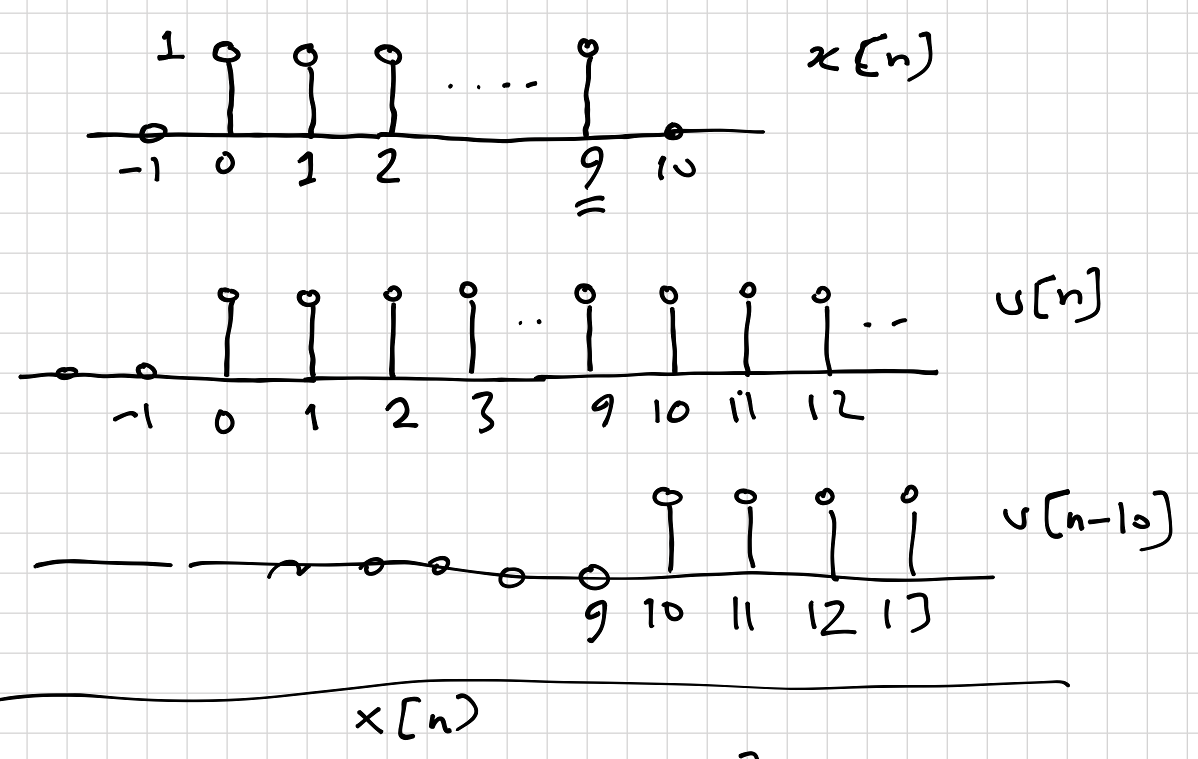
$x(t) = x_1(t) - x_2(t)$

$$\boxed{x(t) = A \cdot \{u(t + 0.5) - u(t - 0.5)\}}$$

Ex

$$x[n] = \begin{cases} 1, & 0 \leq n \leq 9 \\ 0, & \text{otherwise} \end{cases}$$

Express $x[n]$ as a weighted sum of two unit step function?

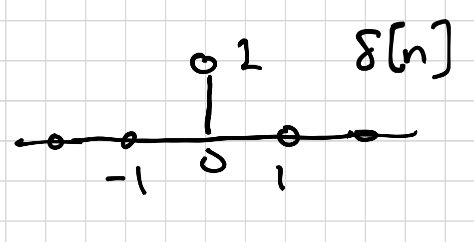


$$x[n] = u[n] - u[n - 10]$$

③ Impulse Function (Dirac-Delta Func.)

DT unit impulse function

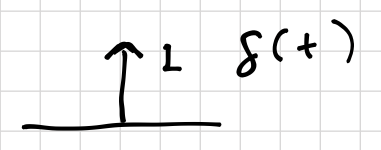
$$\delta[n] = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases}$$

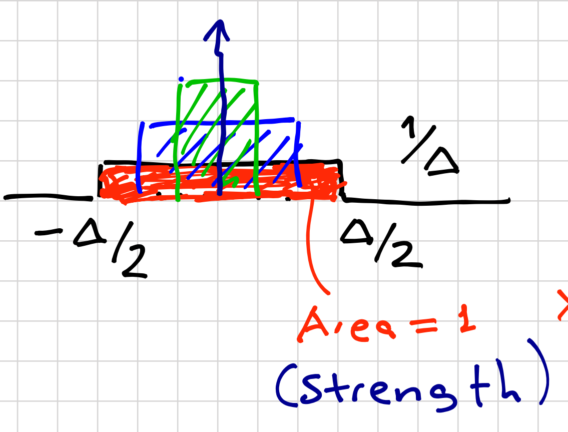


CT CT unit impulse function is defined by the following two relations

① $\delta(t) = 0 \qquad t \neq 0$

② $\displaystyle\int_{-\infty}^{+\infty} \delta(t)\, dt = 1$



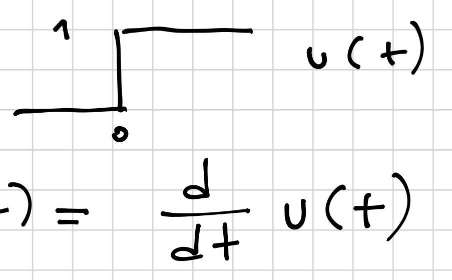


$x_\Delta(t)$

$$\boxed{\delta(t) = \lim_{\Delta \to 0} x_\Delta(t)}$$

/* $3\delta(t)$ has a strength of 3 */

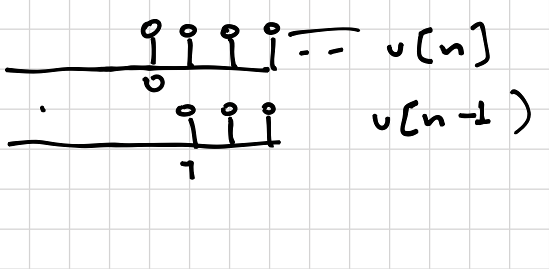
⊛ $\delta(t)$ and $u(t)$ are related to each other



$$\delta(t) = \frac{d}{dt} u(t)$$

$$u(t) = \int_{-\infty}^{t} \delta(\tau)\, d\tau$$

⊛ $\delta[n]$ and $u[n]$ are related to each other.



$$\delta[n] = u[n] - u[n-1]$$

$$u[n] = \sum_{k=-\infty}^{n} \delta[k]$$

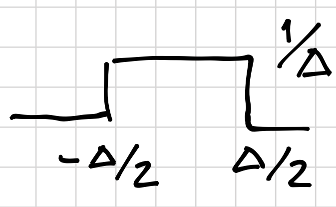
(*) $\delta(t)$ and $\delta[n]$ are even functions.

$$\delta(t) = \delta(-t)$$
$$\delta[n] = \delta[-n]$$

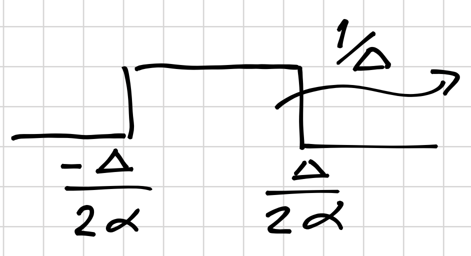
(*)
$$\left.\begin{aligned}\int_{-\infty}^{+\infty} x(t)\,\delta(t-\underline{\underline{t_0}})\,dt &= x(t_0)\\ \sum_{n=-\infty}^{+\infty} x[n]\,\delta[n-n_0] &= x[n_0]\end{aligned}\right\}$$

(*) Time-scaling property

$$\delta(\alpha t) = \frac{1}{\alpha}\,\delta(t)\ ,\quad \alpha>0$$



$$\lim_{\Delta\to 0} x_\Delta(t) = \delta(t)$$

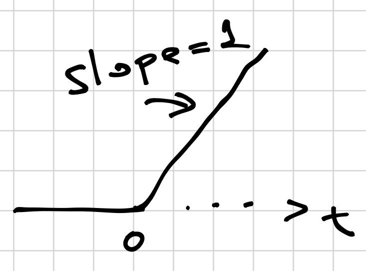


$$\lim_{\Delta\to 0} x_\Delta(\alpha t) = \frac{1}{\alpha}\,\delta(t)$$

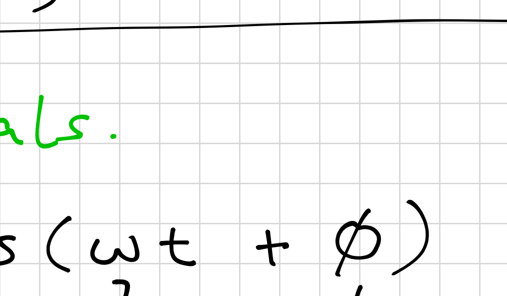
(4) Ramp Function

CT Unit ramp function

$$r(t) = \begin{cases} t, & t \geq 0 \\ 0, & t<0 \end{cases}$$



DT
$$r[n] = \begin{cases} n, & n \geq 0 \\ 0, & n<0 \end{cases}$$



$$\left.\begin{aligned} r[n] &= n\cdot u[n] \\ r(t) &= t\cdot u(t)\end{aligned}\right\}$$

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(5) Sinusoidal Signals.

CT
$$x(t) = \underbrace{A}_{\text{amplitude}} \cdot \cos(\underbrace{\omega}_{\substack{\text{angular freq.}\\ (\text{rad/sec})}} t + \underbrace{\phi}_{\text{phase angle (rad)}})$$

CT sinusoidal signals are periodic.

Period $\boxed{T = \frac{2\pi}{\omega}}$

DT
$$x[n] = \underbrace{A}_{\text{amplitude}} \cdot \cos(\underbrace{\Omega}_{\text{frequency}} n + \underbrace{\phi}_{\text{phase angle}})$$

DT sinusoidal signals MAY or MAY NOT be periodic.

In order for $x[n]$ to be periodic there must be an integer N which satisfy the following for all $\underline{n}$

$$x[n] = x[n+N] \quad \forall n$$

$$x[n+N] = A\cdot\cos(\Omega n + \boxed{\Omega N} + \phi)$$

/* If $\cos(a+b) = \cos(a)$, b is an integer multiple of 2π */

ΩN must be an integer multiple of 2π.

$$\boxed{\Omega N = 2\pi m \qquad m\in\mathbb{Z}^+}$$

Ther should at least be one (m,N) integer pairs in order for $x[n]$ to be periodic.

$$\Omega = 2\pi\left(\frac{m}{N}\right)$$

Ex
$$x[n] = \underline{\underline{\sin}}[5\pi n]$$
$$\Omega = 5\pi$$

$$\Omega = 5\pi = 2\pi\left(\frac{m}{N}\right) \qquad \frac{m}{N} = \frac{5}{2} \qquad (m,N) = (5,2)$$

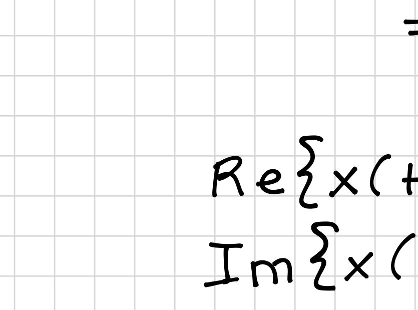
The equation is satisfied.

Ex
$$x[n] = \sin[2n]$$

$$\Omega = 2 \qquad 2 = 2\pi\cdot\left(\frac{m}{N}\right)$$

we cannot find an integer pair (m,N) that satisfies the equation. So, $x[n]$ is NOT periodic.

Relation Between Sinusoidal and Complex Exponential Signals.



$$\boxed{\begin{aligned}&\text{Euler's Identity}\\ &e^{j\theta} = \cos\theta + j\sin\theta\end{aligned}}$$

CT
$$\begin{aligned} x(t) &= A\cdot e^{j\omega t} \\ &= A\left[\cos(\omega t) + j\sin(\omega t)\right]\end{aligned}$$

$$\mathrm{Re}\{x(t)\} = A\cdot\cos(\omega t)$$
$$\mathrm{Im}\{x(t)\} = A\cdot\sin(\omega t)$$

DT
$$x[n] = A\ e^{j\Omega n}$$
$$\mathrm{Re}\{x[n]\} = A\cos(\Omega n)$$
$$\mathrm{Im}\{x[n]\} = A\sin(\Omega n)$$

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(6) Exponentially Damped Sinusoidal Signals

CT
$$x(t) = A\ e^{-\alpha t}\sin(\omega t + \phi)$$

$\alpha > 0$

$\alpha < 0$