Fourier Representation of Signals.

	DT: SIGNALS	CT
→ periodic	DT Fourier Series (DTFS)	FS Fourier Series
non Periodic	DT Fourier Transform (DTFT)	FT Fourier Transform

Periodic Signals (Fourier Series)

– Representing periodic signals as weighted superposition of complex sinusoids.

– Each sinusoid in the representation must have the same period as the signal.
∴ The fundamental frequency of each sinusoid must be integer multiple of the fundamental frequency of $x(t)$

– $x[n]$ is a DT signal with period N, we want to represent $x[n]$ by

$$\hat{x}[n] = \sum_{k} A[k] \cdot e^{jk\Omega_0 n} \quad k \in \mathbb{Z}.$$

$$\Omega_0 = \frac{2\pi}{N}$$

$k\Omega_0$: integer multiple of Ω_0 frequency of k^{th} sinusoid.

$A[k]$: Its weight

$e^{jk\Omega_0 n}$: harmonic

Each of these signals have a common period N

CT

$x(t)$ T: fundamental period

$\omega_0 : \frac{2\pi}{T}$ fundamental frequency

Represent $x(t)$ by

$$\hat{x}(t) = \sum_{k} A[k] e^{jk\omega_0 t}$$

$k\omega_0$: frequency of k^{th} sinusoid.

$A[k]$: weight

$e^{jk\omega_0 t}$: k^{th} harmonic.

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* How many harmonics for DTFS?

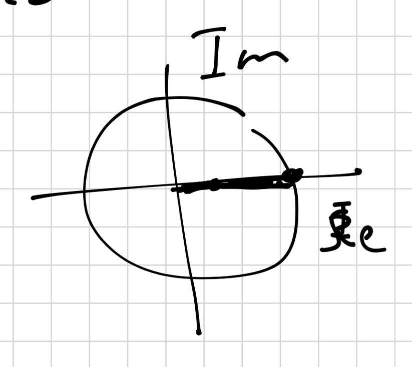
There can only be N distinct complex sinusoids in the form of $e^{jk\Omega_0 n}$

$e^{jk\Omega_0 n}$ is periodic. One of its periods is $N = \frac{2\pi}{\Omega_0}$

$$e^{j(k+N)\Omega_0 n} = e^{jk\Omega_0 n}$$

$$= e^{jk\Omega_0 n} \cdot e^{jkN\Omega_0 n}$$

$$= e^{jk\Omega_0 n} \cdot \underbrace{e^{jk2\pi n}}_{=1} \rightarrow \cos(k2\pi n) + j\sin(k2\pi n)$$



$k=0 \quad n=0$

$\cos(k2\pi n) = 1$

$\sin(k2\pi n) = 0$

$k = 0, 1, \ldots, N-1$

$$\hat{x}[n] = \sum_{k=0}^{N-1} A[k] \cdot e^{jk\Omega_0 n}$$

$$= \sum_{k=\langle N \rangle} A[k] e^{jk\Omega_0 n}$$

– FS (CT signals)

The CT complex sinusoids $e^{jk\omega_0 t}$ with distinct frequencies $k\omega_0$ are always distinct.

So, there could be an infinite number of distinct terms ∴

$$\hat{x}(t) = \sum_{k=-\infty}^{+\infty} A[k] \cdot e^{jk\omega_0 t}$$

We want to find coefficients $A[k]$ such that $\hat{x}[n]$ and $\hat{x}(t)$ are good aproximations to $x[n]$ and $x(t)$

We use a measure called Mean-Squre Error (MSE) – MSE should be minimized for this purpose

$$\text{DT} \quad MSE = \frac{1}{N} \sum_{n=0}^{N-1} |x[n] - \hat{x}[n]|^2$$

$$\text{CT} \quad MSE = \frac{1}{T} \int_{0}^{T} |x(t) - \hat{x}(t)|^2 dt$$

NONPERIODIC SIGNALS

~ There are no restrictions on the period of the complex sinusoids.

~ The complex sinusoids will include a continuum of frequencies.

$$\text{CT} \quad \hat{x}(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} X(j\omega) e^{j\omega t} dt$$

$$\text{DT} \quad \hat{x}[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\Omega}) e^{j\Omega t} dt$$

DT Periodic Signals – DTFS

$$(n, N) \xleftrightarrow{DTFS} (\Omega, k)$$

$x[n]$ is periodic, N is the fundamental period. ($\Omega_0 = \frac{2\pi}{N}$

DTFS of $x[n]$

$$x[n] = \sum_{k=0}^{N-1} \underbrace{X[k]}_{\text{DTFS coeffients.}} \cdot e^{jk\Omega_0 n}$$

$$\underbrace{X[k]}_{\text{Freqency domain representation.}} = \frac{1}{N} \sum_{n=0}^{N-1} x[n] \cdot e^{-jk\Omega_0 n}$$

Each DTFS coefficient is associated with a different frequency.