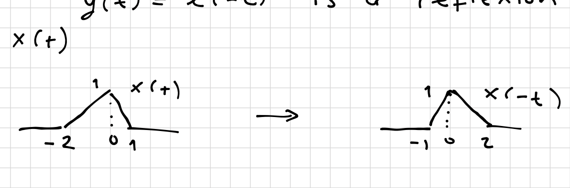
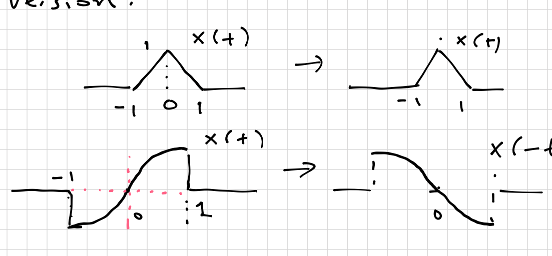
Reflection

$x(t)$

$y(t) = x(-t)$ is a reflexion of $x(t)$



- Even signals : $x(t)$ is the same as its reflected version.
- Odd signals : $x(t)$ is the same as the negative of its reflected version.

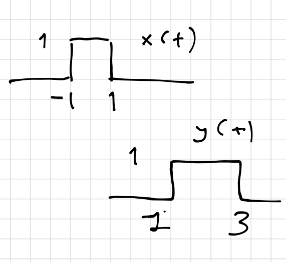


Time Shifting

$y(t) = x(t - t_0) \quad t_0 \in \mathbb{R}$

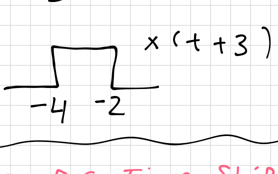
If $t_0 > 0$ then the output is obtained by shifting the input signal by t_0 toward right

If $t_0 < 0$ then the output is obtained by shifting the input signal Left by t_0



$y(t) = x(t-2)$
$y(0) = x(-2)$
$y(1) = x(-1)$
$y(3) = x(3-2)$
$x(1)$

$x(t+3)$



Precedence Rule for Time-Shifting and Time-Scaling

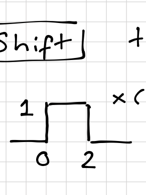
$y(t) = x(\alpha t - \beta)$

$v(t) = x(t - \beta)$ } SHIFT

$y(t) = v(\alpha t) = x(\alpha t - \beta)$ } SCALE

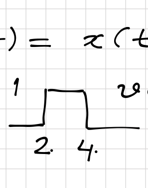
$\boxed{\text{Shift}}$ then $\boxed{\text{scale}}$

Ex



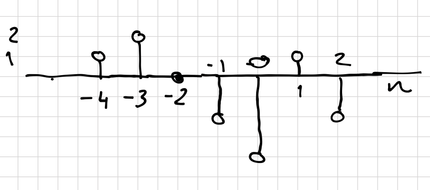
$y(t) = x(2t+2)$

① $v(t) = x(t-2)$



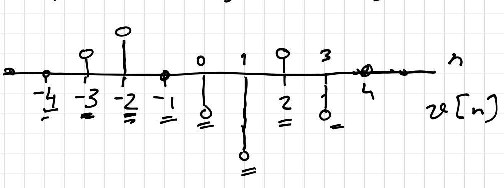
② $y(t) = v(2t) = x(2t-2)$

Ex $x[n]$

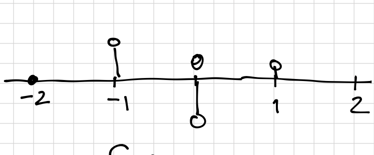


$y[n] = x[2n-1]$

① Shift $v[n] = x[n-1]$



② Scale $y[n] = v[2n]$



$y[n] = x[2n-1]$

$$y[n] = \begin{cases} 2, & n=-1 \\ -2, & n=0 \\ 1, & n=1 \\ 0, & \text{otherwise.} \end{cases}$$

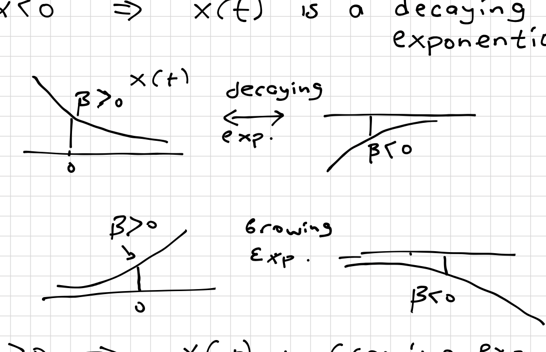
ELEMANTARY SIGNALS

① Exponential Signals

CT

$x(t) = \beta \cdot e^{\alpha t}$ (β: amplitud.) $\quad \beta, \alpha \in \mathbb{R}$

If $\alpha < 0 \Rightarrow x(t)$ is a decaying exponential.



$\alpha > 0 \Rightarrow x(t)$: Growing expon.

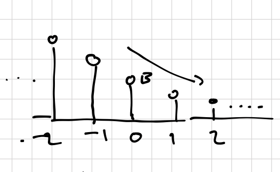
DT

$x[n] = B \cdot r^n \quad (r = e^{\alpha})$

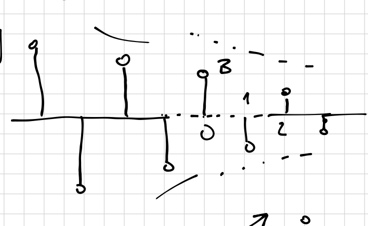
$0 < |r| < 1 \rightarrow$ decaying exp.

$|r| > 1 \rightarrow$ growing "

$\boxed{0 < r < 1}$:

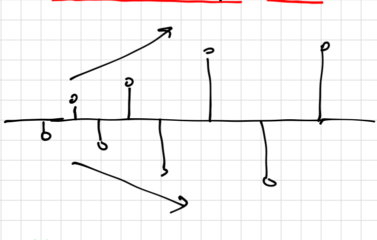


$\boxed{-1 < r < 0}$



$\boxed{r > 1}$

$\boxed{r < -1}$

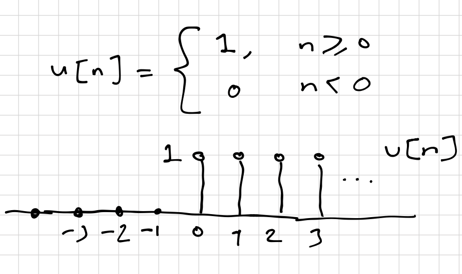


② Step Function

DT

A DT unit step function

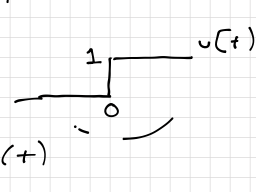
$$u[n] = \begin{cases} 1, & n \geq 0 \\ 0 & n < 0 \end{cases}$$



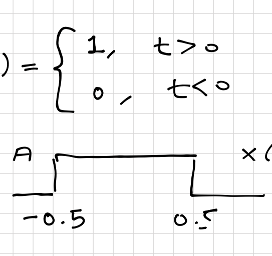
CT

CT unit step function.

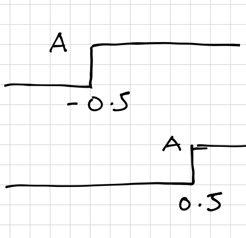
$$u(t) = \begin{cases} 1, & t > 0 \\ 0, & t < 0 \end{cases}$$



Ex



Express $x(t)$ as a weghted sum of to step functions.



$x_1(t) = A\,u(t+0.5)$

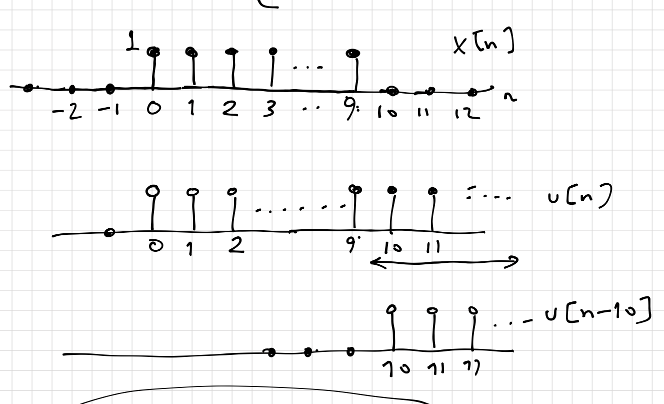
$x_2(t) = A\,u(t-0.5)$

$x(t) = x_1(t) - x_2(t)$

$x(t) = A\{u(t+0.5) - u(t-0.5)\}$

Ex

$$x[n] = \begin{cases} 1, & 0 \leq n \leq 9 \\ 0, & \text{otherwise} \end{cases}$$



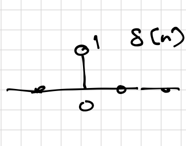
$x[n] = u[n] - u[n-10]$

③ Impulse Function (Dirac-Delta Fn).

DT

unit impulse function.

$$\delta[n] = \begin{cases} 1, & n=0 \\ 0, & n \neq 0 \end{cases}$$

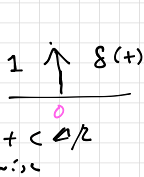


CT

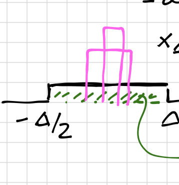
CT unit impulse function is defined by the following two relations.

① $\delta(t) = 0 \quad t \neq 0$

② $\int_{-\infty}^{+\infty} \delta(t)\,dt = 1$



$$x_\Delta(t) = \begin{cases} 1/\Delta, & -\Delta/2 < t < \Delta/2 \\ 0, & \text{otherwise} \end{cases}$$

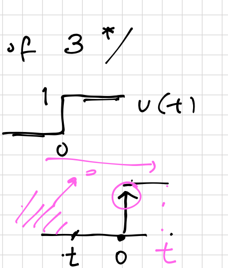


$\delta(t) = \lim_{\Delta \to 0} x_\Delta(t)$

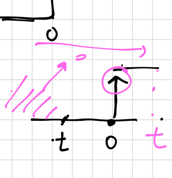
Area = ① → strength.

/* $3\delta(t)$ has a strength of 3 */

(*) $\delta(t) = \frac{d}{dt} u(t)$



(*) $u(t) = \int_{-\infty}^{t} \delta(\tau)\,d\tau$



(*) $\delta[n] = u[n] - u[n-1]$

(*) $u[n] = \sum_{k=-\infty}^{n} \delta[k]$

(*) $\delta(t)$ and $\delta[n]$ are even signals.

$\delta(t) = \delta(-t)$
$\delta[n] = \delta[-n]$

(*) $\int_{-\infty}^{+\infty} x(t)\,\delta(t-t_0)\,dt = x(t_0)$

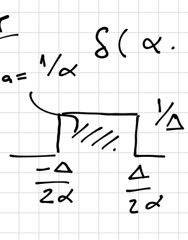
(*) $\sum_{n=-\infty}^{+\infty} x[n]\,\delta[n-n_0] = x[n_0]$

Time-scaling Property

CT

$\delta(\alpha \cdot t) = \frac{1}{\alpha} \cdot \delta(t) \quad \alpha > 0$

Area $= 1/\alpha$



$x_\Delta(\alpha t)$

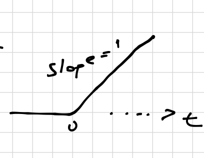
$\lim_{\Delta \to 0} x_\Delta(\alpha t) = \frac{1}{\alpha}\,\delta(t)$

④ Ramp Function

CT

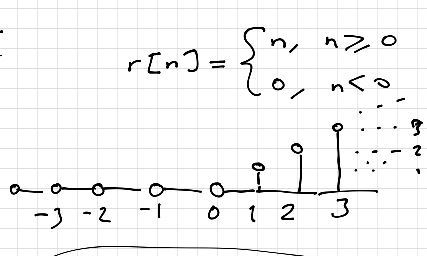
Unit ramp function

$$r(t) = \begin{cases} t, & t \geq 0 \\ 0, & t < 0 \end{cases}$$



DT

$$r[n] = \begin{cases} n, & n \geq 0 \\ 0, & n < 0 \end{cases}$$



$r[n] = n \cdot u[n]$
$r(t) = t \cdot u(t)$

⑤ Sinusoidal Signals.

CT

$x(t) = A \cdot \cos(\omega t + \phi)$

A: Amplitude; ω: angular freq. (rad/sec); ϕ: phase angle (rad)

CT sinusoidal signals are periodic!

Period $T = \frac{2\pi}{\omega}$

DT

$x[n] = A \cdot \cos(\Omega n + \phi)$

A: amplitude; Ω: frequency (rad); ϕ: phase angle (rad.)

DT sinusoidal signals may or may not be periodic.

For a peridic signal :

$x[n] = x[n+N] \quad \forall n$,

$x[n+N] = A \cdot \cos(\Omega n + \Omega N + \phi)$

/* $\cos(a+b) = \cos(a) \Rightarrow b$ is an integer multiple of 2π */

ΩN must be integer multiple of 2π

$\Omega N = 2\pi \cdot m, \quad m \in \mathbb{Z}^+$

There should at least be one (m, N) integer pair in order for $x[n]$ to be periodic.

$\Omega = 2\pi \left(\frac{m}{N}\right)$

Ex

$x[n] = \sin[5\pi \cdot n]$

$\Omega = 5\pi = 2\pi \left(\frac{m}{N}\right)$ $\quad m = 5$, $n = 2$ $\therefore x[n]$ is periodic.

Ex

$x[n] = \sin[2n]$

$\Omega = 2 = 2\pi \cdot \left(\frac{m}{N}\right)$

no (m, N) integer pair exist

$\therefore x[n]$ is not periodic.