Signal Processing / Fall 2020

— mergen. btk...

— Textbook "Signals and Systems", Simon Haykin & Barry Van Veen Wiley, 2nd ed.

Türkçe : 1) Oppenheim, "Sinyaller ve Sistemler"

2) Schaum's outlines "

What is a signal

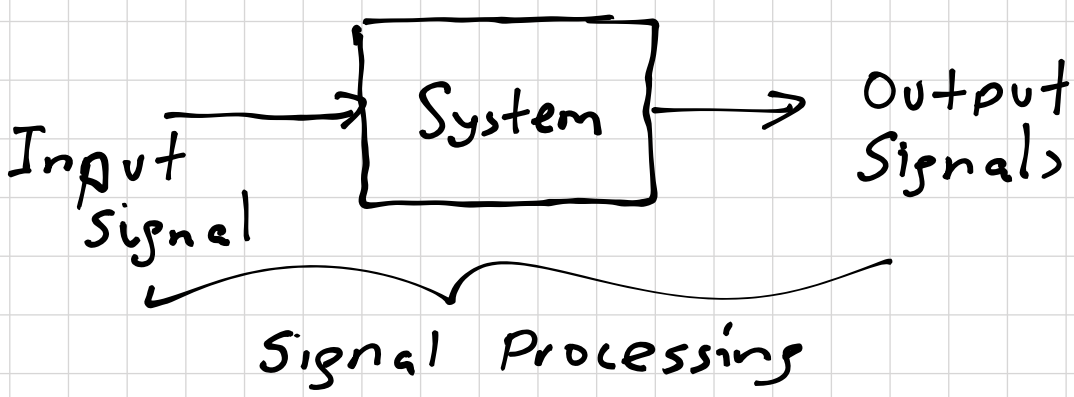
- Speech Signals.
- Emails
- Heartbeat
- Radio waves
- Stock prices

Sinyal, bir fiziksel olgunun doğasında bilgi ileten bir veya daha fazla değişkenin fonksiyonudur.

"A signal is a function of one or more variables that conveys information on the nature of a physical phenomenon"

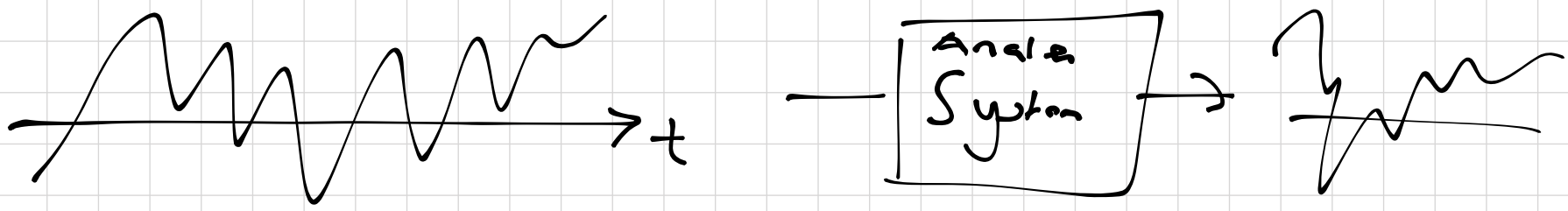
What is a system

A system is an entity that manipulates one or more signals to accomplish a function, therefore yielding new signals.

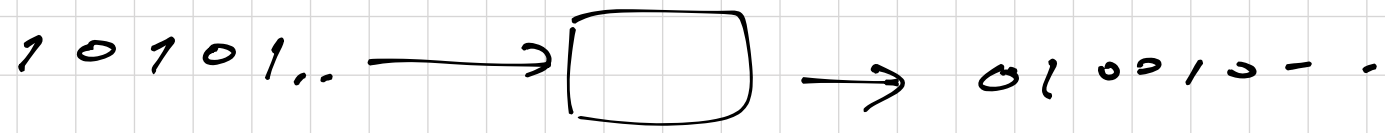


Analog vs Digital Signal Processing

- Analog signal processing involves continuous signals.



- Digital Signal Processing } Discrete & quantized



Classification of Signals

- In this class we will focus on one dimensional single valued signals.

- we will represent the signals as a function time.

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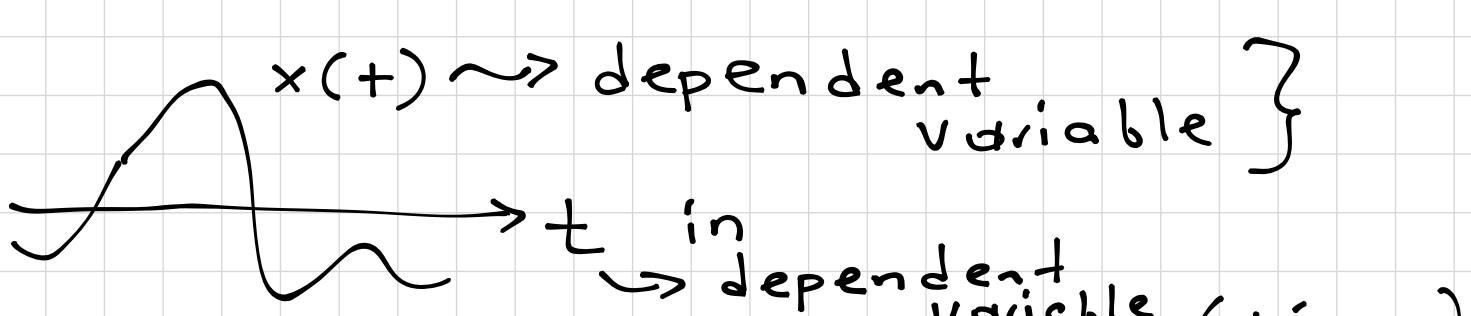
① Continuous-Time (CT) vs.
Discrete-Time (DT) Signals

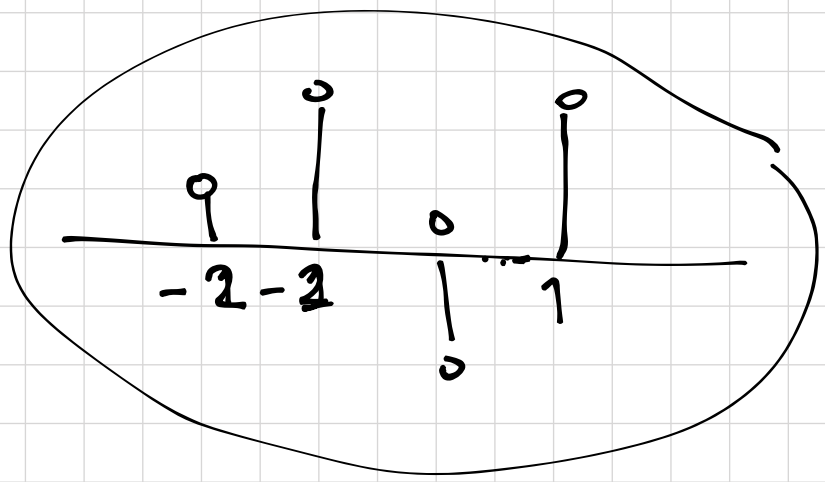


- A CT signal, x(t), is defined for all time, t.

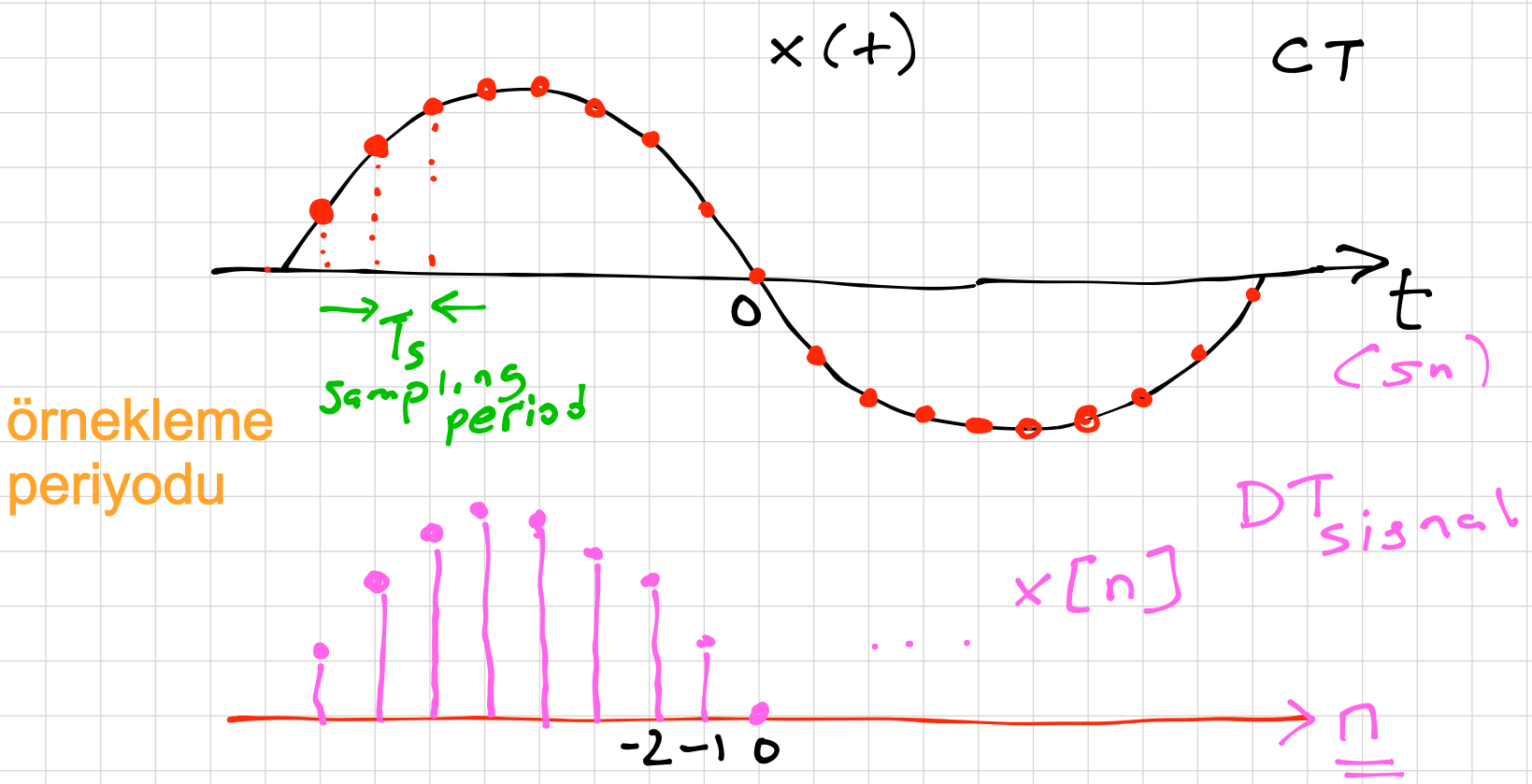
x (t) → time

parantheses are used

- A DT signal is defined only on discrete instants of time.



Sampling — is used to derive DT signals from a CT signal by taking the values at a uniform rate.

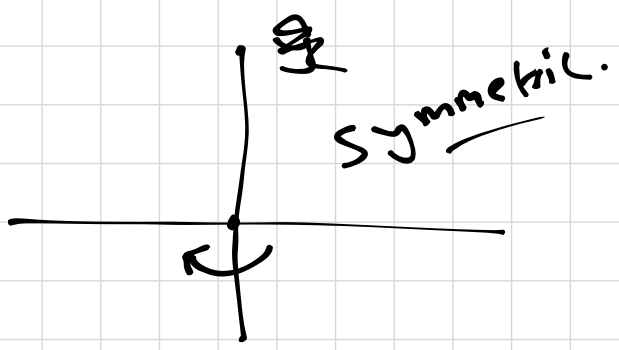


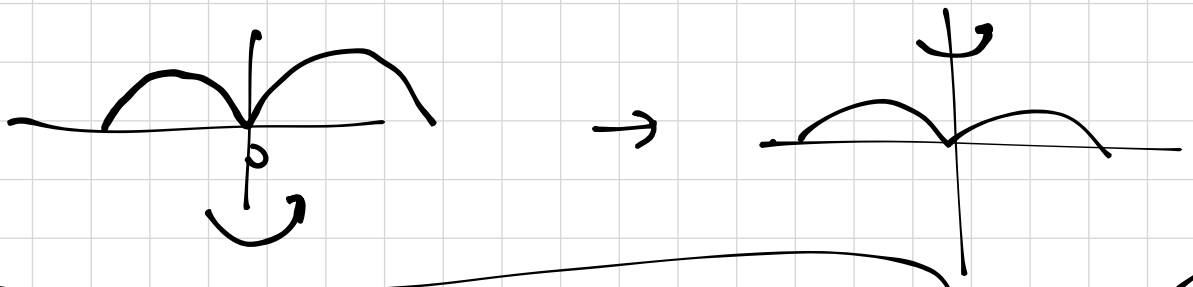
$$x[n] = x[nT_s], \quad n = 0, \mp 1, \mp 2 \cdots$$

independent variable

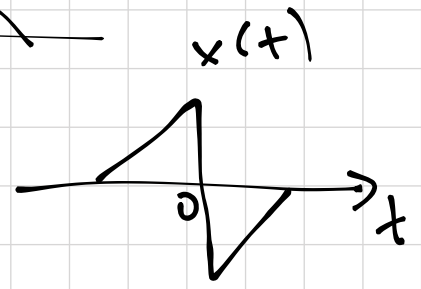
② Even and Odd Signals

Even $\quad x(-t) = x(t)$





Odd $\quad x(-t) = -x(t)$



(antisymmetrical) about y axis.

Ex

$$x(t)=\begin{cases}\sin(\pi t/T), & -T\le t<T\\ 0, & \text{otherwise}\end{cases}$$

$$x(-t)=\begin{cases}\sin(-\pi t/T), & -T\le -t<T\\ 0, & \text{otherwise}\end{cases}$$

$$=\begin{cases}-\sin\left(\frac{\pi t}{T}\right), & -T\le t\le T\\ 0, & \text{otherwise}\end{cases}$$

$x(-t)=-x(t)$: ODD signal

$-T\le -t\le T$

$T\ge t\ge -T$

Even-Odd Decomposition

① $x(t)=\overbrace{x_e(t)}^{\text{even component}}+\underbrace{x_o(t)}_{\text{odd component}}$

$x_e(t)$ is an even signal

$x_o(t)$ " " odd "

$$x_e(t)=x_e(-t)$$

$$x_o(t)=-x_o(-t)$$

+

② $x(t)=x_e(t)+x_o(t)=x_e(-t)-x_o(-t)$

$x(-t)=x_e(t)-x_o(t)$

$$x_e(t)=\frac{1}{2}[x(t)+x(-t)]$$

$$x_o(t)=\frac{1}{2}[x(t)-x(-t)]$$

Ex Find the even and odd components of the following signal.

$$x(t) = e^{-2t} \cdot \cos(t)$$

$$x(-t) = e^{2t} \cdot \cos(-t)$$

$$= e^{2t} \cos(t)$$

$$x_e(t) = \frac{1}{2}\left[e^{-2t}\cos t + e^{2t}\cos t\right]$$

$$= \cos t \cdot \underline{\frac{1}{2}\left[e^{-2t} + e^{2t}\right]}$$

$$x_e(t) = \cosh(2t)\cos(t)$$

$$x_o(t) = -\sinh(2t)\cos t$$

Symmetry for complex signal

A complex-valued signal, $x(t)$ is said to be "conjugate symmetric" if

(conjugate: eşlenik)

$$x(-t) = \underline{x^*(t)}$$

/*

$$x(t) = a(t) + j \cdot b(t)$$

$$x^*(t) = a(t) - j\, b(t)$$

*/

If $x(t)$ conjugate symmetric

$$a(-t) + j\, b(-t) = a(t) - j\, b(t)$$

($a(-t)$: EVEN; $j\,b(-t)$: odd)

- The real part of $x(t)$ is EVEN
- The imaginary " " $x(t)$ is ODD

③ Periodic and non-periodic signals

CI

If $x(t)$ is a periodic signal

then $\boxed{x(t) = x(t+T), \quad \forall t}$

where T is a positive constant.

— If the condition is satisfied for let's say $T = T_0$ then it will also be satisfied for $T = 2T_0, 3T_0, \cdots$

$$x(t) = x(t+T_0) = x(t+2T_0) = \cdots$$

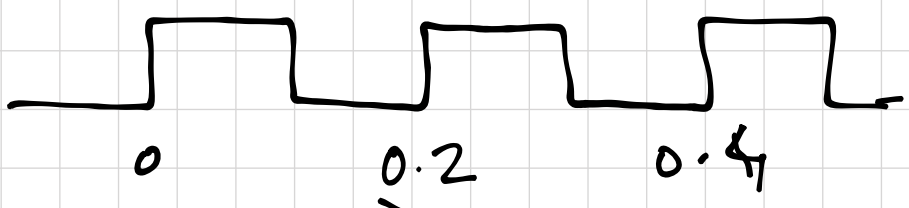
— The smallest valve of T that satisfies the condition is called "the fundamental period" T (seconds)

— frequency $f = \frac{1}{T}$ (Hz)

— Angula frequency

$$\omega = 2\pi f = \frac{2\pi}{T} \quad \text{(rad/sec)}$$

Ex



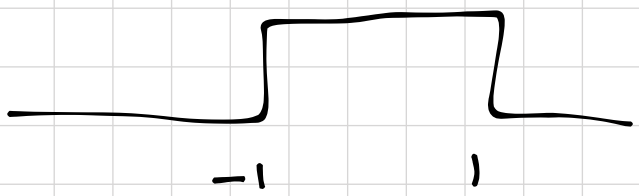
$x(t)$

$T = 0.2$ s.

$f = \frac{1}{0.2} = 5$ Hz

$\omega = 2\pi 5 = 10\pi$ rad/s

Ex



non-periodic

DT

$x[n]$ is periodic if

$$x[n] = x[n+N] \quad \text{for all } n$$

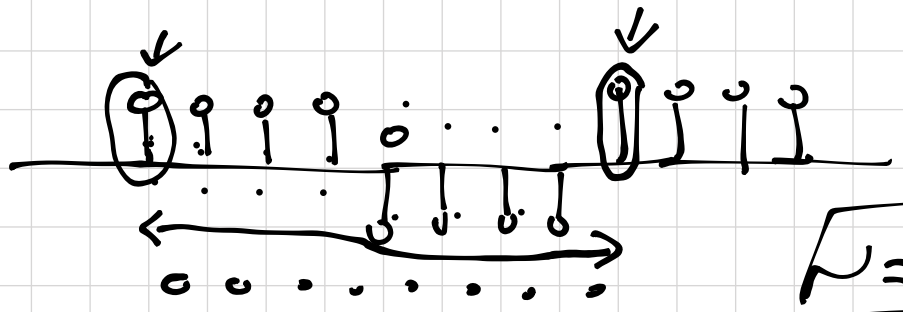
where N is a positive integer.

The smallest N that satisfies this condition is called "the fundamental period"

fundamental frequency } $\Omega = \frac{2\pi}{N}$ (radians)

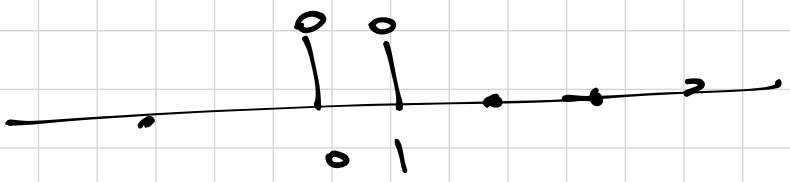
$N \rightarrow$ integer!

Ex



$N = 8$ $\quad \Omega = \frac{2\pi}{8} = \frac{\pi}{4}$ rad.

Ex



non-periodic.