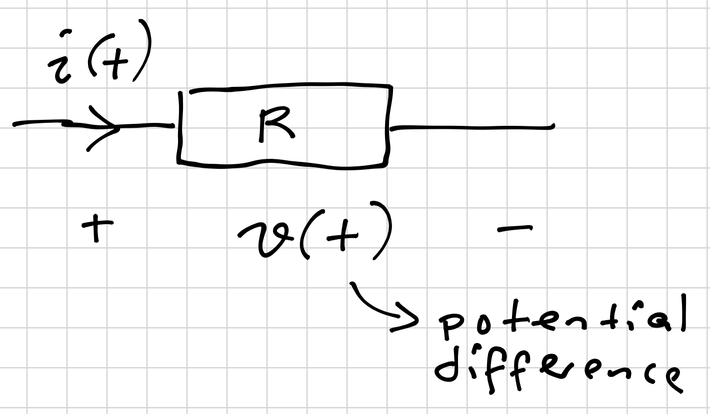
④ Deterministic vs Random Signals

- A <u>deterministic signal</u> is a signal about which there's no uncertainty with respect to its value at any time.

- A <u>random signal</u> is that about which there's uncertainty before it occurs.

⑤ Energy vs Power Signals.



$$i(t) = \frac{v(t)}{R}$$

$$v(t)\, i(t) = v(t) \frac{i(t)}{R}$$

The instantenous power dissipated in the resistor is

$$p(t) = \frac{v^2(t)}{R} = R \cdot i^2(t)$$

If $R=1$ then $p(t) = v^2(t) = i^2(t)$

Let's define instantenous power of a signal $x(t)$

$$p(t) \triangleq x^2(t)$$

Total energy

$$E = \lim_{T \to \infty} \int_{-T/2}^{T/2} x^2(t)\, dt$$

$$E = \int_{-\infty}^{+\infty} x^2(t)\, dt$$ → total energy.

Average power

$$P = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} x^2(t)\, dt$$

If $x(t)$ is periodic with the fundametal period T then the average (time-averaged) power is

$$\boxed{P = \frac{1}{T} \int_{\langle T \rangle} x^2(t)\, dt}$$ average power

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For a DT signal

$x[n]$ -

Total Energy $E = \sum_{n=-\infty}^{+\infty} x^2[n]$

Average Power $P = \lim_{N \to \infty} \frac{1}{2N} \sum_{n=-N}^{N} x^2[n]$

If $x[n]$ is periodic with fundamental period N

$$P = \frac{1}{N} \sum_{n=0}^{N-1} x^2[n]$$

- A signal is referred to as an energy signal if

$\boxed{0 < E < \infty}$ → Energy

- It is a power signal if

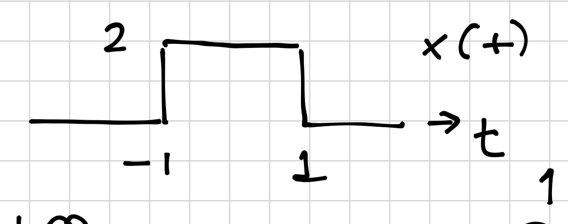
$\boxed{0 < P < \infty}$ → Power signal

Energy signals have zero <u>average power</u>

Power signals have <u>infinite energy</u>

A <u>non-zero signal</u> <u>CANNOT</u> be both Energy and Power signal

Ex



$E = ?$
$P = ?$

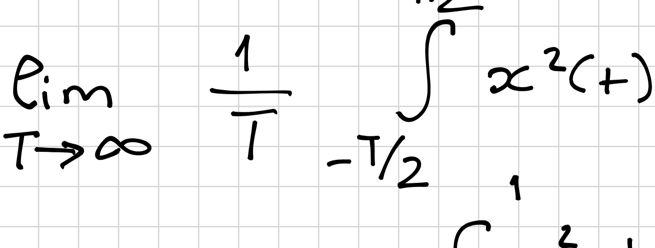
$$E = \int_{-\infty}^{+\infty} x^2(t)\, dt = \int_{-1}^{1} 2^2\, dt = 4t \Big|_{-1}^{1} = \underline{8}$$

$0 < E < \infty$ → $\boxed{\text{ENERGY signal}}$

$$P = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} x^2(t)\, dt$$

$$= \lim_{T \to \infty} \frac{1}{T} \int_{-1}^{1} 2^2\, dt = \lim_{T \to \infty} \frac{1}{T} \cdot 8 = \underline{0}$$

Ex

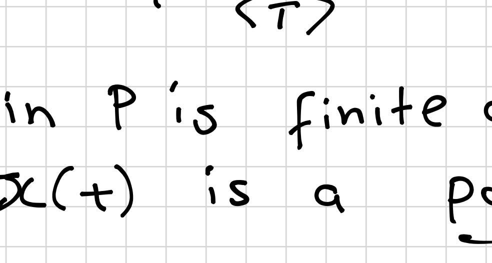


$$P = \frac{1}{T} \int_{\langle T \rangle} x^2(t)\, dt = \frac{1}{2} \int_0^2 2^2\, dt = 4$$

Sin P is finite and non-zero $x(t)$ is a <u>power signal</u>.

It has infinite <u>energy</u>.

Ex



$$E = \sum_{n=-\infty}^{+\infty} x^2[n] = 2^2 + 2^2 = 8$$

It is an energy signal!

<u>$P = 0$</u>

Basic Operations on Signals

$x[n]$ $x(t)$ → independent variables

dependent variable

Operations Performed on the Dependent Variable

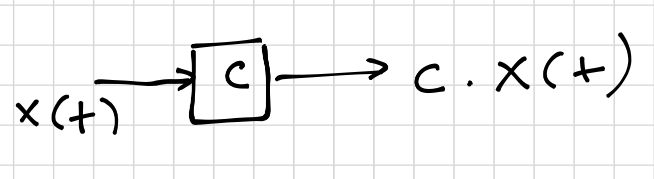
- Amplitude Scaling

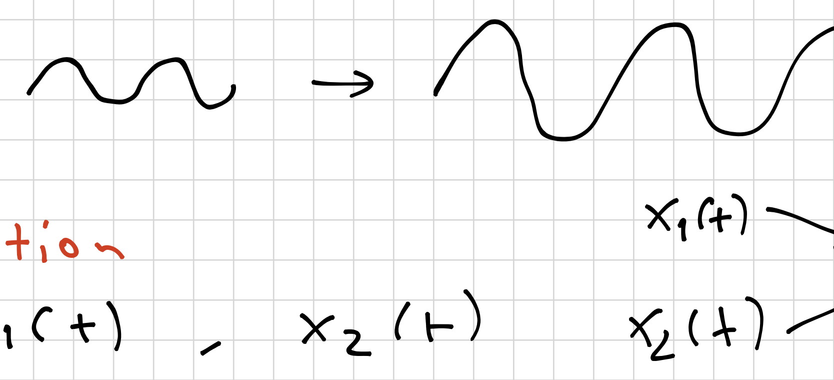
$x(t)$

$y(t) = c \cdot x(t)$

c → real constant

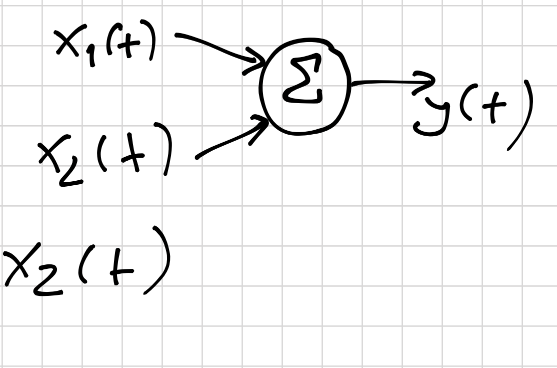
→ Same for DT





- Addition

$x_1(t)$, $x_2(t)$

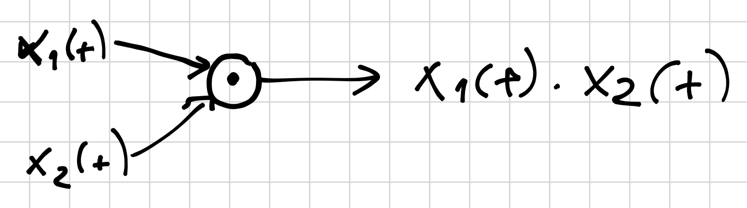


$$y(t) = x_1(t) + x_2(t)$$

Multiplication

$x_1(t)$, $x_2(t)$

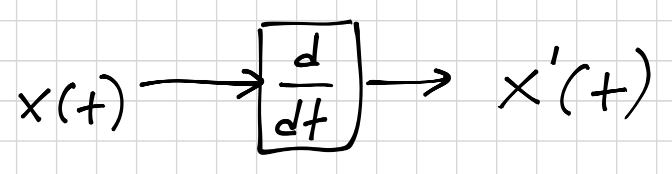
$$y(t) = x_1(t) \cdot x_2(t)$$ } same for DT



Differentiation (only applies to CT)

$x(t)$

$$y(t) = \frac{d}{dt} x(t)$$



Integration (CT)

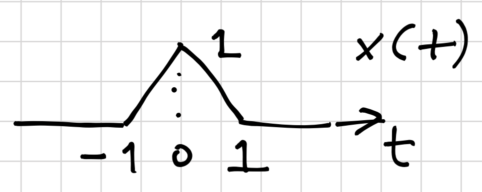
$$y(t) = \int_{-\infty}^{t} x(z)\, dz$$

operations performed on the Independent Variable

Time Scaling }

$$y(t) = x(a \cdot t) \quad a \in \mathbb{R}^+$$

Example

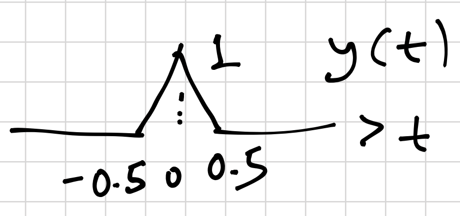


$y(t) = x(2t)$

$y(-0.5) = x(2 \cdot -0.5) = x(-1) = 0$

$y(0) = x(2 \times 0) = 1$

$y(1) = x(2 \times 0.5) = 0$

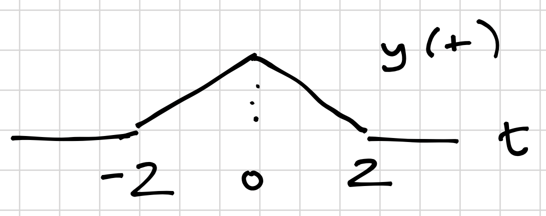


If $a > 1$ $\Rightarrow$ The output is the COMPRESSED version of $x(t)$

Ex $a = 0.5$

$y = x(0.5 \cdot t)$

$x(-1) = x(0.5 \times -2) = y(-2) = 0$
$x(0) = x(0.5 \times 0) = y(0) = 1$
$x(1) = x(0.5 \times 2) = y(2) = 0$

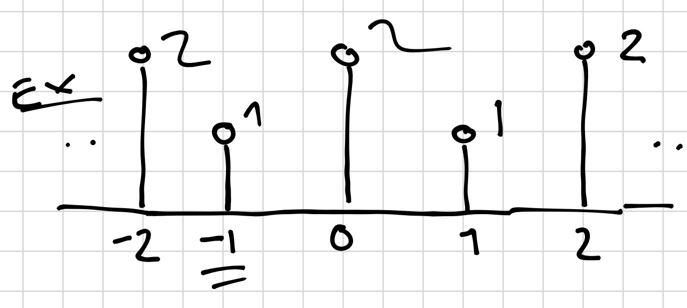


If $0 < a < 1$ then the output is a $\boxed{\text{stretched}}$ version of $x(t)$

for DT signals

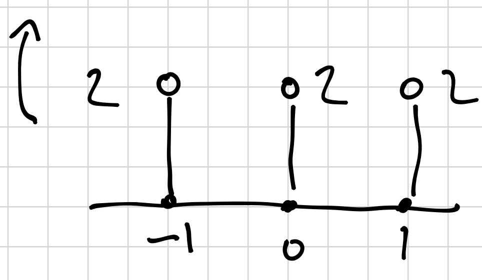
$$y[n] = x[k \cdot n] \qquad k \in \mathbb{Z}^+$$

Ex



$y[n] = x[2 \cdot n]$

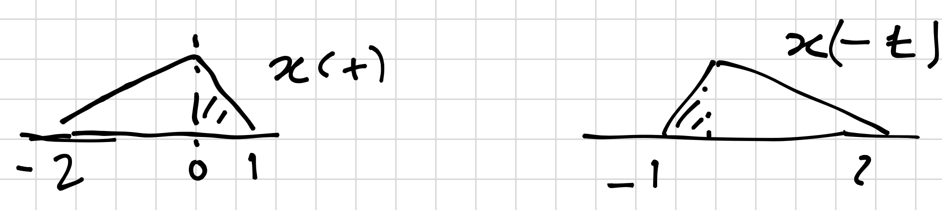
$x[-2] = x[2 \cdot (-1)] = y[-1] = 2$
$x[1] = x[2 \times \_\_]$ ✗
$x[0] = x[2 \cdot 0] = y[0] = 2$
$x[1] = x[2 \cdot \_\_]$ ✗
$x[2] = x[2 \cdot 1] = y[1] = 2$



Reflection

$x(t)$ is a CT signal.

$y(t) = x(-t)$ is a reflection of $x(t)$



For

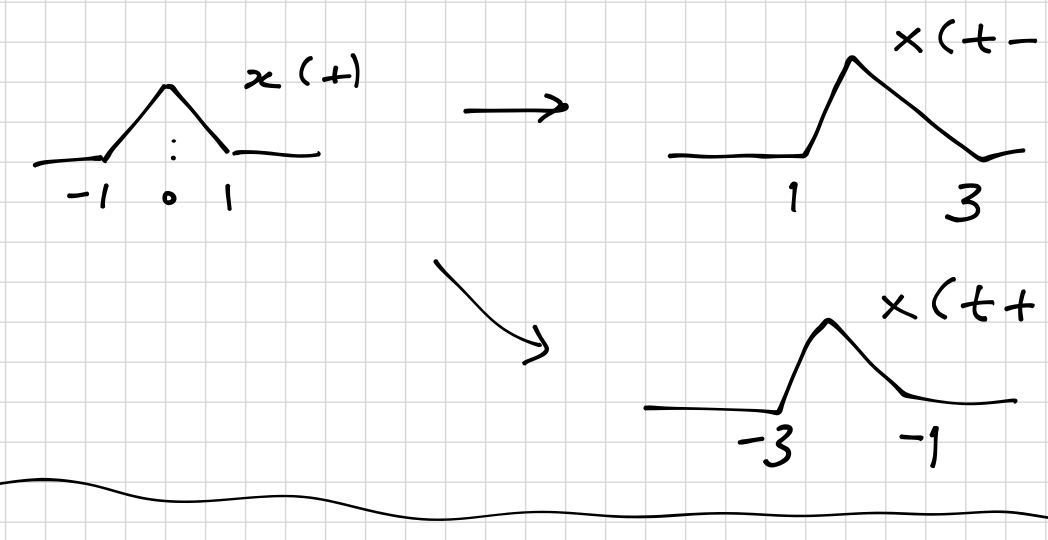
- Even signals : $x(t)$ is the same as its reflected version
- Odd signals $\rightarrow$ $x(t)$ is the same as the negative of its reflected version.

Time Shifting }

$$y(t) = x(t - t_0) \qquad t_0 \in \mathbb{R}$$

If $t_0 > 0$ the output is obtained by shifting the input signal by t toward right.

If $t_0 < 0$ $\rightarrow$ shift left



Precedence Rule for Time-shifting and Time-scaling

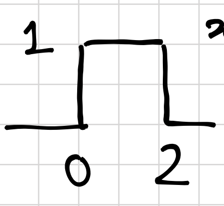
- $y(t) = x(\alpha t - \beta)$
- Let's define an intermediate signal

$$v(t) = x(t - \beta) \quad \} \text{ SHIFT}$$

$$y(t) = v(\alpha t) = x(\alpha t - \beta) \quad \} \text{ SCALE}$$

$\boxed{\text{Shift First} \rightarrow \text{Scale Second}}$

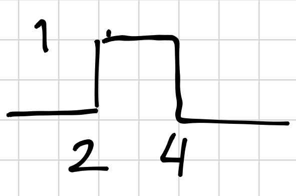
Ex



$x(t)$

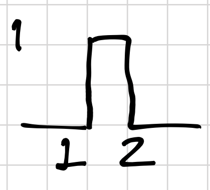
$$y(t) = x(2t-2)$$

① $v(t) = x(t-2)$



$v(t)$

② $y(t) = v(2t)$



$v(2t) = x(2t-2)$