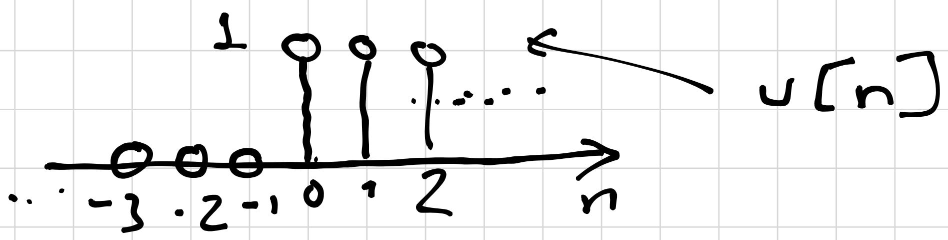
(2) Step Function

DT The DT unit step function is defined by

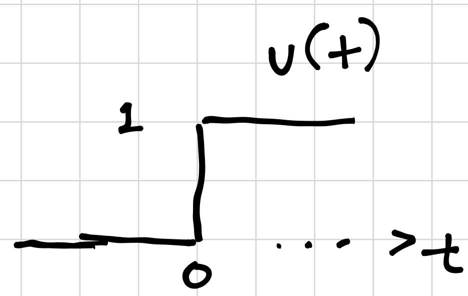
$$u[n] = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}$$



CT

CT unit step function

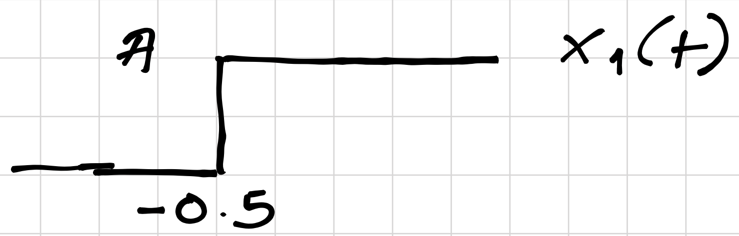
$$u(t) = \begin{cases} 1, & t > 0 \\ 0, & t < 0 \end{cases}$$



at $t=0$ unit step fn. is undefined.

Ex

Express $x(t)$ as a weighted sum of two step functions



$$x_1(t) = A \cdot u(t + 0.5)$$

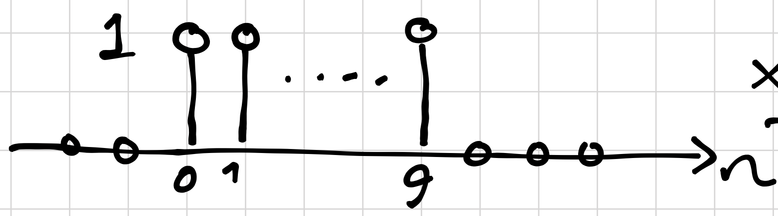
$$x_2(t) = A \cdot u(t - 0.5)$$

$$x(t) = x_1(t) - x_2(t)$$

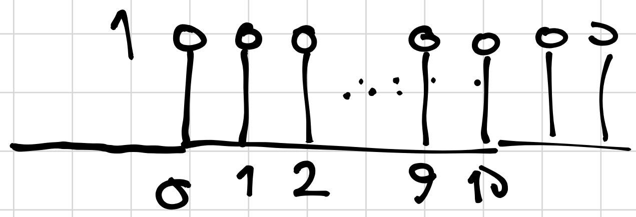
$$\{ x(t) = A\, u(t + 0.5) - A\, u(t - 0.5) \} *$$

Ex

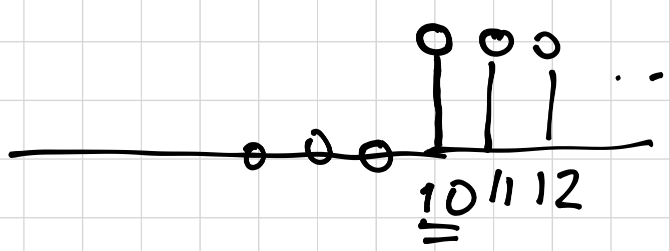
$$x[n] = \begin{cases} 1, & 0 \le n \le 9 \\ 0, & \text{otherwise} \end{cases}$$



$x[n]$



$u[n]$



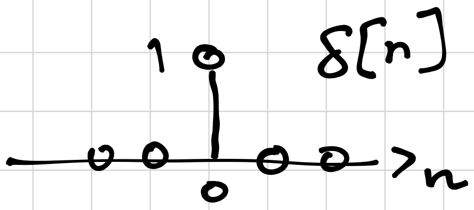
$u[n-10]$

$$x[n] = u[n] - u[n-10]$$

③ Impulse Function (Dirac - Delta Function)

DT   DT unit impulse function

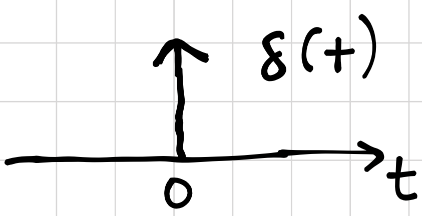
$$\delta[n] = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases}$$



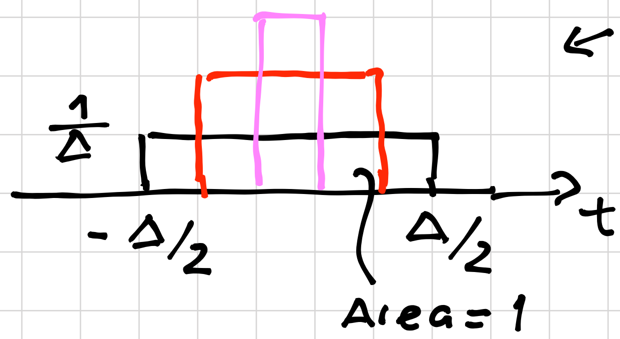
CT   CT unit impulse function is defined by the following two relations

① $\delta(t) = 0 \;, \; t \neq 0$

② $$\int_{-\infty}^{+\infty} \delta(t)\, dt = 1$$



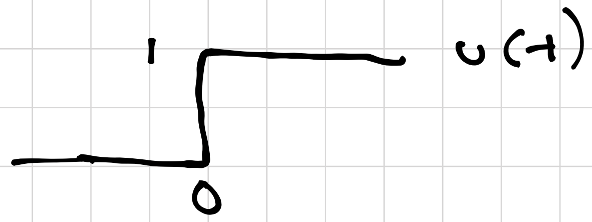
Let's define



$$\delta(t) = \lim_{\Delta \to 0} x_\Delta(t)$$

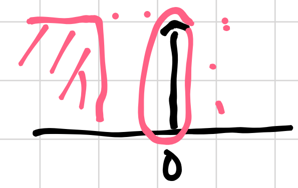
/* $3\,\delta(t)$ has a strength of 3 */

* $\delta(t)$ and $u(t)$ are related to each other.

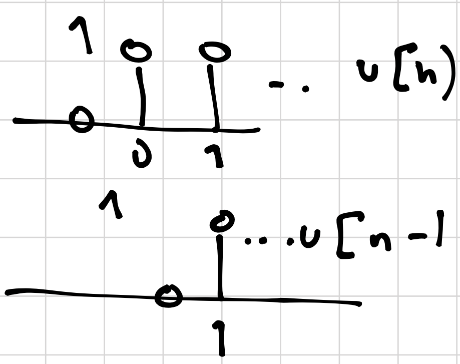


$$\delta(t) = \frac{d}{dt} u(t)$$

$$u(t) = \int_{-\infty}^{t} \delta(\tau)\, d\tau$$

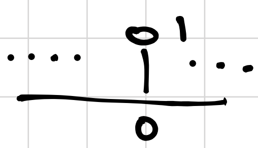


* $\delta[n]$ and $u[n]$ relationship.



$$\delta[n] = u[n] - u[n-1]$$

$$u[n] = \sum_{k=-\infty}^{n} \delta[k]$$



* $\delta(t)$ and $\delta[n]$ are even functions

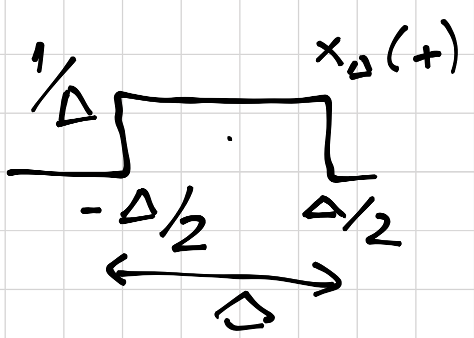
$$\delta(t) = \delta(-t)$$
$$\delta[n] = \delta[-n]$$

* $$\int_{-\infty}^{+\infty} x(t) \cdot \delta(t-t_0)\, dt = x(t_0)$$

$$\sum_{n=-\infty}^{+\infty} x[n] \cdot \delta[n-n_0] = x[n_0]$$

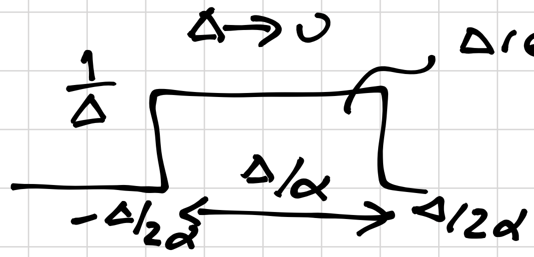
* Time scaling property

$$\delta(\alpha \cdot t) = \frac{1}{\alpha} \cdot \delta(t), \quad \alpha > 0$$



$$\lim_{\Delta \to 0} x_\Delta(t) = \delta(t)$$

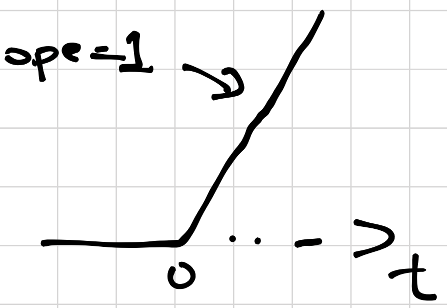
area $= 1/\alpha$



4 Ramp Function

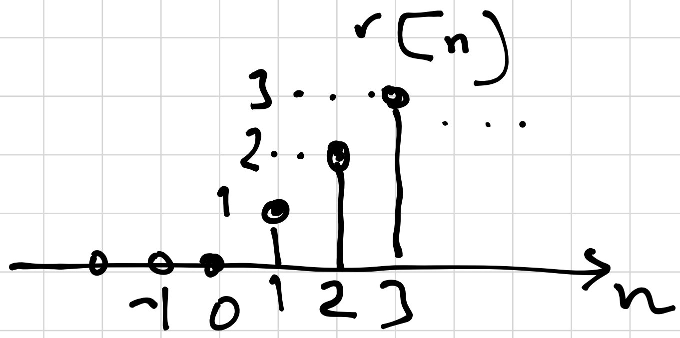
CT Unit ramp function

$$r(t) = \begin{cases} t, & t \geq 0 \\ 0, & t < 0 \end{cases}$$



DT

$$r[n] = \begin{cases} n, & n \geq 0 \\ 0, & n < 0 \end{cases}$$



$$r[n] = n \cdot u[n]$$

$$r(t) = t \cdot u(t)$$

(Real)

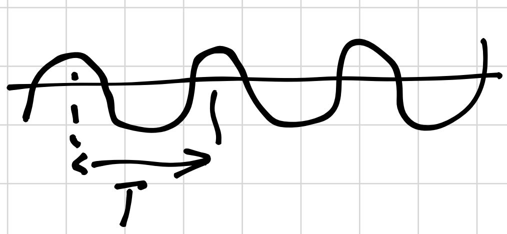
5 Sinusoidal Signals

CT $x(t) = A \cdot \cos(\omega t + \phi)$

- A: amplitude
- ω: angular freq. (rad/sec)
- ϕ: phase angle (rad)

CT sinusoidal signals are periodic.

Period $T = \dfrac{2\pi}{\omega}$



DT

$x[n] = A \cos(\Omega n + \phi)$

- A: amplitude
- Ω: (frequency)!

DT sinusoidal signal MAY or MAY NOT be sinusoidal.

In order for $x[n]$ to be periodic there must be an integer N that satisfy the following for all n!!

$$x[n] = x[n+N]$$

$$x[n+N] = A \cdot \cos\left(\Omega n + \underline{\underline{\Omega N}} + \phi\right)$$

if ΩN is an integer multiple of 2π

:

$$\underline{\Omega N} = 2\pi m \quad , m \in \mathbb{Z}^+$$

/* $\cos(a) = \cos(a + 2\pi m)$ */

There should be at least on (m, N) integer pairs.

$$\Omega = 2\pi \frac{m}{N}$$

Ex

$$x[n] = \sin[5\underline{\underline{\pi}} n]$$

$$\Omega = 5\pi = 2\pi \frac{m}{N} \qquad \frac{m}{N} = 5/2 \qquad \boxed{\begin{matrix} m = 5 \\ N = 2 \end{matrix}}$$

The equation is satisfied. Period is $\underline{\underline{2}}$

Ex

$$x[n] = \sin[2n]$$

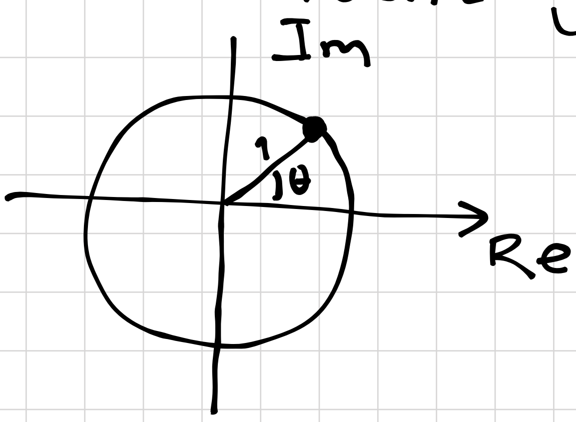
$$\Omega = 2 \qquad 2 = 2\pi \cdot \frac{m}{N}$$

we cannot find an integer pair (m, N) $\therefore$ NOT PERIODIC.

Relation Between Sinusoidal and Complex Exponential signals.

$j = \sqrt{-1} = i$

Euler's identity $\quad e^{j\theta} = \underline{\cos\theta + j\sin\theta}$



CT $x(t) = A \cdot e^{j\omega t}$

$$= A \cdot [\cos(\omega t) + j \sin(\omega t)]$$

$$\left.\begin{aligned} Re\{x(t)\} &= A \cdot \cos(\omega t) \\ Im\{x(t)\} &= A \cdot \sin(\omega t) \end{aligned}\right\}$$

DT

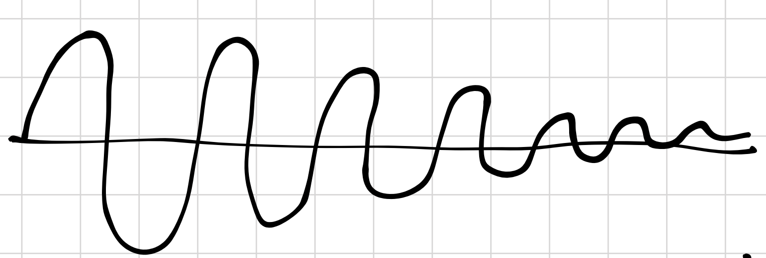
$$x[n] = A \cdot e^{j\Omega n}$$

$$Re\{x[n]\} = A \cos(\Omega n)$$

$$Im\{x[n]\} = A \sin(\Omega n)$$

- Exponentially Damped Sinusoidal Signals

CT



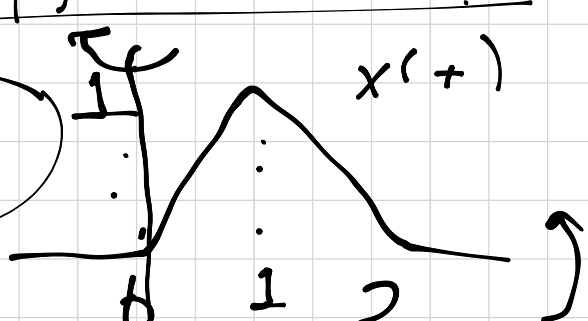
$$x(t) = A \cdot e^{-\alpha t} \cdot \sin(\omega t + \phi) \quad , \alpha > 0$$

Ex Pr 1.9 @ p 25

$x(t) = -x(-t)$?

$$x(t) = \begin{cases} t, & 0 \leq t \leq 1 \\ 2-t & 1 \leq t \leq 2 \\ 0, & \text{otherwise} \end{cases}$$

- CT



- neither even nor odd
- deterministic

- Periodic ? No

$$E = \int_{-\infty}^{+\infty} x^2(t)\, dt = \int_0^1 t^2\, dt + \int_1^2 (2-t)^2\, dt$$

$$= \frac{t^3}{3}\Big|_0^1 - \frac{(2-t)^3}{3}\Big|_1^2 = \frac{1}{3} + \frac{1}{3} = 2/3$$

$0 < E < \infty$ $\therefore$ It is an energy signal

$\therefore P = 0$