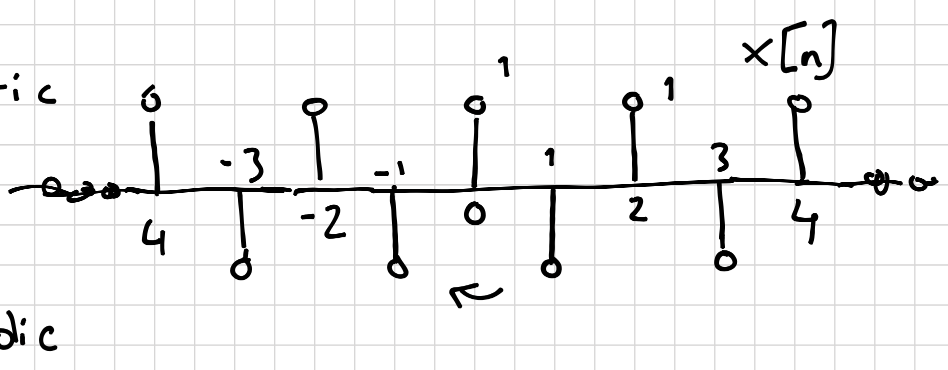
Ex pr 1.19 @ page 25

g)

$$x[n] = \begin{cases} \cos(\pi n) = (-1)^n, & -4 \leq n \leq 4 \\ 0, & \text{otherwise} \end{cases}$$

•) Deterministic

•) Periodic ?



non-periodic

•) Energy ? Power ?

$$E = \sum_{n=-4}^{4} \left[(-1)^n\right]^2 = 9$$

therefore

E is finite and nonzero $\therefore$ $x[n]$ is an energy signal.

Since it is an energy signal
It cannot be a power signal.

It's average power is zero.

•) It's even because it is symmetrical around the vertical axis.

$$x[-n] = \begin{cases} \cos(-\pi n), & 4 \geqslant -n \geqslant -4 \\ 0, & \text{otherwise} \end{cases}$$

$-4 \leq n \leq 4$

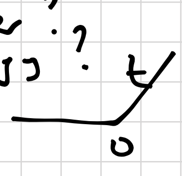
$$= x[n]$$

$\therefore$ It's an even signal.

Ex

$$r(t) = \begin{cases} t, & t \geqslant 0 \\ 0, & \text{elsewhere} \end{cases}$$

power ? energy ?



$$E = \int_0^{\infty} t^2 \, dt = \frac{t^3}{3}\Big|_0^{\infty} = \infty$$

$\therefore$ NOT an energy signal !

$$P = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} t^2 \, dt$$

$$= \lim_{T \to \infty} \frac{1}{T} \left[\frac{t^3}{3}\right]_{-T/2}^{T/2}$$

$$= \lim_{T \to \infty} \frac{T^2}{6} = \infty$$

$\therefore$ NOT a power signal.

Homework

$x(t) = \sqrt{t}\, u(t)$??

Ex Given two periodic CT signals, $x_1(t)$ and $x_2(t)$, of which the fundamental periods are T_1 and T_2 respectively,

$$x(t) = x_1(t) + x_2(t)$$

Is $x(t)$ periodic ? If so what is the fundamental period ?

$$x_1(t) = x_1(t + T_1) = x_1(t + mT_1) \quad m \in \mathbb{Z}^+$$

$$x_2(t) = x_2(t + T_2) = x_2(t + k.T_2), \; k \in \mathbb{Z}^+$$

$$x(t) = \boxed{x_1(t + mT_1) + x_2(t + kT_2)} \quad m, k \in \mathbb{Z}^+$$

Let's say $x(t)$ is periodic and its fundamental period is T

$$x(t) = x(t + T)$$
$$= x_1(t) + x_2(t)$$
$$= x_1(t + T) + x_2(t + T)$$
$$= x_1(t + mT_1) + x_2(t + kT_2)$$

For periodicity

$$T = mT_1 = kT_2$$

$$\frac{T_1}{T_2} = \frac{k}{m}$$

$\therefore$ $\frac{T_1}{T_2}$ must be a rational number.

We, then, can find at least one $(k, m) \in \mathbb{Z}^+$ pair

– Fundamental period — Find the smallest (k, m) integer pair

$$T = mT_1 = kT_2$$

If we cannot find an $(m, k) \in \mathbb{Z}^+$ pair, $x(t)$ is not periodic !

Ex

$x_1[n]$ is periodic, N_1
$x_2[n]$ is " , N_2

Under what condititon $x[n] = x_1[n] + x_2[n]$ is periodic ?

$$x_1[n] = x_1[n + N_1] = x_1[n + mN_1]$$
$$+ \; x_2[n] = x_2[n + N_2] = x_2[n + kN_2]$$

$$x[n] = x[n + N]$$
$$= x_1[n + N] + x_2[n + N]$$
$$= x_1[n + mN_1] + x_2[n + kN_2]$$

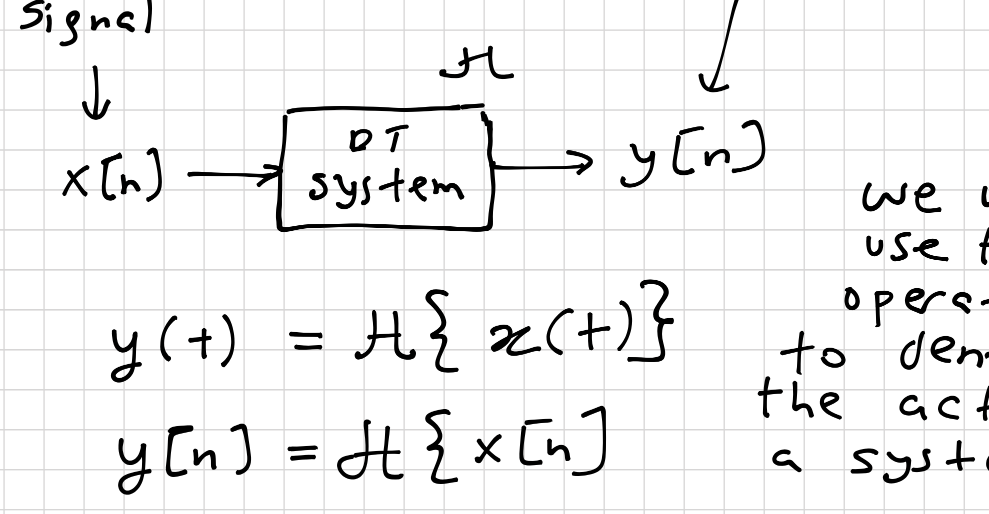
$$N = mN_1 = kN_2$$

$\frac{m}{k} = \frac{N_2}{N_1}$ must be rational.

This is always rational !
$\therefore$ $x[n]$ is always periodic !

Find the smallest $(m, k) \in \mathbb{Z}^+$ pair — $\boxed{N = mN_1 = kN_2}$

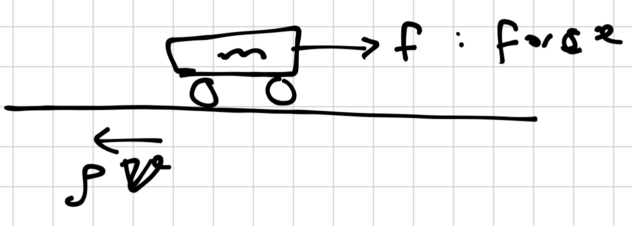
SYSTEMS



$$y(t) = \mathcal{H}\{x(t)\}$$
$$y[n] = \mathcal{H}\{x[n]\}$$

We will use the operator $\mathcal{H}$ to denote the action of a system.

Ex



v: velocity ρ : frictional force

m: mass

$$f(t) \longrightarrow v(t)$$

$$\frac{d}{dt} v(t) = \frac{1}{m} \left[f(t) - \rho v(t) \right]$$

$\vdots$

$$\underbrace{v(t)}_{\text{output}} = \underbrace{\mathcal{H}}_{\text{system}} \{ \underbrace{f(t)}_{\text{input}} \}$$

PROPERTIES of SYSTEMS

① Stability.

A system is bounded-input bounded-output (BIBO) stable if and only if every bounded input results in a bounded output.

$$y(t) = \mathcal{H}\{x(t)\}$$

$\mathcal{H}$ is BIBO-stable

if $|y(t)| \leq M_y < \infty$, $\forall t$ (M_y is some finite positive number)

when $|x(t)| \leq M_x < \infty$, $\forall t$ (M_x is some finite positive number.)

/* Same applies to DT systems */

$\boxed{\text{Ex}}$ Let's say that a DT system $\mathcal{H}$, has the following input-output relationship

$$y[n] = \mathcal{H}\{x[n]\} = \frac{1}{3}\left(x[n-1] + x[n] + x[n+1] \right)$$

Is $\mathcal{H}$ stable?

Assume $|x[n]| \leq M_x < \infty$, $\forall n$

$$|y[n]| = \left| \frac{1}{3} \left(x[n-1] + x[n] + x[n+1] \right) \right|$$

$$|y[n]| \leq \frac{1}{3} \left(\underbrace{|x[n+1]|}_{< M_x} + \overbrace{|x[n]|}^{< M_x} + \underbrace{|x[n-1]|}_{< M_x} \right)$$

/* $a + b = c$
$|a| + |b| \geq |c|$ */

$$\leq \frac{1}{3} \left(M_x + M_x + M_x \right)$$

$$= M_x$$

$$|y[n]| \leq M_x < \infty$$

$y[n]$ is bounded for all n

$\mathcal{H}$ is stable

Ex Let $y[n] = \mathcal{H}\{x[n]\} = r^n x[n]$

Is $\mathcal{H}$ stable when $r > 1$?

- Assum that $|x[n]| \leq M_x < \infty$, $\forall n$

$$|y[n]| = |r^n| \, |x[n]| \leq |r^n| \cdot M_x$$

If $r > 1 \Rightarrow |r^n|$ will diverge as n increases. We cannot say $y[n]$ is bounded when $x[n]$ is bounded.

$\therefore$ For $r > 1$, $\mathcal{H}$ is not stable