

Learning Process

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Under the classical BS model, interest rate used in deriving the stochastic process of stock price is fixed. This assumption deviates from the reality that interest rate is fluctuating. Thus, it's feasible to construct a stochastic process for short rate (instantaneous rate). In HW5, the Hull-White model is applied to construct sample paths for short rate. The model has the following dynamics

$$dr(t) = (\theta(t) - ar(t))dt + \sigma dW(t)$$

a can be described as the speed for interest rate to return to its mean value. θ_t can be directly derived from the initial yield curve.

After construction via Monte Carlo method, each sample path is then incorporated into the differential equation to derive process for stock price. The differential equation is given below

$$\Delta S = S(r(t)\Delta t + \sigma\sqrt{\Delta t}N(t))$$

, where $N(t) \sim N(0,1)$. The equation is simply a discrete form of GBM SDE (stochastic differential equation).

$$\frac{dS}{S} = r(t)dt + \sigma W_t.$$