

Unruh temperatures in circular and drifted Rindler motions

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Abstract

We study the temperatures for the circular and drifted Rindler motions by employing the Unruh-DeWitt detector method. In the circular motion, the temperature is increasing along the radius of the circular motion until it reaches the maximum, and then it is decreasing and eventually vanishing at the limit to the radius where the proper acceleration is infinite. In fact, the temperature is proportional to the proper acceleration quadratically near the origin of the circular motion as compared to the usual Unruh effect depending on the linear proper acceleration. On the other hand, in the drifted Rindler motion, the observer moves with a relative velocity in the direction transverse to the acceleration. If the detector is moving slowly in the transverse direction with a finite proper acceleration, then the temperature behaves like the usual Unruh temperature, while it vanishes for the speed of light in the transverse direction according to the infinite proper acceleration. Consequently, it turns out that the temperatures behave nonlinearly with respect to the proper acceleration and the infinite proper acceleration would not always permit the divergent temperature.

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I. INTRODUCTION

A detector in an accelerated frame measures excitations related to fluctuations of the quantum vacuum, which results in the remarkable prediction of the Unruh temperature in quantum field theory [1]. The detector is defined to move through a region permeated by a quantum scalar field and is weakly coupled to the scalar field in its motion, which is known as Unruh-DeWitt detector [2–4]. Self-correlations of the scalar field on a trajectory cause the detector to observe excitations. The detector undergoing a uniform linear acceleration α within the Minkowski space perceives the Unruh temperature as

$$T_U = \frac{\hbar}{k_B c} \frac{\alpha}{2\pi}. \quad (1)$$

The Unruh temperature of the Rindler observer is equivalent to the temperature measured by a locally fiducial observer near the event horizon of the Schwarzschild black hole in the large mass limit [1]. It is worth noting that the experimental verification of the Unruh effect would be difficult, since the linear acceleration of $2.6 \times 10^{22} \text{cm/s}^2$ is required to produce the temperature of 1K [5].

Related to the phenomena of the linearly uniform acceleration, one might wonder what would be the case for a circular motion having the constant magnitude of a proper acceleration with a constant speed. The circular motion is interesting not only from the theoretical point of view but also the experimental verification [6, 7]. So, many efforts have been devoted to studying the rate of the response function for the circular motion [8–14], where the Unruh-DeWitt detector is set as a rotating monopole detector for simplifying calculations. Despite this simplification, it would be difficult to calculate the rate of response function explicitly. As a slightly different approach, the Wightman function was written by a sum over the normal modes, and then the rate of response function was nicely calculated in Ref. [14].

On the other hand, one can consider a similar motion to the circular one such as the drifted Rindler motion, where the detector is uniformly accelerated and at the same time moves at a constant speed in the direction perpendicular to the acceleration. The similarity between the circular motion and the drifted Rindler motion has been discussed in several literatures [11, 15–17], where the circular motion for the large radius is actually corresponding to the drifted Rindler motion. For the drifted Rindler detector, it is also difficult to calculate the

rate of the response function, so that it was treated numerically and the result is compatible with the Planck spectrum [9].

It is commonly expected for the Unruh temperature to be proportional to the linear acceleration of the frame. So, one might wonder what happens for the circular and the drifted Rindler motions. At first glance, one might expect that the temperatures would still be proportional to the proper acceleration defined in the circular and drifted Rindler motions.

In this paper, we would like to study the temperatures for the circular motion and the drift Rindler motion, respectively, by using the Unruh-DeWitt detector method. In the circular motion, the temperature vanishes not only when the proper acceleration becomes zero but also when the radius approaches a limit to the radius where the proper acceleration diverges and the magnitude of the tangential velocity becomes the speed of light. On the other hand, in the drifted Rindler motion, if the detector is moving slowly in the transverse direction with a finite proper acceleration, then the temperature behaves like the usual Unruh temperature. If the transverse velocity becomes the speed of light, then the proper acceleration for the drifted Rindler motion goes to infinity and thus the temperature vanishes very similarly to the case of the circular motion. Therefore, the temperatures depend on the proper accelerations nonlinearly in the circular and drifted Rindler motions, and the infinite proper acceleration would not always permit that the temperature diverges.

In Sec. II, we encapsulate the Unruh-DeWitt detector method for the Unruh temperature. Then, in Sec. III, we obtain the temperature for the circular motion by employing the Unruh-DeWitt detector method. In Sec. IV, we also calculate the temperature for the drifted Rindler motion. Finally, conclusion and discussion will be given in Sec. V. We will set $\hbar = c = k_B = 1$ for simplicity.

II. UNRUH-DEWITT DETECTOR METHOD

Let us first recapitulate the Unruh-Dewitt detector method for the calculations of the Unruh temperatures presented in Refs. [2–4]. One should assume a detector moving through a region permeated by a quantum scalar field $\Phi(x)$ along a trajectory $x^\mu(\tau)$ in the Minkowski spacetime with a proper time τ . The Lagrangian of the minimal interaction between the detector and the scalar field is written as $L = \kappa \mathcal{D}(\tau) \Phi(x(\tau))$ with a small coupling constant

κ , where the detector operator $\mathcal{D}(\tau)$ is defined as $\mathcal{D}(\tau) = e^{iH_0\tau}\mathcal{D}_0e^{-iH_0\tau}$.

As the detector accelerates, it will measure the energy transition from the energy \mathcal{E}_i of the ground state to the energy \mathcal{E} of an excited state. Then, the first order amplitude $\mathcal{A}^{(1)}$ is given by

$$\mathcal{A}^{(1)} = i\kappa\langle\mathcal{E}|\mathcal{D}_0|\mathcal{E}_i\rangle \int_{\tau_i}^{\tau} d\tau e^{i\tau(\mathcal{E}-\mathcal{E}_i)} \langle\psi|\Phi(x)|0\rangle. \quad (2)$$

Here, the Minkowski vacuum and the excited state are denoted as $|0\rangle$ and $|\psi\rangle$ respectively. So, the transition probability defined as $\mathcal{P} = \int d\mathcal{E}|\mathcal{A}^{(1)}|^2$ is calculated as

$$\mathcal{P} = \kappa^2 \int d\mathcal{E} |\langle\mathcal{E}|\mathcal{D}_0|\mathcal{E}_i\rangle|^2 \mathcal{R}(\mathcal{E} - \mathcal{E}_i), \quad (3)$$

where the response function \mathcal{R} is

$$\mathcal{R}(\Delta\mathcal{E}) = \int_{\Delta\tau_i^+}^{\Delta\tau^+} d\Delta\tau^+ \int_{\Delta\tau_i^-}^{\Delta\tau^-} d\Delta\tau^- e^{-i\Delta\tau^- \Delta\mathcal{E}} \mathcal{G}^+(\Delta\tau^-) \quad (4)$$

with the energy difference $\Delta\mathcal{E} = \mathcal{E} - \mathcal{E}_i$ and the time differences $\Delta\tau^\pm = \tau \pm \tau'$. Note that the positive frequency Wightman function \mathcal{G}^+ is defined as

$$\mathcal{G}^+(\Delta\tau^-) = \langle 0|\Phi(\tau)\Phi(\tau')|0\rangle. \quad (5)$$

From the transition probability (3), we can obtain the transition probability per unit time as

$$\dot{\mathcal{P}} = \kappa^2 \int d\mathcal{E} |\langle\mathcal{E}|\mathcal{D}_0|\mathcal{E}_i\rangle|^2 \dot{\mathcal{R}}(\Delta\mathcal{E}) \quad (6)$$

with the rate of the response function

$$\dot{\mathcal{R}}(\Delta\mathcal{E}) = \int_{-\infty}^{\infty} d\Delta\tau^- e^{-i\Delta\tau^- \Delta\mathcal{E}} \mathcal{G}^+(\Delta\tau^-), \quad (7)$$

where the integration range of $\Delta\tau^-$ is extended up to $\pm\infty$.

Finally, the Unruh temperature can be read off from the relation of

$$\dot{\mathcal{R}} = \frac{\Delta\mathcal{E}}{2\pi} \frac{1}{(e^{\Delta\mathcal{E}/T} - 1)} \quad (8)$$

between the Planck distribution and the rate of the response function [2–4]. The rate of the response function is closely related to the temperature of the system in the sense that it reduces to $\dot{\mathcal{R}} = T/(2\pi)$ for $\Delta\mathcal{E}/T \ll 1$.

III. TEMPERATURE IN THE CIRCULAR MOTION

By using the Unruh-DeWitt detector method, we calculate the temperature measured by the rotating detector. Let us set the detector to rotate around the z -axis with a radius ρ and a constant angular velocity Ω assumed to be positive finite. The timelike Killing vector for the rotational motion is given by [9, 10, 18, 19]

$$\xi^\mu = (\gamma, -\gamma\Omega y, \gamma\Omega x, 0), \quad (9)$$

which is generating the rotational trajectory as

$$x^\mu = (\gamma\tau, \rho \cos(\gamma\Omega\tau), \rho \sin(\gamma\Omega\tau), 0), \quad (10)$$

where $\gamma = (1 - v^2)^{-1/2}$ is the Lorentz factor and the velocity v is tangent to the circular orbit as defined by $v = \rho\Omega$.

On the other hand, the rotating frame is described by the line element,

$$ds^2 = -(1 - \Omega^2\rho^2)dt^2 + 2\Omega\rho^2 d\varphi dt + d\rho^2 + \rho^2 d\varphi^2 + dz^2. \quad (11)$$

Note that there is a limit to the radius at $\rho_H = 1/\Omega$ [20]. The proper acceleration for the rotating motion is calculated as $a_{\text{cir}} = \gamma^2\rho\Omega^2$, which is also simplified as $a_{\text{cir}} = \Omega v(1 - v^2)^{-1}$. The proper acceleration vanishes for $v = 0$ corresponding to $\rho = 0$, while the proper acceleration is infinite for $v \rightarrow 1$ corresponding to $\rho \rightarrow \rho_H$.

In the local field theory, the Lagrangian density of the massless free scalar field Φ is written as

$$\mathcal{L} = -\frac{1}{2}\Phi(x)(-\square)\Phi(x), \quad (12)$$

and the positive frequency Wightman function is obtained as

$$\mathcal{G}^+(x, x') = \int \frac{d^3k}{(2\pi)^3 2\omega} e^{ik_\mu(x^\mu - x'^\mu)} \quad (13)$$

$$= \frac{1}{4\pi^2} \frac{1}{\Delta x^\mu \Delta x_\mu} \quad (14)$$

with $\Delta x^\mu = x^\mu(\tau) - x^\mu(\tau')$. Plugging the rotating trajectory (10) into Eq. (14), one can get the explicit form of the Wightman function as

$$\mathcal{G}_{\text{cir}}^+(\Delta\tau^-) = \frac{1}{4\pi^2} \frac{1}{-(\gamma\Delta\tau^-)^2 + \frac{4v^4\gamma^4}{a_{\text{cir}}^2} \sin^2\left(\frac{a_{\text{cir}}}{2\gamma}\Delta\tau^-\right)}. \quad (15)$$

In order to get the temperature, one should calculate the rate of response function (7) from Eq. (15). However, it appears to be non-trivial to evaluate the integral (7) and so the rate of the response function will be obtained in a slightly different fashion along the line of Refs. [12–14].

We rewrite the Wightman function by using the cylindrical polar coordinates in order to get the rate of the response function (7). The momentum is expressed in terms of the cylindrical polar coordinates of θ and $k = \sqrt{k_x^2 + k_y^2}$ as

$$k^\mu = (\omega, k \cos(\theta), k \sin(\theta), k_z). \quad (16)$$

Substituting the circular trajectory (10) and the momentum (16) into Eq. (13), we can rewrite the positive frequency Wightman function as

$$\mathcal{G}_{\text{cir}}^+(\Delta\tau^-) = \int_0^\infty \frac{k dk}{(2\pi)^2} \int_{-\infty}^\infty \frac{dk_z}{2\omega} e^{-i\gamma\omega\Delta\tau^-} J_0 \left(\left| 2k\rho \sin \left(\frac{\gamma\Omega}{2} \Delta\tau^- \right) \right| \right), \quad (17)$$

where we used the integral representation of the Bessel function of zeroth order as $J_0(x) = (2\pi)^{-1} \int_{-\pi}^\pi d\theta e^{-ix \sin(\theta)}$. Then, the Wightman function (17) can be essentially expressed as a sum over the normal modes as [8, 10, 12–14]

$$\mathcal{G}_{\text{cir}}^+(\Delta\tau^-) = \sum_{m=-\infty}^\infty \int_0^\infty \frac{k dk}{(2\pi)^2} \int_{-\infty}^\infty \frac{dk_z}{2\omega} e^{-i\gamma(\omega-m\Omega)\Delta\tau^-} J_m^2(k\rho) \quad (18)$$

by using the relation of

$$J_0 \left(\left| 2k\rho \sin \left(\frac{\gamma\Omega}{2} \Delta\tau^- \right) \right| \right) = \sum_{m=-\infty}^\infty e^{i\gamma m\Omega\Delta\tau^-} J_m^2(k\rho), \quad (19)$$

where $J_m(x)$ denotes the Bessel function of order m . Before performing the summation and the integrals in Eq. (18), we evaluate the integral over the differential proper time $\Delta\tau^-$ of the rate of response function as

$$\begin{aligned} \dot{\mathcal{R}}_{\text{cir}}(\Delta\mathcal{E}) &= \int_{-\infty}^\infty d\Delta\tau^- e^{-i\Delta\tau^- \Delta\mathcal{E}} \mathcal{G}_{\text{cir}}^+(\Delta\tau^-) \\ &= \sum_{m=-\infty}^\infty \int_0^\infty \frac{k dk}{2\pi} \int_{-\infty}^\infty \frac{dk_z}{2\omega} J_m^2(k\rho) \delta(\Delta\mathcal{E} + \gamma(\omega - m\Omega)), \end{aligned} \quad (20)$$

where $\omega = \sqrt{k^2 + k_z^2}$ is always positive. If we set the energy difference $\Delta\mathcal{E}$ as a positive definite value, the delta function in Eq. (20) is always zero for the summation over m when $m < \Delta\mathcal{E}/(\gamma\Omega)$. Thus, the integral over k_z is evaluated as

$$\dot{\mathcal{R}}_{\text{cir}}(\Delta\mathcal{E}) = \sum_{m \geq \Delta\mathcal{E}/(\gamma\Omega)}^\infty \int_0^{m\Omega - \Delta\mathcal{E}/\gamma} \frac{k dk}{2\pi\gamma} \frac{J_m^2(k\rho)}{\sqrt{(m\Omega - \Delta\mathcal{E}/\gamma)^2 - k^2}} \quad (21)$$

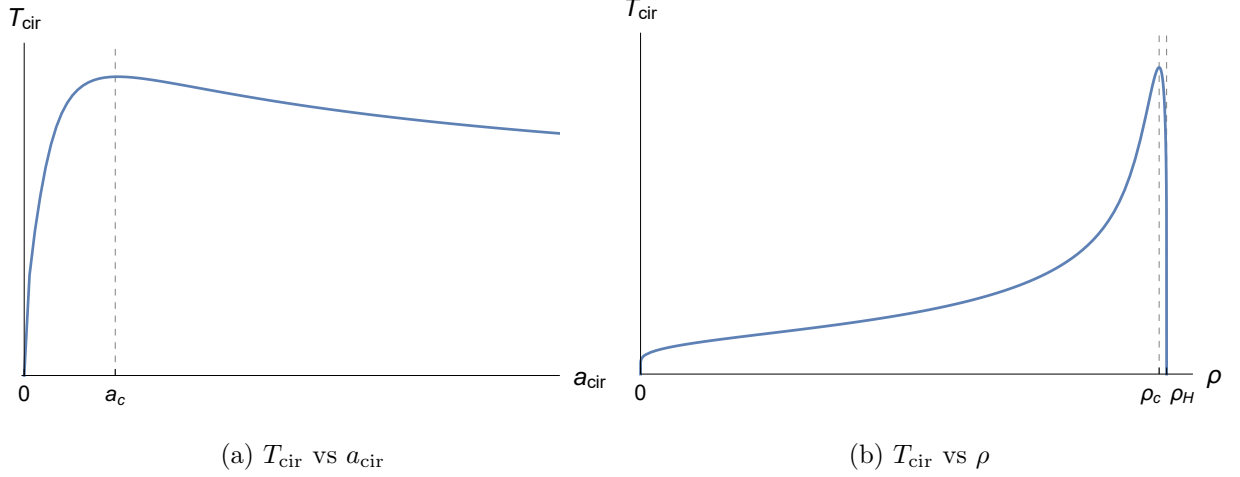


FIG. 1. The temperature (23) is plotted with respect to the proper acceleration a_{cir} in Fig. (a), where the temperature vanishes for $a_{\text{cir}} \rightarrow \infty$ as well as $a_{\text{cir}} = 0$. The behavior of the temperature is also plotted with respect to the radius of the circular motion in Fig. (b) in order to exhibit the whole profile of the temperature easily. The temperature vanishes at $\rho = \rho_H$ and $\rho = 0$. The maximum temperature appears at the critical point a_c and ρ_c . Note that the parameters are fixed as $\Delta\mathcal{E} = 1$ and $\Omega = 1$ for convenience.

with $K = \sqrt{(m\Omega - \Delta\mathcal{E}/\gamma)^2 - k^2}$. Note that there exists an upper bound on k as $k \leq m\Omega - \Delta\mathcal{E}/\gamma$ to make K a real value. Finally, the rate of the response function (21) is rewritten by using the hypergeometric function denoted by ${}_1F_2[\{\mu\}, \{\nu, \lambda\}, x]$ as [14]

$$\begin{aligned} \dot{\mathcal{R}}_{\text{cir}}(\Delta\mathcal{E}) = & \sum_{m \geq \Delta\mathcal{E}/(\gamma\Omega)}^{\infty} \frac{\rho^{2m}}{2\pi\gamma} \left(m\Omega - \frac{\Delta\mathcal{E}}{\gamma} \right)^{2m+1} \\ & \times \frac{{}_1F_2[\{m + \frac{1}{2}\}, \{m + \frac{3}{2}, 2m + 1\}, -\rho^2(m\Omega - \Delta\mathcal{E}/\gamma)^2]}{\Gamma(2m + 2)}. \end{aligned} \quad (22)$$

Now, the temperature can be read off from Eqs. (8) and (22) as

$$T_{\text{cir}} = \frac{\Delta\mathcal{E}}{\ln \left(1 + \frac{\Delta\mathcal{E}}{2\pi\dot{\mathcal{R}}_{\text{cir}}(\Delta\mathcal{E})} \right)}, \quad (23)$$

which is plotted in Fig. 1. Note that the Unruh temperature (1) for the Rindler motion is proportional to the uniform linear acceleration. However, the behavior of the temperature (23) shows that the temperature vanishes at both $a_{\text{cir}} = 0$ ($\rho = 0$) and $a_{\text{cir}} \rightarrow \infty$ ($\rho = \rho_H$) as seen from Fig. 1. According to this fact, it appears to be natural to exist a maximum

temperature in Fig. 1(a) in contrast to the conventional behavior of the acceleration of Unruh temperature for the Rindler motion.

In order to discuss the behavior of the temperature (23) analytically near the center of the circular motion and the limit to the radius, let us take the leading orders of the response function (22) and the temperature (23) in the IR limit of $\Delta\mathcal{E} \rightarrow 0$. For the center of the circular motion as $0 < \rho \ll \rho_c$, the leading order of the rate of the response function (22) is written as

$$\dot{\mathcal{R}}_{\text{cir}} = \frac{\Omega}{12\pi} \frac{\rho^2}{\rho_H^2}, \quad (24)$$

where the angular velocity is fixed. By using Eq. (8), one can identify the temperature in the IR limit as

$$T_{\text{cir}} = \frac{v}{6} \frac{a_{\text{cir}}}{\gamma} \sim a_{\text{cir}}^2, \quad (25)$$

which is proportional to the proper acceleration quadratically. So, the temperature for the circular motion is more or less different from that of the Rindler motion in the IR limit. On the other hand, for the limit to the radius as $\rho_c \ll \rho < \rho_H$, the leading order of the rate of the response function (22) in the IR limit is obtained as

$$\dot{\mathcal{R}}_{\text{cir}} = \frac{\mathcal{B}}{2\pi} \frac{\Omega}{\gamma}, \quad (26)$$

where $\mathcal{B} = \sum_{m=1}^{\infty} (\Gamma(2m+2))^{-1} m^{2m+1} {}_1F_2 \left[\left\{ m + \frac{1}{2} \right\}, \left\{ m + \frac{3}{2}, 2m+1 \right\}, -m^2 \right] \simeq 17.0161$. Then the temperature is written as

$$T_{\text{cir}} = \mathcal{B} \frac{a_{\text{cir}}}{v\gamma^3} \sim \frac{1}{\sqrt{a_{\text{cir}}}}, \quad (27)$$

which is proportional to the inverse square root of the proper acceleration. This feature is very different from the standard Unruh effect in that the temperature is no longer proportional to the regular powers of the proper acceleration. In the next section, the temperature for the drifted Rindler motion will be explored how it is different from the Unruh temperature.

IV. TEMPERATURE IN THE DRIFTED RINDLER MOTION

If the radius of the circular motion is very large, the motion looks like the linear accelerated motion with a high speed in the direction perpendicular to the acceleration. So, we

investigate the temperature for the drifted Rindler motion in order to compare it with the temperature for the circular motion.

Let us consider the detector moving with a relative velocity in the direction transverse to the acceleration. The world line of the drifted Rindler motion is represented by the Rindler coordinates $\tilde{x}^\mu = (\eta, \xi, \tilde{y}, \tilde{z})$ [11, 21, 22],

$$\eta(\tau) = \frac{\gamma\tau}{1 + \alpha\xi_0}, \quad \xi(\tau) = \xi_0, \quad \tilde{y}(\tau) = v\gamma\tau, \quad \tilde{z}(\tau) = 0, \quad (28)$$

where v is the velocity of the translational motion along the y -direction. The parameter α means the proper acceleration when the velocity v vanishes. The constant ξ_0 will be fixed as $\xi_0 = 0$ for simplicity. From the trajectory (28), the drifted Rindler motion in the Minkowski coordinates is described by

$$t(\tau) = \frac{1}{\alpha} \sinh(\alpha\gamma\tau), \quad x(\tau) = \frac{1}{\alpha} \cosh(\alpha\gamma\tau), \quad y(\tau) = v\gamma\tau, \quad z(\tau) = 0, \quad (29)$$

which recovers the familiar standard Rindler motion for $v = 0$. The line element of the drifted Rindler motion is the same as that of the Rindler one as

$$ds^2 = -(1 + \alpha\xi)^2 d\eta^2 + d\xi^2 + d\tilde{y}^2 + d\tilde{z}^2, \quad (30)$$

where the horizon is located at $\xi_H = -1/\alpha$. By using the trajectory (28) and the metric (30), the proper acceleration is calculated as $a_{\text{drift}} = \alpha\gamma^2$. It is worth noting that the proper acceleration a_{drift} diverges when v approaches the speed of light for a finite non-vanishing linear acceleration of α .

Plugging the drifted Rindler trajectory (29) into the positive frequency Wightman function (14) in the local field theory, we can explicitly calculate the Wightman function as [9, 11]

$$\mathcal{G}_{\text{drift}}^+(\Delta\tau^-) = \frac{1}{4\pi^2} \frac{1}{v^2\gamma^2(\Delta\tau^-)^2 - \frac{4}{\alpha^2} \sinh^2\left(\frac{\alpha\gamma}{2}\Delta\tau^-\right)}, \quad (31)$$

and then the rate of the response function is formally written as [9]

$$\dot{\mathcal{R}}_{\text{drift}}(\Delta\mathcal{E}) = \frac{1}{4\pi^2} \int_{-\infty}^{\infty} d\Delta\tau^- \frac{e^{-i\Delta\tau^- \Delta\mathcal{E}}}{v^2\gamma^2(\Delta\tau^-)^2 - \frac{4}{\alpha^2} \sinh^2\left(\frac{\alpha\gamma}{2}\Delta\tau^-\right)}. \quad (32)$$

For $v = 0$, the rate of the response function was exactly calculated and thus the Unruh temperature was identified with $T = \alpha/(2\pi)$ [1–4]. Unfortunately, it appears to be impossible to evaluate the rate of response function (32) exactly for a finite velocity.

We now express the rate of the response function (32) for the special limit of $\alpha \gg \gamma$ with a finite v or $v \sim 1$ with a finite α , which eventually results in $a_{\text{drift}} = \alpha\gamma^2 \gg 1$. Then, the second term in the denominator in Eq. (32) will be dominant, and the leading term of the rate of response function is approximately calculated as

$$\dot{\mathcal{R}}_{\text{drift}}(\Delta\mathcal{E}) \sim \frac{\alpha}{(2\pi)^2\gamma}. \quad (33)$$

From Eq. (8), we obtain the temperature as

$$T_{\text{drift}} \sim \frac{\Delta\mathcal{E}}{\ln\left(1 + \frac{2\pi\gamma\Delta\mathcal{E}}{\alpha}\right)}. \quad (34)$$

For $\alpha \gg \gamma$ with a finite v and a finite ΔE , the temperature for the drift motion behaves like

$$T_{\text{drift}} \sim \frac{1}{\gamma}T_{\text{U}}, \quad (35)$$

which reduces to the Unruh temperature, $T_{\text{drift}} = T_{\text{U}}$ for $v = 0$, as it should be. However, for $v \sim 1$ with a finite α , the temperature (34) goes to zero, which can be found in the circular motion for the limit of $\rho \sim \rho_{\text{H}}$ corresponding to $a_{\text{cir}} \gg 1$ as seen from Eq. (27) and Fig. 1. This feature is compatible with the fact that the motion for the linear accelerated motion with a high speed in the direction perpendicular to the acceleration looks like the circular motion with a large radius; precisely, the linear acceleration is related to the angular velocity as $\alpha \sim \Omega$ from Eqs. (26) and (33).

V. CONCLUSION AND DISCUSSION

We calculated the temperatures for the circular and the drifted Rindler motions. For the circular motion, the temperature is increasing starting from the zero temperature at the origin of $\rho = 0$ corresponding to $a_{\text{cir}} = 0$, and then it reaches the maximum value of the temperature. At last, it goes to zero at the limit to the radius of $\rho = \rho_{\text{H}}$ corresponding to $a_{\text{cir}} \rightarrow \infty$. It is interesting to note that the temperature is proportional to the proper acceleration quadratically near the origin of the circular motion as compared to the usual Unruh effect depending on the linear proper acceleration. The asymptotic behavior of the temperature for the limit of $a_{\text{cir}} \rightarrow \infty$ is also non-trivial in that the temperature (27) vanishes in spite of divergent proper acceleration. On the other hand, we also investigated the temperature in the drifted Rindler motion for the two special limits satisfying the condition

of $a_{\text{drift}} \gg 1$. First, for $\alpha \gg \gamma$ with a small v , the temperature behaves like the Unruh temperature (1) because the spacetime reduces to the Rindler spacetime for $v = 0$. Second, for $v \rightarrow 1$ with a finite α , the temperature (34) goes to zero. Therefore, the infinite proper acceleration does not always give an infinite temperature since the Unruh-DeWitt detector could not measure any excitations for the circular and drifted Rindler motions as long as the velocity approaches the speed of light.

On the other hand, in the case of the circular motion, the Killing vector is null at the spherical surface of $\rho = \rho_{\text{H}}$, where it is timelike inside the surface and spacelike beyond the surface [8]. Thus, no object can be at rest relative to the rotating frame beyond the surface, since there is no object faster than the speed of light. So, the region beyond $\rho = \rho_{\text{H}}$ is similar to the region inside the ergosphere surrounding a rotating black hole [23]. It is worth noting that the particle creation occurs by not only the event horizon but also the ergosphere [24, 25]. So, it has been suggested that the Unruh effects for the detector undergoing the circular motion might be closely related to the ergoregion effect of the rotating black hole [15, 16] like the relation between the Unruh effect of the Rindler motion and the local thermal effects near the event horizon of the Schwarzschild black hole [1]. So, it would be interesting to study the temperature near the ergosphere of the rotating black hole in connection with the results presented in this work. This issue deserves further attention.

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