

Charged Black Hole in Gravity's Rainbow: Violation of Weak Cosmic Censorship

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Abstract

We investigate the validity of the weak cosmic censorship conjecture for charged black holes in the presence of gravity's rainbow under charged particle absorption. The rainbow effect is shown to play an important role when the rainbow black hole is modified by a particle carrying energy and electric charge. Remarkably, we prove that the rainbow charged black hole can be overspun beyond the extremal condition under charged particle absorption. Further, it is demonstrated that the second law of thermodynamics and cosmic censorship conjecture are violated owing to the rainbow effect.

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1 Introduction

Black holes are one of the interesting topics related to various phenomena, such as, gravitational wave and gamma-ray burst. Further, a black hole has a very different structure compared with other stellar objects. This difference originates from the event horizon covering the inside of black holes. No particle, not even light, can escape once it reaches the inside of a black hole by passing through the horizon. When a particle enters a black hole, its irreducible mass increases as an extensive property [1, 2], which is an energy distributed on the surface of the horizon [3]. On the contrary, the black hole has a reducible energy in the form of rotational and electric energies. The reducible energy can decrease by a specific process, such as, Penrose process [4, 5]. The extensive behavior of an irreducible mass is now understood in terms of black hole thermodynamics, and its square is proportional to the Bekenstein-Hawking entropy of a black hole [6, 7]. Further, as a conjugate variable of the entropy, the Hawking temperature is obtained from an emission through a quantum effect on the horizon [8, 9]. Hence, the black hole can be studied as a thermal system following the laws of thermodynamics.

There is an interesting conjecture related to the internal structure of a black hole. At the center of the black hole spacetime, a singularity is located that is covered by the horizon, and thus it cannot be seen by an external observer. However, in absence of the horizon, the naked singularity is exposed, and causality breaks down. To prevent this situation, the weak cosmic censorship conjecture ensures that the horizon always stably covers the singularity located inside a black hole [10, 11]. However, there is no general proof on the validity of the conjecture for black holes; therefore, we need to test the conjecture for each type of black hole. The first investigation was about a Kerr black hole, which cannot be overspun beyond the extremal condition by adding a particle [12]. Further, the self-force effect of a particle is shown to be important in ensuring the stability of the horizon in the Kerr black hole [13, 14]. Moreover, charged black hole was also treated with importance in the weak cosmic censorship conjecture, because a backreaction was considered in [15, 16]. Several tests on various black holes included in modified gravity theories are still being performed [17–30]. In addition, we found that the validity of the cosmic censorship conjecture is closely related to the first and second law of thermodynamics for a given black hole system [31]. Hence, if the first law is assumed to have the thermodynamic volume and pressure term in a charged anti-de Sitter black hole, the second law and weak cosmic censorship conjecture are significantly affected [32].

On the other hand, much attention has been paid to modified dispersion relations (MDRs) not only in threshold anomalies in ultra-high cosmic rays and TeV photons [33–42], but also in loop quantum gravity from the semi-classical point of view [43–47]. Subsequently, many efforts have been devoted to studying various aspects of MDRs [48–55]. In this context, it was suggested that the Minkowski spacetime should be deformed by the energy of a particle laid on the spacetime. Gravity’s rainbow is a generalization of this deformation to present MDRs to curved spacetimes, such as, in black holes and cosmology. Hence, a geometry is distorted by the energy of the test particle moving in it. The concepts of Schwarzschild black hole and Friedmann-Robertson-Walker cosmology were first modified in terms of gravity’s rainbow [56]. Gravity’s Rainbow has also been studied to investigate various aspects of black holes [57–68] and cosmology [69–76]. However, the weak cosmic censorship conjecture

has not been thoroughly studied for rainbow black holes.

In this work, we investigate the validity of the weak cosmic censorship conjecture for charged black holes in the presence of gravity's rainbow under charged particle absorption. We set a specific pair of rainbow functions, which provide compatible results with quantum-spacetime-phenomenology perspective and loop-quantum-gravity approach [33, 34, 43–47]. Because the metric of the black hole is deformed owing to the rainbow effect, we expect that the deformation affects the weak cosmic censorship conjecture and thermodynamics in the black hole. The conjecture is investigated for a charged particle entering the black hole. Interestingly, the initial state is the rainbow charged black hole deformed by gravity's rainbow originated from the charged particle, but the final state is the charged black hole without the rainbow effect, because there is no particle in the final state. Non-extremal black holes and extremal black hole are considered in their initial states. Further, we determine whether the extremal black hole can spin up beyond the extremal condition under charged particle absorption in the presence of gravity's rainbow.

This paper is organized as follows. In section 2, we review charged black holes in Einstein's gravity coupled with Maxwell field. Then, gravity's rainbow and rainbow charged black holes are introduced. In section 3, we obtain a solution to Hamilton-Jacobi equations for a charged particle in the rainbow charged black hole. Then, changes in the outer horizon and Bekenstein-Hawking entropy are investigated in cases of non-extremal black holes. In section 4, we describe the investigation of the weak cosmic censorship conjecture for extremal black holes. In section 5, we briefly summarize our results.

2 Charged Black Holes in Gravity's Rainbow

In this work, we consider the charged black hole spacetime modified by the energy of a charged particle. Without consideration of rainbow effects of the particle, charged black holes are asymptotically flat solutions to Einstein's gravity coupled with Maxwell field $F_{\mu\nu}$. The action is given as

$$S = \frac{1}{16\pi} \int d^4x \sqrt{-g} (R - F_{\mu\nu} F^{\mu\nu}), \quad (1)$$

where R is the curvature. The field equations are

$$\nabla F^{\mu\nu} = 0, \quad R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 2 \left(F_{\mu\rho} F_{\nu}^{\rho} - \frac{1}{4} g_{\mu\nu} F_{\lambda\rho} F^{\lambda\rho} \right). \quad (2)$$

The charged black hole is a spherical symmetric solution to Eq. (2). The metric of the charged black hole is of mass M and electric charge Q as

$$ds^2 = -H(r)dt^2 + \frac{1}{H(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2), \quad H(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2}, \quad A = -\frac{Q}{r}dt, \quad (3)$$

where A is the electric potential. The charged black hole has inner and outer horizons expressed as

$$r_i = M - \sqrt{M^2 - Q^2}, \quad r_h = M + \sqrt{M^2 - Q^2}. \quad (4)$$

In addition, the extremal condition is $M = Q$, where the electric charge is maximized for a given mass. We mainly consider thermodynamics on the outer horizon, and hence all thermodynamic properties

are defined on it. Then, the surface area of the outer horizon A_h and Bekenstein-Hawking entropy S_{RN} are related as

$$S_{\text{RN}} = \frac{A_h}{4} = \pi r_h^2. \quad (5)$$

The Hawking temperature T_{RN} and electric potential Φ_{RN} of the charged black hole are given by

$$T_{\text{RN}} = \frac{r_h - M}{2\pi r_h^2}, \quad \Phi_{\text{RN}} = \frac{Q}{r_h}. \quad (6)$$

Then, the thermodynamic variables are related by the first law of thermodynamics as

$$dM = T_{\text{RN}} dS_{\text{RN}} + \Phi_{\text{RN}} dQ, \quad (7)$$

which determines the change in the charged black hole under the infinitesimal variation.

Then, we will consider a charged particle moving in the charged black hole spacetime. Because the energy and momentum of the particle can affect the spacetime structure in consideration of MDRs, we should consider the effects of the MDR to obtain more precise results. Gravity's rainbow is the model taking account of the MDR of the particle. Here, the form of MDR is given by [34, 56]

$$f(E)^2 E^2 - g(E)^2 p^2 = m^2, \quad (8)$$

where E , p , m represent the energy, momentum, and mass of a test particle, respectively. The effect of gravity's rainbow is imposed by functions $f(E)$ and $g(E)$ denoted as rainbow functions. The rainbow functions should be reduced to $\lim_{E \rightarrow 0} f = 1$ and $\lim_{E \rightarrow 0} g = 1$, because the effect of the MDR is consistent with that of the ordinary dispersion relation in the low energy limit. It is worth noting that the modified dispersion relation (8) can be rewritten in the form of the ordinary dispersion relation of $\tilde{E} - \tilde{p} = m^2$ by means of the transformations $\tilde{E} = f(E)E$ and $\tilde{p} = g(E)p$.

The MDR in Eq. (8) is about a particle moving in Minkowski spacetime. Hence, we need to generalize the MDR to that of the curved spacetime of a black hole as done in [56]. Because the MDR of the black hole should be coincident with Eq. (8) in its asymptotically flat limit, the metric of the black hole is also modified to include the rainbow function. Compatible with Eq. (8), the metric of a black hole with gravity's rainbow is obtained under the transformation [56]

$$t = \frac{\tilde{t}}{f(E)}, \quad r = \frac{\tilde{r}}{g(E)}, \quad G = \frac{\tilde{G}}{g(E)}. \quad (9)$$

Under the transformation in Eq. (9), the metric of the charged black hole in Eq. (3) becomes

$$ds^2 = -\frac{F(r)}{f(E)^2} dt^2 + \frac{1}{F(r)g(E)^2} dr^2 + \frac{r^2}{g(E)^2} (d\theta^2 + \sin^2 \theta d\phi^2), \quad (10)$$

where we omit the tilde signs to avoid confusion. The function $F(r)$ is $H(r)$ of Eq. (3) modified by the rainbow effect. The function $F(r)$ and electric potential are expressed as

$$F(r) = 1 - \frac{2GM}{r} + \frac{g(E)GQ^2}{r^2}, \quad A = -\frac{g(E)}{f(E)} \frac{Q}{r} dt. \quad (11)$$

Because the rainbow charged black hole in Eq. (10) includes energy dependence on its metric, various thermodynamic properties are imposed on the rainbow effect. The location of the inner and outer horizons r_I and r_H are given by

$$r_I = GM - \sqrt{G^2 M^2 - GQ^2 g(E)}, \quad r_H = GM + \sqrt{G^2 M^2 - GQ^2 g(E)}. \quad (12)$$

Then, in the rainbow charged black hole, the extremal condition and the horizon become

$$M = \sqrt{\frac{g(E)}{G}} Q, \quad r_H = GM. \quad (13)$$

The Bekenstein-Hawking entropy of the rainbow charged black hole is calculated as [68]

$$S_H = \frac{\pi r_H^2}{g(E)^2 G}. \quad (14)$$

We set $G = 1$ to avoid confusion as follows.

The rainbow effect will be realized when we consider forms of the rainbow functions. Various forms of rainbow functions are allowed in gravity's rainbow. Here, we consider the black hole spacetime modified by the rainbow effect originated from the charged particle. Hence, from a quantum-spacetime-phenomenology perspective [33, 34], the MDR is appropriate in our case. The form of the MDR is given as [33, 34, 42]

$$m^2 \approx E^2 - p^2 + \eta \frac{E^n}{E_P^n} p^2, \quad (15)$$

where E_P denotes the Planck energy, η is a positive free parameter, and n is a positive integer. Further, the MDR is compatible with results from the loop-quantum-gravity approach [43–47]. By comparison with Eqs. (8) and (15), the rainbow functions are expressed as

$$f(E) = 1, \quad g(E) = \sqrt{1 - \eta \frac{E^n}{E_P^n}}, \quad (16)$$

where $n = 1$ will be chosen for analytic calculations.

3 Charged Particle Absorption in Rainbow Charged Black Hole

We consider variations of the rainbow charged black hole caused by charged particle absorption. When a particle enters a black hole, its conserved quantities will be transferred to those of the black hole. The mass and electric charge of the black hole are assumed to change as much as the energy and electric charge carried by the particle at the outer horizon. The conserved quantities of the particle will be attained in terms of a dispersion relation obtained by solving the equations of motion of the particle. To derive the dispersion relation of the particle in the rainbow charged black hole, we will apply the Hamilton-Jacobi method and separate variable. The Hamiltonian of a particle having an electric charge q in the electric potential A_μ is expressed as

$$\mathcal{H} = \frac{1}{2} g^{\mu\nu} (p_\mu - q A_\mu) (p_\nu - q A_\nu), \quad (17)$$

where the four momentum p_μ of the particle is defined as

$$p_\mu = \partial_\mu \mathcal{I}. \quad (18)$$

The Hamilton-Jacobi action \mathcal{I} of the charged particle is given by

$$\mathcal{I} = \frac{1}{2}m^2\lambda - Et + \mathcal{I}_r(r) + \mathcal{I}_\theta(\theta) + L\phi. \quad (19)$$

in which λ is the affine parameter. According to translation symmetries to t and ϕ coordinates in the metric Eq. (10), the corresponding conserved quantities are defined as the energy and angular momentum, E and L , respectively. Then, from Eqs. (17) and (19), the Hamiltonian equation becomes

$$\begin{aligned} -2\frac{\partial \mathcal{I}}{\partial \lambda} &= -\frac{f(E)^2}{F(r)} \left(-E + \frac{g(E)}{f(E)} \frac{qQ}{r} \right)^2 + g(E)^2 F(r) (\partial_r \mathcal{I}_r)^2 + \frac{g(E)^2}{r^2} (\partial_\theta \mathcal{I}_\theta)^2 + \frac{g(E)^2}{r^2 \sin^2 \theta} L^2 \\ &= -m^2. \end{aligned} \quad (20)$$

Using a separate variable K , the Hamiltonian equation in Eq. (20) is divided into radial and θ -directional equations [77].

$$K = -\frac{m^2 r^2}{g(E)^2} + \frac{r^2 f(E)^2}{g(E)^2 F(r)} \left(-E + \frac{g(E)}{f(E)} \frac{qQ}{r} \right)^2 - r^2 F(r) (\partial_r \mathcal{I}_r)^2, \quad K = (\partial_\theta \mathcal{I}_\theta)^2 + \frac{1}{\sin^2 \theta} L^2. \quad (21)$$

The Hamilton-Jacobi action in Eq. (19) now becomes

$$\mathcal{I} = \frac{1}{2}m^2\lambda - Et + \int dr \sqrt{R} + \int d\theta \sqrt{\Theta} + L\phi, \quad (22)$$

where

$$\begin{aligned} \mathcal{I}_r &= \int dr \sqrt{R}, \quad R = \frac{1}{r^2 F(r)} \left(-K - \frac{m^2 r^2}{g(E)^2} + \frac{r^2 f(E)^2}{g(E)^2 F(r)} \left(-E + \frac{g(E)}{f(E)} \frac{qQ}{r} \right)^2 \right), \\ \mathcal{I}_\theta &= \int d\theta \sqrt{\Theta}, \quad \Theta = K - \frac{1}{\sin^2 \theta} L^2. \end{aligned} \quad (23)$$

By solving Eq. (22), the radial and θ -directional angular momentum of the charged particles for a given location are obtained as

$$\begin{aligned} p^r &= g(E)^2 \sqrt{-\frac{F(r)}{r^2} K + \frac{F(r)}{g(E)^2} m^2 + \frac{f(E)^2}{g(E)^2} \left(-E + \frac{g(E)}{f(E)} \frac{qQ}{r} \right)^2}, \\ p^\theta &= \frac{g(E)^2}{r^2} \sqrt{K - \frac{1}{\sin^2 \theta} L^2}. \end{aligned} \quad (24)$$

By removing K in Eq. (24), the momenta and conserved quantities of the charged particle are related to the dispersion relation as

$$f(E)^2 \left(-E + \frac{qQg(E)}{rf(E)} \right)^2 = \frac{(p^r)^2}{g(E)^2} + F(r) \left(-m^2 + \frac{r^2}{g(E)^2} (p^\theta)^2 + \frac{g(E)^2}{r^2 \sin^2 \theta} L^2 \right). \quad (25)$$

When the charged particle passes through the outer horizon r_H , it is assumed to be completely absorbed by the black hole, because the particle is indistinguishable from the black hole as seen by an observer outside the horizon. Hence, at the outer horizon, the energy and electric charge of the particle contribute to the black hole. In the limit to the outer horizon, the dispersion relation (25) becomes

$$E - \frac{g(E) qQ}{f(E) r_H} = \frac{1}{f(E)g(E)} |p^r|, \quad (26)$$

where we choose the positive sign in front of the kinetic term $|p|$ in Eq. (26). This choice positively relates the energy of the particle to its kinetic energy without the electric potential of the Q term in the positive flow of time [1, 2]. The total energy of the particle can be negative with the contribution of the electric potential, when the electric attraction acts on the particle.

We now investigate the variation of the rainbow charged black hole under charged particle absorption. The energy and electric charge of the charged particle change the mass and electric charge of the black hole. Hence, we assume that

$$dM = E, \quad dQ = q. \quad (27)$$

The energy and electric charge are related by Eq. (26). Hence, changes in the mass and electric charge of the black hole are also constrained by the relation

$$f(dM)dM = \frac{g(dM)Q}{r_H} dQ + \frac{1}{g(dM)} |p^r|. \quad (28)$$

Because the horizon r_H includes the rainbow function $g(dM)$ in its form as shown in Eq. (12), the change in the mass dM takes the form of an expansion in the first-order equation owing to $dM, dQ, |p^r| \ll M, Q$. Then, the dispersion relation in Eq. (28) becomes

$$dM = |p^r| + \frac{Q}{M + \sqrt{M^2 - Q^2}} dQ, \quad (29)$$

which has the same form as the ordinary dispersion relation of the charged black hole without the rainbow effects: $f(dM) = 1$ and $g(dM) = 1$. However, qualitatively, Eq. (29) is different from the ordinary one, because, here, $M = Q$ is not the extremal condition. The extremal condition of Eq. (29) is at $M = g(dM)Q$. The variables are defined at the rainbow charged black hole.

The outer horizon covers the singularity inside the black hole. Hence, the location of the outer horizon plays an important role in the weak cosmic censorship conjecture. If the location of the outer horizon is irreducible, we ensure that the horizon cannot disappear and the weak cosmic censorship conjecture is valid. Under Eq. (28), the initial state (M, Q) changes to the final state $(M + dM, Q + dQ)$. In this context, an important issue should be mentioned. Here, the initial state is assumed to be a non-extremal black hole with gravity's rainbow; hence, the rainbow charged the black hole, because the charged particle may induce the rainbow effect to the charged black hole. However, once the particle is absorbed into the black hole, there is no particle to introduce the rainbow effect in the spacetime. Hence, the final state should be ordinary charged black hole without gravity's rainbow. Thus, the initial and final outer horizons $r_{H,\text{initial}}$ and $r_{H,\text{final}}$ are solutions to

$$F(r_{H,\text{initial}}) = 0, \quad H(r_{H,\text{final}}) = 0, \quad (30)$$

where $F(r)$ and $H(r)$ are functions of g^{rr} with and without gravity's rainbow, respectively. When we assume that the final state is also a non-extremal black hole, the location of the horizon changes to

$$\begin{aligned} dr_H &= r_{H,\text{final}} - r_{H,\text{initial}} \\ &= (M + dM)^2 + \sqrt{(M + dM)^2 - (Q + dQ)^2} - \left(M^2 + \sqrt{M^2 - g(dM)Q^2} \right) \\ &= \frac{4M - \eta Q^2 + 4\sqrt{M^2 - Q^2}}{4\sqrt{M^2 - Q^2}} |p^r| - \frac{Q^3 \eta}{4 \left(M^2 - Q^2 + M\sqrt{M^2 - Q^2} \right)} dQ, \end{aligned} \quad (31)$$

which is obtained in the first order of variables. Interestingly, the change in the outer horizon depends on its charge q . Hence, if the particle has a sufficiently large charge, the outer horizon can decrease due to particle absorption. Further, when such decrease occurs in an extremal black hole, there is a possibility that the weak cosmic censorship conjecture can be invalid owing to overcharging. This aspect will be investigated in detail in the case of an extremal black hole. Here, it should be noted that the decrease in r_H is due to the rainbow effect. In the limit of $\eta \rightarrow 0$ without the rainbow effect, the change in the outer horizon is only proportional to $|p^r|$ and always increases under particle absorption. The condition under which the particle decreases the outer horizon is obtained in terms of the inequality

$$\frac{q}{|p^r|} > \frac{-4Q^2 + \left(M + \sqrt{M^2 - Q^2} \right) (8M - \eta Q^2)}{\eta Q^3}, \quad (32)$$

where we assume that the charge Q of black hole is positive. The behavior of dr_H is shown in detail in Fig.1 The negative regions for dr_H increase for a large η , as shown in Fig. 1. However, even if η

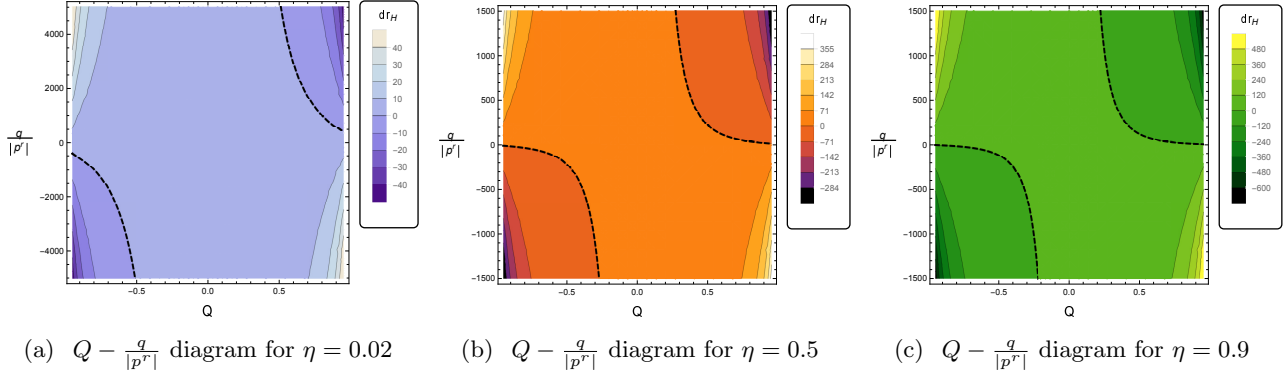


Figure 1: Changes in the outer horizon dr_H in $Q - \frac{q}{|p^r|}$ diagrams of $M = 1$ for a given η .

is small, the negative regions do not vanish, because the rainbow effect causes the decrease of the horizon. The outside of the black dashed lines are obtained according to Eq. (32)

According to Eq. (31), the horizon radius depends on the radial momentum and electric charge of the particle in the presence of the rainbow effect. However, the entropy is expected to be irreducible, because the second law of thermodynamics ensures that the entropy increases in an irreversible process,

such as, particle absorption. Here, we investigate the change in the entropy with the rainbow effect. For the same reason stated in the previous analysis, the initial state of the black hole (M, Q) is assumed to be the metric of Eq. (10), including the rainbow effect. Then, the entropy of the initial black hole S_i is given by Eq. (14). After the black hole absorbs the particle, there is no particle in the spacetime, and hence the rainbow effect should also be removed in the final state. The final black hole is now of the form $(M + dM, Q + dQ)$ according to Eq. (28), and its metric follows Eq. (3). Further, this also changes the final entropy S_f to Eq. (5) without the rainbow effect. The change in the entropy is written as

$$dS = S_f(M + dM, Q + dQ, r_H + dr_H) - S_i(M, Q, r_H). \quad (33)$$

Then, the change in the entropy is obtained as

$$\begin{aligned} dS &= \pi(r_H + dr_H)^2 - \frac{\pi r_H^2}{g(dM)^2} \\ &= \frac{\pi |p^r|}{2Q^2 - 2M \left(M + \sqrt{M^2 - Q^2} \right)} \left[8\eta M^4 + Q^2 \left(\eta Q^2 + \left(4 - \eta Q \frac{dQ}{|p^r|} \right) \sqrt{M^2 - Q^2} \right) \right. \\ &\quad - 4M^2 \left(2\eta Q^2 + \left(4 - \eta Q \frac{dQ}{|p^r|} \right) \sqrt{M^2 - Q^2} \right) + MQ^2 \left(-3\eta Q \frac{dQ}{|p^r|} + 12 - 4\eta \sqrt{M^2 - Q^2} \right) \\ &\quad \left. + 4M^3 \left(\eta Q \frac{dQ}{|p^r|} - 4 + 2\eta \sqrt{M^2 - Q^2} \right) \right]. \end{aligned} \quad (34)$$

Although particle absorption is an irreversible process, Eq. (34) implies that the entropy can increase or decrease in accordance with the radial momentum and electric charge of the particle. Without the rainbow effect, $\eta \rightarrow 0$, the change in the entropy becomes irreducible. Hence, the rainbow effect occurs

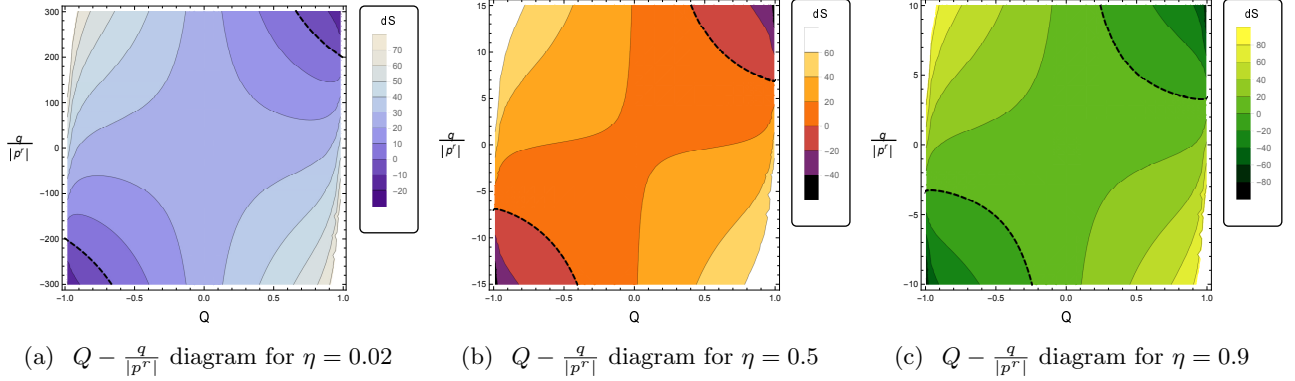


Figure 2: Changes in the entropy dS in $Q - \frac{q}{|p^r|}$ diagrams of $M = 1$ for a given η .

with the violation of the second law of thermodynamics under charged particle absorption. Because the change in the entropy is divergent when the black hole approaches the extremal black hole, we should investigate detailed behaviors of the extremal black hole by a different method. Further, we can obtain the range of the radial momentum and electric charge of the particle to decrease the entropy

of the system.

$$\frac{q}{|p^r|} > \frac{4}{\eta Q} + \frac{8M^4 - 8M^2Q^2 + Q^4 + \sqrt{M^2 - Q^2}(8M^3 - 4MQ^2)}{Q(-4M^3 + 3MQ^2 - (4M^2 - Q^2)\sqrt{M^2 - Q^2})}. \quad (35)$$

where Q is assumed to be positive. The detailed changes in the entropy are shown in Fig. 2. In a small rainbow effect proportional to η in Fig. 2(a), the negative regions of dS correspond to relatively large values of q and Q . Even if we consider an infinitesimally small rainbow effect, such as, $\eta \ll 1$, the entropy of an extremal black hole has a possibility to decrease under particle absorption including a large electric charge. This implies that the second law of thermodynamics can be violated due to the rainbow effect. As the rainbow effect η increases, the violation appears in boarder regions in Fig. 2(b) and (c). In other words, denoted as boundaries of negative changes of the entropy, the black dashed lines in Fig. 2 occupy broader areas for large values of η . Further, the divergence of dS in Eq. (34) is also presented at the region of $|Q| \sim 1$ in Fig. 2. Because the violation of the second law of thermodynamics implies the decrease of the horizon area, if the horizon of the extremal black hole becomes small, it can disappear owing to overcharging beyond the extremal condition. Thus, we need to investigate the extremal black hole in the context of the weak cosmic censorship conjecture.

4 Violation of Weak Cosmic Censorship in Extremal Black Hole

The extremal black hole has the maximum charge for a given mass. Hence, by adding a particle, if the electric charge becomes greater than the mass of the black hole, the black hole is overcharged beyond the extremal condition. Further, the horizons will disappear, and the weak cosmic censorship conjecture will be invalid. To investigate the validity of the conjecture, we estimate the final state from the initial state of an extremal black hole under particle absorption. However, we cannot use the same method applied in Eqs. (31) and (34), because the final state can be a naked singularity where Eqs. (31) and (34) are unavailable. To estimate the final state, we have to focus on the analytical structure of the metric function $F(r)$, which is well defined in the cases of both black hole and naked singularity.

The function $F(r)$ of the extremal black hole (M, Q) has only one minimum point located at the horizon r_e , which satisfies

$$F(M, Q, r)|_{r=r_e} = 0, \quad \partial_r F(M, Q, r)|_{r=r_e} = 0, \quad (\partial_r)^2 F(M, Q, r)|_{r=r_e} > 0. \quad (36)$$

This implies that the inner and outer horizons are coincident and located at the minimum point, as shown in Fig. 3(a). Further, the minimum value of $F(r)$ is zero in the initial state. Under charged particle absorption, the function $F(r)$ of the initial state becomes $H(r)$ of the final state in $(M + dM, Q + dQ)$, because there is no particle providing the rainbow effect in the final state. Hence, the minimum point and value are also moved in the final state. From the moved minimum value, we can estimate the final state, as shown in Fig. 3(b). When the minimum value is positive in the final state, the function $F(r)$ has no solution corresponding to the inner or outer horizon, as indicated by the red line in Fig. 3(b). Then, the singularity is not covered by a horizon, and the conjecture is invalid. If the

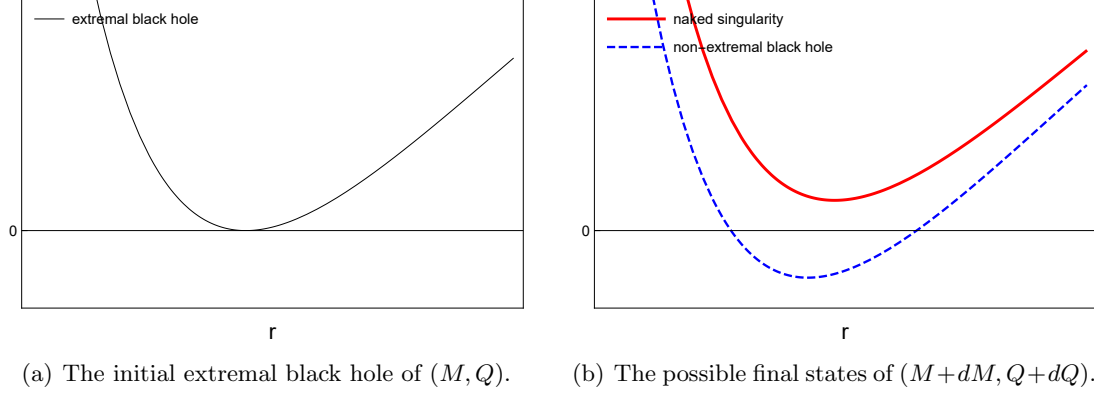


Figure 3: The function $F(r)$ in initial and final states.

final state is still a black hole, the function $F(r)$ has a negative minimum value and horizons as shown by the blue line in Fig. 3(b). Thus, we can estimate the final state from the sign of the minimum value. The change in the minimum value is

$$\begin{aligned}
 dF_{\min} &= H(M + dM, Q + dQ, r_e + dr_e) - F(M, Q, r_e) \\
 &= -\frac{2|p^r|}{r_e} + \frac{\eta}{2}(dQ + |p^r|),
 \end{aligned} \tag{37}$$

where we consider up to the first order of expansion. Without the rainbow effect $\eta = 0$ in Eq. (37), the minimum value has a negative sign, which implies non-extremal black hole, as shown by the blue dashed line in Fig. 3(b). This is already reported in previous literature, and the weak cosmic censorship is valid. However, with the rainbow effect, $\eta \neq 0$, interestingly, the sign of the minimum value depends on the radial momentum and electric charge of the particle. The change of the minimum value becomes positive, when

$$\frac{q}{|p^r|} > \frac{4}{\eta r_e} - 1, \tag{38}$$

where we assume $\eta \ll 1$. Remarkably, the radial momentum increases the minimum value with the rainbow effect. Further, the role of the radial momentum is opposite to that in the case without the rainbow effect. This implies that the charged particle of high kinetic energy causes overcharging of the extremal black hole. Thus, the weak cosmic censorship conjecture is *invalid*. According to Eq. (37), the change in the minimum value dF_{\min} is shown in detail in Fig. 4 with respect to $\frac{q}{|p^r|}$ of the particle. As we expected, overcharging beyond the extremal condition becomes broader for a larger value of η in comparison with Figs. 4 (a), (b), and (c). The largely charged particle tends to overcharge the extremal black hole. However, for a massive extremal black hole, overcharging can be caused by a particle with relatively small charge. The decrease of the black dashed line representing Eq. (38) clearly shows this behavior in Fig. 4. This is based on the dependence on r_e and η in Eq. (37). The negative contribution of $|p^r|$ becomes small in a massive extremal black hole, but its positive contribution is substantial under the rainbow effect. As a result, the massive extremal black hole becomes a naked singularity for

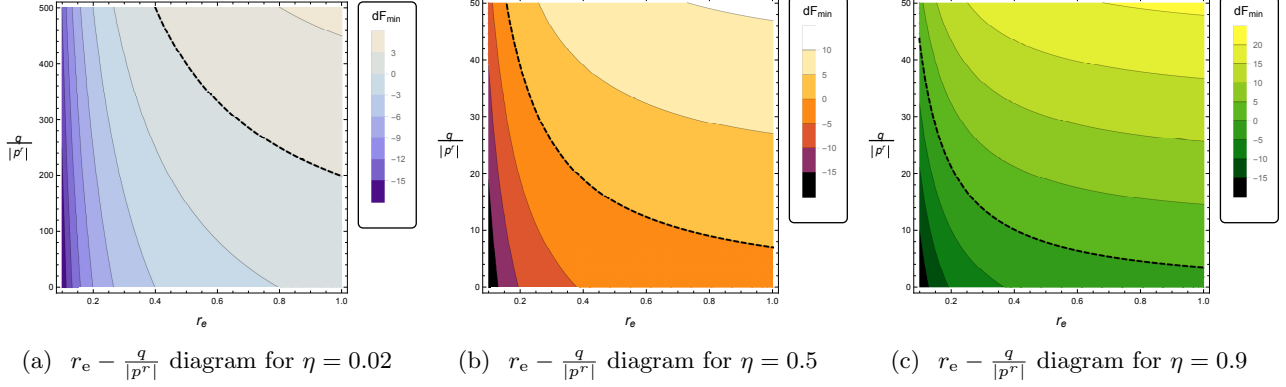


Figure 4: Changes in the minimum value of the function dF_{\min} in $r_e - \frac{q}{|p^r|}$ diagrams for a given η .

a relatively small charge of the particle for a given η . Therefore, the rainbow effect plays an important role in overcharging an extremal black hole.

5 Summary

We have investigated violations in the second law of thermodynamics and weak cosmic censorship conjecture in a charged black hole with gravity's rainbow. Among the sets of rainbow functions representing various aspects of MDRs, we choose one that is well consistent with the quantum-spacetime-phenomenology perspective [33, 34], such as, the loop-quantum-gravity approach [43–47]. Considering the rainbow functions, the charged black hole is modified to impose the rainbow effect from the MDR on its metric. Then, we have studied infinitesimal variations of the rainbow charged black hole caused by a charged particle to understand the effect of the MDR. The particle including the rainbow effect is assumed to change the mass and charge of the black hole as much as its own energy and charge, when it passes through the outer horizon. Here, there is a remarkable point about charged particle absorption. Because the rainbow effect is presented by the charged particle, the initial state is assumed to be the rainbow black hole, but the final state does not include the rainbow effect in its spacetime, because the particle does not exist due to absorption into the black hole. This concept differs from previous studies without gravity's rainbow. However, changes in the outer horizon and Bekenstein-Hawking entropy depend on the radial momentum and charge of the particle with the rainbow effect and which do not occur without the rainbow effect. Hence, owing to the rainbow effect, the second law of thermodynamics is violated by the absorption of a largely charged particle. Further, because the change diverges when the initial state is assumed to be an extremal black hole, we investigated the conjecture for the case of extremal black hole. In consideration of the minimum value of the function $F(r)$, we have proven that the extremal black hole can be transformed to a naked singularity by a largely charged particle. Here, the rainbow effect plays an important role to establish the invalidity of the cosmic censorship conjecture. It is worth noting that violations of the second law of thermodynamics and cosmic censorship conjecture in the presence of gravity's rainbow are demonstrated for

the first time in this study.

Acknowledgments

This work was supported by the National Research Foundation of Korea (NRF) grant funded by the Korea government (MSIT) (NRF-2018R1C1B6004349). YG was supported by the National Research Foundation of Korea(NRF) grant funded by the Korea government(MSIP) (2017R1A2B2006159).

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