

## Something special at the event horizon

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We revisit the free-fall energy density of scalar fields semiclassically by employing the trace anomaly on a two-dimensional Schwarzschild black hole with respect to various black hole states in order to clarify whether something special at the horizon happens or not. For the Boulware state, the energy density at the horizon is always negative divergent, which is independent of initial free-fall positions. However, in the Unruh state the initial free-fall position is responsible for the energy density at the horizon and there is a critical point to determine the sign of the energy density at the horizon. In particular, a huge negative energy density appears when the freely falling observer is dropped just near the horizon. For the Hartle–Hawking state, it may also be positive or negative depending on the initial free-fall position, but it is always finite. Finally, we discuss physical consequences of these calculations.

*Keywords:* Black holes; Hawking radiation; free fall.

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### 1. Introduction

Hawking radiation has invoked some issues which are of relevance to not only information loss problem in quantum gravity theory<sup>1,2</sup> but also black hole complementarity.<sup>3–5</sup> The latter states that there are no contradictory physical observations between a freely falling observer and a distant observer since the two descriptions are complementary. The presence of Hawking radiation indicates that the rest observer at infinity sees the flux of particles. By the way, Unruh has argued that “a geodesic detector near the horizon will not see the Hawking flux of

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particles”<sup>6</sup> and then showed that the infalling negative energy flux can exist near the horizon.<sup>7</sup> Moreover, it has been shown that the finite flux at the horizon can be found even in the freely falling frame by studying the Green’s function on the Schwarzschild black hole.<sup>8</sup> Recently, it has been claimed that a freely falling observer finds something special at the event horizon called the firewall and burns up because of high energy quanta,<sup>9</sup> and subsequently much attention has been paid to the firewall issue.<sup>10–15</sup> A similar prediction referred to as an energetic curtain has also been done based on different assumptions.<sup>16</sup> However, it has also been proposed that there is no apparent need for firewalls since the unitary evolution of black hole entangles a late mode located outside the event horizon with a combination of early radiation and black hole states, instead of either of them separately,<sup>17</sup> and argued that the remaining set of nonsingular realistic states do not have firewalls but yet preserve information.<sup>18</sup>

On the other hand, there has been some interests in radiation in freely falling frames in its own right and it has been widely believed that the equivalence principle tells us that the free-falling observer cannot see any radiation. This fact is based on the classical argument of locality but it may not be true in quantum regime such that a freely falling observer can find quantum-mechanical radiation and temperature.<sup>19–23</sup> Recently, it has been claimed that the freely falling observer dropped at the horizon necessarily encounters the infinite negative energy density, when the observer passes through the horizon.<sup>24</sup> On general grounds, one may regard this phenomenon as a very special feature such a simplified model<sup>25</sup> that Hawking temperature happens to be independent of the black hole mass. Otherwise, is there any special choice of vacuum to give rise to the infinite energy density at the horizon? If the existence of the infinite energy density at the horizon turns out to be true, one may wonder what happens at the horizon when the frame is dropped far from the horizon.

Now, we would like to study the quantum-mechanical energy densities measured by the freely falling observer on the two-dimensional Schwarzschild black hole background where Hawking temperature explicitly depends on the black hole mass. The trace anomaly for massless scalar fields will be employed to calculate the energy–momentum tensors along with covariant conservation law of the energy–momentum tensors. Then, the energy density will be characterized by three states;<sup>26</sup> the Boulware, Unruh and Hartle–Hawking states in order to investigate what state is relevant to the infinite energy density at the horizon. If there exists such a nontrivial effect at the horizon, then this fact will be tantamount to the failure of no drama condition which has been one of the assumptions for black hole complementarity, and this work will be the quantum field theoretic realization of nontrivial effect at the horizon.<sup>16</sup>

Now, in Sec. 2, we encapsulate how to formulate the freely falling frame by solving the geodesic equation of motion and present the explicit form of the corresponding energy density. First, the simplest Boulware state will be studied in Sec. 3

where the energy density is always negative and divergent at the horizon, which is interestingly irrespective of the initial free-fall position  $r_s$ . In Sec. 4, we shall find much more nontrivial effects in the Unruh state such that the observer finds positive radiation during free fall as long as  $r_s > r_0$ , where  $r_0$  is a point for the free-fall energy density to vanish. By the way, there is a critical point  $r_c$  for the observer to see only negative radiation during free fall for  $r_s < r_c$ . The closer the initial free-fall position approaches the horizon, the larger negative energy density can be found, so that it can be divergent at the horizon eventually. There is also an intermediate region of  $r_c < r_s < r_0$  for the observer to see negative radiation initially and to find positive radiation finally at the horizon. Finally, we explain the reason why the infinite energy density at the horizon appears in our calculations. In Sec. 5 for the Hartle–Hawking state, some similar behaviors to the case of the Unruh state will be reproduced; however, the crucial difference comes from the fact that the energy density at the horizon is always finite. After all, the calculation in this work will show that the nontrivial effect measured by the freely falling observer in the semiclassical argument is sensitive to the initial free-fall position and the black hole states. Finally, we will discuss physical consequences of this work in Sec. 6.

## 2. Freely Falling Frame

Let us start with the two-dimensional Schwarzschild black hole governed by,<sup>6,27</sup>

$$ds^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2, \quad (1)$$

where the metric function is given by  $f(r) = 1 - 2M/r$  and the horizon is defined at  $r_H = 2M$ . Solving the geodesic equation for the metric (1), the proper velocity of a particle can be obtained as<sup>28</sup>

$$u^\mu = \left( \frac{dt}{d\tau}, \frac{dr}{d\tau} \right) = \left( \frac{k}{f(r)}, \pm \sqrt{k^2 - f(r)} \right), \quad (2)$$

where  $\tau$  and  $k$  are the proper time and the constant of integration, respectively. The  $k$  can be identified with the energy of a particle per unit mass for  $k > 1$ , which can be written as  $k = 1/\sqrt{1 - v^2}$  with  $v = dr/dt$  at the asymptotic infinity. In this case, the motion of the particle is unbounded, so that the particle lies in the range of  $r \geq r_H$ . For  $0 \leq k \leq 1$ , the motion of the particle is bounded such that there is a maximum point  $r_{\max}$  where the particle lies in the range of  $r_H \leq r \leq r_{\max}$ . We are going to consider a freely falling frame starting at  $r_s = r_{\max}$  with zero velocity toward the black hole, which can be shown to be the latter case by identifying  $k = \sqrt{f(r_s)}$  in Eq. (2), and thus the proper velocity of a free-falling observer can be written as

$$u^\mu = \left( \frac{dt}{d\tau}, \frac{dr}{d\tau} \right) = \left( \frac{\sqrt{f(r_s)}}{f(r)}, -\sqrt{f(r_s) - f(r)} \right). \quad (3)$$

If the observer starts to fall into the black hole at the spatial infinity, then  $f(r_s) = 1$  while  $f(r_h) = 0$  for the observer to fall into the black hole just at the horizon. Then, the radial velocity with respect to the Schwarzschild time becomes  $v = -f(r)\sqrt{f(r_s) - f(r)}/\sqrt{f(r_s)}$  which vanishes both at the initial free-fall position and the horizon, and the maximum speed occurs at  $r = 6Mr_s/(4M + r_s)$ . The proper time from  $r_s$  to  $r_H$  is also obtained as

$$\tau = 2M \frac{\sqrt{f(r_s)(1 - f(r_s))} + \sin^{-1} \sqrt{f(r_s)}}{(1 - f(r_s))^{3/2}}, \quad (4)$$

which is finite except for the case of the initial free fall at the asymptotic infinity. So, it will take finite proper time to reach the event horizon when free fall begins at finite distance.

Now, in the light-cone coordinates defined by  $\sigma^\pm = t \pm r^*$  through  $r^* = r + 2M \ln(r/M - 2)$  the proper velocity (3) can be written as

$$u^+ = \frac{1}{\sqrt{f(r_s)} + \sqrt{f(r_s) - f(r)}}, \quad (5)$$

$$u^- = \frac{\sqrt{f(r_s)} + \sqrt{f(r_s) - f(r)}}{f(r)}, \quad (6)$$

where  $u^\pm = u^t \pm u^r/f(r)$  and the energy-momentum tensors are expressed as<sup>27</sup>

$$\langle T_{\pm\pm} \rangle = -\frac{N}{48\pi} \left( \frac{2Mf(r)}{r^3} + \frac{M^2}{r^4} \right) + \frac{N}{48} t_\pm, \quad (7)$$

$$\langle T_{+-} \rangle = -\frac{N}{48\pi} \frac{2M}{r^3} f(r), \quad (8)$$

where  $N$  is the number of massless scalar fields and  $t_\pm$  are functions of integration to be determined by boundary conditions. The general covariance is guaranteed by covariant conservation law of the energy-momentum tensors. The two-dimensional trace anomaly for scalar fields was employed to get the nontrivial vacuum expectation value of the energy-momentum tensors,<sup>27</sup> so that the two differential equations and one anomaly equation determine the explicit form of the energy-momentum tensors with two unknown.

Now, the energy density measured by the free-falling observer can be calculated as,<sup>28,29</sup>

$$\epsilon = \langle T_{\mu\nu} \rangle u^\mu u^\nu, \quad (9)$$

by using the proper velocity and the energy-momentum tensor. In connection with Hawking radiation, the fields are quantized on the classical background metric in such a way that nontrivial radiation will appear and the energy density (9) will not vanish even in the freely falling frame. Substituting Eqs. (5)–(8) into Eq. (9), the energy density can be calculated as

$$\begin{aligned} \epsilon(r|r_s) = & -\frac{N}{48\pi r^4 f(r)} \left[ 8Mr f(r_s) + 4M^2 \left( \frac{f(r_s)}{f(r)} - \frac{1}{2} \right) \right. \\ & - \pi r^4 \left( \sqrt{\frac{f(r_s)}{f(r)}} - \sqrt{\frac{f(r_s)}{f(r)} - 1} \right)^2 t_+ \\ & \left. - \pi r^4 \left( \sqrt{\frac{f(r_s)}{f(r)}} + \sqrt{\frac{f(r_s)}{f(r)} - 1} \right)^2 t_- \right], \end{aligned} \quad (10)$$

and it is reduced to

$$\epsilon(r_s|r_s) = -\frac{N}{48\pi r_s^4 f(r_s)} [8Mr_s f(r_s) + 2M^2 - \pi r_s^4 (t_+ + t_-)], \quad (11)$$

at the special limit of  $r = r_s$  where observation is done at the moment when free fall begins. Now, let us investigate characteristics for the energy density measured by the free-falling observer for the Boulware, Unruh and Hartle–Hawking states, respectively in what follows.

### 3. Boulware State

The Boulware state is obtained by choosing  $t_{\pm} = 0$ , where the energy density (10) reads as

$$\epsilon_B(r|r_s) = -\frac{NM^2}{12\pi r^4 f(r)} \left[ \frac{2rf(r_s)}{M} + \frac{f(r_s)}{f(r)} - \frac{1}{2} \right], \quad (12)$$

which is always negative. So the freely falling observer encounters more and more negative energy density and then eventually negative divergent one at the hori-

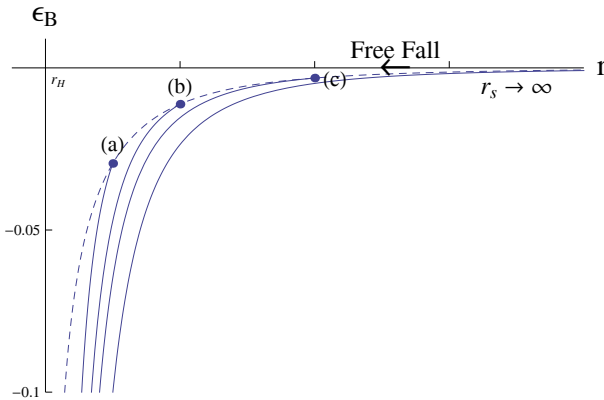


Fig. 1. The energy densities in the Boulware state are plotted by simply choosing as  $N = 124$ ,  $M = 1$ . They are always negative no matter where the initial free-fall positions are. The solid curves represent  $\epsilon_B(r|r_s)$  and the dotted curve does  $\epsilon_B(r_s|r_s)$  where three energy densities at  $r_s$  are denoted by the black dots (a), (b) and (c).

zon which is independent of the initial free-fall position. If the observation is done at the moment when free fall begins, the energy density is reduced to  $\epsilon_B(r_s|r_s) = -N[4Mr_sf(r_s) + M^2]/[24\pi r_s^4 f(r_s)]$ , so that the observer who starts at the horizon finds the divergent energy immediately and asymptotically vanishes without Hawking radiation as shown in Fig. 1.

#### 4. Unruh State

The Unruh state is characterized by choosing functions of integration as  $t_+ = 0$  and  $t_- = 1/(16\pi M^2)$  in Eq. (10), which yields the energy density as

$$\epsilon_U(r|r_s) = -\frac{NM^2}{12\pi r^4 f(r)} \left[ \frac{2rf(r_s)}{M} + \frac{f(r_s)}{f(r)} - \frac{1}{2} - \frac{r^4}{64M^4} \left( \sqrt{\frac{f(r_s)}{f(r)}} + \sqrt{\frac{f(r_s)}{f(r)} - 1} \right)^2 \right]. \quad (13)$$

The energy density measured by the free-falling observer at the horizon who falls into the black hole from  $r_s$  is simplified as  $\epsilon_U(2M|r_s) = (N(63r_s^2 - 320Mr_s + 384M^2))/[3072\pi M^2 r_s(r_s - 2M)]$ , which is not always positive definite. In other words, the initial free-fall position is crucial to determine the sign of the energy density at the horizon in contrast to the Boulware case. Specifically, the energy density at the horizon is indeed positive for  $r_s > r_0$  where  $r_0$  is the initial free-fall position for the energy density to vanish. For instance, it is positive finite as seen from the case (c) in Fig. 2, and it becomes  $\epsilon_U(2M|\infty) = 21N/(1024\pi M^2)$  where the free-fall frame is dropped at the spatial infinity. On the other hand, there is a critical point  $r_c = 8(20M + \sqrt{22}M)/63$  which is defined by the point where the observer

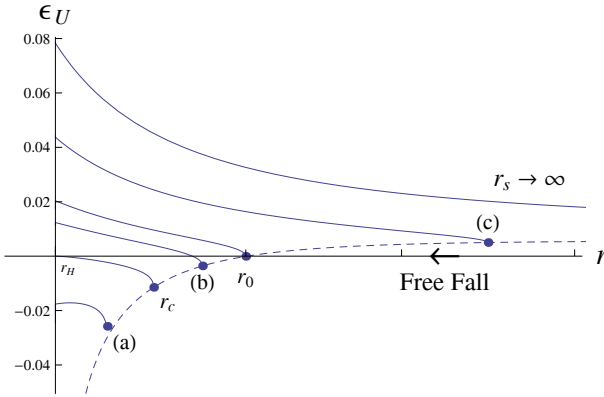


Fig. 2. The energy densities for the Unruh state are plotted by setting  $N = 12$ ,  $M = 1$ . The critical point appears at  $r_c \approx 3.1M$  and the energy density vanishes at  $r_0 \approx 4.2M$ . The solid curves are for  $\epsilon_U(r|r_s)$  and the dotted curve represents  $\epsilon_U(r_s|r_s)$  such that there are largely three free-fall cases: (a) is for  $r_s < r_c$ , (b) is for  $r_c < r_s < r_0$  and (c) is for  $r_s > r_0$ .

finds the zero energy at the horizon, so that the observer will see the positive energy at the horizon as long as  $r_s > r_c$ . For  $r_c < r_s < r_0$ , there appears a transition from the negative energy density to the positive energy density, which can be seen from the case (b) in Fig. 2. For  $r_s < r_c$ , the observer will see only negative radiation at the horizon like the case (a). When the initial free-fall position approaches the horizon closer, the larger negative energy density appears.

Using Eq. (11), the energy density can be obtained at the moment when the free fall just begins, then the corresponding energy density is given as  $\epsilon_U(r_s|r_s) = -NM^2[4r_sf(r_s)/M+1-r_s^4/(32M^4)]/[24\pi r_s^4 f(r_s)]$  which is described by the dotted curve in Fig. 2. Note that the energy density at the horizon  $\epsilon_U(2M|2M)$  is negative divergent whereas it is positive finite  $\epsilon_U(\infty|\infty) \rightarrow \pi(N/12)T_H^2$  at the asymptotic infinity, where  $T_H$  is the Hawking temperature.

In particular, we would like to explain why the freely falling observers who are moving slowly with respect to the black hole when they pass through the horizon should see very high (negative) energy density. Actually, the conventional wisdom is that the freely falling observer near the horizon cannot see any outgoing Hawking radiation as  $\langle T_{--} \rangle = 0$ . It can be easily understood from the Unruh effect<sup>6</sup> which tells us that the frame near the horizon can be described as the local-flat metric in terms of the Kruskal coordinates on the Schwarzschild black hole, so that the corresponding observer can be regarded as the freely falling observer while the fiducial observer sees radiation because the observer is now on the accelerated frame. Moreover, in the collapsing black hole described by the Unruh state, it was shown that the energy flow across the future horizon is seen to be negative,  $\langle T_{++} \rangle < 0$ , since the corresponding positive energy would flow out to infinity.<sup>7</sup> All these facts can also be confirmed by using Eqs. (7) and (8).

In this work, we employed the energy density which consists of three components of  $\langle T_{++} \rangle$ ,  $\langle T_{--} \rangle$ ,  $\langle T_{+-} \rangle$ , while the energy-momentum tensors related to the fluxes have been discussed in the previous works. Explicitly, the energy density (9) can be reduced to  $\epsilon = \langle T_{++} \rangle u^+ u^+$  at the horizon since  $\langle T_{--} \rangle = \langle T_{+-} \rangle = 0$  there. Note that it does not vanish but also is negative because of nonvanishing ingoing negative flux as  $\langle T_{++} \rangle = -N/(768\pi M^2) < 0$  from Eq. (7). At the horizon, the nonvanishing energy density is related to the nonvanishing ingoing energy-momentum tensor as it should be. To explain the reason why the high energy density appears near the horizon for a very slowly falling frame, let us rewrite the free-fall energy density (9) as  $\epsilon = \langle T_{tt} \rangle u^t u^t$  in the normal coordinates where the radial velocity is fixed as  $u^r = 0$  for convenience when the observer is dropped from rest at  $r = r_s$ . Note that the time component of the velocity at the starting point of  $r_s$  by definition becomes  $u^t = dt/d\tau = 1/\sqrt{f(r_s)}$  which is larger than one because the function  $f(r_s)$  is less than one near the horizon, so that  $dt > d\tau$  where  $dt$  is a time measured by the fiducial observer and  $d\tau$  is a proper time measured by the freely falling observer. Moreover, it shows that the gravitational time dilation effect is much more significant when the observer is dropped close to the horizon. By the way, as a corollary to this fact, the frequency in the freely falling frame is higher than that

in the fixed frame, so that this factor contributes to the energy density. Therefore, it eventually becomes the high energy density of  $\epsilon(r_s|r_s) = -N/(768\pi M^2 f(r_s))$  near the horizon, where  $r_s$  represents the starting position when the observer is dropped from rest. On the other hand, if the freely falling observer starts with the nonzero initial velocity at a certain point from the horizon, then the observer can see the positive energy at that instant because  $\langle T_{tr} \rangle$  with  $u^r \neq 0$  gives rise to the positive contribution to the energy density.

One more thing to be mentioned in this section is that, we could calculate the free-fall energy density not only at any finite distance but also near the horizon and at infinity in the simplified context which is one of the advantages of the two-dimensional model, so that we could further discuss the critical point to characterize the positive energy zone and the negative energy zone by solving the exact geodesic equation analytically. The result shown in Fig. 2 is physically compatible with the previous one that the positive energy flux would flow out to infinity while a corresponding amount of negative energy flux would flow down the black hole,<sup>7</sup> so that the area of horizon decreases at a rate expected positive energy flux at infinity.<sup>8</sup>

## 5. Hartle–Hawking State

For the Hartle–Hawking state, let us take  $t_{\pm} = 1/(16\pi M^2)$  in Eq. (10), then the energy density can be obtained as

$$\epsilon_{\text{HH}}(r|r_s) = -\frac{NM^2}{12\pi r^4 f(r)} \left[ \frac{2rf(r_s)}{M} - \left( \frac{r^4}{16M^4} - 1 \right) \left( \frac{f(r_s)}{f(r)} - \frac{1}{2} \right) \right]. \quad (14)$$

The free-falling observer at  $r_s$  toward the black hole will find the finite energy density at the horizon of  $\epsilon_{\text{HH}}(2M|r_s) = N(r_s - 3M)/(48\pi M^2 r_s)$ . In particular, it becomes  $\epsilon_{\text{HH}}(2M|\infty) = N/(48M^2\pi)$  when the observer is dropped at spacial infinity at rest. There is the point  $r_0$  where the energy density measured in the free-falling frame vanishes; however, the crucial difference from the Unruh case is that the freely falling observer starting at  $r_s > r_0$  may encounter alternatively the positive energy and the negative energy density during the free fall as shown in the case (b) in Fig. 3, whereas only the positive energy density appears in the Unruh state. There is also the critical point  $r_c$  to characterize the sign of the energy density at the horizon as has been done in the Unruh case, so that the freely falling observer at the horizon will see the positive energy density for  $r_s > r_c$  and the negative energy density for  $r_s < r_c$ . Moreover, the observer will necessarily find a transition from the positive energy density to the negative energy density for  $r_0 < r_s < r_c$ . Note that  $r_0$  and  $r_c$  in the Hartle–Hawking state are smaller than those in the Unruh state, respectively, which is shown in Fig. 3.

At  $r = r_s$ , the energy density in the Hartle–Hawking state from Eq. (11) becomes  $\epsilon_{\text{HH}}(r_s|r_s) = -N[8Mr_s f(r_s) + 2M^2 - r_s^4/(8M^2)]/[48\pi r_s^4 f(r_s)]$ , where the behavior of the energy density is described by the dotted curve in Fig. 3. Explicitly, when



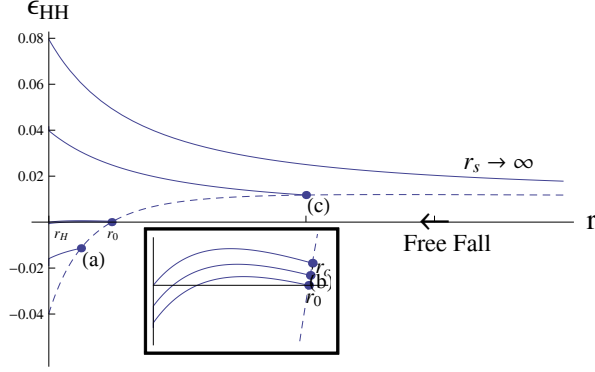


Fig. 3. The energy densities in the Hartle–Hawking state are plotted by setting  $N = 12$ ,  $M = 1$ . The solid curves describe  $\epsilon_{\text{HH}}(r|r_s)$  and the dotted curve represents  $\epsilon_{\text{HH}}(r_s|r_s)$ . There are largely three free-fall cases (a) is for  $r_s < r_0$ , (b) in the box is for  $r_0 < r_s < r_c$  and (c) is for  $r_s > r_c$ , where  $r_0 \approx 2.98M$  and  $r_c = 3M$ .

$r_s = 2M$ , it becomes negative finite as  $\epsilon_{\text{HH}}(2M|2M) \rightarrow -N/(96\pi M^2)$  which is contrast to the infinite energy density in the Unruh state. At the asymptotic infinity, it is finite  $\epsilon_{\text{HH}}(\infty|\infty) \rightarrow \pi N T_{\text{H}}^2/6$ , and the energy density in the Hartle–Hawking state is two times that of the Unruh state, i.e.  $\epsilon_{\text{HH}}(\infty|\infty) = 2\epsilon_{\text{U}}(\infty|\infty)$ .

## 6. Discussion

At first sight, it seemed that the freely falling observer could not find any radiation based on the intuitive argument that the surface gravity translated into the radiation temperature vanishes in local inertial frames; however, the explicit calculation shows that radiation can exist even in freely falling frames and moreover it depends on the free-fall position unequivocally for certain black hole states. For the Boulware state, the energy density is always negative divergent at the horizon, which is independent of the initial free-fall positions. For the Hartle–Hawking state, the energy density at the horizon is negative finite when free fall toward the black hole begins at  $r_s < r_c$  while it is positive finite at the horizon for  $r_s > r_c$ , where  $r_c = 3M$  is the critical initial free-fall position to determine the sign of the energy density at the horizon. In particular, the Unruh state describing the collapsing black hole gives a slightly larger critical value of  $r_c \approx 3.1M$  compared to that of the Hartle–Hawking state. As expected, the energy density at the horizon in the Unruh state behaves as mixed between the Boulware state and the Hartle–Hawking state, in that the energy density at the horizon can be either divergent or finite. Therefore, the energy density measured by the freely falling observer in the semiclassical black hole is characterized by mainly two conditions of both the initial free-fall position and the black hole state.

In connection with black hole complementarity, let us drop the particle at a finite distance near the horizon such that it will take finite proper time for the freely

falling observer to reach the horizon. Subsequently, the particle will reach the origin without any drama at the horizon. On the other hand, the distant observer using the Schwarzschild coordinates feels that the particle is still at rest at the horizon so that the observer cannot see the particle crossing the horizon forever. But, the particle will appear awkwardly through Hawking radiation eventually. To resolve this problem, the notions of the membrane paradigm and the stretched horizon with black hole complementarity are helpful in understanding the relation between the infalling particle and Hawking radiation.<sup>3-5</sup> However, the present calculation will modify black hole complementarity since the freely falling observer can perceive radiation throughout free fall especially at the event horizon. It means that when the observer passes through the horizon, he/she sees that the infalling particle may be either excited or annihilated by the energy density around the black hole.

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