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Black hole complementarity in gravity's rainbow

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Abstract. To see how the gravity's rainbow works for black hole complementary, we evaluate the required energy for duplication of information in the context of black hole complementarity by calculating the critical value of the rainbow parameter in the certain class of the rainbow Schwarzschild black hole. The resultant energy can be written as the well-defined limit for the vanishing rainbow parameter which characterizes the deformation of the relativistic dispersion relation in the freely falling frame. It shows that the duplication of information in quantum mechanics could not be allowed below a certain critical value of the rainbow parameter; however, it might be possible above the critical value of the rainbow parameter, so that the consistent formulation in our model requires additional constraints or any other resolutions for the latter case.

Keywords: modified gravity, quantum black holes

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Contents

1 Introduction

The discovery of Hawking's thermal radiation from a black hole [1, 2] has raised the information loss paradox when the black hole evaporates completely [3]. This paradox could be solved for the fixed observer outside the horizon if the Hawking radiation carried the information of infalling matter state so that the quantum-mechanical evolution could be unitary. For the consistency between general relativity and quantum mechanics, it requires black hole complementarity [4–6] which tells us that the infalling observer (Alice) sees nothing strange when passing through the horizon based on the equivalence principle while the external fixed observer (Bob) outside the horizon sees the information reflected at the stretched horizon based on quantum mechanics. The two observers of Alice and Bob can only detect the information inside and outside the black hole, respectively, but never both simultaneously. Bob measures the information of the infalling state during a certain time which amounts to at least the Page time [7, 8] and then jumps into the horizon, he cannot receive the message from Alice since the available time to send the message to him is too short. It means that the required energy derived from the Heisenberg's uncertainty principle exceeds the mass of the black hole so that black hole complementarity turns out to be valid [5]. Moreover, black hole complementarity is marginally safe for the scrambling time [9, 10], and the recent application of the membrane paradigm appears in ref. [11].

On the other hand, it has been claimed that there is a puzzle referred to as the firewall paradox of quantum black holes based on the monogamy principle in quantum mechanics and semiclassical quantum field theory [12, 13]. The consistency of quantum mechanics requires that the freely falling observer should burn up when crossing the horizon in virtue of the firewall, which means that the probing inside the black hole is forbidden. A similar prediction called the energetic curtain around the black hole from different assumptions was also studied in ref. [14]. Subsequently, much attention has been paid to study the firewall issue along with not only resolutions from various viewpoints but also including some arguments of the absence of the firewall [15–33]. Essentially, most issues on black hole complementarity are related to the interplay between the Hawking radiation from the quantum mechanics and the geometry from general relativity. Now, one might consider additional element that is the interaction between the infalling apparatus and Hawking radiation plus geometry. In fact, the effect of the infalling apparatus on the gravity background is assumed to be small compared to the black hole mass M for the definite explanation [5]. For a further test for black hole complementarity, one might want to find a convenient setting to take into account this effect of the test particle or the apparatus on the black hole.

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On the other hand, it is worth noting that, based on not only experimental explanation for the threshold anomalies in ultra high cosmic rays and Tev photons [34–41] but also theoretic points of view in the semi-classical limit of loop quantum gravity [42–45], there have been a number of applications in the framework of gravity's rainbow in order to explore some effects of a test particle on black holes and cosmology [46–66]. Some years ago, Amelino-Camelia proposed that the modified dispersion relation could come from a deformation of the classical relativity called the doubly special relativity [67, 68] which is the extended version of the Einstein's special theory of relativity in which the Plank length is also required to be invariant under any inertial frames along with the invariant speed of light. And then the most common illustration was presented by Magueijo and Smolin [69–71], which indeed gives rise to the modified dispersion relation. This notion was promoted to the curved spacetime in which the general spacetime background felt by a test particle would depend on its energy so that the energy of the test particle deforms the background geometry. Recently, the information loss paradox in gravity's rainbow was discussed in ref. [72] by obtaining the finite coordinate time for the asymptotic observer along the geodesic curve, and some subtleties of specification of the event horizon were pointed out.

In this work, we would like to study information loss paradox in the rainbow Schwarzschild black hole in the context of duplication of information. For this purpose, we will take into account the effect of probing particles on the black hole by employing the special class of the rainbow metric, and calculate the required energy in the freely falling frame for Alice to send the message to Bob who jumped into the black hole at the Page time whether the energy of quanta will be able to become an enormous scale or not by calculating the critical value of the rainbow parameter in the gravity's rainbow. Let us first recapitulate the formalism of gravity's rainbow and introduce the rainbow Schwarzschild metric incorporated with the effect of the test particle in the self-contained manner in section 2. Next, assuming the Heisenberg uncertainty relation, the energy associated with the proper time will be calculated on the particular rainbow functions [73] in order for the definite illustration in section 3. The deformed metric from the test particle effect will be characterized by a rainbow parameter η. The energy uncertainty will reproduce the well-known limit presented by Susskind and Thorlacius for the vanishing rainbow parameter [5]. Furthermore, it turns out that duplication of information by Alice could be still impossible below a certain critical rainbow parameter while the duplication of information might be possible above the critical rainbow parameter. The latter case is inconsistent with the assumptions of black hole complementarity and thus no-cloning theorem of quantum information will be violated. Finally, conclusion and discussion will be given in section 4.

2 Rainbow gravity

Let us start with the modified dispersion relation in the doubly special relativity [67, 68] which makes both the speed of light and the Planck length invariant under the non-linear Lorentz transformation in the momentum space, which is compactly given as [69, 70]

$$f(E/E_p)^2E^2 - g(E/E_p)^2c^2p^2 = m^2c^4, \tag{2.1}$$

where the Planck energy is $E_p = \sqrt{c^5\hbar/G}$, and E and m are the energy and the mass of the test particle, respectively. In this formulation, the rainbow functions $f(E/E_p)$ and $g(E/E_p)$ satisfy $\lim_{E\to 0} f = 1$ and $\lim_{E\to 0} g = 1$ so that the modified dispersion is reduced to the conventional one when the energy of the test particle vanishes. The metric based on the modified equivalence principle could also be expressed in terms of a one-parameter family of orthonormal frame fields [71],

$$g^{\mu\nu}(E/E_p) = \eta^{ab} e_a^{\mu}(E/E_p) e_b^{\nu}(E/E_p), \tag{2.2}$$

where the energy dependent frame fields are written as $e_0(E/E_p) = f^{-1}(E/E_p)\tilde{e}_0$ and $e_i(E/E_p) = g^{-1}(E/E_p)\tilde{e}_i$ and $\tilde{e}$ is the ordinary energy-independent one. According to this fact, the Einstein field equations are also redefined as

$$G_{\mu\nu}(E/E_p) = 8\pi G(E/E_p) T_{\mu\nu}(E/E_p), \tag{2.3}$$

where $G(E/E_p)$ is the energy dependent Newton constant which becomes the conventional Newton constant as $G = G(0)$ for $E = 0$. From eq. (2.3), the energy-dependent Schwarzschild solution can be obtained as [71]

$$ds^2 = -\frac{1}{f^2(E/E_p)}\left(1 - \frac{2G(0)M}{r}\right) dt^2 + \frac{1}{g^2(E/E_p)\left(1 - \frac{2G(0)M}{r}\right)} dr^2 + \frac{r^2}{g^2(E/E_p)} d\Omega^2, \tag{2.4}$$

where the spacetime coordinates are chosen as the energy-independent coordinates. In the next section, after deriving the generic form of the energy relation, we will choose specific rainbow functions [34] which can also be obtained from loop quantum gravity approach [42–45] as

$$f(E/E_p) = 1, \qquad g(E/E_p) = \sqrt{1 - \eta\left(\frac{E}{E_p}\right)^n}, \tag{2.5}$$

where η is the rainbow parameter and $n = 2$ is chosen in order for analytic calculations.

3 η-dependent energy for duplication of information

Based on the calculations in refs. [4, 5], we can calculate the required energy for two copies of information on the Schwarzschild black hole which is described by using the Kruskal-Szekeres coordinates of $ds^2 = -32M^3 r^{-1} e^{-r/2M} dU dV$ where $U = -e^{-(t-r^*)/4M}$, $V = e^{(t+r^*)/4M}$ and $r^* = r + 2M \ln(|r - 2M|/2M)$. For convenience, the constants are set to $G = \hbar = c = k_B = 1$; however, these will be recovered as necessary. After Alice passes through the stretched horizon, Bob will essentially begin to observe the information through the Hawking radiation at the Page time $t_{\text{Page}} \sim M^3$ [7]. Let us assume that Alice passes through the stretched horizon at $V_A = 1$, then Bob should go through the stretched horizon at least after the Page time, so $V_B \sim e^{M^2}$. To send the information by Alice before Bob arrives at the curvature singularity, Alice should send the information before $U_A = 1/V_B \sim e^{-M^2}$, where the curvature singularity appears at $UV = 1$. Next, the interval of the proper time $\Delta\tau$ which is nothing but the free-fall time for Alice near $V_A = 1$ is given as $\Delta\tau \sim Me^{-M^2}$. By using the Heisenberg's energy-time uncertainty principle of $\Delta\tau\Delta E \geq 1/2$, one can eventually obtain $\Delta E \sim M^{-1}e^{M^2}$ which is indeed larger than the mass of black hole, and the information should be encoded into the radiation with super-Planckian scale of the quanta. Therefore, Bob cannot see the duplication of Alice's information physically, so that black hole complementarity is valid.

Let us now calculate the energy for the duplication of information by using the rainbow Schwarzschild metric (2.4) along the argument in ref. [5]. First, the metric (2.4) is written as

$$ds^2 = -\frac{4r_H^3}{g^2 r} e^{-r/r_H} dU dV, \tag{3.1}$$

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in terms of the rainbow Kruskal-Szekeres coordinates defined as $U = -e^{-((g/f)t - r^*)/2r_H}$, $V = e^{((g/f)t + r^*)/2r_H}$ and $r^* = r + r_H \ln(|r - r_H|/r_H)$. The rainbow metric and the conformal transformations are reduced to the conventional ones for $f = g = 1$ where the probing energy E goes to zero.

To get the information retention time for the rainbow black hole, we consider the Stefan-Boltzmann law written as [8]

$$\frac{dM}{dt} = -A\sigma T^4, \tag{3.2}$$

where A denotes the area of a black body and $\sigma = \pi^2 k_B^4/(60\hbar^3 c^2)$ is the Stefan-Boltzmann constant, and the Hawking temperature T is also calculated as

$$T = \frac{\kappa_{\rm H}}{2\pi} = \frac{1}{8\pi GM}\frac{g(E/E_p)}{f(E/E_p)}, \tag{3.3}$$

where κ_H is the surface gravity at the horizon. Using the explicit form of the rainbow functions (2.5), the black hole temperature can be written as [63]

$$T_{\rm H} = \frac{1}{8\pi GM}\sqrt{1 - \eta\left(\frac{E}{E_p}\right)^n}. \tag{3.4}$$

In order to eliminate the dependence of the particle energy in the black temperature (3.4), one can use the heuristic method in ref. [74]. It shows that the Heisenberg uncertainty principle gives a relation between the momentum of the Hawking particle p emitted from the black hole and the mass of black hole M as $p = \Delta p \sim 1/(2GM)$ by regarding $\Delta x \sim 2GM$ [74]. Next, for a definite illustration, we choose $n = 2$. So the energy for the massless particle can be easily solved as $E = E_p/(\sqrt{\eta + 4G^2E_p^2M^2})$. Plugging this into the temperature (3.4), one can obtain the rainbow Hawking temperature as [75]

$$T_{\rm H} = \frac{1}{8\pi GM}\sqrt{\frac{4GM^2}{4GM^2 + \eta}}, \tag{3.5}$$

where it reproduces the well-known Hawking temperature for the large black hole. Next, from the Stefan-Boltzmann law (3.2), the Page time when the black hole has emitted half of its initial Bekenstein-Hawking entropy can be calculated as [8]

$$t_{\rm Page} \sim M^3 + \mathcal{O}(\eta), \tag{3.6}$$

where the black hole was assumed to be very large even after the Page time.

Based on gedanken experiments, let us now suppose that Alice passes through the stretched horizon at $V_A(t_A, r_A^*) = 1$ in figure 1 and then after a period of the Page time (3.6) outside the black hole Bob will jump into the stretched horizon at $V_B(t_B, r_B^*)$ with the information which has been gathered from Hawking radiation, i.e., $t_B = t_A + t_{\rm Page}$. It means that Bob starts to move toward the horizon at $V_B = e^{((g/f)t_B + r_B^*)/2r_H} = e^{((g/f)(t_A + t_{\rm Page}) + r_A^*)/2r_H} = e^{((g/f)t_{\rm Page})/(2r_H)}V_A$. Note that $r_A^* = r_B^*$ and $V_A = 1$. Thus Alice should send her messages before $U_A = 1/V_B = e^{-(g/f)M^2}$, so that from eq. (3.1) the proper time measured by Alice near the horizon $r \sim r_H$ can be written as

$$\Delta\tau^2 = \frac{4r_H^2}{g^2}\, e^{-1}(U_A - 0)\Delta V_A \tag{3.7}$$

$$\sim \frac{1}{g^2}\, M^2 e^{-(g/f)M^2}, \tag{3.8}$$

where we assumed that ΔV_A is a nonvanishing finite value near $V = 1$ [5].

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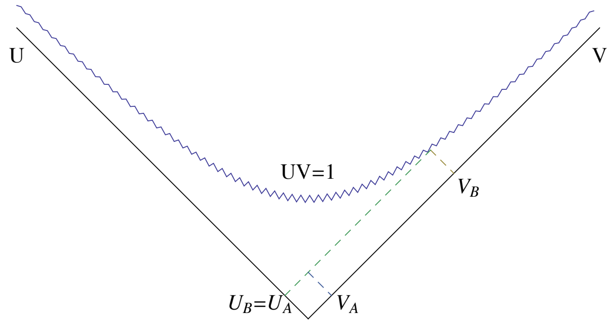

Figure 1. The wiggly curve denotes the black hole singularity where $UV = 1$. Alice passes through the horizon at V_A and then Bob also jumped into the horizon at V_B just after the Page time. Alice should send the signal to Bob before the message hits the singularity.

Next, one can find the appropriate energy-time uncertainty relation in the local rainbow inertial frame. For this purpose, we assume that the usual Heisenberg uncertainty principle is valid in the local inertial frame where there is no rainbow effect. Then the uncertainty relation between the energy and time in our case should be written as

$$\Delta\tau\Delta E \geq \frac{1}{2f} \tag{3.9}$$

by taking into account the rainbow effect. It tells us that the energy uncertainty becomes $\Delta E \sim g/f$ since the proper time is proportional to $1/g$ from eq. (3.8), which is compatible with the structure of the modified dispersion relation (2.1). Consequently, using eqs. (3.8) and (3.9), the energy uncertainty is obtained as

$$\Delta E^2 \sim \frac{M_p^4 c^4 \left(1 - \eta\left(\frac{\Delta E}{E_p}\right)^2\right)}{M^2} e^{\frac{M^2}{M_p^2}\sqrt{1-\eta\left(\frac{\Delta E}{E_p}\right)^2}} \tag{3.10}$$

by recovering dimensional constants. Note that in the absence of the rainbow effect of $\eta \to 0$, the energy uncertainty is consistent with the standard result of the Susskind-Thorlacius limit of $\Delta E \sim (M_p^2 c^2/M)e^{M^2/M_P^2}$ whose value could be beyond the Planckian scale for the large black hole [5]. From eq. (3.10), one can see that $d(\Delta E(\eta))/d\eta$ for an arbitrary M is always negative, so that it is monotonically decreasing as shown in figure 2. Instead of solving the closed form of eq. (3.10) perturbatively, we provide a criterion for information cloning, and define a critical case where the energy uncertainty at the Page time amounts to the black hole mass of $\Delta E \sim M$, which is given as

$$\eta_c = \frac{M^4 - 4W^2(Z)}{M^6} \tag{3.11}$$

by means of the Lambert W function defined as $Z = W(Z)e^{W(Z)}$ where $Z = M^4/2$. Therefore, no-cloning theorem of quantum information is valid as long as $\eta \ll \eta_c$ since the required energy of the quanta for the cloning exceeds the mass of the black hole like the result in ref. [5], while it might be violated for $\eta \gg \eta_c$.

4 Conclusion and discussion

The required energy in the freely falling frame for Alice to send the message to Bob who jumped into the black hole at the Page time was calculated in the rainbow Schwarzschild black

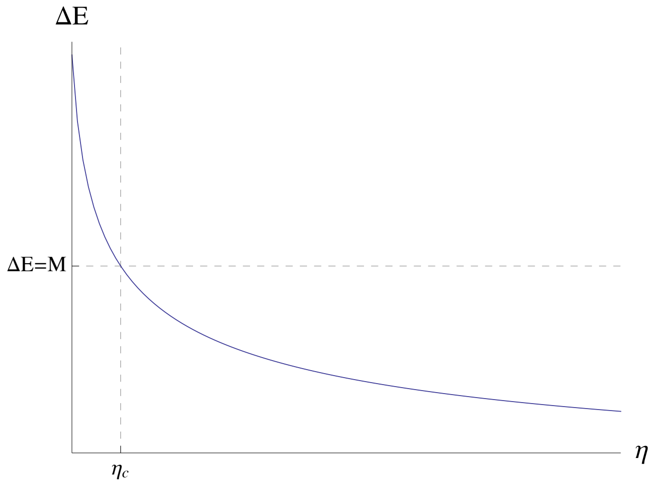

Figure 2. The energy uncertainty vs the rainbow parameter is plotted by setting M=2 for convenience. It is indeed monotonically decreasing function for the arbitrary M.

hole in order to investigate how black hole complementarity works in the rainbow gravity. It tells us that the rainbow parameter should be much less than the critical rainbow parameter to maintain the unitarity in quantum mechanics so that black hole complementarity can be safe. If the rainbow parameter were much larger than the critical one, then the energy uncertainty would be made smaller than the Planckian scale of the energy. However, it does not mean that the generic rainbow gravity can save black hole complementarity since the present calculation is based on the particular rainbow functions.

As for the case to violate the unitarity, it is necessary to introduce additional devices or resolutions to protect unitarity in the rainbow Schwarzschild black hole. So we would like to comment on some various speculations to resolve this problem. i) A certain rainbow-improved black hole complementarity might exist and it should be reduced to the conventional black hole complementarity when the rainbow parameter vanishes. ii) The firewall like objects or something else, for example, the brick wall [76] might exist to protect the unitarity for the generic non-vanishing rainbow parameter. However, it was shown that the brick wall can be eliminated due to the modification of the density of states by choosing appropriate rainbow functions [77]. iii) As was claimed by Hawking [78], if there were no event horizon behind which information is lost, then information could be preserved during the evaporation. It was also stressed that the event horizons are inappropriate to describe the physical black holes [79]. Recent numerical calculations by taking into account the quantum back reaction of the geometry show that a star stops collapsing a finite radius larger than its horizon [80, 81]. If this were true, then it could be analogously applied to this rainbow gravity. However, it was claimed that the radial distance between the event horizon and the apparent horizon is much smaller than the Planck length for the large black hole [82]. So the information loss paradox seems to be still ongoing issue in the rainbow gravity.

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