Previous Year Questions
Fluid Mechanics (2008-22)
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Kinematics of Fluid in Motion

1. If the velocity potential of a fluid is $\phi = \frac{1}{r^3} z \tan^{-1}\left(\frac{y}{x}\right), r^2 = x^2 + y^2 + z^2$, then show that the stream lines lie on the surfaces $x^2 + y^2 + z^2 = c(x^2 + y^2)^{\frac{2}{3}}$, c being a constant.
2. Show that $\phi = xf(r)$ is a possible form for the velocity potential for an incompressible fluid motion. If the fluid velocity $q \to 0\, as\, r \to \infty$,find the surfaces of constant speed.
3. Given the velocity potential $\phi = \frac{1}{2}\log\left(\frac{(x+a)^2+y^2}{(x-a)^2+y^2}\right)$, determine the streamlines.
4. In an axis symmetric motion, show that stream function exists due to equation of continuity. Express the velocity components of the stream function. Find the equation satisfied by the stream function if the flow is irrotational.
5. If the velocity of an incompressible fluid at the point (x, y, z) is given by $\left(\frac{3xz}{r^5}, \frac{3yz}{r^5}, \frac{3z^2-r^2}{r^5}\right), r^2 = x^2 + y^2 + z^2$, then prove that the liquid motion is possible and that the velocity potential is $\frac{z}{r^3}$. Further, determine the streamlines.
6. For an incompressible fluid flow, two components of velocity (u, v, w) are given by $u = x^2 + 2y^2 + 3z^2, v = x^2y - y^2z + zx$. Determine the third components w so that they satisfy the equation of continuity. Also find the z −components of acceleration.
7. For a two-dimensional potential flow, the velocity potential is given by $\phi = x^2y - xy^2 + \frac{1}{3}(x^3 - y^3)$. Determine the velocity components along the directions x *and* y. Also, determine the stream function ψ and check whether ϕ represents a possible case of flow or not.
8. A velocity potential in two-dimensional fluid flow is given by $\phi(x,y) = xy + x^2 - y^2$. Find the stream function for this flow.
9. Show that $\vec{q} = \frac{\lambda(-y\hat{\imath}+x\hat{\jmath})}{x^2+y^2}, (\lambda = cosntant)$ is a possible incompressible fluid motion. Determine the streamlines. Is the kind of the motion potential? If yes, then find the velocity potential.
10. The velocity components of an incompressible fluid in spherical polar coordinates (r, θ, ψ) are $(2Mr^{-3}\cos\theta, Mr^{-2}\sin\theta, 0)$, where M is a constant. Show that the velocity is

of the potential kind. Find the velocity potential and the equations of the streamlines. (10, 2022)

Equation of Motion

1. An infinite mass of fluid is acted on by a force $\frac{\mu}{r^{\frac{3}{2}}}$ per unit mass directed to the origin. If initially the fluid is at rest and there is a cavity in the form of the sphere $r = C$ in it, show that the cavity will be filled up after an interval of time $\left(\frac{2}{5\mu}\right)^{\frac{1}{2}} C^{\frac{5}{4}}$.
2. A stream is rushing form a boiler through a conical pipe, the diameters of the ends of which are D *and* d. If V *and* v be the corresponding velocities of the stream and if the motion is assumed to steady and diverging from the vertex of the cone, then prove that $\frac{v}{V} = \frac{D^2}{d^2} e^{\frac{u^2-v^2}{2K}}$ where K is the pressure divided by the density and is constant.
3. A sphere of radius R, whose centre is at rest, vibrates radially in an infinite incompressible fluid of density ρ, which is at rest at infinity. If the pressure at infinity is Π, so that the pressure at time t is $\Pi + \frac{1}{2}\rho\left\{\frac{d^2R^2}{dt^2} + \left(\frac{dR}{dt}\right)^2\right\}$

Sources and Sinks

1. Let the fluid fills the region $x \geq 0$ (right half of 2d plane). Let a source α be $(0, y_1)$ and equal sink at $(0, y_2)$, $y_1 > y_2$. Let the pressure be same as pressure at infinity i.e., P_0. Show that the resultant pressure on the boundary (y axis) is $\frac{\pi\rho\alpha^2(y_1-y_2)^2}{2\, y_1\, y_2\, (y_1+y_2)}$, ρ being the density of the fluid.
2. Two sources, each of strength m are placed at the point $(-a, 0), (a, 0)$ and a sink of strength 2m is at the origin. Show that the stream lines are the curves $(x^2 + y^2)^2 = a^2(x^2 - y^2 + \lambda xy)$ where λ is a variable parameter. Show also that the fluid speed at any point is $\frac{(2ma^2)}{(r_1 r_2 r_3)}$ *where* r_1, r_2 *and* r_3 are the distance of the points from the source and the sink.
3. A simple source of strength m is fixed at the origin O in a uniform stream of incompressible fluid moving with velocity $U\vec{\imath}$. Show that the velocity potential ϕ at any point P of the stream is $\frac{m}{r} - Ur\cos\theta$ *where* $OP = r$ and θ is the angle which OP makes with the direction $\vec{\imath}$. Find the differential equation of the streamlines and show that they lie on the surfaces $Ur^2\sin^2\theta - 2m\cos\theta = constant$.
4. What arrangement of sources and sinks can have the velocity potential $w = \ln\left(z - \frac{a^2}{z}\right)$? Draw the corresponding sketch of the streamlines and prove that the two of them subdivide into the circle $r = a$ and the axis of y.

5. Show that for the complex potential $\tan^{-1} z$, the streamlines and equipotential curves are circles. Find the velocity at any point and check the singularity at $z = \pm i$.
6. Two sources of strength $m/2$ are placed at $(\pm a, 0)$. Show that at any point on the circle $x^2 + y^2 = a^2$, the velocity is parallel to y-axis and is inversely proportional to y.

Vortex Motion

1. In an incompressible fluid the vorticity at every point is constant in magnitude and direction; show that the components of velocity u, v, w are solution of Laplace's equation.
2. When a pair of equal and opposite rectilinear vortices is situated in a long circle cylinder at equal distance from its axis, show that the path of each vortex is given by the equation $(r^2 \sin^2\theta - b^2)(r^2 - a^2)^2 = 4a^2b^2r^2\sin^2\theta, \theta$ being measured from the line through the centre perpendicular to the joint of the vortices.
3. An infinite row of the equidistant rectilinear vortices are at distance $'a'$ apart. The vortices are of the same numerical strength K but they are alternately of opposite signs. Find the Complex function that determines the velocity potential and the stream function.
4. Prove that the necessary and sufficient conditions that the vortex lines may be at right angles to the stream lines are $u, v, m = \mu\left(\frac{\partial\phi}{\partial x}, \frac{\partial\phi}{\partial y}, \frac{\partial\phi}{\partial z}\right)$ where μ *and* ϕ are function of x, y, z, t .
5. If n rectilinear vortices of the same strength K are symmetrically arranged as generators of a circular cylinder of radius a in an infinite liquid, prove that the vortices will move round the cylinder uniformly in time $\frac{8\pi^2 a^3}{(n-1)K}$. Find the velocity at any point of the liquid.
6. Does a fluid with velocity $\vec{q} = \left[z - \frac{2x}{r}, 2y - 3z - \frac{2y}{r}, x - 3y - \frac{2z}{r}\right]$ possess vorticity, where $\vec{q}(u, v, w)$ is the velocity in the Cartesian frame $\vec{r}(x, y, z)$ and $r^2 = x^2 + y^2 + z^2$? What is the circulation in the circle $x^2 + y^2 = 9, z = 0$?
7. Two point vortices each of strength k are situated at $(\pm a, 0)$ and a point vortex of strength $-\frac{k}{2}$ is situated at the origin. Show that the fluid motion is stationary and also find the equations of streamlines. If the streamlines, which pass through the stagnation points, meet the $x - axis$ at $(\pm b, 0)$, then show that $3\sqrt{3}\,(b^2 - a^2)^2 = 16a^3b$. (20, 2022)
8. Verify that $w = ik\log\left\{\frac{z-ia}{z+ia}\right\}$ is the complex potential of a steady flow of fluid about a circular cylinder, where the plane $y = 0$ is a rigid boundary. Find also the force exerted by the fluid on unit length of the cylinder. (20, 2022)

Motion of Cylinder and Sphere

1. A rigid sphere of radius a is placed in a stream of fluid whose velocity in the undisturbed state is V. Determine the velocity of the fluid at any point of the disturbed stream.
2. Consider a uniform flow U_0 in the positive x- direction. A cylinder of radius a is located at the origin. Find the stream function and the velocity potential. Find also the stagnation points.
3. The space between two concentric spherical shells of radii a, b $(a < b)$ is filled with a liquid of density ρ. If the shells are set in motion, the inner one with velocity U in the x-direction and the outer one with velocity V in the y-direction, then show that the initial motion of the liquid is given by velocity potential $\phi = \frac{\left\{a^3 U\left(1+\frac{1}{2} b^3 r^{-3} x\right) - b^3 V\left(1+\frac{1}{2} a^3 r^{-3} y\right)\right\}}{b^3 - a^3}$ *where* $r^2 = x^2 + y^2 + z^2$, the coordinate being rectangular. Evaluate the velocity at any point of the liquid.

Navier-Stokes equation

1. Find Navier-Stokes equation for steady laminar flow of a viscous incompressible fluid between two infinite parallel plates.