NEXT IAS

MATHEMATICS OPTIONAL

ORDINARY DIFFERENTIAL EQUATIONS (ASSIGNMENT)

By
Avinash Singh
(Ex IES, B.Tech IIT Roorkee)

DELHI CENTRE
27-B, Pusa Road, Metro Pillar No. 118,
Near Karol Bagh Metro, New Delhi-110060

BHOPAL CENTRE
Plot No. 46 Zone - 2, M.P Nagar,
Bhopal - 462011

Phone: 8081300200, 8827664612 | E-mail : info@nextias.com | Web: www.nextias.com

Tutorial Sheet - I

(1) **Solve :** $y - x\left(\frac{dy}{dx}\right) = a\left(y^2 + \frac{dy}{dx}\right) . \left[(x-a)(1-ay) = cy\right]$

(2) **Solve :** $\sqrt{(1+x^2+y^2+x^2y^2)} + xy\left(\frac{dy}{dx}\right) = 0$

Ans. $\log x - \log\left\{1+(1+x^2)^{1/2}\right\} + (1+x^2)^{1/2} + (1+y^2)^{1/2} = c$

(3) $(x+y)(dx-dy) = dx+dy$; $\left[x-y+c = \log(x+y)\right]$

(4) Solve $(x+2y-1)\, dx = (x+2y+1)dy$

Ans. $3x+6y-1 = ce^{3(x-y)/2}$

(5) $x\cos\left(\frac{y}{x}\right)(ydx+xdy) = y\sin\left(\frac{y}{x}\right)(xdy-ydx)$

Ans. $xy\cos\left(\frac{y}{x}\right) = c$

(6) $(4y+3x)dy + (y-2x)dx = 0$

Ans. $c(x^2-2xy-2y^2) = \left\{\frac{(\sqrt{3}+1)x+2y}{(\sqrt{3}-1)x-2y}\right\}^{1/2\sqrt{3}}$

(7) $(x^2-4xy-2y^2)dx + (y^2-4xy-2x^2)dy = 0$

Ans. $y^3 - 6xy^2 - 6x^2y + x^3 = c$

(8) $\frac{dy}{dx} = (x+2y-3)/(2x+y-3)$

(9) $dy/dx = (x+y+4)/(x-y-6)$

Ans: $(x-1)^2 + (y+5)^2 = ce^2\tan^{-1}\left\{\frac{(y+5)}{(x-1)}\right\}$

(10) $(2x^2+3y^2-7)\,xdx - (3x^2+2y^2-8)\,ydy = 0$

Ans: $(x^2-y^2-1)^5 = c(x^2+y^2-3)$

(11) $(x+y)^2dx - (y^2-2xy-x^2)dy = 0$

Ans: $x^3 - y^3 + 3xy(x+y) = c$

(12) Solve $(1+e^{x/y})dx + e^{x/y}\left\{1-\left(\frac{x}{y}\right)\right\}dy = 0$

Ans: $x + ye^{x/y} = c$

(13) Solve $xdx + ydy + \frac{xdy - ydx}{x^2 + y^2} = 0$

Ans: $x^2 + y^2 - 2\tan^{-1}(x / y) = c$

(14) $y\sin 2xdx - (1 + y^2 + \cos^2 x)dy = 0$

Ans: $3y\cos 2x + 6y + 2y^3 = c$

(15) Show $(4x+3y+1)dx+(3x+2y+1)dy=0$ is a family of hyperbolas with a common axis and tangent at the vertex.

(16) Find the values of constant λ such that $(2xe^y+3y^2)\ (dy/dx)\ +(3x^2+\lambda e^y)=0$ is exact. further for this value of λ, Solve the equation.

Ans: λ= 2 , $x^3 + 2e^x + y^3 = c$

(17) **Solve:** $(x^3y^3 + x^2y^2 + xy + 1)ydx + (x^3y^3 - x^2y^2 - xy + 1)xdy = 0$

Ans: $xy^{-}\left({}^{1}\!/_{xy} \right) - 2\log y = c$

(18) Solve $(2ydx + 3xdy) + 2xy(3ydx + 4xdy) = 0$

Ans: $x^2y^3+2x^3y^4=c$

(19) $(2x^2y^2 + y)dx - (x^3y - 3x)dy = 0$

Ans: $4x^{10/7}\,y^{-5/7} - 5x^{-4/7}\,y^{-12/7} = c$

(20) Sin$x$ $\left({}^{dy}\!/_{dx} \right)$ $+3y=cosx$

Ans: $\left(y + \frac{1}{3} \right)\ \tan^3\left({}^{x}\!/_{2} \right) = 2\tan\left({}^{x}\!/_{2} \right) - x + c$

(21) $(x + 2y^3)\left({}^{dy}\!/_{dx} \right) = y$

Ans: ${}^{x}\!/_{y} = y^2 + c$

(22) $(1 + y^2)dx = (\tan^{-1} y - x)dy$

Ans: $x = \tan^{-1} y - 1 + ce^{\tan^{-1} y}$

(23) ${}^{dy}\!/_{dx} + y\cos x = \left({}^{1}\!/_{2} \right)\sin 2x$

Ans: $y = ce^{-\sin x} + \sin x - 1$

(24) Solve $\frac{dz}{dx}+\frac{z}{x}\log z=\frac{z}{x^2}(\log z)^2$

Ans: $\frac{1}{x(\log z)}=\frac{1}{2x^2}+c$

(25) $(x^2-2x+2y^2)dx+2xydy=0$

Ans: $y^2x^2=\frac{2x^3}{3}-\frac{x^4}{4}+c$

(26) $(xy^2+e^{-1/x^3})dx-x^2ydy=0$

Ans: $y^2/x^2=\left(2/3\right)e^{-1/x^3}+c$

(27) Solve $x(dy/dx)+y=y^2\log x$

Ans: $\frac{1}{y}=\log x+1+cx$

(28) $(x^3y^2+xy)dx=dy$

Ans: $y^{-1}=(2-x^2)+ce^{-\frac{x^2}{2}}$

(29) $(x^2y^3+xy)\left(dy/dx\right)=1$

Ans: $\frac{1}{x}=2-y^2+ce^{-\frac{y^2}{2}}$

(30) Find the curve for which the position of y axis cut off between the origin and tangent varies as the cube of the abscissa of the point of contact.

Ans: $2y=-kx^3+cx$

(31) Find the Cartesian equation of the curve in which the perpendicular from the foot of the ordinate on the tangent is of constant length.

Ans: $y = k\cosh\{(x+c)/k\}$

(32) Find the family of curves whose tangent form an angle $\pi/4$ will the hyperbola $xy=c$

Ans: $y=x-2\sqrt{c}\tan^{-1}\left(x/\sqrt{c}\right)+c'$

(33) Show that the Curve in which the angle between the tangent and the radius vector at any point is half of the vectorial angle is a cardioid.

Ans: $kr - 1 = rce^{\theta}$

(34) If the population of country doubles in 50 years in how many years will treble under the assumption that rate of increase is proportional to the number of inhabitants.

Ans: 78.25 yrs

(35) A metal bar at temperature of 100°F is placed at a constant temp of 0°F. If after 20 Minutes the temperature of the bar is half , find an expression for the temperature of the bar at any time.

Ans: $T = 100\ e^{(-0.035)t}$

(36) Show that the only curves having constant curvatures are circles and straight lines.

(37) Find the curve for which sum of the reciprocal of the radius vector and the polar subtangent is costant.

Tutorial Sheet - II

(38) Find the orthogonal trajectories of the family of curves $3xy=x^3-a^3$ a being parameter of the family.

Ans. $x^2 = y - \left(\frac{1}{2}\right)e^{2y} + c$

39) Find the orthogonal trajectories of $x^2+y^2=2ax$.

Ans. $x^2 + y^2 = cy$, c being parameter.

(40) Find the orthogonal trajectories of the family of circles $x^2 + y^2 + 2fy + 1 = 0$, where f is parameter

Ans) $x^2 + y^2 + 2gx - 1 = 0$

(41) Find the orthogonal trajectories of family of parabolas $y^2 = 4a(x + a)$, where a is parameter.

Ans. $y = 2x\left(\frac{dy}{dx}\right) + y\left(\frac{dy}{dx}\right)^2$

(42) Find the orthogonal trajectories of the family of curves $\frac{x^2}{(a^2 + \lambda)} + \frac{y^2}{(b^2 + \lambda)} = 1$, where λ is a parameter.

(Ans) $\{x + y\left(\frac{dy}{dx}\right)\}\left\{x - y\left(\frac{dx}{dy}\right)\right\} = a^2 - b^2$

(43) Find the orthogonal trajectories of r=a(1+cosnφ)

Ans. $r^{n^2} = b(1 - \cos n\varphi)$

(44) Find the equation of the family of oblique trajectories which cut the family of concentric circles at 30°.

(45) $p^3 + 2xp^2 - y^2p^2 - 2xy^2p = 0$

Ans. $(y - c)(y + x^2 - c)[x + \left(\frac{1}{y}\right) - c] = 0$

(46) $p(p + y) = x(x + y)$

(47) $x^2p^2 - 2xyp + 2y^2 - x^2$

Ans. $x = ce^{\pm\sin^{-1}\left(\frac{y}{x}\right)}$

(48) $p^2 + 2py\cot x = y^2$

Ans. $(y - c\sec^2 x/2)\left(y - c\operatorname{cosec}^2 x/2\right) = 0$

(49) $x^2p^2 - 2xyp + y^2 - x^2y^2 - x^4 = 0$

Ans. $\begin{Bmatrix} y - x\sinh(c+x) \\ y - x\sinh(c-x) \end{Bmatrix} = 0$

(50) $y = 2px + p^2y$

Ans. $2xc - y^2 + c = 0$

(51) $y^2 \log y = xpy + p^2$

Ans. $\log y = cx + c^2$

(52) $x = y + a \log p$

Ans. $\begin{Bmatrix} y = c - a\log(1-p) \\ x = c - a\log(1-p) + a\log p \end{Bmatrix}$

(53) $x = 4(p + p^3)$

Ans. $\begin{Bmatrix} y = 2p^2 + 3p^4 + c \\ x = 4(p + p^3) \end{Bmatrix}$

(54) $y + px = x^4p2$

Ans. $xy + c = c^2x$

(55) $y = yp^2 + 2px$

Ans. $c^2y^2 = 4(1 + x)$

(56) $y = 2\,px + f(xp^2)$

Ans. $y = 2cx^{1/2} + f(c^2)$

(57) $y = 2\,px - p^2$

Ans. $x = (2/3)p + cp^{-2}$, $y = (1/3)p^2 + 2cp^{-1}$

(58) $y = 3px + 4p^2$

Ans. $x = -(8/5)p + cp^{-3/2}$

$y = 3cp^{-1/2} - (4/5)p^2$

(59) $p = \tan(px - y)$

(60) $\sin px \cos y = \cos px.\sin y + p$

(61) Solve $p^2x(x-2) + p(2y - 2xy - x + 2) + y^2 + y = 0$

(62) $x^2(y - px) = yp^2$

(63) $(px - y)(py + x) = h^2p$

(64) $y^2(y - xp) = x^4p^2 \ (x = \frac{1}{u}, y = \frac{1}{v})$

(65) $y = 2px + y^2p^3$

(66) $xp^2 - 2yp + x + 2y = 0. \ (y - x = v \ \& \ x^2 = u)$

(67) $(px^2 + y^2)(px + y) = (p + 1)^2$

(68) $(x^2 + y^2)(1 + p)^2 - 2(x + y)(1 + p)(x + yp) + (x + yp)^2 = 0$

Tutorial Sheet - III

69. Find the differential equation of the family of circles $x^2+y^2+2cx+2c^2-1=0$ (c is arbitrary) Determine singular solution of the differential equation.

Ans. $2y^2p^2+2xyp+x^2+y^2+1=0, x^2+2y^2-2=0$

70. Find the solution of the differential equation $y=2xp-yp^2$ where $p=\dfrac{dy}{dx}$ also find the singular solution.

Ans. $y^2=2cx-c^2;\ \begin{matrix} x-y=0 \\ x+y=0 \end{matrix}$

71. Find general and singular solutions of $3xy=2px^2-2p^2 or\ y=(2x/3)p-(2/3x)p^2$

Ans. $(3y+2c)^2=4cx^3, x^3-6y=0$

72. Solve the differential equation $y=x-2ap+ap^2$. Find the singular solution and interpret it geometrically.

Ans. $4a(y-c)=x^2+c^2-2xc;\quad y-x+a=0$

73. Find the complete primitive (general solution) and singular solution of $(xp-y)^2=p^2-1$.

Ans. $x^2-y^2=1$

74. Reduce the equation $xyp^2-p(x^2+y^2-1)+xy=0$ to clairaut's form by the substitutions $x^2=u$ and $y^2=v$. Hence, show that the equation represents a family of conics touching the four sides of a square.

Ans. $y^2=cx^2-\dfrac{c}{c-1}$

$x+y+1=0, x+y-1=0, x-y+1=0, x-y-1=0$

75. Solve $(D^2+a^2)y=\cot ax$

Ans. $c_1\cos ax+c_2\sin ax+\left(\dfrac{1}{a^2}\right)\sin ax\log\tan\left(\dfrac{ax}{2}\right)$

76. $(4D^2+12D+9)y=144e^{-3x}$

77. $(9D^2-12D+4)y=e^{\frac{2x}{3}}$

Ans. $(c_1+c_2x)e^{\frac{2x}{3}}+(\dfrac{1}{18})x^2e^{\frac{2x}{3}}$

78. $(D-1)(D^2-2D+2)y=e^x$

Ans. $e^x(c_1+c_2\cos x+c_3\sin x+x)$

79. $(D^4 - 2D^3 + 5D^2 - 8D + 4)y = e^x$

Ans. P.I. = $x^2 e^{\frac{x}{10}}$

80. $(D^2 - 6D + 8)y = (e^{2x} + 1)^2$

Ans. $c_1 e^{2x} + c_2 e^{4x} + \left(\frac{1}{8}\right)(4xc^{4x} - 8xe^{2x} + 1)$

81. $\left(\frac{d^4 y}{dx^4}\right) - m^4 y = \sin mx$

82. Find the solution of $\left(\frac{d^2 y}{dx^2}\right) + 4y = 8\cos 2x$, given that $y = 0$ and $\frac{dy}{dx} = 0$, when $x = 0$

83. $(D^3 + 3D^2 + 2D)y = x^2$

84. $(D^3 - D^2 - 6D)y = x^2 + 1$

Ans. $c_1 + c_2\, e^{3x} + c_3\, e^{-2x} - (1/18)(x^3 - x^2/2 + 25x/6)$

85. $(D^2 - 6D + 9)y = x^2 e^{3x}$

Ans. $(c_1 + c_2 x)e^{3x} + (1/12)x^4 e^{3x}$

86. Solve $(D^3 - 3D - 2)y = 540x^3 e^{-x}$

87. $(D^2 - 1)y = \cosh x . \cos x$

Ans. $c_1 e^x + c_2 e^{-x} + (2/5)\sin x \sinh x - (2/5)\cos x \cosh x$.

88. $(D^4 + D^2 + 1)y = e^{-x/2}\cos(x\sqrt{3}/2)$

89. Solve $(D^2 - 4D + 4)y = 8x^2 e^{2x}\sin 2x$

Ans. $(c_1 + c_2 x)e^{2x} + e^{2x}(3\sin 2x - 4x\cos 2x - 2x^2 \sin 2x)$

90. $d^2y/dx^2 - 5\,dy/dx + 6y = e^{4x}(x^2 + 9)$

Ans. $c_1 e^{2x} + c_2 e^{3x} + (1/4)e^{4x}(2x^2 - 6x + 25)$

91. $(D^4 + 2D^2 + 1)y = x^2 \cos x$

Ans. $(c_1 + c_2 x)\cos x + (c_3 + c_4 x)\sin x - (1/4)[(x^4/12 - \frac{3x^2}{4})\cos x - (x^3/3)\sin x]$

92. Solve $(D^2 + 1)^2 = 24x\cos x$ given $y = Dy = D^2 y = 0$ and $D^3 y = 12$. When $x = 0$'.

Ans. y = $c_1x^2 + c_2x^{(5+\sqrt{21})/2} + x^{(5-\sqrt{21})/2} - x^3/5$

97. Find the values of λ for which all solution of $x^2\left(d^2y/dx^2\right) - 3x\left(dy/dx\right) - \lambda y = 0$ tends to zero as $x \to \infty$

Ans. $-1 \le \lambda < 0$

98. $(x^2D^2 + 4xD + 2)y = e^x$

Ans. $y = c_1x^{-2} + c_2x^{-1} + x^{-2}e^x$

99. Solve $(x^3D^3 + 2x^2D^2 + 2)y = 10(x + 1/x)$

Ans. $c_1x^{-1} + x[c_2\cos\log x + c_3\sin\log x] + 5x + 2x^{-1}\log x$.

100. $\left\{(x+1)^4D^3 + 2(x+1)^3D^2 - (x+1)^2D + (x+1)\right\}y = \dfrac{1}{(x+1)}$

Ans. $y = \left\{c_1 + c_2\log(1+x)\right\}(1+x) + c_3(1+x)^{-1} - \left(\dfrac{1}{9}\right)(1+x)^{-2}$

101. $(1+2x)^2\dfrac{2d^2y}{dx^2} - 6(1+2x)\dfrac{dy}{dx} + 16y = 8(1+2x)^2$

Ans. $y(x) = (1+2x)^2\log(1+2x)[1+\log(1+2x)]$

102. $(3-x)y'' - (9-4x)y' + (6-3x)y = 0$

Ans. $c_2e^x + (1/8)c_1e^{3x}(4x^3 - 42x^2 + 150x - 183)$

103. Solve: $x^2y'' + xy' - y = 0$, given that $(x + 1/x)$ is one integral by using the method of reduction of order.

Ans. $y = c_2(x + 1/x) + c_1'(1/x)$

104. $\sin^2 x\left(d^2y/dx^2\right) = 2y$,given that $y = \cot x$ is a solution.

Ans. $\cot x[c_2] + c_1(1 - x\cot x)$

105. $\left(d^2y/dx^2\right) - (1+x)\,dy/dx + xy = x$

Ans. $c_1e^x\int e^{-x-x^2/2}\,dx + c_2e^x + 1$ (Left in integral form).

106. Make use of transformation $y(x)=V(x)\sec x$ to obtain the solution of $y''-2y'\tan x+5y=0$, y(0)=0, y′(0)=$\sqrt{6}$

Ans. $y=\sec x\,\sin\left(x\sqrt{6}\right)$

107. Solve $y''-4xy'+(4x^2-1)y=-3e^{x^2}\sin 2x$

Ans. $y=e^{x^2}(c_1\cos x+c_2\sin x+\sin 2x)$

108. Solve $y''+2xy'+(x^2+1)y=x^3+3x$

Ans. $x+e^{-x^2/2}(c_1x+c_2)$

109. $\dfrac{d^2y}{dx^2}+\dfrac{L}{x^{1/3}}\dfrac{dy}{dx}+\left(\dfrac{1}{4x^{2/3}}-\dfrac{1}{6x^{4/3}}-\dfrac{6}{x^2}\right)y=0$

Ans. $y=e^{\left(-\frac{3}{4}\right)x^{2/3}}(c_1x^3+c_2x^{-2})$

110. Solve $y''+(4\operatorname{cosec} 2x)y'+(2\tan^2 x)y=e^x\cot x$ by changing the dependent variable.

Ans. $\cot x(c_1e^{x\sqrt{2}}+c_2e^{-x\sqrt{2}}-e^x)$

111. Solve $\sin^2 xy''+\sin x\cos x\,y'+4y=0$

Ans. $c_1\cos\left\{2\log\tan\left(\dfrac{x}{2}\right)\right\}+c_2\sin\left(2\log\tan\dfrac{x}{2}\right)$

112. Solve $(1+x^2)^2y''+2x(1+x^2)y'+4y=0$

Ans. $(1+x^2)y=c_1(1-x^2)+2c_2x$

113. Solve $xy''-y'+4x^3y=x^5$

Ans. $c_1\cos x^2+c_2\sin x^2+x^2/4$

114. $\cos x\,y''+y'\sin x-2y\cos^3 x=2\cos^5 x$

Ans. $c_1e^{\sqrt{2}\sin x}+c_2e^{-\sqrt{2}\sin x}+\sin^2 x$

115. If $y=y_1(x)$ and $y=y_2(x)$ are the two solutions of the equation $\dfrac{d^2y}{dx^2}+p(x)\left(\dfrac{dy}{dx}\right)+\phi(x)y=0$ where $p(x),Q(x)$ are continuous function of x, Prove that $y_1 \frac{dy_2}{dx} - y_2 \frac{dy_1}{dx} = ce^{-\int p dx}$, c arbitrary cont.

116. Apply the method of variation of parameters to solve the equation

$(x+2)y_2 - (2x+5)y_1 + 2y = (x+1)e^x$

Ans. $c_1(2x+5) + c_2 e^{2x} - e^x$

Tutorial Sheet - IV

117. Solve by the method of variation of parameters $\frac{d^2y}{dx^2}+(1+\cot x)\frac{dy}{dx}-y\cot x=\sin^2 x.$

Ans. $y=c_1(\sin x-\cos x)+c_2e^{-x}-(1/10)(\sin 2x-2\cos 2x)$

118. Solve by the method of variation of parameters $x^2y''-2x(1+x)y'+2(x+1)y=x^3$

Ans. $y=c_1x+c_2xe^{2x}-\left(x^2/2\right)-\left(x/4\right)$

119. $y_2+n^2y=\sec nx$ (use of vop)

Ans. $y=c_1\cos nx+c_2\sin nx+\left(1/n^2\right)\cos nx\log\cos nx+(x/n)\sin nx$

Laplace Transform

120. Find out the Laplace transform of

(a) $\cos^2 at$

(b) $\sin^3 2t$

(c) $\dfrac{1}{\sqrt{\pi t}}$

(d) $\begin{cases} \sin t, 0 < t < \pi \\ 0 \qquad t > \pi \end{cases}$

(e) $f(x) = \begin{cases} x/a\,, 0 < x < a \\ 1 \quad , \quad x > a \end{cases}$

(f) $f(x) = \begin{cases} 0 & 0 < t < 1 \\ t & 1 < t < 2 \\ 0 & t > 2 \end{cases}$

(g) Find $L.\{\sin\sqrt{t}$

(h) $L.\left\{\dfrac{\cos\sqrt{t}}{\sqrt{t}}\right\} = \sqrt{\dfrac{\pi}{5}}\, e^{-1/4s}$

Ans. $\dfrac{\sqrt{\pi\, e^{-\frac{1}{4s}}}}{2\, s^{3/2}}$

(i) $(t+3)^2 e^t$

Ans. $\dfrac{9s^2 - 12s + s}{(s-1)^3}$

(j) Show $L\{(1+te^{-t})^3\} = \dfrac{1}{s} + \dfrac{3}{(s+1)^2} + \dfrac{6}{(s+2)^3} + \dfrac{6}{(s+3)^4}$

$$G(t) = \begin{cases} \sin\left(t - \dfrac{\pi}{3}\right) & t > \dfrac{\pi}{3} \\ 0 & t < \dfrac{\pi}{3} \end{cases}$$

(l) If $L\left\{2\sqrt{\dfrac{t}{\pi}}\right\} = \dfrac{1}{s^{3/2}}$, show that $\dfrac{1}{s^{1/2}} = L\left\{\dfrac{1}{\sqrt{\pi t}}\right\}$.

(m) Find L f (t), if $f''(t) + 3f'(t) + 2f(t) = 0\,, f(0) = 1, f'(0) = 2$.

(n) $L(t\sin at), L((\sin 2t/t)$

(o) Find the replace transform of (sin at)/t. Does the replace transform of (cos at)/t exist ?

(p) Prove that $L\left\{\dfrac{\cos at - \cos bt}{t}\right\} = \dfrac{1}{2}\log\dfrac{s^2 + b^2}{s^2 + a^2}$

(q) $\int_0^\infty \dfrac{\sin^2 t}{t^2}\, dt = \dfrac{\pi}{2}$

(r) Evaluation $L\left[{}_0\int^t \frac{\sin x}{x}dx\right]$

Ans. $\frac{\cot^{-1}s}{s}$

(s) Evaluate L(f(t), if f(t) has a period 2π and is given by $f(t)=\begin{cases}\sin t & 0<t<\pi\\ 0 & \pi<t<2\pi\end{cases}$

Ans. $\frac{1}{1+s^2}(1-e^{-s\pi})$

(t) Find the replace of square wave given by

$f(t)=\begin{cases}E & 0\le t\le T/2\\ -E & T/2\le t\le T\end{cases}$ and $f(t+T)=f(t)$

Ans. $\frac{E}{S}\tanh\frac{ST}{4}$

(u) ${}_0\int^{\infty} te^{-2t}\cos t\,dt = 3/25$

(v) ${}_0\int^{\infty} e^{-t}\frac{\sin t}{t}dt = \pi/4$

(w) To Prove that $L\left\{erf(\sqrt{t})\right\}=\frac{1}{s\sqrt{s+1}}$

121. Evaluate L^{-1} of.

(a) $\frac{6}{2S-3}-\frac{3+4S}{9S^2-16}+\frac{8-6S}{16S^2+9}$

Ans. $3e^{3t/2}-\frac{1}{4}\sinh\frac{4t}{3}-\frac{4}{9}\cosh\frac{4t}{3}+\frac{2}{3}\sin\frac{3t}{4}-\frac{3}{8}\cos\frac{3t}{4}$

(b) $\frac{1}{s}\cos\frac{1}{s}=1-\frac{t^2}{(2!)^2}+\frac{t^4}{(4!)^2}-\frac{t^6}{(6!)^2}+....$

(c) $\int\limits_0^{\infty}\cos x^2dx=\frac{1}{2}(\pi/2)^{1/2}$

(d) $\int\limits_0^{\infty}e^{-x^2}dx=\frac{\sqrt{\pi}}{2}$

(e) $L^{-1}\left\{\frac{2s^2-6s+5}{s^3-6s+11s-6}\right\}$ = $e^{-t}+2e^{-2t}+3e^{-3t}$

(f) $L^{-1}\dfrac{s}{(s^2+a^2)(s^2+b^2)}=\dfrac{1}{a^2-b^2}(\cos bt-\cos at)$

(g) $\dfrac{s-a}{(s-a)^2+b^2}$ (h) $\dfrac{1}{s^2(s+1)^2}$

(i) $L^{-1}\left\{\dfrac{s}{s^4+4a^4}\right\}=\dfrac{1}{2a^2}$ *sinh* at . sin at

(j) $L^{-1}\left[\dfrac{1}{(s-1)^5(s+2)}\right]=e^t\left[\dfrac{t^4}{72}-\dfrac{t^3}{54}+\dfrac{t^2}{54}-\dfrac{t}{81}+\dfrac{1}{243}-\dfrac{1}{243}e^{-2t}\right]$

(k) $L^{-1}\left\{\dfrac{e^{4-3s}}{(s+4)^{5/2}}\right\}$

(l) $L^{-1}\left\{\dfrac{se^{-\frac{2\pi s}{3}}}{s^2+9}\right\}$

(m) If $L^{-1}\left\{\dfrac{e^{-1/s}}{s^{1/2}}\right\}=\dfrac{\cos 2\sqrt{t}}{\sqrt{\pi t}}$

Find $L^{-1}\left\{\dfrac{e^{-a/s}}{s^{1/2}}\right\}=\left(\dfrac{\cos 2\sqrt{ta}}{\sqrt{\pi t}}\right)$

(m) $L^{-1}\left\{\dfrac{s}{(s^2+a^2)^2}\right\}$

(n) $L^{-1}\left\{\log\left(\dfrac{1+s}{s}\right)\right\}$ **Ans.** $\dfrac{1-e^{-t}}{t}$

(o) $L^{-1}\left\{\log\left(1-\dfrac{1}{s^2}\right)\right\}$ **Ans.** $\dfrac{2(1-\cosh t)}{t}$

(p) $L^{-1}\left\{\tan^{-1}(1/s)\right\}$ **Ans.** $\left(\dfrac{1}{t}\right)\sin t$

(q) $L^{-1}\left\{\dfrac{1}{s}\log\dfrac{s+2}{s+1}\right\}$

122. Apply convolution theorem to show that ${}_0\int^t \sin u.\cos(t-u)du=\dfrac{1}{2}t\sin t$

123. $L^{-1}\left\{\frac{s}{(s^2+2s)^2}\right\}$

124. Solve $y''(t)+y(t)=t$ with $y'(0)=1, y(\pi)=0$

Ans. $y=\pi\cos t+t$

125. Solve $(D^2+n^2)y=a\sin(nt+\alpha)$ if $y=Dy=0$ when t=0

Ans. $\frac{a}{2n^2}[\cos\alpha\sin nt-nt\cos(\alpha+nt)]$

126. Solve $[tD^2+(1-2t)D-2]y=0, y(0)=1, y'(0)=2$

Ans. $y=e^{2t}$

127. Solve $y''-ty'+y=1$, if $y(0)=1, y'(0)=2$

Ans. $y(t)=1+2t$