Tutorial Sheet: Statics II-Virtual Work

1. Five weightless rods of equal lengths are joined together so as to form a rhombus ABCD with one diagonal BD. If a weight W be attached to C and the system be suspended from A, show that there is a thrust in BD equal to $W/\sqrt{3}$.

2. Four rods of equal weight w form a rhombus ABCD, with smooth hinges at the joints. This frame is suspended by the point A, and a weight W is attached to C. A stiffening rod of negligible weight joins the middle points of AB and AD, keeping these inclined at α to AC. Show that the thrust in this stiffening rod is $(2W + 4w)\tan\alpha$.

3. Four equal uniform rods, each of weight W, are joined to form rhombus ABCD, which is placed in a vertical plane with AC vertical and A resting on a horizontal plane. The rhombus is kept in the position in which $\angle BAC = \theta$ by a light string joining B and D. Find the tension of the string.

4. A string of length a forms the shorter diagonal of a rhombus formed of four uniform rods, each of length b and weight W, which are hinged together. If one of the rods be supported in a horizontal position. Prove that the tension of the string is $\frac{2W(2b^2-a^2)}{b\sqrt{4b^2-a^2}}$.

5. A regular hexagon ABCDEF consists of six equal rods which are each of weight W and are freely joined together. The two opposite angles C and F are connected by a string, which is horizontal, AB being in contact with horizontal plane. A weight W' is placed at the middle point of DE. Show that the tension of the string is $\frac{3W+W'}{\sqrt{3}}$.

6. A step ladder has a pair of legs which are joined by a hinge at the top, and are connected by a cord attached at one-third of the distance from the lower end of the rod. If the weight of each leg be W_1 and acts at their middle points and if a man of weight of W is two-third the way up the ladder, show by the principle of virtual work, that the tension in the cord is $\frac{1}{2}\left(W + \frac{3}{2}W_1\right)\tan\alpha$, α being the inclination of the each leg to the vertical.

7. A solid hemisphere is supported by a string, fixed to a point on its rim and to a point on a smooth vertical wall with which the curved surface of the hemisphere is in contact. If θ and φ are the inclinations of the string and the plane base of the hemisphere to the vertical, prove that $\tan\varphi = \frac{3}{8} + \tan\theta$.

8. A uniform beam of length of *2a*, rests in equilibrium against a smooth vertical wall and upon a smooth peg at a distance b from the wall. Show that in the position of equilibrium the beam is inclined to the wall at an angle $\sin^{-1}\left(\frac{b}{a}\right)^{\frac{1}{3}}$.

9. A heavy uniform rod of length 2a, rests with its end in contact with two smooth inclined planes, of inclination $\alpha\ and\ \beta$ to the horizon. If θ be the inclination of the rod to the horizon, prove that by principle of virtual work, that $\tan\theta = \frac{1}{2}(\cot\alpha - \cot\beta)$.

10. Two equal rods, *AB* and *AC*, each of length *2b* , are freely joined at A and rest on a smooth vertical circle of radius a. Show that if 2θ be the angle between them, then $b\sin^3\theta = a\cos\theta$.

11. A heavy elastic string, whose natural length is *2πa* is placed round a smooth cone whose axis is vertical and whose semi vertical angle is α. If *W* be the weight and λ the modulus of elasticity of the string, prove that it will be in equilibrium when in the form of circle whose radius is $a(1 + \frac{W}{2\lambda\pi}\cot\alpha)$.

12. A smooth parabolic wire is fixed with its axis vertical and vertex downwards, and in it is placed a uniform rod of length *2l* with its ends resting on the wire. Show that, for equilibrium, the rod is either horizontal, or makes with the horizontal an angle θ given by $\cos^2\theta = \frac{2a}{l}$, *4a* being the latus rectum of the parabola.