Tutorial Sheet: Dynamics-I

1. A particle is moving with a constant velocity parallel to the axis y and a velocity proportional to y parallel to the x- axis, prove that it will describe a parabola.
2. Prove that the angular acceleration of the direction of motion of a point moving in a plane is $\frac{v}{\rho}\frac{dv}{ds} - \frac{v^2}{\rho^2}\frac{d\rho}{ds}$.
3. The line joining two points A, B is of constant length a and the velocities of A and B are in directions which make angles α and β respectively with AB. Prove that the angular velocity of AB is $\frac{u\sin(\alpha-\beta)}{a\cos\beta}$, where u is velocity of A.
4. The velocities of a particle along and perpendicular to a radius vector from a fixed origin are λr^2 and $\mu\theta^2$, where λ and μ are constants; find the polar equation of the path of the particle and also it's radial and transverse acceleration in terms of r and θ only.
5. Show that the path of a point P which possesses two constant velocities u and v, the first of which is in a fixed direction and the other is perpendicular to the radius OP drawn from a fixed O, is a conic, whose focus is O and eccentricity is u/v.
6. A small ring is at rest on a smooth straight horizontal rod of length a at distance b from one end of the rod. The rod is then suddenly set rotating in a horizontal plane about the end O with constant angular velocity ω .Prove that the ring will leave the rod with velocity $\omega\sqrt{2a^2 - b^2}$ after a time $\frac{1}{\omega}\cosh^{-1}\left(\frac{a}{b}\right)$.
7. A small bead slides with constant speed v on a smooth wire in the shape of cardioid $r = a(1 + \cos\theta)$. Show that the angular velocity is $\frac{v}{2a}\sec\frac{\theta}{2}$ and that the radial component of the acceleration is constant.
8. A point describe the cycloid $s = 4a\sin\psi$ with uniform speed v. Find its acceleration at any point.
9. A point moves in a plane curve, so that its tangential and normal accelerations are equal and the angular velocity of the tangent is constant. Find the curve.
10. A particle moves in the curve $y = a\log\sec(x/a)$ in such a way that the tangent to the curve rotates uniformly; prove that the resultant acceleration of the particle varies as the square of the radius of the curvature.

Rectilinear Motion

1. A point moving in a straight line with uniform acceleration describes distances a, b feet in successive intervals of t_1 , t_2 seconds. Prove that the acceleration is $\frac{2(t_1b-t_2a)}{[t_1t_2(t_1+t_2)]}$.
2. For $1/m$ of the distance between two stations a train is uniformly accelerated and for $1/n$ of the distance it is uniformly retarded; it starts from rest at one station and cones to rest at the other. Prove that the ratio of its greatest velocity to average velocity is $1+\frac{1}{m}+\frac{1}{n}:1$.
3. Prove that the shortest time from rest to rest in which a steady load of P tones can lift a weight of W tons through a vertical distance h feet is $\sqrt{\frac{2h}{g}\frac{P}{P-W}}$ sec.
4. A load W is to be raised by a rope from rest to rest, through a height h; the greatest tension which the rope can safely bear is nW. Show that the least time in which the ascent can be made is $\left[\frac{2nh}{(n-1)g}\right]^{\frac{1}{2}}$.
5. A particle is performing a SHM of period T about a centre O and it passes through a point P where OP=b with velocity v in the direction OP; prove that the time which elapses before it turns to P is $\frac{T}{\pi}\tan^{-1}\left(\frac{vT}{2\pi b}\right)$.
6. A point moving in the straight line with SHM has velocities v_1 *and* v_2 when its distances from the centre are x_1 *and* x_2. Show that the period of motion is $2\pi\sqrt{\frac{x_1^2-x_2^2}{v_2^2-v_1^2}}$.
7. A body is attached to one end of an inelastic string, and the other end moves in a vertical line with SHM of amplitude a, making n oscillations per seconds. Show that the string will not remain tight during the motion unless $n^2<\frac{g}{4\pi^2 a}$
8. A particle of mass m is attached to a light wire which is stretched tightly between two fixed points with a tension T. If a, b be the distance of the particle from the two ends, prove that the period of small transverse oscillations of mass m is $2\pi/\sqrt{\left\{\frac{T(a+b)}{mab}\right\}}$.
9. Two light elastic strings are fastened to a particle of mass m and their other ends to fixed points so that the strings are taut. The modulus of each is λ, the tension T, and length a and b. Show that the period of an oscillations along the line of the strings is $2\pi\left[\frac{mab}{(T+\lambda)(a+b)}\right]^{\frac{1}{2}}$.
10. A mass m hangs form a light spring and is given a small vertical displacement. If l is length of the spring when the system is in equilibrium and n the number of oscillations per sec, show that the natural length of the spring is $l-\left(\frac{g}{4\pi^2n^2}\right)$.
11. A light elastic string of natural length l has one extremity fixed at point O and the other attached to a stone, the weight of which in equilibrium would extend the string to a length l_1. Show that if the stone be dropped from rest at O, it will come to instantaneous rest at a depth $\sqrt{l_1^2-l^2}$ below the equilibrium position.

12. A heavy particle is attached to one end of an elastic string, the other end of which is fixed. The modulus of elasticity of the string is equal to the weight of the particle. The string is drawn vertically down till it is four times of its natural length and then let go. Show that the particle will return to this point in time $\sqrt{\left(\frac{a}{g}\right)}\left[\frac{4\pi}{3}+2\sqrt{3}\right]$, where a is natural length of the string.

13. If the earth's attraction varies inversely as the square of the distance from its centre and g be its magnitude at the surface, the time of falling from the height h above the surface to the surface is $\sqrt{\frac{a+h}{2g}}\left[\sqrt{\frac{h}{a}}+\frac{a+h}{a}\sin^{-1}\sqrt{\frac{h}{a+h}}\right]$, where a is the radius of the earth.

14. A particle is projected vertically upwards form the surface of earth with a velocity just sufficient to carry it to the infinity. Prove that the time it takes to reach a height h is $\frac{1}{3}\sqrt{\frac{2a}{g}}\left[\left(1+\frac{h}{a}\right)^{\frac{3}{2}}-1\right]$, where a is the radius of the earth.

15. Assuming that a particle falling freely under gravity can penetrate the earth without facing any resistance, show that a particle falling from rest at a distance b(b>a) from the centre of the earth would on reaching on reaching the centre acquire a velocity $\sqrt{\left[\frac{ga(3b-2a)}{b}\right]}$ and the time to travel form the surface to the centre of the earth is $\sqrt{\frac{a}{g}}\sin^{-1}\sqrt{\frac{b}{3b-2a}}$, where a is the radius of the earth and g is acceleration due to gravity on the earth's surface.

16. A particle whose mass is m is acted upon by a force $m\mu\left[x+\frac{a^4}{x^3}\right]$ towards origin; if it starts from rest at a distance a, show that it will arrive at origin in time $\frac{\pi}{4\sqrt{\mu}}$.

17. A particle starts from rest at a distance b from a fixed point, under the action of a force through the fixed point , the law of which at a distance x is $\mu\left[1-\frac{a}{x}\right]$ towards the point when $x>a$ but $\mu\left[\frac{a^2}{x^2}-\frac{a}{x}\right]$ from the same point when $x<a$; prove that the particle will oscillate through a space $\left[\frac{b^2-a^2}{b}\right]$.

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