Previous Year Questions: Statics
(2008-22)

Equilibrium of System of Particles

1. A solid right circular cone whose height is h and radius of whose base is r, is placed on an inclined plane and it is prevented from sliding. If the inclination θ of the plane (to the horizontal) be gradually increased, find when the cone will topple over. For a cone whose semi- vertical angle is 30°, determine the circular value of θ which when exceeded, the cone will topple over.
2. A ladder of weight 10 kg rests on a smooth horizontal ground leaning against a smooth vertical wall at an inclination $\tan^{-1} 2$ with the horizontal and is prevented from slipping by a string attached at it's lower end and to the junction of the floor and the wall. A body of weight 30 kg begins to ascend the ladder. If the string can bear a tension of 10 kg-wt, how far along the ladder can the boy rise with safety?
3. A uniform rod AB is movable about a hinge at A and rests with one end in contact with a smooth vertical wall. If the rod is inclined at an angle of 30° with the horizontal, find the reaction at the hinge in magnitude and direction.
4. A ladder of weight W rests with one end against a smooth vertical wall and the other end rest on a smooth floor. If the inclination of the ladder to the horizon is 60°, find the horizontal force that they must be applied to the lower end to prevent the ladder from slipping down.
5. Two equal ladders of weight 4kg each are placed so as to lean at A against each other with their end resting on a rough floor, given the coefficient of friction is μ. The ladders at A make an angle 60° with each other. find what weight on the top would cause them to slip.
6. A uniform rod AB of length $2a$ movable about a hinge at A rests with other end against a smooth vertical wall. If α is the inclination of the rod to the vertical, prove that the magnitude of reaction of the hinge is $\frac{1}{2}W\sqrt{4 + \tan^2 \alpha}$ where W is the weight of the rod.
7. A beam AD rests on two supports B and C, where $AB = BC = CD$. It is found that the beam will tilt when a weight of p kg is hung from A or when a weight of q kg is hung from D. Find the weight of the beam.

8. A uniform rod, in vertical position, can turn freely about one of its ends and is pulled aside from the vertical by a horizontal force acting at the other end of the rod and equal to half its weight. At what inclination to the vertical will the rod rest?
9. Two weights P and Q are suspended from a fixed point O by strings OA, OB and are kept apart by a light rod AB. If the strings OA and OB make angles α and β with the rod AB show that the angle θ which the rod makes with the vertical is given by $\tan\theta = \frac{P+Q}{P\cot\alpha - Q\cot\beta}$
10. A square $ABCD$, the length of whose side is a, is fixed in a vertical plane with two of its sides horizontal. An endless string of length $l(> 4a)$ passes over four pegs at the angle of the board and through a ring of weight W which is hanging vertically. Show that the tension of the string is $\frac{W(l-3a)}{2\sqrt{l^2-6la+8a^2}}$.
11. A rod of 8kg is movable in a vertical plane about a hinge at one end, another end is fastened a weight equal to half of the rod, this end is fastened by a string of length l to a point at a height b above the hinge vertically. Obtain the tension in the string.
12. Two rods LM and MN are joined rigidly at the point M such that $(LM)^2 + (MN)^2 = (LN)^2$ and they are hanged freely in equilibrium from a fixed point L. let ω be the weight per unit of length of both the rods which are uniform. Determine the angle, which the rod LM makes with the vertical direction, in terms of lengths of the rods. (10)
13. A heavy string, which is not of uniform density, is hung up from two points. Let T_1, T_2, T_3 be the tensions at the intermediate points A, B, C of the catenary respectively where its inclinations to the horizontal are in arithmetic progressions with common difference β. Let ω_1 *and* ω_2 be the weights of the parts AB and BC of the string respectively. Prove that

    a. Harmonic mean of T_1, T_2 *and* $T_3 = \frac{3T_2}{1+2\cos\beta}$

    b. $\frac{T_1}{T_3} = \frac{\omega_1}{\omega_2}$ (20)

Virtual Work

1. Solid hemisphere is supported by a string fixed to point on its rim and to a point on a smooth vertical wall with which the curved surface of the hemisphere is in contact. If θ and ϕ are the inclination of the string and the plane base of the hemisphere to the vertical, prove by using the principal of virtual work that $\tan\phi = \frac{3}{8} + \tan\theta$
2. Six equal rods AB, BC, CD, DE, EF and FA are each of weight w and are freely jointed at their extremities so as to from a hexagon; the rod AB is fixed in a horizontal position and the middle points of AB and DE are joined by a string. Find the tension in the string.

3. A regular pentagon ABCDE, formed of equal heavy uniform bars jointed together is suspended from the joint A, and is maintained in from by a light road joining the middle points of BC and DE. Find the stress in this rod.
4. Two equal uniform rods AB and AC, each of length l are freely jointed at A and rest on a smooth fixed vertical circle of radius r. If 2θ is the angle between the roads, then find the relation between l, r and θ by using the principal of virtual work.
5. A square framework formed of uniform heavy rods of equal weight W jointed together, is hung up by one corner. A weight W is suspended from each of the three lower corners, and the shape of the square is preserved by a light rod along the horizontal diagonal. Find the thrust of the light rod.
6. A chain of n equal uniform rods is smoothly jointed together and suspended from its one end A_1. A horizontal force $\vec{P}$ is applied to the other end A_{n+1} of the chain. Find the inclinations of the rod to the downward vertical line in the equilibrium configuration. (15, 2022)

Common Catenary

1. The end links of a uniform chain slide along a fixed rough horizontal rod. Prove that the ratio of the maximum span to the length of the chain is $\mu \, log \left[\frac{1+\sqrt{1+\mu^2}}{\mu}\right]$ where μ the coefficient of friction is.
2. Find the length of an endless chain which will hang over a circular pulley of radius a so as to be in contact with two third of the circumference of the pulley.
3. A cable of weight w per unit length and length $2l$ hangs from two points P and Q in the same horizontal line. Show that the span of the cable is $2l\left(1-\frac{2h^2}{3l^2}\right)$, where h is the sag in the middle of the tightly stretched position. (20, 2022)

Stability of Equilibrium

1. A heavy hemispherical shell of radius a has a particle attached a point on the rim, and rests with the curved surface in contact with a rough sphere of radius b at the highest point. Prove that if $\frac{b}{a} > \sqrt{5} - 1$ the equilibrium is stable, whatever be the weight of the particle.
2. A body consists of a cone and underlying hemisphere. The base of the cone and the top of the hemisphere, have same radius a. The whole-body rests on a rough horizontal table with hemisphere in contact with the table. Show that the greatest height of the cone, so that the equilibrium may be stable, is $\sqrt{3}a$.

3. A uniform solid hemisphere rests on a rough plane, inclined to the horizon at an angle ϕ, with its curve surface touching the plane. Find the greatest admissible value of the inclination ϕ for equilibrium. If ϕ be less than this value, is the equilibrium stable?
4. Suppose a cylinder of any cross section is balanced on another fixed cylinder, the contact of curved surface being rough and the common tangent line horizontal. Let $\rho \; and \; \rho'$ be radii of curvature of the two cylinders at the point of contact and h be the height of centre of gravity of the upper cylinder above the point of contact. Show that the upper cylinder is balanced in stable equilibrium if $h < \frac{\rho\rho'}{\rho+\rho'}$. (15, 2022)

Friction

1. One end of a heavy uniform rod AB can slide along a rough horizontal rod AC, to which it is attached by a ring. B and C are joined by a string. When the rod is on the point of sliding, then $AC^2 - AB^2 = BC^2$. If θ is the angle between AB and the horizontal line, then prove that the coefficient of friction is $\frac{\cot\theta}{2+\cot^2\theta}$.
2. A straight uniform beam of length '$2h$' rests in limiting equilibrium in contact with a rough vertical wall of height 'h' with one end on a rough horizontal plane and with the other end projecting beyond the wall. If both the wall and the plane be equally rough, prove that 'λ' the angle of friction, is given by $sin\ 2\lambda = sin\ \alpha\ sin\ 2\alpha$, '$\alpha$' being the inclination of the beam to the horizon.
3. A uniform ladder of weight W rests at an angle of 45° with the horizontal with its upper extremity against a rough vertical wall and its lower extremity on the ground. If μ and μ' are the coefficients of limiting friction between the ladder and the ground and wall respectively, then find the minimum horizontal force required to move the lower end of the ladder towards the wall.
4. The base of an inclined plane is 4 metres in length and the height is 3 metres. A force of 8kg acting parallel to the plane will just prevent a weight of 20 kg from sliding down. Find the coefficient of friction between the plane and the weight.

Equilibrium of Forces in Three Dimensions

1. On a rigid body the forces $10(\hat{\imath} + 2\hat{\jmath} + 2\hat{k})N, 5(-2\hat{\imath} - \hat{\jmath} + 2\hat{k})N \; and \; 6(2\hat{\imath} + 2\hat{\jmath} - \hat{k})N$ are acting at points with position vector $\hat{\imath} - \hat{\jmath}$, $2\hat{\imath} + 5\hat{k}$, and $4\hat{\imath} - \hat{k}$ respectively. Reduce this system to a single force $\vec{R}$ acting at the point $4\hat{\imath} + 2\hat{\jmath}$ together with a couple $\vec{G}$ whose axis passes through this point. Does the point $4\hat{\imath} + 2\hat{\jmath}$ lie on the central axis.