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Tutorial Sheet-II

Constrained Motion

1. A heavy particle of weight W, attached to a fixed point by a light inextensible string, describes a circle in a vertical plane. The tension in the string has the values mW and nW respectively when the particle is at the highest and lowest point in the path. Show that $n = m + 6$.
2. A heavy particle hangs from a fixed point O, by a string of length a. It is projected horizontally with a velocity $v^2 = (2 + \sqrt{3})ag$; show that the string becomes slack when it has described an angle $\cos^{-1}\left(-\frac{1}{\sqrt{3}}\right)$.
3. A particle inside and at the lowest point of a fixed smooth hollow sphere of radius a is projected horizontally with velocity $\sqrt{\frac{7}{2}\,ag}$. Show that it will leave the sphere at a height $\frac{3}{2}a$ above the lowest point and its subsequent path meets the sphere again at the point of projection.
4. Find the velocity with which a particle must be projected along the interior of a smooth vertical hoop of radius a from the lowest point in order that it may leave the hoop at an angular distance of 30° from the vertical. Show that it will strike the hoop again at an extremity of the horizontal diameter.
5. A particle is free to move on a smooth vertical circular wire of radius a. It is projected from the lowest point with velocity just sufficient to carry it to the highest point. Show that the reaction between the particle and the wire is zero after a time $\sqrt{\frac{a}{g}}\log(\sqrt{5} + \sqrt{6})$.
6. A heavy bead slides on a smooth circular wire of radius a. It is projected from the lowest point with a velocity just sufficient to carry it to the highest point, prove that the radius through the bead in time t will turn through an angle $2\tan^{-1}\left[\sinh\left\{t\sqrt{\frac{g}{a}}\right\}\right]$. And that the bead will take infinite time to reach the highest point.
7. A particle is placed on the outside of a smooth vertical circle. If the particle starts from a point whose angular distance is α from the highest point of circle, show that it will fly off the curve when $\cos\theta = \frac{2}{3}\cos\alpha$.
8. A particle is placed at the highest point of a smooth vertical circle of radius a and is allowed to slide down starting with a negligible velocity. Prove that it will leave the circle after describing

vertically a distance equal to one third of the radius. Find the position of the directrix and the focus of the parabola subsequently described and show that its latus rectum is $\frac{16}{27}\,a$.

9. A heavy particle slides down a smooth cycloid starting from rest at the cusp, the axis being vertical and vertex downwards, prove that the magnitude of the acceleration is equal to g at every point of the path and the pressure when the particle arrives at the vertex is equal to twice the weight of the particle.
10. Prove that for a particle, sliding down the arc and starting from the cusp of a smooth cycloid whose vertex is lowest, the vertical velocity is maximum when it has described half the vertical height.
11. A particle starts from the rest at the cusp of a smooth cycloid whose axis is vertical and vertex downwards. Prove that when it has fallen through half the distance measured along the arc to the vertex, two third of the time of descent will have elapsed.
12. A particle is projected with velocity V from the cusp of a smooth inverted cycloid down the arc, show that the time of reaching the vertex is $2\sqrt{\frac{a}{g}}\tan^{-1}[\sqrt{4ag}\,/V\,]$.
13. A particle is placed very near the vertex of a smooth cycloid whose axis is vertical and vertex upwards, and is allowed to run down the curve. Prove that it will leave the curve when it has fallen through half the vertical height of the cycloid.