Tutorial Sheet IV- Stable and Unstable Equilibrium

1. A hemisphere rests in equilibrium on a sphere of equal radius; show that the equilibrium is unstable when the curved, and stable when the flat surface of the hemisphere rests on the sphere.

2. A uniform cubical box of edge a is placed on the top of a fixed sphere, the centre of of the face of the cube is being in contact with the highest point of the sphere. What is the least radius of the sphere for which the equilibrium will be stable?

3. A heavy uniform cube balances on the highest point of a sphere whose radius is r. If the sphere is rough enough to prevent sliding and the side of the cube be $\frac{\pi r}{2}$, show that the cube can rock through a right angle without falling.

4. A body, consisting of a hemisphere and a cone on the same base, rests on a rough horizontal table; the hemisphere being in contact with the table. Show that the greatest height of the cone so that the equilibrium may be stable is $\sqrt{3}$ times the radius of the hemisphere.

5. A uniform solid hemisphere rests in equilibrium upon a rough horizontal plane with its curved surface in contact with the plane and a particle of mass m is fixed at the centre of the plane face. Show that for any value of m, the equilibrium is stable.

6. A heavy spherical shell of radius r has a particle attached to a point on the rim and rests with the curved surface in contact with a rough sphere of radius R at the highest point. Prove that if $\frac{R}{r} > \sqrt{5} - 1$, the equilibrium is stable, whatever be the weight of the particle.

7. A solid hemisphere rests on a plane inclined to the horizontal at an angle $\alpha < \sin^{-1}(3\backslash 8)$, and the plane is rough enough to prevent any sliding. Find the position of equilibrium and show that it is stable.

8. A heavy uniform beam of length $2a$, rests with its end in contact with two smooth inclined planes, of inclination $\alpha \; and \; \beta \; (\alpha > \beta)$ to the horizon. If θ be the inclination of the rod to the horizon, prove that $\tan \theta = \frac{1}{2}(\cot \alpha - \cot \beta)$ and show that the beam is unstable in this position.

9. A uniform rod rests with one end against a smooth vertical wall and with a point in its length resting on a smooth peg. Find the position of equilibrium and show that it is unstable.

10. A uniform rod of length *2l*, is attached by smooth rings at both the ends of a parabolic wire, fixed with its axis vertical and vertex downwards, and latus rectum *4a*. Show that the angle θ which the rod makes with the horizontal in a slanting position of equilibrium is given by $\cos^2\theta = \frac{2a}{l}$, and that, if these positions exist, they are stable. Show also that the positions in which the rod is horizontal are stable or unstable according as the rod is below or above the focus.

Friction

1. A uniform rod rests in limiting equilibrium within a rough hollow sphere; if the rod subtends angle 2α at the centre of the sphere, and if λ be the angle of friction, show that the angle the angle of inclination of the rod to the horizon is $\tan^{-1}\left[\frac{\sin 2\lambda}{2\cos(\alpha+\lambda)\cos(\alpha-\lambda)}\right]$

2. A beam AB rests with one A in contact with rough horizontal floor, and the other end B in contact with a rough vertical wall; to discuss its equilibrium, the inclination of the beam to the horizontal being given.

3. Two uniform rods *AB, BC* of weights *W and W'*, are smoothly jointed at B and are placed so as to be in a straight line on a rough horizontal table; the end A is acted on by a gradually increasing force P in the direction perpendicular to the rods. Find how the equilibrium is broken.