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Tutorial Sheet - II

1. Find a complete integral of $q = (z + px)^2$
2. Find a complete integral of $16p^2z^2 + 9q^2z^2 + 4z^2 - 4 = 0$
3. Find a complete integral of $(p^2 + q^2)x = pz$ and deduce the solution which pass through the curve $x = 0,\ z^2 = 4y$.
4. Find complete and singular integrals of $2xz - px^2 - 2qxy + pq = 0$.
5. Find a complete integral of
   a. $p^2 + q^2 - 2px - 2qy + 1 = 0.$
   b. $p^2 + q^2 - 2px - 2qy + 2xy = 0$
   c. $p^2x + q^2y = z$
   d. $2(z + px + qy) = yp^2$
   e. $z^2 = pqxy$
   f. $px + qy = z(1 + pq)^{\frac{1}{2}}$
   g. $z = \left(\frac{1}{2}\right)(p^2 + q^2) + (p - x)(q - y)$
   h. $2x\{z^2q^2 + 1\} = zp$
   i. $p^2x(x - 1) + 2pqxy + q^2y(y - 1) - 2pxz - 2qyz + z^2 = 0$
   j. $2(y + zq) = q(xp + yq)$
   k. $pq = x^m y^n z^l$
   l. $p^m \sec^{2m} x + z^l q^n cosec^{2n}\, y = z^{\frac{lm}{m-n}}$
   m. $(x + y)(p + q)^2 + (x - y)(p - q)^2 = 1$
   n. $(x^2 + y^2)(p^2 + q^2) = 1$
   o. $2(y + zq) = q(xp + yq).$
   p. $2q(z - px - qy) = 1 + q^2$
   q. $p^2x + q^2y = (z - 2px - 2qy)^2$
   r. $4xyz = pq + 2px^2y + 2qxy^2$
   s. $9(p^2z + q^2) = 4$
   t. $p^3 + q^3 - 3pqz = 0$
   u. $z^2(p^2 + q^2 + 1) = k^2$

v. $yzp^2 = q$

w. $2p^2q^2 + 3x^2y^2 = 8x^2q^2(x^2 + y^2)$

x. $2x(z^2q^2 + 1) = pz$

6. Find the complete integral, general integral and singular solution of $pq = 4xy$. Show that the equation is satisfied by $z = 2xy + C, C$ being arbitrary constant.
7. Find the complete integral, general integral and singular solution of $p^3 + q^3 = 27z$.
8. Find the characteristics of the equation $pq = z$ and determine the integral surface which passes through the parabola $x = 0, y^2 = z$.

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