Tutorial Sheet-III

1. Solve:
   a. $25r - 40s + 16t = 0$
   b. $(D^4 + D'^4)Z = 0$
   c. $r + s - 2t = (2x + y)^{\frac{1}{2}}$
   d. $r - t = x - y$
   e. $(D_x^3 - 7D_xD_y^2 - 6D_y^3)z = \sin(x + 2y) + e^{3x+y}$
   f. $\frac{\partial^3 z}{\partial x^2\,\partial y} - 2\frac{\partial^3 z}{\partial x\,\partial^2 y} + \frac{\partial^3 z}{\partial y^3} = \frac{1}{x^2}$
   g. $(D^2 - 2DD' - 15D'^2)z = 12xy$
   h. $(D^2 - DD' - 2D'^2)z = (2x^2 + xy - y^2)\sin xy - \cos xy$
   i. $r + s - 6t = y\cos x$
   j. $(D^2 + DD' - 6D'^2)z = x^2\sin(x + y)$
   k. $(D^3 + D^2D' - DD'^2 - D'^3)z = e^y\cos 2x$
2. Find a real function V of x and y, satisfying $(D^2 + D'^2)V = -4\pi(x^2 + y^2)$ and reducing to zero, when $y = 0$.
3. Find the solution of the equation $(D^2 + D'^2)z = e^{-x}\cos y$ which $\to 0$ as $x \to \infty$ and has the value $\cos y$ when $x = 0$.
4. Find a surface satisfying $r - 2s + t = 6$ and touching the hyperbolic paraboloid $z = xy$ along its section by the plane $y = x$.
5. A surface is drawn satisfying $r + t = 0$ and touching $x^2 + z^2 = 1$ along its section by $y = 0$. Obtain its equation in the form $x^2(x^2 + z^2 - 1) = y^2(x^2 + z^2)$
6. Solve:
   a. $r + 2s + t + 2p + 2q + z = 0$
   b. $(D - D'^2)z = 0$
   c. $(D - 2D' - 1)(D - 2D'^2 - 1)z = 0$
   d. $(DD' + aD + bD' + ab)z = e^{mx+ny}$
   e. $(D^2 - 4DD' + D - 1)z = e^{3x-2y}$
   f. $(D^2 + DD' + D' - 1)z = \sin(x + 2y)$
   g. $(D - D'^2)z = \cos(x - 3y)$
   h. $(D^2 - D')z = 2y - x^2$
   i. $(3D^2 - 2D'^2 + D - 1)z = 4e^{x+y}\cos(x + y)$
   j. $(D^2 + DD' + D' - 1)z = 4\sinh x$
7. Find a surface satisfying $r + s = 0$ and touching the elliptic paraboloid $z = 4x^2 + y^2$ along its section by the plane $y = 2x + 1$.