Tutorial Sheet: Statics III- Common Catenary

1. Derive the intrinsic equation of common catenary.

2. If T be tension at any point P of a catenary, and T_0 that at the lowest point A, prove that $T^2 - T_0^2 = W^2$, W being eight of the arc AP of the catenary.

3. Show that for a common catenary $x = c\log(\frac{y+s}{c})$.

4. A rope of length $2l$ feet is suspended between two points at the same level, and the lowest point of the rope is b feet below the points of suspension. Show that the horizontal component of the tension is $(l^2 - b^2)/2b$, w being the weight of the rope per foot of its length.

5. A uniform chain of length l, which can just bear a tension of n times its weight, is stretched between two points on the same horizontal line. Show that the least possible sag in the middle is $l\left\{n - \sqrt{n^2 - \frac{1}{4}}\right\}$.

6. A given length, $2s$, of a uniform chain has to be hung between two points at the same level and the tension has not to exceed the weight of a length b of the chain. Show that the greatest span is $\sqrt{b^2 - s^2}\log\frac{b+s}{b-s}$.

7. The end links of a uniform chain slide along a fixed rough horizontal rod. Prove that the ratio of maximum span to the length of the chain is $\mu\log\left\{\frac{1+\sqrt{1+\mu^2}}{\mu}\right\}$ where μ is the coefficient of friction.

8. A weight W is suspended from a fixed point by a uniform string of length l and weight w per unit length. It is drawn aside by a horizontal force P. Show that in the position of equilibrium, the distance of W from the vertical through the fixed point is $\frac{P}{w}\left\{\sinh^{-1}\left(\frac{W+lw}{P}\right) - sinh^{-1}\left(\frac{W}{P}\right)\right\}$.

9. A uniform chain, of length l and weight W, hangs between two fixed points at the same level, and a weight W' is attached at the middle point. If k be the sag in the middle, prove that the pull on either point of support is $\frac{k}{2l}W + \frac{l}{4k}W' + \frac{l}{8k}W$.

10. A heavy string of uniform density and thickness is suspended from two given points in the same horizontal plane. A weight, an nth that of the string, is attached to its lowest point. Show that if θ, φ be the inclinations to the vertical of the tangents at the highest and lowest points of the string $\tan\varphi = (1+n)\tan\theta$

11. Find the length of an endless chain which will hang over a circular pulley of radius a so as to be in contact with two third of the circumference of the pulley. $\left[a\left\{\frac{3}{log(2+\sqrt{3}}+\frac{4\pi}{3}\right\}\right]$

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