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Tutorial Sheet-I

1. Show that the differential equation of all cones which have vertex at the origin is $px + qy = z$. Verify that $yz + zx + xy = 0$ is a surface satisfying the above equation.
2. From PDE: $2z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$
3. Eliminate a, b, c from $z = a(x + y) + b(x - y) + abt + c$
4. Form the PDE by eliminating the arbitrary constants a and b from $\log(az - 1) = x + ay + b$.
5. Find a PDE by eliminating a, b, c from $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$.
6. Find the PDE of all planes which are at a constant distance 'a' from the origin.
7. Form a PDE by eliminating the arbitrary functions f and F from $z = f(x + iy) + F(x - iy)$, where $i^2 = -1$.
8. Find the DE of all surfaces of revolution having z- axis as the axis of rotation.
9. Equation of all the cone with vertex at *P (a,b,c)* is of the form $f\left(\frac{x-a}{z-c}, \frac{y-b}{z-c}\right) = 0$. Find the differential equation of the cone.
10. Solve $p \tan x + q \tan y = \tan z$
11. Solve $z(z^2 + xy)(px - qy) = x^4$
12. Solve $xzp + yzq = xy$
13. Solve $\left\{\frac{b-c}{a}\right\} yzp + \left\{\frac{c-a}{b}\right\} zxq = \left\{\frac{a-b}{c}\right\} xy$
14. Solve $(z^2 - 2yz - y^2)p + (xy + zx)q = xy - zx$. If the solution of this represents a sphere, what will be the coordinates of it's centre.
15. Solve $(y^3x - 2x^4)p + (2y^4 - x^3y)q = 9z(x^3 - y^3)$.
16. Solve $x^2p + y^2q = nxy$
17. Solve $(3x + y - z)p + (x + y - z)q = 2(z - y)$.
18. Solve $x(x^2 + 3y^2)p - y(3x^2 + y^2)q = 2z(y^2 - x^2)$
19. Solve $z(x + 2y)p - z(y + 2x)q = y^2 - x^2$
20. Solve $y^2(x - y)p + x^2(y - x)q = z(x^2 + y^2)$
21. Solve $(x^2 - y^2 - z^2)p + 2xyq = 2xz$
22. Solve$(x^2 - yz)p + (y^2 - zx)q = z^2 - xy$

23. Solve $\cos(x+y)\,p + \sin(x+y)\,q = z$

24. Solve $xp + yq = z - a\sqrt{x^2+y^2+z^2}$

25. Solve $(x^3+3xy^2)p + (y^3+3x^2y)q = 2z(x^2+y^2)$

26. Solve $(2x^2+y^2+z^2-2yz-zx-xy)p + (x^2+2y^2+z^2-yz-2zx-xy)q = x^2+y^2+2z^2-yz-zx-2xy$.

27. Solve $\{my(x+y)-nz^2\}p - \{lx(x+y)-nz^2\}q = (lx-my)z$

28. Solve $px(z-2y^2) = (z-qy)(z-y^2-2x^2)$.

29. Solve $(x+y-z)(p-q) + a(px-qy+x-y) = 0$.

30. Find the integral surface of the PDE $(x-y)p + (y-x-z)q = z$ through the circle $z=1, x^2+y^2=1$.

31. Find the general integral of the PDE $(2xy-1)p + (z-2x^2)q = 2(x-yz)$ and also the particular integral which passes through the line $x = 1\,, y = 0$.

32. Find the integral surface of $x^2p + y^2q + z^2 = 0$ which passes through the hyperbola $xy = x+y,\ z = 1$.

33. Find the integral surface of the PDE $yp + xq = z-1$ which passes through the curve $z = x^2+y^2, y = 2x$.

34. Write down the system of equations for obtaining the general equation of surfaces orthogonal to the family given by $x(x^2+y^2+z^2) = C_1y^2$

35. Find the surface which is orthogonal to the one parameter system $z = cxy(x^2+y^2)$ which passes through the hyperbola $x^2-y^2=a^2, z=0$.

36. Find the family of surfaces orthogonal to the family of surfaces given by the differential equation $(y+z)p + (z+x)q = x+y$.

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