Tutorial Sheet: IV Projectiles

1. A body is projected at an angle α to the horizontal so as to clear two walls of equal height a at a distance $2a$ from each other. Show that the range is equal to $2a\cot\frac{\alpha}{2}$.
2. If R be the horizontal range and h the greatest height of a projectile, prove that the initial velocity is $\left[2g\left(h+\frac{R^2}{16h}\right)\right]^{\frac{1}{2}}$.
3. A particle is projected from O at an elevation α and after t seconds it appears to have an elevation β as seen from the point of elevation. Prove that the initial velocity was $\frac{gt\cos\beta}{2\sin(\alpha-\beta)}$.
4. A projectile aimed at a mark which is in a horizontal plane through the point of projection, falls a meter short of it when the elevation is α and goes b meters too far when the elevation is β. Show that if the velocity of projection be same in all cases, the proper elevation is $\frac{1}{2}\sin^{-1}\frac{a\sin 2\beta+b\sin 2\alpha}{a+b}$.
5. A particle is thrown over a triangle from one end of a horizontal base and grazing over the vertex falls on the other end of the base. If A and B be the base angles of the triangle and α the angle of projection, prove that $\tan\alpha = \tan A + \tan B$.
6. A gun is fired from the moving platform and the ranges of the shots are observed to be R and S when the platform is moving forward and backward respectively with velocity v. prove that the elevation of the gun is $\tan^{-1}\left\{\frac{g}{4v^2}\frac{(R-S)^2}{R+S}\right\}$.
7. A shot is fired with velocity V at an elevation θ, strikes a point P on the horizontal plane through the point of projection. If the point P is receding from the gun with velocity v, show that the elevation must be changed to ϕ, where $\sin 2\phi = \sin 2\theta + \frac{2v}{V}\sin\phi$.
8. The distance between the axle-trees of the front and hind wheels of a carriage of radii a and b respectively is c. A particle of mud, driven from the highest point of the hind wheel, alights on the highest point of the front wheel. Show that the velocity of the carriage is $\left[\frac{g(c+a-b)(c+b-a)}{4(b-a)}\right]^{\frac{1}{2}}$.

9. If α *and* β are the two possible directions to hit a given point (a, b), then show that $\tan(\alpha + \beta) = -a/b$.
10. Determine the least velocity with which a ball can be thrown to reach the top of a cliff 40 meters high and $40\sqrt{3}$ meters away from the point of projection.
11. A shell bursts at a horizontal distance a from the foot of a hill of height h. Fragments of the shell fly in all directions with a velocity upto V. Find how long a man on the top of the hill will be in danger.
12. A particle is projected with velocity u from a point on a plane inclined at an angle β to the horizontal. If r *and* r' are its maximum ranges up and down the plane, prove that $\frac{1}{r} + \frac{1}{r'}$ is independent of the inclination of the plane.
13. For a given velocity of projection, the maximum range down an inclined plane is three times the maximum range up the inclined plane; show that the inclination of plane to the horizontal is 30°.
14. A particle is projected at an angle α with the horizontal from the foot of the plane, whose inclination to the horizontal is β. Show that it will strike the plane at right angles if $\cot\beta = 2\tan(\alpha - \beta)$.
15. A fort is on the edge of the cliff of height h. Show that there is an annular region of area $8\pi hk$ in which the fort is out of the range of the ship, but the ship is not out of the range of the fort, where $\sqrt{2gk}$ is the velocity of the shells used by both.

Work, Energy and Impulse

1. If a light elastic string, whose natural length is that of a uniform rod, be attached to the rod at both the ends and suspended by the middle point, show that the rod will descend until each of the two portions of the string is inclined to the horizon at an angle θ, given by the equation $\cot^3\frac{\theta}{2} - \cot\frac{\theta}{2} = 2n$, the modulus of elasticity of the string being n times the weight of the rod.
2. A spider hangs from the ceiling by a thread of modulus of elasticity equal to its weight. Show that it can climb to the ceiling with an expenditure of work equal to only three quarters of what would be required if the thread were inelastic.
3. A particle is set moving with kinetic energy E straight up an inclined plane of inclination α and coefficient of friction μ. Prove that the work done against friction before the particle comes to rest is $\frac{E\mu\cos\alpha}{\sin\alpha + \mu\cos\alpha}$.

4. A uniform string of mass M and length $2a$ is placed symmetrically over a smooth peg and has particles of massed m and m' attached to its ends $(m > m')$. Show that the string runs off the peg when its velocity is $\sqrt{\left[\frac{M+2(m-m')}{M+m+m'}\,ag\right]}$.
5. A gun is mounted on a gun carriage, movable on a smooth horizontal plane, and the gun is elevated at an angle α to the horizon. A shot is fired and leaves the gun in a direction inclined at an angle θ to the horizon. If the mass of the gun and its carriage be n times that of the shot, prove that $\tan\theta = \left(1+\frac{1}{n}\right)\tan\alpha$.
6. Assuming that in a canon, the force on the ball depends only on the volume of the gas generated by the gun powder, show that the ratio of the final velocity of the ball when the gun is free to recoil to its velocity when the gun is fixed is $\sqrt{\left[\frac{M}{m+M}\right]}$, where M and m are the masses of the gun and the ball respectively.
7. A body of mass $m_1 + m_2$ moving in a straight line is split into two parts of masses m_1 *and* m_2 by an internal explosion which generates kinetic energy E. Show that if after the explosion, the two parts move in the same line as before, their relative speed is $\sqrt{\left[\frac{2E(m_1+m_2)}{m_1 m_2}\right]}$.
8. A shot of mass m fired horizontally, penetrates a thickness s of a fixed plate of mass M; prove that if M is free to move the thickness penetrated is $\frac{Ms}{M+m}$.
9. A block of M rests on a smooth horizontal table and a bullet of mass m is fired into it. The penetration of the bullet is opposed by a constant resisting force. If the experiment is repeated with the block firmly fixed, show that the depth of the penetration of the bullet and the time which elapses before the bullet is at rest relatively to the block are in each case increased in the ratio $\left(1+\frac{m}{M}\right) : 1$.
10. A hammer of mass M falls freely from a height h on the top of an inelastic pile of mass m which is driven into the ground a distance a feet. Assuming that, the resistance of the ground is constant, find its value and show that the time during which the pile is in motion is given by $\frac{a(M+m)}{M}\left(\frac{2}{gh}\right)^{\frac{1}{2}}$. Find also, what fraction of kinetic energy is lost by the impact.