Previous Year Questions: Dynamics (2008-22)

Rectilinear Motion

1. One end of light elastic string of natural length l and modulus of elasticity 2mg is attached to a fixed-point O and the other end to a particle of mass m. The particle initially held at rest is bit fall. Find the greatest extension of the string during the motion and show that the particle will reach O again after the time

$$(\pi + 2 - \tan^{-1} 2)\sqrt{\frac{2l}{g}}$$

2. The velocity of train increase from 0 to v at a constant acceleration f_1 and then remains constant for an interval and again decreases to 0 at a constant retardation f_2. If the total distance described is x, find the total time taken.
3. A mass of 560 kg moving with velocity 240m/s strikes a fixed target and is brought to rest in 1/100 sec. Find the impulse of the blow on the target and assuming the resistance to be uniform throughout the time taken by the body in coming to the rest, find the distance to which it penetrates. (20)
4. After a ball has been falling under the gravity for 5 sec, it passes through a pane of glass and loses half its velocity. If it now reaches the ground in 1 sec, find the height of glass above the ground. (10)
5. A particle of mass m moves on a straight line under an attractive force mn^2x towards a point O on the line, where x is distance from O. If $x = a$ and $\frac{dx}{dt} = u$ when $t=0$, find $x(t)$ for any time $t>0$.(10)
6. A particle moves with an acceleration $\mu\left(x + \frac{a^4}{x^3}\right)$ towards the origin. If it starts from the rest at a distance a from the origin, find its velocity when it distance from the origin is $\frac{a}{2}$. (12)
7. A particle P moves in a plane such that it is acted on by two constant velocities u and v respectively along the direction OX and along the direction perpendicular to OP where O is some fixed point, that is the origin. Show that the path traversed by P is a conic section with focus at O and eccentricity u/v.
8. A particle moves in a straight line. Its acceleration is directed towards a fixed point O in the line and is always equal to $\mu\sqrt[3]{\frac{a^5}{x^2}}$ when it is at a distance x from O. If it starts from rest at a distance a from O, then find the time, the particle will arrive at O.

9. A particle is acted on by a force parallel to the axis of Y whose acceleration (always towards the axis of x) is μy^{-2} and when $y = a$, it is projected parallel to the axis of X with velocity $\sqrt{\frac{2a}{\mu}}$. Find the parametric equation on the path of the particle. Here μ is constant.

10. A particle moving along the y-axis has an acceleration Fy towards the origin, where F is a positive and even function of y. The periodic time, when the particle vibrates between y = -a and y = a, is T. Show that $\frac{2\pi}{\sqrt{F_1}} < T < \frac{2\pi}{\sqrt{F_2}}$ where F_1 *and* F_2 are the greatest and the least values of F within the range [-a, a]. Further, show that when a simple pendulum of length l oscillates through 30° on either side of the vertical line, T lies between $2\pi\sqrt{\frac{l}{g}}$ and $2\pi\sqrt{\frac{l}{g}}\sqrt{\frac{\pi}{3}}$.

Simple Harmonic Motion

1. A body is performing S.H.M in a straight line OPQ. Its velocity is zero at points P and Q whose distances from O are x and y respectively and its velocity is υ at the mid-point between P and Q. Find the time of one complete oscillation. (10)

2. A particle is performing SHM of period T about a centre O with amplitude a and it passes through a point P, when OP =b in the direction OP. Prove that the time which elapses before it returns to P is $\frac{T}{\pi}\cos^{-1}\frac{b}{a}$. (10)

3. A body moving under SHM has an amplitude a and time period T, If the velocity is trebled, when the distance from mean position is $\frac{2}{3}a$,the period being unaltered. Find the new amplitude.

4. A particle moving with simple harmonic motion in a straight line has velocities v_1 and v_2 at distances x_1 and x_2 respectively from the centre of its path. Find the period of its motion. (12)

Motion in Plane

1. A shot fired with a velocity V at an elevation α strikes a point P in a horizontal plane through the point of projection. If the point P is receding from the gun with velocity v, show that the elevation must be changed to θ where

$$\sin 2\theta = \sin 2\alpha + \frac{2v}{V}\sin\theta$$

2. A particle is projected with velocity V from the cusp of a smooth inverted cycloid down the arc. Show that the time of reaching the vertex is $2\sqrt{\frac{a}{g}}\cot^{-1}\frac{V}{2\sqrt{ag}}$

3. A particle slides down the arc of a smooth cycloid whose axis is vertical and vertex lowest. Prove that the time occupied in falling down the first half of the vertical height is equal to the time of falling down the second half.

Projectile

1. A projectile aimed at a mark which is in the horizontal plane through the point of projection falls x meter short of it when the angle of projection is α and goes y meter beyond when the angle of projection is β. If the velocity of projection is assumed same in all cases, find the correct angle of projection.
2. If v_1, v_2, v_3 are the velocities at three points A, B, C of the path of a projectile, where the inclinations to the horizon are $\alpha, \alpha-\beta, \alpha-2\beta$ and if t_1, t_2 are the times of describing the arcs AB, BC respectively. Prove that $v_3 t_1 = v_1 t_2$ and $\frac{1}{v_1}+\frac{1}{v_3}=\frac{2\cos\beta}{v_2}$
3. A particle projected from a given point on the ground just clears a wall of height h at a distance d from the point of projection. If the particle moves in a vertical plane and if the horizontal range is R, find the elevation of the projection.
4. A particle is projected from the base of a hill whose slope is that of a right circular cone whose axis is vertical. The projectile grazes the vertex and strikes the hill again at point a on the base. If the semi vertical angle of the cone is 30°, h is height, determine the initial velocity u of the projection and its angle of projection.
5. Describe the motion and path of a particle of mass m which is projected in vertical plane through a point of projection with velocity u in a direction making an angle θ with the horizontal direction. Further, if particles are projected from that point in the same vertical plane with velocity $4\sqrt{g}$, then determine the locus of vertices of their paths. (15M, 2021)

Constrained Motion

1. A particle of mass 2.5 kg hangs at the end of a string 0.9 m long, the other end of which is attached to a fixed point. The particle is projected horizontally with a velocity 8 m/sec. Find the velocity of the particle and tension in the string when the string is (i) horizontal (ii) vertically upward.
2. A particle of mass m, hanging vertically from a fixed point by a light inextensible cord of length l is struck by a horizontal blow which imparts of it a velocity $2\sqrt{gl}$. Find the velocity and height of the particle from of its initial position when the cord becomes slack.

3. A particle is free to move on a smooth vertical circular wire of radius a. At time $t = 0$, it is projected along the circle from its lowest point A with velocity just sufficient to carry it to the highest point B. Find the time T at which the reaction between the particle and the wire is zero.

4. A smooth parabolic tube is placed with vertex downwards in vertical plane. A particle slides down the tube from rest under the influence of gravity. Prove that in any position, the reaction of the tube is equal to $2w\frac{h+a}{\rho}$, where 'w' is the weight of the particle, 'ρ' the radius of curvature of the tube, '$4a$' its latus rectum and 'h' the initial vertical height of the particle above the vertex of the tube.

5. A heavy particle hangs by an inextensible string of length 'a' from a fixed point and is then projected horizontally with a velocity $\sqrt{2ag}$. If $\frac{5a}{2} > h > a$, then prove that circular motion ceases when the particle has reached the height $\frac{1}{3}(a + 2h)$ from the point of projection. Also, prove that the greatest height ever reached by the particle above the point of projection is $\frac{(4a-h)(a+2h)^2}{27a^2}$. (15M, 2021)

Central Orbit

1. A particle of mass m moves under a force $\{3au^4 - 2(a^2 - b^2)u^5\}, u = \frac{1}{r}$, a, b and μ (> 0) being given constants. It is projected from an apse at a distance $a + b$ with velocity $\frac{\sqrt{\mu}}{a+b}$. Show that its orbit is given by the equation $r = a + b \cos \theta$, where (r, θ) are the plane polar coordinates of a point.

2. A body is describing an ellipse of eccentricity e under the action of a central force directed towards a focus and when at the nearer apse, the centre of force is transferred to the other focus. Find the eccentricity of the new orbit in terms of the eccentricity of the original orbit.

3. A particle moves with a central acceleration $\mu(r^5 - 9r)$ being projected from an apse at a distance $\sqrt{3}$ with velocity $3\sqrt{2\mu}$. Show that the path is the curve
$$x^4 + y^4 = 9$$

4. A particle moves in a plane under a force, towards a fixed centre, proportional to the distance. If the path of the particle has apsidal distance a, b $(a > b)$, then find the equation of the path.

5. A mass starts from rest at a distance 'a' from the center of force which attract inversely as the distance. Find the time of arriving at the center.

6. A particle starts at a great distance with velocity V. let p be the length of the perpendicular from the centre of a star on the tangent to the initial path of the particle.

Show that the least distance of the particle from the centre of the star is λ ,where $V^2\lambda = \sqrt{\mu^2 + p^2V^4} - \mu$. Here μ is constant.

7. A particle moves with a central acceleration, which varies inversely as the cube of the distance. If it is projected from an apse at a distance a from the origin with a velocity which √2 is times the velocity for a circle of radius 'a', then find the equation to the path.

8. Prove that the path of a planet, which is moving so that its acceleration is always directed to a fixed point (star) and is equal to $\frac{\mu}{(dislance)^2}$, is a conic section. Find the conditions under which the path becomes (i) ellipse, (ii) parabola and (iii) hyperbola.

9. If a planet, which revolves around the Sun in circular orbit, is suddenly stopped in its orbit, then find the time in which it would fall into the Sun. Also find the ratio of its falling time to the period of revolution of the planet. (10M, 2021)

Work, Energy and Impulse

1. A four-wheeled railway truck has a total mass M, the mass and radius of gyration of each pair of wheels and axel are m and k respectively, and the radius of each wheel is r. Prove that if the truck is propelled along a level track by a force P, the acceleration is $\frac{P}{M+\frac{2mk^2}{r^2}}$, and find the horizontal force exerted on each axle by the truck. The axle friction and wind resistance are to be neglected.

2. A shell lying in a strength smooth horizontal tube suddenly breaks into two portions of masses m_1 and m_2 if s be the distance between the two masses inside the tube after time t, show that the work done by the explosion can be written as equal to $\frac{1}{2}\frac{m_1m_2}{m_1+m_2}\frac{s^2}{t^2}$

3. A spherical shot of W gm weight and radius r cm, lies at the bottom of cylindrical bucket of radius R cm. The bucket is filled with water up to a depth of h cm (h > 2r). Show that the minimum amount of work done in lifting the shot, just clear of the water, must be $\left[W\left(h-\frac{4r^3}{3R^2}\right)+W'\left(r-h+\frac{4r^3}{3R^2}\right)\right]$ cm gm. W' gm is the weight of water displaced by the shot.

4. A fixed wire is in the shape of the Cardioid $r = a(1 + cos\,\theta)$, the initial line being the downward vertical. A small ring of mass m can slide on the wire and is attached to the point r = 0 of the Cardioid by an elastic string of natural length a and modulus of elasticity 4 mg. The string is released from rest when the string is horizontal. Show by using the laws of conservation of energy that $a\theta^2(1 + cos\,\theta) - g\,cos\,\theta\,(1 - cos\,\theta) = 0$, g being the acceleration due to gravity.

5. The force of attraction of a particle by the earth is inversely proportional to the square of its distance from the earth's centre. A particle, whose weight on the surface of the

earth is W, falls to the surface of the earth form a height $3h$ above it. Show that the magnitude of work done by the earth's attraction force is $\frac{3}{4}hW$, where h is the radius of the earth.

6. A heavy ring of mass m, slides on a smooth vertical rod and is attached to a light string which passes over a small pulley distance a from the rod and has a mass M $(> m)$ fastened to its other end. Show that if the ring be dropped from a point in the rod in the same horizontal plane as the pulley, it will descend a distance $\frac{2Mm}{M^2-m^2}a$ before coming to rest.

7. A light rigid rod ABC had three particles each of mass m attached to it at A, B and C. The rod is struck by a blow P at right angles to it at a point distant from A equal to BC. Prove that the kinetic energy set up is $\frac{1}{2}\frac{P^2}{m}\frac{a^2-ab+b^2}{a^2+ab+b^2}$, where $AB = a \, and \, BC = b$.

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