Tutorial Sheet III - Central Orbits

1. A particle describes the curve $r^n = a^n \cos n\theta$ under a force to the pole. Find the law of force.
2. Find the law of force towards the pole under which the curve $r^2 = 2ap$ is described.
3. Show that the only law for a central attraction for which the velocity in a circle at any distance is equal to the velocity acquired in falling from infinity to the distance is that of inverse cube.
4. A particle moves under a force $m\mu\{3au^4 - 2(a^2 - b^2)u^5\}, a > b$ and is projected from an apse at a distance $(a + b)$ with velocity $\sqrt{\mu}\,/(a + b)$. Show that the equation of its path is $r = a + b\cos\theta$.
5. A particle is moving with central acceleration $\mu(r^5 - c^4 r)$ being projected from an apse at a distance c with velocity $c^3\sqrt{\frac{2\mu}{3}}$, show that its path is the curve $x^4 + y^4 = c^4$.
6. If the law of force be $\mu\left(u^4 - \frac{10}{9}\,au^5\right)$ and the particle be projected from an apse at a distance $5a$ with a velocity equal to $\sqrt{\frac{5}{7}}$ of that in a circle at the same distance, show that the orbit is the limacon $r = a(3 + 2\cos\theta)$.
7. In a central orbit, the force is $\mu u^3(3 + 2a^2u^2)$; if the particle be projected at a distance a with a velocity $\sqrt{\frac{5\mu}{a^2}}$ in a direction making an angle $\tan^{-1}\left(\frac{1}{2}\right)$ with the radius vector, show that the equation to the path is $r = a\tan\theta$.
8. A particle moves in a plane under a central force which varies inversely as the square of the distance from the fixed point, find the orbit.
9. If v_1 *and* v_2 are the linear velocities of a planet when it is respectively nearest and farthest from the sun, prove that $(1 - e)v_1 = (1 + e)v_2$.
10. A particle describes an ellipse under a force to the focus S. When the particle is at one extremity of the minor axis, its kinetic energy is doubled without any change in the direction of motion. Prove that the particle proceeds to describe a parabola.

11. A body is describing an ellipse of eccentricity e under the action of a force tending to a focus and when at the nearer apse the centre of the force is transferred to the other focus. Prove that the eccentricity of the new orbit is $e(3+e)/(1-e)$.

12. If a planet were suddenly stopped in its orbit supposed circular, show that it would fall into the sun in a time which is $\sqrt{2}/8$ times the period of the planet's revolution.