Tutorial Sheet: II

Equations of Motion of Inviscid fluids

1. Derive Euler's dynamical equations of motion in cartesian coordinates.

2. A liquid is contained between two parallel planes, the free surface is circular cylinder of radius 'a' whose axis is perpendicular to the planes. All the liquid within a concentric circular cylinder of radius 'b' is suddenly annihilated; prove that if Π be the pressure at the outer surface, the initial pressure at any point on the liquid distant 'r' from the center is $\Pi\left(\frac{\log r-\log b}{\log a-\log b}\right)$.

3. A mass of liquid of density ρ whose external surface is a long circular cylinder of radius a which is subject to a constant pressure Π, surrounds a coaxial long circular cylinder of radius b. The internal cylinder is suddenly destroyed; show that if v is the velocity at the internal surface, when the radius is r, then $v^2=\left(\frac{2\Pi(b^2-r^2)}{\rho r^2\left\{\log\frac{(r^2+a^2-b^2)}{r^2}\right\}}\right)$.

4. A sphere is at rest in an infinite mass of homogeneous liquid of density ρ, the pressure at infinity being P. If the radius R of the sphere varies in such a way that $R=a+b\cos nt\,, where\ b>a,$ show that pressure at the surface of the sphere at any time is $P+\frac{bn^2\rho}{4}(b-4a\cos nt-5b\cos 2nt)$.

5. A steady inviscid incompressible fluid flow has a velocity field $u=fx, v=-fy, w=0,$ where f is a constant. Derive an expression for the pressure field $p(x,y,z)$ if the pressure $p(0,0,0)=p_0$ and $\boldsymbol{F}=-gz\boldsymbol{\hat{k}}$.

6. An infinite mass of fluid is acted on by a force $\frac{\mu}{r^{\frac{3}{2}}}$ per unit mass directed to the origin. If initially, the fluid is at rest and there is cavity in the form of the sphere $r=c$ in it, show that the cavity will be filled up after an interval of time$\left(\frac{2}{5\mu}\right)^{\frac{1}{2}}c^{\frac{5}{4}}$.

7. An infinite fluid in which a spherical hollow of radius a is initially at rest under the action of no forces. If a constant pressure Π is applied at infinity, show that the time of filling up the cavity is $a\left(\frac{\pi\rho}{6\Pi}\right)^{\frac{1}{2}}\frac{\Gamma\left(\frac{5}{6}\right)}{\Gamma\left(\frac{4}{3}\right)}$.

8. A mass of fluid of density ρ and volume $\left(\frac{4}{3}\right)\pi c^3$ is in the form of spherical shell. A constant pressure Π is exerted on the external surface of the shell. There is no pressure on the internal surface and no other forces act on the liquid. Initially the liquid is at rest and the internal radius of the shell is 2c. Prove that the velocity of the internal surface when its radius is c, is $\left(\frac{14\Pi}{3\rho}\frac{2^{\frac{1}{3}}}{2^{\frac{1}{3}}-1}\right)^{\frac{1}{2}}$.

A mass of perfect incompressible fluid of density ρ is bounded by concentric spherical surfaces. The outer surface is contained by a flexible envelope which exerts continuously uniform pressure Π and contracts from radius R_1 *to radius* R_2. The hollow is filled with a gas obeying Boyle's law, its radius contracts from c_1 *to* c_2 and the pressure of the gas is initially, p_1. Initially the whole mass is at rest. Prove that, neglecting the mass of the gas, the velocity v of the inner surface when the configuration R_2, c_2 is reached is given by $\frac{v^2}{2} = \frac{c_1^3}{c_2^3}\left\{\frac{1}{3}\left(1-\frac{c_2^3}{c_1^3}\right)\frac{\Pi}{\rho}-\frac{p_1}{\rho}\log\frac{c_1}{c_2}\right\}/(1-\frac{c_2}{R_2})$