Mathematics Optional Foundation Course

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Previous Year Questions: Linear Programming (2008-2022)

Formulation of LPP

1. A paint factory produces both interior and exterior paint from two raw materials M_1 and M_2.The basic data is as follows:

	Tons of Raw material per ton of		Maximum daily availability
	Exterior Paint	Interior Paint	
Raw Material M_1	6	4	24
Raw Material M_2	1	2	6
Profit per Ton (1000)	5	4	

A market survey indicates that the daily demand of interior paint cannot exceed that of exterior paint by more than 1 ton. The maximum daily demand of interior paint is 2 tons. The factory wants to determine the optimum product mix of interior and exterior paint that maximizes daily profits. Formulate the LP problem for this situation.

2. For each hour per day that Ashok studies mathematics, it yields him 10 marks and for each hour that he studies physics, it yields him 5 marks. He can study at most 14 hours a day and he must get at least 40 marks in each. Determine graphically how many hours a day he should study mathematics and physics each, in order to maximize his marks?
3. An agricultural firm has 180 tons of nitrogen fertilizer, 250 tons of phosphate and 220 tons of potash. It will be able to sell a mixture of these substances in their respective ratio 3: 3: 4 at a profit of Rs.1500 Per ton and a mixture in the ratio 2: 4: 2 at a profit of Rs. 1200 per ton. Pose a linear programming problem to show how many tons of these two mixtures should be prepared to obtain the maximum profit.
4. How many basic solutions are there in the following linearly independent set of equations? Find all of them.

   $2x_1 - x_2 + 3x_3 + x_4 = 6;\ 4x_1 - 2x_2 - x_3 + 2x_4 = 10$

5. UPSC maintenance section has purchased sufficient number of curtain cloth pieces for curtain requirement of its building. The length of each piece is 17 feet. The requirement according to curtain length is as follows:

Curtain Length (in feet)	Number Required
5	700
9	400
7	300

The width of all curtains is same as that of available pieces. Form a linear programming problem in standard from that decide the number of pieces cut in different ways so that the total trim loss is minimum. Also give a basic feasible solution to it.

Graph Method

1. Solve Graphically $Maximise\ Z = 6x_1 + 5x_2$

$$Subject\ to: \begin{array}{c} 2x_1 + x_2 \le 16, \\ x_1 + x_2 \le 11 \\ x_1 + 2x_2 \ge 6 \\ 5x_1 + 6x_2 \le 90 \\ x_1, x_2 \ge 0 \end{array}$$

2. Find the maximum value of $5x + 2y$ with constraints $x + 2y \ge 1, 2x + y \le 1, x \ge 0\ and\ y \ge 0$ by graphical method.
3. Using graphical method, find the maximum value of $2x + y$ Subject to $4x + 3y \le 12$, $4x + y \le 8$, $4x - y \le 8$, $x, y \ge 0$
4. Using graphical method, solve the linear programming problem.

$$Maximise\ Z = 3x_1 + 2x_2$$

$$\text{Subject to } \begin{array}{c} x_1 - x_2 \ge 1, \\ x_1 + x_3 \ge 3 \\ x_1, x_2, x_3 \ge 0 \end{array}$$

Simplex Method

1. $Maximise\ Z = 3x_1 + 5x_2 + 4x_3$

$$Subject\ to: \begin{array}{c} 2x_1 + 3x_2 \le 8, \\ 3x_1 + 2x_2 + 4x_3 \le 15 \\ 2x_2 + 5x_3 \le 10 \\ x_1, x_2, x_3 \ge 0 \end{array}$$

2. Solve by simplex method, the following LP Problem:

$$Maximise\ Z = 5x_1 + 3x_2$$

$$Subject\ to: \begin{array}{c} 3x_1 + 5x_2 \le 15, \\ 5x_1 + 2x_2 \le 10 \\ x_1, x_2 \ge 0 \end{array}$$

3. $Maximise \;\; Z = 2x_1 + 3x_2 - 5x_3$

$$Subject\ to: \begin{array}{c} x_1 + x_2 + x_3 = 7, \\ 2x_1 - 5x_2 + x_3 \geq 10 \\ x_1, x_2, x_3 \geq 0 \end{array}$$

4. $Minimise\ Z = 5x_1 - 4x_2 + 6x_3 - 8x_4$, subject to the constraints, $\begin{array}{c} x_1 + 2x_2 - 2x_3 + 4x_4 \leq 40 \\ 2x_1 - x_2 + x_3 + 2x_4 \leq 8 \\ 4x_1 - 2x_2 + x_3 - x_4 \leq 10 \\ x_1, x_2, x_3, x_4 \geq 0 \end{array}$

5. Find all *optimal solutions* of the following linear programming problem by the *simplex method*:

$$Maximise \;\; Z = 30x_1 + 24x_2$$

$$Subject\ to: \begin{array}{c} 5x_1 + 4x_2 \leq 200, \\ x_1 \leq 32 \\ x_2 \leq 40 \\ x_1, x_2 \geq 0 \end{array}$$

6. Consider the following linear programming problem:

   Maximise: $x_1 + 2x_2 - 3x_3 + 4x_4$

   Subject to $\begin{array}{c} x_1 + x_2 + 2x_3 + 3x_4 = 12 \\ x_2 + 2x_3 + x_4 = 8 \\ x_1, x_2, x_3, x_4 \geq 0 \end{array}$

   a. Using the definition, find its all *basic solutions*. Which of these are *degenerate basic feasible solutions* and which *are non-degenerate basic feasible solutions*?
   b. Without solving the problem, show that it has an optimal solution and which of the *basic feasible solution(s)* is/are *optimal*?

7. $Maximise \;\; Z = 2x_1 + 3x_2 + 6x_3$

$$Subject\ to: \begin{array}{c} 2x_1 + x_2 + x_3 \leq 5, \\ 3x_2 + 2x_3 \leq 6 \\ x_1, x_2, x_3 \geq 0 \end{array}$$

   Is the optimum solution unique? Justify the answer.

8. Solve the following linear programming problem by simplex method:

$$Maximise \;\; Z = 3x_1 + 5x_2 + 4x_3$$

$$Subject\ to: \begin{array}{c} 2x_1 + 3x_2 \leq 8, \\ 2x_2 + 5x_3 \leq 10 \\ 3x_1 + 2x_2 + 4x_3 \leq 15 \\ x_1, x_2, x_3 \geq 0 \end{array}$$

9. Solve the following liner programming problem by Big M-method and show that the problem has finite optimal solutions. Also Find the value of the objective Function:

   $Min\ Z = 3x_1 + 5x_2$, subject to the constraints, $\begin{array}{c} x_1 + 2x_2 \geq 8 \\ 3x_1 + 2x_2 \geq 12 \\ 5x_1 + 6x_2 \leq 60 \\ x_1, x_2 \geq 0 \end{array}$

NEXT IAS

10. Solve the linear programming problem using Simplex Method

Minimise: $x_1 + 2x_2 - 3x_3 - 2x_4$

Subject to
$$\begin{gathered} x_1 + 2x_2 - 3x_3 + x_4 = 4 \\ x_1 + 2x_2 - x_3 + 2x_4 = 4 \\ x_1, x_2, x_3, x_4 \geq 0 \end{gathered}$$

11. Solve the linear programming problem using simplex method:

$$Minimise\ \ Z = -6x_1 - 2x_2 - 5x_3$$

$$Subject\ to: \begin{gathered} 2x_1 - 3x_2 + x_3 \leq 14, \\ -4x_1 + 4x_2 + 10x_3 \leq 46 \\ 2x_1 + 2x_2 - 4x_3 \leq 37 \\ x_1 \geq 2, x_2 \geq 1, x_3 \geq 3 \end{gathered}$$

12. Solve the following linear Programming problem using Big M method:

$$Maximise\ \ Z = 4x_1 + 5x_2 + 2x_3$$

$$Subject\ to: \begin{gathered} 2x_1 + x_2 + x_3 \geq 10, \\ x_1 + 3x_2 + x_3 \leq 12 \\ x_1 + x_2 + x_3 = 6 \\ x_1, x_2, x_3 \geq 0 \end{gathered}$$

13. Use two phase method to solve the following linear programming problem:

Minimise $z = x_1 + x_2$

Subject to
$$\begin{gathered} 2x_1 + x_2 \geq 4 \\ x_1 + 7x_2 \geq 7 \\ x_1, x_2 \geq 0 \end{gathered}$$

Duality

1. Find the dual of the following linear programming problem:

$$Max\ Z = 2x_1 - x_2 + x_3$$

$$Subject\ to: \begin{gathered} x_1 + x_2 - 3x_3 \leq 8 \\ 4x_1 - x_2 + x_3 = 2 \\ 2x_1 + 3x_2 - x_3 \geq 5 \\ x_1, x_2, x_3 \geq 0 \end{gathered}$$

2. Construct the dual of the primal problem: $Maximize\ Z = 2x_1 + x_2 + x_3$, subject to the constraints, $\begin{array}{c} x_1 + x_2 + x_3 \geq 6 \\ 3x_1 - 3x_2 + 2x_3 = 3 \\ -4x_1 + 3x_2 - 6x_3 = 1 \\ x_1, x_2, x_3 \geq 0 \end{array}$

3. Write down the dual of the following LP problem and hence solve it by graphical method

   $Min\ Z = 6x_1 + 4x_2$, subject to the constraints, $\begin{array}{c} 2x_1 + x_2 \geq 1 \\ 3x_1 + 4x_2 \geq 1.5 \\ x_1, x_2 \geq 0 \end{array}$

4. Solve the following linear programming problem by the *simplex method*. Write its dual. Also, write the optimal solution of the dual from the optimal table of the given problem:

   $Maximize\ Z = 2x_1 - 4x_2 + 5x_3$, subject to the constraints, $\begin{array}{c} x_1 + 4x_2 - 2x_3 \leq 2 \\ -x_1 + 2x_2 + 3x_3 \leq 1 \\ x_1, x_2, x_3 \geq 0 \end{array}$

5. Consider the following LPP,

   Maximise: $2x_1 + 4x_2 + 4x_3 - 3x_4$

   Subject to $\begin{array}{c} x_1 + x_2 + x_3 = 4 \\ x_1 + 4x_2 + x_4 = 8 \\ x_1, x_2, x_3, x_4 \geq 0 \end{array}$

   Use the dual problems to verify that the basic solution (x_1, x_2) is not optimal.

6. Convert the following LPP into dual LPP:

   $Minimize\ Z = x_1 - 3x_2 - 2x_3$, subject to the constraints, $\begin{array}{c} 3x_1 - x_2 - 2x_3 \leq 7 \\ 2x_1 - 4x_2 \geq 12 \\ -4x_1 + 3x_2 + 8x_3 = 10 \\ x_1, x_2, x_3 \geq 0 \end{array}$

   Where $x_1, x_2 \geq 0\ and\ x_3$ is unrestricted in sign.

7. Solve the following linear programming problem by the simplex method. Write its dual. Also, write the optimal solution of the dual from the optimal table of the given problem:

   $Maximise\ \ Z = x_1 + x_2 + x_3$

   $Subject\ to: \begin{array}{c} 2x_1 + x_2 + x_3 \leq 2, \\ 4x_1 + 2x_2 + x_3 \leq 2 \\ 3x_1 + 2x_2 + 4x_3 \leq 15 \\ x_1, x_2, x_3 \geq 0 \end{array}$ (15, 2022)

Transport Problem

1. Solve the following transportation problem

		Destination						
		D_1	D_2	D_3	D_4	D_5	D_6	**Availability**
Factories	F_1	2	1	3	3	2	5	50
	F_2	3	2	2	4	3	4	40
	F_3	3	5	4	2	4	1	60
	F_4	4	2	2	1	2	2	30
Demand		30	50	20	40	30	10	

by finding the initial solution by Matrix Minima Method.

2. Determine an optimal transportation programme so that the transportation cost of 340 tons of a certain type of material from three factories to five warehouses W_1, W_2, W_3, W_4, W_5 is minimized. The five warehouses must receive 40 tons, 50 tons, 70 tons, 90 tons and 90 tons respectively. The availability of the material at F_1, F_2, F_3, is 100 tons, 120 tons, 120 tons respectively. The transportation costs per ton from factories to warehouses are given in the table below:

	W_1	W_2	W_3	W_4	W_5
F_1	4	1	2	6	9
F_2	6	4	3	5	7
F_3	5	2	6	4	8

Use Vogel's Approximation Method to obtain the initial basic feasible solution.

3. By the method of Vogel, determine an initial basic feasible solution for the following transportation problem: Products P_1, P_2, P_3 & P_4 have to be sent of destinations D_1, D_2 *and* D_3. The cost of sending product P_i to destinations D_j is C_{ij}, where the matrix

$$[C_{ij}] = \begin{bmatrix} 10 & 0 & 15 & 5 \\ 7 & 3 & 6 & 15 \\ 0 & 11 & 9 & 13 \end{bmatrix}$$

The total requirements of destinations D_1, D_2 *and* D_3 are given by 45, 45, 95 respectively and the availability of the products P_1, P_2, P_3 & $P_4$are respectively 25, 35, 55 and 70.

4. Find the initial basic feasible solution to the following transportation problem by Vogel's approximation method. Also, find its optimal solution and the minimum transportation cost.

		D_1	D_2	D_3	D_4	**Supply**
origins	O_1	6	4	1	5	14
	O_2	8	9	2	7	16
	O_3	4	3	6	2	5
	Demand	6	10	15	4	

5. Find the initial basic feasible solution of the following transportation problem using Vogel's approximation methods and find the cost.

		D_1	D_2	D_3	D_4	D_5	**Supply**
	O_1	4	7	0	3	6	14
origins	O_2	1	2	-3	3	8	9
	O_3	3	-1	4	0	5	17
	Demand	8	3	8	13	8	

6. Find the initial basic feasible solution of the following transportation problem by Vogel's approximation method and use it to find the optimal solution and the transportation cost of the problem.

		D_1	D_2	D_3	D_4	**Supply**
Sources	S_1	10	0	20	11	15
	S_2	12	8	9	20	25
	S_3	0	14	16	18	10
	Demand	5	20	15	10	

7. Find the initial basic feasible solution of the following transportation problem by Vogel's approximation method and use it to find the optimal solution and the transportation cost of the problem:

Destination

		A	B	C	D	
Source	S_1	21	16	25	13	11
	S_2	17	18	14	23	13
	S_3	32	27	18	41	19
	Requirement	6	10	12	15	43

(20, 2022)

Assignment Problem

1. Solve the minimum time assignment problem:

Machines

	M_1	M_2	M_3	M_4
J_1	3	12	5	14
J_2	7	9	8	12
J_3	5	11	10	12
J_4	6	14	4	11

2. Solve the assignment problem to maximise the sale:

Territories

		I	**II**	**III**	**IV**	**V**
Salesmen	**A**	3	4	5	6	7
	B	4	15	13	7	6
	C	6	13	12	5	11
	D	7	12	15	8	5
	E	8	13	10	6	9

3. In a factory there are five operators O_1, O_2, O_3, O_4, O_5 and five machines M_1, M_2, M_3, M_4, M_5 . The operating costs are given when the operator O_i operates the M_j machine. There are restrictions that O_3 cannot be allowed to operate the M_3 and O_2 cannot be allowed to operate the M_5. The cost matrix is given below. Find the optimal assignment and the optimal assignment cost also.

Machines

	M_1	M_2	M_3	M_4	M_5
O_1	24	29	18	32	19
O_2	17	26	34	22	21
O_3	27	16	28	17	25
O_4	22	18	28	30	24
O_5	28	16	31	24	27

4. A department of company has five employees with five jobs to be performed. The time (in hours) that each man takes to perform each job is given in the effectiveness matrix. Assign all the jobs to these five employees to minimize the total processing time:

		Employees				
		I	**II**	**III**	**IV**	**V**
Jobs	**A**	10	5	13	15	16
	B	3	9	18	13	6
	C	10	7	2	2	2
	D	7	11	9	7	12
	E	7	9	10	4	12

NEXT IAS