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Previous Year Questions: ODE (2008-22)

Formation of DE

1. Find the Wronskian of the set of functions $\{3x^3, |x^3|\}$ on the interval [-1,1] and determine whether the set is linearly dependent on [-1,1]. (12)

2. Find the differential equation of the family of circles in the xy plane passing through $(-1,1)$ and $(1,1)$. (10)

3. Determine the orthogonal trajectory of a family of curves represented by the polar equation $r = a(1 - \cos\theta), (r, \theta)$ being the polar coordinates of any point. (10)

4. Find the orthogonal trajectories of family of curves $x^2 + y^2 = ax$. (12)

5. Obtain the equation of the orthogonal trajectory of the family of curves represented by $r^n = a \sin n\theta\,, (r, \theta)$ being the plane polar coordinates. (10)

6. Find the curve for which the part of the tangent cut off by the axes is bisected at the point of tangency. (10)

7. Show that the family of parabolas $y^2 = 4cx + 4c^2$ is self-orthogonal. (10)

8. Find the DE representing all the circles in the xy plane. (10)

9. If the growth rate of the population of bacteria at any time t is proportional to the amount present at that time and population doubles in one week, then how much bacterias can be expected after 4 weeks? (08)

10. Find the orthogonal trajectories of the family of circles passing through the points (0, 2) and (0,-2). (10)

11. Find the orthogonal trajectories of the family of confocal conics $\frac{x^2}{a^2+\lambda} + \frac{y^2}{b^2+\lambda} = 1; a > b > 0$ are constants, and λ is a parameter. Show that the given family of curves is self- orthogonal. (10M, 2021)

12. Show that the orthogonal trajectories of the system of parabolas: $x^2 = 4a(y + a)$ belong to the same system. (10, 2022)

First Order and First Degree

1. Solve the differential equation $ydx + (x + x^3y^2)dy = 0$. (12)

2. Consider the differential equation $y' = \alpha x, x > 0$ where α is constant. Show that

   i) If $\phi(x)$ is any solution and $\psi(x) = \phi(x)e^{-\alpha x}$, then $\psi(x)$ is a constant;

   ii) If $\alpha < 0$, then every solution tends to zero as $x \to \infty$. (12)

3. Show that the differential equation $(3y^2 - x) + 2y(y^2 - 3x)y' = 0$ admits an integrating factor which is a function of $(x + y^2)$. Hence solve the equation. (12)

4. Verify that $\frac{1}{2}(Mx + Ny)d(\ln xy) + \frac{1}{2}(Mx - Ny)\, d\ln\frac{x}{y} = Mdx + Ndy$. Hence show that –

   i) If the differential equation $Mdx + Ndy = 0$ is homogeneous, then $Mx + Ny$ is an integrating factor unless $Mx + Ny = 0$;

   ii) If the differential equation $Mdx + Ndy = 0$ is not exact but is of the form $f_1(xy)ydx + f_2(xy)xdy = 0$ then $(Mx - Ny)^{-1}$ is an integrating factor unless $Mx - Ny = 0$. (20)

5. Obtain the solution of ordinary differential equation $\frac{dy}{dx} = (4x + y + 1)^2,\ if\ y(0) = 1$. (10)

6. Solve $\frac{dy}{dx} = \frac{2xye^{\left(\frac{x}{y}\right)^2}}{y^2\left(1+e^{\left(\frac{x}{y}\right)^2}\right)+2x^2e^{\left(\frac{x}{y}\right)^2}}$ (12)

7. Show that the DE $(2xy \log y)dx + \left(x^2 + y^2\sqrt{y^2+1}\right)dy = 0$ is not exact. Find an integrating factor and hence, the solution of the equation. (20)

8. y is a function of x such that the differential coefficient $\frac{dy}{dx}$ is equal to $\cos(x + y) + \sin(x + y)$. Find out a relation between x and y, which is free from any derivative. (10)

9. Solve the DE $(5x^3 + 12\,x^2 + 6y^2)\,dx + 6xy\,dy = 0$. (10)

10. Justify that a DE of the form: $[y + xf(x^2 + y^2)\,]dx + [yf(x^2 + y^2) - x]\,dy = 0$, where $f(x^2 + y^2)$ is an arbitrary function of $(x^2 + y^2)$, is not exact DE and $\frac{1}{x^2+y^2}$ is an integrating factor for it. Hence solve this DE for $f(x^2 + y^2) = (x^2 + y^2)^2$. (10)

11. Find the sufficient condition for the DE $M\,dx + N\,dy = 0$ to have an integrating factor as a function of *(x+y)*. What will be the IF in that case? Hence find the IF for the DE $(x^2 + xy)\,dx + (y^2 + xy)\,dy = 0$ and solve it. (15)

12. Solve the DE $x\cos x\frac{dy}{dx}+y\,(x\sin x+\cos x)=1.$ (10)

13. Solve the DE $(2xy^4e^y+2xy^3+y)\,dx+(x^2y^4e^y-x^2y^2-3x)dy=0.$ (10)

14. Find the constant a such that $(x+y)^a$ is the IF of $(4x^2+2xy+6y)dx+(2x^2+9y+3x)dy=0$ and hence solve the DE. (12)

15. Solve $\frac{dy}{dx}=\frac{1}{1+x^2}(e^{\tan^{-1}x}-y)$ (10)

16. Solve : $\{y(1-x\tan x)+x^2\cos x\}dx-xdy=0$ (10)

17. Solve the following simultaneous linear DEs: $(D+1)y=z+e^x$ *and* $(D+1)z=y+e^x$(8)

18. Find α *and* β such that $x^\alpha y^\beta$ is an integrating factor of $(4y^2+3xy)dx-(3xy+2x^2)$ $dy=0$ and solve the equation.

19. Find $f(y)$ such that $(2xe^y+3y^2)dy+\left(3x^2+f(y)\right)dx=0$ is exact and hence solve. (12)

20. Solve the DE $(2y\sin x+3y^4\sin x\cos x)\,dx-(4y^3\cos^2 x+\cos x)dy=0.$ (10)

21. Solve the DE (10)

$$x\cos\left(\frac{y}{x}\right)(ydx+xdy)=y\sin\left(\frac{y}{x}\right)(xdy-ydx)$$

22. Show that the general solution of the differential equation $\frac{dy}{dx}+Py=Q$ can be written in the form $y=\frac{Q}{P}-e^{-\int Pdx}\left\{C+\int e^{Pdx}d\left(\frac{Q}{P}\right)\right\}$, where P and Q are non-zero functions of x and C, an arbitrary constant. (10, 2022)

First order but not of first Degree

1. Solve the equation $y-2xp+yp^2=0.$ (15)

2. Solve $\frac{dy}{dx}=\frac{y^2(x-y)}{3xy^2-x^2y-4y^3}$,*y(0)=1.* (20)

3. Obtain the Clairaut's form of the differential equation $\left(x\frac{dy}{dx}-y\right)\left(y\frac{dy}{dx}+y\right)=a^2\frac{dy}{dx}.$ Also find its general solution.

4. Solve the DE: $x=py-p^2$.

5. Consider the DE $xyp^2-(x^2+y^2-1)p+xy=0.$ Substituting $u=x^2$ *and* $v=y^2$ reduce the equation to Clairaut's form. Hence or otherwise solve the equation. (10)

6. Solve $\left(\frac{dy}{dx}\right)^2y+2\frac{dy}{dx}x-y=0$

7. Find all the possible solutions of DE: $y^2\log y=xy\frac{dy}{dx}+\left(\frac{dy}{dx}\right)^2.$ (15M)

Singular Solution

1. Obtain the singular solution of the DE

$$\left(\frac{dy}{dx}\right)^2\left(\frac{y}{x}\right)^2\cot^2\alpha-2\left(\frac{dy}{dx}\right)\left(\frac{y}{x}\right)+\left(\frac{y}{x}\right)^2 cosec^2\alpha=1$$

2. Also find the complete primitive of the given DE. Give the geometrical interpretations of the complete primitive and singular solution. (15)

3. Find the general and singular solutions of the DE: $9p^2(2-y)^2=4(3-y)$. (10)

4. Find the general and singular solutions of the differential equation: $(x^2-a^2)p^2-2xyp+y^2+a^2=0$, where $p=\frac{dy}{dx}$. Also find the geometric relation between the general and singular solutions. (10, 2022)

Second Order and higher but constant Coefficient

1. Obtain the general solution of the 2nd order ODE $y''-2y'+2y=x+e^x\cos x$. (15)

2. Find the general solution of the equation $y'''-y''=12x^2+6x$. (20)

3. Find a particular integral of $\frac{d^2y}{dx^2}+y=e^{\frac{x}{2}}\sin\frac{x\sqrt{3}}{2}$. (10)

4. Using the method of variation of parameters, solve the DE $(D^2+2D+1)y=e^{-x}\log x$. (15)

5. Find the initial value differential equation: $20y''+4y'+y=0, y(0)=3.2\ and\ y'(0)=0$. (7)

6. Using the method of variation of parameters, solve the DE (08)

$$\frac{d^2y}{dx^2}-\frac{dy}{dx}-2y=44-76x-48x^2.$$

7. Solve : $y''-y=x^2e^{2x}$ (10)

8. Solve : $y'''-6y''+12y'-8y=12e^{2x}+27e^{-x}$ (10)

9. Solve: $y''+16y=32\sec 2x$. (13)

10. Solve the IVP :

$$y''-5y'+4y=e^{2t}, y(0)=\frac{19}{12}\ , y'(0)=\frac{8}{3}$$

11. Determine the complete solution of the DE $\frac{d^2y}{dx^2}-4\frac{dy}{dx}+4y=3x^2e^{2x}\sin 2x$. (10)

12. Solve the differential equation: $\frac{d^2y}{dx^2}+2y=x^2e^{3x}+e^x\cos 2x$ (10M)

13. Solve the following differential equation: $(3x+2)^2 \frac{d^2y}{dx^2} + 5(3x+2)\frac{dy}{dx} - 3y = x^2 + x + 1$.

(10, 2022)

Variable Coefficient

1. Solve the differential equation $x^3y'' - 3x^2y' + xy = \sin \ln x + 1$. (15)

2. Use the method of variation of parameters to find the general solution of $x^2y'' - 4xy' + 6y = -x^4 \sin x$. (12)

3. Use the method of undetermined coefficients to find the particular solution of $y'' + y = \sin x + (1 + x^2)e^x$ and hence find its general solution. (20)

4. Using the method of variation of parameters, solve the 2nd order DE $\frac{d^2y}{dx^2} + 4y = \tan 2x$.

(15)

5. Solve the ODE $x(x-1)y'' - (2x-1)y' + 2y = x^2(2x-3)$ (20)

6. Using the method of variation of parameters, solve the DE $\frac{d^2y}{dx^2} + a^2y = \sec ax$. (10)

7. Find the general solution of the equation $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = \ln x \sin \ln x$. (15)

8. Solve by method of variation of parameters: $\frac{dy}{dx} - 5y = \sin x$. (10)

9. Solve the ODE $x^3\frac{d^3y}{dx^3} + 3x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + 8y = 65 \cos \log x$. (20)

10. Solve the ODE: $x\frac{d^2y}{dx^2} - 2(x+1)\frac{dy}{dx} + (x+2)y = (x-2)e^{2x}$, when e^x is a solution to its corresponding homogeneous DE.

11. Solve $x^4\frac{d^4y}{dx^4} + 6x^3\frac{d^3y}{dx^3} + 4x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} - 4y = x^2 + 2\cos \ln x$.

12. Find the general solution of the equation $x^2\frac{d^3y}{dx^3} - 4x\frac{d^2y}{dx^2} + 6\frac{dy}{dx} = 4$. (15)

13. Solve the DE $x\frac{d^2y}{dx^2} - \frac{dy}{dx} - 4x^3y = 8x^3 \sin x^2$. (9)

14. Solve $(1+x)^2y'' + (1+x)y' + y = 4\cos\log(1+x)$. (13)

15. Solve the DE $\frac{d^2y}{dx^2} + (3\sin x - \cot x)\frac{dy}{dx} + 2y\sin^2 x = e^{-\cos x}\sin^2 x$.

16. Find the linearly independent solution of the corresponding homogeneous DE of the equation $x^2y'' - 2xy' + 2y = x^3 \sin x$ and then find the general solution of the given equation by the method of the variation of parameters. (15)

17. Using the method of variation of parameters, solve the DE (20)

$$y'' + (1 - \cot x)y' - y\cot x = \sin^2 x\,, if\ y = e^{-x}\ is\ one\ solution\ of\ CF.$$

18. Solve the following DE: (10)

$$(x+1)^2y'' - 4(x+1)y' + 6y = 6(x+1)^2 + \sin\log(x+1).$$

19. Solve the equation: $\frac{d^2y}{dx^2} + (\tan x\ - 3\cos x)\frac{dy}{dx} + 2y\cos^2 x = \cos^4 x$ completely by demonstrating all the steps involved. (15M)

20. Find the general solution of the differential equation $x^2\frac{d^2y}{dx^2} - 2x(1+x)\frac{dy}{dx} + 2(1+x)y = 0$. Hence solve the differential equation $x^2\frac{d^2y}{dx^2} - 2x(1+x)\frac{dy}{dx} + 2(1+x)y = x^3$ by the Method of variation of Parameters. (10M)

21. Solve the following differential equation by using the method of variation of parameters: $(x^2-1)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = (x^2-1)^2$, given that $y = x$ is one solution of the reduced equation. (15, 2022)

<u>Laplace Transform</u>

1. Using Laplace Transform, solve the initial value problem $y'' - 3y' + 2y = 4t + e^{3t}$ with $y(0) = 1, y'(0) = -1$. (15)

2. Find the inverse Laplace Transform of $F(s) = \ln\frac{s+1}{s+5}$. (20)

3. Use Laplace Transform method to solve the following initial value problem:

   $\frac{d^2x}{dt^2} - 2\frac{dx}{dt} + x = e^t, x(0) = 2$ and $\frac{dx}{dt}(0) = -1$. (15)

4. Using Laplace transform, solve the IVP $y'' + 2y' + y = e^{-t}, y(0) = -1, y'(0) = 1$. (12)

5. Using Laplace transform, solve the DE $(D^2 + n^2)x = a\sin(nt + \alpha)$, subject to initial conditions $x = 0\ and\ \frac{dx}{dt} = 0\ at\ t = 0$ in which a, n, α are constants.

6. Solve the IVP: $\frac{d^2y}{dt^2} + y = 8e^{-2t}\sin t\,, y(0) = 0, y'(0) = 0$ by using Laplace transform. (15)

7. Obtain the Laplace Inverse transform of $\left\{\ln\left(1 + \frac{1}{s^2}\right) + \frac{s}{s^2+25}\,e^{-\pi s}\right\}$.

8. Using Laplace Transform, solve $y'' + y = t, y(0) = 1, y'(0) = -2$. (12)

9. Using Laplace transform solve the following

   $y'' - 2y' - 8y = 0, y(0) = 3, y'(0) = 6$. (10)

10. Solve the following initial value problem using Laplace Transform:

$$\frac{d^2y}{dx^2} + 9y = r(x), y(0) = 0, y'(0) = 4$$

where $r(x) = \begin{Bmatrix} 8\sin x & if\ 0 < x < \pi \\ 0 & if\ x \geq \pi \end{Bmatrix}$ (17)

11. Find the Laplace transform of $f(t) = \frac{1}{\sqrt{t}}$. (5)

12. Find the inverse Laplace transform of $\frac{5s^2-3s-16}{(s-1)(s-2)(s+3)}$ (05)

13. Find the Laplace Transform of $t^{-\frac{1}{2}}$ and $t^{\frac{1}{2}}$. Prove that Laplace Transform of $t^{n+\frac{1}{2}}$, where $n \in N$, is

$$\frac{\Gamma\left(n+1+\frac{1}{2}\right)}{s^{n+1+\frac{1}{2}}}$$

14. Using Laplace Transform, solve the IVP $ty'' + 2ty' + 2y = 2; y(0) = 1\ and\ y'(0)$ is arbitrary. Does this problem has a unique solution. (10)

15. Solve the initial value problem $\frac{d^2y}{dx^2} + 4y = e^{-2x}\sin 2x, y(0) = y'(0) = 0$ using Laplace Transform method.

16. Solve the following initial value problem by using Laplace's transformation $\frac{d^2y}{dt^2} - 3\frac{dy}{dt} + 2y = h(t)$, where $h(t) = \begin{cases} 2, & 0 < t < 4 \\ 0, & t > 4 \end{cases}$ $y(0) = 0, y'(0) = 0.$ (15, 2022)

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