NEXT IAS

Tutorial Sheet: Statics- I

Introduction

1. The greatest resultant which two forces can have is P and the least is Q. Show that if they act at an angle, the resultant is of the magnitude $\sqrt{P^2\cos^2\frac{\theta}{2}+Q^2\sin^2\frac{\theta}{2}}$.
2. Forces P and Q act at O and have a resultant R. If any transversal cuts their line of action at A, B, C respectively, then show that $\frac{P}{OA}+\frac{Q}{OB}=\frac{R}{OC}$.
3. A string of length l is fastened to two points A and B at the same level at a distance a apart. A ring of weight W can slide on the string, and a horizontal force F is applied to it such that the ring is in equilibrium vertically below B, prove that $F=aW/l$ and that the tension in the string is $W(l^2+a^2)/2l^2$.
4. One end of a light inextensible string of length of length l is fastened to the highest point of smooth circular wire of radius a kept fixed in vertical plane. The other end of the string is attached to a small heavy ring of weight W which slides on the wire. Find the tension of the string and the reaction of the wire.
5. ABC is a triangle. Forces *P, Q, R* acting along the lines OA, OB, OC are in equilibrium. Prove that if O is the orthocenter of the triangle ABC, then $\frac{P}{a}=\frac{Q}{b}=\frac{R}{c}$.

Equilibrium of Rigid Body

1. Three forces *P, 2P, 3P* act along the sides AB, BC, CA of an equilateral triangle ABC; find the magnitude and direction of their resultant, and find also the point in which its line of action meet the side BC.
2. Three forces *P, Q, R* act along the sides BC, CA and AB of an equilateral triangle ABC, taken in order. If their resultant passes through the incentre of $\Delta\, ABC$, then prove that $P+Q+R=0$.
3. Three forces *P, Q, R* act along the sides of a triangle formed by the line *x+y=1, y-x=1 and y=2.* Find the equation of the line of action of the resultant.
4. Weights W_1, W_2 are fastened to a light inextensible string ABC at the points B, C, the end A being fixed. Prove that, if a horizontal force P is applied at C and in equilibrium AB and BC are inclined at angles θ, ϕ to the vertical, then $P=(W_1+W_2)\tan\theta=W_2\tan\phi$.

5. Three equal uniform rods, each of weight W, are smoothly joined so as to form an equilateral triangle. If the system be supported at the middle point of one of the rods, show that the action at the lowest angle is $\sqrt{3}\,W/6$ and that at each of the other is $W\sqrt{\frac{13}{12}}$.
6. A heavy rod AB rest with its two ends on two smooth inclined planes which face each other and are inclined at angles α and β to the horizontal. The center of gravity of the rod divides it into two parts a and b. Find the inclination of the rod to the horizontal and the reactions of the planes.
7. Two uniform rods AB, BC rigidly joined at B so that angle ABC is a right angle, hang freely in equilibrium from a fixed-point A. The lengths of rods are a and b, and their weights are wa and wb. Prove that if AB makes an angle θ with the vertical $\tan\theta = \frac{b^2}{a^2+2ab}$.
8. A uniform bar AB, of weight 2W and length a, is free to turn about a smooth hinge at its upper end A, and a horizontal force is applied to the end B so that bar is in equilibrium with B at a distance l from the vertical through A. Prove that the reaction at the hinge is equal to $W\left[\frac{4a^2-3l^2}{a^2-l^2}\right]^{\frac{1}{2}}$.
9. One end of a uniform rod of weight W is hinged and the other is tied by a string to a point in the same horizontal line as the hinge. If the rod and the string both be inclined at an angle θ to the horizon, in the position of equilibrium, prove that the reaction of the hinge is $(\mathrm{W}/4)\sqrt{9+cot^2\theta}$.
10. A hemisphere of radius a and weight W is placed with its curved surface on a smooth table and a string of length l ($< a$) is attached to a point on its rim and to a point on the table. Find the position of equilibrium and prove that the tension of the string is $\frac{3W}{8}\frac{(a-l)}{\sqrt{2al-l^2}}$.
11. A heavy uniform rod of length $2a$ rests partially inside and partially outside a fixed hemispherical bowl of radius r, the rim of the bowl is horizontal, and one point of the rod is in contact with the rim. If θ be the inclination of the rod to the horizontal, show that $a \cos\theta = 2r cos 2\theta$. Also show that the greatest value of the θ is $\sin^{-1}(\frac{\sqrt{3}}{3})$.
12. A heavy right cone of semi vertical angle α rests in limiting equilibrium with its plane base upon an inclined plane of inclination θ to the horizon. Show that the cone will topple over or not as $\tan\theta > or < 4\tan\alpha$. Examine the case when $\tan\theta = 4\tan\alpha$.