By Avinash Singh (Ex IES, B.Tech IITR)

Tutorial Sheet: Fluid Mechanics-III

Sources and Sinks

1. Prove that the radius of curvature R at any point of the streamline $\psi = const$ is given by $R = \frac{(u^2+v^2)^{\frac{3}{2}}}{\left|u^2\left(\frac{\partial v}{\partial x}\right)-2uv\frac{\partial u}{\partial x}-v^2\frac{\partial u}{\partial y}\right|}$ where u, v are respectively the velocity components of a fluid motion along OX and OY.
2. Show that $u = -\omega y, v = \omega x, w = 0$ represents a possible motion of inviscid fluid. Find the stream function and sketch streamlines. What is basic difference between this motion and one represented by the potential $\phi = A\log r$, where $r = (x^2 + y^2)^{\frac{1}{2}}$.
3. A two-dimensional flow field is given by $\psi = xy$.
   a. Show that the flow is irrotational.
   b. Find the velocity potential.
   c. Verify that ψ *and* ϕ satisfy the Laplace equation
   d. Find the streamlines and potential lines.
4. Find the stream function of the two-dimensional motion due to two equal sources and an equal sink situated midway between them.

Vortex Motion

1. Verify that the stream function ψ and velocity potential ϕ of a two-dimensional vortex flow satisfies the Laplace equation.
2. If two vertices are of same strength and the spin is same in both, show that the relative stream lines are given by $\log(r^4 + a^4 - 2a^2r^2\cos 2\theta) - \left(\frac{r^2}{2a}\right) = const, \theta$ being measured from the join of vertices, the origin being its middle point, 2a being the distance between the vortices.

3. Three parallel rectilinear vortices of the same strength k and in the same sense meet any plane perpendicular to them in an equilateral triangle of side a. show that the vortices all move round the same cylinder with uniform speed in time $(4\pi^2a^2)/3k$.

4. Two point vortices each of strength k are situated at $(\pm a, 0)$ and a point vortex of strength $-k/2$ is situated at the origin. Show that the fluid motion is stationary and find the equations of streamlines. Show that the streamline which passes through the stagnation point meet the x -axis at $(\pm b, 0)$ where $3\sqrt{3}\,(b^2 - a^2)^2 = 16a^3b$.

5. An infinite long line vortex of strength m, parallel to the axis of z, is situated in infinite liquid bounded by a rigid wall in the plane $y = 0$. Prove that, if there be no field of force, the surfaces of equal pressure are given by $\{(x-a)^2 + (y-b)^2\}\{(x-a)^2 + (y+b)^2\} = c\{-(x-a)^2 + (y+b)^2\}$, where (a, b) are the coordinates of the vortex and c is a parametric constant.

6. A vortex pair is situated within a cylinder. Show that it will remain at rest if the distance of either from the centre is given by $\left(\sqrt{5}-2\right)^{\frac{1}{2}} a$, where a is the radius of the cylinder.