# Tutorial Sheet: Fluid Mechanics-I

1. For a two-dimensional flow the velocities at a point in a fluid may be expressed in the Eulerian coordinates by $u = x + y + 2t \text{ and } v = 2y + t$. Determine the Lagrange coordinates as the functions of the initial positions $x_0$ and $y_0$ and the time t.

2. If the velocity distribution is $\boldsymbol{q} = Ax^2y\hat{\boldsymbol{\imath}} + By^2zt\hat{\boldsymbol{\jmath}} + Czt^2\hat{\boldsymbol{k}}$ **,** where A, B, C are constants, then find the acceleration and velocity components.

3. The particles of a fluid move symmetrically in space with regard to a fixed centre; prove that the equation of continuity is $\frac{\partial\rho}{\partial t} + u\frac{\partial\rho}{\partial r} + \frac{\rho}{r^2}\frac{\partial(r^2u)}{\partial r} = 0$, where $u$ is the velocity at distance $r$.

4. A mass of fluid moves in such a way that each particle describes a circle on one plane about a fixed axis; show that the equation of continuity is $\frac{\partial\rho}{\partial t} + \frac{\partial\rho\omega}{\partial\theta} = 0$, where $\omega$ is the angular velocity of a particle whose azimuthal angle is $\theta$ at time $t$.

5. A mass of fluid is in motion so that the lines of motion lie on the surface of co-axial cylinders. Show that the equation of continuity is $\frac{\partial\rho}{\partial t} + \frac{1}{r}\frac{\partial\rho u}{\partial\theta} + \frac{\partial\rho v}{\partial z} = 0$, where $u$,$v$ are the velocity perpendicular and parallel to $z$.

6. Consider a two-dimensional incompressible steady flow field with velocity components in spherical coordinates $(r, \theta, \phi)$ given by $v_r = c_1\left(1 - \frac{3}{2}\frac{r_0}{r} + \frac{1}{2}\frac{r_0^3}{r^3}\right)\cos\theta, v_\phi = 0, v_\theta = -c_1\left(1 - \frac{3}{4}\frac{r_0}{r} - \frac{1}{4}\frac{r_0^3}{r^3}\right)\sin\theta, r \geq r_0 > 0$ where $c_1$ and $r_0$ are arbitrary constants. Is the equation of continuity satisfied?

7. Liquid flows through the pipe whose surface is the surface of revolution of the curve $y = a + \frac{kx^2}{a}$ about the x axis $(-a \leq x \leq a)$. If the liquid enters at the end $x = -a$ of the pipe with velocity V, show that the time taken by a liquid particle to traverse the entire length of the pipe from $x = -a \text{ to } x = a$ is $\left\{\frac{2a}{V(1+k^2)}\right\}\left\{1 + \frac{2k}{3} + \frac{k^2}{5}\right\}$. Assume that $k$ is so small that the fluid remains one dimensional throughout.

8. Show that the surface $\frac{x^2}{a^2k^2t^4} + kt^2\left(\frac{y^2}{b^2} + \frac{z^2}{c^2}\right) = 1$ is a possible form of boundary surface of a liquid at time $t$.

9. Determine the restrictions on $f_1, f_2, f_3$ if $\left(\frac{x^2}{a^2}\right) f_1(t) + \left(\frac{y^2}{b^2}\right) f_2(t) + \left(\frac{z^2}{c^2}\right) f_3(t) = 1$ is a possible boundary surface of a liquid.

10. Show that $\left(\frac{x^2}{a^2}\right) \tan^2 t + \left(\frac{y^2}{b^2}\right) \cot^2 t = 1$ is a possible form for the bounding surface of a liquid, and find an expression for the normal velocity.

11. Find the equation of the streamlines for the flow $\boldsymbol{q} = -(3y^2)\hat{\imath} - 6x\hat{\jmath}$ at the point (1,1).

12. Determine the streamlines and path lines of the particle when the components of velocity field are given by $u = \frac{x}{1+t}, v = \frac{y}{2+t}$ $and$ $w = \frac{z}{3+t}$. Also state the condition for which the streamlines are identical with path lines.

13. If the velocity of an incompressible fluid at the point $(x, y, z)$ is given by $\left(\frac{3xz}{r^5}, \frac{3yz}{r^5}, \frac{3z^2 - r^2}{r^5}\right)$, prove that the liquid motion is possible and that the velocity potential is $\frac{\cos\theta}{r^2}$. Also determine the streamlines.

14. Show that if the velocity potential of an irrotational fluid motion is equal to $A(x^2 + y^2 + z^2)^{-\frac{3}{2}} z \tan^{-1}\left(\frac{y}{x}\right)$, the lines of flow will be on the series of the surfaces $x^2 + y^2 + z^2 = c^{\frac{2}{3}}(x^2 + y^2)^{\frac{2}{3}}$.

If the fluid be in motion with a velocity potential $\phi = z \log r$, and if the density at a point fixed in space be independent of the time, show that the surfaces of equal density are of the forms $r^2 \left\{\log r - \frac{1}{2}\right\} - z^2 = f(\theta, \rho)$, $where$ $\rho$ is the density at $(z, r, \theta)$.