# Previous Year Questions: Linear Algebra (2008-22)

## Vector Space

1. Let S be a non-empty set and let V denote the set of all functions from S into R. Show that V is vector space with respect to the vector addition $(f + g)(x) = f(x) + g(x)$ and scalar multiplication $(c.f)(x) = cf(x)$
2. Find the dimension of the subspace of $R^4$ spanned by the set $\{(1,0,0,0)\ (0,1,0,0)\ (1,2,0,1)\ (0,0,0,1)\}$. Hence find a basis for the subspace. (15)
3. Prove that the set V of the vectors $(x_1, x_2, x_3, x_4)$ in $R_4$ satisfy the equation $x_1 + x_2 + x_3 + x_4 = 0$ and $2x_1 + 3x_2 - x_3 + x_4 = 0$ is a subspace of $R^4$. What is the dimension of this subspace? Find one of its bases. (12)
4. Prove that the set V of all $3 \times 3$ real symmetric matrices form a linear subspace of the space of all $3 \times 3$ real matrices. What is the dimension of this subspace? Find at least of the bases for V.
5. In the space $R^n$. Determine whether or not the set $\{e_1 - e_2, e_2 - e_3, \ldots\ldots, e_{n-1} - e_n, e_n - e_1\}$ is linearly independent.
6. Let $T$ be a linear transformation from a vector space V over real's into V such that
   $T - T^2 = I$. Show that $T$ is invertible.
7. Show that the subspaces of $R^3$ spanned by two sets of vectors $\{(1,1,-1),(1,0,1)\}\ and\ \{(1,2,-3),(5,2,1)\}$ are identical. Also find the dimension of this subspace.
8. Prove or disapprove the following statement: if $B = \{b_1, b_2, b_3, b_4, b_5\}$ is a basis for $R^5$ and V is a two-dimensional subspace of $R^5$, then V has a basis made of two members of B.
9. Let V be the vector space of all $2 \times 2$ matrices over the field of real numbers. Let W be the set consisting of all matrices with zero determinant. Is W a subspace of V? Justify your answer. (8)
10. Show that the vectors $X_1 = (1, 1 + i, i), X_2 = (i, -i, 1 - i)$ and $X_3 = (0, 1 - 2i, 2 - i)$ in $C^3$ are linearly independent over the field of real numbers but are linearly dependent over the field of complex numbers.

11. Let V and W be the following subspaces of $R^4$ : $V = \{(a, b, c, d) : b - 2c + d = 0\}$ and $W = \{(a, b, c, d): a = d, b = 2c\}$. Find a basis and the dimension of $V, W, V \cap W$.

12. The vectors $V_1 = (1, 1, 2, 4), V_2 = (2, -1, -5, 2), V_3 = (1, -1, -4, 0)$ and $V_4 = (2, 1, 1, 6)$ are linearly independent. Is it true? Justify your answer.

13. Find the dimension of the subspace of $R^4$, spanned by the set $\{(1, 0, 0, 0), (0, 1, 0, 0), (1, 2, 0, 1), (0, 0, 0, 1)\}$. Hence find its basis.

14. If $w_1 = \{(x, y, z) | x + y - z = 0\}$, $w_2 = \{(x, y, z) | 3x + y - 2z = 0\}, w_3 = \{(x, y, z) | x - 7y + 3z = 0\}$ , then find $dim(w_1 \cap w_2 \cap w_3)$ and $dim(w_1 + w_2)$.

15. Suppose $U$ and $W$ are distinct four-dimensional subspaces of a vector space $V$, when $\dim V = 6$. Find the possible dimensions of subspace $U \cap W$.

16. Consider the set $V$ of all $n \times n$ real magic squares. Show that $V$ is a vector space over $R$. Give examples of two distinct $2 \times 2$ magic squares.

17. Show that $S = \{(x, 2y, 3x): x, y \text{ } are \text{ } real \text{ } numbers\}$ is a subspace of $R^3(R)$. Find two bases of S. Also find the dimension of S.

18. Provet that any set of $n$ linearly independent vectors in a vector space V of dimension $n$ constitutes a basis for V. (10, 2022)

# Linear Transformation

1. Show that $B = \{(1, 0, 0), (1, 1, 0), (1, 1, 1)\}$ is a basis of $R^3$. Let $T: R^3 \to R^3$ be a linear transformation such that $T(1, 0, 0) = (1, 0, 0)$, $T(1, 1, 0) = (1, 1, 1)$ and $T(1, 1, 1) = (1, 1, 0)$. Find $T(x, y, z)$. (15)

2. Let $\beta = \{(1, 1, 0) (1, 0, 1), (0, 1, 1)\}$ and $\beta' = \{(2, 1, 1), (1, 2, 1), (-1, 1, 1)\}$ be the two ordered bases of $R^3$. Then find a matrix representing the linear transformation $T : R^3 \to R^3$ which transforms $\beta$ into $\beta'$. Use this matrix representation to find $T(x)$, where $x = (2, 3, 1)$.

3. Let $L: R^4 \to R^3$ be a linear transformation defined by $L(x_1, x_2, x_3, x_4) = (x_3 + x_4 - x_1 - x_2, x_3 - x_2, x_4 - x_1)$. Then, find the rank and nullity of $L$. Also, determine null space and range space of $L$. (20)

4. What is the null space of the differential transformation $\frac{d}{dx}: P_n \to P_n$ where $P_n$ is the space of all polynomials of degree $\leq n$ over the real numbers? What is the null space of the second derivatives as a transformation of $P_n$? What is the null space of the $k^{th}$ derivative $P_n$? (12)

5. Let $M = \begin{bmatrix} 4 & 2 & 1 \\ 0 & 1 & 3 \end{bmatrix}$. Find the unique linear transformation: $R_3 \rightarrow R_2$, so that $M$ is the matrix of $T$ with respect to the basis $\beta = \{v_1 = (1,0,0),\ v_2 = (1,1,0), v_3 = (1,1,1)\}$ *of* $R^3$ *and* $\beta' = \{w_1 = (1,0),\ w_2 = (1,1)\}$ of $R_2$. Also find $T(x,y,z)$. (20)

6. Find the nullity and a basis of the null space of the linear transformation $A : R^4 \rightarrow R^4$, given by the matrix $A = \begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$.

7. Show that the vectors $(1,1,1), (2,1,2)$ *and* $(1,2,3)$ are linearly independent in $R^3$. Let $R^3 \rightarrow R^3$ be a linear transformation defined by $T(x,y,z) = (x + 2y + 3z,\ x + 2y + 5z,\ 2x + 4y + 6z)$. Show that the images of above vectors under T are linearly dependent. Give the reason for the same. (10)

8. Let $T : R^3 \rightarrow R^3$ be the linear transformation defined by $T(\alpha,\beta,\gamma) = (\alpha + 2\beta - 3\gamma,\ 2\alpha + 5\beta - 4\gamma,\ \alpha + 4\beta + \gamma)$. Find a basis and the dimension of the image of $T$ and the kernel of $T$. (12)

9. Consider the mapping $f: R^2 \rightarrow R^2$ *by* $f(x,y) = (3x + 4y, 2x - 5y)$. Find the matrix $A$ relative to the basis $(1,0), (0,1)$ and the matrix $B$ relative to the basis $(1,2), (2,3)$.

10. Let $P_n$ denote the vector space of all real polynomials of degree at most $n$ and $T : P_2 \rightarrow P_3$ be linear transformation given by $T(f(x)) = \int_0^x p(t)dt$ , $p(x) \in P_2$. Find the matrix of $T$ with respect to the bases $\{1, x, x^2\}$ *and* $\{1, x, 1 + x^2, 1 + x^3\}$ of $P_2$ and $P_3$ respectively. Also find the null space of $T$. (10)

11. Let V be an $n$-dimensional vector space and $T : V \rightarrow V$ be an invertible linear operator. If $\beta = \{X_1, X_2, \dots X_n\}$is a basis of V, show that $\beta' = \{TX_1, TX_2 \dots TX_n\}$ is also a basis of V. (8)

12. Let $V = R^3$ and $T \in A(V)$, for all $a_i \in A(V)$, be defined by $T(a_1, a_2, a_3) = (2a_1 + 5a_2 + a_3, -3a_1 + a_2 - a_3, a_1 + 2a_2 + 3a_3)$. What is the matrix $T$ relative to the basis $V_1 = (1,0,1),\ V_2 = (-1,2,1),\ V_3 = (3,-1,1)$ ?

13. If $M_2(R)$ is space of real matrices of order $2 \times 2$ and $P_2(x)$ is the space of real polynomials of degree at most 2, then find the matrix representation of $T: M_2(R) \rightarrow P_2(x)$ such that $T = \left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = a + b + c + (a - d)x + (b + c)x^2$, with respect to the standard bases of $M_2(R)$ and $P_2(x)$. Further find the null space of $T$.

14. If $T: P_2(x) \rightarrow P_3(x)$ is such that $T(f(x)) = f(x) + 5\int_0^x f(t)dt$, then choosing $\{1, 1 + x, 1 - x^2\}$ and $\{1, x, x^2, x^3\}$ as bases of $P_2(x)$ and $P_3(x)$ respectively, find the matrix of T. (10)

15. If $A = \begin{bmatrix} 1 & -1 & 2 \\ -2 & 1 & -1 \\ 1 & 2 & 3 \end{bmatrix}$ is the matrix representation of a linear transformation $T: P_2(x) \to P_2(x)$ with respect to the bases $\{1 - x, x(1 - x), x(1 + x)\}$ *and* $\{1, 1 + x, 1 + x^2\}$ , then find T. (18)

16. Consider the matrix mapping $A: R^4 \to R^3$, where $A = \begin{bmatrix} 1 & 2 & 3 & 1 \\ 1 & 3 & 5 & -2 \\ 3 & 8 & 13 & -3 \end{bmatrix}$. Find a basis and dimension of the image of $A$ and those of kernel $A$.

17. Let $T : R^2 \to R^2$ be a linear map such that $T(2, 1) = (5, 7)$ and $T(1, 2) = (3, 3)$. If $A$ is the matrix corresponding to $T$ with respect to the standard bases $(e_1, e_2)$, then find *Rank (A)*.

18. Let $M_2(R)$be the vector space of all $2 \times 2$ real matrices. Let $B = \begin{bmatrix} 1 & -1 \\ -4 & 4 \end{bmatrix}$. Suppose $T : M_2(R) \to M_2(R)$ is a linear transformation defined by $T(A) = BA$. Find the rank and nullity of $T$. Find a matrix $A$ which maps to the null matrix.

19. Let $F$ be a subfield of complex numbers and $T$ is a function from $F^3 \to F^3$ defined by $T(x_1, x_2, x_3) = (x_1 + x_2 + 3x_3,\ 2x_1 - x_2, -3x_1 + x_2 - x_3)$.What are the conditions on $a$, $b$, $c$, such that $(a, b, c)$ be in the null space of $T$? Find the nullity of $T$.

20. Find the matrix associated with the linear operator on $V_3(R)$ defined by $T(a, b, c) = (a + b, a - b, 2c)$, with respect to ordered basis $B = \{(0,1,1), (1,0,1), (1,1,0)\}$. (10,2021)

21. Let $T: R^2 \to R^3$ be a linear transformation such that $T\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ and $T\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} -3 \\ 2 \\ 8 \end{pmatrix}$. Find $T\begin{pmatrix} 2 \\ 4 \end{pmatrix}$. (10,2022).

# Matrices

1. Show that the matrix $A$ is invertible if and only if the $adj\ (A)$ is invertible. Hence find $|adj\ (A)|$. (12)

2. Let $A$ be a non-singular matrix. Show that if $I + A + A^2 + \dots + A^n = 0$, then $A^{-1} = A^n$ .(15)

3. Find a Hermitian and skew-hermitian matrix each, whose sum is the matrix $\begin{bmatrix} 2i & 3 & -1 \\ 1 & 2 + 3i & 2 \\ -i + 1 & 4 & 5i \end{bmatrix}$

4. Find a $2 \times 2$ real matrix A which is both orthogonal and skew-symmetric. Can there exist a $3 \times 3$ real matrix which is both orthogonal and skew-symmetric? Justify your answer. (20)

5. If $\lambda_1, \lambda_2, \lambda_3$ are the Eigen values of matrix $A = \begin{bmatrix} 26 & -2 & 2 \\ 2 & 21 & 4 \\ 44 & 2 & 28 \end{bmatrix}$, show that $\sqrt{\lambda_1^2 + \lambda_2^2 + \lambda_3^2} \leq \sqrt{1949}$. (12)

6. Let A and B be $n \times n$ matrices over reals. Show that $I - BA$ is invertible if $I - AB$ is
invertible. Deduce that $AB$ and $BA$ have same Eigen values. (20)

7. Let A be a non-singular $n \times n$, square matrix. Show that $A.(adjA) = |A|I_n$. Hence show that $|adj(adjA)| = |A|^{(n-1)^2}$.

8. Let $\lambda_1, \lambda_2, \ldots, \lambda_n$ be the Eigen values of a $n \times n$ square matrix A with corresponding Eigen vectors $X_1, X_2, \ldots, X_n$. If B is a matrix similar to A, show that the Eigen values of $B$ are same as that of A. Also find the relation between the Eigen vectors of B and Eigen vectors of A. (10)

9. Let $A = \begin{bmatrix} 2 & -2 & 2 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$ and $C$ be a non-singular matrix of order $3 \times 3$. Find the Eigen values of the matrix $B^3$ where $B = C^{-1}AC$. (10)

10. If $\lambda$ is a characteristic root of a non-singular matrix $A$ then prove that $|A|/\lambda$ is a characteristic root of $Adj.A$

11. Let $H = \begin{bmatrix} 1 & i & 2+i \\ -i & 2 & 1-i \\ 2-i & 1+i & 2 \end{bmatrix}$ be a Hermitian matrix. Find a non-singular matrix $P$ such that $D = P^T H \bar{P}$ is diagonal.

12. Let A be a square matrix and $A^*$ be its adjoint, show that the Eigen values of matrices $AA^*$ and $A^*A$ are real. Further show that $Trace\,(AA^*) = Trace\,(A^*A)$.

13. Let $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{bmatrix}$ where $\omega\ (\neq 1)$ is a cube root of unity. If $\lambda_1, \lambda_2, \lambda_3$, denote the eigenvalues of $A^2$, show that $|\lambda_1| + |\lambda_2| + |\lambda_3| \leq 9$.

14. Find the rank of the matrix $A = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 5 & 8 & 12 \\ 3 & 5 & 8 & 12 & 17 \\ 3 & 5 & 8 & 17 & 23 \\ 8 & 12 & 17 & 23 & 30 \end{bmatrix}$.

15. Let $A$ be a Hermitian matrix having all distinct eigenvalues $\lambda_1, \lambda_2, \ldots, \lambda_n$. If $X_1, X_2, \ldots X_n$ are corresponding Eigen vectors then show that the $n \times n$ matrix C whose $k^{th}$ column consists of the vector $X_k$ is non singular. (8)

16. Using elementary row or column operations, find the rank of the matrix $\begin{bmatrix} 0 & 1 & -3 & -1 \\ 0 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$

17. Verify Cayley – Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$ and hence find its inverse. Also find the matrix representation $A^5 - 4A^4 - 7A^3 + 11A^2 - A - 10I$.

18. Let $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$. Find the Eigen values of A and the corresponding Eigen vectors.

19. Prove that Eigen values of a unitary matrix have absolute value 1.

20. Reduce the following matrix to row echelon form and hence find its rank: $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 5 \\ 1 & 5 & 5 & 7 \\ 8 & 1 & 14 & 17 \end{bmatrix}$

21. If matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$, then find $A^{30}$.

22. Find the Eigen values and Eigen vectors of the matrix $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$.

23. Using elementary row operations, find the inverse of $A = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 3 & 2 \\ 1 & 0 & 1 \end{bmatrix}$.

24. If $A = \begin{bmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{bmatrix}$, then find $A^{14} + 3A - 2I$.

25. Using elementary row operations, find the condition that the linear equations have a solution: $x - 2y + z = a$; $2x + 7y - 3z = b$; $3x + 5y - 2z = c$. (7)

26. If $A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, then find the Eigenvalues and Eigenvectors of $A$.

27. Prove that Eigen values of a Hermitian matrix are all real.

28. Let $A = \begin{bmatrix} 2 & 2 \\ 1 & 3 \end{bmatrix}$. Find a non-singular matrix $P$ such that $P^{-1}AP$ is diagonal matrix.

29. Show that similar matrices have the same characteristic polynomial.

30. Prove that the distinct non-zero eigen vectors of a matrix are linearly independent.

31. Let A be a $3 \times 2$ matrix and B a $2 \times 3$ matrix. Show that $C = A.B$ is a singular matrix.

32. Let A and B be two orthogonal matrices of same order and $det\, A + det\, B = 0$, show that $A + B$ is a singular matrix.

33. Let $A = \begin{bmatrix} 5 & 7 & 2 & 1 \\ 1 & 1 & -8 & 1 \\ 2 & 3 & 5 & 0 \\ 3 & 4 & -3 & 1 \end{bmatrix}$

a. Find the rank of the matrix A.

b. Find the dimension of the subspace $V = \left\{ (x_1, x_2, x_3, x_4) \in R^4 \,\middle|\, A \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = 0 \right\}$

34. $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$, State the Cayley-Hamilton theorem. Use this theorem to find $A^{100}$.

35. Define an $n \times n$ matrix as $A = I - 2u\,u^T$, where $u$ is a unit column vector

(i) Examine if $A$ is symmetric.

(ii) Examine if $A$ is orthogonal.

(iii) Show that $trace\,(A) = n - 2$

(iv) Find $A_{3\times 3}$ when $u = \begin{bmatrix} 1/3 \\ 2/3 \\ 2/3 \end{bmatrix}$

36. If $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$, then show that $A^2 = A^{-1}$ $(without\ finding\ A^{-1})$. (10, 2021)

37. Prove that the eigen vectors, corresponding to two distinct eigne values of a real symmetric matrix, are orthogonal. (8, 2021)

38. For two square matrices A and B of order 2, show that $trace\,(AB) = trace\,(BA)$. Hence show that $AB - BA \neq I_2, where\ I_2$ is identity matrix is order 2. (7, 2021)

39. Reduce the following matrix to a row reduced echelon form and hence, also, find its rank.

$\begin{bmatrix} 1 & 3 & 2 & 4 & 1 \\ 0 & 0 & 2 & 2 & 0 \\ 2 & 6 & 2 & 6 & 2 \\ 3 & 9 & 1 & 10 & 6 \end{bmatrix}$ (10, 2021)

40. Find the eigen values and corresponding eigen vectors of the matrix $A = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$, over the complex-number field. (10, 2021)

41. Let the set $P = \left\{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \,\middle|\, x - y - z = 0 \text{ and } 2x - y + z = 0 \right\}$ be the collection of vectors of a vector space $R^3(R)$.Then

a. Prove that P is subspace of $R^3$

b. Find the basis and dimension of P. (10+10)

42. Find a linear map $T: R^2 \to R^2$ which rotates each vector of $R^2$ by an angle $\theta$. Also prove that for $\theta = \frac{\pi}{2}$, T has no eigenvalue in R. (15, 2022)

# Solution of System of Linear Equations

1. Let $= \begin{bmatrix} 1 & 0 & -1 \\ 3 & 4 & 5 \\ 0 & 6 & 7 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} 2 \\ 6 \\ 5 \end{bmatrix}$. Solve the system of equations given by $AX = B$. Using the above, also solve the system of equations $A^T X = B$ where $A^T$ denotes the transpose matrix of A.(10)

2. Find the dimension and a basis for the space W of all solutions of the following homogeneous system using matrix notation:
$$x_1 + 2x_2 + 3x_3 - 2x_4 + 4x_5 = 0$$
$$2x_1 + 4x_2 + 8x_3 + x_4 + 9x_5 = 0$$
$$3x_1 + 6x_2 + 13x_3 + 4x_4 + 14x_5 = 0$$

3. Find the inverse of matrix $A = \begin{bmatrix} 1 & 3 & 1 \\ 2 & -1 & 7 \\ 3 & 2 & -1 \end{bmatrix}$ by using elementary row operations.

   Hence solve the system of linear equations
$$x + 3y + z = 10$$
$$2x - y + 7z = 12$$
$$3x + 2y - z = 4$$

4. Investigate the values of $\lambda$ and μ so that the equations $x + y + z = 6$, $x + 2y + 3z = 10$, $x + 2y + \lambda z = \mu$ have

a. No solution

b. Unique solution

c. An infinite number of solutions

5. Consider the following system of equation in x, y, z
$$x + 2y + 2z = 1$$
$$x + ay + 3z = 3$$
$$x + 11y + az = b$$

a. For which values of '$a$' does that system have a unique?

b. For which of values $(a, b)$ does the system have more than one solution?

6. If $A = \begin{bmatrix} 1 & 2 & 1 \\ 1 & -4 & 1 \\ 3 & 0 & -3 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 1 & 1 \\ 1 & -1 & 0 \\ 2 & 1 & -1 \end{bmatrix}$. Then, show that $AB = 6I_3$. Use this result to solve the following system of equations: $2x + y + z = 5$; $x - y = 0$; $2x + y - z = 1$;

7. For the system of linear equations $x + 3y - 2z = -1$; $5y + 3z = -8$; $x - 2y - 5z = 7$,

a. determine the following statements, which are true or false:

# Solution of System of Linear Equations

1. Let $= \begin{bmatrix} 1 & 0 & -1 \\ 3 & 4 & 5 \\ 0 & 6 & 7 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} 2 \\ 6 \\ 5 \end{bmatrix}$. Solve the system of equations given by $AX = B$. Using the above, also solve the system of equations $A^T X = B$ where $A^T$ denotes the transpose matrix of A.(10)

2. Find the dimension and a basis for the space W of all solutions of the following homogeneous system using matrix notation:

$$x_1 + 2x_2 + 3x_3 - 2x_4 + 4x_5 = 0$$
$$2x_1 + 4x_2 + 8x_3 + x_4 + 9x_5 = 0$$
$$3x_1 + 6x_2 + 13x_3 + 4x_4 + 14x_5 = 0$$

3. Find the inverse of matrix $A = \begin{bmatrix} 1 & 3 & 1 \\ 2 & -1 & 7 \\ 3 & 2 & -1 \end{bmatrix}$ by using elementary row operations.

   Hence solve the system of linear equations

$$x + 3y + z = 10$$
$$2x - y + 7z = 12$$
$$3x + 2y - z = 4$$

4. Investigate the values of $\lambda$ and μ so that the equations $x + y + z = 6$, $x + 2y + 3z = 10$, $x + 2y + \lambda z = \mu$ have

a. No solution

b. Unique solution

c. An infinite number of solutions

5. Consider the following system of equation in x, y, z

$$x + 2y + 2z = 1$$
$$x + ay + 3z = 3$$
$$x + 11y + az = b$$

a. For which values of *'a'* does that system have a unique?

b. For which of values *(a, b)* does the system have more than one solution?

6. If $A = \begin{bmatrix} 1 & 2 & 1 \\ 1 & -4 & 1 \\ 3 & 0 & -3 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 1 & 1 \\ 1 & -1 & 0 \\ 2 & 1 & -1 \end{bmatrix}$. Then, show that $AB = 6I_3$. Use this result to solve the following system of equations: $2x + y + z = 5$; $x - y = 0$; $2x + y - z = 1$;

7. For the system of linear equations $x + 3y - 2z = -1$; $5y + 3z = -8$; $x - 2y - 5z = 7$,

a. determine the following statements, which are true or false:

i. The system has no solution

ii. The system has unique solution

iii. The system has infinitely many solutions

8. Let $A = \begin{bmatrix} 1 & 0 & 2 \\ 2 & -1 & 3 \\ 4 & 1 & 8 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 & 2 \\ 2 & -1 & 3 \\ 4 & 1 & 8 \end{bmatrix}$

a. Find $AB$

b. Find $det(A)$ and $det(B)$

c. Solve the system of linear equations $x + 2z = 3; 2x - y + 3z = 3;\ 4x + y + 8z = 14.$

9. Find all solution to the following system of equations by row reduced method:

$$\begin{aligned} x_1 + 2x_2 - x_3 &= 2 \\ 2x_1 + 3x_2 + 5x_3 &= 5 \\ -x_1 - 3x_2 + 8x_3 &= -1 \end{aligned}$$ (15, 2022)

i. The system has no solution

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$$\begin{aligned} x_1 + 2x_2 - x_3 &= 2 \\ 2x_1 + 3x_2 + 5x_3 &= 5 \\ -x_1 - 3x_2 + 8x_3 &= -1 \end{aligned} \quad (15,\ 2022)$$