Probability Theory and Probability Distributions

§ 2.1. INTRODUCTION

There are two types of situations generally we face, first type of situation is such that we know the results of that situation. For example, if we throw a ball in the sky definitely the ball will come down on the earth, *i.e.*, we are 100% sure that the ball comes down. This type of situation is called deterministic phenomenon. But the second type of situation in which we can not say about the result but consider only some possibility. For example, we toss a coin then we have two possibilities either head or tail. If we throw a die there are six possibilities and we cannot say 100% surely that which one face will come. These types of events are called the probabilitistic events.

In this chapter we deal with the second type of situation *i.e.*, probabilistic phenomenon. Our interest here is to know how much is the chance of occurrence of a particular event. So in this chapter we learn about how to calculate the chance of this type of situation.

Before going into deep, we will take a short survey on the history of probability theory.

The theory of probability has been developed by mathematicians as well as statisticians for the game of chance for gamblers. A great contribution is given by Galileo, French mathematician, **B. Pascal** and **P. Fermat, J. Bernoulli, De Moivre, Thomas Bayes, P.S. Laplace, R.A. Fisher, A. Kolmogorov** and many others. Kolmogorov axiomised the theory of probability and his book "foundations of probability" published in 1933 introduced probability as a set function which is regarded as a classic approach.

Starting with game of chance, today probability has become one of the most important tools of statistics. Actually probability and statistics are so inter-related that without one, another is incomplete. The knowledge of probability and its various types of distributions helps in the development of probabilistic decision models.

Now we shall define the probability in a simple way. The probability is the quantitative measure of uncertainty. It is a number which lies between zero and one including both zero and one.

§ 2.2. DEFINITIONS OF VARIOUS TERMS

In this section we shall define some terms which are related to the probability theory and without understanding them, we can not obtain a deeper understanding of probability. These are :

(a) Random Experiment or Trial. An activity whose results or otucomes are not likely to be known until its completion is known as random experiment or trial. For example, tossing of a coin or throwing of die.

(b) Events. The results of various trials are not unique, and depend on all the possible outcomes of a trial. These outcomes are called events or cases. For example,

(i) If we are tossing a coin, outcomes are head (H) and tail (T). So the head and the tail are the two events and tossing of a coin is one trial or random experiemnt.

(ii) Drawing a card from a pack of well-shuffled cards is a trial and getting of an ace is an event.

(iii) Throwing of a die is a trial and getting 6 (or 1, 2, 3, 4, 5) is an event.

(c) Mutually Exclusive Events. Two or more events are said to be mutually exclusive events if they cannot occur simultaneously in a single trial of a random experiment. In other words two or more events are mutually exclusive or incompatible if the occurrence of one of them rules out the occurrence of the other. For example,

(i) In tossing of a coin events head and tail are mutually exclusive, since both head and tail cannot occur simultaneously in the same trial.

(ii) In throwing of a die all the 6 faces (1 to 6) are mutually exclusive.

(d) Exhaustive Events. The totality of all the possible events or outcomes in a trial is known as exhaustive events. For example,

(i) In throwing of a die, there are 6 (six) exhaustive cases or events which are 1, 2, 3, 4, 5, 6.

(ii) In tossing of a coin there are two exhaustive cases or events such as head or tail.

(e) Equally Likely Events. Events are said to be equally likely if one cannot be prefered to others. In other words if possibility of occurrence of each event is same then events are known as equally likely events. For example,

(i) If an unbiased coin is tossed, there is no reason of the pereference to be given to either head or tail. Thus, getting a head or tail are equally likely events.

(ii) If an unbiased die is thrown then getting 1 or 2 or 3 or 4 or 5 or 6 are equally likely events.

(f) Favourable Events. The number of cases favourable to an event in an experiment or trial is the number of outcomes which entail the happening of the event. For example :

(i) In the throwing of a die the number of cases favourable to getting the multiple of 3 are two *i.e.*, 3 and 6.

(ii) If a pair of fair dice is tossed then the number of cases favourable to getting a sum 7 is 6 *i.e.*, (1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1).

(g) Independent Events. Two or more events are said to be independent events if the happening if one event is not affected by the other event. For example, in throwing a fair die the event of getting a six in the first throw is independent of getting a six in the second, third or in subsequent throws.

§ 2.3. DEFINITION OF PROBABILITY

Here we shall discuss two definitions of probability :

(i) Mathematical or Classical or 'a prior' probability, and

(ii) Statistical or empirical probability.

§ 2.4. MATHEMATICAL OR CLASSICAL DEFINITION OF PROBABILITY

If there are n exhaustive, mutually exclusive and equally likely events then the probability of occurrence of an event A is defined as the ratio of favourable number of cases or events (m) to the exhaustive number of cases or events (n). If we denote the probability of event A by $P(A)$, then

$$P(A) = \frac{\text{Number of favourable cases or events}}{\text{Number of Exhaustive cases or events}} = \frac{m}{n}.$$

Thus the probability is a concept which measures numerically the degree of certainty or uncertainty of the occurrence of an event.

Remarks. (i) From the above formula it is clear that $P(A)$ is a positive number which lies in the interval [0, 1], *i.e.*, $P(A)$ cannot be greater than one and cannot be less than zero. Thus $0 \le P(A) \le 1$.

(ii) If the probability of an event is zero then it is known as impossible event and if the probability of an event is one then it is known as certain event.

Example 1. *A fair coin is tossed. What is the probability of getting a head ?*

Solution. The number of favourable cases is one *i.e.*, head and the number of exhaustive cases is two *i.e.*, head and tail. Thus the probability of getting a head

$$= \frac{\text{Number of favourable cases } (m)}{\text{Number of exhaustive cases } (n)}$$

$$= \frac{1}{2}.$$

Example 2. *When a fair die is thrown, what is the probability of getting an even number ?*

Solution. Here the no. of exhaustive cases is 6 *i.e.*, (1, 2, 3, 4, 5, 6) and the number of favourable cases is 3, *i.e.*, (2, 4, 6). Thus the probability of getting an even number is $\frac{3}{6} = \frac{1}{2}$.

Example 3. *Two unbiased coins are tossed what is the probability of getting two heads ?*

Solution. Here the exhaustive number of cases $n = 4$, *i.e.*, (HT, TH, TT, HH) and the number of favourable cases $m = 1$ *i.e.*, (HH). Thus the probability of getting two heads

$$= \frac{m}{n} = \frac{1}{4}.$$

Example 4. *Two dice are thrown, what is the probability that the sum of the upper face values is 8 ?*

Solution. Here the no. of exhaustive cases $(n) = 6 \times 6 = 36$

and the number of favourable cases $(m) = 5$ *i.e.*, (2, 6), (3, 5), (4, 4), (5, 3), (6, 2).

Thus the probability of sum of the upper face values being $8 = \frac{m}{n}$

$$= \frac{5}{36}.$$

Example 5. *Find the chance of occurring a king when a card is drawn from a pack of 52 cards.*

Solution. We know that there are 4 kings and 52 cards in a pack of cards. Thus the number of exhaustive cases $(n) = 52$. And the number of favourable cases $(m) = 4$. Thus the probability of occurring a king

$$= \frac{m}{n} = \frac{4}{52} = \frac{1}{13}.$$

Example 6. *A bag contains 6 white, 8 red and 5 black balls. A ball is drawn at random. Find the probability that it is white.*

Solution. Here the number of exhaustive cases $(n) = 6 + 8 + 5 = 19$ and the number of favourable cases $(m) = 6$

$$\therefore \qquad p = \frac{m}{n} = \frac{6}{19}.$$

Example 7. *An ordinary die is thrown. What is the probability that the number appearing on the die is* 3 ?

Solution. The number on the faces of the die are 1, 2, 3, 4, 5 and 6. The chance of any one of these appearing on the upper face of the die is same.

Let the event of number 3 appearing on the upper face of the die be denoted by A. Then

$$P(A) = \frac{\text{number of favourable cases}}{\text{number of exhaustive cases}} = \frac{1}{6}.$$

Example 8. *What is the chance that a leap year selected at random will contain* 53 *Wednesdays* ?

Solution. A leap year consists of 366 days in which there are 52 complete weeks and 2 days are more. 52 weeks consist 52 Wednesday and so we are to find the probability of being one Wednesday out of two remaining days. These two remaining days may make the following seven combinations :

(i) Tuesday and Wednesday
(ii) Wednesday and Thursday
(iii) Thursday and Friday
(iv) Friday and Saturday
(v) Saturday and Sunday
(vi) Sunday and Monday
(vii) Monday and Tuesday.

Thus, here number of favourable cases are 2 ((i) and (ii) only) and the exhaustive number of cases are 7. Hence the required probability $= \frac{2}{7}$.

Example 9. *What is the probability that a non-leap year selected at random will contain* 53 *Sundays ?*

Solution. A non-leap year contains 365 days in which there are 52 complete weeks and one day more. 52 week contain 52 sundays and so we are to find the probability of being one Sunday out of one remaining day. This one remaining day may be any one of the following seven days : (i) Sunday, (ii) Monday, (iii) Tuesday, (iv) Wednesday, (v) Thursday, (vi) Friday, (vii) Saturday.

Thus out of 7 likely cases only one is favourable. Hence the required probability $= \frac{1}{7}$.

Example 10. *Find the probability that there occurs* 3 *or more when a dice is thrown.*

Solution. Here the number of exhaustive cases = 6. And the number of favourable cases = 4 (*i.e.*, 3, 4, 5, 6). Thus the required probability $= \frac{4}{6} = \frac{2}{3}$.

Example 11. *From a pack of 52 cards two cards are drawn at random. Find the probability of the following events :*

(*i*) *Both cards are of spade.*

(*ii*) *One card is of spade and one card is of diamond.*

Solution. The total number of ways in which 2 cards can be drawn out of 52 cards number $(S) = {}^{52}C_2 = \frac{52 \times 51}{2 \times 1} = 1326.$

i.e., exhaustive number of cases = 1326.

(i) Let A be the event that both cards are of spade, then number (A) = number of ways in which 2 cards can be drawn out of 13 cards of spade

$$= {}^{13}C_2 = \frac{13 \times 12}{2 \times 1} = 78.$$

Thus the required probability $= \frac{\text{number } (A)}{\text{number } (S)} = \frac{78}{1326} = \frac{1}{17}$.

(ii) Let B be the event that one card is of spade and one card is of diamond, then

number (B) = Number of ways in which one card can be drawn out of 13 cards of spade and one card can be drawn out of 13 cards of diamond.

$$= {}^{13}C_1 \times {}^{13}C_1 = 13 \times 13.$$

Thus the required probability $= \frac{\text{number } (B)}{\text{number } (S)} = \frac{13 \times 13}{1326} = \frac{13}{102}$.

Example 12. *From a pack of* 52 *cards,* 5 *cards are drawn at random. Find the probability of the following events :*

(*i*) *There are* 2 *red and* 3 *black cards.*

(*ii*) *There are* 2 *kings and* 3 *queens.*

Solution. The total number of ways in which 5 cards can be drawn from a pack of 52 cards number $(S) = {}^{52}C_5$

$$= \frac{52 \times 51 \times 50 \times 49 \times 48}{5 \times 4 \times 3 \times 2 \times 1} = 2598960.$$

$\therefore$ Number of exhaustive case number $(S) = 2598960$

(i) In a pack of cards, there are 26 red cards and 26 black cards. Let A be the event that 2 cards are red and 3 cards are black. Then

number (A) = Number of ways in which 2 red and 3 black cards can be drawn

$$= {}^{26}C_2 \times {}^{26}C_3 = \frac{26 \times 25}{2 \times 1} \times \frac{26 \times 25 \times 24}{3 \times 2 \times 1}$$

$$= 845000.$$

$\therefore$ The required probability $P(A) = \frac{\text{number } (A)}{\text{number } (S)} = \frac{845000}{2598960}$.

(ii) In a pack of cards, there are 4 kings and 4 queens.

Let B be the event that 2 kings and 3 queens are drawn.

Then number (B) = Number of ways in which 2 kings and 3 queens can be drawn

$$= {}^4C_2 \times {}^4C_3 = \frac{4 \times 3}{2 \times 1} \times \frac{4 \times 3 \times 1}{3 \times 1} = 24.$$

$\therefore$ The required probability $= \dfrac{\text{number }(B)}{\text{number }(S)} = \dfrac{24}{2598960}$.

Example 13. *From a beg containing 5 white, 7 red and 4 black balls, a man draws 3 balls at random. Find the probability of all being white.*

Solution. The total number of balls in the beg = 5 + 7 + 4 = 16.

Therefore the total number of ways in which 3 balls can be drawn out of 16 balls number

$$(S) = {}^{16}C_3 = \frac{16 \times 15 \times 14}{3 \times 2 \times 1} = 560.$$

Let A be the event that all the three balls being white. The total number of white ball is 5. So the number of ways in which 3 balls can be drawn

$$= {}^5C_3 = \frac{5 \times 4 \times 3}{3 \times 2 \times 1} = 10 = \text{number }(A).$$

$\therefore$ The required probability is given by

$$P(A) = \frac{\text{number }(A)}{\text{number }(S)} = \frac{10}{560} = \frac{1}{56}.$$

Example 14. *A card is drawn from a well shuffled pack of 52 cards. Find the probability that it is either a diamond or a king.* **[Indore M.B.A. 2002]**

Solution. The total number of ways in which one card can be drawn from a pack of 52 cards number $(S) = {}^{52}C_1 = 52$.

Now in a pack of 52 cards, there are 13 cards of diamond and 4 cards of king. Let A be the event that one card drawn is either a diamond or a king. Then

number $(A) = 13 + 3 = 16$, because a diamond king can be counted only one time.

$\therefore$ The required probabiilty $P(A) = \dfrac{\text{number }(A)}{\text{number }(B)} = \dfrac{16}{52} = \dfrac{4}{13}$.

Example 15. *From 15 tickets marked 1 to 15, one ticket is drawn at random. Find the chance that the number on it is a multiple of 4.*

Solution. Here we are given that, total number of exhaustive cases *i.e.*, number $(S) = 15$.

Let A be the event that a number is a multiple of 4. So the favourable cases are (4, 8, 12). Thus we have 3 favourable cases *i.e.*, number $(A) = 3$.

$\therefore$ The required probability $= \dfrac{\text{number }(A)}{\text{number }(S)} = \dfrac{3}{15} = \dfrac{1}{5}$.

Example 16. *Two letters are taken at random from the word 'RANDOM'. Find the probability that*

(*i*) *Both the letters are vowels.*

(*ii*) *It least one is vowel.*

(*iii*) *One of the letter is 'N'.*

Solution. There are 6 letters in the word 'RANDOM' out of which A and O are the two vowels. Now, out of 6 letters, two letters can be choosen is ${}^6C_2$ ways *i.e.*, number $(S) = {}^6C_2 = 15$.

The total number of exhaustive events number $(S) = 15$

$RA, RN, RD, RO, RM, AN, AD, AO, AM, ND, NO, NM, DO, DM, OM.$

(i) Let A be the event that both letters are vowel *i.e.*, AO.

So the favourable number of cases number $(A) = 1$.

$\therefore$ The required probability $P(A) = \dfrac{\text{number }(A)}{\text{number }(S)} = \dfrac{1}{15}$.

(ii) Let B be the event that at least one letter is vowel, *i.e.*, RA, RO, AN, AD, AO, AM, NO, DO, OM. So the favourable number of cases number $(B) = 9$.

$\therefore$ The required probability $P(B) = \dfrac{9}{15} = \dfrac{3}{5}$.

(iii) Let C be the event that one of the letter is 'N', *i.e.*, RN, AN, ND, NO, NM. So the favourable number of cases number $(C) = 5$.

$\therefore$ The required probability $P(C) = \dfrac{5}{15} = \dfrac{1}{3}$.

Example 17. *What are the odds in favour of getting a six in a single throw of a die ?*

Solution. Here the total number of exhaustive events are 6, *i.e.*, number $(S) = 6 = n$. Let A be the event of getting a six.

So the favourable case is only one, *i.e.*, number $(A) = 1 = m$.

$\therefore$ The required probability $P(A) = \dfrac{\text{number }(A)}{\text{number }(S)} = \dfrac{1}{6}$.

And the required odds in favour of the event $A = m : n - m = 1 : 6 - 1 = 1 : 5$.

Example 18. *What are the odds against of throwing a six in a single throw of a die ?*

Solution. Here the total number of exhaustive events are 6, *i.e.*, no. $(S) = 6 = n$. Let A be the event of throwing a six.

So the favourable case is only one, *i.e.*, number $(A) = 1 = m$.

$\therefore$ The required odds against the event $A = n - m : n = 5 : 6$.

Example 19. *In a single throw with two dice, what is the probability of throwing two ones ?*

Solution. Here the total number of exhaustive cases, *i.e.*, number $(S) = 6 \times 6 = 36$.

Let A be the event of throwing two ones in a single throw of two dice. Thus the favourable number of cases *i.e.*, number $(A) = 1$.

$\therefore$ The required probability $= \dfrac{\text{number }(A)}{\text{number }(S)} = \dfrac{1}{36}$.

Remark. The classical definition of probability deals with cases where all events are equally likely. Thus in the case of biased coin; unfair die, etc. this definition fails. Thus classical definition also fails when exhaustive number of cases in a experiment or trial is infinite.

To overcome the above limitations, Von Mises gave the following definition of probability.

Statistical or Empirical Probability :

If a large number of trials performed under some essentially homogeneous and identical conditions, the limit of the ratio of number of happening of an event to the total number of trials is unique and finite, then this limit is known as the probability of happening of that event.

If an event A occurs m times in a series of n trials, then the probability of happening of the event A is given by

$$P(A) = \lim_{n \to \infty} \frac{m}{n}.$$

Example 20. *If* 1000 *tosses of a coin result in* 529 *heads (H), then the probability of getting head is* $\frac{529}{1000} = 0.529$. *If another* 1000 *tosses result in* 481 *heads, then probability of head is* $\frac{529+481}{2000} = 0.505$. *Again if another* 1000 *tosses result in* 497 *heads, then the probability of heads is* $\frac{529+481+497}{3000} = 0.5023$ *etc.*

Solution. Clearly this limit tends to 0.5 *i.e.*, 0.5290, 0.5050, 0.5023, ... 0.5. Hence by statistical definition, the probability of getting head is 0.5.

§ 2.5. SOME DEFINITIONS

(a) Complementary Event. Let A be an event then 'not happening of the event A' is called the complementary of A and is denoted by $\overline{A}$. For example, in a pack of cards there are only two types of cards red and black. Let if A be the event of getting red card then $\overline{A}$ is the event of getting black card.

Remark. If A is an event and $\overline{A}$ its complementary event then

$$P(A) + P(\overline{A}) = 1.$$

(b) Simple Event. If there is only one favourable outcome of an experiment then this event is called a simple or elementary event. For example, In tossing of a coin, the event of getting head is a simple or elementary event.

(c) Compound Event. If there are more than one favourable outcomes of an experiment then these events are called compound events. For example, in throwing of a die, the event of getting an even number is {2, 4, 6}. So this event is called compound event.

Remark. Every compound event can be represented as the union of simple events.

§ 2.6. COMPOSITION OF EVENTS

Following are three fundamental rules to composite two or more events by the help of set notations. Let S be a sample space and A and B be its two events.

(a) Union of Two Events. The union of two events A and B is represented by $A \cup B$ or $A + B$. The union of two events means either A or B happens *i.e.*, the event $A \cup B$ includes all those elements (or outcomes) which A or B contain. The union of A and B is represented by the shaded area in the Ven diagram if Fig. 1.

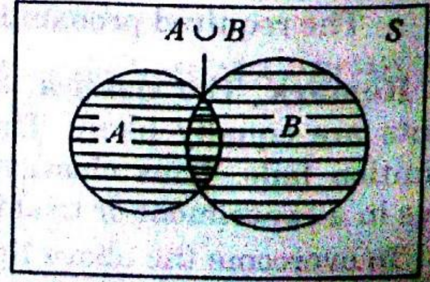


Fig. 1

(b) Intersection of two Events. The intersection of two events A and B is represented by $A \cap B$ or $A \,.\, B$. The intersection of two events means, both events A and B happen *i.e.*, the event $A \cap B$ includes all those elements which are common to both A and B. The intersection of A and B is represented by the shaded area in the Ven diagram in Fig. 2.

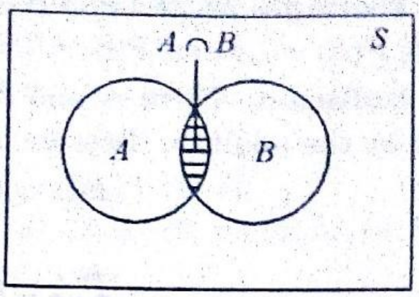


Fig. 2

(c) Complement of an event A. The complement of an event A is represented by $\overline{A}$ or A^C. The complement of an event A means the event A does not happen.

The complement of event A is represented by the shaded area in the Ven diagram in Fig. 3.

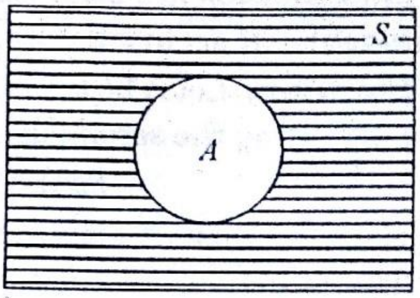


Fig. 3

§ 2.7. ADDITION THEOREM OF PROBABILITY

Statement. *If A and B are any two events then*

$$P(A \cup B) = P(A) + P(B) - P(A \cap B).$$

Proof. Let S be the sample space and n be the number of elements in the sample space S. Again let l be the number of elements in event A and m be the number of elements in the event B, *i.e.*,

$$\text{number } (S) = n, \ \text{number } (A) = l \text{ and number } (B) = m.$$

If the events A and B are not mutually exclusive then $A \cap B \neq \phi$ (phi). Let number $(A \cap B) = r$.

Then clearly number $(A \cup B) = l + m - r$.

Now the probability of happening $A \cup B$ is given by

$$P(A \cup B) = \frac{\text{number } (A \cup B)}{\text{number } (S)}$$

$$= \frac{l + m - r}{n} = \frac{l}{n} + \frac{m}{n} - \frac{r}{n}$$

$$= \frac{\text{number } (A)}{\text{number } (S)} + \frac{\text{number } (B)}{\text{number } (S)} - \frac{\text{number } (A \cap B)}{\text{number } (S)}$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B).$$ **Hence proved.**

Remark. If A and B are mutually exclusive events then

$$A \cap B = \phi \ \text{ and } \ \text{number } (\phi) = 0$$

$\Rightarrow$ $$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$\Rightarrow$ $$P(A \cup B) = P(A) + P(B) \qquad [\because \ P(\phi) = 0]$$

Example 21. *For any two events A and B given that* $P(A) = 0.4$ *and* $P(B) = 0.5$, $P(A \cap B) = 0.2$. *Find* $P(A \cup B)$.

Solution. We know the addition theorem of probability

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$\therefore$ $$P(A \cup B) = 0.4 + 0.5 - 0.2$$

$$P(A \cup B) = 0.7.$$

Example 22. *If A and B are two mutually exclusive events then find* $P(A \cup B)$, *while* $P(A) = 0.4$ *and* $P(B) = 0.3$.

Solution. When A and B are mutually exclusive events then $P(A \cap B) = 0$.
Now by the addition theorem of probability

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$= 0.4 + 0.3 - 0$$
$$P(A \cup B) = 0.7.$$

Example 23. *The probabilities that a student of a class will get a first, second and third division are* 0.30, 0.45 *and* 0.25 *respectively. What is the probability that the student will get at least second division ?*

Solution. Let A be the event of the student getting first division and B be the event of getting the second division. Thus, we have

$$P(A) = 0.30 \text{ and } P(B) = 0.45.$$

Here the event A and B are mutually exclusive events.

Therefore the probability of getting at least second division is given by $P(A \text{ or } B)$.

Thus
$$P(A \text{ or } B) = P(A \cup B)$$
$$= P(A) + P(B)$$
$$= 0.30 + 0.45 \qquad \text{[By addition theorem of probability]}$$
$$= 0.75.$$

Example 24. *Two cards are drawn at random from a pack of* 52 *cards. Find the probability that both of these cards are either of red colour or both kings.*

Solution. The total number of ways in which 2 cards can be drawn out of 52 cards $= {}^{52}C_2 = 1326$

$$\therefore \qquad \text{number } (S) = 1326.$$

Let A be the event that both cards are of red colour. Since there are 26 red cards in the pack of 52 cards.

$$\therefore \qquad \text{number } (A) = {}^{26}C_2 = 325.$$

Again let B be the event that both cards are kings. Since there are 4 kings in the pack of 52 cards.

$$\therefore \qquad \text{number } (B) = {}^{4}C_2 = 6.$$

But in the two events A and B, we have that two red kings are common. So

$$\text{number } (A \cap B) = {}^{2}C_2 = 1.$$

Therefore the required probability is given by

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$= \frac{\text{number } (A)}{\text{number } (S)} + \frac{\text{number } (B)}{\text{number } (S)} - \frac{\text{number } (A \cap B)}{\text{number } (S)}$$
$$= \frac{325}{1326} + \frac{6}{1326} - \frac{1}{1326}$$
$$= \frac{330}{1326} = \frac{55}{221}$$
$$P(A \cup B) = \frac{55}{221}.$$

Example 25. *If two dice are thrown once. What is the probability of getting an odd number on the first die or a total of 7 on both dice ?*

Solution. The total number of exhaustive cases number $(S) = 6 \times 6 = 36$.

Let A be the event that the number on the first die is odd.

$\therefore \quad A = \{(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (3, 1), (3, 2), (3, 3),$
$(3, 4), (3, 5), (3, 6), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)\}$

i.e., number $(A) = 18$.

$$\therefore \quad P(A) = \frac{\text{number }(A)}{\text{number }(S)} = \frac{18}{36} = \frac{1}{2}.$$

Again let B be the event that the sum of numbers on the both dice is 7.

$$\therefore \quad B = \{(1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)\}$$

i.e., number $(S) = 6$

$$\therefore \quad P(B) = \frac{\text{number }(B)}{\text{number }(S)} = \frac{6}{36} = \frac{1}{6}.$$

Here A and B are not mutually exclusive events. There are three outcomes $\{(1, 6), (3, 4), (5, 2)\}$ which are common in both A and B events.

i.e., number $(A \cap B) = 3$.

$$\therefore \quad P(A \cap B) = \frac{\text{number }(A \cap B)}{\text{number }(S)} = \frac{3}{36} = \frac{1}{12}.$$

Therefore the required probability can be calculated as

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{1}{2} + \frac{1}{6} - \frac{1}{12} = \frac{6+2-1}{12} = \frac{7}{12}$$

$$P(A \cup B) = \frac{7}{12}.$$

Example 26. *In a household, probability of having a television set is* 0.80, *probability of having washing machine is* 0.55 *and probability of having both television set and washing machine is* 0.45. *what is the probability that the family has television set or washing machine ?*

Solution. Let A be the event of having washing machine and B be the event of having television set.

Therefore $(A \cap B)$ be the event of having both washing machine and television set.

Thus $\quad P(A) = 0.55$ and $P(B) = 0.80, P(A \cap B) = 0.45$.

Therefore the required probability is given by

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= 0.55 + 0.80 - 0.45$$

$$P(A \cup B) = 0.90.$$

Example 27. *If* 0.80 *be the probability of happening at least one of the two events A and B and* 0.40 *be the probability of happening the both together then find* $P(\overline{A}) + P(\overline{B})$.

Solution. The probability of happening at least one of the two events A and $B = P(A \cup B)$. And the probability of the happening both events $= P(A \cap B)$. Thus we are given that

$$P(A \cup B) = 0.80 \text{ and } P(A \cap B) = 0.4.$$

Then by the addition theorem of probability, we have that

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\Rightarrow \quad P(A) + P(B) = P(A \cup B) + P(A \cap B)$$
$$= 0.8 + 0.4$$
$$= 1.20.$$

Now
$$P(\overline{A}) + P(\overline{B}) = \{1 - P(A)\} + \{1 - P(B)\}$$
$$= 2 - \{P(A) + P(B)\}$$
$$= 2 - 1.20 = 0.80$$

$$P(\overline{A}) + P(\overline{B}) = 0.80.$$

§ 2.8. CONDITIONAL PROBABILITY

If we have two events A and B and the happening of event A depends upon the happening of event B, then the probability of A is called conditional probability and is denoted by $P(A / B)$. Thus $P(A/B)$ denotes the conditional probability for the event A when the event B has already happened.

§ 2.9. MULTIPLICATION THEOREM OF PROBABILITY

Statement. *If A and B are two events with respective known probabilities P(A) and P(B) than the probability that both will happen simultaneously is given by the product of the probability of A and the conditional probability of B when A has already happened, i.e.,*

$$P(A \cap B) = P(A) \,.\, P(B/A)$$

or
$$P(A \cap B) = P(B) \,.\, P(A/B).$$

Proof. Let S be a sample space and A and B be two events. Then suppose that event A has happened and $B \neq \phi$. Sicne, $A \subset S$ and the event A has happened, therefore all elements of S cannot occur and only those elements of S which belong to A can occur. Again if the event B happened, then all elements of B cannot occur but only those elements of B can occur which belong to A. Thus the set of common elements is $A \cap B$. Hence, the probability of event B when A has happened *i.e.*, $P(B/A)$ is given by

$$P(B/A) = \frac{\text{number } (A \cap B)}{\text{number } (A)}$$

$$= \frac{\text{number } (A \cap B)/\text{number } (S)}{\text{number } (A)/\text{number } (S)}$$

[Dividing Nr. and Dr. by number (S)]

$$P(B/A) = \frac{P(A \cap B)}{P(A)}$$

$$\therefore \quad P(A \cap B) = P(A) \,.\, P(B/A).$$

Similarly we can prove that

$$P(A \cap B) = P(B) \,.\, P(A/B).$$

§ 2.10. INDEPENDENT AND DEPENDENT EVENTS

Let A and B be two events of a sample space S then A and B are called independent events if

$$P(A \cap B) = P(A) \,.\, P(B).$$

Otherwise if $P(A \cap B) \neq P(A) \,.\, P(B)$ then A and B are called dependent events.

Example 28. *If A and B are two events, where* $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{3}$ *and* $P(A \cap B) = \frac{1}{4}$. *Then calculate the following :*

(i) $P(A / B)$ (ii) $P(B / A)$

(iii) $P(A \cup B)$

(iv) *If A and B are independent then find* $P(A \cup B)$.

Solution. (i) If A and B are any two events then

$$P(A \cap B) = P(A) \,.\, P(B/A) = P(B) \,.\, P(A/B)$$

$$\therefore \quad P(A / B) = \frac{P(A \cap B)}{P(B)} = \frac{1/4}{1/3} = \frac{3}{4}$$

$$P(A / B) = \frac{3}{4}.$$

(ii)

$$P(B / A) = \frac{P(A \cap B)}{P(A)} = \frac{1/4}{1/2} = \frac{2}{4} = \frac{1}{2}$$

$$P(B / A) = \frac{1}{2}.$$

(iii)

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{1}{2} + \frac{1}{3} - \frac{1}{4} = \frac{6 + 4 - 3}{12} = \frac{7}{12}$$

$$P(A \cup B) = \frac{7}{12}.$$

(iv) If A and B are independent then

$$P(A \cup B) = P(A) + P(B) - P(A) \,.\, P(B)$$

$$= \frac{1}{2} + \frac{1}{3} - \frac{1}{2} \times \frac{1}{3}$$

$$= \frac{3 + 2 - 1}{6} = \frac{4}{6}$$

$$P(A \cup B) = \frac{2}{3}.$$

Example 29. *A pair of dice is thrown once. What is the probability of getting a total of* 10 *while the digit on the first die is* 4 ?

Solution. Let A be the event of getting a total of 10 on both dice and B be the event of getting 4 on the first die.

$\therefore$ $A = \{(4, 6), (5, 5), (6, 4)\}$ *i.e.*, number $(A) = 3$

$B = \{(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6)\}$ *i.e.*, number $(B) = 6$.

Now $A \cap B = \{(4, 6)\}$ *i.e.*, number $(A \cap B) = 1$.

Now the probability of getting a total of 10 while the digit on the first die is $4 = P(A/B)$

$$= \frac{P(A \cap B)}{P(B)} = \frac{\text{number } (A \cap B)/\text{number } (S)}{\text{number } (B)/\text{number } (S)}$$

$$= \frac{\text{number } (A \cap B)}{\text{number } (B)} = \frac{1}{6}.$$

Example 30. *Probability that A speaks the truth is $\frac{3}{5}$ and B speaks the truth is $\frac{5}{8}$. In what percentage of cases are they are likely to contradict each other ?*

[Indore M.B.A. 2004]

Solution. Let A be the event that A spaeks truth and B be the event that B speaks truth.

$$\therefore \quad P(A) = \frac{3}{5} \text{ and } P(B) = \frac{5}{8}.$$

The probability that they will contradict each other

$$= P\,[(A \cap \overline{B}) \cup (\overline{A} \cap B)]$$

$$= P(A \cap \overline{B}) + P(\overline{A} \cap B) \quad [\because (A \cap \overline{B}) \text{ and } (\overline{A} \cap B) \text{ are mutually exclusive}]$$

$$= P(A)\,.\,P(\overline{B}) + P(\overline{A})\,.\,P(B) \quad [\because A \text{ and } B \text{ are independent}]$$

$$= P(A)\,.\,\{1 - P(B)\} + \{1 - P(A)\}\,.\,P(B)$$

$$= \frac{3}{5} \times \left(1 - \frac{5}{8}\right) + \left(1 - \frac{3}{5}\right) \times \frac{5}{8}$$

$$= \frac{3}{5} \times \frac{3}{8} + \frac{2}{5} \times \frac{5}{8}$$

$$= \frac{9}{40} + \frac{10}{40} = \frac{19}{40}.$$

Example 31. *In a college 25% students in mathematics, 15% students in physics and 10% student in mathematics and physics both are failed, A student is selected random*

(i) if he is failed in physics, then find the chance of his failure in mathematics,

(ii) if he is failed in mathematics, then find the chance of his failure in physics,

(iii) find the chance of his failure in mathematics or physics.

Solution. Let A be the event of failure in mathematics and B be the event of failure in physics.

Suppose that the total number of students appearing in the examination were 100. So number $(S) = 100$.

Since 25% students are failed in mathematics, so number $(A) = 25$.

$$\therefore \quad P(A) = \frac{\text{number }(A)}{\text{number }(S)} = \frac{25}{100} = \frac{1}{4}.$$

Since 15% students are failed in physics, so number $(B) = 15$.

$$\therefore \quad P(B) = \frac{\text{number }(B)}{\text{number }(S)} = \frac{15}{100} = \frac{3}{20}.$$

Again 10% students are failed in physics and mathematics both, so number $(A \cap B) = 10$

$$\therefore \quad P(A \cap B) = \frac{\text{number }(A \cap B)}{\text{number }(S)} = \frac{10}{100} = \frac{1}{10}.$$

(i) The chance of failure in mathematics while he is failed in physics is given by

$$P(A / B) = \frac{P(A \cap B)}{P(B)} = \frac{1/10}{3/20} = \frac{2}{3}.$$

and $P[X = 0] = \dfrac{\quad}{0\,!} = e^{-m} = e^{-2} = 0.135.$

§ 2.18. NORMAL DISTRIBUTION

The Normal distribution was first discovered by De-Moivre (an English Mathematician) in 1773. De Moivre obtained this continuous distribution as a limiting case of Binomial distribution and applied it to the problems of game of chance. It was credited to Gauss (1809) who used normal curve to describe the theory of accidental errors of measurements involved in the calculation of orbits of heavenly bodies.

Normal distribution is a continuous probability distribution. It has been observed that a vast number of continuous random variables arising in studies of social, natural, biological, economic, psychological phenomena follow normal distribution. In theory of statistics, this distribution is very widely applied and thus plays a very important role.

Definition. A random variable X is said to have a normal distribution with parameters m and σ^2 if its density function is given by the probability law,

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left\{\frac{x-m}{\sigma}\right\}^2\right] \quad \ldots(1)$$

$$-\infty < x < \infty, \sigma > 0, -\infty < \mu < \infty$$

where m is called mean and σ^2 is called variance and defined by the normal law [Eq. (1)] is denoted by $X \sim N(\mu, \sigma^2)$.

If X is a normal variate with mean m and variance σ^2 or standard deviation σ, then the random variable

$$Z = \frac{X-m}{\sigma}$$

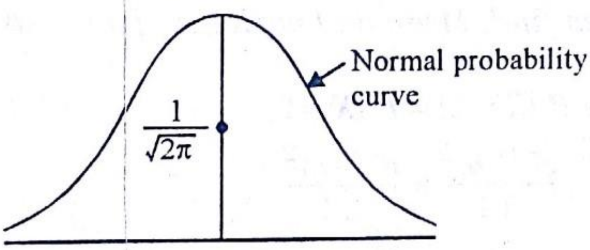
is called the standard normal variate which has the normal distribution with mean 0 and standard deviation unity and is given by

$$f(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2}; \ -\infty \le z \le \infty.$$

Properties of the Normal Distribution or Curve :

The normal probability curve with mean m and standard deviation σ is given by the equation

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-m)^2/2\sigma^2}; \ -\infty < x < \infty.$$



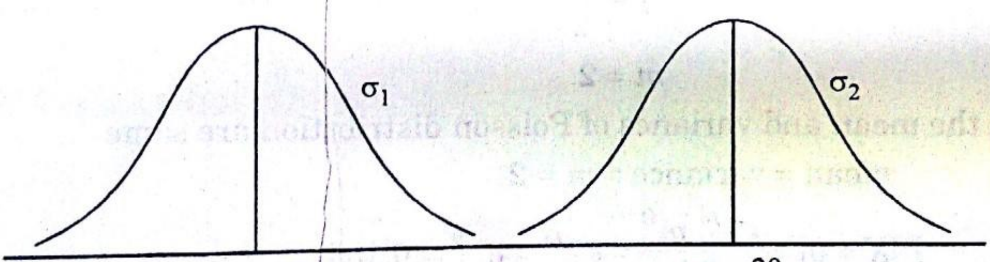


Fig. 5

Two normal curve with $m_1 \neq m_2$ and $\sigma_1 = \sigma_2$.

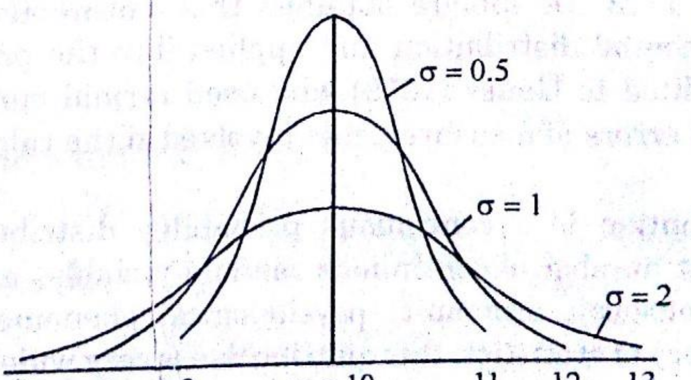


Fig. 6

Three normal curves with same mean but different standard deviations.

1. The curve is bell shaped and symmetrical about the line $x = m$, *i.e.*, mean and is asymptotic to X-axis *i.e.*, it is closer to x-axis but never touches it.

2. Mean, Median and Mode of normal distribution are equal and coincide at $x = m$.

3. The mean and variance are m and σ^2, respectively.

4. Since there is only one maximum point on the curve. So the normal curve is unimodel *i.e.*, only one mode.

5. The mean deviation about the mean of normal distribution is about 4/5 of its standard deviation.

6. The curve has two points of inflextion at $x = \mu \pm \sigma$. Both of these points are equidistance from the mean.

7. The total area under the normal curve and above the horizontal X-axis from $-\infty$ to ∞ is unity.

8. The most important property of the normal curve is the area property.

Area of normal curve between $m - \sigma$ and $m + \sigma$ is 68.27%

Area of normal curve between $m - 2\sigma$ and $m + 2\sigma$ is 95.45%

Area of normal curve between $m - 3\sigma$ and $m + 3\sigma$ is 99.73%

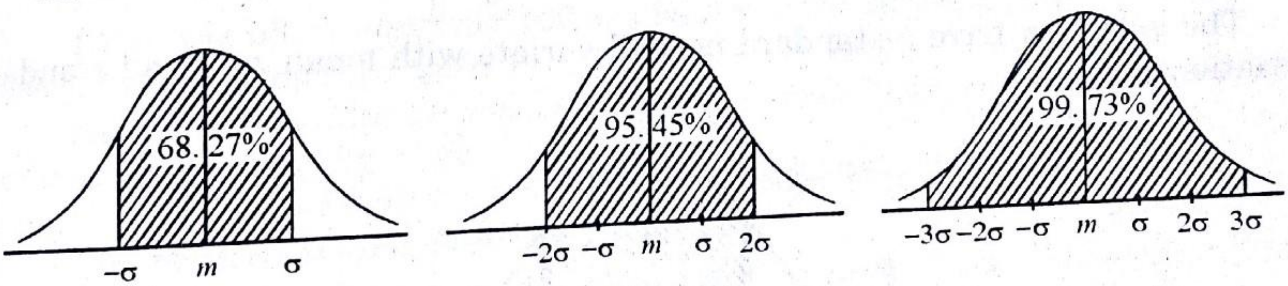


Fig. 7

9. The most probable limits of for a normal variate are $m \pm 3\sigma$, *i.e.*,

$$P[m - 3\sigma \leq X \leq m + 3\sigma] = 0.9973.$$

10. The sum and difference of independent normal variables also has a normal distribution.

Standard Form of Normal Curve :

The standard normal probability curve with mean 0 and standard deviation unity is given by equation

$$f(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2}; \; -\infty \leq z < \infty$$

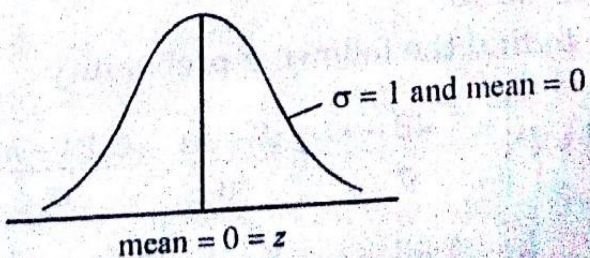


Fig. 8

How to Find out the Area under the Normal Curve :

If the mean m, standard deviation σ and two ordinates $x = a$ and $x = b$ are known. We have to find $P[a \le x \le b]$ or area under the two ordinates shown in given curve.

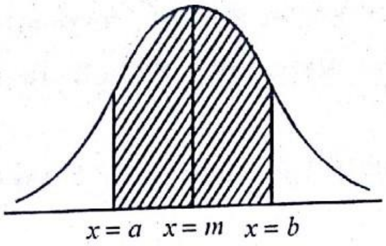


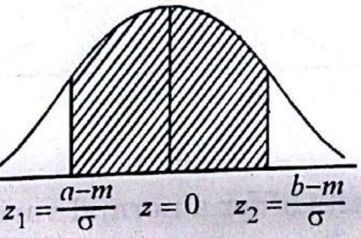
Fig. 9

First of all we transform normal distribution into the standard normal variate with the help of the following transformation

$$z = \frac{x - m}{\sigma}$$

i.e.,
$$P[a \le x \le b] = P\left[\frac{a - m}{\sigma} \le \frac{x - m}{\sigma} \le \frac{b - m}{\sigma}\right]$$
$$= P\left[\frac{a - m}{\sigma} \le z \le \frac{b - m}{\sigma}\right].$$

The variate z have a standard normal variate with mean zero and standard deviation one.



Fig, 10

The shaded area under the standard normal curve from 0 to z is given in the table of normal distribution.

The following points should be kept in mind to calculate the area.

The ordinate $z = 0$ divides the area under the standard normal curve into two equal parts. Thus the area on the right and left of $z = 0$ is 0.5.

or
$$P[0 \le z \le \infty] = P[-\infty \le z \le 0] = 0.5$$

The total area under the standard normal curve is one.

Example 52. *For a normal distribution with mean* 1 *and S.D.* 3. *Find the probability that* $-1.43 \le X \le 6.19$.

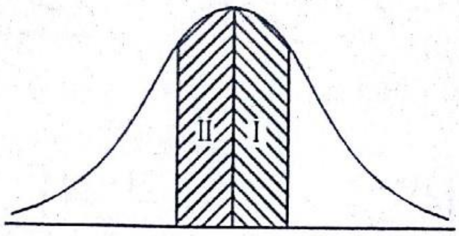
Solution. We have to find the following probability

$$P[-1.43 \le X \le 6.19] = P\left[\frac{-1.43 - m}{\sigma} \le \frac{X - m}{\sigma} \le \frac{6.19 - m}{\sigma}\right]$$

$$= P\left[\frac{-1.43 - 1}{3} \le \frac{X - 1}{3} \le \frac{6.19 - 1}{3}\right]$$ [Since $m = 1$ and $\sigma = 3$]

$$= P\left[\frac{-2.43}{3} \le z \le \frac{5.19}{3}\right]$$

$$= P\,[-0.81 \le z \le 1.73]$$



Fig, 11

We divided the whole area into two parts I and II between $z = 0$ to 1.73 and $z = -0.81$ to 0 respectively.

The area of the first part will be directly obtained. But the area of the second part $z = -0.81$ to 0 is equivalent to area of the part $z = 0$ to 0.81 because of area of right and left at ordinate $z = 0$ are symmetric.

Now $= P\,[0 < z < 1.73] + P\,[0 < z < 0.81]$

$= 0.4573 + 0.2910$ [From Table of Normal distribution]

$= 0.7483.$

Example 53. *A sample of* 100 *dry battery cells, tested to find the length of life produced the following results* $\bar{x} = 12$ *hours,* $\sigma = 3$ *hours.*

Assuming the data to be normally distributed, what percentage of the battery cells are expected to have life.

(i) *less than* 6 *hours* *(ii)* *more than* 15 *hours*

(iii) *between* 10 *and* 14 *hours.*

Solution. Let X be a random variable, denoting the life in hours of battery cells and follows normal distribution

then $\bar{x} = \text{mean } (m) = 12$ and $\sigma = 3$

(i) $P\,[X < 6] = P\left[\frac{X-m}{\sigma} < \frac{6-m}{\sigma}\right]$

$$= P\left[z < \frac{6-m}{\sigma}\right]$$

$$= P\left[z < \frac{6-12}{3}\right]$$

$$= P\,[z < -2]$$

$$= P\,[z > 2]$$

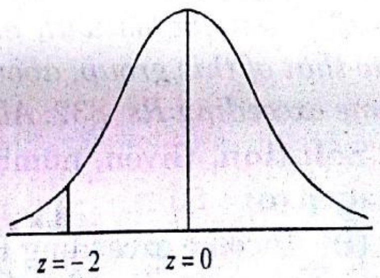
[because of symmetric]

$$= P\,[0 < z < \infty] - P\,[0 < z < 2]$$

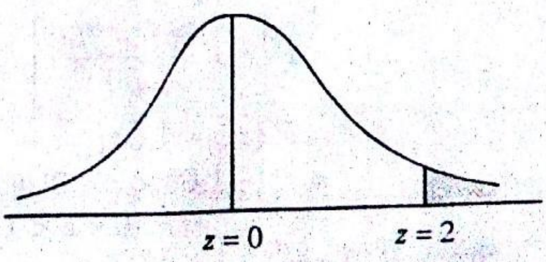
$$= 0.5 - 0.4772$$

[From normal table]

$$= 0.228 \text{ or } 2.28\%.$$



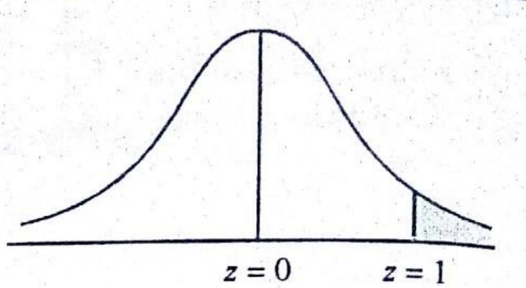
Fig, 12



Fig, 13

(ii) $P(X > 15) = P\left[\frac{X-m}{\sigma} > \frac{15-m}{\sigma}\right]$

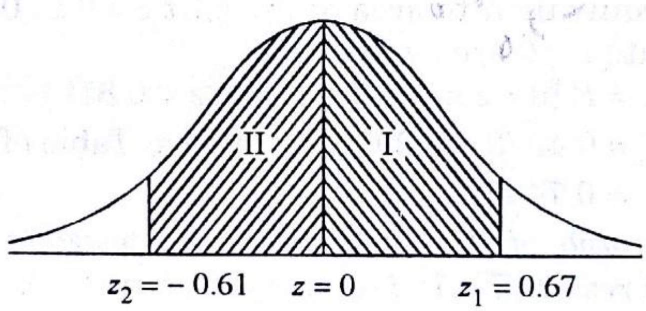
$$= P\left[z > \frac{15-12}{3}\right]$$
$$= P\,[z > 1]$$
$$= P\,[0 < z < \infty] - P\,[0 < z < 1]$$
$$= 0.5 - 0.3413$$
$$= 0.1587 \text{ or } 15.87\%.$$



Fig, 14

(iii) $P\,[10 < X < 14]$

$$= P\left[\frac{10-m}{\sigma} < \frac{X-m}{\sigma} < \frac{14-m}{\sigma}\right]$$
$$= P\left[\frac{10-12}{3} < z < \frac{14-12}{3}\right]$$
$$= P\,[-0.67 < z < 0.67]$$
$$= 2P\,[0 < z < 0.67]$$



Fig, 15

Since $P\,[0 < z < a] = P\,[-a < z < 0]$

here $\quad P\,[0 < z < 0.67] = P\,[-0.67 < z < 0]$

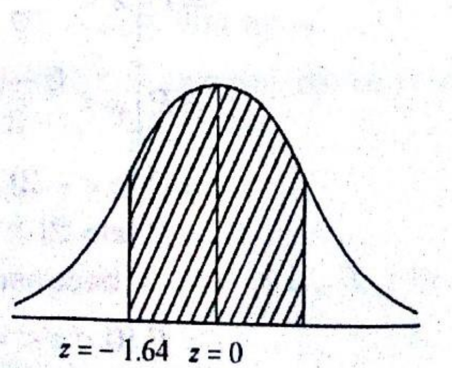
$$= 2 \times 0.2386$$
$$= 0.4972 \text{ or } 49.72\%$$

Example 54. *The income of a group of 10,000 persons was found to be normally distributed with mean Rs. 750 P. M. and standard deviation of Rs. 50. Show that of this group, about 95% had income exceeding Rs. 668 and only 5% had income exceeding Rs. 832. Also find the lowest income among the richest 100.*

Solution. Given, number of persosns = 1000, mean $(m) = 750$ and standard deviation $(\sigma) = 50$.

(i) Income exceeding Rs. 668

$$\therefore\ P\,[X > 668] = P\left[\frac{X-m}{\sigma} > \frac{668-m}{\sigma}\right]$$
$$= P\left[z > \frac{668-750}{50}\right]$$
$$= P\,[z > -1.64]$$
$$= P\,[-1.64 < z < 0] + P\,[0 < z < \infty]$$
$$= P\,[0 < z < 1.64] + 0.5$$



Fig, 16

Since $P\,[0 < z < \infty] = 0.5$

and $\quad P\,[-a < z < 0] = P\,[0 < z < a]$

$\therefore \quad P\,[z > -1{\cdot}64] = 0.4495 + 0.5$ [From normal table]

$$= 0.9495.$$

$\therefore \quad$ Required percentage $= 94.95\% \approx 95\%$.

(ii) Income exceeding Rs. 832

$$P[X > 832] = P\left[\frac{x-m}{\sigma} > \frac{832-m}{\sigma}\right]$$

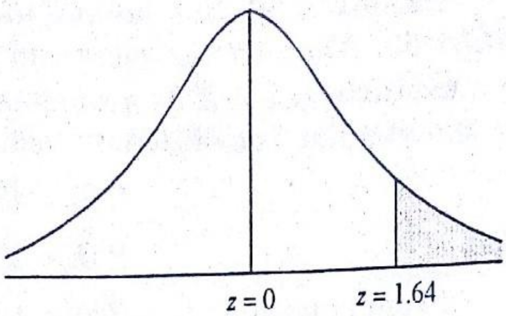
$$= P\left[z > \frac{832-750}{50}\right]$$

$= P[z > 1.64]$

$= P[0 < z < \infty] - P[0 < z < 1.64]$

$= 0.5 - 0.4495$ [From normal table]

$= 0.0505.$



Fig, 17

$\therefore$ Required percentage = 5.05% ≈ 5%.

(iii) Now 100 persons out of 1000 persons $= \frac{100}{10000} = 0.01.$

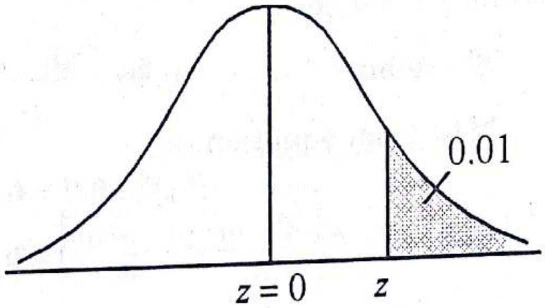
Let Rs. x denote the lowest income among the richest 100 persons then

$$P[X > x] = 0.01$$

$$\Rightarrow \quad P\left[\frac{X-m}{\sigma} > \frac{x-m}{\sigma}\right] = 0.01$$

$$\Rightarrow \quad P\left[z > \frac{x-m}{\sigma}\right] = 0.01$$

$$\Rightarrow \quad P\left[z > \frac{x-750}{50}\right] = 0.01.$$



Fig, 18

The remaining area 0 to z is $0.5 - 0.01 = 0.49$. Now we search the area 0.49 in normal table and there corresponding value of z will be found out, which is equal to $z = 0.71$.

Therefore $\quad z = 0.71 = \frac{x-750}{50}$

$\Rightarrow \quad x = 750 + 0.71 \times 50 = 750 + 35.5$

$= 785.5.$

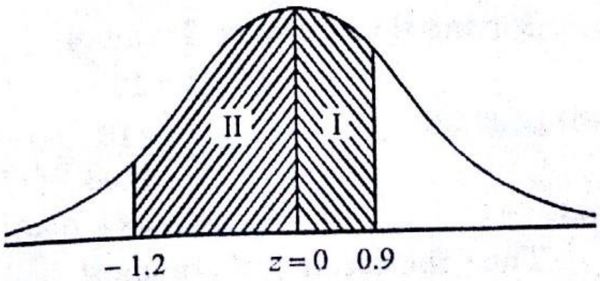
$\therefore$ Required lowest income = Rs. 785.5.

Example 55. *A random variable X is normally distributed with mean* = 12 *and standard deviation* 2. *Find* $P[9.6 < x < 13.8] = ?$

Solution. $P[9.6 < x < 13.8]$

$$= P\left[\frac{9.6-m}{\sigma} < \frac{x-m}{\sigma} < \frac{13.8-m}{\sigma}\right]$$

$$= P\left[\frac{9.6-12}{2} < \frac{x-12}{2} < \frac{13.8-12}{2}\right]$$



Fig, 19

$= P[-1.2 < z < 0.9]$

$$= \underbrace{P[0 < z < 0.9]}_{\text{I}} + \underbrace{P[-1.2 < z < 0]}_{\text{II}}$$

$= P[0 < z < 0.9] + P[0 < z < 1.2]$ [By symmetry]

$= 0.3159 + 0.3849$ [From normal table]

$= 0.7008.$

PROBLEM SET

1. If one coin is tossed, what is the probability of a head ?
2. What is the probability of an odd number in one throw of a die ?
3. What is the probability of getting more than 4 in a single throw of a die ?
4. What is the probability of throwing a sum of 9 in a single throw of two dice ?
5. If two coins are tossed, what is the probability of getting one head and one tail ?
6. In a class of 30 students 18 are boys and 12 are girls what is the chance that a student selected at random will be a boy ?
7. What is chance that a leap year selected at random will contain 53 sundays ?
8. A beg contains 9 red, 7 black and 6 white balls. Find the probability of drawing a white ball.
9. An urn contains 7 white, 6 green and 5 black balls. Two balls are drawn at random. Find the probability that both are white balls.
10. From a set of 17 lottery tickets, numbered 1, 2, ..., 17. One is drawn at random. Show that the chance of drawing a number which is divisible by 3 or 5 or 7 is 9/17.
11. A card is drawn at random from a pack of 52 cards. Find the probability of the following events.
    (i) Only one ace. (ii) A black card.
12. If A and B are two events such that $P(A) = \frac{1}{3}, P(B) = \frac{1}{4}$ and $P(A \cap B) = \frac{1}{12}$, then evaluate $P(A/B)$, $P(B/A)$ and $P(A \cup B)$.
13. A problem is given to three students, A, B and C whose chance of solving it are $\frac{1}{3}, \frac{1}{4}$ and $\frac{1}{5}$, respectively. What is the probability that the problem will be solved ?
14. The probability of A living for next 5 years is $\frac{1}{3}$ and that of B living for next 5 years is $\frac{1}{4}$. Find the probability that
    (i) both will be living for next 5 years, and
    (ii) at least one of them will be living for next 5 years.
15. In a pack of well shuffled cards, two cards are drawn and first card is replaced before drawing the second card. What is the probability of both being red ?
16. Two begs contain 3 white, 5 black and 5 white, 3 black balls respectively. One ball is drawn from each beg. What is the probability that they are white ?
17. Two dice are thrown. What is the probability that sum of the numbers appearing on the dice is 7, if 4 appears on the first die ?
18. The probability of two events A and B are 0.15 and 0.60 respectively. The probability of both events to happen together is 0.23. Then find the probability that neither A nor B happen.

43. Suppose the number of telephone calls on an operator received from 9.00 A.M. to 9.05 A.M. follows a Poisson distribution with mean 3. Find the probability that

(i) The operator will receive no call in that time interval tomorrow.

(ii) In the next three days the operator will receive a total of 1 call in that time interval. ($e^{-3} = 0.04978$)

44. In a sample of 1000 cases, the mean of a certain test is 14 and standard deviation is 2.5. Assuming the distribution to be normal. Find

(i) How many students score between 12 and 15 ?

(ii) How many students score above 18 ?

(iii) How many students score below 18 ?

(iv) How many students score 16 ?

45. For a normal distribution with mean 2 and standard deviation 3, find the value of a variate such that the probability of the interval from the mean to that value is 0.4115.

46. In a normal distribution 31% of the items are under 45 and 8% are over 64. Find the mean and standard deviation of the distribution.

47. The mean height of 500 students is 151 cm. and the S.D. is 15 cm. Assuming that the heights are normally distributed. Find how many students lie between 120 and 155 cm.

48. A minimum height is to be prescribed for eligibility to a government service such that 60% of the young men will have a fair chance of coming upto that standard. The height of youngmen are normally distributed with mean 60.6 inches and S.D. 2.55 inches. Determine the minimum specification.

49. Fit a normal distribution to the following data :

Mid point :	100	95	90	85	80	75	70	65	60	55	50	45
Frequency :	0	1	3	2	7	12	10	9	5	3	2	0

50. Fit a normal distribution to the following data :

Class :	60–62	63–65	66–68	69–71	72–74
Frequency :	5	18	42	27	8