Minimization of DFA examples using Partition Method or Equivalence Theorem:

Minimization of DFA means reduction of states.If X & Y are two states in a DFA, we can combine these two states into single state {X,Y} if they are not distinguishable i.e. equivalent or indistinguishable. Two states are said to be indistinguishable or equivalent state if δ (X, w) and δ (Y, w) are going to accepting /final states or going to non-accepting /non-final states.It is also called as State minimization.

Symbolically this can be represented as

1. $\delta(X, w) \in F$ and $\delta(Y, w) \in F$

   OR

2. $\delta(X, w) \notin F$ and $\delta(Y, w) \notin F$

Minimization of DFA steps/rules of minimization of DFA in automata:

Minimization of DFA questions or problems using partition method can be solved by following steps(How to do minimization of DFA):

Step1: Try to delete all the states to which we cannot reach from initial state (unreachable state)

Step2: Draw state transition table

Step3: Find out equivalent set

Step4: Draw / Construct minimized DFA

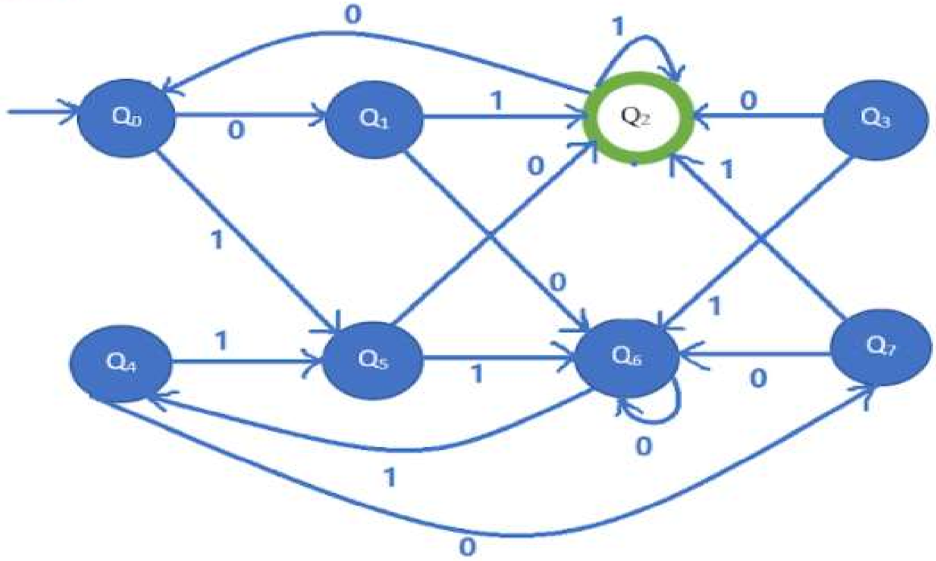
Note:

0-Equivalent Set: Try to separate non-final states from final states.

n-Equivalent Set: We take information only from previous equivalent set i.e.(n-1)-Equivalent

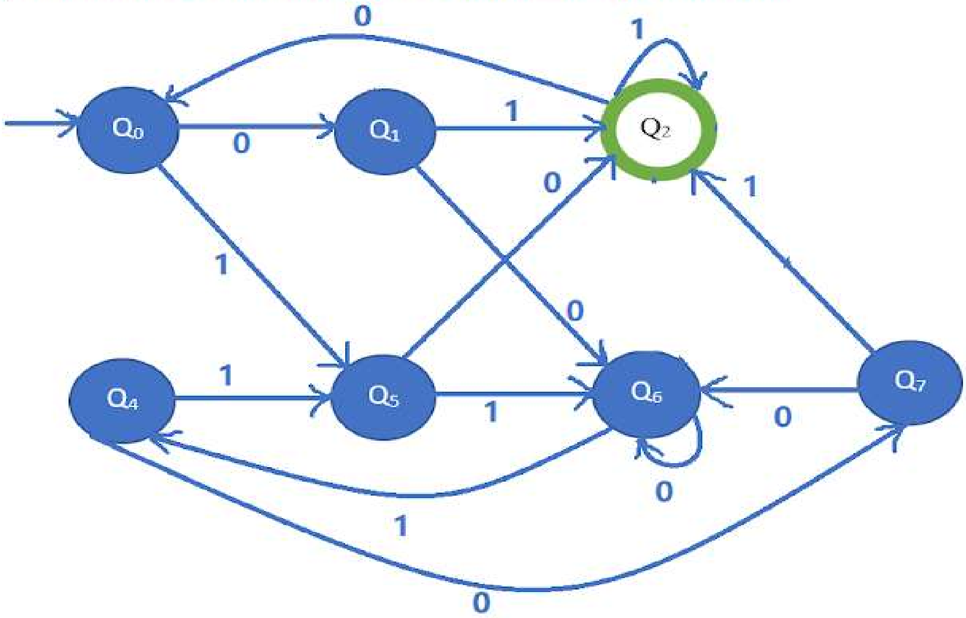
Minimization of DFA Examples:

Example-2: Minimize the given DFA using partition method or construct minimized DFA using Equivalence method



Given DFA

Step-1: Try to delete all the states to which we cannot reach from initial state



DFA without unreachable state

Step-2: Draw state transition table of DFA:

Present State	Next State	
	Input a	Input b
→ Q_0	Q_1	Q_5
Q_1	Q_6	*Q_2
*Q_2	Q_0	*Q_2
Q_4	Q_7	Q_5
Q_5	*Q_2	Q_6
Q_6	Q_6	Q_4
Q_7	Q_6	*Q_2

Step-3: Find out equivalent sets:

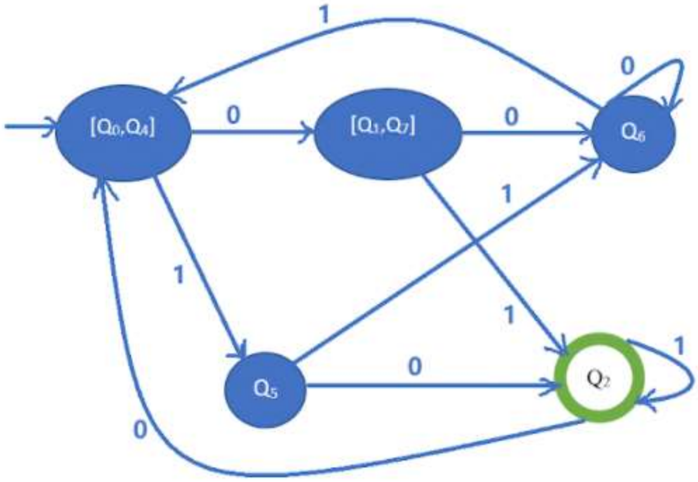
0-Equivalent Set: [Q_0, Q_1, Q_4, Q_5 , Q_6, Q_7] [Q_2]

1-Equivalent Set: [Q_0, Q_4, Q_6] [Q_1, Q_7] [Q_5] [Q_2]

2-Equivalent Set: [Q_0, Q_4] [Q_6] [Q_1, Q_7] [Q_5] [Q_2]

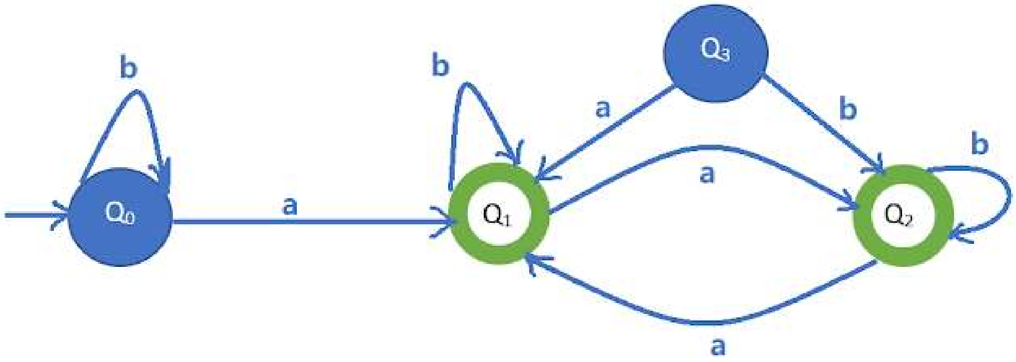
3-Equivalent Set: [Q_0, Q_4] [Q_6] [Q_1, Q_7] [Q_5] [Q_2]

Step-4: Draw minimized DFA:



Minimization of DFA Examples:

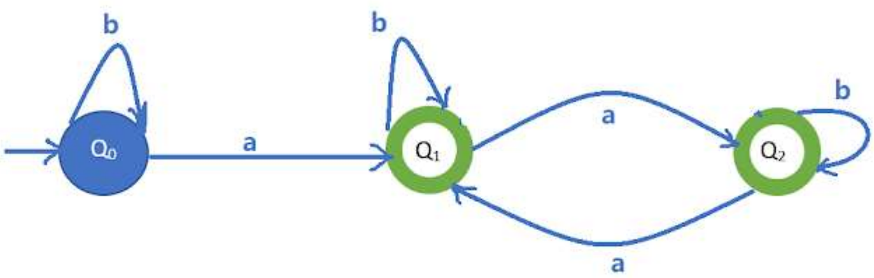
Example-3: Minimize the given DFA using partition method or construct minimized DFA using Equivalence method



Given DFA

Solution:

Step-1: Try to delete all the states to which we cannot reach from initial state



Step-2: Draw state transition table of DFA:

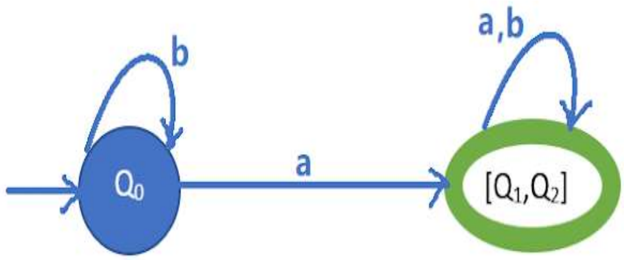
Present State	Next State	
	Input a	Input b
→ Q_0	*Q_1	Q_0
*Q_1	*Q_2	*Q_1
*Q_2	*Q_1	*Q_2

Step-3: Find out equivalent sets:

0-Equivalent Set: [Q_0] [Q_1, Q_2]

1-Equivalent Set: [Q_0] [Q_1, Q_2]

Step-4: Draw minimized DFA:



Example 1:Construct a DFA, that accepts set of all strings over ∑={a,b} of length 2 i.e. |w|=2

Example 2:Construct a DFA, that accepts set of all strings over ∑={a,b} of length at least 2 i.e. |w|>=2

Example 3: Construct a DFA, that accepts set of all strings over ∑={a,b} of length at most 2 i.e. |w|<=2

Example 4: Construct a DFA, that accepts string 'ab' over ∑={a,b}

Example 5: Construct a DFA,accepting all strings ending with 'ab' over ∑={a,b}

Example 6: Design DFA which checks whether a given binary number is divisible by 3
Example 7: Design DFA to accepts L , where L is set of strings in which 'a' always appears trippled over ∑={a,b}

Example 8: Construct minimal DFA over ∑={a,b} which checks whether given

a) Binary number is even

b) Binary number is odd

Example 9: Construct minimal DFA which accepts set of all strings over ∑={a,b} in which every 'a' should never be followed by 'bb".

Example 10: Construct minimal DFA over ∑={a,b} which accepts L={$a^n b^n$ |n, m>=1}

Example 11: Construct minimal DFA over ∑={a,b} which accepts L={$a^n b^n$| n, m>=0}

Example 12: Construct a transition system which can accept strings over the alphabet a,b,......,z containing either cat or rat.

Example 13: Design DFA for the following languages shown below over ∑={a,b}

a) L={w |w does not contains substring ab}
b) L={ w |w contains neither substring ab or ba}
c) L={ w |w is any string that does not contain exactly two a's}
d) L={ w |w is any string except a & b}

Example 14: Consider below transition and verify whether the following strings will be accepted or not. Explain

a. 010101
b. 0011
c. 111100

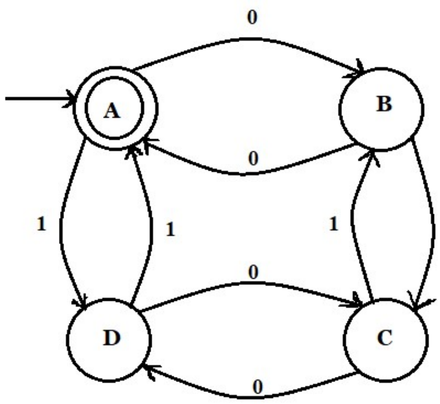


Fig: Tranaition diagram

Example 1: Construct a DFA, that accepts set of all strings over $\sum=\{a,b\}$ of length 2 i.e. $|w|=2$

SOLUTION:

$\Sigma=\{a,b\}$

L = {All the strings of length 2 }

L = { aa, ab, ba, bb }

- So DFA can be $Q=\{q_0, q_1, q_2\}$, $\Sigma=\{a,b\}$, $q_0=\{q_0\}$, $F=\{q_2\}$ and δ is given by the table

1)Transition diagram:

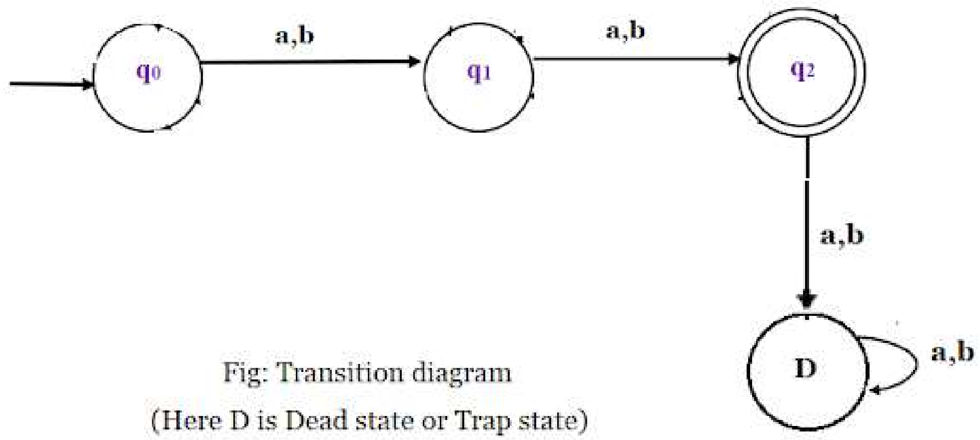


Fig: Transition diagram

(Here D is Dead state or Trap state)

2)Transition Table:

Present State	Next State	
	Input a	Input b
→ q_0	q_1	q_1
q_1	q_2	q_2
* q_2	D	D
D	D	D

3)Transition function:

$\delta(q_0, a) = q_1$, $\delta(q_0, b) = q_1$

$\delta(q_1, a) = q_2$, $\delta(q_1, b) = q_2$

$\delta(q_2, a) = D$, $\delta(q_2, b) = D$

$\delta(D, a) = D$, $\delta(D, b) = D$

Construct a DFA, that accepts set of all strings over $\Sigma=\{a,b\}$ of length at least 2 i.e. $|w|>=2$

SOLUTION:

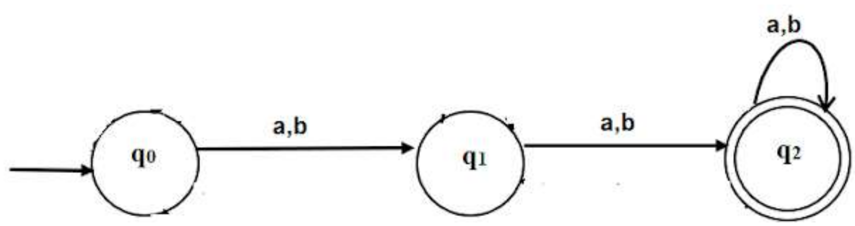
$\Sigma=\{a,b\}$

L = {All the strings of length at least 2 }

L = {aa, ab,ba,bb,aaa,.......,bbb,..........}

➢ So DFA can be $Q=\{q_0, q_1, q_2\}$, $\Sigma=\{a,b\}$, $q_0=\{q_0\}$, $F=\{q_2\}$ and δ is given by the table

1)Transition diagram:



2)Transition Table:

Present State	Next State	
	Input a	Input b
→ q_0	q_1	q_1
q_1	q_2	q_2
* q_2	q_2	q_2

3)Transition function:

$\delta(q_0, a) = q_1$, $\delta(q_0, b) = q_1$

$\delta(q_1, a) = q_2$, $\delta(q_1, b) = q_2$

$\delta(q_2, a) = q_2$, $\delta(q_2, b) = q_2$

Construct a DFA, that accepts set of all strings over $\Sigma=\{a,b\}$ of length at most 2 i.e. $|w|<=2$

SOLUTION:

$\Sigma=\{a,b\}$

L = {All the strings of length at most 2 }

L = {ϵ, a, b, aa, ab, ba, bb}

➢ So DFA can be $Q=\{q_0, q_1, q_2\}$,$\Sigma=\{a,b\}$, $q_0=\{q_0\}$,$F=\{q_0, q_1, q_2\}$ and δ is given by the table

1)Transition diagram:

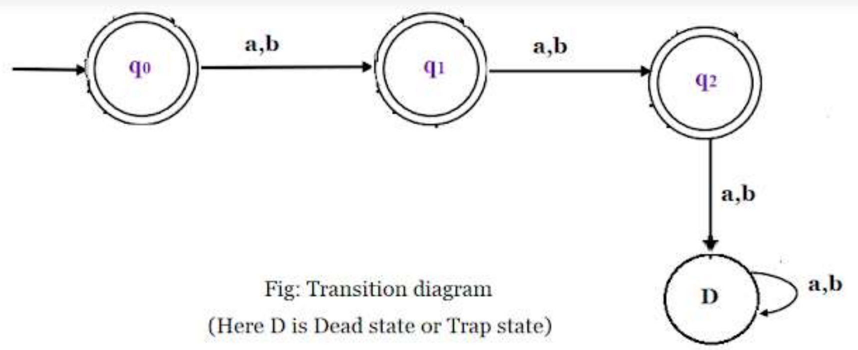


Fig: Transition diagram

(Here D is Dead state or Trap state)

2)Transition Table:

Present State	Next State	
	Input a	Input b
$\rightarrow *q_0$	q_1	q_1
$* q_1$	q_2	q_2
$* q_2$	D	D
D	D	D

3)Transition function:

$\delta(q_0, a)= q_1$,$\delta(q_0,b)= q_1$

$\delta(q_1, a)= q_2$,$\delta(q_1,b)= q_2$

$\delta(q_2, a)= D$,$\delta(q_2,b)= D$

$\delta(D, a)= D$,$\delta(D, b)= D$